Constraining the low-density neutron star equation of state from heavy-ion collisions

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Acknowledgments:
Overview

- Theoretical model: Sub-saturation EoS with light clusters at CCSN/HIC conditions
- Experimental data analysis
- Fit of exp data to theory
Why are these clusters important?

- They influence supernova properties: the clusters can modify the neutrino transport, affecting the cooling of the proto-neutron star and/or binary and accreting systems.
Many EoS models in literature, like e.g. phenomenological models, whose parameters are fitted to nuclei properties, such as RMF, or Skyrme.

Solution: Need Constraints (Experiments, Observations, Microscopic calculations)
**EoS Constraints**

- **VEoS**: only depends on exp. B and scattering phase shifts. Correct zero-density limit for finite T EoS.

- **Kc from HIC**: cluster formation observed in HIC.

- **PREXII** (Adhikari et al, PRL 126, 172502 (2021): Rn-Rp(208Pb)=0.283 ± 0.071 fm.
  - Reed et al, PRL 126, 172503: L = 106 ± 37 MeV;
  - Yue et al, PRR 4, L022054: L = 85.5 ± 22.2 MeV;
  - Essick et al, PRL 127, 192701: L = 53^{+14}_{-15} MeV

  - Reinhard et al, PRL 127, 232501 (2021), get r(skin)=0.19±0.02 fm, with L = 54 ± 8 MeV.

- **Spectra of charged pions** (Estee et al, PRL 126, 162701 (2021)): 42<L<117 MeV.

- **CREX** (Adhikari et al, PRL 129, 042501 (2022): Rn-Rp(48Ca)=0.121 ± 0.026 ±0.024: seems to indicate the L could be smaller...
EoS Constraints

- Talks along the week!

A few examples:

- NASA’s Neutron star Interior Composition Explorer (NICER) X-ray telescope:
  - PSR J0030+0451:
    - Riley et al., ApJL 887, L21 (2019): $M = 1.34^{+0.15}_{-0.16} \ M_\odot$; $R = 12.71^{+1.14}_{-1.19} \ km$
    - Miller et al., ApJL 887, L24 (2019): $M = 1.44^{+0.15}_{-0.14} \ M_\odot$; $R = 13.02^{+1.24}_{-1.06} \ km$
  - PSR J0740+6620:
    - Riley et al., ApJL 918, L27 (2021): $M = 2.072^{+0.067}_{-0.066} \ M_\odot$; $R = 12.39^{+1.30}_{-0.98} \ km$
    - Miller et al., ApJL 918, L28 (2021): $M = 2.08 \pm 0.07 \ M_\odot$; $R = 13.7^{+2.6}_{-1.5} \ km$

Astrophysical Observations:


Others followed:

- GW190814 (Abbott et al, ApJL 896, L44 (2020): BH+2.5–2.6M\odot object (not ruled out yet to be NS)...
EoS Constraints

In a near future:

• **ATHENA**, an X-ray high-precision determination observatory for NS mass and radius to be launched in 2028.

• New data on NS systems will heavily increase when **SKA**, the world’s largest radio telescope, will be in full power.

• The radio telescope **FAST** has started operating, and will give information on the NS mass.

• The Einstein Telescope (**ET**), an underground infrastructure to host a 3G gravitational-wave observatory, foresees the beginning of construction in 2026 with the goal to start observations in 2035...

• ...

• On the experimental side, **FAIR** will put more constraints on the high-density behaviour of nuclear matter.

• Results of **INDRA-FAZIA** experiment.

• ...
Supernova EoS with light clusters

• The SN EoS should incorporate: all relevant clusters, (mean-field) interaction between nucleons and clusters, and a suppression mechanism of clusters at high densities.

• Different methods: nuclear statistical equilibrium, quantum statistical approach, and

• RMF approach: clusters as new degrees of freedom, with effective mass dependent on density.
• In-medium effects: cluster interaction with medium described via the meson couplings, or effective mass shifts, or both
• Constrains are needed to fix the couplings:
  low densities: Virial EoS
  high densities: cluster formation has been measured in HIC
In-medium effects

- Binding energy of each cluster: \( B_j = A_j m^* - M_j^* \), \( j = d, t, h, \alpha \)

with \( m^* = m - g_s \phi_0 \) the nucleon effective mass and

\[
M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j)
\]

the cluster effective mass.

\( g_{sj} = x_{sj} A_j g_s \)

the scalar cluster-meson coupling

binding energy shift
• The Binding energy of each cluster then becomes:

\[ B_j = A_j g_s \phi_0 (x_{sj} - 1) + B_j^0 + \delta B_j. \]

• \( x_{sj} \) can vary from 0 to 1 so for the two extreme cases, we have:

\[
B_j = B_j^0 + \delta B_j, \text{ if } x_{sj} = 1,
\]
\[
B_j = B_j^0 + \delta B_j - A_j g_s \phi_0, \text{ if } x_{sj} = 0.\]

• This implies that a larger \( x_{sj} \) corresponds to a larger \( B_j \), and that the cluster dissolution density will occur at larger densities.

\( x_{sj} \) needs to be determined from exp. constraints
In-medium effects - \( \delta B_j \)

**binding energy shift** → Responsible for dissolution of clusters

\[
\delta B_j = \frac{Z_j}{\rho_0} (\epsilon_p^* - m \rho_p^*) + \frac{N_j}{\rho_0} (\epsilon_n^* - m \rho_n^*)
\]

energetic counterpart of classical ExV mechanism

\[
\epsilon_j^* = \frac{1}{\pi^2} \int_{p_{F_j}^{(\text{gas})}}^{} p^2 e_j(p) (f_{j+}(p) + f_{j-}(p)) dp
\]

\[
\rho_j^* = \frac{1}{\pi^2} \int_{p_{F_j}^{(\text{gas})}}^{} p^2 (f_{j+}(p) + f_{j-}(p)) dp
\]

Lowest energy levels of the gas energy density and density

the energy states occupied by the gas are excluded in the calculation of \( B_j \):

double counting avoided!
Contribution of $\delta B_j$

- $\delta B_j$ completely negligible in the low-density range
- but rises fast for larger densities

However, this does not give a complete picture of the in-medium effects and cluster dissolution mechanism: the particle fractions are affected in a complex way due to the self-consistency of the approach, since the equations of motion for the meson fields are modified by $\delta B_j$. 
Cluster fractions - effect of $\delta B_j$

- Note that at finite $T$, the clusters dissolve at a $\rho$ well above the one for which $B_j \sim 0$.

$\delta B_j$, important for dissolution of clusters!
Exp Constraint: Equilibrium constants

• In Qin et al., PRL 108, 172701 (2012), Kc were calculated with data from HIC:

\[ K_c[j] = \frac{\rho_j}{N_j} \frac{Z_j}{\rho_n \rho_p} \]

• At the time, unique existing constraint on in-medium modifications of light clusters at finite T.

• This analysis was performed using ideal gas considerations.
Exp Constraint: Equilibrium constants

- Yellow bands: exp data from Qin et al
- Red points: RMF model calculated at (T, rho, yp) of exp data with $x_s = 0.85 \pm 0.05$

- $x_s$ first fitted to the Virial EoS, model-ind constraint, only depends on exp B and scattering phase shifts. Provides correct zero-density limit for finite-T EoS.

- Our theoretical model describes quite well experimental data, except for deuteron
Experimental chemical equilibrium constants with INDRA data

- Experimental data includes 4He, 3He, 3H, 2H, and 6He.
- 3 experimental systems: 136Xe+124Sn, 124Xe+124Sn, and 124Xe+112Sn.

\[ \text{V}_{\text{surf}} \text{ is the velocity of the emitted particles at the nuclear surface, so fastest particles correspond to earliest emission times.} \]

\[ \text{The temperature, proton fraction and density as a function of V}_{\text{surf}}, \text{ for the intermediate mass system.} \]
Experimental determination of chemical equilibrium constants

- Weak point: T and density are NOT directly measured, but deduced from experimental multiplicities, using analytical expressions that assume the physics of an ideal gas...

- Since we are in thermodynamical equilibrium, the free volume occupied by the clusters should be the same!

- But they aren't....

- This is not surprising because we are using an expression for the volume where we consider an ideal gas of classical clusters..
Considering in-medium effects

- How to solve this problem?

- We should take into account the interactions between clusters:

- We introduce a correction factor that modifies the binding energies of the clusters:

\[ B^{\text{tot}}_{AZ} = B_{AZ} - \Delta_{AZ}, \quad \Delta_{AZ} = a_1 A^{a_2} + a_3 |I|^{a_4} \]
Considering in-medium effects

- a1, a2, a3, and a4 parameters are random variables that need to be determined.
- How to do that? Bayesian analysis.

- They are going to be calculated such that the volumes of the clusters,

\[ V_f^{(AZ)} = h^3 R_{np}^{\frac{A-Z}{A-1}} \exp \left[ \frac{B_{AZ} - \Delta_{AZ}}{T(A-1)} \right] \cdot \left( \frac{2J_{AZ} + 1}{2^A} \tilde{Y}_{11}^A (\tilde{p}) \right)^{\frac{1}{A-1}} \]

are the same, so that the thermodynamical conditions are fulfilled.

- The posterior distribution is obtained by imposing the volume observation with a likelihood probability:

\[ P_{post}(\tilde{a}) = \mathcal{N} \exp \left( -\sum_{AZ} (V_f^{(AZ)}(\tilde{a}) - \bar{V}_f(\tilde{a}))^2 \right) \frac{1}{2\bar{V}_f(\tilde{a})^2} \]

- To minimize assumptions, we take flat priors

\[ P_{prior}(\tilde{a}) = \theta(\tilde{a}_{\text{min}} - \tilde{a}_{\text{max}}) \]
Experimental chemical equilibrium constants with INDRA data

- The points show the posterior expectation values for the volumes:

- When we apply the correction, the volumes converge.
We obtain densities larger than the ideal gas limit.
The 3 data systems are compatible.
Equilibrium constants and data from INDRA

- This work shows that there are in-medium effects:
- We obtain a higher $x_s$ as compared to a previous fit of Qin et al data:
- The higher the $x_s$, the bigger the binding energies (and the smaller effect of the medium), and the higher the dissolution densities of the clusters.
Later, we used other RMF models to fit to the same INDRA data. The values of $x_s$ found for FSU2R and DDME2 were very close to the previous one found using the FSU model.
Mass fractions and dissolution density

- Looking at the mass fractions, we find that different models predict similar abundances.
- $\text{rho}_{\text{diss}}(\text{FSU}2R) = [0.0758:0.0857] \text{ fm}^{-3}$
- $\text{rho}_{\text{diss}}(\text{DDME2}) = [0.0741:0.0851] \text{ fm}^{-3}$

In a near future:
- New data is expected where similar analysis can be implemented.
- PhD students Tiago Custódio and Alex Rebillard-Soulié working on the subject from the theoretical (EoS) and experimental (data analysis) points of view, respectively.
Conclusions

- Our model reproduces both virial limit and Kc from HIC data.
- INDRA data was analysed based on a new method, with in-medium effects.
- By fitting to a theoretical RMF model, a larger scalar coupling than the one found in a previous study, NOT including in-medium effects in the data analysis, was found.
- This implies bigger binding energies => larger melting densities => MORE clusters in CCSN matter!!

- Clusters are relevant and should be explicitly included in EoS for CCSN simulations and NS mergers.

Thank you!