Universal or not? EFT insights into two-neutron halos and ^{6}Li

Daniel Phillips

with Matthias Göbel, Hans-Werner Hammer, Chloë Hebborn, and Carl Brune

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Halo EFT

Bertulani, Hammer, van Kolck, NPA (2003);
Bedaque, Hammer, van Kolck, PLB (2003);
Reviews: Hammer, Ji, DP, J. Phys. G 44, 103002 (2017);

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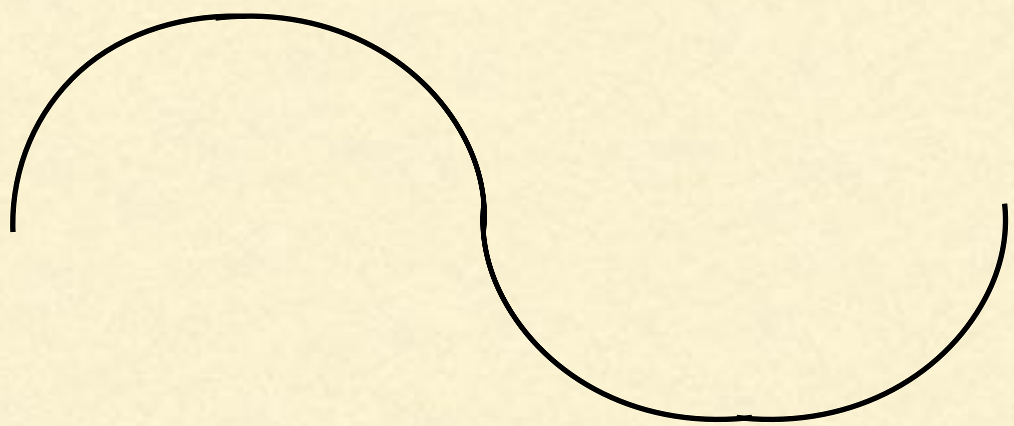

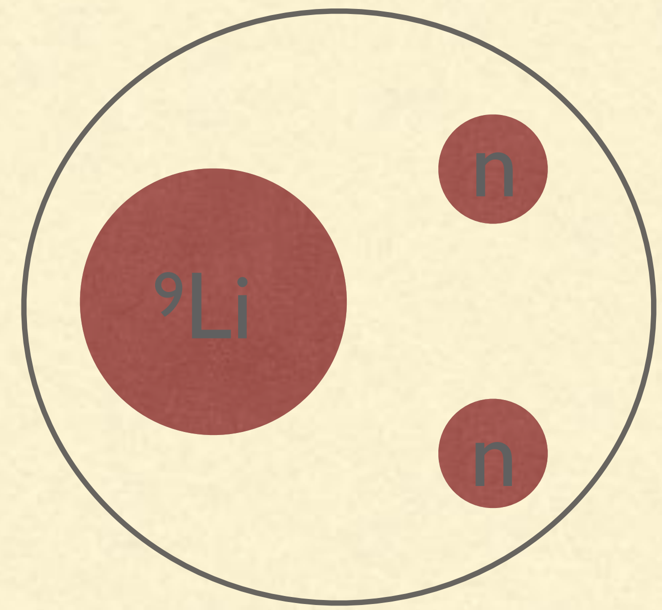

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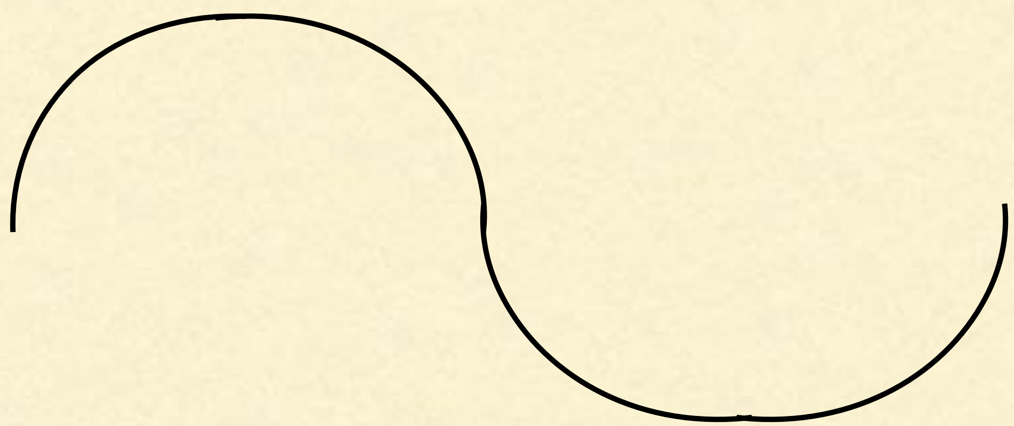
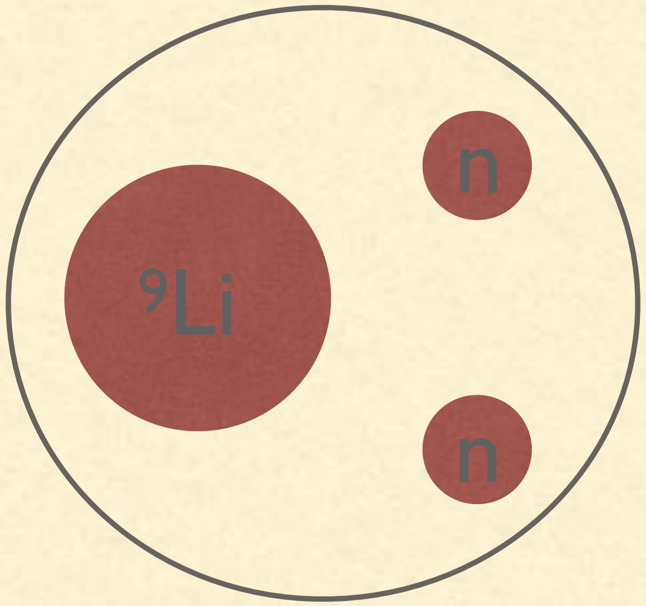
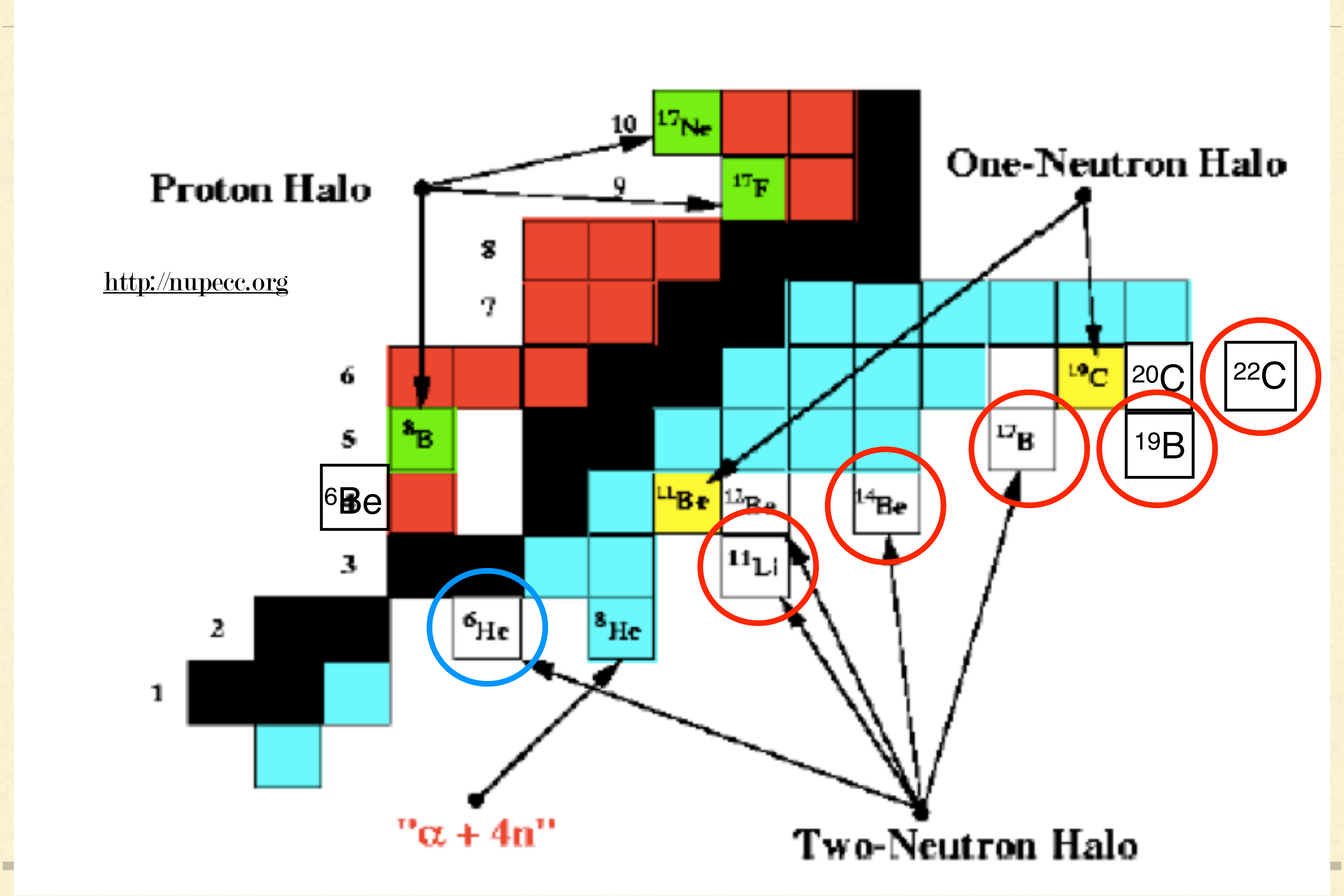




- Define $R_{halo}=\langle r^2\rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}. Valid for $\lambda \lesssim R_{halo}$
- Typically $R \equiv R_{core} \sim 2$ fm. Since $\langle r^2\rangle$ is related to the neutron separation energy we seek systems with neutron separation energies less than 1 MeV
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT
- ^{22}C, ^{11}Li, ^{12}Be, ^{19}B, ^{62}Ca (hypothesized), and ^{3}H: all s-wave 2n halos

Halo nuclei: examples



Outline

- What is Halo EFT and what does it do for us?
- Halo EFT for Borromean s-wave 2n halos
- Measuring nn relative-momentum distributions using fast breakup
- The unitary limit in momentum distributions of 2n halos
- The surprisingly small uncertainty of d + ^{4}He→^{6}Li + γ at low energies
- What it teaches us about ^{6}Li

Two-body scattering amplitude in Halo EFT

$$t_0^{2B}(E) = -\frac{2\pi}{m_R}\frac{1}{k\cot\delta(E) - ik}; \quad k = \sqrt{2m_R E}$$

$$k\cot\delta(E) = -\frac{1}{a} + \frac{1}{2}rk^2 + O(k^4R^3)$$

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Elastic scattering: this is effective-range theory with built-in UQ

Lagrangian: shallow S- and P-states

$$
\begin{aligned}
\mathcal{L} \;\; = \;\; & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M}\right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m}\right) n \\
& + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_0\right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_1\right] \pi_j \\
& - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c\right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \, i\overleftrightarrow{\nabla}_j \, c) + (c^\dagger \, i\overleftrightarrow{\nabla}_j \, n^\dagger) \pi_j\right] \\
& - \frac{g_1}{2} \frac{M-m}{M_{nc}} \left[\pi_j^\dagger \, i\overrightarrow{\nabla}_j \, (nc) - i\overleftrightarrow{\nabla}_j \, (n^\dagger c^\dagger) \pi_j\right] + \ldots \, ,
\end{aligned}
$$

- c, n: "core", "neutron" fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings
- Additional EM couplings at sub-leading order

But it’s more than just s-wave nn & nc scattering

- So not just two-body scattering: also EM processes

Chen, Rupak, Savage (1999); Hammer, DP (2011)

- And other partial waves

Bertulani, Hammer, van Kolck (2003); Bedaque, Hammer, van Kolck (2003); Brown & Hale (2005); Braun et al. (2018); Ando (2016-present)

- Extension to pp, p-core, and cluster-cluster scattering

Kong & Ravndal (1999); Higa, Hammer, van Kolck (2008); Ryberg, Forssén, Hammer, Platter (2014, 2016)

- Expansion around limit of a bound or unbound state near threshold. Include higher-order effects in ERE in proportion to their importance. Expansion in kR_{core}, where R_{core} is scale of unresolved core physics

- Extends to three-body states at cost of one additional parameter (S_{2n})

Bedaque, Hammer, van Kolck (1999); Hammer & Mehen (2001); Bedaque et al. (2002); Ji, Platter, DP (2009)

- Then predictive for four-body states (bosons or distinguishable particles) at LO accuracy

Platter, Hammer, Meißner (2005); Bazak, Kirscher, König, Pavon Valderrama, Barnea, van Kolck (2018)

Equations for s-wave 2n halo

Canham, Hammer (2008)

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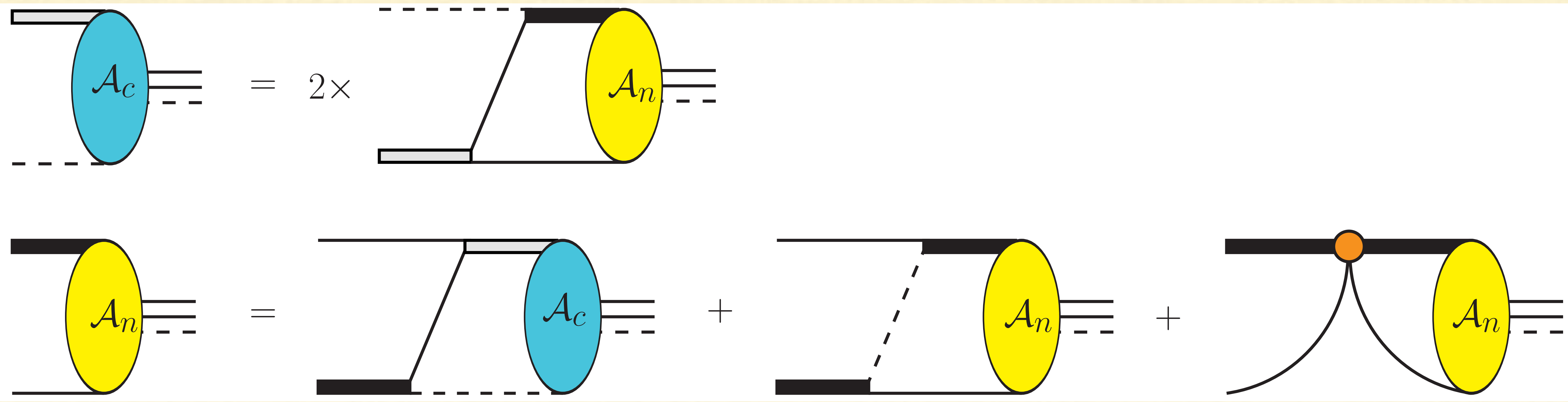
- Core-n and n-n contact interactions at leading order: solve 3B problem



- (cn)-n contact interaction to stabilize three-body system

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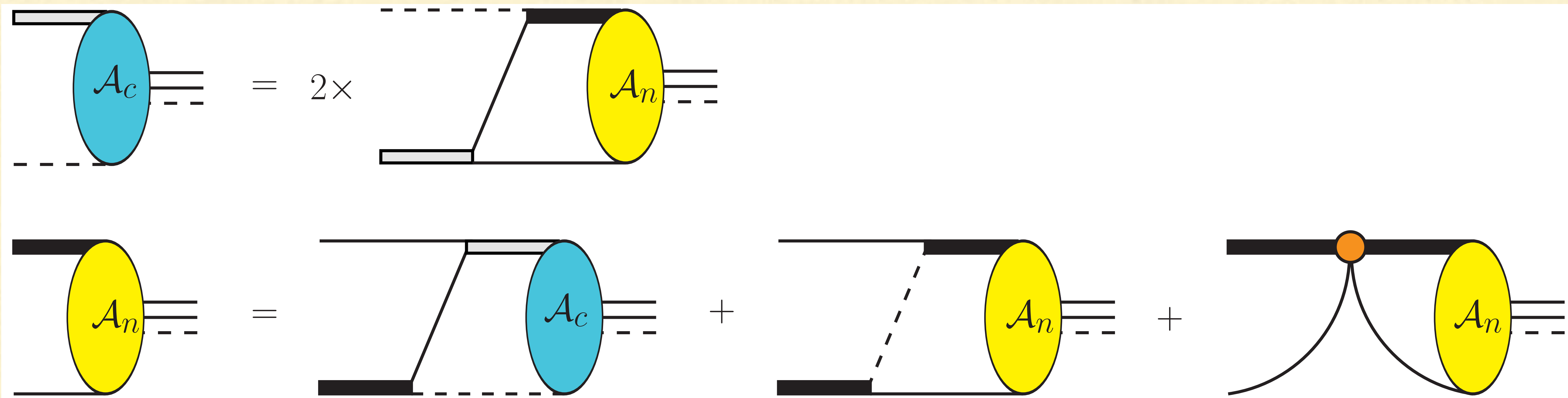
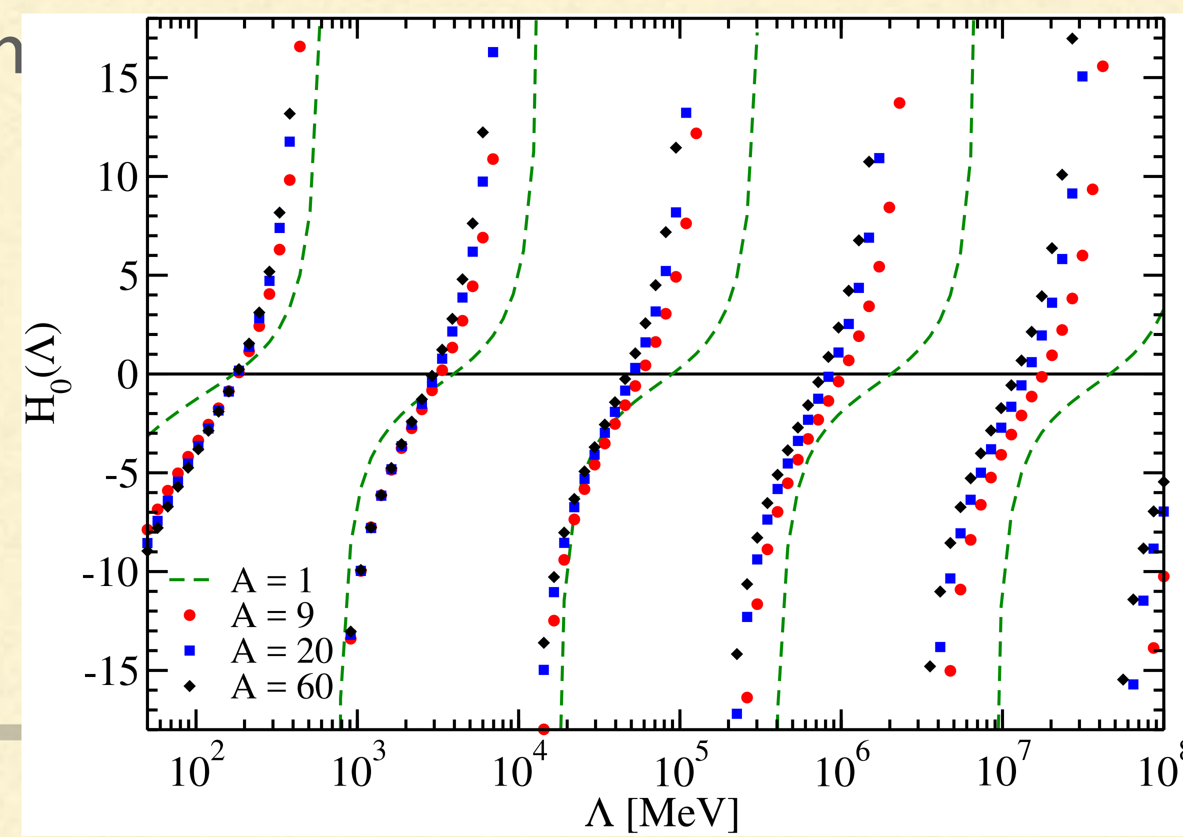


- (cn)-n contact interaction to stabilize th

- Efimov-Thomas effects



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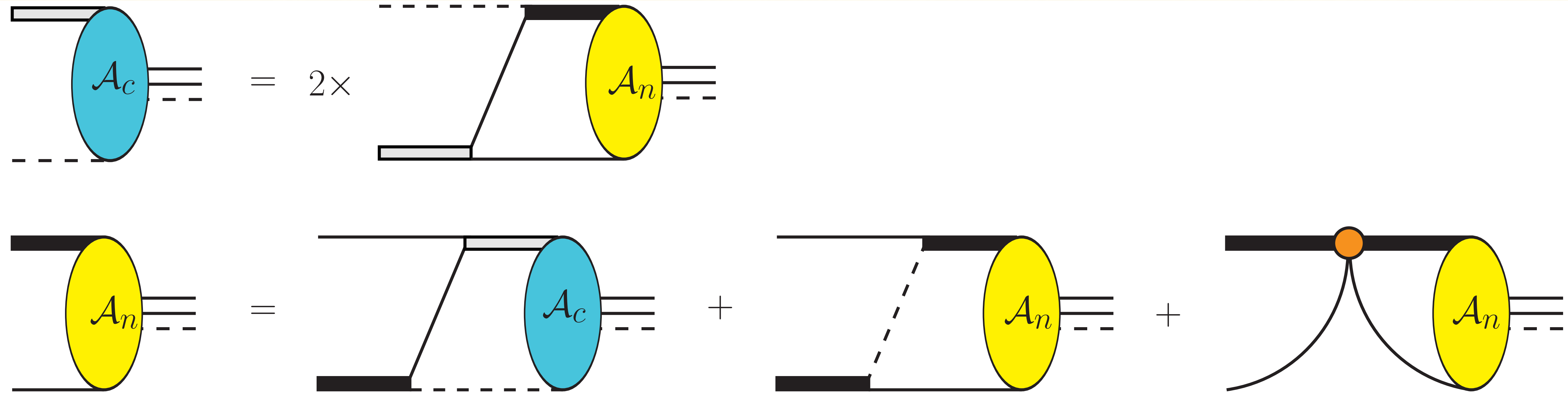

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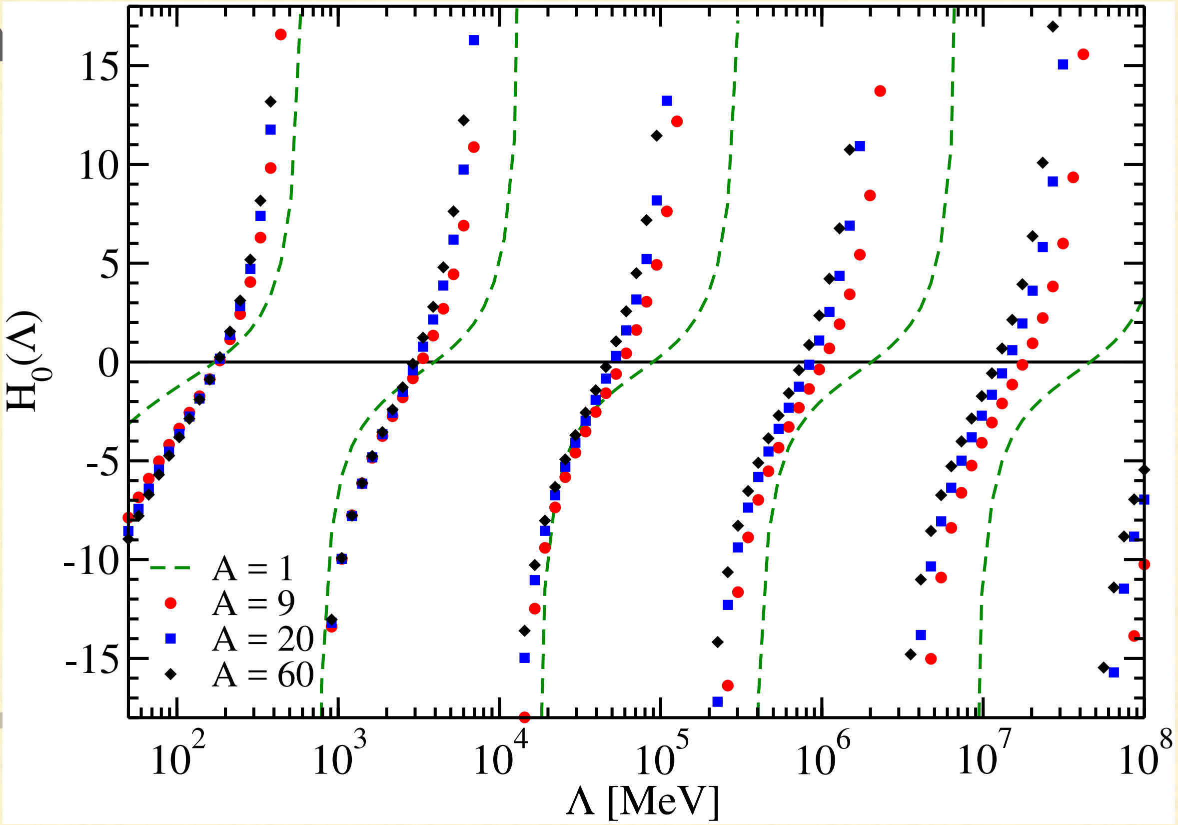
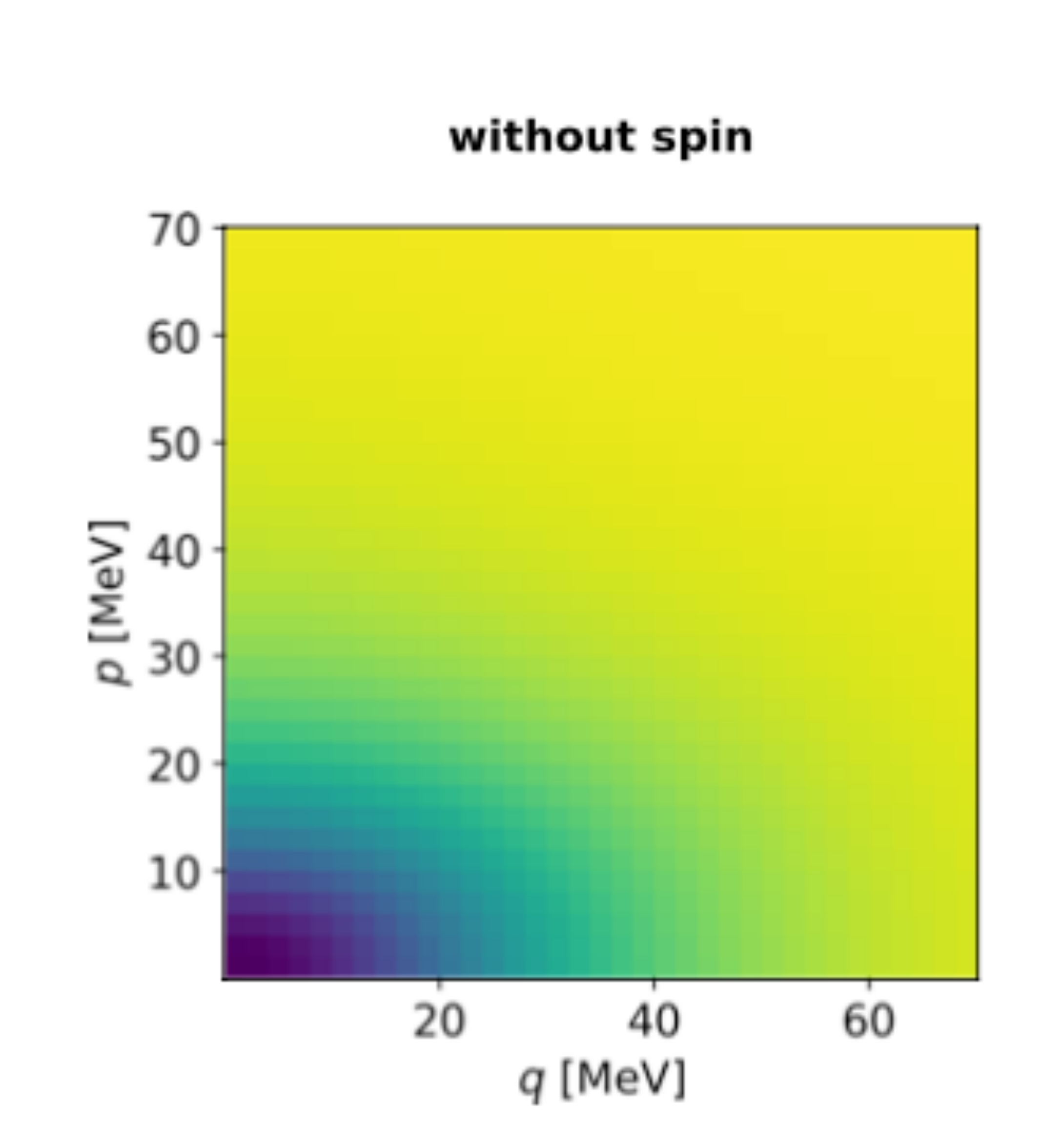


- Efimov-Thomas effects
- Inputs: $E_{nn}=1/(m\, a_{nn}^2)$, E_{nc}, S_{2n} (=B)
- Output: everything; up to $O(R_{core}/R_{halo})$

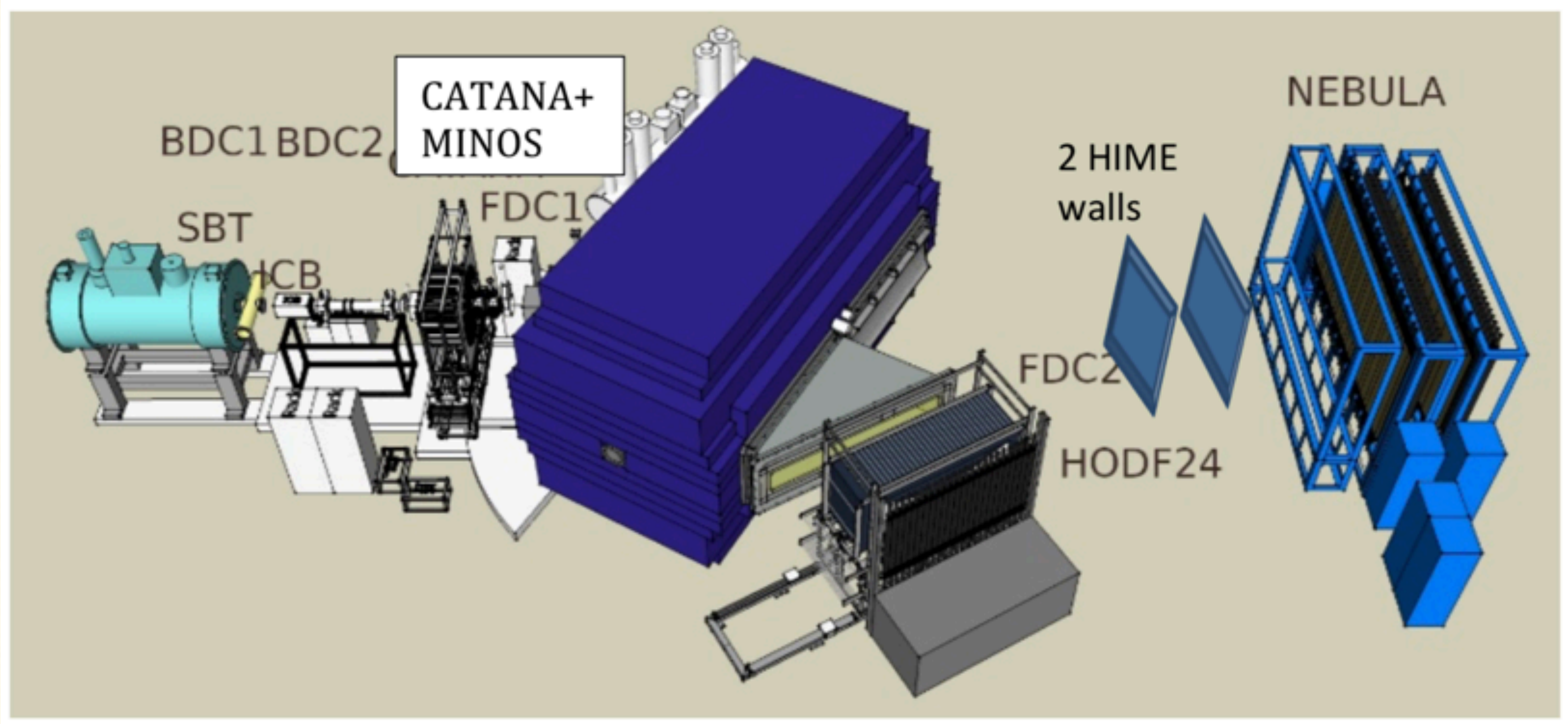
^{11}Li as a 2n halo

- a_{nn}=-18.7 fm, E_{nc}=0.026 MeV
- S_{2n}=369 keV
- Calculations done with a cutoff of 470 MeV, but results checked for a cutoff of 700 MeV
- Here results with a spin-0 core, but we also examined case of spin-3/2 core
- Results identical if spin-1 and spin-2 nc interactions have equal strength

^{11}Li wave function



RIKEN experiment with ^{6}He beam

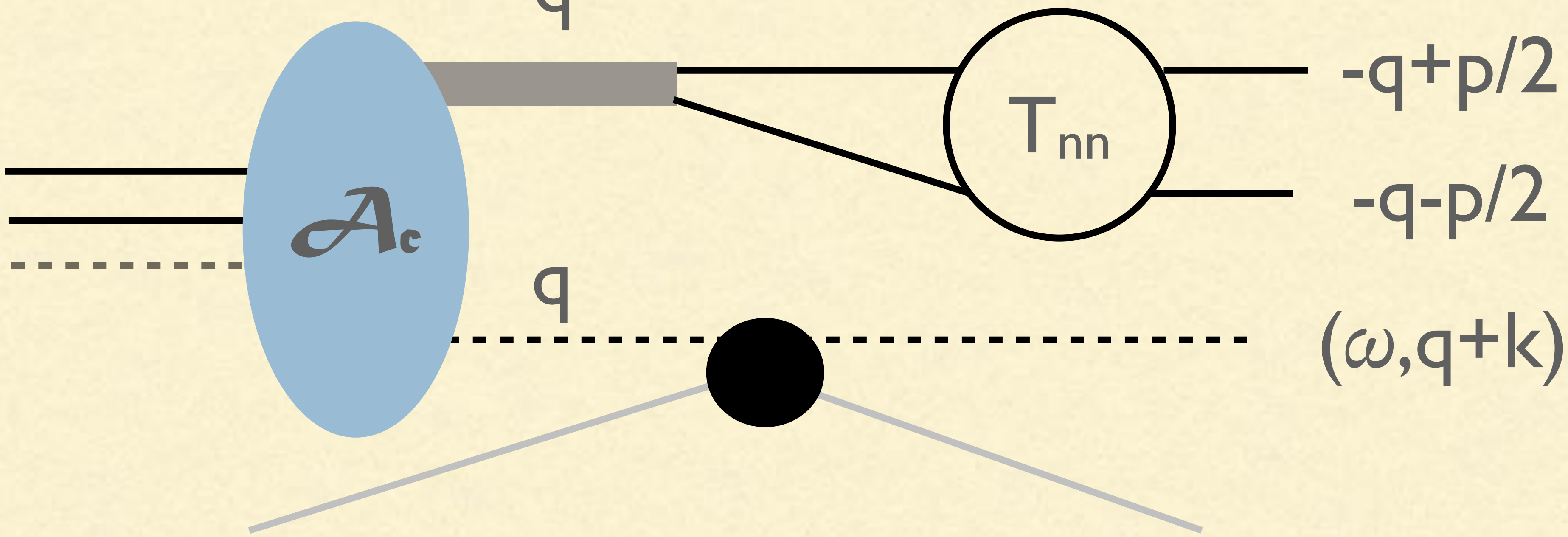


Tom Aumann spokesperson

- Detect proton and alpha in TPC
- Detect neutrons in HIME + NEBULA: excellent energy resolution

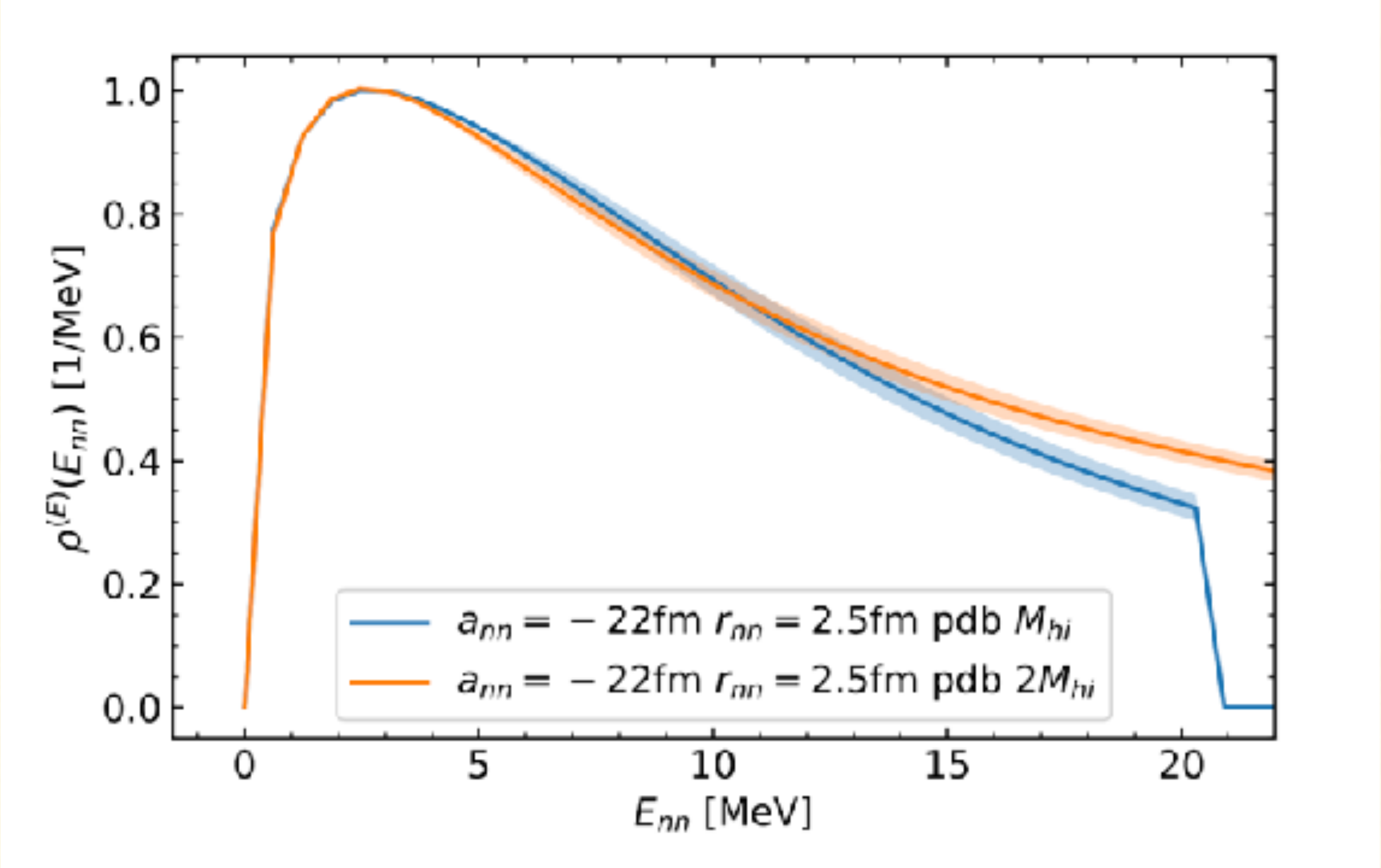
^{6}He(p,p'α) and the nn scattering length

Göbel, Aumann, Bertulani, Frederico, Hammer, Phillips, PRC (2021)



- Quasi-free alpha-particle knockout can leave nn pair almost at rest
- Final-state interaction then generates significant dependence of neutron relative-energy spectrum $f(p^2/m_n)$ on a_{nn}
- ^{6}He acts as a “holder” for low-momentum neutrons
- Neutrons actually move fast in lab. frame: inverse kinematics

Neutron energy distribution in ^{6}He



No FSI included at first

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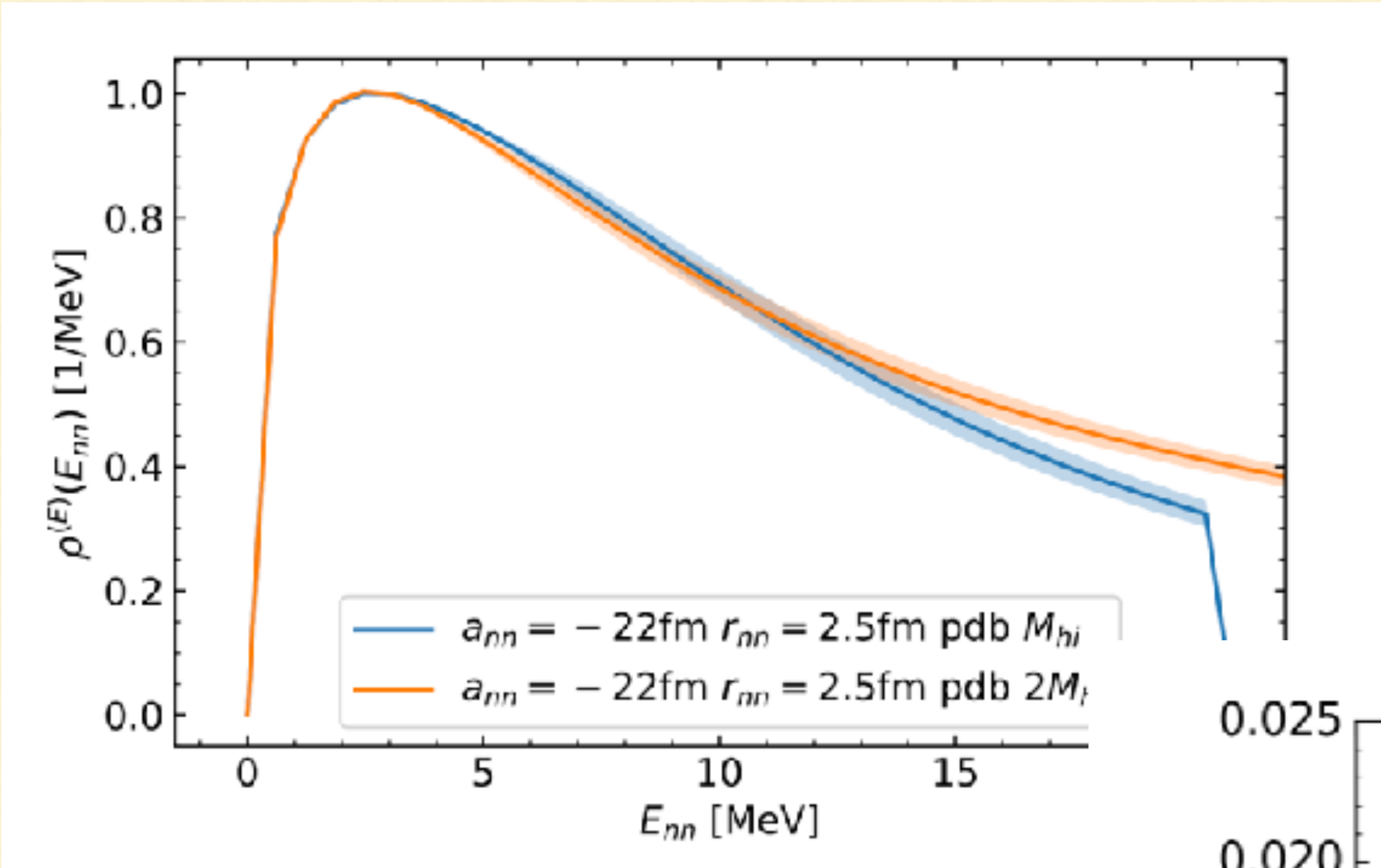

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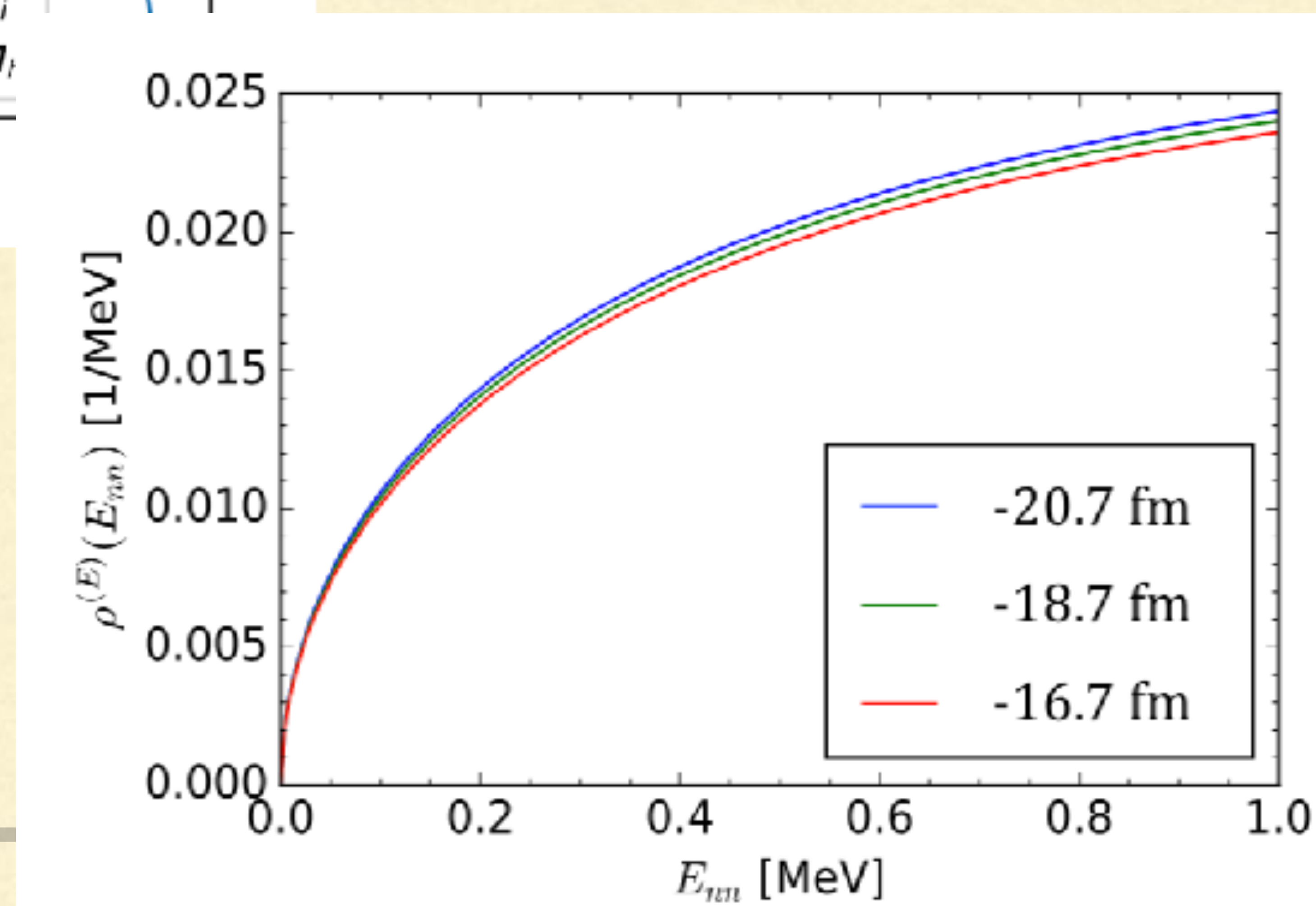

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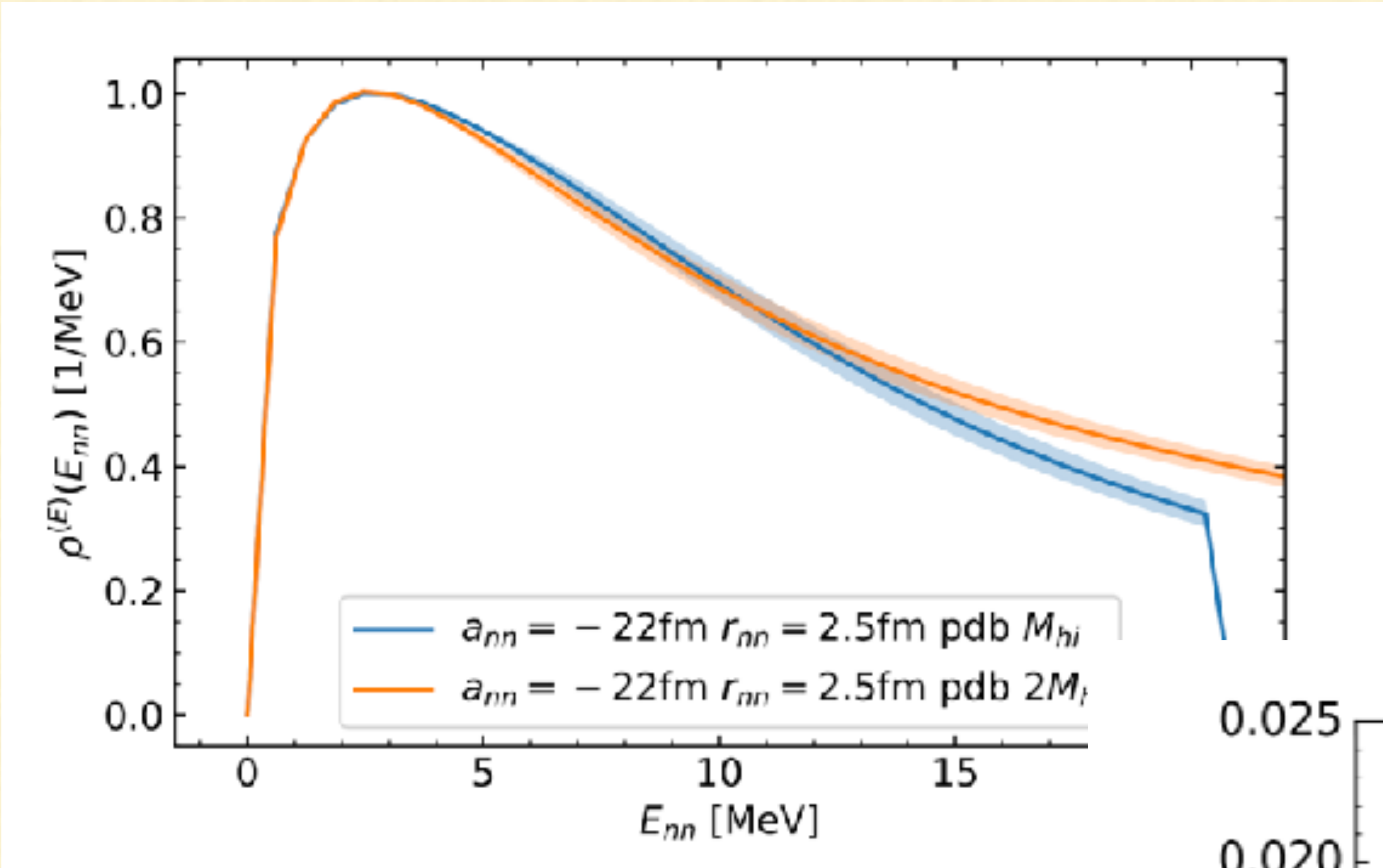

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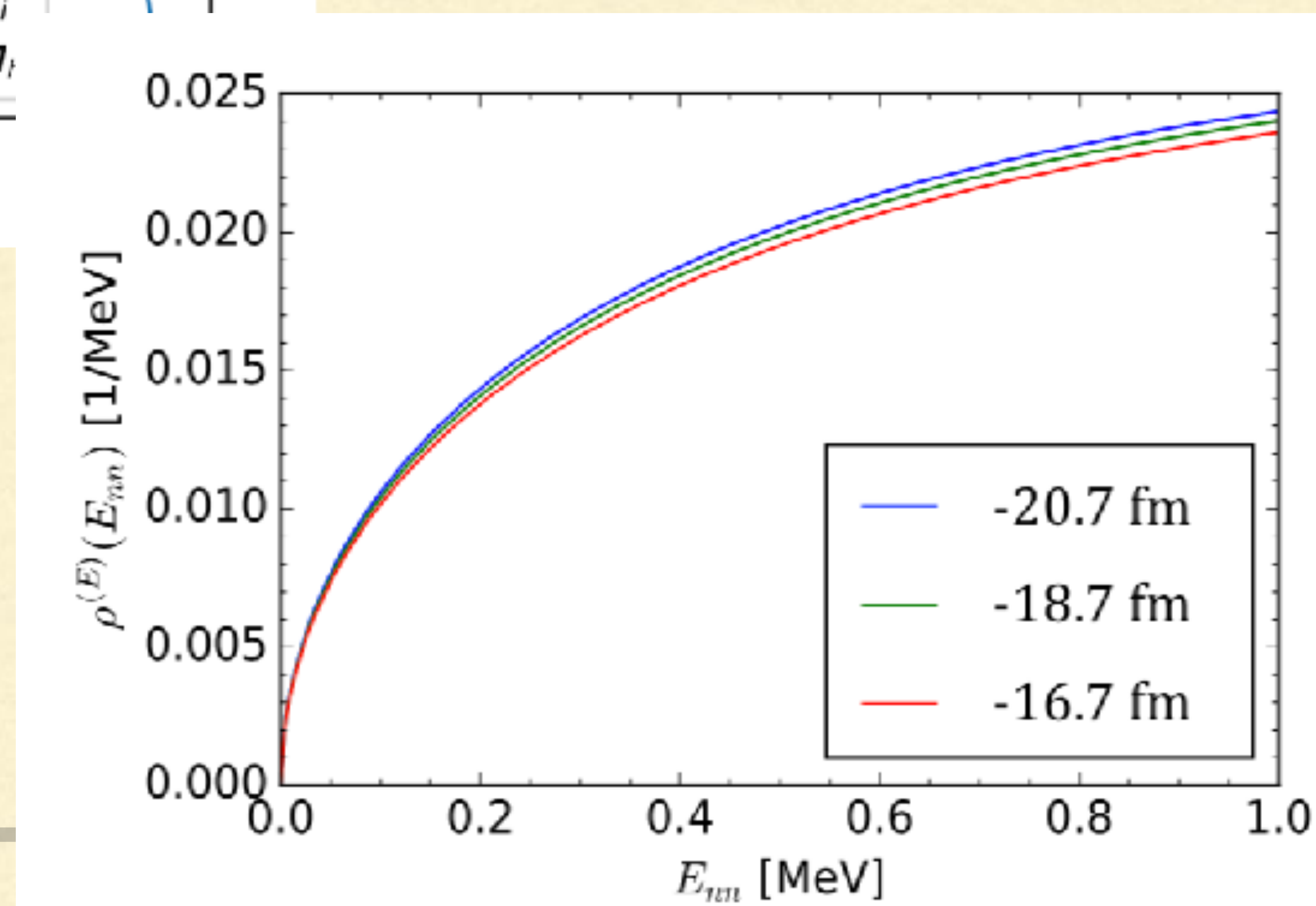
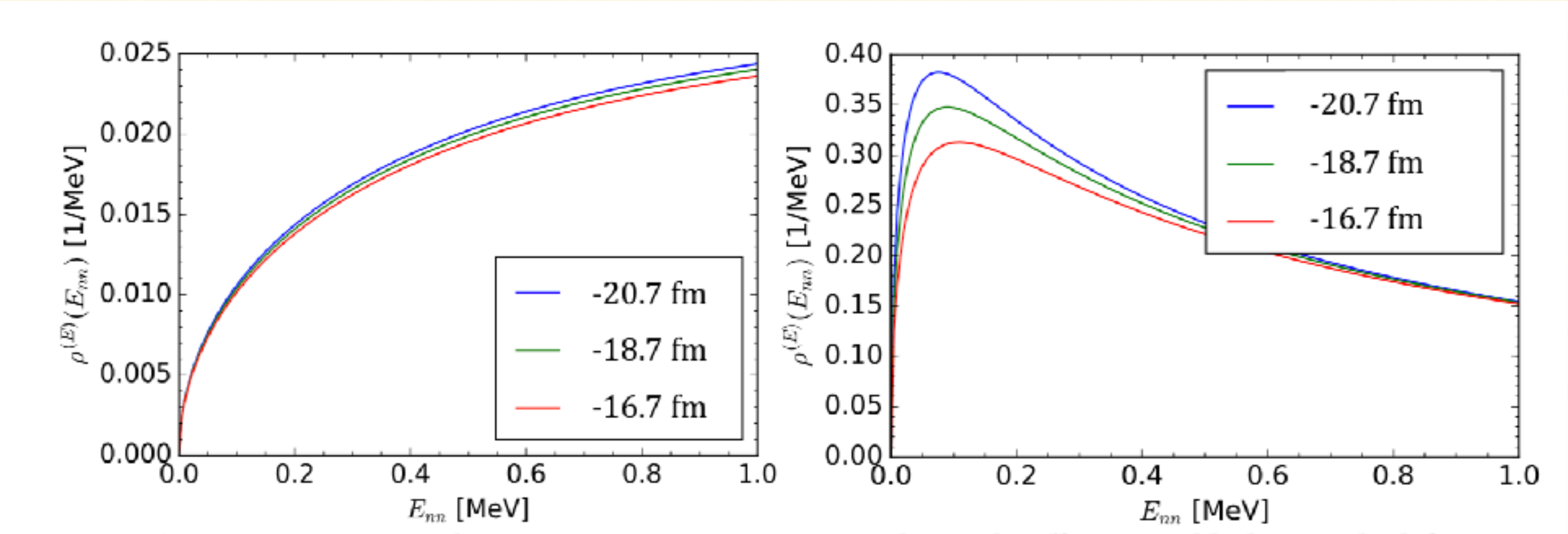


^{6}He structure at low momentum not significantly affected by cutoff or a_{nn} (or r_{nn})

Sensitivity to a_{nn} and (not) r_{nn}



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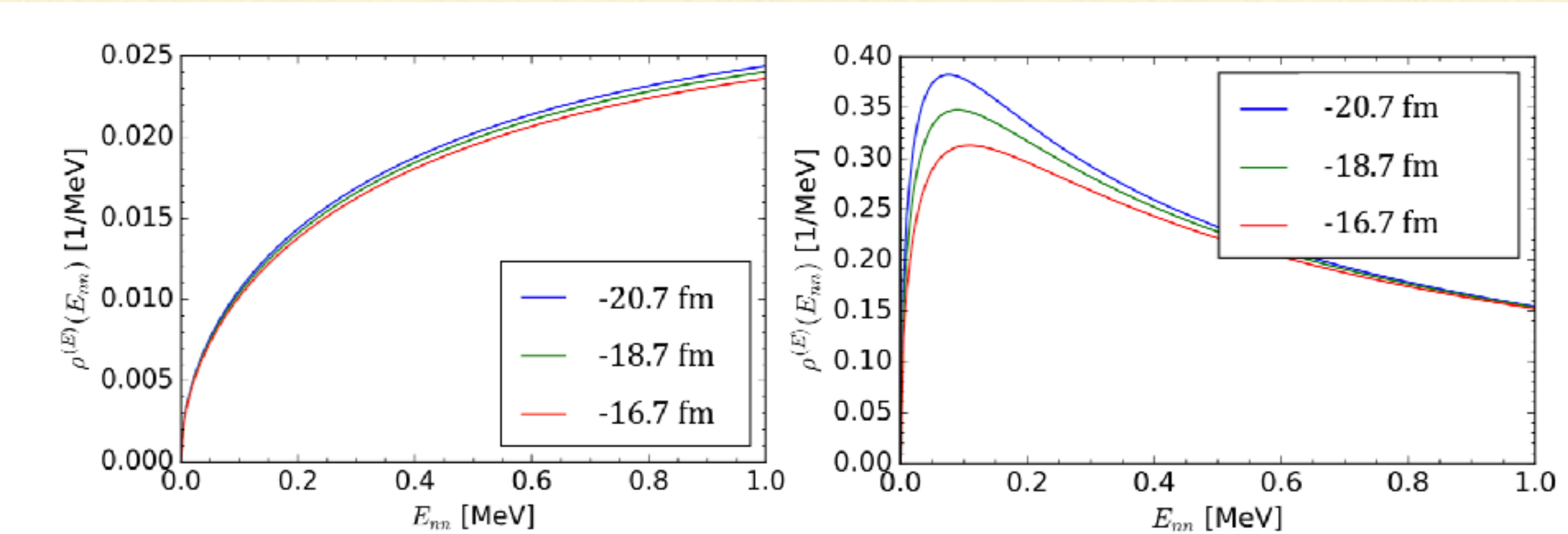

Note that since this is not an absolute measurement we need to decide how to normalize the spectra

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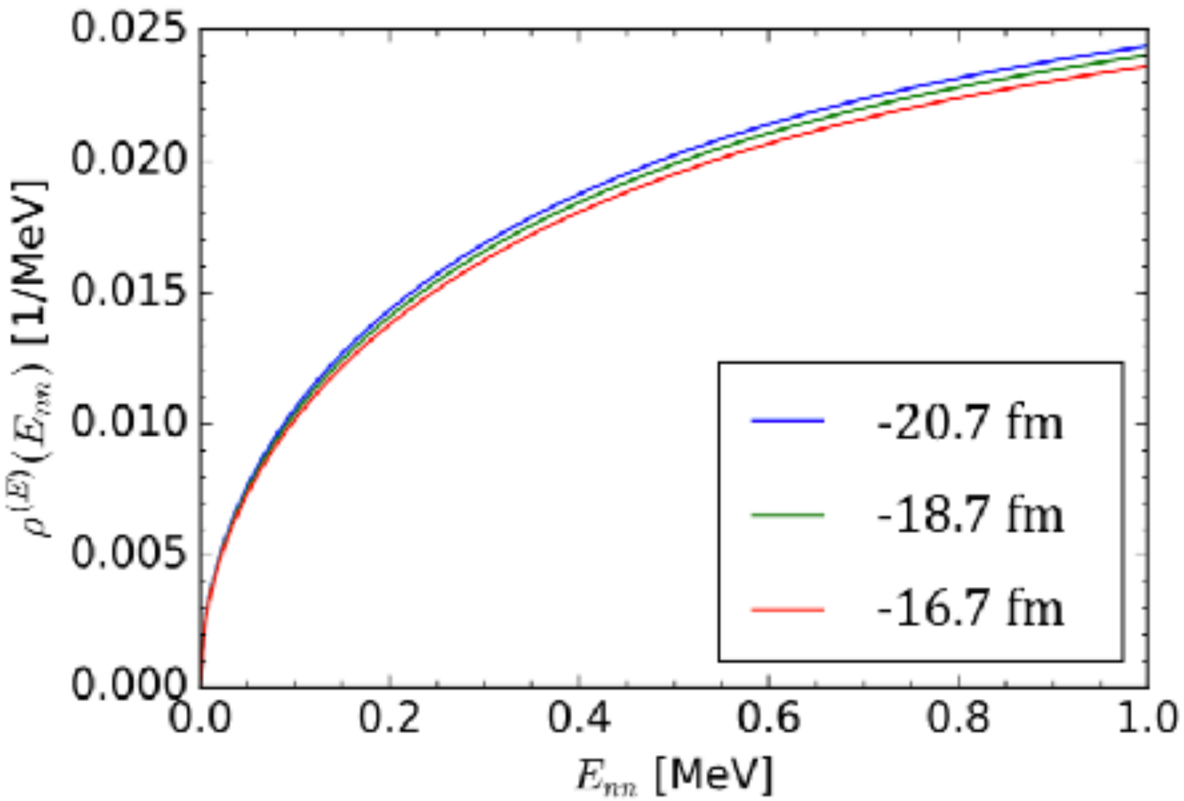

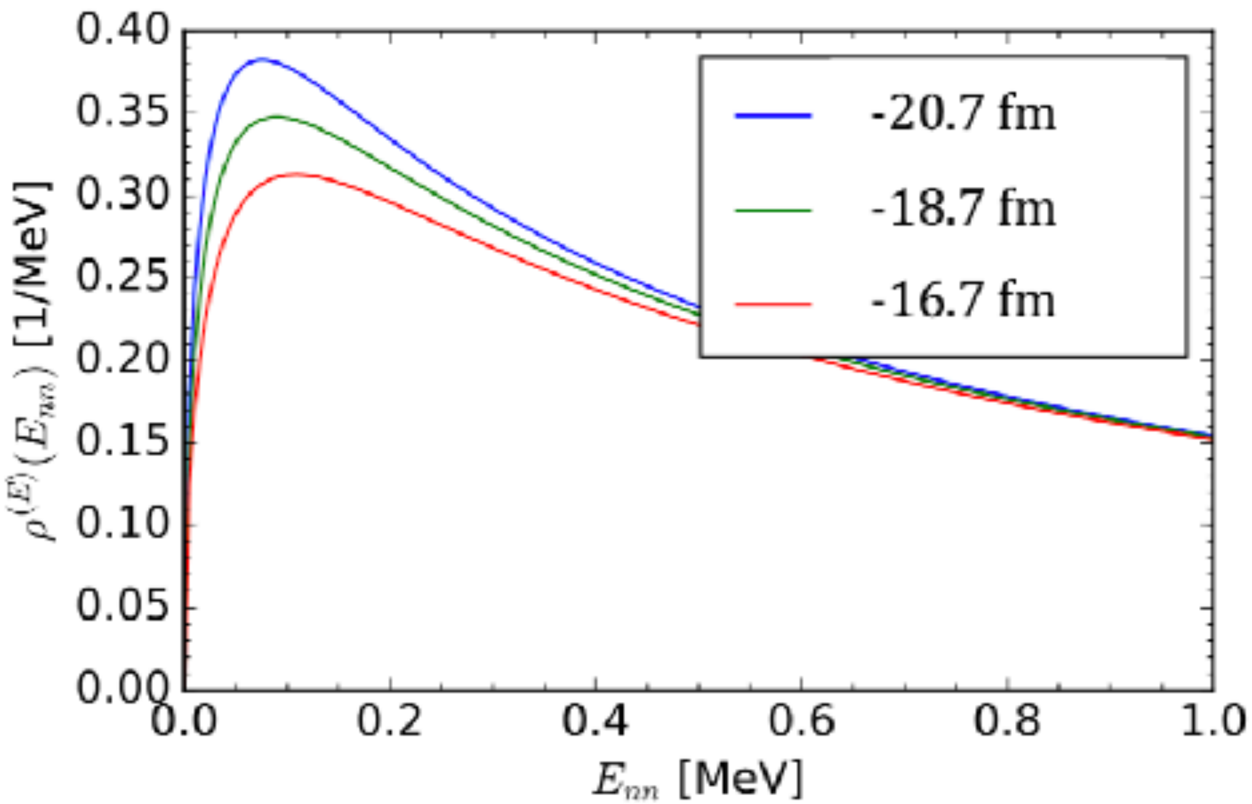

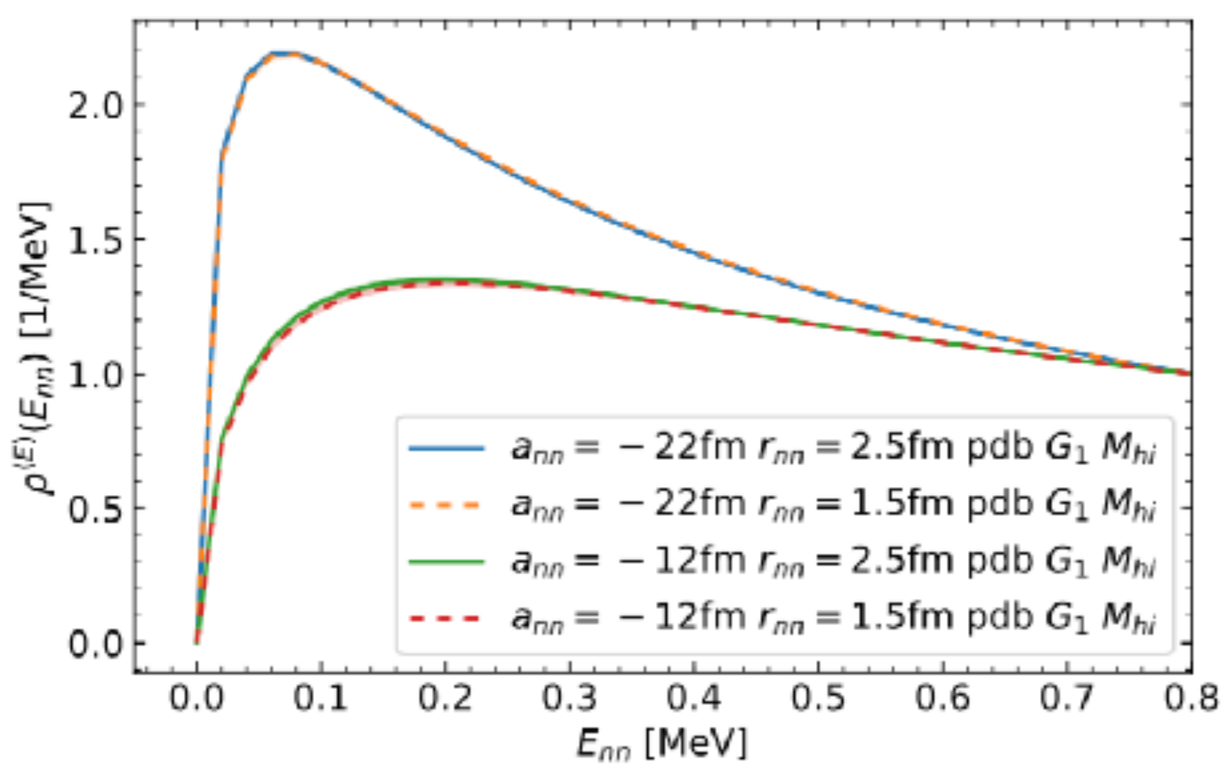

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What we learn from ^{6}He

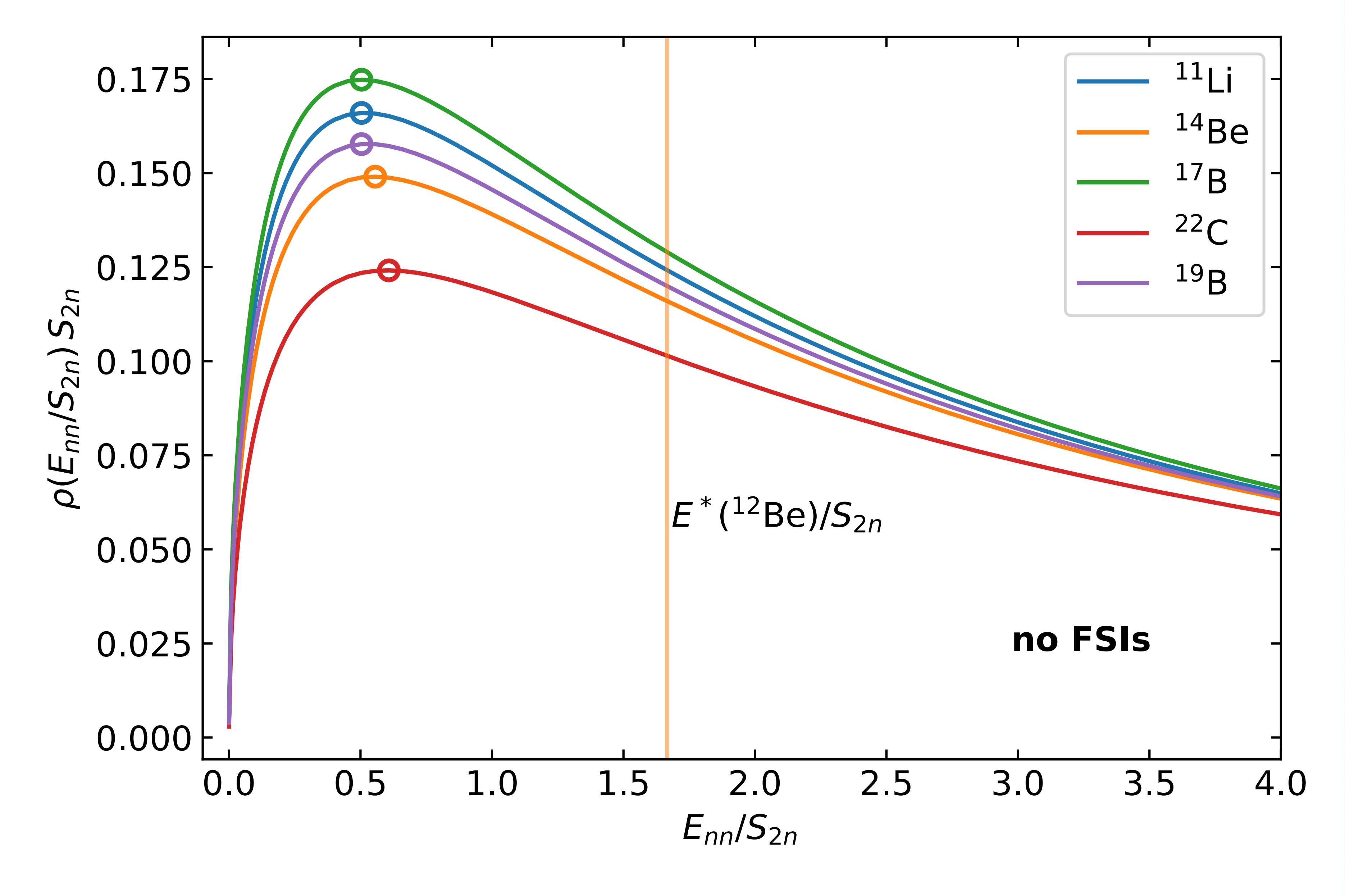
So ^{6}He relative-momentum distribution work shows:

- Very little sensitivity to a_{nn} in "structure part"
- NLO corrections to structure part should be small (not this talk)
- Even less sensitivity to r_{nn}
- Strong a_{nn} modification from FSI
- This modification can be well described by an enhancement factor

$$\rho^{\text{full}}(E_{nn}) \approx G(E_{nn}, a_{nn}, r_{nn})\rho^{gs}(E_{nn})$$

nn momentum distributions for s-wave 2n halos

Göbel, Hammer, DP, PRC (2024)



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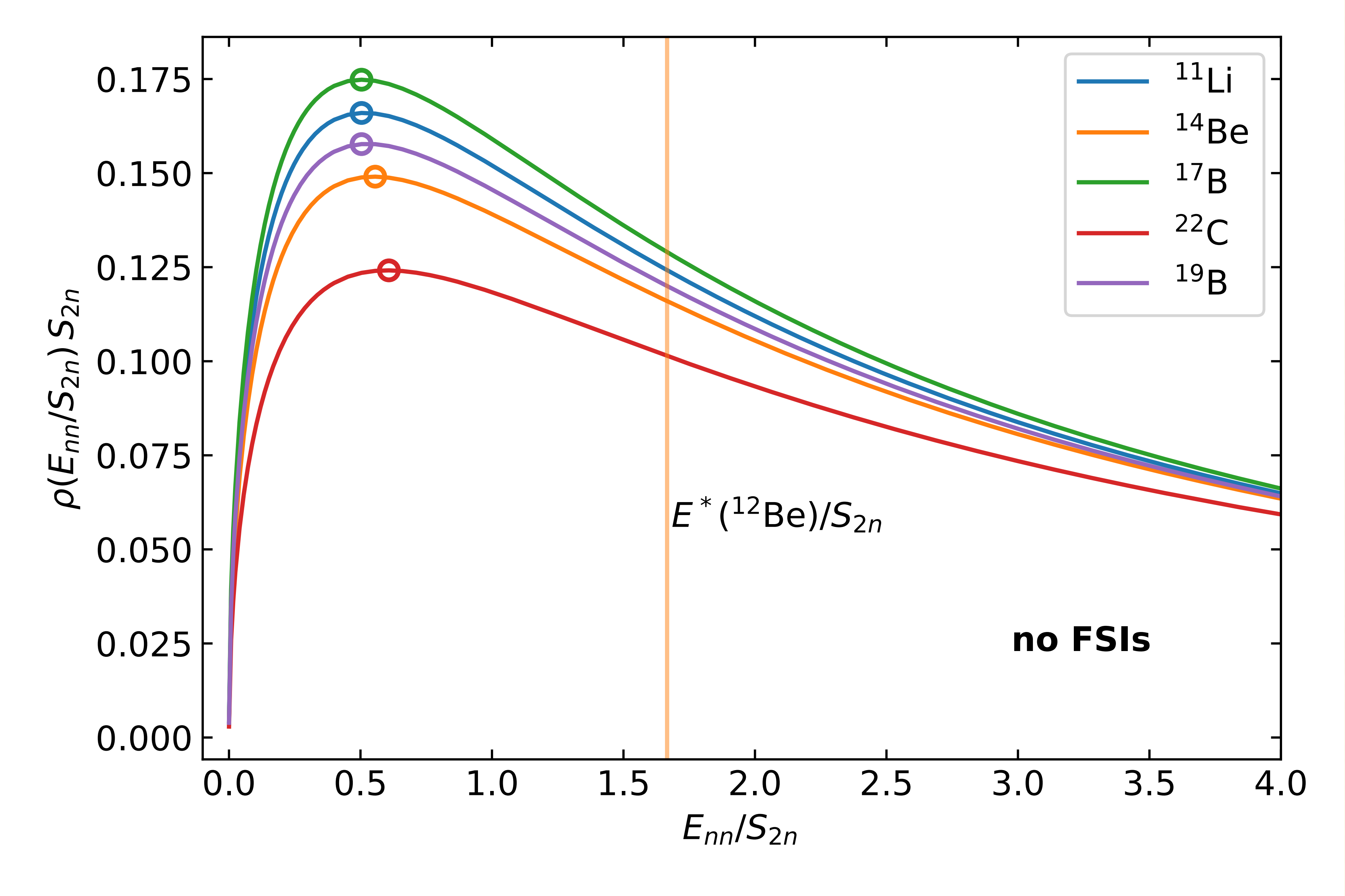

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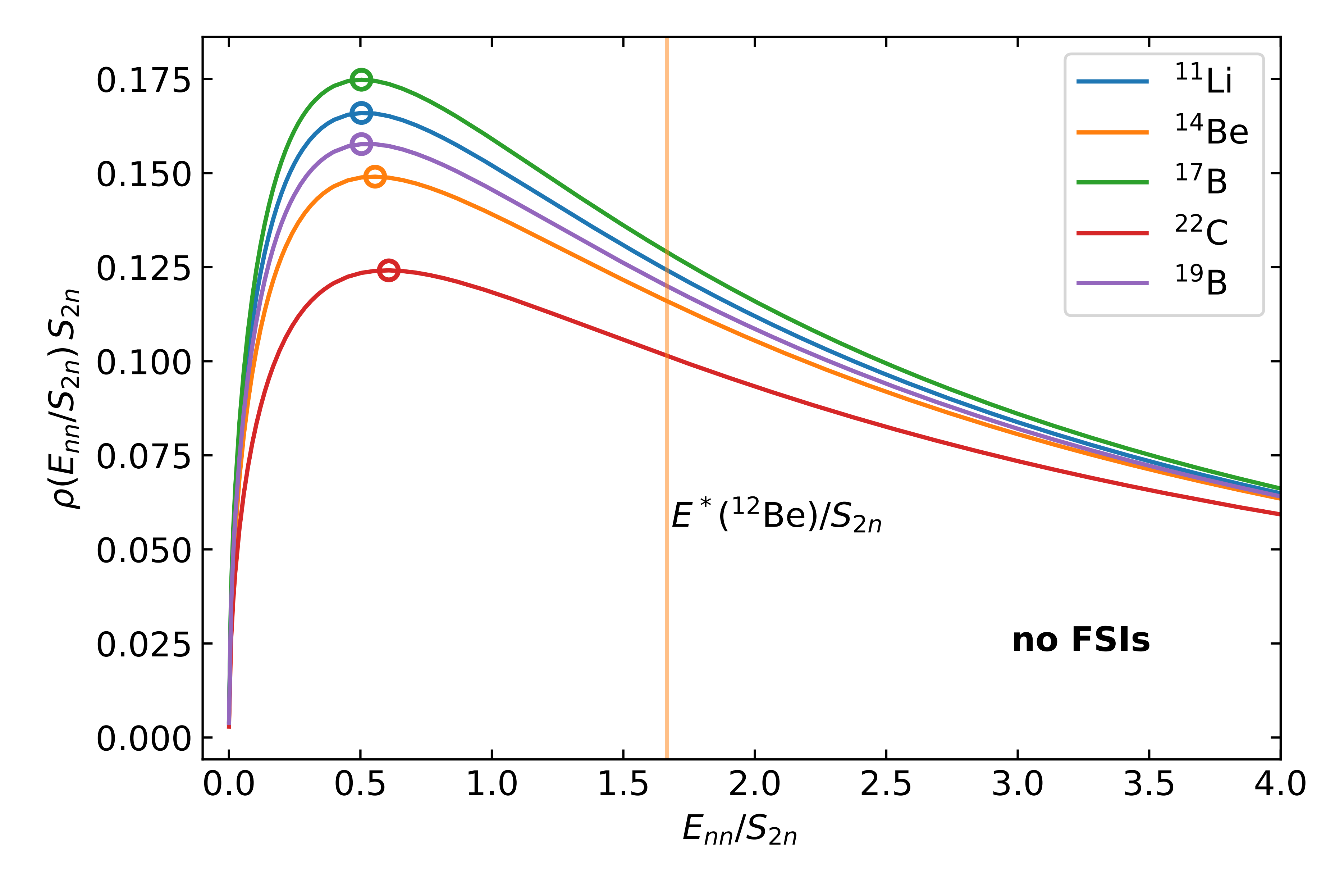

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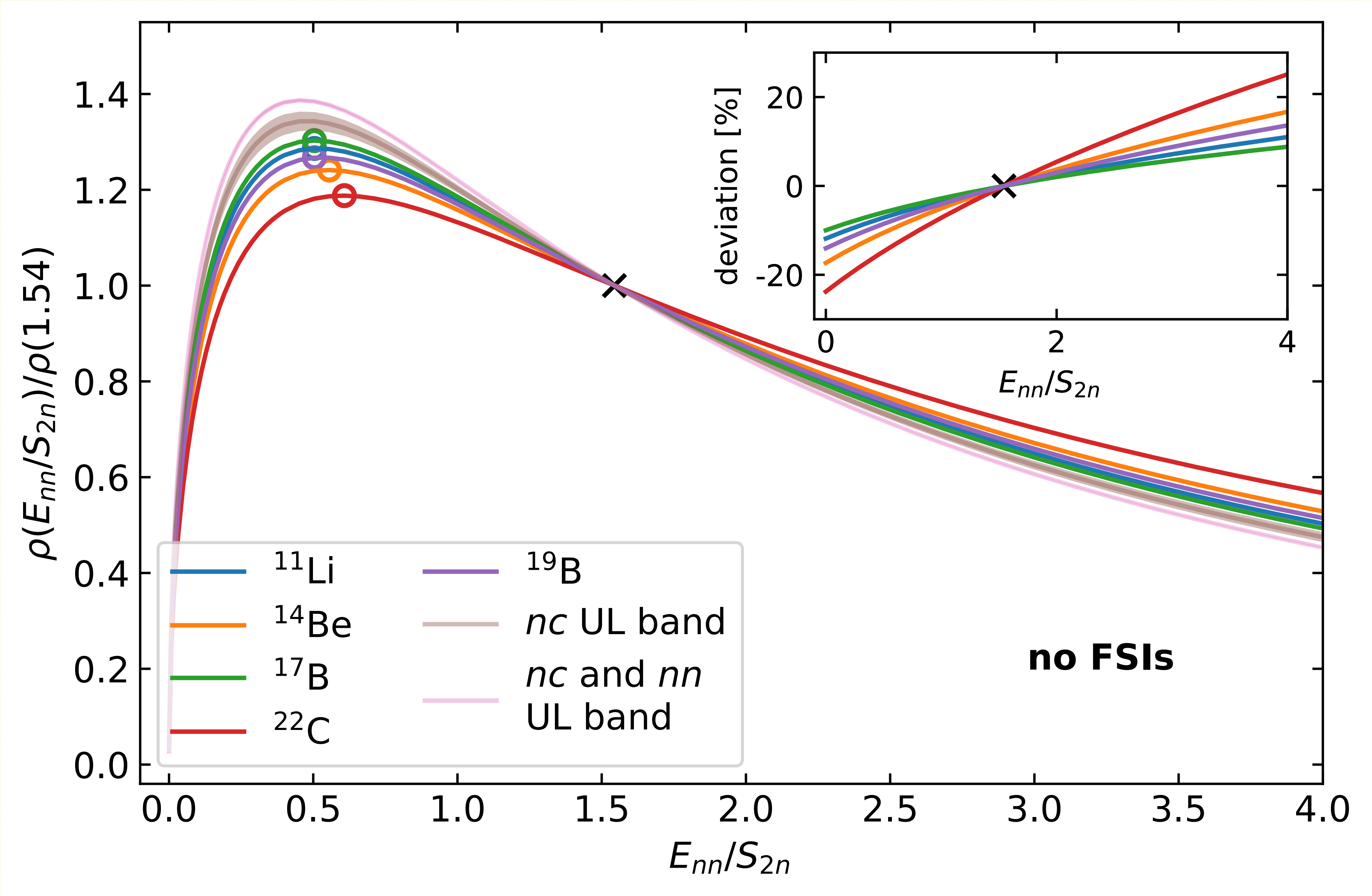

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Going to the unitary limit

- The "unitary limit" is another limit on top of LO Halo EFT: $|a| \to \infty$
- The 2B state is then right at threshold. No scales left: $r \to 0$, $|a| \to \infty$.
- 2B amplitude: $t^{2B}(E = k^2/m_R) \sim \dfrac{1}{ik}$, 2B problem has conformal invariance
- Efimov effect in 3B system: infinite tower of bound states $E^{(n)}/E^{(n-1)} = 515$
- Ratio of 4B and 3B binding energies $E^{4B,n}/E^{3B,n} = 4.6$ + excited tetramer

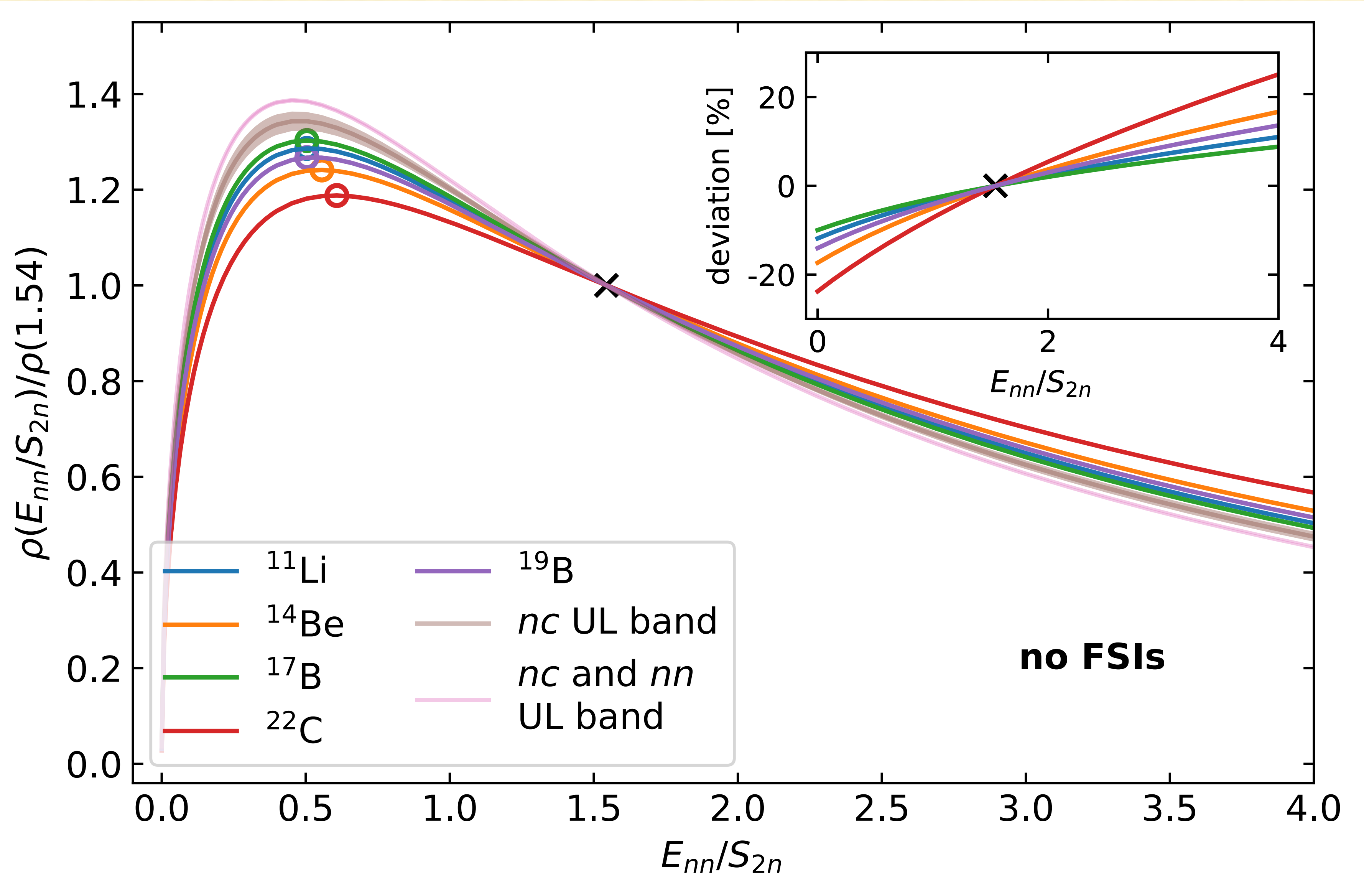
  Platter & Hammer (2007); Deltuva (2012)
- Scaling dimension of multi-neutron momentum distributions calculable

  Son & Hammer (2022); Chowdry, Mishra, Son (2023)
- **This talk:** momentum distribution of nn relative-momentum distributions in Borromean s-wave 2n halos

The unitary limit can be seen in 2n halos

Cf. for ^{19}B: Hiayma, Lazauskas, Marqués, Carbonell (2019); Hiyama, Lazauskas, Carbonell, Frederico (2023)

$$\rho^{g.s.}(E_{nn}/S_{2n}; V_{nn}, V_{nc}, S_{2n}, A) \approx \rho^{g.s.}(E_{nn}/S_{2n})$$



i.e., ρ is the same function for all halos to better than 20%

- Works because halos are sufficiently bound that precise values of a_{nn} and a_{nc} do not matter.
- A dependence also goes away

But can it be measured?

Results for other 2n halos after FSI modification

- Use Møller operator to include nn FSI:

$$\psi_c^{(\mathrm{wFSI})}(p,q) = < p,q;\zeta_c,\xi_c\,|\,(1+t_{nn}(E_p)G_0(E_p))\,|\,\Psi\rangle$$

- Relative energy distribution: $\rho(E_{nn}) = \sqrt{\frac{m_n}{4E_{nn}}}\int_0^\Lambda dq\, q^2\,|\,\Psi_c(p_{nn},q)\,|^2\, p_{nn}^2$

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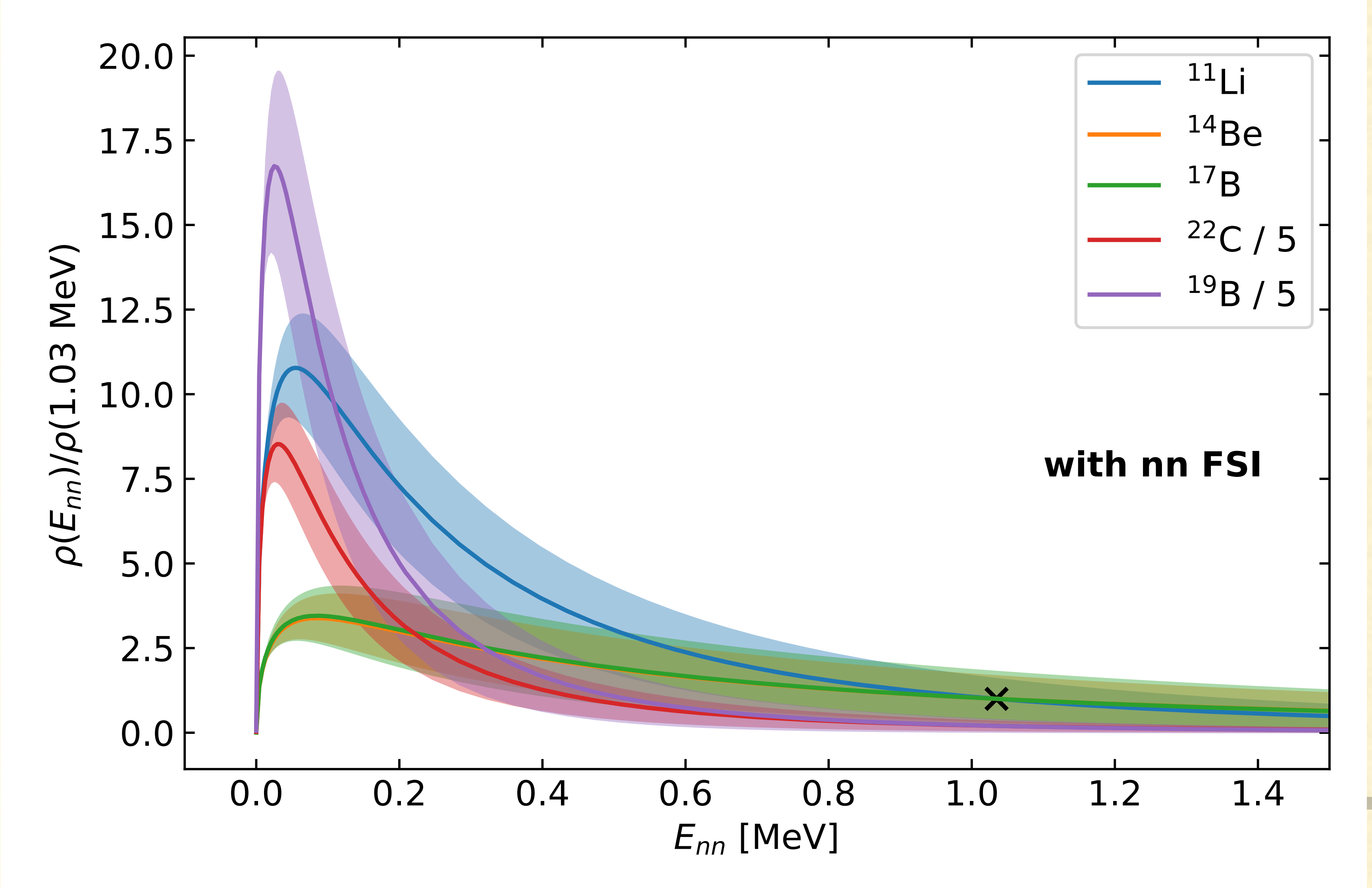

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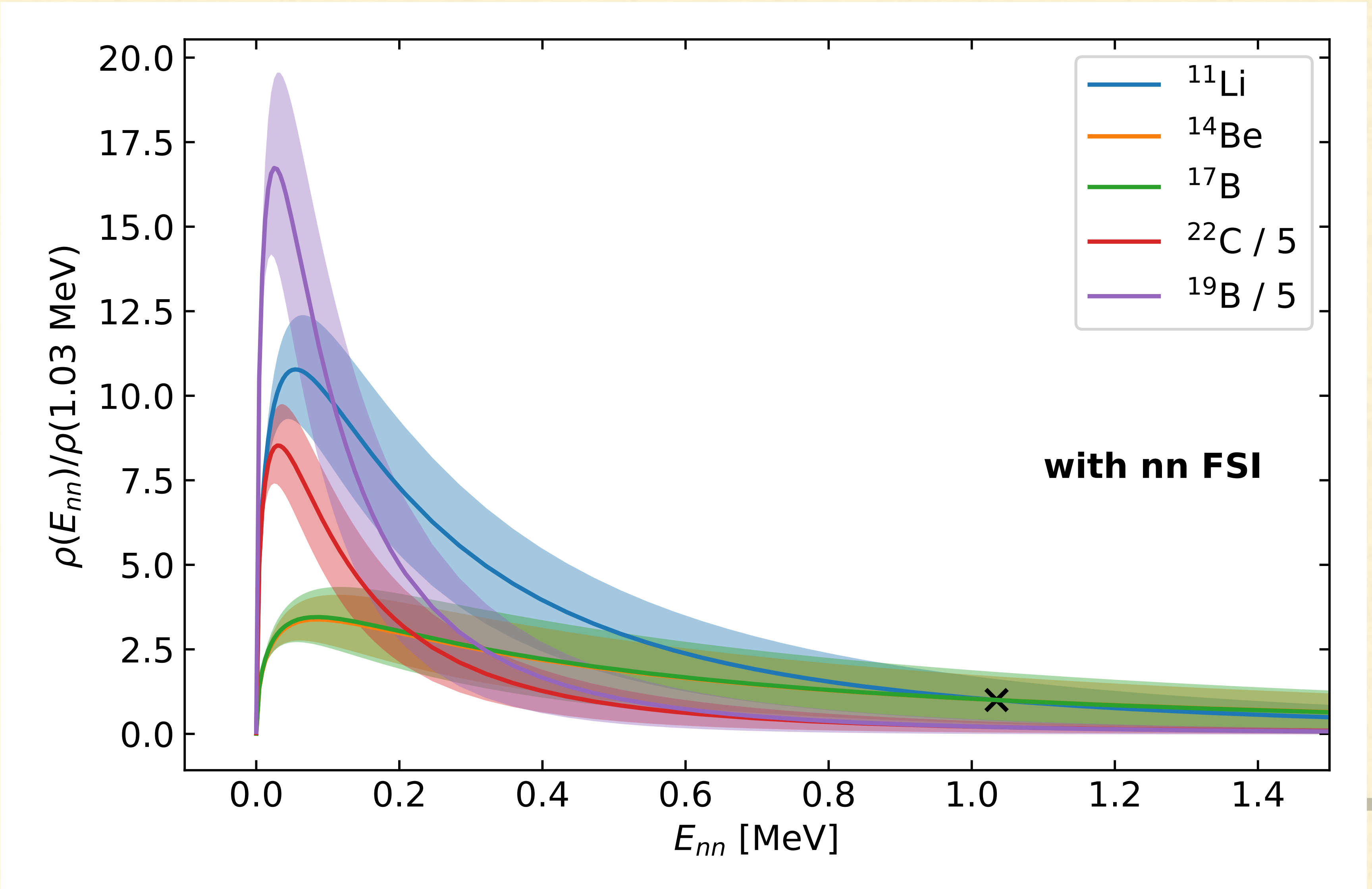

nn interaction produces variation on scale $1/(m_n a_{nn}^2)$

Ground-state distribution varies on scale S_{2n}

Divide out by FSI factor

Hypothesis: $\rho^{(\mathrm{wFSI})}(E_{nn}/S_{2n}; a_{nn}, r_{nn}) \approx G(E_{nn}; a_{nn}, r_{nn})\rho^{g.s.}(E_{nn}/S_{2n})$

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So we plot: $\rho(E_{nn}/S_{2n}) = \dfrac{\rho^{\text{full LO Halo EFT}}(E_{nn}/S_{2n}; a_{nn})}{G(E_{nn}; a_{nn}, r_{nn})}$

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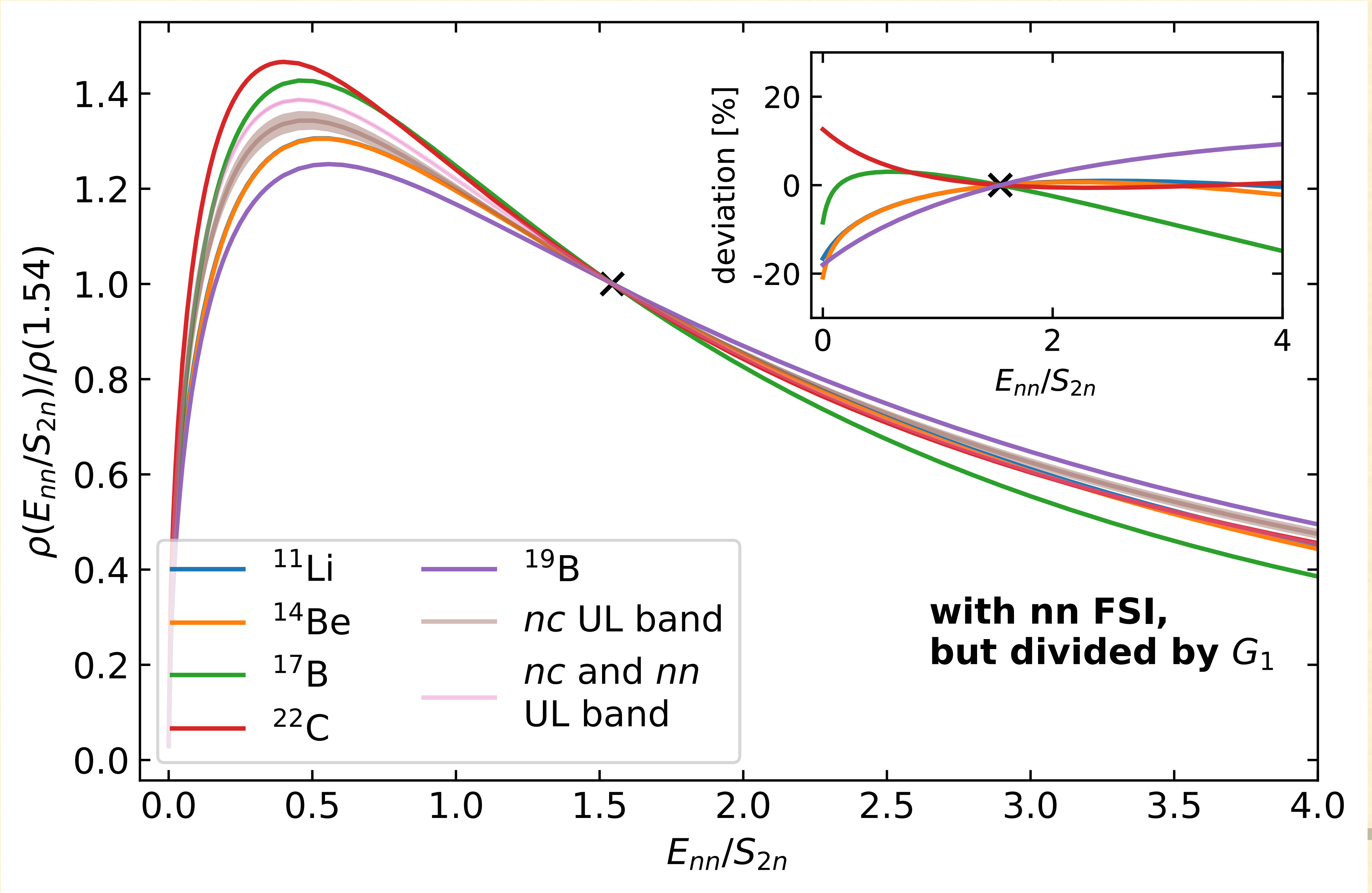

Distributed ±20% around UL result for ρ

Summary and outlook: part 1

Summary:

- Halo EFT describes the low-momentum physics of halo nuclei
- There is a unique nn momentum distribution in (s-wave) 2n halos
- Approximately the unitary limit momentum distribution: nothing about the nn and nc interactions matters except that they're strong
- This claim can be checked by measuring the nn relative energy distribution on several halos and dividing out the effects of FSI

To do:

- NLO corrections & comparison to ab initio calculations
- ^3H and other non-Borromean halos?

Analyzing ab initio calculations of ^{6}Li in Halo EFT

Hebborn, Brune, Phillips, in preparation

- Want to describe $\alpha(d,\gamma)^6$Li (motivated by Big Bang production of ^{6}Li). 6Li has a deuteron separation energy of 1.5 MeV: comparable to the deuteron binding energy of 2.2 MeV, cf. proton separation energy in $\alpha \approx 20$ MeV.

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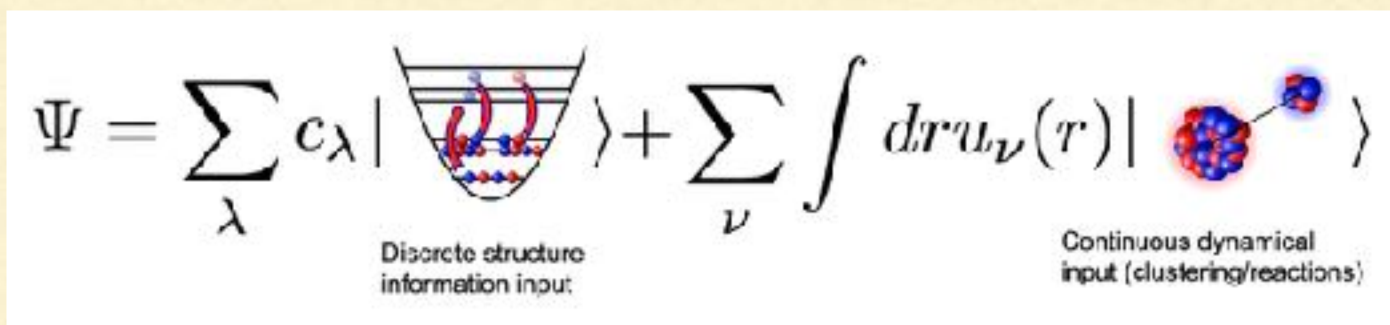
- NCSMC calculation: diagonalization of nuclear Hamiltonian using an over-complete basis



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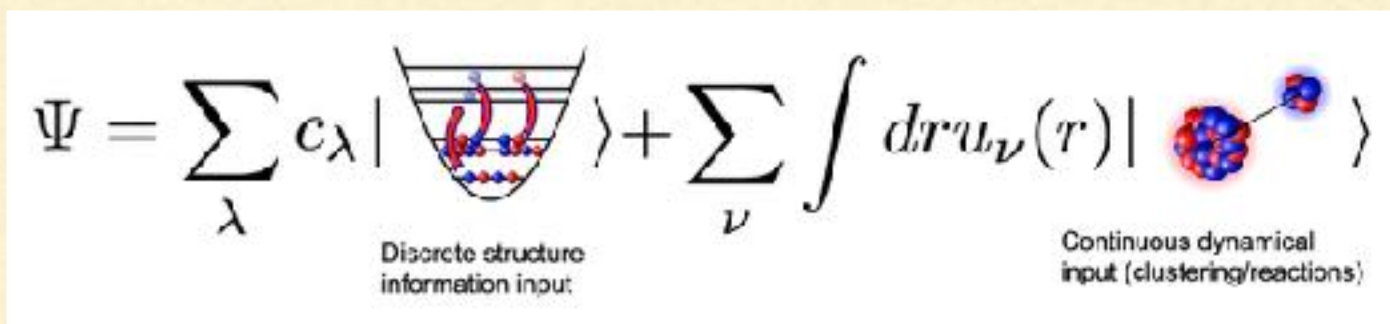
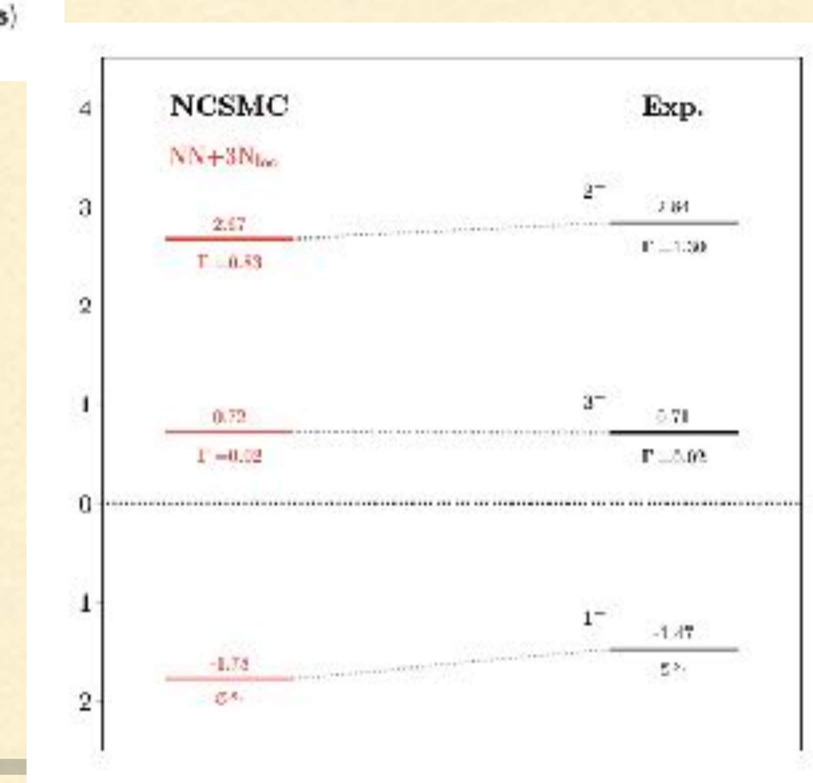


- Convergence with 10 positive parity and 5 negative parity ^{6}Li states, and deuteron ground state + 8 pseudo states for continuum at N_{max}=11



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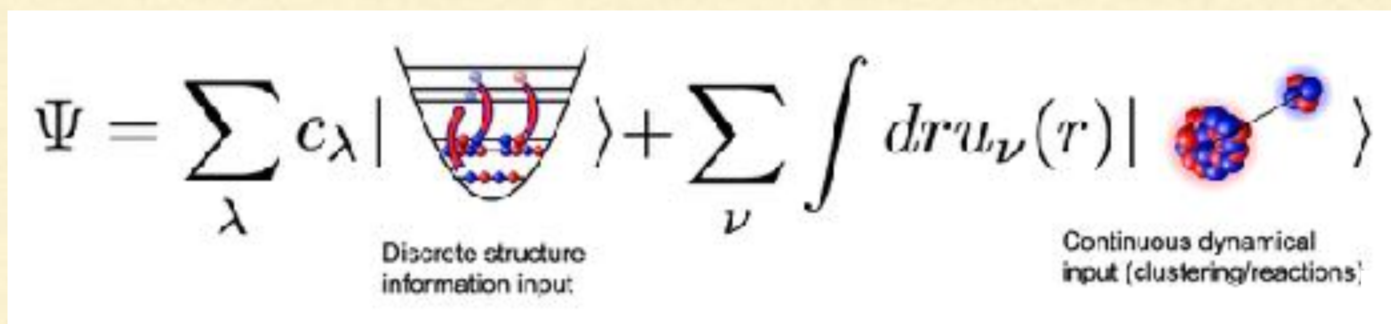

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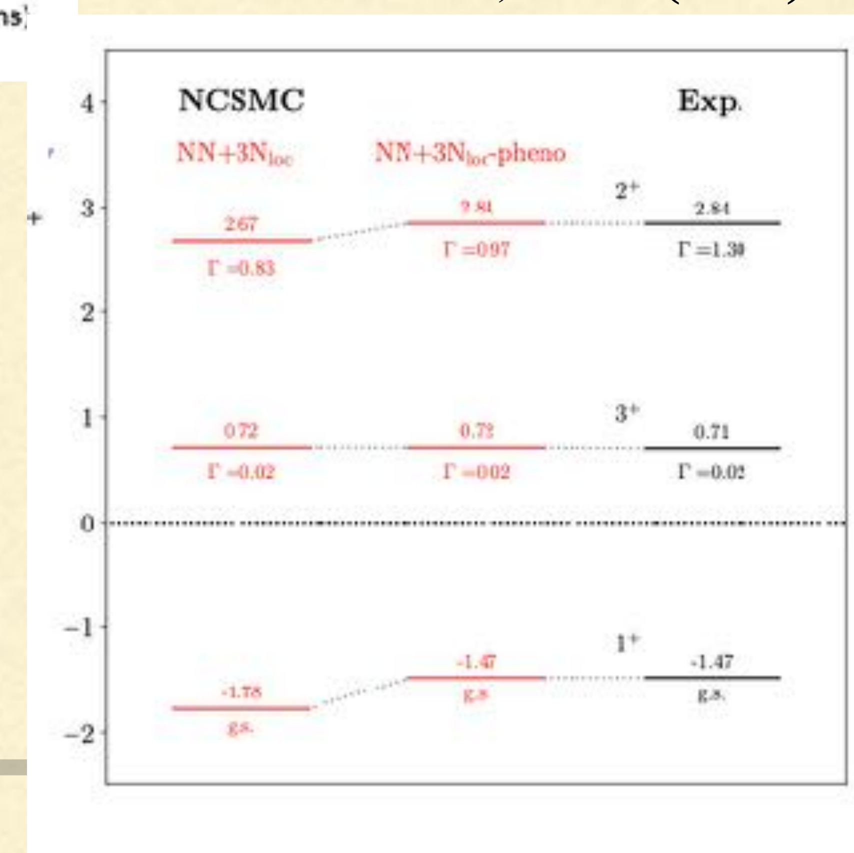
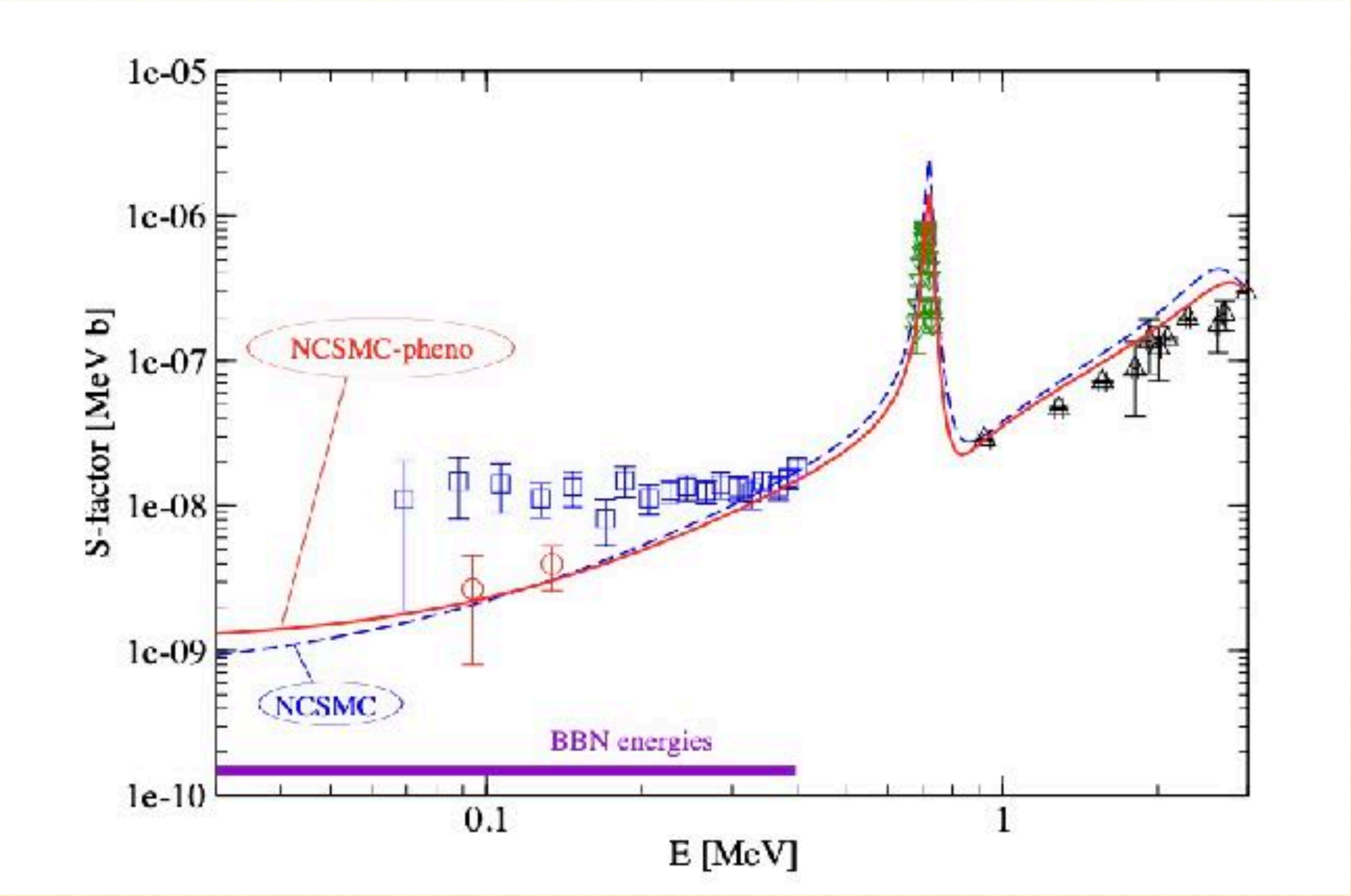


- Convergence with 10 positive parity and 5 negative parity ^{6}Li states, and deuteron ground state + 8 pseudo states for continuum at $N_{max}=11$

- Pheneomenological correction to NCSMC Hamiltonian to shift energies of ground and first excited state so they agree with experiment

NCSMC results



- Excellent agreement with data
- Small uncertainties due to chiral force and N_{max} thanks to pheno adjustment

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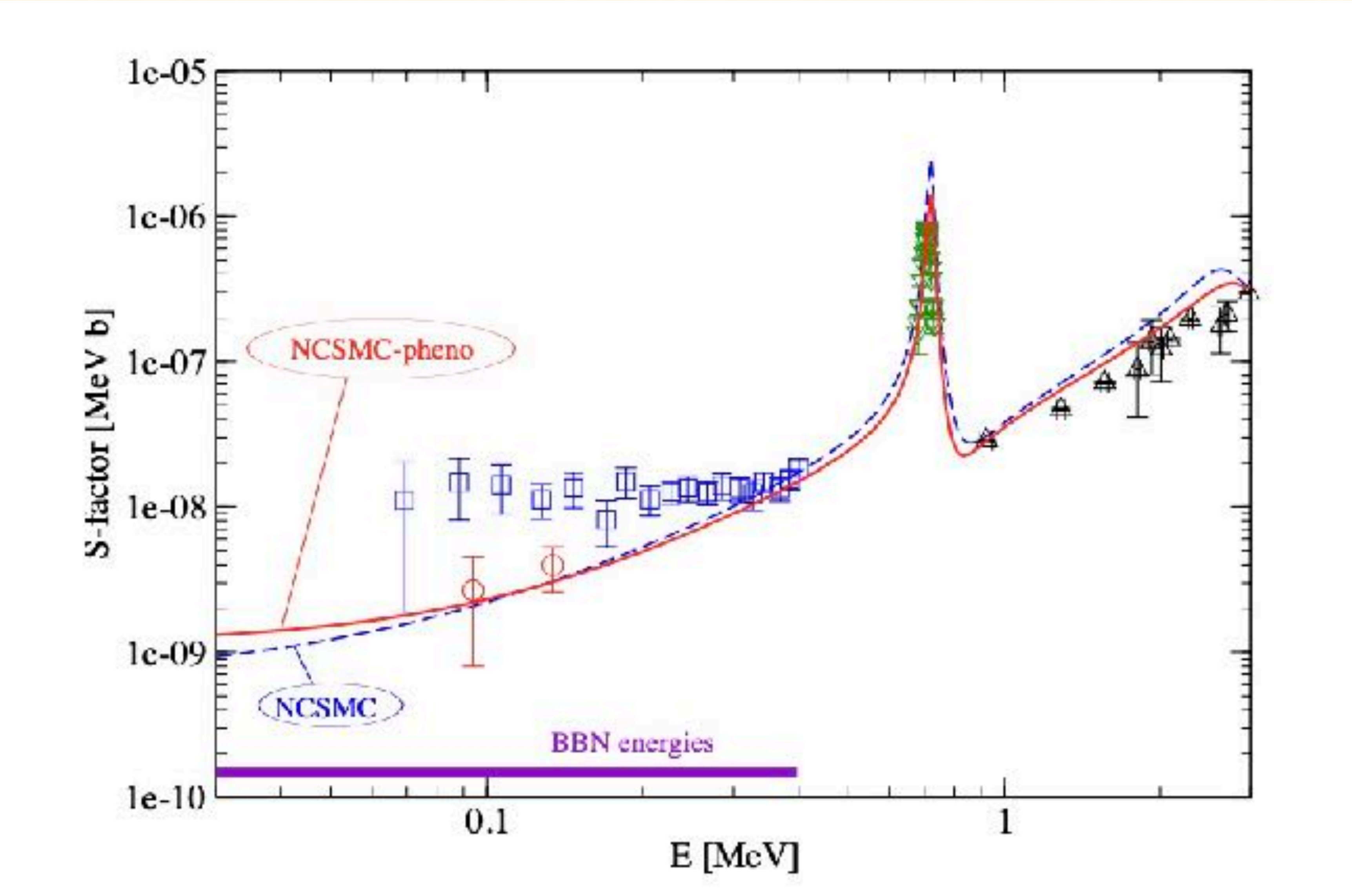

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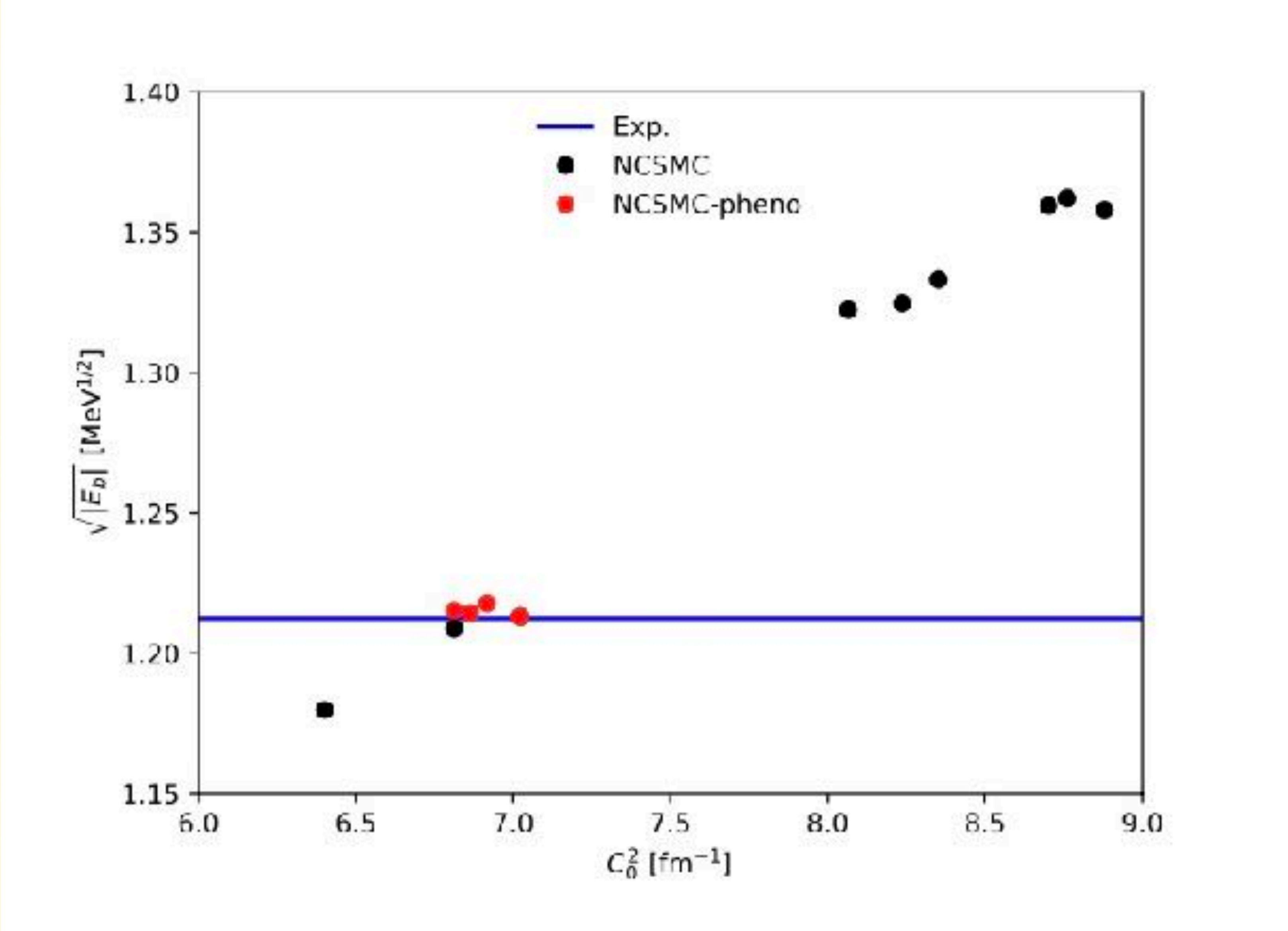
ANC?

Is ANC of bound state (or width for resonance) stable against variations of force, etc., once deuteron separation energy, S_d, has been adjusted?

- Results indicate a one-parameter correlation between C_0^2 and the binding momentum $\sqrt{S_d}$
- $C_0^2 = \dfrac{2\gamma_0}{1-\gamma_0 r_0}$ so C_0^2 should scale approximately linearly with $\sqrt{S_d}$



Is this correlation indicative of universality?

Fitting *ab initio* phase shifts to CMERE

Bethe (1949)
Sparenberg, Capel, Baye (2010)

$$\frac{2\pi\eta}{e^{2\pi\eta}-1}k\cot(\delta) + 2k_C\mathrm{Re}[H(\eta)] \equiv K(k^2)$$

$$k_C = m_R Z_1 Z_2 \alpha_{\mathrm{em}}$$

$$\eta = k_C/k$$

$$K(k^2) = -\frac{1}{a_0} + \frac{1}{2}r_0k^2 + \frac{1}{4}P_0k^4 + \frac{1}{8}Q_0k^6 + \frac{1}{16}R_0k^8 + \ldots$$

- CMERE=Coulomb Modified Effective Range Expansion
- $K(k^2)$ is real and analytic in k^2 within a radius of convergence defined by the first (non-Coulomb) analytic structure
- Extrapolate Coulomb Modified Effective Range Theory amplitude to $k^2 < 0$ to find zero of inverse amplitude

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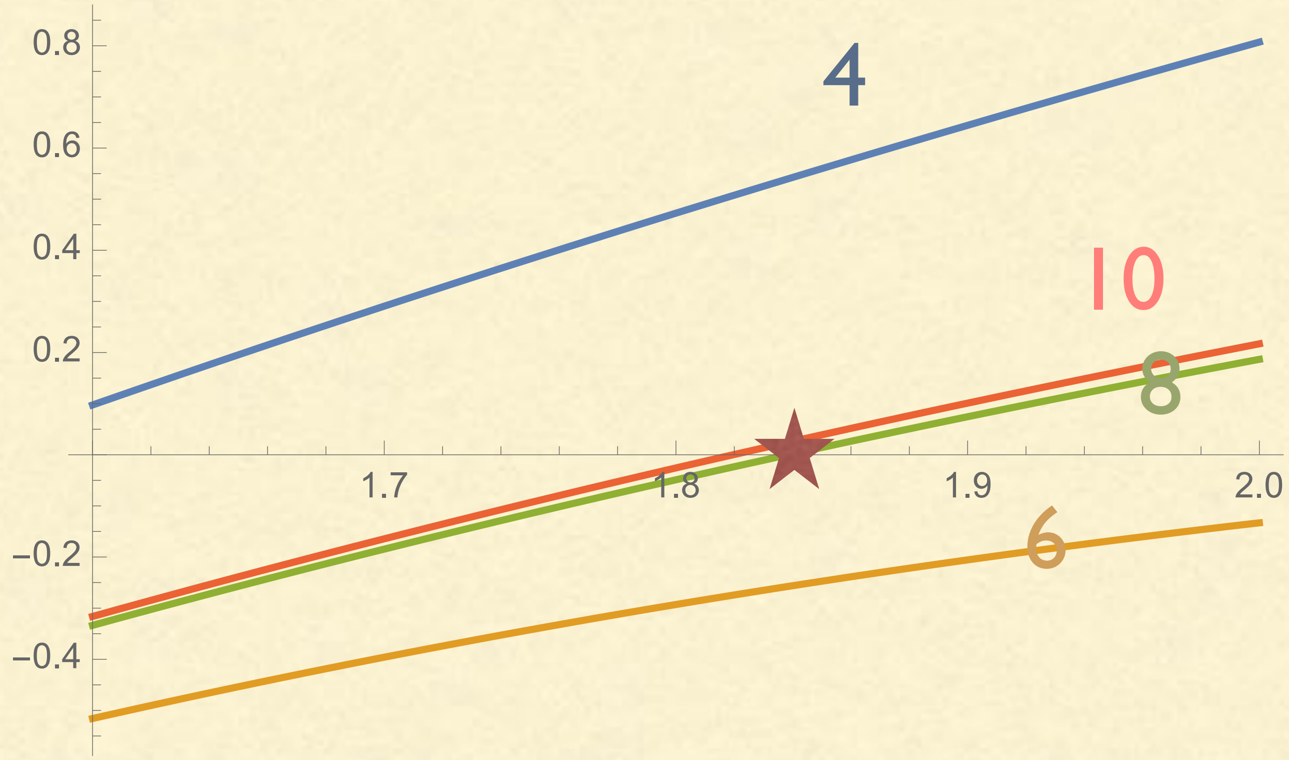
Fit to E_{max}=3 MeV of NN-only phase shifts



Constrained CM-ERE

$$K(k^2) = 2k_C H(\eta(-S_d)) + \frac{1}{2}\rho_0(k^2+\gamma_0^2) + \frac{1}{4}P_0(k^2+\gamma_0^2)^2 + \ldots \qquad \gamma_0^2 = 2m_R S_d$$

- Expand around bound-state pole
- Then $C_0^2 = 6k_C \frac{\Gamma(1+|\eta(S_d)|)^2}{\tilde{H}(-\eta(S_d)) - 3\rho_0 k_C}$
- C_0^2=9.38 fm^{-1} at sixth order; 8.46 fm^{-1} at eight order; 8.53 fm^{-1} at tenth order.
- Not bad, but high order required
- Cf. $6k_C$=0.56 fm^{-1} and $2\gamma_0$=0.69 fm^{-1}. Fine tuned?
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Ryberg et al. (2016)
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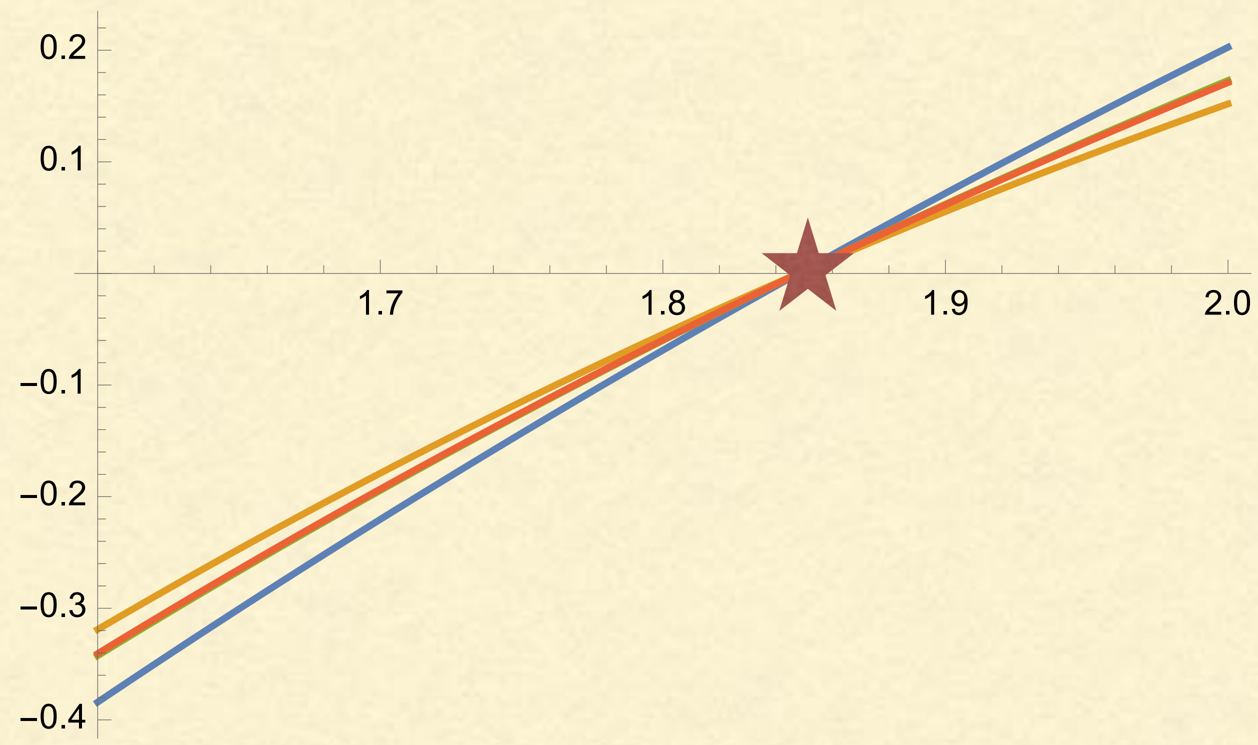

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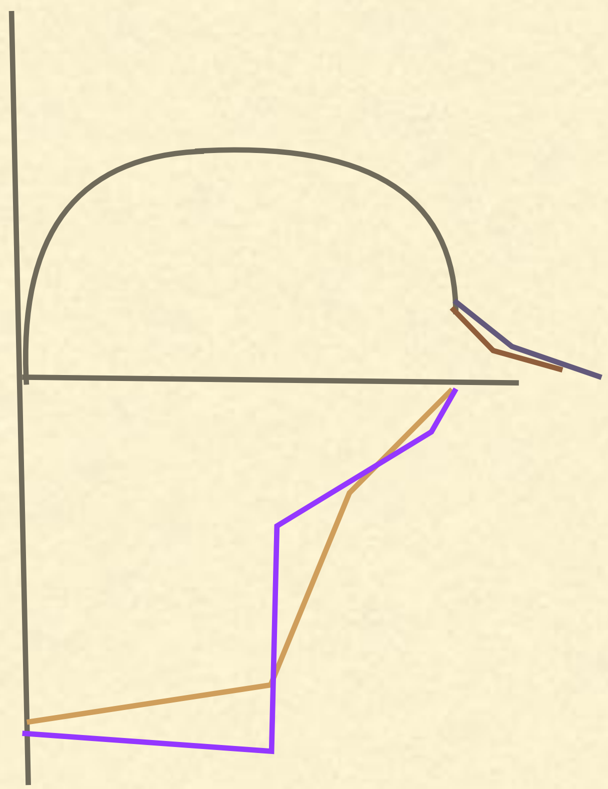

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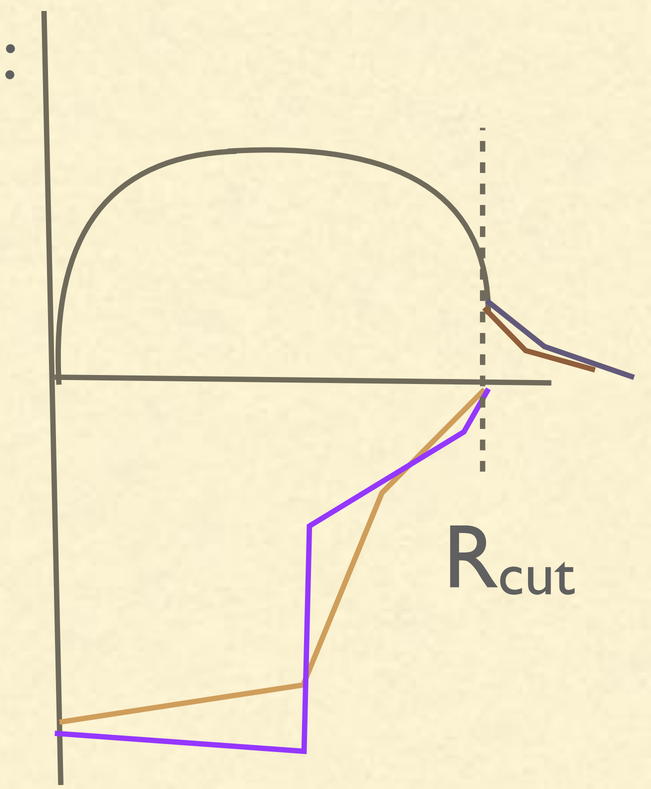

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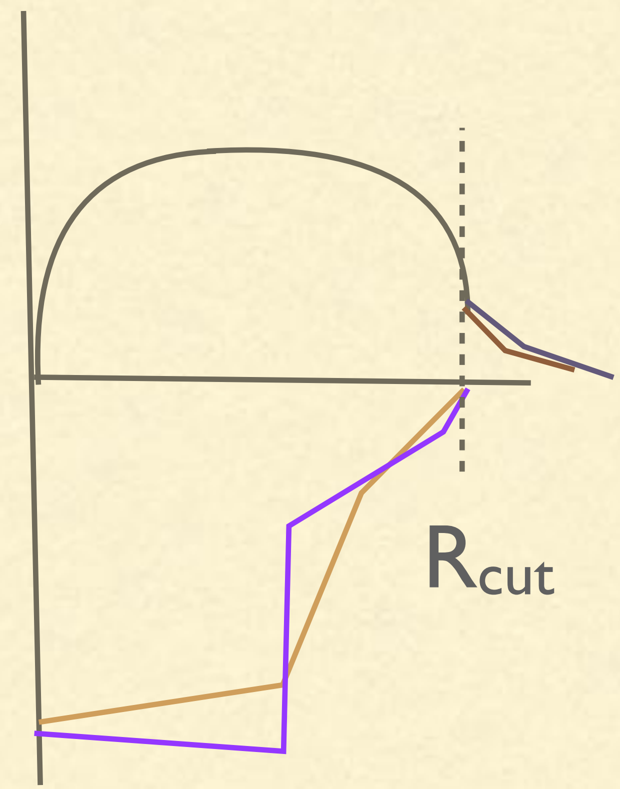

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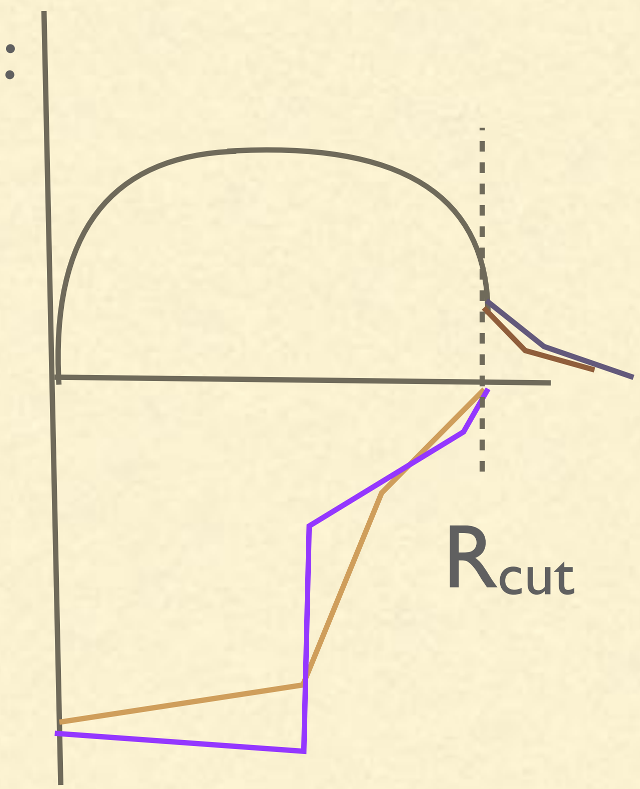

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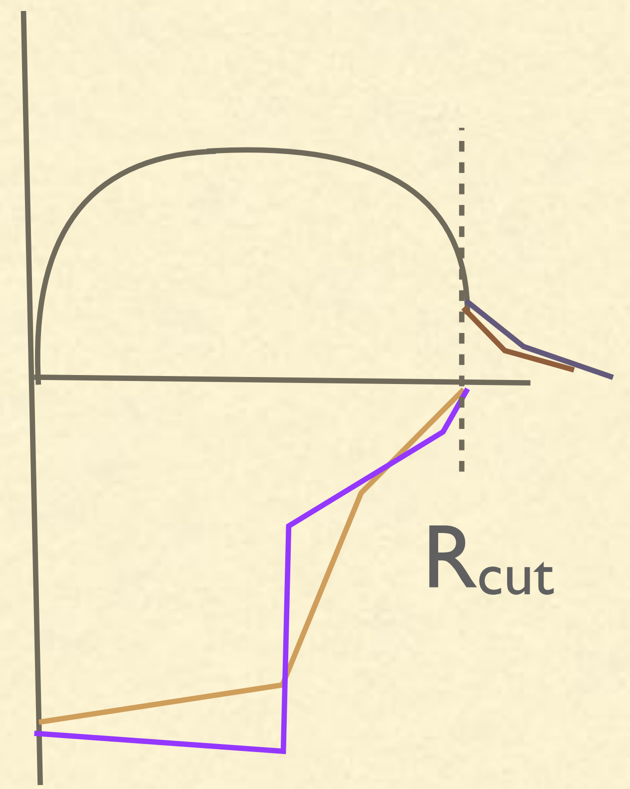

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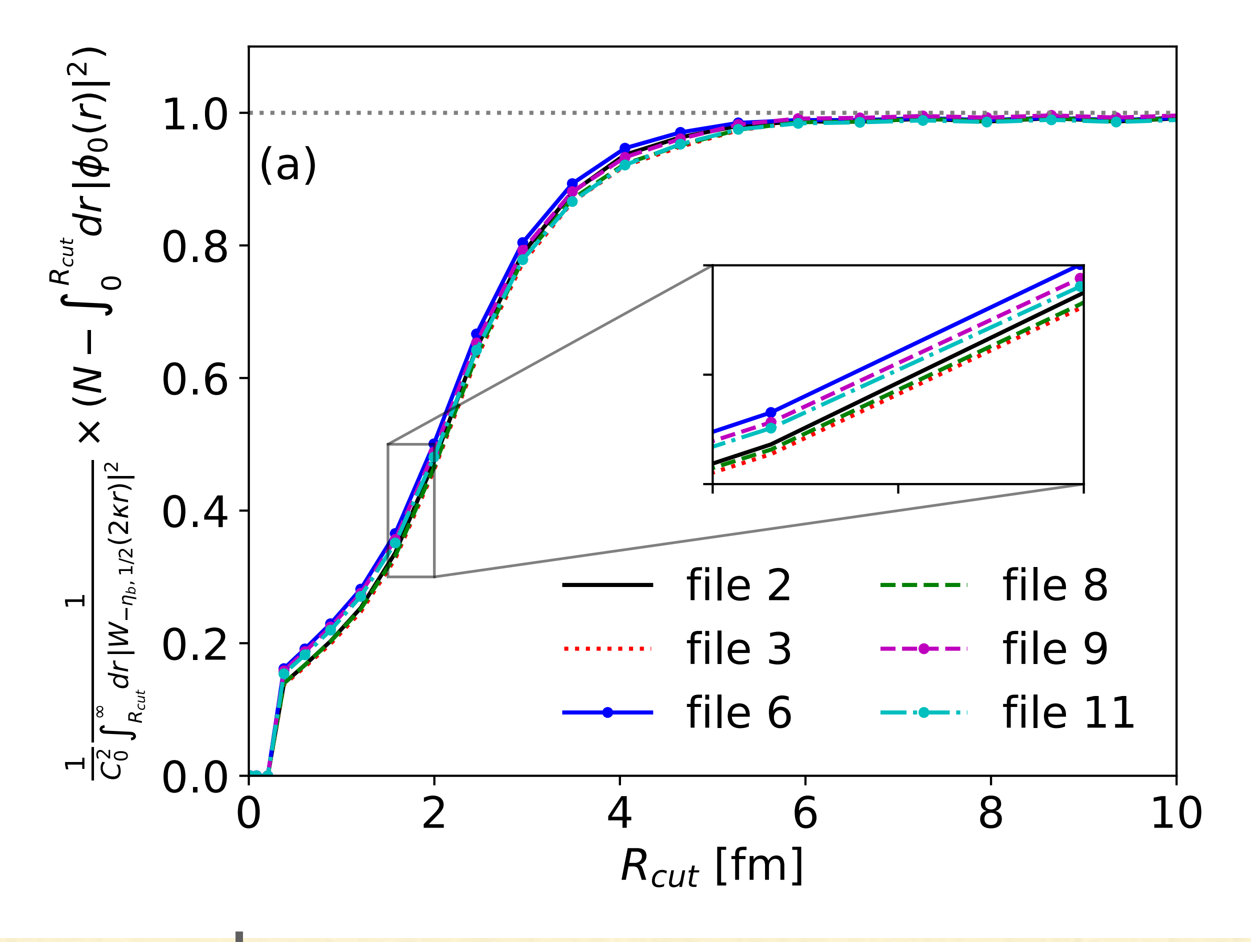
- So if f(x) becomes independent of x then $C_{0,j}^2$ will grow linearly with $\gamma_{0,j}$ in that region

Getting on the scaling curve

- How far do we have to go in R_{cut} for $f(\gamma_0 R_{cut})$ to become R_{cut} independent?

- Calculate exterior probability via:
  1. N-I(R_{cut})
  2. $\int_{R_{cut}}^{\infty} dr\, C_0^2 W_{-\eta,1/2}(2\gamma r)$



- R_{cut} in asymptotic region once they're equal

- By this measure asymptotic wave function reached already at 5 fm

- Scaling region! $f(\gamma_0 R_{cut})$ independent of R_{cut}

Summary and outlook: part 2

- NCSMC calculations yield consistent scattering and bound-state results
- Offer the possibility to compute ANCs and separation energies *ab initio*
- But getting the proton (or deuteron or neutron or …) separation energy of halos accurately is hard: NNLO ChiEFT only predicts it to~a few hundred keV
- NCSMC-pheno adjusts last few hundred keV of separation energy
- So can be expected to also get ANC right if correlation is one parameter
- ^{6}Li is a halo (fine tuned) nucleus: one-parameter ANC-S_d correlation expected from independence of short-distance part of wave function to fine tuning
- Offers possibility to perform halo calculations in smaller model spaces or without three-body forces and then pheno adjust to reproduce separation energy and ANC