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Status and Update on V_{us} determination

Emilie Passemar
passemar@ific.uv.es
IFIC Valencia/Indiana University

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GENERALITAT VALENCIANA

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Outline

1. Introduction and Motivation
2. Why Cabbibo angle anomaly?
3. Prospects
4. Can Tau physics help?
5. Conclusion and Outlook

1. Introduction and Motivation

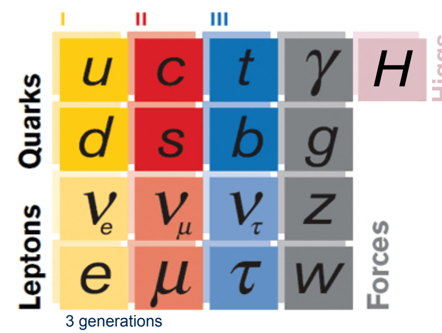
1.1 Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

    Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}$$

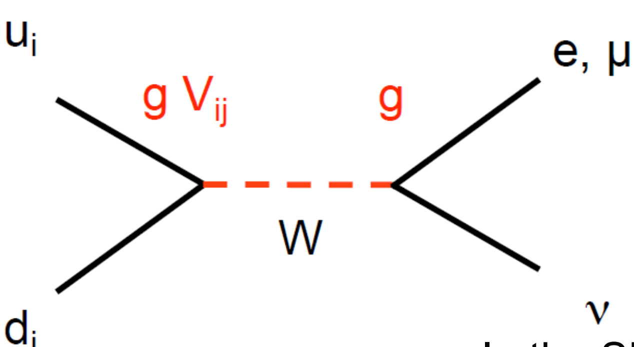
Unitary matrix



  - Check unitarity of the first row of the CKM matrix:

Cabibbo Universality: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$

Negligible ~2x10^{-5} (B decays)



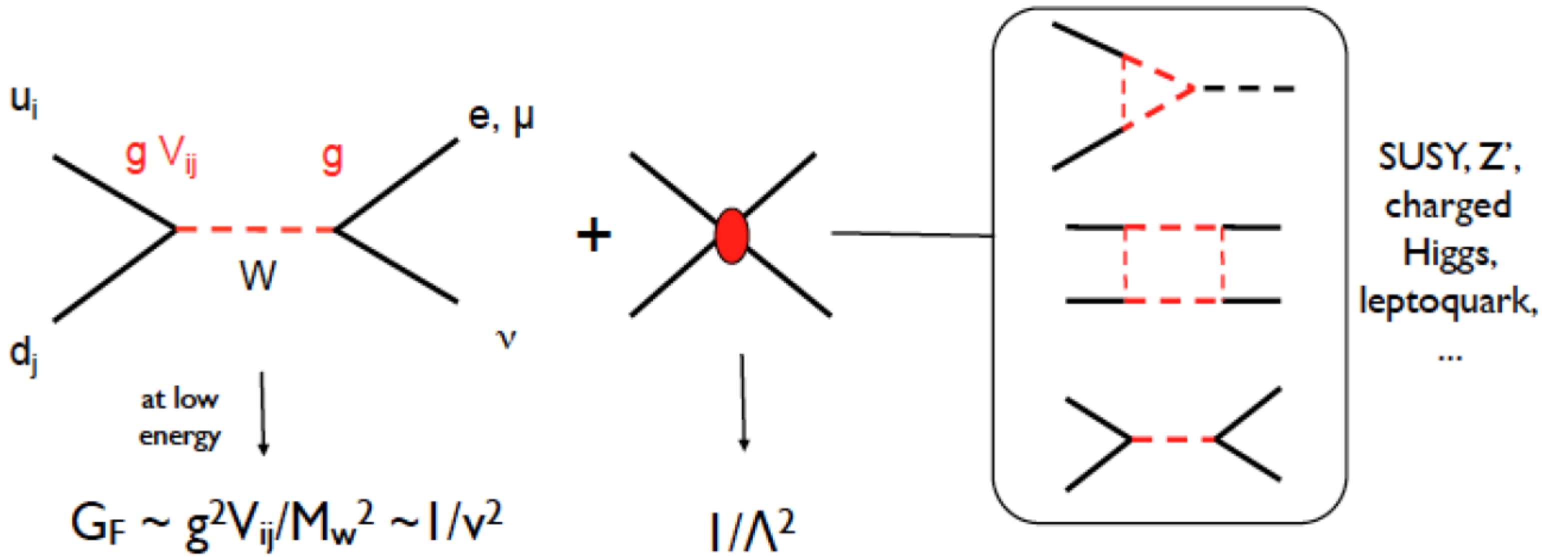
$$|V_{ud}| = \cos\theta_C \quad \text{and} \quad |V_{us}| = \sin\theta_C$$

In the SM: W exchange ⇒ V – A structure only

1.2 Constraining New Physics

- BSM: sensitive to tree-level and loop effects of a large class of models

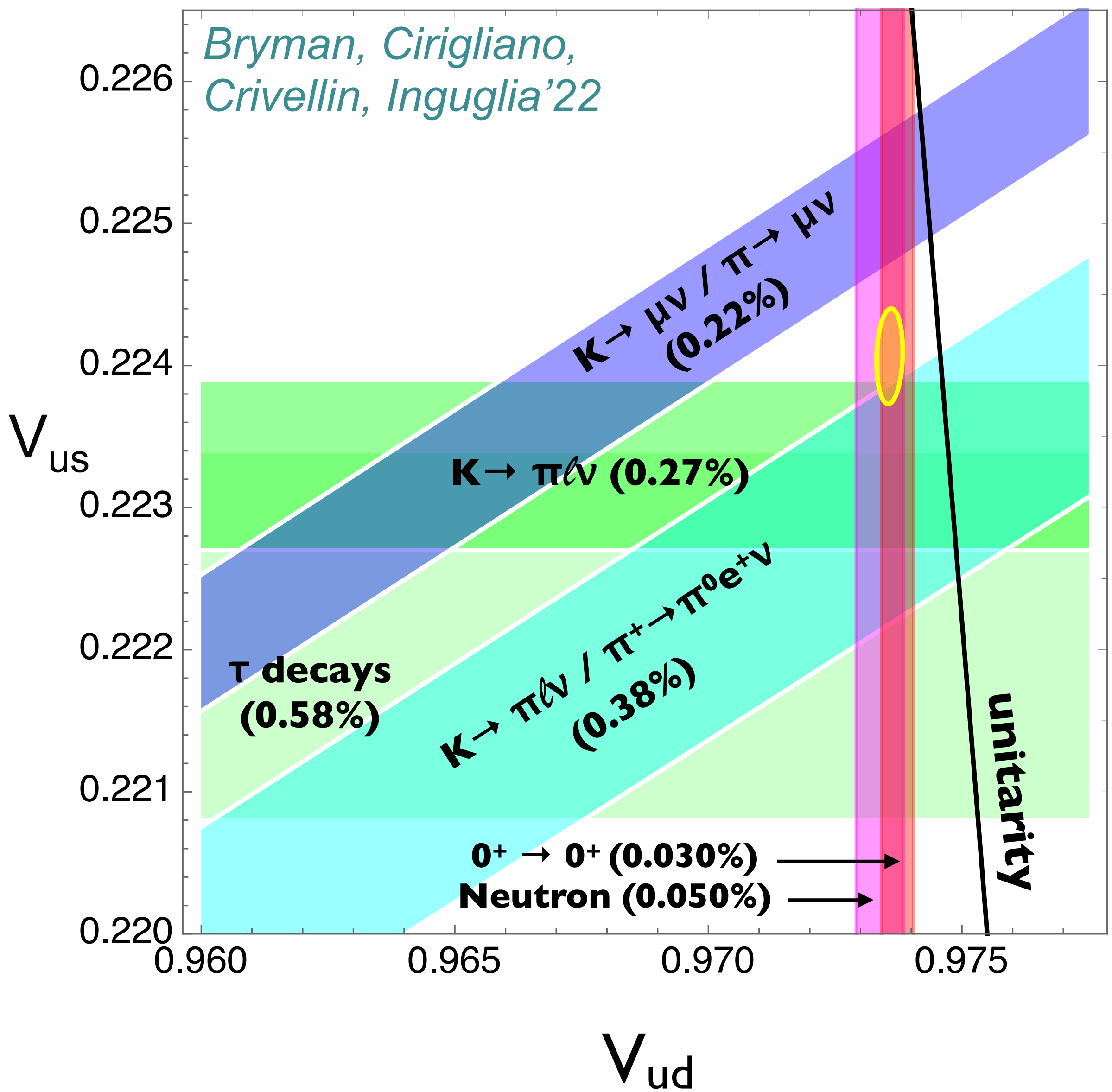
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$$



BSM effects : $\Delta \sim \frac{c_n}{g^2}\frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \longleftrightarrow \Lambda \sim 1\text{-}10$ TeV

1.3 Cabibbo angle anomaly

Moulson & E.P.@CKM2021



$|V_{ud}| = 0.97373(31)$

$|V_{us}| = 0.2231(6)$

$|V_{us}|/|V_{ud}| = 0.2311(5)$

Fit results, no constraint

$V_{ud} = 0.97365(30)$

$V_{us} = 0.22414(37)$

$\chi^2/\text{ndf} = 6.6/1\ (1.0\%)$

$\Delta_{CKM} = -0.0018(6)$

−2.7σ

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$$

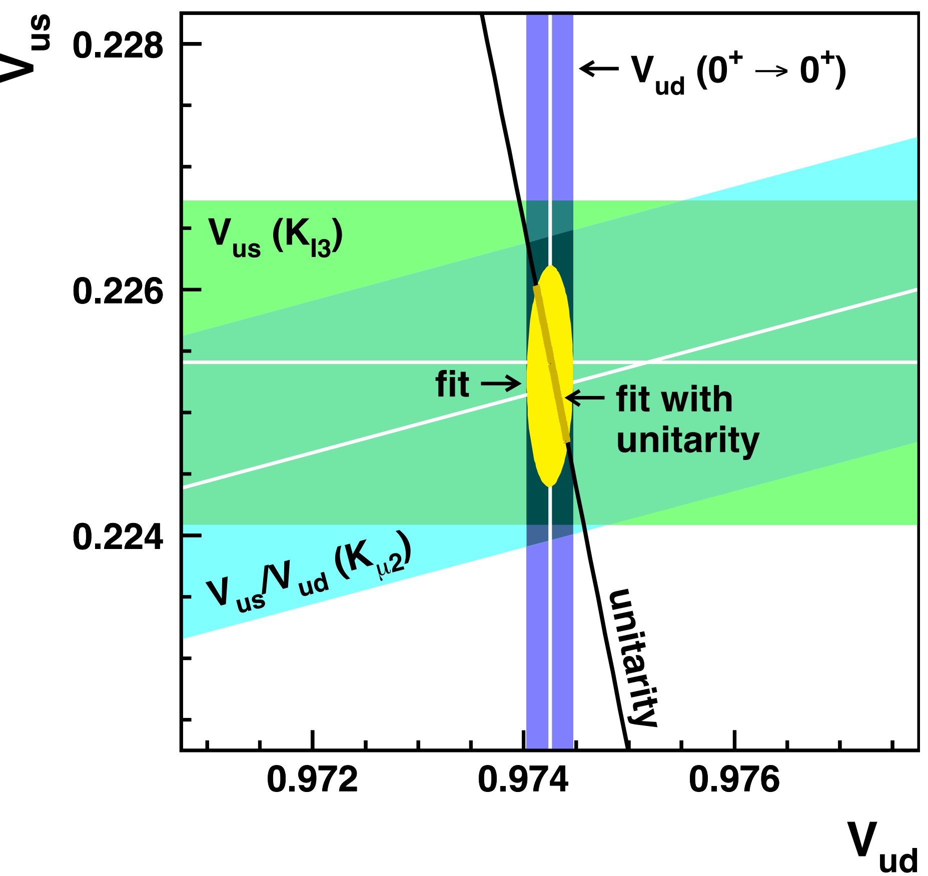
Negligible ~2×10^{-5} (B decays)

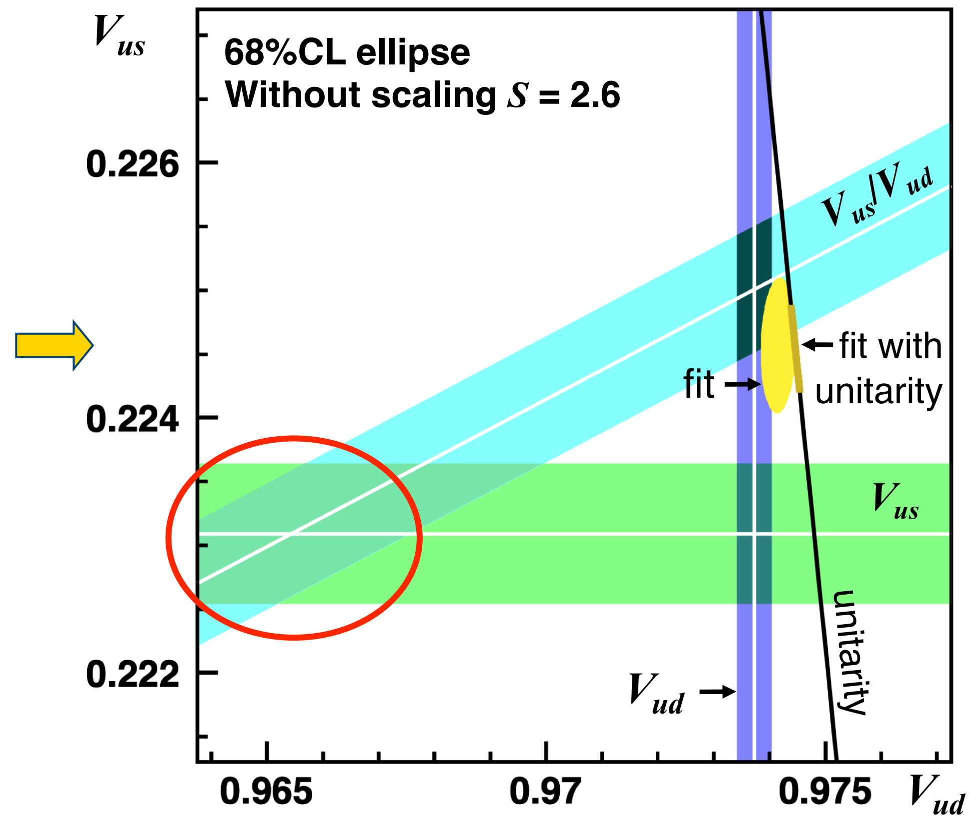
2. Why the Cabbibo angle anomaly?

2.1 Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*

Moulson & E.P.@CKM2021





2.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{RC}) \times F_{\text{kin}}$$

Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

2.2 Changes on V_{us} and V_{ud} since 2011

- Almost no change on the experimental side since 2011

Flavianet Kaon WG: *Antonelli et al'11*

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Radiative corrections

- Changes in *theoretical* inputs:
  - Impressive progress on hadronic matrix element computations from *lattice QCD* for V_{us} and V_{us}/V_{ud} extraction from Kaon decays

FLAG'24

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Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

- Changes in *theoretical* inputs:
  - Impressive progress on hadronic matrix element computations from lattice QCD for V_{us} and V_{us}/V_{ud} extraction from Kaon decays
  - Radiative corrections for V_{ud} extraction from dispersive methods and EFTs but also for V_{us} extraction (+ lattice QCD)

Seng et al.'18'19, Gorshteyn'18, Cirigliano et al.'22,'24

⇒ see talks this week

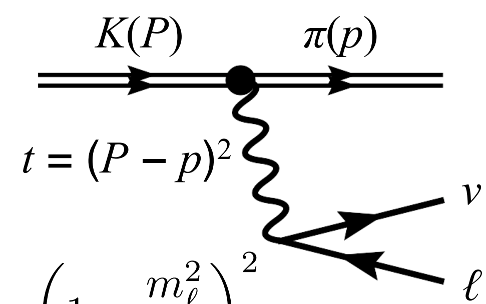
2.3 V_{us} extraction from K_{l3} decays

- Master formula for $K \rightarrow \pi l \nu_l$: $K = \{K^+, K^0\}$, $l=\{e,\mu\}$

$$\Gamma\left(K \to \pi l \nu [\gamma]\right) = \frac{Br(K_{l3})}{\tau} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl} \left(1 + 2\Delta_{\rm EM}^{Kl} + 2\Delta_{\rm SU(2)}^{K\pi}\right)$$

Hadronic matrix element:

$$\left\langle \pi^-(p)\right| \bar{s}\gamma_\mu u \left|K^0(P)\right\rangle = f_+^{K^0\pi^-}(0)\left[\left(P+p\right)_\mu \bar{f}_+^{K^0\pi^-}(t) + \left(P-p\right)_\mu \bar{f}_-^{K^0\pi^-}(t)\right]$$



- Phase space integrals: $$I_{K\ell} = \frac{2}{3}\int_{m_\ell^2}^{t_0} \frac{dt}{M_K^8}\, \bar{\lambda}^{3/2} \left(1+\frac{m_\ell^2}{2t}\right)\left(1-\frac{m_\ell^2}{2t}\right)^2 \times \left(\bar{f}_+^2(t) + \frac{3m_\ell^2 \Delta_{K\pi}^2}{(2t+m_\ell^2)\bar{\lambda}}\bar{f}_0^2(t)\right),$$

- In K_{e3} decays: only vector FF $\bar{f}_+^{K^0\pi^-}(t)$

- In $K_{\mu 3}$ decays, also need the scalar FF $$\bar{f}_0(t) = \bar{f}_+(t) + \frac{t}{m_K^2 - m_\pi^2}\bar{f}_-(t)$$

- For V_{us}, need integral over phase space of squared matrix element: Parameterize form factors and fit distributions in t (or related variables)

$K\pi$ form factor parametrizations

- Parametrizations based on Taylor expansion:

$$\bar{f}_{+,0}(t) = 1 + \lambda_{+,0}\left(\frac{t}{m^2_{\pi^\pm}}\right) \quad \text{or} \quad \bar{f}_{+,0}(t) = 1 + \lambda'_{+,0}\left(\frac{t}{m^2_{\pi^\pm}}\right) + \lambda''_{+,0}\left(\frac{t}{m^2_{\pi^\pm}}\right)^2$$

Very simple parametrization but limited in energy range and not physically motivated: many parameters and strong correlations between them

⇒ unstable fits

- Physically motivated parametrizations:
  - Pole parametrization

$$\bar{f}_{+,0}(t) = \left(\frac{M^2_{V,S}}{M^2_{V,S} - t}\right)$$

Well motivated for the vector (K* resonance)
But for the scalar M_S?

  - Dispersive parametrization

Bernard, Oertel, E.P., Stern'06,'09

$$\bar{f}_{+}(t) = \exp\left[\frac{t}{m^2_{\pi}}\left(\Lambda_{+} - H(t)\right)\right] \quad \text{and} \quad \bar{f}_{0}(t) = \exp\left[\frac{t}{m^2_{K} - m^2_{\pi}}\left(\ln C - G(t)\right)\right]$$

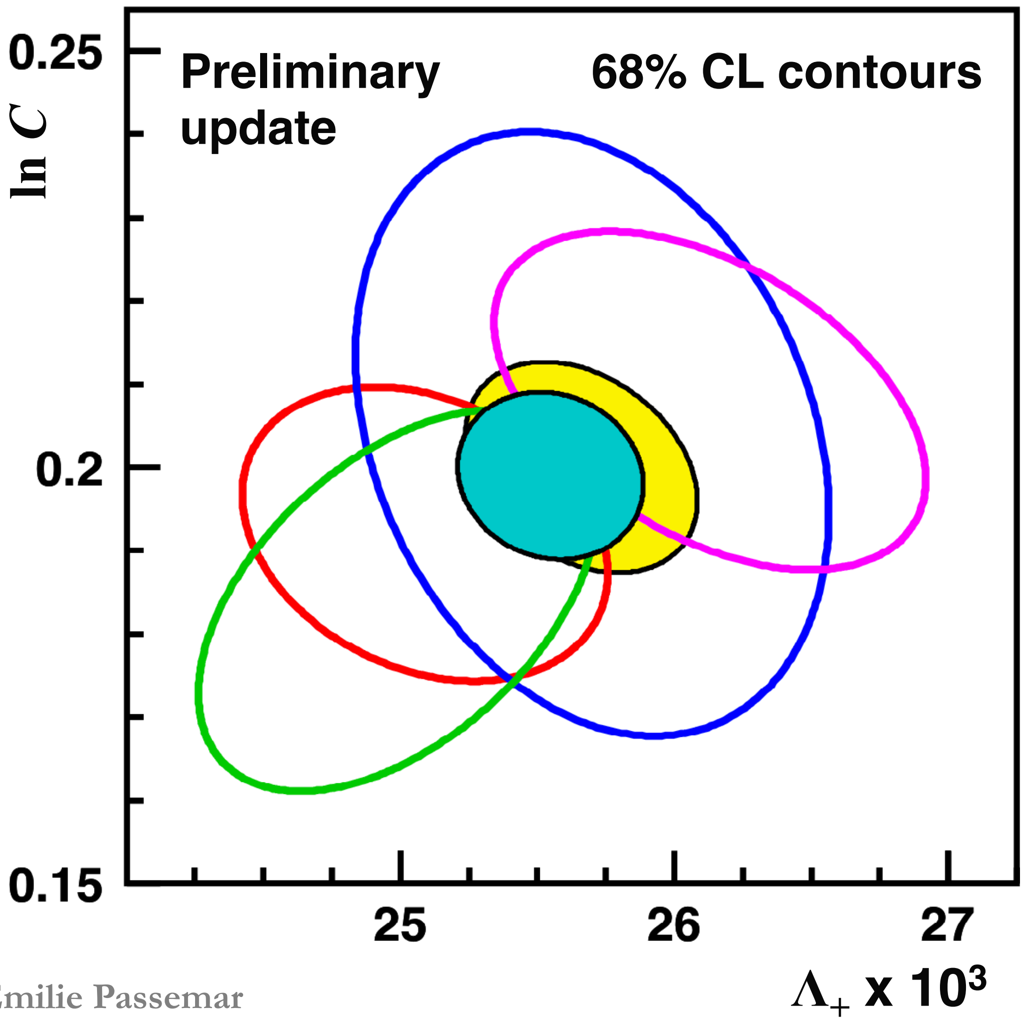
Kπ scattering phase

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from KTeV KLOE ISTRA+ NA48/2

NA48 K_{e3} data included in fits but not shown

2010 fit | Update



$\Lambda_+ \times 10^3$	=	**25.55 ± 0.38**
$\ln C$	=	**0.1992(78)**
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Integrals

Mode	Update	2010
$K^0{}_{e3}$	**0.15470(15)**	0.15476(18)
$K^+{}_{e3}$	**0.15915(15)**	0.15922(18)
$K^0{}_{\mu 3}$	**0.10247(15)**	0.10253(16)
$K^+{}_{\mu 3}$	**0.10553(16)**	0.10559(17)

Only tiny changes in central values

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

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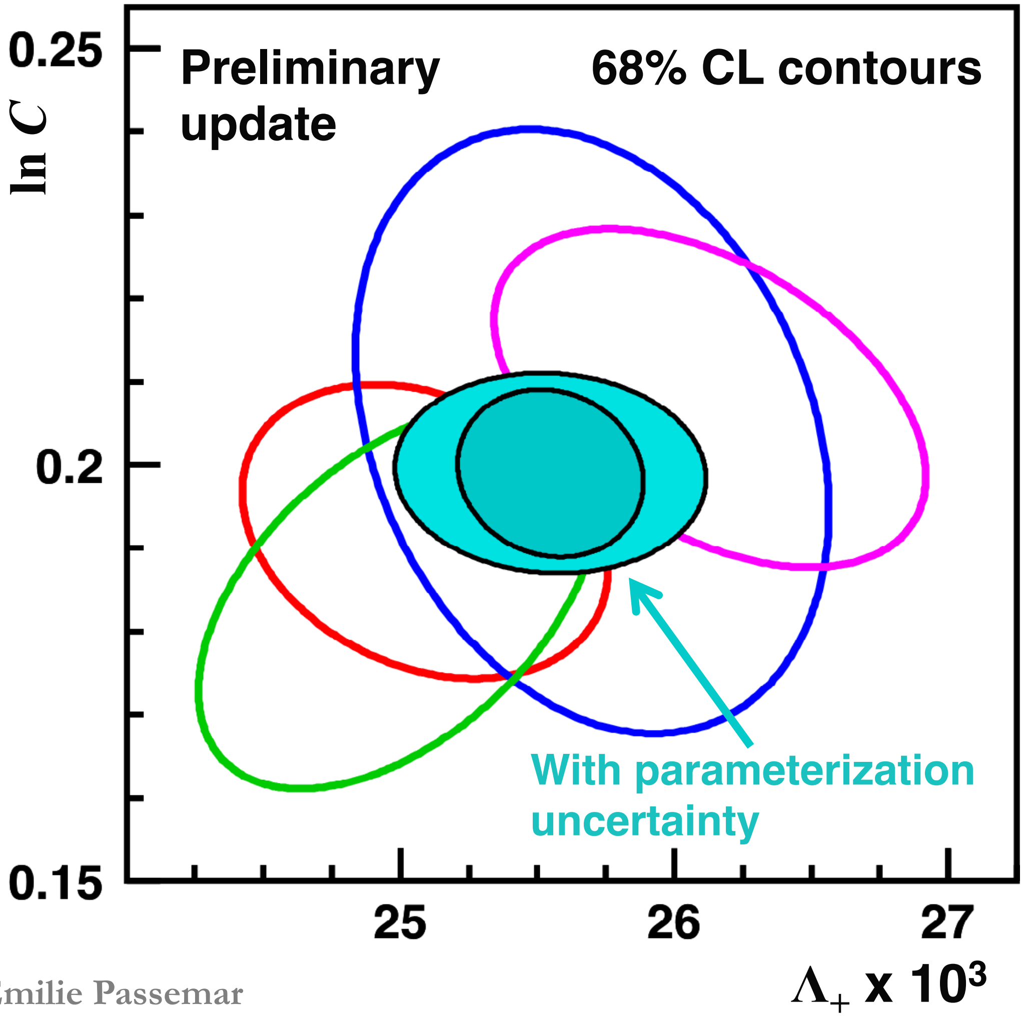

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χ^2/ndf	=	7.5/7 (38%)

Fit results include common uncertainty from $H(t)$, $G(t)$:

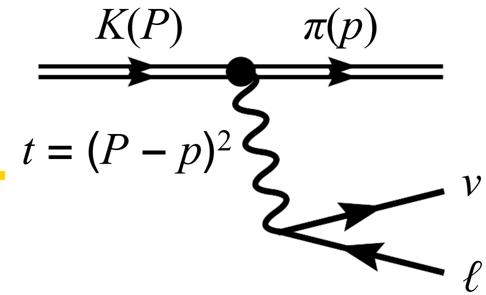
$\sigma_{param}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{param}(\ln C) = 0.0040$

KTeV, Bernard et al.'09

Confidence ellipses shown **without** common uncertainty (except as indicated)

2.3 V_{us} from K_{l3} ($K \to \pi l \nu_l$)



- Master formula for $K \to \pi l \nu_l$: $K = \{K^+, K^0\}$, $l=\{e,\mu\}$

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3})/\tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K |V_{us}|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1 + 2\Delta_{EM}^{Kl} + 2\Delta_{SU(2)}^{K\pi}\right)$$

Average and work by Flavianet Kaon WG *Antonelli et al'11* and then by *M. Moulson*, see e.g. *Moulson&E.P.@CKM2021*

Theoretically

- Possible update on S_{EW}? Based on *Cirigliano et al.'23, Gorbahn et al.'25*
- Update on long-distance EM corrections *Seng et al.'21*
- Improvement on Isospin breaking evaluation due to more precise dominant input: quark mass ratio from $\eta \to 3\pi$ + lattice QCD *Colangelo et al.'18, FLAG'21*
- Progress from lattice QCD on the $K \to \pi$ FF

$$\langle \pi^-(p) | \bar{s}\gamma_\mu u | K^0(P) \rangle = f_+^{K^0\pi^-}(0)\left[(P+p)_\mu \bar{f}_+^{K^0\pi^-}(t) + (P-p)_\mu \bar{f}_-^{K^0\pi^-}(t)\right]$$

Electromagnetic and isospin breaking corrections

- Master formula for

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = \frac{Br(K_{l3})}{\tau} C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K |V_{us}|^2 \left| f_+^{K^0\pi^-}(0)\right|^2 I_{Kl} \left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

- Short distance electroweak correction *Sirlin'82*

$$S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_S}{4\pi}\right)\log\frac{m_Z}{m_\rho} + O\left(\frac{\alpha\alpha_S}{\pi^2}\right) \Rightarrow S_{\mathrm{ew}} = 1.0232(3)$$

Resummation of large logarithms at NLL possible, see *Cirigliano et al.'23, Gorbahn et al.'25*

$\Rightarrow$ see talk by *Vincenzo*

Electromagnetic and isospin breaking corrections

- Master formula for

$$\Gamma\left(K\to\pi l\nu[\gamma]\right)=\frac{Br(K_{l3})}{\tau}C_K^2\frac{G_F^2 m_K^5}{192\pi^3}S_{EW}^K\left|V_{us}\right|^2\left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1+2\Delta_{\rm EM}^{Kl}+2\Delta_{\rm SU(2)}^{K\pi}\right)$$

- Long distance EM corrections: $\Delta_{\rm EM}^{K\ell}$ Computed in ChPT at $O(p^2e^2)$

Cirigliano, Giannotti, Neufeld'08

New calculation by *Seng et al.'21* using hybrid current algebra and ChPT with resummation of largest terms to all chiral orders:

- Reduced uncertainties at $O(e^2p^4)$
- Lattice evaluation of QCD contributions to γW box diagrams

	Cirigliano et al. '08	**Seng et al. '21**
$\Delta^{\rm EM}(K^0_{e3})$ [%]	0.50 ± 0.11	**0.580 ± 0.016**
$\Delta^{\rm EM}(K^+_{e3})$ [%]	0.05 ± 0.12	**0.105 ± 0.023**
$\Delta^{\rm EM}(K^+_{\mu3})$ [%]	0.70 ± 0.11	**0.770 ± 0.019**
$\Delta^{\rm EM}(K^0_{\mu3})$ [%]	0.01 ± 0.12	**0.025 ± 0.027**

Electromagnetic and isospin breaking corrections

- Master formula for

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = \frac{Br(K_{l3})}{\tau} C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

- Isospin breaking : $\Delta_{\mathrm{SU(2)}}^{K\pi} = \dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$

Gasser & Leutwyler'85

Computed in ChPT at O(p^4): $\Delta_{\mathrm{SU(2)}}^{K\pi} = \dfrac{3}{4}\dfrac{1}{Q^2}\left[\dfrac{\overline{M}_K^2}{\overline{M}_\pi^2} + \dfrac{\chi_{p^4}}{2}\left(1 + \dfrac{m_s}{\hat{m}}\right)\right] = \mathbf{2.61(17)\%}$

$\left[\hat{m} \equiv \dfrac{m_d + m_u}{2}\right]$ and $Q^2 \equiv \dfrac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$

Inputs from lattice QCD and from $\eta \to 3\pi$ analysis for Q

Colangelo et al.'18 + FLAG'21

Determination of $f_+(0)$

- SU(3) breaking in $f_+(0)$
  - CVC + Ademollo-Gatto theorem: $f_+^{K^0\pi^-}(0) - 1 = O\left((m_s - m_u)^2\right)$

$$f_+^{K^0\pi^-}(0) = 1 + f_{p^4} + f_{p^6} + \ldots$$

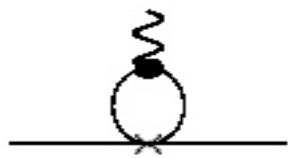
f_{p^4}: $O(m_q)$ $\quad$ f_{p^6}: $O(m_q^2)$

chiral expansion

Gasser & Leutwyler'85

- f_{p4}:

→ One loop graph :



→ First order in m_q, 2nd order in $(m_s - m_u)$ ⇒ $f_{p^4} \sim \frac{(m_s - m_u)^2}{m_s}$

→ No local operators, UV finite, free of uncertainties

⇒ $f_{p^4} = -0.0227$

Determination of $f_+(0)$

- SU(3) breaking in $f_+(0)$
  - CVC + Ademollo-Gatto theorem: $f_+^{K^0\pi^-}(0) - 1 = O\left((m_s - m_u)^2\right)$

$$f_+^{K^0\pi^-}(0) = 1 \; + \; f_{p^4} \; + \; f_{p^6} \; + \ldots$$

chiral expansion

f_{p^4}: $O(m_q)$ $\quad$ f_{p^6}: $O(m_q^2)$

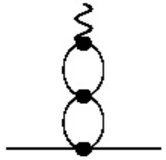
  - f_{p6}:

Bijnens & Talavera'02

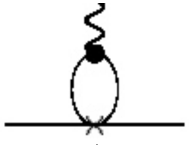
$$f_{p^6} = f_{p^6}^{2-\mathrm{loops}}(\mu) + f_{p^6}^{\mathrm{L_i}\times\mathrm{loop}}(\mu) + f_{p^6}^{\mathrm{tree}}(\mu)$$

$$f_{p^6}^{2-\mathrm{loops}}(M_\rho) \;=\; 0.0113$$

$$f_{p^6}^{L_i\times\mathrm{loop}}(M_\rho) \;=\; -0.0020$$



Large positive chiral loop cont.

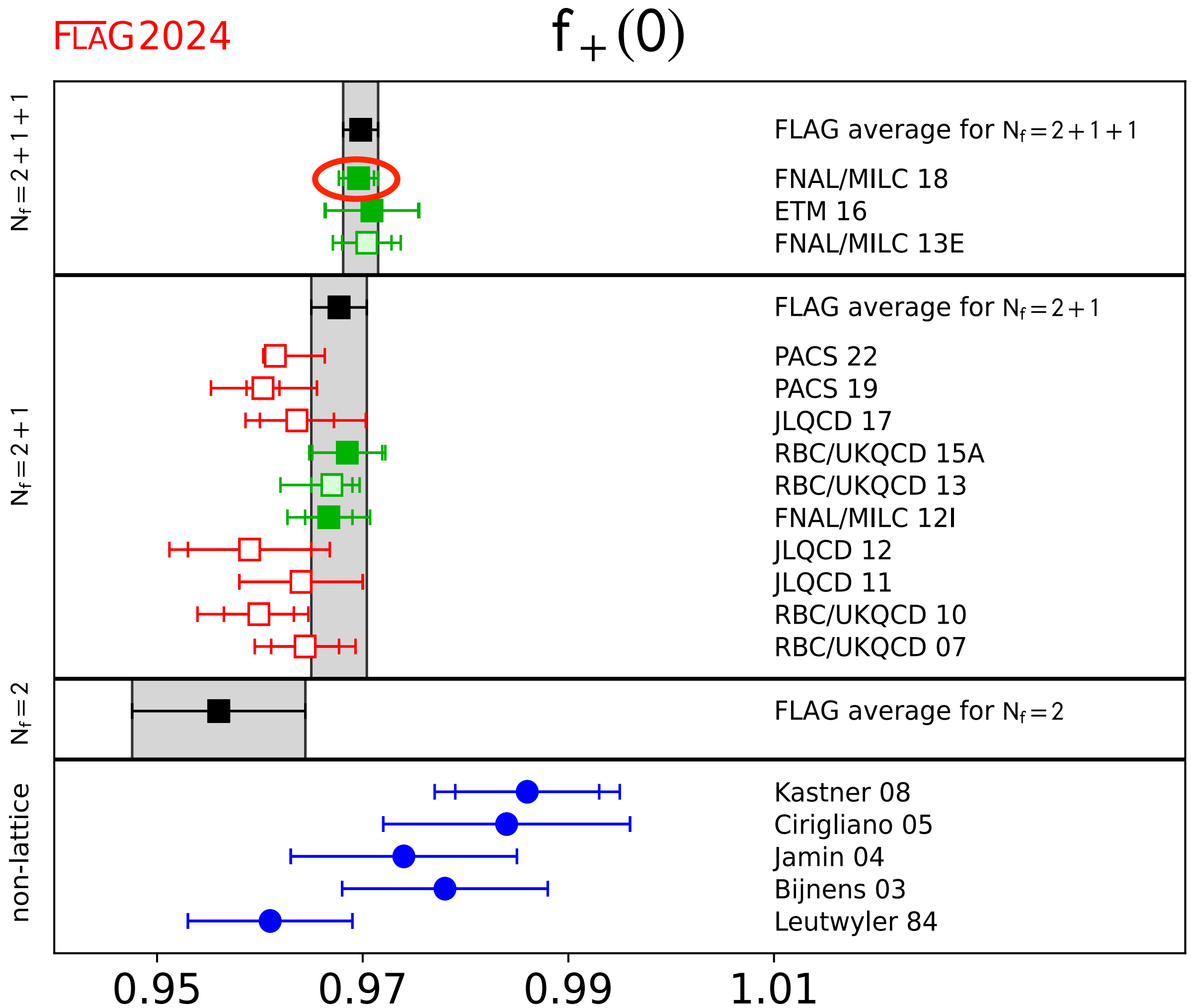


$$8\frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2}\left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho)\right]$$

LECs not fixed by chiral symmetry: quark model, large-Nc estimates, *LQCD*

$f_+(0)$ from lattice QCD

- Recent progress on Lattice QCD for determining $f_+(0)$



$$f_+(0)^{FLAG24}_{N_f=2+1+1} = 0.9698(17)$$

0.18% uncertainty

to be compared to

$$f_+(0)^{FLAG16}_{N_f=2+1+1} = 0.9704(32)$$

$$f_+(0)^{2010}_{N_f=2+1} = 0.959(50)$$

Uncertainty divided by ~2 w/ 2016 and by 25 w/ 2011!

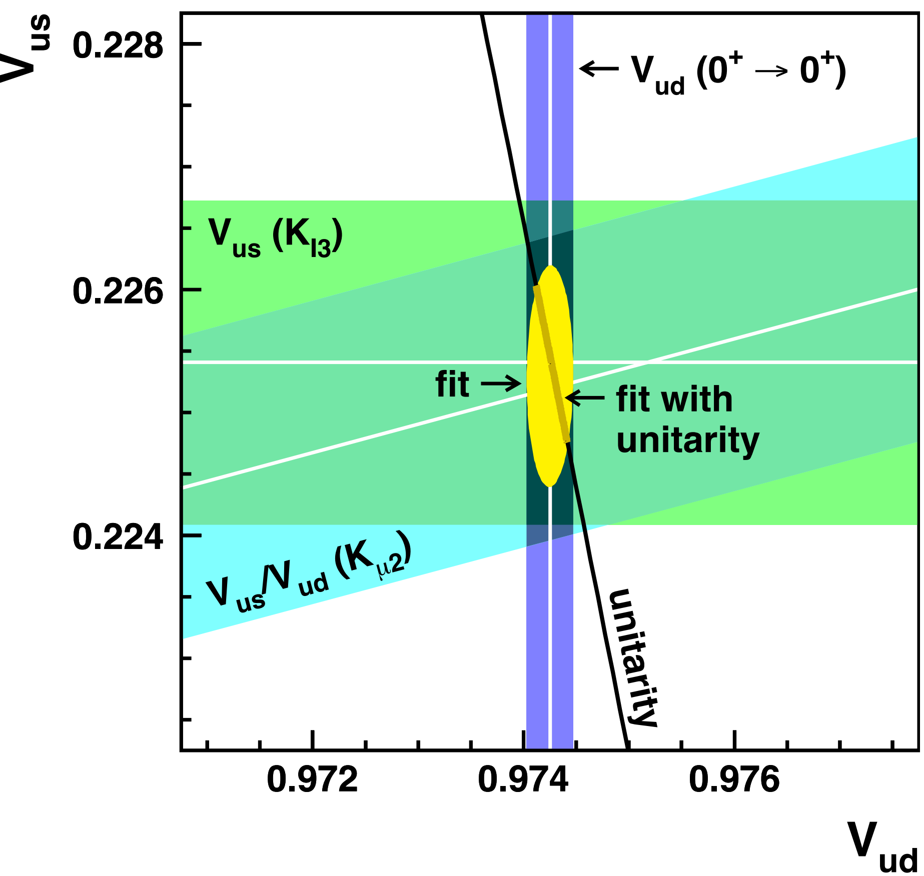
➡ Lattice uncertainties at the same level as exp.

-3.2σ away from unitarity!

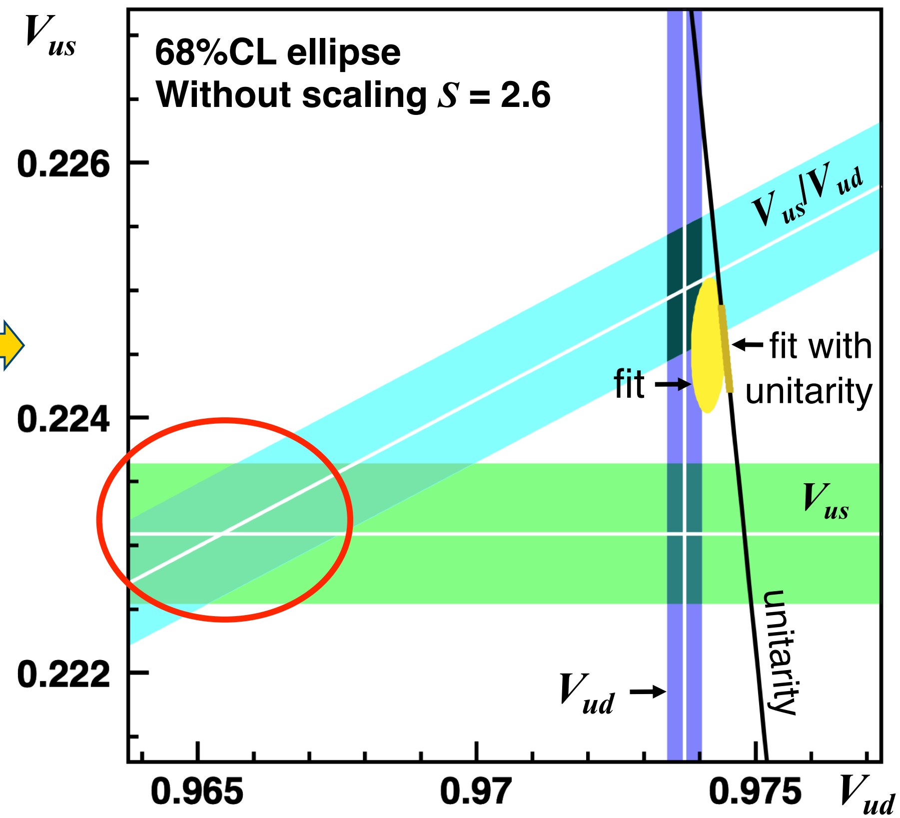
2011: $V_{us} = 0.2254(5)_{exp}(11)_{lat}$ → $V_{us} = 0.2231(4)_{exp}(4)_{lat}$

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*



Moulson & E.P.@CKM2021



2.4 V_{us}/V_{ud} from K_{l2}/π_{l2}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

- Recent progress on radiative corrections computed on lattice: *Giusti et al.'17, Di Carlo et al.'19, Boyle et al.'21*

⇨ see talk by *Xin-Yu Tuo*

- Main hadronic input: f_K/f_π

- In 2011: V_{us}/V_{ud} = 0.2312(4) $_{exp}$(12)$_{lat}$

- In 2021: V_{us}/V_{ud} = 0. 2311(3)$_{exp}$(4)$_{lat}$ the lattice error is reducing by a factor of 3 compared to 2011! It is now of the same order as the experimental uncertainty.

-1.8σ away from unitarity

2.4 V_{us}/V_{ud} from K_{12}/π_{12}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

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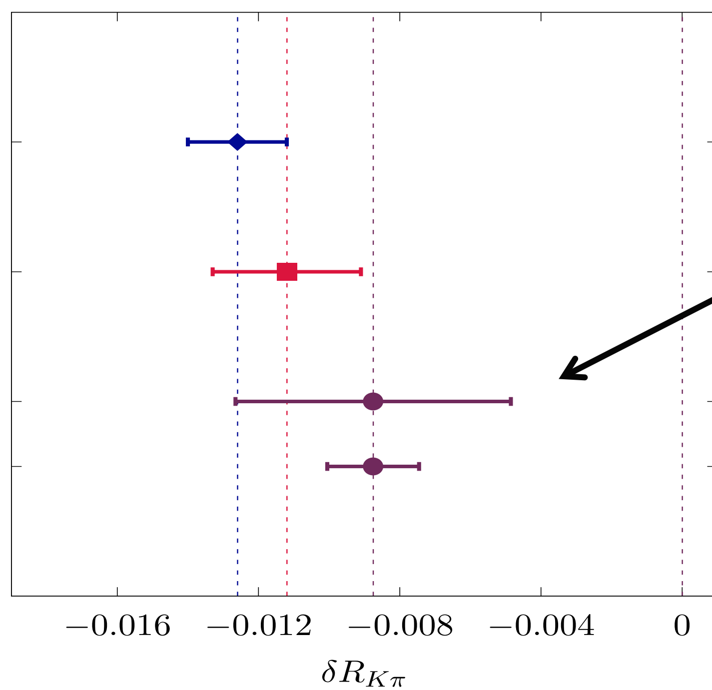
Portelli@CKM25



$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)$ (RM123S (2019))

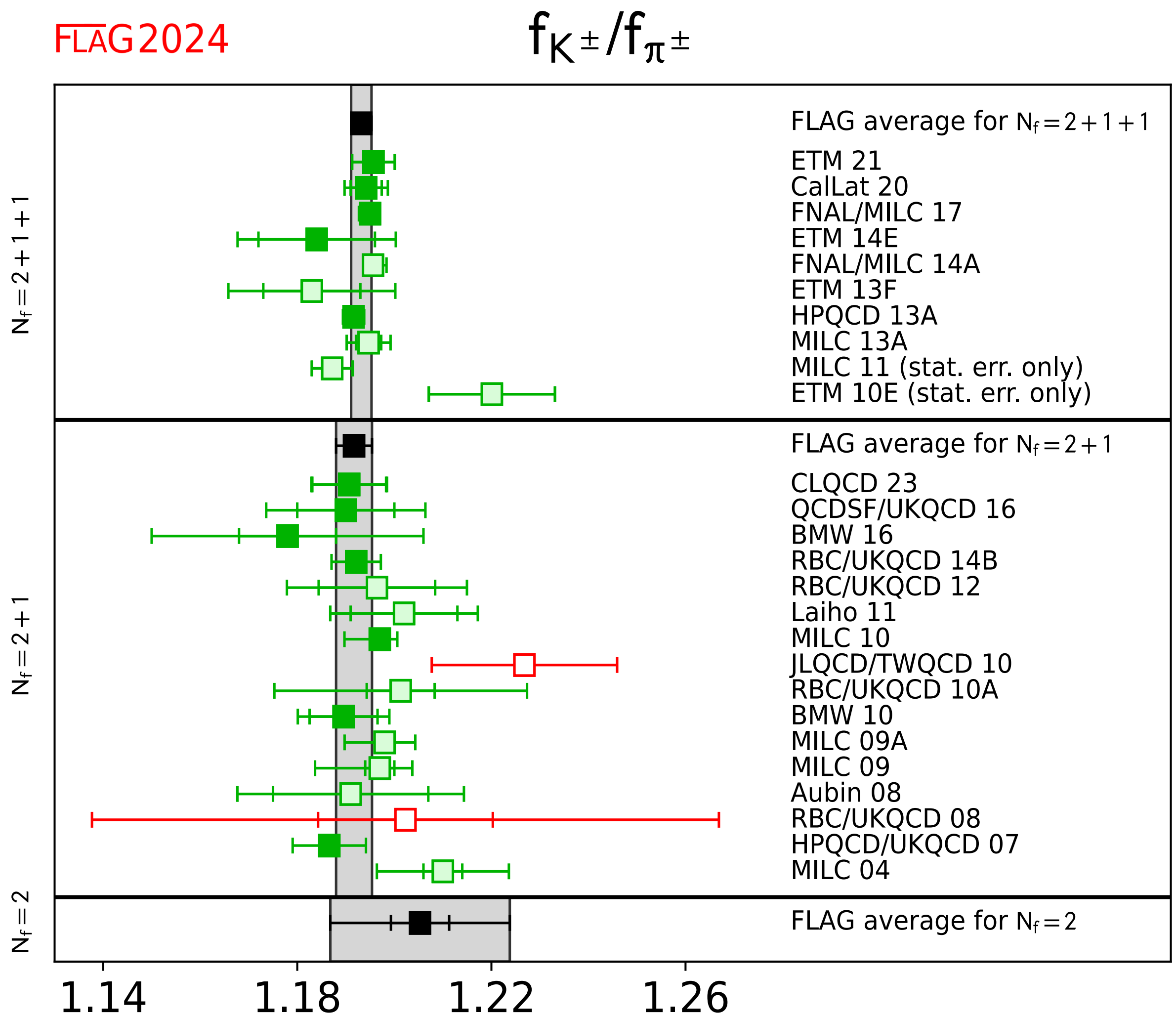
$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)$ (χPT (2011))

$\delta_{SU(2)} + \delta_{\mathrm{EM}}$ $= -0.0086(41)$ (RBC-UKQCD (2023))

Non negligible finite volume effects (FVE) found

2.2 f_K/f_π from lattice QCD

Progress since 2018: ➡ new results from *ETM'21* and *CalLat'20*



Now Lattice collaborations include SU(2) IB corr.
For N_f=2+1+1, FLAG2024

$$f_{K^+}/f_{\pi^+} = 1.1932(21)$$

0.18% uncertainty

Results have been stable over the years

For average substract IB corr.

$$f_K/f_\pi = 1.1967(18)$$

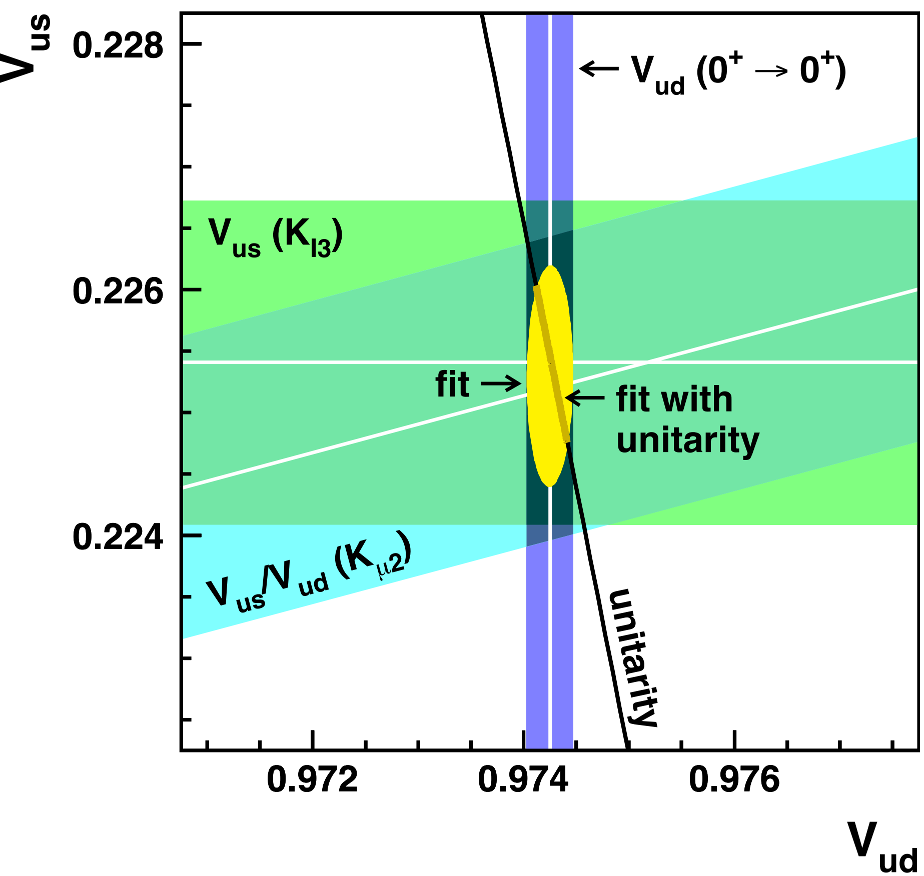
In 2011: $f_K/f_\pi = 1.193(6)$

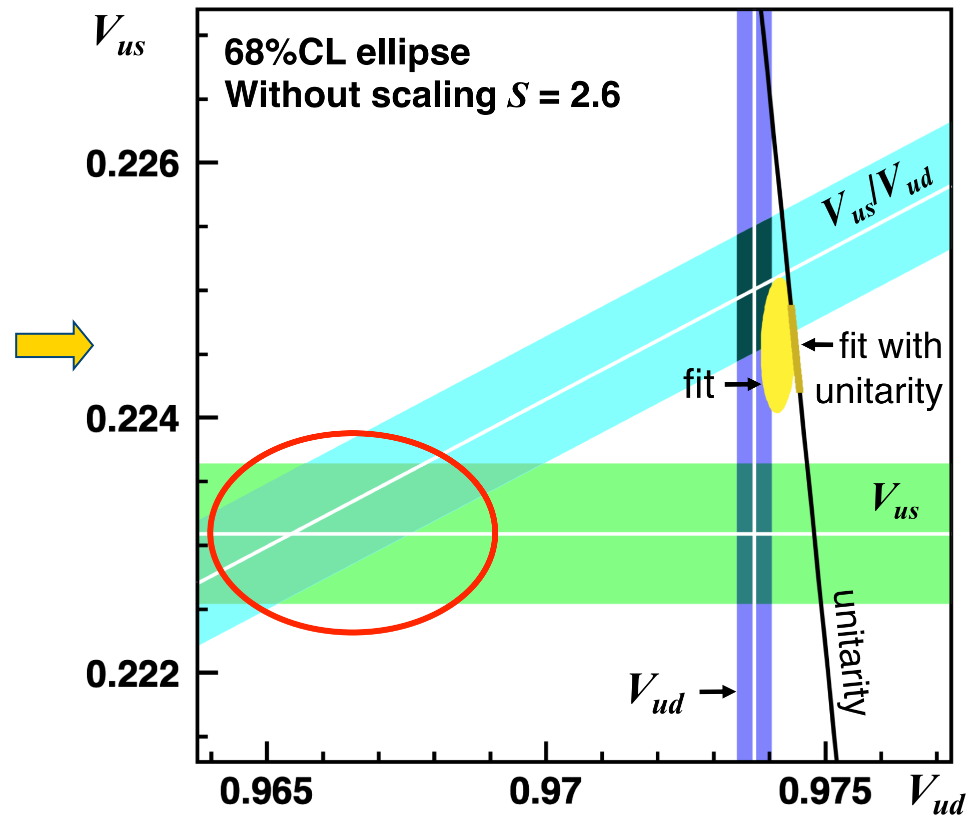
➡ V_{us}/V_{ud} = 0. 23108(29)$_{exp}$(42)$_{lat}$

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*

Moulson & E.P.@CKM2021





3. Prospects

3.1 Experimental Prospects for V_{us}

On Kaon side

Cirigliano et al'22

- *NA62* could measure several BRs: $K_{\mu3}/K_{\mu2}$, $K \to 3\pi$, $K_{\mu2}/K \to \pi\pi$
- Note that the high precision measurement of BR($K_{\mu2}$) (0.3%) comes only from a single experiment: KLOE. It would be good to have another measurement at the same level of accuracy

  In progress ! ⇨ See talk by *Victor Shang*

- *LHCb* : could measure BR($K_S \to \pi\mu\nu$) at the < 1% level?
  $K_S \to \pi\mu\nu$ measured by KLOE-II but not competitive
  τ_S known to 0.04% (vs 0.41% for τ_L, 0.12% for $\tau_\pm$)

3.2 V_{us} from Hyperon decays

V_{us} can be measured from Hyperon decays:

- $\Lambda \to p e \nu_e$ Possible measurement at *BESIII, Super τ-Charm factory?*

- Possibilities at *LHCb*?

Channel	$\mathcal{R}$	ϵ_L	ϵ_D	$\sigma_L(\mathrm{MeV}/c^2)$	$\sigma_D(\mathrm{MeV}/c^2)$
$K^0_S \to \mu^+\mu^-$	1	1.0 (1.0)	1.8 (1.8)	~ 3.0	~ 8.0
$K^0_S \to \pi^+\pi^-$	1	1.1 (0.30)	1.9 (0.91)	~ 2.5	~ 7.0
$K^0_S \to \pi^0\mu^+\mu^-$	1	0.93 (0.93)	1.5 (1.5)	~ 35	~ 45
$K^0_S \to \gamma\mu^+\mu^-$	1	0.85 (0.85)	1.4 (1.4)	~ 60	~ 60
$K^0_S \to \mu^+\mu^-\mu^+\mu^-$	1	0.37 (0.37)	1.1 (1.1)	~ 1.0	~ 6.0
$K^0_L \to \mu^+\mu^-$	~ 1	$2.7\ (2.7)\ \times 10^{-3}$	0.014 (0.014)	~ 3.0	~ 7.0
$K^+ \to \pi^+\pi^+\pi^-$	~ 2	$9.0\ (0.75)\ \times 10^{-3}$	$41\ (8.6)\ \times 10^{-3}$	~ 1.0	~ 4.0
$K^+ \to \pi^+\mu^+\mu^-$	~ 2	$6.3\ (2.3)\ \times 10^{-3}$	0.030 (0.014)	~ 1.5	~ 4.5
$\Sigma^+ \to p\mu^+\mu^-$	~ 0.13	0.28 (0.28)	0.64 (0.64)	~ 1.0	~ 3.0
$\Lambda \to p\pi^-$	~ 0.45	0.41 (0.075)	1.3 (0.39)	~ 1.5	~ 5.0
$\Lambda \to p\mu^-\bar{\nu}_\mu$	~ 0.45	0.32 (0.31)	0.88 (0.86)	–	–
$\Xi^- \to \Lambda\mu^-\bar{\nu}_\mu$	~ 0.04	$39\ (5.7)\ \times 10^{-3}$	0.27 (0.09)	–	–
$\Xi^- \to \Sigma^0\mu^-\bar{\nu}_\mu$	~ 0.03	$24\ (4.9)\ \times 10^{-3}$	0.21 (0.068)	–	–
$\Xi^- \to p\pi^-\pi^-$	~ 0.03	0.41(0.05)	0.94 (0.20)	~ 3.0	~ 9.0
$\Xi^0 \to p\pi^-$	~ 0.03	1.0 (0.48)	2.0 (1.3)	~ 5.0	~ 10
$\Omega^- \to \Lambda\pi^-$	~ 0.001	$95\ (6.7)\ \times 10^{-3}$	0.32 (0.10)	~ 7.0	~ 20

R = ratio of production

ϵ = ratio of efficiencies

- To be able to extract V_{us} one needs to compute form factors precisely

  ⇒ Lattice effort from *RBC/UKQCD*

3.3 Theoretical Prospects for V_{us} from Kaon and Baryon decays

- Lattice Progress on hadronic matrix elements: decay constants, FFs: Only 1 result at per mile accuracy for $f_+(0)$ from lattice QCD

  ⇨ It would be great to have other determinations

- Full QCD**+**QED decay rate on the lattice,for **Leptonic decays of kaons and pions** ⇨ Inclusion of EM and IB corrections :
  - Perturbative treatment of QED on lattice established
  - Formalism for K_{l2} worked out but non negligible finite volume effects found

- Application of the method for **semileptonic Kaon** (K_{l3}) **and Baryon decays**

- Theoretical analytical program for Radiative corrections using lattice inputs

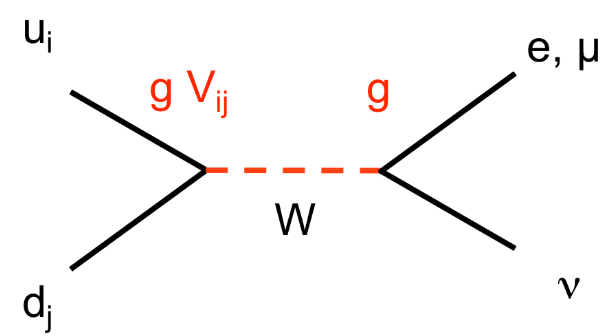
  ⇨ **Aim: Per mille level within 10 years**

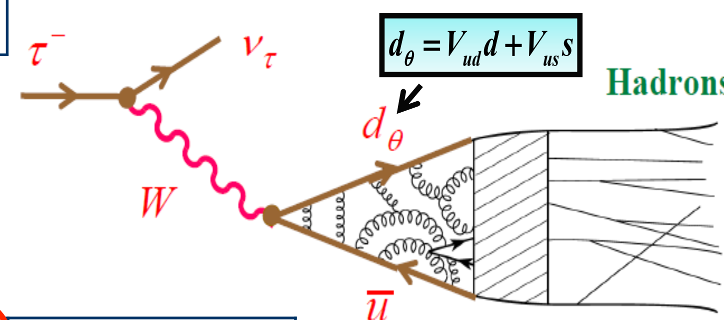
4. Can Tau physics help?

Path to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$





- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi \nu_\tau$	$\tau \to h_{NS} \nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K \nu_\tau$	$\tau \to h_S \nu_\tau$ (inclusive)

4.1 V_{us} from $\tau \to K\nu_\tau$ / $\tau \to \pi\nu_\tau$ decays

- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

$$\frac{\Gamma(\tau \to K\nu[\gamma])}{\Gamma(\tau \to \pi\nu[\gamma])} = \frac{\left(1 - m_{K^\pm}^2/m_\tau^2\right)}{\left(1 - m_{\pi^\pm}^2/m_\tau^2\right)} \frac{f_K^2}{f_\pi^2} \frac{|V_{us}|^2}{|V_{ud}|^2} \left(1 + \delta_{\mathrm{LD}}\right)$$

- Main input hadronic input: f_K/f_π as for Kaon physics

From Tau physics: $V_{us}/V_{ud} = 0.2289(18)_{exp}(4)_{lat}$ *HFLAV'23* *-2.1σ* away from unitarity

to be compared to $V_{us}/V_{ud} = 0.2311(3)_{exp}(4)_{lat}$ ⇨ Need important exp. improvement !

Inclusive determination of V_{us}

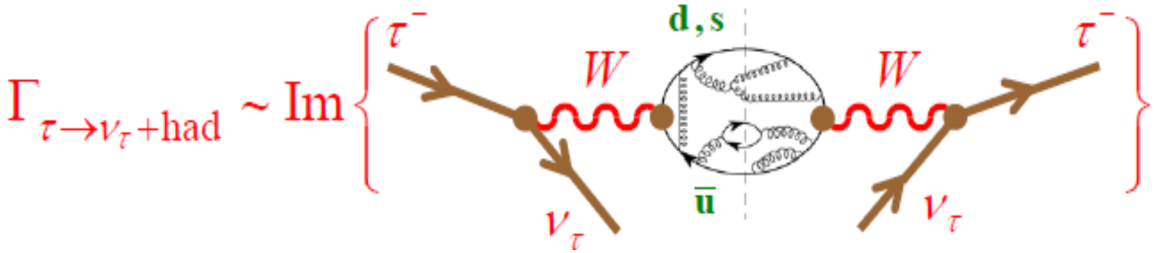
- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

–

Inclusive τ-decays

Braaten, Narison, Pich'92



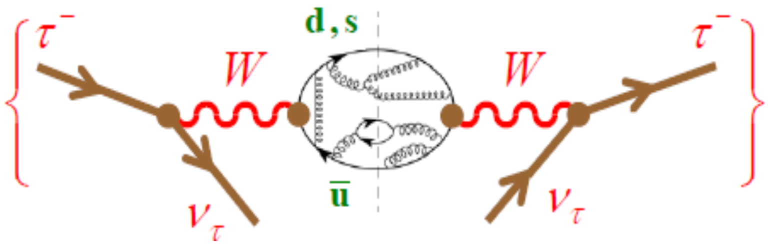
- Quantity of interest : $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \textbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}$$

3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\left\{ \right.$$



$$\Rightarrow \quad R_\tau(m_\tau^2) = 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon)+\text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

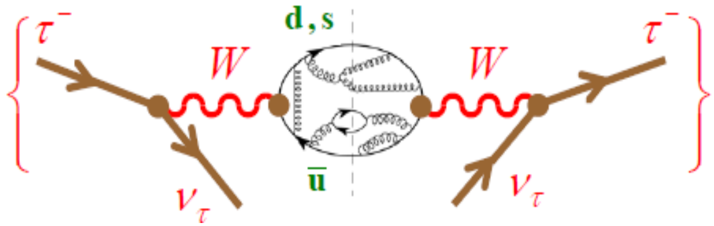
$$\Pi^{(J)}(s) \equiv |V_{ud}|^2\left(\Pi^{(J)}_{ud,V}(s)+\Pi^{(J)}_{ud,A}(s)\right)+|V_{us}|^2\left(\Pi^{(J)}_{us,V}(s)+\Pi^{(J)}_{us,A}(s)\right)$$

J

$$\Pi^{\mu\nu}_{ij,V/A}(q) = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi^{(1)}_{ij,V/A}(q^2) + q^\mu q^\nu\,\Pi^{(0)}_{ij,V/A}(q^2)$$

3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ: $\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}$ 

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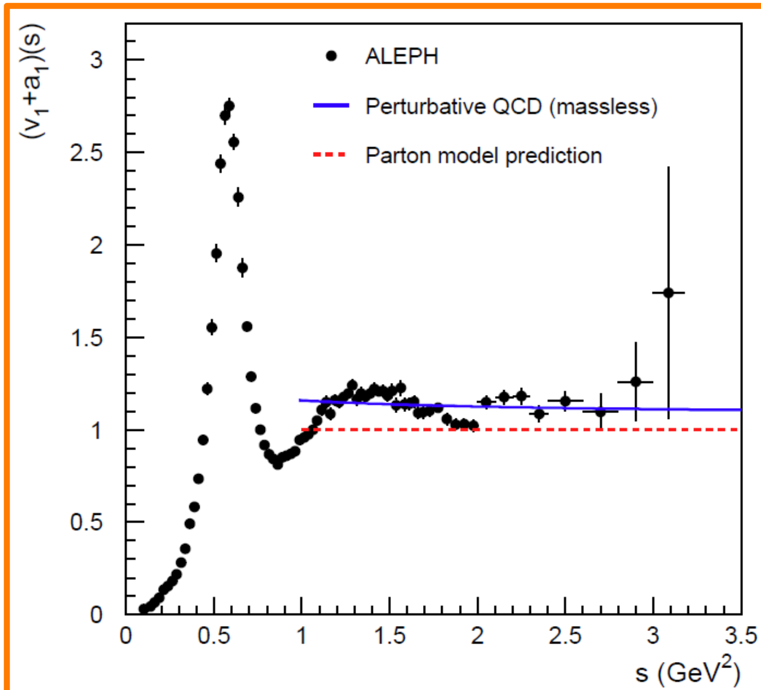
- Spectral functions:

$$\text{Im}\Pi^{(1)}_{\bar{u}d,V/A}(s) = \frac{1}{2\pi}v_1/a_1(s)$$

- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system

$\Rightarrow$ mix of non-perturbative and perturbative effects



Measurements

- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} = ?$$

- Decomposition as a function of observed and separated final states:

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

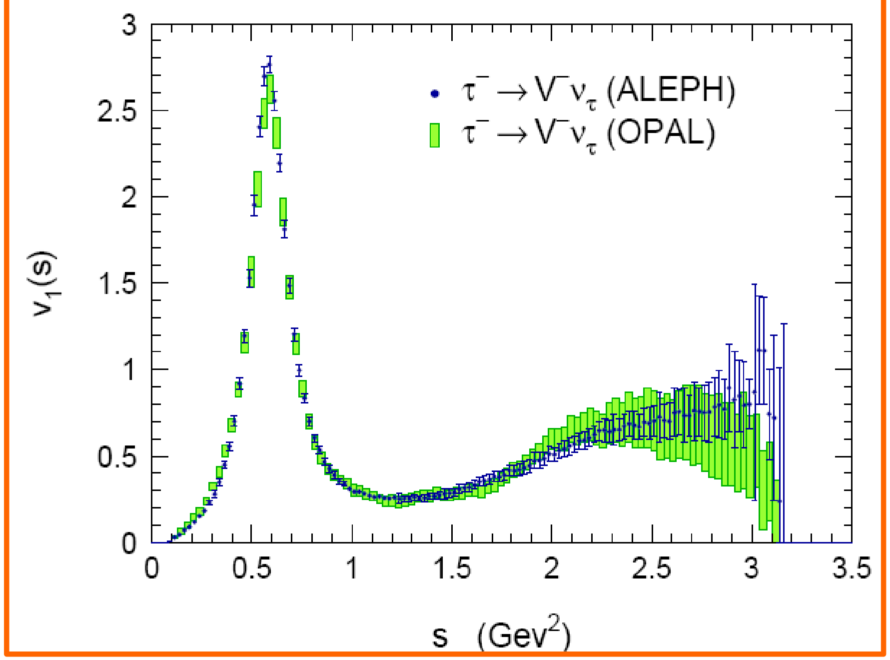
$R_{\tau,V}$ ⇨ $\tau^- \to \nu_\tau + h_{v,s=0}$

(even number of pions)

$R_{\tau,A}$ ⇨ $\tau^- \to \nu_\tau + h_{A,s=0}$

(odd number of pions)

$R_{\tau,S}$ ⇨ $\tau^- \to \nu_\tau + h_{V+A,s=1}$



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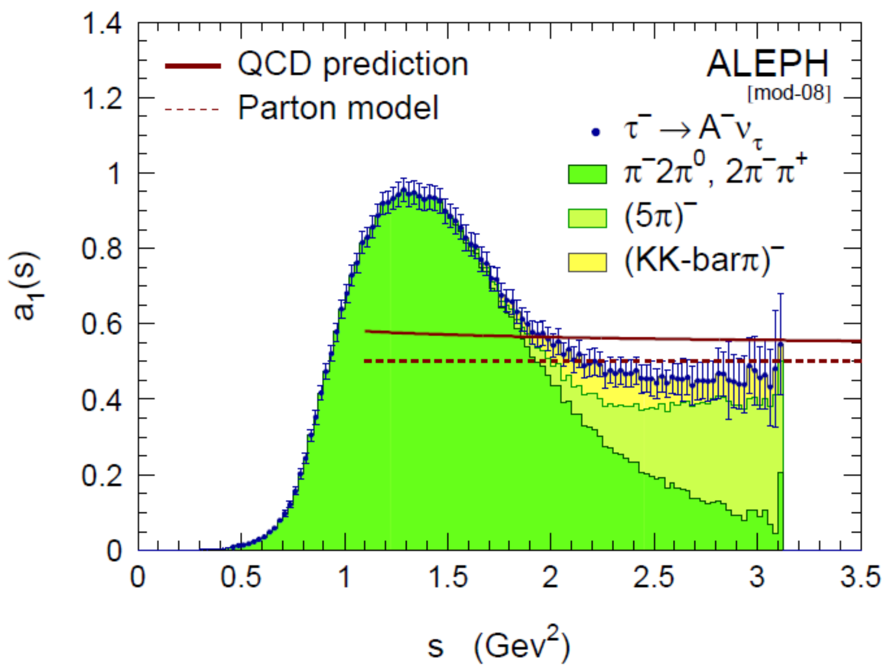

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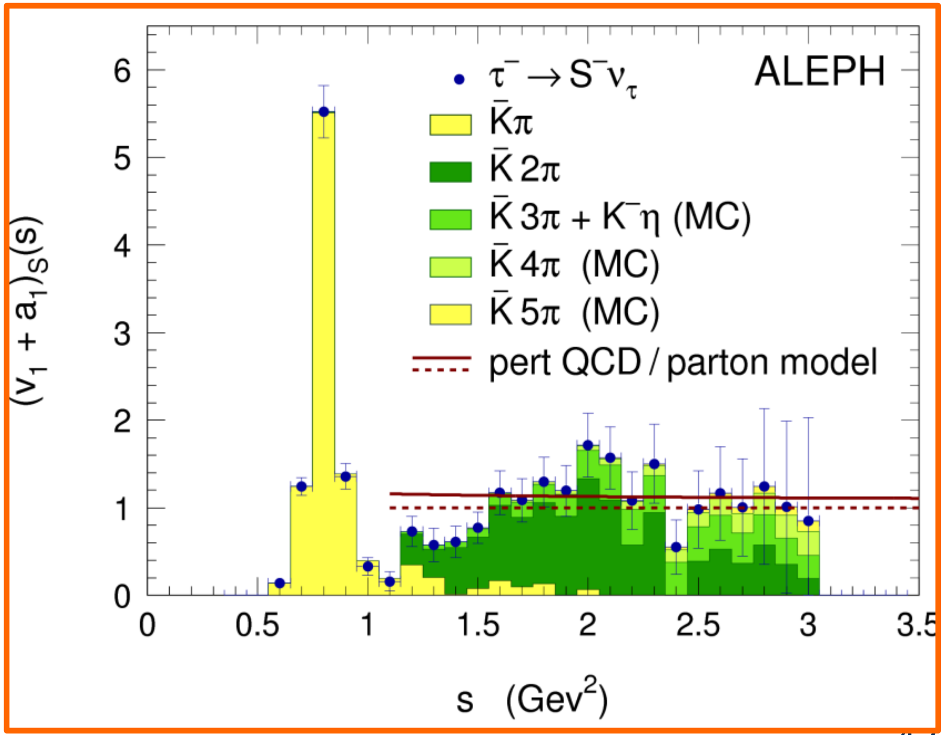

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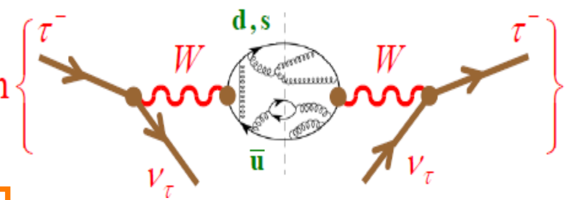


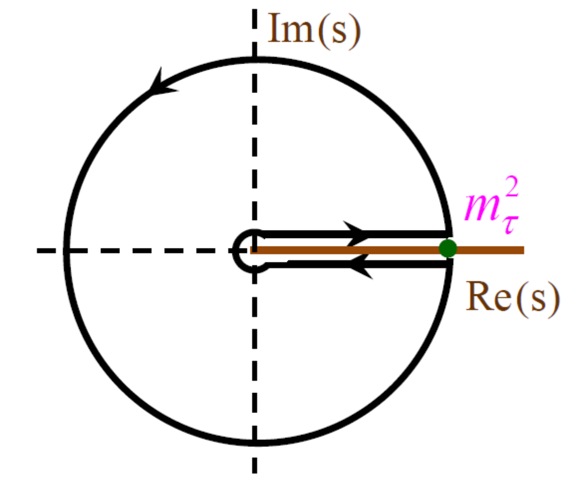
$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

  ⇨ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

3.3 Operator Product Expansion

$$\Pi^{(J)}(s) = \sum_{D=0,2,4...} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s,\mu) \langle O_D(\mu) \rangle$$

Wilson coefficients — Operators

μ separation scale between short and long distances

- D=0: Perturbative contributions
- D=2: Quark mass corrections
- D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\left\langle m_j \bar{q}_i q_i \right\rangle$
- D=6: 4 quarks operators, $\left\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \right\rangle$
- D≥8: Neglected terms, supposed to be small…

$$\Rightarrow \quad R_{\tau,V}(s_0) = \frac{3}{2} |V^{ud}|^2 S_{EW} \left(1 + \delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V} \right)$$

similar for $R_{\tau,A}(s_0)$ and $R_{\tau,S}(s_0)$

Perturbative Part

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ Marciano &Sirlin'88, Braaten & Li'90, Erler'04

- Perturbative part (D=0):

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

$$\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%$$

Baikov, Chetyrkin, Kühn'08

Non-perturbative part

- Calculation of R_τ:

Braaten, Narison, Pich'92

$$R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0):

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Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for R_τ^{NS} $\left(\propto m_u, m_d\right)$ but not for R_τ^S $\left(\propto m_s\right)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

  ⇨ Use of weighted distributions

Ex: In the non-strange sector: $\delta_{NP}^{NS} = -0.0064(13)$ *Davier et al.'14*

Inclusive determination of V_{us}

- With QCD on: $$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}$$

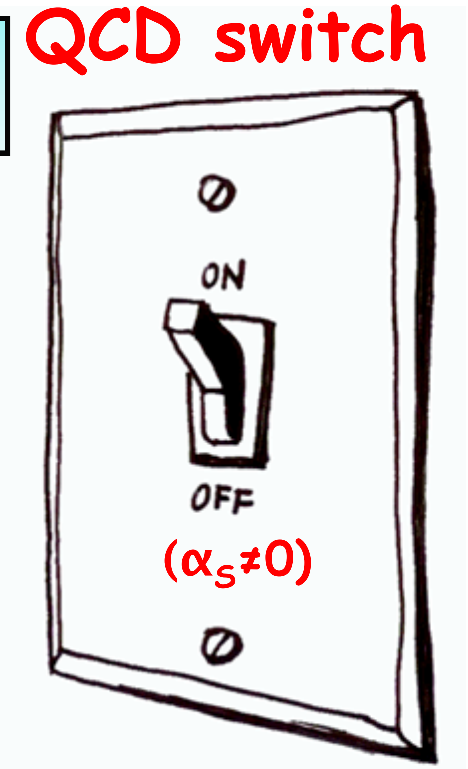
- Use OPE: $$R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)$$

$$R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)$$

- $$\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0238(33)$$ *Gamiz et al'07, Maltman'11*



$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

HFLAV'23

$R_{\tau,S} = 0.1615(28)$

$R_{\tau,NS} = 3.4650(84)$

$|V_{ud}| = 0.97373(32)$

$\Rightarrow$ $$|V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0010_{\text{th}}$$

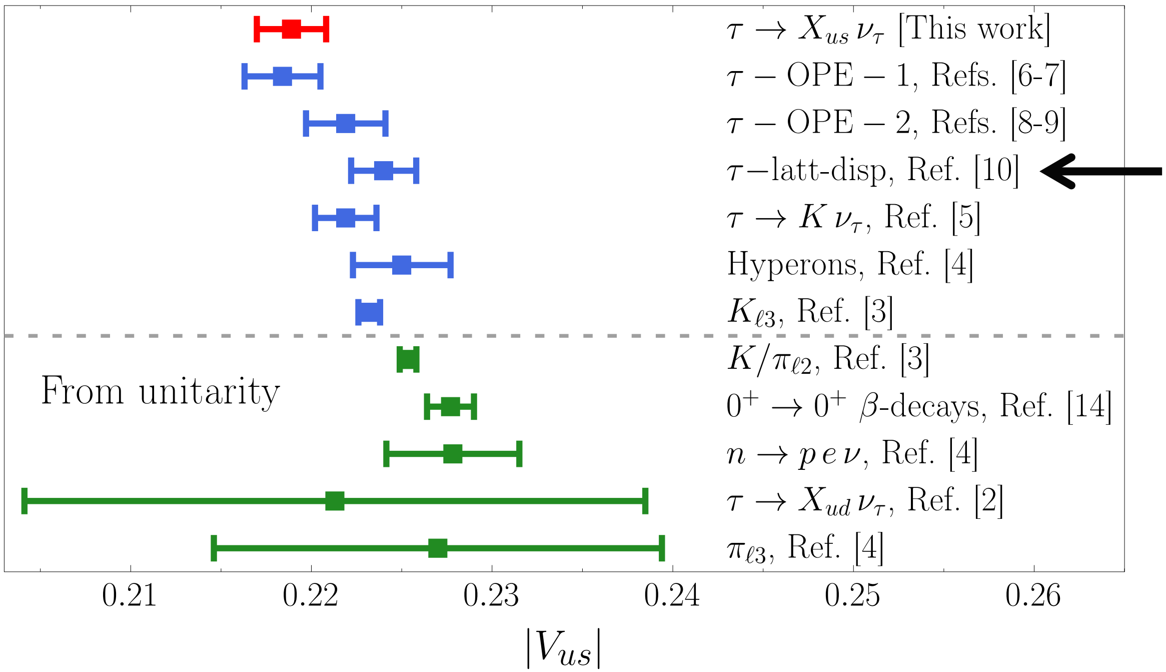
-3.7σ away from unitarity!

A. Lusiani@Tau'25

Inclusive determination of V_{us}

- See recent lattice work by *ETMC'24* *Gagliardi@Tau25*

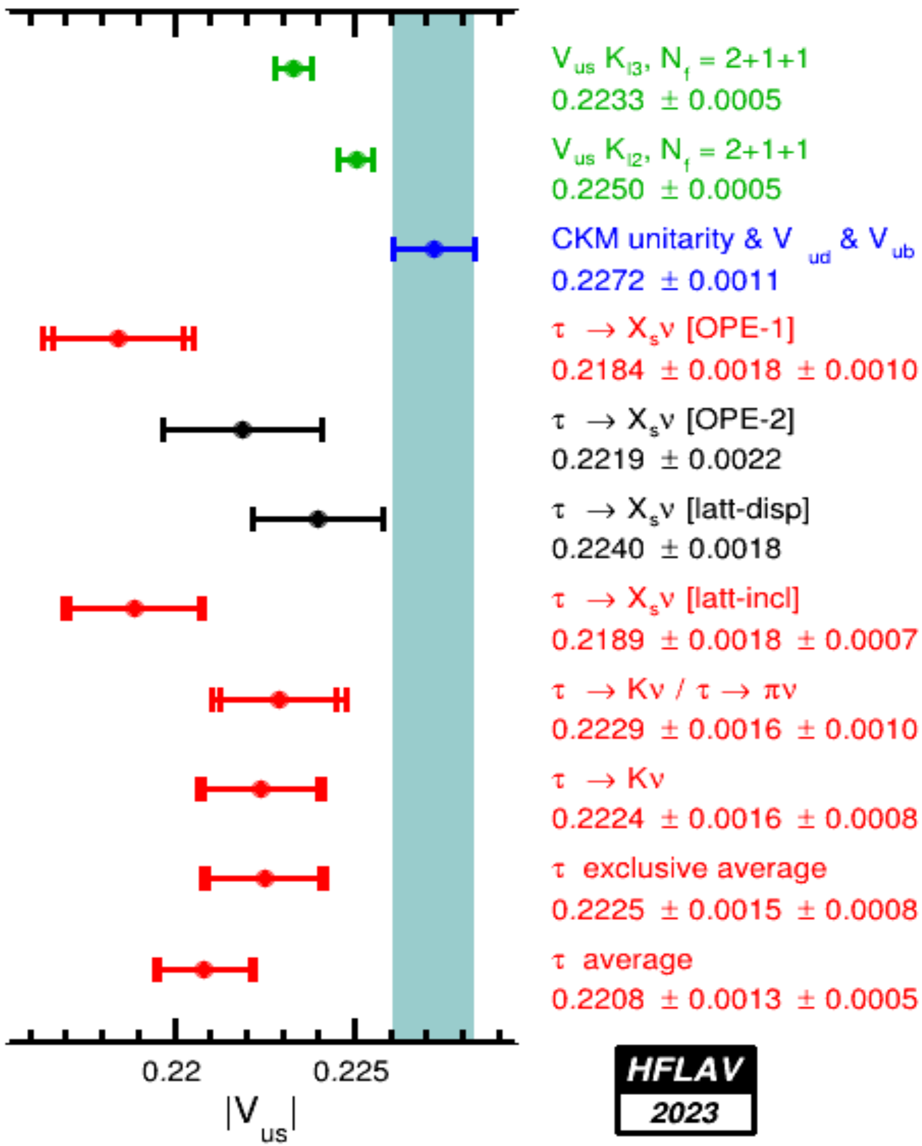
$$R^{\tau}_{\rm us}/|V_{\rm us}|^2 = 3.407\ (22)$$ [0.6% uncertainty]



$$|V_{us}|_{\tau-\rm latt-incl} = 0.2189(7)_{\rm th}(18)_{\rm exp}$$

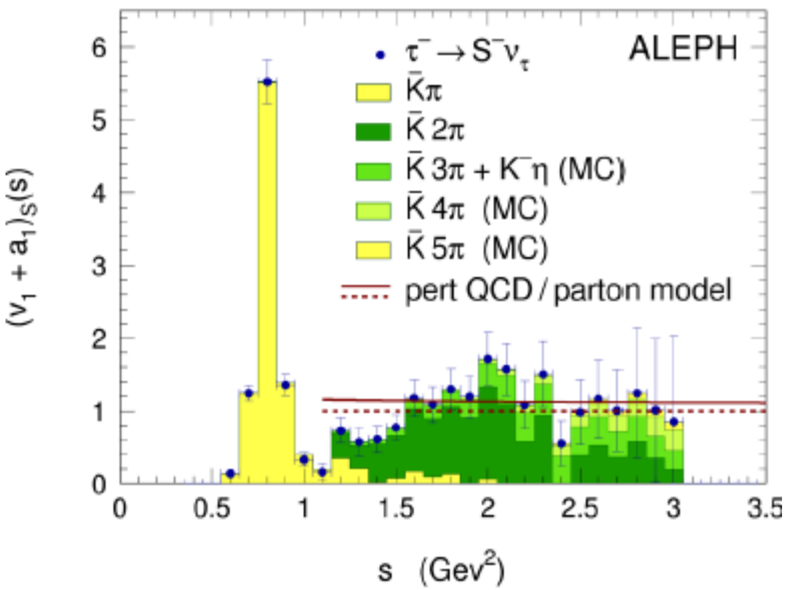
- Aim: Reach per mile level precision on R_{us}/V_{us}

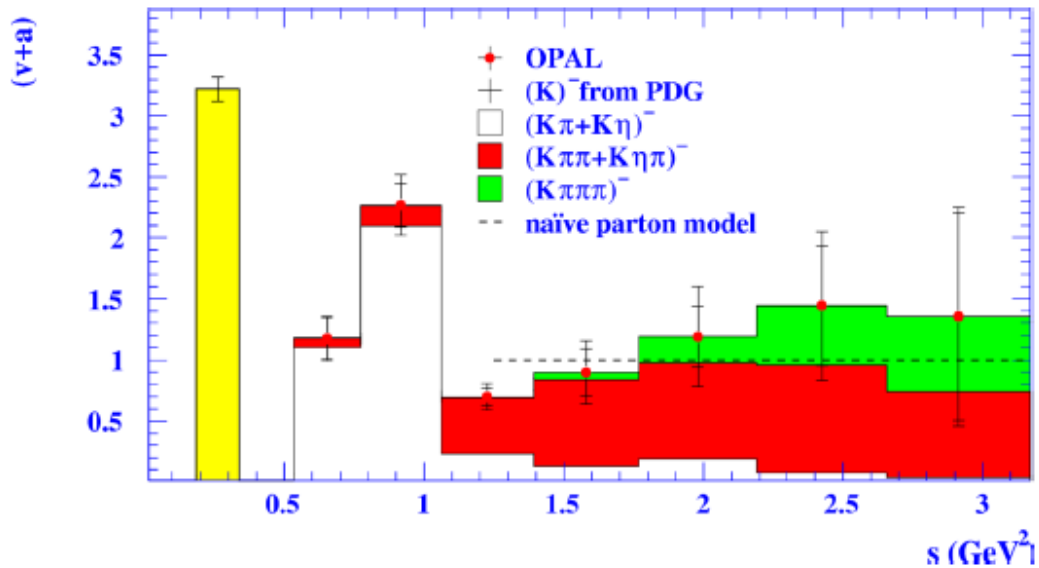
3.4 V_{us}: summary



3.4 Prospects : τ strange Spectral functions

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

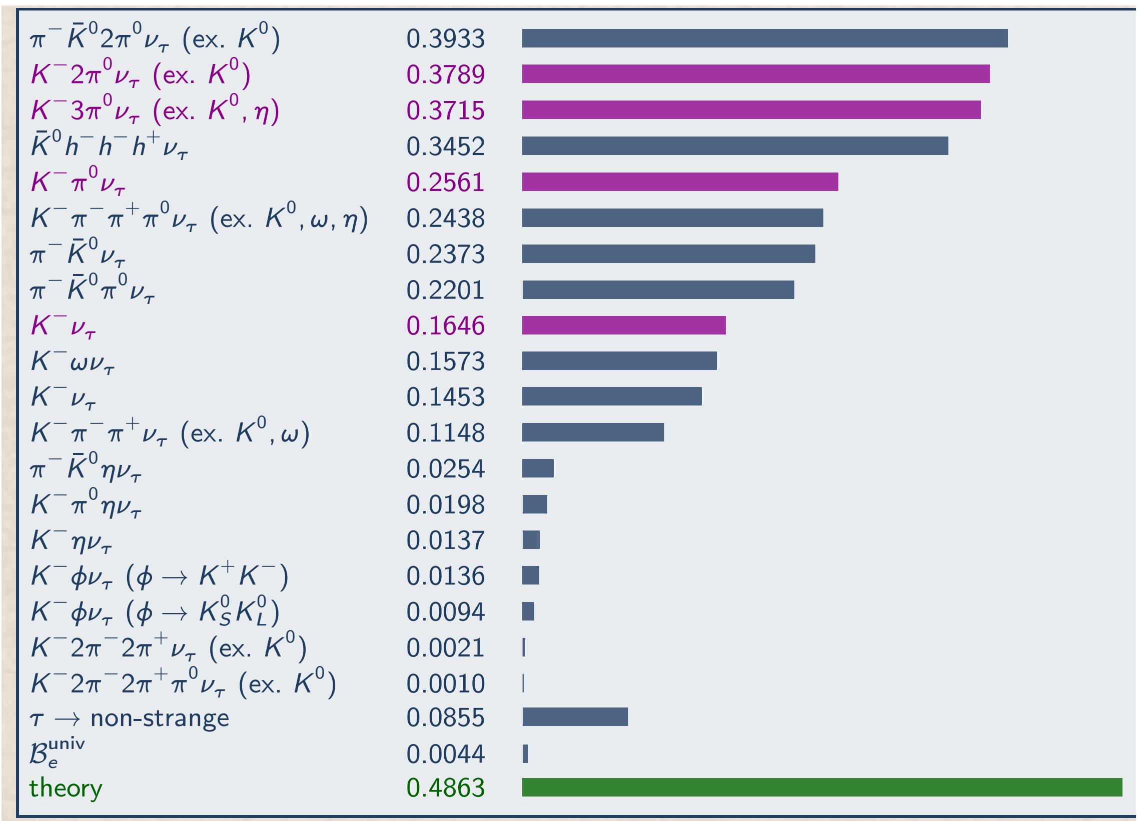
Smaller $\tau \to$ K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller V_{us}

$$R_\tau^S\big|_{old} = 0.1686(47) \Rightarrow R_\tau^S\big|_{new} = 0.1615(28)$$

$$|V_{us}|_{old} = 0.2214 \pm 0.0031_{exp} \pm 0.0010_{th} \Rightarrow |V_{us}|_{new} = 0.2176 \pm 0.0019_{exp} \pm 0.0010_{th}$$

3.4 Prospects : τ strange BRs

- Very interesting quantity to extract V_{us}: QCD part completely independent from form factors or decay constants ⇨ Use OPE
- Experimentally very challending since all Brs need to be measured



A. Lusiani@Tau'25

⇨ *Belle II @ 50 ab^{-1}* + Creativity from young physicists

5. Conclusion and Outlook

Conclusion and Outlook

- Recent precision determinations of V_{us} and V_{ud} enable unprecedented tests of the SM and constraints on possible NP models
- Tensions in unitarity of 1st row of CKM matrix have reappeared!
- We need to work hard to understand where they come from:
  - On experimental side:
    For V_{us}, new measurements in kaons (*NA62:* $K_{\mu3}/K_{\mu2}$, *LHCb?*)
    and in tau decays from *Belle II*
    V_{us} from hyperon decays? ⇨ *BESSIII*, *LHCb*?
  - On theory side:
    Calculate very precisely radiative corrections, isospin breaking effects and matrix elements
    Be sure that the uncertainties are under control
  - If these tensions are confirmed ⇨ what do they tell us?
- Interesting time ahead of us!

6. Back-up

Electromagnetic and isospin breaking corrections

- Master formula for

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = \frac{Br(K_{l3})}{\tau} C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl} \left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

- Short distance electroweak correction *Sirlin'82*

$$S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_S}{4\pi}\right)\log\frac{m_Z}{m_\rho} + O\left(\frac{\alpha\alpha_S}{\pi^2}\right) \Rightarrow S_{\mathrm{ew}} = 1.0232(3)$$

Cirigliano, Giannotti, Neufeld'08

- Long distance EM corrections: $\Delta_{\mathrm{EM}}^{K\ell}$ Computed in ChPT at $O(p^2e^2)$

- Isospin breaking : $\Delta_{\mathrm{SU(2)}}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$ *Gasser & Leutwyler'85* $\left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$

Computed in ChPT at O(p⁴): $\Delta_{\mathrm{SU(2)}}^{K\pi} = \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{\hat{m}}\right)\right] = 2.61(17)\%$

Inputs from lattice QCD and from $\eta \to 3\pi$ analysis for Q $\quad Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$

1.1 Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

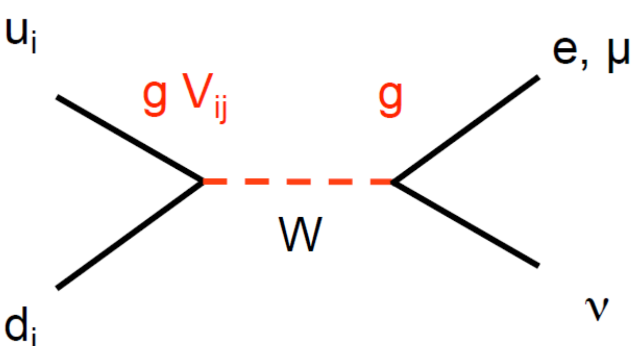
    Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}$$

Gauge coupling

  - Universality: Is G_F from μ decay equals to G_F from π, K, nuclear β decay?

$$G_\mu^2 = (g_\mu g_e)^2 / M_W^4 \quad \stackrel{?}{=} \quad G_{\text{CKM}}^2 = (g_q g_\ell)^2 \left(|V_{ud}|^2 + |V_{us}|^2 \right) / M_W^4$$



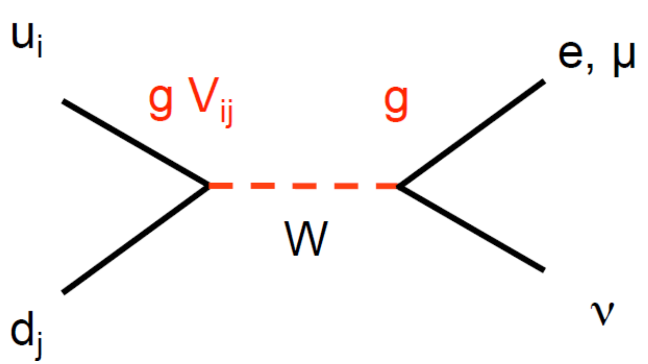
1.2 Constraining New Physics

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

    Description of the weak interactions :

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{eL} + \bar{\mu}_L \gamma^\alpha \nu_{\mu L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau L} \right) + \text{h.c.}$$

- Look for *new physics*
  - In the Standard Model : W exchange ⇒ only V-A structure



2.3 V_{us}/V_{ud} from K_{l2}/π_{l2}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

- Recent progress on radiative corrections computed on lattice:

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- $N_f = 2+1+1$ Twisted-mass Wilson fermions

Giusti et al.'18

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)}$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)}$

Update, extended description, and systematics of Giusti et al.

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)}$

Di Carlo et al.'19

2.1 V_{us} from K_{l3}

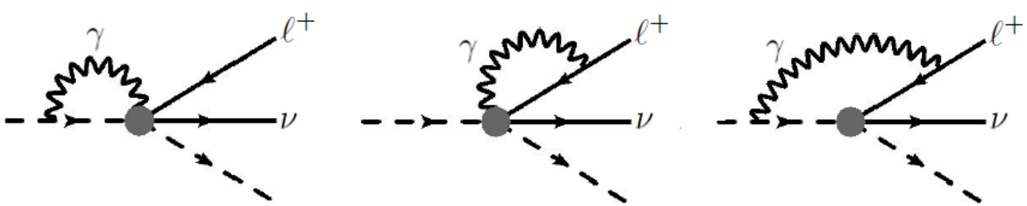
Matthew Moulson,
Chien Yeah Seng

Progress since 2018:

- First experimental measurement of BR of $K_S \to \pi\mu\nu$
  BR($K_S \to \pi\mu\nu$) = (4.56 ± 0.20) ×10^{-4}

KLOE-2
PLB 804 (2020)

- Theoretically update on long-distance EM corrections:



Up to now computation at fixed order e^2p^2 + model estimate for the LECs

Cirigliano et al. ’08

New calculation of complete EW RC using hybrid current algebra and ChPT (Sirlin’s representation) with resummation of largest terms to all chiral orders

– Reduced uncertainties at O(e^2p^4)
– Lattice evaluation of QCD contributions to γW box diagrams

Seng et al.’21

2.1 V_{us} from K_{l3}

Matthew Moulson,
Chien Yeah Seng

Progress since 2018:

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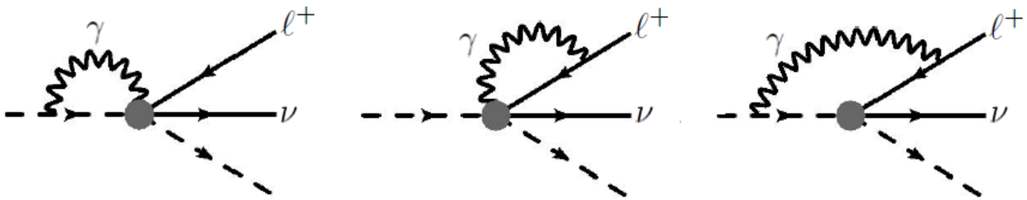


Only K_{e3} at present
For $K_{\mu 3}$ modes continue to use
Cirigliano et al. ’08

	Cirigliano et al. ’08	Seng et al. ’21
$\Delta^{EM}(K^0{}_{e3})$ [%]	0.50 ± 0.11	**0.580 ± 0.016**
$\Delta^{EM}(K^+{}_{e3})$ [%]	0.05 ± 0.13	**0.105 ± 0.024**
ρ	+0.081	**–0.039**

Progress since 2018:

- Theoretical progress on isospin breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_{p}{}^{4}$ = 0.252

NLO in strong interaction

O(e^2p^2) term $\varepsilon_{EM}{}^{(4)} \sim 10^{-6}$

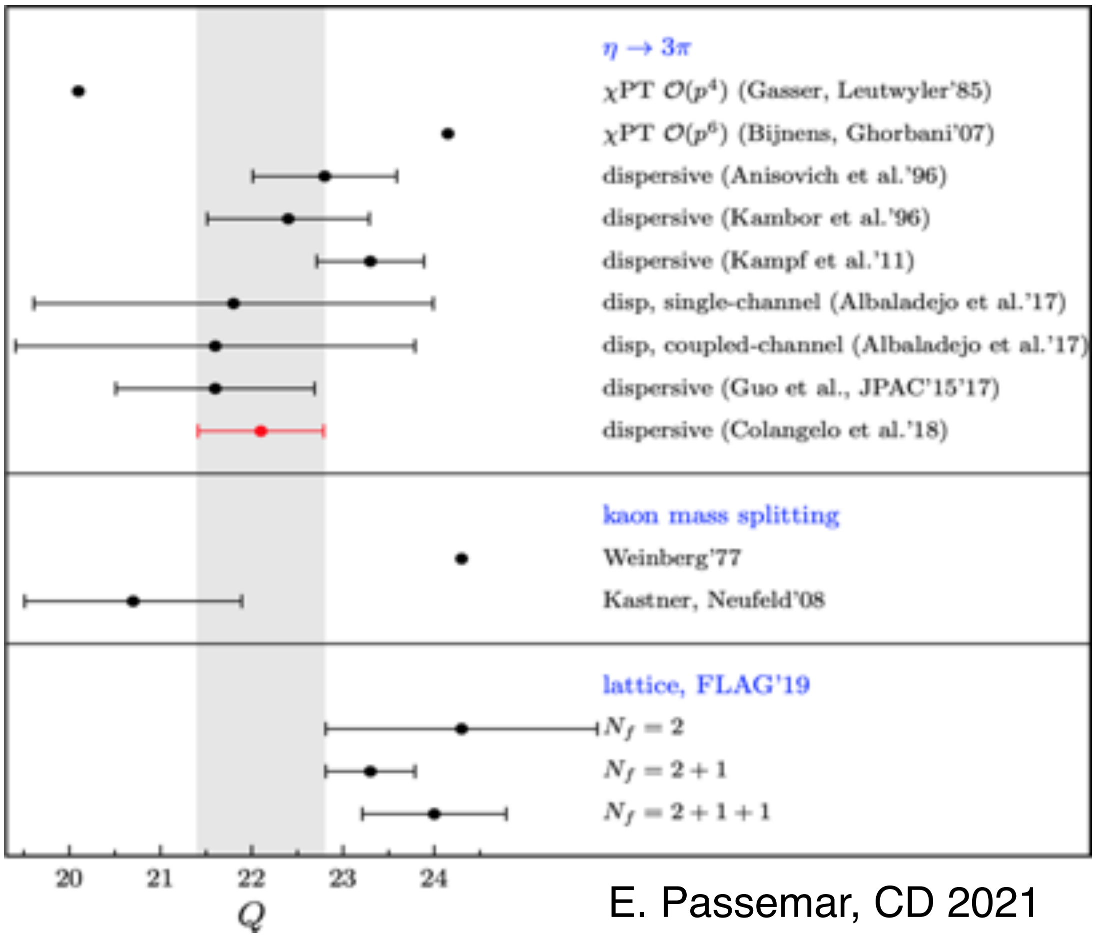
Cirigliano et al., ’02; Gasser & Leutwyler, ’85

$=$ **+2.61(17)%** **Calculated using:**

Q = 22.1(7)	Colangelo et al. ’18, avg. from $\eta \to 3\pi$
$m_s/\hat{m}$ = 27.23(10)	FLAG ’20, N_f = 2+1+1 avg.
M_K = 494.2(3) M_π = 134.8(3)	Isospin-limit meson masses from FLAG ’17

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require $\Delta^{SU(2)}$ **= 2.86(34)%**

Previous to recent results for Q, uncertainty on $\Delta^{SU(2)}$ was leading contributor to uncertainty on V_{us} from $K^{\pm}$ decays



E. Passemar, CD 2021

Reference value of Q from dispersion relation analyses of $\eta \to 3\pi$ Dalitz plots

Colangelo et al., '18

Q = 22.1 ± 0.7

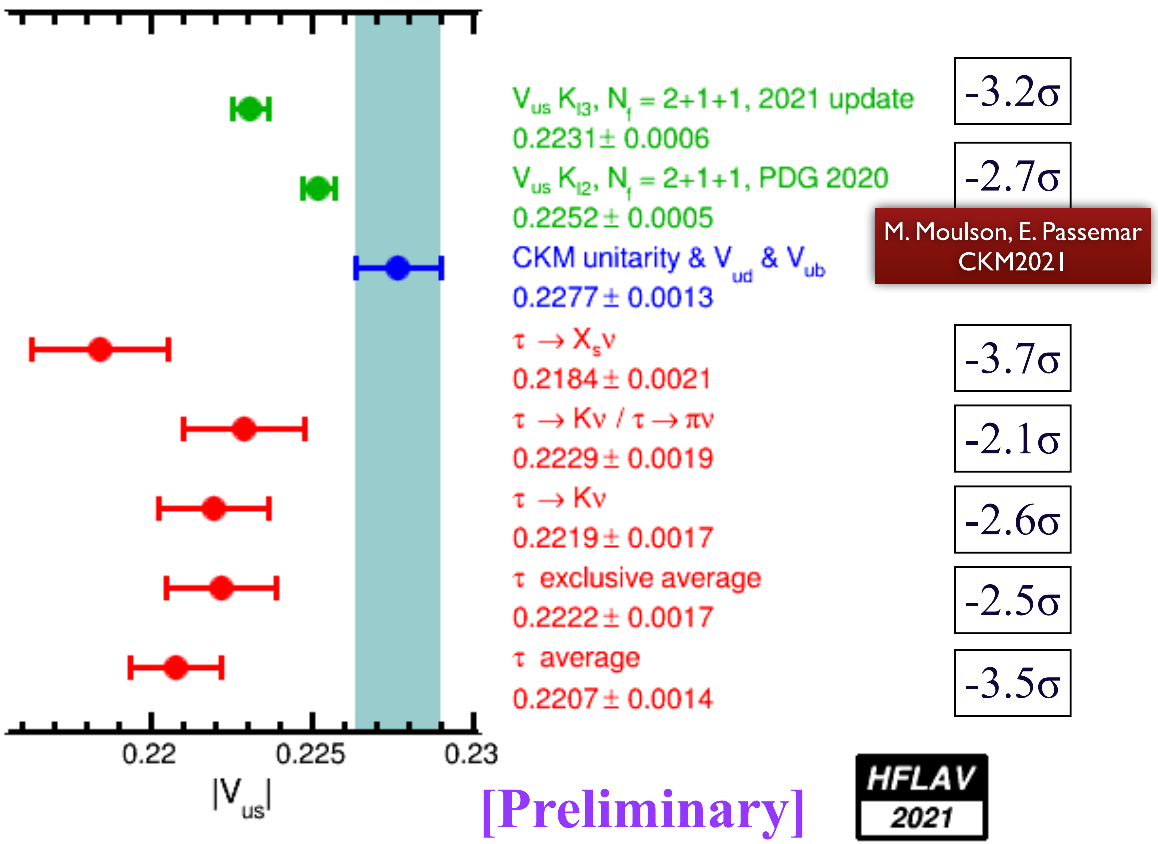
Lattice results for Q somewhat higher than analytical results

But, lattice results have finite correction to LO expectation:

$$Q_M^2 \equiv \frac{\hat{M}_K^2}{\hat{M}_\pi^2} \frac{\hat{M}_K^2 - \hat{M}_\pi^2}{\hat{M}_{K^0}^2 - \hat{M}_{K^+}^2}$$

Low-energy theorem: Q has no correction at NLO

V_{us} from Tau decays



- Belle II with 50 ab^{-1} and ~4.6 x 10^{10} τ pairs will improve V_{us} extraction
- Inclusive measurement is an opportunity to have a complete independent measurement of V_{us} ⇨ not easy as you have to measure many channels

V_{us} from Tau decays

e 13: HFLAV 2021 τ branching fractions to strange final states.

Branching fraction	HFLAV 2021 fit (%)
$K^-\nu_\tau$	0.6957 ± 0.0096
$K^-\pi^0\nu_\tau$	0.4322 ± 0.0148
$K^-2\pi^0\nu_\tau$ (ex.K^0)	0.0634 ± 0.0219
$K^-3\pi^0\nu_\tau$ (ex.K^0, η)	0.0465 ± 0.0213
$\pi^-\overline{K}^0\nu_\tau$	0.8375 ± 0.0139
$\pi^-\overline{K}^0\pi^0\nu_\tau$	0.3810 ± 0.0129
$\pi^-\overline{K}^0 2\pi^0\nu_\tau$ (ex.K^0)	0.0234 ± 0.0231
$\overline{K}^0 h^- h^- h^+\nu_\tau$	0.0222 ± 0.0202
$K^-\eta\nu_\tau$	0.0155 ± 0.0008
$K^-\pi^0\eta\nu_\tau$	0.0048 ± 0.0012
$\pi^-\overline{K}^0\eta\nu_\tau$	0.0094 ± 0.0015
$K^-\omega\nu_\tau$	0.0410 ± 0.0092
$K^-\phi(K^+K^-)\nu_\tau$	0.0022 ± 0.0008
$K^-\phi(K^0_S K^0_L)\nu_\tau$	0.0015 ± 0.0006
$K^-\pi^-\pi^+\nu_\tau$ (ex.K^0, ω)	0.2924 ± 0.0068
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex.K^0, ω, η)	0.0387 ± 0.0142
$K^-2\pi^-2\pi^+\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$X_s^-\nu_\tau$	2.9076 ± 0.0478

HFLAV'21

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C$$

parton model prediction

$$\delta R_\tau \equiv \frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \frac{R_{\tau,S}}{\left|V_{us}\right|^2}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32)$$

Gamiz et al'07, Maltman'11

$$\left|V_{us}\right|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \delta R_{\tau,th}}$$

2.9σ away from unitarity!

$$\left|V_{us}\right| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}$$