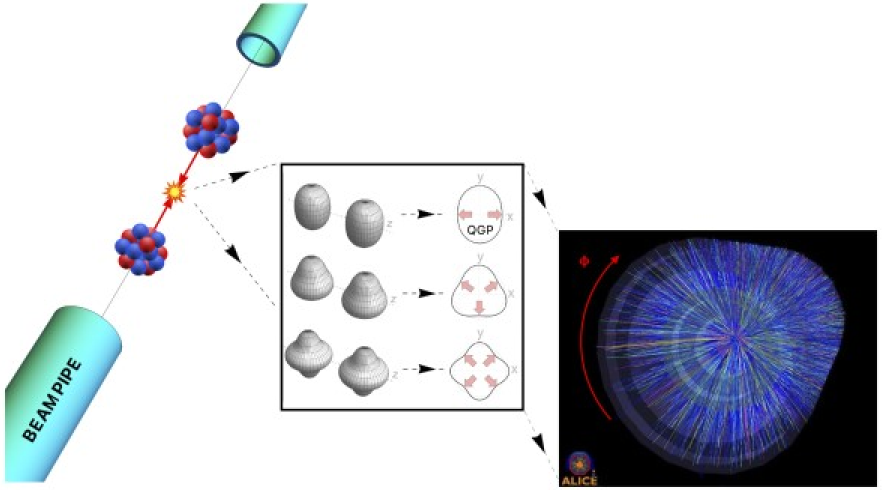
Bayesian analyses of heavy-ion collisions: status and prospects

Jean-François Paquet

January 25, 2023

INT Program INT-23-1a
Intersection of nuclear structure and high-energy nuclear collisions

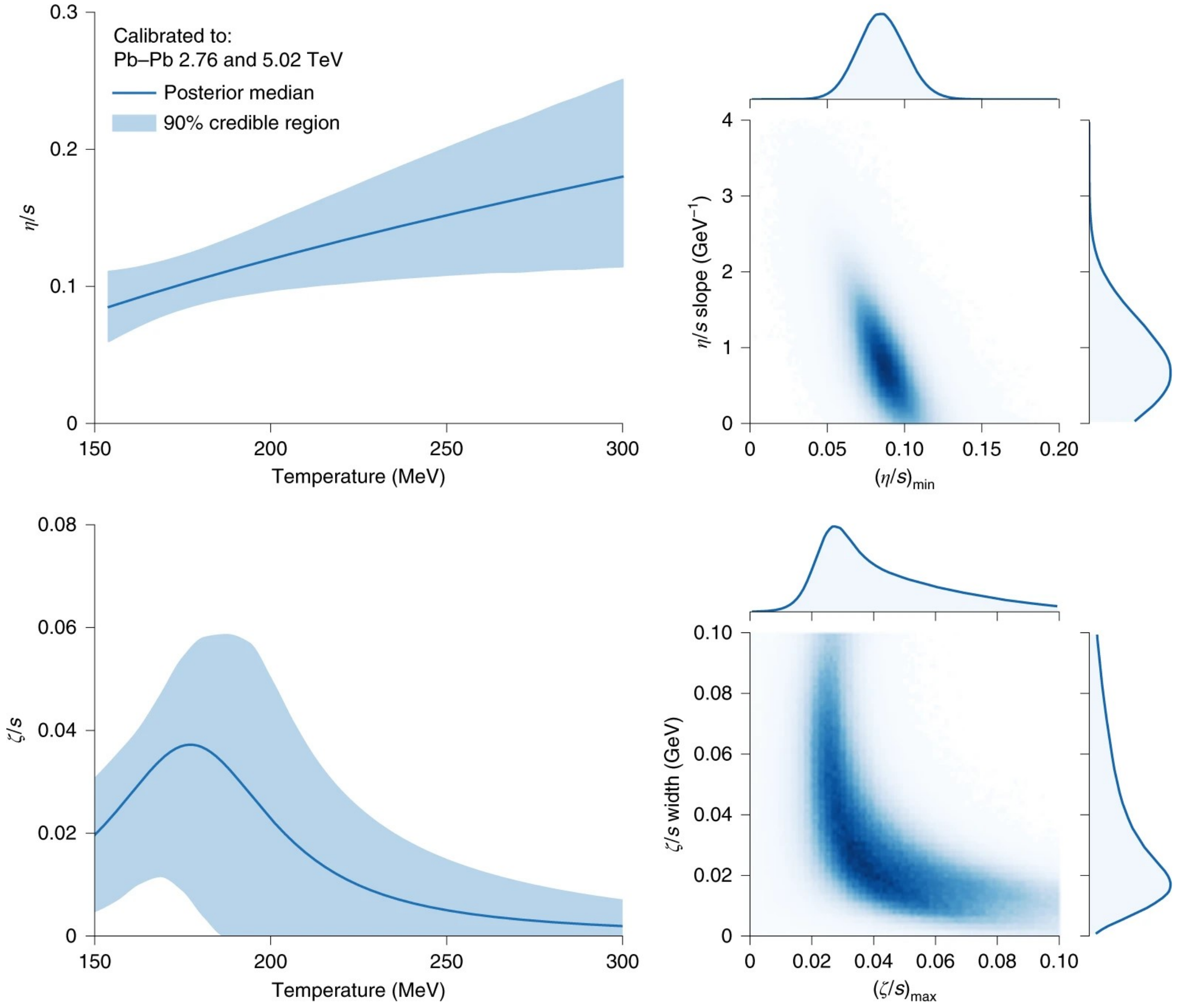


VANDERBILT UNIVERSITY

Bayesian inference

- Statistical approach for model-to-data comparison
- Often associated with:
  - Large-scale model-to-data comparison
  - Uncertainty quantification
  - Emulation
    (PCA, Gaussian process, ...)

Bernhard, Moreland, Bass (2019) Nature Phys.



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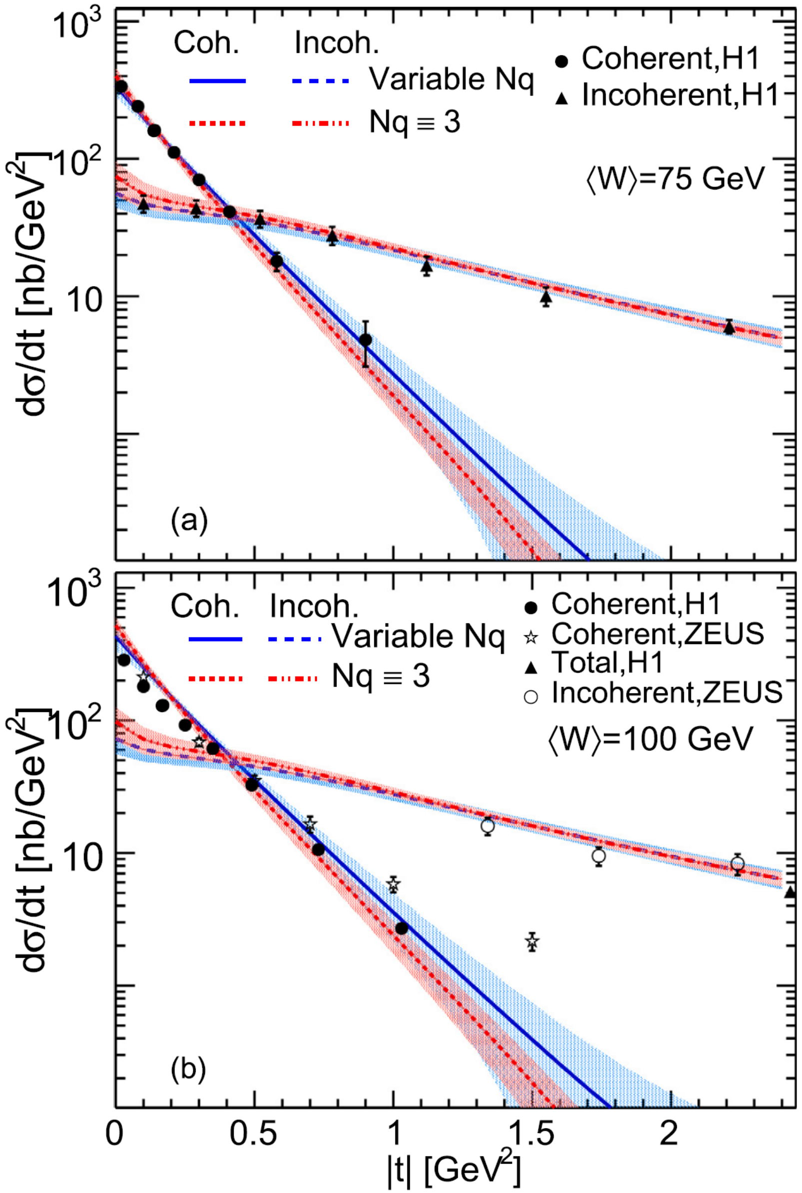
Mäntysaari, Schenke, Shen, Zhao (2022) PLB



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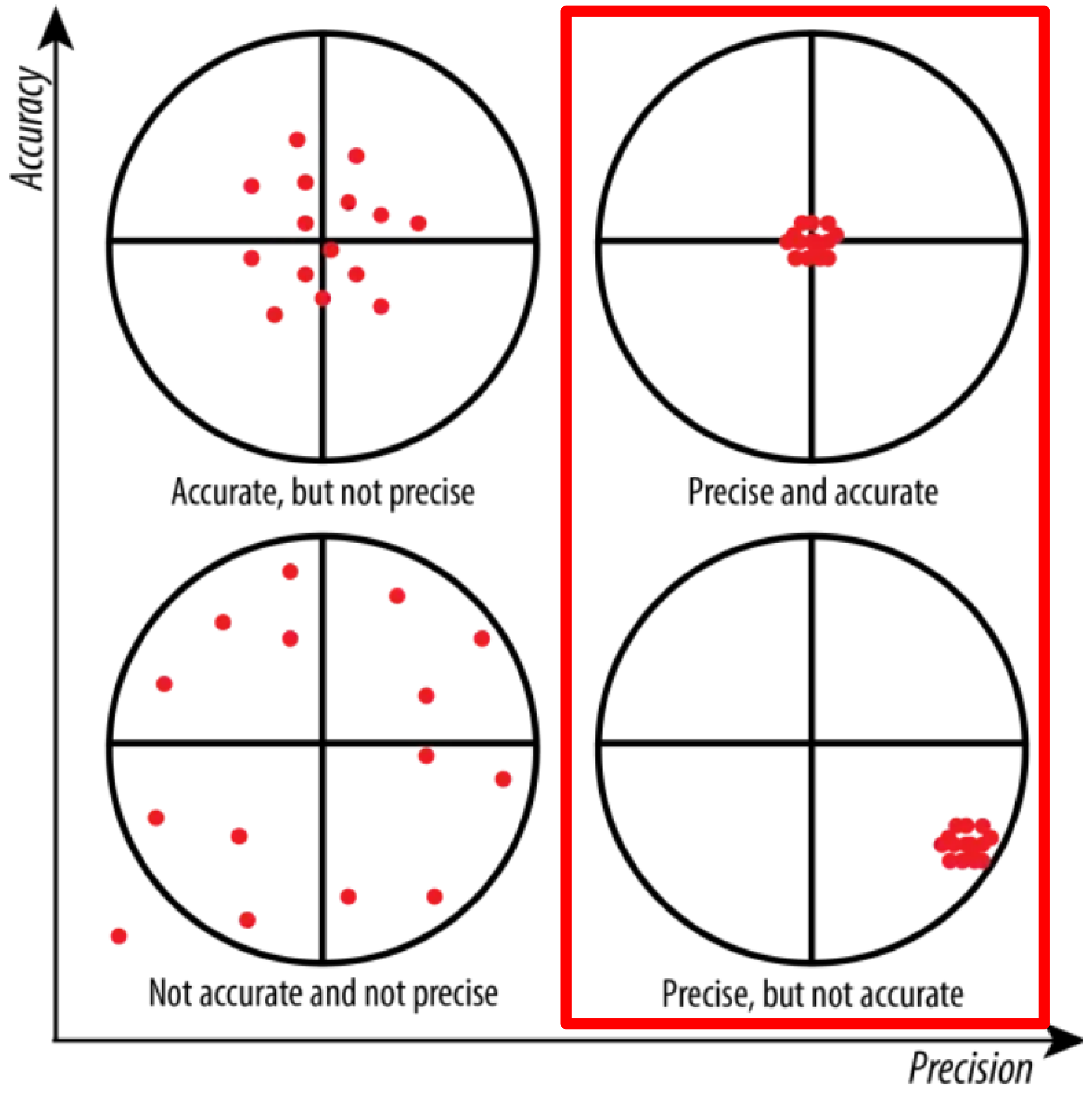

Ref.: https://wp.stolaf.edu/it/gis-precision-accuracy/

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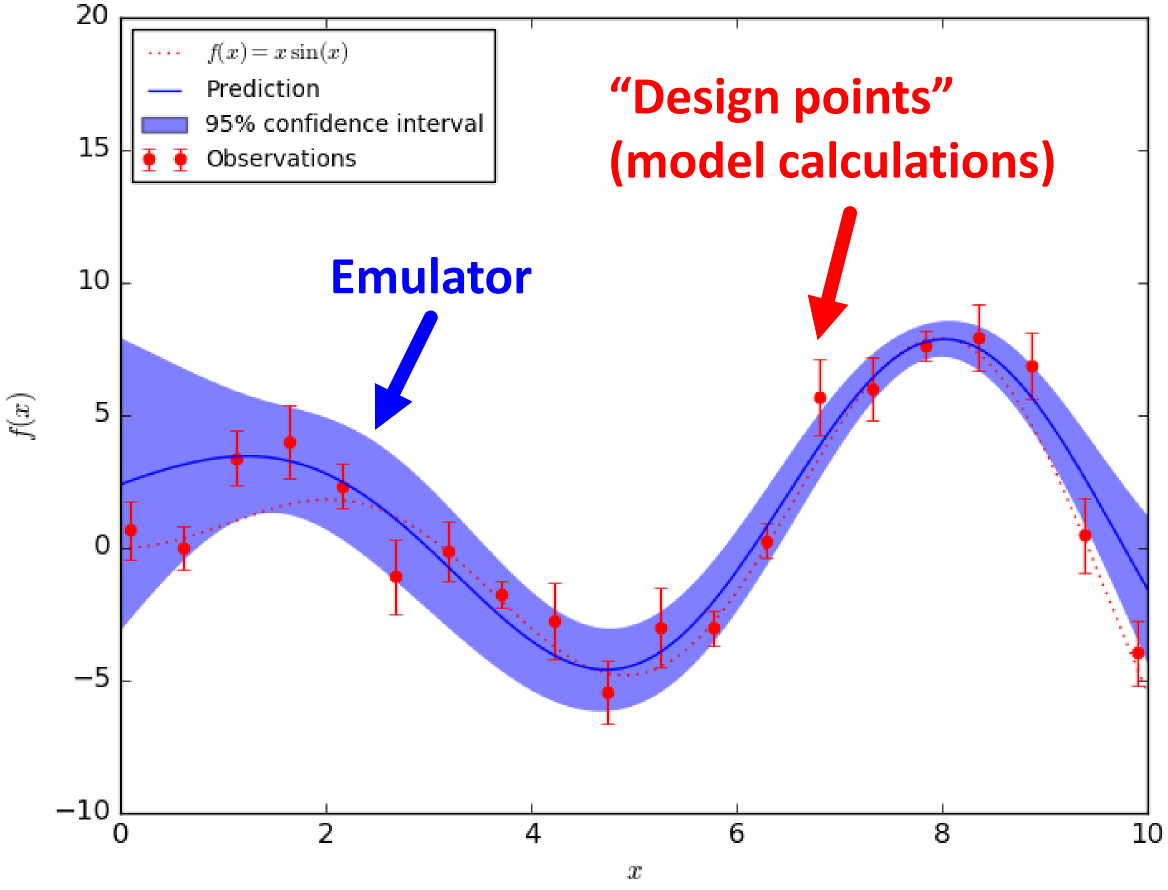
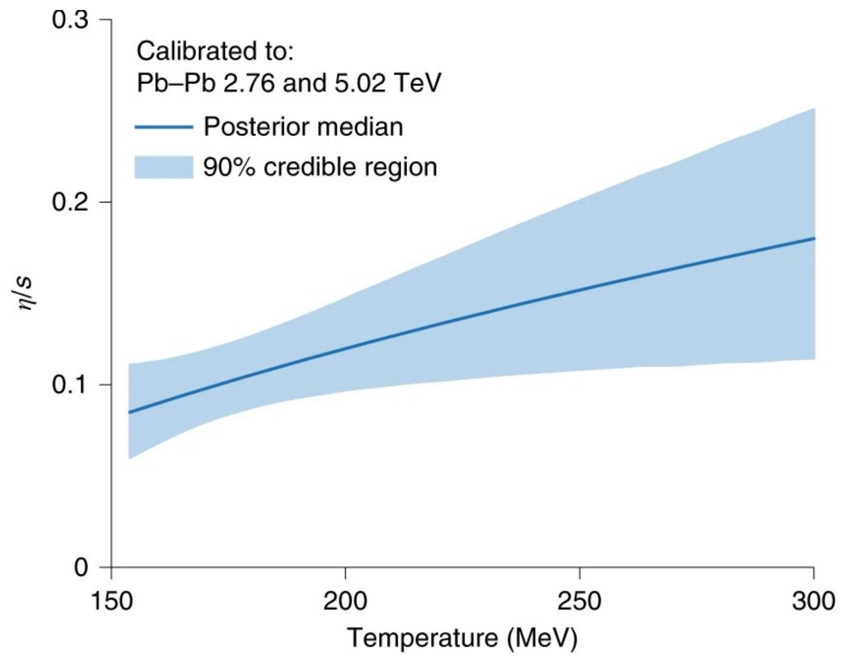
Fig. ref.: https://scikit-learn.org/0.17/auto_examples/gaussian_process/plot_gp_regression.html



Applications of Bayesian inference in heavy-ion collisions

- Most applications have been for:
  - Soft sector: transport coefficients and initial conditions
  - Hard sector: parton energy loss



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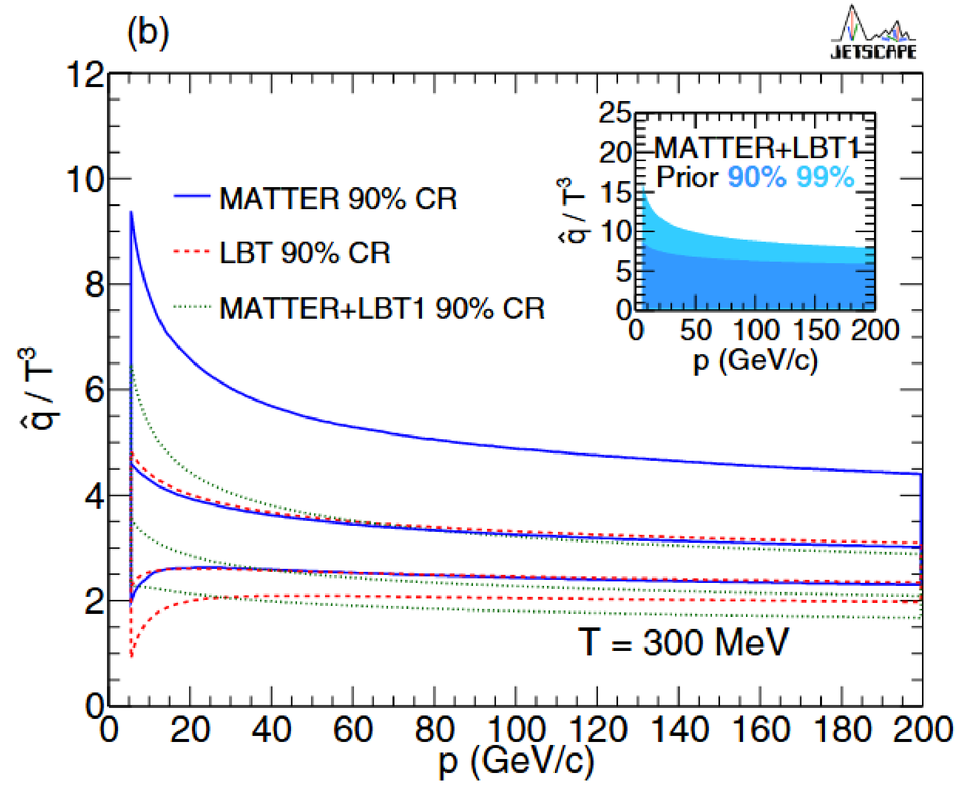
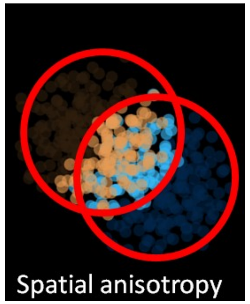
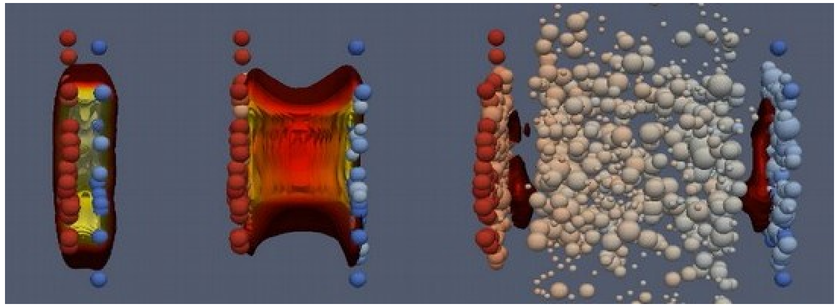
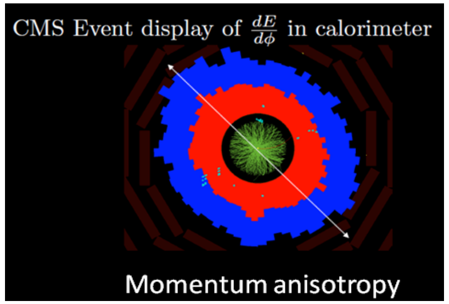


JETSCAPE Collaboration (2021) PRC 104, 024905







OVERVIEW OF BAYESIAN INFERENCE

Bayes' theorem

Adapted from:
Bayesian Methods in Cosmology, edited by Michael P. Hobson, et al., Cambridge University Press, 2009.
Chapter "Foundations and algorithms", by John Skilling

$$\mathbf{prob(p)} \times \mathbf{prob}(\boldsymbol{d}|\boldsymbol{p}) = \boldsymbol{prob(p,d)} = \boldsymbol{prob(d)} \times \boldsymbol{prob(p|d)}$$

$$\underbrace{\text{Prior} \times \text{Likelihood}}_{\text{Inputs}} = \text{Joint} = \underbrace{\text{Evidence} \times \text{Posterior}}_{\text{Outputs}}$$

- The posterior is (generally) what we are after: probability of model parameters given data

$$\boldsymbol{Posterior} \propto \boldsymbol{Prior} \times \boldsymbol{Likelihood}$$

Bayesian parameter inference w/ Gaussian likelihood

Probabilistic constraints on parameters

Mean value of **measurements**

Experimental and theoretical **uncertainties**

$$\mathbf{posterior}(\boldsymbol{p}|\boldsymbol{D}) \propto \boldsymbol{prior}(\boldsymbol{p}) \times \boldsymbol{exp}\left(-\frac{1}{2}\left(\boldsymbol{D}-\boldsymbol{Model}(\boldsymbol{p})\right)^{\boldsymbol{T}} \boldsymbol{Cov}^{-1}\left(\boldsymbol{D}-\boldsymbol{Model}(\boldsymbol{p})\right)\right)$$

External constraints on parameters:
positivity or range of allowed values, "probabilistic theoretical constraints", ...

Prediction of model for given set of parameters (η/s, ζ/s, initial conditions, ...)

Bayesian parameter inference w/ Gaussian likelihood

Probabilistic constraints on parameters

$$\mathbf{posterior}(\boldsymbol{p}|\boldsymbol{D}) \propto \boldsymbol{prior}(\boldsymbol{p}) \times \boldsymbol{exp}\left(-\frac{\boldsymbol{\chi}^2}{2}\right)$$

External constraints on parameters:
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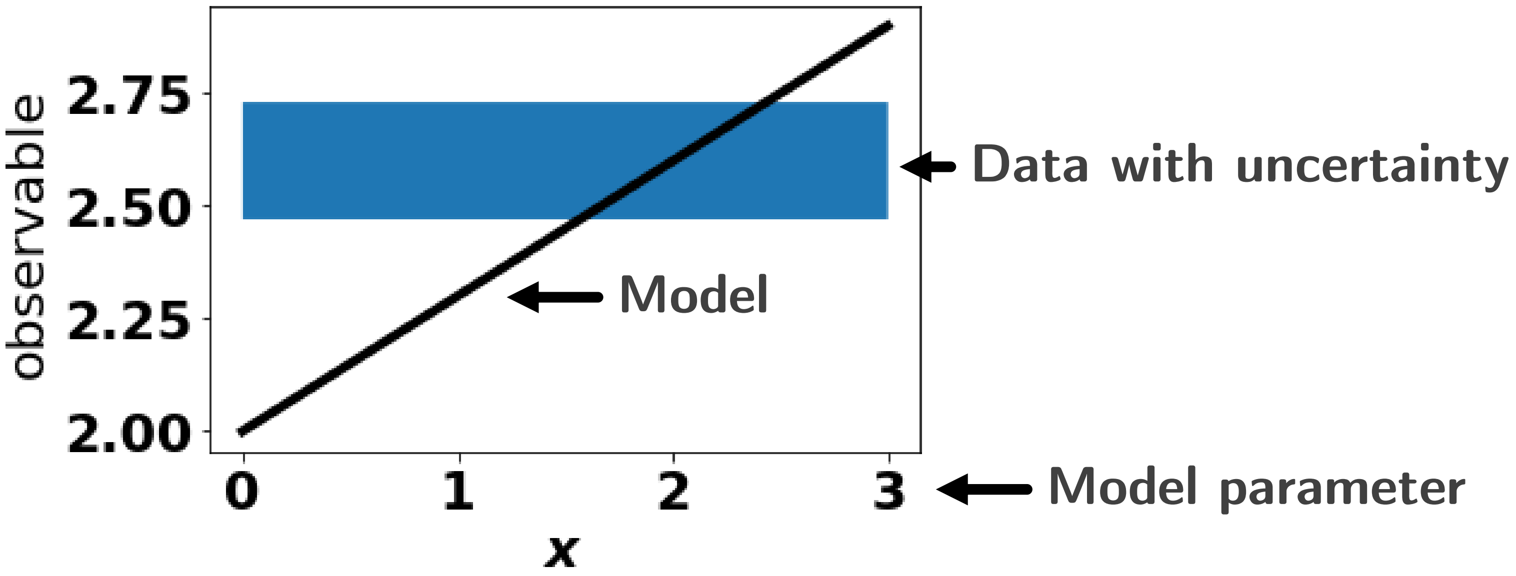
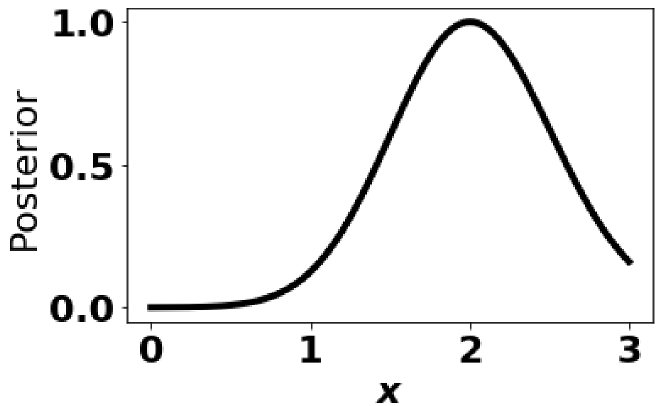
Simple example (with Gaussian likelihood)



- Say your model is $observable(x) = 2 + 0.3\,x$
- Say the measured observable is $d = 2.6 \pm 0.13$ (5% relative uncertainty)

- Result:



$$\boldsymbol{exp}\left[-\frac{(\boldsymbol{d}-\mathbf{2}-\mathbf{0.3}\,\boldsymbol{x})^2}{\mathbf{2}(\mathbf{0.05}\boldsymbol{d})^2}\right] = \boldsymbol{exp}\left[-\frac{\left((\boldsymbol{d}-\mathbf{2})/\mathbf{0.3}-\boldsymbol{x}\right)^2}{\mathbf{2}(\mathbf{0.05}\boldsymbol{d}/\mathbf{0.3})^2}\right]$$

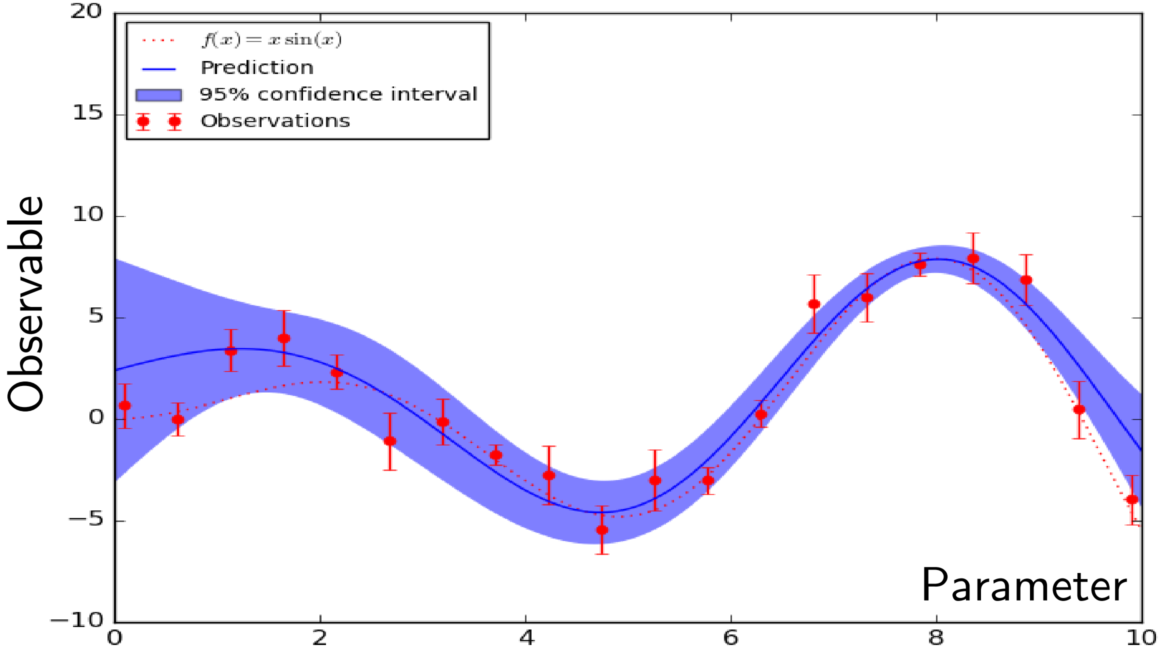
- **Uncertainty on x:** $\dfrac{(\textbf{exp.uncert})}{(\boldsymbol{slope\ of\ model})} = \dfrac{\mathbf{0.05}\,\boldsymbol{d}}{\mathbf{0.3}}$

Bayesian inference in practice

- Models are non-linear, numerical, expensive, stochastic

Experimental and theoretical **uncertainties**

$$\mathbf{posterior}(p|D) \propto prior(p) \times exp\left(-\frac{1}{2}(Data - Model(p))^{T} Cov^{-1} (Data - Model(p))\right)$$



Generally needs to replace model prediction by fast "surrogate"/emulator

Bayesian inference in practice

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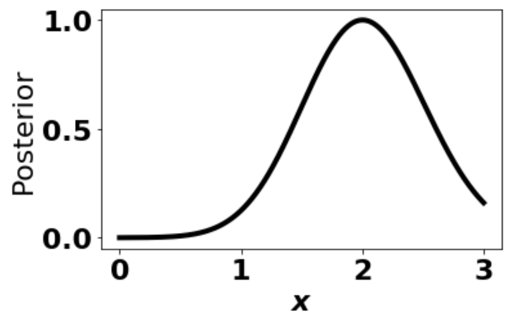
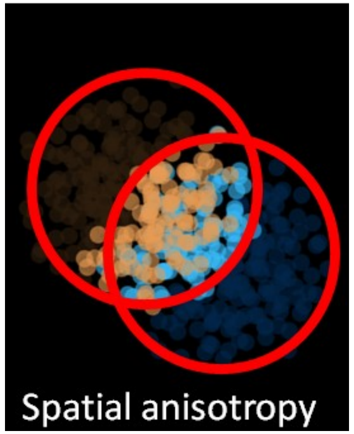
$$\textbf{posterior}(p|D) \propto prior(p) \times exp\left(-\frac{1}{2}\left(Data - Model(p)\right)^T Cov^{-1}\left(Data - Model(p)\right)\right)$$

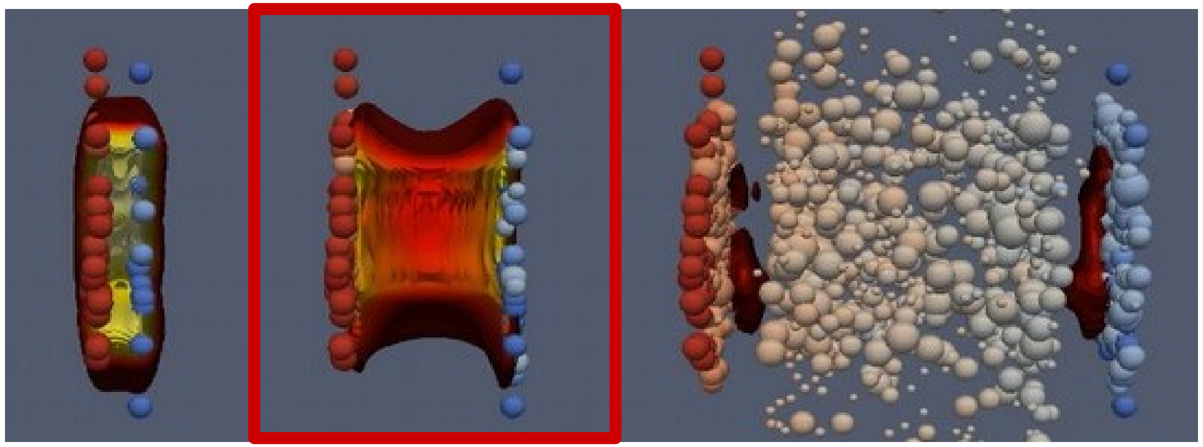
- Bayesian inference in practice:
  - Choose a model and a set of parameters
  - Choose priors for parameters, and prepare emulator for model over prior range
  - Choose data set
  - Compute and study the posterior and the evidence

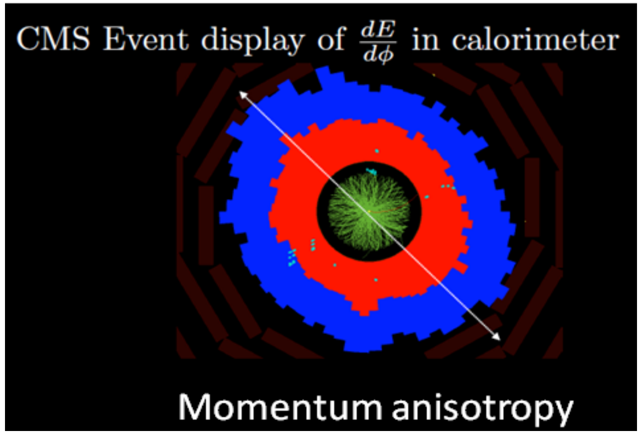


Multistage simulations of heavy ion collisions

Based on figures by Derek Teaney, CMS Coll., MADAI, H. Elfner and J. Bernhard







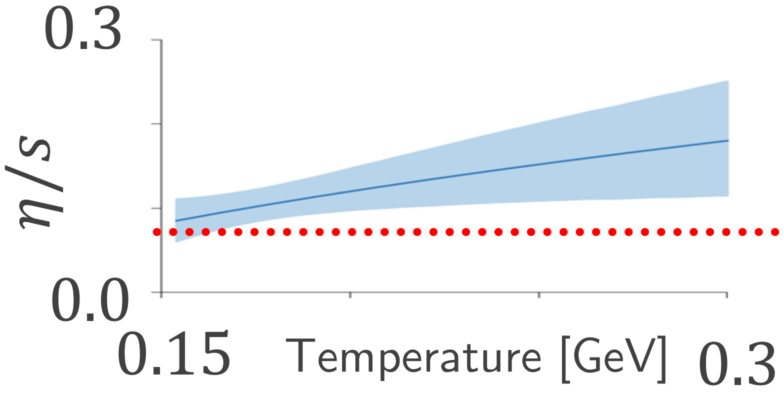
Energy deposition → Early dynamics → Hydrodynamics → Hadronic transport

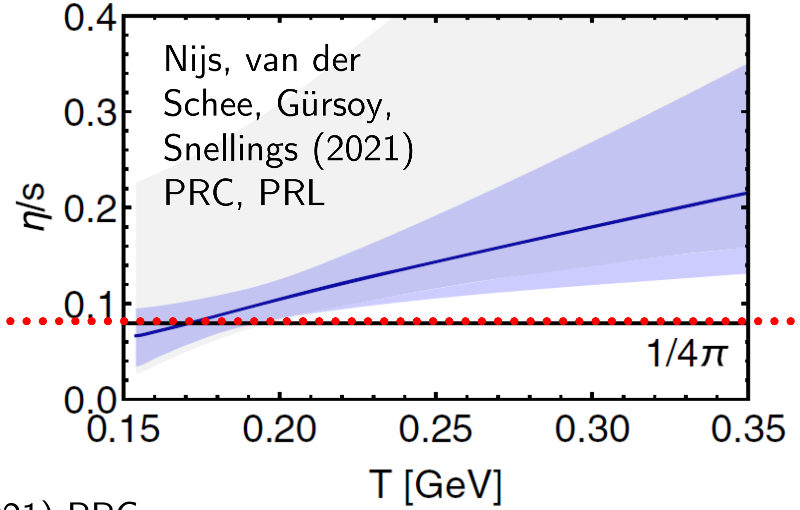
- Energy-momentum tensor of plasma: $T^{\mu\nu} = \epsilon u^\mu u^\nu - (P(\epsilon) + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$
- Conservation of energy and momentum: $\partial_\nu T^{\mu\nu} = 0$
- Mueller-Israel-Stewart-type relativistic viscous hydrodynamics

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta}\dot{\pi}^{\alpha\beta} + \boxed{\pi^{\mu\nu} = 2\,\eta(T)(\partial^\mu u^\nu +}\cdots) + (2^{\text{nd}}\text{ order}); \quad \tau_\Pi \dot{\Pi} + \boxed{\Pi = -\zeta(T)\,\partial_\mu} u^\mu + (2^{\text{nd}}\text{ order});$$

Shear viscosity calibrations

Bernhard, Moreland, Bass (2019) Nat.Phys.





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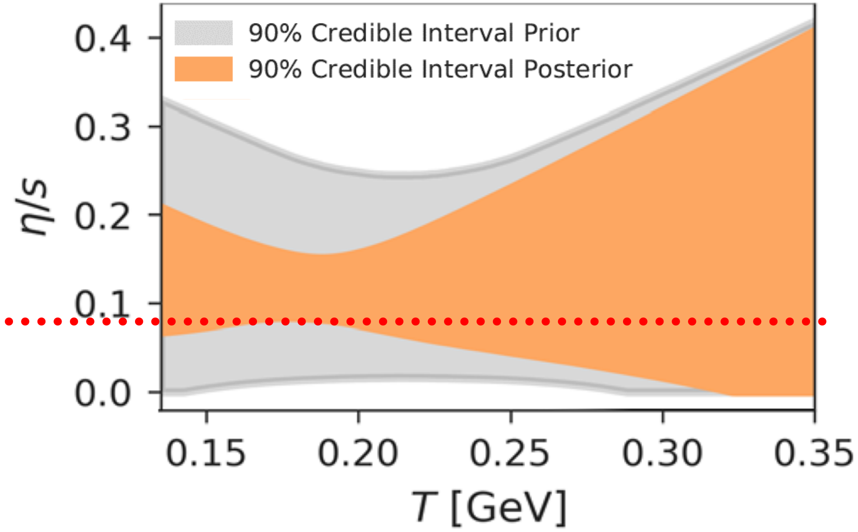

Similar results from Parkkila, Onnerstad, Kim (2021) PRC

Bulk viscosity calibrations

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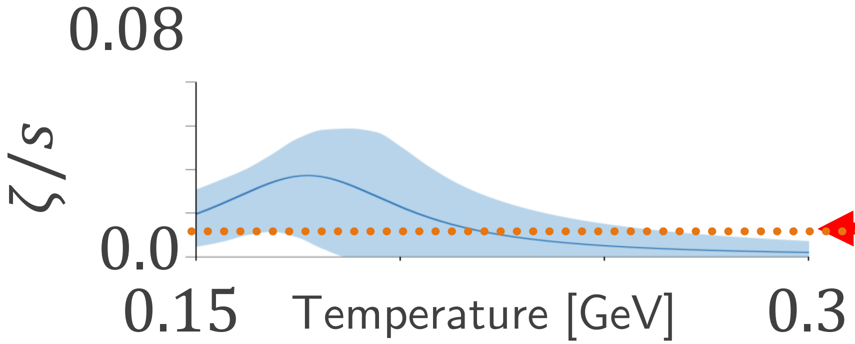
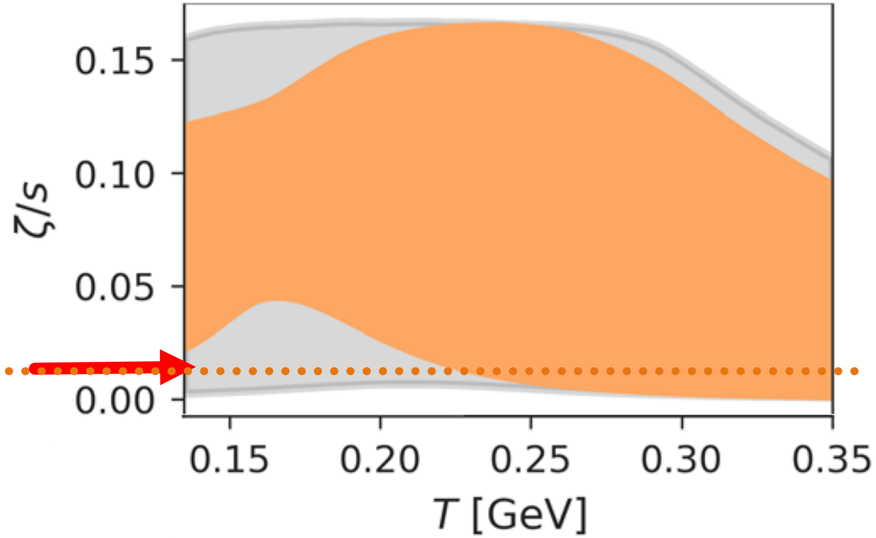
Bernhard, Moreland, Bass (2019) Nat.Phys.



Nijs, van der Schee, Gürsoy, Snellings (2021) PRC, PRL

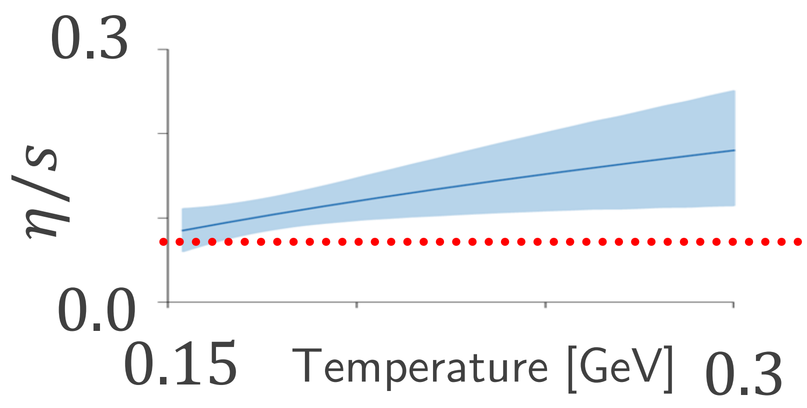
$\zeta/s < 0.01$



Parkkila, Onnerstad, Kim (2021) PRC: $\zeta/s < 0.03$

Shear viscosity calibrations

Bernhard, Moreland, Bass (2019) Nat.Phys.



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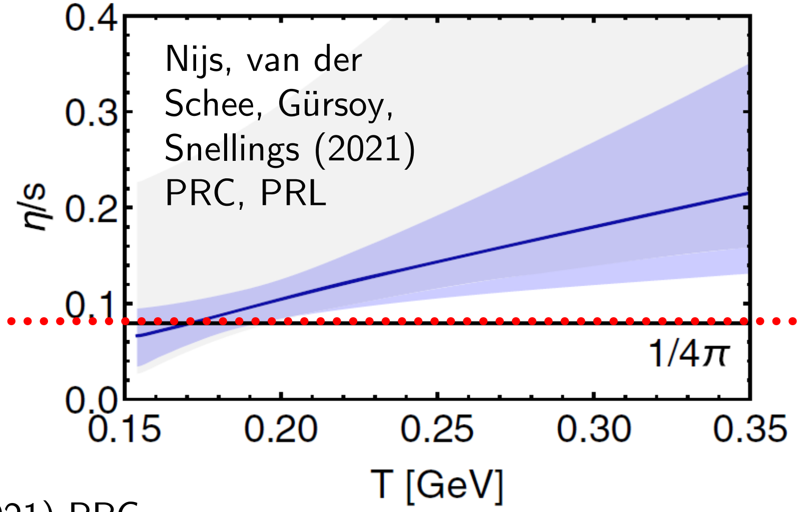

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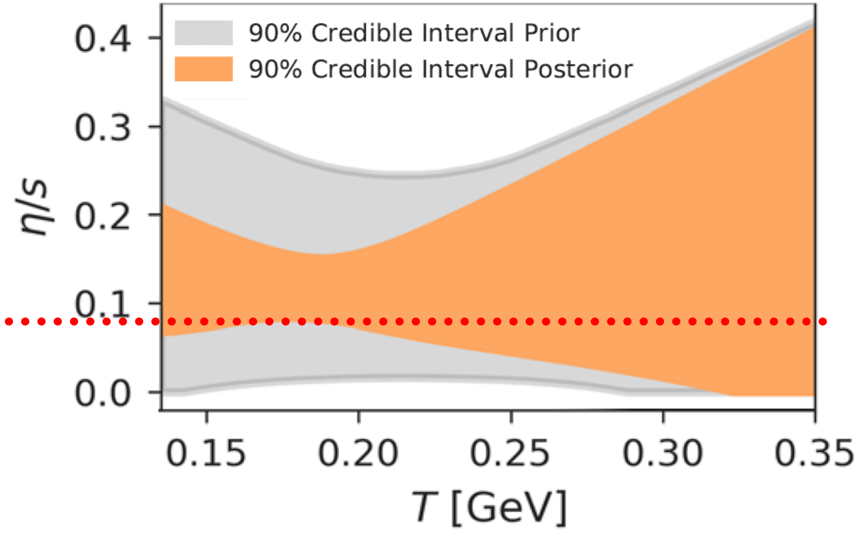

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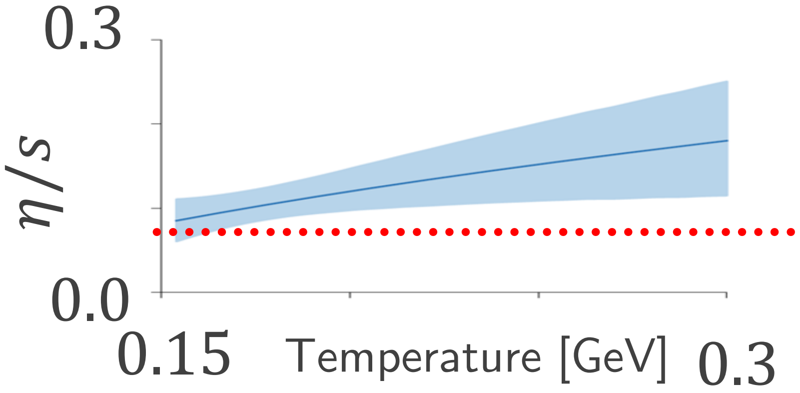

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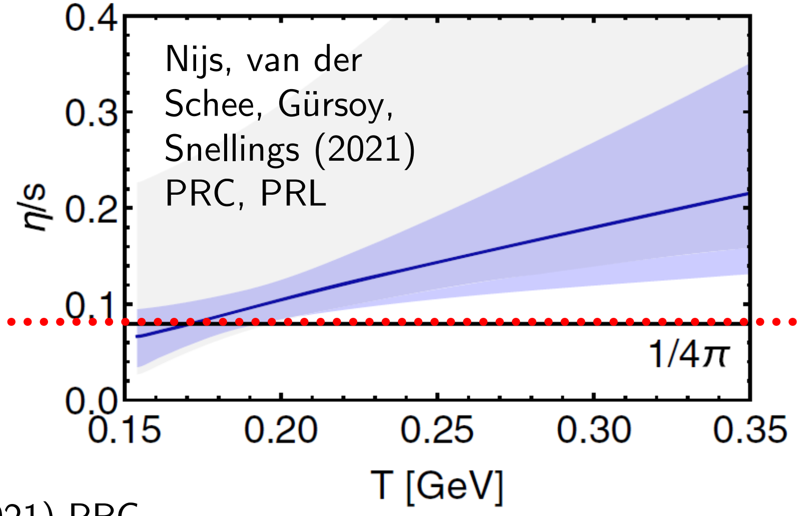

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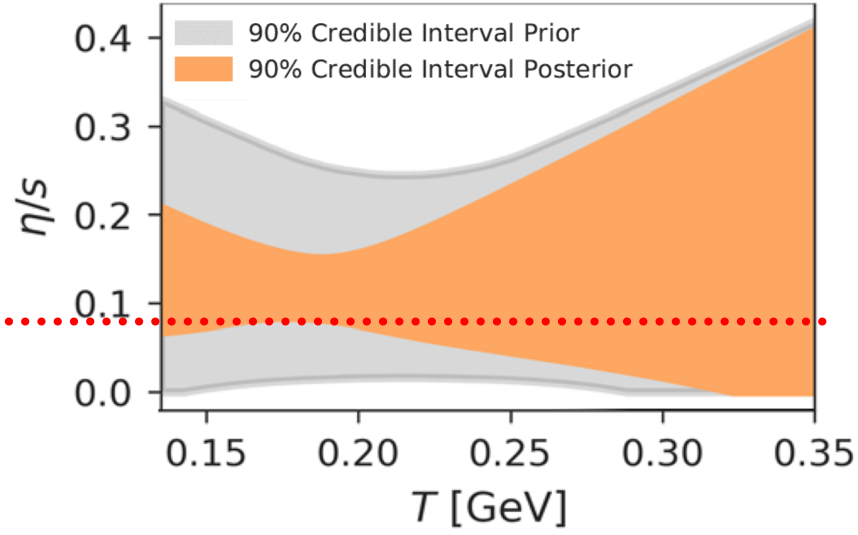

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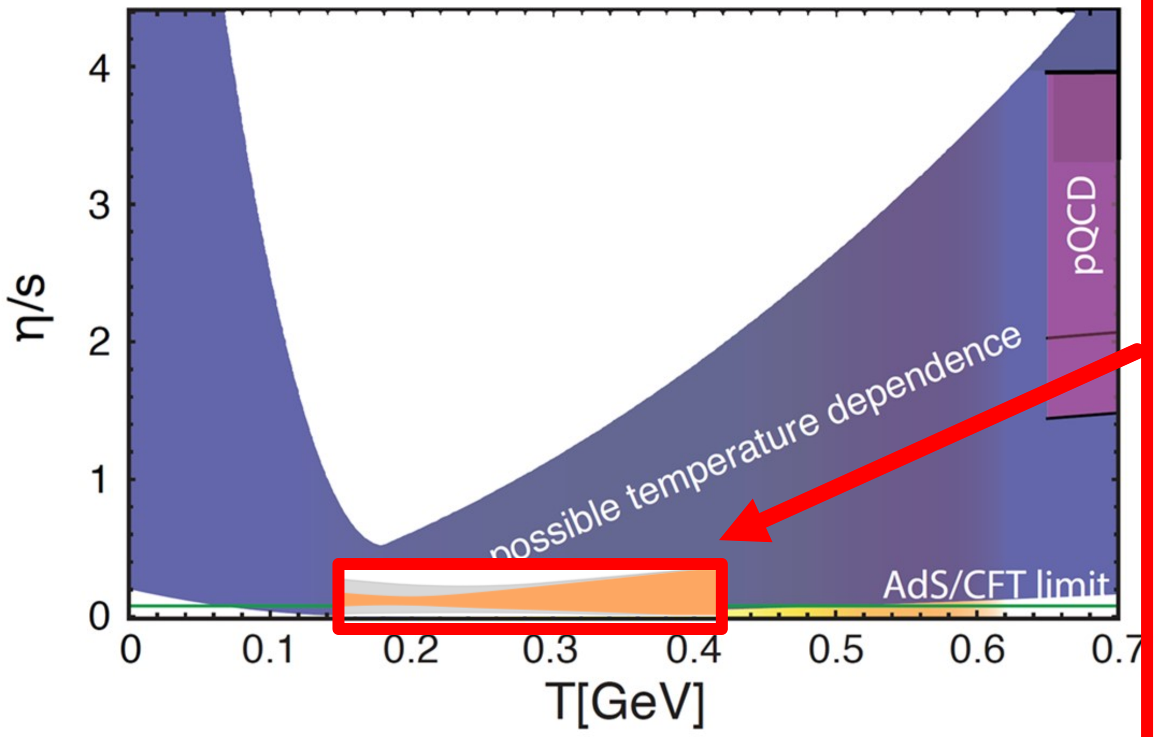
- Why do results differ?
  - Models (initial stage+hydrodynamics+hadronic afterburner): similar but not identical
  - Differences in priors but also in parametrizations
  - Different selection of data

Keeping everything in perspective

Modified from the Hot QCD White Paper 2015



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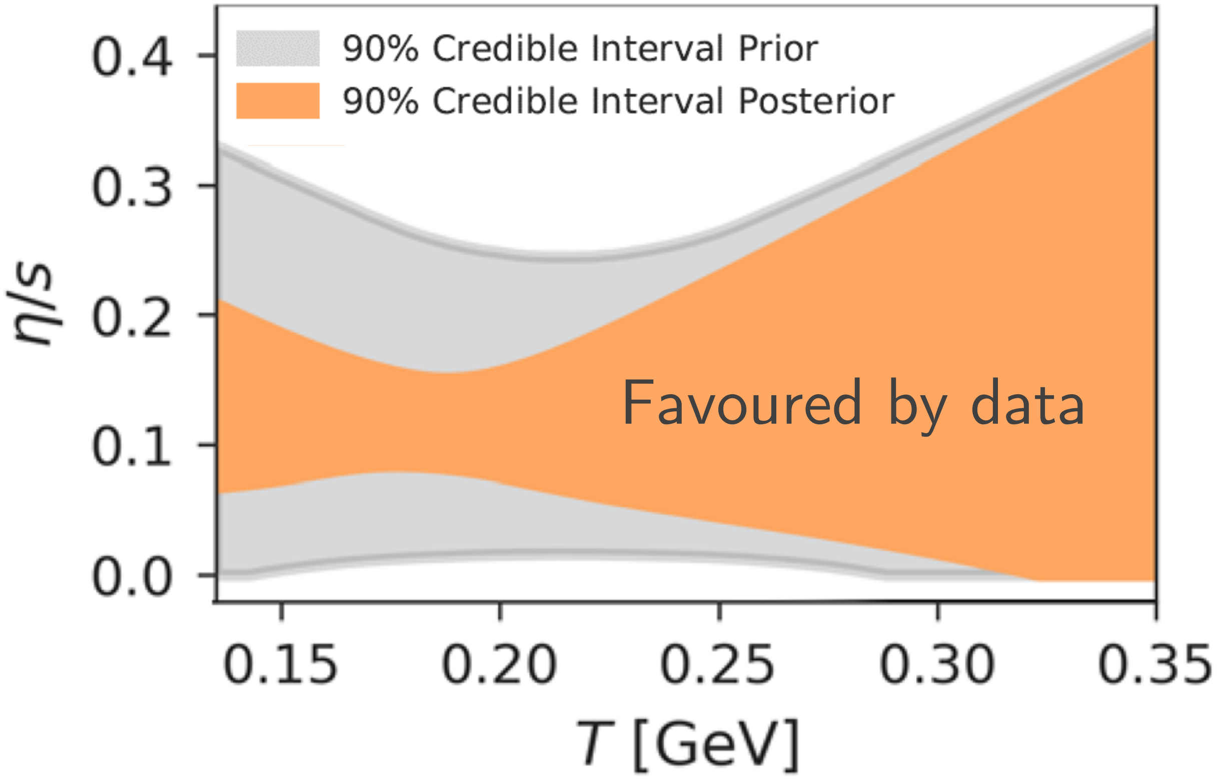

Pros and cons of Bayesian inference

Benefits:

- Systematic and <u>reproducible</u> constraints on model parameters
- Propagation of uncertainties (experimental, theoretical; covariance)
- Scales well to large number of measurements and model parameters
- Model selection/comparison
- Model mixing
- Experimental design

Challenges:

- Expensive numerically
- Emulation introduces additional uncertainty (complicating experimental design and interpretation of uncertainties)
- Communicating meaning of uncertainties; "precision vs accuracy"
- Communication meaning of parameters

Why do calibrations differ, and what to learn from it?

- Why do results differ?
  - **Models**
  - Differences in priors but also in parametrizations
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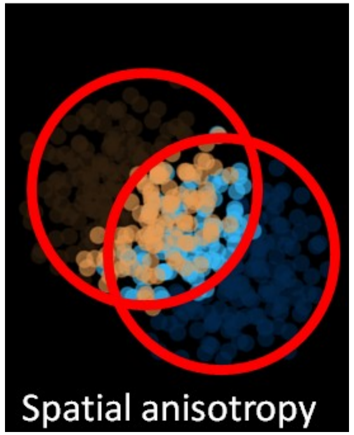
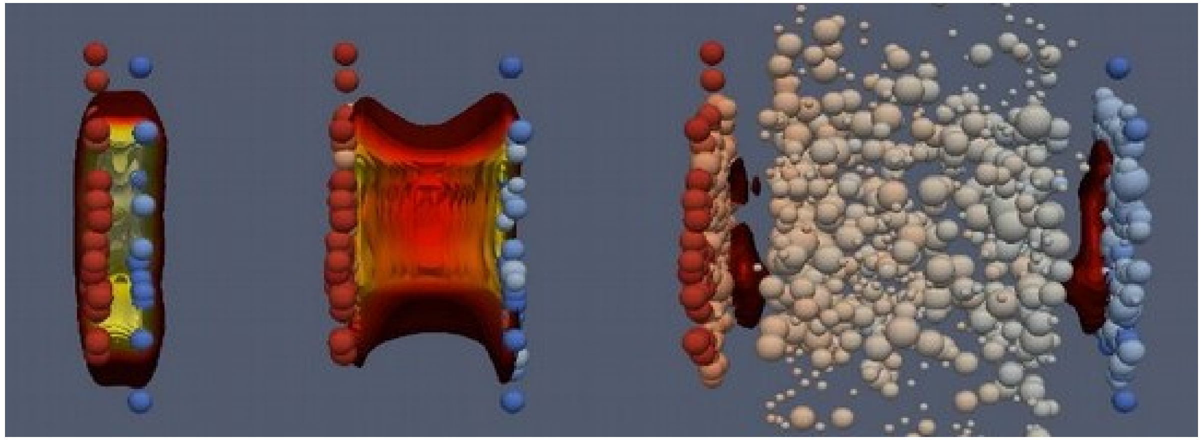
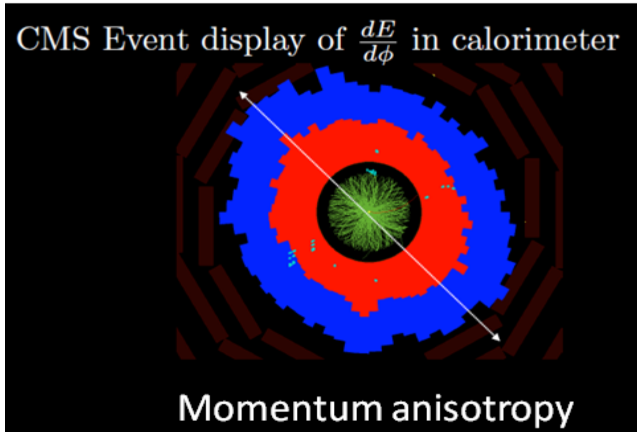
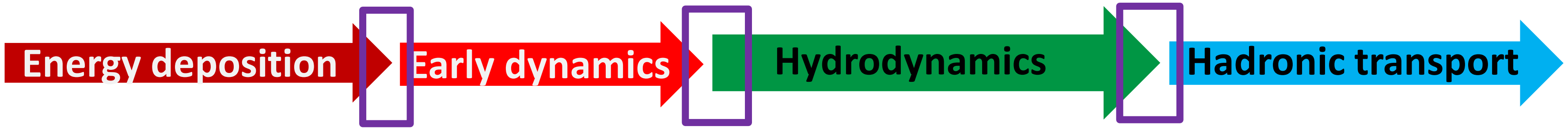
Model differences are often a reflection of "theoretical uncertainty"

Based on figures by Derek Teaney, CMS Coll., MADAI, H. Elfner and J. Bernhard









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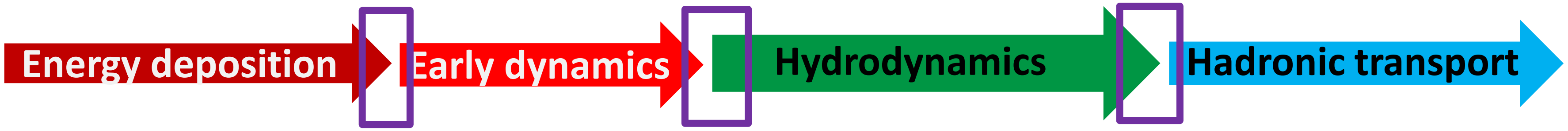
Model differences should reduce as theoretical description of heavy-ion collisions improve



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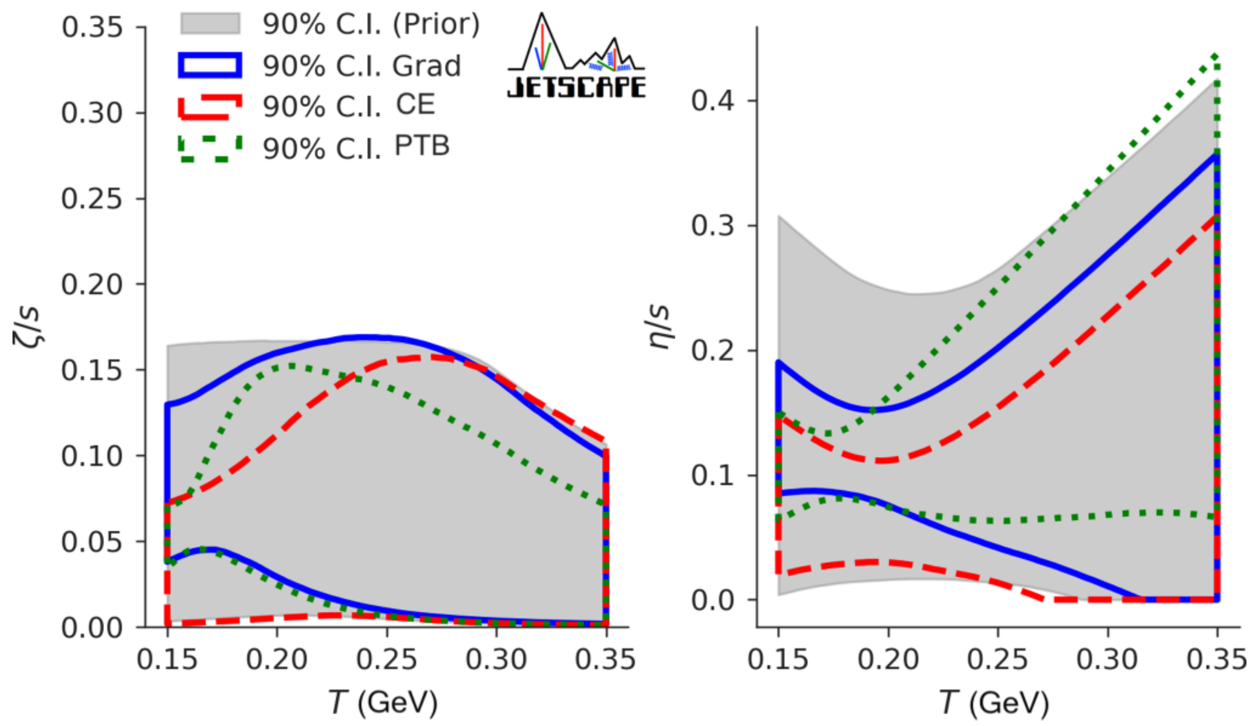

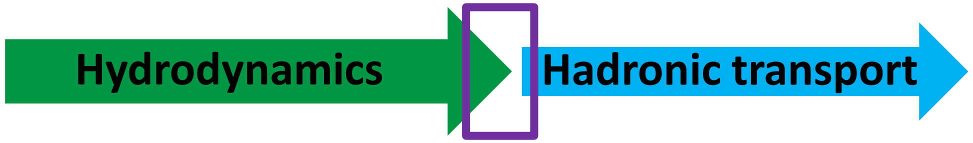

$$T^{\mu\nu} = \sum_n g_n \int \frac{d^3k}{(2\pi)^3 K^0} K^\mu K^\nu f_n(K)$$
$$= \epsilon u^\mu u^\nu - (g^{\mu\nu} - u^\mu u^\nu)(P + \Pi) + \pi^{\mu\nu}$$

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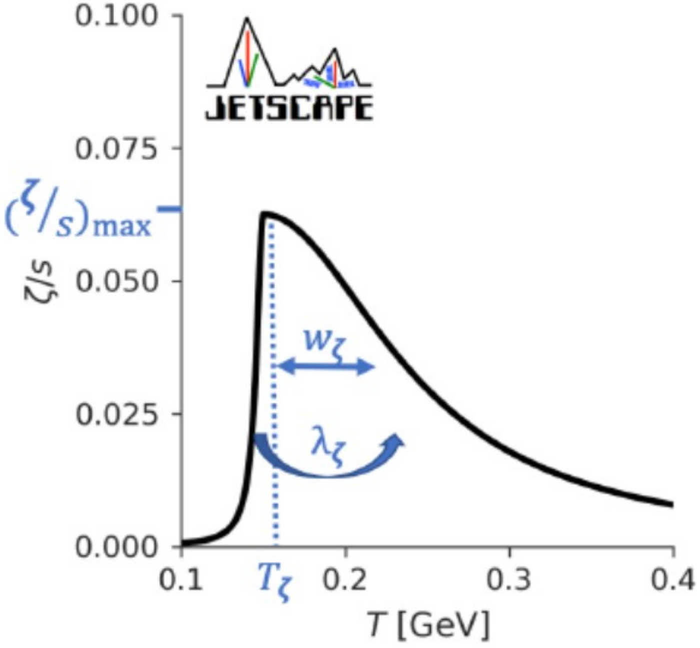

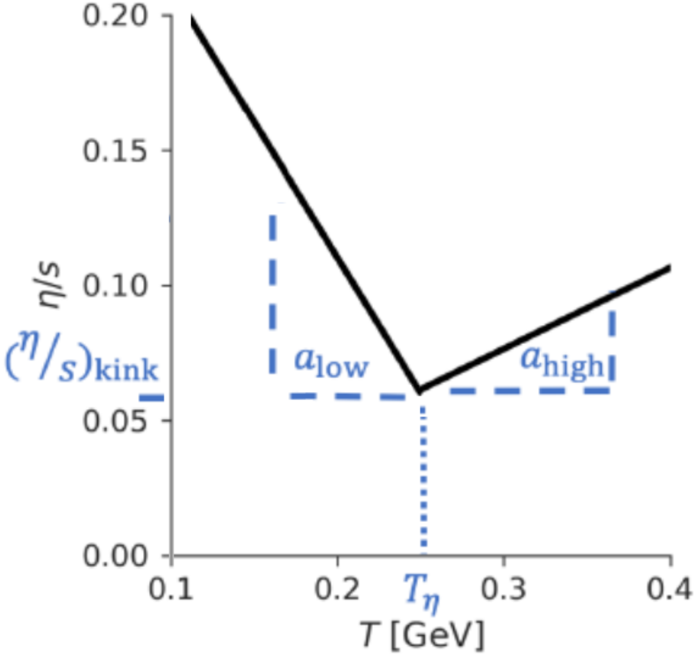

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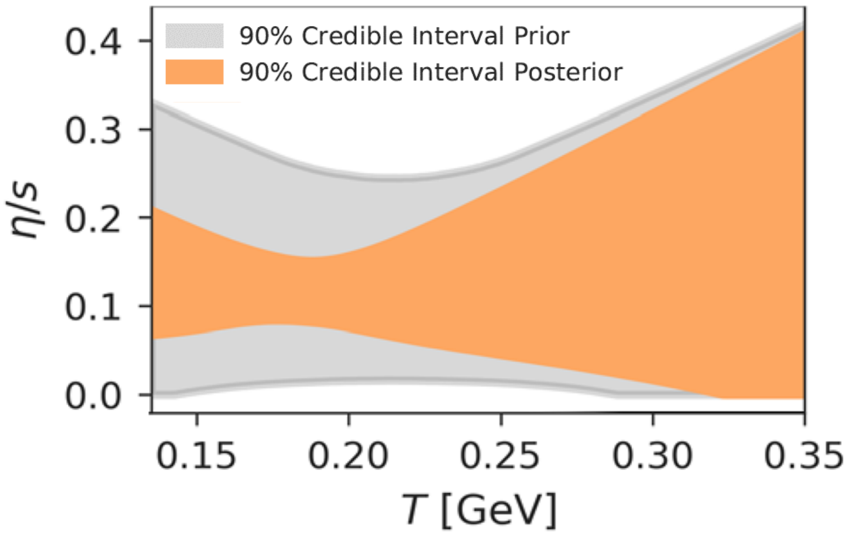

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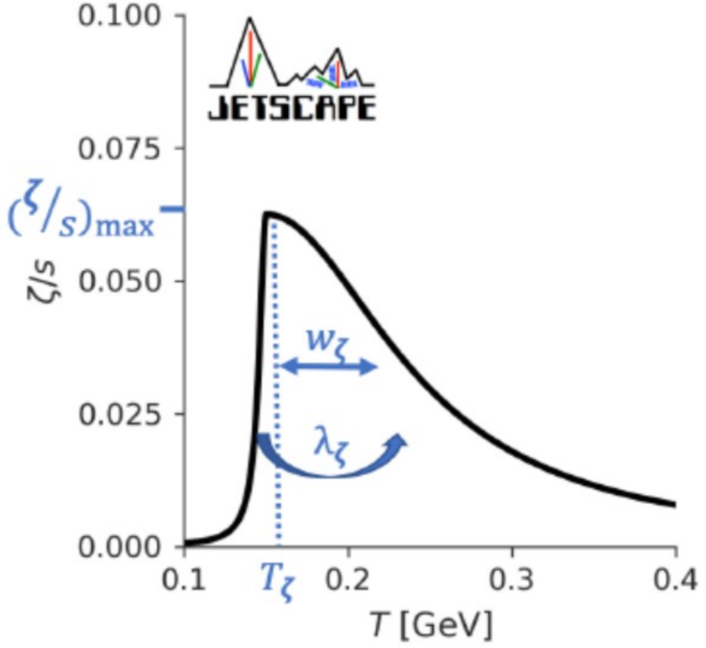

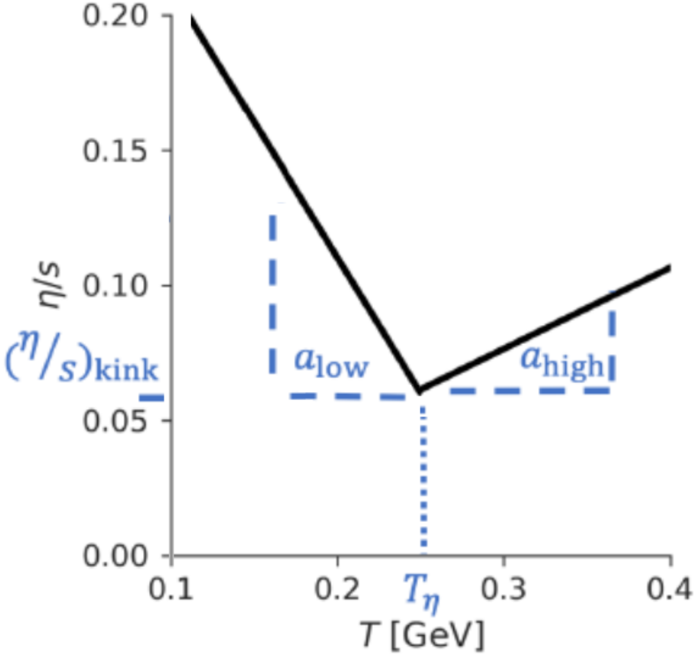

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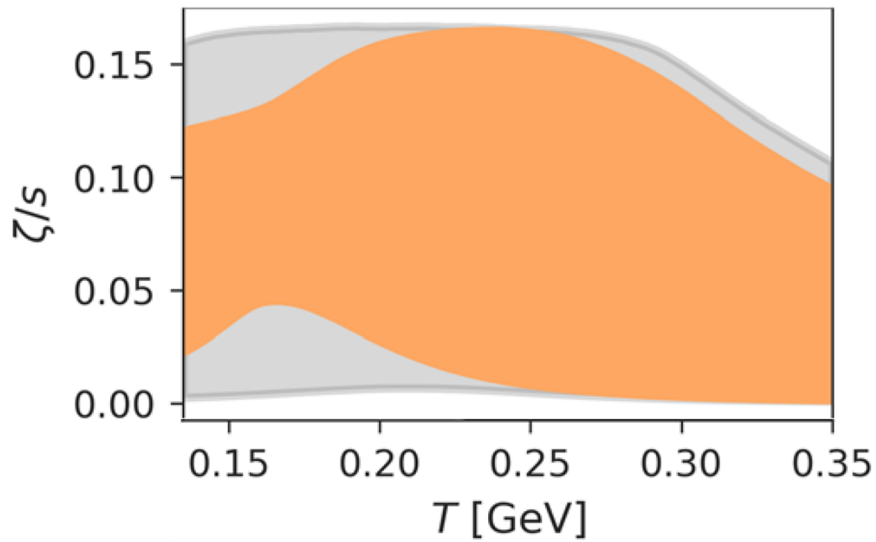

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- Why do results differ?
  - Models
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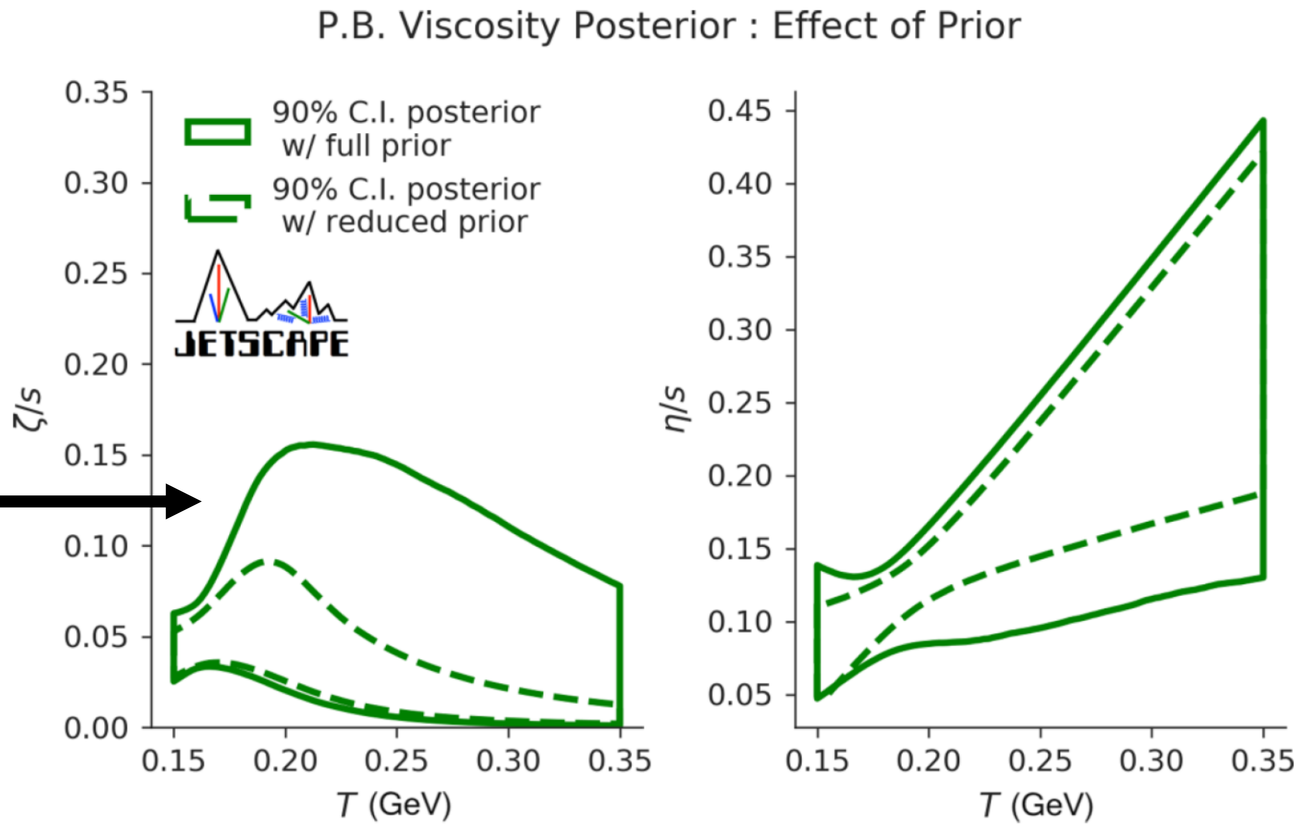
  - Different selection of data

Dashed line is posterior (90% credible interval) using prior from a previous analysis

Solid line is posterior using broader prior



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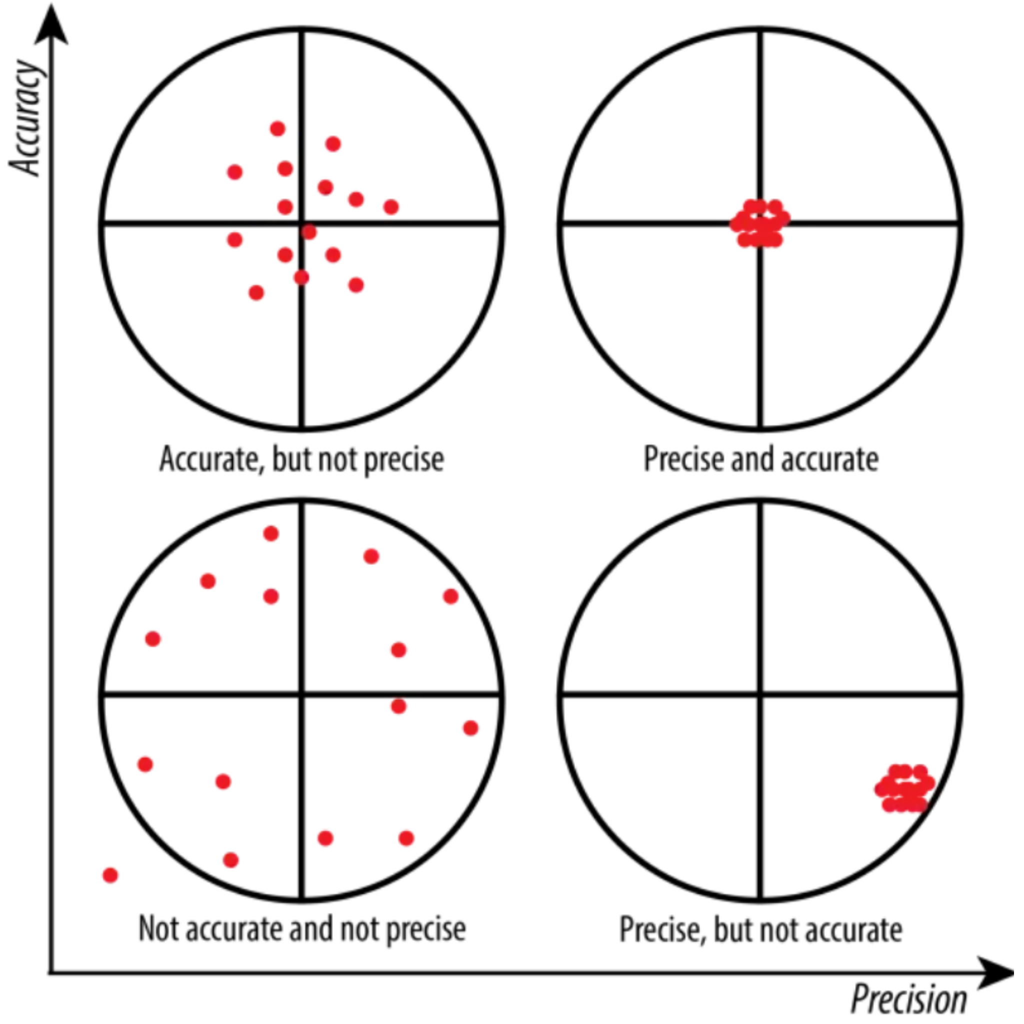
- Why do results differ?
  - Models
  - Differences in priors but also in parametrizations
  - **Different selection of data**

More data $\neq$ Better results
(if model quality varies by observables)

But picking and choosing data is risky,
and comparison with large data sets is desirable

Model selection can help

Ref.: https://wp.stolaf.edu/it/gis-precision-accuracy/



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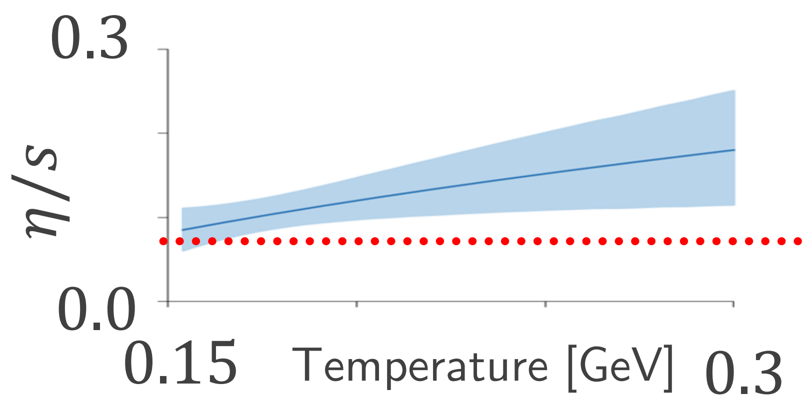

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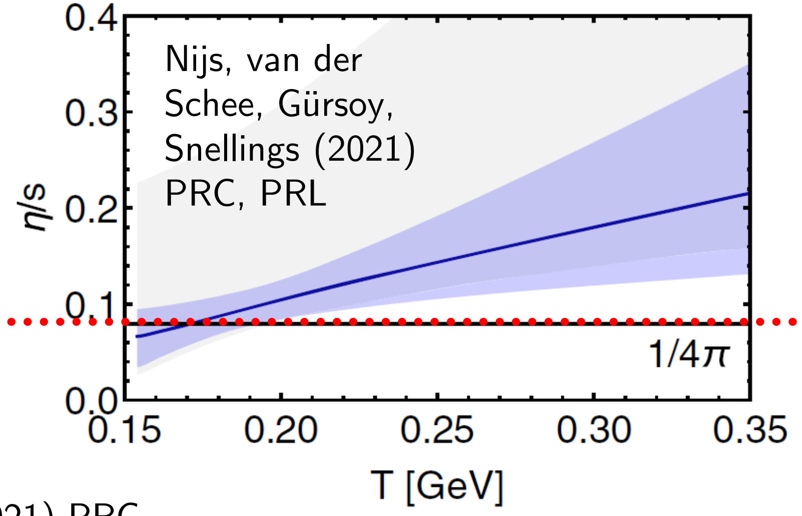

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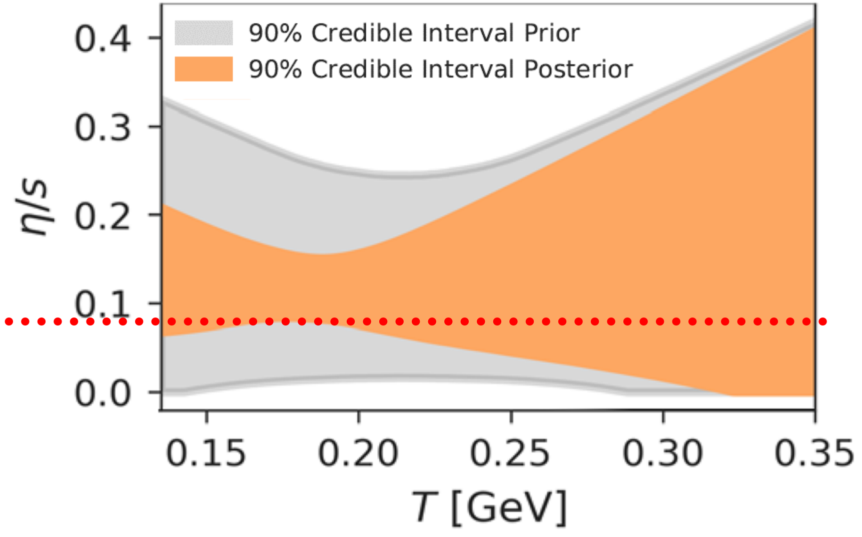
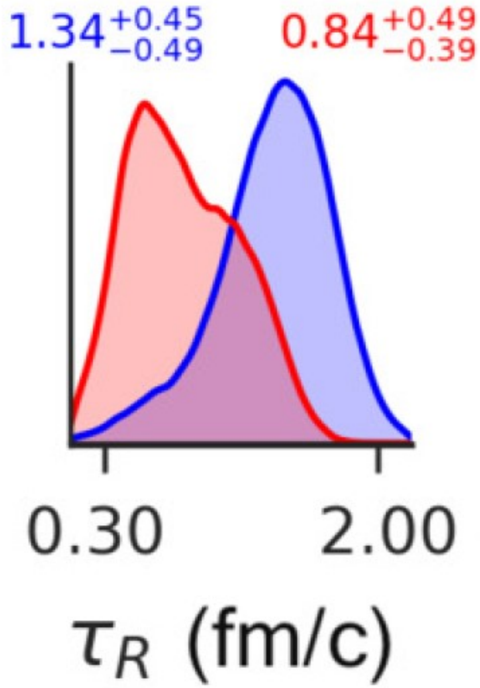
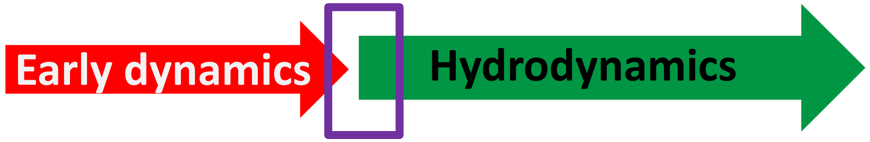


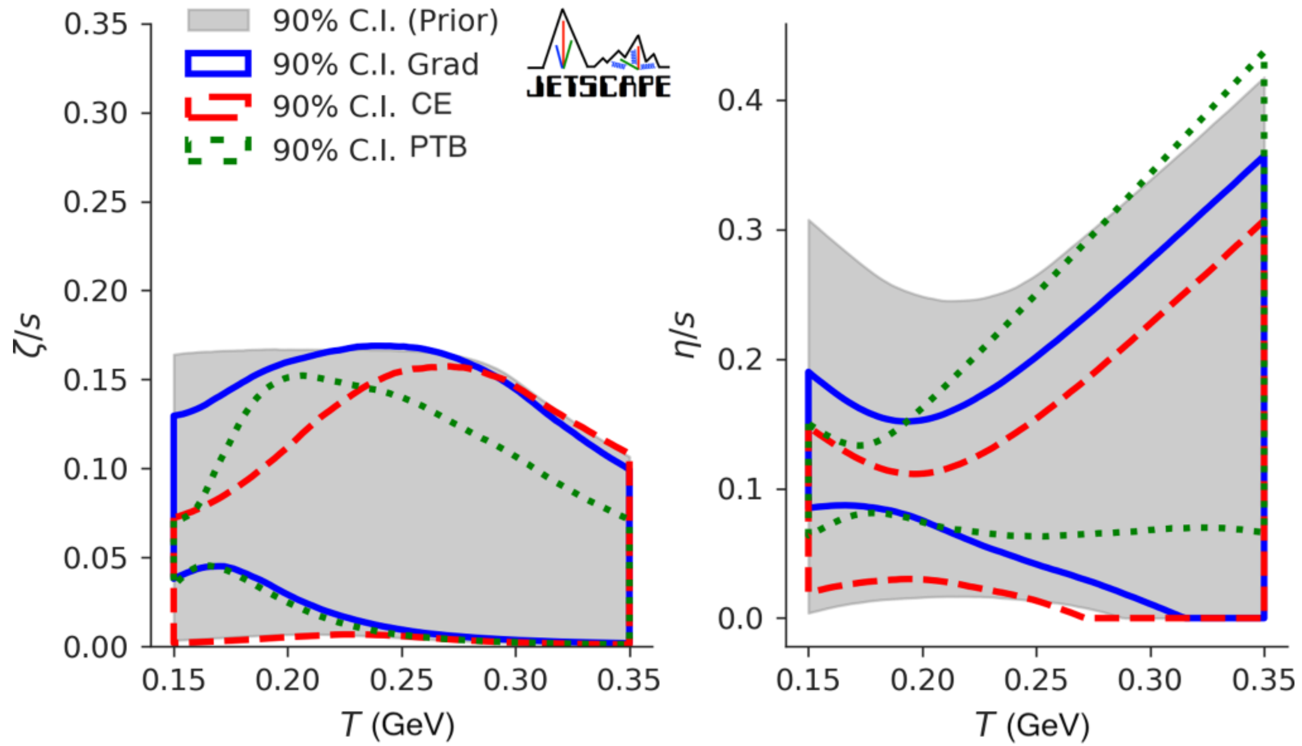
JETSCAPE Collaboration, (2021) PRC, PRL

- Why do results differ?
  - Models: differences in model, but also in uncertainties that are quantified
  - Differences in priors & parametrizations: communicating the results
  - Data selection

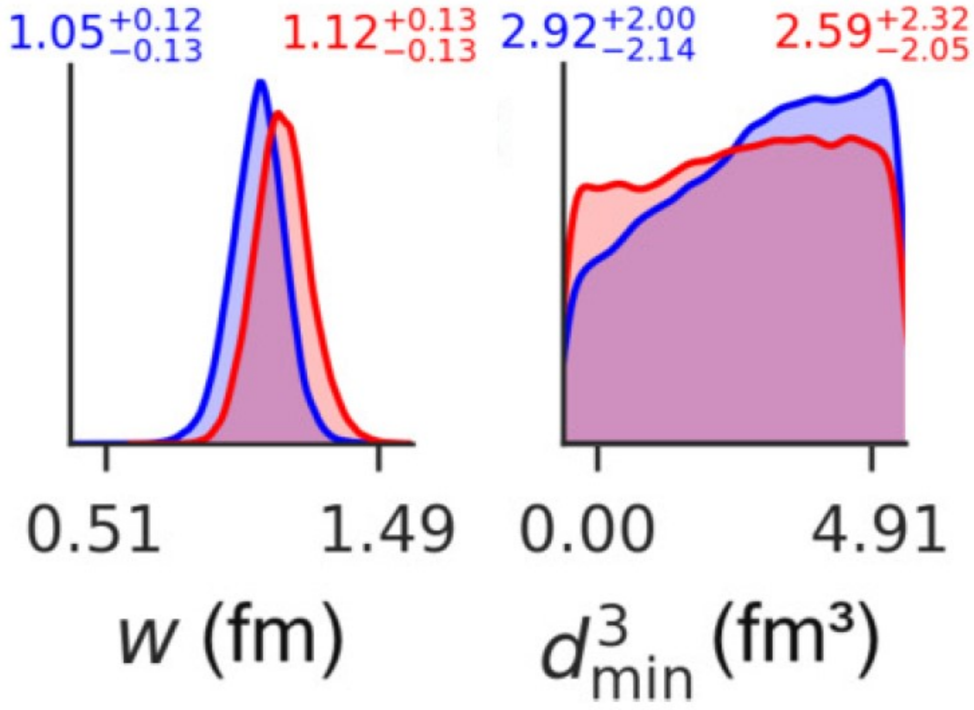
Communicating the results

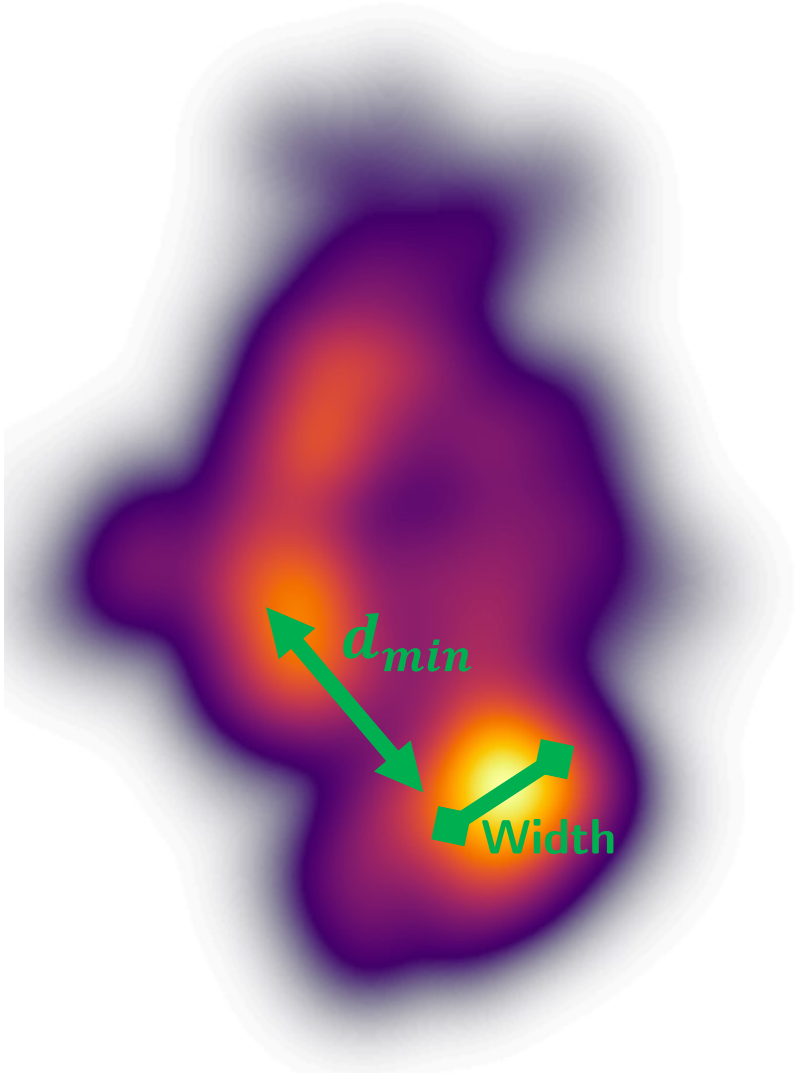


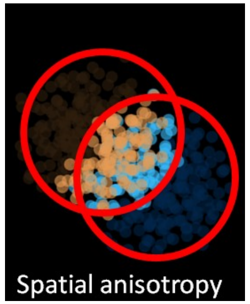


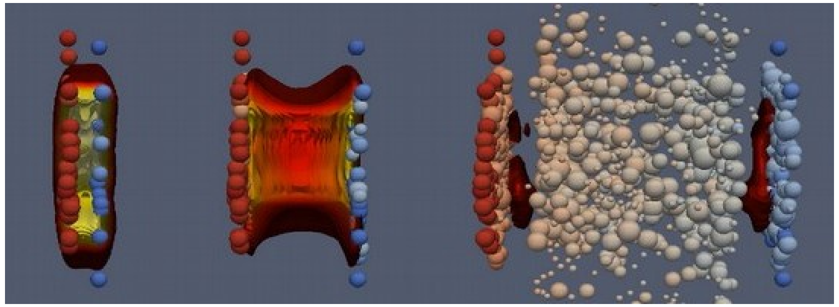


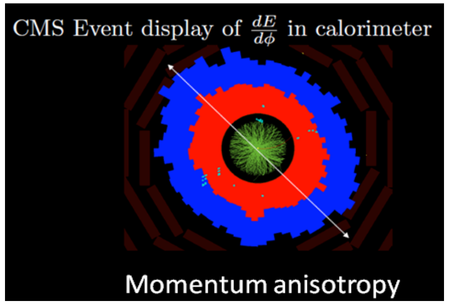
Communicating the results











OPPORTUNITIES AND OUTLOOK

Pros and cons of Bayesian inference

Benefits:

- Systematic and reproducible constraints on model parameters
- Propagation of uncertainties (experimental, theoretical; covariance)
- Scales well to large number of measurements and model parameters
- Model selection/comparison
- Model mixing
- **Experimental design**

Challenges:

- Expensive numerically
- Emulation introduces additional uncertainty (complicating experimental design and interpretation of uncertainties)
- Communicating meaning of uncertainties; "precision vs accuracy"
- Communication meaning of parameters

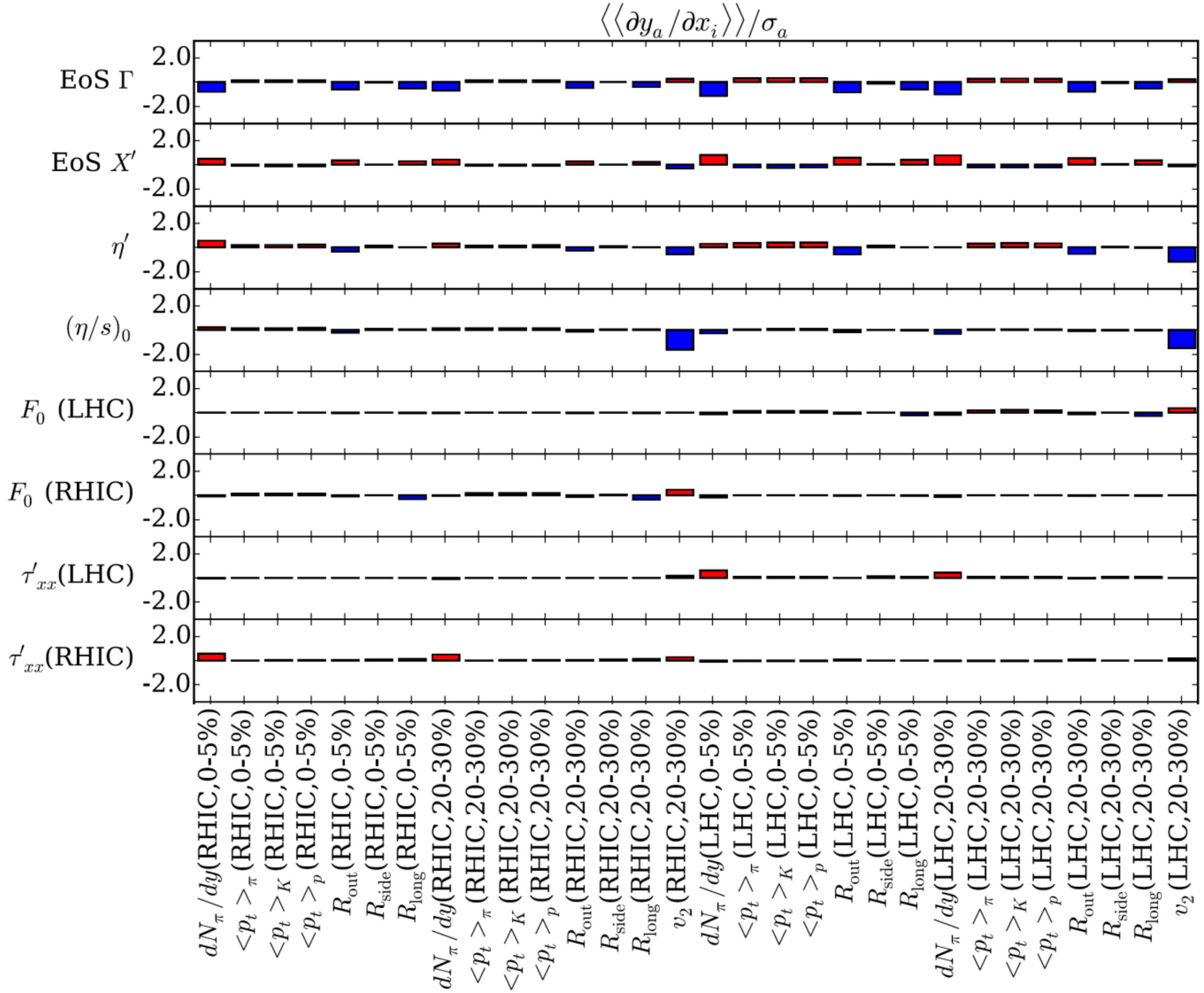
Experimental design

Model responses of an observable with respect to a given parameter

Toward a deeper understanding of how experiments constrain the underlying physics of heavy-ion collisions

Evan Sangaline and Scott Pratt

Phys. Rev. C **93**, 024908 – Published 10 February 2016



Experimental design

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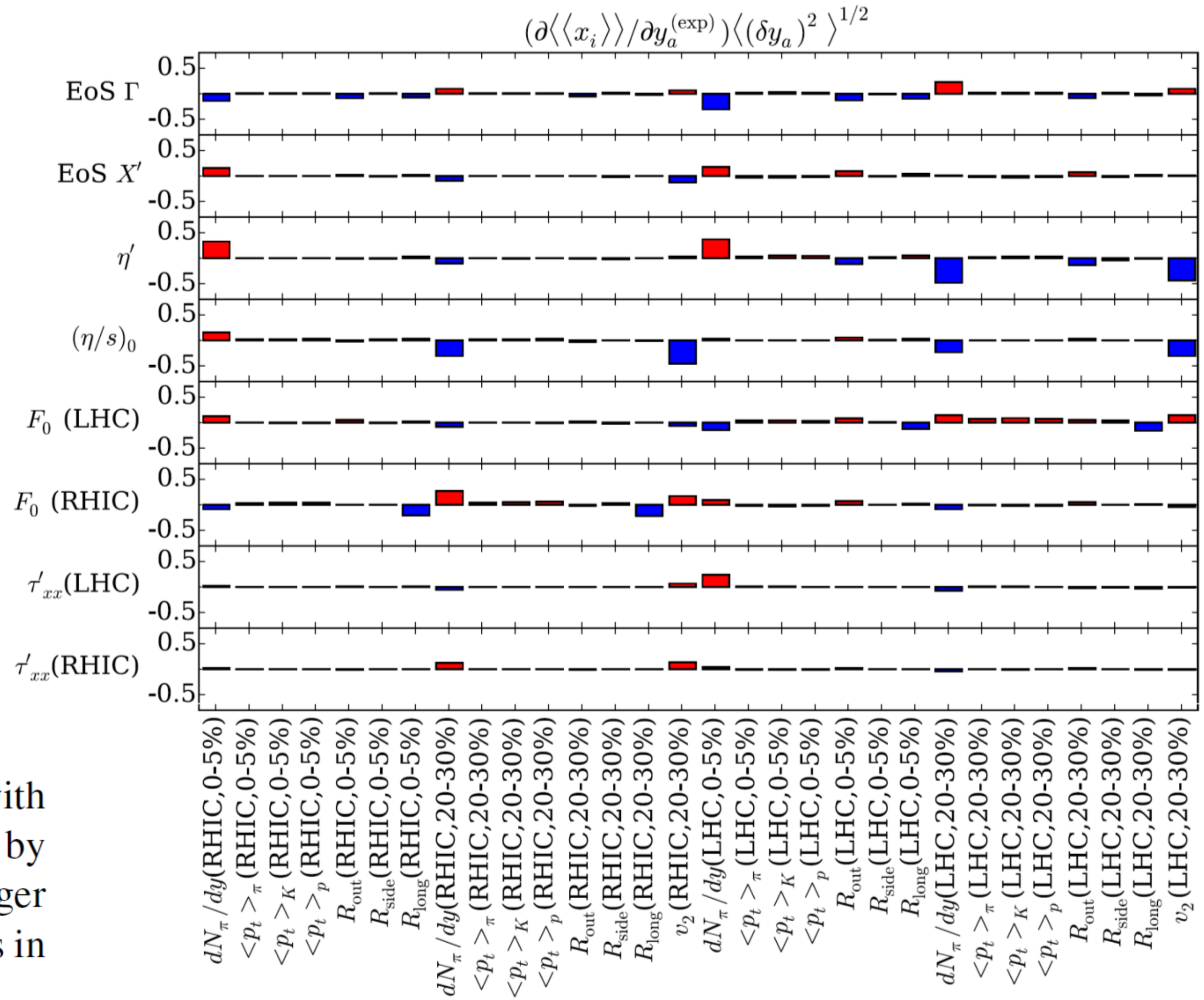

FIG. 5. The change of the inferred value of a parameter with respect to changes in a measurement, $\partial\langle\langle x_i\rangle\rangle/\partial y_a^{(\mathrm{exp})}$, are scaled by the spread of model values throughout the prior, $\langle\delta y_a^2\rangle^{1/2}$. Larger absolute values point to measurements which play important roles in constraining that parameter.

Experimental design

Predictions and postdictions for relativistic lead and oxygen collisions with the computational simulation code TRAJECTUM

Govert Nijs and Wilke van der Schee
Phys. Rev. C **106**, 044903 – Published 12 October 2022

V. BAYESIAN ANALYSIS USING OO SIMULATED DATA

In the previous section, we showed several predictions for the oxygen runs at the LHC and RHIC. While this is very interesting and important because it allows for a good test of the current model, one can answer an additional question. Assuming, as we have been so far, that the soft sector of OO collisions can be described by the same hydrodynamical model as for PbPb, one can wonder whether the addition of OO data can improve the constraints on the parameters such as those obtained from PbPb alone.

Model mixing

- What if a model is better at predicting certain observables than others?
- What if a model works better in certain regions of the parameter space (e.g. at small viscosity)

- Model mixing can take the "best" out of different models

See e.g. document from BAND collaboration for a discussion
https://arxiv.org/pdf/2012.07704.pdf

Reducing numerical cost

- Multiple methods are being investigated:
  - Transfer learning, multifidelity emulation

  Yi Ji et al, arXiv:2209.13748; Liyanage et al (2022) PRC
  - Adaptive sampling
  - Optimizing statistical and interpolation uncertainties

Pros and cons of Bayesian inference

Benefits:

- Systematic and <u>reproducible</u> constraints on model parameters
- Propagation of uncertainties (experimental, theoretical; covariance)
- Scales well to large number of measurements and model parameters
- Model selection/comparison
- Model mixing
- **Experimental design**

Challenges:

- Expensive numerically
- Emulation introduces additional uncertainty (complicating experimental design and interpretation of uncertainties)
- Communicating meaning of uncertainties; "precision vs accuracy"
- Communication meaning of parameters

QUESTIONS?