

New phases and the CEP in QCD phase structure: checks and balances

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Universität Heidelberg & ExtreMe Matter Institute

The QCD Critical Point: Are We There Yet?

INT Seattle, November 3rd 2025



fqcd-collaboration.github.io



STRUCTURES
CLUSTER OF
EXCELLENCE



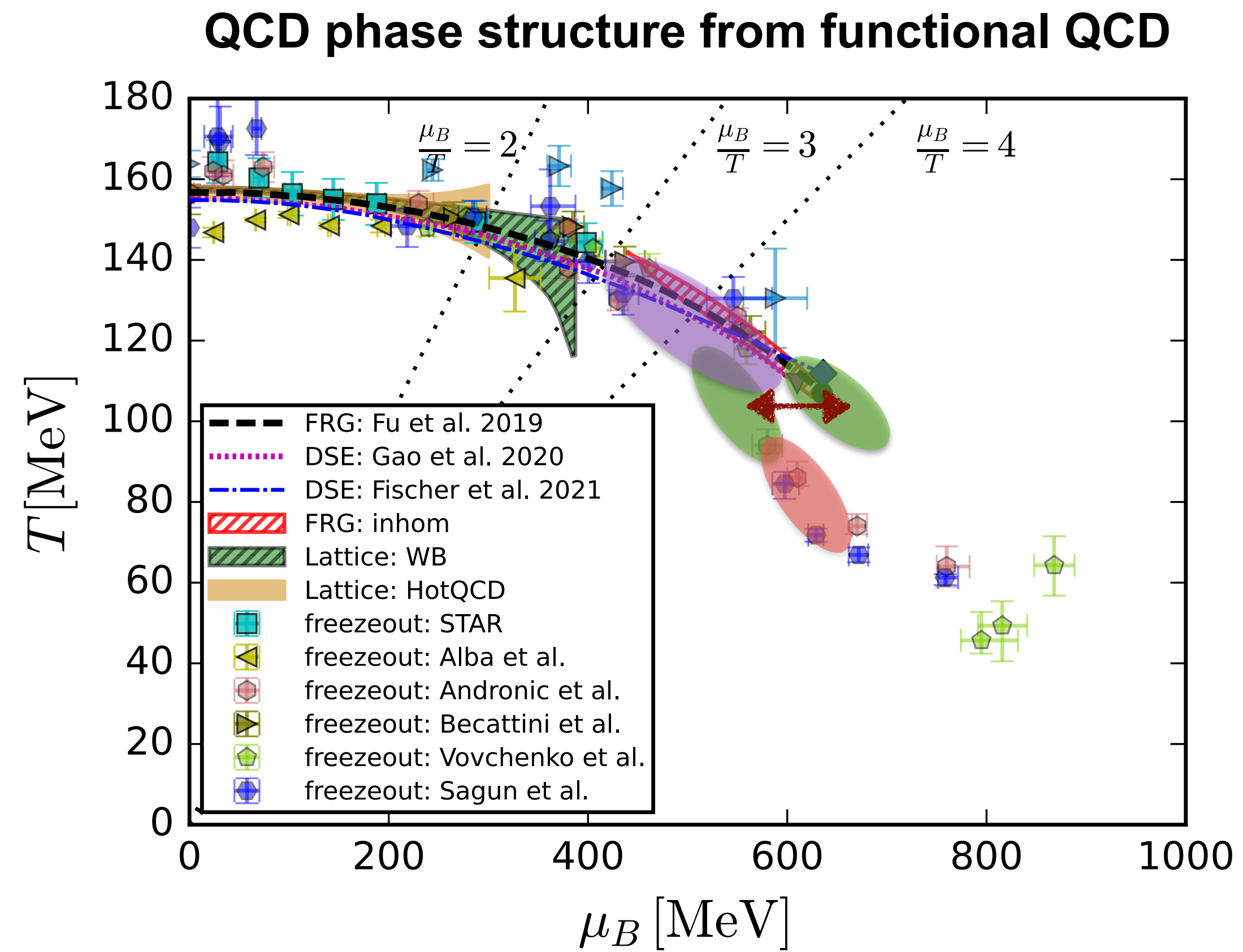
UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386



Outline

- **Critical end point and new phases: where do we stand?**
- **Checks and balances**
- **Supplement**

Critical end point and new phases: where do we stand?



Experimental & Theoretical Landscape

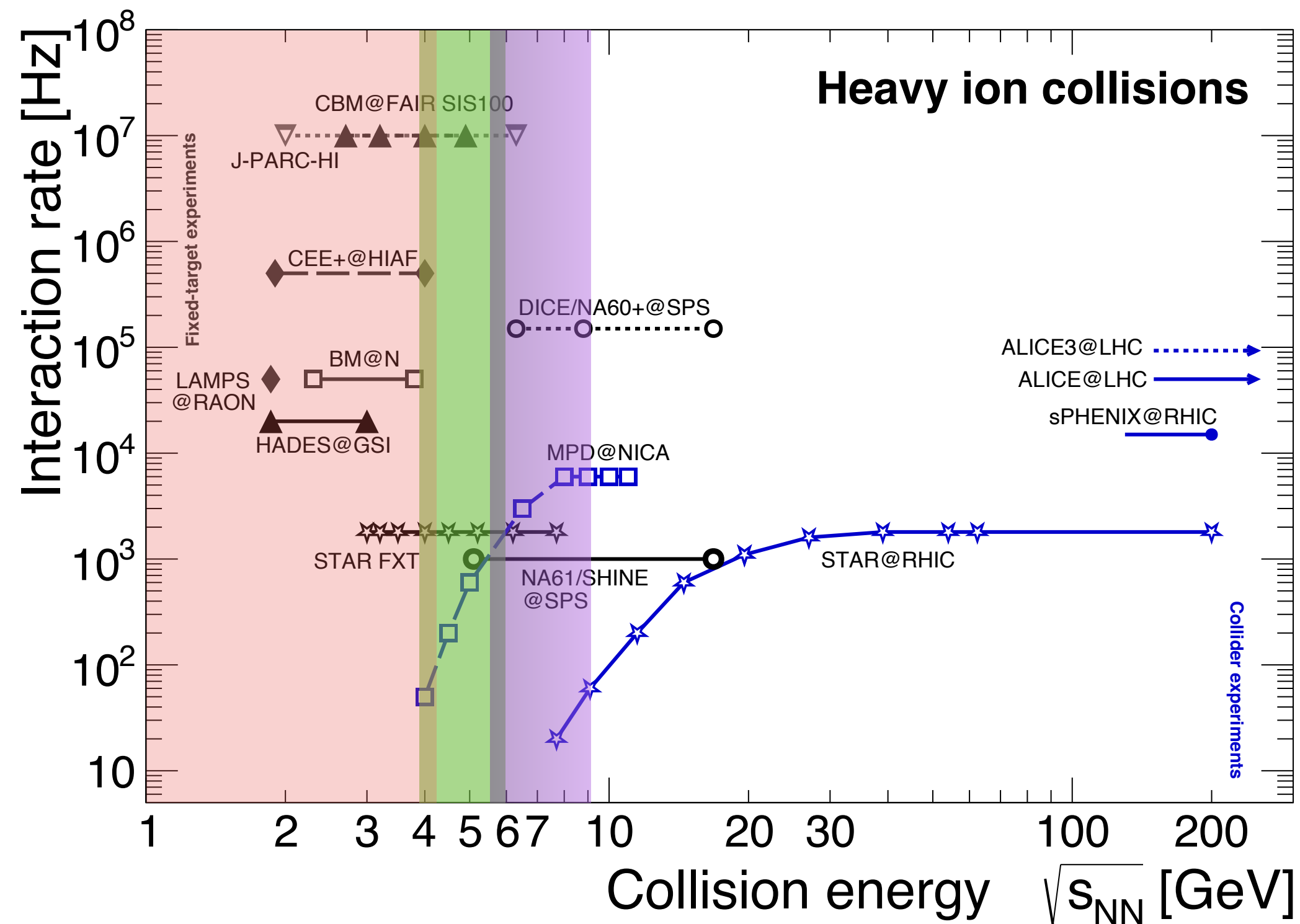
Experimental landscape

Theoretical landscape

Onset of new phases on the chiral transition line (CEP)

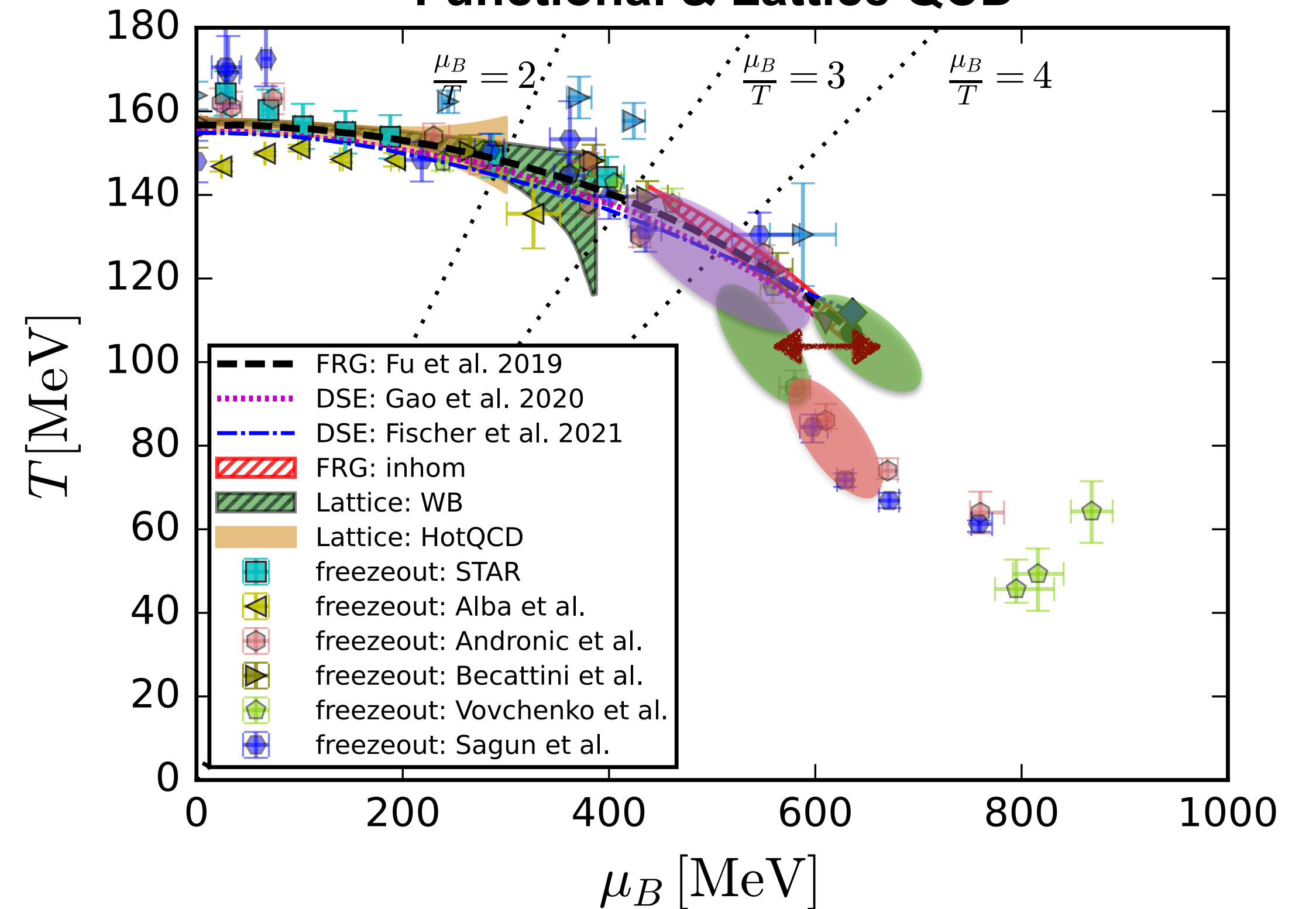
New phases

Transition regime



Peak of kurtosis on the freeze-out line

Functional & Lattice QCD



Galatyuk, A982 (2019) update 2025; CBM, EPJA 53 3 (2017) 60

fQCD
fqcd-collaboration.github.io

Fu, JMP, Rennecke, PRD 101 (2020) 054032

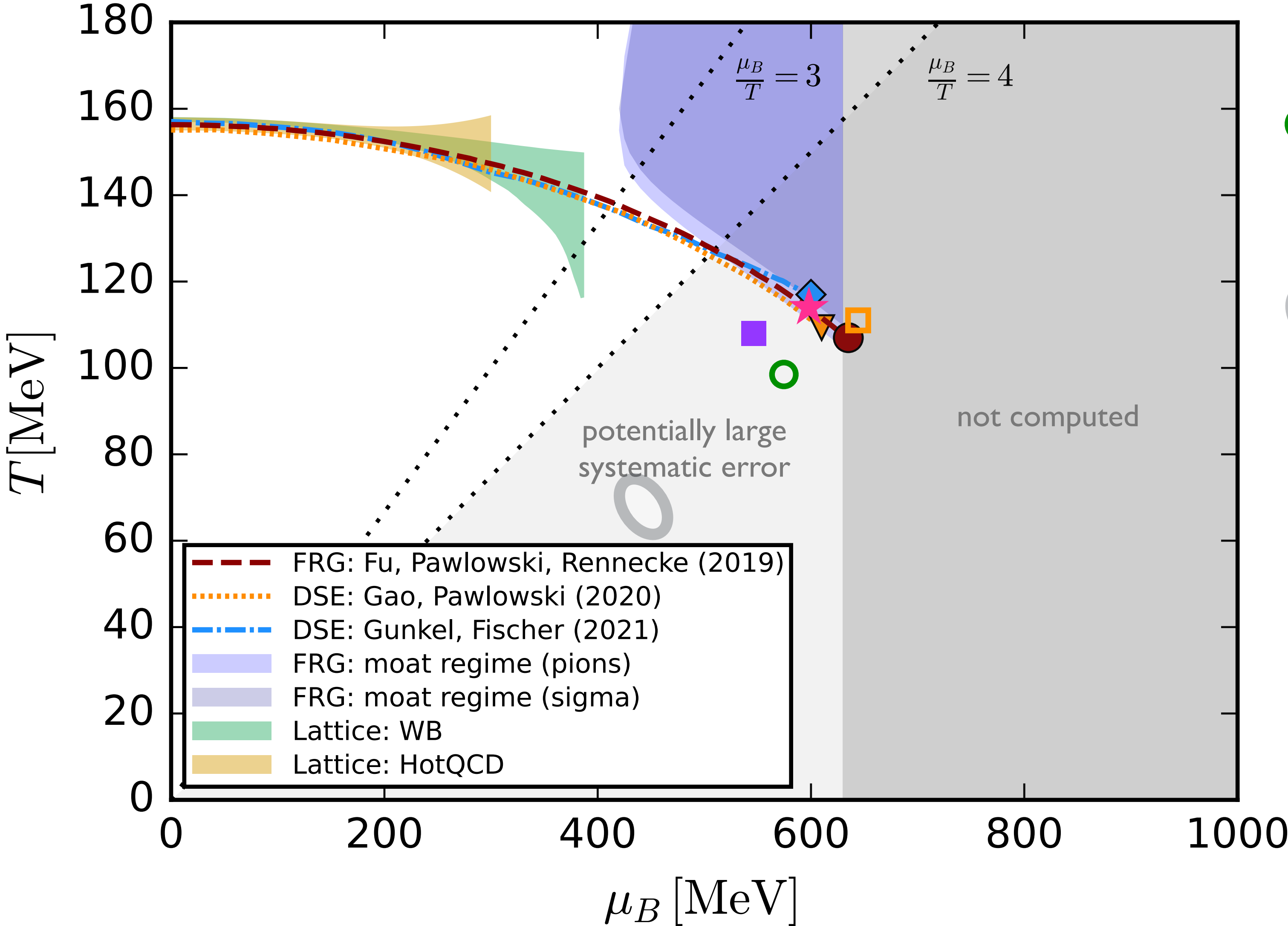
Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

QCD PHASE DIAGRAM & THE CEP

Modified from Fabian Rennecke's
plenary talk @ QM 2025

FRG & DSE results corroborated by subsequent "extrapolations" of lattice data



using Yang-Lee edge singularities:

- conformal Padé [Basar, 2312.06952] (green circle)
- multi-point Padé [Clarke et al., 2405.10196] (orange square)
 $N_\tau = 6, 8$ results + continuum estimate [Schmidt, 2504.00629]
- very recent lattice extrapolation ($16^3 \times 8$) [Adam et al, 2507.13254] (grey circle)

using thermodynamics:

- constant entropy density [Shah et al. 2410.16206] (pink star)

model-based extrapolation (tiny selection):

- holography [Hippert et al., 2309.00579] (purple square)
(agrees with [Cai et al., 2201.02004])

CEP location well constrained
by now. And it's in **FAIR** range!

$\sqrt{s_{NN}} \approx 3.6 - 4.1 \text{ GeV}$



Direct computations and the minimal point of view

Those are my interpretations,
and if you don't like them....
well, I have others

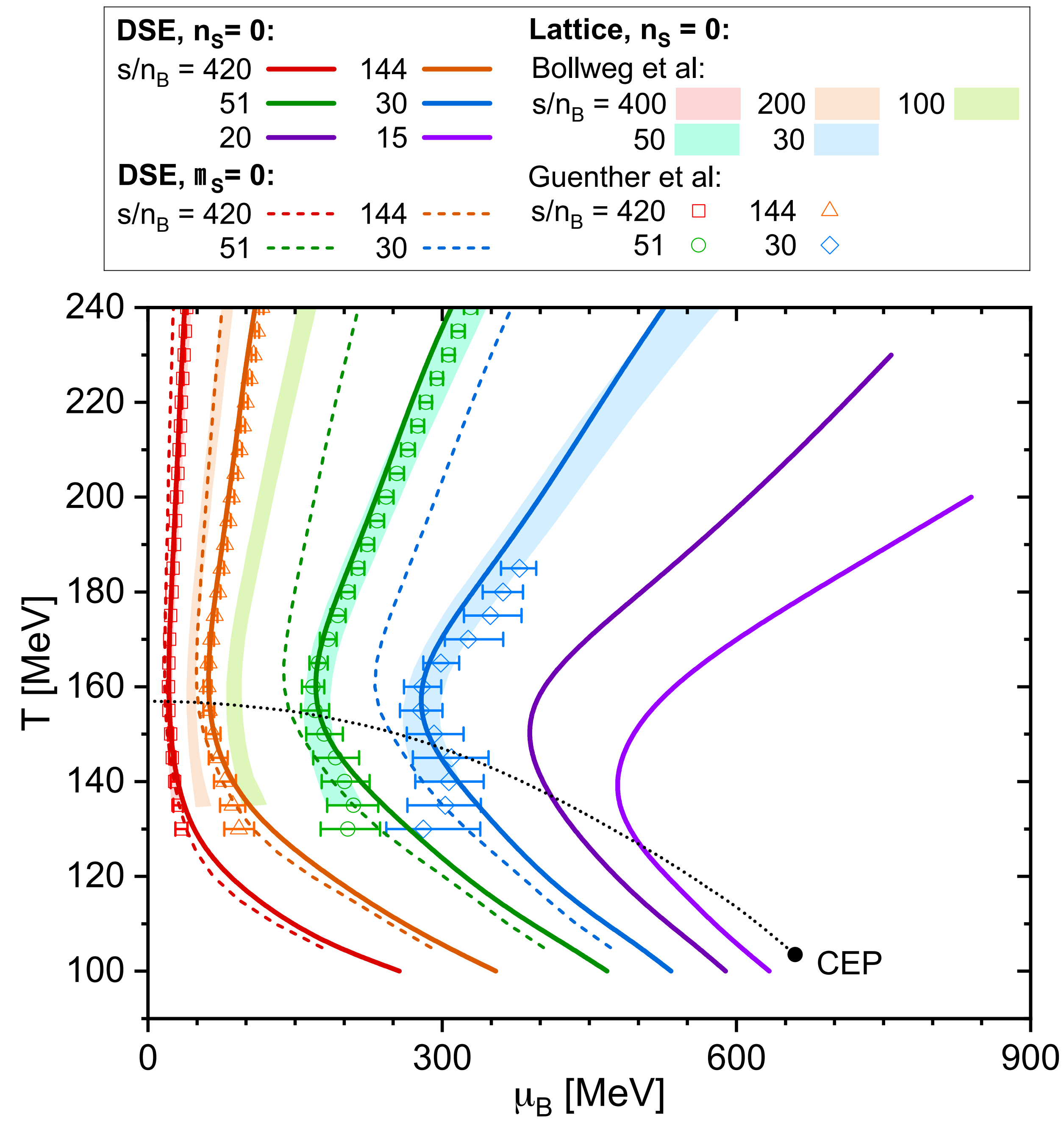
- Self-consistent truncations to functional relations define analytic functions in μ_B , eg:

$$\partial_t \left\langle q(x) \bar{q}(y) \right\rangle (\mu_B) = \text{Loop} \left[\left\langle q(x) \bar{q}(y) \right\rangle (\mu_B), \left\langle q(x) A_\mu(y) \bar{q}(z) \right\rangle (\mu_B), \dots ; \mu_B \right]$$

Unique: QCD-based analytic continuations
that satisfy the lattice benchmarks
at small chemical potential.

- ? Corroborated?

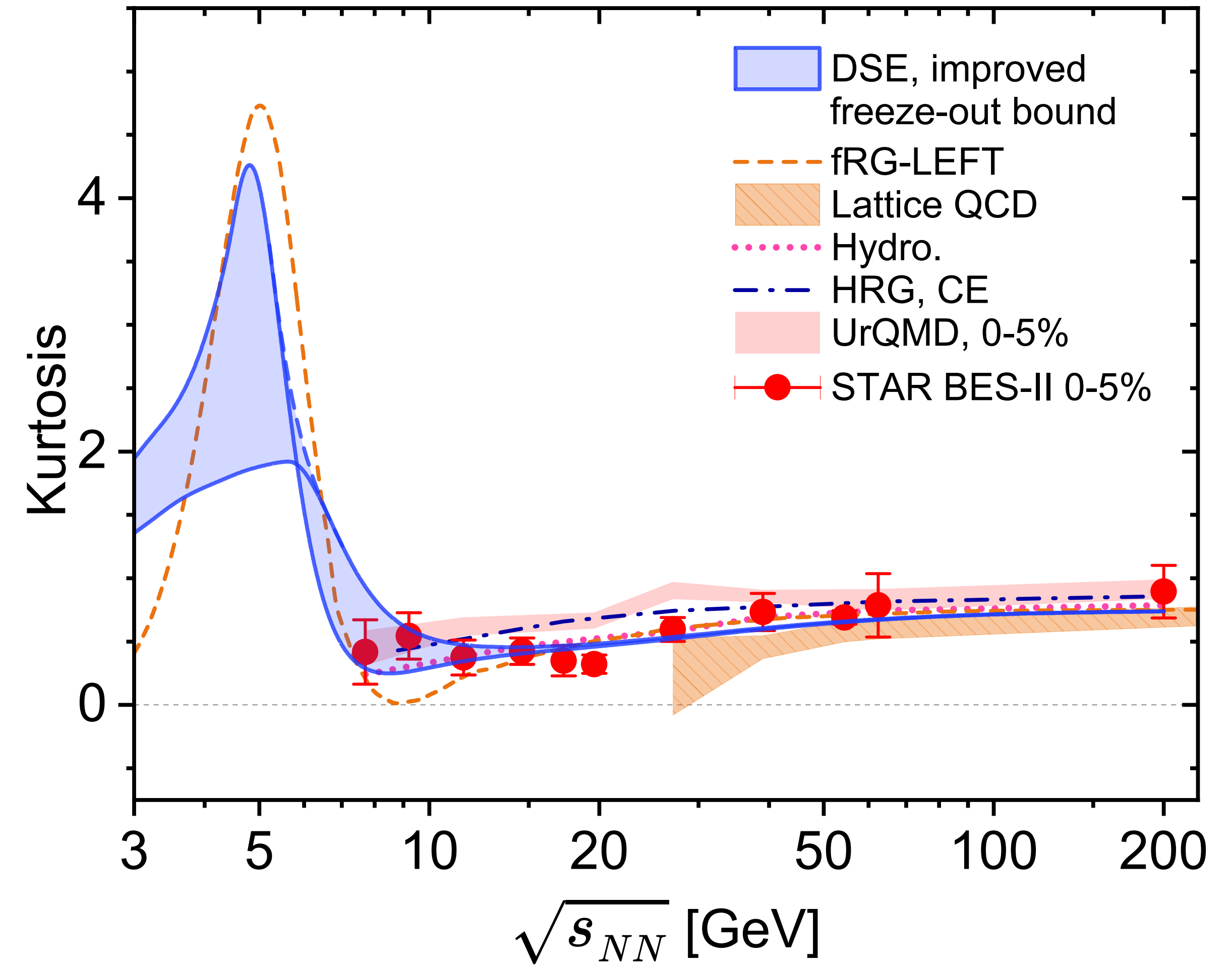
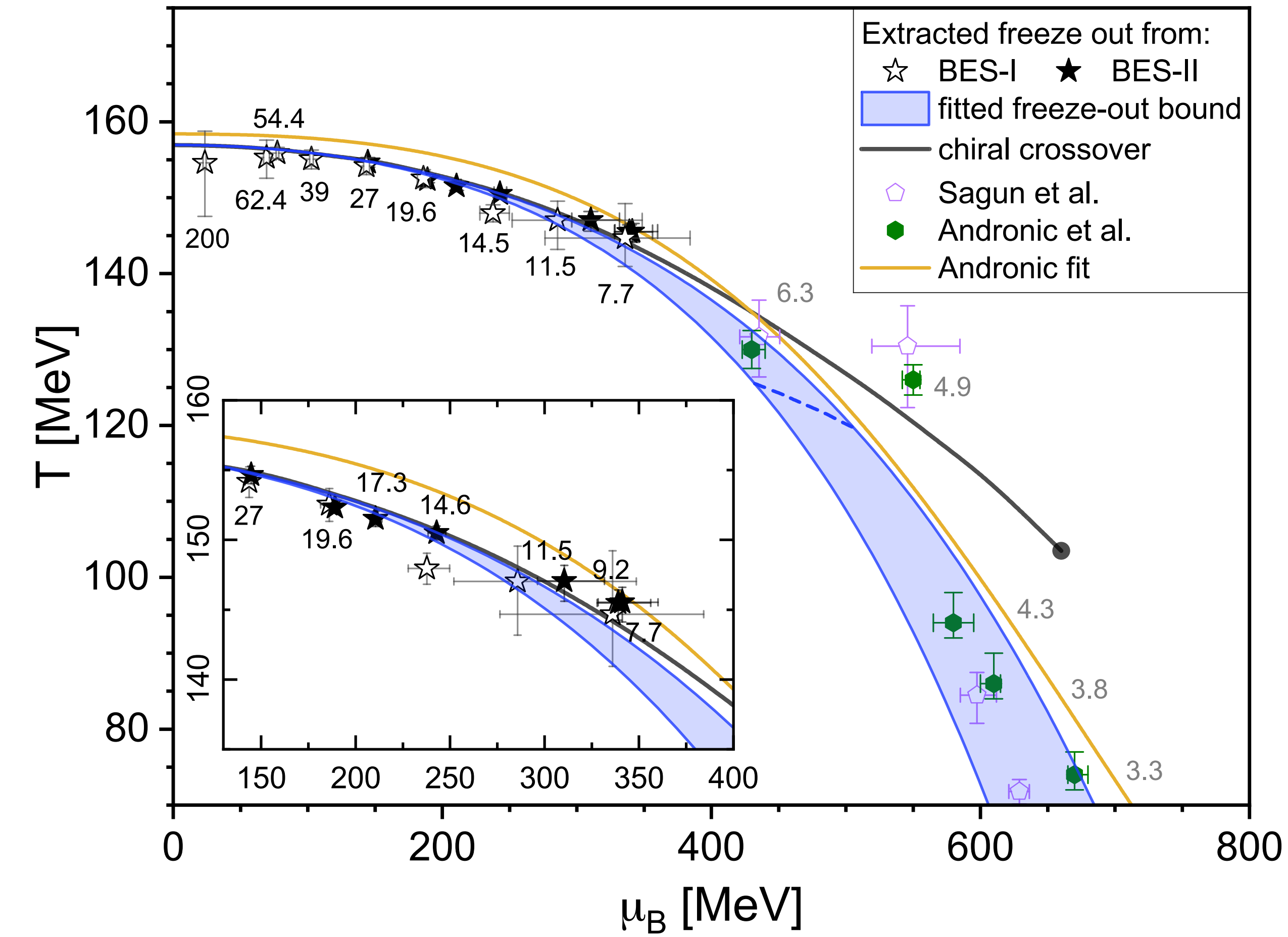
By definition, LEFTs (including extrapolations) cannot assess
the existence or absence of emergent phenomena!



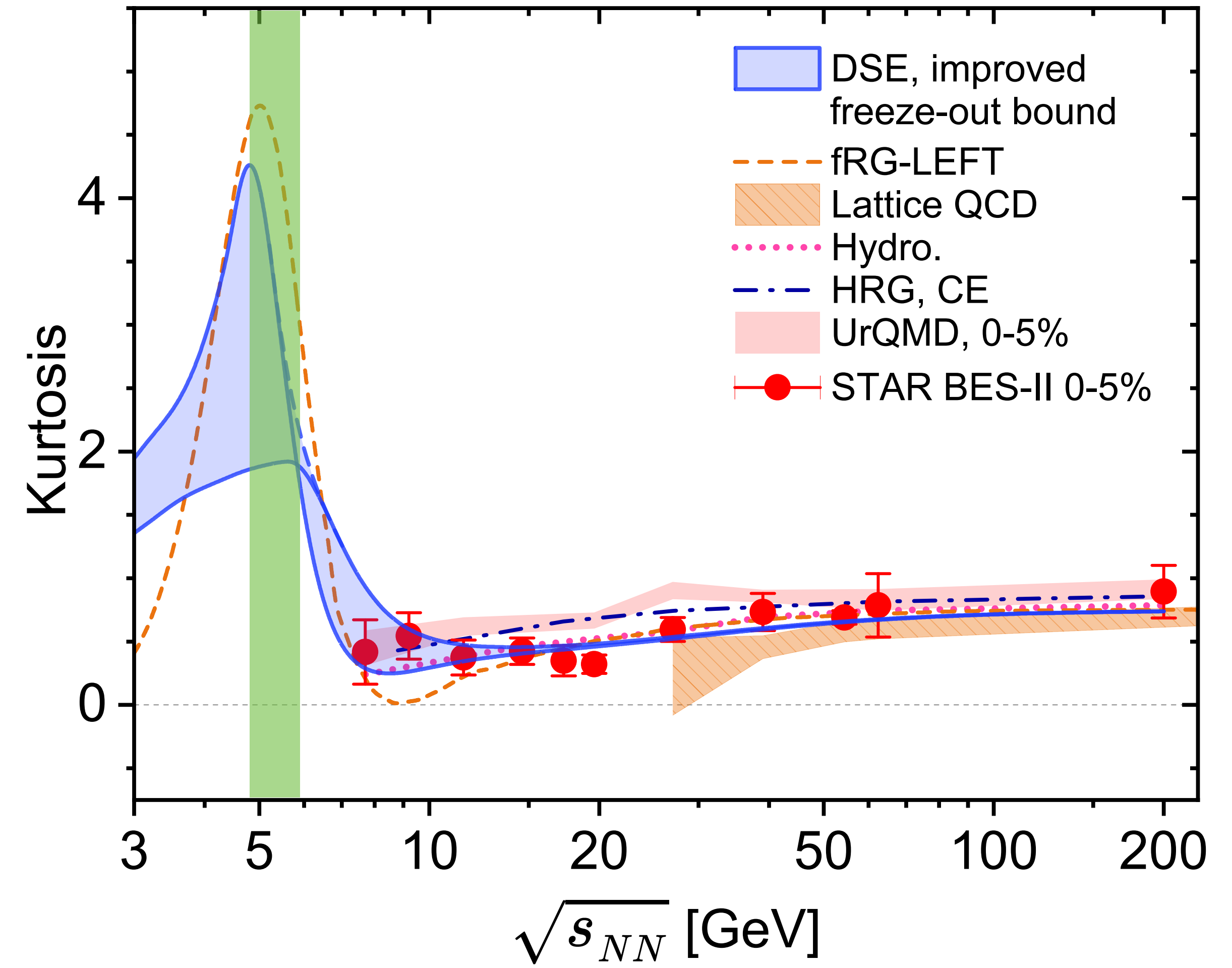
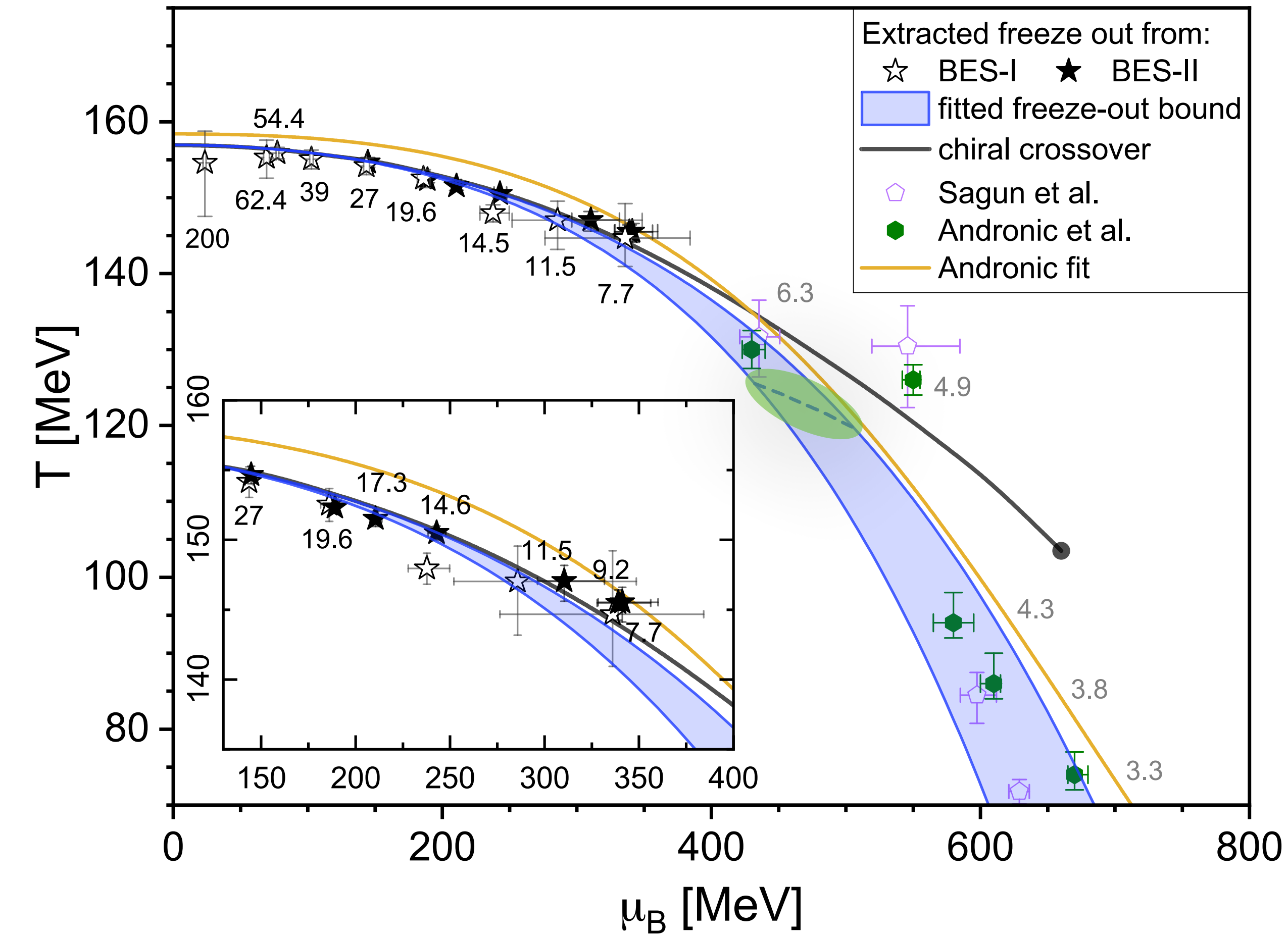
Functional EoS

preliminary

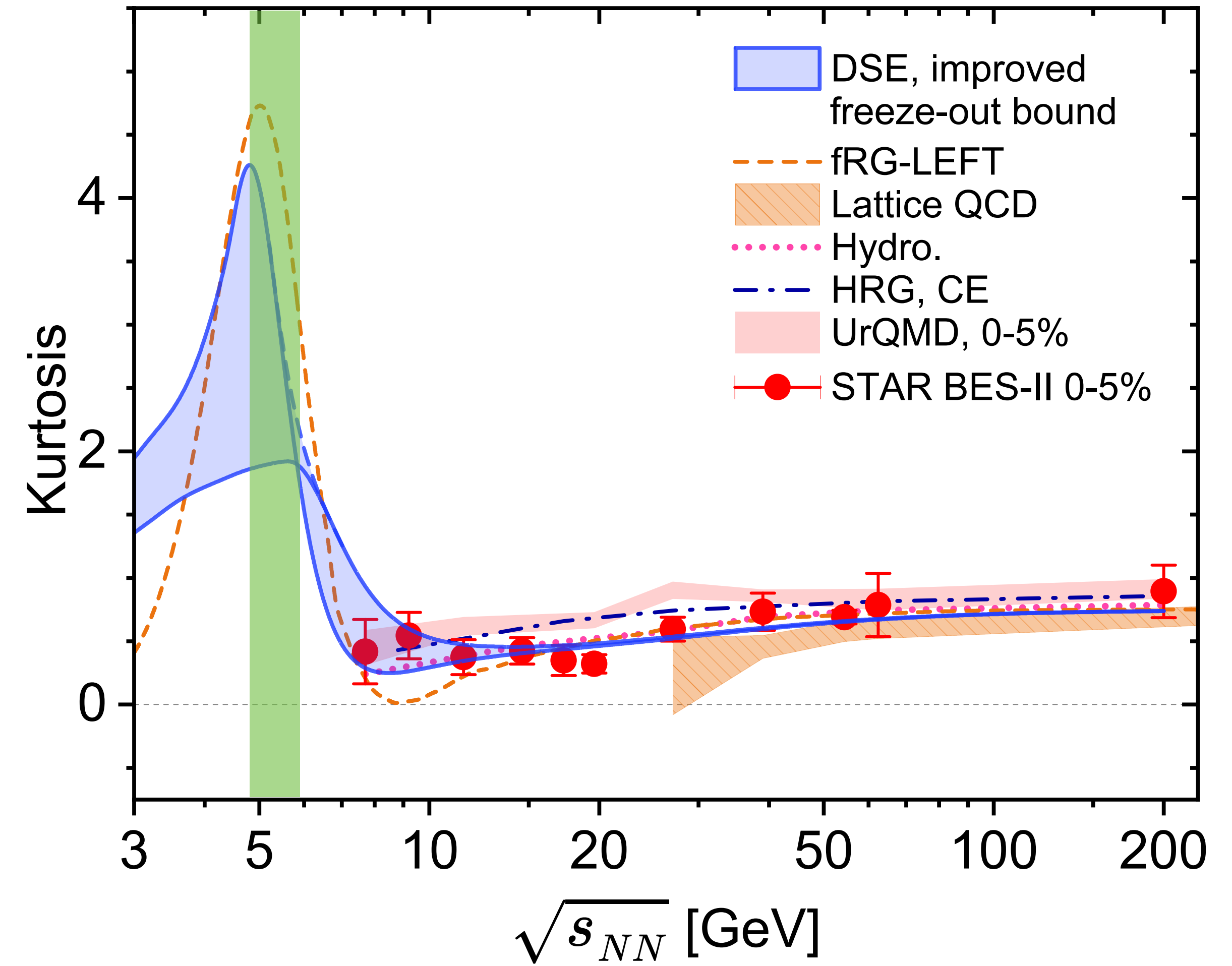
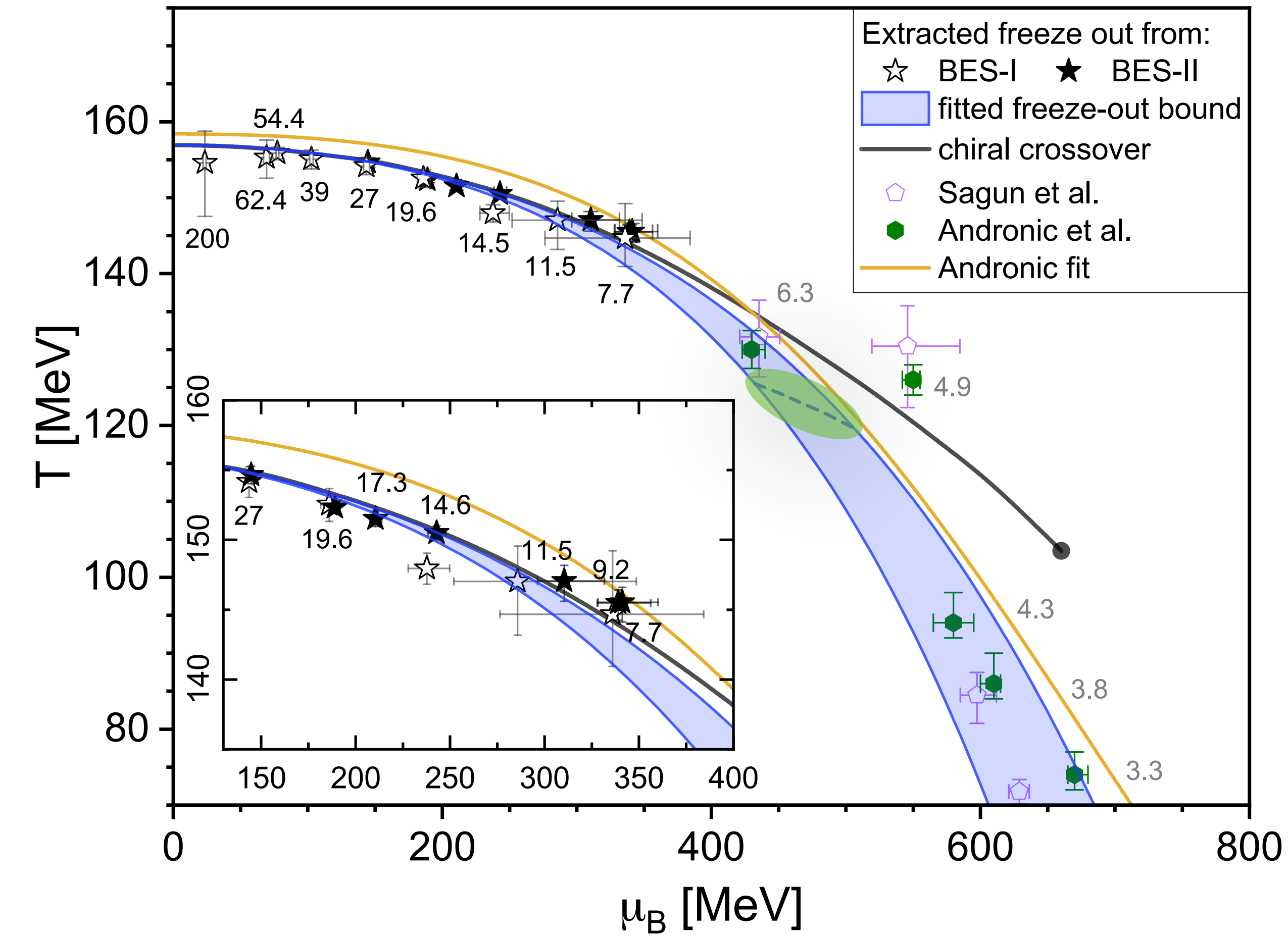
Functional Freeze-out



Functional Freeze-out



Functional Freeze-out



Demand: build in dynamics

preliminary

Strongly correlated physics

‘Confinement’ related ‘crossovers’ at $T_\chi \lesssim T_{\text{conf}} \lesssim 2 T_\chi$

Functional QCD

Lattice QCD

Variants of the
traced Polyakov loop

$$P = \frac{1}{N_c} \text{tr} \, e^{2\pi i \varphi}$$

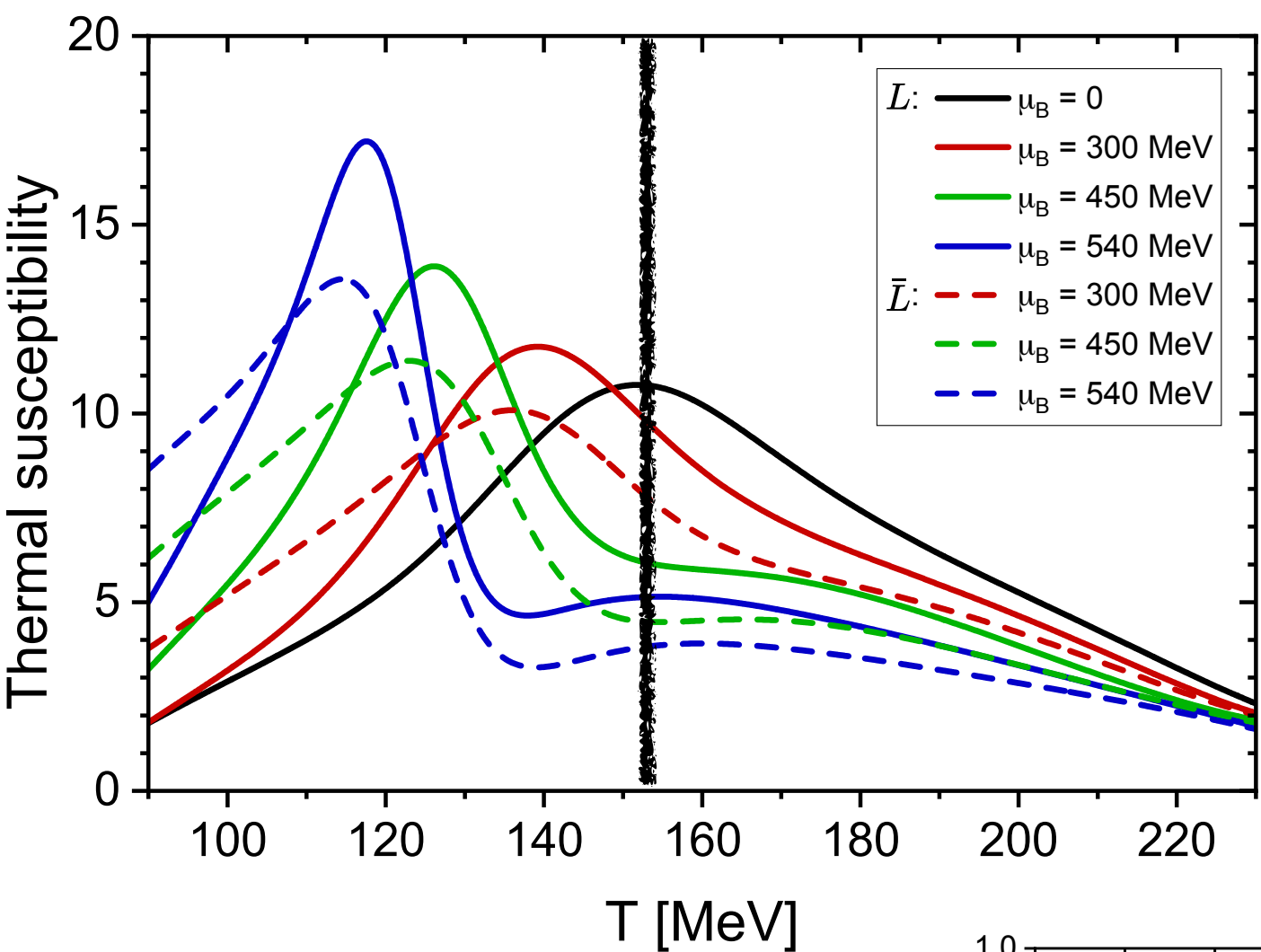
Braun, Gies, JMP, PLB 684 (2010) 262-267
⋮
Lu, Gao, Liu, JMP, arxiv:2504.05099

⋮
Borsanyi, Fodor, Hoelbling, Katz, Krieg, Ratti, Szabo, JHEP 09, 073 (2010)
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Borsányi, Fodor, Guenther, Kara, Paolo Parotto, Pásztor, Pirelli, Wong, PRD 110 (2024) 114507

Strongly correlated physics

‘Confinement’ related ‘crossovers’ at $T_\chi \lesssim T_{\text{conf}} \lesssim 2 T_\chi$

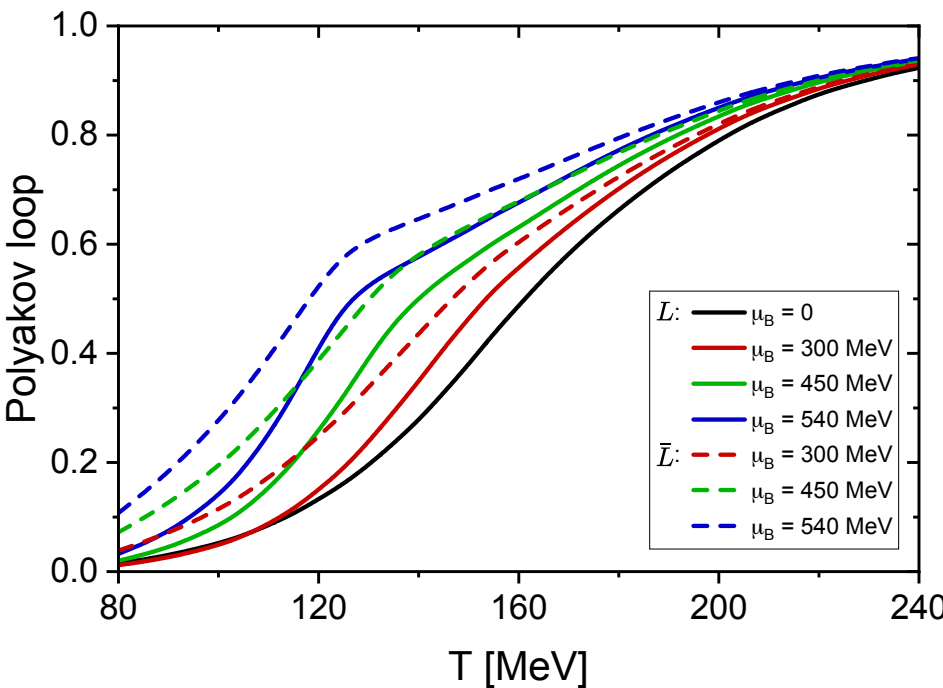
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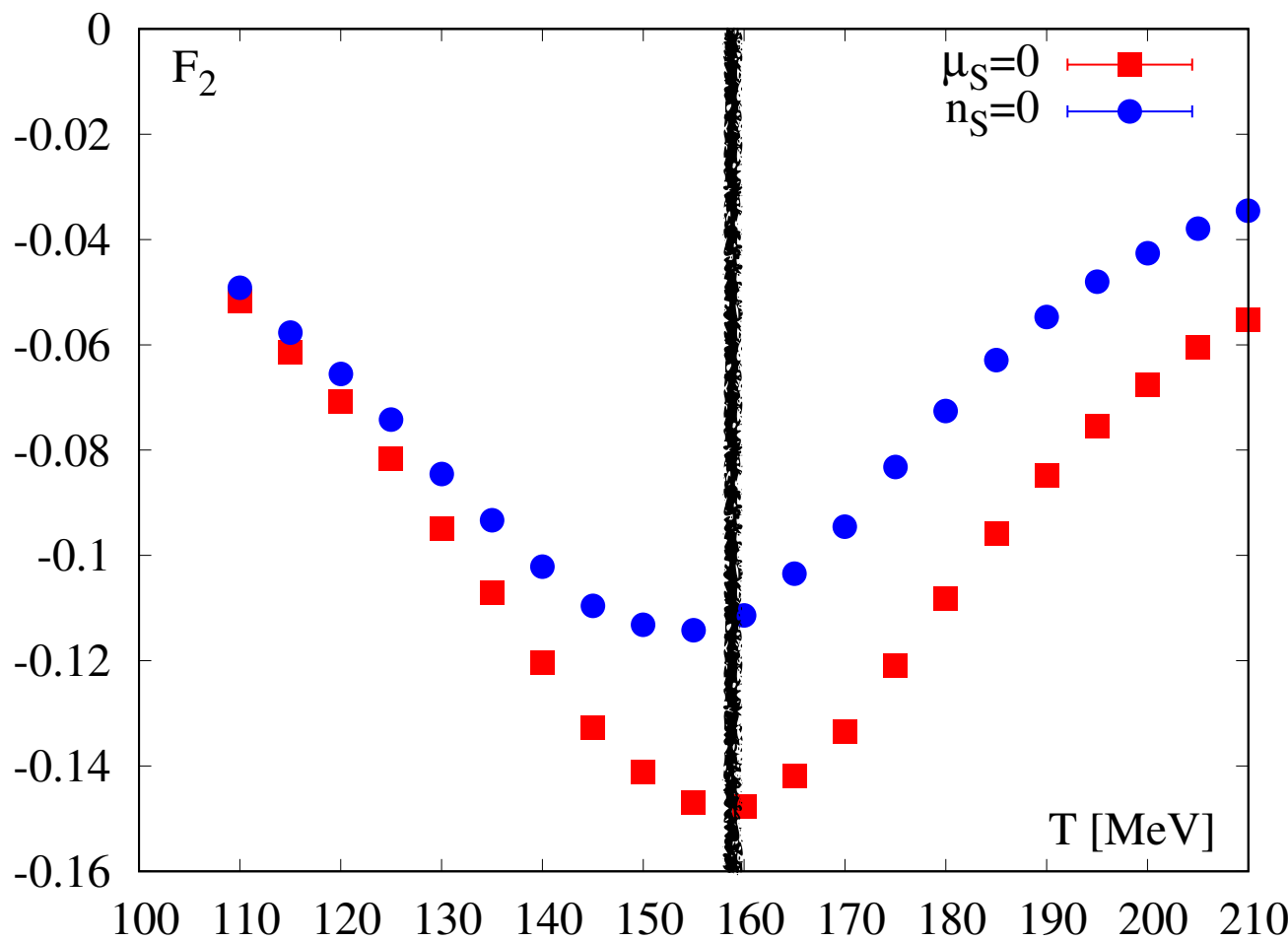
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$$\propto P(\langle \text{ev}(\varphi) \rangle)$$



Lattice QCD



$$\propto T \frac{\partial^2 \log |\langle P \rangle|}{(\partial \mu_B / T)^2}$$

Braun, Gies, JMP, PLB 684 (2010) 262-267

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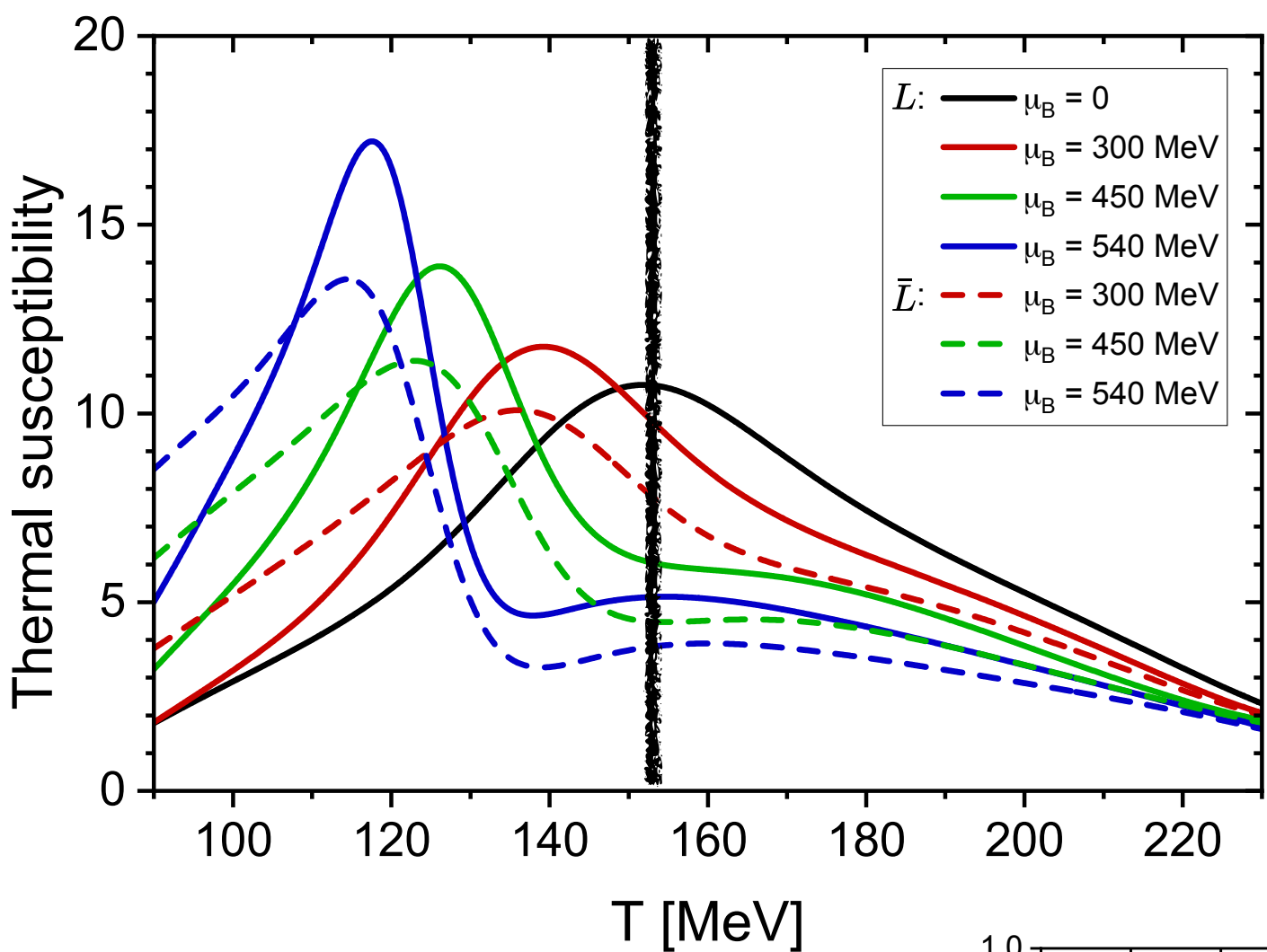
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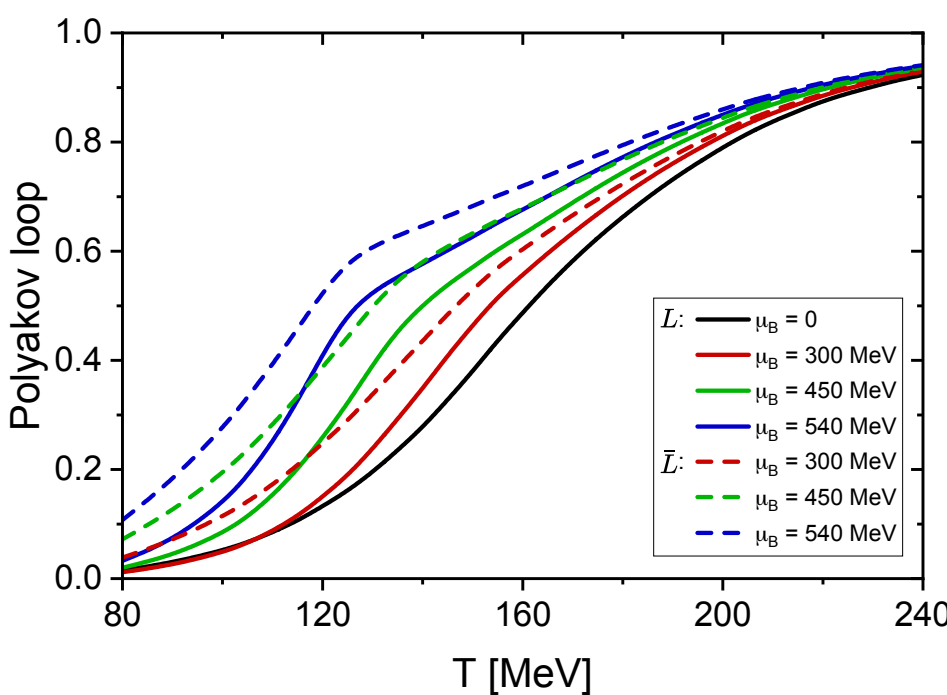
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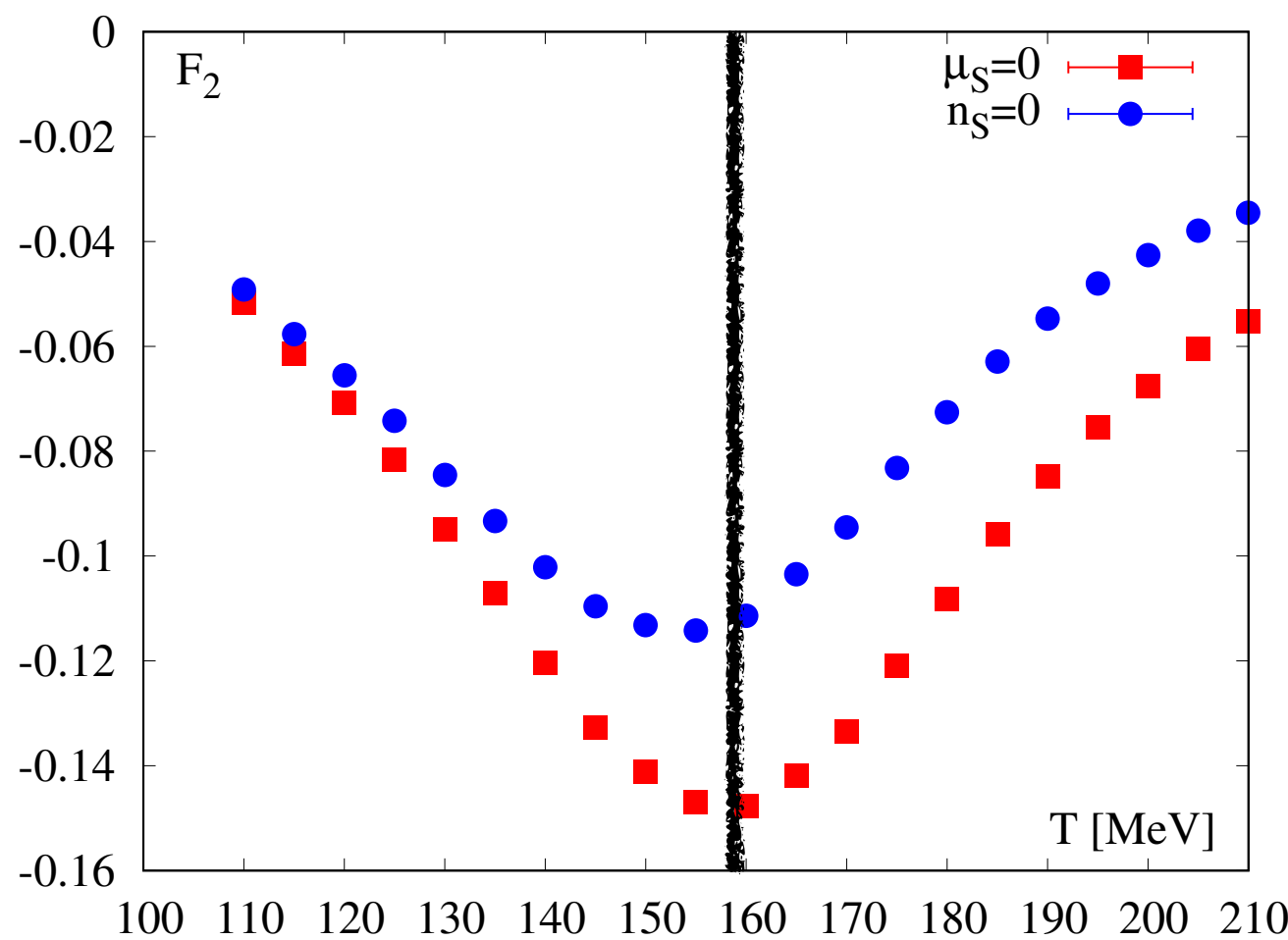
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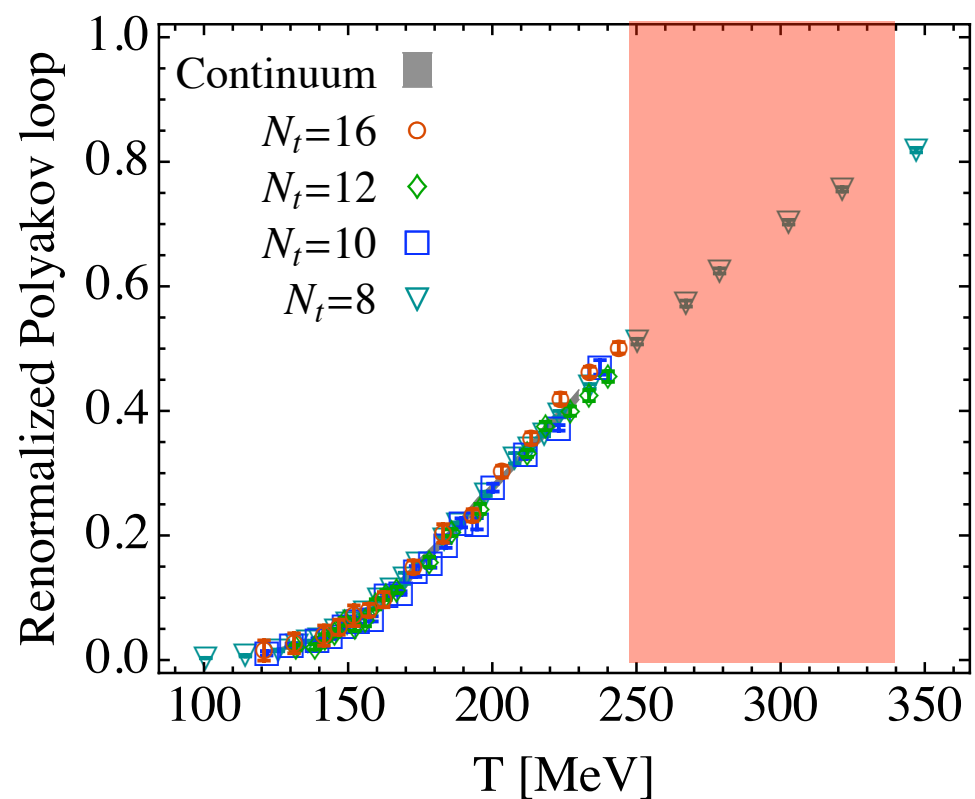


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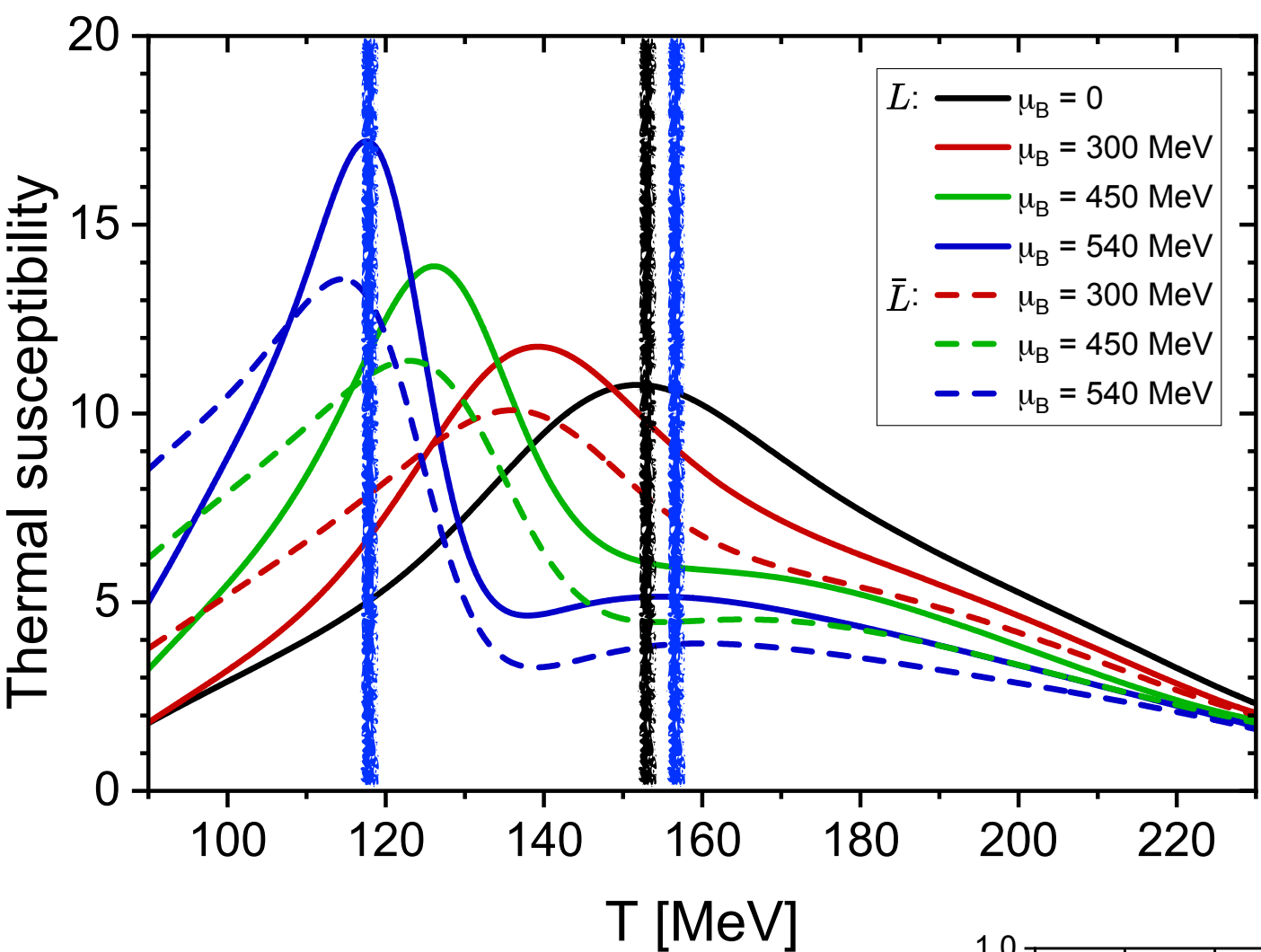
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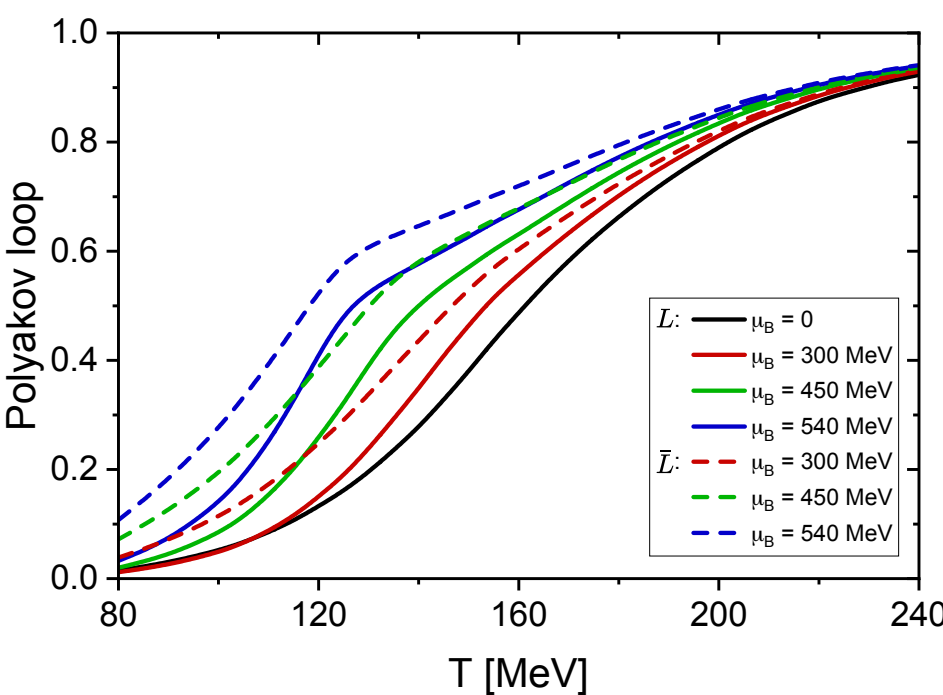
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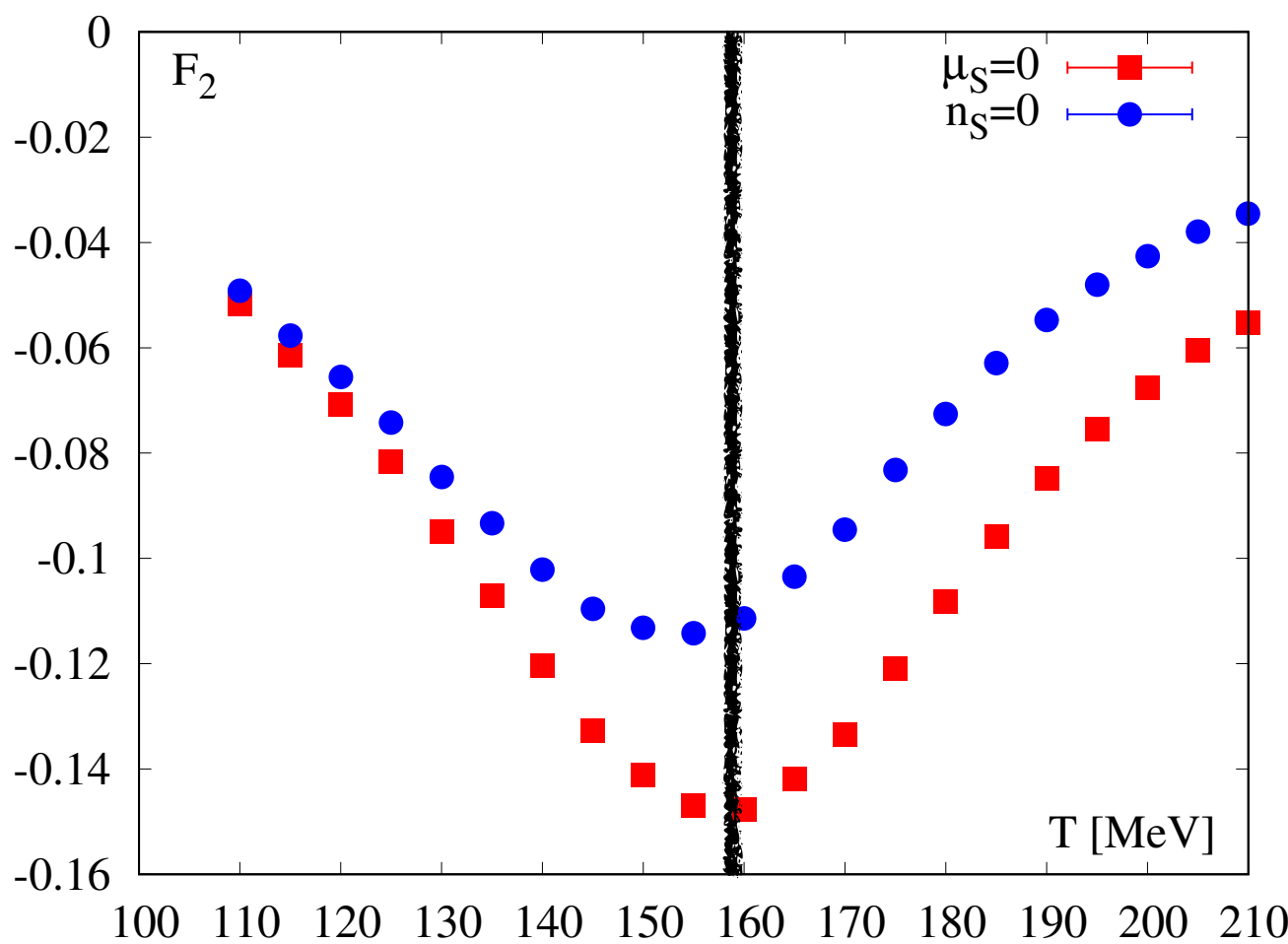
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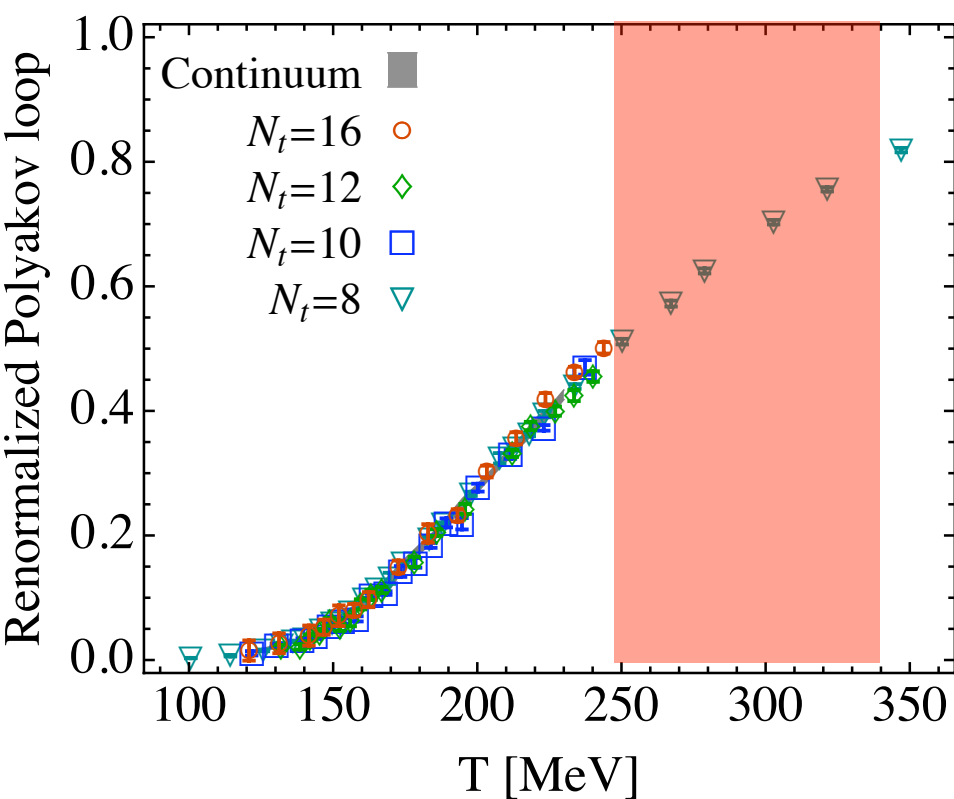


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Borsányi, Fodor, Guenther, Kara, Paolo Parotto, Pásztor, Pirelli, Wong, PRD 110 (2024) 114507

CEP exclusion regime

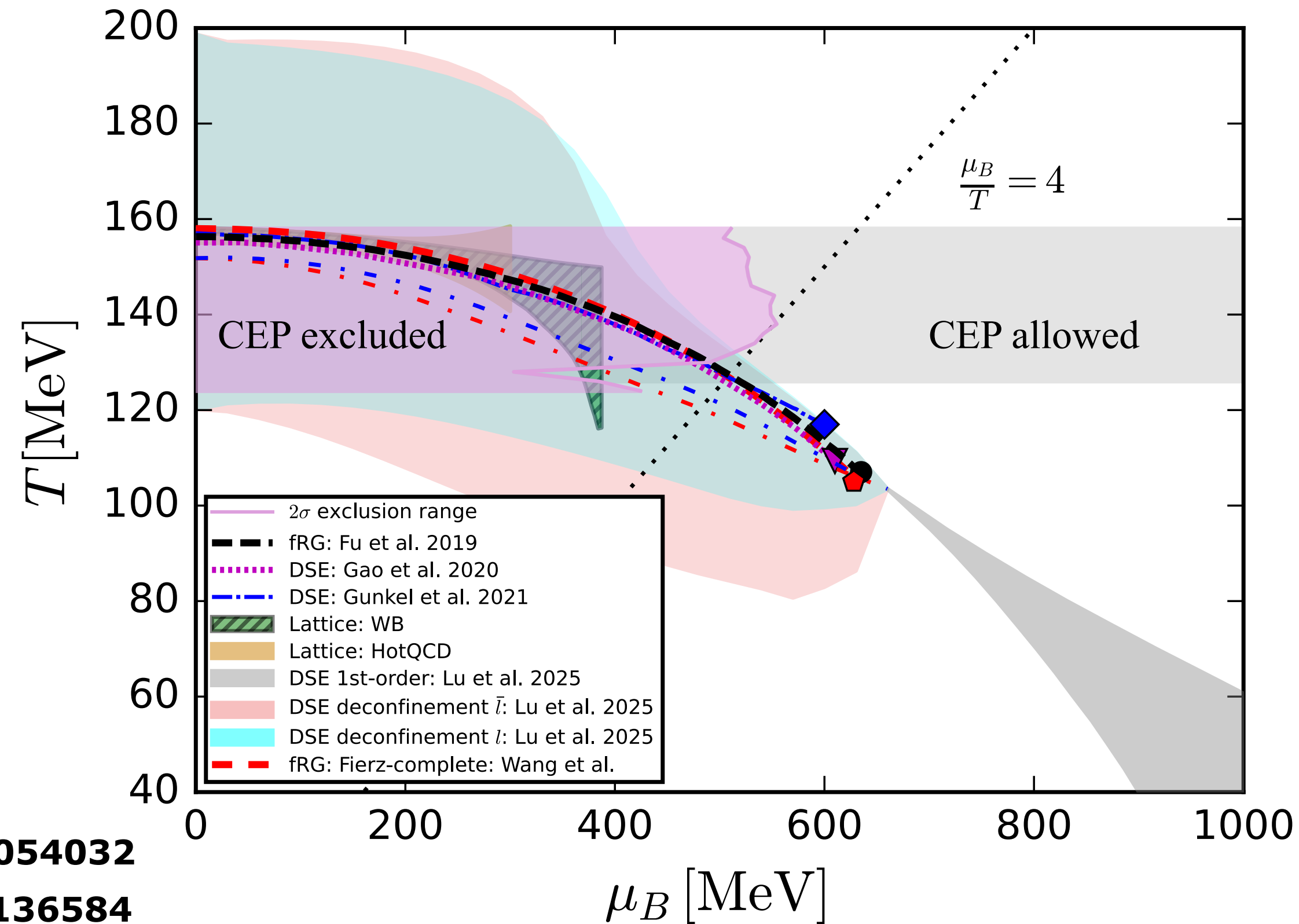
No **chiral** critical end point for $\mu_B \lesssim 600$ MeV

Functional QCD

Functional QCD CEP+ lattice QCD exclusion plot

Lattice QCD

$\mu_B \lesssim 450$ MeV



Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

Lu, Gao, Liu, JMP, arXiv:2504.05099

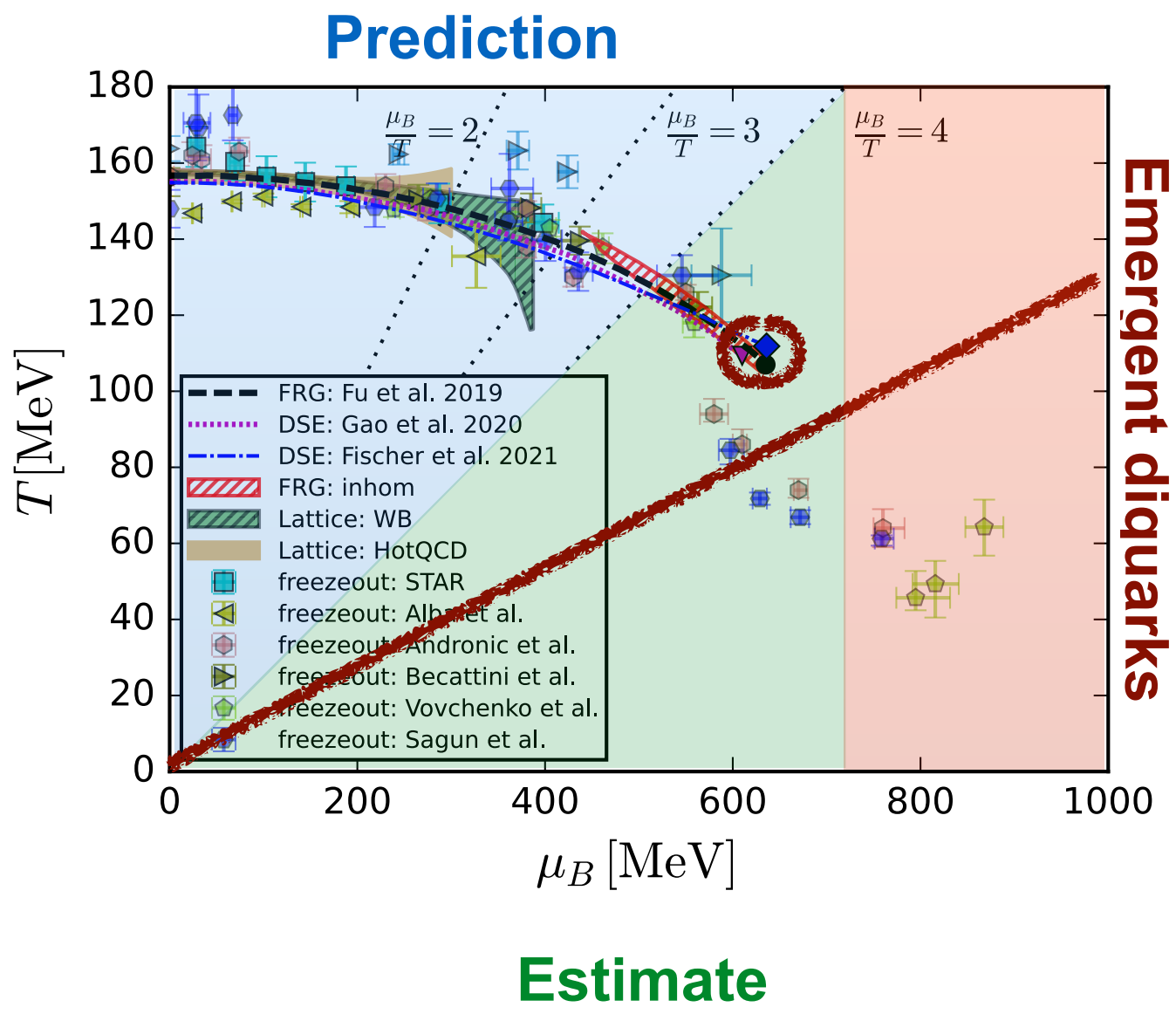
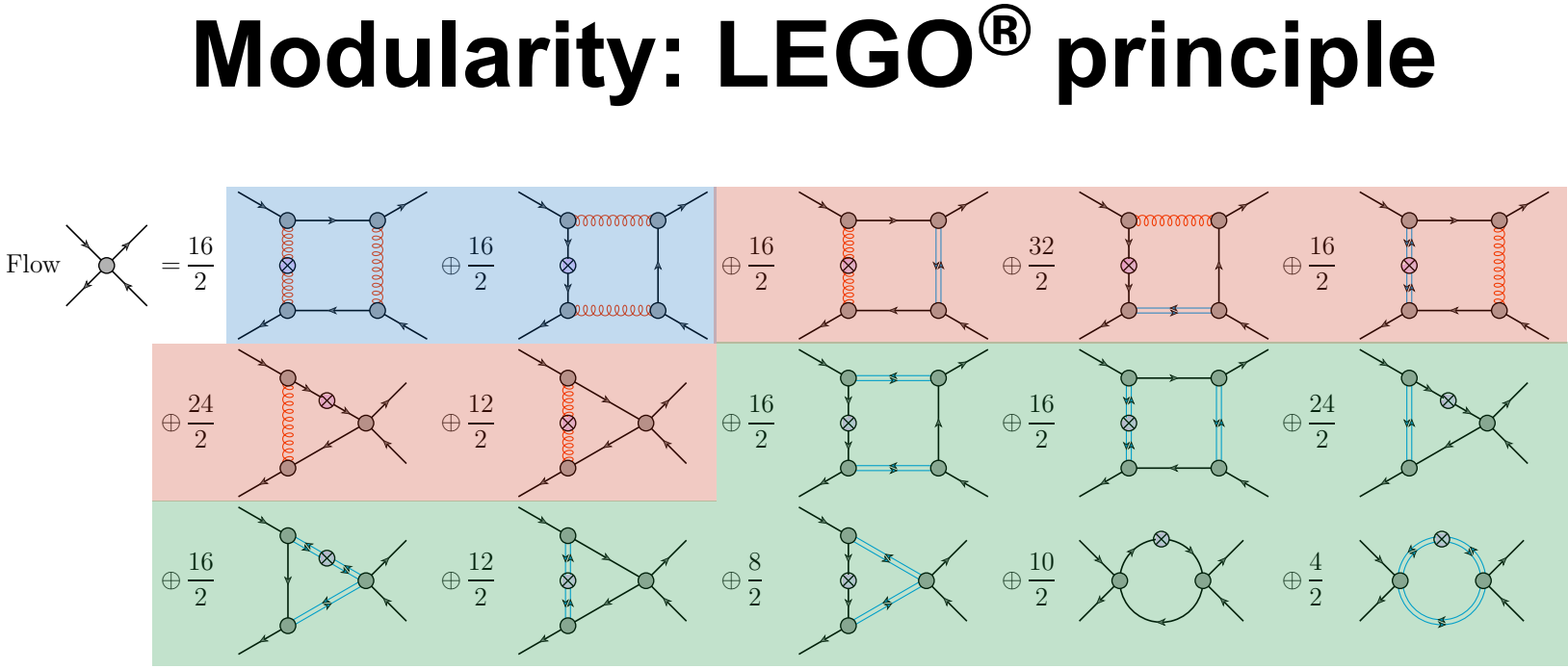
Lu, Gao, Liu, arXiv:2509.02974

Wang et al, in preparation

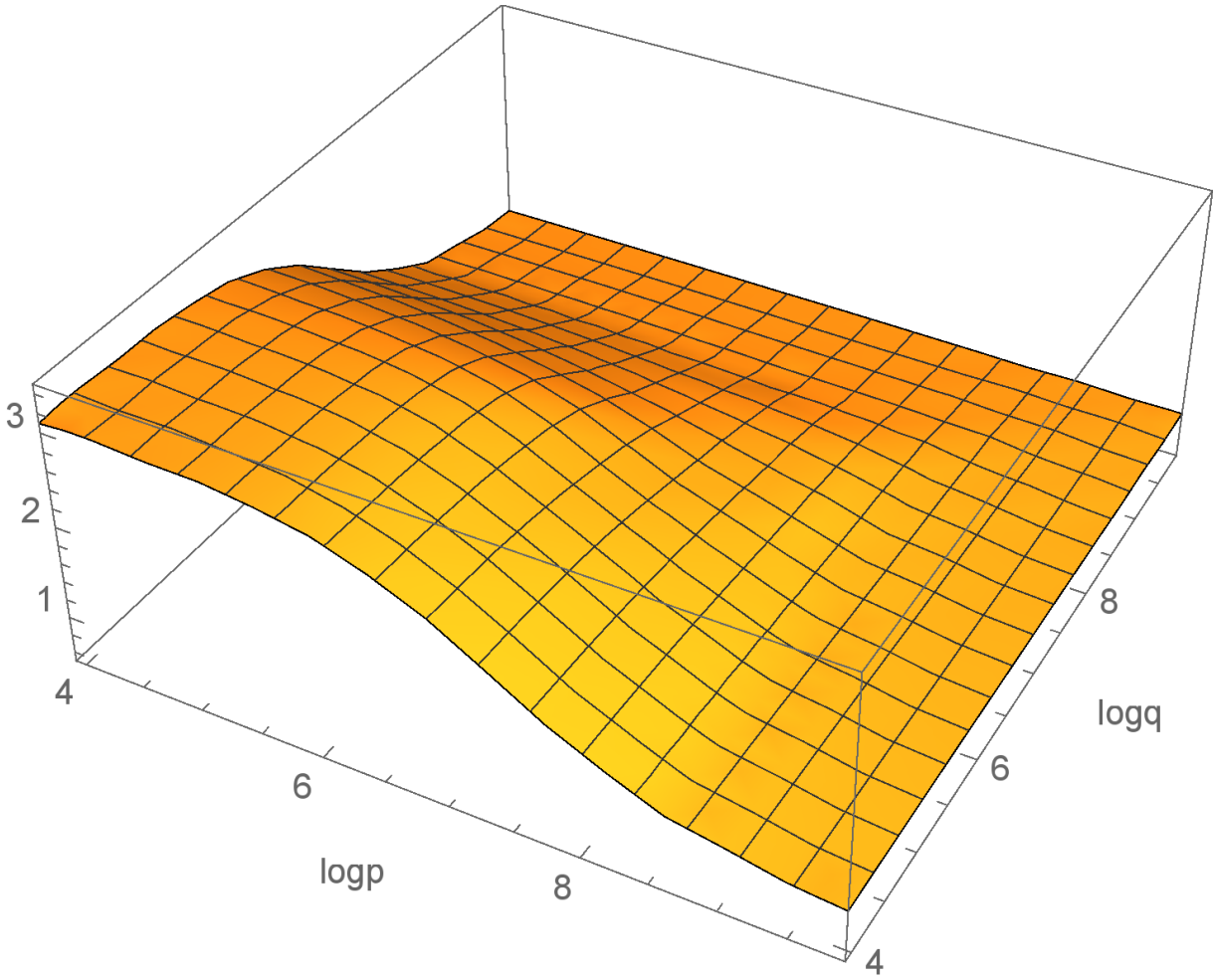
Borsanyi, Fodor, Guenther, Parotto, Pasztor,
Ratti, Vovchenko, Wong, arxiv: 2502.10267

Checks and balances

Systematic error estimates

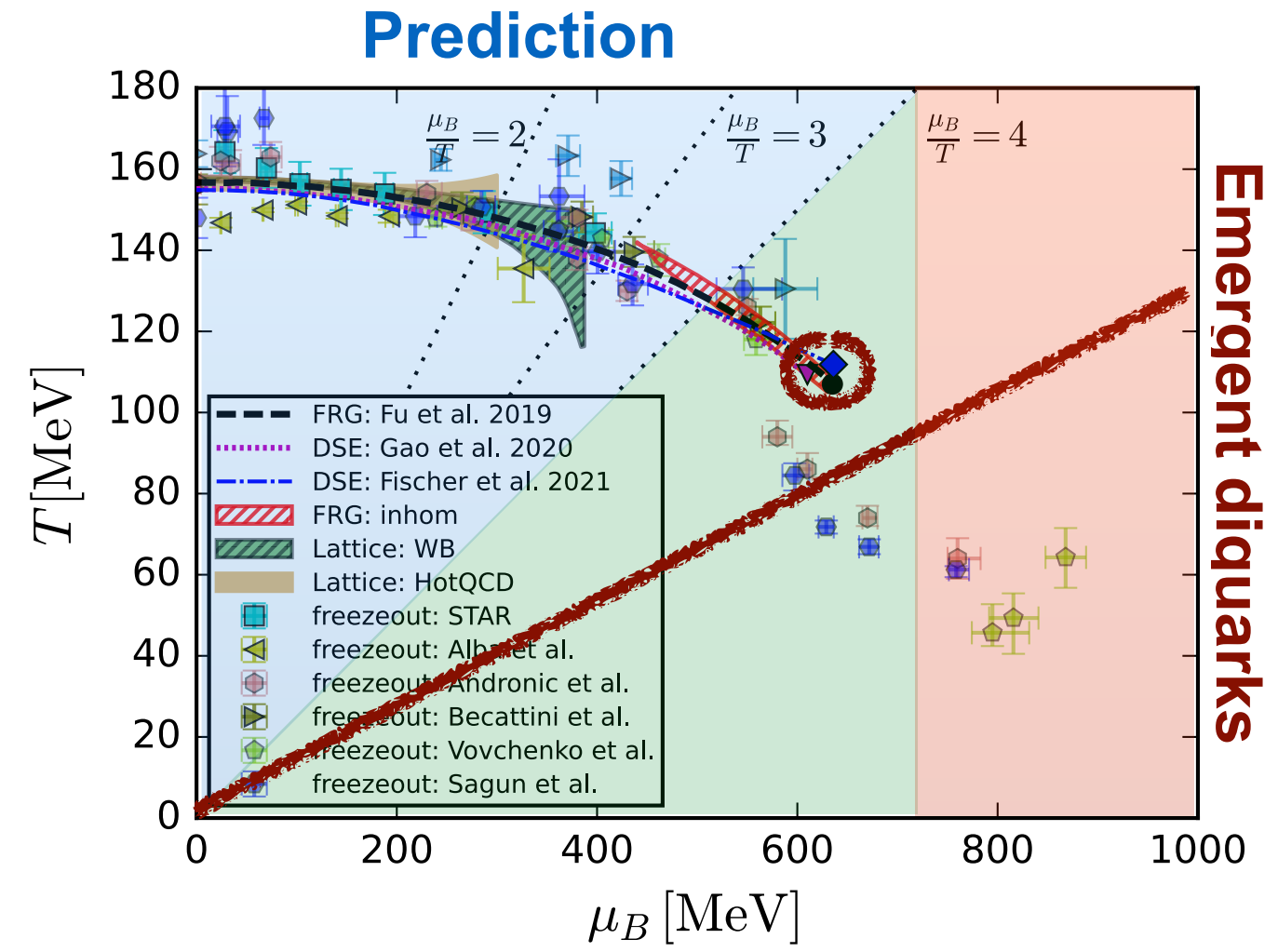


Apparent convergence



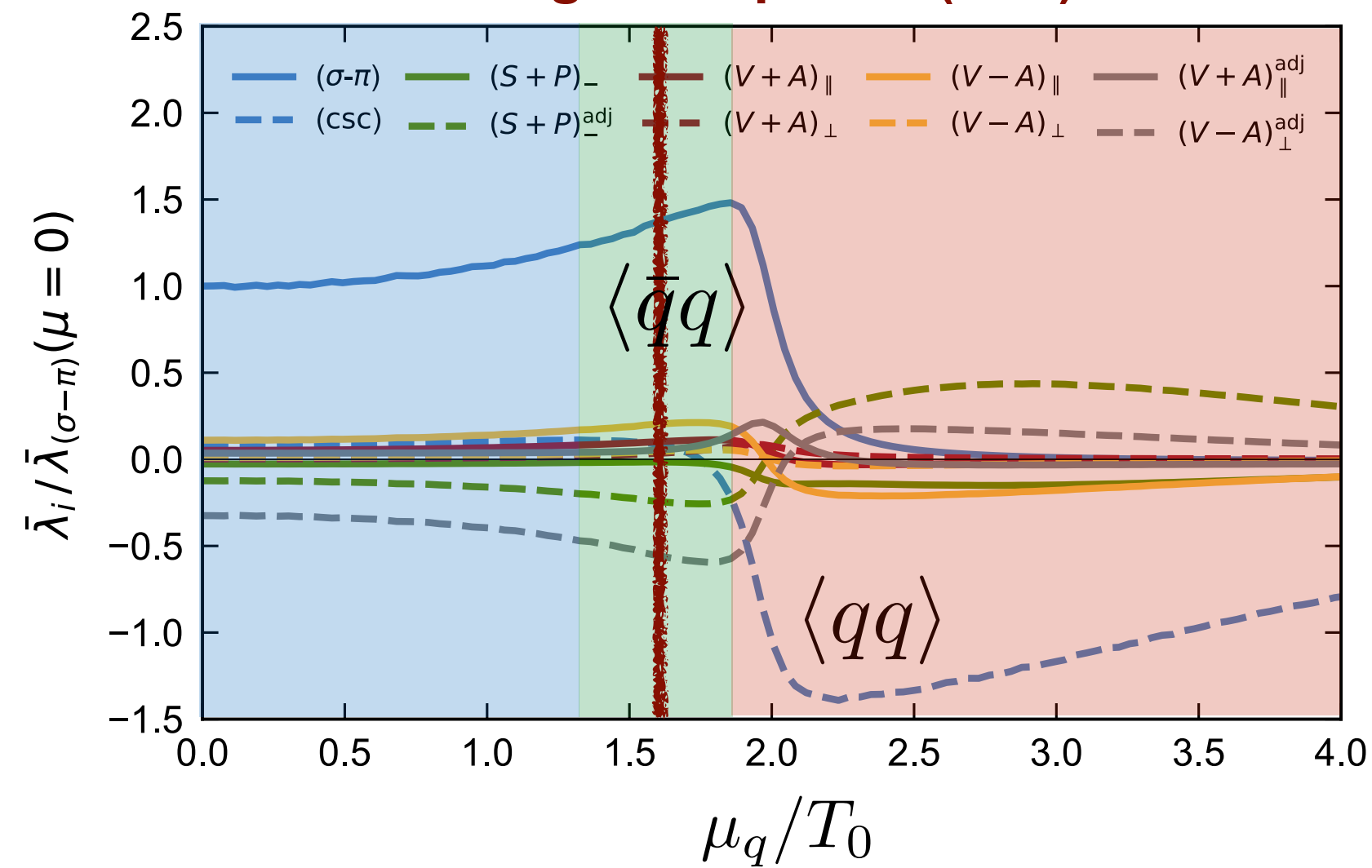
Checks and balances

Emergent diquarks



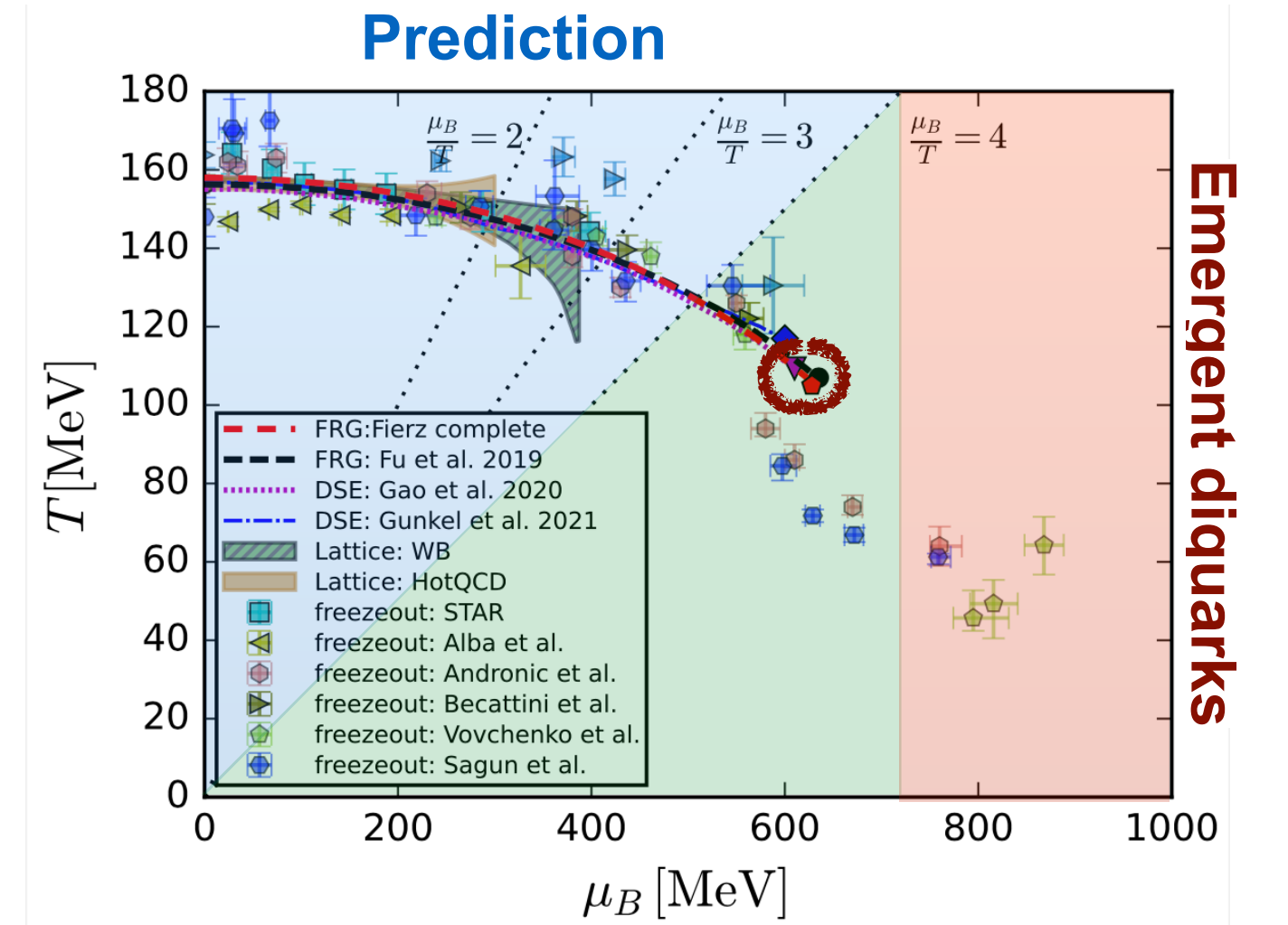
Regime of quantitative reliability
of
current best truncation

Emergent diquarks (fRG)



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

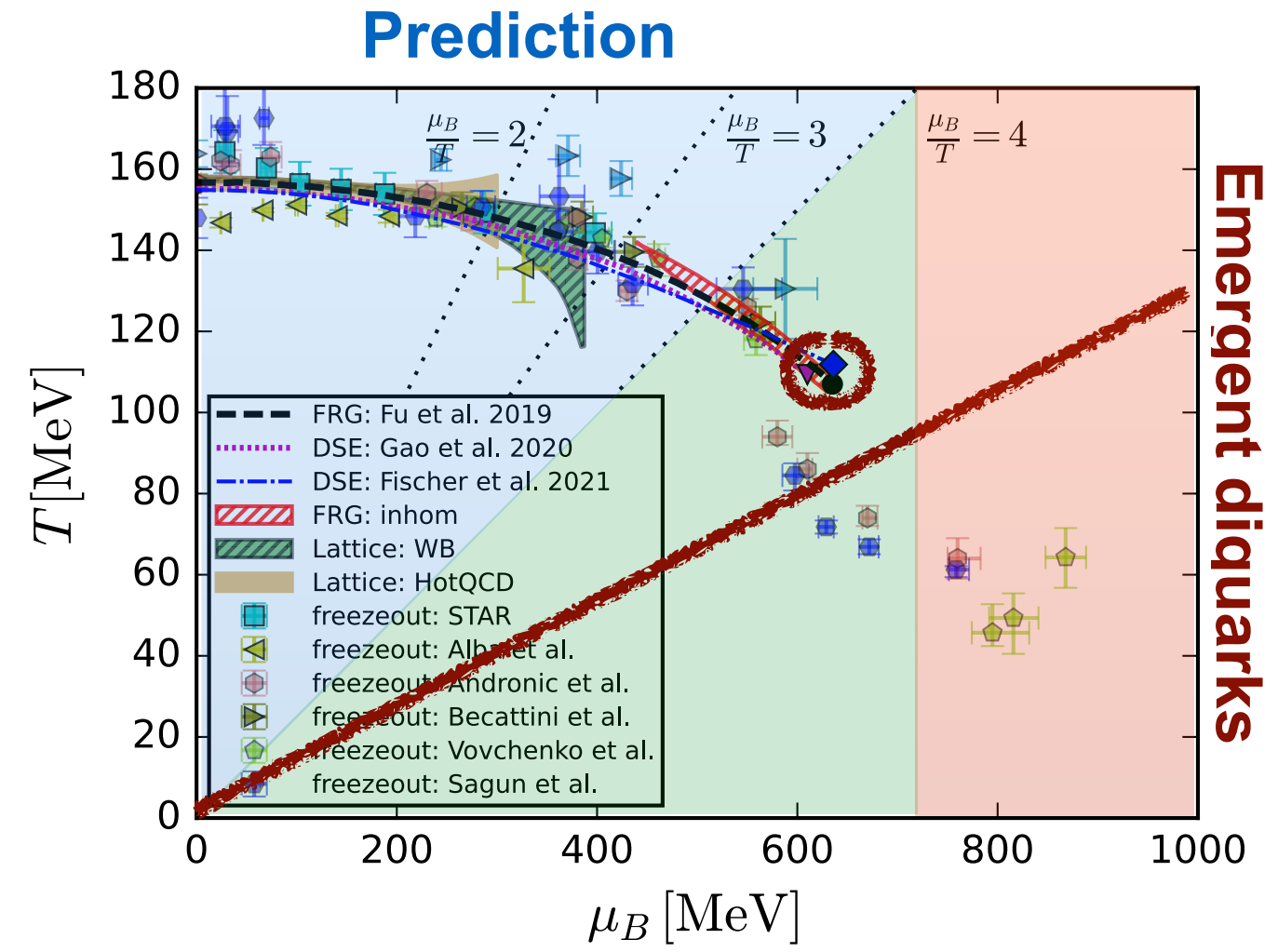
Fierz-complete phase structure



Wang et al, in preparation

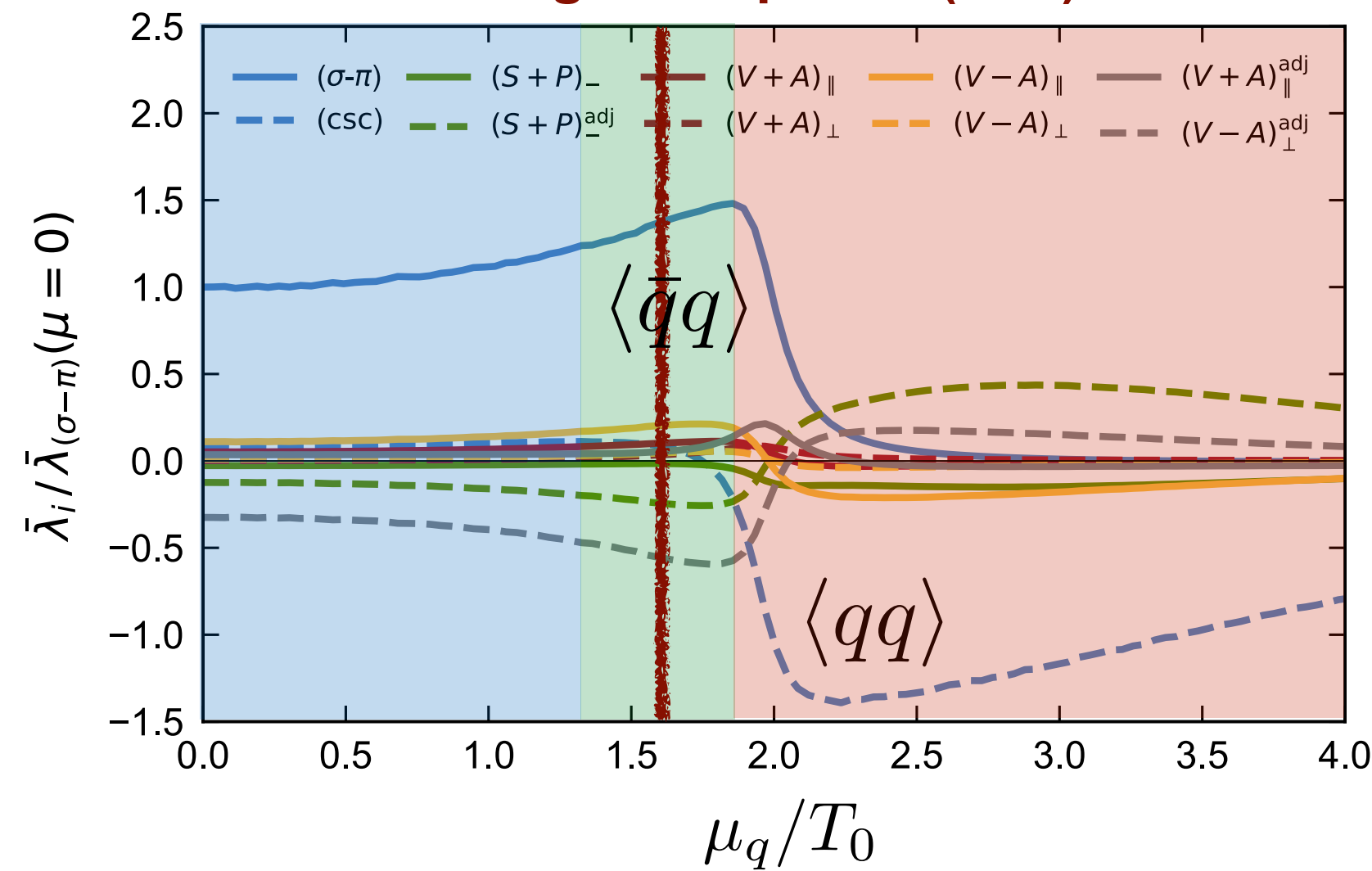
Checks and balances

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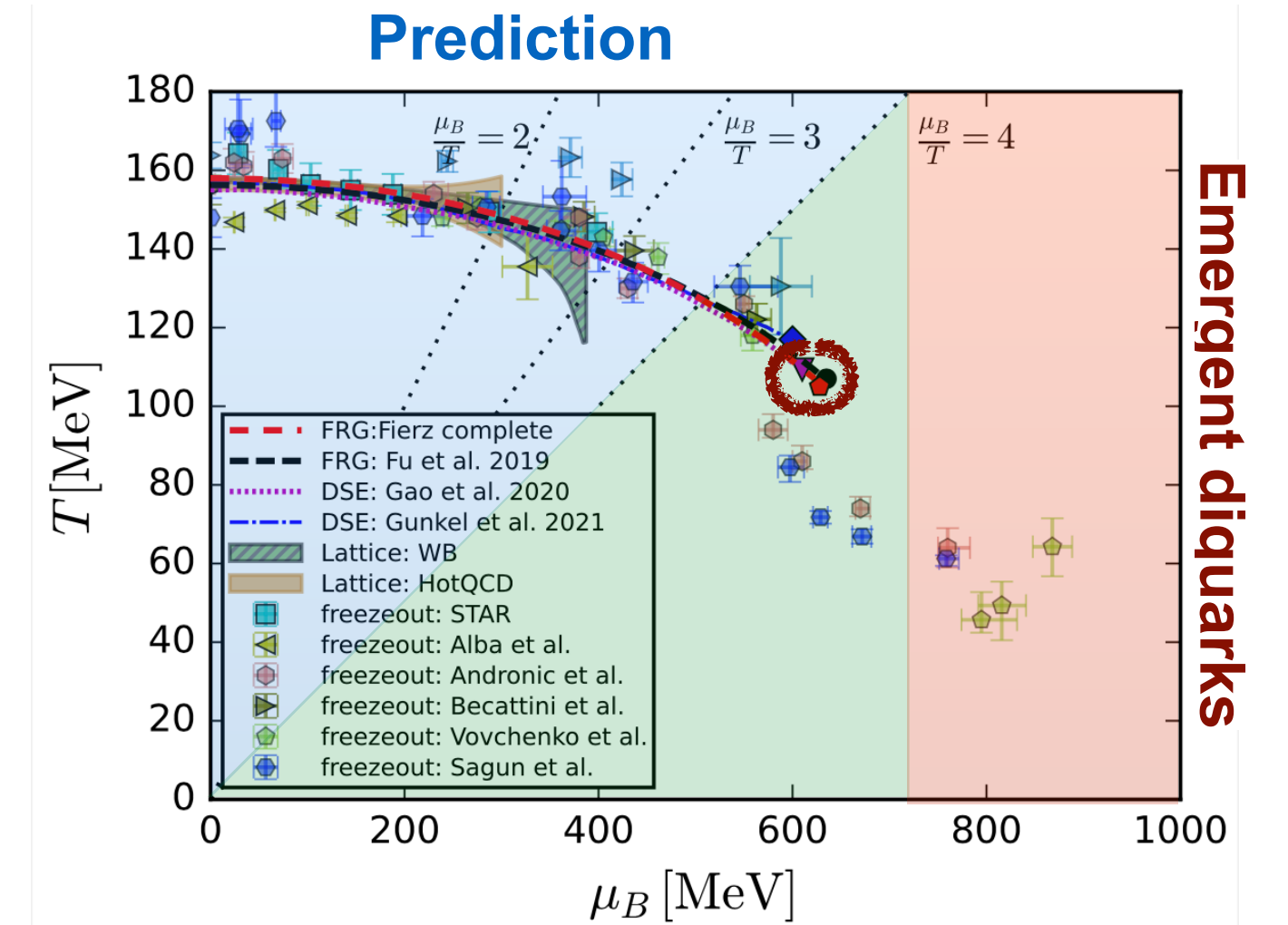


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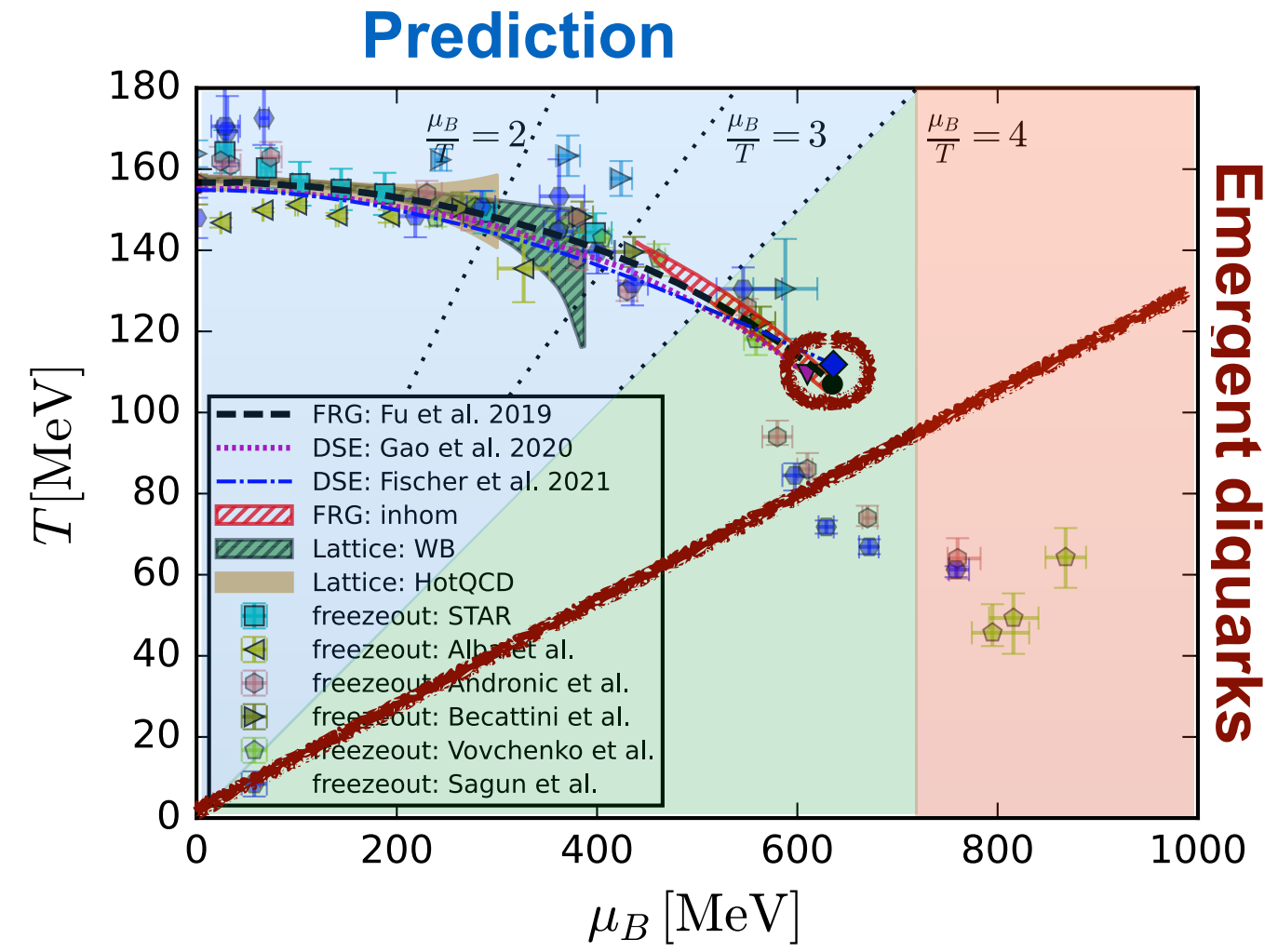


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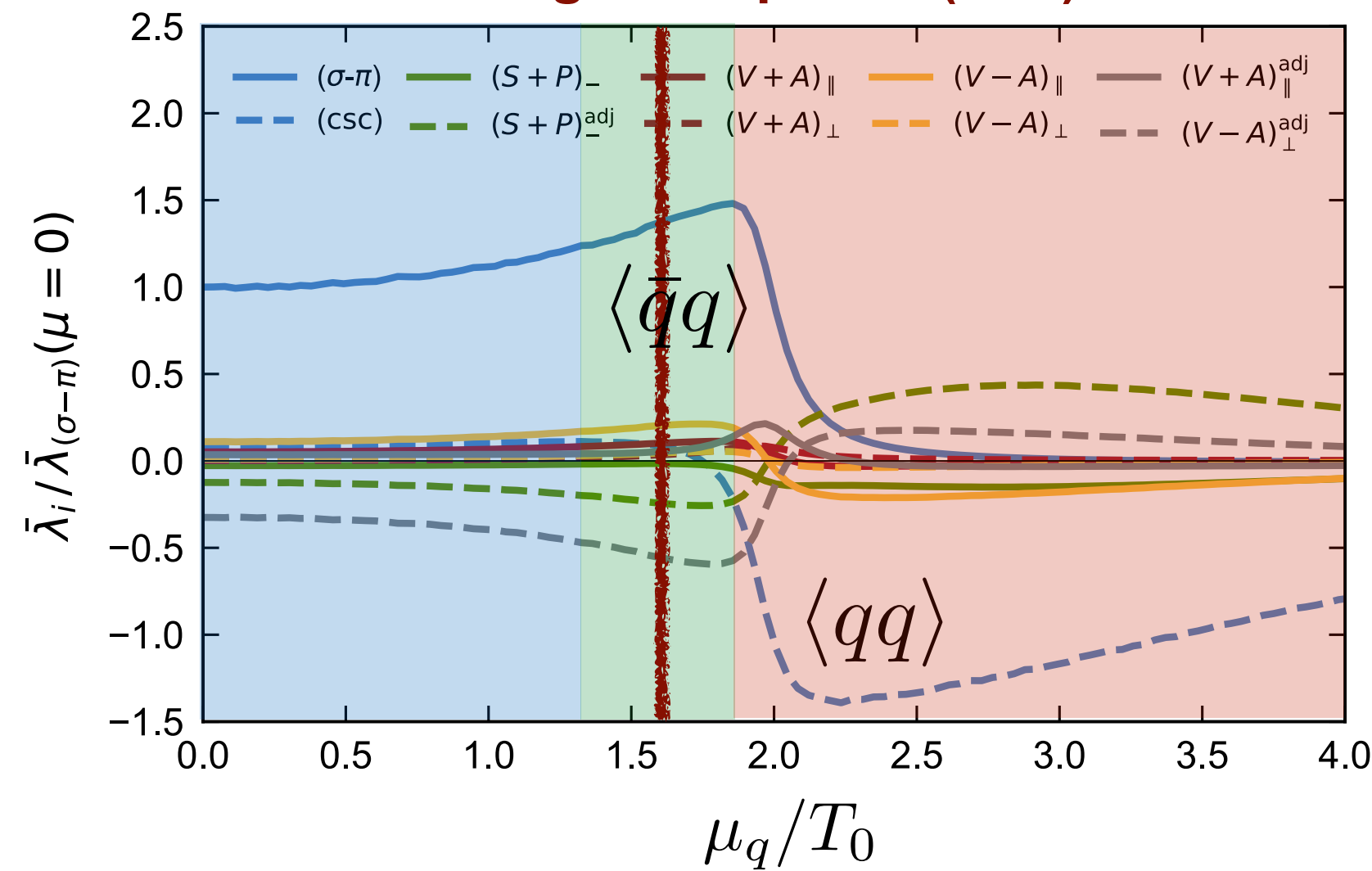
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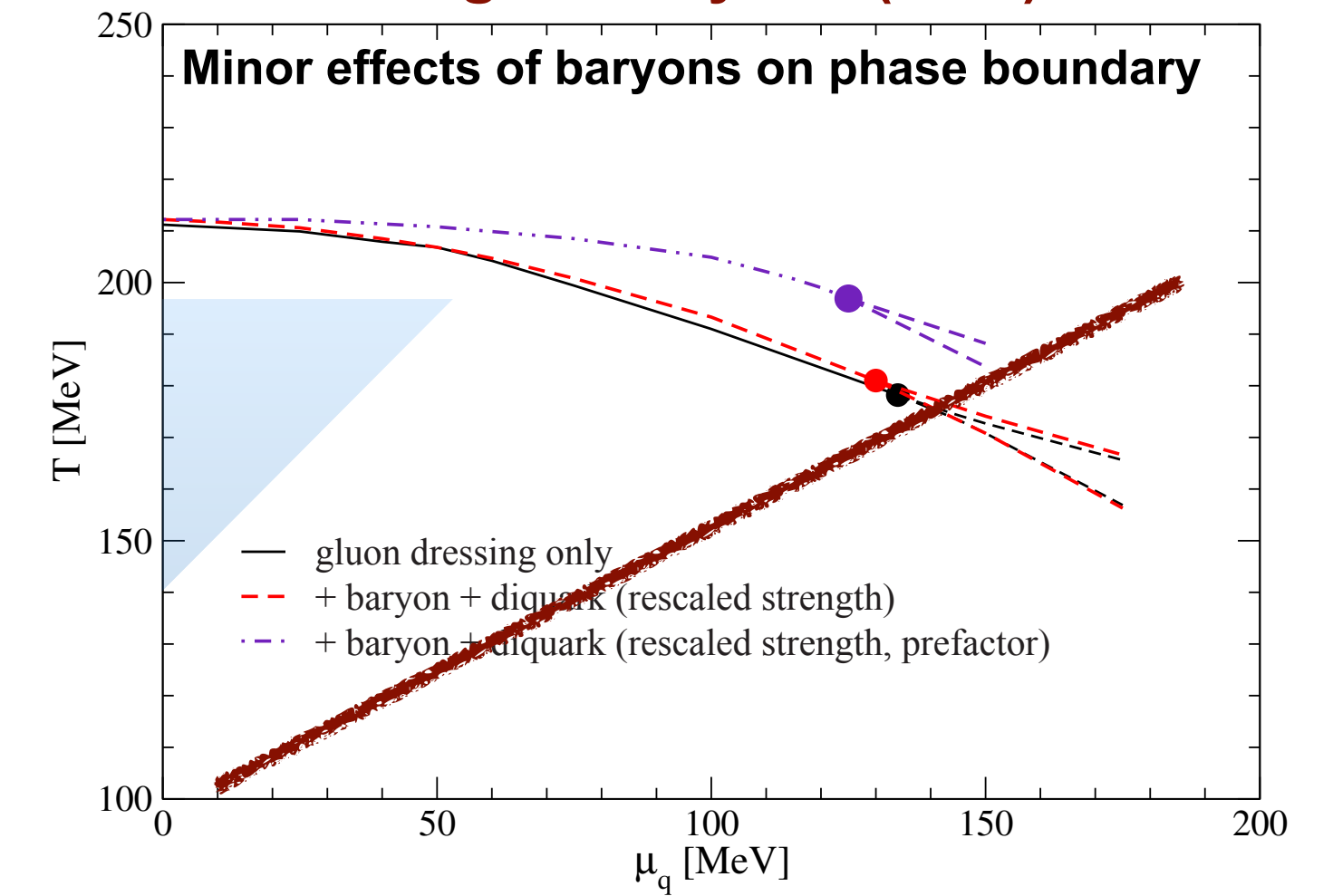


Regime of quantitative reliability
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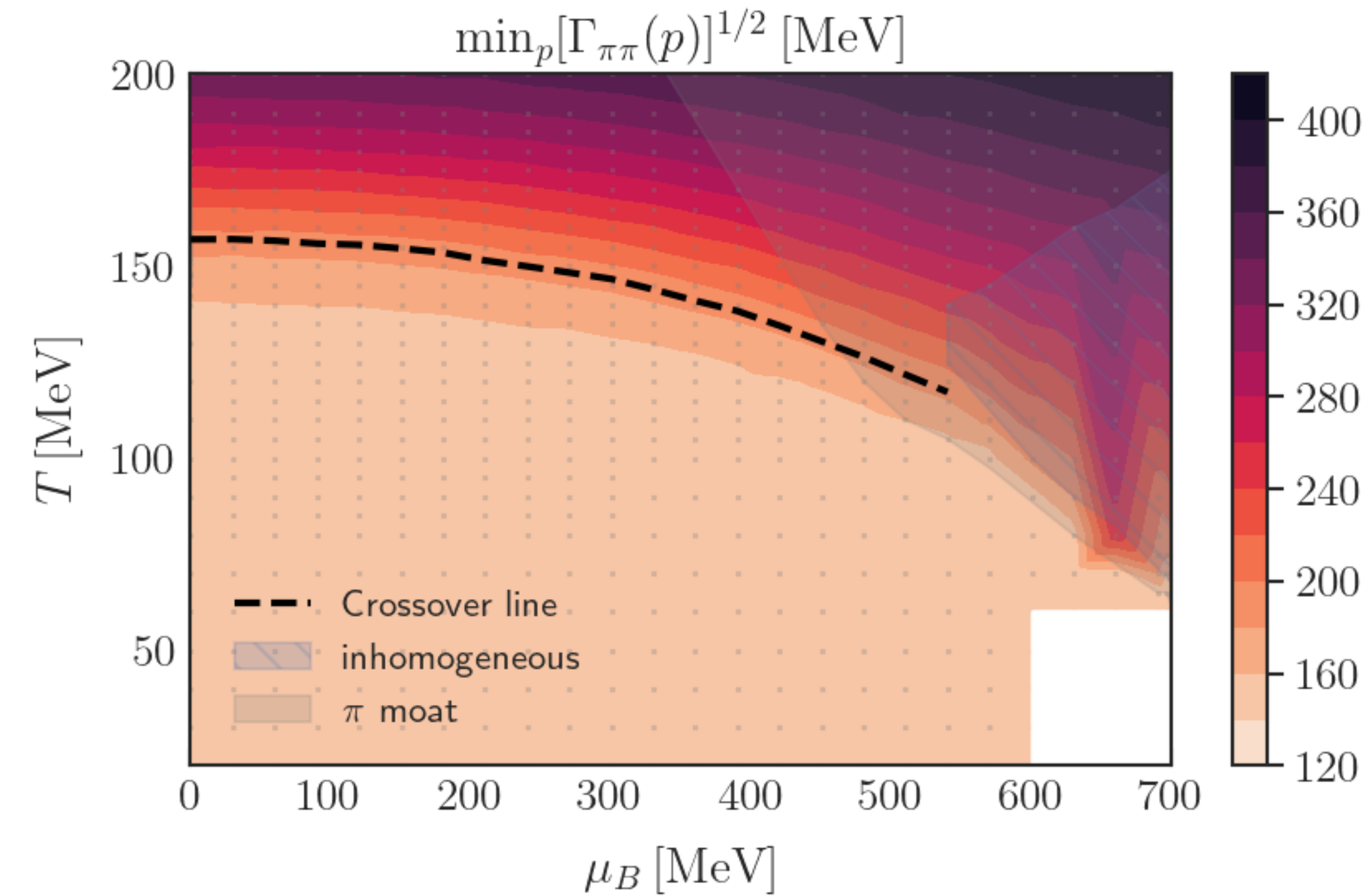


Emergent baryons (DSE)

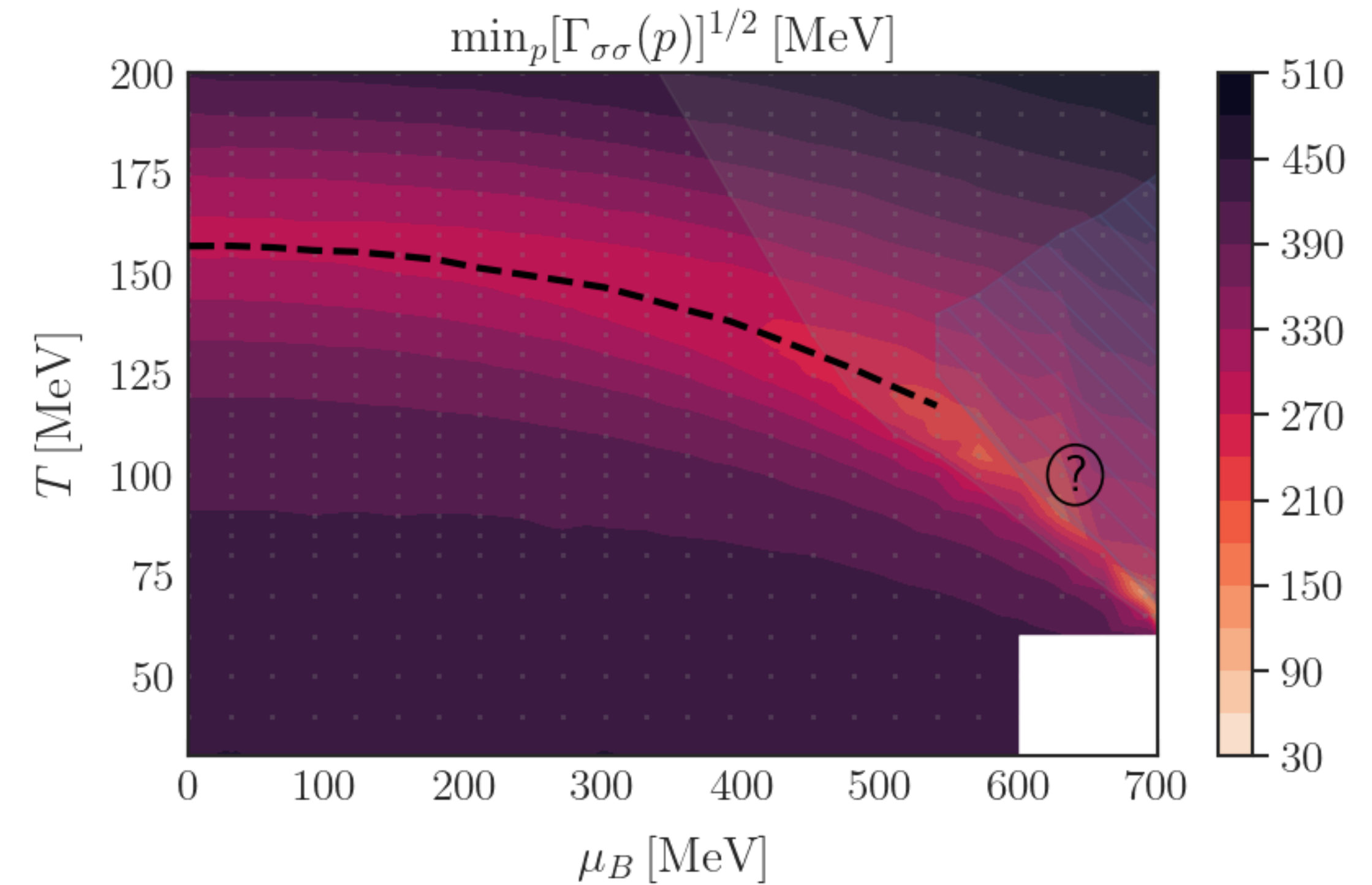


The QCD Moat

Pion ‘minimum’ mass



Scalar mode ‘minimum’ mass

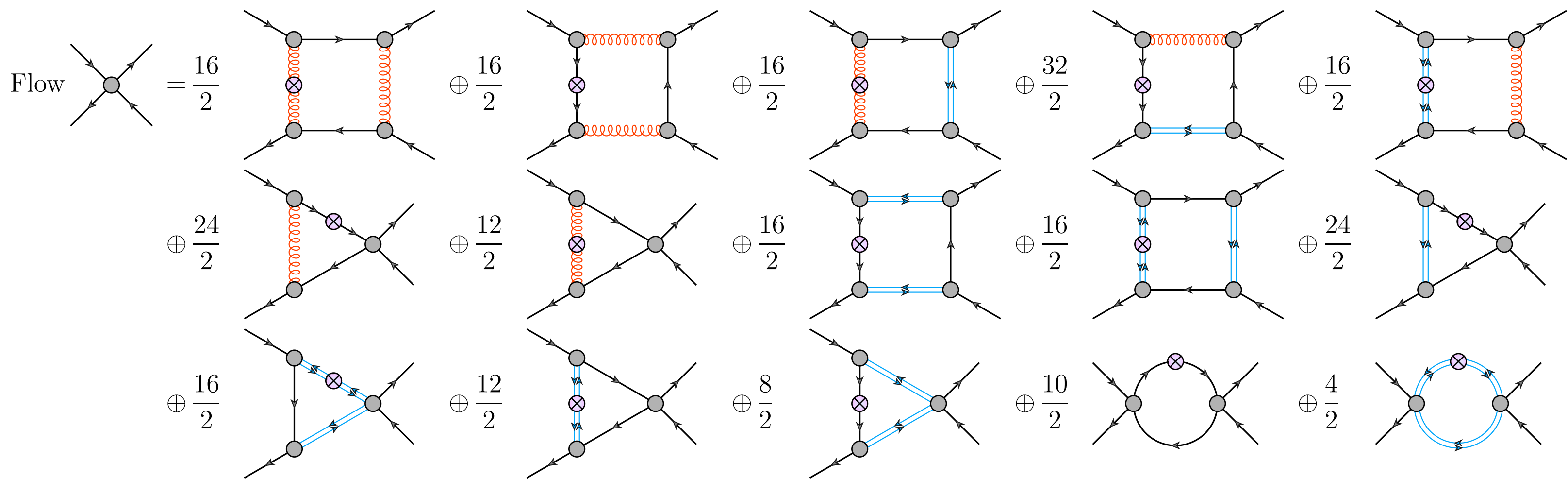
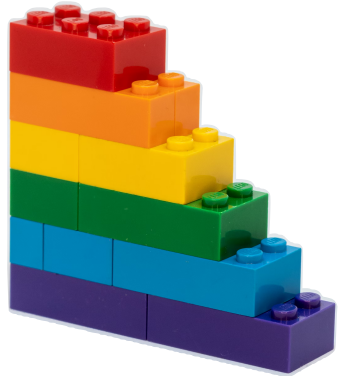


preliminary

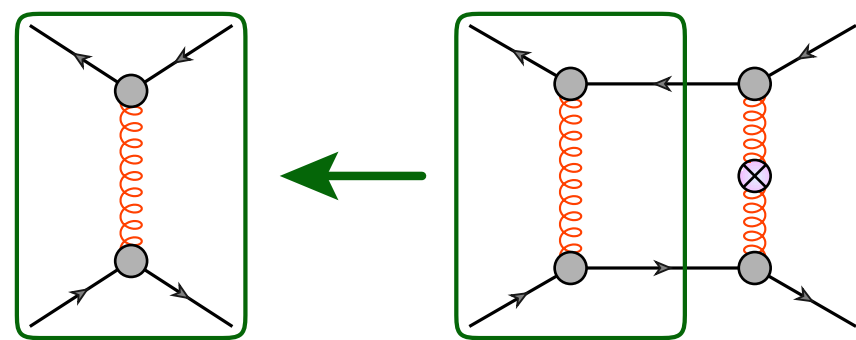
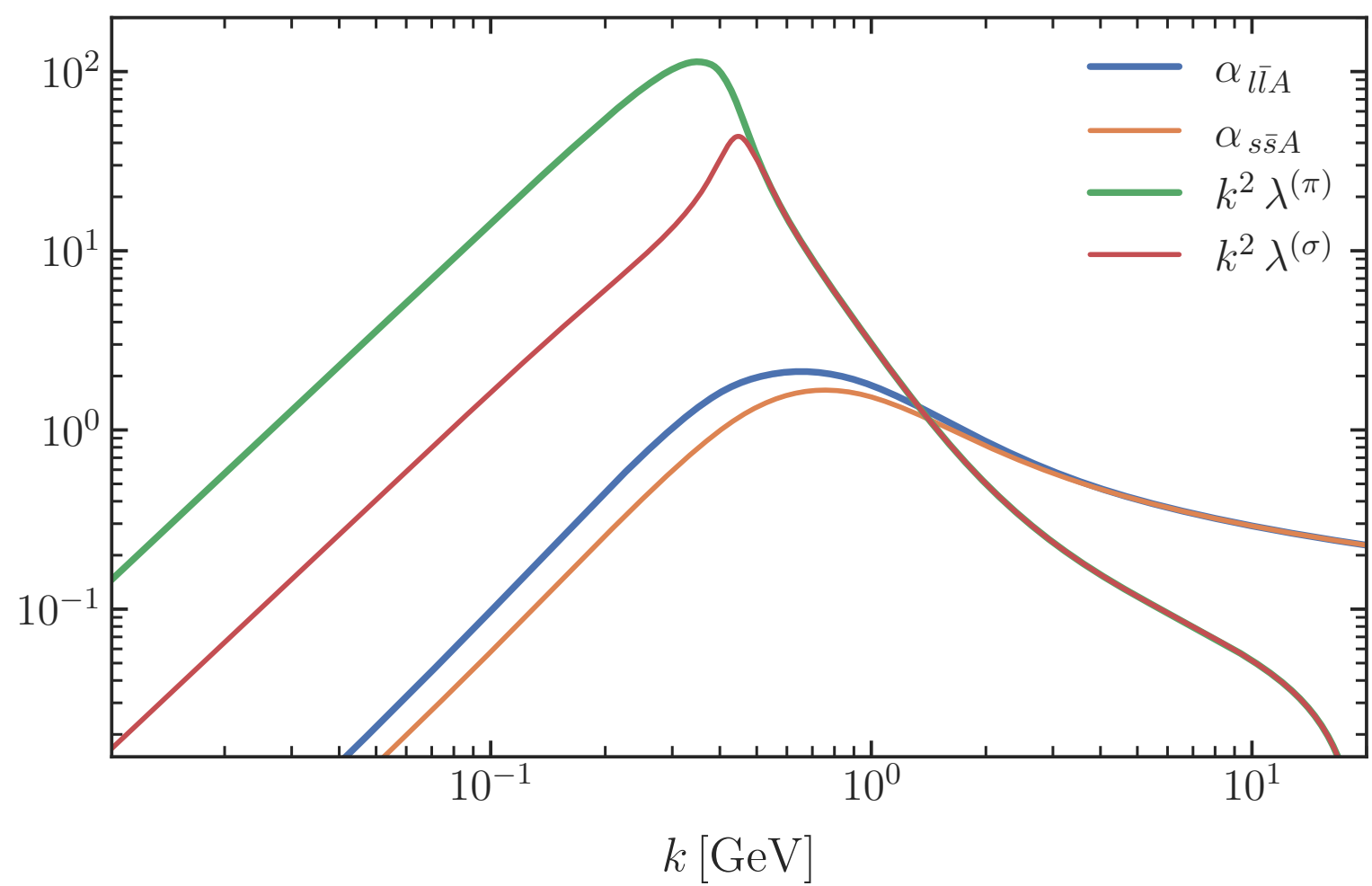
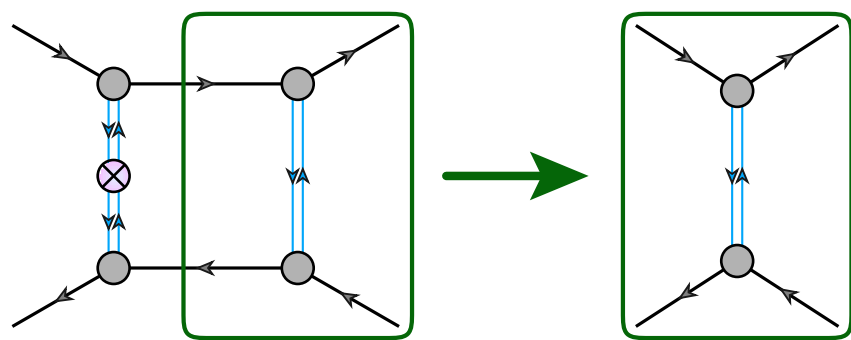
Supplementary material

Phase structure from functional QCD: how to

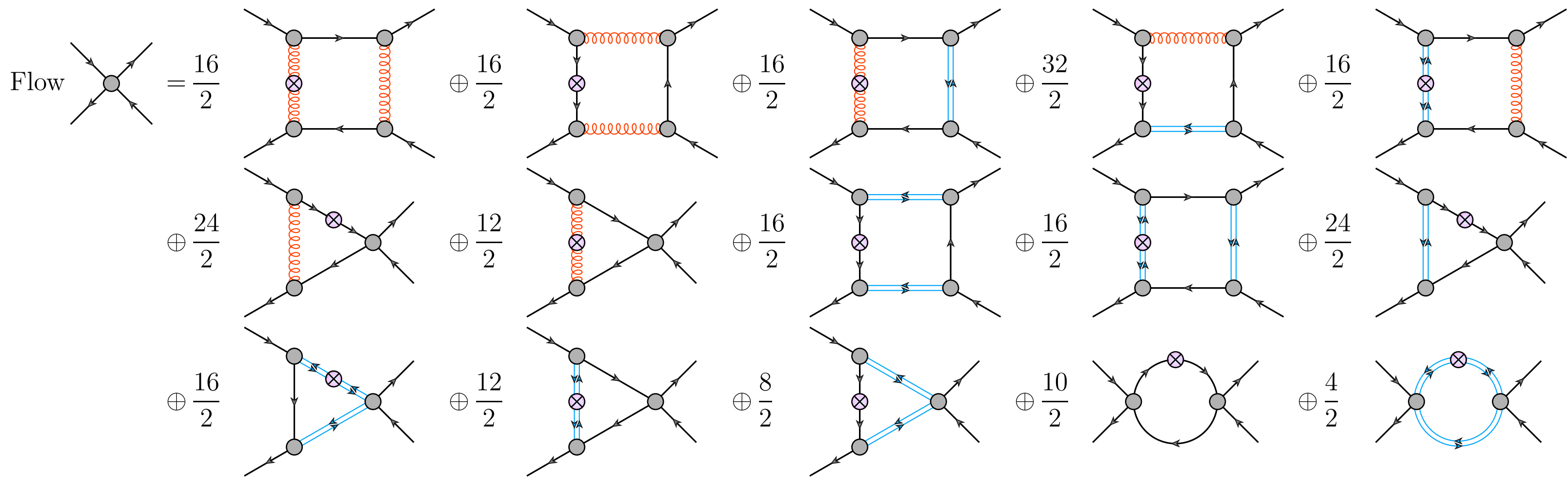
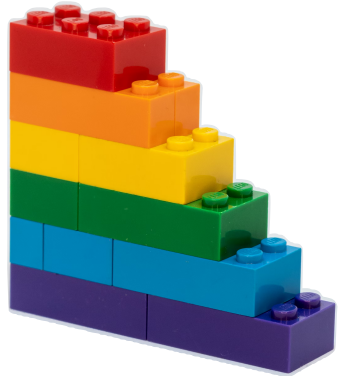
How to: systematic error estimates & the LEGO[®] principle



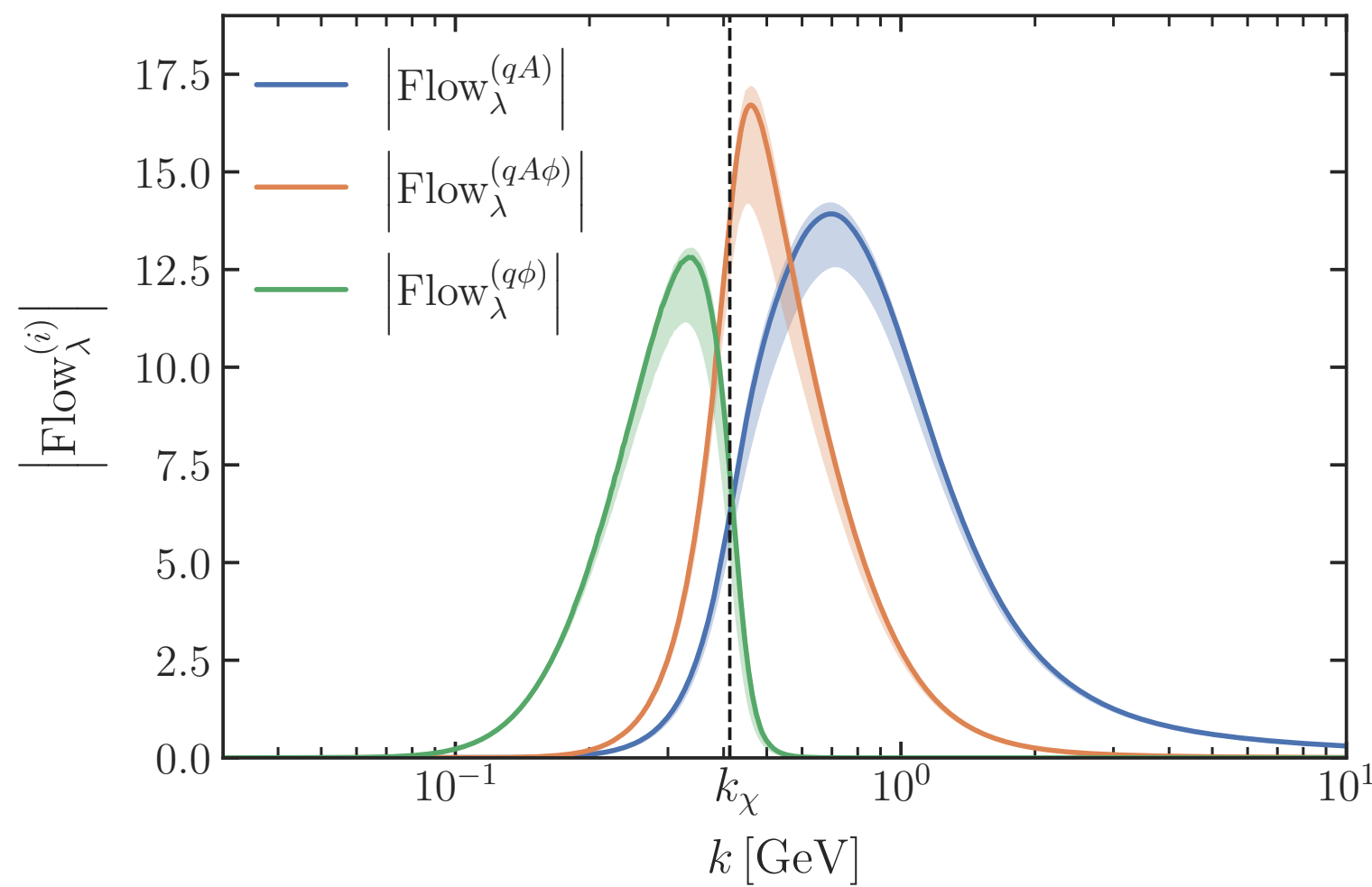
Example: 4-quark scattering vertex



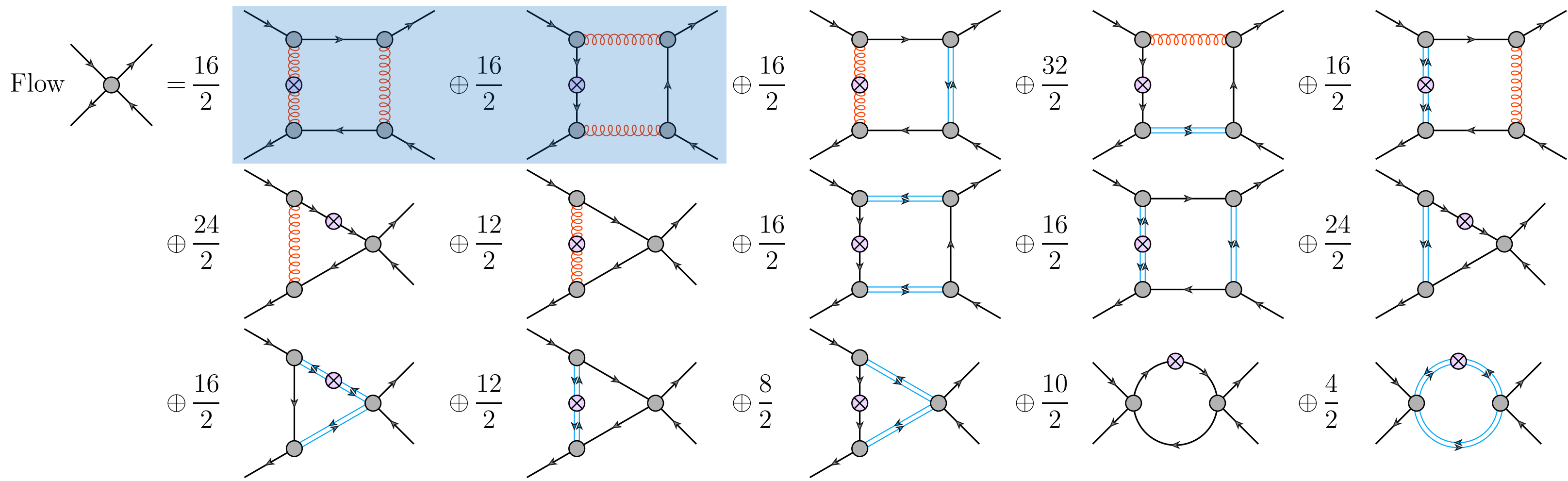
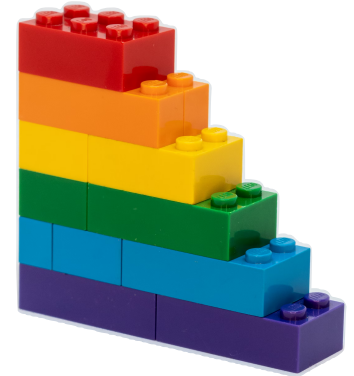
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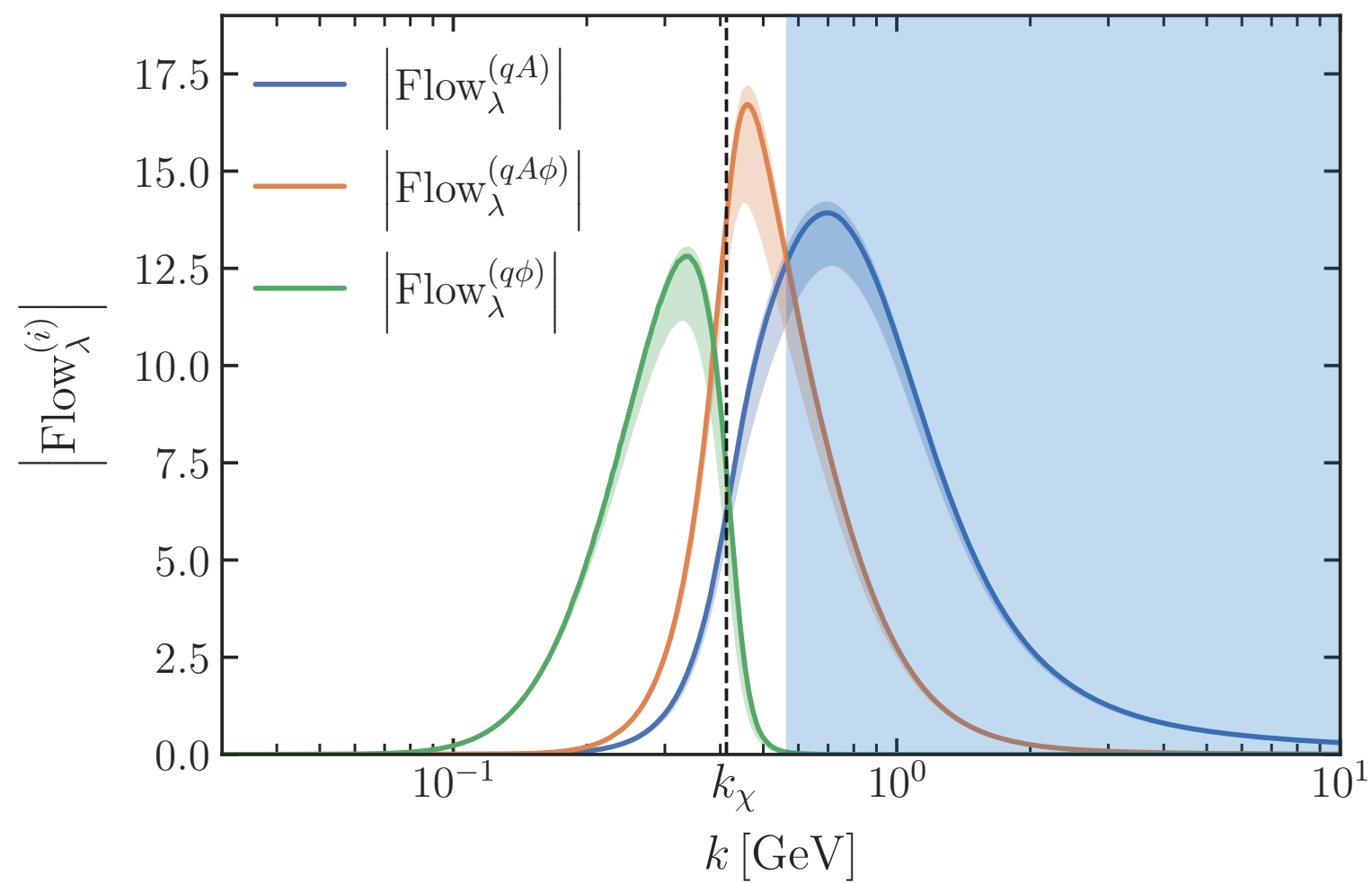
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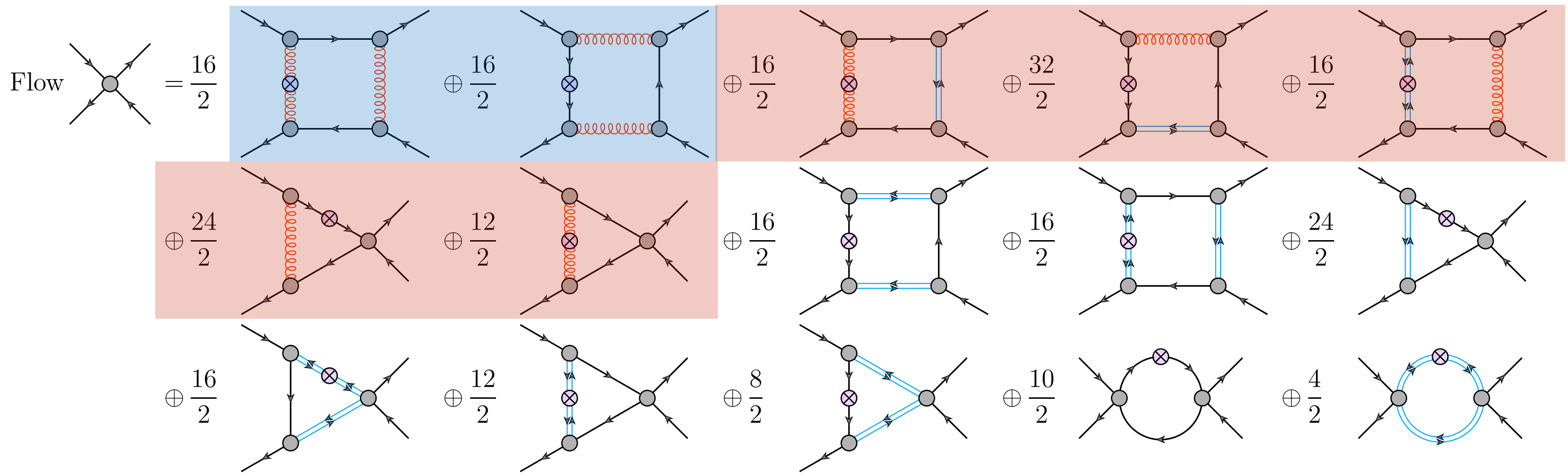
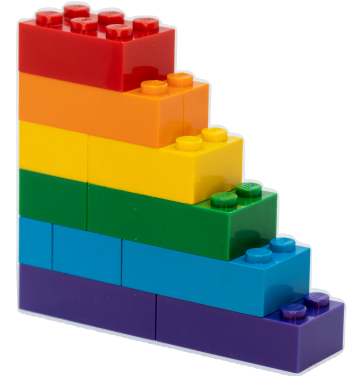
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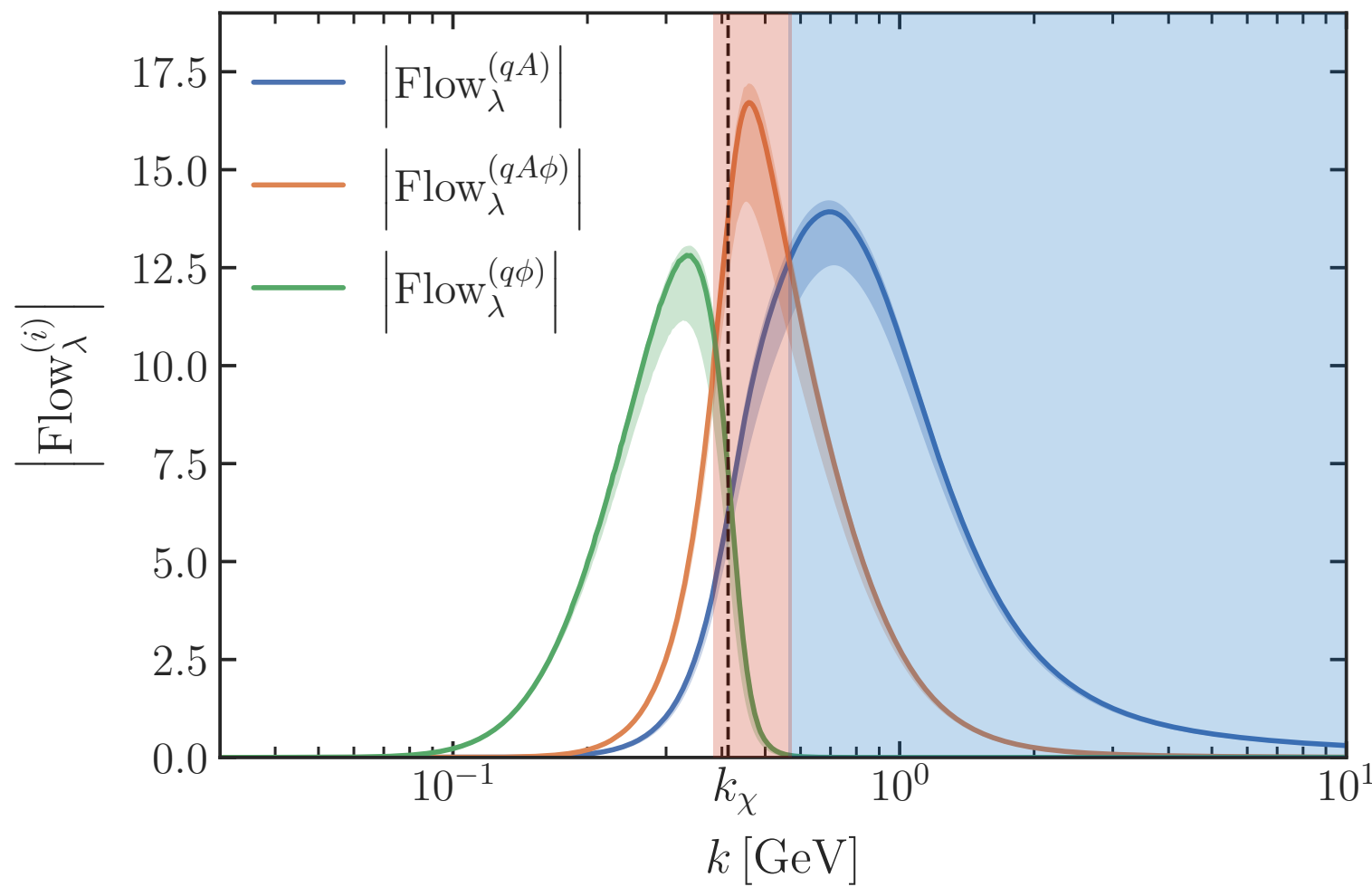
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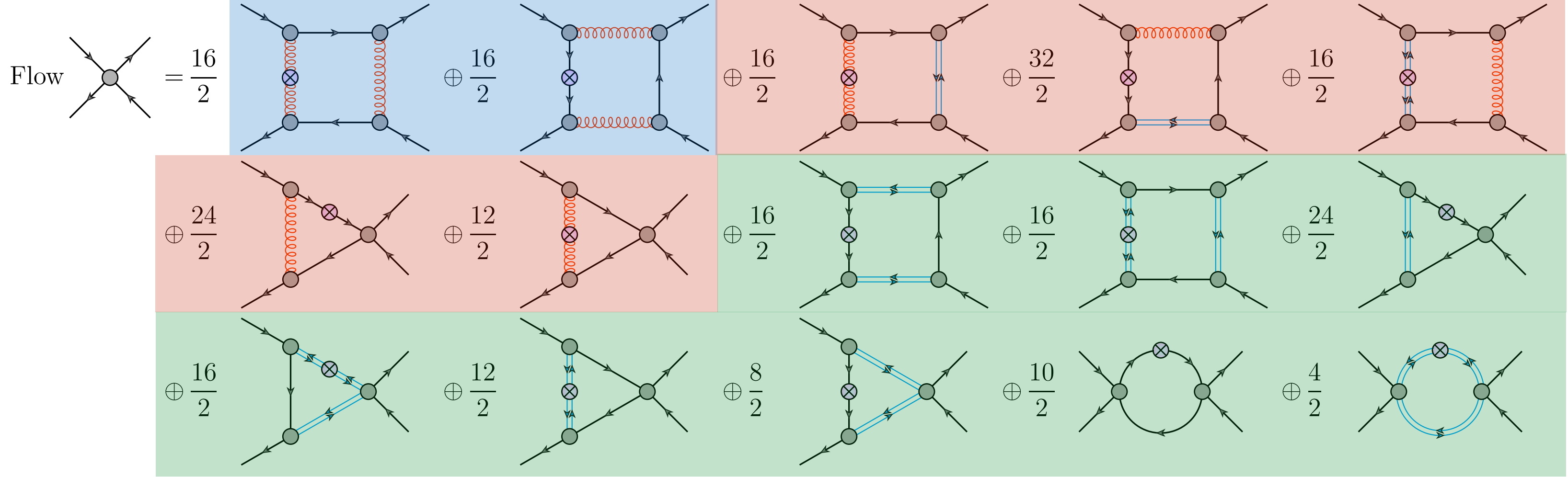
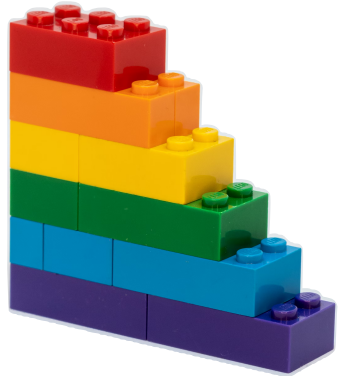
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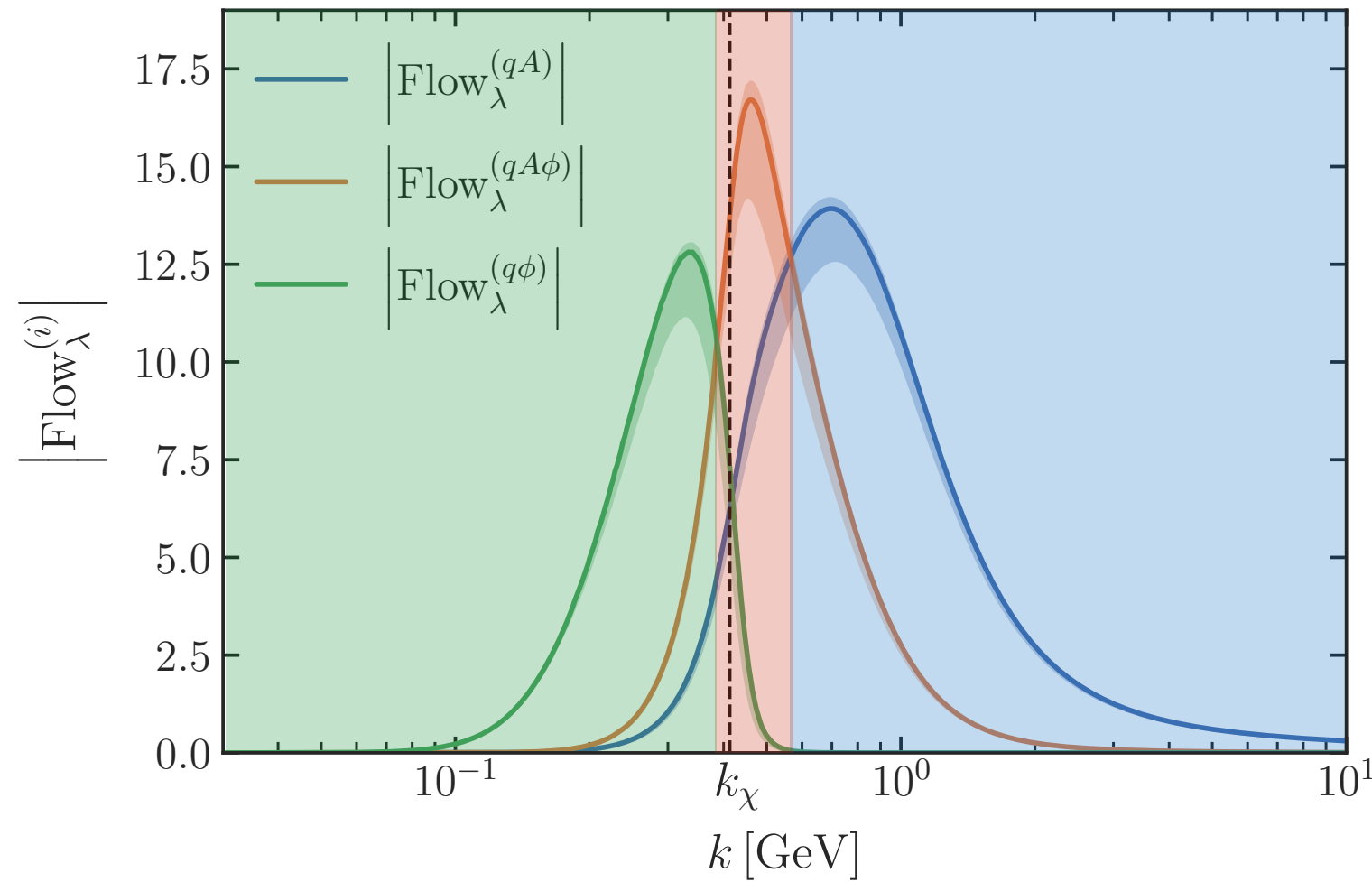
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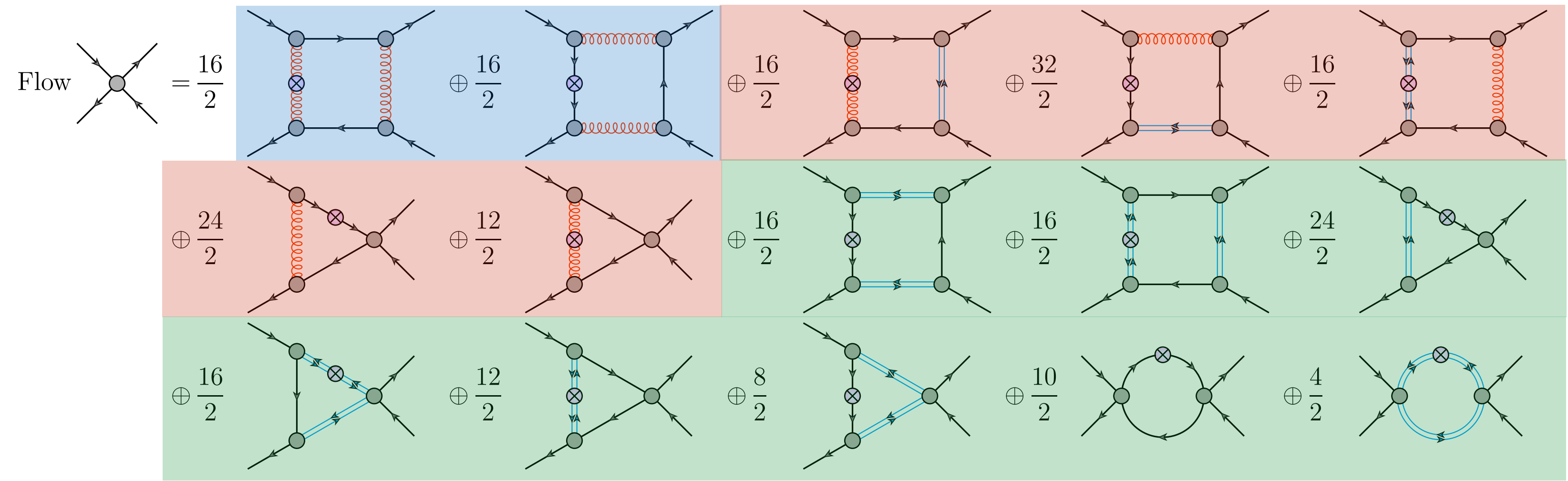
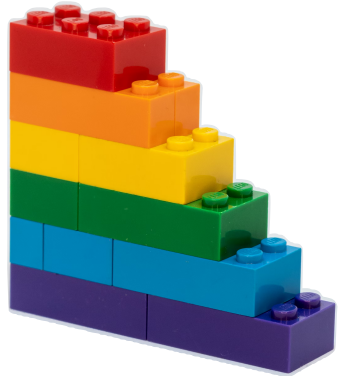
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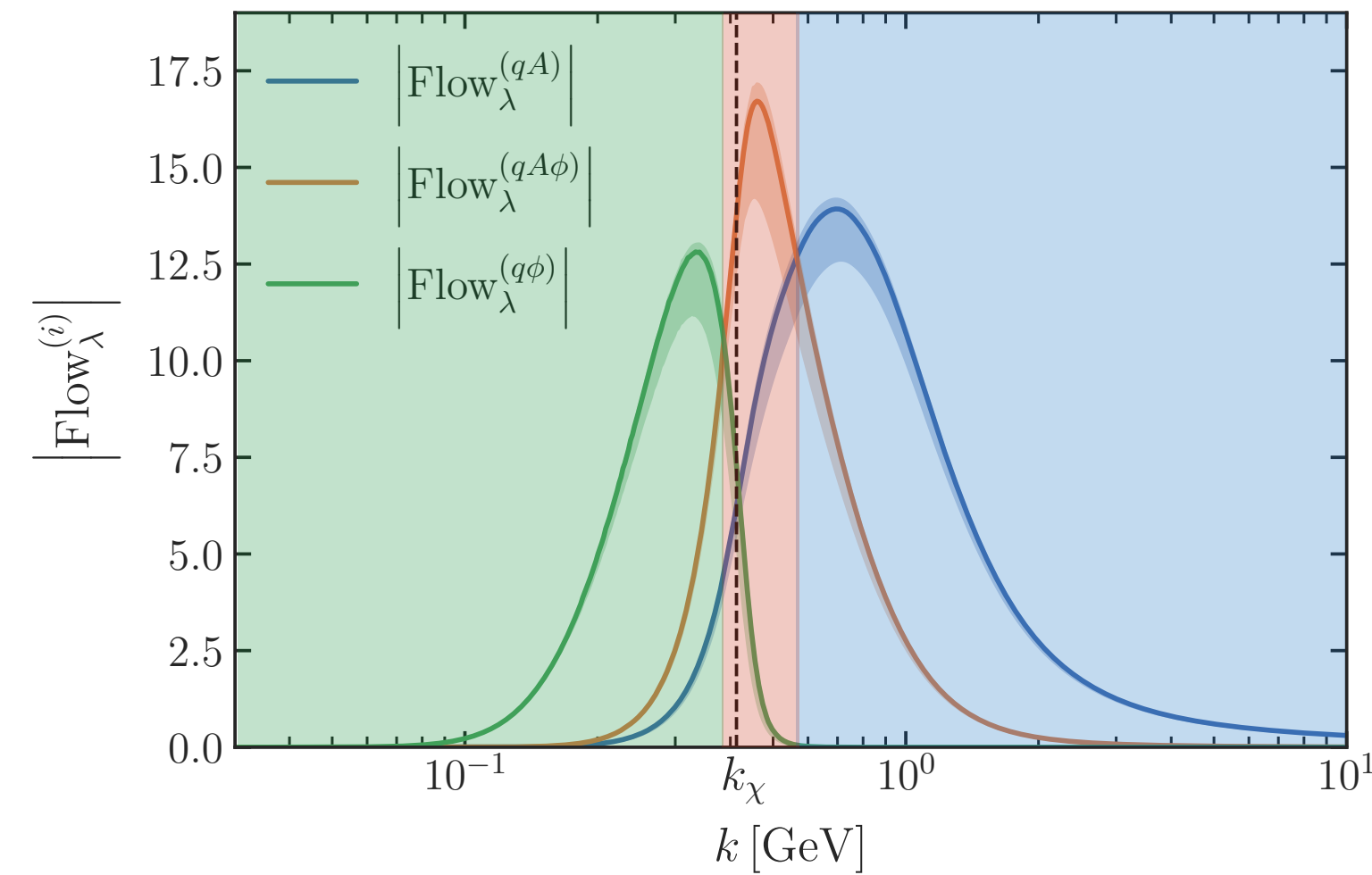
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How to: systematic error estimates & the LEGO[®] principle



The unreasonable effectiveness of low energy effective theories



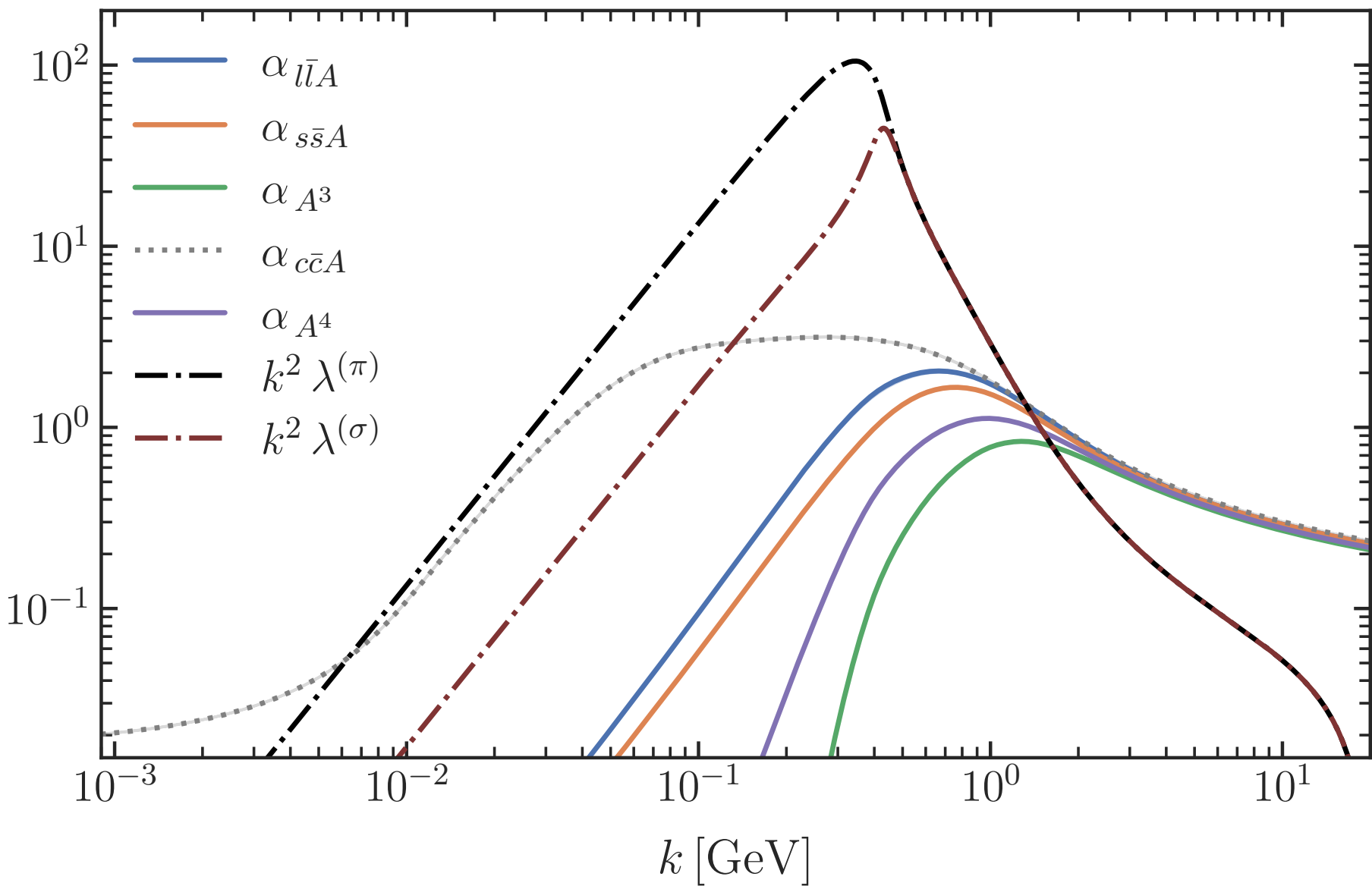
Access and combined use of
error estimates
from functional QCD & LEFTs

The unreasonable effectiveness of low energy effective theories

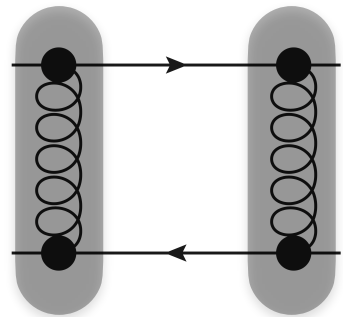
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dashed loop} - \text{solid loop} + \frac{1}{2} \text{blue loop} \right)$$

fQCD

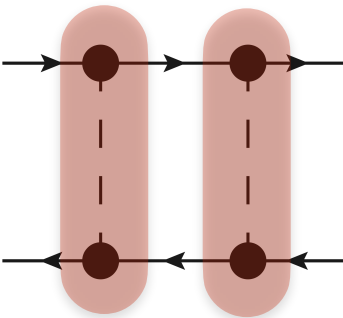
Sequential decoupling of gluon, quark, sigma, pion fluctuations



$$\frac{g_{lAl}^2}{g_{\bar{s}As}^2}$$



$$k^2 \lambda^{(\pi, \sigma)}$$



Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

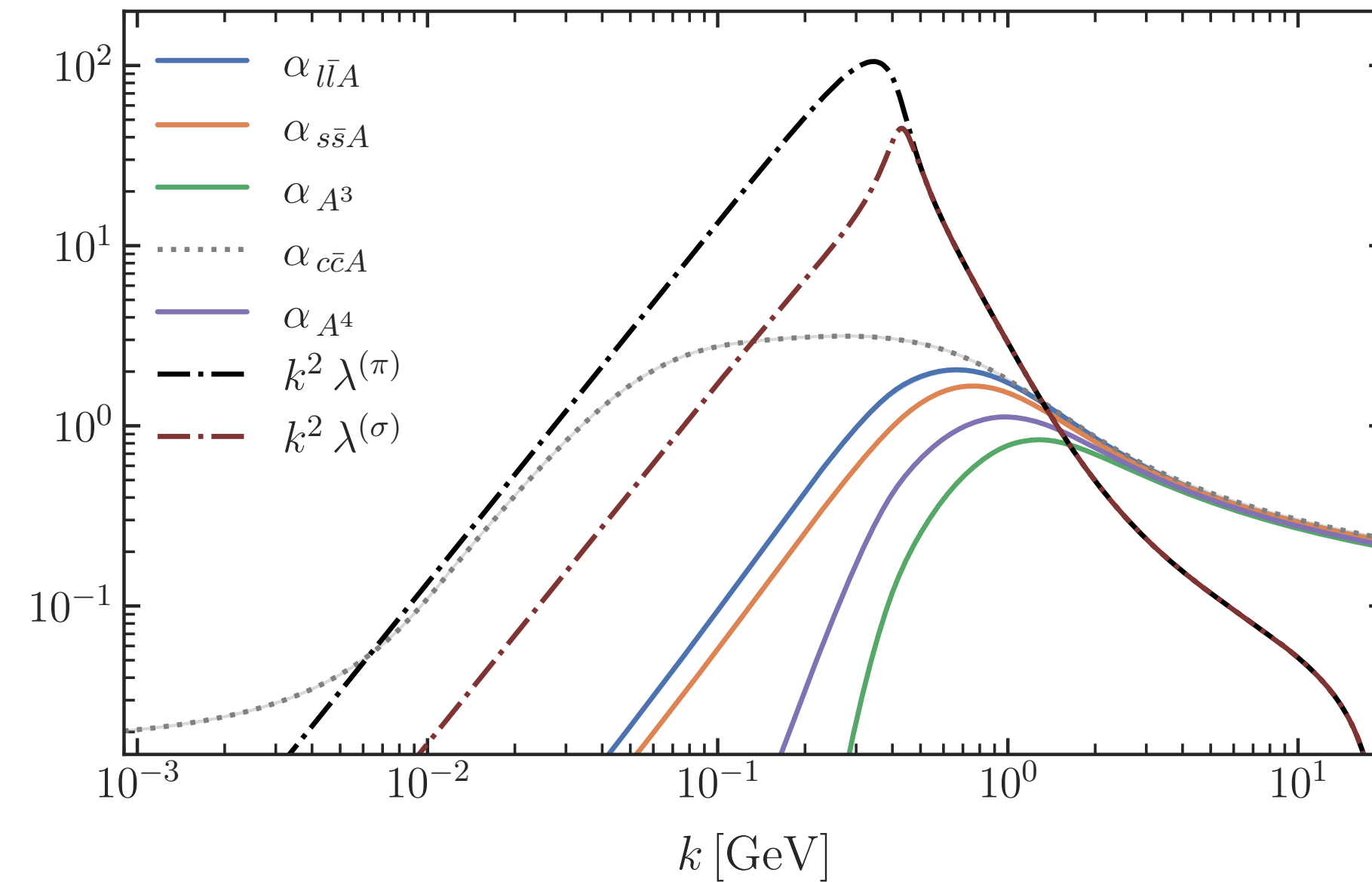
Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

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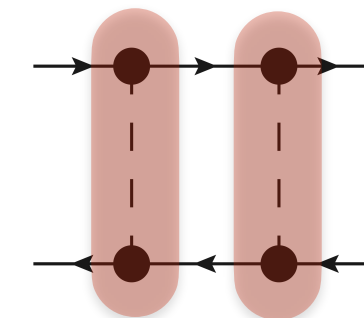
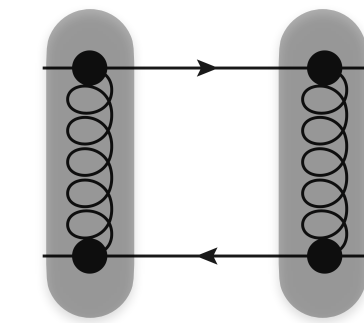
fQCD

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$$k^2 \lambda^{(\pi, \sigma)}$$



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Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

PQM-model

$$\frac{1}{2} \text{orange loop} - \text{dashed loop} + \text{solid loop} + \frac{1}{2} \text{blue loop}$$

PNJL-model

$$\frac{1}{2} \text{orange loop} - \text{dashed loop} + \text{solid loop}$$

QM-model

$$-\text{solid loop} + \frac{1}{2} \text{blue loop}$$

NJL-model

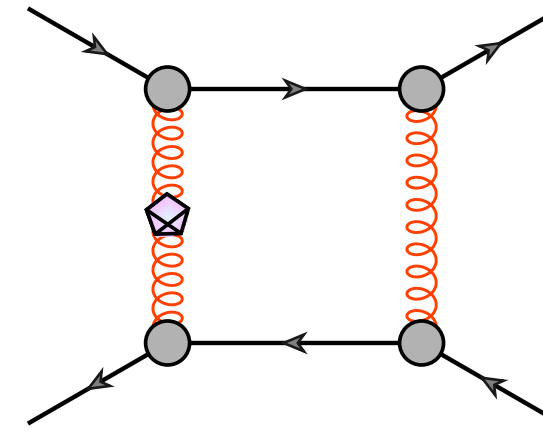
$$-\text{solid loop}$$

QCD-assisted low energy effective theories

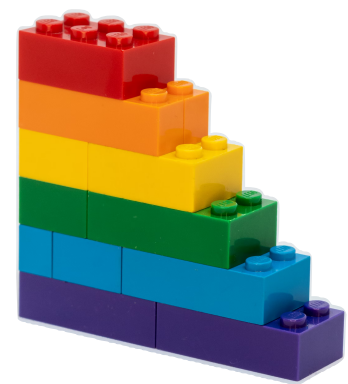
How to: systematic error estimates & the LEGO[®] principle



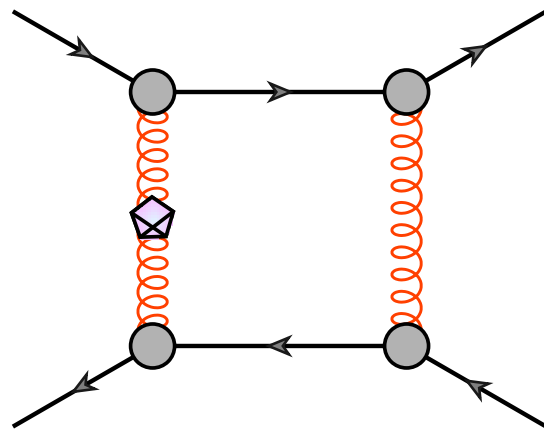
Example: Disect quark-gluon box



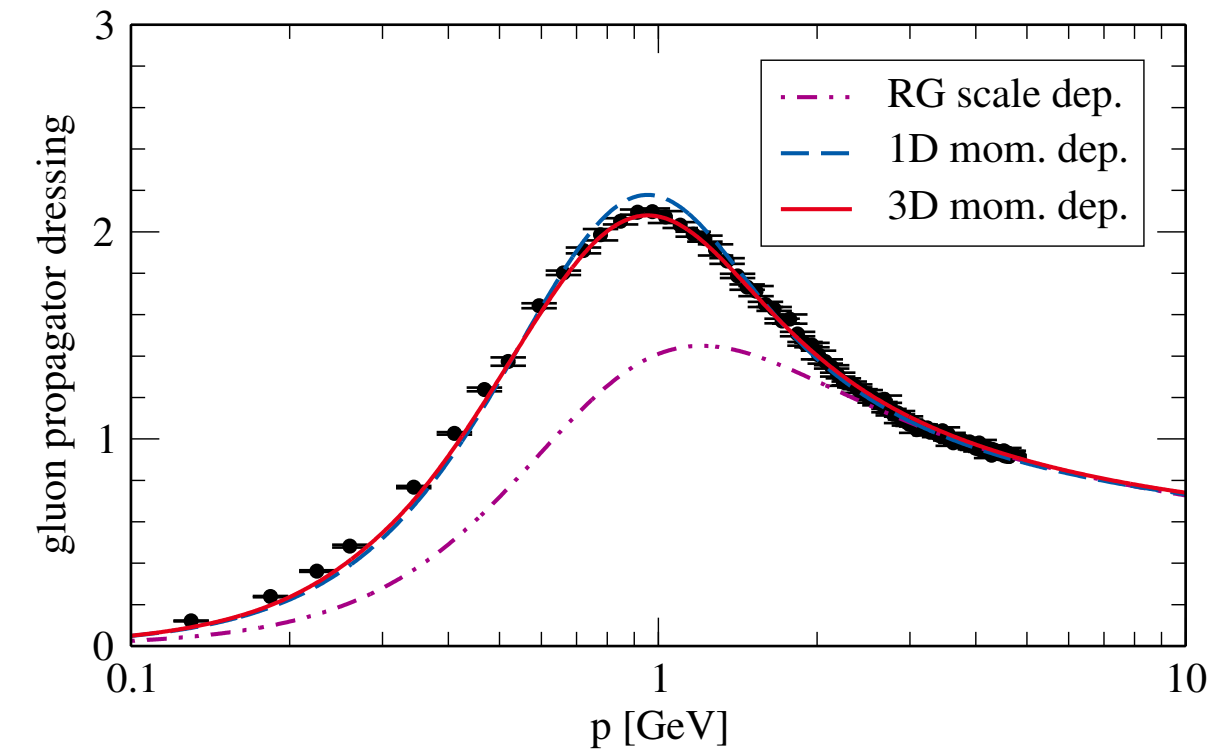
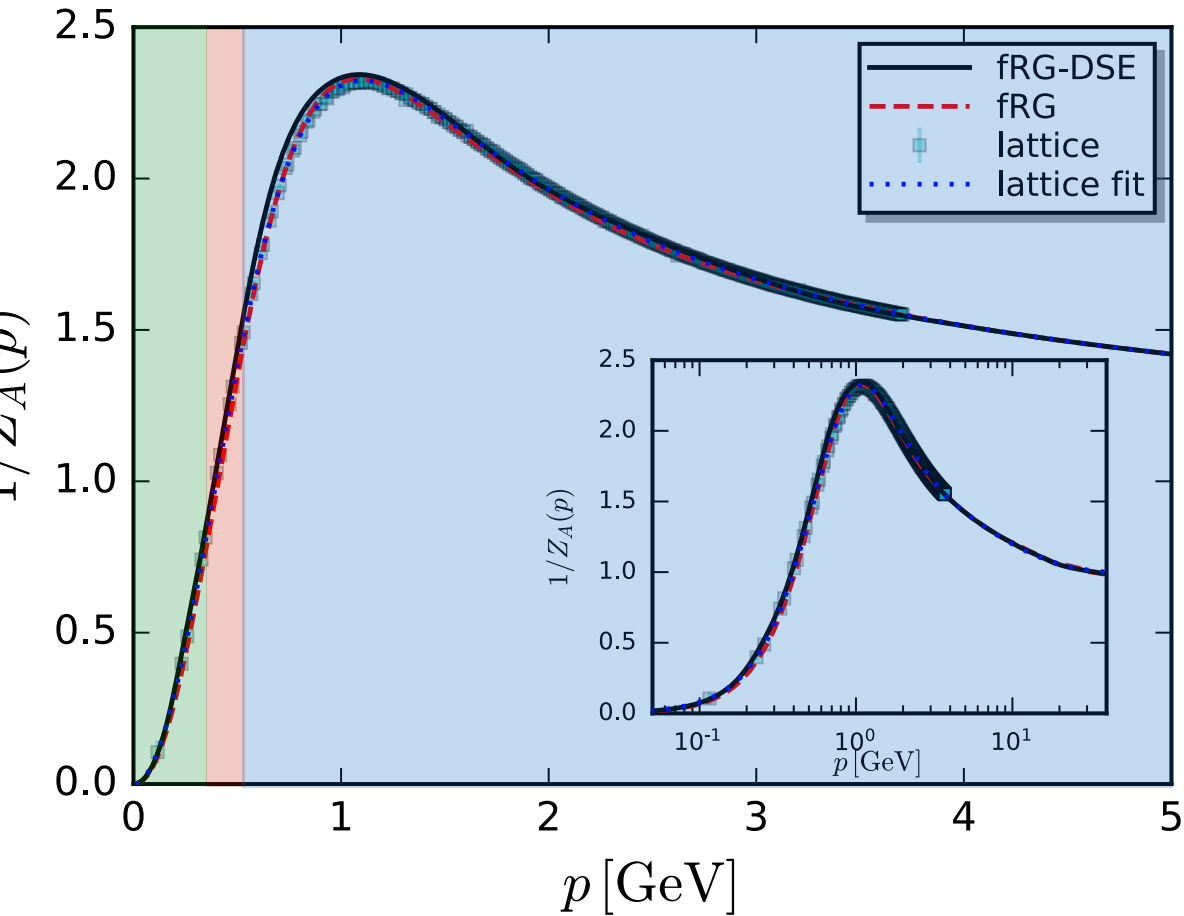
How to: systematic error estimates & the LEGO[®] principle



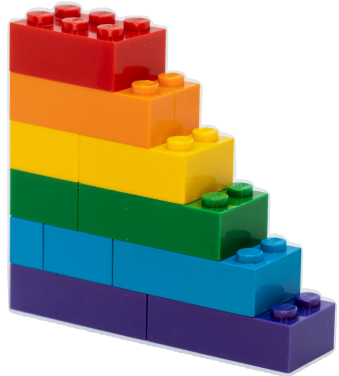
Example: Disect quark-gluon box



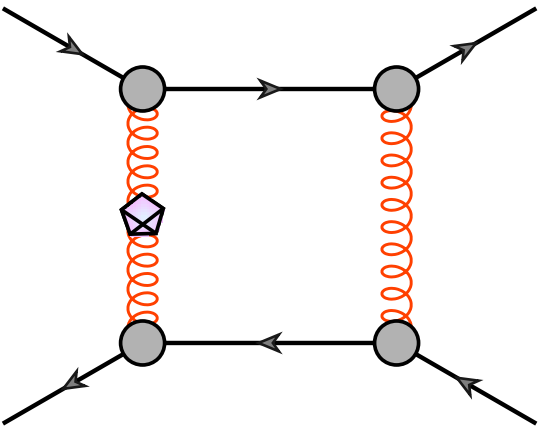
gluon two-point correlator



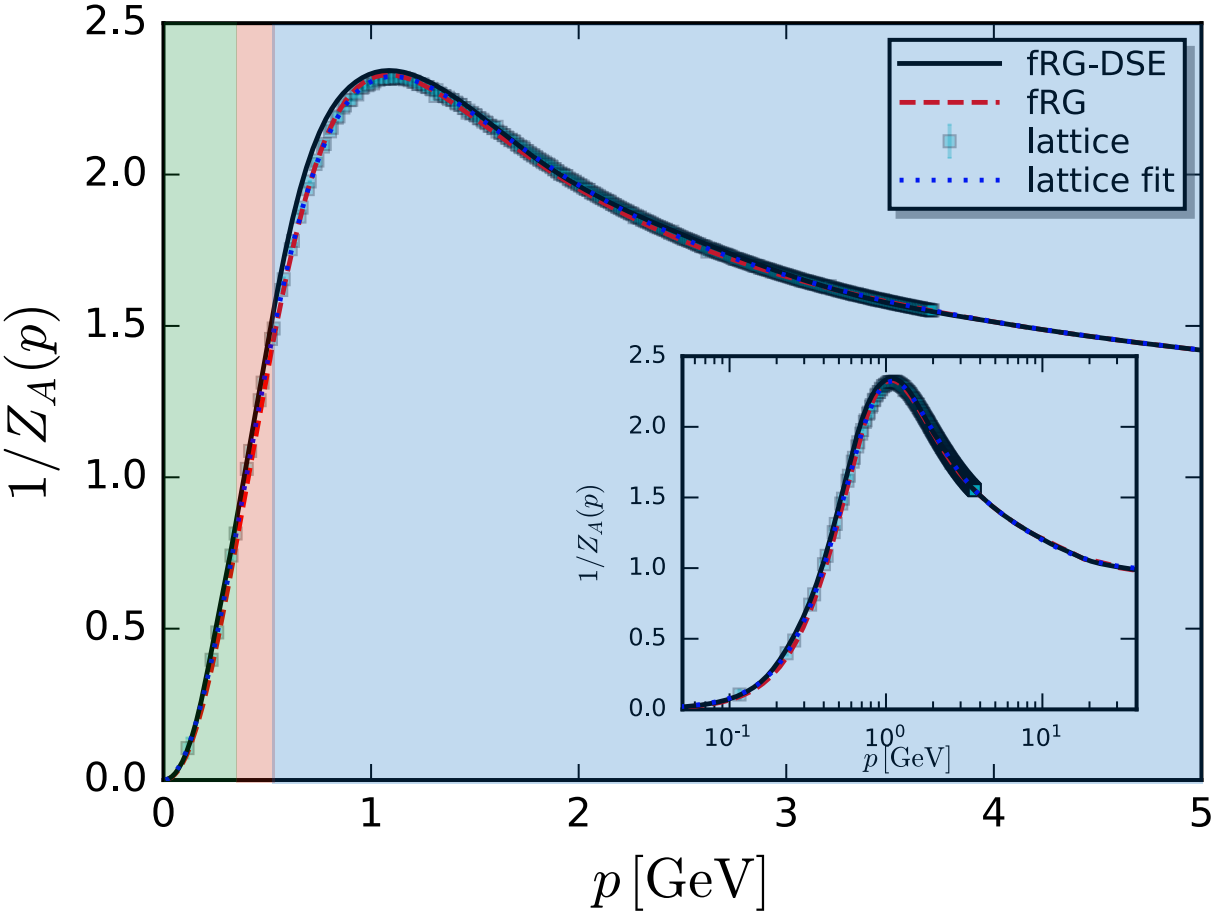
How to: systematic error estimates & the LEGO[®] principle



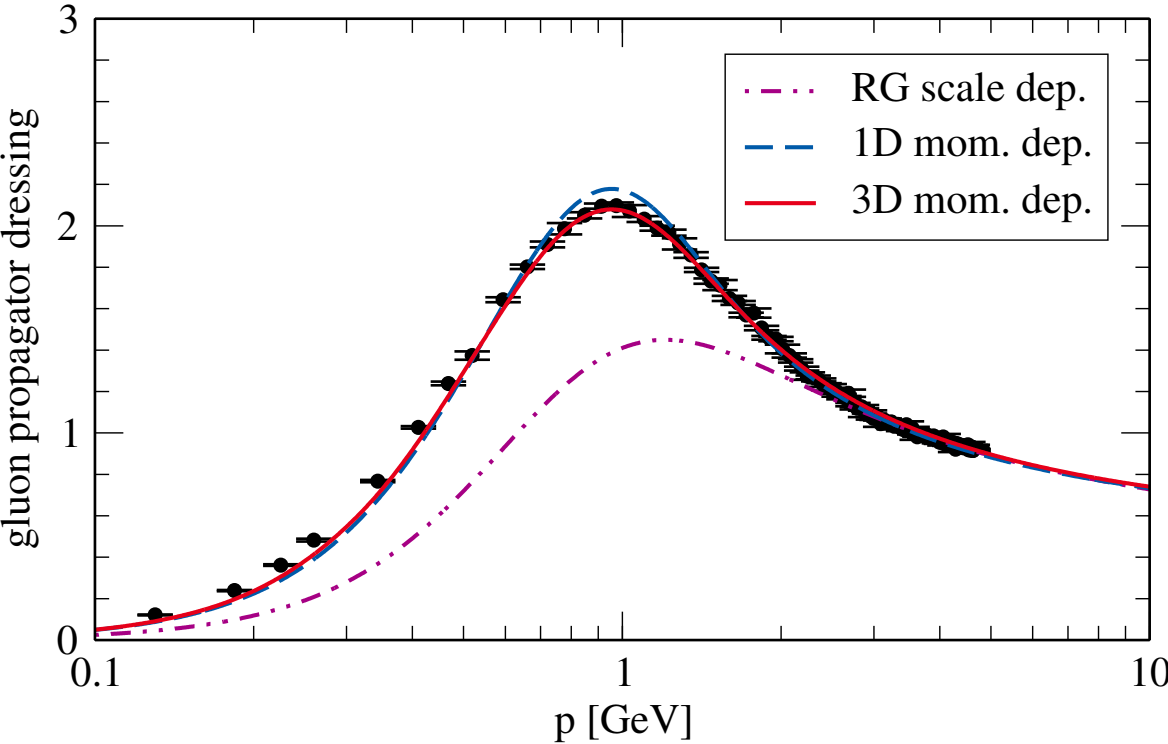
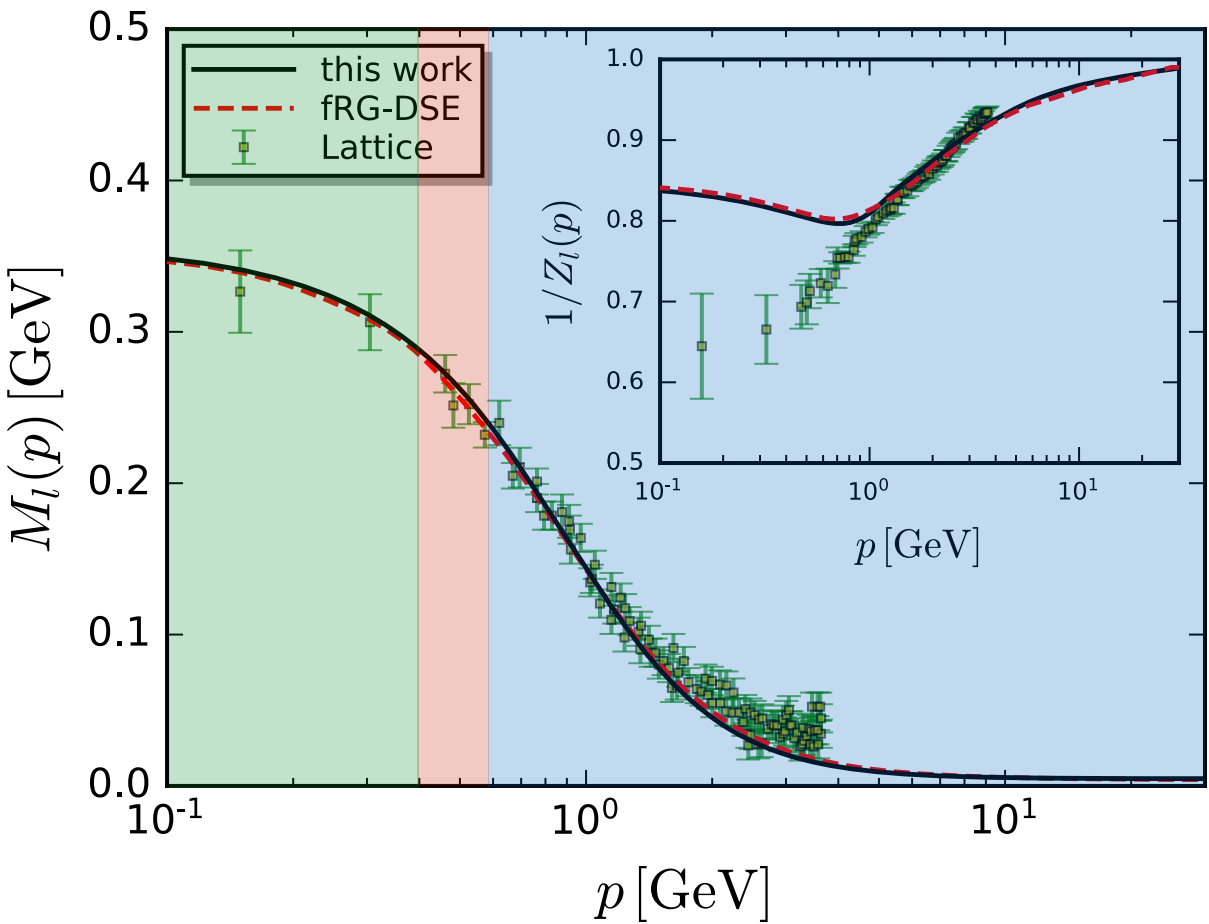
Example: Disect quark-gluon box



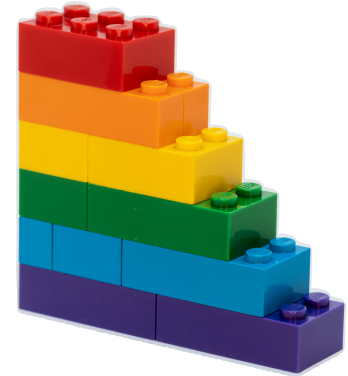
gluon two-point correlator



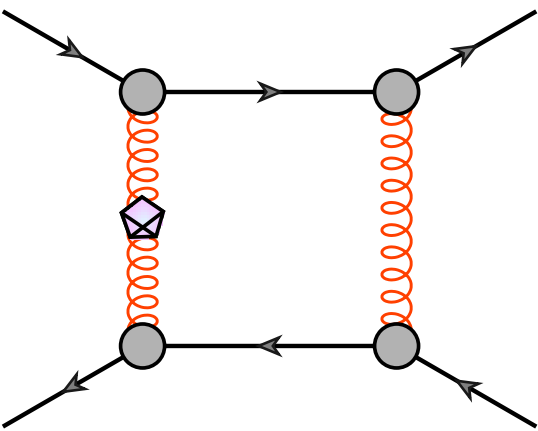
quark two-point correlator



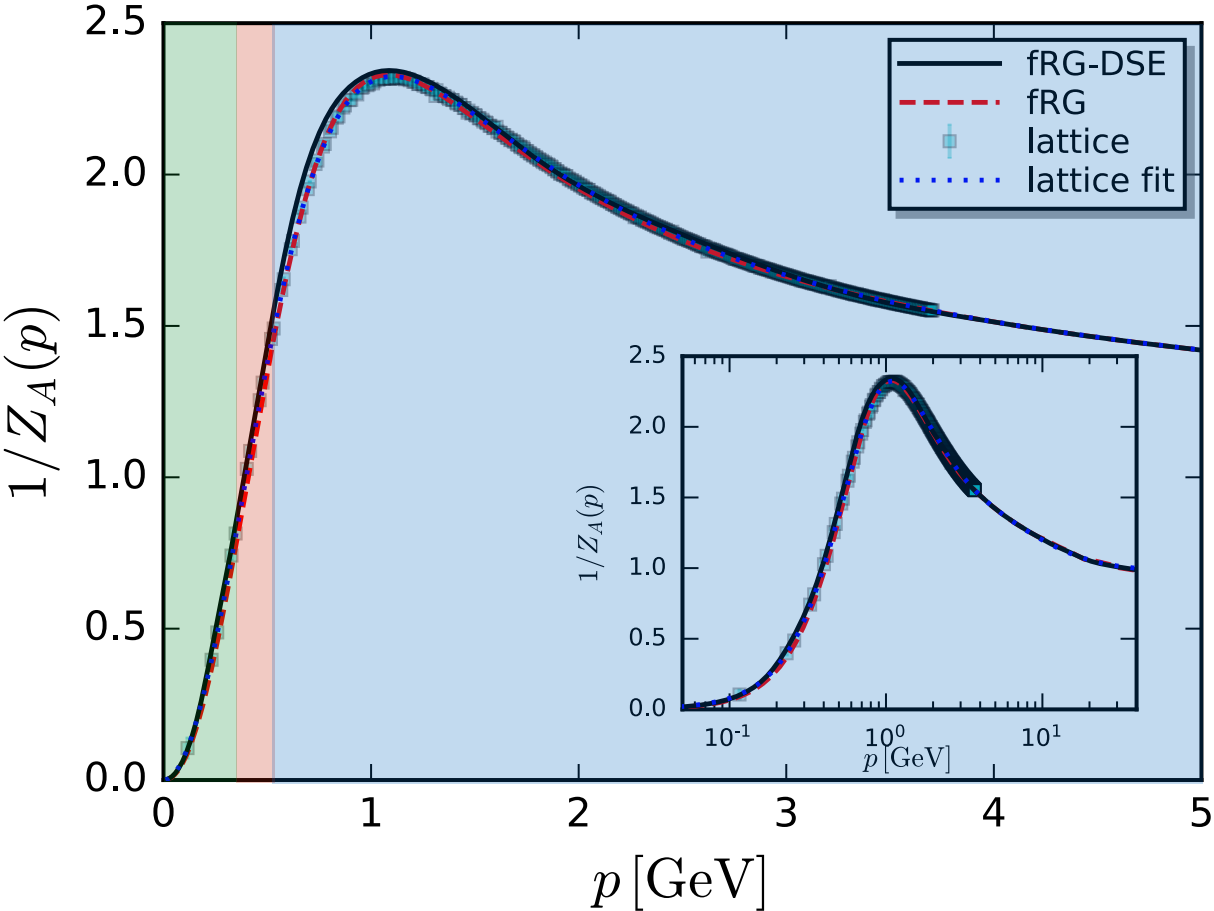
How to: systematic error estimates & the LEGO[®] principle



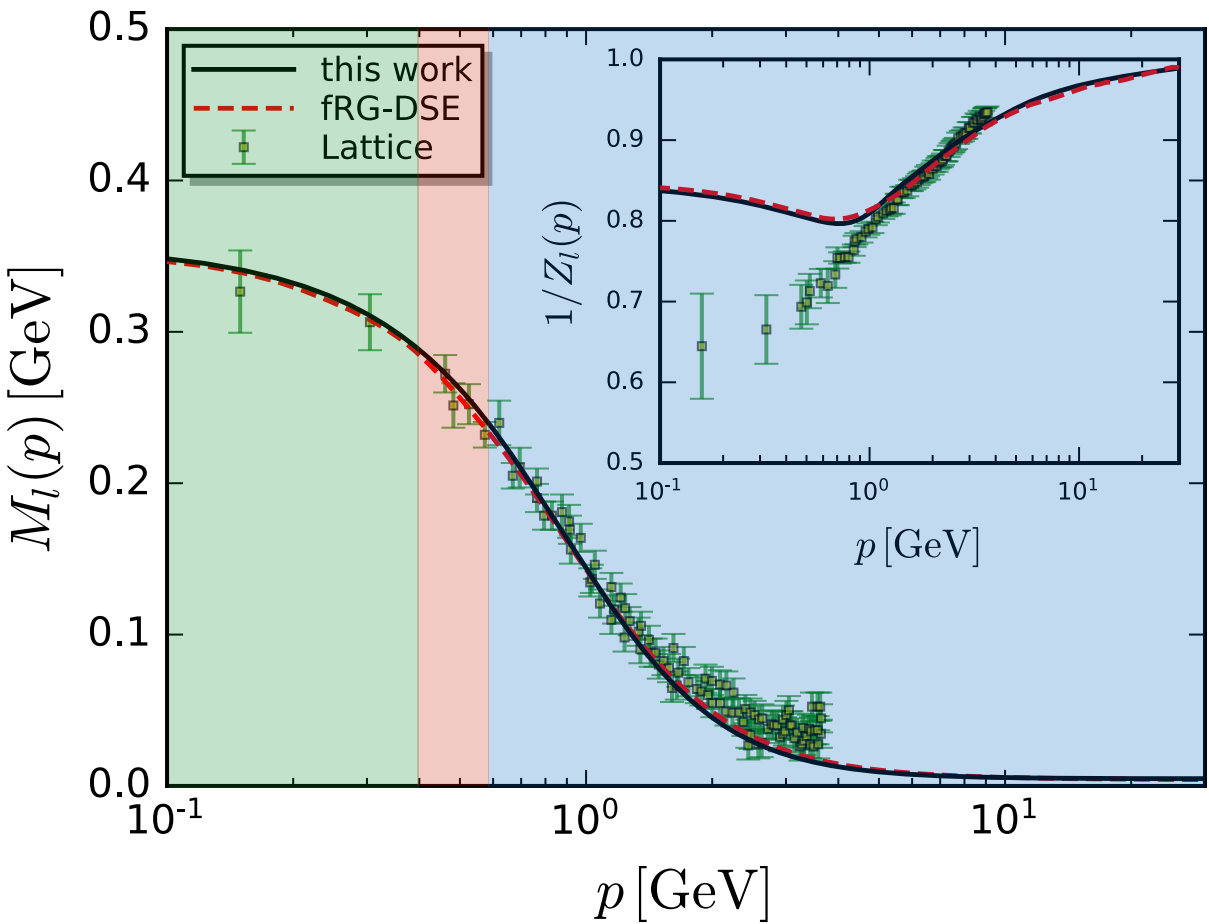
Example: Disect quark-gluon box



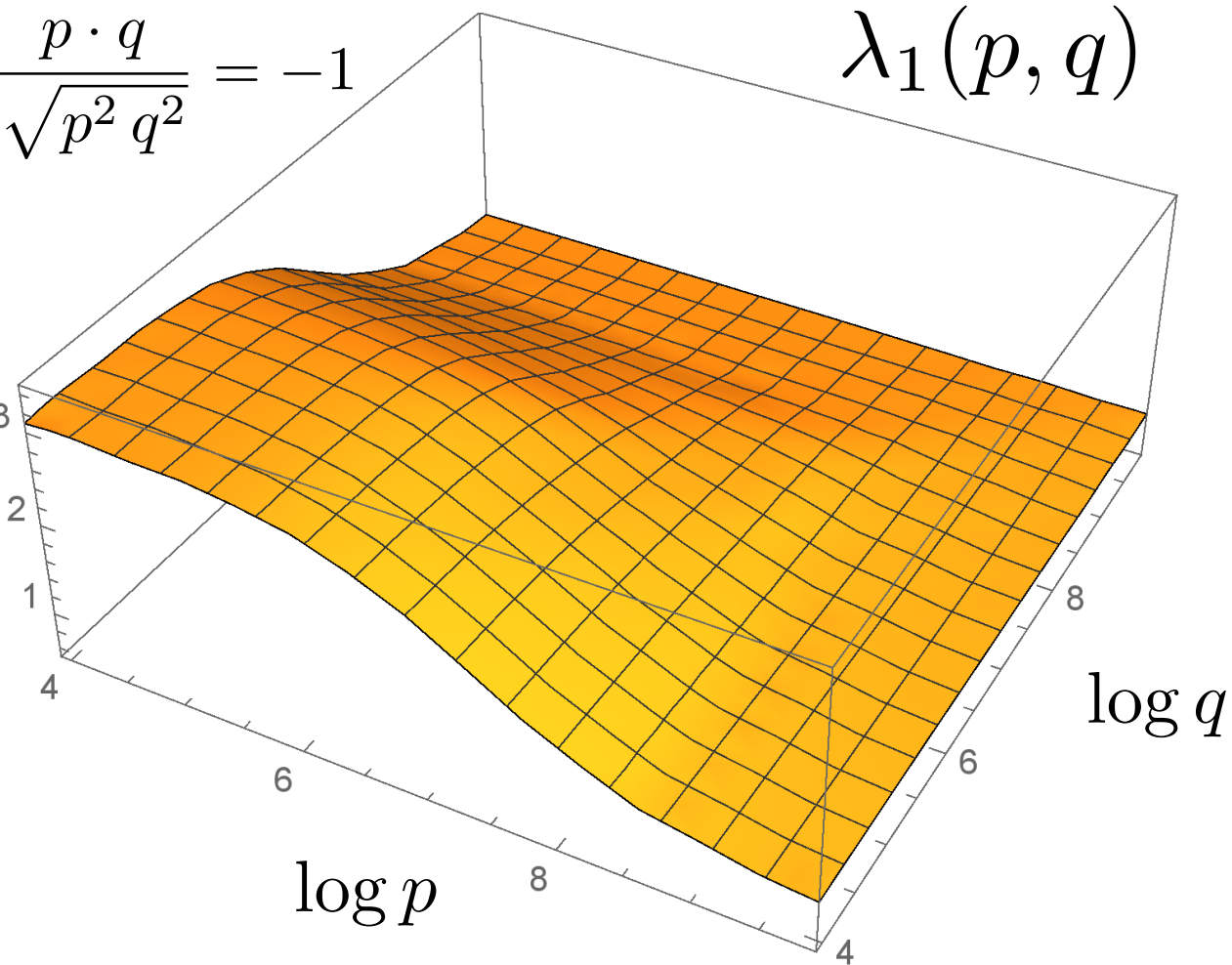
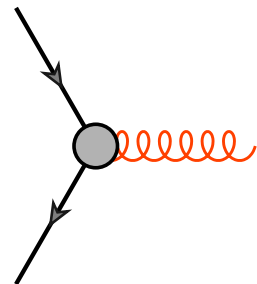
gluon two-point correlator



quark two-point correlator

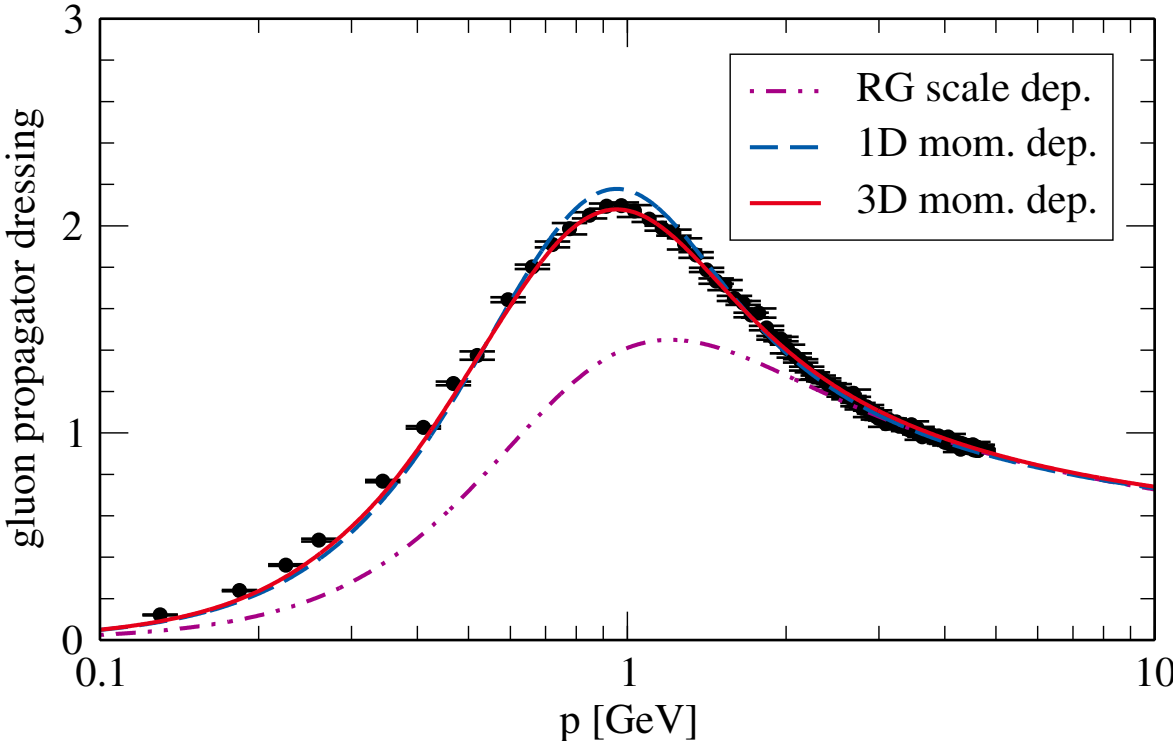


quark-gluon scattering



$$\lambda_1(p, q), \dots, \lambda_8(p, q)$$

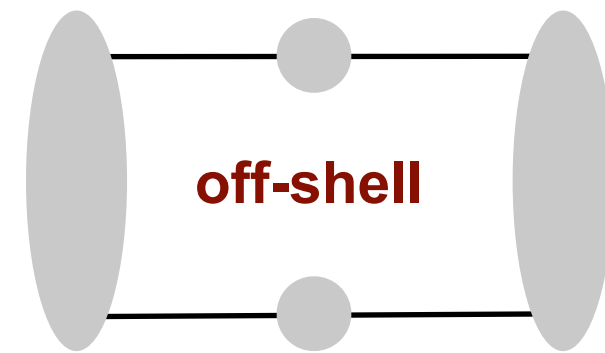
Relevant: $\lambda_1, \lambda_4, \lambda_7$



Three remarks on Functional Methods for QCD

- off-shell representation of thermodynamic observables

e.g. $\text{Tr} \langle q(x) \bar{q}(x) \rangle$

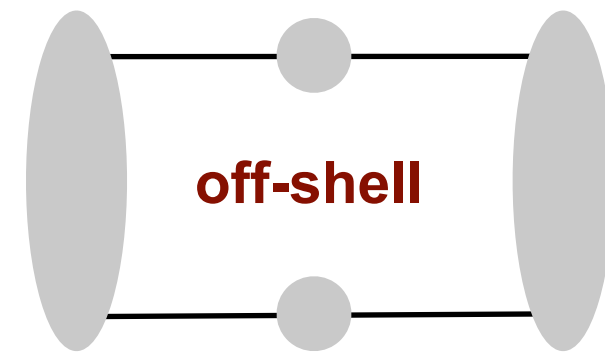


**pressure, trace anomaly,
fluctuations, volume flucs., ...**

Three remarks on Functional Methods for QCD

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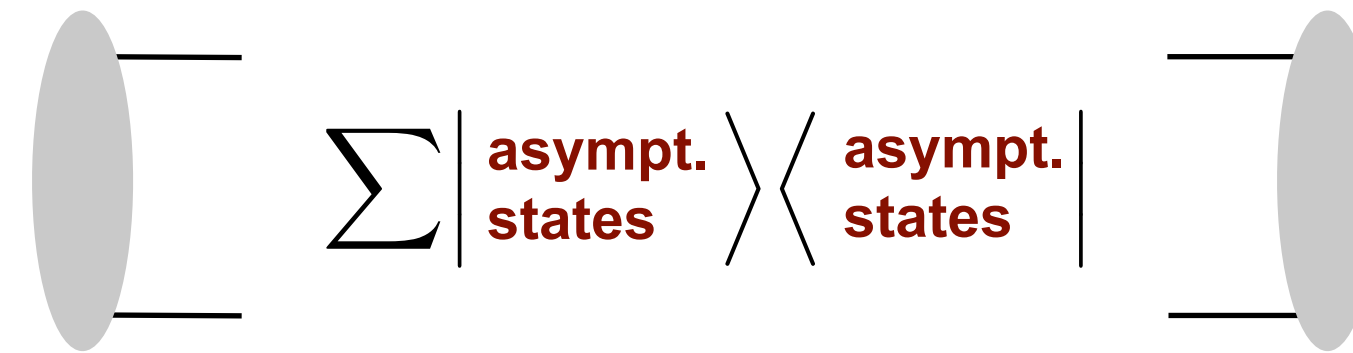
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on-shell

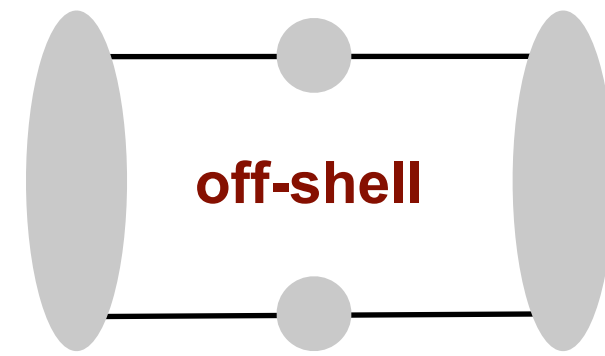


e.g. hadron resonances

Three remarks on Functional Methods for QCD

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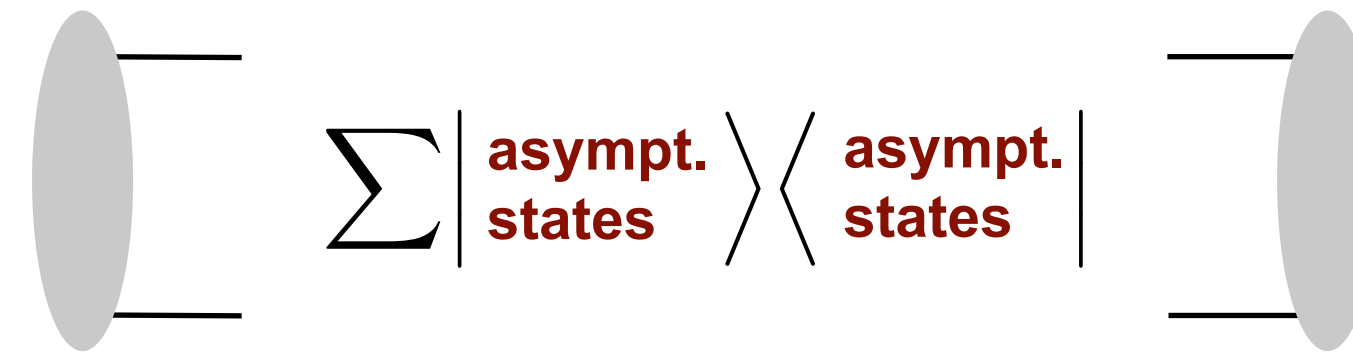
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on-shell



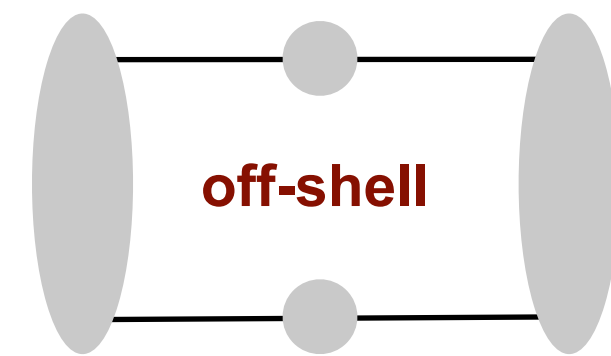
e.g. hadron resonances

- gauge fixing = parameterisation

Three remarks on Functional Methods for QCD

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on-shell



e.g. hadron resonances

- gauge fixing = parameterisation

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$

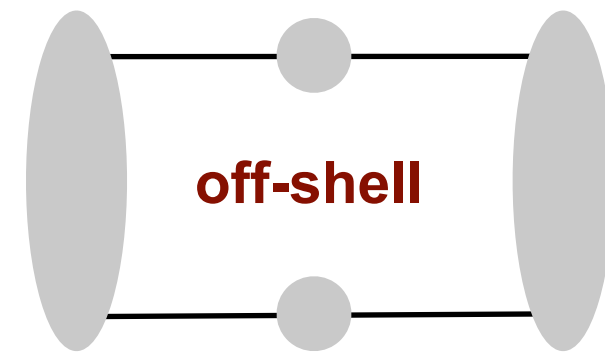
Consequences

I: simple correlations

Three remarks on Functional Methods for QCD

- off-shell representation of thermodynamic observables

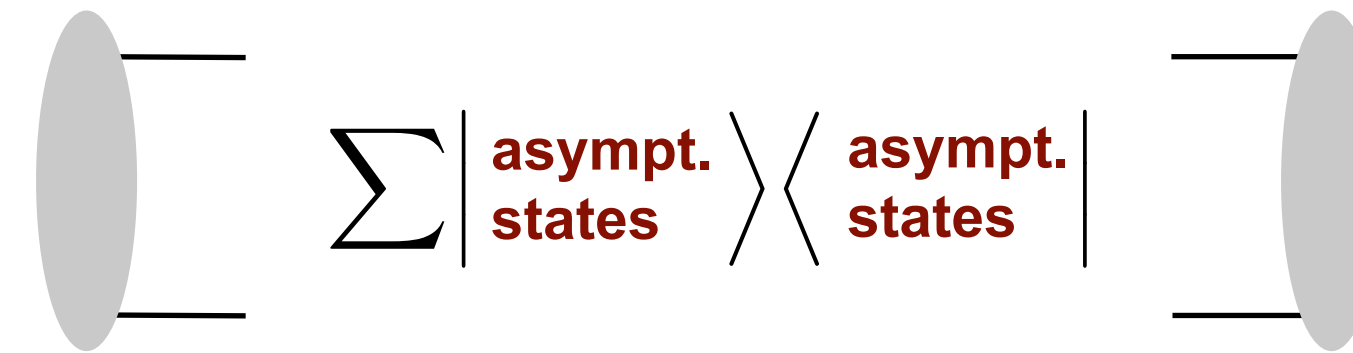
e.g. $\text{Tr} \langle q(x) \bar{q}(x) \rangle$



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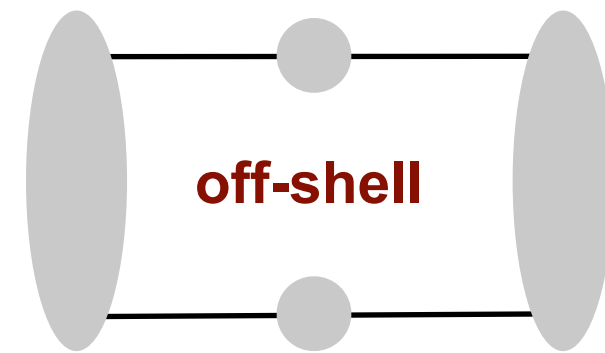
I: simple correlations

II: Difficult access to some observables
'No free lunch theorem'

Three remarks on Functional Methods for QCD

- off-shell representation of thermodynamic observables

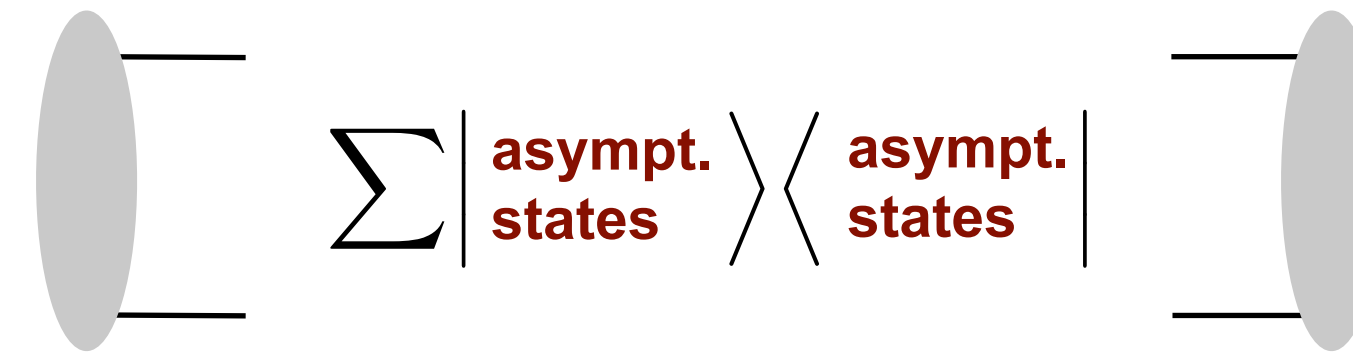
e.g. $\text{Tr} \langle q(x) \bar{q}(x) \rangle$



pressure, trace anomaly,
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on-shell



e.g. hadron resonances

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'No free lunch theorem'

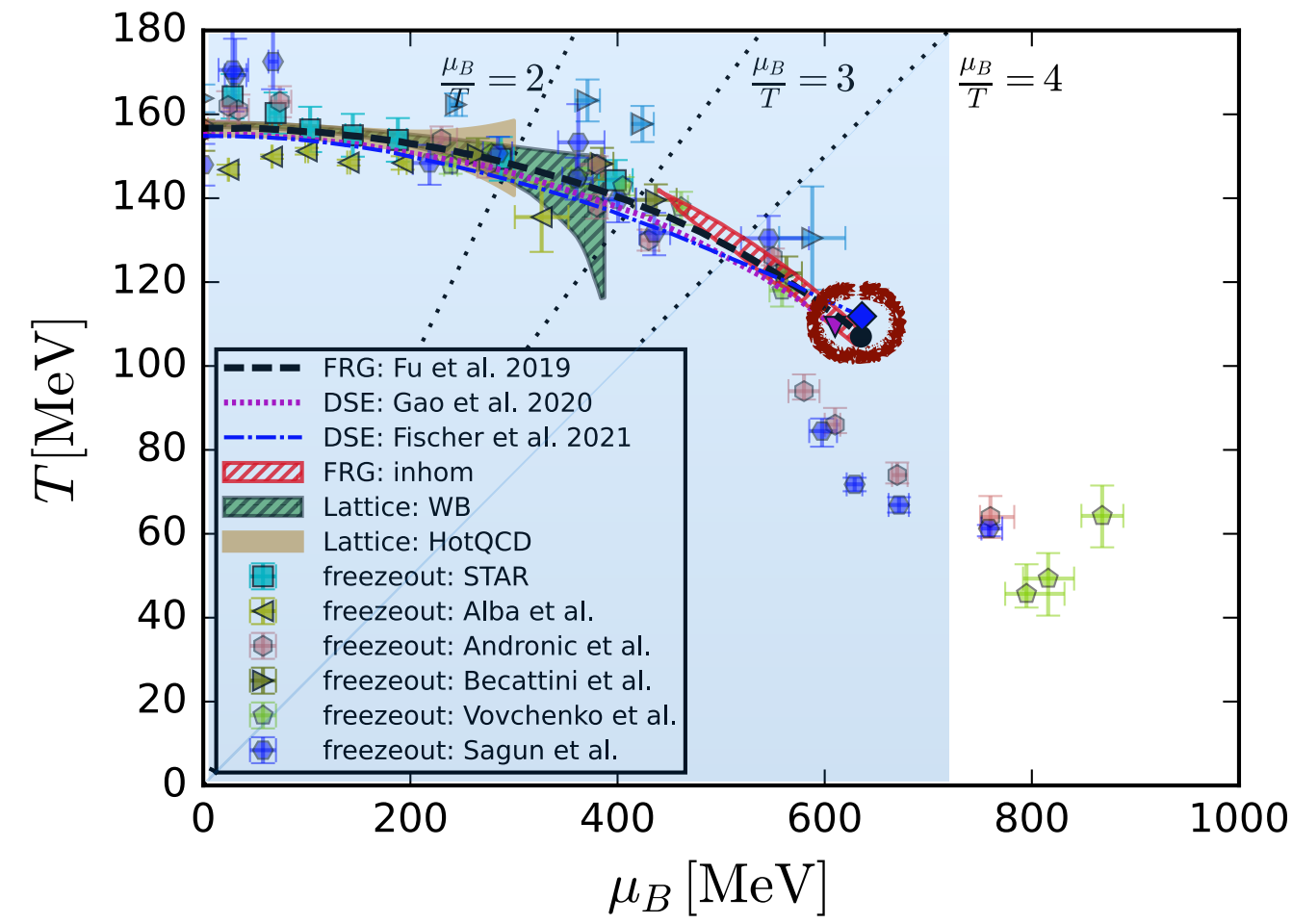
- 'Your mean field is not my mean field'

$$\left. \frac{\delta S_{\text{cl}}[\phi]}{\delta \phi} \right|_{\phi=\bar{\phi}} = 0$$

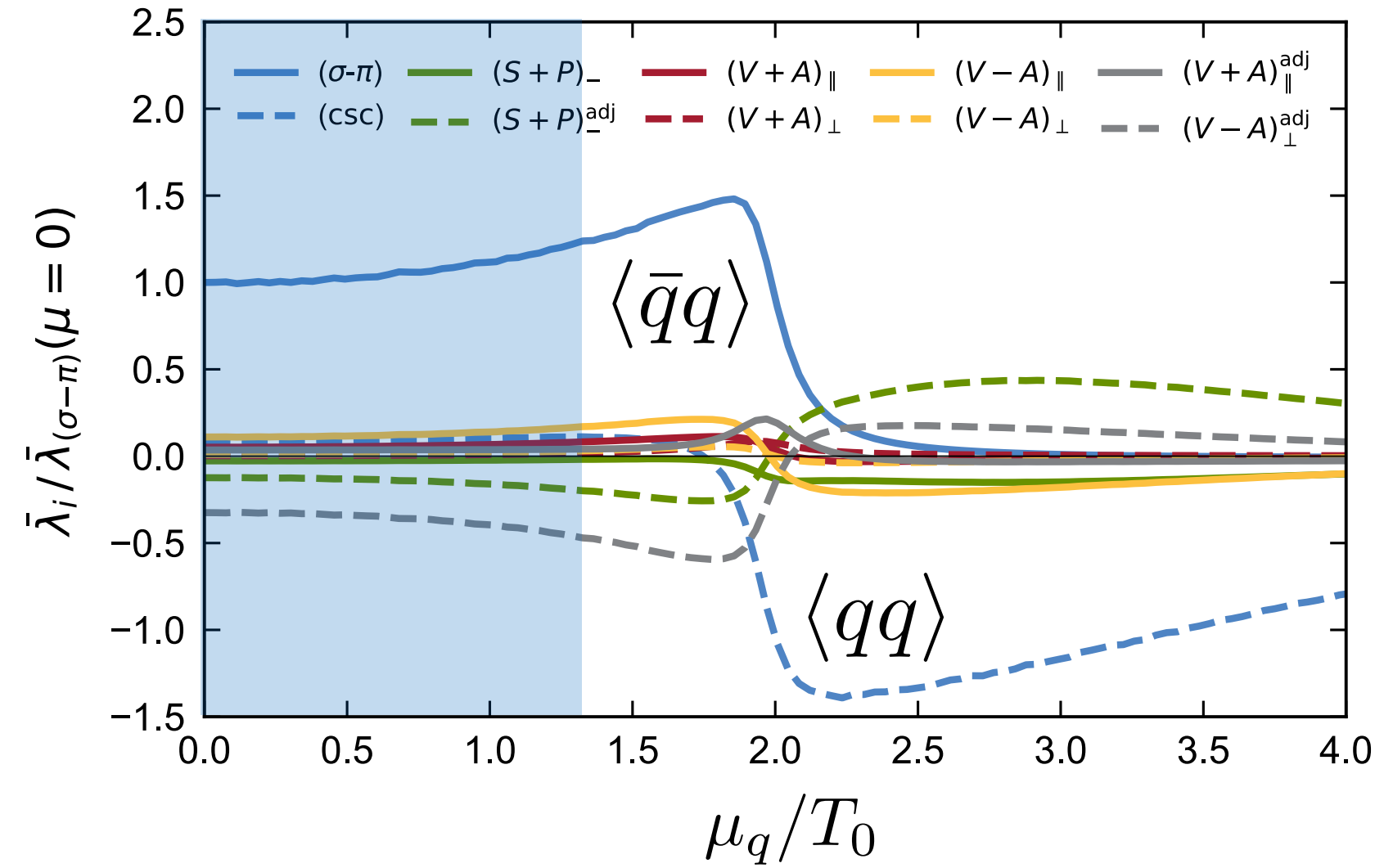
$$\left. \frac{\delta \Gamma[\phi]}{\delta \phi} \right|_{\phi=\bar{\phi}_{\text{quant}}} = 0$$

Phase structure from functional QCD: Predictions & estimates

Dominance of scalar-pseudoscalar fluctuations
Pions & sigma mode

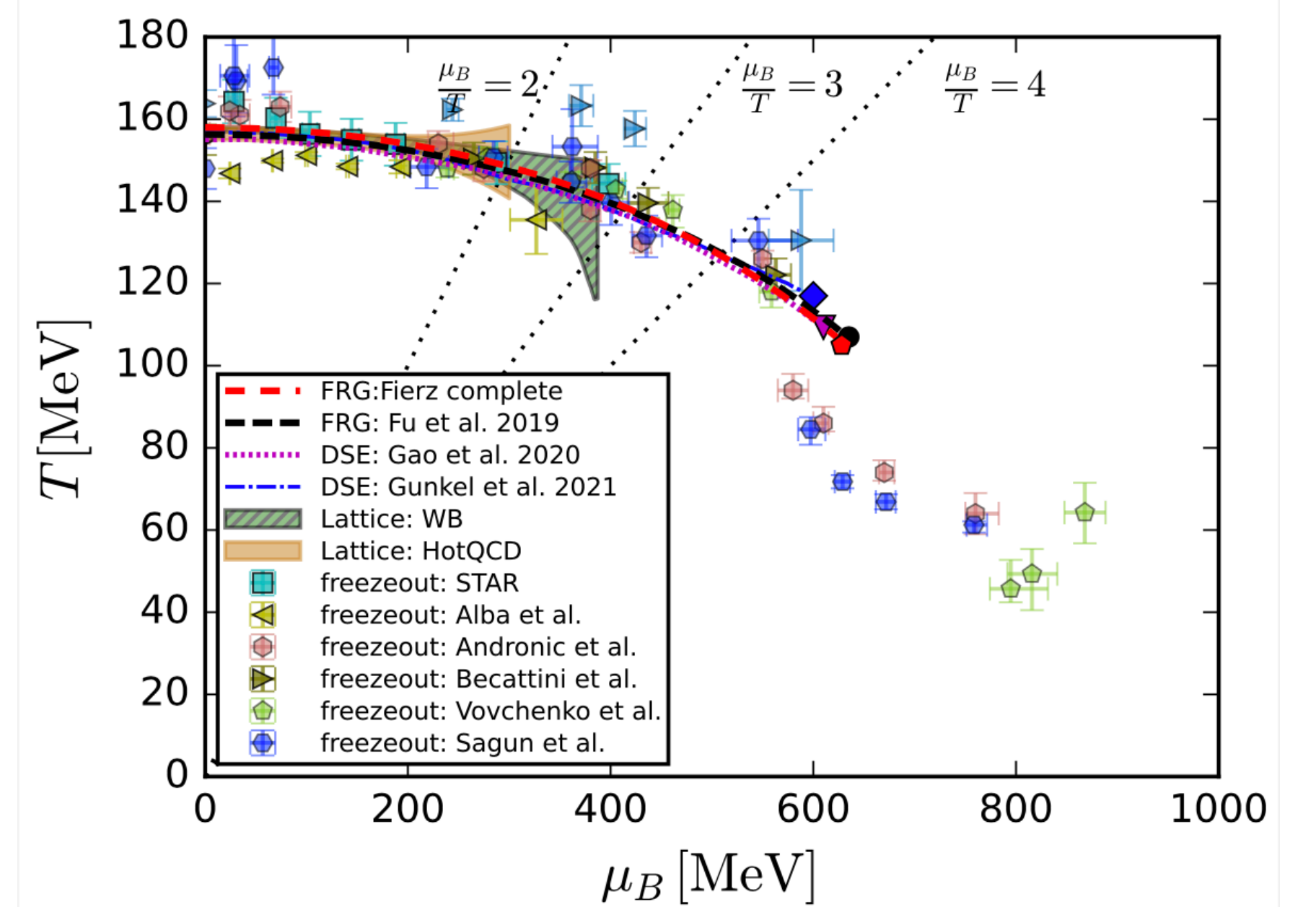


Four-quark scattering channels



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Fierz-complete phase structure

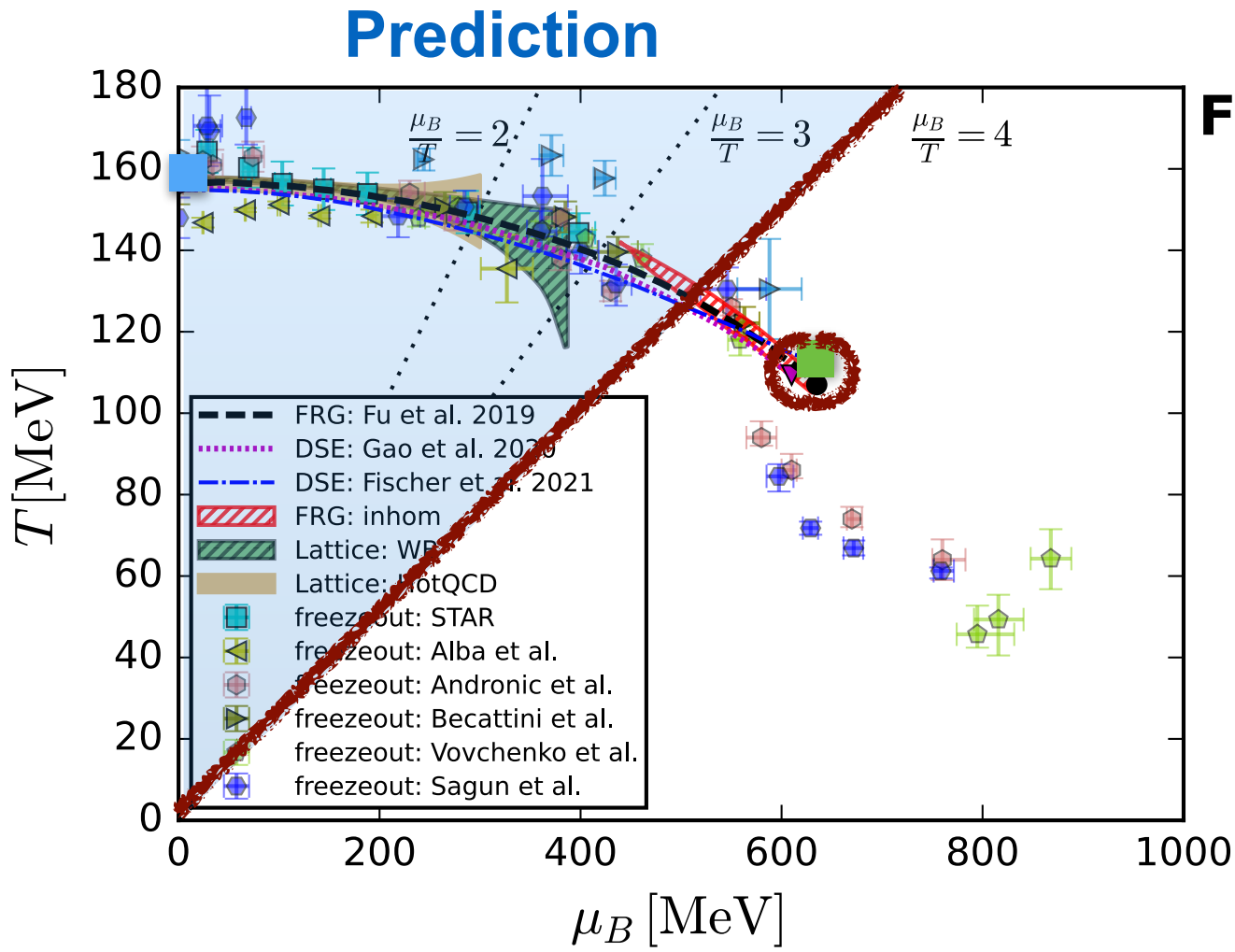


Wang et al, in preparation

Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302



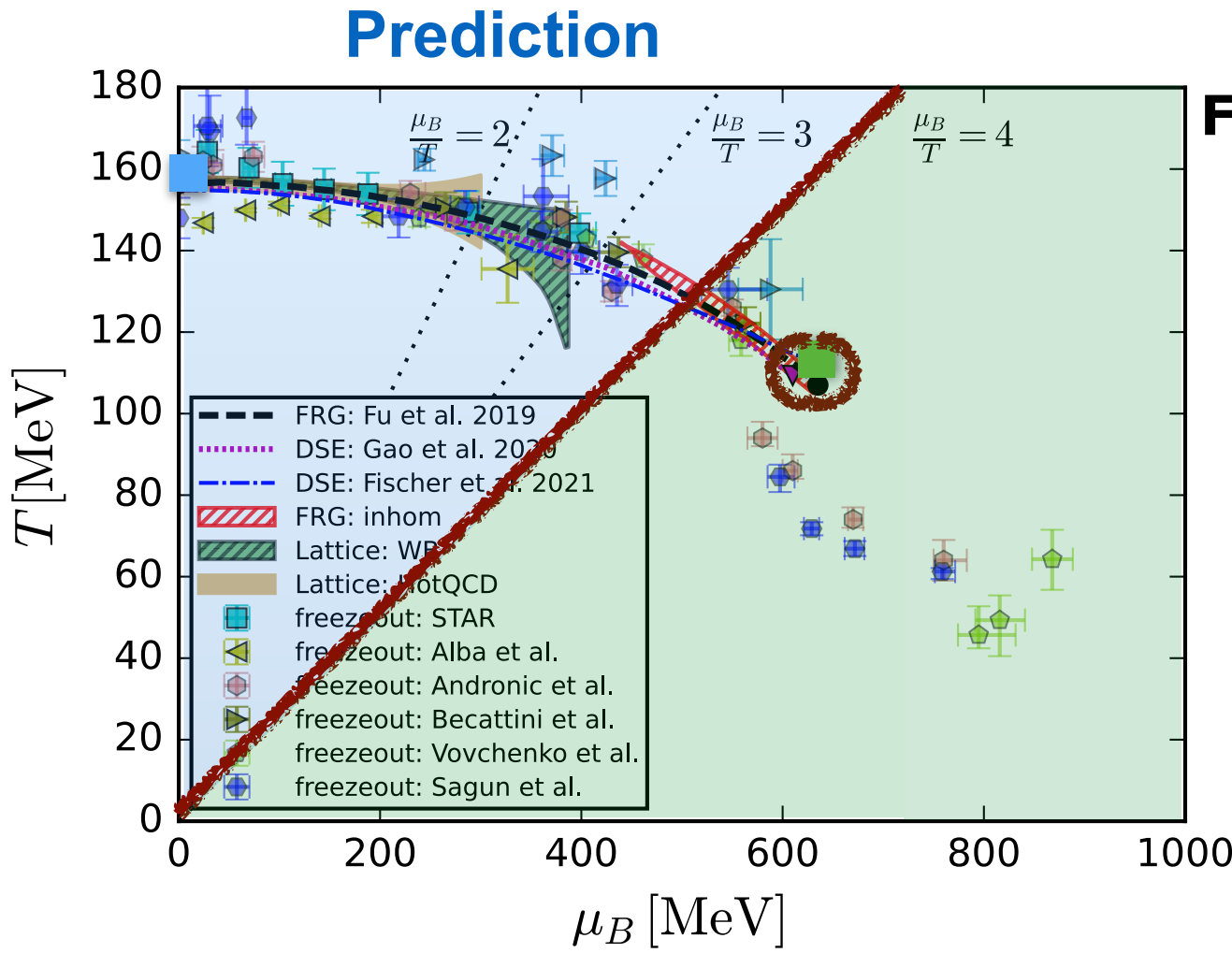
Fu, JMP, Rennecke, PRD 101 (2020) 054032

**Regime of quantitative reliability
of
current best truncation**

Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

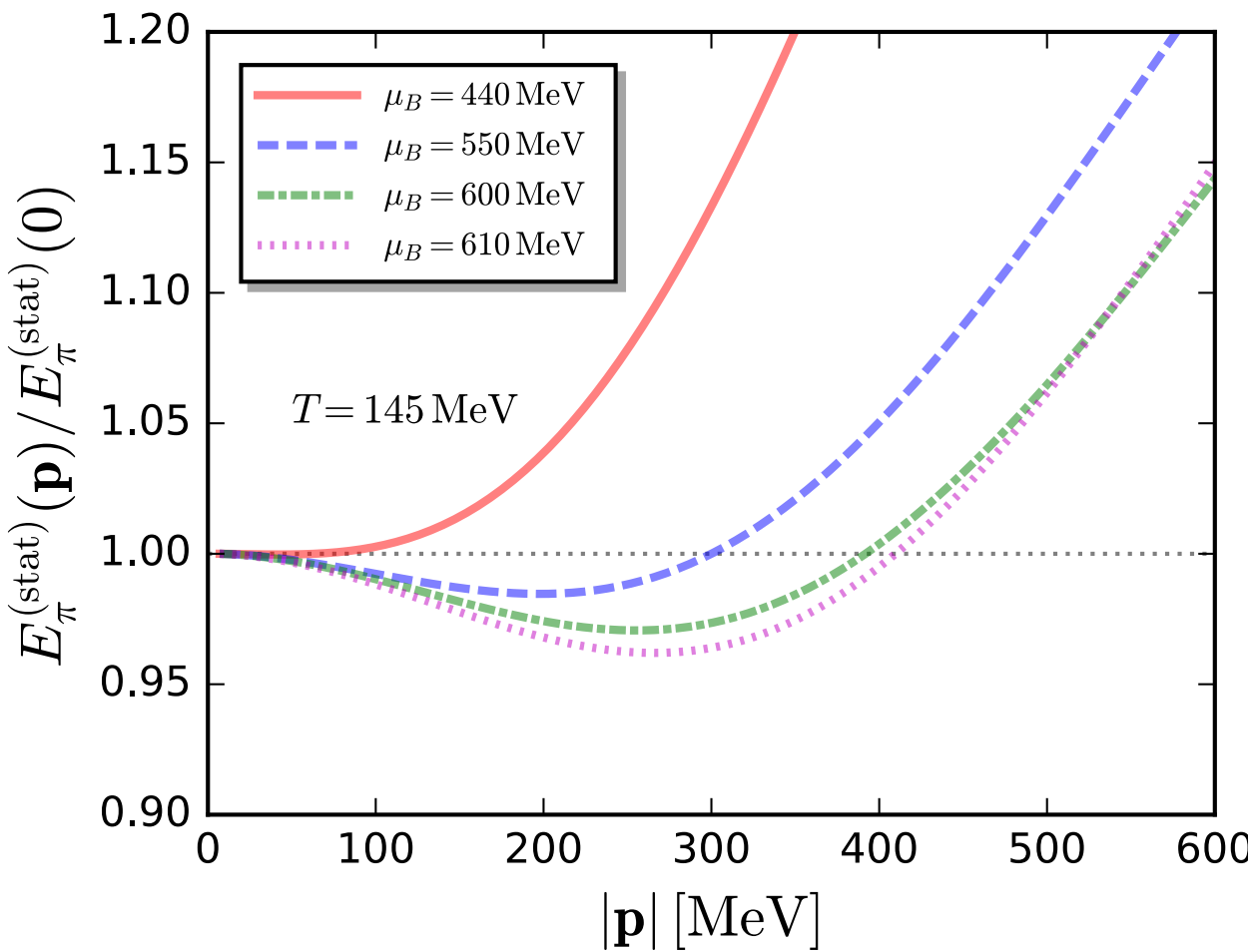


Fu, JMP, Rennecke, PRD 101 (2020) 054032

Regime of quantitative reliability
of
current best truncation

Estimate

Moat regime is not captured quantitatively



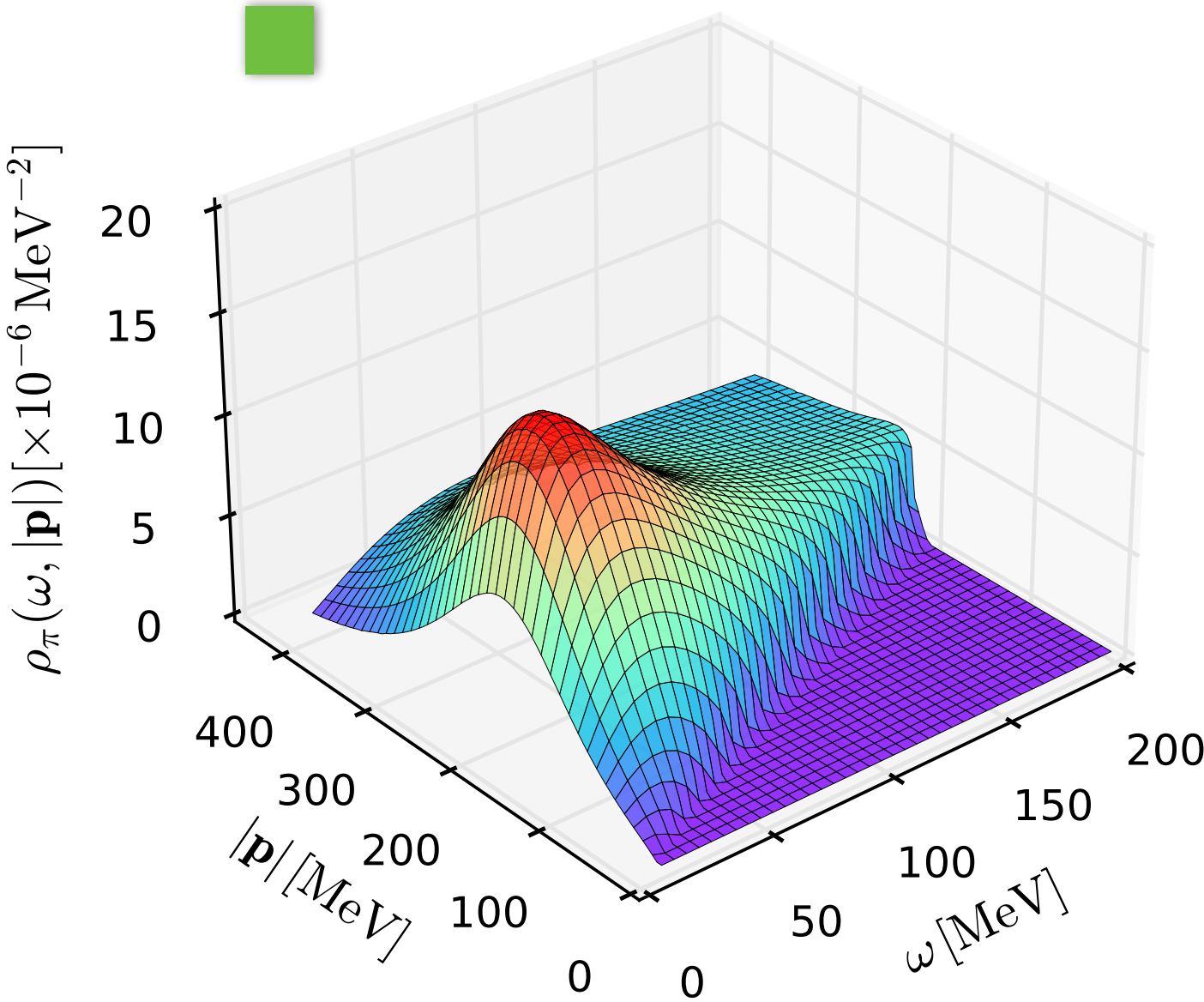
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

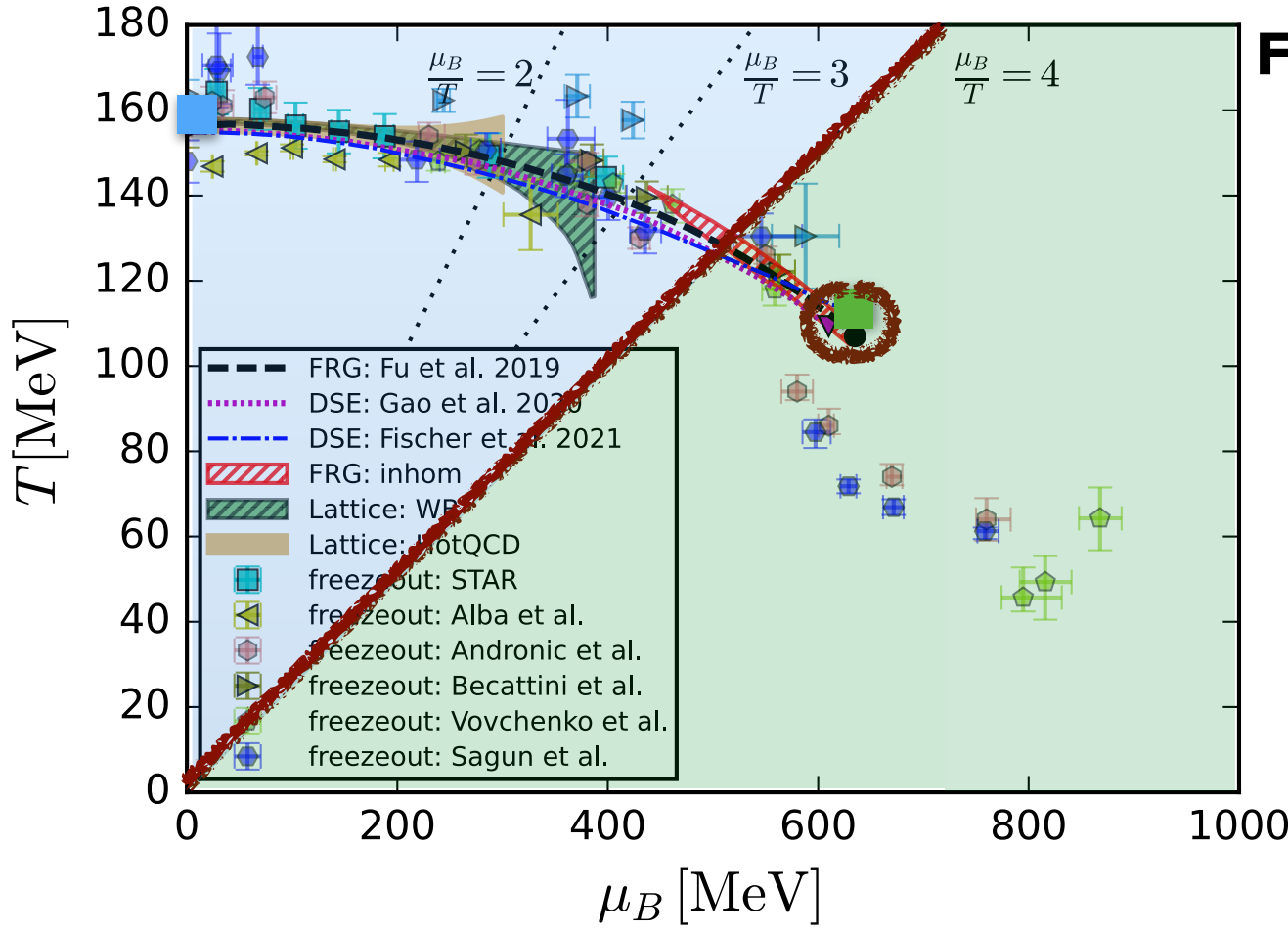
Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

T=114 MeV & $\mu_B=630$ MeV



Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

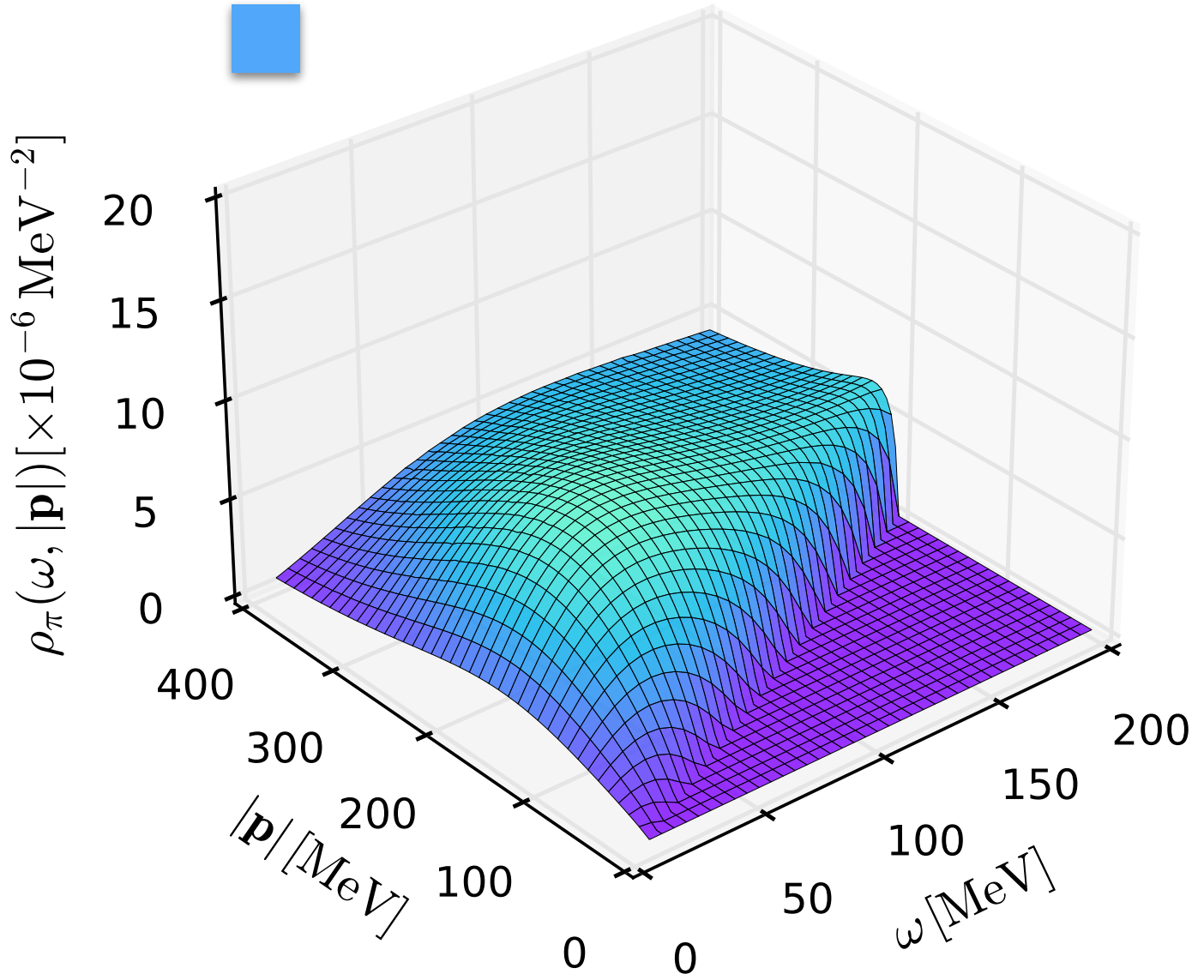
Regime of quantitative reliability
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Estimate

Moat regime is not captured quantitatively

Pion spectral functions

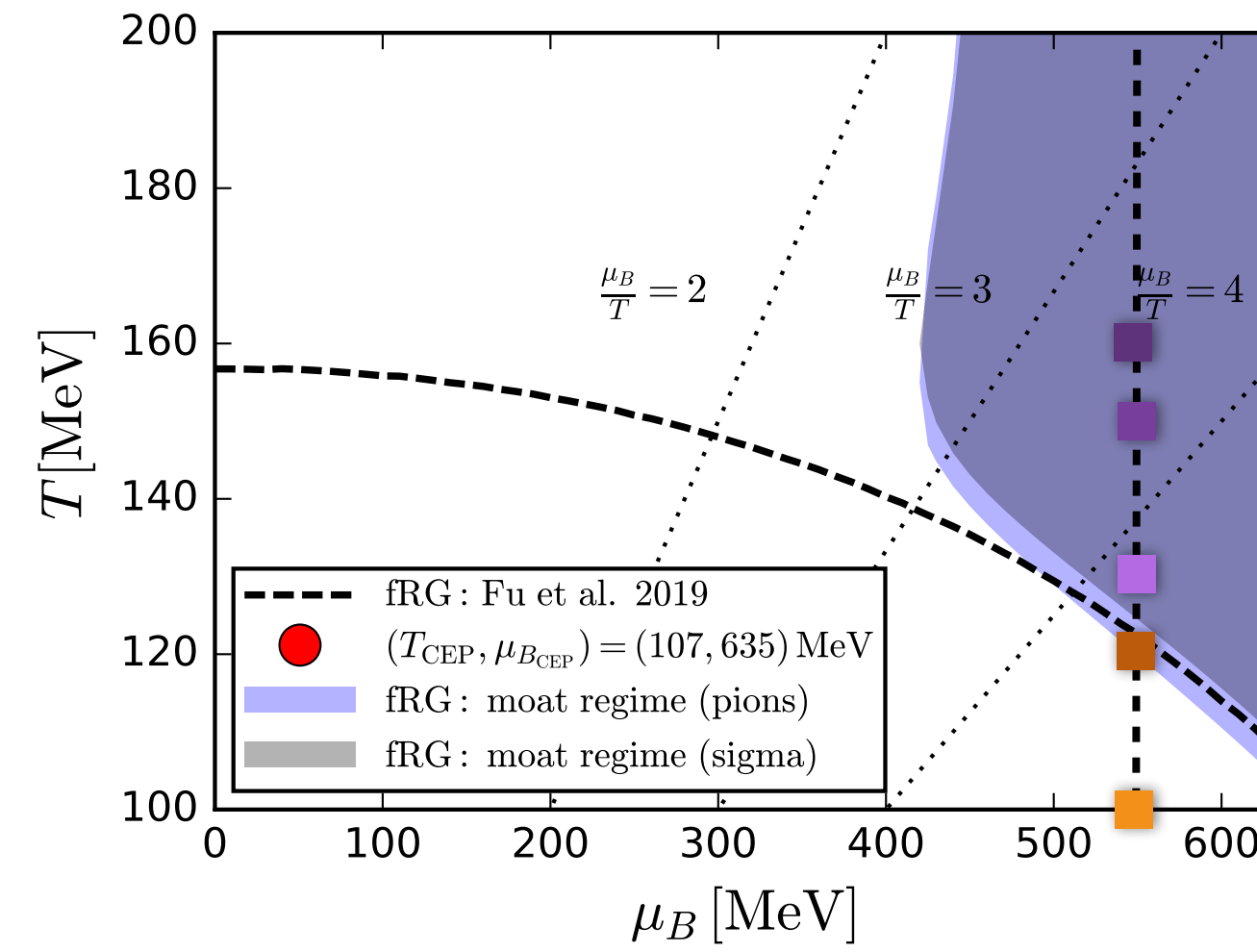
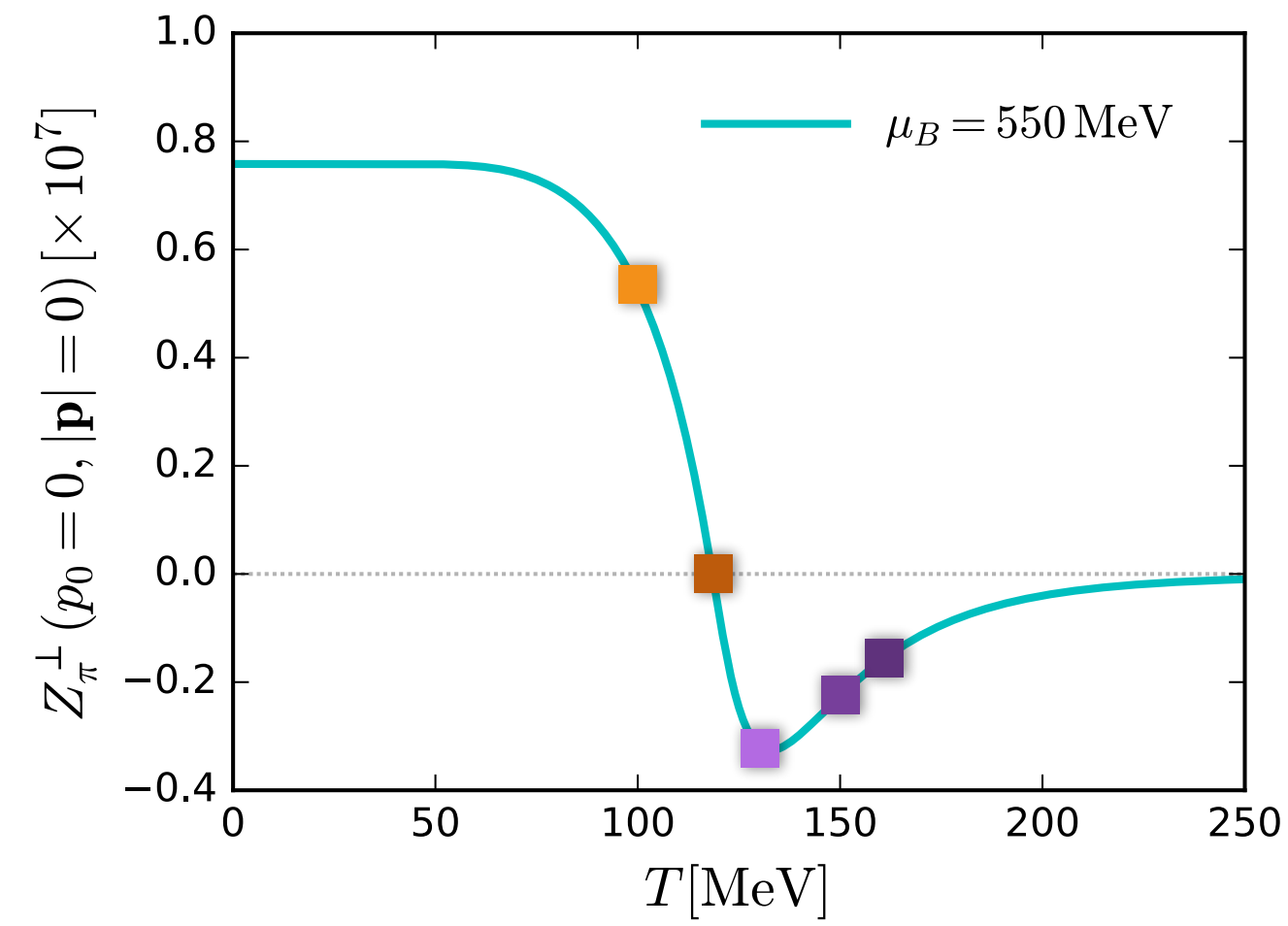
T=160 MeV & $\mu_B=0$ MeV



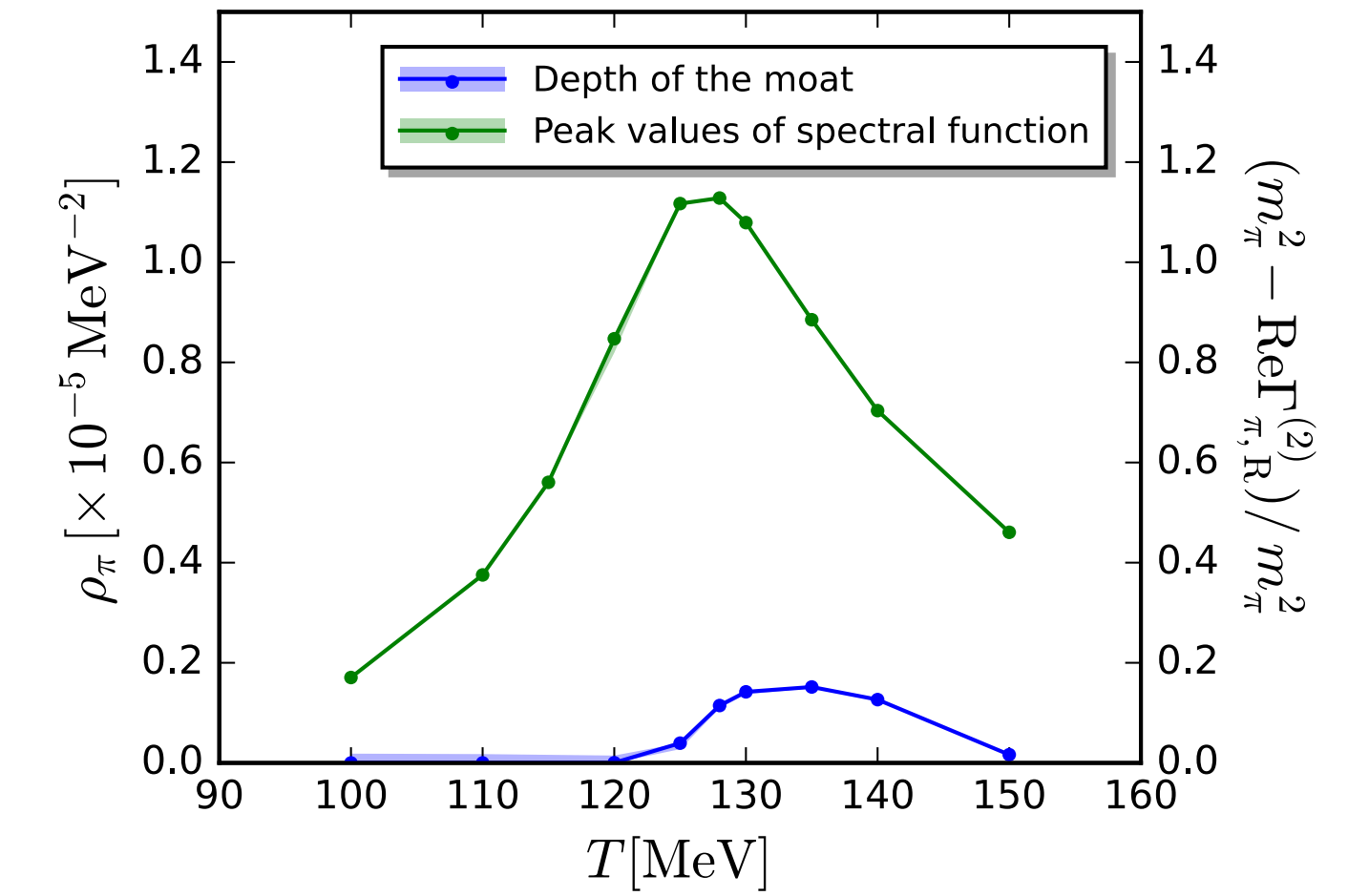
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Spatial wave function of the pion at p=0



Depth of the moat & spectral peak of the pion



Dissecting the moat & the moaton

$\mu_B = 650$ MeV

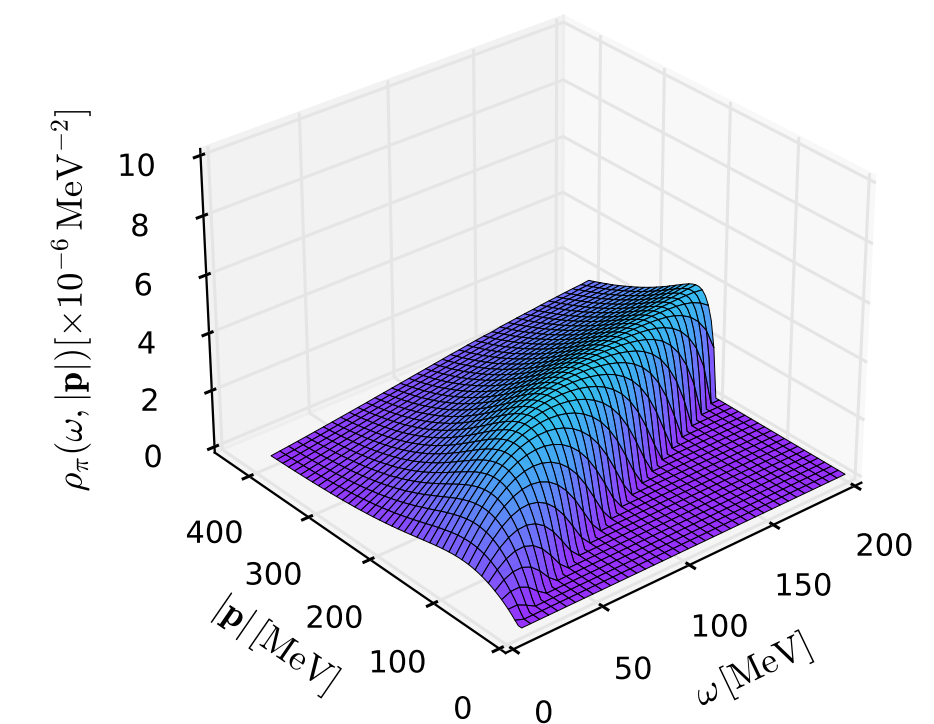
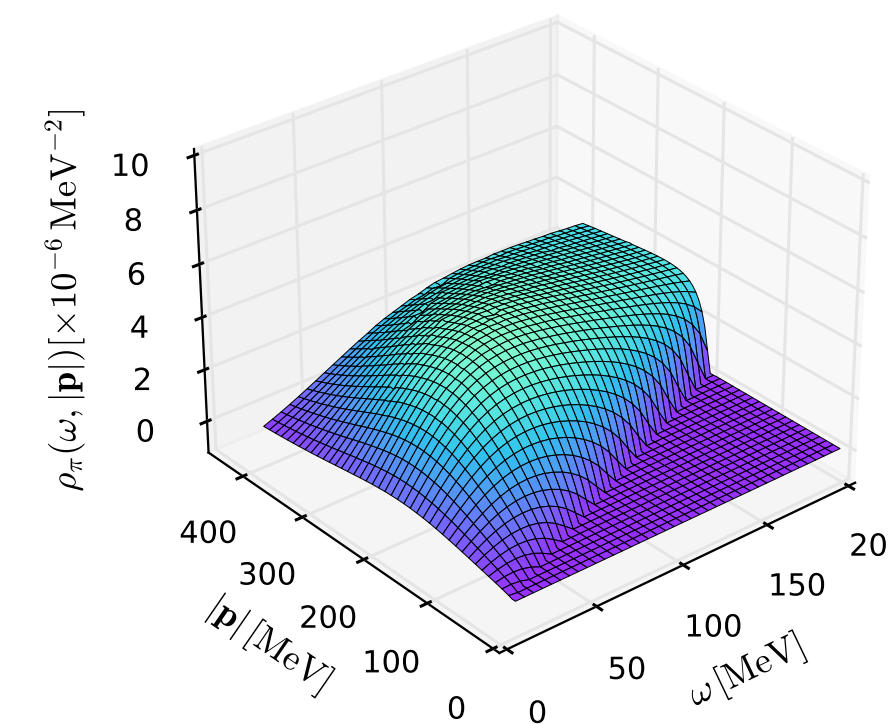
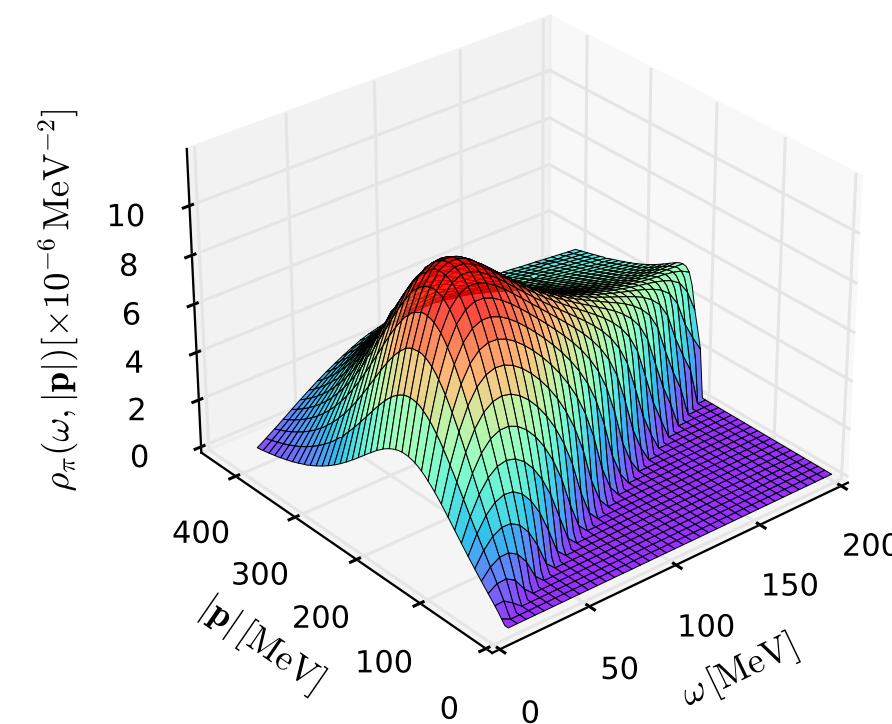
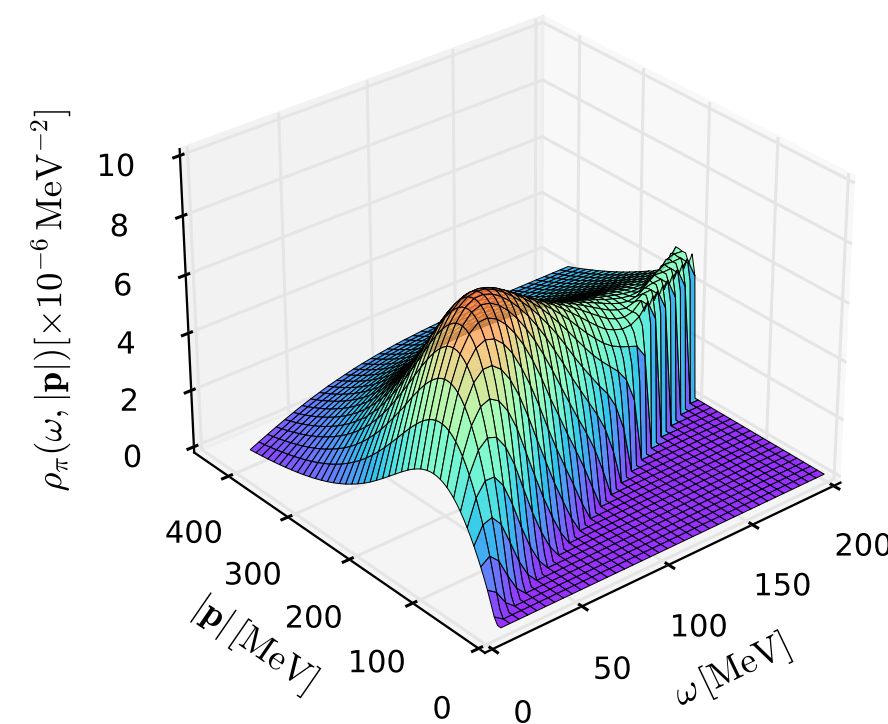
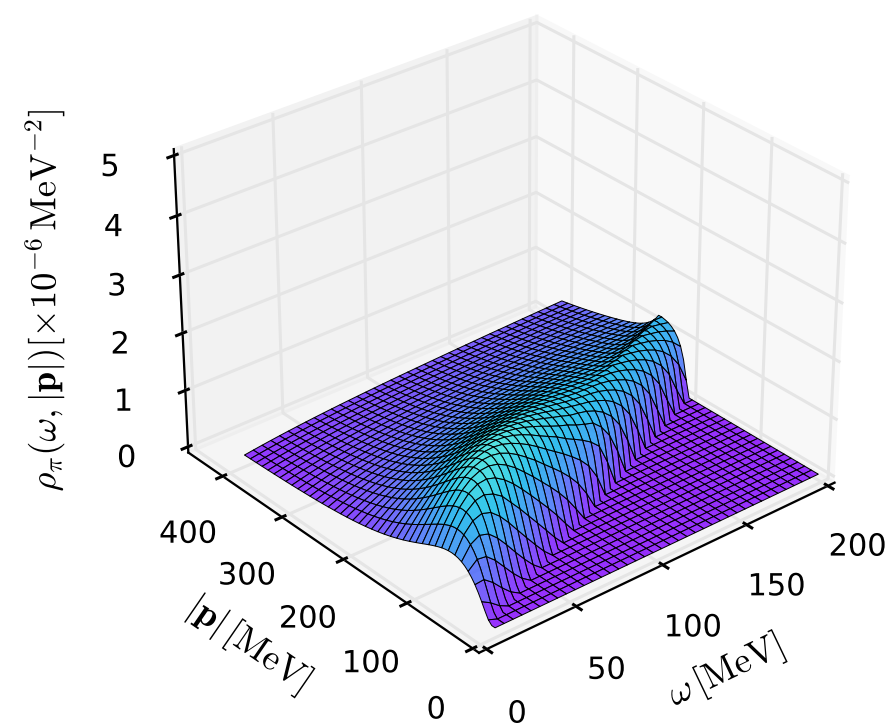
■ T=100 MeV

■ T=120 MeV

■ T=130 MeV

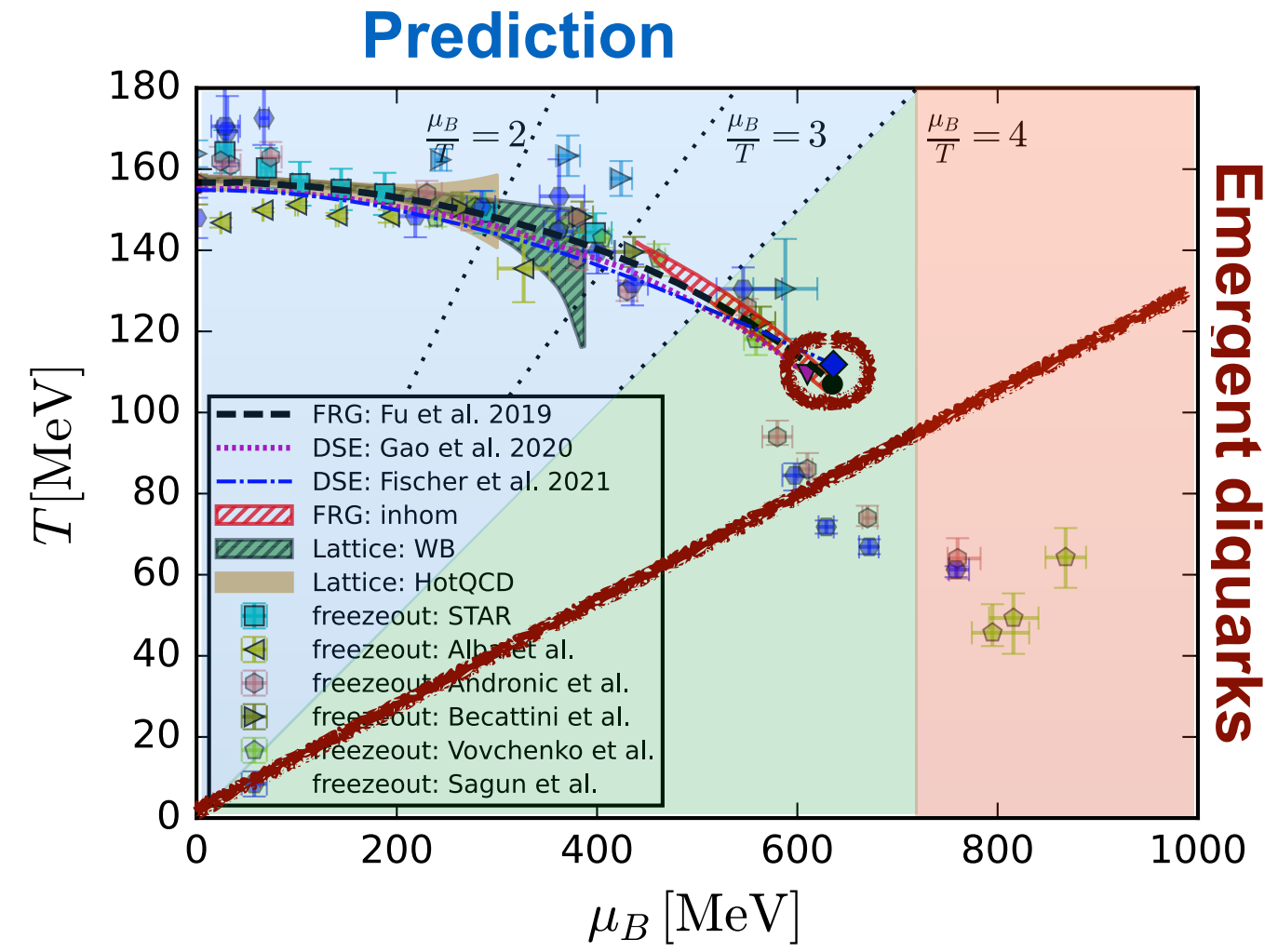
■ T=150 MeV

■ T=160 MeV



Predictions & estimates

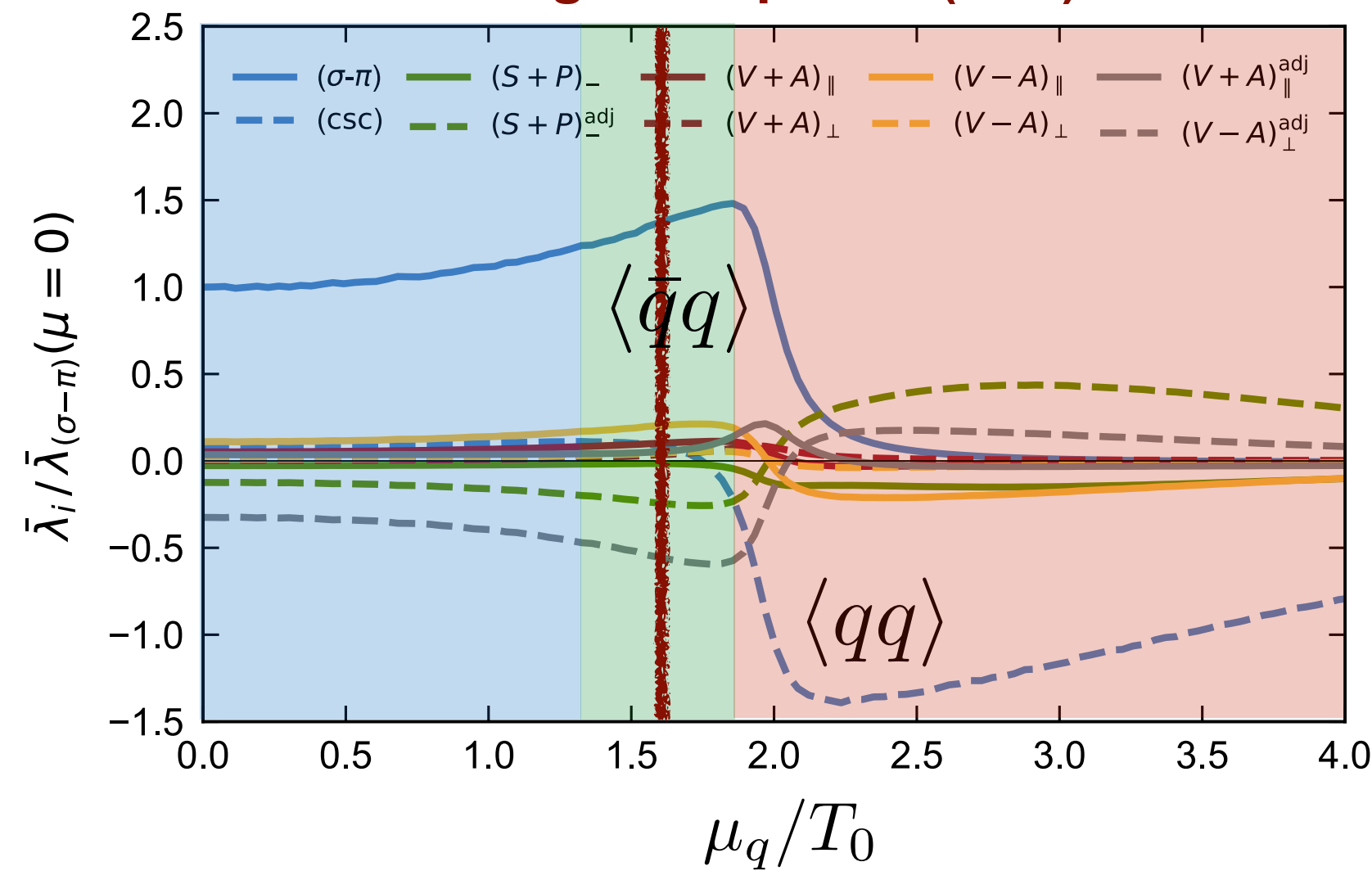
Emergent diquarks



Estimate

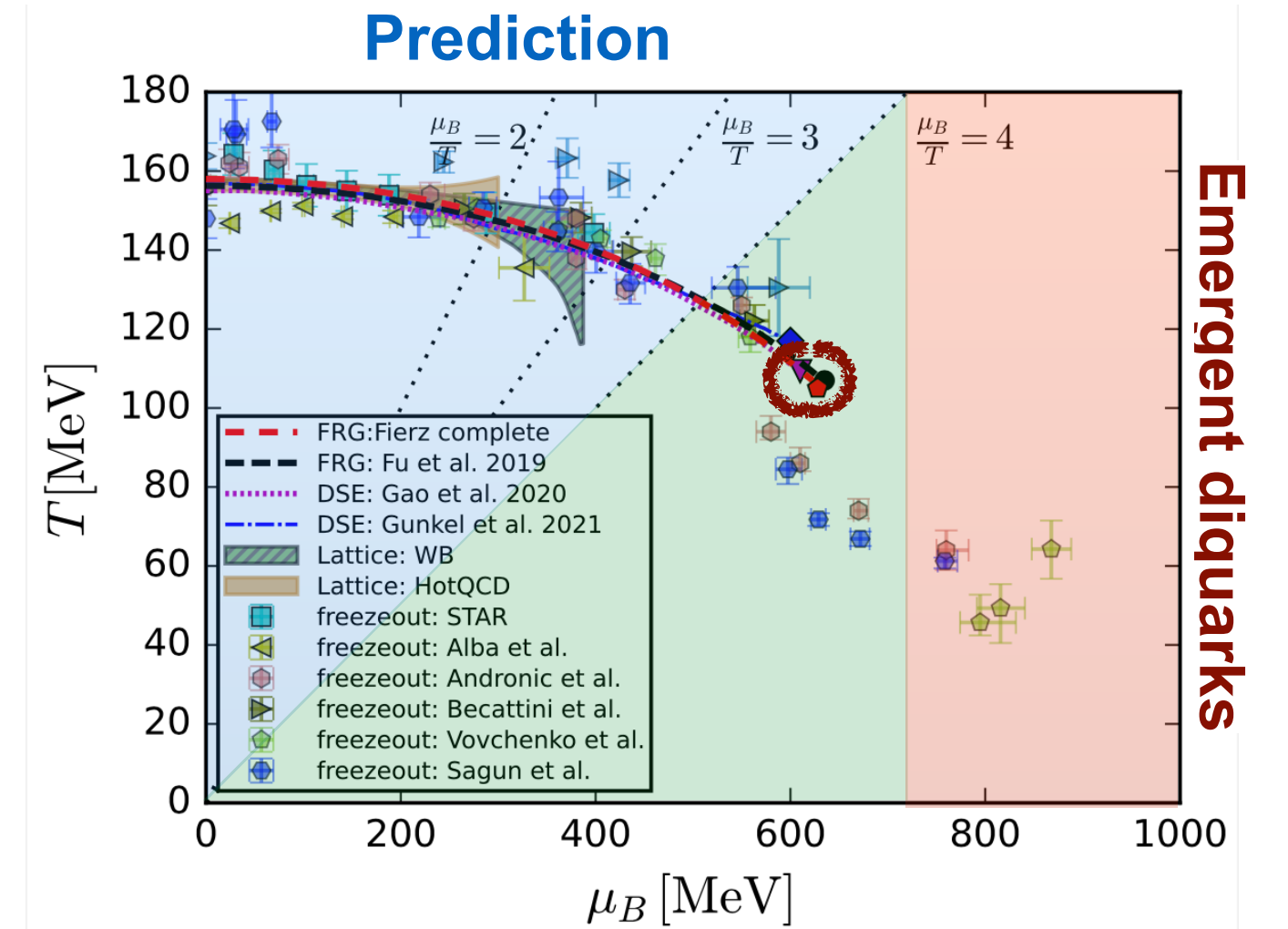
Regime of quantitative reliability
of
current best truncation

Emergent diquarks (fRG)



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Fierz-complete phase structure



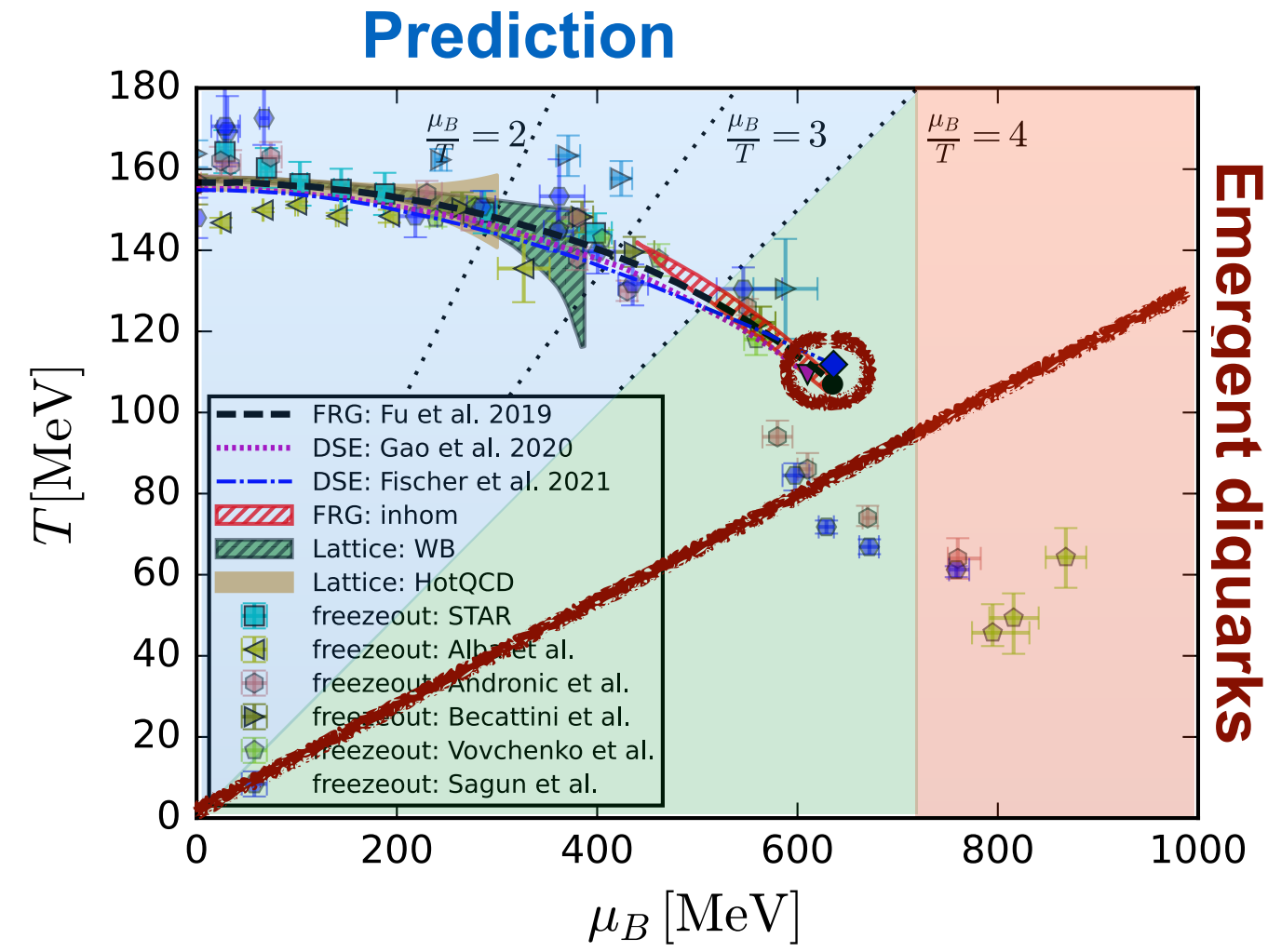
Estimate



Wang et al, in preparation

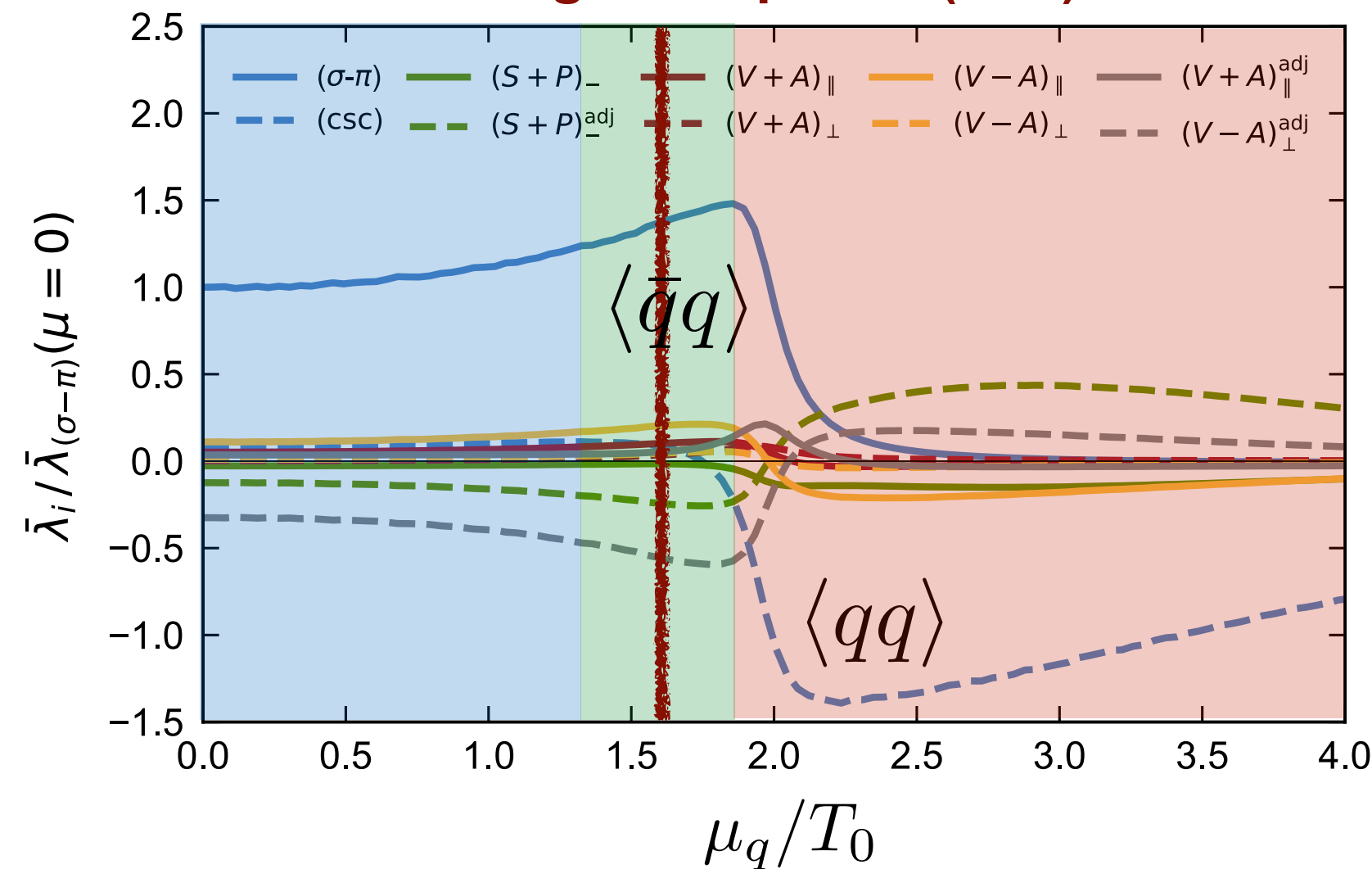
Predictions & estimates

Emergent diquarks

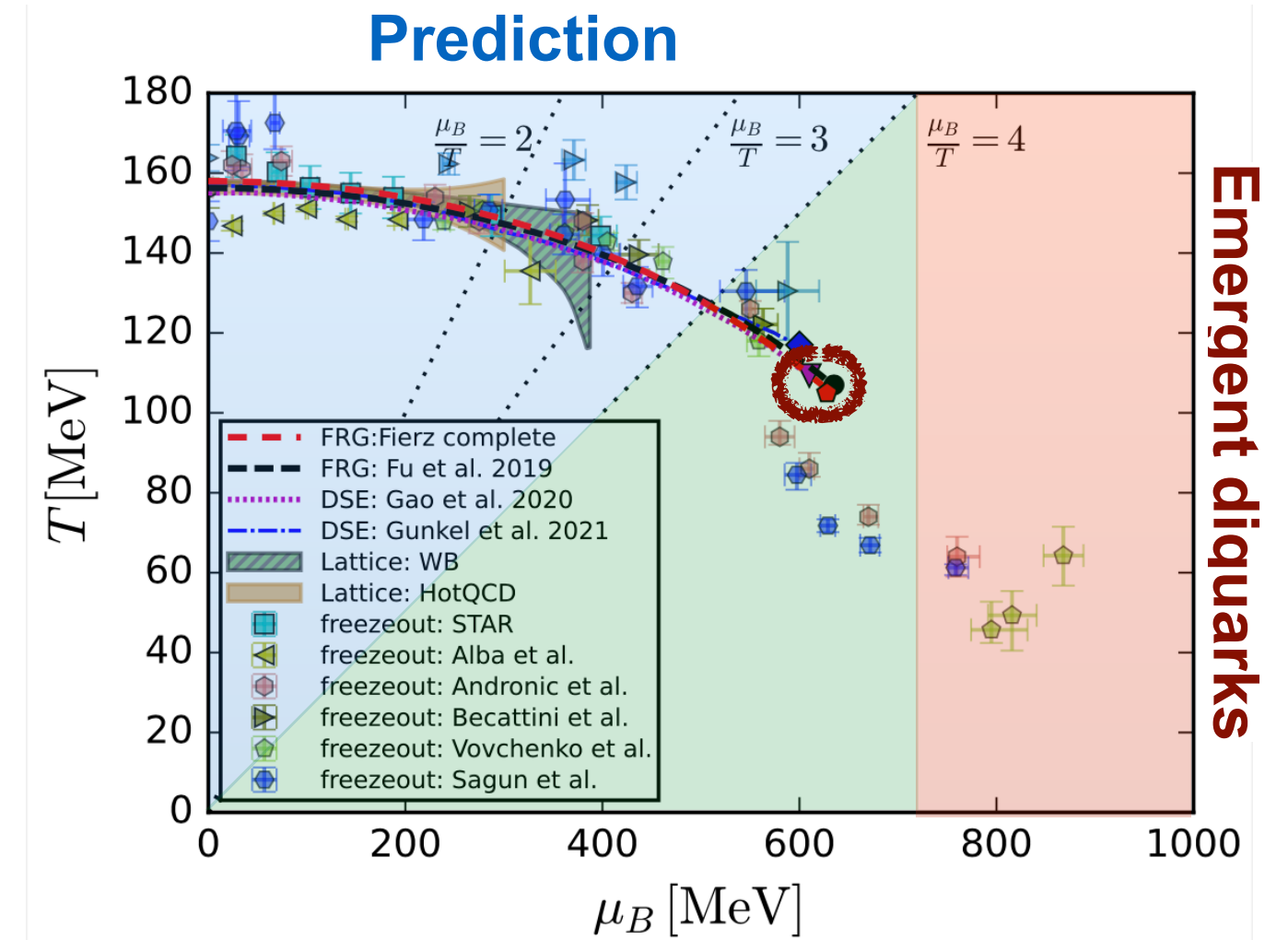


Regime of quantitative reliability
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Emergent diquarks (fRG)

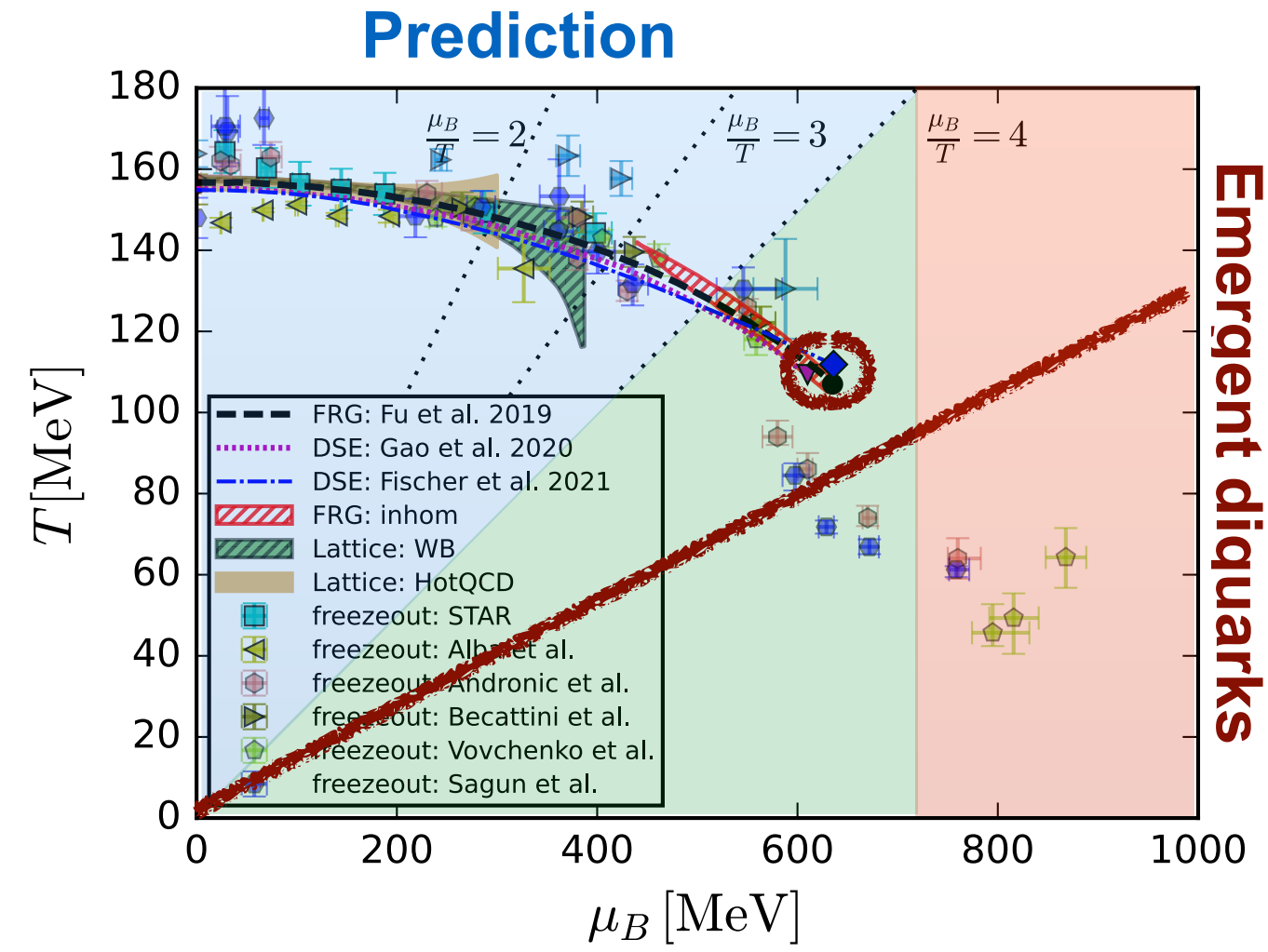


Fierz-complete phase structure



Predictions & estimates

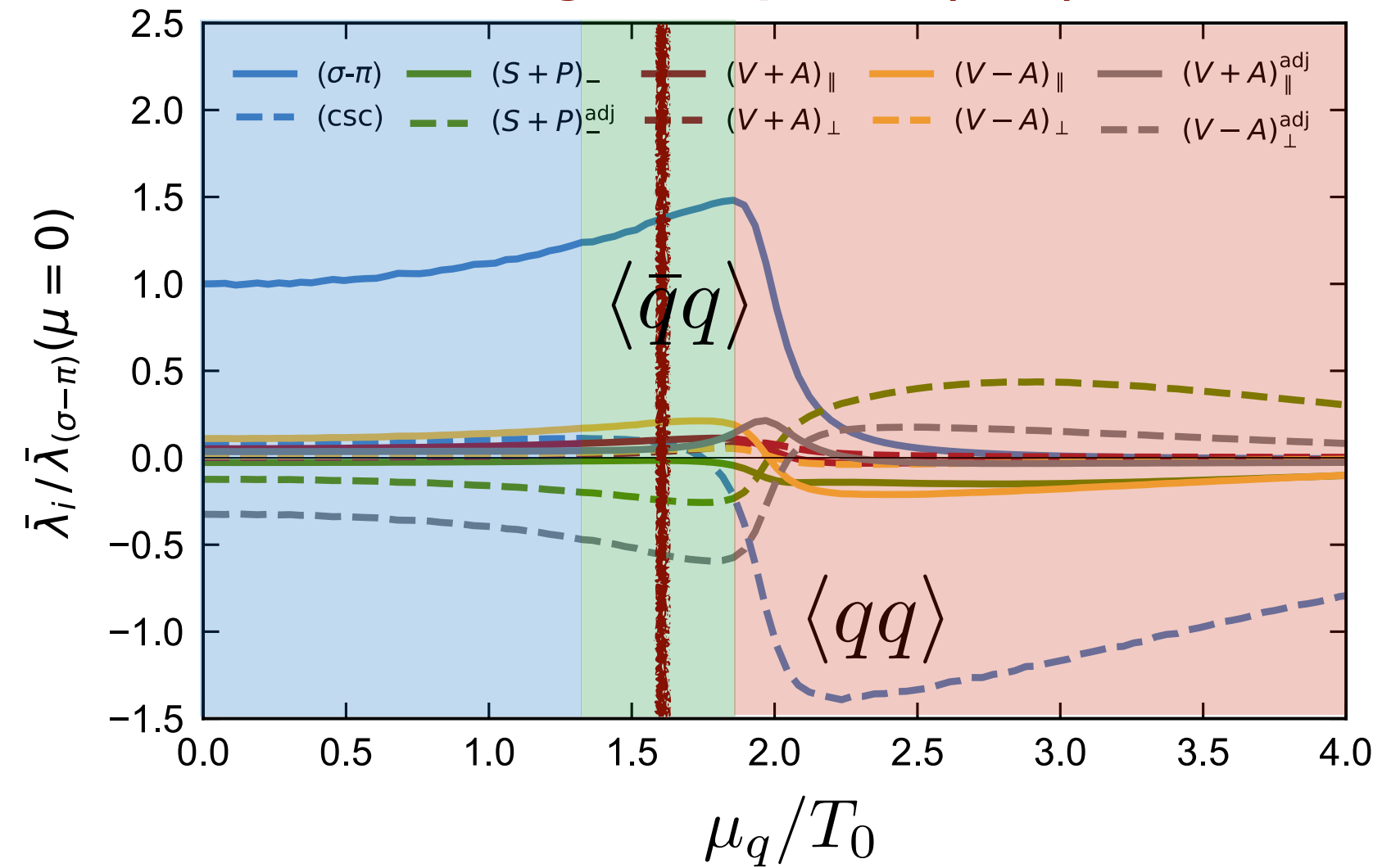
Emergent diquarks



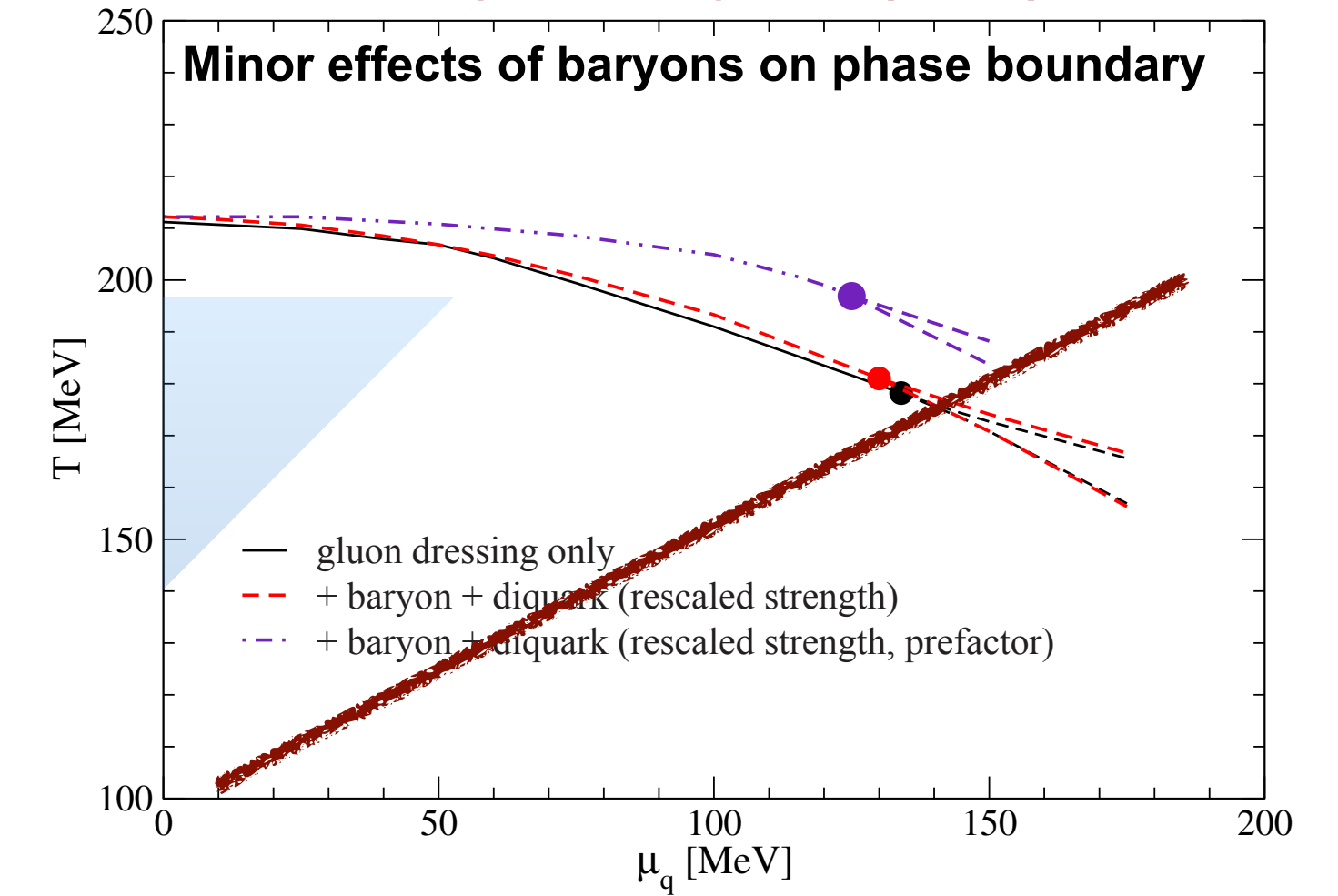
Estimate

Regime of quantitative reliability
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current best truncation

Emergent diquarks (fRG)



Emergent baryons (DSE)

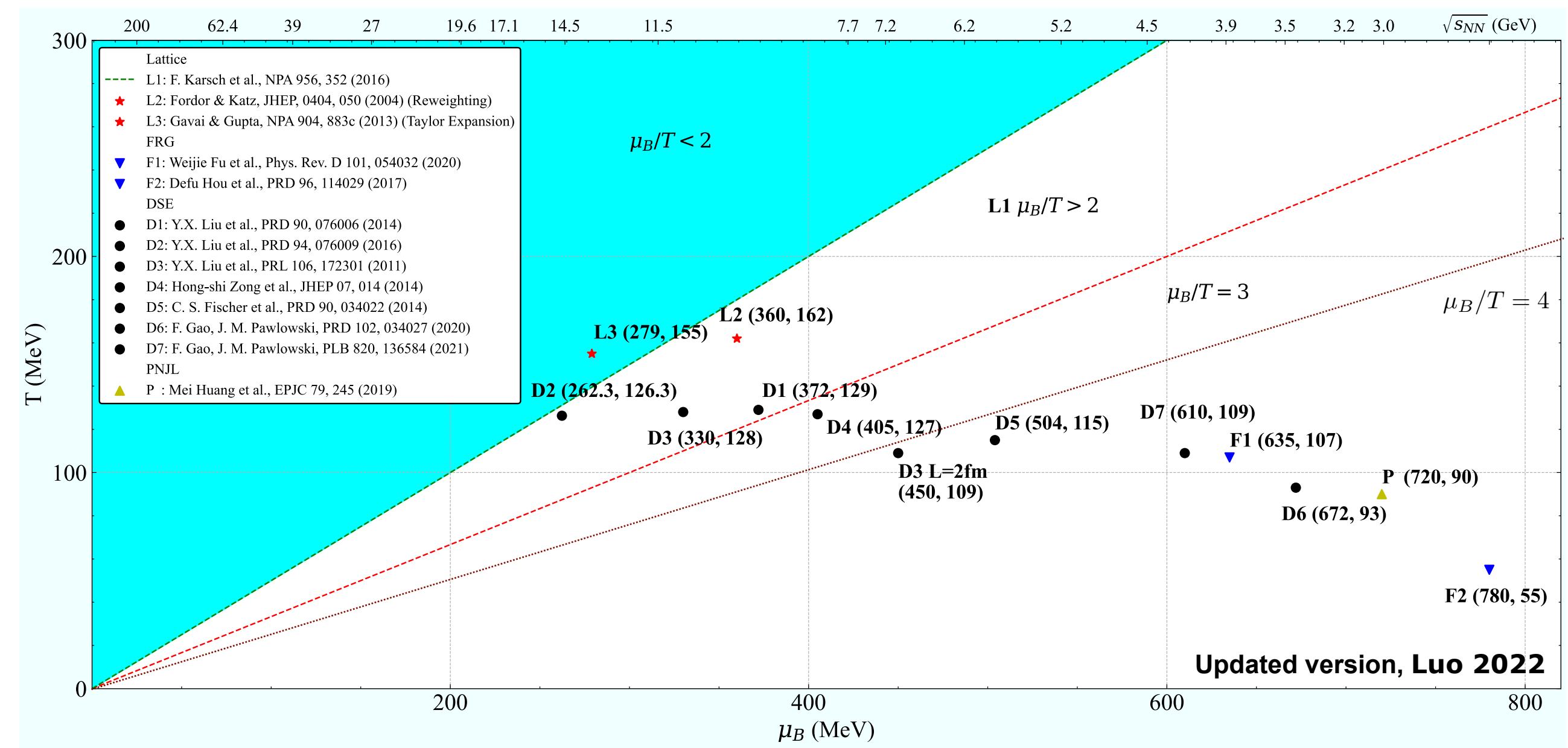


Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

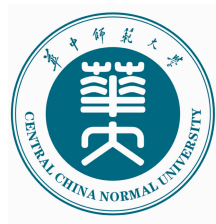
Disclaimer

Most functional computations
(LEFT or QCD) have not been
set-up for CEP-predictions!

Lack of predictive power
for CEP-predictions
is no quality measure!

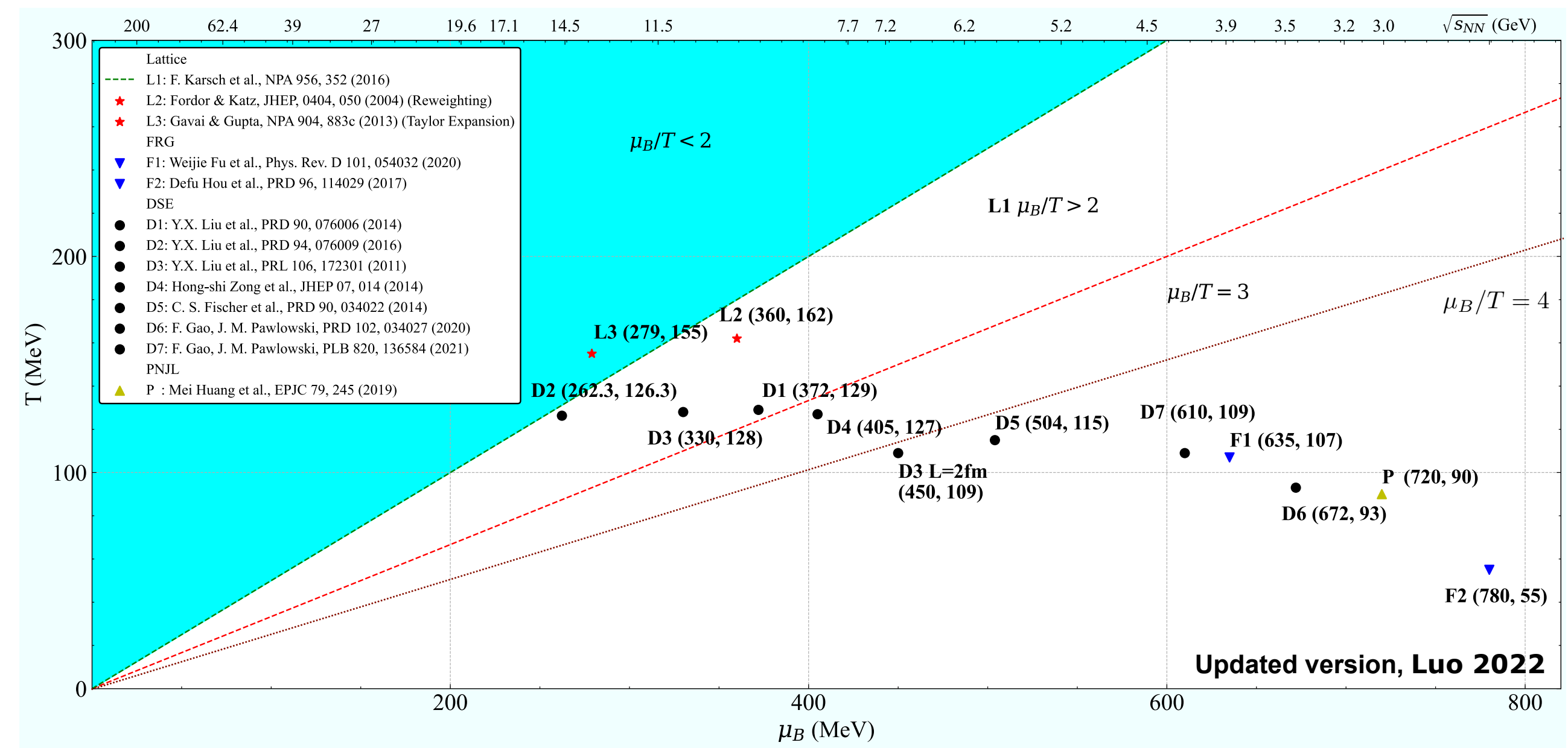
CEP is standing for
'regime with new physics'

Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

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Common folklore
since ~2004

Large uncertainties for the estimation of CP location.

Xiaofeng Luo

The 10th RHIC BES theory and experiment online seminar, Oct. 6th, 2020

9

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

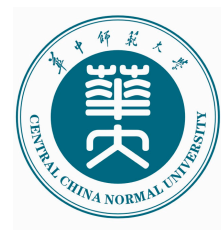
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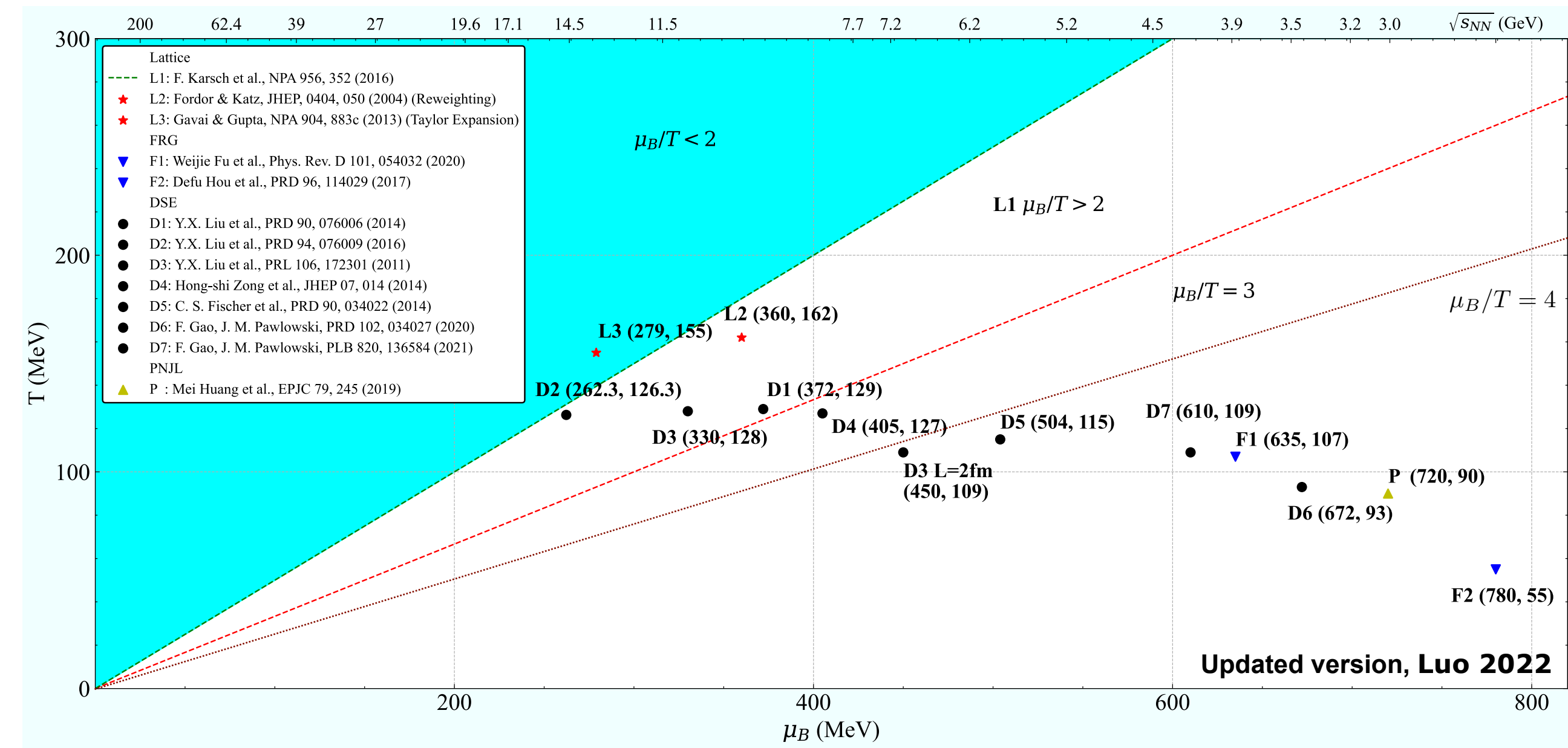
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Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

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Large uncertainties for the estimation of CP location.

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) ‘old’ CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

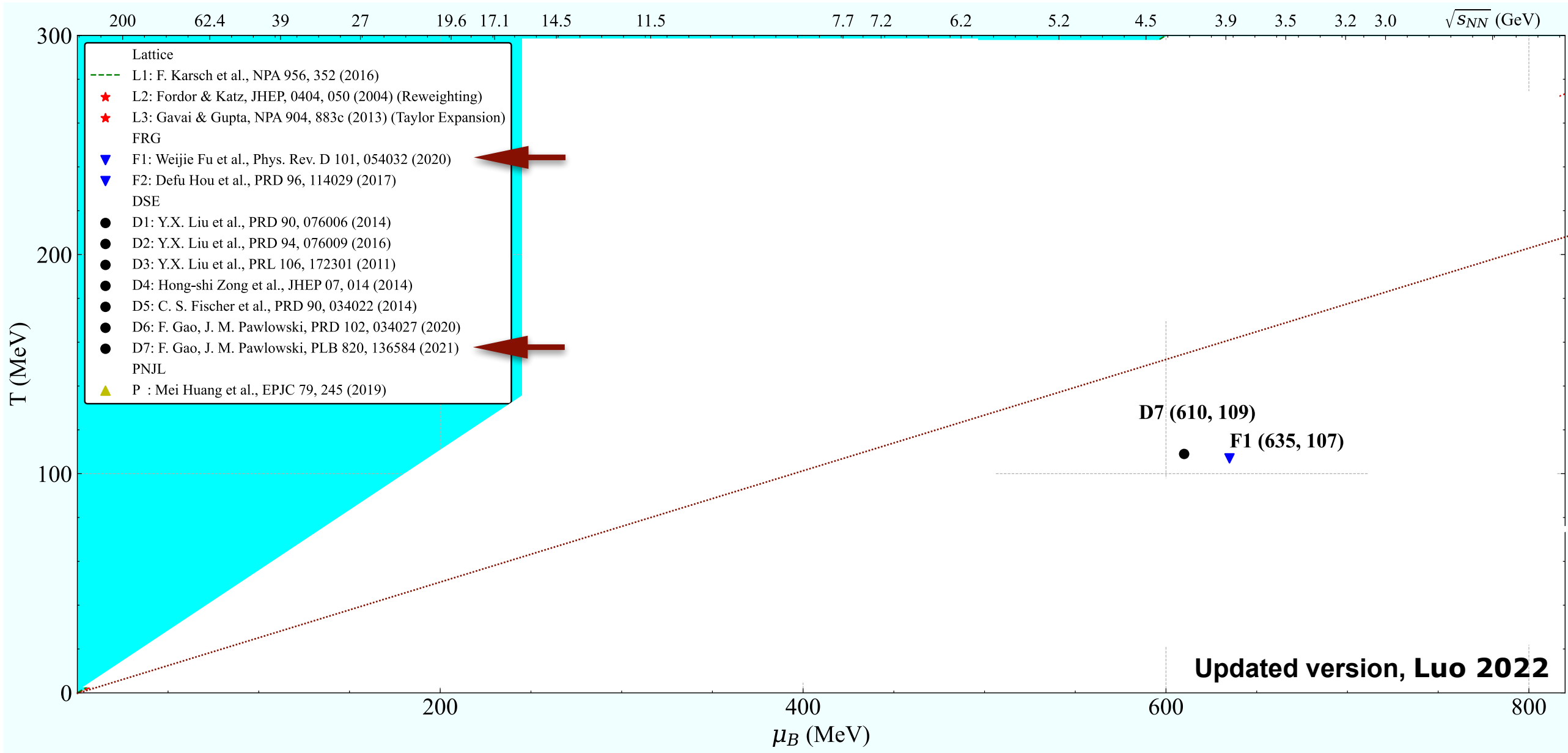
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Functional QCD



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Predictions, estimates & extrapolations and how to judge them

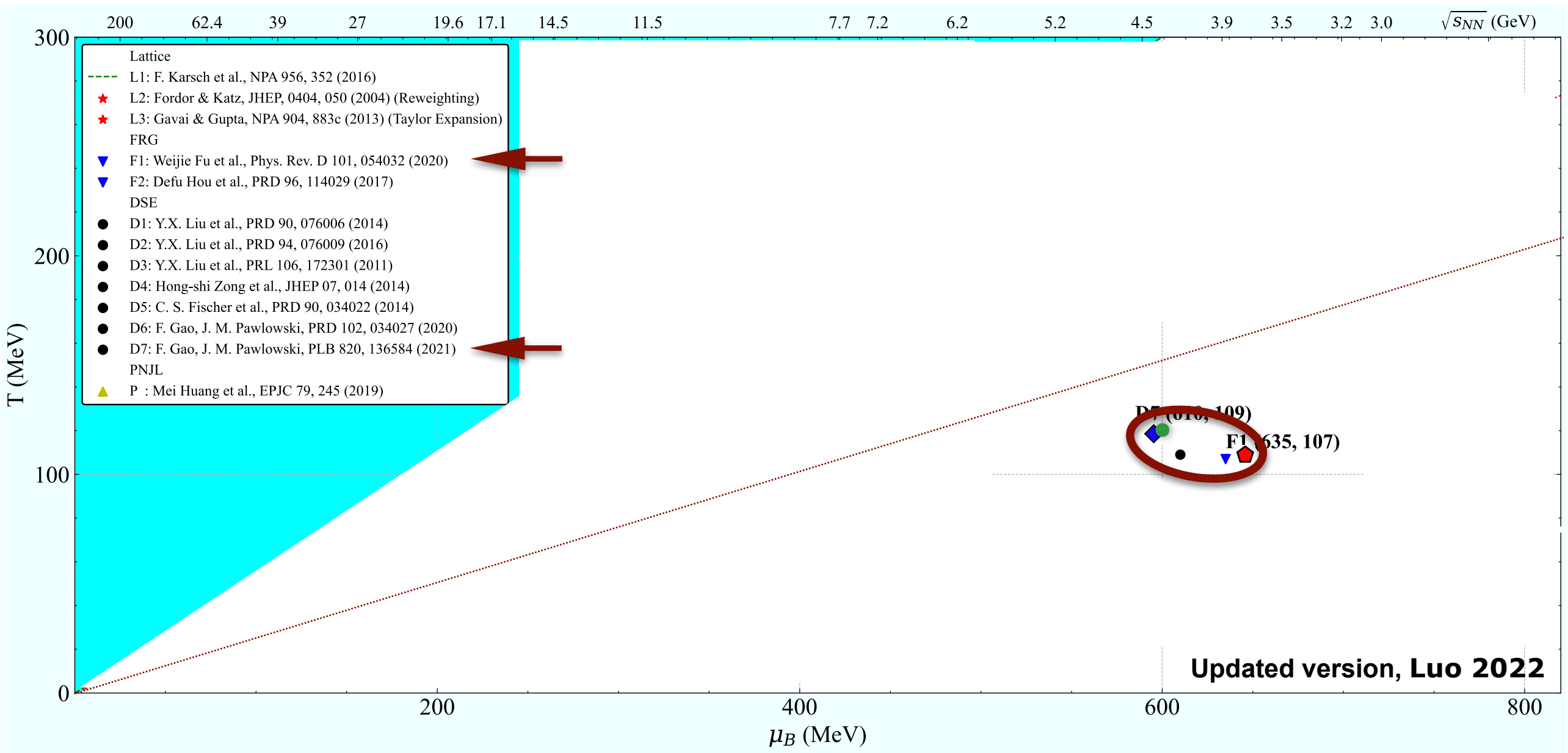


Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

Functional QCD

- ◆ Gao, Lu, JMP, Schneider, in prep (DSE)
- ◆ Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)



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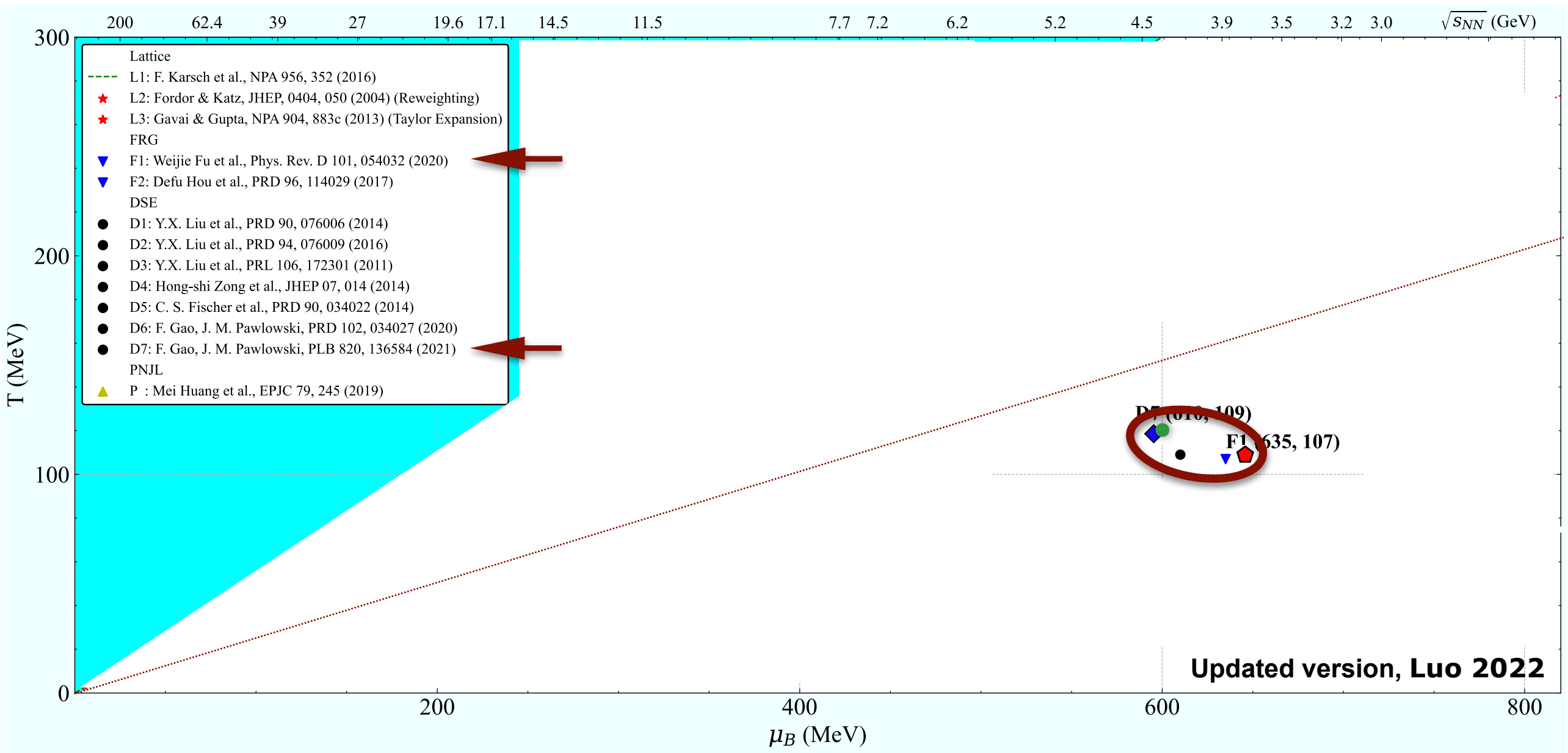


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Still small uncertainties for the estimation of **CP location / Onset of new phases**

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

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Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

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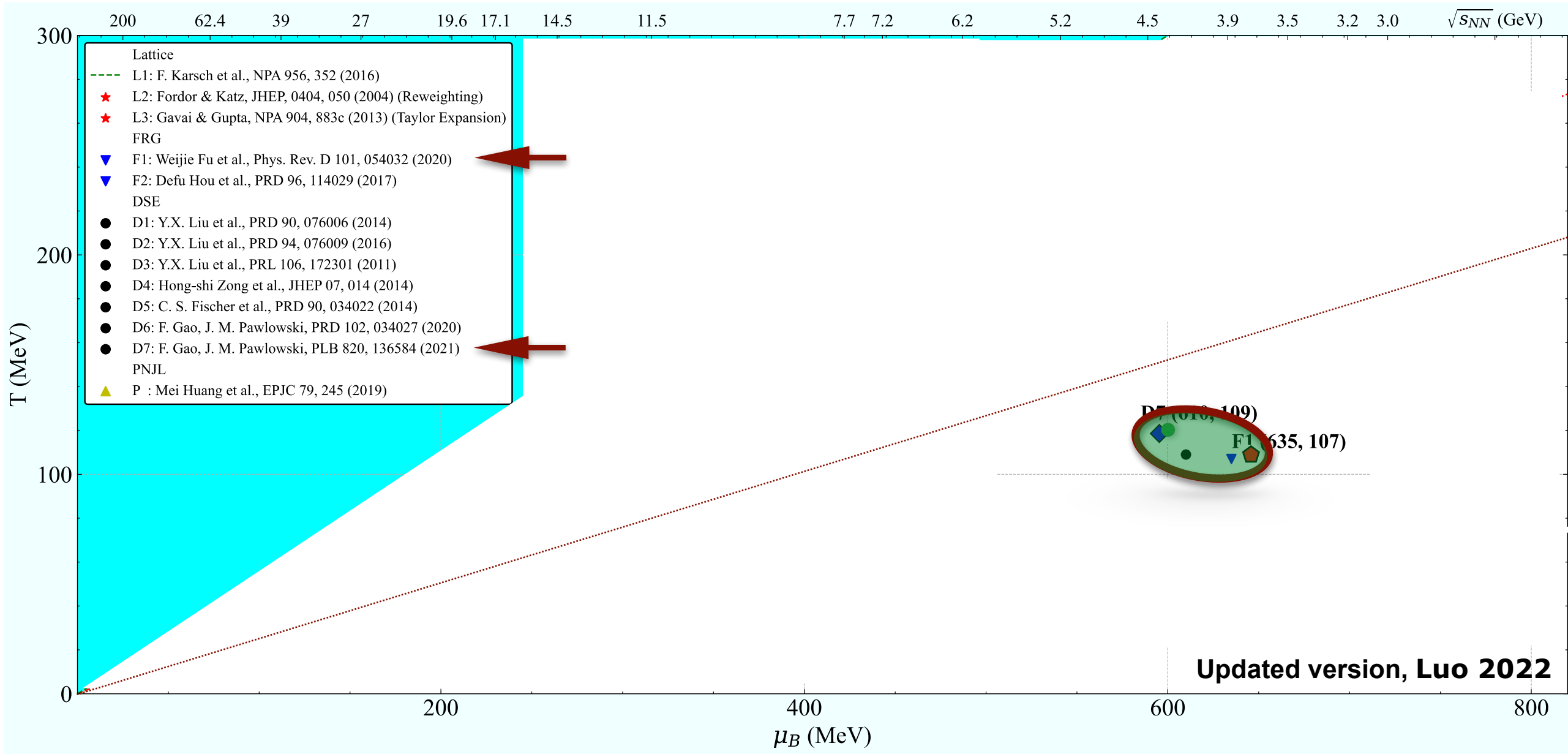
Extrapolations

Lattice extrapolations: Basar, PRC 110 (2024) 015203
Bielefeld-Parma, arXiv:2405.10196

⋮

Holographic models: Hippert, Grefa, Manning, Noronha,
Noronha-Hostler, Portillo Vazquez, Ratti,
Rougemont, Trujillo, arXiv: 2309.00579

⋮



Still small uncertainties for the estimation of **CP location / Onset of new phases**

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

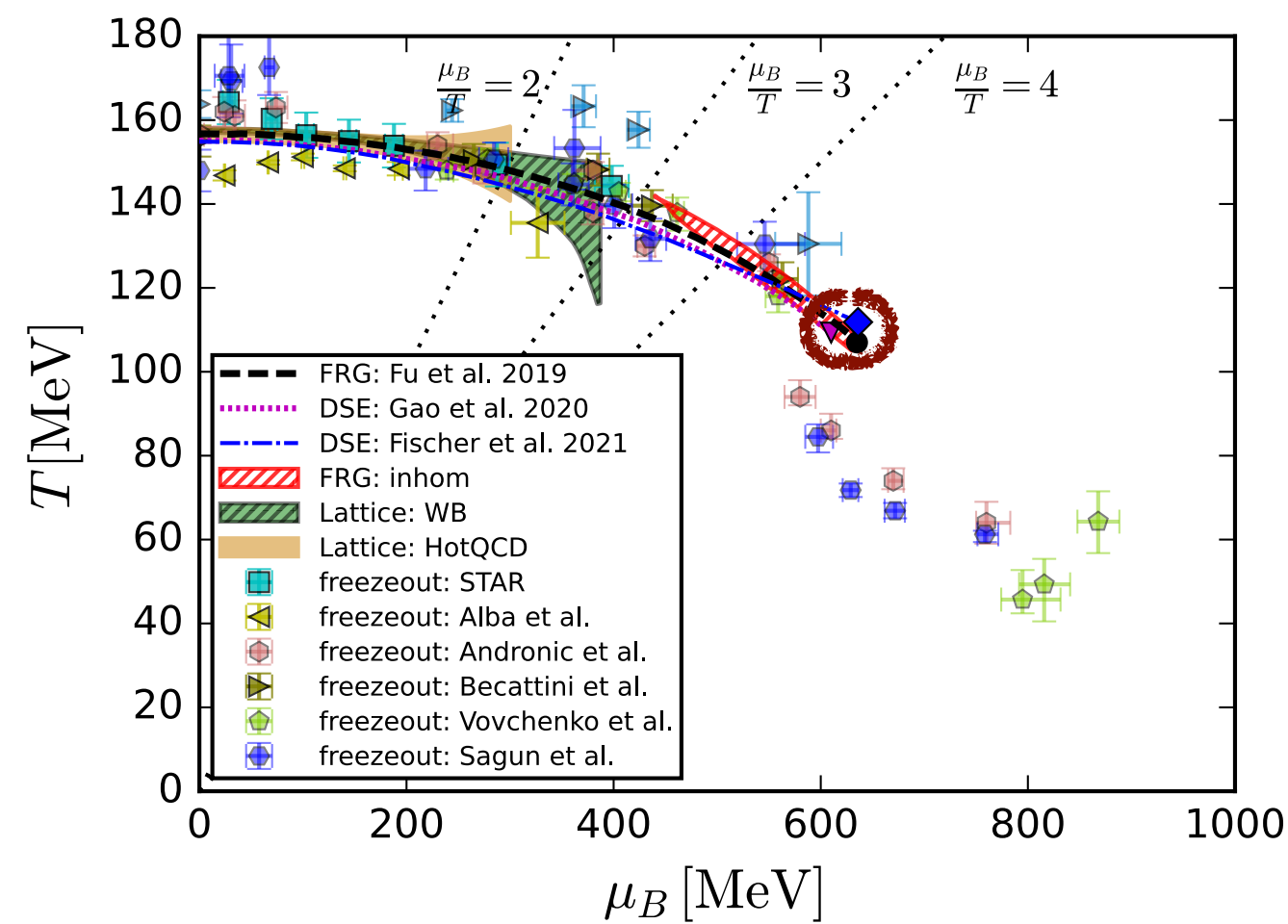
(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$

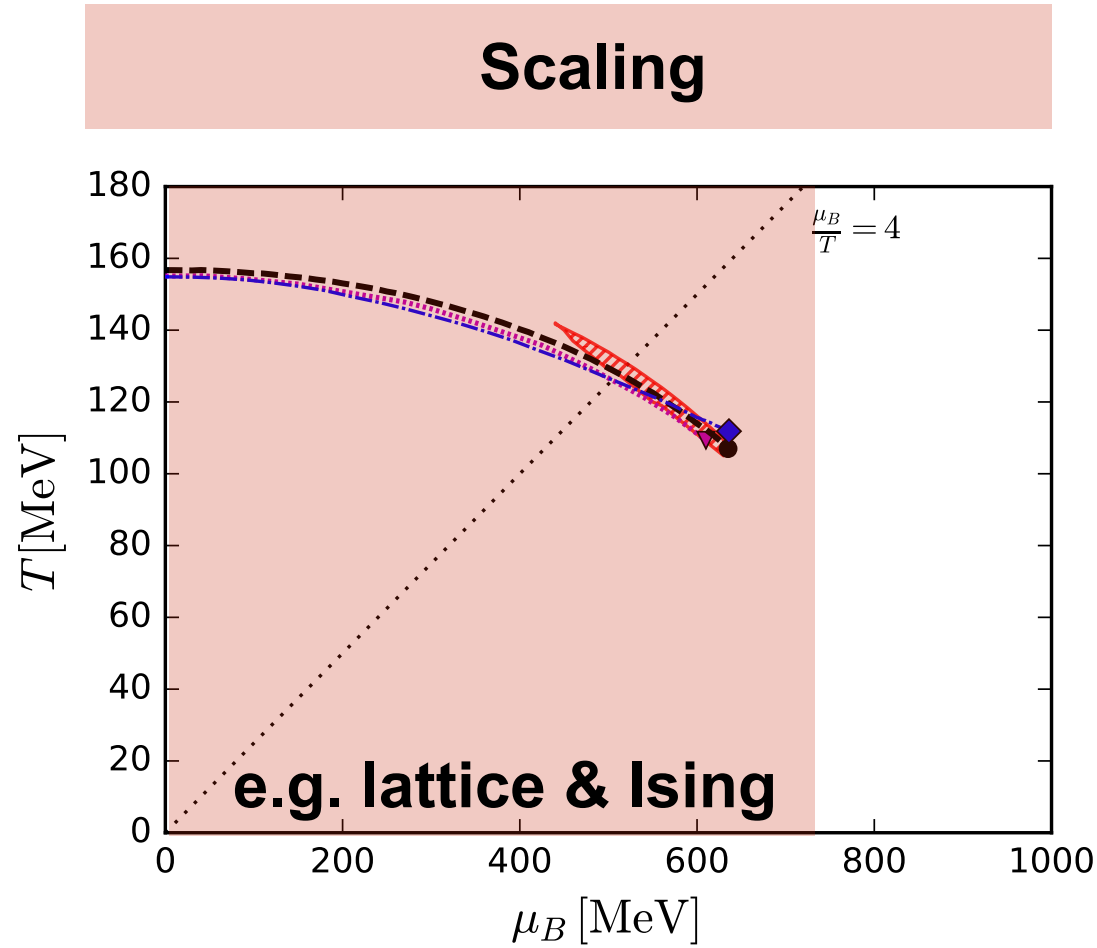
(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Predictions, estimates & extrapolations and how to use them

Predictions, estimates & extrapolations and how to use them



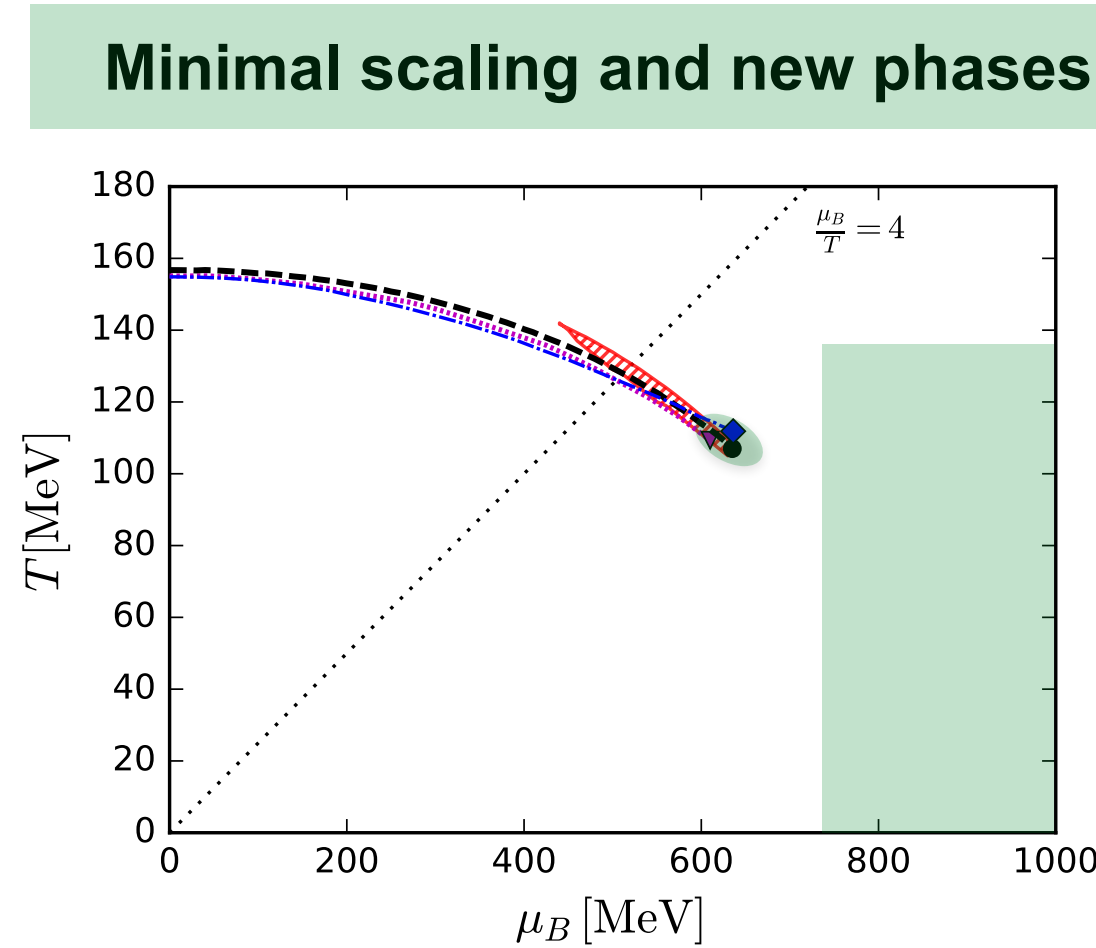
Scenario I



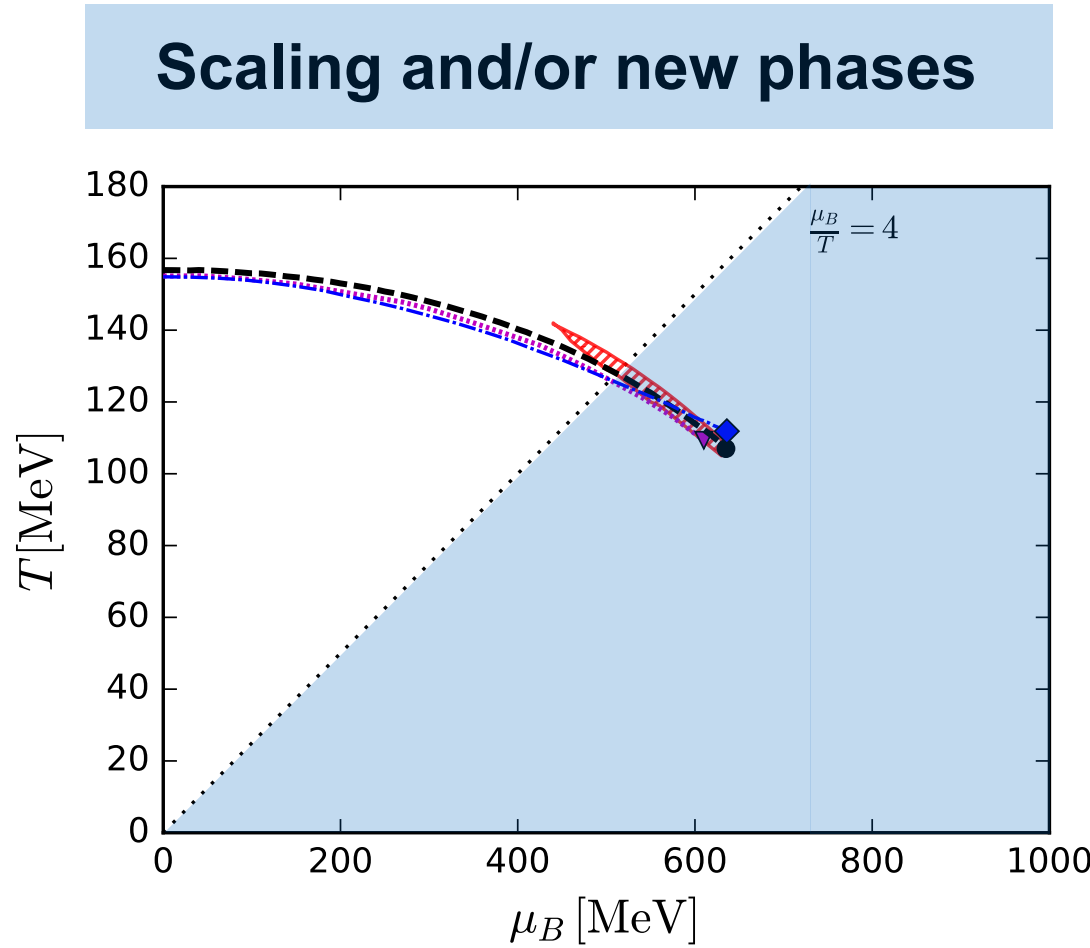
Extrapolations
for
Pheno



Scenario II



Scenario III



Predictions, estimates & extrapolations and how to use them

Out by the LEGO[®] principle

Fu, JMP, Rennecke, PRD 101 (2020) 054032

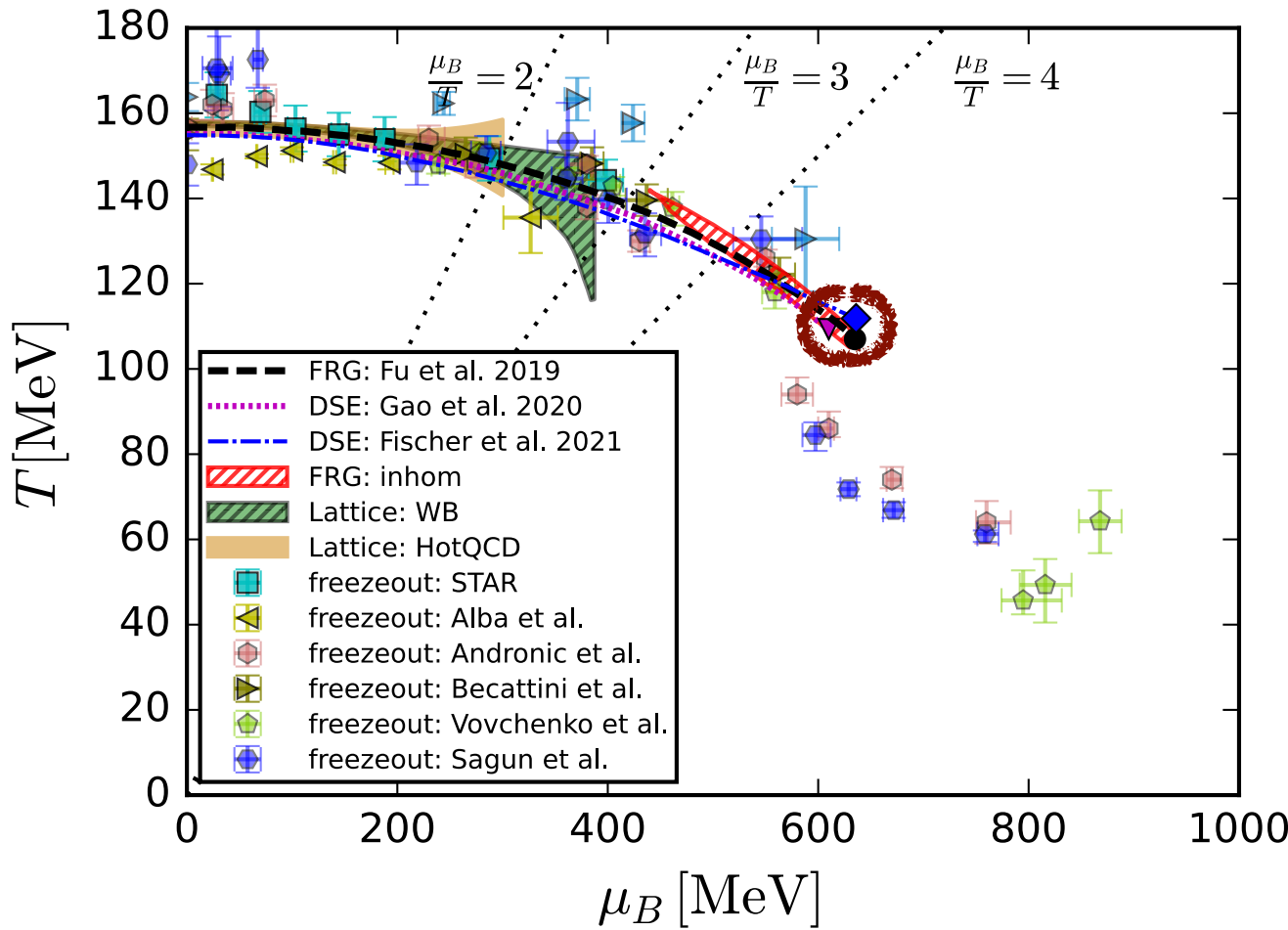
+

Size of scaling regime in LEFTs

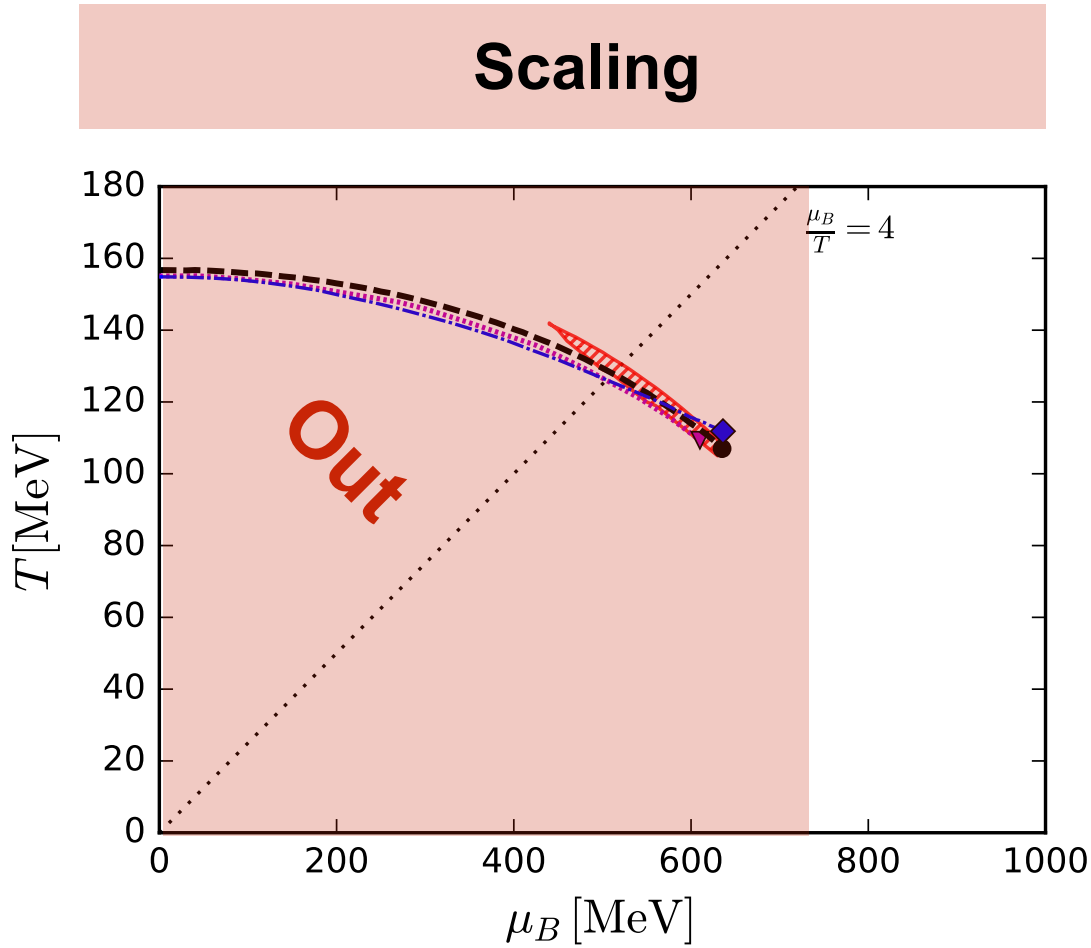
Schaefer, Wambach, PRD 75 (2007) 085015

Braun, Klein, Piasecki, EPJC 71 (2011) 1576

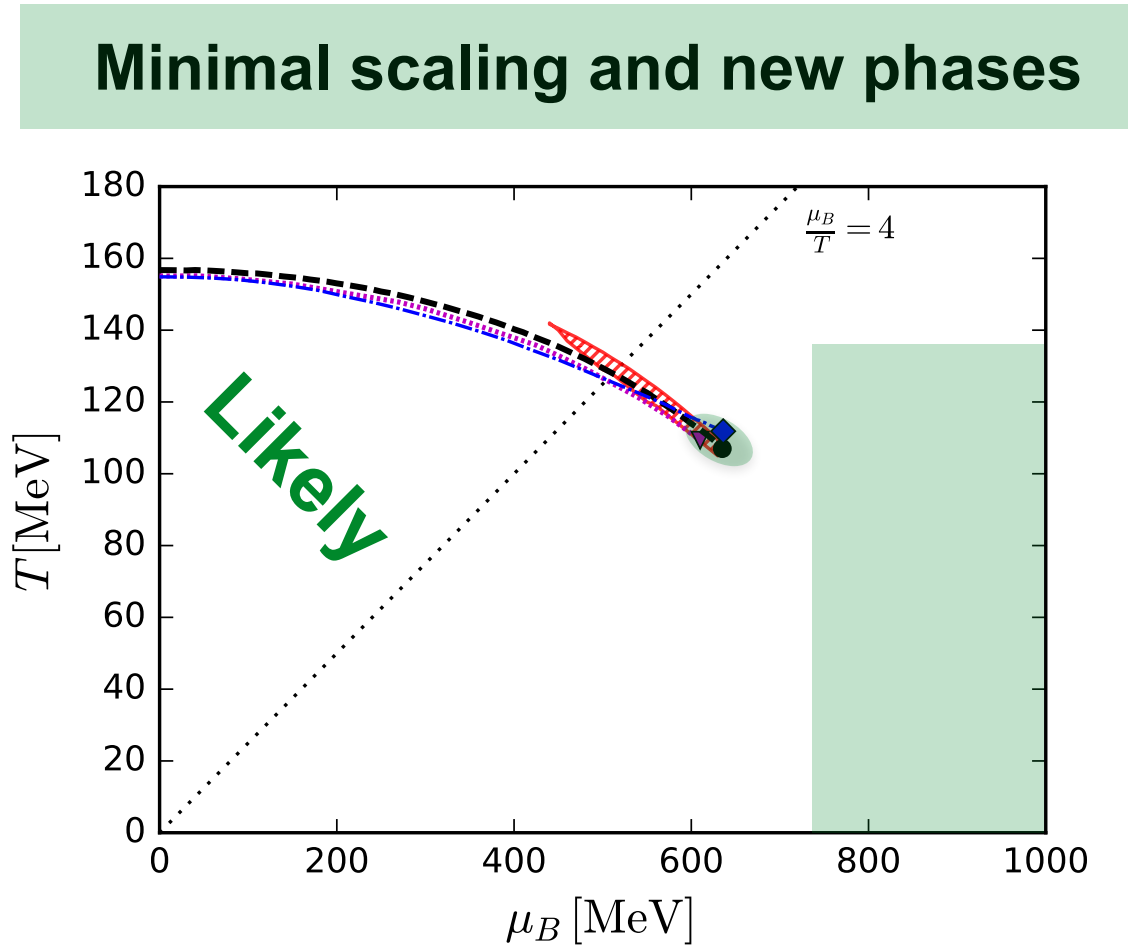
⋮



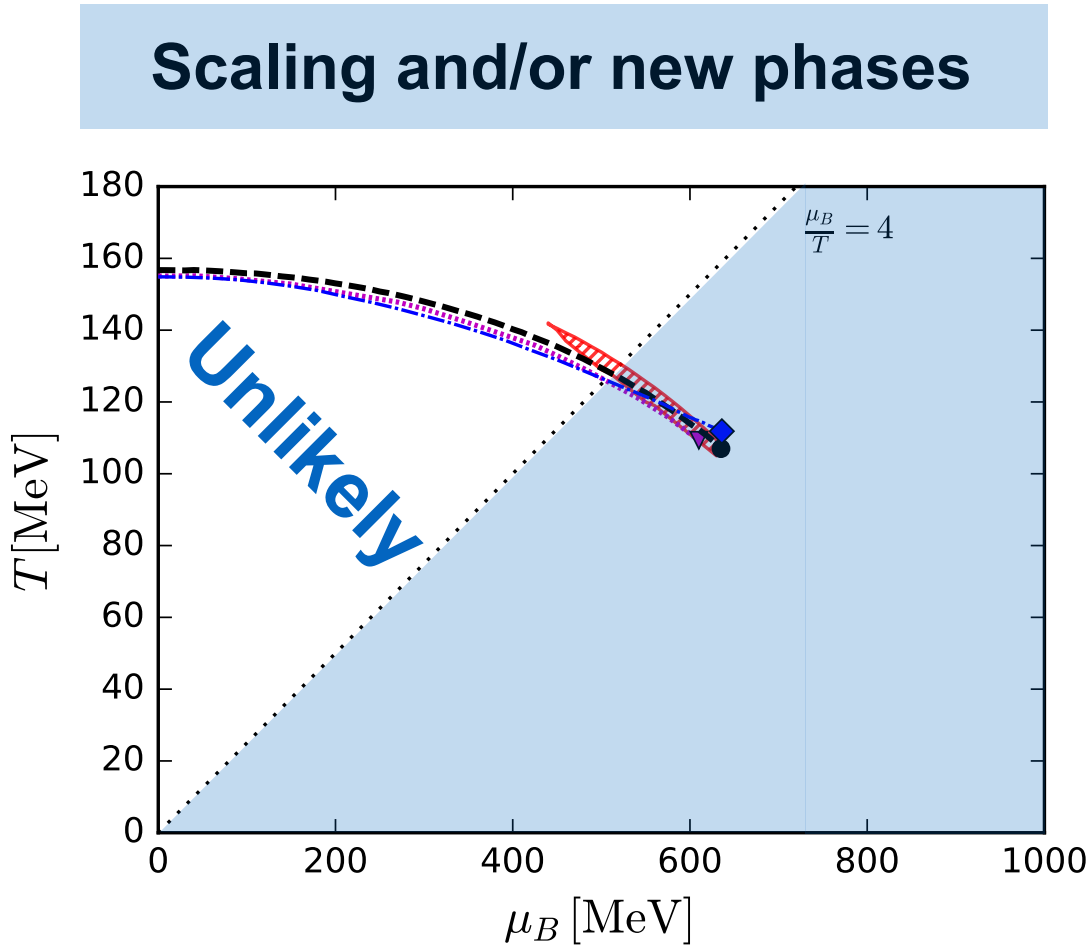
Scenario I



Scenario II



Scenario III



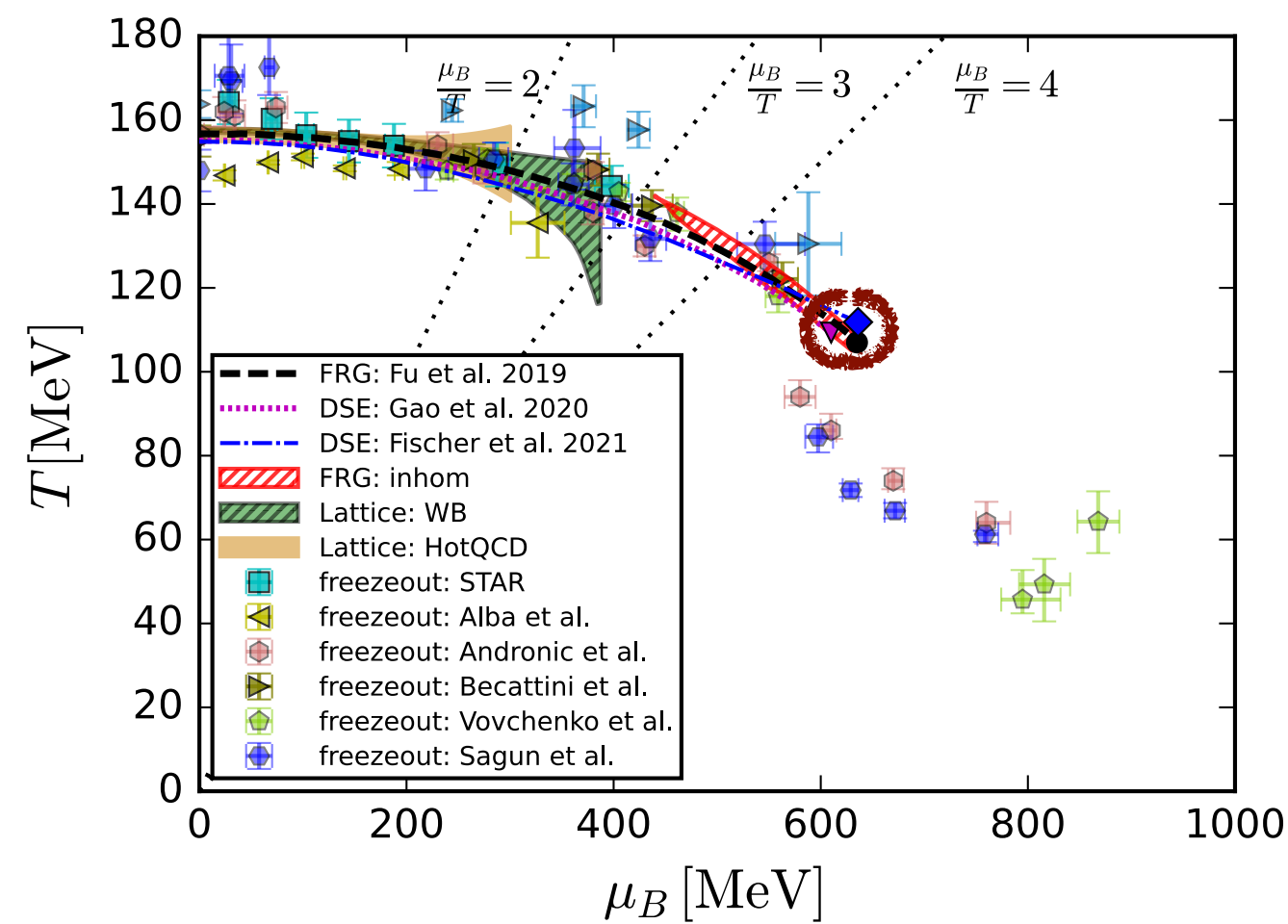
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010
Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853

**+ many results in dynamical
low energy effective theories**

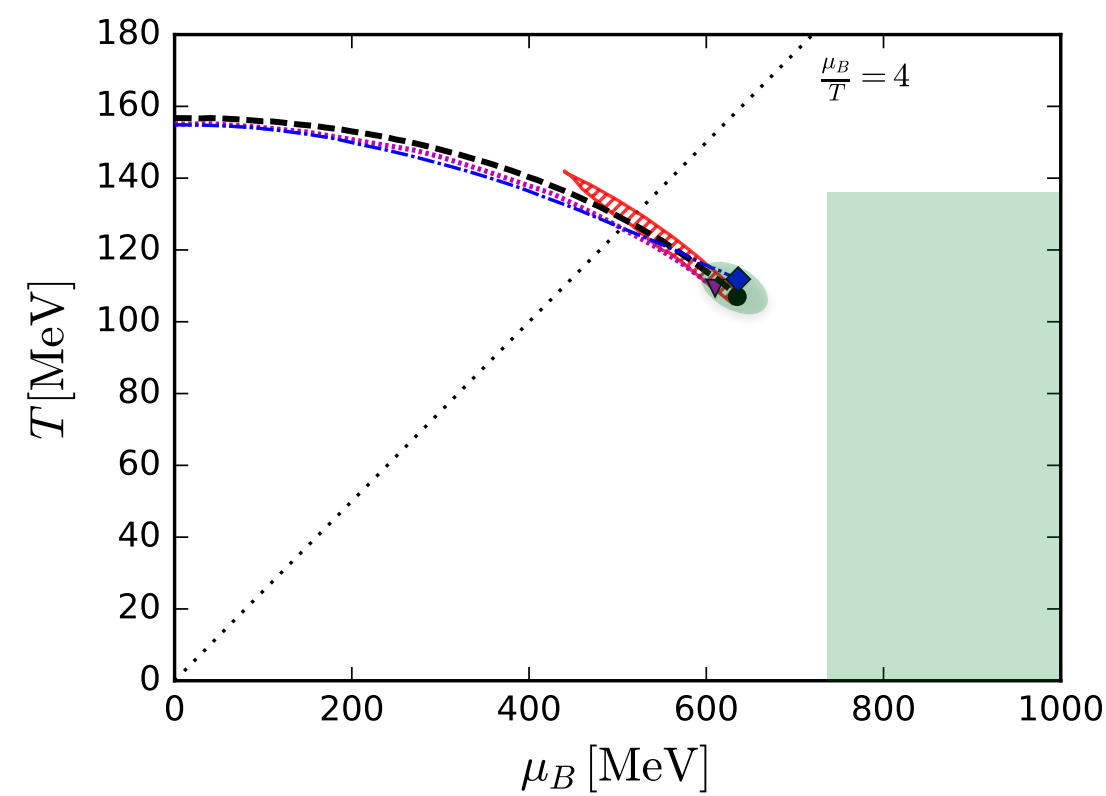
LEGO[®] principle

Predictions, estimates & extrapolations and how to use them



Scenario II

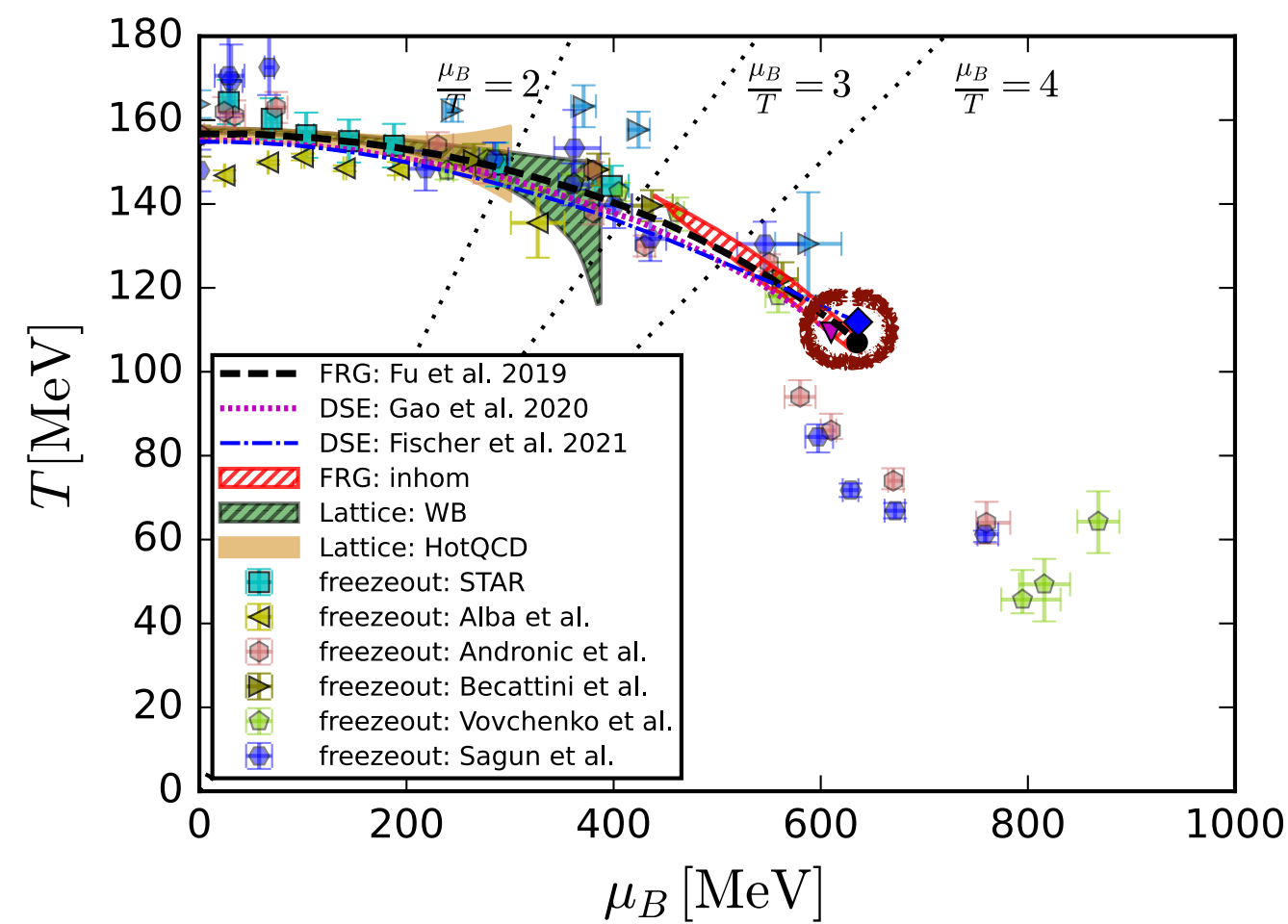
Minimal scaling and new phases



Extrapolations
for
Pheno

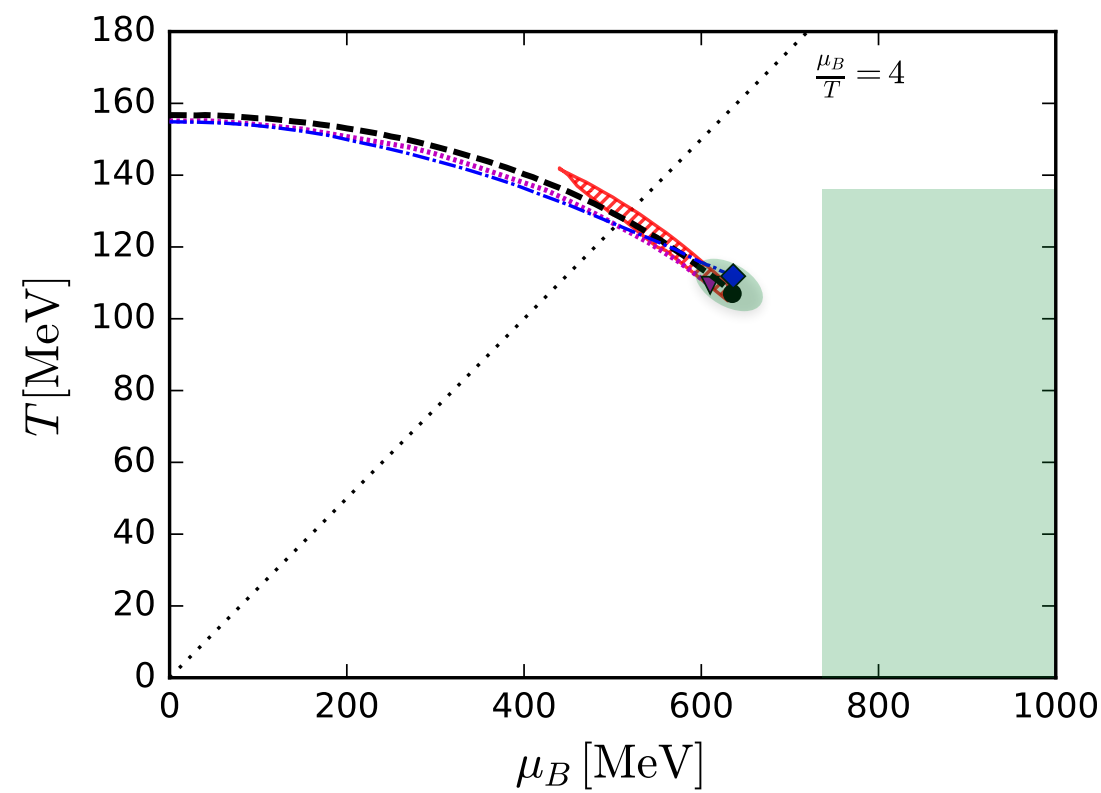


Predictions, estimates & extrapolations and how to use them



Scenario II

Minimal scaling and new phases

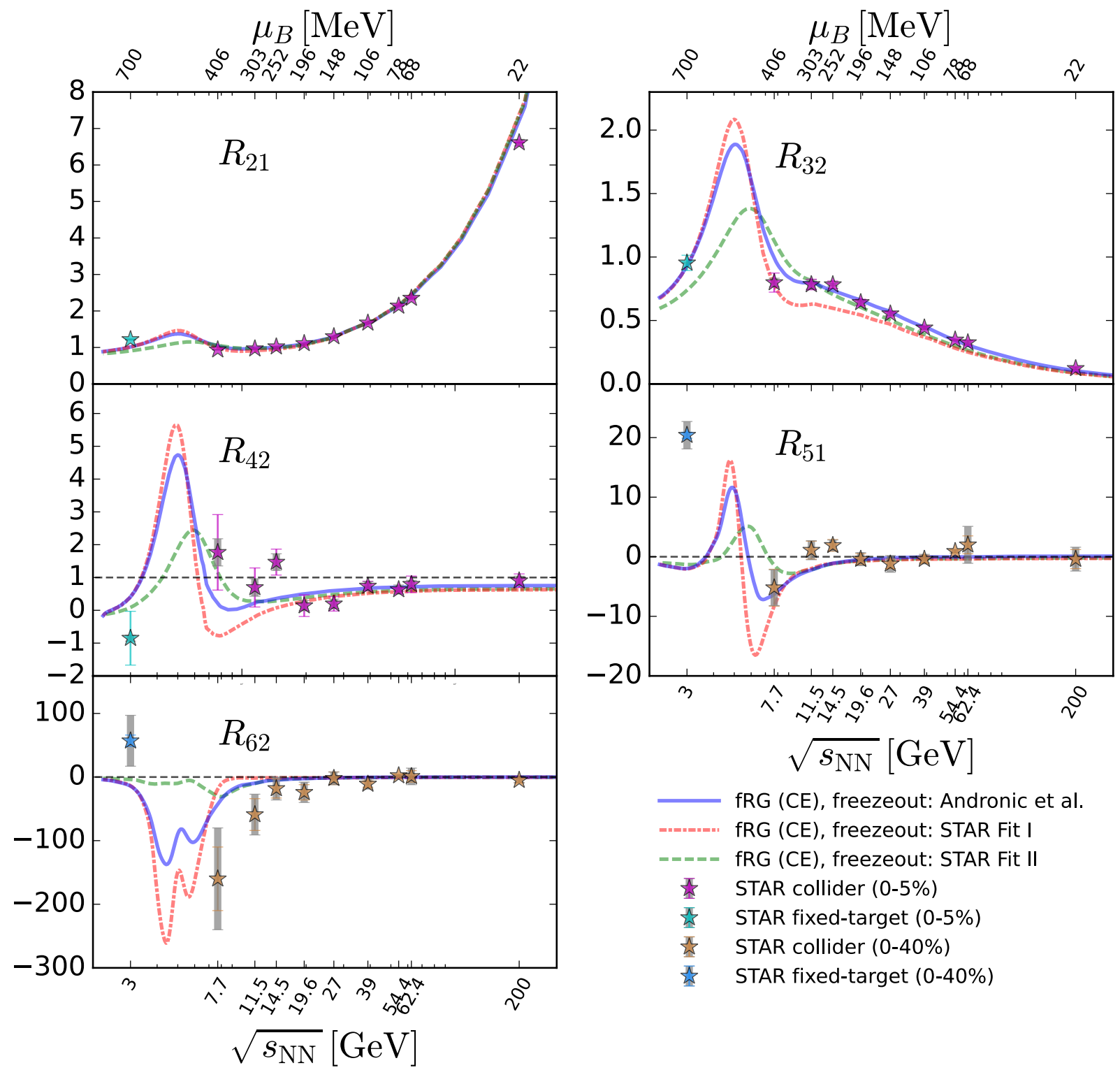


Extrapolations
for
Pheno



Ripples of the critical end point

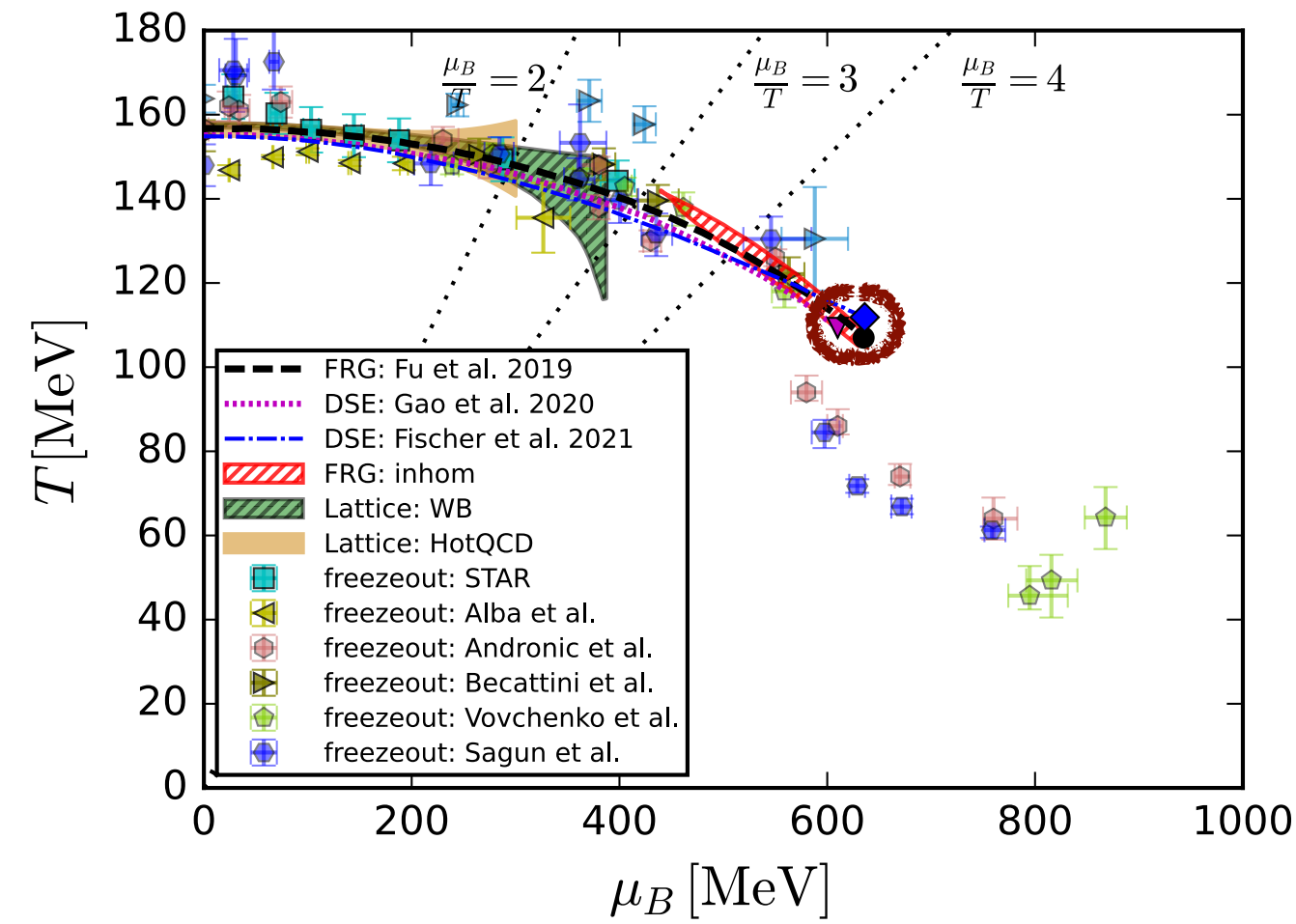
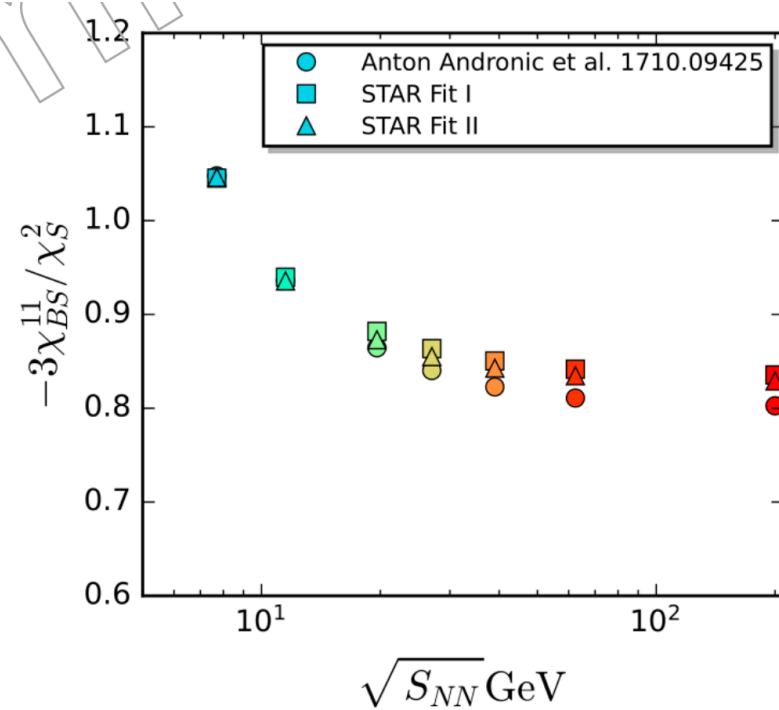
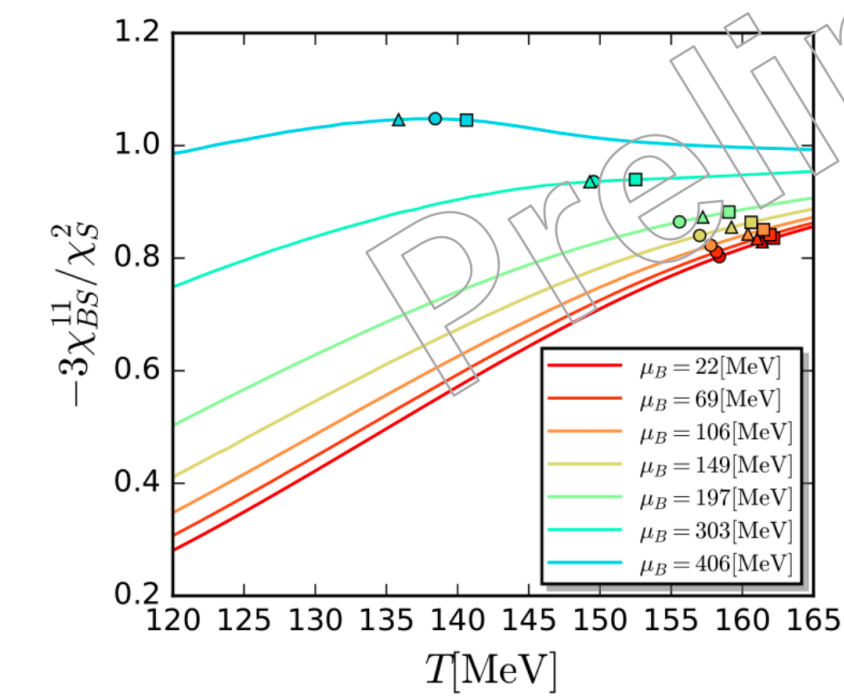
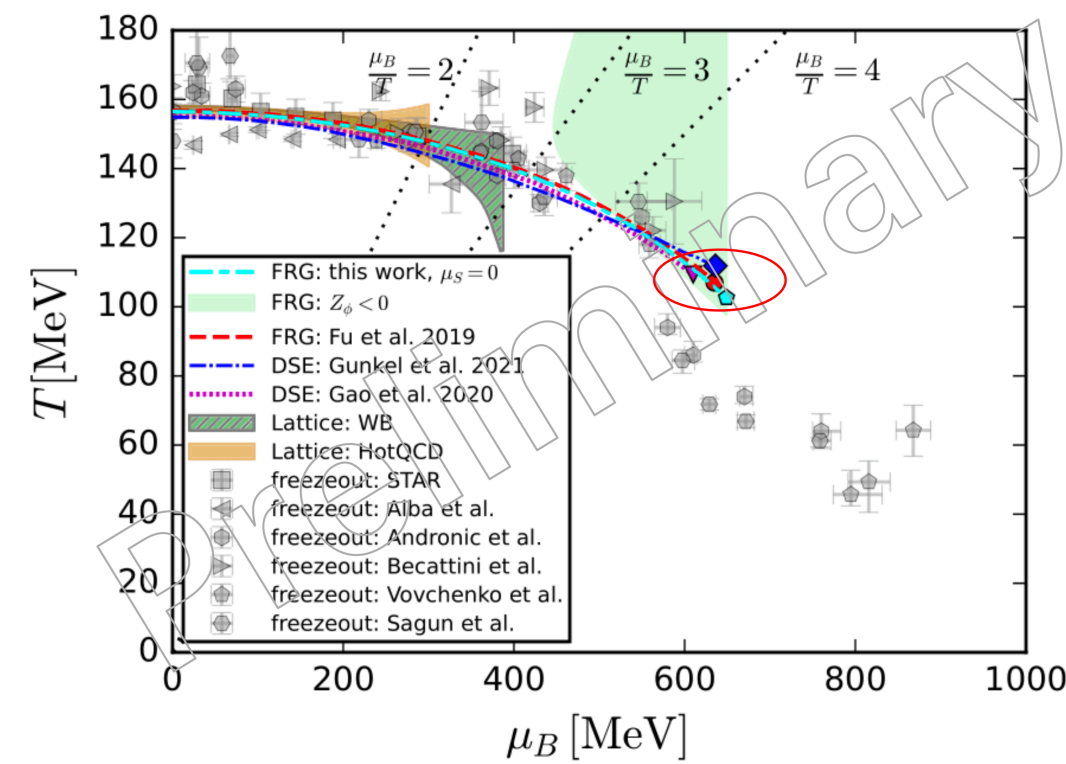
baryon & proton number fluctuations



Predictions, estimates & extrapolations and how to use them

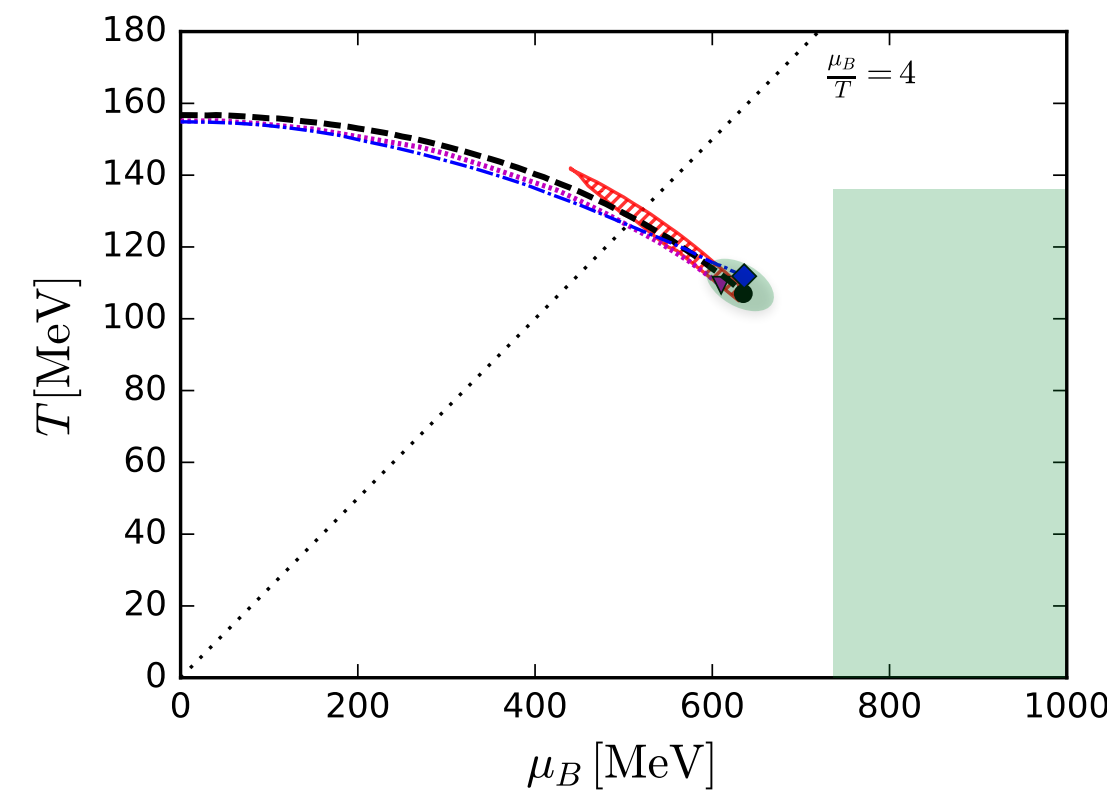
Strangeness neutrality

baryon & proton number fluctuations



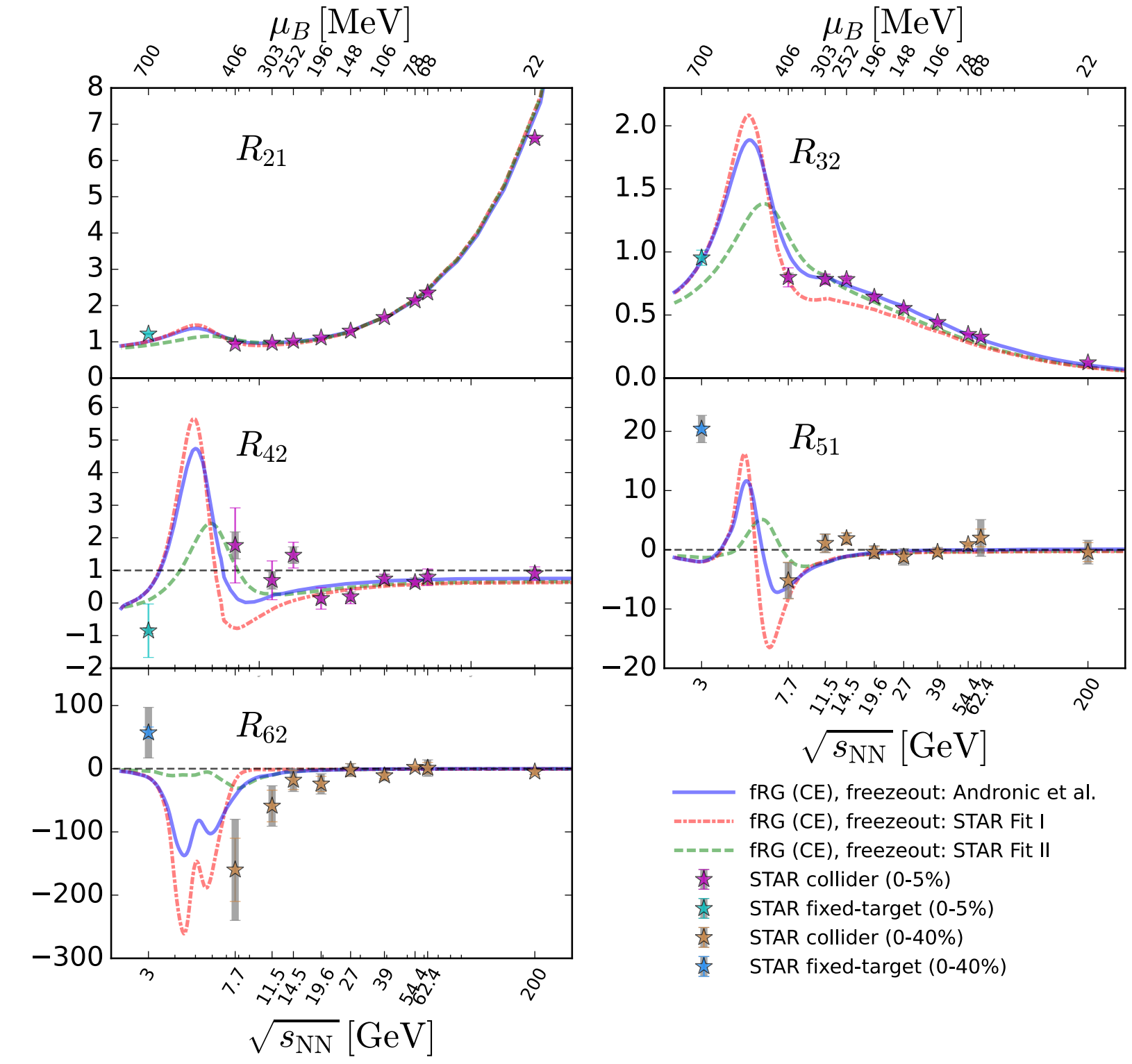
Scenario II

Minimal scaling and new phases



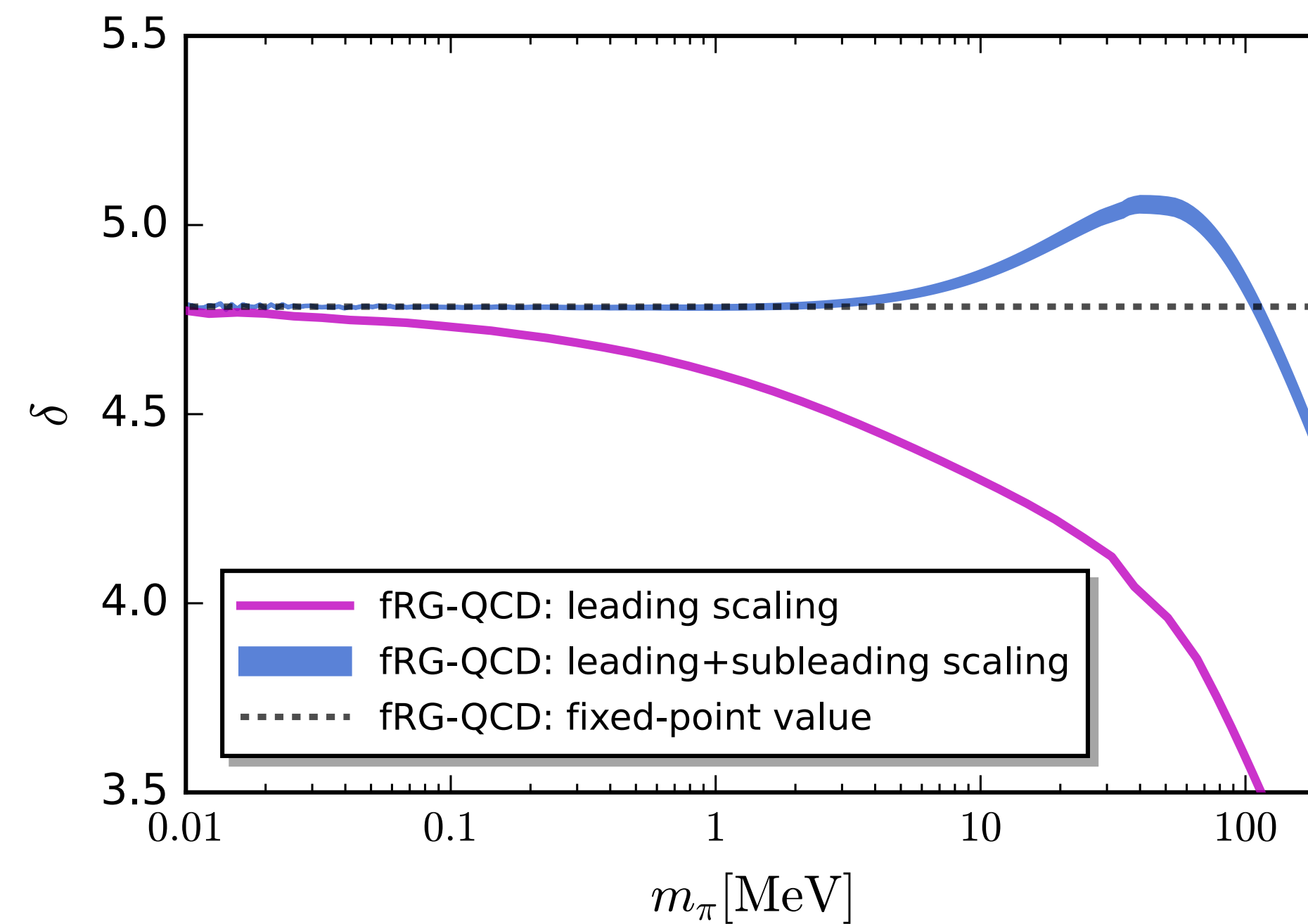
Ripples of the critical end point

baryon & proton number fluctuations



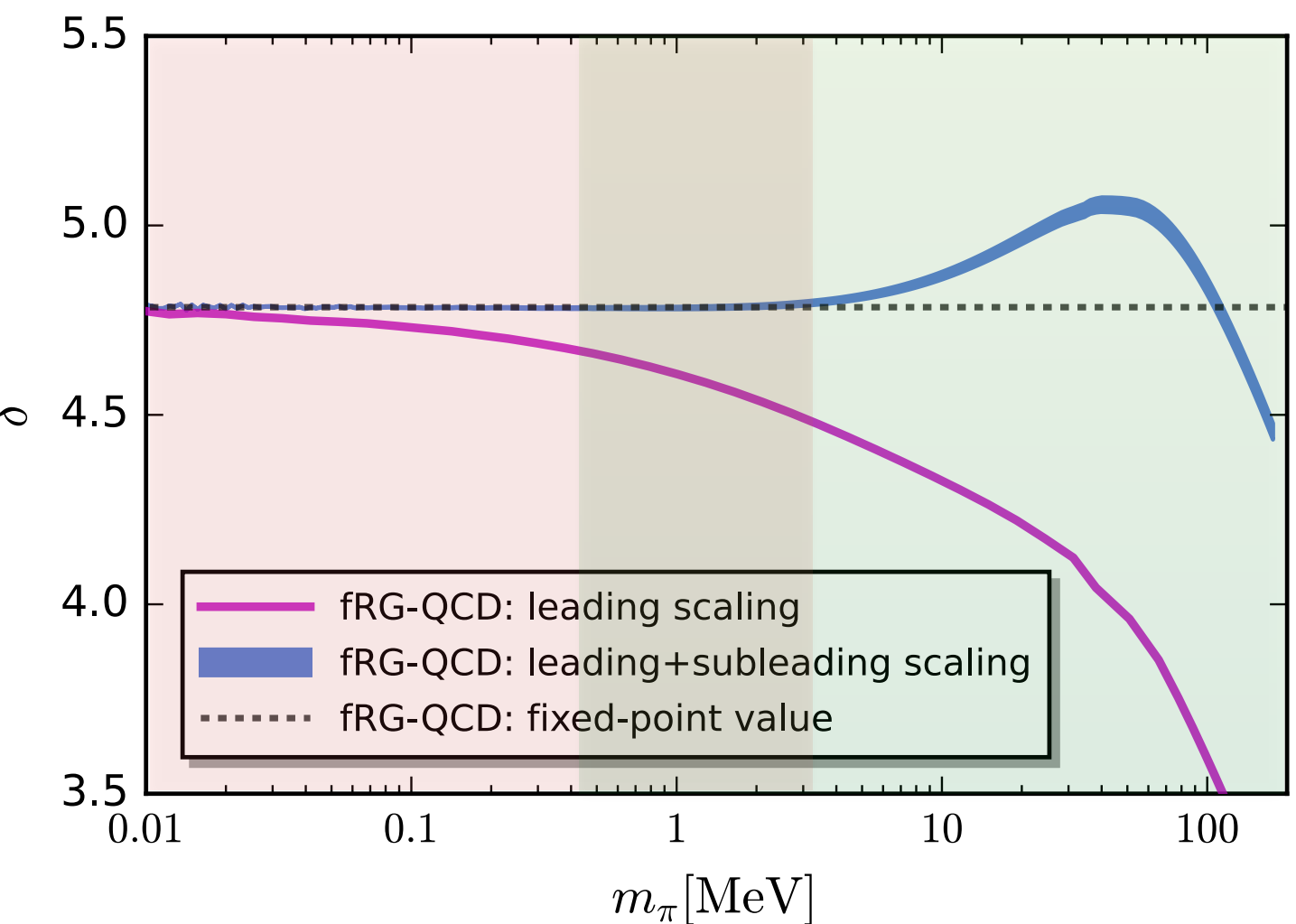
The physics of soft modes

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta} [1 + a_m m_\pi^{2\theta_H} + \dots]$$



Chiral dynamics & quasi-massless modes in the chiral limit

Scaling coefficient as function of the pion mass



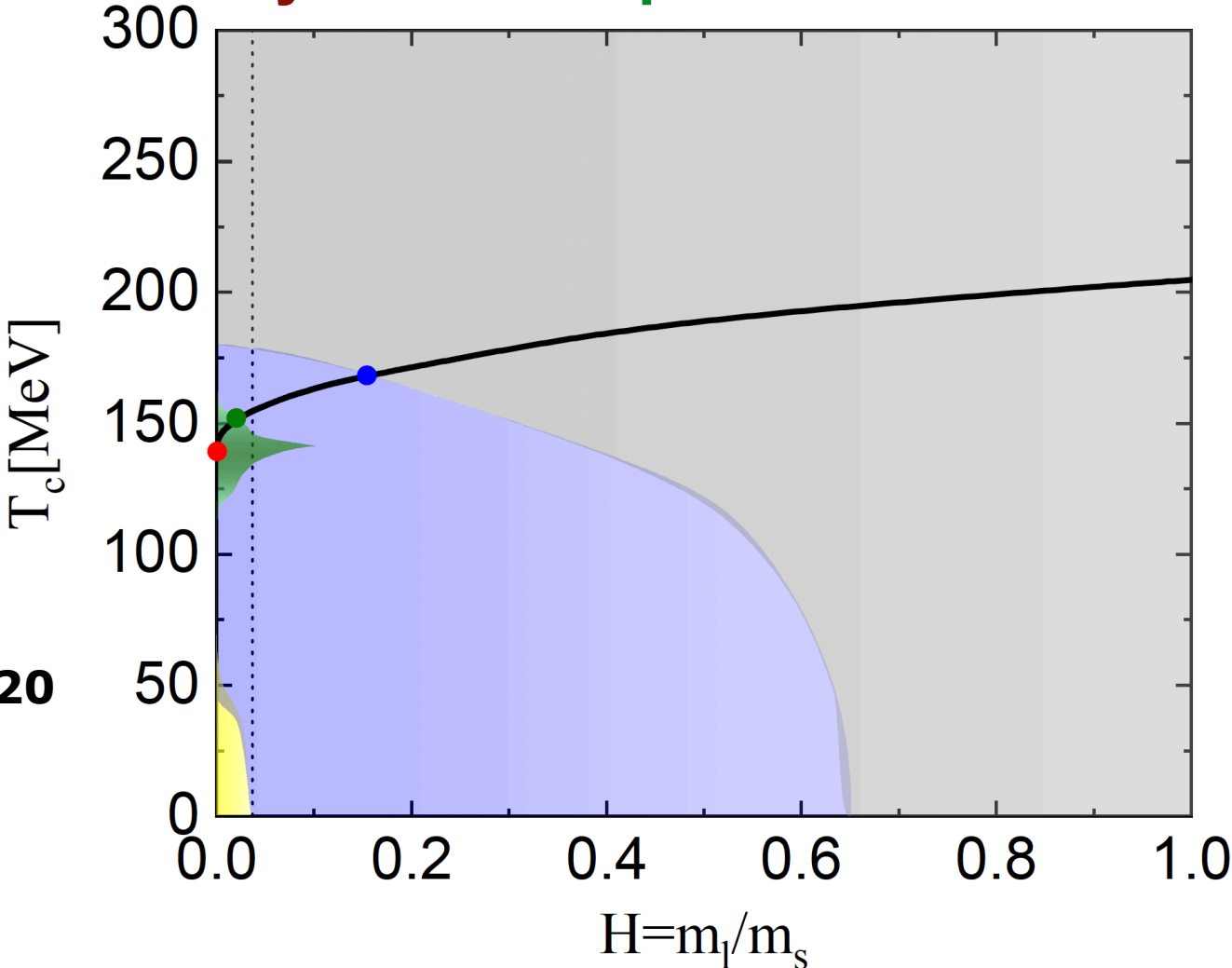
Critical O(4) scaling

‘chiral scaling’
Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta} [1 + a_m m_\pi^{2\theta_H} + \dots]$$

Small chiral scaling regime

Chiral dynamics & quasi-massless modes



‘Non-critical chiral scaling’

Far away from the critical regime for $m_\pi \gtrsim 1$ MeV

$$\Delta_l(T, H) \approx \Delta_{l,\chi}(0) \left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H \right)$$

$$V_\chi(\Delta_l) \propto \begin{matrix} \updownarrow & \updownarrow & \updownarrow \\ \Delta_l^6 & \Delta_l^4 & \Delta_l^2 \end{matrix}$$

Critical scaling ●

Gao, JMP, PRD 105 (2022) 094020

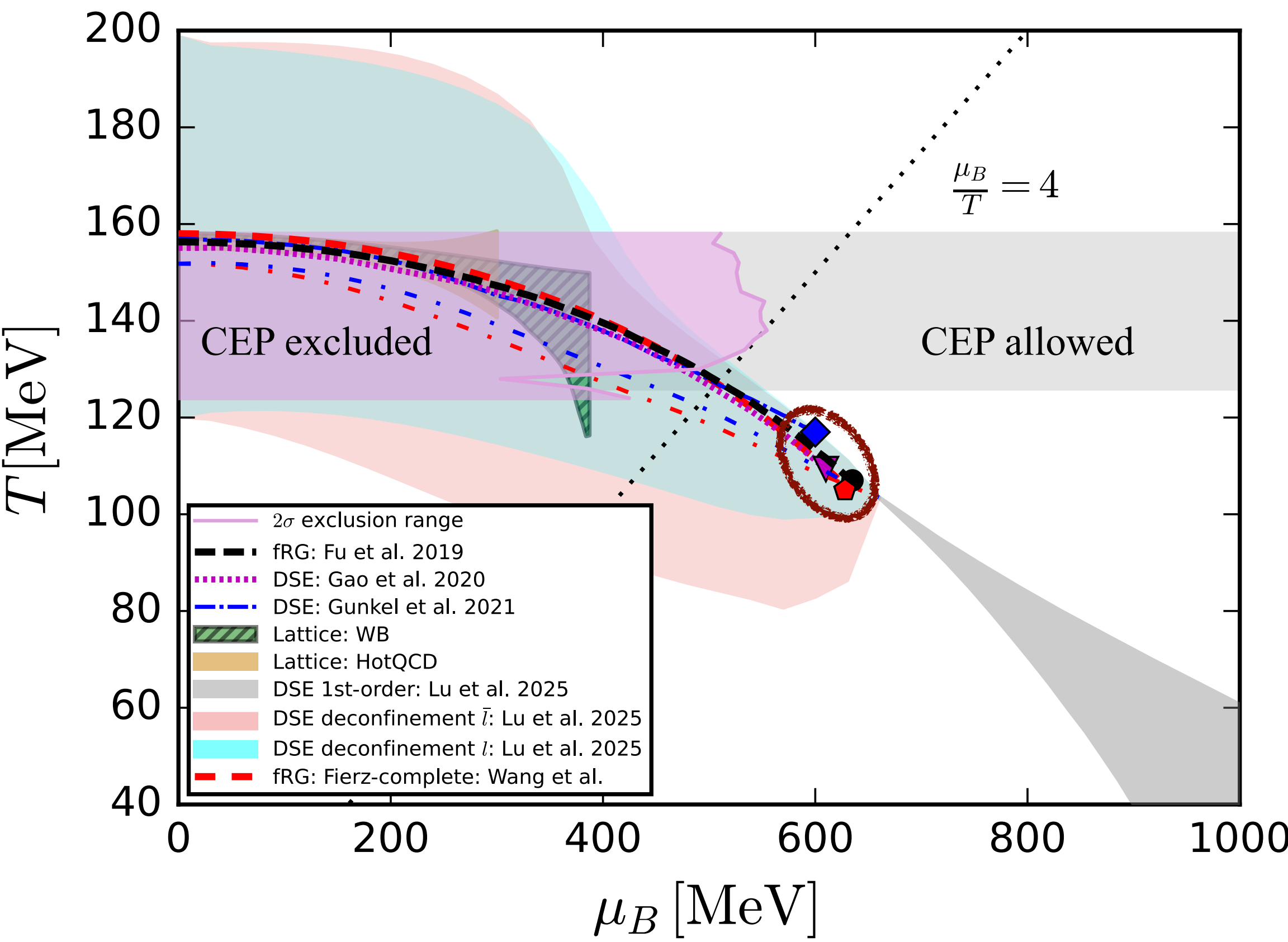
Criticality & quasi-massless modes around the CEP



Fu, JMP, Rennecke, PRD 101 (2020) 054032
 Wang, Zhou, Huang, Wen, Yin, Fu, in preparation

Small scaling regime around potential CEP

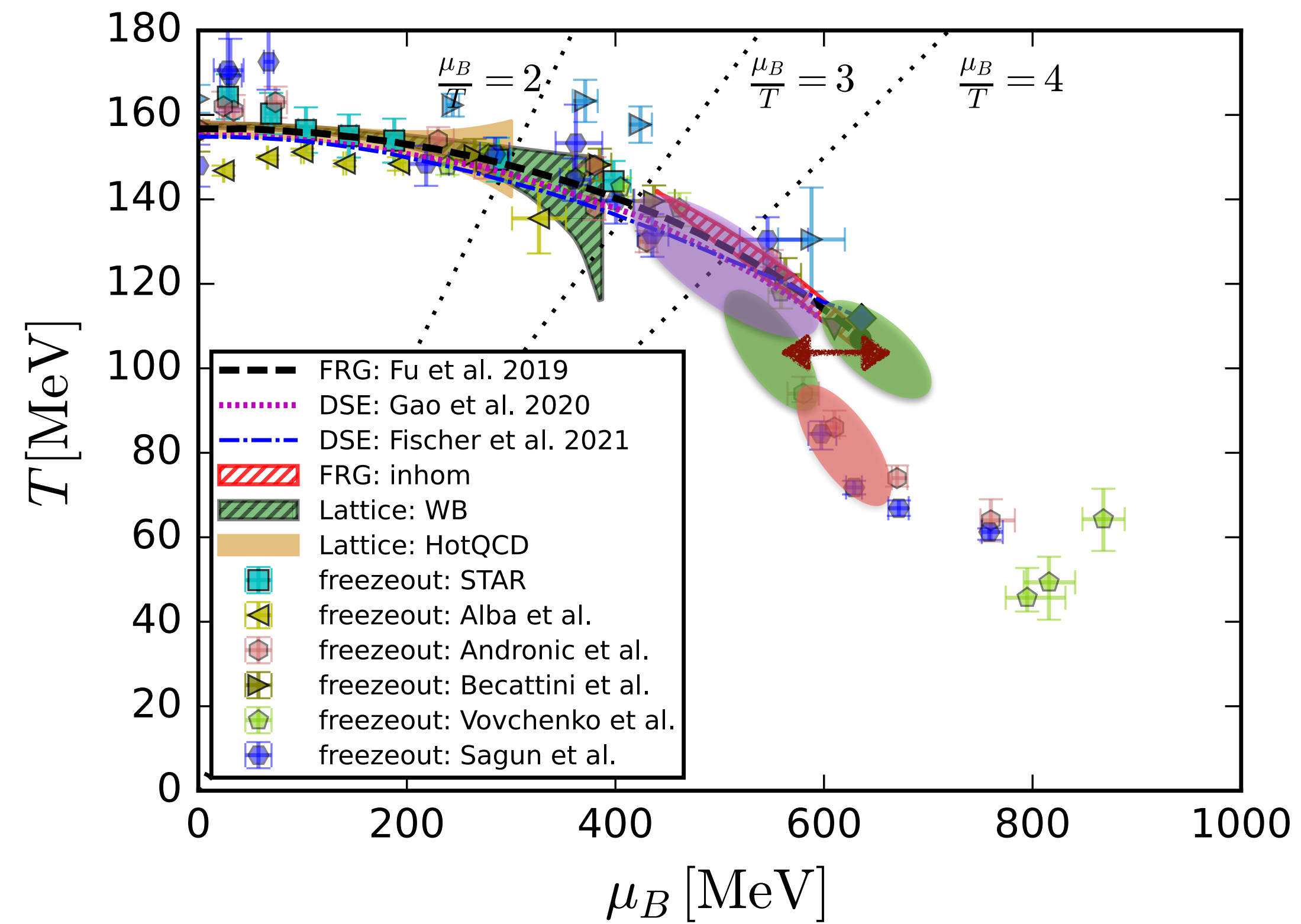
!!Great News!!
 Location of CEP/New phase accessible via combination
 of precision measurements & computations



Ripples of the critical point

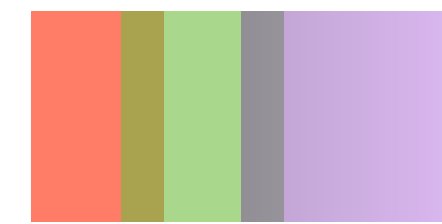
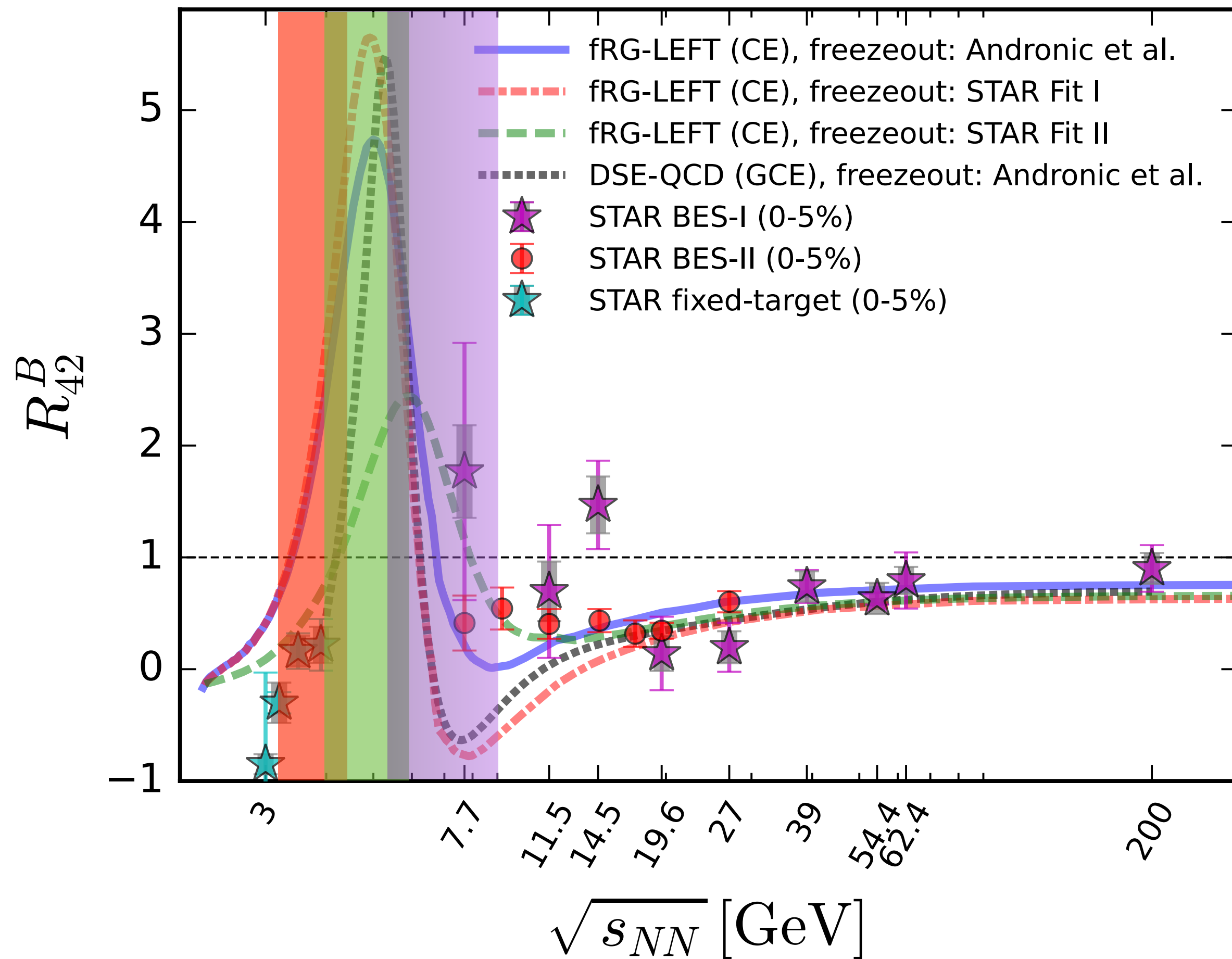
Onset of new phases at $600 \text{ MeV} \lesssim \mu_B^{\text{onset}} \lesssim 700 \text{ MeV}$

High density phase structure from functional QCD



Ripples of the critical point

net-baryon fluctuations in QCD vs net-proton fluctuations at STAR



Dominated by non-critical soft modes

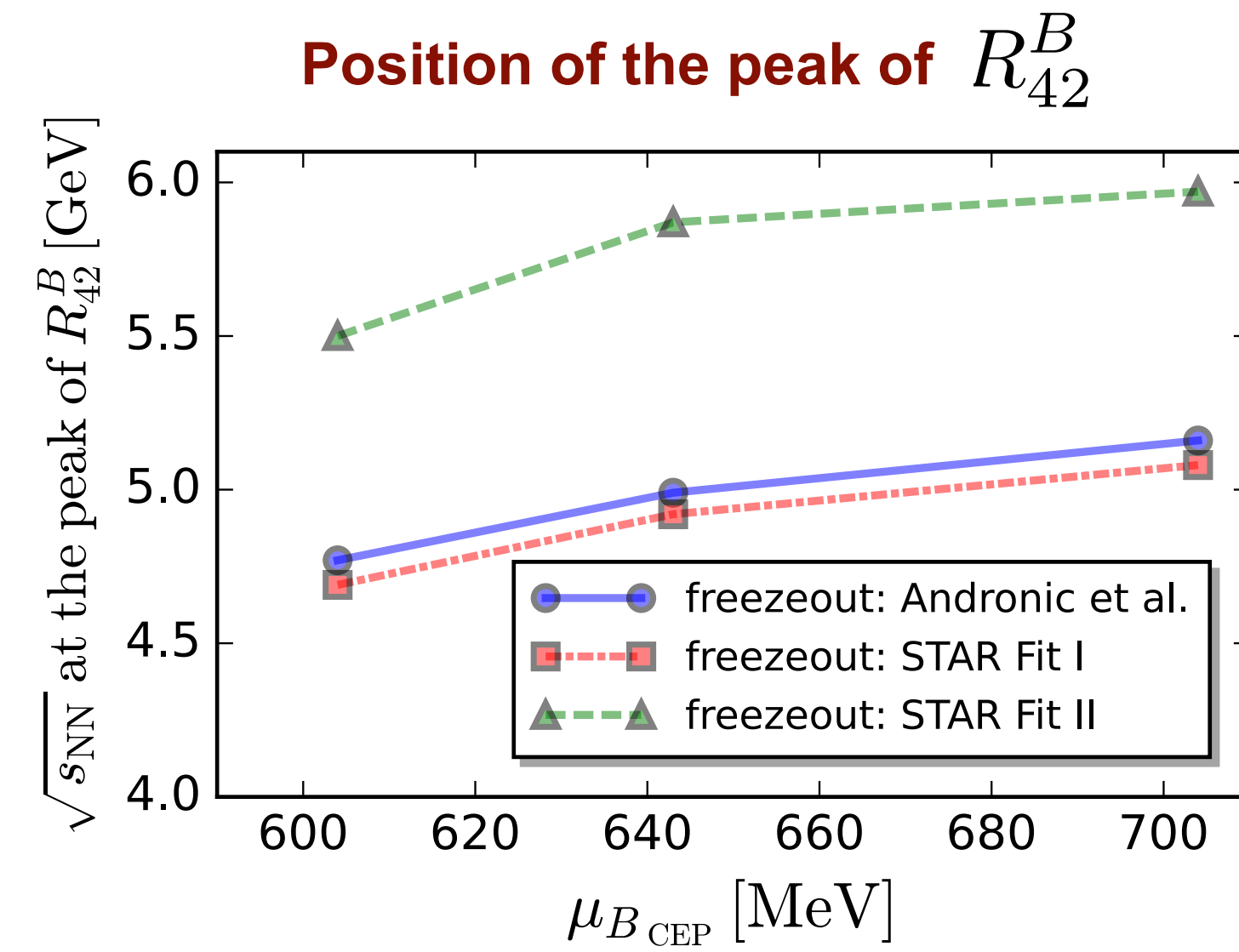
Results:
 1st principles functional QCD computations
 & low energy effective theories/extrapolations
 explicit & implicit assumptions

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

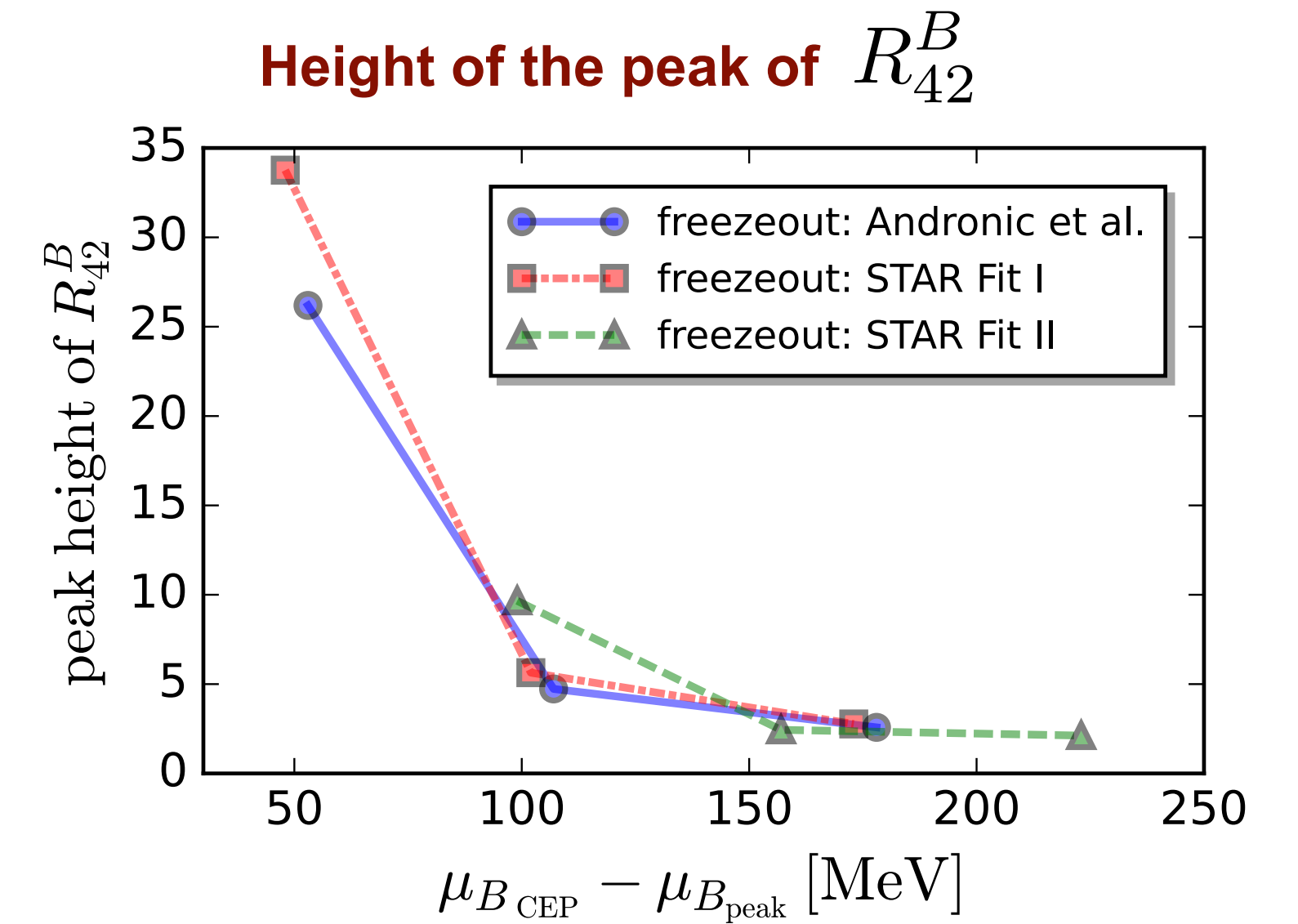
Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

Lu, Gao, Liu, JMP, arXiv:2504.05099

Ripples of the critical point



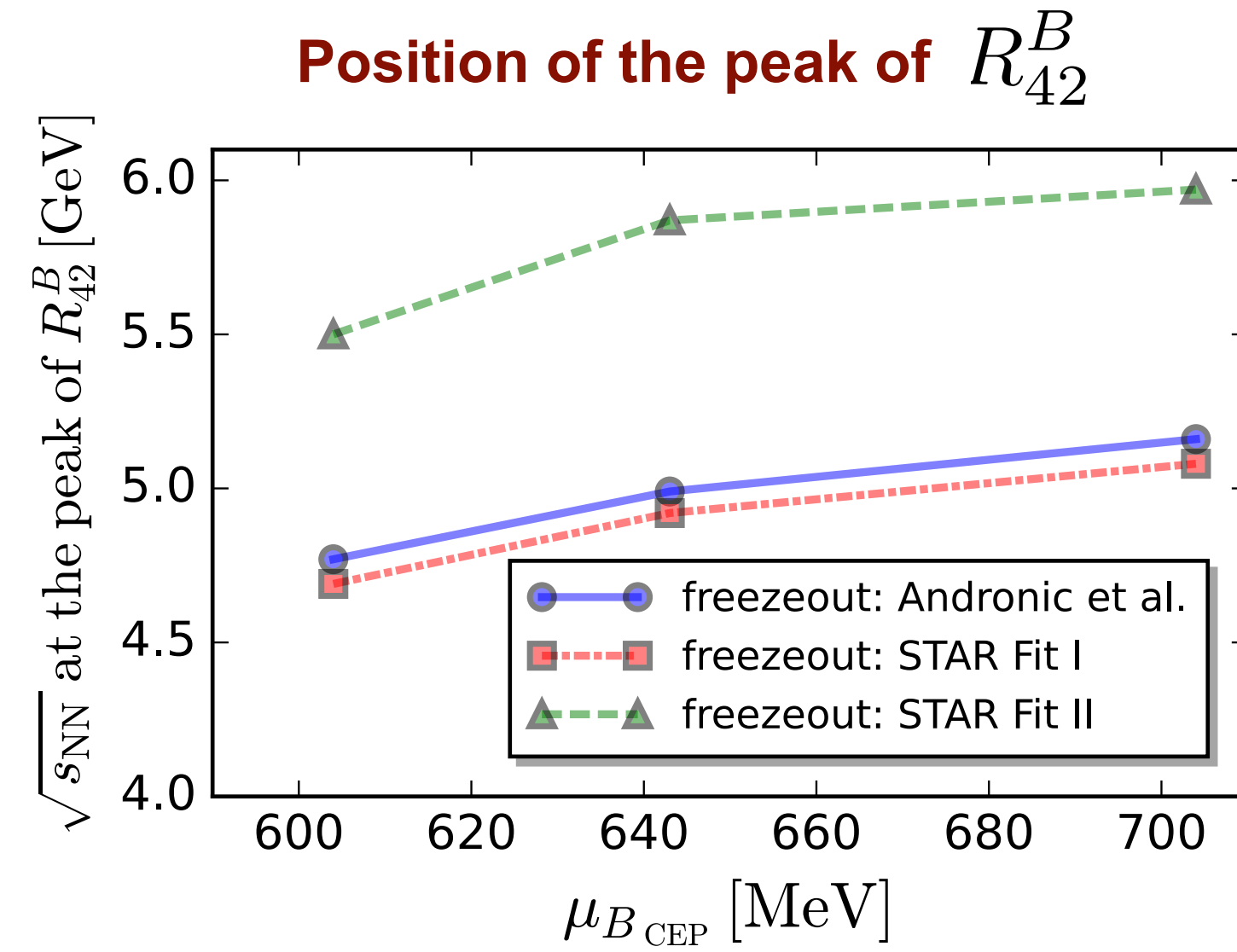
Reconstructing the CEP



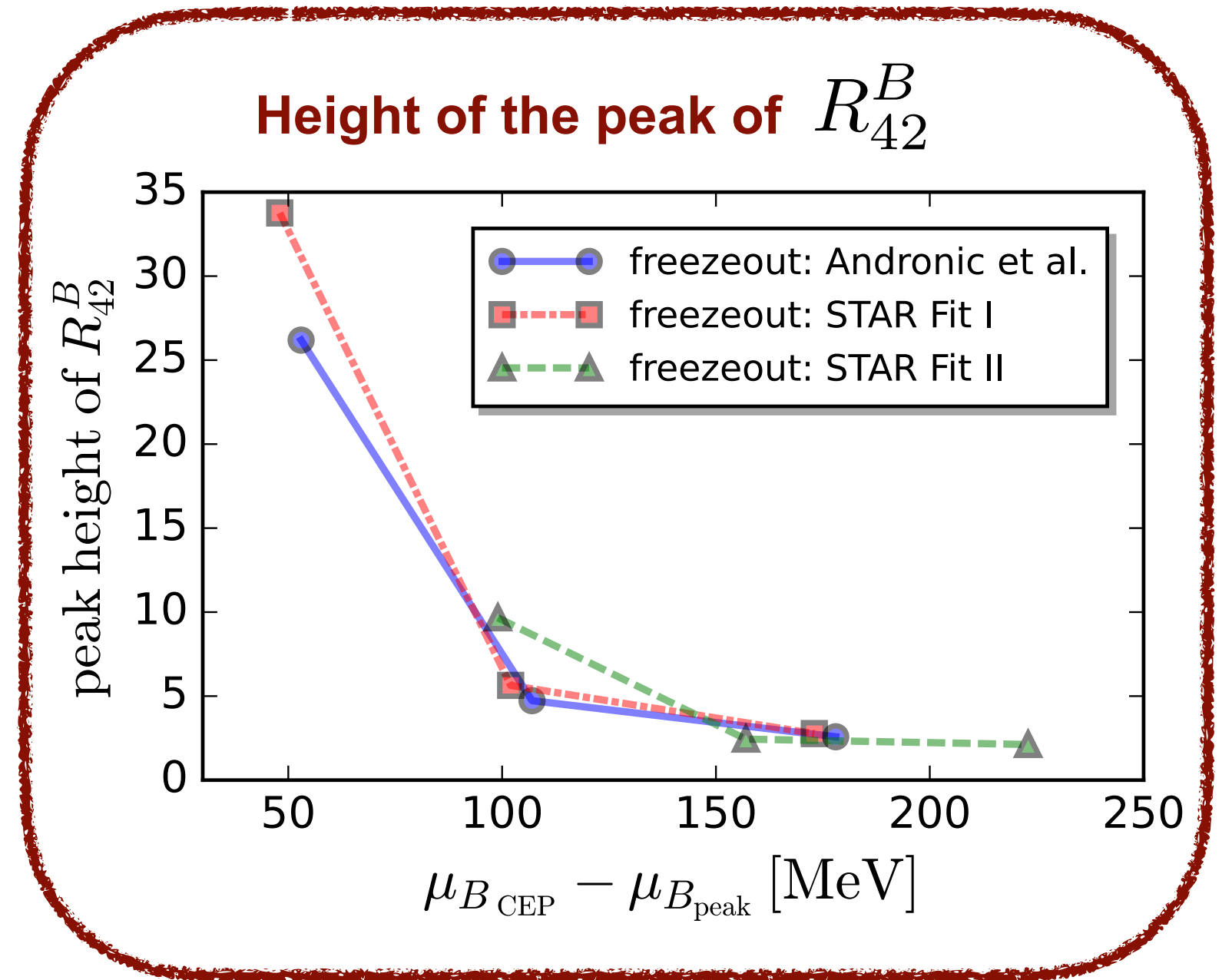
Dominated by non-critical soft modes

Demand: Precise freeze-out line at high density

Ripples of the critical point



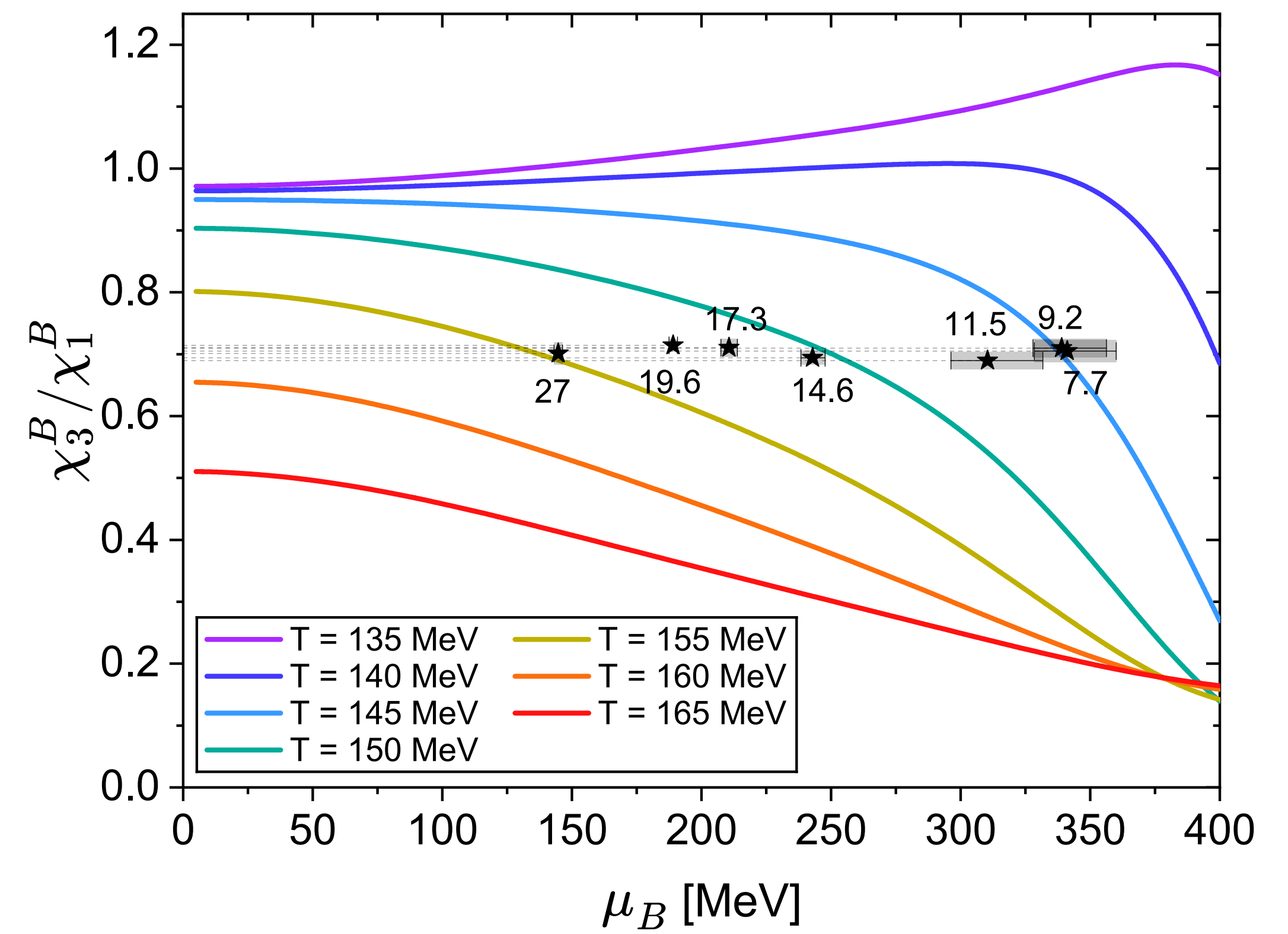
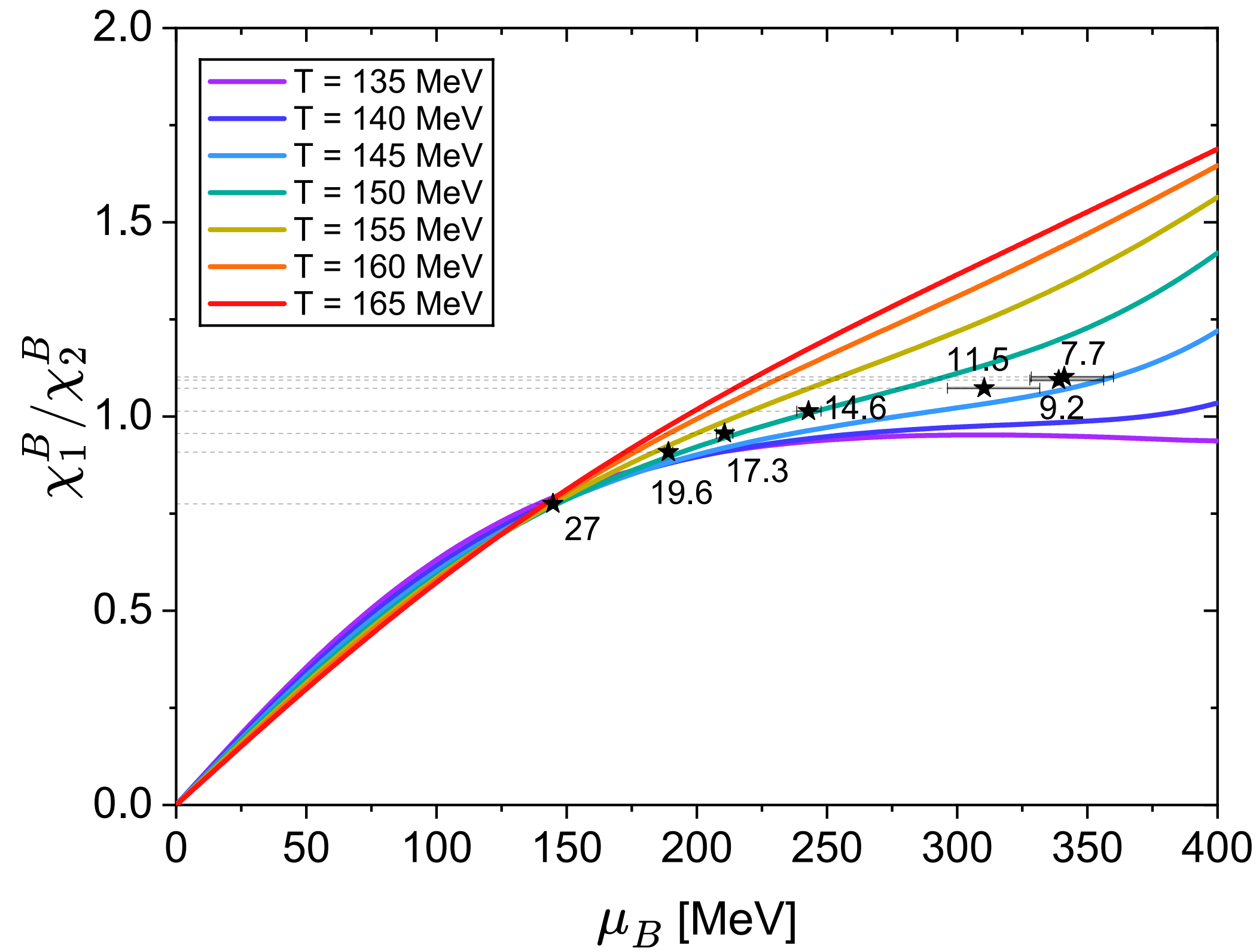
Reconstructing the CEP



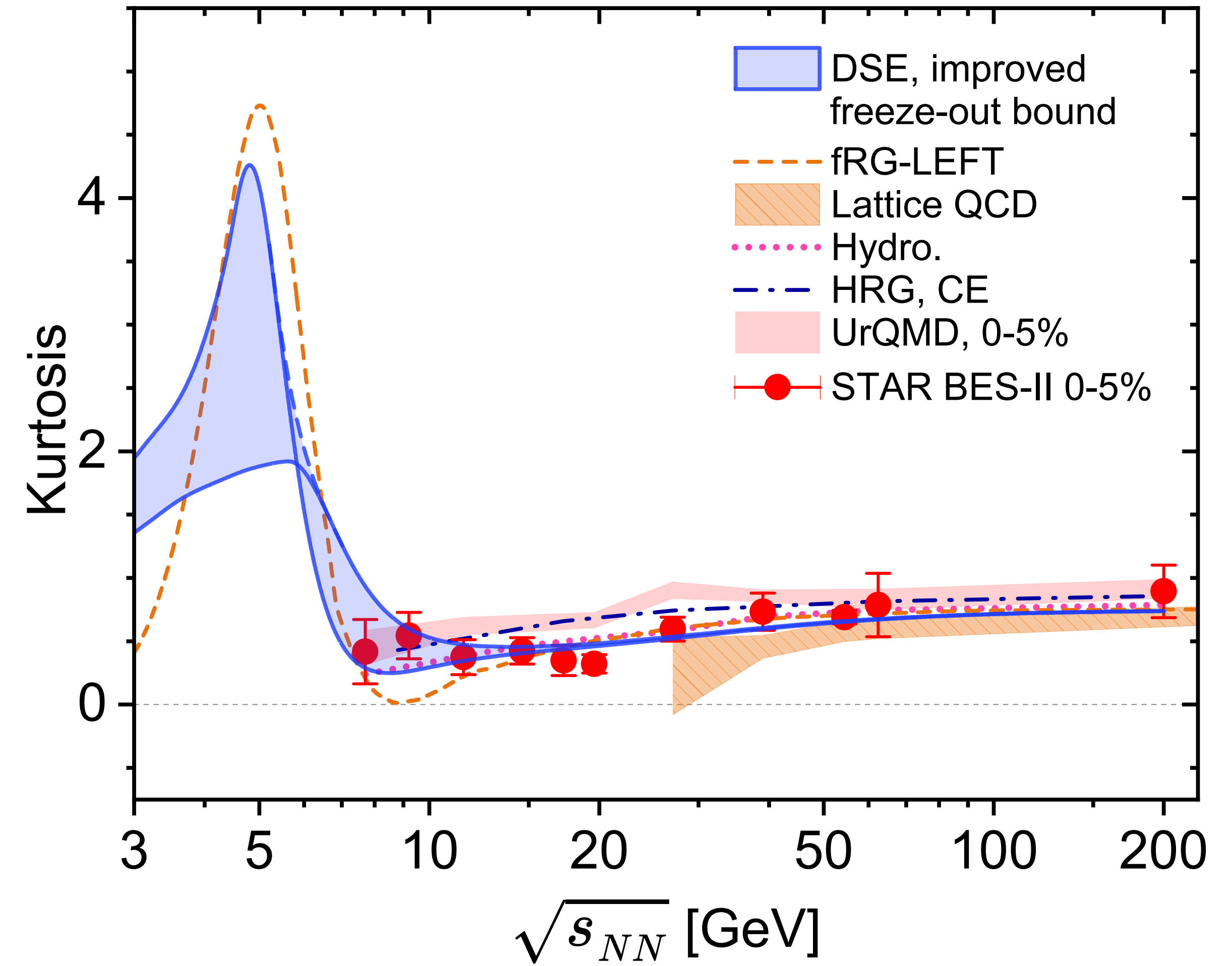
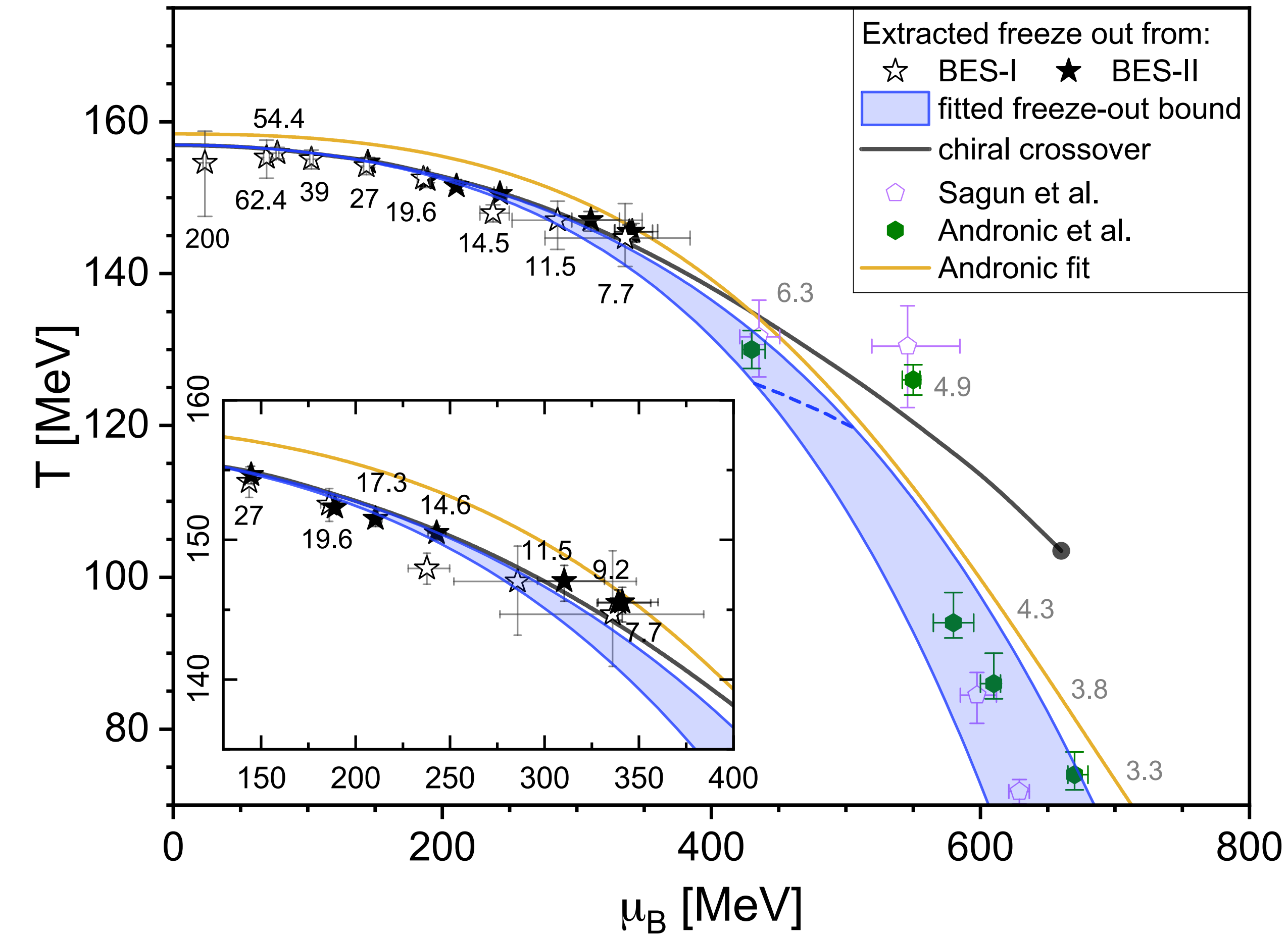
Dominated by non-critical soft modes

Functional Freeze-out

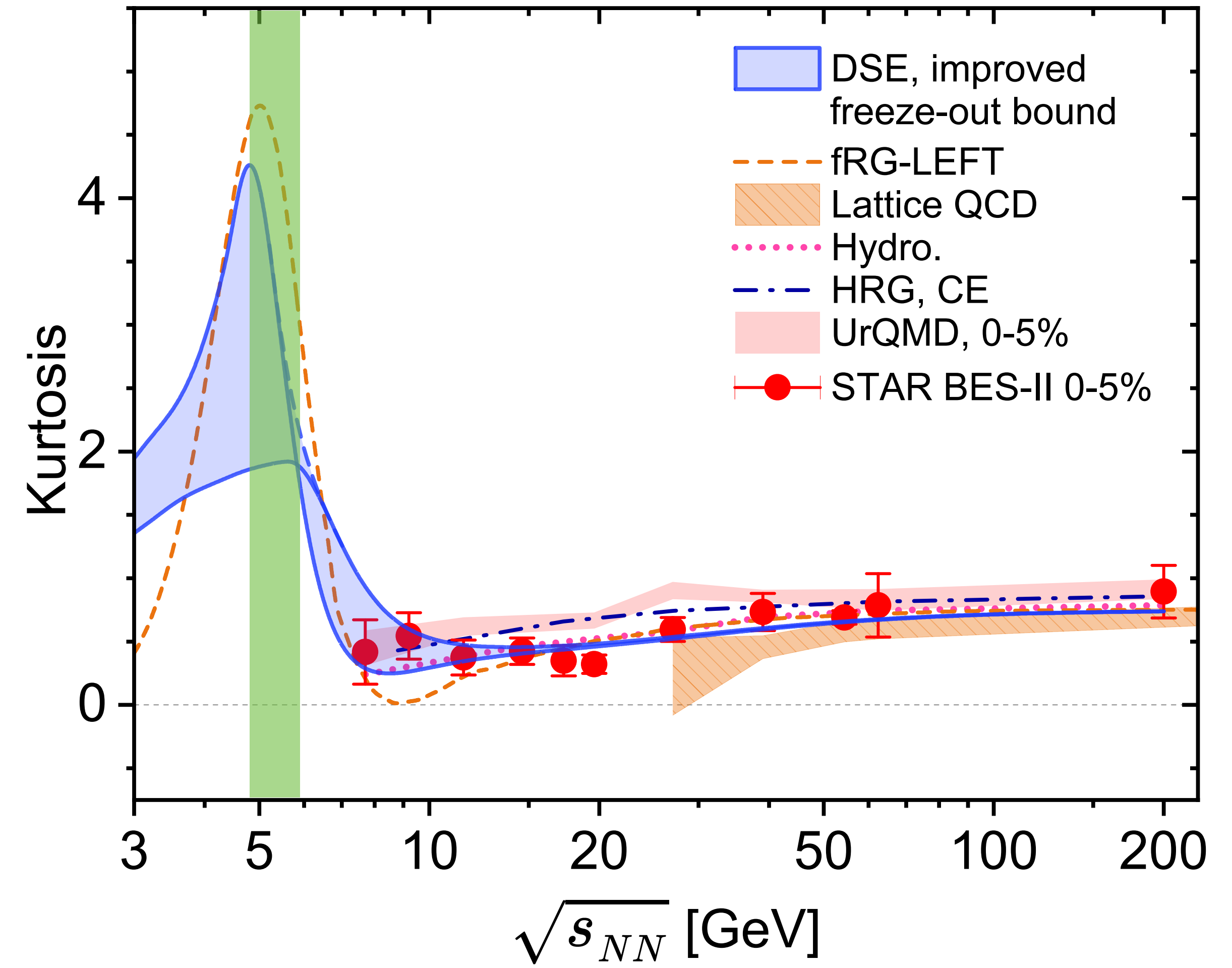
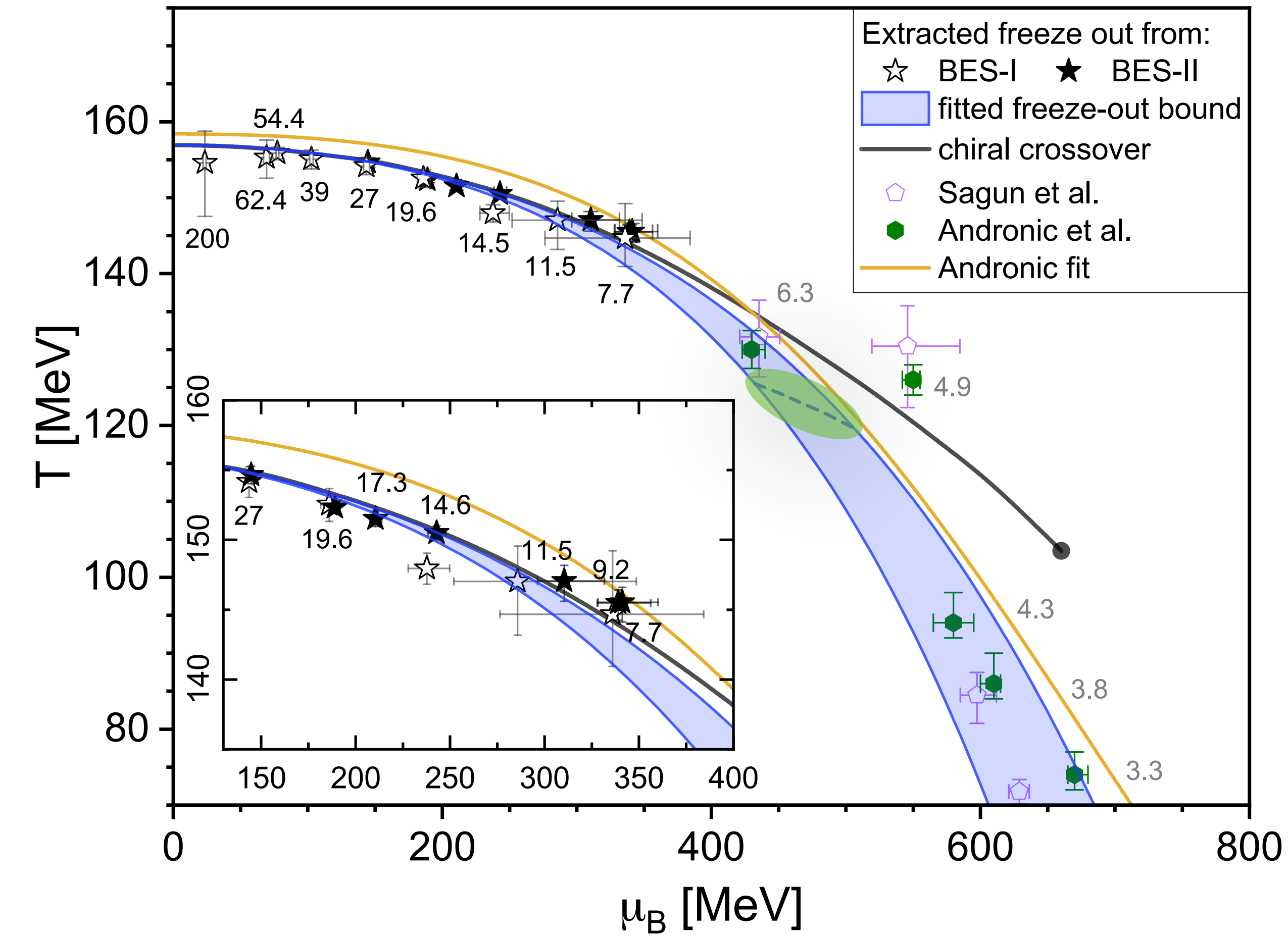
$$(T_f, \mu_{Bf}) \longleftrightarrow \sqrt{s}$$



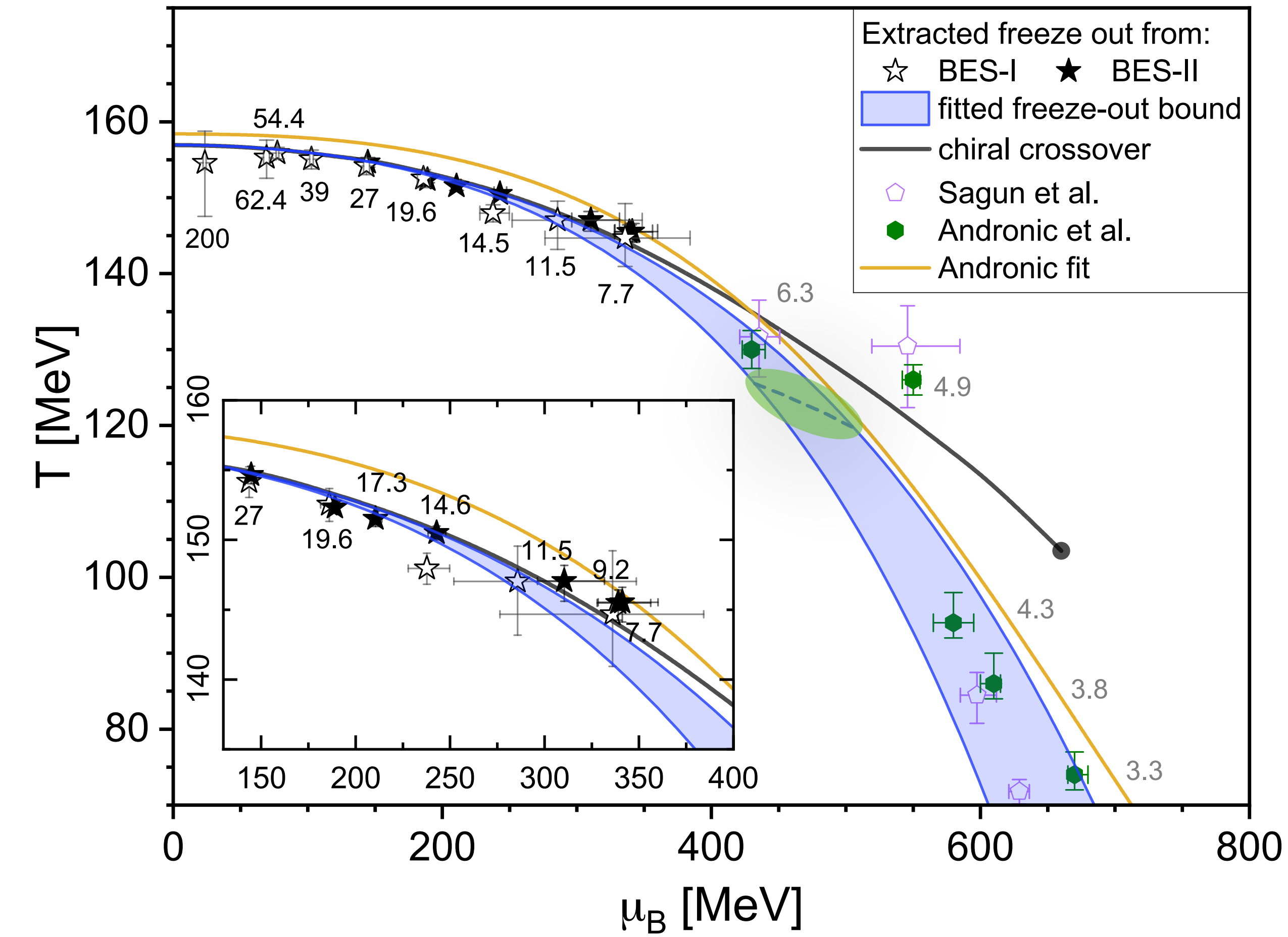
Functional Freeze-out



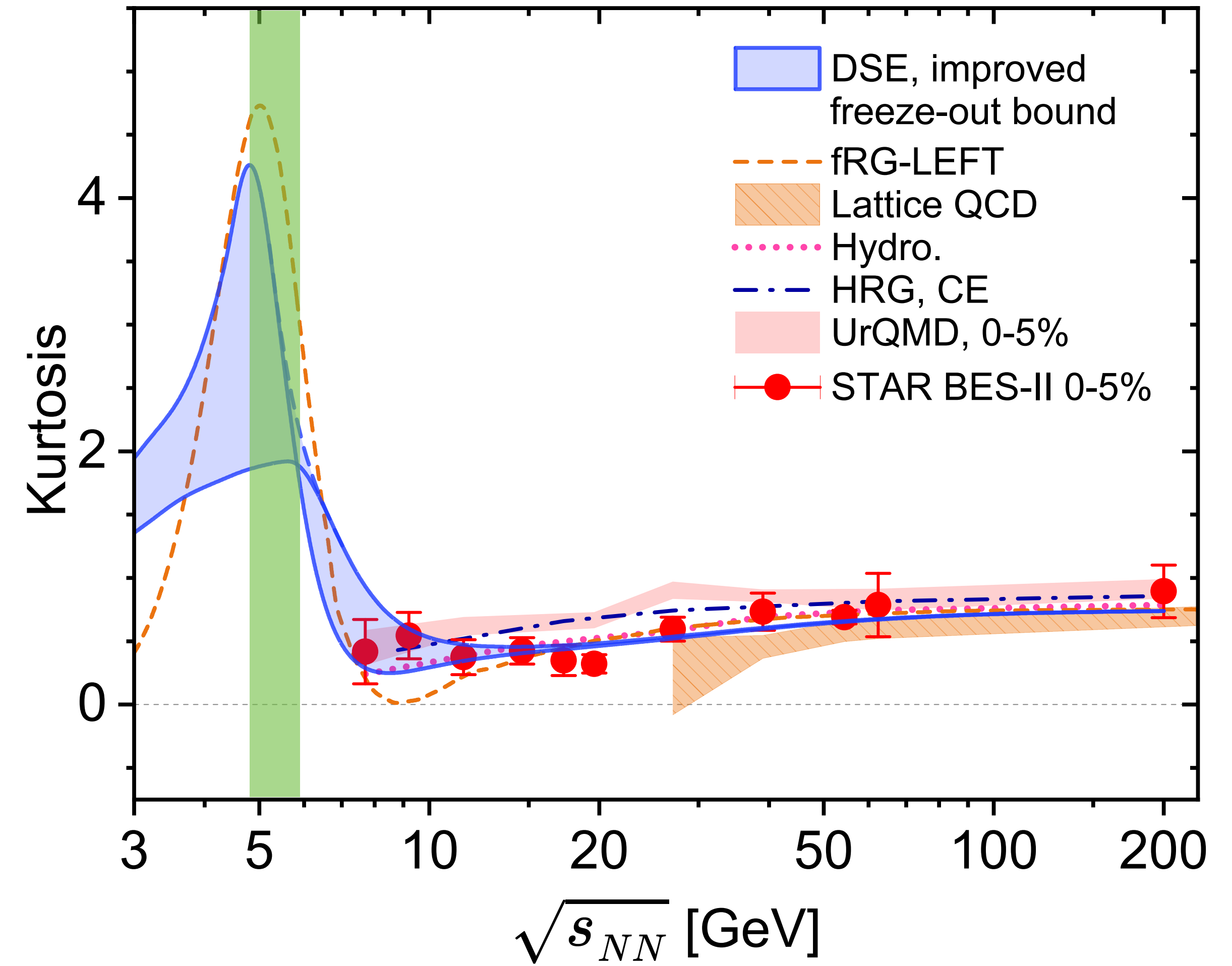
Functional Freeze-out



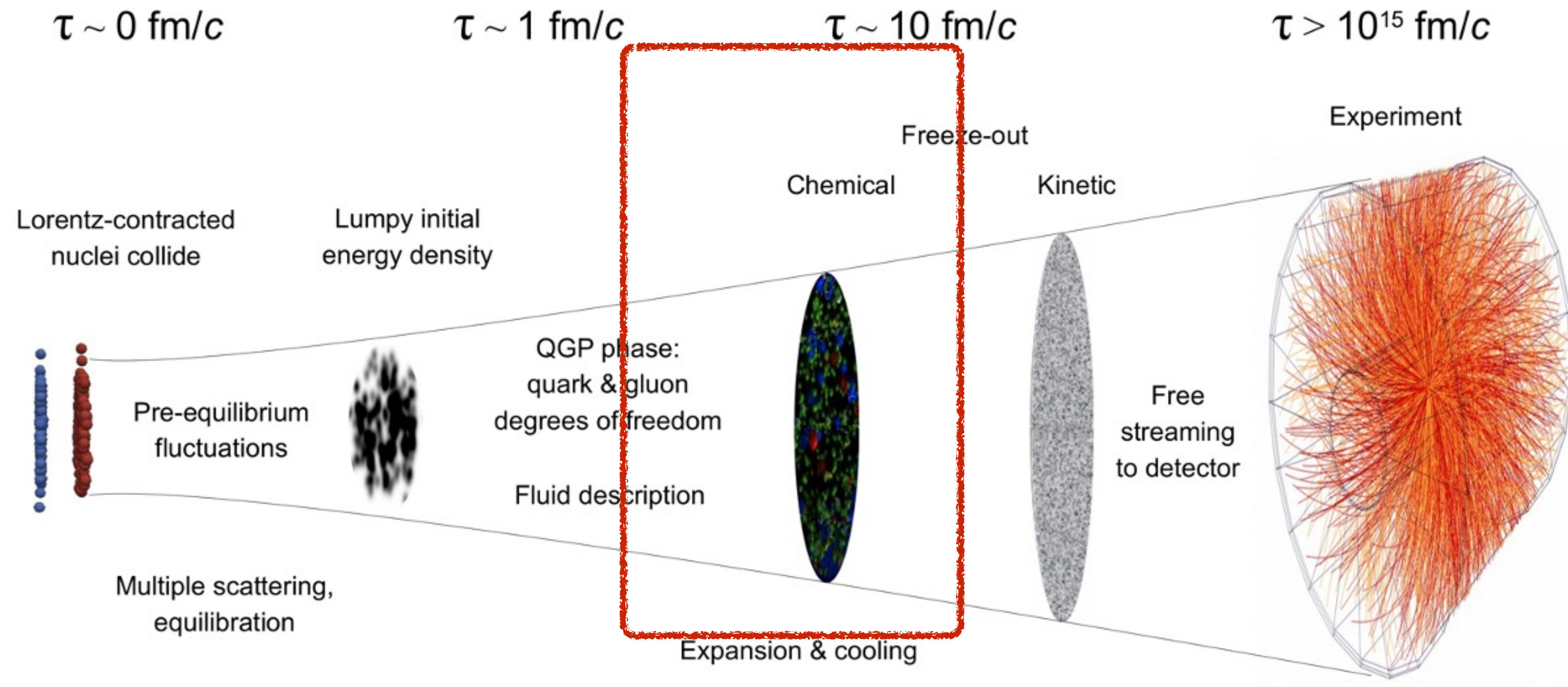
Functional Freeze-out



Demand: build in dynamics



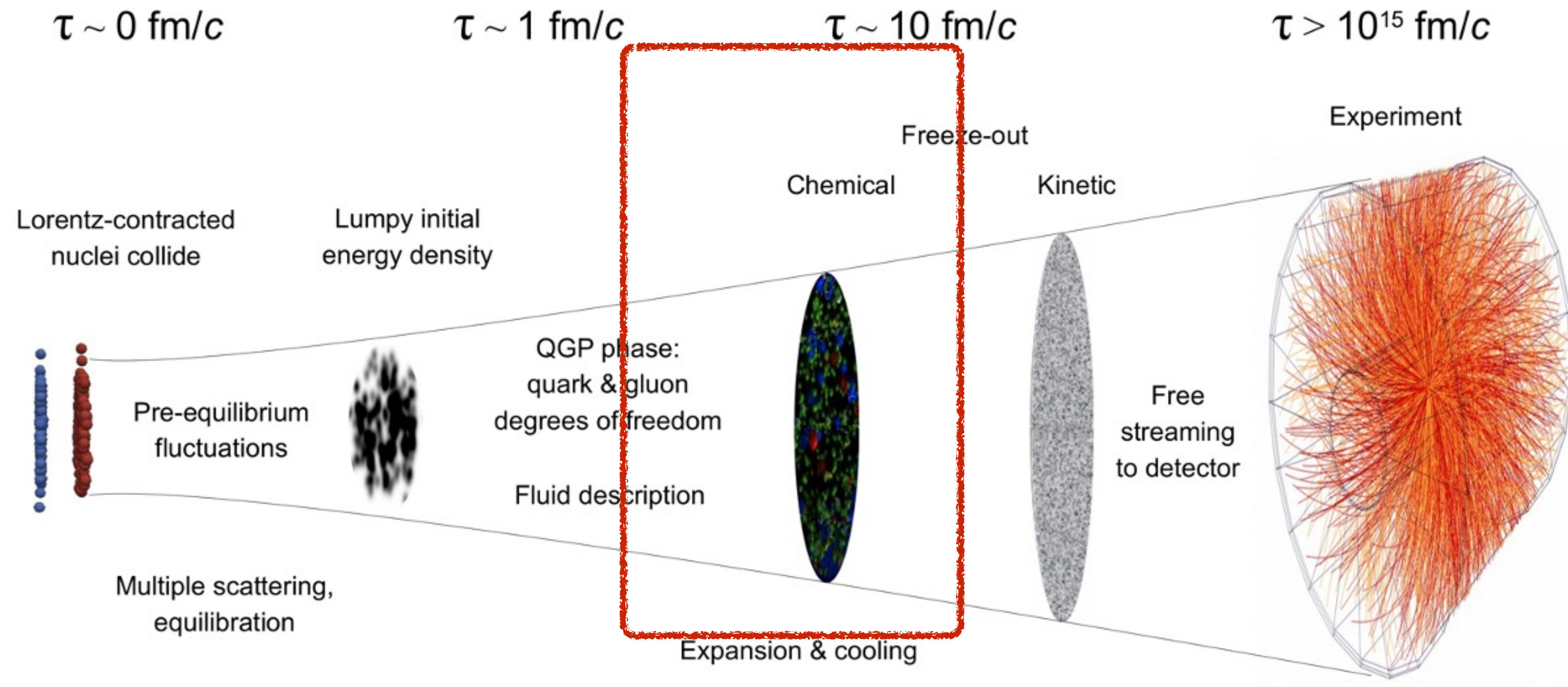
Unfolding strongly correlated QCD with heavy ion collisions (extreme QCD)



How to unfold: inverse problem!

Demand: unfolding HICs

Unfolding strongly correlated QCD with heavy ion collisions (extreme QCD)



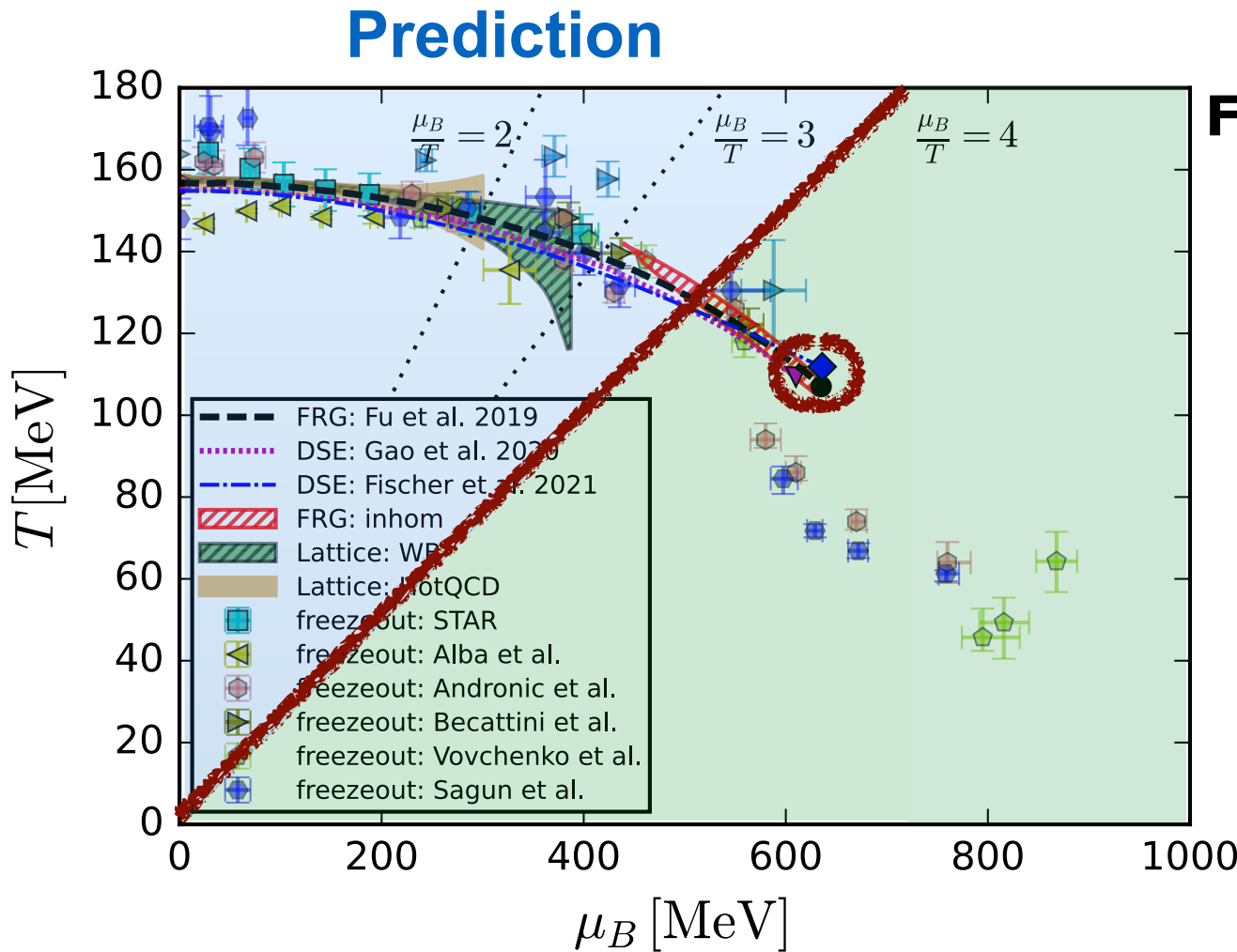
How to unfold: inverse problem!

Demand: unfolding HICs

The QCD Moat

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

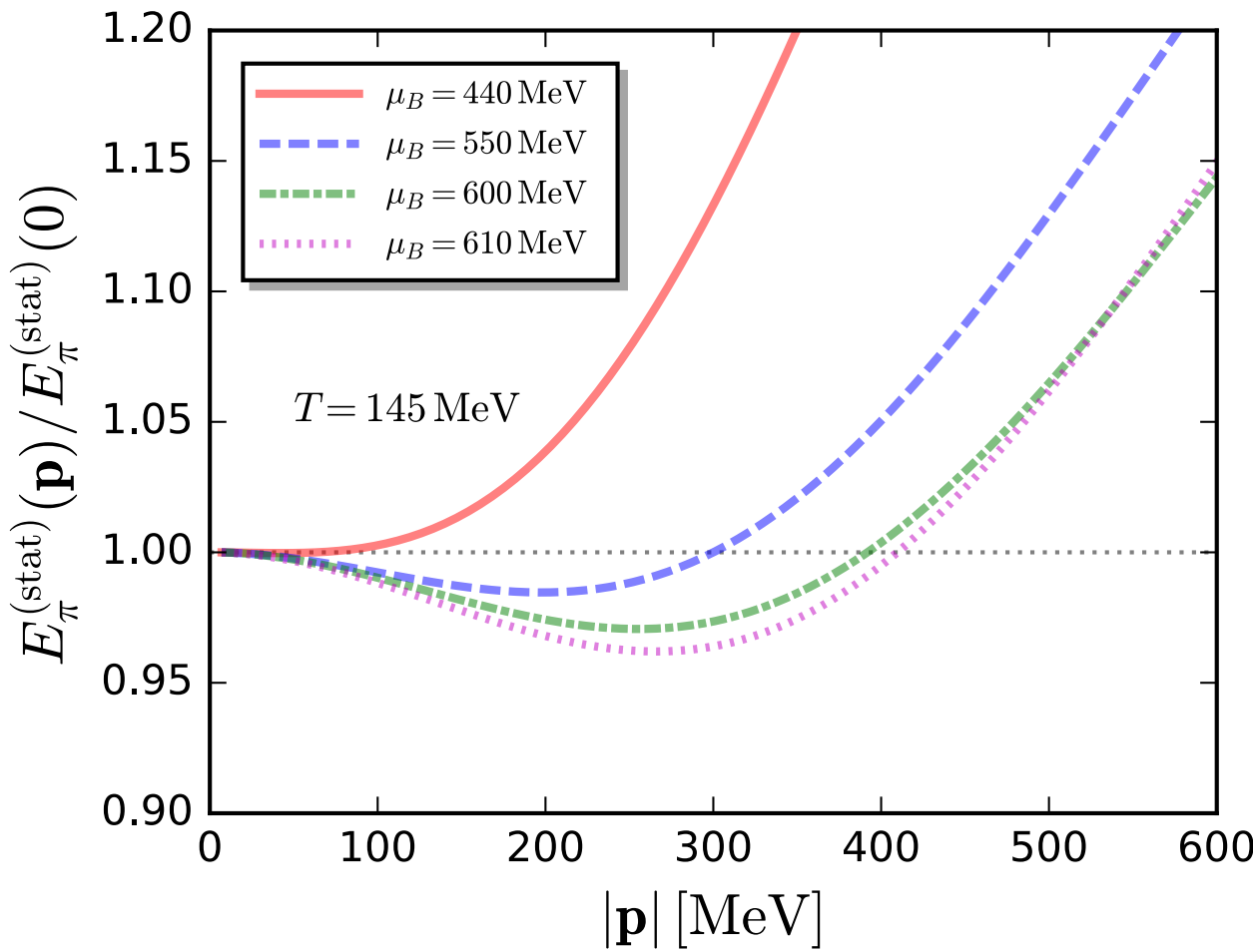


Fu, JMP, Rennecke, PRD 101 (2020) 054032

Regime of quantitative reliability
of
current best truncation

Estimate

Moat regime is not captured quantitatively



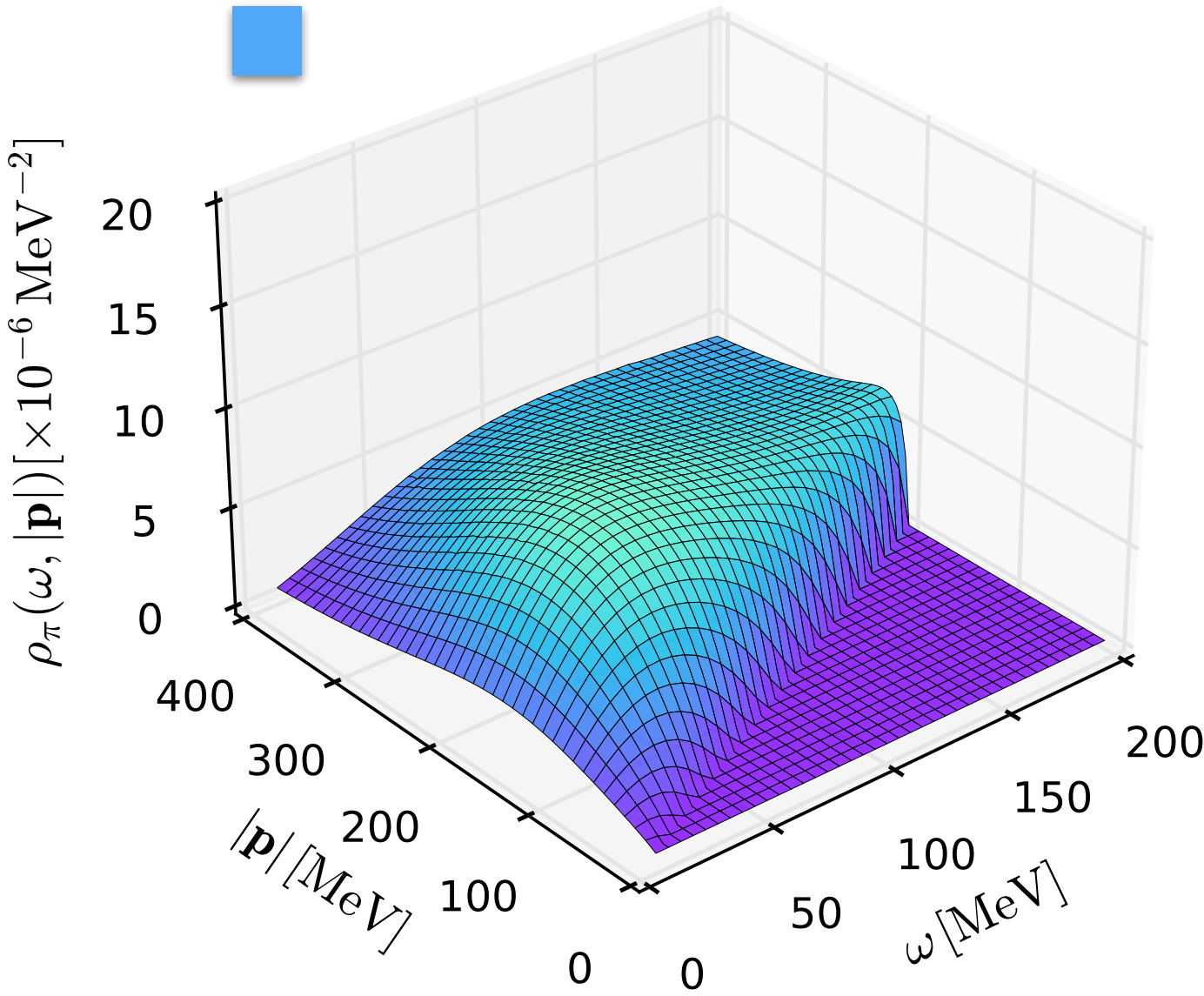
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

The QCD Moat

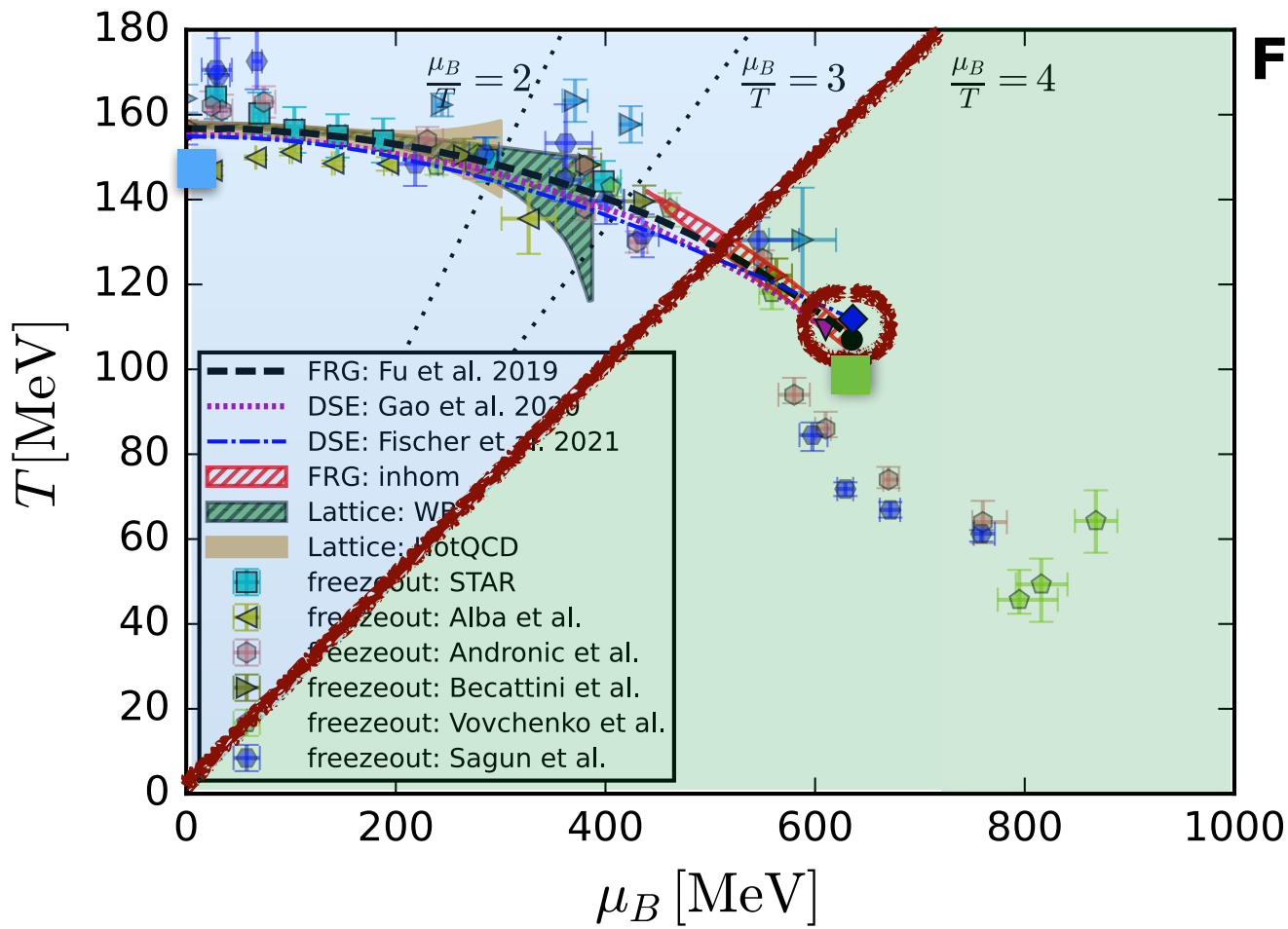
Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

T=160 MeV & $\mu_B=0$ MeV



Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032

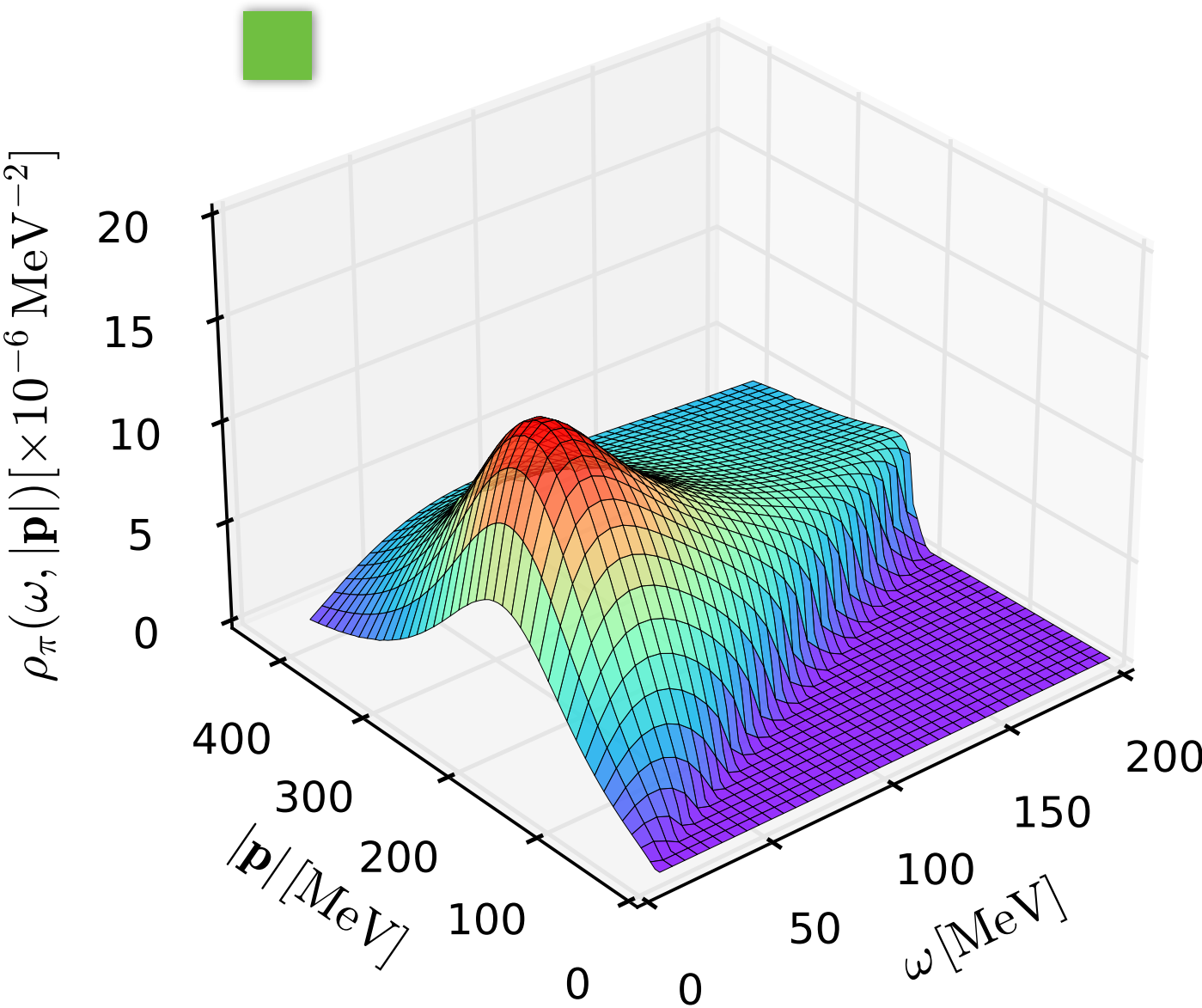
Regime of quantitative reliability
of
current best truncation

Estimate

Moat regime is not captured quantitatively

Pion spectral functions

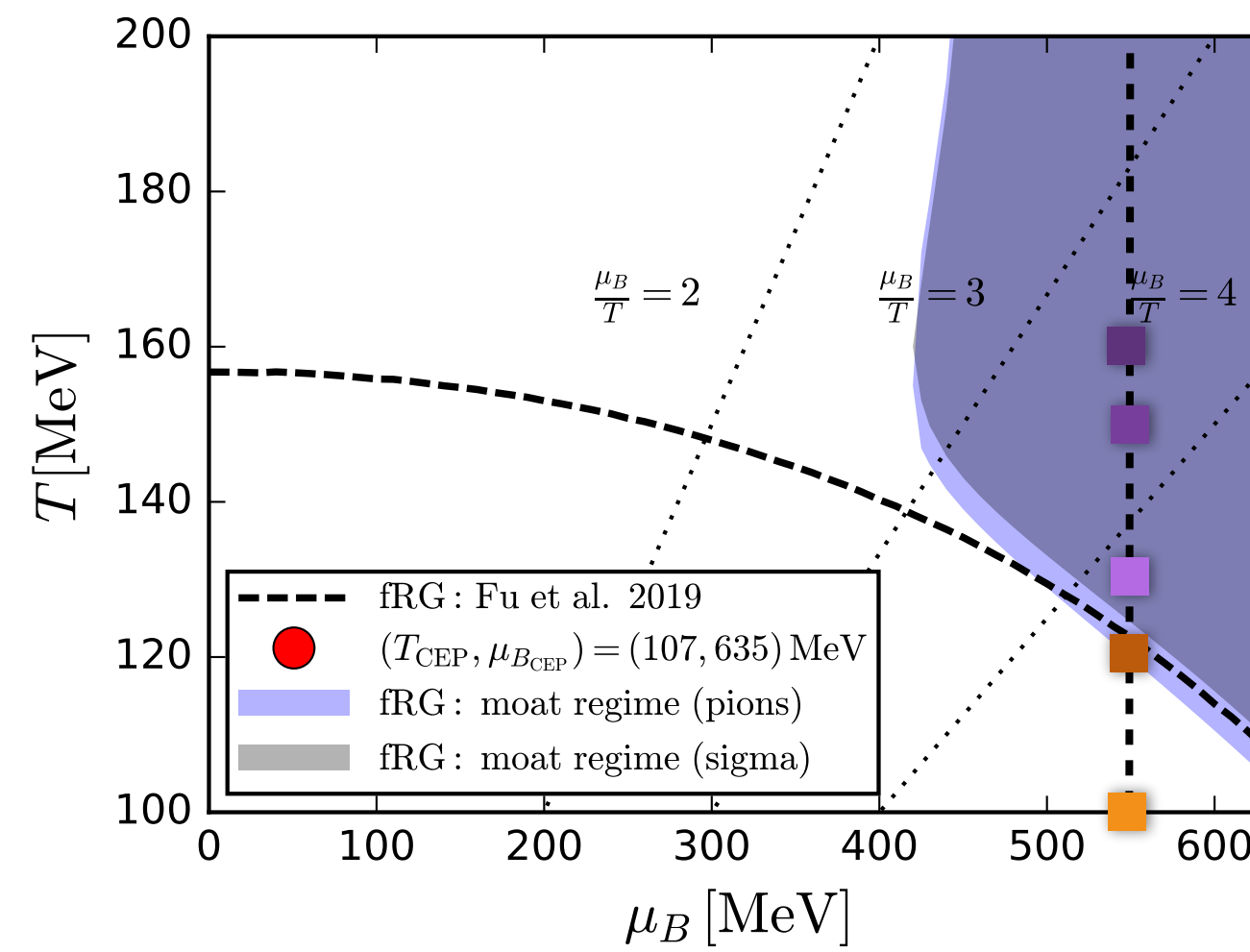
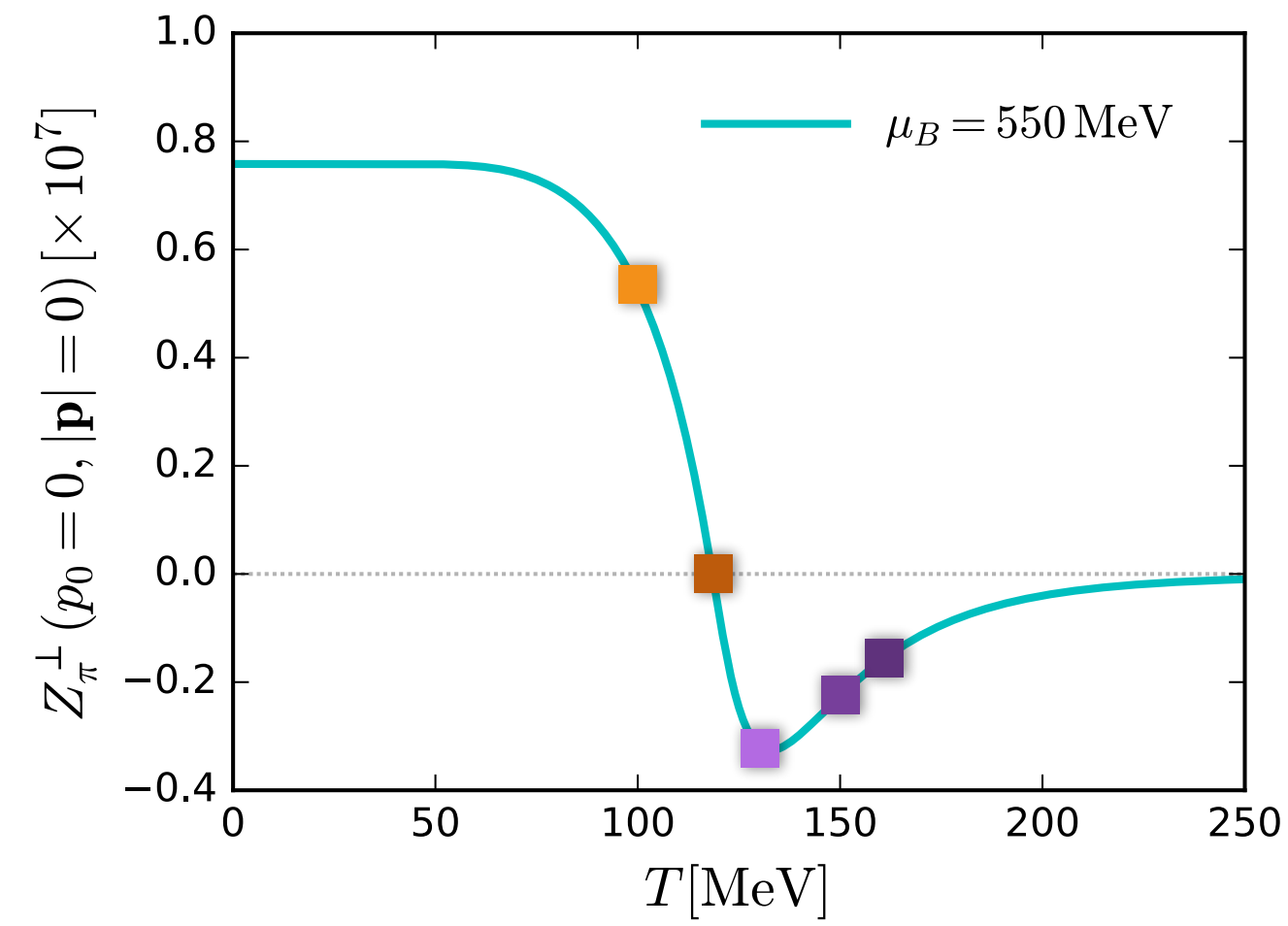
T=114 MeV & $\mu_B=630$ MeV



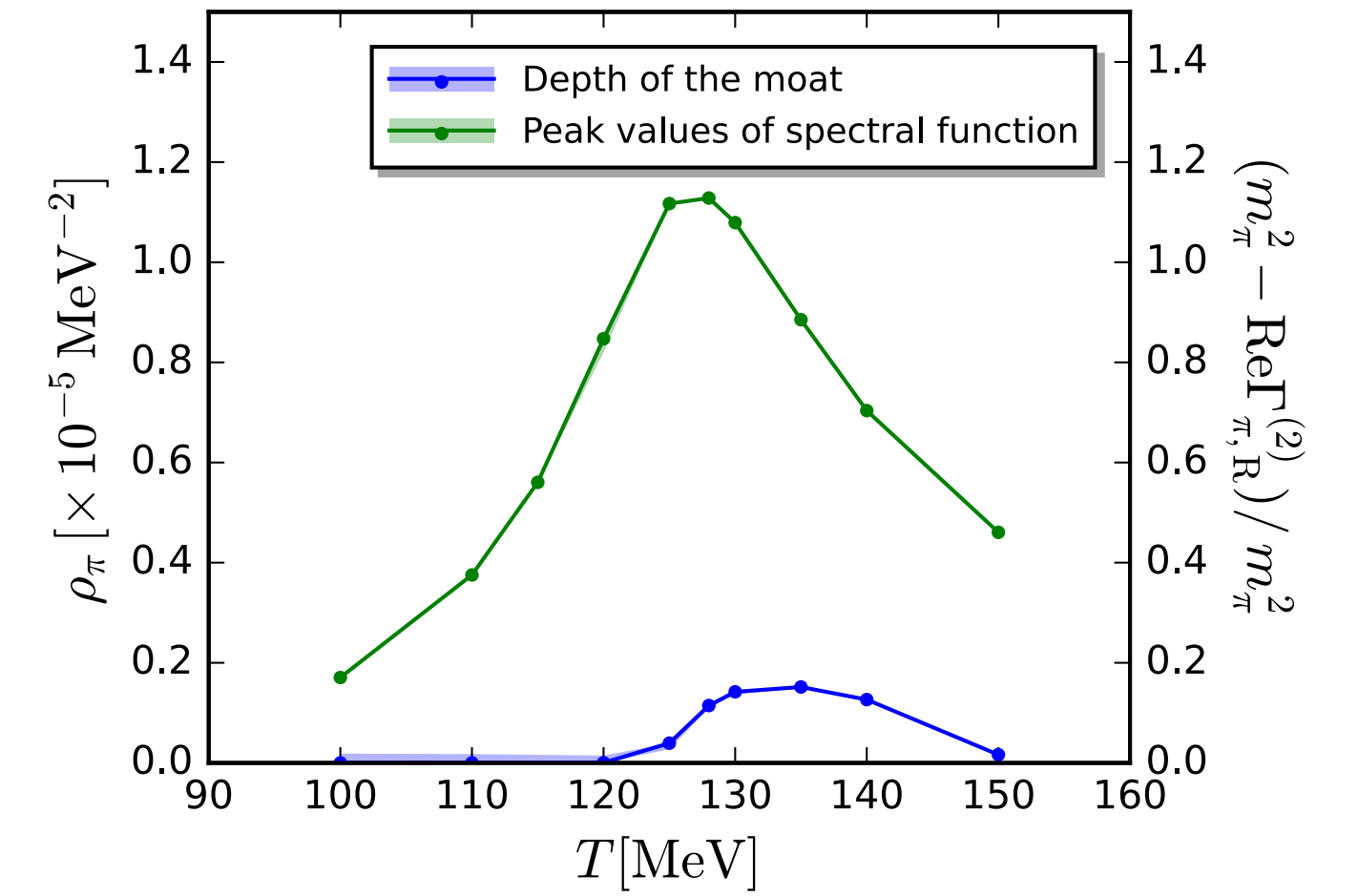
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

The QCD Moat

Spatial wave function of the pion at $p=0$



Depth of the moat & spectral peak of the pion



Dissecting the moat & the moaton

$\mu_B = 650 \text{ MeV}$

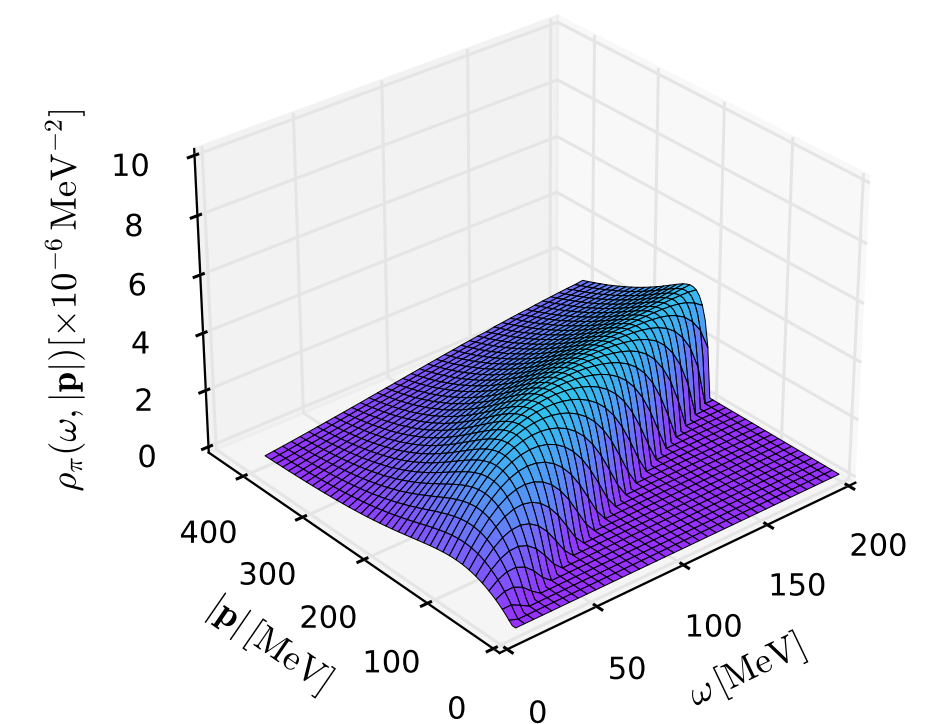
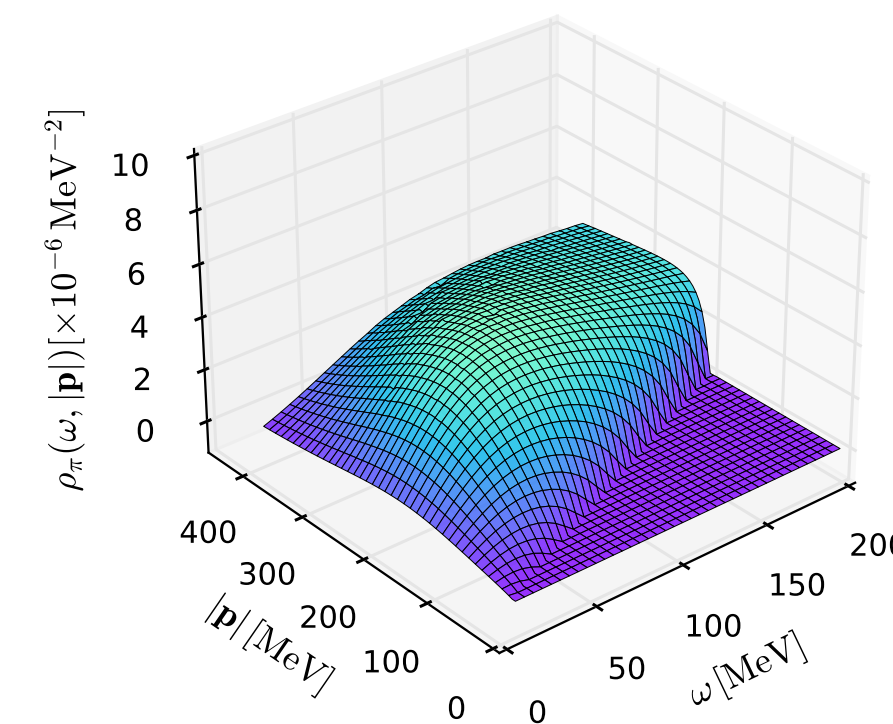
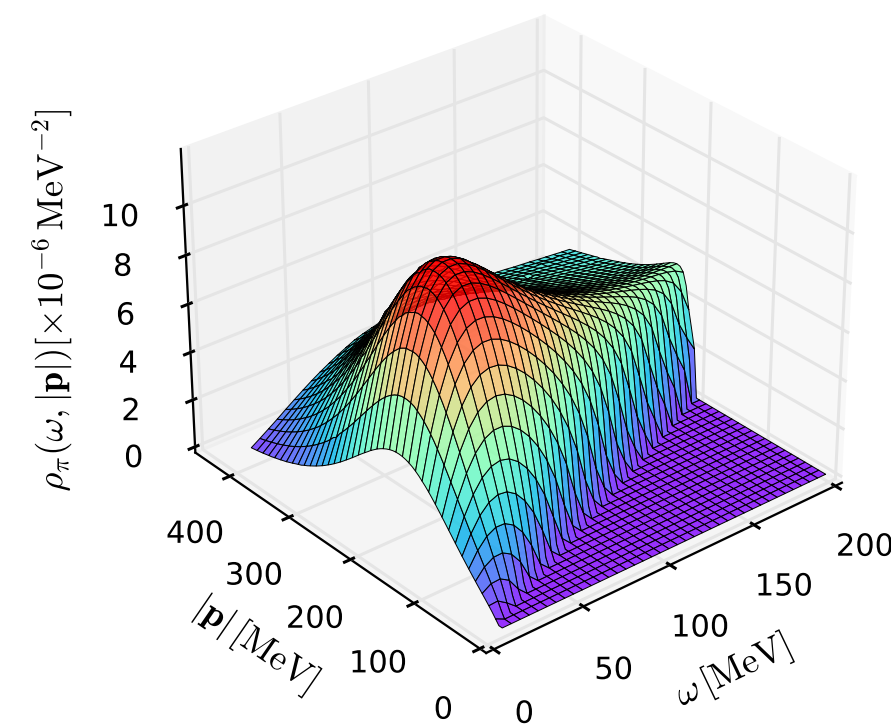
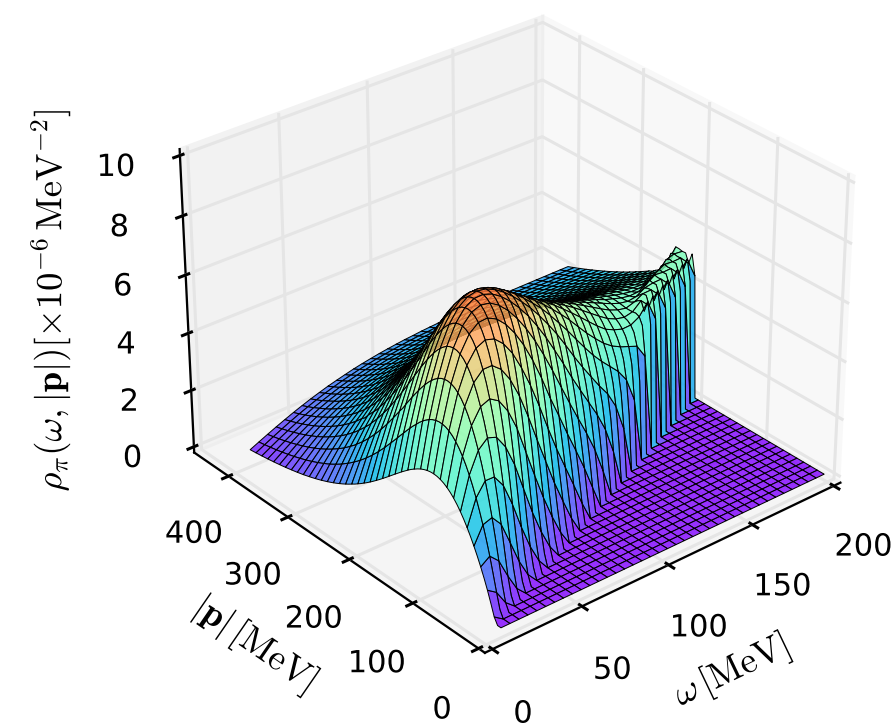
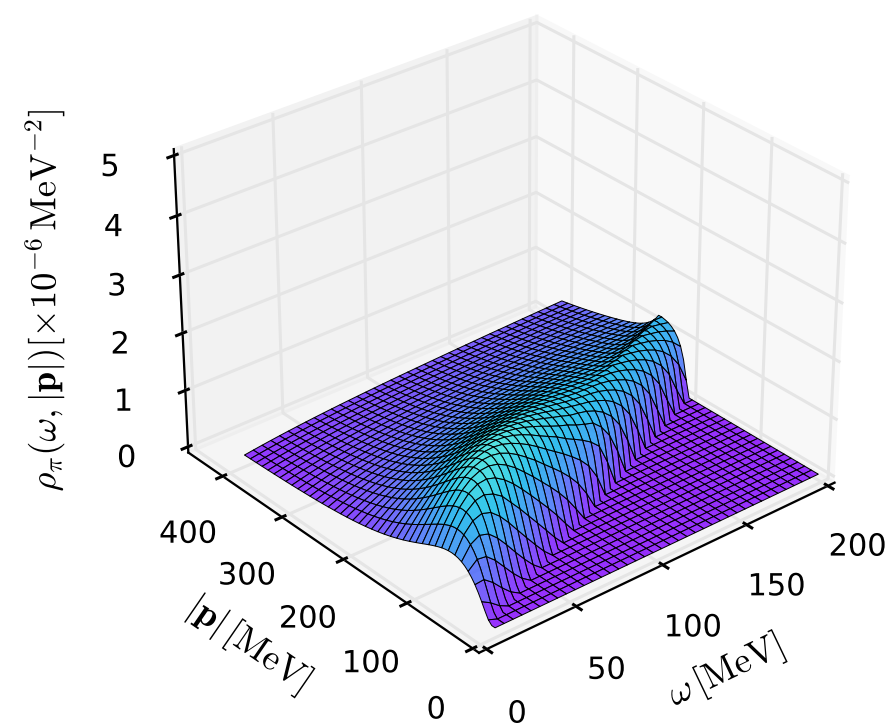
■ T=100 MeV

■ T=120 MeV

■ T=130 MeV

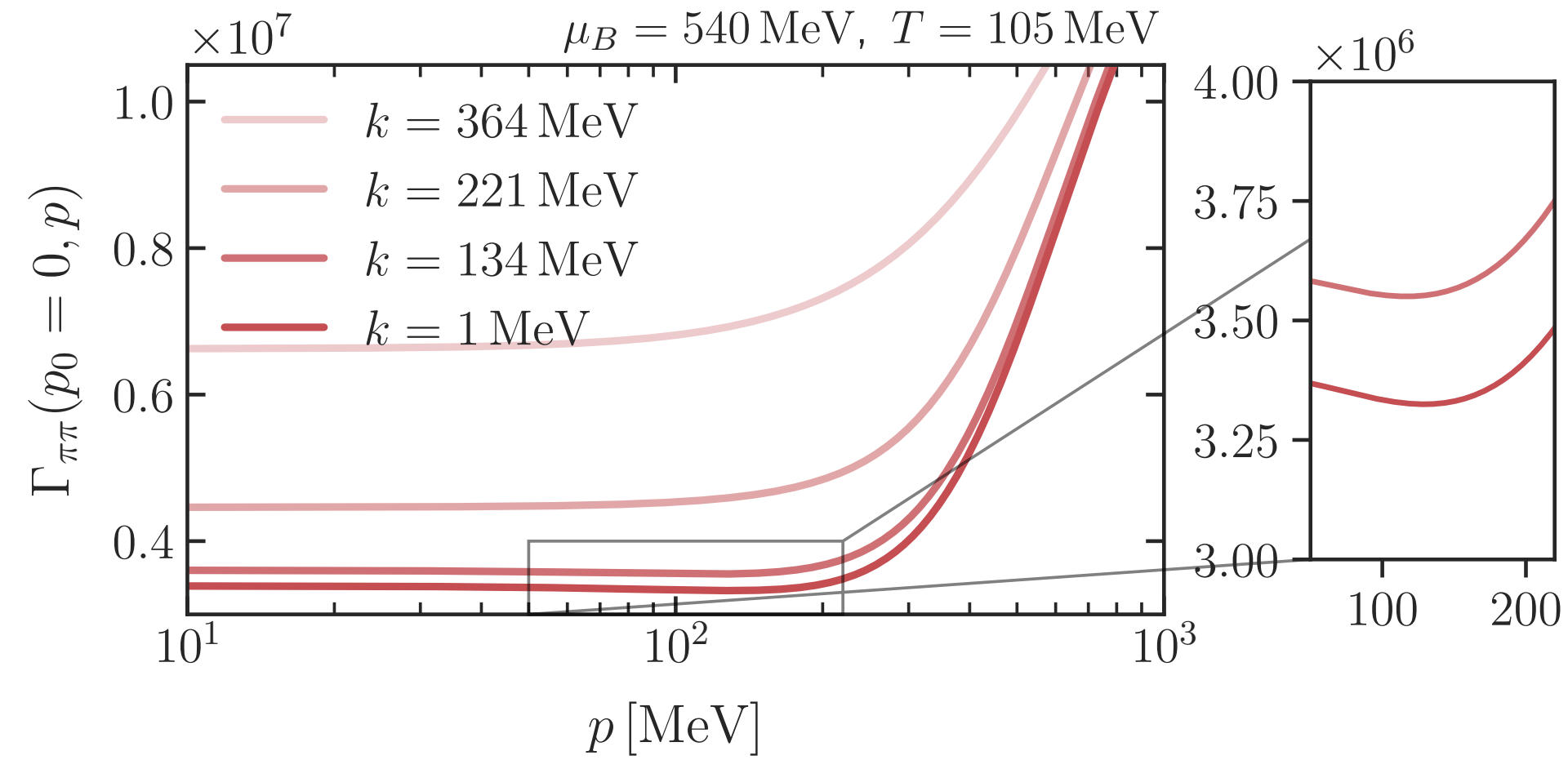
■ T=150 MeV

■ T=160 MeV

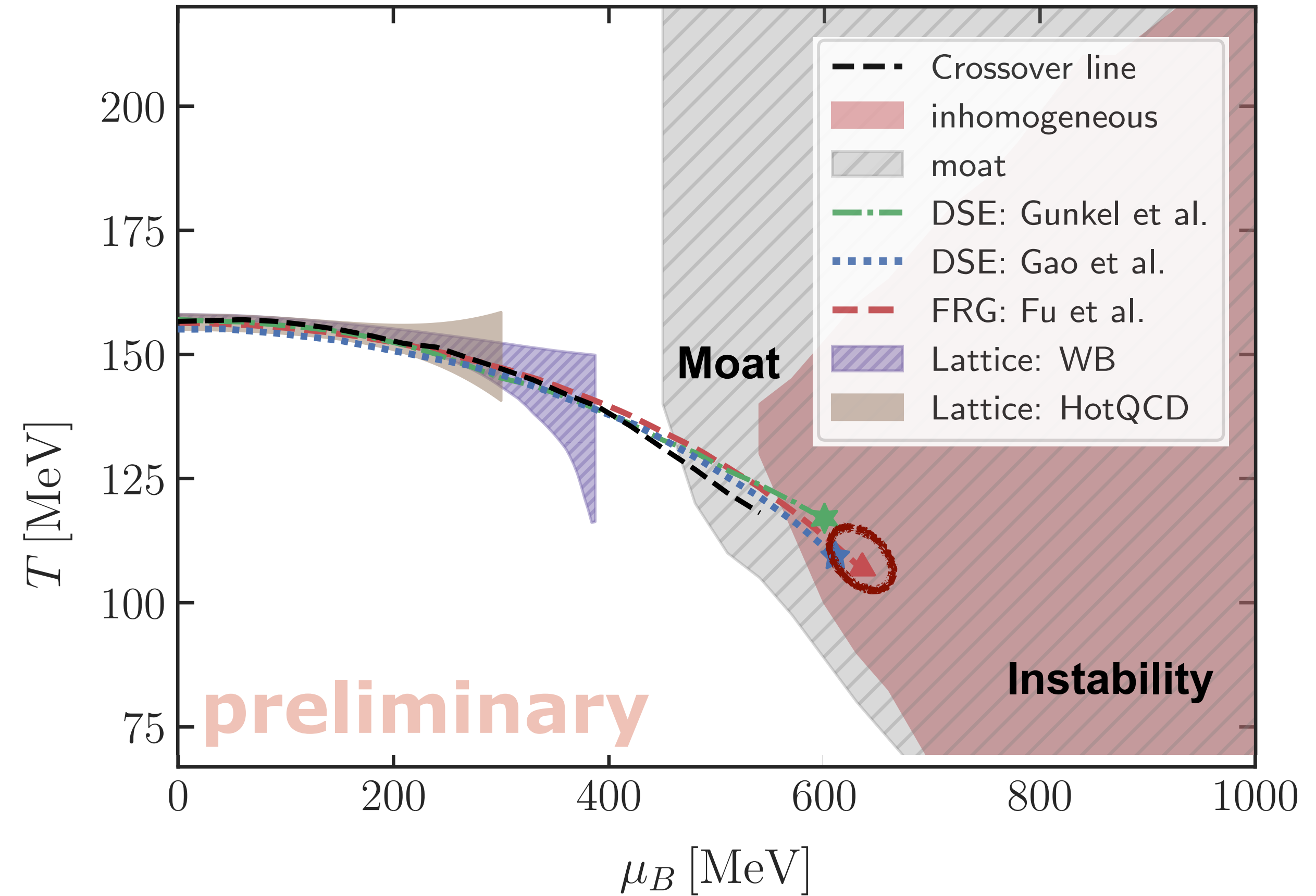
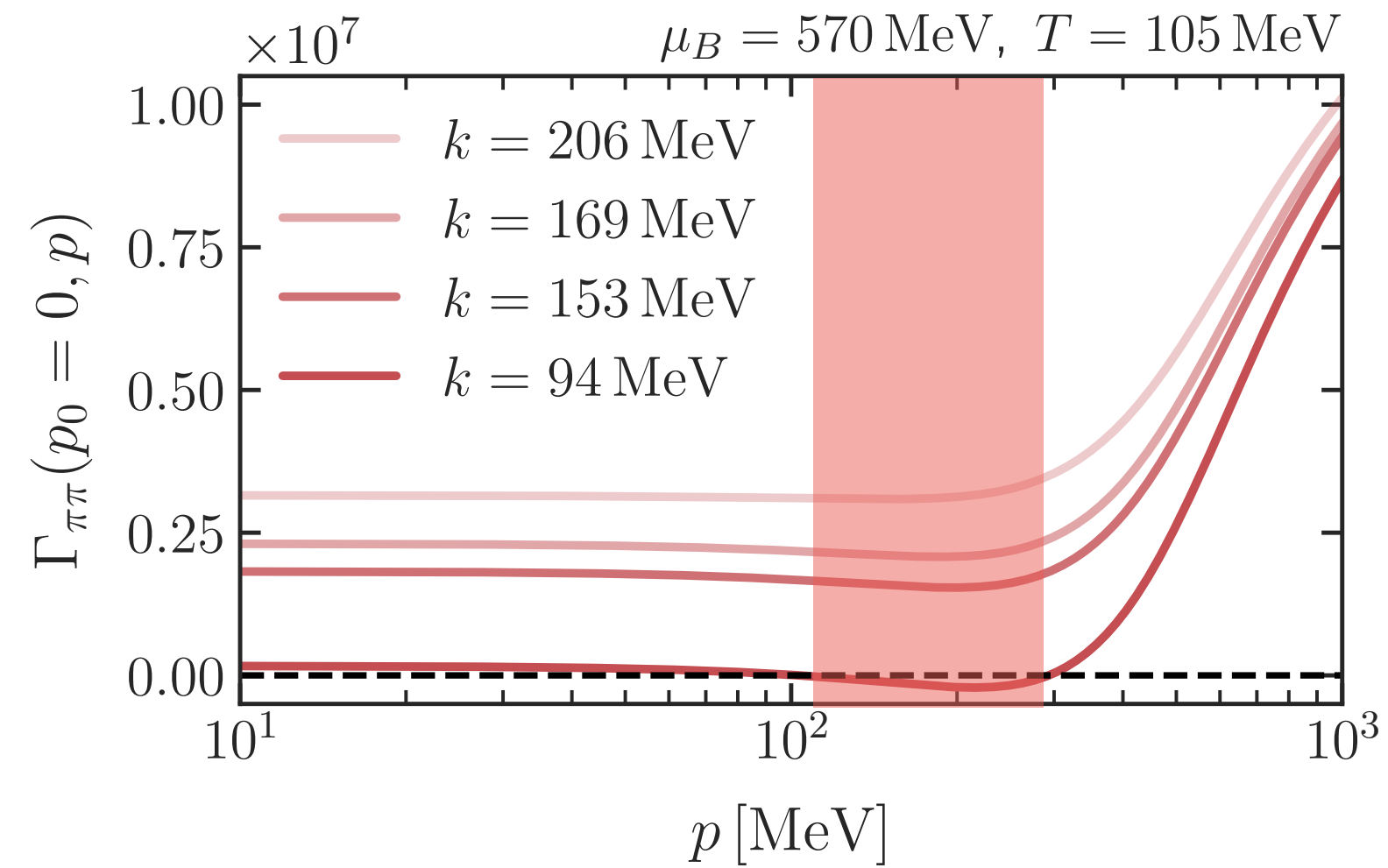


The QCD Moat

Moat



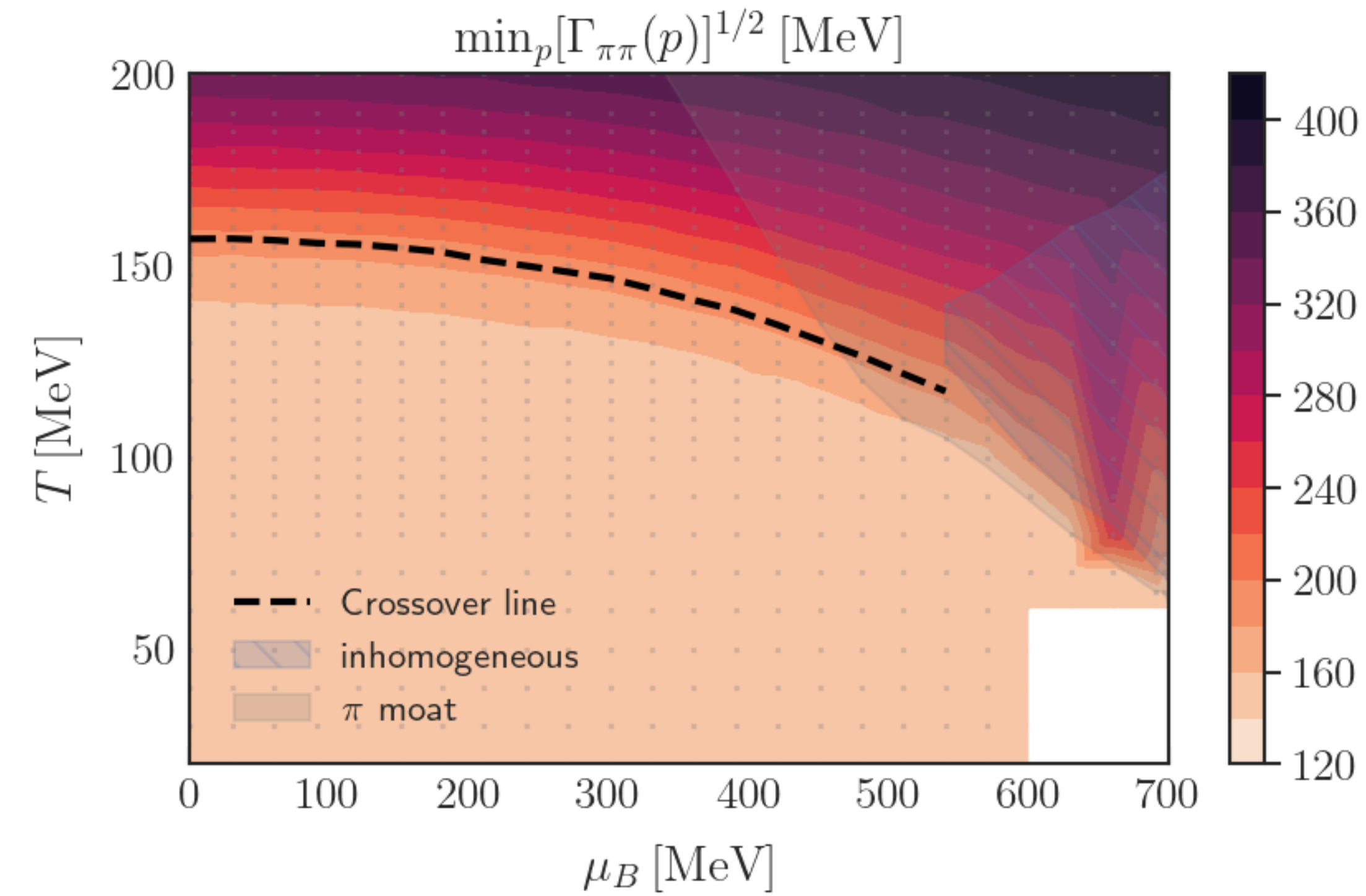
Instability



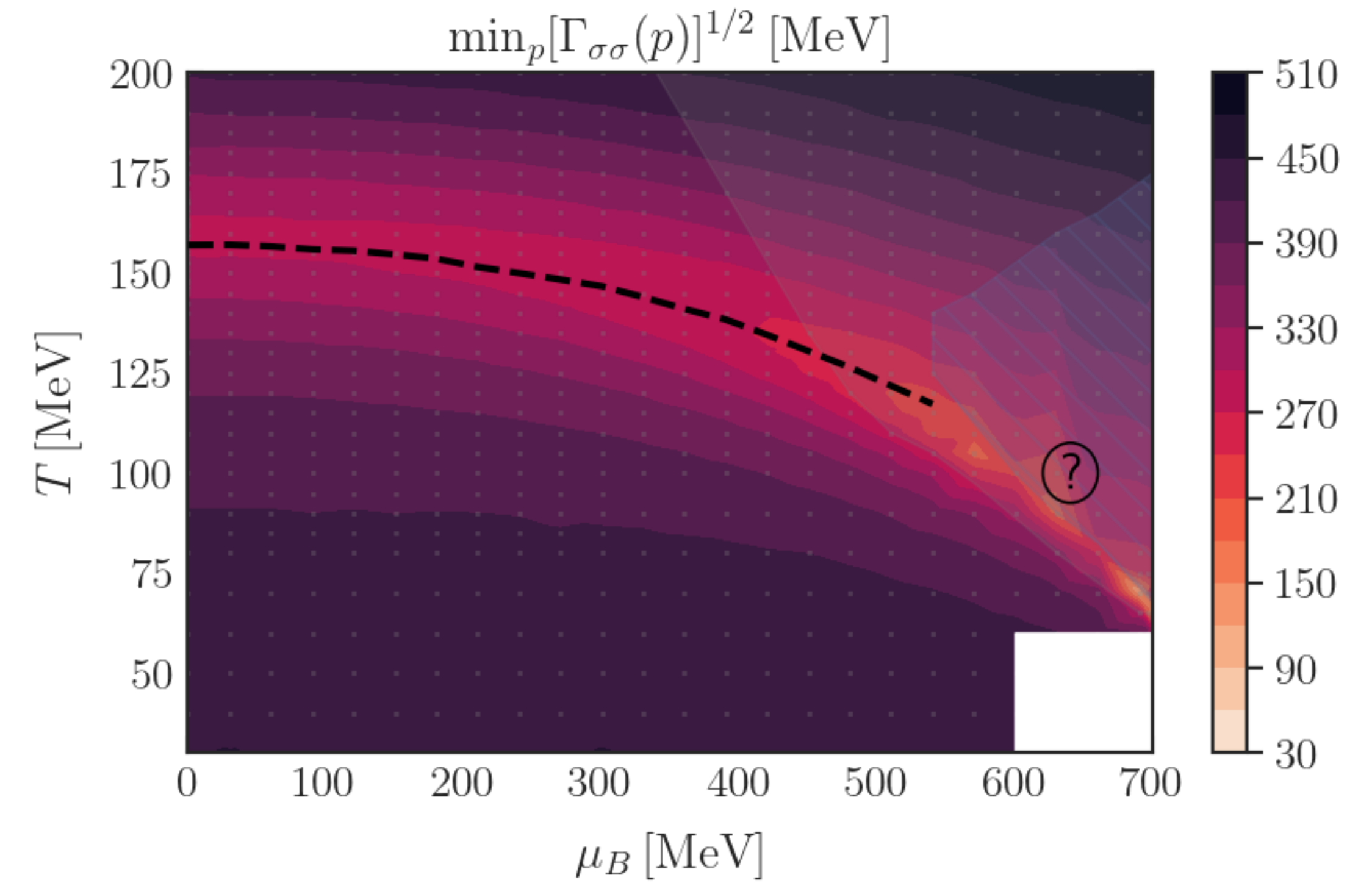
- No full computation deep inside the instability regime
- Estimated location of CEP

The QCD Moat

Pion ‘minimum’ mass



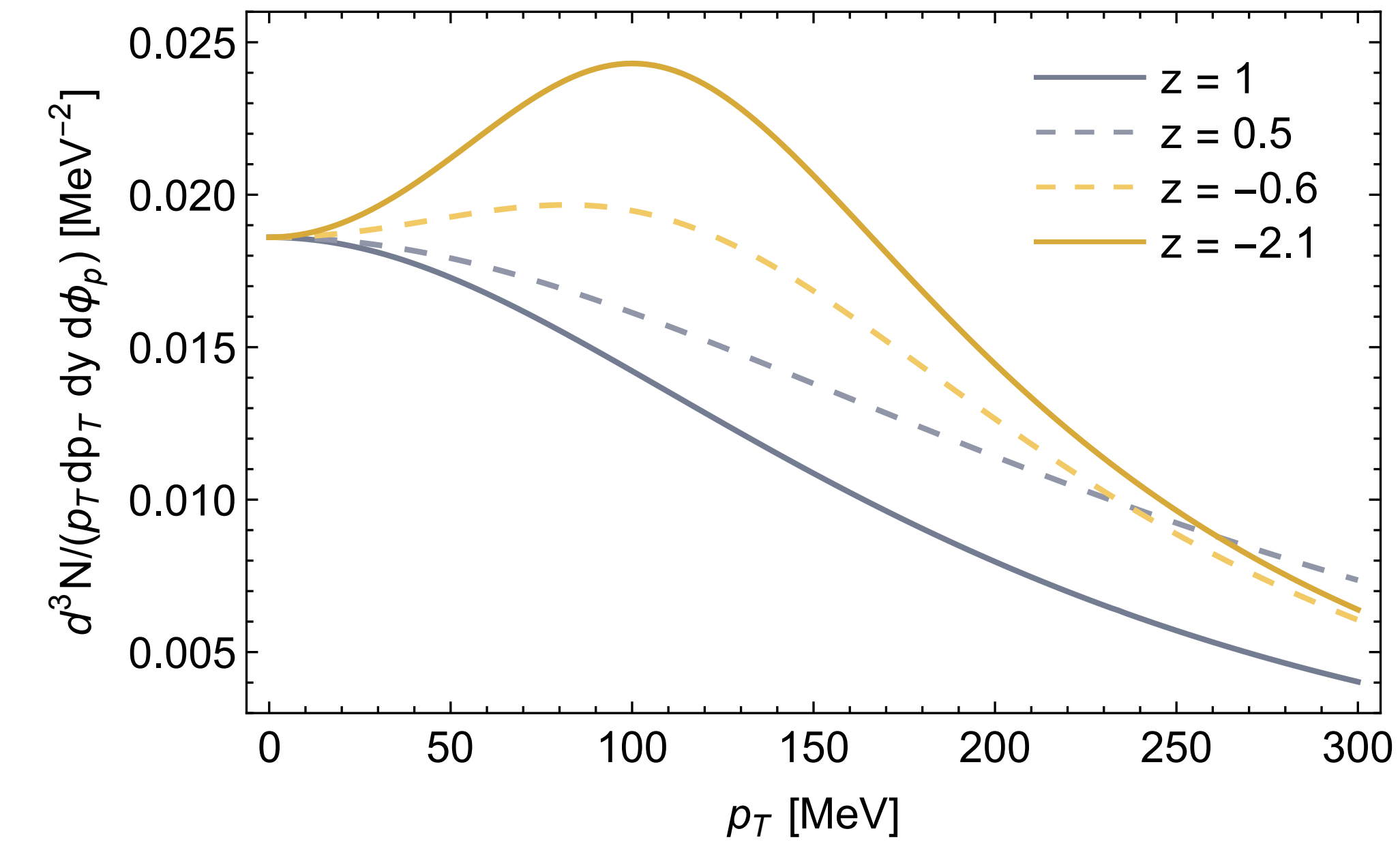
Scalar mode ‘minimum’ mass



preliminary

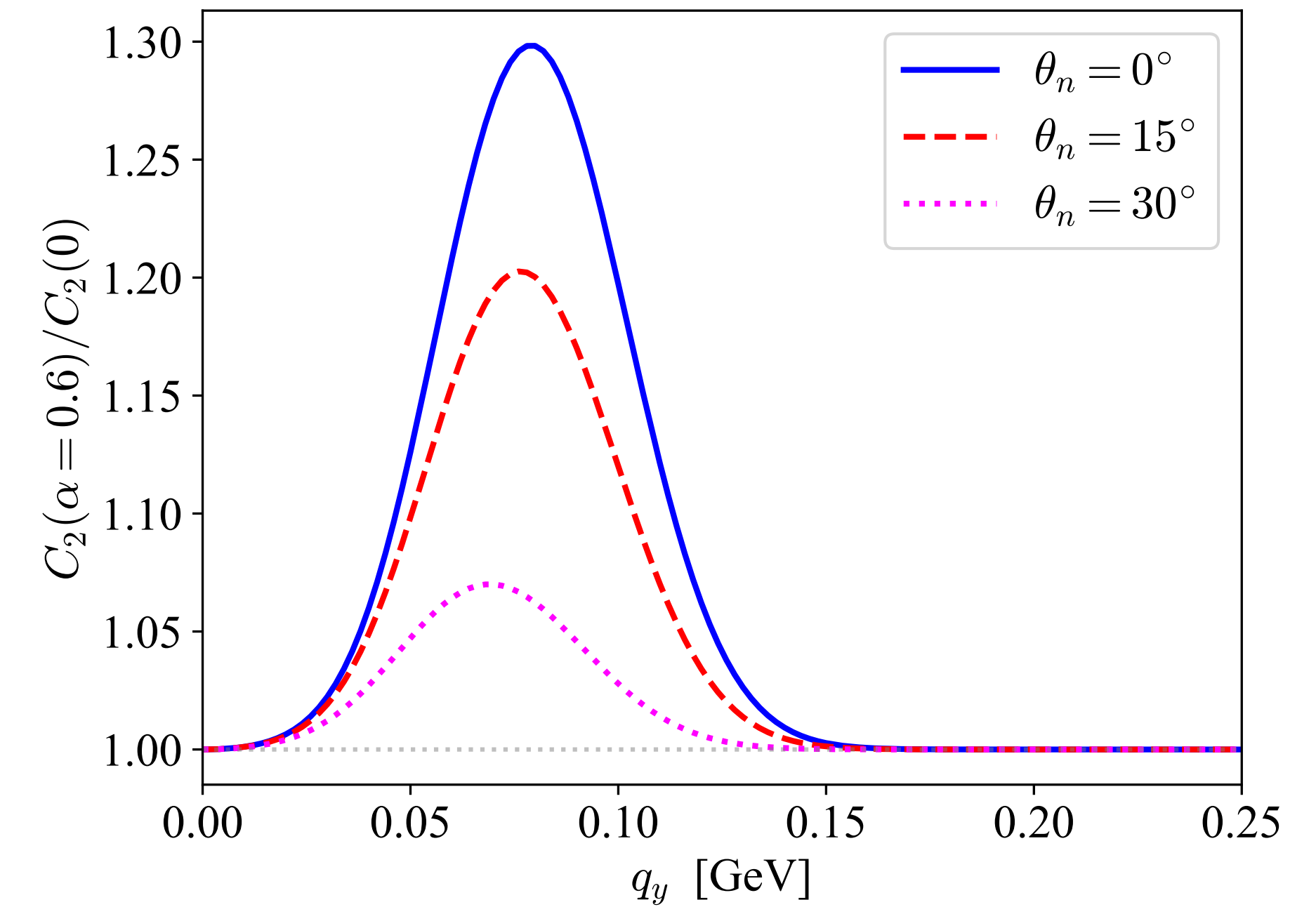
The QCD Moat

Experimental signatures



Rennecke, Pisarski, Rischke, PRD 107 (2023) 116011

HBT Interferometry



**Fukushima, Hidaka, Inoue, Shigaki, Yamaguchi,
PRC 109 (2024) L051903**

Demand: QCD predictions & further signatures