In-medium parton showers with overlapping emissions

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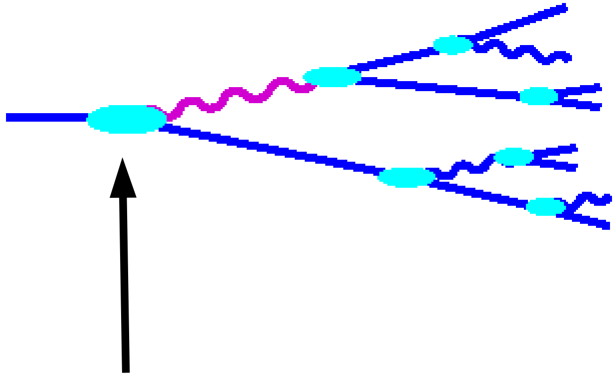
Reporting (eventually) on recent work with

Shahin Iqbal Omar Elgedawy

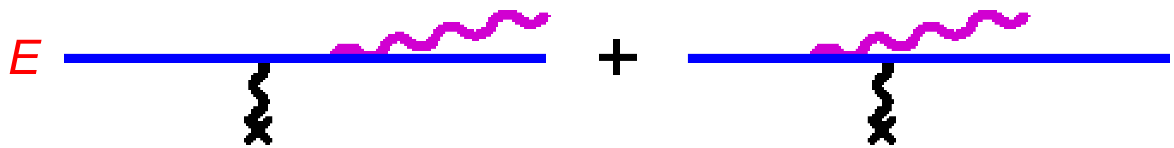
letter: 2212.08086
details: 2302.10215

and work in preparation

Medium-induced showering

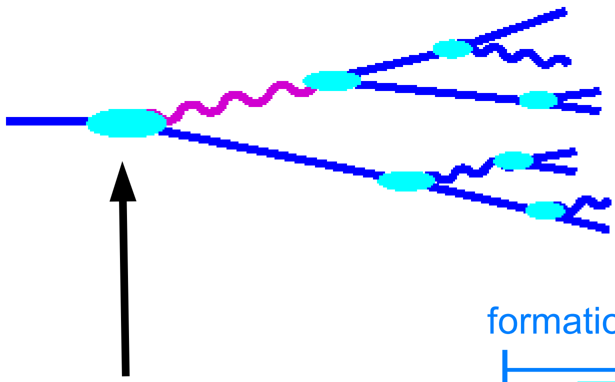


Naively:



Prob. of brem ~ α per collision with medium
(up to logs)

Medium-induced showering



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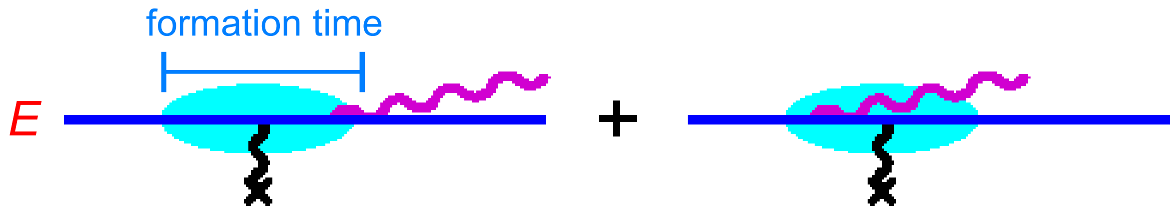
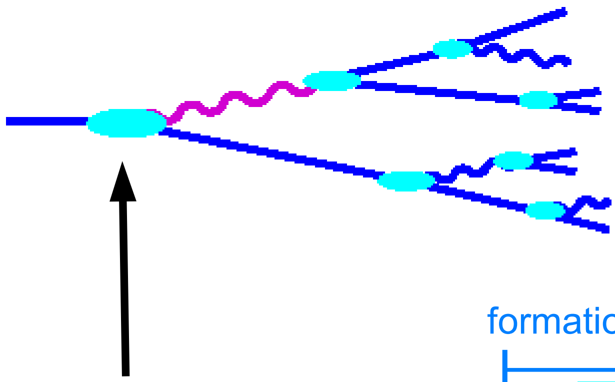


Formation time means quantum duration of splitting process.

Formation time grows with energy E.

Medium-induced showering



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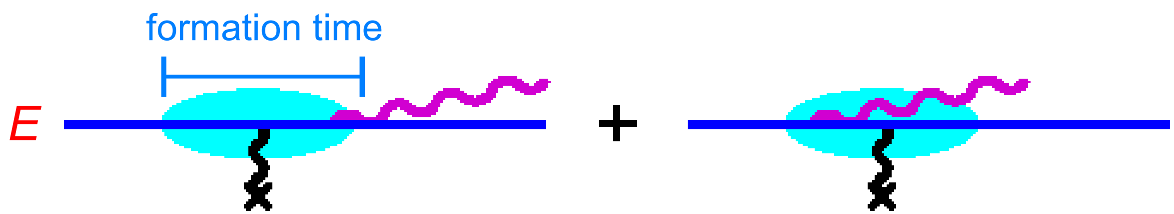
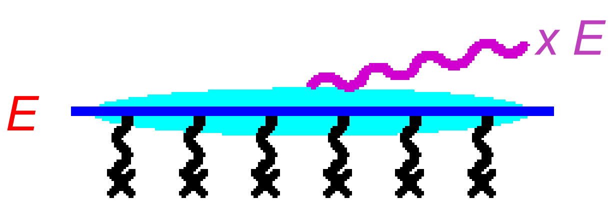


Formation time means quantum <u>duration</u> of splitting process.

Formation time grows with energy *E*.

LPM Effect:

What happens when formation time $\gg$ mean free time between collisions w/ medium?



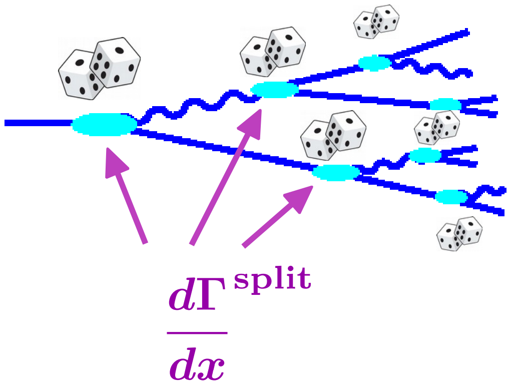
Prob. of brem ~ α per formation time

QED (1950s): LPM [Landau-Pomeranchuk & Migdal]

QCD (1990s): BDMPS-Z + many later variations

→ calculation of splitting rates $\dfrac{d\Gamma^{\text{split}}}{dx}$

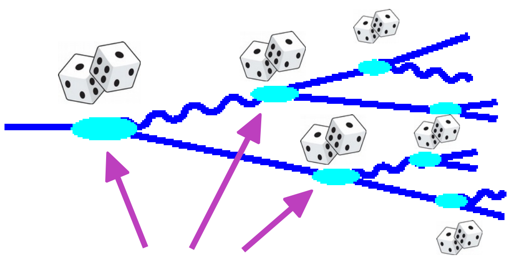
Can we then describe in-medium shower development by



(LPM splitting rates)

?

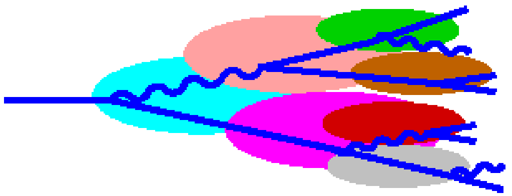
Or can splittings overlap?



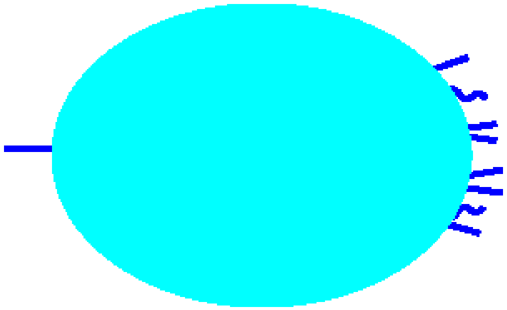
$$\frac{d\Gamma^{\text{split}}}{dx}$$

(LPM splitting rates)

vs.



vs.



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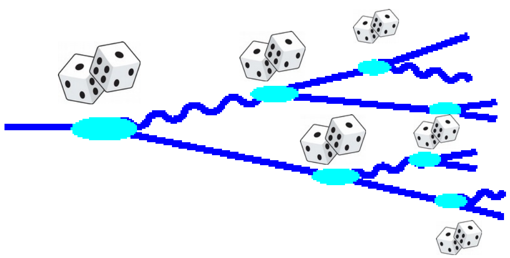
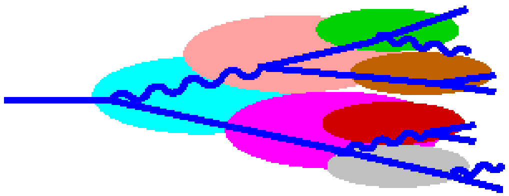
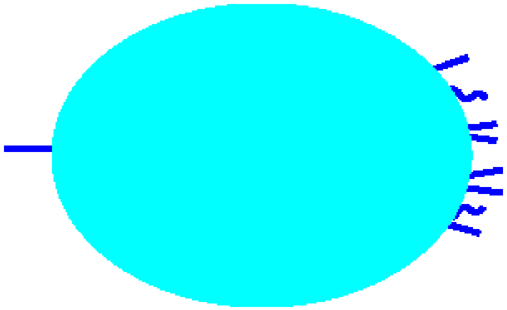


vs.



vs.



Prob. of brem ~ $\alpha(\mu)$ per formation time

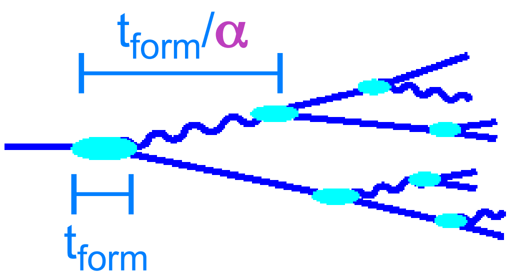
⇨ Prob. two consecutive splittings overlap ~ $\alpha(\mu)$

a scale depending on energy

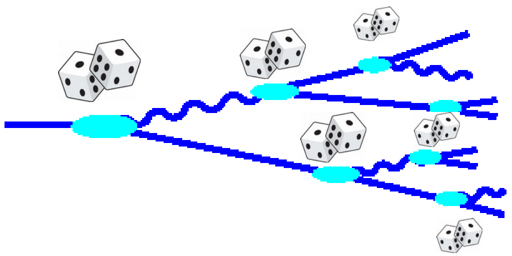
Not the $\alpha(T)$ of the QGP

All depends on how big $\alpha(\mu)$ is!

For small α, there is a hierarchy of scales that (typically) separates the splittings:



Summary so far



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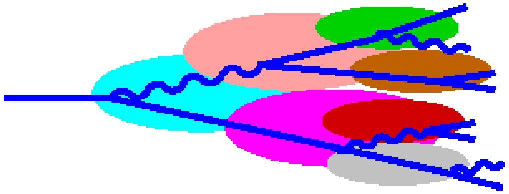
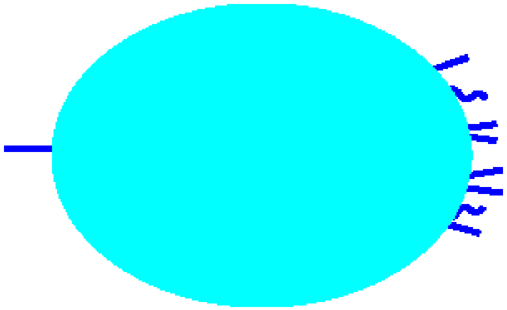


vs.



$\alpha_s(\mu)$ small

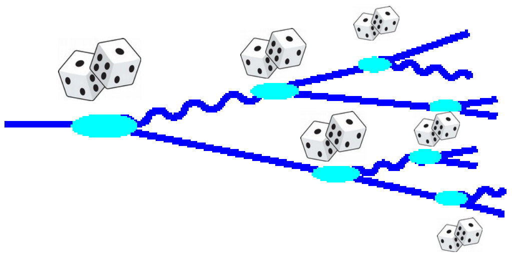
a “standard” picture
of a shower

$\alpha_s(\mu)$ big

HELP!

Turn to AdS/CFT for
qualitative insight

How do we tell if



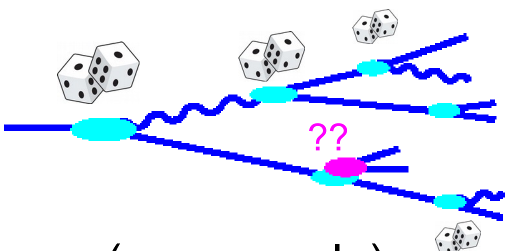
is a good or bad picture for reasonable values of $\alpha_s(\mu)$?

Two approaches

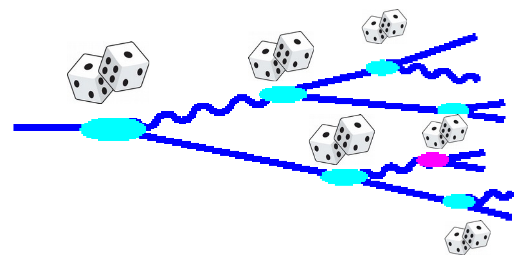
(1) EXTERNAL VALIDATION: Confront w/ experiment.
But…. many confounding factors.

(2) INTERNAL CONSISTENCY: Test with theory!

Question:
Are the first corrections



to



(an example)

small for reasonable values of $\alpha_s(\mu)$?

Perks for theorists:

- May avoid confounding factors by testing in simplified situations.
- Can test on simple shower characteristics not accessible to experiment.

So…

A theorist thought experiment

Simplifying assumptions

- Treat elastic scattering w/ medium in the $\hat{q}$ approximation:

$$\langle (\text{change in } p_\perp)^2 \rangle = \hat{q} \cdot (\text{distance traveled})$$

- A static, homogeneous, “infinite”-size QGP

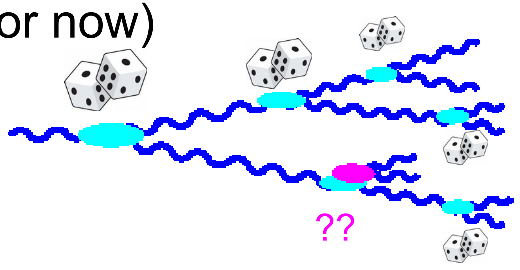
In that case, the scale μ for $\alpha_s(\mu)$ is $\quad \mu \sim (\hat{q}E)^{1/4}$

and formation times are $\quad t_{\rm form} \sim \sqrt{E/\hat{q}}$

- Start with a parton that is (approx.) on-shell.

- Study gluon-initiated showers in large-N_c limit (w/ N_f fixed for now)

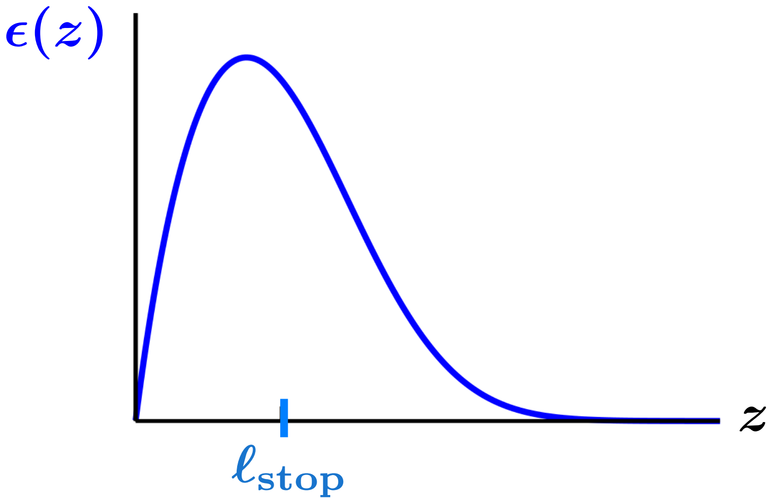
⇨ Only g→gg splittings consider (so far!)



A theorist thought experiment

Something theorists could "observe":

(statistically averaged) distribution of energy deposited by shower as a function of distance *z*



$\ell_{\text{stop}} \equiv \langle z \rangle$ (1st moment of energy deposition distribution)

$$\ell_{\text{stop}} \sim \frac{t_{\text{form}}}{\alpha} \sim \frac{1}{\alpha}\sqrt{\frac{E}{\hat{q}}}$$

Note: ℓ_{stop} depends on $\hat{q}$

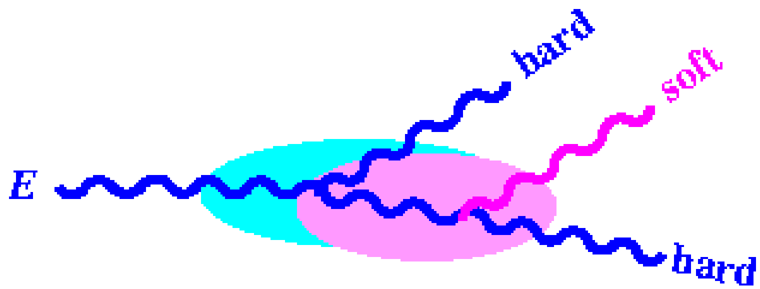
How big are the overlap corrections to ε(z)?

Answer: **BIG !** … which has been know since

Iancu (2014)
Blaizot and Mehtar-Tani (2014)
Wu (2014)

[building on radiative corrections to $\hat{q}$ found by Liou, Mueller, Wu (2013)]

(1) BIG because there is a double-log enhancement coming from SOFT radiation:

Prob. of overlap 

suppressed by (in my application)

$$\alpha_s(\mu)\,\ln^2\!\left(\frac{E}{T}\right)$$ ⇨ BIG result for large *E*

(2) But these BIG soft-radiation effects can be absorbed into an effective value of $\hat{q}$:

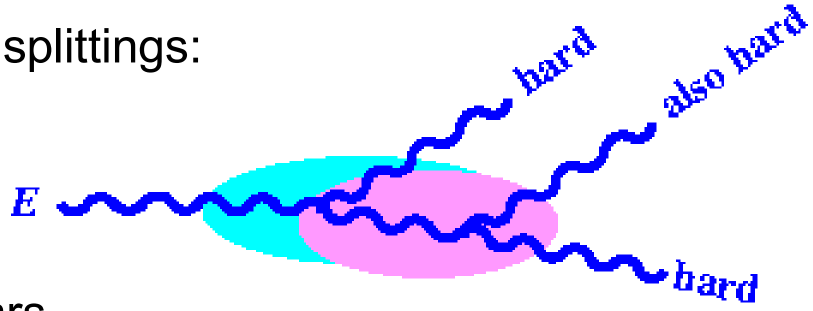
$$\hat{q} \longrightarrow \hat{q}_{\rm eff}(E) = \hat{q}\left[1 + \#\alpha_s \ln^2\!\left(\frac{E}{T}\right)\right]$$

Can even be re-summed at leading log to all orders in α_s

A REFINED QUESTION

How big are overlap effects that cannot be absorbed in $\hat{q}$?

(1) Need to calculate overlap of two hard splittings:



Extremely difficult calculation.

After lots of QFT and many (!!) years ...

Completed (for gluons) in 2022 with S. Iqbal and Tyler Gorda

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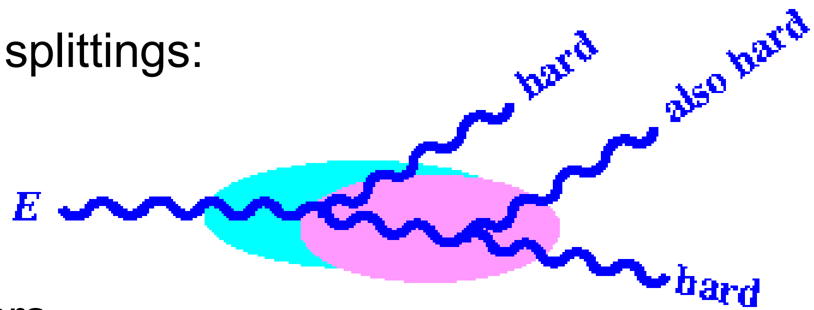

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After lots of QFT and many (!!) years ...

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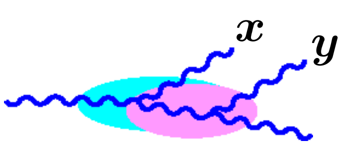
Technical note

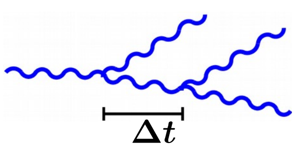
The drawing above is short-hand for what we call



$\Delta \frac{d\Gamma}{dx\,dy} \equiv$ the overlap correction to two independent splittings

$$= \Bigg[\underbrace{\left\langle \left| \int_0^\infty d(\Delta t) \right.\right.}_{\text{full calculation of double splitting rate}} \Bigg]$$

full calculation of double splitting rate

$\left\langle \left| \int_0^\infty d(\Delta t) \right.\right.$  $\left.\left. + \cdots \right|^2 \right\rangle_{\substack{\text{medium}\\\text{avg}}}$ $-$ [pretending the two splittings are independent dice roles $\frac{d\Gamma^{\rm split}}{dx}$ and $\frac{d\Gamma^{\rm split}}{dy}$]

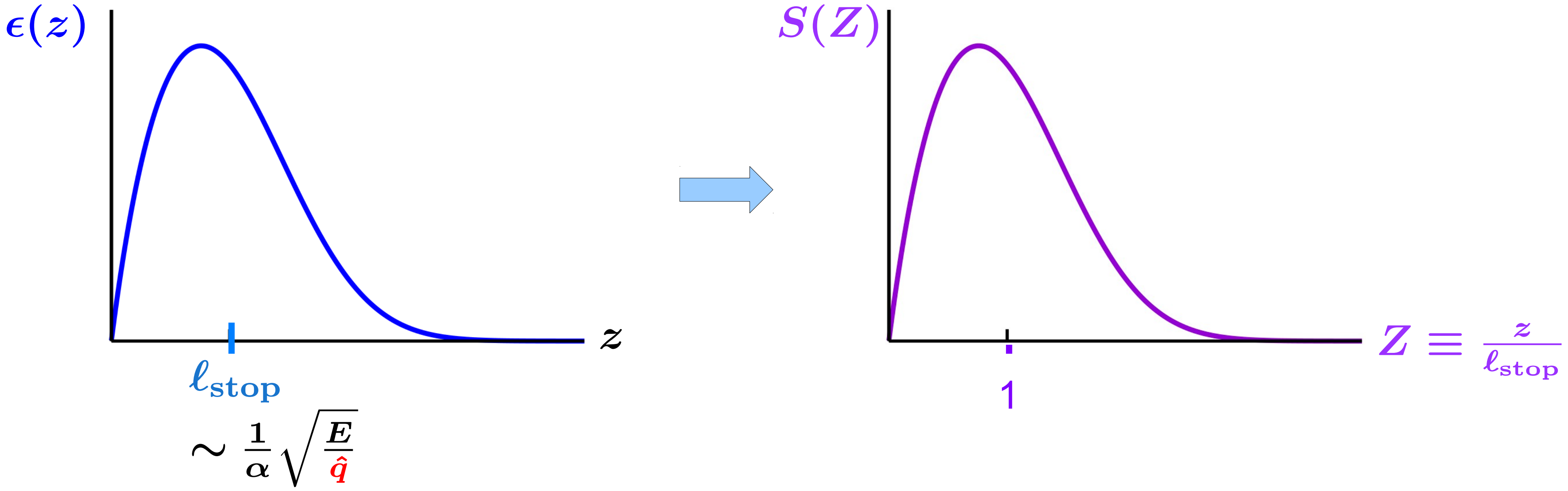
which cancels except for contributions from splittings separated by $\Delta t \lesssim t_{\rm form}$

A REFINED QUESTION

How big are overlap effects that cannot be absorbed in $\hat{q}$?

(2) Choose a theorist observable that is insensitive to $\hat{q}$:

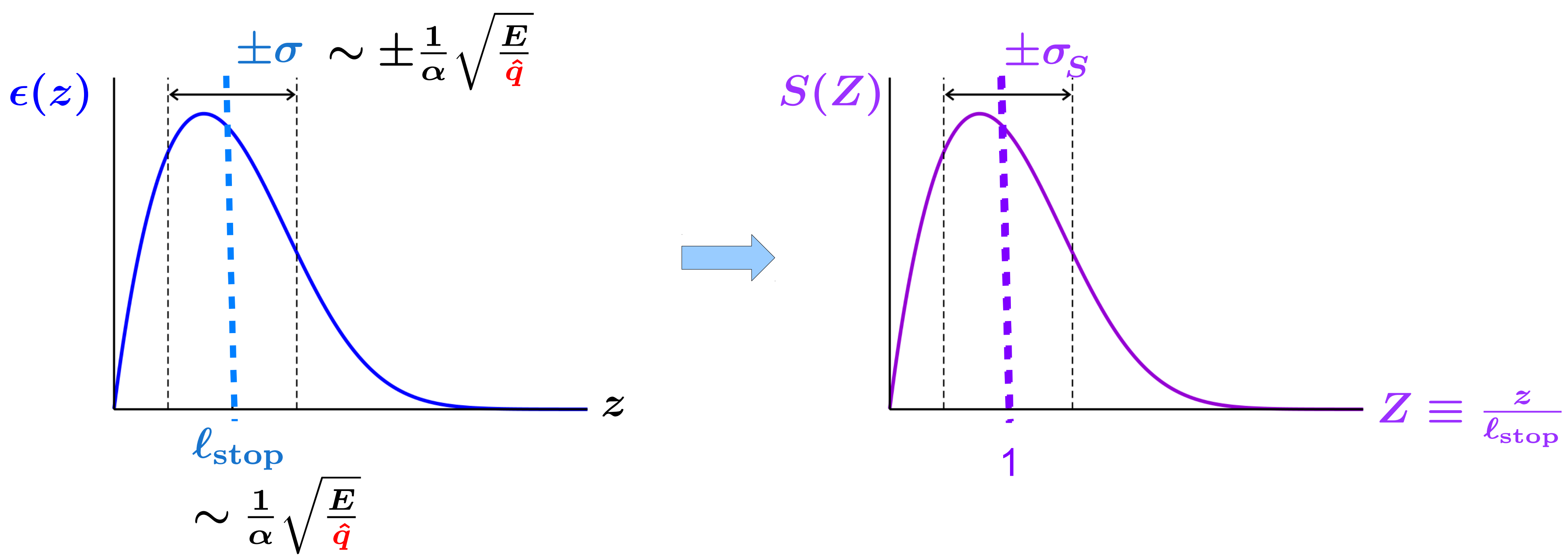
consider the shape $S(Z)$ of the energy deposition distribution:



A REFINED QUESTION

How big are overlap effects that <u>cannot</u> be absorbed in $\hat{q}$?

<u>Example</u>

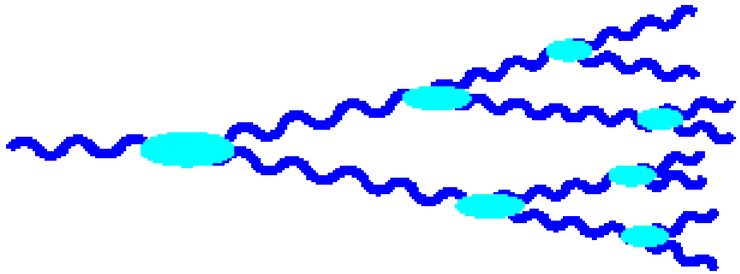


$$\sigma_S = \frac{\sigma}{\ell_{\mathrm{stop}}} \text{ is independent of } \hat{q}$$ *

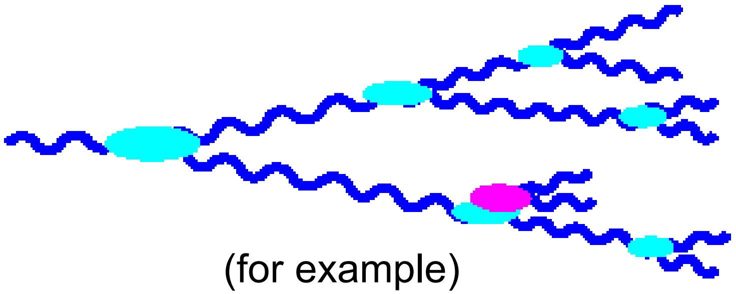
* Important, interesting, and resolvable caveats that I'll explain later.

How to account for overlaps in showers

Think of

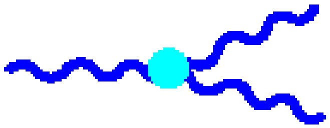


+



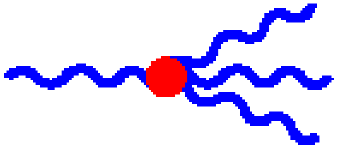
(for example)

as “standard” shower development with independent splittings but two types of localized, independent vertices:



1→2 (normal LPM)

$$\frac{d\Gamma^{\rm split}}{dx}$$



1→3 (overlap correction)

$$\Delta \frac{d\Gamma}{dx\,dy}$$

Then treat these “splitting” probabilities as purely classical.

RESULTS

To start: the width of the shape *S*(*Z*) of energy deposition

Large-N_f QED [2018 w/ S. Iqbal]:

"LO" means "ignoring overlaps"

charge deposition
$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.87\, N_{\rm f}\,\alpha(\mu)\right]$$

Large-N_c QCD (gluons only) [2022 w/ S. Iqbal and O. Elgedawy]:

energy deposition
$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 + \text{???}\, N_{\rm c}\,\alpha_{\rm s}(\mu)\right]$$

DRUM ROLL
PLEASE

RESULTS

To start: the width of the shape S(Z) of energy deposition

Large-N_f QED [2018 w/ S. Iqbal]:

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charge deposition

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energy deposition

$$\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - 0.02\, N_{\rm c}\,\alpha_{\rm s}(\mu)\right]$$

RESULTS

To start: the width of the shape S(Z) of energy deposition

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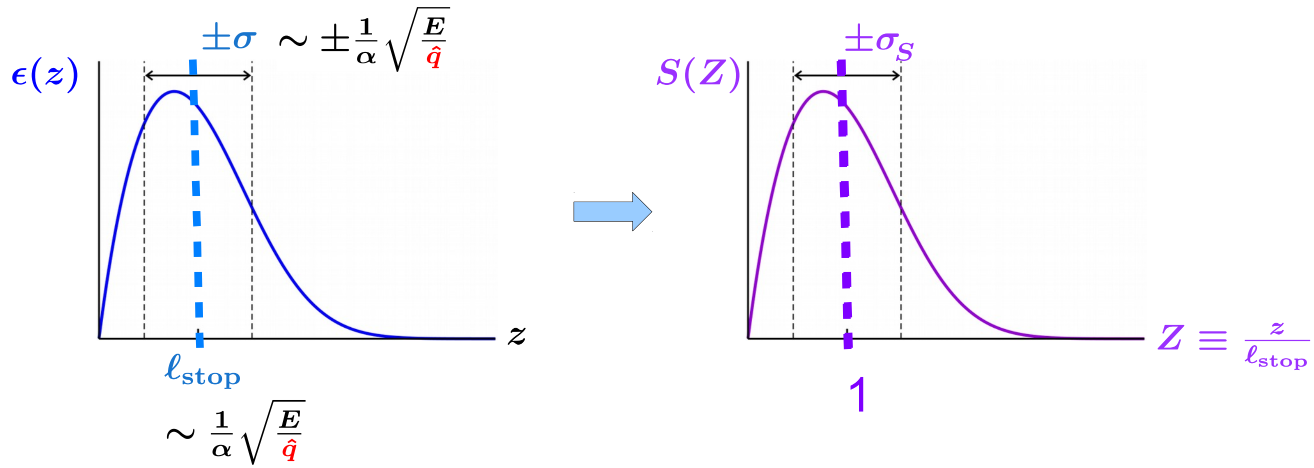
Conclusion for this test

Overlap corrections that cannot be absorbed into $\hat{q}$ are negligible.

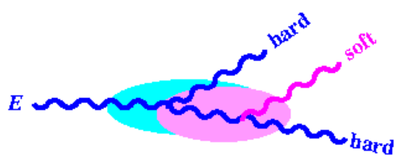
I half-lied about something

Remember



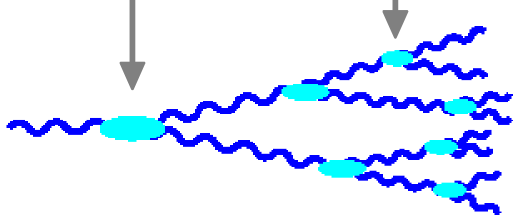
$\sigma_S = \frac{\sigma}{\ell_{\rm stop}}$ is independent of $\hat{q}$ *

and why we did that:



$$\hat{q} \longrightarrow \hat{q}_{\rm eff}(E) = \hat{q}\left[1 + \#\alpha_{\rm s} \ln^2\left(\frac{E}{T}\right)\right]$$

But then $\hat{q}_{\rm eff}(E)$ is different *here* and *there*.



Those difference don't quite cancel in $\sigma_S = \sigma/\ell_{\rm stop}$ and $S(Z)$.
They cancel at leading log but leave behind BIG single-log corrections to σ_S and $S(Z)$:

overlap corrections $\sim \alpha_{\rm s}(\mu) \ln\left(\frac{E}{T}\right)$

Factorization

Remember that soft radiation can be absorbed into $\hat{q}$.

When factorizing away some IR or UV physics in QFT, we must introduce a *factorization scale* to do NLO calculations.

Examples

UV divergences absorbed into couplings:	renormalization scale μ
Collinear divergences absorbed into PDFs:	factorization scale $M_{\rm fac}$

Such factorization scales appear explicitly inside logarithms in NLO results.

- Set them to the appropriate physics scale for the process.
- Check sensitivity to the precise choice of scale.

Our problem

To factorize *all* the soft radiation effects into $\hat{q}_{\rm eff}$,
we introduce an energy factorization scale

$\Lambda_{\rm fac} \sim$ (min energy of daughters of

)

The result shown earlier was for

$$\Lambda_{\rm fac} \sim \# x(1-x)E \quad \text{with} \quad \# = 1$$

Now showing dependence on the normalization # of the factorization scale:

$$\boldsymbol{\sigma_S = \frac{\sigma}{\ell_{\rm stop}} = \left(\frac{\sigma}{\ell_{\rm stop}}\right)_{\rm LO} \left[1 - (0.02 + 0.001 \ln \#) N_c \alpha_s(\mu)\right]}$$

Extremely weak dependence on factorization scale.

The QED and gluon shower results are very different – Why?

relative size of overlap effect on value of $\sigma_{\rm S} = \sigma/\ell_{\rm stop}$

Large-N_f QED:

e^- initiated charge deposition $\quad -0.85\, N_f\, \alpha$

Large-N_c QCD (gluons only):

g initiated energy deposition $\quad -0.02\, N_c\, \alpha_s$

- Accidental cancellation in gluon case?
- Something special about shape of energy deposition?
- Something about absence of fermions?
- … ??

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e^- initiated charge deposition	$-0.85\, N_f\, \alpha$
e^- initiated energy deposition	$+1.13\, N_f\, \alpha$
γ initiated energy deposition	$+0.98\, N_f\, \alpha$

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Large-N_c QCD (gluons only):	
g initiated energy deposition	$-0.02\ N_c\ \alpha_s$
$N_f >> N_c >> 1$ QCD:	
q initiated charge deposition	$+0.01\ N_f\ \alpha$
q initiated energy deposition	$-0.01\ N_f\ \alpha$
g initiated energy deposition	$-0.01\ N_f\ \alpha$

- Accidental cancellation in gluon case?
- Something special about shape of energy deposition? **NO**
- Something about absence of fermions? **NO**
- … ??

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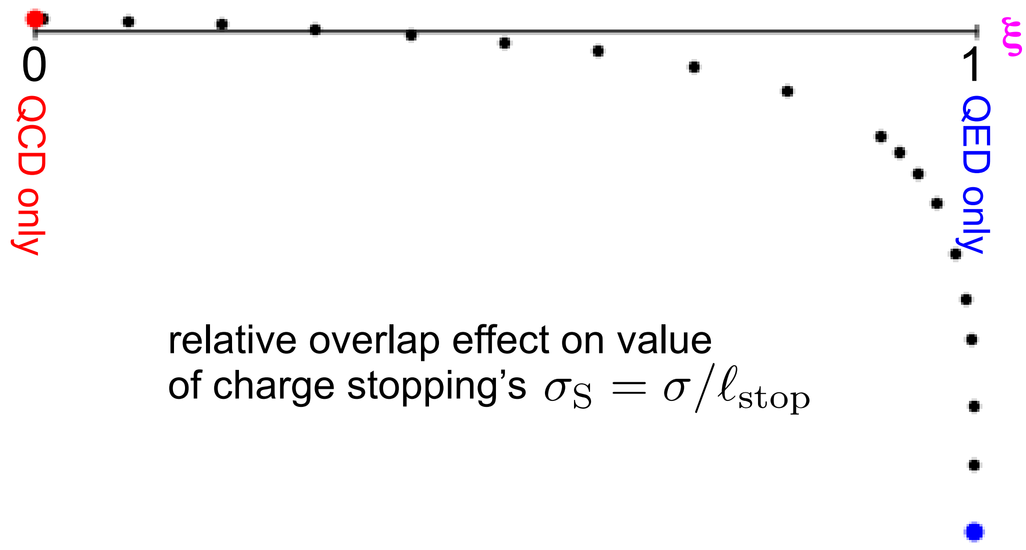
Large-N_c QCD (gluons only):	
g initiated energy deposition	$-0.02\, N_c\, \alpha_s$
$N_f \gg N_c \gg 1$ QCD:	
q initiated charge deposition	$+0.01\, N_f\, \alpha$
q initiated energy deposition	$-0.01\, N_f\, \alpha$
g initiated energy deposition	$-0.01\, N_f\, \alpha$

- Accidental cancellation in gluon case? **unlikely**
- Something special about shape of energy deposition? **NO**
- Something about absence of fermions? **NO**
- … ??

Which is the weird one: small QCD results or large QED results?

A hybrid model

- Give quarks electric charge (all the same)
- Imagine that you could adjust the relative importance "ξ" of QED vs QCD contributions to $\hat{q}$



The reason QED and QCD are different (we believe)

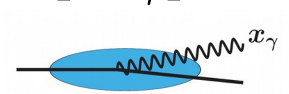
Explanation: Part 1

Leading order (i.e. BDMPS-Z) splitting rates $\propto \sqrt{\hat{q}/E}$

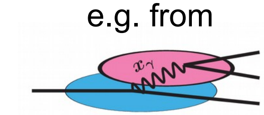
→ Overlap (NLO) corrections that have the form

$$\left[\frac{d\Gamma}{dx_\gamma}\right]^{\rm LO} \longrightarrow \left[\frac{d\Gamma}{dx_\gamma}\right]^{\rm LO} \Big(1 + \underbrace{{\rm const}\ N\alpha}_{\text{e.g. from}}\Big)$$

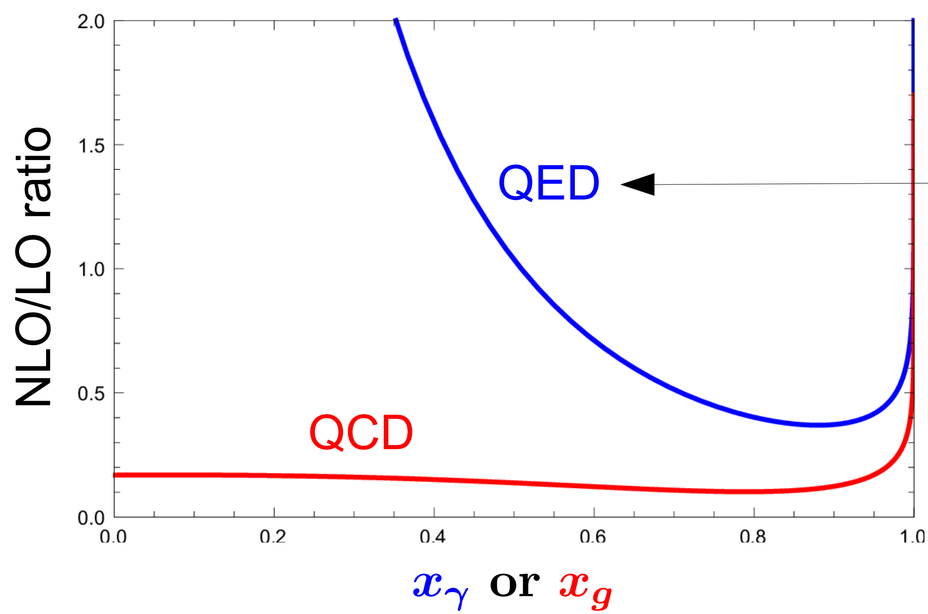
e.g.



e.g. from



can be absorbed into the value of $\hat{q}$



deviates much more from "constant"

But why?

Ultimate Reason

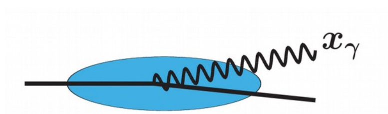
a) Because gluons (having color) scatter easily from a QCD medium,
but photons (having no charge) do not scatter easily from a QED medium.

Explanation: Part 2

and b) Because the LPM effect depends on the collinearity of high-energy splittings.

Soft brem gluons: scatter easily → less collinearity
→ less LPM suppression
→ higher splitting rates

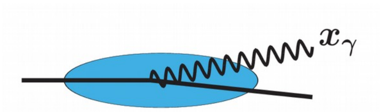
Soft brem photons: none of that



QED: $$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO} \sim \alpha\sqrt{\frac{\hat{q}}{x_\gamma E}}$$

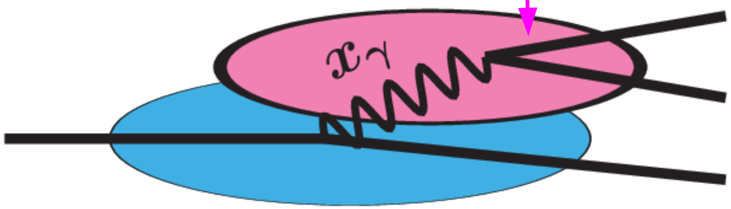
QCD: $$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO} \sim \alpha_{\rm s}\sqrt{\frac{\hat{q}}{x_g^3 E}}$$

rate grows faster!



QED: $\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO} \sim \alpha\sqrt{\frac{\hat{q}}{x_\gamma E}}$ QCD: $\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO} \sim \alpha_s\sqrt{\frac{\hat{q}}{x_g^3 E}}$

But, in QED, if you have overlapping



Now the γ has converted to an e^+e^- pair, which can effectively scatter from medium

→ less collinearity

→ less LPM suppression

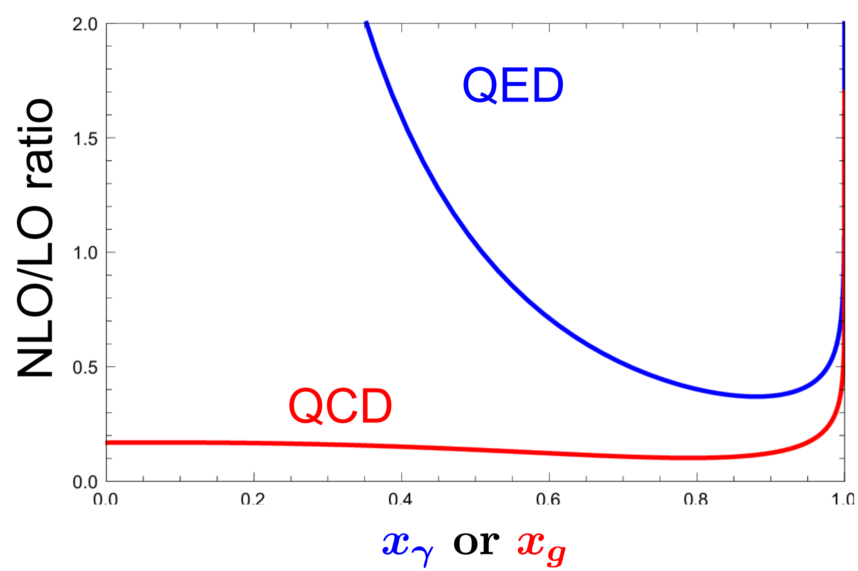
→ higher overall splitting rate

QED and QCD: $\left[\frac{d\Gamma}{dx_{\gamma,g}}\right]_{\rm NLO} \sim \alpha^2\sqrt{\frac{\hat{q}}{x_{\gamma,g}^3 E}}$

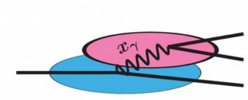
But then the ratio (for small x_γ)

$$\frac{\rm NLO}{\rm LO} \sim \begin{cases} \frac{\alpha}{x_\gamma} & \text{QED} \\ \alpha & \text{QCD} \end{cases}$$

This explains part of



Explanation: Part 3

The  rate is actually

$$\left[\frac{d\Gamma}{dx_{\gamma,g}}\right]_{\rm NLO} \sim \begin{cases} \alpha^2 \sqrt{\dfrac{\hat{q}}{x_\gamma^3 E}} \ln x_\gamma & \textbf{QED} \\ \alpha^2 \sqrt{\dfrac{\hat{q}}{x_g^3 E}} & \textbf{QCD} \end{cases}$$

so that

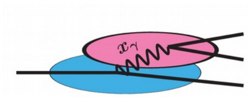
$$\frac{\rm NLO}{\rm LO} \sim \begin{cases} \dfrac{\alpha}{x_\gamma} \ln x_\gamma & \textbf{QED} \\ \alpha & \textbf{QCD} \end{cases}$$

More specifically,

QED: $$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi} \left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO} \left\{ \frac{\frac{3}{4}\ln x_\gamma}{x_\gamma} + \frac{\# + O(\sqrt{x_\gamma})}{x_\gamma} \right\}$$

QCD: $$\left[\frac{d\Gamma}{dx_g}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi} \left[\frac{d\Gamma}{dx_g}\right]_{\rm LO} \{\# + O(\sqrt{x_g})\}$$

Explanation: Part 3

The  rate is actually

$$\left[\frac{d\Gamma}{dx_{\gamma,g}}\right]_{\rm NLO} \sim \begin{cases} \alpha^2\sqrt{\dfrac{\hat q}{x_\gamma^3 E}}\ \ln x_\gamma & \textbf{QED} \\ \alpha^2\sqrt{\dfrac{\hat q}{x_g^3 E}} & \textbf{QCD} \end{cases}$$

so that

$$\frac{\rm NLO}{\rm LO} \sim \begin{cases} \dfrac{\alpha}{x_\gamma}\ln x_\gamma & \textbf{QED} \\ \alpha & \textbf{QCD} \end{cases}$$

More specifically,

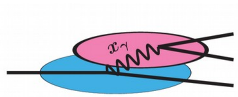
QED: $$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi}\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO}\left\{\frac{\frac{3}{4}\ln x_\gamma}{x_\gamma} + \frac{\# + O(\sqrt{x_\gamma})}{x_\gamma}\right\}$$

QCD: $$\left[\frac{d\Gamma}{dx_g}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi}\left[\frac{d\Gamma}{dx_g}\right]_{\rm LO}\{\# + O(\sqrt{x_g})\}$$

This term accounts for 92% of the difference between QED and QCD overlap effects for $\sigma_{\rm S} = \sigma/\ell_{\rm stop}$

Where does it come from?

Explanation: Part 3

The  rate is actually

$$\left[\frac{d\Gamma}{dx_{\gamma,g}}\right]_{\rm NLO} \sim \begin{cases} \alpha^2 \sqrt{\dfrac{\hat{q}}{x_\gamma^3 E}}\ \ln x_\gamma & \text{QED} \\ \alpha^2 \sqrt{\dfrac{\hat{q}}{x_g^3 E}} & \text{QCD} \end{cases}$$

so that

$$\frac{\rm NLO}{\rm LO} \sim \begin{cases} \dfrac{\alpha}{x_\gamma} \ln x_\gamma & \text{QED} \\ \alpha & \text{QCD} \end{cases}$$

More specifically,

many years of work (incl. numerics)

"easy" to calculate analytically

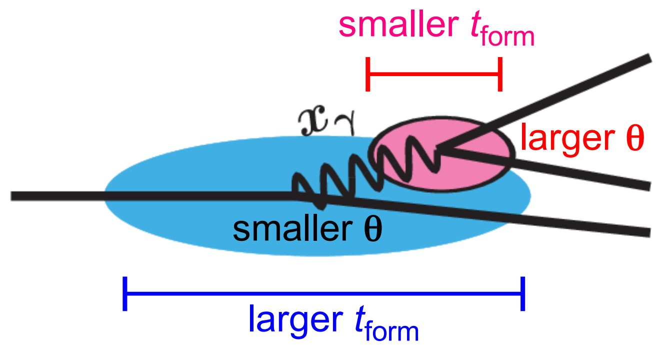
QED: $$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi} \left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO} \left\{ \frac{\frac{3}{4}\ln x_\gamma}{x_\gamma} + \frac{\# + O(\sqrt{x_\gamma})}{x_\gamma} \right\}$$

QCD: $$\left[\frac{d\Gamma}{dx_g}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi} \left[\frac{d\Gamma}{dx_g}\right]_{\rm LO} \left\{ \# + O(\sqrt{x_g}) \right\}$$

This term accounts for 92% of the difference between QED and QCD overlap effects for $\sigma_{\rm S} = \sigma/\ell_{\rm stop}$

Where does it come from?

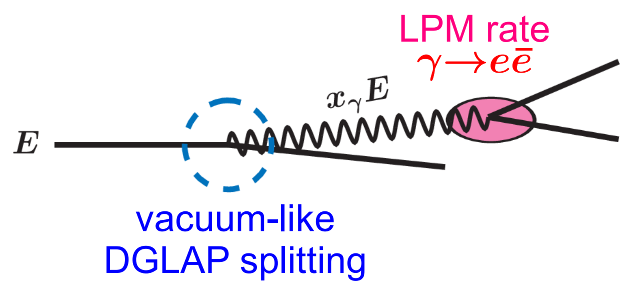
For small x_γ :



Larger θ disrupts LPM more

⇒ think of $\gamma \to e\bar{e}$ as the "underlying" process

and think of $e \to e\gamma$ as an initial-state DGLAP-like correction



$$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm NLO} \approx \frac{\alpha}{2\pi} P_{e\to\gamma}(x_\gamma) \ln\left(\frac{\Delta E_{\rm max}}{\Delta E_{\rm min}}\right) \times \Gamma^{\gamma\to e\bar{e}}_{\rm LPM}(x_\gamma E)$$

$$\approx \frac{\alpha}{2\pi} P_{e\to\gamma}(x_\gamma) \ln\left(\frac{1/t^{\rm LPM}_{\rm form}(\gamma{\to}e\bar{e})}{1/t^{\rm LPM}_{\rm form}(e{\to}e\gamma)}\right) \times \Gamma^{\gamma\to e\bar{e}}_{\rm LPM}(x_\gamma E)$$

The logarithm! $\sim \ln x_\gamma$

QCD: No logarithm. Because gluon has color,
$t^{\rm LPM}_{\rm form}(g{\to}q\bar{q}) \sim t^{\rm LPM}_{\rm form}(q{\to}qg)$

Review of why QED different from QCD

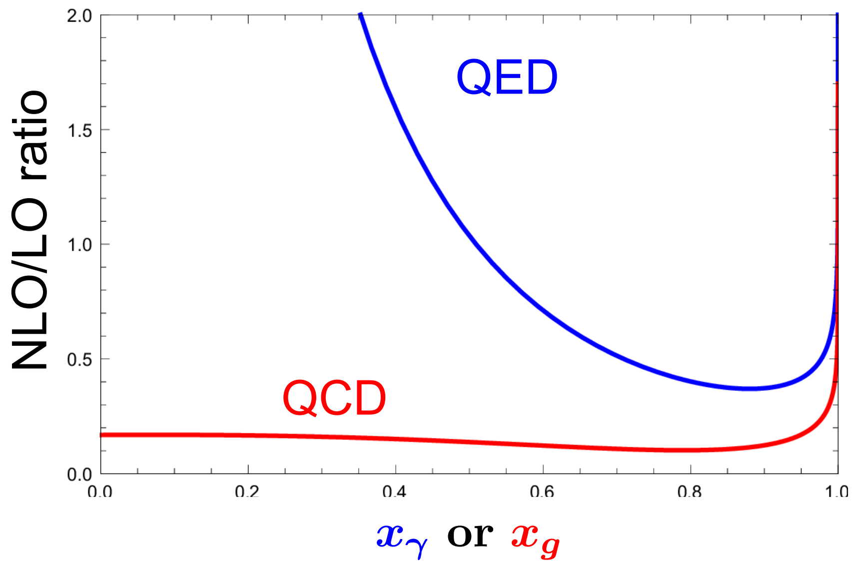
$$\frac{\mathbf{NLO}}{\mathbf{LO}} \sim \begin{cases} \frac{\alpha}{x_\gamma} \ln x_\gamma & \text{QED} \\ \alpha & \text{QCD} \end{cases}$$

More specifically,

QED: $$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi} \left[\frac{d\Gamma}{dx_\gamma}\right]_{\rm LO} \left\{ \frac{\frac{3}{4}\ln x_\gamma}{x_\gamma} + \frac{\# + O(\sqrt{x_\gamma})}{x_\gamma} \right\}$$

QCD: $$\left[\frac{d\Gamma}{dx_g}\right]_{\rm NLO} \underset{x_\gamma \ll 1}{\simeq} \frac{N_{\rm f}\alpha}{2\pi} \left[\frac{d\Gamma}{dx_g}\right]_{\rm LO} \left\{ \# + O(\sqrt{x_g}) \right\}$$

This term accounts for 92% of the difference between QED and QCD overlap effects for $\sigma_{\rm S} = \sigma/\ell_{\rm stop}$



Summary

- Overlap effects *that cannot be absorbed into* $\hat{q}$ appear negligible for in-medium QCD showers.
- We understand why QED gave a much(!) larger result (for the same value of $N\alpha$).

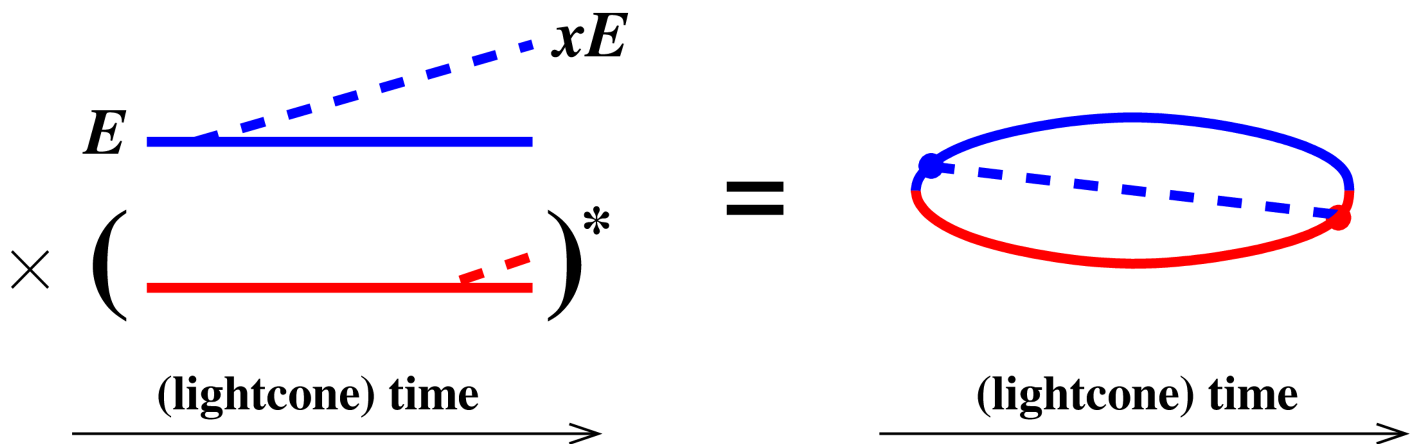
Caveats:

- Our tests were restricted to studying energy and charge deposition distributions.
- Our tests were restricted to infinite volume, on-shell particles initiating the shower, and measurements that do not depend on tracking $p_\perp$ evolution.
- We've only tested the large-N_f and N_f=0 (pure glue) limits.
- We only have (complete) results in the limit N_c>>1.

Backup Slides

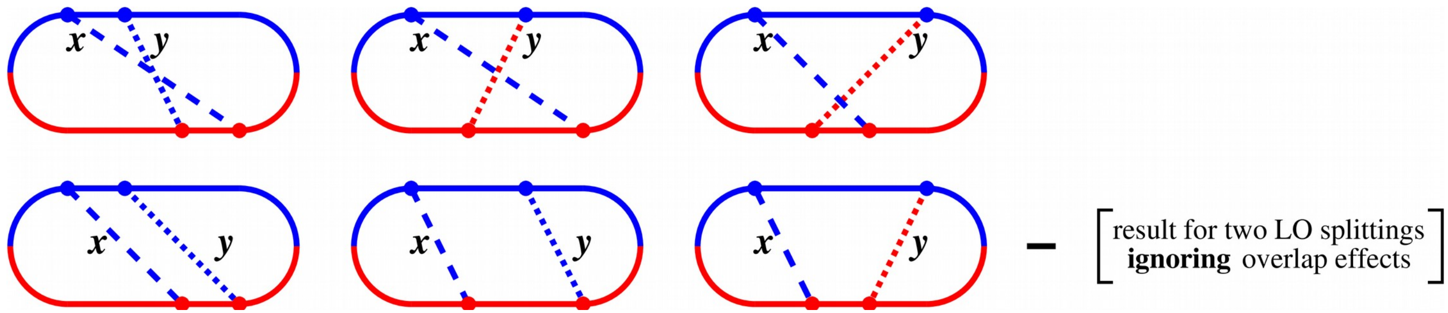
Examples of Diagrams

Leading-order (BDMPS-Z) $g \rightarrow gg$

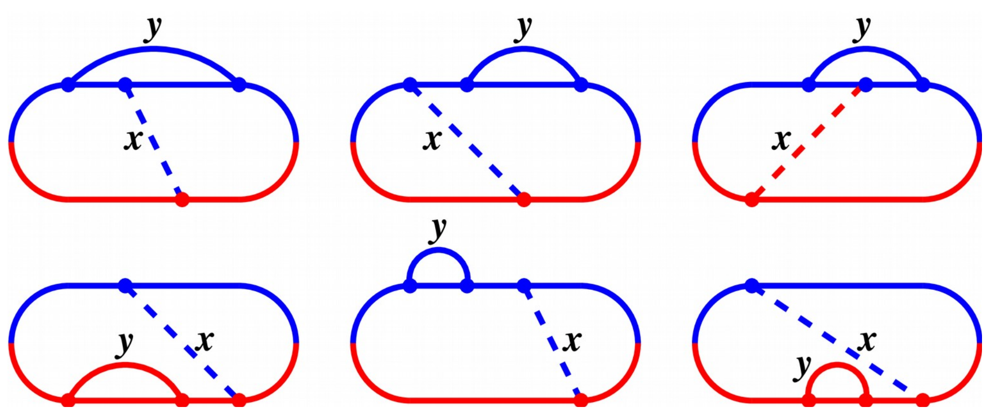


Examples of Diagrams

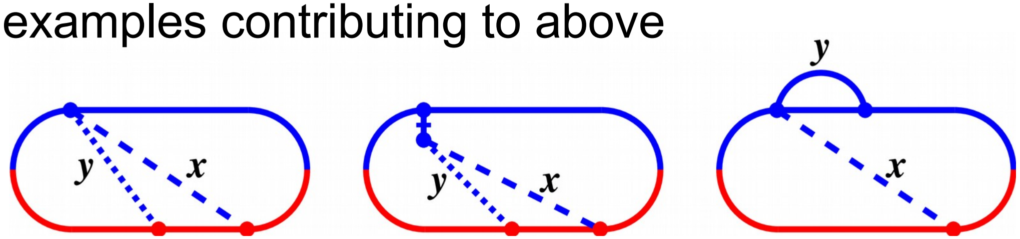
Overlapping double splitting *g→gg→ggg*

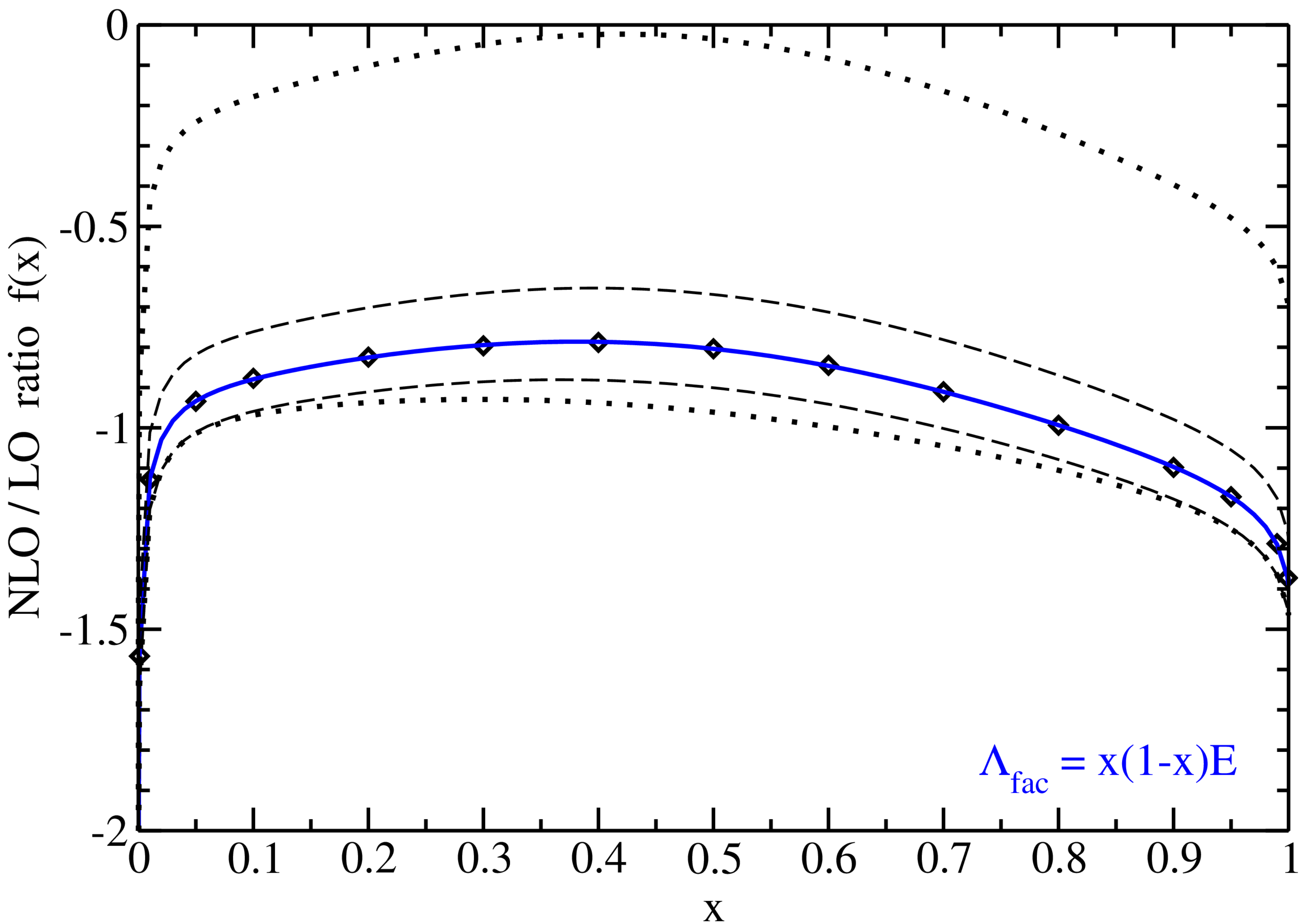


Virtual corrections to single splitting *g→gg*



Some other examples contributing to above





0
-0.5
-1
-1.5
-2
NLO / LO ratio f(x)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
$\Lambda_{fac} = x(1-x)E$