INT Workshop 20R-77
Origin of the Visible Universe Unraveling the Proton Mass

On the definition of electromagnetic and gravitational local spatial densities for composite spin-1/2 systems

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Based on:
Epelbaum, Gegelia, Lange, Meißner, Polyakov [arXiv:2201.02565, to appear in PRL] (2022)
and
Panteleeva, Epelbaum, Gegelia, Meißner [arXiv:2205.15061, prepared for PRD] (2022)

15 Juni 2022

Outline

- historical aspects of the definition of spatial densities
- electromagnetic current density
- traditional current densities in static approximation
- new definition of the current densities
- IMF densities, interpretation of new current densities
- comparing the new and traditional densities
- gravitational spatial densities for spin-1/2
- features of new gravitational spatial densities and comparing with the densities in IMF
- mass and energy distribution

Motivation

[Hofstadter et. all,
Rev. Mod. Phys. 30, 482 (1958)]

momentum transfer

$$F(Q_L^2) = \int_0^\infty d^3 r \rho(\mathbf{r}) e^{i\vec{Q}_L\cdot\vec{r}}$$

for non-relativistic systems

Breit frame:

[Sachs,
Phys. Rev.126, 2256-2260 (1962)]

$$Q^2 = -q^2$$

$$\rho(r) = \int \frac{d^3Q}{(2\pi)^3} G_E(Q^2) e^{-i\vec{Q}\cdot\vec{r}}$$

[M.V.Polyakov,
Phys. Lett.B 555, 57 (2003)]

$$T_{\mu\nu}(\mathbf{r},\mathbf{s}) = \frac{1}{2E}\int \frac{d^3Q}{(2\pi)^3} e^{i\vec{Q}\cdot\vec{r}} \langle p', s' | \hat{T}_{\mu\nu}(0) | p, s\rangle$$

- **Breit frame assumed delocalised wave packets**
- **charge distribution governed by the size of wave packet rather than the intrinsic size**

[M. Burkardt
Phys. Rev. D 66 (2002), 119903(E),

G. Miller
Phys. Rev. Lett. 99, 112001 (2007)
Phys. Rev. C 79, 055204 (2009)
Ann. Rev. Nucl. Part. Sci. 60 (2010), 1-25
Phys. Rev. C99, no.3, 035202 (2019),

A. Freese and G. Miller
Phys. Rev. D103, 094023 (2021)]

connection? ⟷

$$\rho(r) \equiv \langle \Psi | \hat{\rho}(\mathbf{r},0) | \Psi \rangle$$

- **using a Gaussian wave packet and FF $F(q^2)$**
- **the connection <u>is valid</u> for $\Delta \gg R \gg 1/m$**
- **<u>not valid</u> for light hadrons**
- **dependence on wave packet for $\Delta \sim 1/m$**

[R.L.Jaffe,
Phys. Rev. D103 no.1, 016017 (2021)]

Construction of electromagnetic densities for a spin-1/2 particle

$F_1(0) = 1,\ F_2(0) = \kappa/m$

$q = p' - p$

Matrix element of electromagnetic current operator at t=0:

$$\langle p', s' | \hat{j}^\mu(\mathbf{r},0) | p, s\rangle = e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{r}}\, \bar{u}(p', s') \left[\gamma^\mu F_1(q^2) + \frac{1}{2} i\sigma^{\mu\nu} q_\nu F_2(q^2)\right] u(p, s)$$

Normalisable Heisenberg-picture state with the center of mass position X:

$$|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}} \phi(s, \mathbf{p})\, e^{-i\mathbf{p}\cdot\mathbf{X}} |p, s\rangle$$

Profile function: **spherically symmetric**

$\phi(s, \mathbf{p}) = \phi(\mathbf{p}) = R^{3/2}\tilde{\phi}(R|\mathbf{p}|)$

Size of the packet

$\int d^3p\, |\phi(s, \mathbf{p})|^2 = 1$

$$j^\mu_\phi(\mathbf{r}) = \langle \Phi, \mathbf{X}, s' | \hat{j}^\mu(\mathbf{r},0) | \Phi, \mathbf{X}, s\rangle$$

$\mathbf{X} = \mathbf{0}$

ZAMF - zero average momentum frame, where $\langle \mathbf{p}\rangle = 0$ for $|\Phi, \mathbf{X}, s\rangle$

$\mathbf{P} = (\mathbf{p}' + \mathbf{p})/2,\ \mathbf{q} = \mathbf{p}' - \mathbf{p}$

$$j^\mu_\phi(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}} \bar{u}(p', s') \left[\gamma^\mu F_1((E-E')^2 - \mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2} F_2((E-E')^2 - \mathbf{q}^2)\right] u(p, s) \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$E = \sqrt{m^2 + \mathbf{P}^2 - \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4} \qquad E' = \sqrt{m^2 + \mathbf{P}^2 + \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4}$$

Current densities in static approximation

$$j^\mu_\phi(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}} \bar{u}(p',s')\left[\gamma^\mu F_1((E-E')^2-\mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2} F_2((E-E')^2-\mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

the non-vanishing contribution for $R \to 0$ only in the region where $\mathbf{P} = \tilde{\mathbf{P}}/R$

[R.L. Jaffe, 2021]

taking $m \to \infty$ and after that $R \to 0$ using method of dimensional counting one obtains:

[J. Gegelia, at al. ,Theor. Math. Phys.101, 1313-1319 (1994)]

$$J^0_{\text{static}}(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_E(-\mathbf{q}^2) \equiv \rho^{\text{ch}}_{\text{static}}(r)$$

$G_E(0) = 1$ $\qquad \int d^3p\, |\phi(s,\mathbf{p})|^2 = 1$

$$\mathbf{J}_{\text{static}}(\mathbf{r}) = \frac{\nabla_\mathbf{r} \times \boldsymbol{\sigma}}{2m} \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_M(-\mathbf{q}^2) \equiv \frac{\nabla_\mathbf{r} \times \boldsymbol{\sigma}}{2m} \rho^{mag}_{static}(r)$$

$G_M(0) = 1 + \kappa$

- coincide with Breit Frame expressions
- no dependence on wave packet!
- valid for heavy systems with $\Delta \gg R \gg 1/m$
- this approximation is doubtful for light hadrons, $\Delta \lesssim 1/m$

[R.L. Jaffe, Phys. Rev. D103 no.1, 016017, (2021)]

New definition of the current densities

$$j^\mu_\phi(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}} \bar{u}(p',s')\left[\gamma^\mu F_1((E-E')^2-\mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2} F_2((E-E')^2-\mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

the non-vanishing contribution for $R \to 0$ only in the region where $\mathbf{P} \sim 1/R$

taking $R \to 0$ for arbitrary m, using method of dimensional counting:

$$J^0(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \underbrace{\int_{-1}^{+1} d\alpha \frac{1}{2} F_1\left[(\alpha^2-1)\mathbf{q}^2\right]}_{1 \text{ for } \mathbf{q}=\mathbf{0}} \equiv \rho_1(r)$$

★

$$\mathbf{J}(\mathbf{r}) = \frac{\nabla_\mathbf{r}\times\boldsymbol{\sigma}}{2m} \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \underbrace{\int_{-1}^{+1} d\alpha \frac{1}{4}\left(1+\alpha^2\right) m F_2\left[(\alpha^2-1)\mathbf{q}^2\right]}_{2/3\kappa \text{ for } \mathbf{q}=\mathbf{0}} \equiv \frac{\nabla_\mathbf{r}\times\boldsymbol{\sigma}}{2m}\rho_2(r)$$

New definition of the current densities

these densities can be rewritten as: $J^\mu(\mathbf{r}) = \frac{1}{4\pi}\int d\hat{\mathbf{n}}\, J^\mu_{\hat{\mathbf{n}}}(\mathbf{r})$

$$\hat{\mathbf{n}} = \frac{\mathbf{P}}{|\mathbf{P}|}$$

$$J^0_{\hat{\mathbf{n}}}(\mathbf{r}) = \rho_{1,\hat{\mathbf{n}}}(\mathbf{r}) \qquad \mathbf{J}_{\hat{\mathbf{n}}}(\mathbf{r}) = \frac{1}{2m}\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}_\perp\, \rho_{2,\hat{\mathbf{n}}}(\mathbf{r})$$

$$\rho_{i,\hat{\mathbf{n}}}(\mathbf{r}) = \rho_i(r_\perp)\,\delta(r_\parallel)$$

$$\rho_1(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2}\, e^{-i\mathbf{q}_\perp\cdot\mathbf{r}_\perp}\, F_1\left(-\mathbf{q}_\perp^2\right), \qquad \rho_2(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2}\, e^{-i\mathbf{q}_\perp\cdot\mathbf{r}_\perp}\, mF_2\left(-\mathbf{q}_\perp^2\right)$$

- no dependence on wave packet
- **But! Also no dependence on Compton wavelength** $1/m$
  - -> applicable also for light hadrons
  - -> $J^\mu_{\text{static}}(\mathbf{r})$ doesn't emerge from $J^\mu(\mathbf{r})$ by $m \to \infty$
  - -> non-commutativity $R \to 0$ and $m \to \infty$

  [Epelbaum et al. (2022)]

[G.N.Fleming, Physical Reality Math. Descrip., 357 (1974)]

IMF densities

boosted state

$$U(\Lambda_{\mathbf{v}})\,|\,p,s\rangle = \sum_{s_1} D_{s_1 s}\left[W\left(\Lambda_{\mathbf{v}},\frac{\mathbf{p}}{m}\right)\right]|\,\Lambda_{\mathbf{v}}p,s_1\rangle$$

Wigner rotation

boosted packet

$$|\,\Phi,\mathbf{0},s\rangle_{\mathbf{v}} = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\sqrt{\gamma\left(1-\frac{\mathbf{v}\cdot\mathbf{p}}{E}\right)}\,\phi\left(\Lambda_{\mathbf{v}}^{-1}\mathbf{p}\right)\sum_{s_1} D_{s_1 s}\left[W\left(\Lambda_{\mathbf{v}},\frac{\Lambda_{\mathbf{v}}^{-1}\mathbf{p}}{m}\right)\right]|\,p,s_1\rangle$$

boost from ZAMF to moving frame

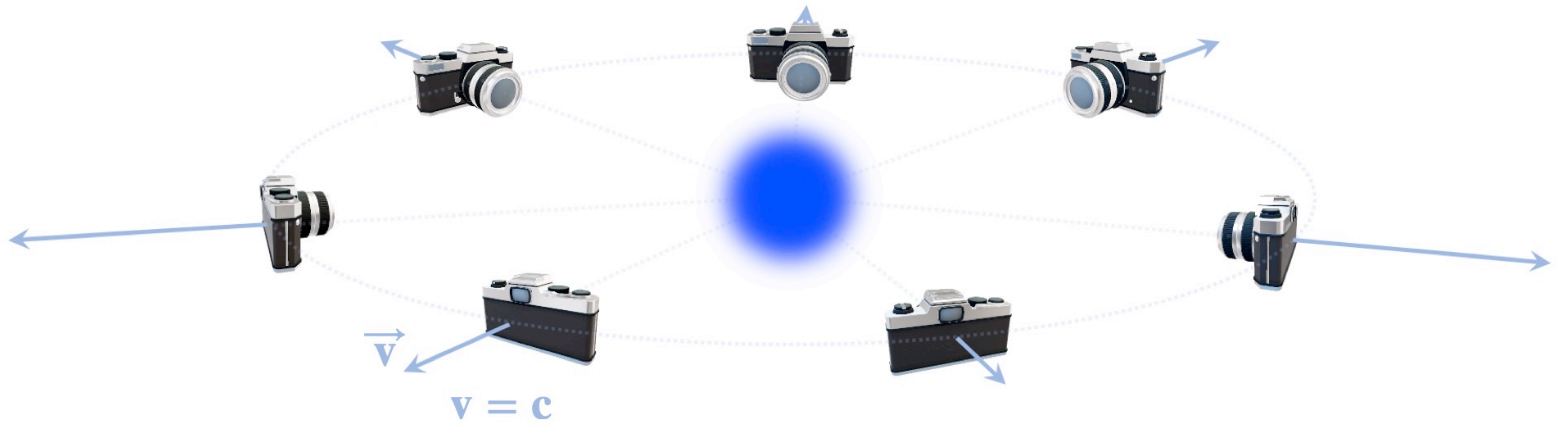
$$j^{\mu}_{\phi,v}(\mathbf{r}) = {}_{\mathbf{v}}\langle\Phi,\mathbf{0},s'\,|\,\hat{j}^{\mu}(\mathbf{r},0)\,|\,\Phi,\mathbf{0},s\rangle_{\mathbf{v}}$$

IMF with $v \to 1,\ \gamma \to \infty$

due to Wigner rotation

$$J^0_{ZAMF}(\mathbf{r}) = \frac{1}{4\pi}\int d\hat{\mathbf{v}}\,J^0_{IMF}(\mathbf{r})\,, \qquad \mathbf{J}_{ZAMF}(\mathbf{r}) = 2\times\frac{1}{4\pi}\int d\hat{\mathbf{v}}\,\mathbf{J}_{IMF}(\mathbf{r})\,.$$

- holographic relationship between ZAMF and IMF
- valid for any systems independently on the Compton wavelength
- described only by intrinsic properties of system

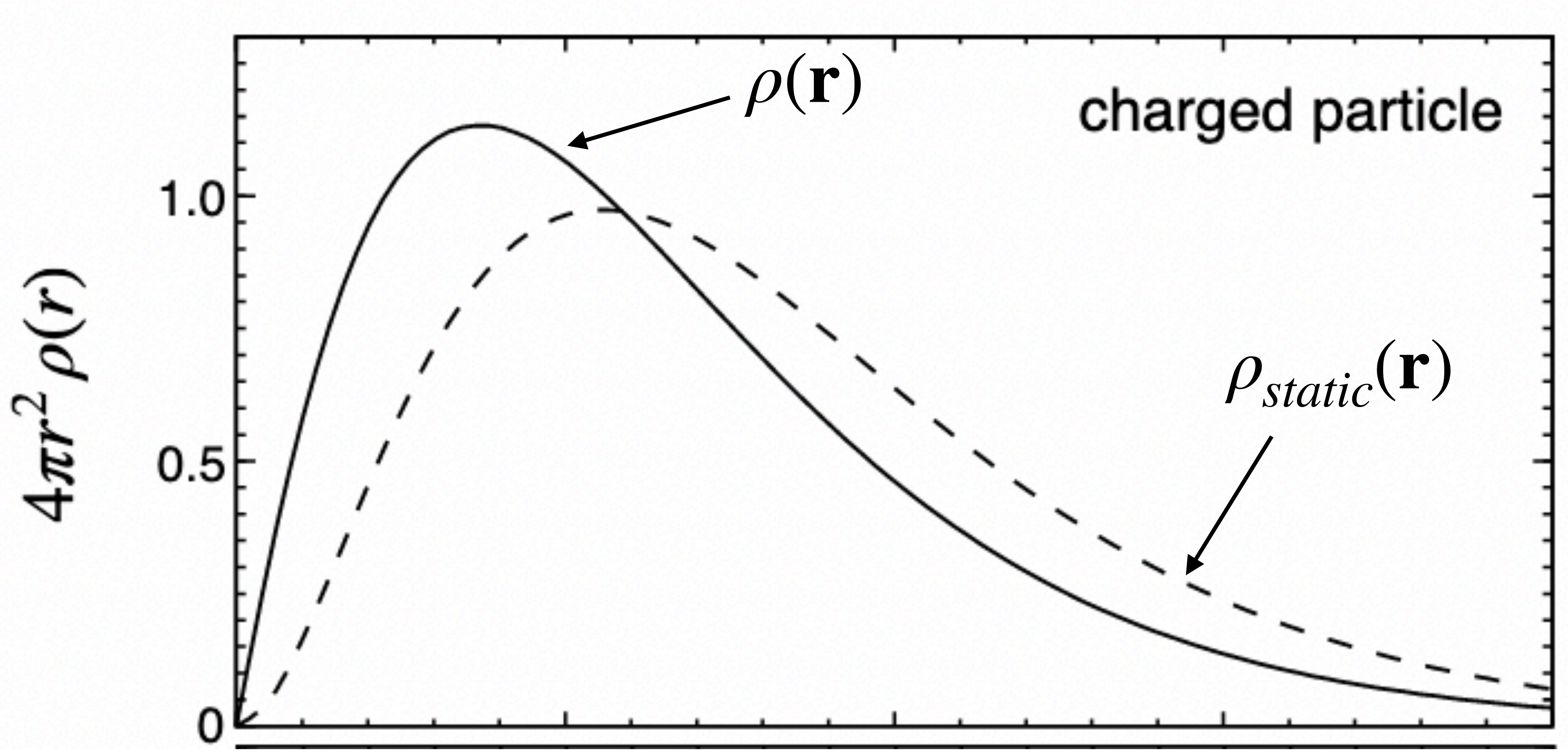


Comparing traditional and new electric charge densities

$$\rho_{static}(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} F_1[\mathbf{q}^2]$$

$$\rho(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{+1} d\alpha \frac{1}{2} F_1\left[(\alpha^2 - 1)\mathbf{q}^2\right]$$

$$F_1(q^2) = (1 - q^2/\Lambda^2)^{-2} \quad \text{with } \Lambda^2 = 0.71 \text{ GeV}$$



[Epelbaum at al. [arXiv:2201.02565] (2022)]

radius related to charge density, defined via sharp localised packet is smaller than in Breit Frame due to the squeezing

$$\sqrt{\langle r^2\rangle_{\text{static}}} = \sqrt{6\left(F_1'(0) + \frac{F_2(0)}{4m}\right)} \simeq 0.8409(4)\,, \quad \sqrt{\langle r^2\rangle} = \sqrt{4F_1'(0)} \simeq 0.62649,$$

$\Delta \gg R \gg 1/m$ $\qquad$ $R \to 0$

New gravitational spatial densities for spin-1/2

[Panteleeva, Epelbaum, Gegelia, Meißner, *'in preparation'*]

$$|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\, \phi(s, \mathbf{p})\, e^{-i\mathbf{p}\cdot\mathbf{X}}\, |p, s\rangle$$

superposition of one-particle states

- an eigenstate of $\hat{Q}$
- not an eigenstate of $\hat{p}^\mu$
- $\langle \Phi, \mathbf{X}, s' | \hat{T}^{\mu\nu}(\mathbf{r},0) | \Phi, \mathbf{X}, s\rangle$ has contributions only of one-particle states

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$$\langle p', s' | \hat{T}_{\mu\nu}(\mathbf{r},0) | p, s\rangle = e^{i\mathbf{q}\cdot\mathbf{r}}\, \bar{u}(p', s') \left[A(q^2)\frac{P_\mu P_\nu}{m} + iJ(q^2)\frac{P_\mu \sigma_{\nu\alpha} q^\alpha + P_\nu \sigma_{\mu\alpha} q^\alpha}{2m} + D(q^2)\frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \right] u(p, s)\,,$$

$$t_\phi^{\mu\nu}(\mathbf{r}) \equiv \langle \Phi, \mathbf{0}, s' | \hat{T}^{\mu\nu}(\mathbf{r},0) | \Phi, \mathbf{0}, s\rangle$$

$$t_\phi^{\mu\nu}(\mathbf{r}) = \int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\, \bar{u}(p', s') \left[A\left((E-E')^2 - \mathbf{q}^2\right) \frac{P_\mu P_\nu}{m} + iJ\left((E-E')^2 - \mathbf{q}^2\right) \frac{P_\mu \sigma_{\nu\alpha} q^\alpha + P_\nu \sigma_{\mu\alpha} q^\alpha}{2m} + D\left((E-E')^2 - \mathbf{q}^2\right) \frac{q^\mu q^\nu - \eta^{\mu\nu} q^2}{4m} \right]$$

$$\times\, u(p, s)\, \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

New gravitational spatial densities for spin-1/2

taking $R \to 0$ for arbitrary m, using method of dimensional counting one obtains:

$$t_\phi^{\mu\nu}(\mathbf{r}) = N_{\phi,\infty} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} \left[\hat{n}^\mu \hat{n}^\nu A\left(-\mathbf{q}_\perp^2\right) + \frac{iJ\left(-\mathbf{q}_\perp^2\right)}{2m} \left(\hat{n}^\mu (\boldsymbol{\sigma}_\perp \times \mathbf{q}_\perp)^\nu + \hat{n}^\nu (\boldsymbol{\sigma}_\perp \times \mathbf{q}_\perp)^\mu + \hat{\mathbf{n}} \cdot (\boldsymbol{\sigma}_\perp \times \mathbf{q}_\perp)(\delta^{\mu 0}\hat{n}^\nu + \delta^{\nu 0}\hat{n}^\mu) \right) \right] e^{-i\mathbf{q}\cdot\mathbf{r}} +$$

$$+ \frac{1}{2} N_{\phi,0} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} \left(\tilde{q}^\mu \tilde{q}^\nu + g^{\mu\nu}\mathbf{q}_\perp^2 \right) D\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$N_{\phi,\infty} = \frac{1}{R} \int d\tilde{P} \tilde{P}^3 |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$

$$N_{\phi,0} = \frac{R}{2} \int d\tilde{P} \tilde{P} |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$

$$(\boldsymbol{\sigma}_\perp \times \mathbf{q}_\perp)^0 = 0$$

$$\tilde{q}^\mu = (q_\parallel, \mathbf{q})$$

$$\hat{\mathbf{n}} = \frac{\tilde{\mathbf{P}}}{|\tilde{P}|}$$

- keeping the leading order terms for each form factor separately!
- the reason: from the comparison with moving frame follows that EMT has two separate contributions
- only overall normalisation of densities depends on the wave packet

Mass and energy distribution

$$t_\phi^{00}(\mathbf{r}) \equiv \langle \Phi, \mathbf{0}, s' | \hat{T}^{00}(\mathbf{r},0) | \Phi, \mathbf{0}, s \rangle$$

For sharply localised packet $R \to 0$ and arbitrary m

$$t_\phi^{00}(\mathbf{r}) = N_{\phi,\infty} \int d\hat{\mathbf{n}} \tilde{A}_{\hat{\mathbf{n}}}(\mathbf{r})$$

Energy distribution

$$\tilde{A}_{\hat{\mathbf{n}}}(\mathbf{r}) = \delta(r_{\parallel})\tilde{A}(r_\perp) \qquad \hat{\mathbf{n}} = \frac{\mathbf{P}}{|\mathbf{P}|}$$

$$\tilde{A}(r_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_\perp} A(-\mathbf{q}_\perp^2)$$

$$N_{\phi,\infty} = \frac{1}{R} \int d\tilde{P} \tilde{P}^3 |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$

Static approximation $(m \to \infty, R \to 0)$: $\quad R \gg 1/m$

$$t_{static}^{00}(\mathbf{r}) = m \int \frac{d^3 q}{(2\pi)^3} A(-\mathbf{q}^2) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

Mass distribution

$m \to \infty \quad R \gg 1/m,$
$\mathbf{P} \sim 1/R \ll m$

$$E = \sqrt{m^2 + \mathbf{P}^2} \simeq m + O(\mathbf{P}^2/(2m))$$

$$\hat{T}^{00} | \Phi \rangle \simeq m | \Phi \rangle$$

Why $t_\phi^{00} \sim N_{\phi,\infty} \sim \infty$?

- the non-vanishing contribution to $\langle \Phi, \mathbf{0}, s' | \hat{T}^{00}(\mathbf{r},0) | \Phi, \mathbf{0}, s \rangle$ for $R \to 0$ is given by $\mathbf{P} \sim 1/R$
- the energy $E = \sqrt{m^2 + \mathbf{P}^2} \sim \dfrac{1}{R}$

Gravitational spatial densities in IMF

in moving frame:

$$t^{\mu\nu}_{\mathbf{v}}(\mathbf{r}) = t^{\mu\nu}(\mathbf{r}) + t^{\mu\nu}_{2}(\mathbf{r})$$

flow tensor   stress tensor

$$t^{00}_{ZAMF}(\mathbf{r}) = \frac{1}{4\pi\gamma}\int d^2\hat{\mathbf{v}}\; t^{00}_{IMF}(\mathbf{r})$$

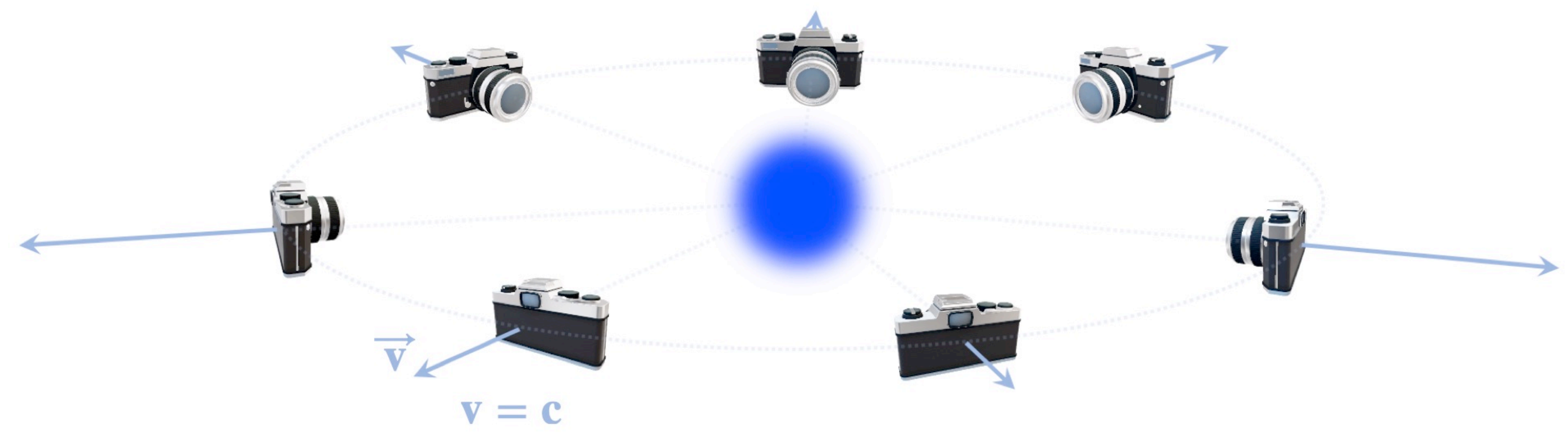
due to Wigner rotation

$$t^{0i}_{ZAMF}(\mathbf{r}) = \frac{2}{4\pi\gamma}\int d^2\hat{\mathbf{v}}\; t^{0i}_{IMF}(\mathbf{r})$$

$$t^{ij}_{ZAMF}(\mathbf{r}) = \frac{\gamma N_\infty}{2\pi}\int d^2\hat{\mathbf{v}}\; t^{ij}_{IMF}(\mathbf{r})$$

- two contributions which characterise the movement of the system as a whole ($A(-\mathbf{q}_\perp^2)$ and $J(\mathbf{q}_\perp^2)$, flow tensor) and contributions corresponding to internal structure ($D(-\mathbf{q}_\perp^2)$, pure stress tensor) 〔A. Freese, G. Miller “2021”〕

- holographic interpretation of densities in ZAMF in terms of densities in IMF

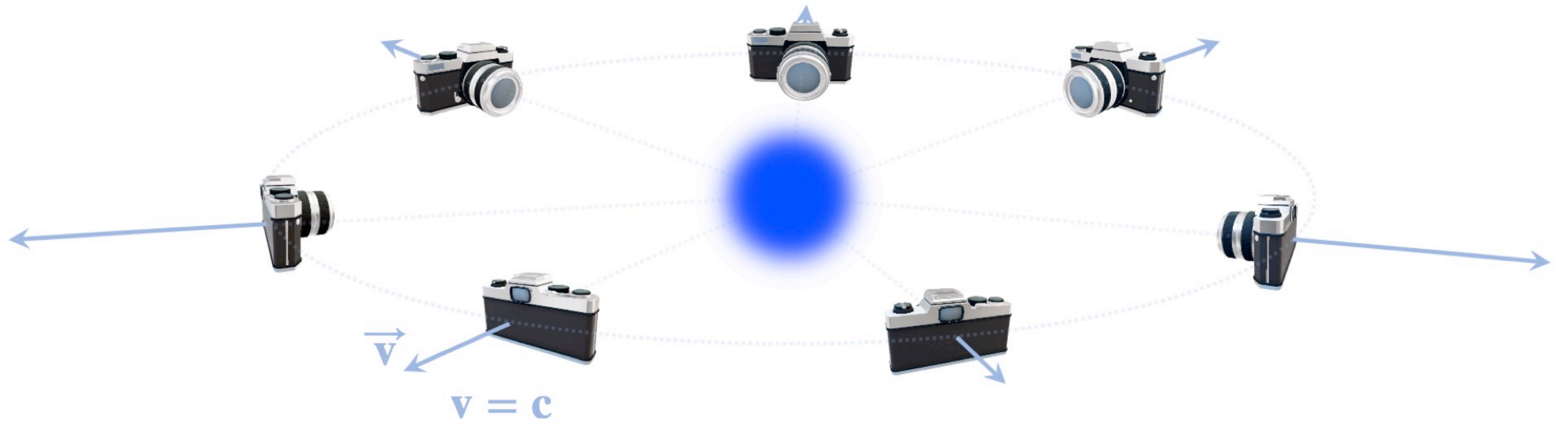


[Panteleeva, Epelbaum, Gegelia, Meißner, *‘in preparation’*]

Conclusion

- New definition of electromagnetic spatial densities in ZAMF using spherically symmetric wave packet
  -> independent on wave packet
  -> applied to any system independent in the relation between Compton wavelength and other length scales
  -> static distributions can not be obtained as a systematic approximation to our alternative distributions

- Generalisation on gravitational density distributions
  -> applicable to any system
  -> dependence on the wave packet as a normalisation factor



- Holographic interpretation of the densities in ZAMF in terms of the IMF densities

- Particular way to define local densities. Other ways: phase-space approach, light-cone⋯