Many-body collective neutrino oscillations

Amol V. Patwardhan

SLAC National Accelerator Laboratory

INT 23-2: Astrophysical neutrinos and the origin of the elements

July 18, 2023

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries PHYSICS FRONTIER CENTER

NSF

SLAC NATIONAL ACCELERATOR LABORATORY

References: 1905.00082, 1905.04386, 1908.03511, 2109.08995, 2202.01865, 2205.09384, 23xx.xxxxx (in preparation)

Collaborators: Michael Cervia, Baha Balantekin, Pooja Siwach, Susan Coppersmith, Calvin Johnson, Denis Lacroix, Xilu Wang, Rebecca Surman

Week 2: several talks on collective neutrino oscillations!

Tuesday, July 25, 2023

Start Time	Presentation Title	Presenter	Presenter Organization	Format	Location
9:00 AM	Global and local simulations of collective neutrino oscillations	Zewei Xiong	GSI	Virtual	C520
9:45 AM	Overview: Neutrino mean-field and QKEs in the early universe and compact objects	Evan Grohs	North Carolina State University	In-person	C520
10:30 AM	Break				
11:00 AM	Numerical Methods for Quantum Kinetics	Sherwood Richers	University of Tennessee Knoxville	In-person	C520
11:30 AM	Global and asymptotic features of fast neutrino-flavor conversion in supernova and binary neutron star merger	Hiroki Nagakura	National Astronomical Observatory of Japan	In-person	C520
12:00 PM	Lunch				
2:00 PM	Thermodynamics of oscillating neutrinos	Luke Johns	UC Berkeley	In-person	C520
2:30 PM	Collisional Flavor Instability in Neutrino Gases	Huaiyu Duan	SLAC	In-person	C520
3:00 PM	Break				
3:30 PM	Workshop Talk Title TBD	Julien Froustey	North Carolina State University / UC Berkeley	In-person	C520

Outline

1. Core-collapse supernovae and neutrinos
2. Collective neutrino oscillations
3. Beyond the effective one-particle description of collective oscillations

Key motivations and definitions

$*$ Neutrinos

- Elementary particles
- No electric charge
- *Not* massless, but **really** small masses
- Don't interact much (only weak interactions)
- Byproduct of some types of nuclear reactions, e.g., decay of a neutron into a proton (but other ways to make them too)

Key motivations and definitions

∗ Supernovae (core-collapse supernovae)

- Explosions of massive stars that are much heavier than our sun
- Final stage in the life cycle of massive stars after multiple stages of nuclear burning
- Triggered by gravitational collapse of stellar core, when it gets so heavy that gravity overcomes pressure support
- Neutrinos have a big part to play

Key motivations and definitions

∗ Neutrino flavor oscillations

- Neutrinos come in three 'flavors': electron, muon, and tau
- Neutrino produced in a well-defined flavor state evolves into a quantum superposition of *all three flavors* as it propagates, with oscillating amplitudes in each flavor
- In environments with dense neutrino streams, neutrinos can undergo collective flavor oscillations driven by ν-ν interactions
- Subsequent interactions depend on flavor composition — critically important for supernovae and nucleosynthesis

Core-collapse supernovae and neutrinos

- Stars with $M_\star \gtrsim 8\, M_\odot$ undergo core collapse & neutronization when core mass exceeds $\sim 1.4\, M_\odot$, i.e., when its gravity surpasses the limit of electron degeneracy pressure support
- Core bounce at nuclear density sends shockwave through infalling material $\rightarrow$ shock eventually loses energy and stalls before it can blow up the star
- Details of the explosion mechanism unknown, but neutrinos expected to play a major role
- CCSNe are neutrino factories: νs are the main carriers of gravitational binding energy ($\sim 99\%$) and lepton number radiated away from the star
    - B.E. $\sim 10^{53}$ ergs $\implies$ $\sim 10^{58}$ νs with $\langle E_\nu \rangle \sim 10$ MeV

Core-collapse supernovae and neutrinos

- Neutrinos depositing $\sim 1\%$ of their energy behind the stalled shock front could revive the shock and explode the star
- ν-induced heating in the aftermath of explosion drives baryonic matter outflows from the surface of the nascent neutron star
- Charged-current weak processes govern the energy deposition and n/p ratio, a crucial input for nucleosynthesis

$$\nu_e + n \longleftrightarrow p + e^-$$
$$\bar{\nu}_e + p \longleftrightarrow n + e^+$$

- Flavor asymmetric processes: thorough understanding of neutrino flavor evolution therefore required

Outline

1. Core-collapse supernovae and neutrinos
2. Collective neutrino oscillations
3. Beyond the effective one-particle description of collective oscillations

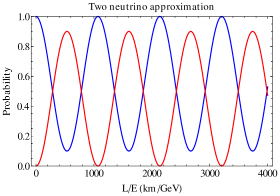
Neutrino oscillations in vacuum

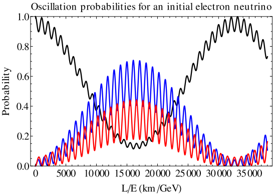
- Neutrino weak-interaction (flavor) eigenstates not aligned with propagation (energy/mass) eigenstates

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$
$$|\nu_x\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

- As neutrinos propagate, mass eigenstates gather quantum mechanical phase at different rates, leading to oscillations

$$P_{ex} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$





Neutrino oscillations in vacuum

- Vacuum oscillations driven by the free-particle Hamiltonian

$$H_{\text{vac}} = \left(\frac{m_1^2}{2p} a_{1\mathbf{p}}^\dagger a_{1\mathbf{p}} + \frac{m_2^2}{2p} a_{2\mathbf{p}}^\dagger a_{2\mathbf{p}} \right) = \omega_\mathbf{p} \, \vec{B} \cdot \vec{J}_\mathbf{p} \ ,$$

where $\omega_\mathbf{p} = \dfrac{\delta m^2}{2p}$, and

$\vec{B} = (0, 0, -1)_{\text{mass}} = (\sin 2\theta, 0, -\cos 2\theta)_{\text{flavor}}$.

- Here we have used the mass-basis 'isospin' operators

$$J_\mathbf{p}^+ = a_{1\mathbf{p}}^\dagger a_{2\mathbf{p}} \ , \qquad J_\mathbf{p}^- = a_{2\mathbf{p}}^\dagger a_{1\mathbf{p}} \ ,$$

$$J_\mathbf{p}^z = \frac{1}{2} \left(a_{1\mathbf{p}}^\dagger a_{1\mathbf{p}} - a_{2\mathbf{p}}^\dagger a_{2\mathbf{p}} \right) \ ,$$

which obey the usual $SU(2)$ commutation relations

$$[J_\mathbf{p}^+, J_\mathbf{q}^-] = 2\delta_{\mathbf{pq}} J_\mathbf{p}^z \ , \qquad [J_\mathbf{p}^z, J_\mathbf{q}^\pm] = \pm \delta_{\mathbf{pq}} J_\mathbf{p}^\pm .$$

Neutrino flavor evolution: matter effects

Matter backgrounds (electrons, nucleons, etc.) modify flavor evolution: "effective mass" through neutrino forward scattering. Mass level crossing $H_{\nu_e\nu_e} = H_{\nu_x\nu_x} \implies$ MSW resonance

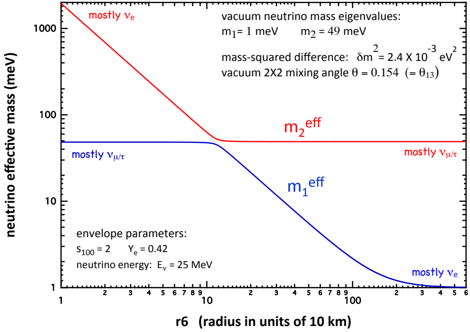


Figure: MSW resonance (figure by George Fuller)

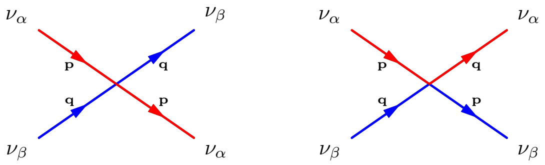
Wolfenstein (1978, '79)
Mikheyev & Smirnov (1985)
Bethe (1986)
Haxton (1986)
Parke (1986)
and so on ...

Neutrino-matter Hamiltonian:

$$H_{\text{mat}} = \lambda\, \vec{L} \cdot \vec{J}_{\mathbf{p}}$$

where $\lambda = \sqrt{2}\, G_F n_e$ and $\vec{L} = (\sin 2\theta, 0, \cos 2\theta)_{\text{mass}}$

Neutrino-neutrino interactions



- Neutrino-neutrino interaction Hamiltonian

$$H_{\nu\nu} = \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}})\, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}} \ .$$

- Note: here, we only consider interactions which preserve or exchange momenta (or equivalently, flavor) [see however: L. Johns, arXiv:2305.04916]

- A many-body coupled quantum system (2^N complex amplitudes) with a complicated geometry on top!

Many-body neutrino Hamiltonian (two-flavor system)

$$H_\nu = \overbrace{\sum_{\mathbf{p}} \omega_{\mathbf{p}} \vec{B} \cdot \vec{J}_{\mathbf{p}}}^{\text{vacuum}} + \overbrace{\lambda \sum_{\mathbf{p}} \vec{L} \cdot \vec{J}_{\mathbf{p}}}^{\text{neutrino-matter}} + \overbrace{\frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}}) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}}^{\text{neutrino-neutrino}},$$

where $\omega_{\mathbf{p}} = \frac{\delta m^2}{2|\mathbf{p}|}$, $\lambda = \sqrt{2} G_F n_e$, $\vec{B} = (0, 0, -1)$, $\vec{L} = (\sin 2\theta, 0, \cos 2\theta)$,
$\vec{J}_{\mathbf{p}}$: neutrino "isospin" operator ($|\uparrow\rangle = |\nu_1\rangle$, $|\downarrow\rangle = |\nu_2\rangle$)

Many-body neutrino Hamiltonian (two-flavor system)

$$H_\nu = \overbrace{\sum_{\mathbf{p}} \omega_{\mathbf{p}} \vec{B}\cdot\vec{J}_{\mathbf{p}}}^{\text{vacuum}} + \overbrace{\lambda \sum_{\mathbf{p}} \vec{L}\cdot\vec{J}_{\mathbf{p}}}^{\text{neutrino-matter}} + \overbrace{\frac{\sqrt{2}G_F}{V}\sum_{\mathbf{p},\mathbf{q}} (1-\cos\vartheta_{\mathbf{pq}})\,\vec{J}_{\mathbf{p}}\cdot\vec{J}_{\mathbf{q}}}^{\text{neutrino-neutrino}},$$

where $\omega_{\mathbf{p}} = \frac{\delta m^2}{2|\mathbf{p}|}$, $\lambda = \sqrt{2}G_F n_e$, $\vec{B} = (0,0,-1)$, $\vec{L} = (\sin 2\theta, 0, \cos 2\theta)$,
$\vec{J}_{\mathbf{p}}$: neutrino "isospin" operator ($|\uparrow\rangle = |\nu_1\rangle$, $|\downarrow\rangle = |\nu_2\rangle$)

"single-angle" approximation $\Downarrow$ neglect neutrino-matter term

$$\boxed{H_\nu = \sum_p \omega_p \vec{B}\cdot\vec{J}_p + \mu(r)\vec{J}\cdot\vec{J}}$$

where $\vec{J} = \sum_{\mathbf{p}} \vec{J}_{\mathbf{p}}$ and $\mu(r) = \frac{\sqrt{2}G_F}{V}\langle(1-\cos\vartheta_{\mathbf{pq}})\rangle$.

Mean-field (random phase) approximation

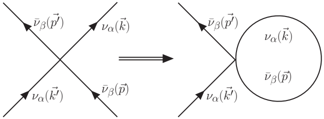


Figure: Volpe *et al.* (2013)

- In an effective one-particle approximation [Sigl and Raffelt (1993), Samuel (1993), Qian and Fuller (1995)], each neutrino considered to interact with an average potential representing all other particles in the medium (including neutrinos)
- Operator product $\mathcal{O}_1\mathcal{O}_2$ approximated as

$$\mathcal{O}_1\mathcal{O}_2 \sim \mathcal{O}_1\langle\mathcal{O}_2\rangle + \langle\mathcal{O}_1\rangle\mathcal{O}_2 - \langle\mathcal{O}_1\rangle\langle\mathcal{O}_2\rangle.$$

  Above expectation values are calculated w.r.t state $|\Psi\rangle$ which satisfies $\langle\mathcal{O}_1\mathcal{O}_2\rangle = \langle\mathcal{O}_1\rangle\langle\mathcal{O}_2\rangle$

Mean-field (random phase) approximation

- This method yields the effective one-particle neutrino interaction Hamiltonian

$$H_{\nu\nu}^{\mathsf{RPA}} = \sum_{\mathbf{p},\mathbf{q}} \mu_{\mathbf{pq}} \left[\vec{J}_{\mathbf{p}} \cdot \langle \vec{J}_{\mathbf{q}} \rangle + \langle \vec{J}_{\mathbf{p}} \rangle \cdot \vec{J}_{\mathbf{q}} - \langle \vec{J}_{\mathbf{p}} \rangle \cdot \langle \vec{J}_{\mathbf{q}} \rangle \right]$$

- Together with the one-body terms (H_{vac} and H_{mat}), Ehrenfest's theorem for the evolution of one-body operator expectation values gives:

$$\boxed{\frac{\mathrm{d}\vec{P}_{\mathbf{q}}}{\mathrm{d}t} = \omega_{\mathbf{q}}\, \vec{B} \times \vec{P}_{\mathbf{q}} + \lambda \vec{L} \times \vec{P}_{\mathbf{q}} + 2 \sum_{\mathbf{p}} \mu_{\mathbf{pq}}\, \vec{P}_{\mathbf{p}} \times \vec{P}_{\mathbf{q}},}$$

where $\vec{P}_{\mathbf{q}} = 2\langle \vec{J}_{\mathbf{q}} \rangle$ is called the neutrino "Polarization vector"

Collective flavor oscillations: synchronized and bipolar

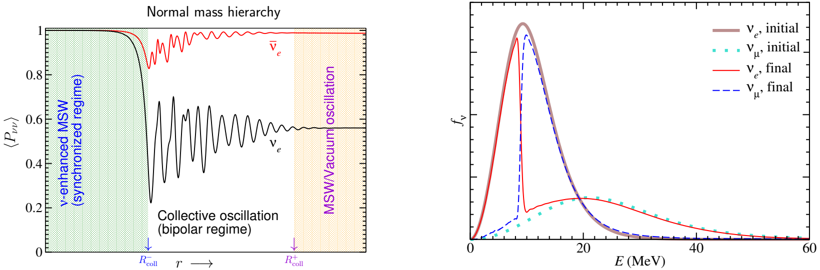
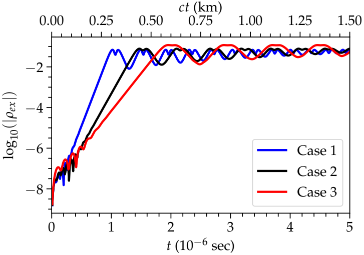


Figure: Taken from Duan et al. (1001.2799). **Left:** regimes for different types of neutrino oscillations in a CCSN environment. **Right:** a neutrino spectral split/swap resulting from collective flavor effects.

'Fast' collective flavor transformations

- Fast collective oscillations — driven by flavor-lepton number crossings in the neutrino angular distributions could cause significant flavor conversion on timescales much shorter than bipolar oscillations, i.e., within $\mathcal{O}(1\text{–}10)$ km from the PNS, making them more relevant for shock reheating and nucleosynthesis
- Recent reviews by Chakraborty et al. (1602.02766), Tamborra & Shalgar (2011.01948), and Richers & Sen (2207.03561)



Neutrino quantum kinetics

- One-particle effective limit of the neutrino Hamiltonian with *only* forward and exchange terms results in the coherent mean-field equations shown above
- In general, other interactions changing neutrino momenta or neutrino number also present $\implies$ collision terms in the one-particle effective description
- Including coherent flavor conversion, collisions, and advection, the evolution equations for the one-body neutrino density matrices in this picture given by

$$i\left(\frac{\partial}{\partial t} + \boldsymbol{v}.\boldsymbol{\nabla}\right)\rho = [H, \rho] + i\mathcal{C}$$

Other cool phenomena in effective one-particle description

- Matter-neutrino resonances
  (Malkus, McLaughlin, Friedland, Wu, Vaananen, Zhu, et al.: 1403.5797, 1507.00946, 1509.08975, 1510.00751, 1607.04671, 1801.07813)

- Collisionally triggered collective flavor instabilities
  (Lucas Johns et al.: 2104.11369, 2206.09225, 2208.11059)

- 'Halo' effect from backscattered neutrinos
  (J. F. Cherry et al.: 1203.1607, 1302.1159, 1908.10594, 1912.11489; V. Cirigliano et al.: 1807.07070)

- Decoherence by wave-packet separation
  (Akhmedov, Kopp, Lindner, Kersten, Smirnov, et al.: 1512.09068, 1702.08338)

Outline

Beyond the mean field, and neutrino entanglement

- The mean-field picture asserts that ν-ν quantum correlations (i.e., entanglement) are strictly prohibited, reducing number of independent amplitudes from 2^N to $2N$ (for two flavors)
- This begs the question as to whether any significant physics is being left out by this approximation
- Early works [Bell et al. (2003), Friedland et al. (2003a, 2003b, 2006), McKellar et al. (2009)] attempted to answer this question using different physical setups. Friedland et al. observed that, in their setup, the build-up of entanglement and the resulting flavor conversion was occurring on timescales suggestive of incoherent effects (and hence unimportant in the large-N limit)

Beyond the mean field, and neutrino entanglement

- Nevertheless, McKellar *et al.* (arXiv:0903.3139) subsequently pointed out that this does not necessary preclude the multiparticle correlations from being significant
- In any case, these early analyses had some notable simplifications, such as omitting the one-body term in the Hamiltonian. It is known, even in the mean-field limit that interplay between H_{vac} and $H_{\nu\nu}$ can give rise to interesting phenomena such as spectral splits
- Renewed interest in re-examining these fundamental questions also partly fueled by ongoing advances in quantum information science & quantum computing capabilities

Some more comments on the many-body formalism

- It was shown in Balantekin & Pehlivan (astro-ph/0607527) that the mean-field solution can also be derived using a saddle-point approximation to the path integral of the full many-body system
- In Volpe *et al.* (arXiv:1302.2374), the many-body evolution is described using a tower of coupled differential equations based on the BBGKY hierarchy, connecting the evolution of one-body density operators to successively higher-order many-body correlations. This provides a potential framework to systematically extend the mean-field evolution equations
- In Pehlivan *et al.* (arXiv:1105.1182), the many-body Hamiltonian was shown to be integrable in the single-angle limit, and the eigenvalue problem was formulated in terms of a set of algebraic Bethe Ansatz equations

Recent work

- Patwardhan, Cervia, Balantekin, Siwach, Coppersmith, Johnson, Lacroix et al.: 1905.04386, 1908.03511, 2109.08995, 2202.01865, 2205.09384;
- Rrapaj, Roggero, Xiong, Martin, Duan, Carlson, Illa, Savage, Yeter-Aydeniz et al.: 1905.13335, 2102.10188, 2102.12556, 2103.11497, 2104.03273, 2111.00437, 2112.12686, 2203.02783, 2207.03189, 2210.08656, 2301.07049 — some of these involve simulating on a quantum computer
- More recently (Shalgar & Tamborra: 2304.13050, Johns: 2305.04916) the suitability of the above studies for judging the efficacy of the mean field has been questioned. Suggested possible paths forward include an open quantum system formulation with wavepackets (S&T), or a unified many-body framework with forward and non-forward scatterings (Johns)

L. Johns et al., arXiv:2305.04916

- The aforementioned many-body literature is predicated on the ν-ν interaction Hamiltonian consisting of only forward and exchange interaction terms. However, forward/exchange interaction should have no special priority in a faithful many-body calculation. The coherent enhancement of forward scattering is in fact a direct consequence of the one-particle effective treatment (quantum kinetics)
- The many-body evolution must not be compared only to the coherent part of the mean-field evolution equations. The $\mathcal{O}(G_F^2)$ terms in the full QKEs do implicitly contain information about the back-reaction of many-body correlations on one-body expectation values

Shalgar and Tamborra, arXiv:2304.13050

- A system of interacting neutrinos in a realistic astrophysical setting is an open quantum system (with finite-sized neutrino wavepackets streaming in and out of any interaction volume), whereas the toy models used in the many-body literature consist of a closed quantum system (with interacting plane waves in a box)

- Even if one considers a many-body Hamiltonian with only forward and exchange interaction terms, it nevertheless contains both coherent and incoherent effects, whereas the mean-field limit of this many-body Hamiltonian retains only the coherent effect

- The evolution of an interacting quantum system driven by incoherent interactions depends on the size of the interacting wavepackets (i.e., the duration of the interactions)

Our framework

- Try to ascertain the potential role of many-body neutrino correlations using simple toy systems. Operating with these toy models necessitates being careful about avoiding sweeping generalizations based on the observed behaviors. Nevertheless, certain patterns or scaling relations could be sought
- Model the neutrino system as N interacting plane waves in a box of volume V, which in general could be time-dependent (to mimic the decreasing density of neutrinos streaming out from a source)
- Examine the evolution of one-body observables (such as expectation values of $\vec{J}_{\mathbf{p}}$) and compare with the mean-field expectation. Additionally, we use entanglement measures (such as bipartite entropy of entanglement) to quantify the degree of multiparticle correlations in the system

Numerical approaches and descriptions

In order to be able to simulate larger and large systems, we have so far explored various numerical methods

- Exploiting the integrability of the single-angle Hamiltonian to diagonalize using Bethe Ansatz solutions [Up to $N \simeq 10$] (1905.04386, 1908.03511)
- Brute force numerical integration using 4th order Runge-Kutta with adaptive time step [Up to $N = 16$] (2109.08995)
- Tensor network calculation using a time-dependent variational principle method [Up to $N \simeq 50$ or $N \simeq 20$ depending on initial state] (2202.01865)
- Approximate phase-space method to evolve a two-beam neutrino system, wherein the neutrinos in each beam are identical to one another [Up to $N \simeq 100$] (2205.09384)

Single-angle limit: Integrability and Bethe Ansatz

- Eigenvalues and eigenstates obtained using procedure derived from Richardson-Gaudin diagonalization (a.k.a. "Bethe-Ansatz" method) — AVP, Cervia, Balantekin, arXiv:1905.04386

- For a system where each neutrino occupies its own energy mode, the eigenproblem can be mapped onto a system of coupled quadratic equations:

$$\tilde{\Lambda}_q^2 + \tilde{\Lambda}_q = \mu \sum_{\substack{p=1 \\ p \neq q}}^{N} \frac{\tilde{\Lambda}_q - \tilde{\Lambda}_p}{\omega_q - \omega_p}$$

$\tilde{\Lambda}_p$ are related to eigenvalues of the invariants h_p of the single-angle Hamiltonian. Bethe-Ansatz equations shown to be equivalent to polynomial relations between invariants h_p
— Cervia, AVP, Balantekin, arXiv:1905.00082

Single-angle limit: adiabatic many-body evolution

- Eigenvalues and eigenvectors facilitate calculating the adiabatic evolution of the many-body neutrino system, starting from any given initial condition, as μ is varied
- Consider an initial many-body state, $|\Psi_0\rangle \equiv |\Psi(\mu_0)\rangle$
  - Example: in the (two-)flavor-basis, $|\nu_e \nu_x \nu_e \nu_e\rangle$
- May be decomposed into the basis of energy eigenstates: $|\Psi(\mu_0)\rangle = \sum_n c_n \, |e_n(\mu_0)\rangle$
- If μ were to change sufficiently slowly then the system adiabatically evolves into

$$|\Psi(\mu)\rangle \simeq \sum_n c_n e^{-i \int_{\mu_0}^{\mu} \frac{E_n(\mu')}{d\mu'/dt} d\mu'} \, |e_n(\mu)\rangle$$

Summary of entanglement measures

Density Matrix, Polarization Vector, & Entanglement Entropy

Consider a pure, many-body neutrino state $\rho = |\Psi\rangle\langle\Psi|$.
Single-neutrino reduced density matrix: $\rho_q \equiv \mathrm{Tr}_{1,\ldots,\widehat{q},\ldots,N}[\rho]$, given by ($\widehat{\ }$ denotes exclusion)

$$\rho_q = \sum_{i_1,\ldots,\widehat{i_q},\ldots,i_N=1}^{2} \langle \nu_{i_1}\ldots\widehat{\nu_{i_q}}\ldots\nu_{i_N}|\rho|\nu_{i_1}\ldots\widehat{\nu_{i_q}}\ldots\nu_{i_N}\rangle\,,$$

- $S(\omega_q)$, Entropy of entanglement between neutrino q and rest:

$$S(\omega_q) = -\mathrm{Tr}[\rho_q \log \rho_q]$$

- "Polarization vector" of neutrino q, $\vec{P}(\omega_q) = 2\,\langle \vec{J}_q\rangle$, related to the reduced density matrix as:

$$\rho_q = \frac{1}{2}\left(\mathbb{I} + \vec{P}(\omega_q)\cdot\vec{\sigma}\right)$$

Relations between entanglement measures

Entanglement entropy has a one-to-one, inverse relationship with the magnitude of the polarization vector

$$S(P_q) = -\frac{1-P_q}{2}\log\left(\frac{1-P_q}{2}\right) - \frac{1+P_q}{2}\log\left(\frac{1+P_q}{2}\right)$$

with $P_q = |\vec{P}(\omega_q)|$

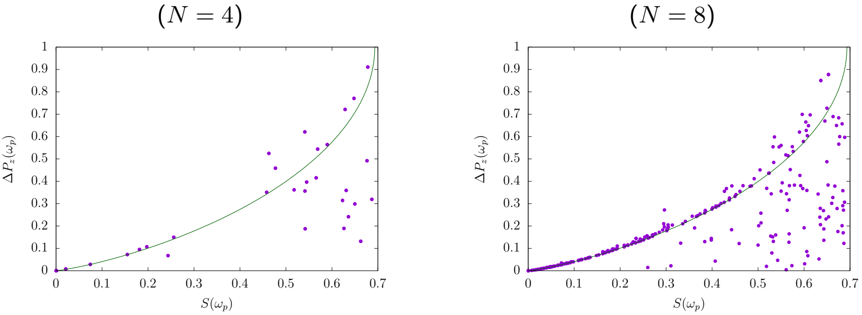
- $P = 1 \iff S = 0$ (Unentangled)
- $P = 0 \iff S = \log(2)$ (*Maximally* Entangled)

Other studies have used different entanglement measures, including bipartite measures such as Negativity, Renyi entropy, and Left-Right entanglement entropy, as well as multipartitie measures such as n-tangle (e.g., Illa and Savage: 2210.08656)

Correlation of P_z-discrepancies and entanglement entropy

Calculate $\Delta P_z(\omega) \equiv |P_z^{\mathrm{MF}}(\omega) - P_z^{\mathrm{MB}}(\omega)|$ at $r \gg R_\nu$ (i.e., $\mu \approx 0$)

- For $N = 4$: all initial conditions with definite flavor ν_e, ν_x (e.g., $|\nu_e, \nu_x, \nu_x, \nu_x\rangle$)
- For $N = 8$: same ICs as $N = 4$, but with four additional ν_e appended to left or right of spectrum

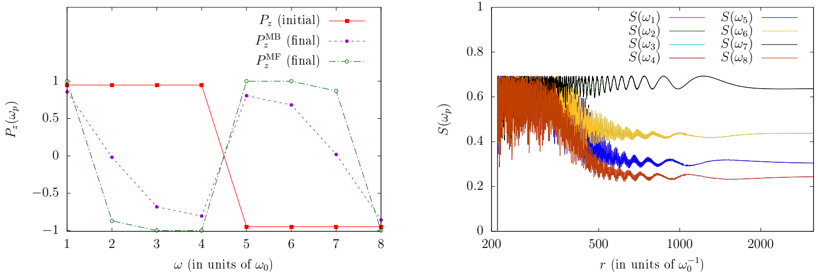


Trendline: $y(S) \equiv P^{\mathrm{MF}}(S) - P^{\mathrm{MB}}(S) = 1 - P(S)$

Example: initial condition with both neutrino flavors

Comparison of final P_z spectra between many-body and mean-field

- Evolve $|\Psi_0\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_x\nu_x\nu_x\nu_x\rangle$ until $r \gg R_\nu$

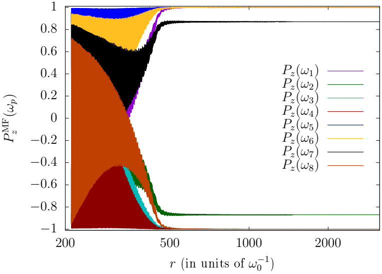


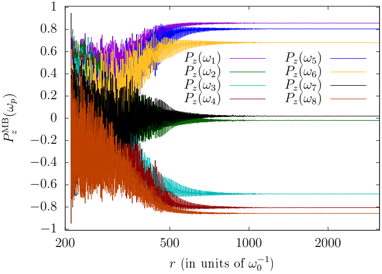
[Cervia, AVP, et al.: 1908.03511]

Spectral split-like features persist in the many-body calculations, but are less sharp relative to mean-field calculations

Comparison of P_z evolution with r

- Same initial conditions, $|\Psi_0\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_x\nu_x\nu_x\nu_x\rangle$





[Cervia, AVP, et al.: 1908.03511]

Spectral splits and entanglement entropy

In [AVP, Cervia, Balantekin: 2109.08995], we extended our calculations to $N = 16$, and noted that the entanglement entropy (and corresponding deviation from the mean-field observables) was seen to be maximum for the neutrinos nearest to the spectral splits. This observation of entanglement localization in certain reigons of the neutrino spectrum motivated the use of tensor networks in a further study (2202.01865).

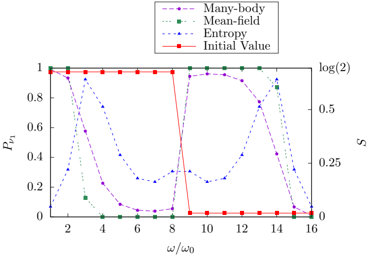


Figure: Initial and final neutrino spectra, along with the respective final state entanglement entropies, for the evolution of initial states $|\nu_e\rangle^{\otimes 8}|\nu_x\rangle^{\otimes 8}$ (left) and $|\nu_e\rangle^{\otimes 12}|\nu_x\rangle^{\otimes 4}$ (right).

Studies by other groups [not an exhaustive list]

Other groups have performed complementary studies and uncovered interesting behaviors

- Presence of collective oscillations on "fast" timescales $[\tau_F \sim \mu^{-1} \log(N)]$ linked to Dynamical phase transitions in the model (Roggero, 2103.11497)

- Simulations with larger N using interacting two-beam models (Xiong, arXiv:2111.00437 & Martin *et al.*, arXiv:2112.12686). For certain initial configurations and mixing angle values, the oscillation behavior was found to converge to the mean-field limit; in other cases, deviations were observed

- Multi-angle model with randomly distributed one- and two-body couplings (Martin et al., 2301.07049) yielded qualitatively different results from the single-angle case . Significant loss of coherence observed in the one- and two-body trace-reduced subsystems, suggesting that the evolution could be approximated as a classical mixture of separable states

Conclusions, Summary and Outlook

- Calculations of collective neutrino flavor evolution typically rely on a 'mean-field', i.e., effective one-particle description, the efficacy of which has been (and continues to be) scrutinized
- Using a simple model of interacting neutrino plane waves in a box, certain deviations from the mean-field behaviour are observed in small systems, which can be attributed to multi-particle entanglement in this class of models
- It has been suggested that these toy models have limited applicability towards robustly evaluating the mean-field approximation. Nevertheless, a number of potentially interesting insights were revealed in these studies, some of which may generalize to more sophisticated future treatments