3-body decay amplitudes in the presence of bound states

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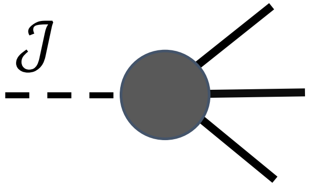
Motivation

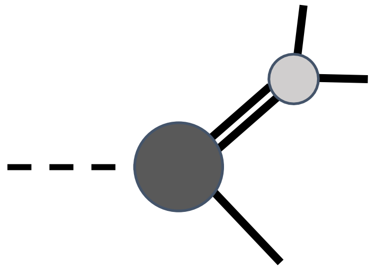
Progress on 3-particle systems : talks by Fernando Romero-López, Michael Doering, Maxim Mai, Md Habib Islam, Sebastian Dawid, …

3-body decay amplitudes



$g-2: \ \gamma * \to \pi\,\pi\,\pi$

3-body decay amplitudes in the presence of bound states



Photoproduction of mesons (GlueX), inform fits beyond isobar model

QC + scattering amplitudes summary

Lüscher Comm. Math. Physics (1986)

2-particle

$$F_2^{-1} + K_2 = 0$$

$$\mathcal{M}_2 = \frac{1}{K_2^{-1} - i\rho}$$

$$s: \; K_2 = -16\pi E^* \tan\delta$$

3-particle

Hansen, Sharpe PRD (2014)

$$F_3^{-1} + K_{df,3} = 0$$

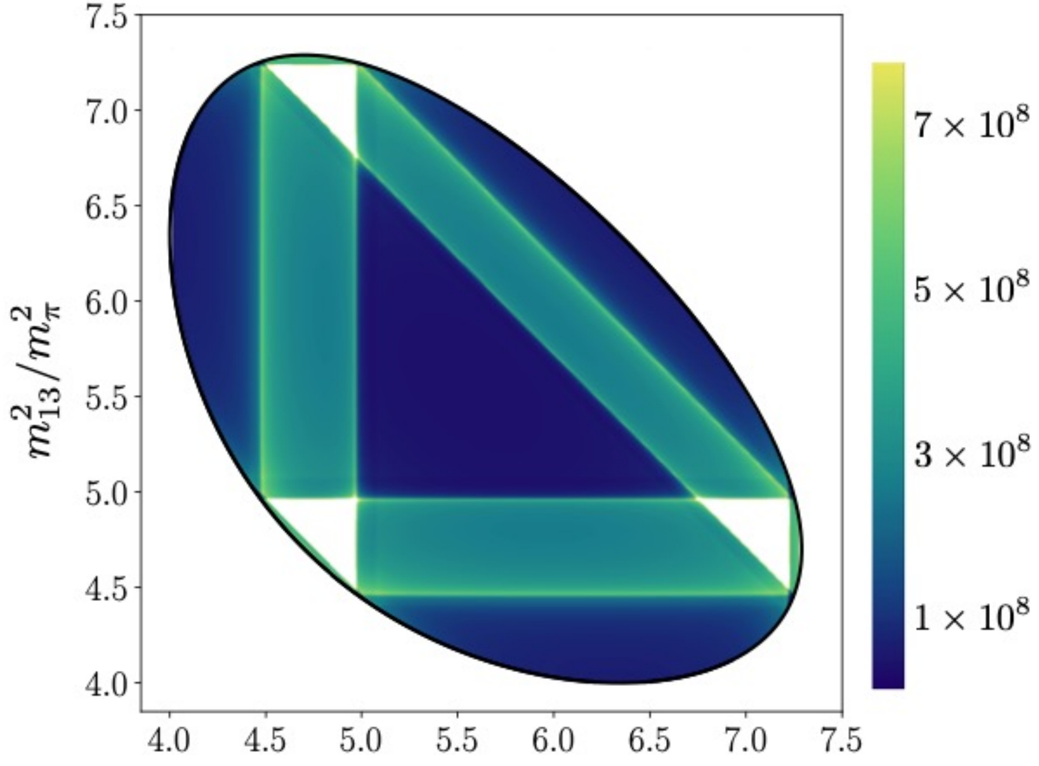
$$\mathcal{M}_3 = \mathcal{D}^{(u,u)} + \mathcal{M}_{df,3}^{(u,u)}$$

$\mathcal{D}^{(u,u)}$ → ladder amplitude

$\mathcal{M}_{df,3}^{(u,u)}$ → short-range 3-body interactions

QC + scattering amplitudes summary

$\pi^+\pi^+\,\pi^+$



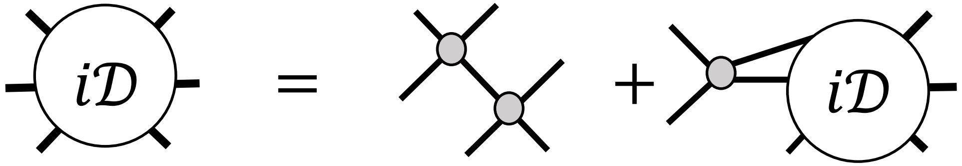
Hansen, Briceño, Edwards, Thomas, Wilson, PRL (2021)

3-particle

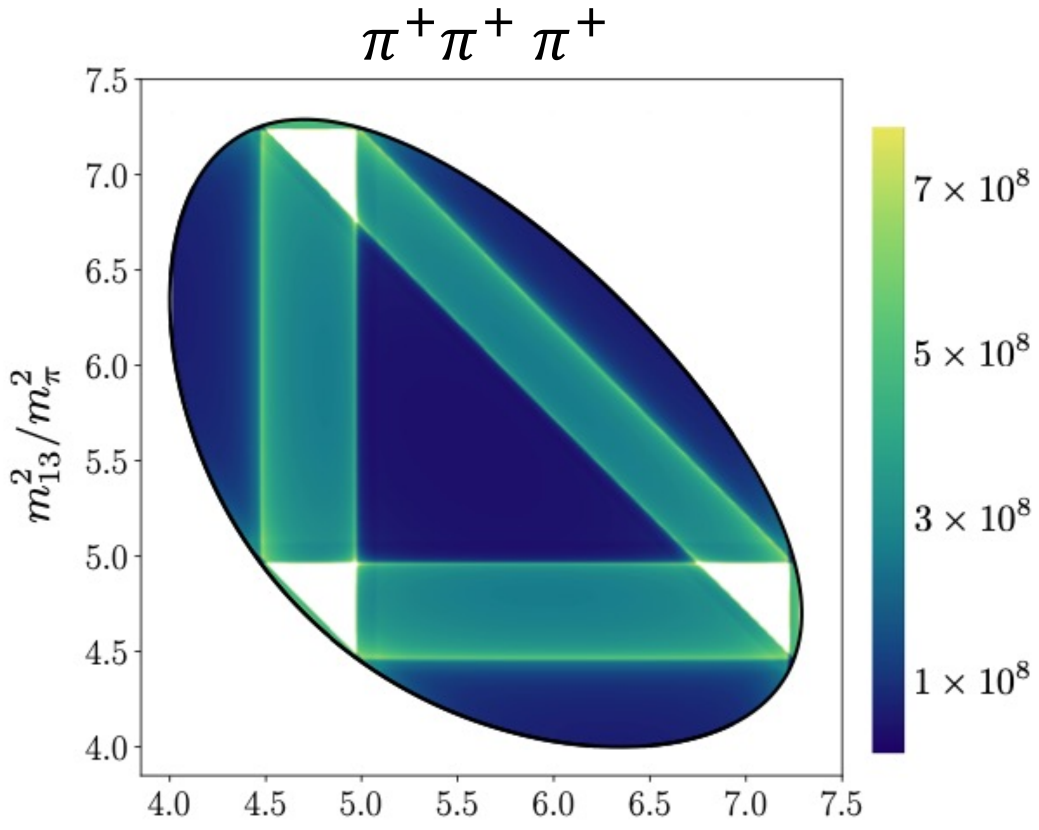
$$F_3^{-1} + K_{df,3} = 0$$

$$\mathcal{M}_3 = \mathcal{D}^{(u,u)} + \mathcal{M}_{df,3}^{(u,u)}$$

$$s:\ \mathcal{M}_2 \xrightarrow[\text{equations}]{\text{integral}} \mathcal{D}^{(u,u)}$$



QC + scattering amplitudes summary



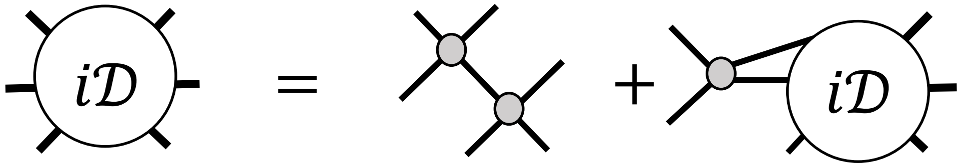
Hansen, Briceño, Edwards, Thomas, Wilson, PRL (2021)

3-particle

$$F_3^{-1} + \cancelto{0}{K_{df,3}} = 0$$

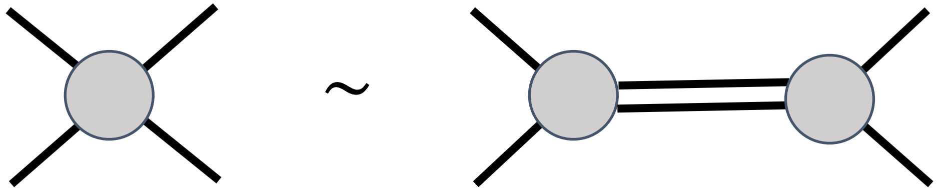
$$\mathcal{M}_3 = \mathcal{D}^{(u,u)} + \cancelto{0}{\mathcal{M}_{df,3}^{(u,u)}}$$

$$s: \ \mathcal{M}_2 \xrightarrow[\text{equations}]{\text{integral}} \mathcal{D}^{(u,u)}$$

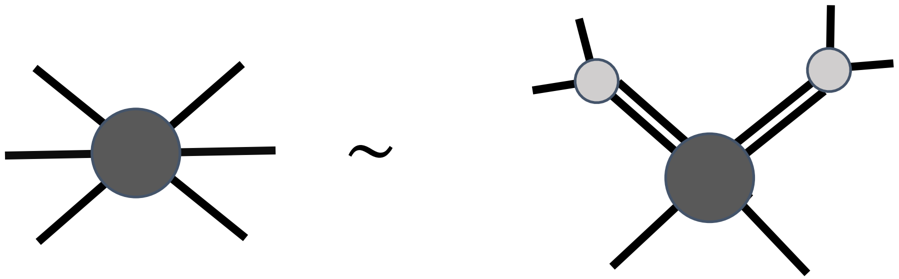


Testing the 3-body formalism – bound states

Thursday talks by:
Md Habib Islam (Digonto)
Sebastian Dawid



$$\sim \quad i\,g\,\frac{i}{s - s_b}\,i\,g$$



$$\sim \quad i\,g\,\frac{i}{s_{2k} - s_b}\,i\,\mathcal{M}_{\varphi b}\,\frac{i}{s_{2p} - s_b}\,i\,g$$

$$\mathcal{M}_{\varphi b} = g^2 \lim_{s_{2k}, s_{2p} \to s_b} d^{(u,u)}(p,k)$$

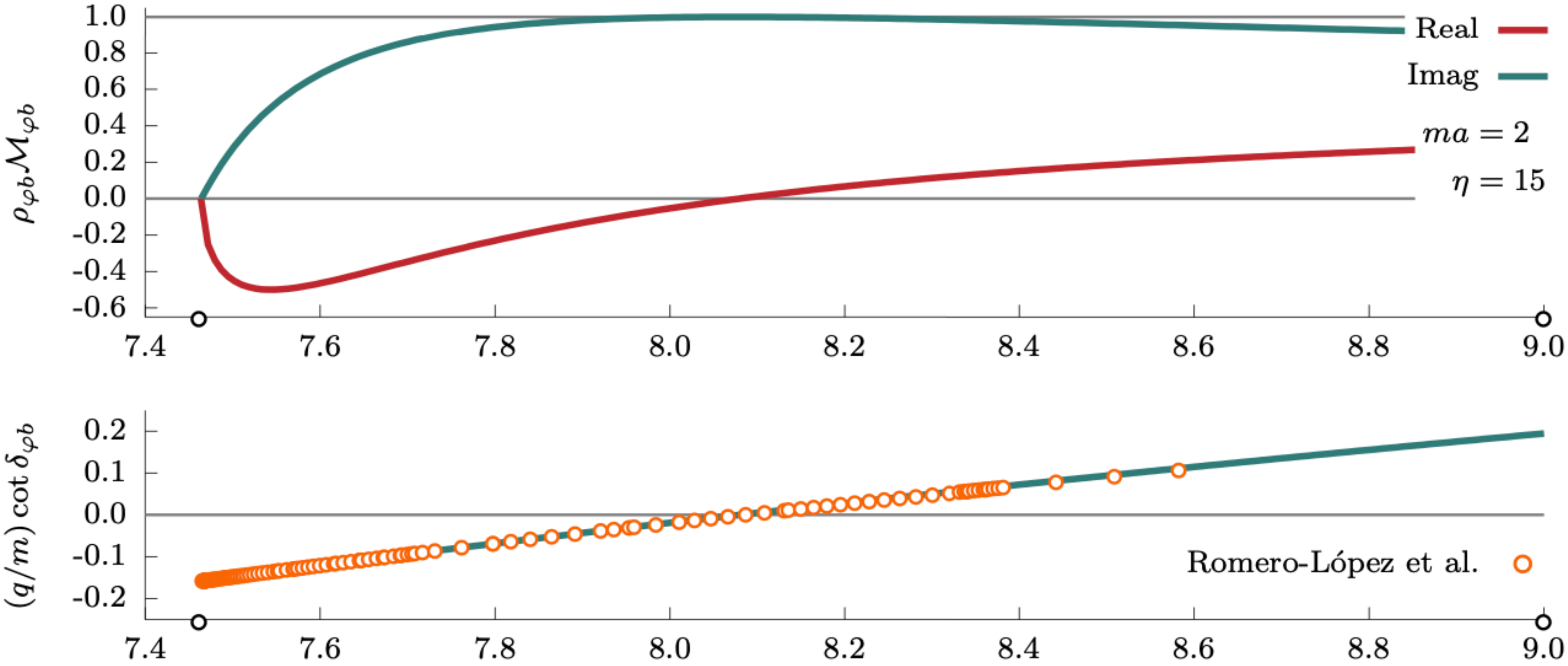
$$\mathcal{D}^{(u,u)}(p,k) = \mathcal{M}_2(p)\,d^{(u,u)}(p,k)\,\mathcal{M}_2(k)$$

Jackura, Briceño, Dawid, Islam, McCarty PRD (2020)

Testing the 3-body formalism – bound states

Jackura, Briceño, Dawid, Islam, McCarty PRD (2020)
Romero-López, Sharpe, Blanton, Briceno, Hansen PRD (2019)

$$F_3^{-1} = 0 \;\rightarrow F_2^{-1} + K_{\varphi b} = 0$$



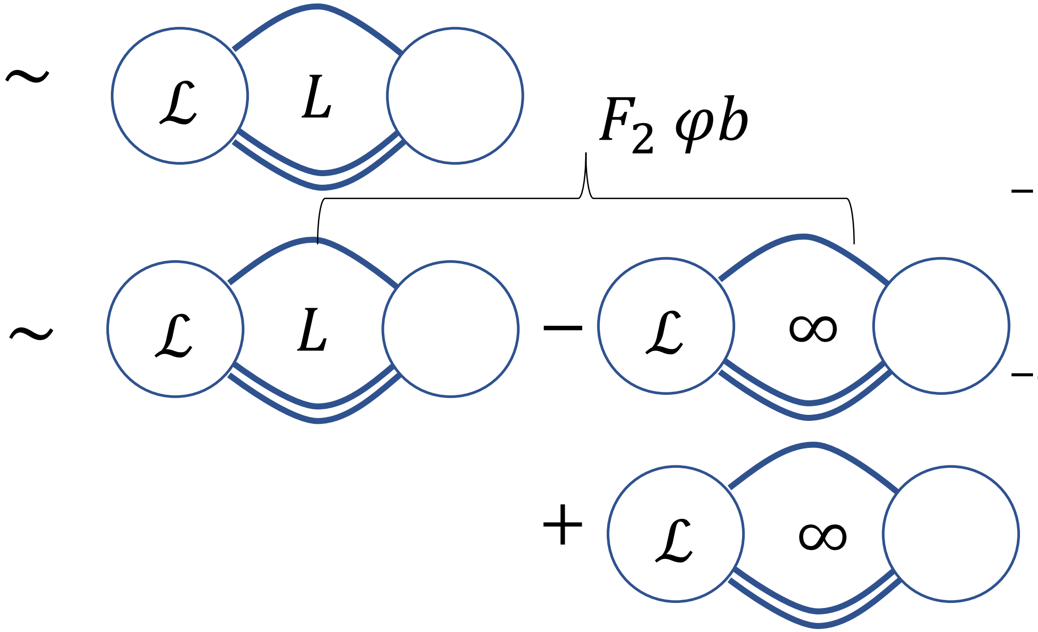
Testing the 3-body formalism – bound states: QC3 → QC2

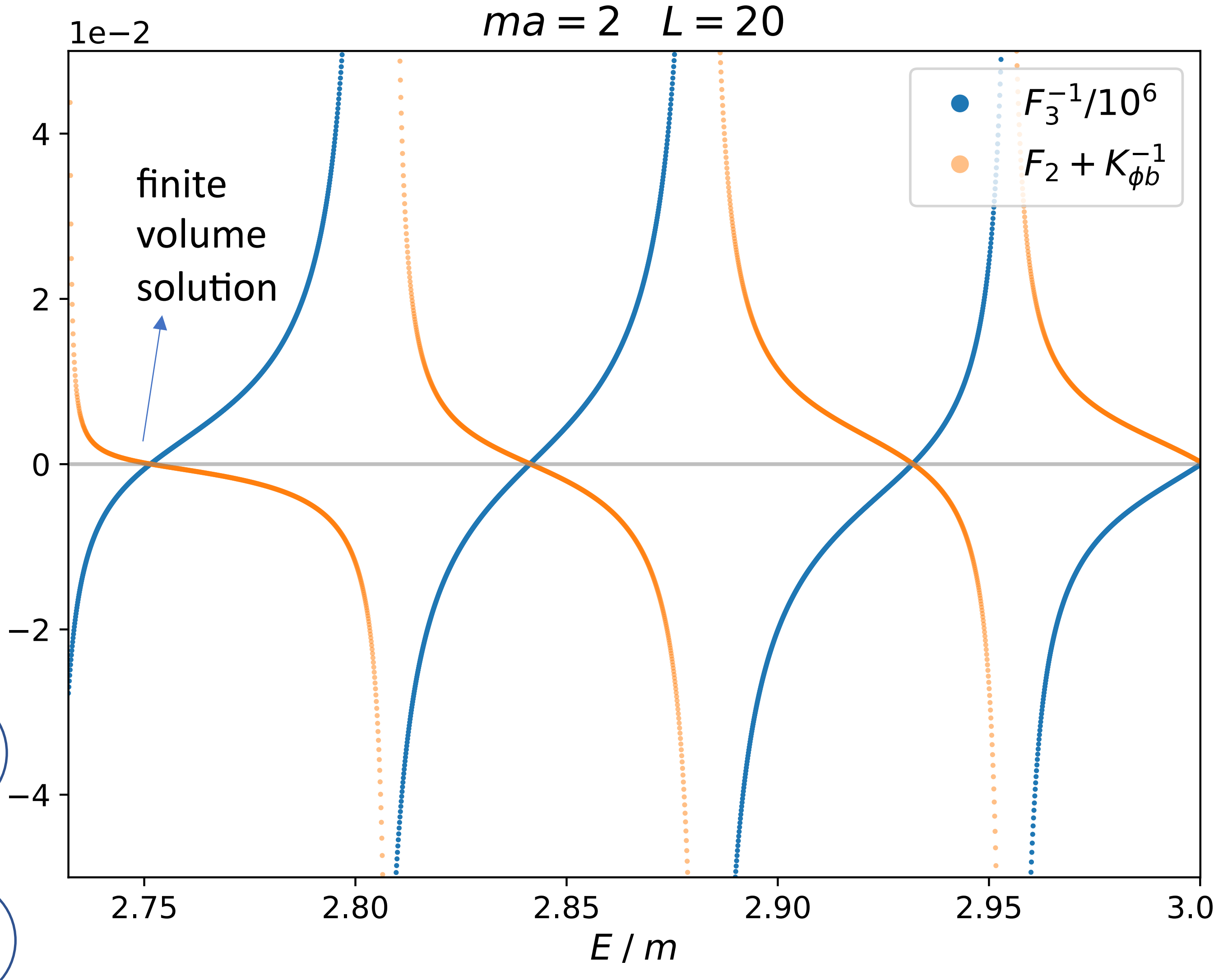
$$F_3^{-1} = 0 \;\rightarrow F_2^{-1} + K_{\varphi b} = 0$$

Analytically?

Briceño, Davoudi
PRD (2012)

$$F_3 \supset -\sum F_2\, \mathcal{M}_2\, F_2$$



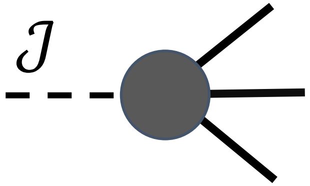


Testing the 3-body formalism – bound states: Decay amplitudes

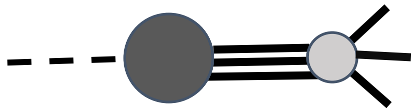
Hansen, Romero-López, Sharpe PRD (2021)

$$T_3 = \mathcal{L}^{(u),iso}(k) A^{iso}$$

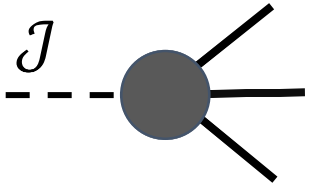
$K_{df,3} = 0 + s$ −wave + isotropic



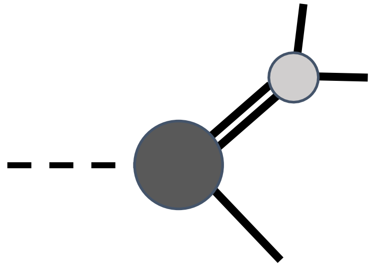
$\sim$



$$\sim \quad \frac{i\, g_t}{s - s_t}$$



$\sim$

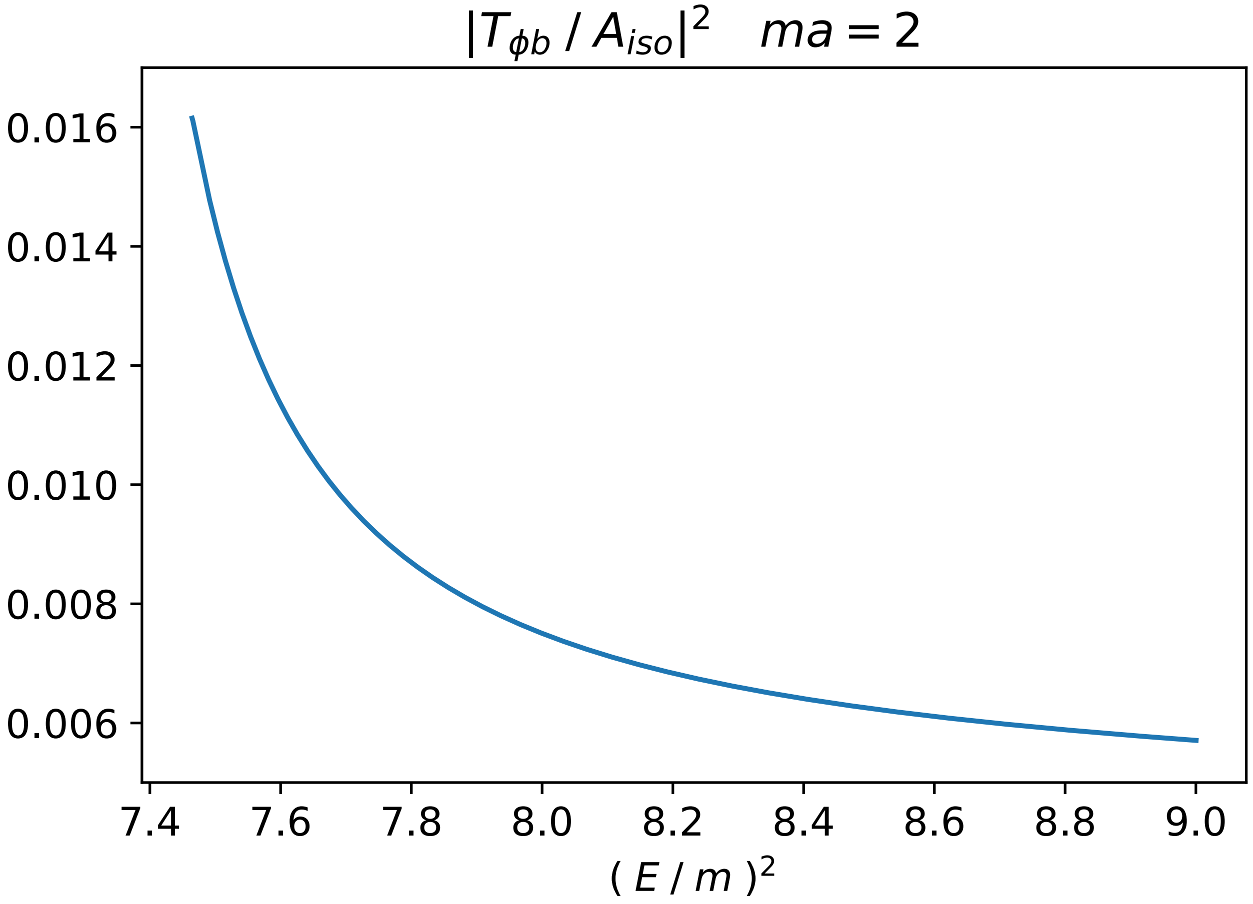


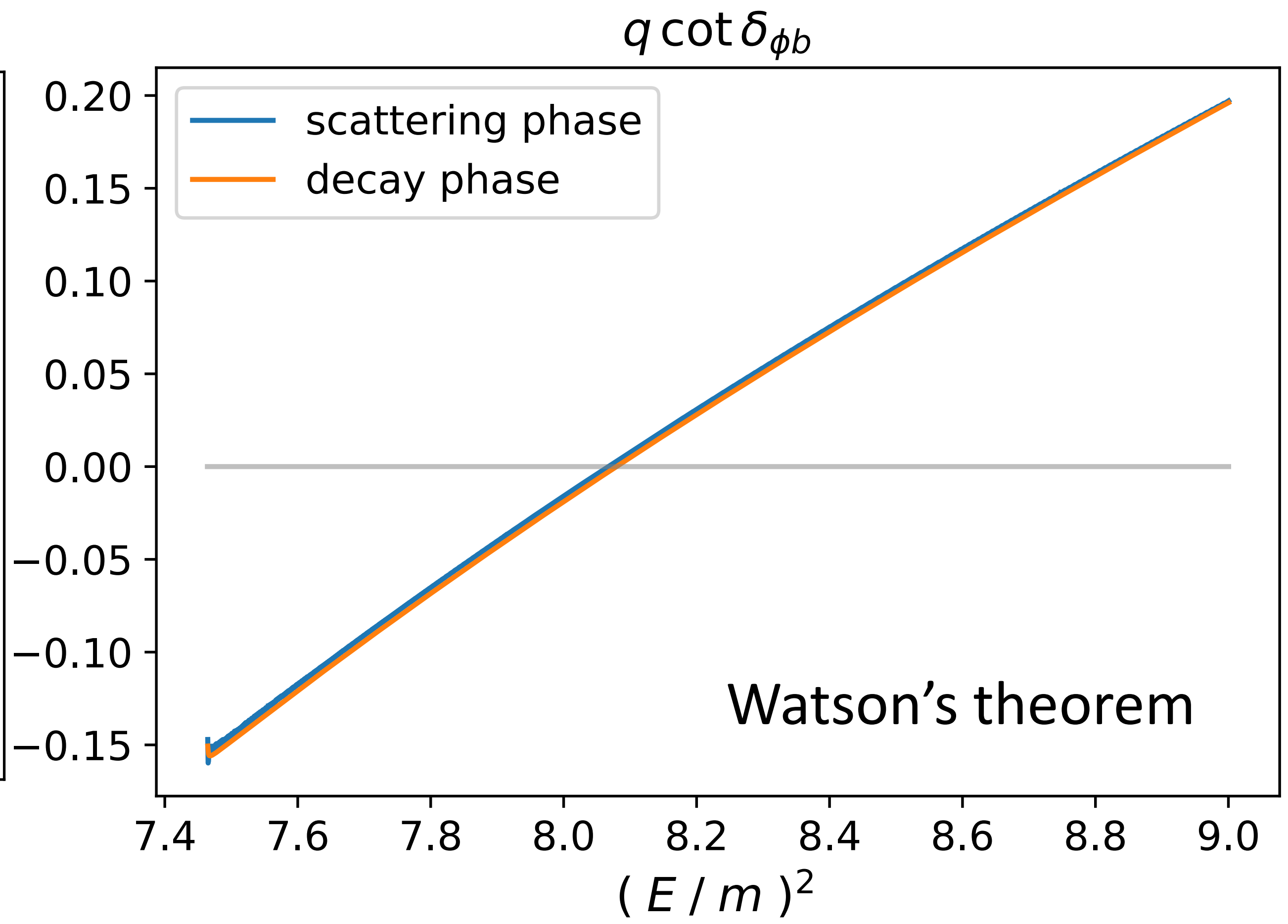
$$\sim \quad i\, T_{\varphi b} \frac{i}{s_{2k} - s_b} i\, g$$

$$T_{\varphi b}\,(E) = \lim_{s_{2k} \to s_b} -g \left[\tilde{\rho}_{PV}(k) + \int \frac{s^2 ds}{\omega_s (2\pi)^3} d^{(u,u)}(k,s)\, \mathcal{M}_2(s) \tilde{\rho}_{PV}(s) \right] A^{iso}$$

same as for $\mathcal{M}_{\varphi b}$

Testing the 3-body formalism – bound states: Decay amplitudes





Testing the 3-body formalism – bound states: Decay amplitudes

Hansen, Romero-López, Sharpe PRD (2021)

$$\left|\frac{T_3}{FV\ ME}\right|^2 \propto \frac{\partial F_3^{iso}(E,L)^{-1}}{\partial E}$$

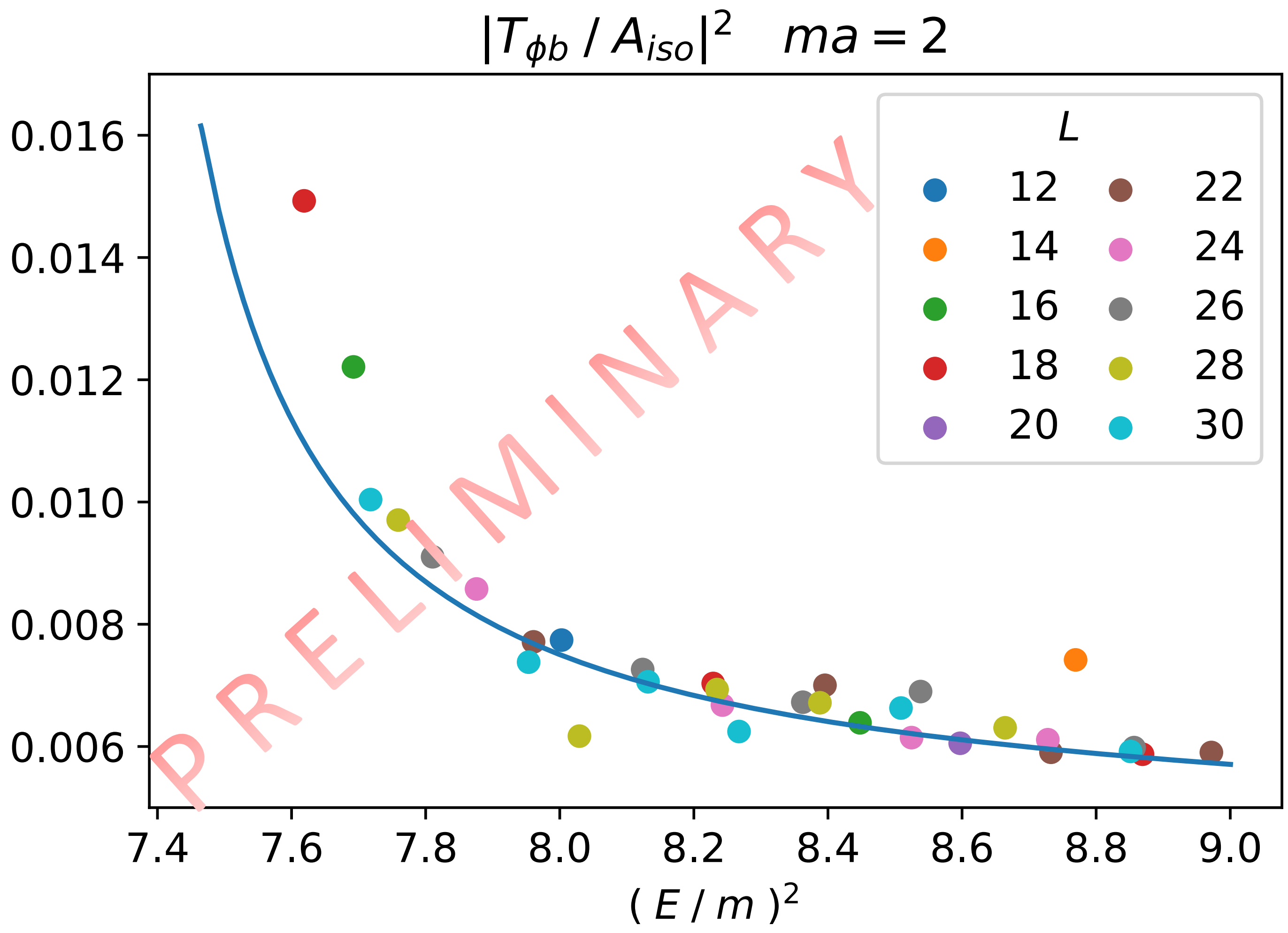
finite volume matrix element

Lellouch, Lüscher Comm. Math. Physics (2001)

$$\left|\frac{T_2}{FV\ ME}\right|^2 \propto \frac{\partial F_2(E,L)^{-1}}{\partial E} + \frac{\partial K_2(E)}{\partial E}$$

$$\left|\frac{T_{\varphi b}}{A^{iso}}\right|^2 = \frac{1}{|1 - i\rho K_{\varphi b}|} \times \left(\frac{\partial F_2{}^{-1}}{\partial E} + \frac{\partial K_{\varphi b}}{\partial E}\right) \times \left(\frac{\partial F_3^{iso\,-1}}{\partial E}\right)^{-1}$$

Testing the 3-body formalism – bound states: Decay amplitudes



$$\left|\frac{T_{\varphi b}}{A^{iso}}\right|^2 = \frac{1}{\left|1 - i\rho K_{\varphi b}\right|} \times \left(\frac{\partial F_2}{\partial E}^{-1} + \frac{\partial K_{\varphi b}}{\partial E}\right) \times \left(\frac{\partial F_3^{iso}}{\partial E}^{-1}\right)^{-1}$$

Summary and future steps

- [x] QC3 reduces to QC2 in the presence of a bound state : analytical result
- [x] Expression for $T_{\varphi b}$ in terms of known ladder amplitude
- [x] Formalism reduces to decay constant of a three-particle bound state in the trimer limit
- [x] 3-particle version of Lellouch-Lüscher factor reduces to 2-particle : numerical evidence
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- [ ] Numerical tests of the deep trimer limit

Summary and future steps

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Thank you!