

The quest for the structure of resonant hadrons

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William & Mary

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In collaboration with:

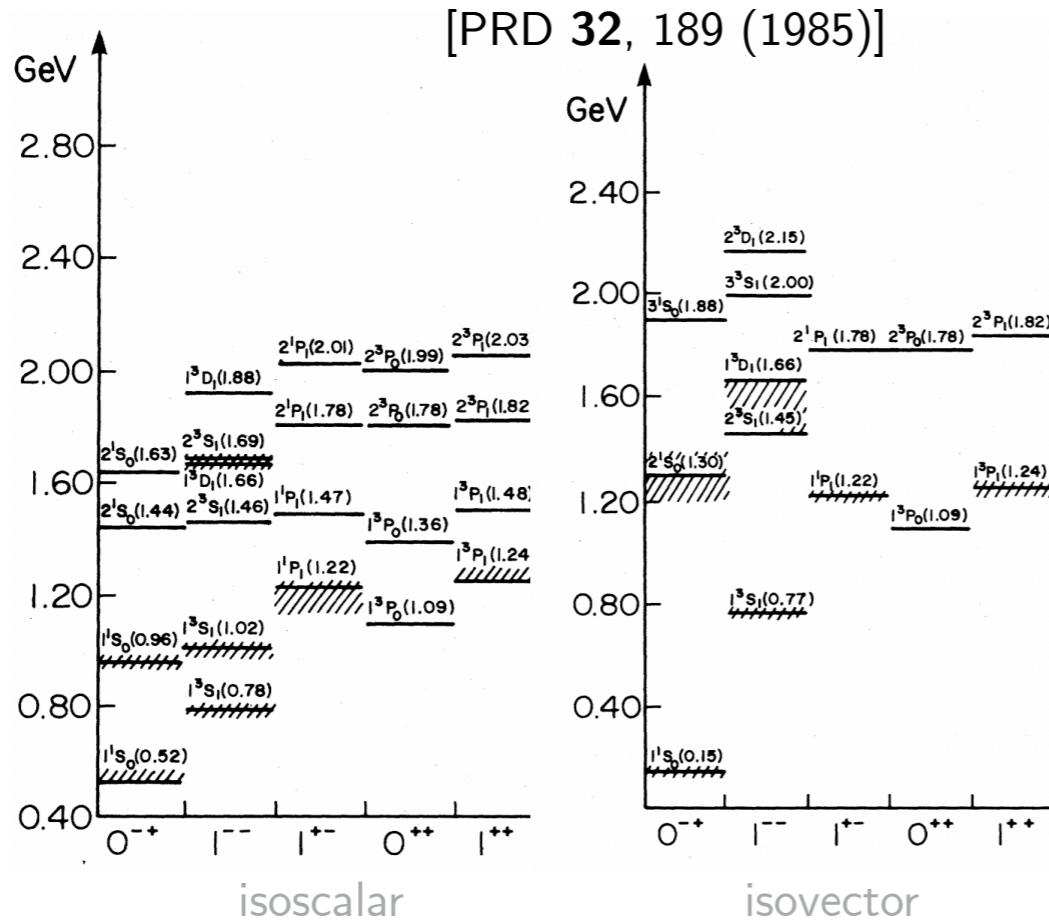
R. Briceño [UC Berkeley], K. Sherman [ODU], A. Jackura [UC Berkeley]



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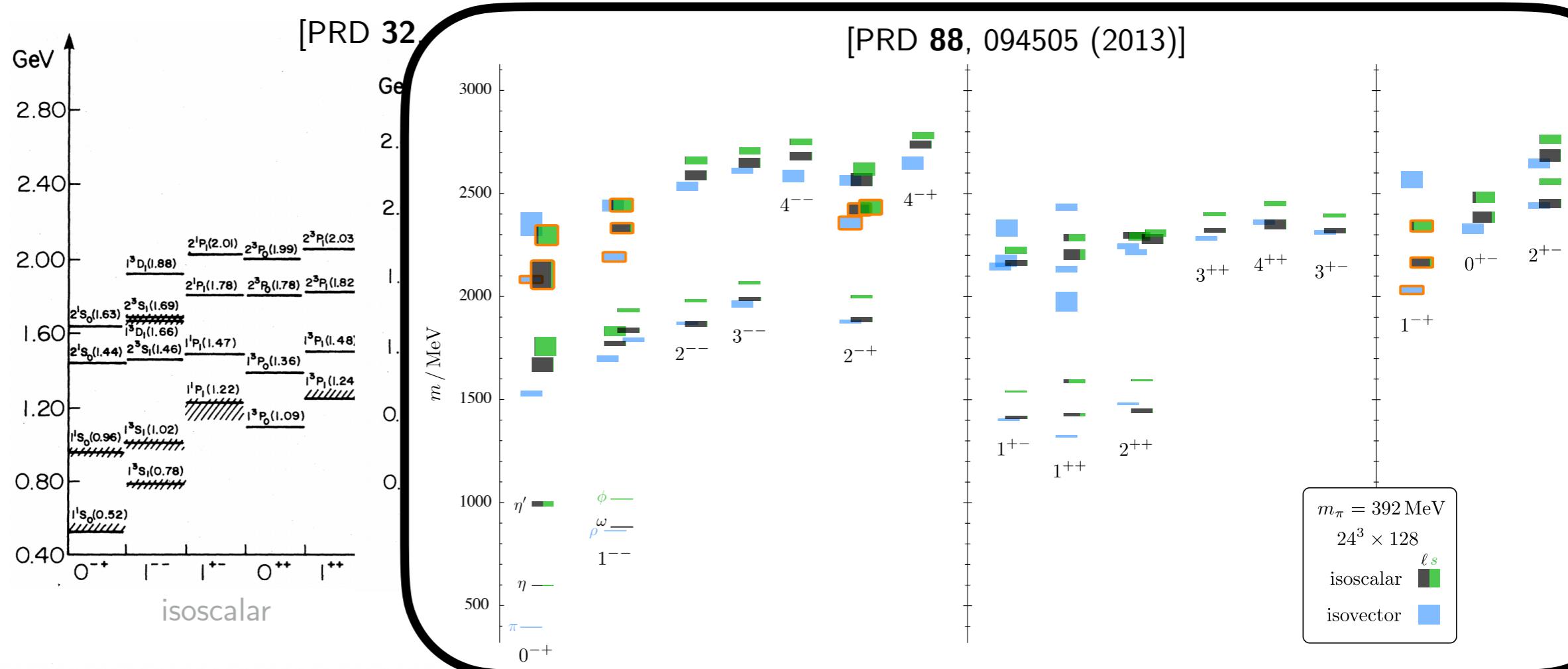
Characterization of hadrons



Meson	Predicted r_E^{-2} (fm 2)	Decay
π^+	$+ (0.66)^2$	$\rho \rightarrow \pi\gamma$
K^+	$+ (0.59)^2$	
K^0	$- (0.30)^2$	$K^{*+} \rightarrow K^+\gamma$

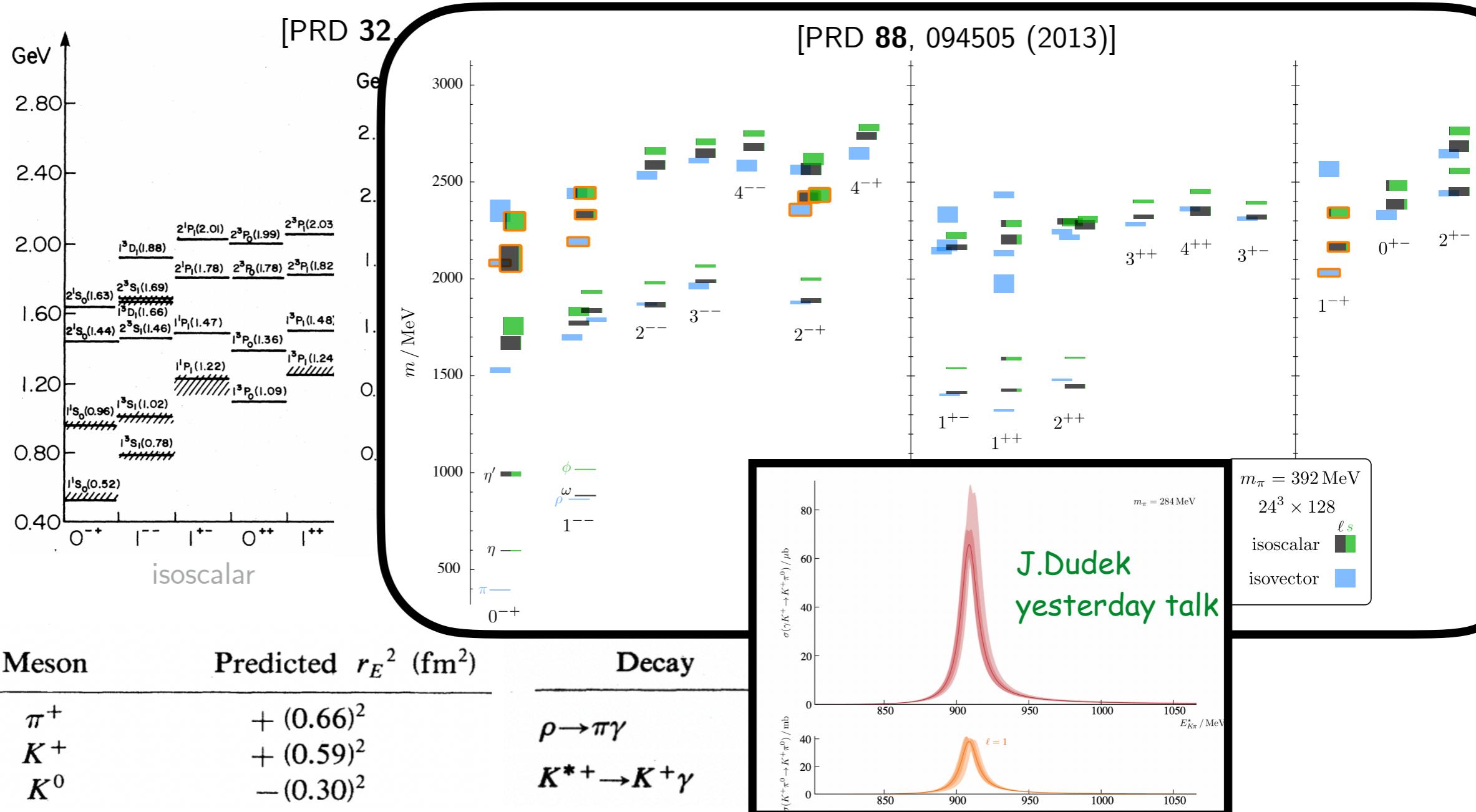
- Couplings to hadronic decay channels
- *Couplings to electroweak currents*

Characterization of hadrons



- Couplings to hadronic decay channels
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Characterization of hadrons

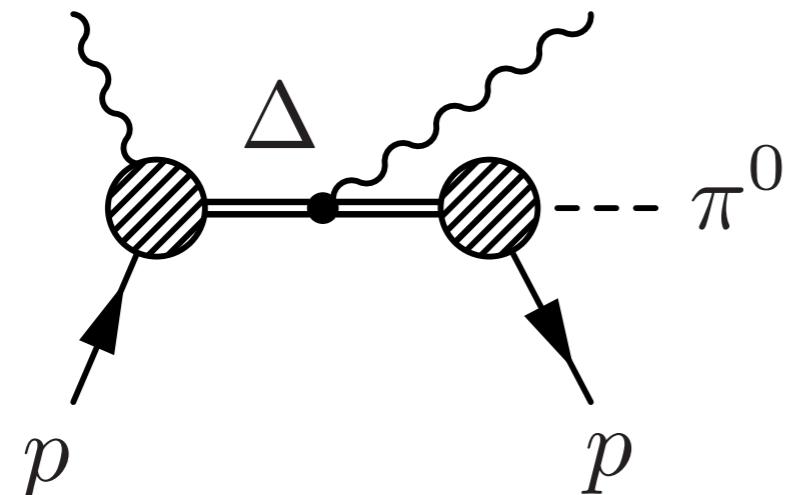


- Couplings to hadronic decay channels

[PRD 106, 114513 (2022)]

- Couplings to electroweak currents*

Structure of resonances

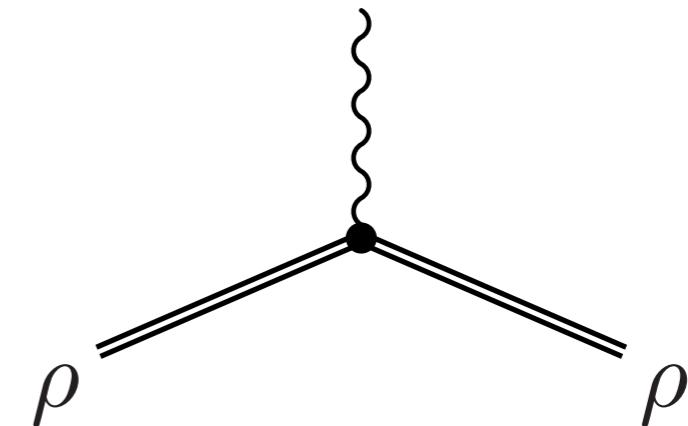


◆ Magnetic Moment of Δ dominated by model unc.

[PRC 64, 065202 (2001), PRL 89, 272001 (2002)]

◆ Beam spin asymmetry ep scattering.

[PRD 96, 113010 (2017)]



◆ Chiral EFT calculation of ρ FFs.

[PL B 730, 115–121 (2014)]

◆ Lattice calculations

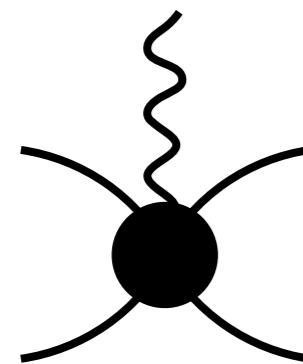
[PRD 91, 114501 (2015), PRD 75, 094504 (2007)]

◆ From e^+e^- BaBar data.

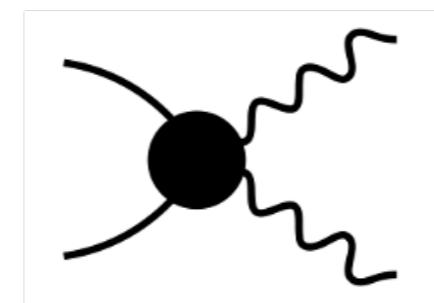
[Int. J. Mod. Phys. A 30, 1550114 (2015)]

Transition amplitudes with multi-hadron states

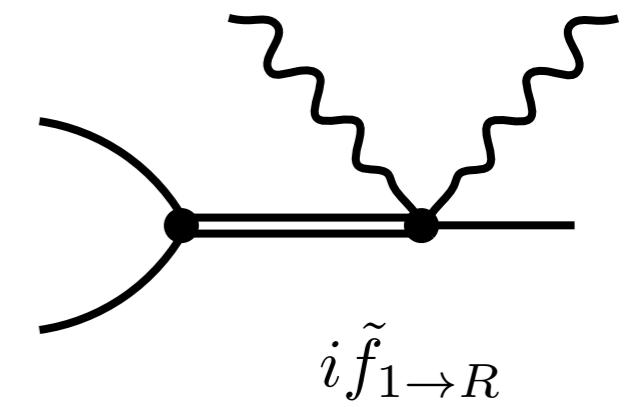
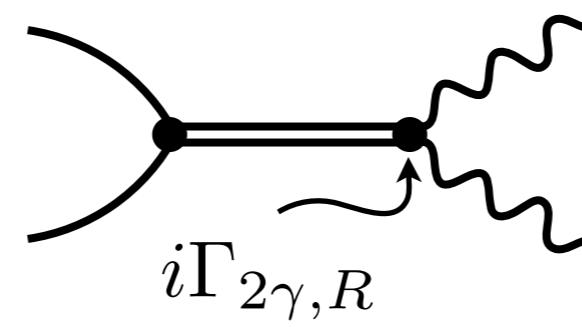
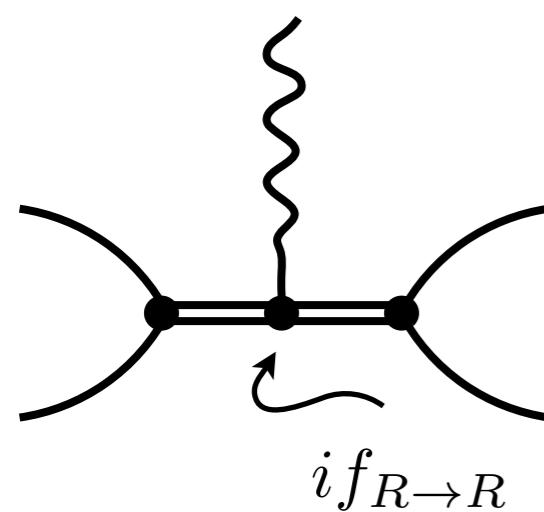
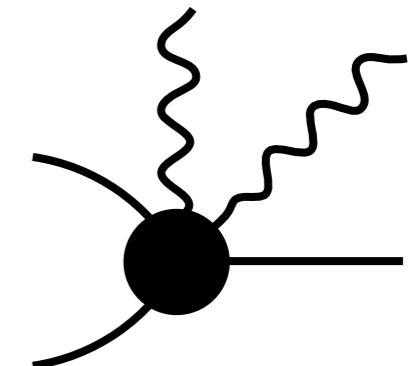
$$2 + \mathcal{J} \rightarrow 2$$



$$\mathcal{J} + \mathcal{J} \rightarrow 2$$



$$1 + \mathcal{J} \rightarrow 2 + \mathcal{J}$$



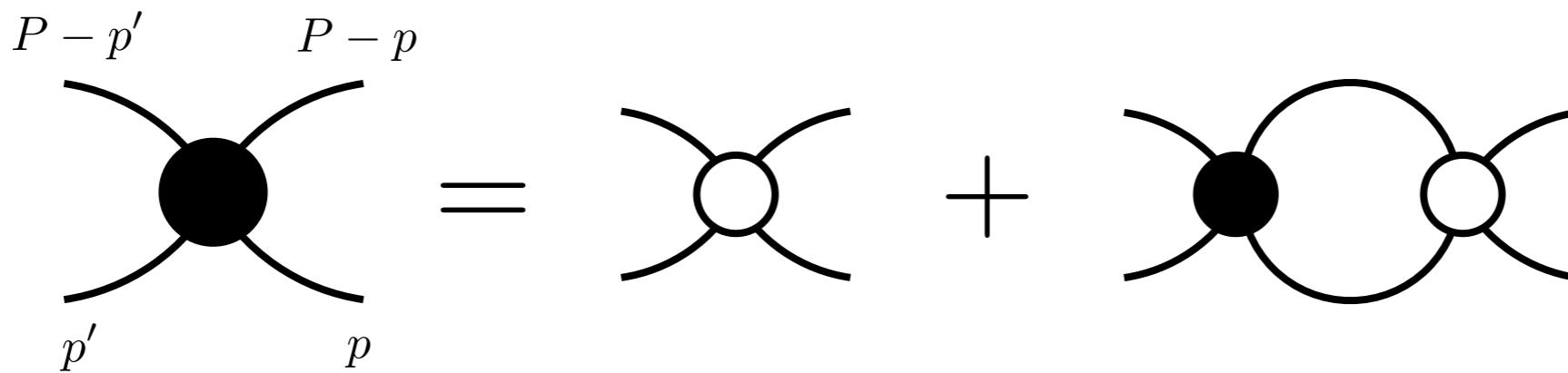
[PRD103 11, 114512 (2021)] FO, et al

[PRD105 11, 11 (2022)] FO, et al.

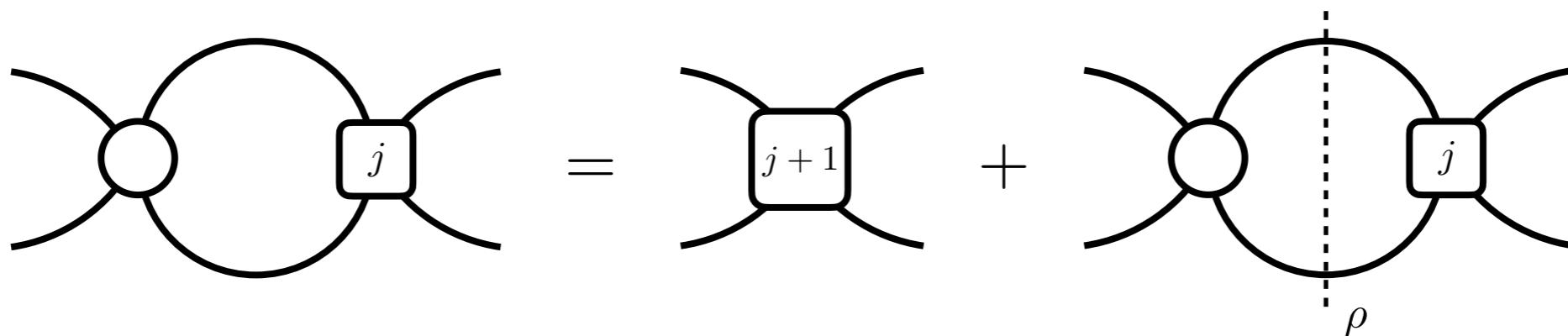
On-shell decomposition of amplitudes

Example: two-to-two scattering

$$i\mathcal{M}(p', p) = i\mathcal{K}_0(p', p) + \int \frac{d^4 k}{(2\pi)^4} i\mathcal{M}(p', k) i\Delta_1(k) i\Delta_2(P - k) i\mathcal{K}_0(k, p)$$



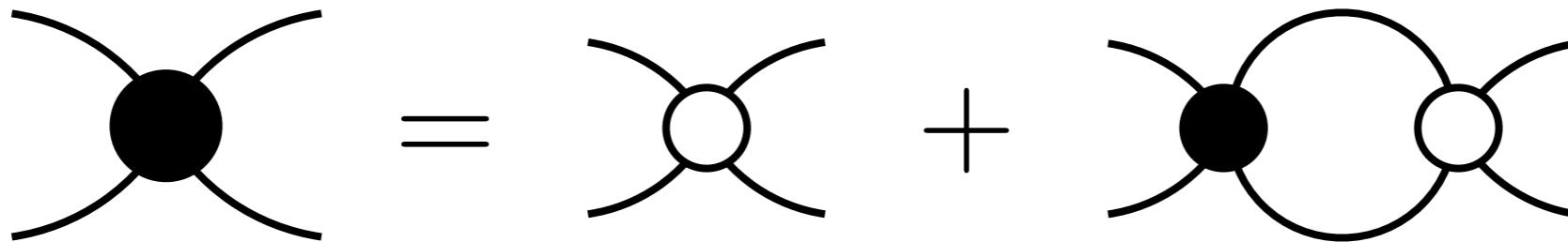
Non-analytic contributions:



$$\rho \propto \sqrt{s - s_{\text{th}}}$$

On-shell decomposition of amplitudes

Example: two-to-two scattering

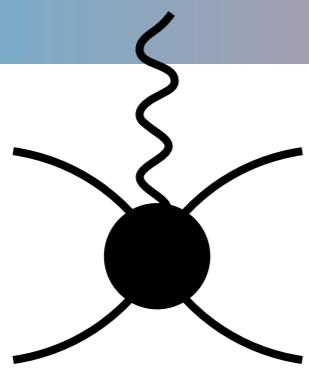


$$\begin{aligned} i\mathcal{M}(p', p) &= \sum_{j=0}^{\infty} i\mathcal{K}_j(p', p) + \sum_{\ell, m_\ell} i\mathcal{M}_{\ell m_\ell}(p') \rho \sum_{j=0}^{\infty} i\mathcal{K}_{j, \ell m_\ell}(p), \\ &\equiv i\mathcal{K}(p', p) + \sum_{\ell, m_\ell} i\mathcal{M}_{\ell m_\ell}(p') \rho i\mathcal{K}_{\ell m_\ell}(p) \end{aligned}$$

\mathcal{K} -matrix parametrization

- Short-distance physics parameterized by a real function.
- Respects unitarity and analyticity.

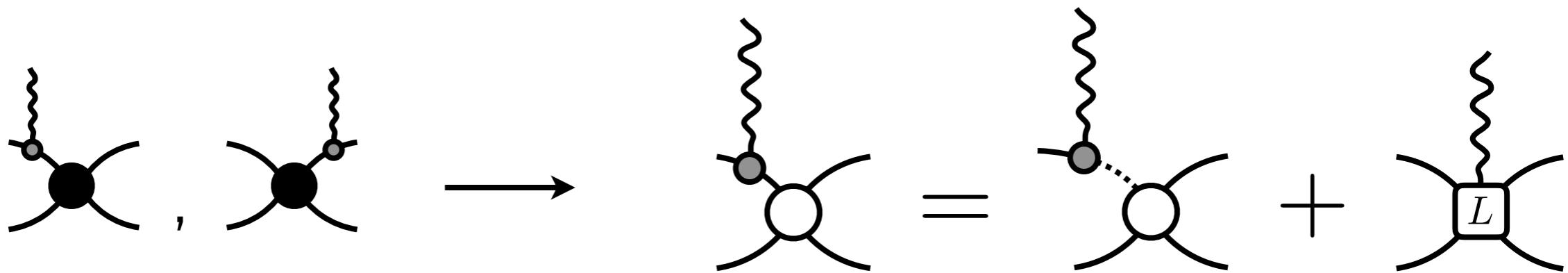
$$i\mathcal{M} = i\mathcal{K} \frac{1}{1 - i\rho\mathcal{K}}$$



On-shell decomposition
 $2 + \mathcal{J} \rightarrow 2$

Single hadron current insertion

- Long range contributions

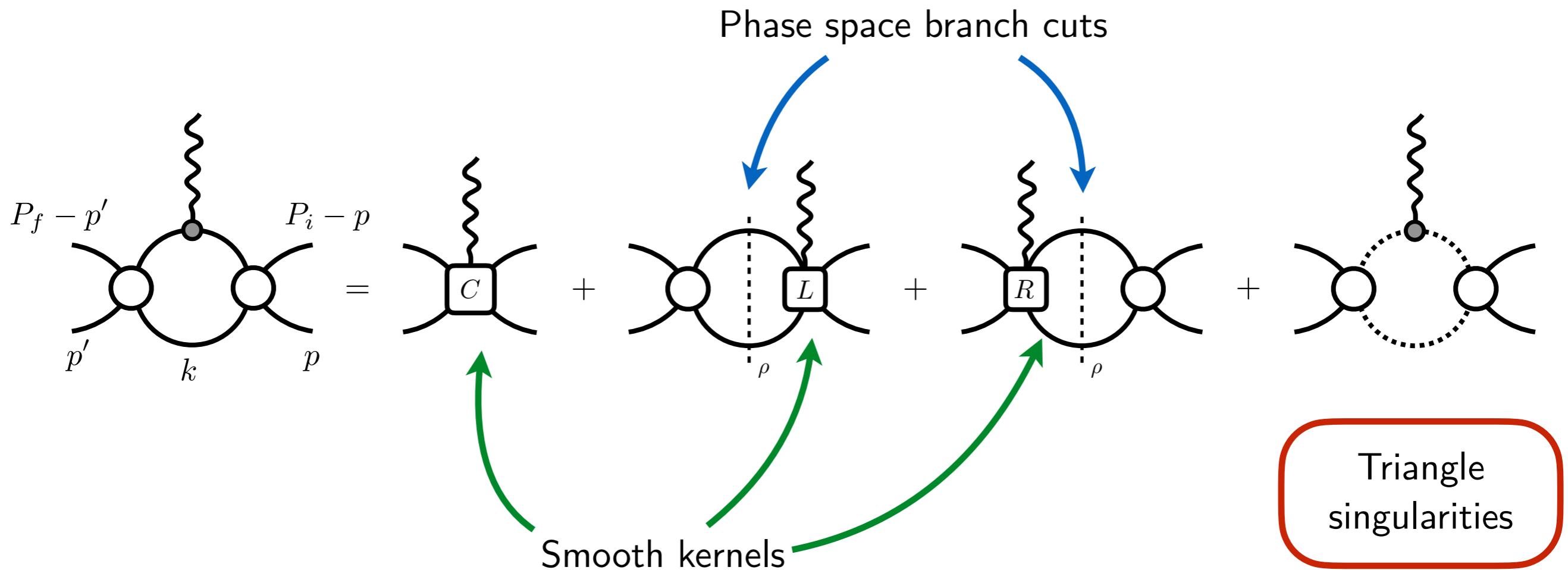


- Triangle diagrams

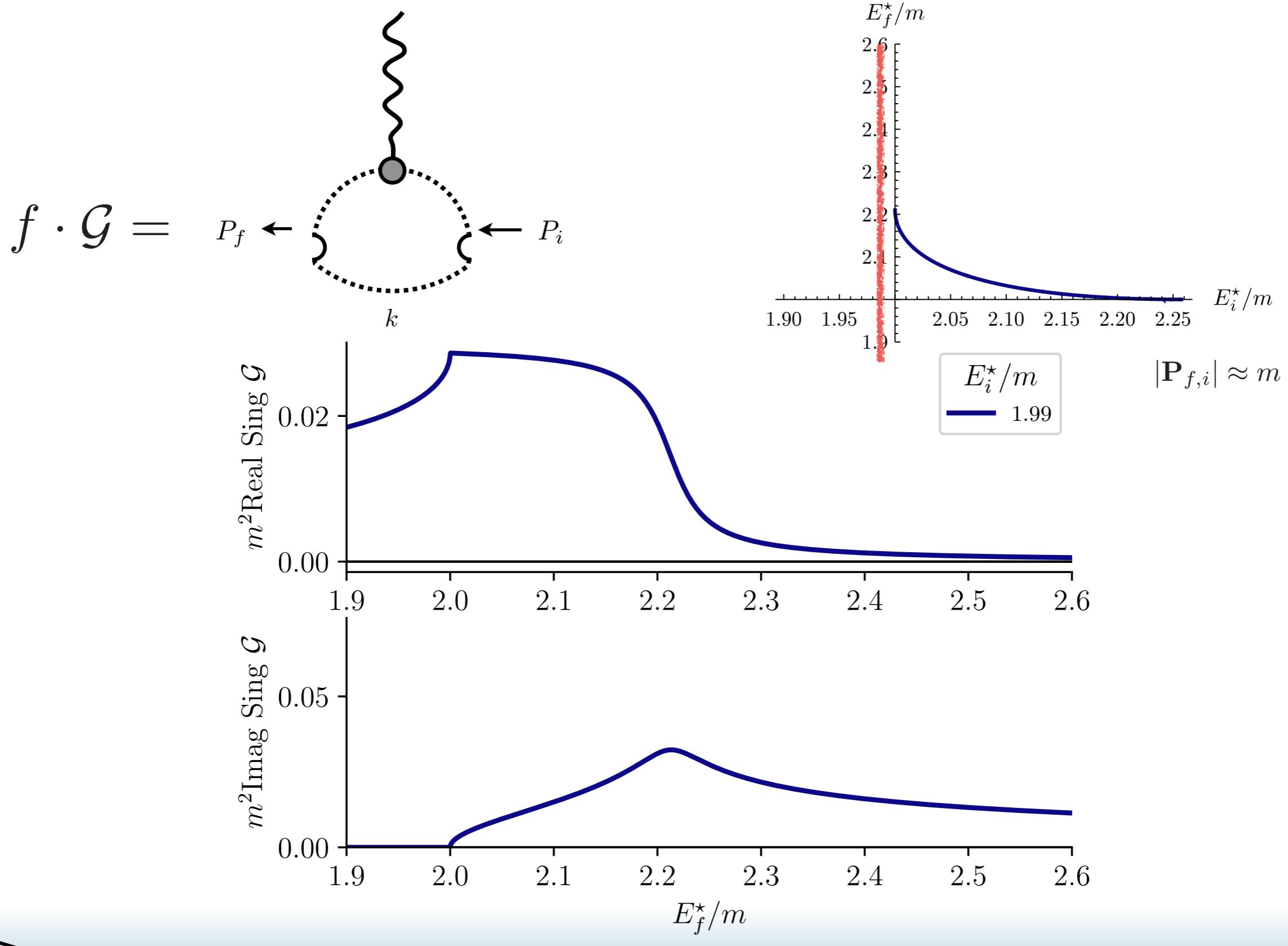


Triangle diagram

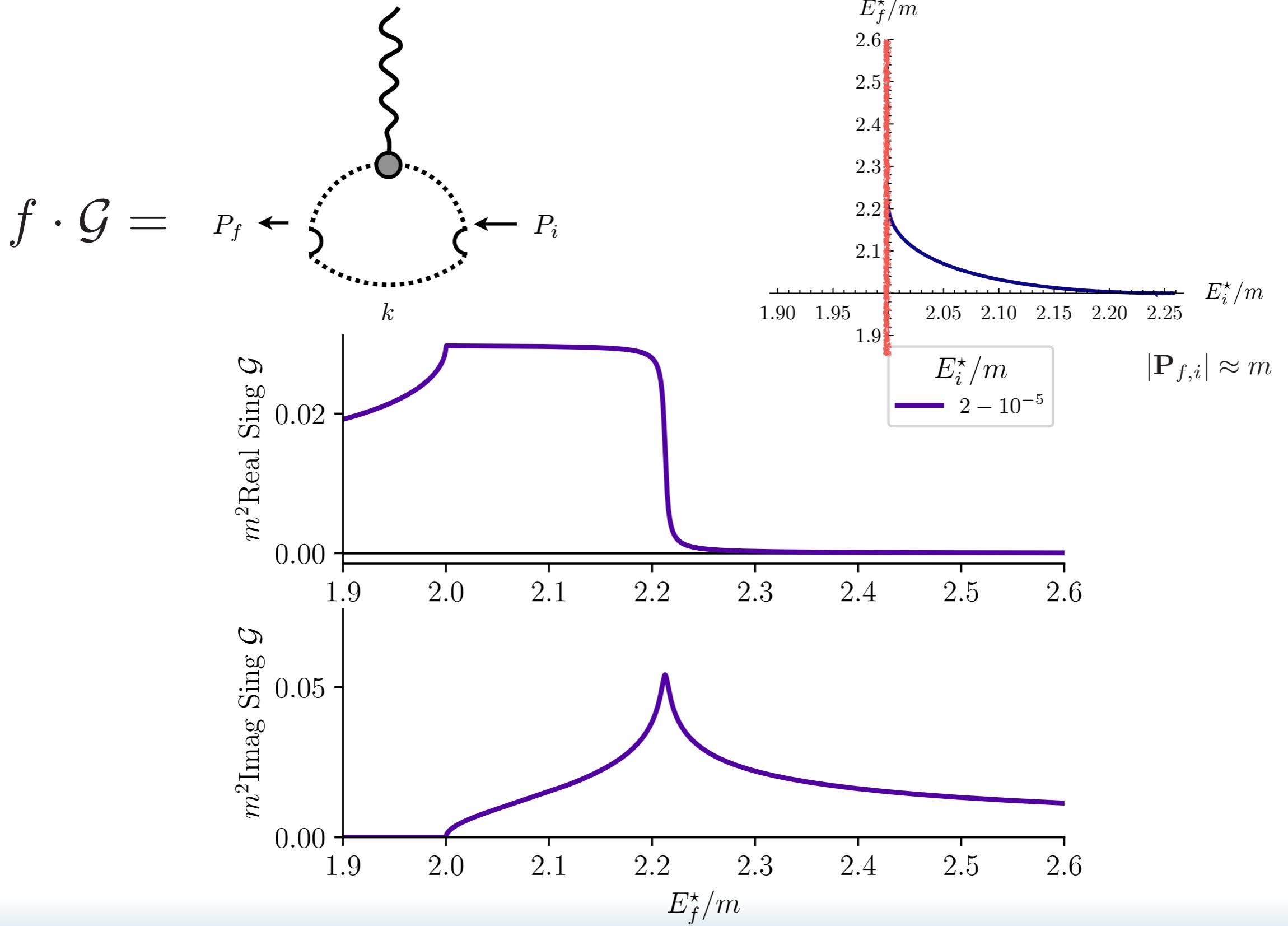
$$f = \text{---} \circlearrowleft$$



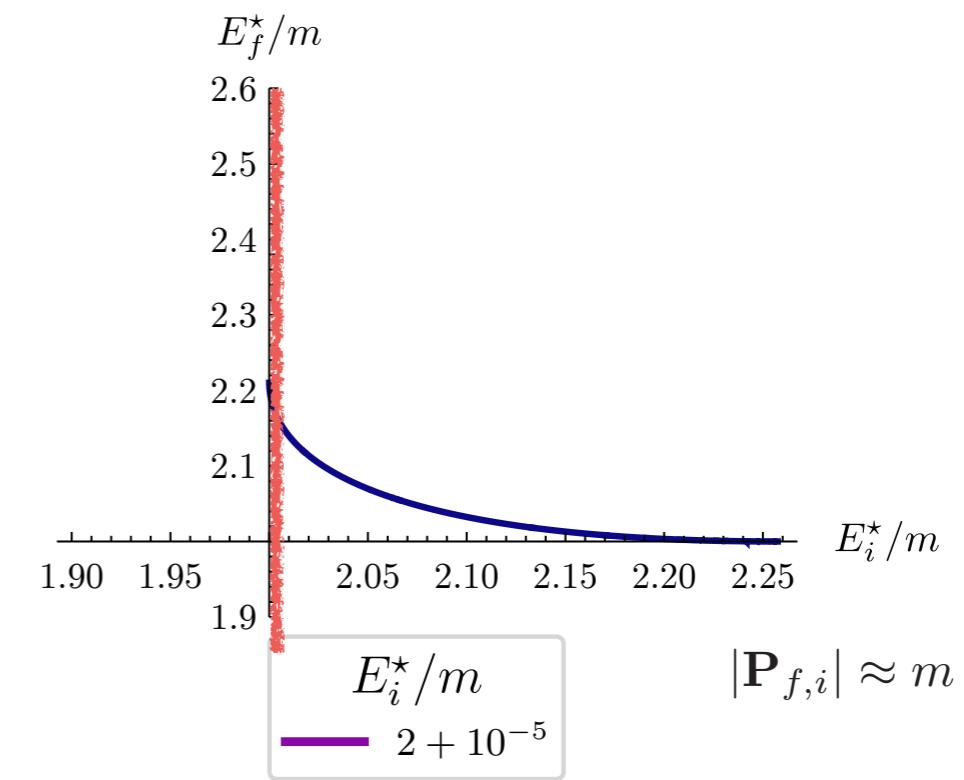
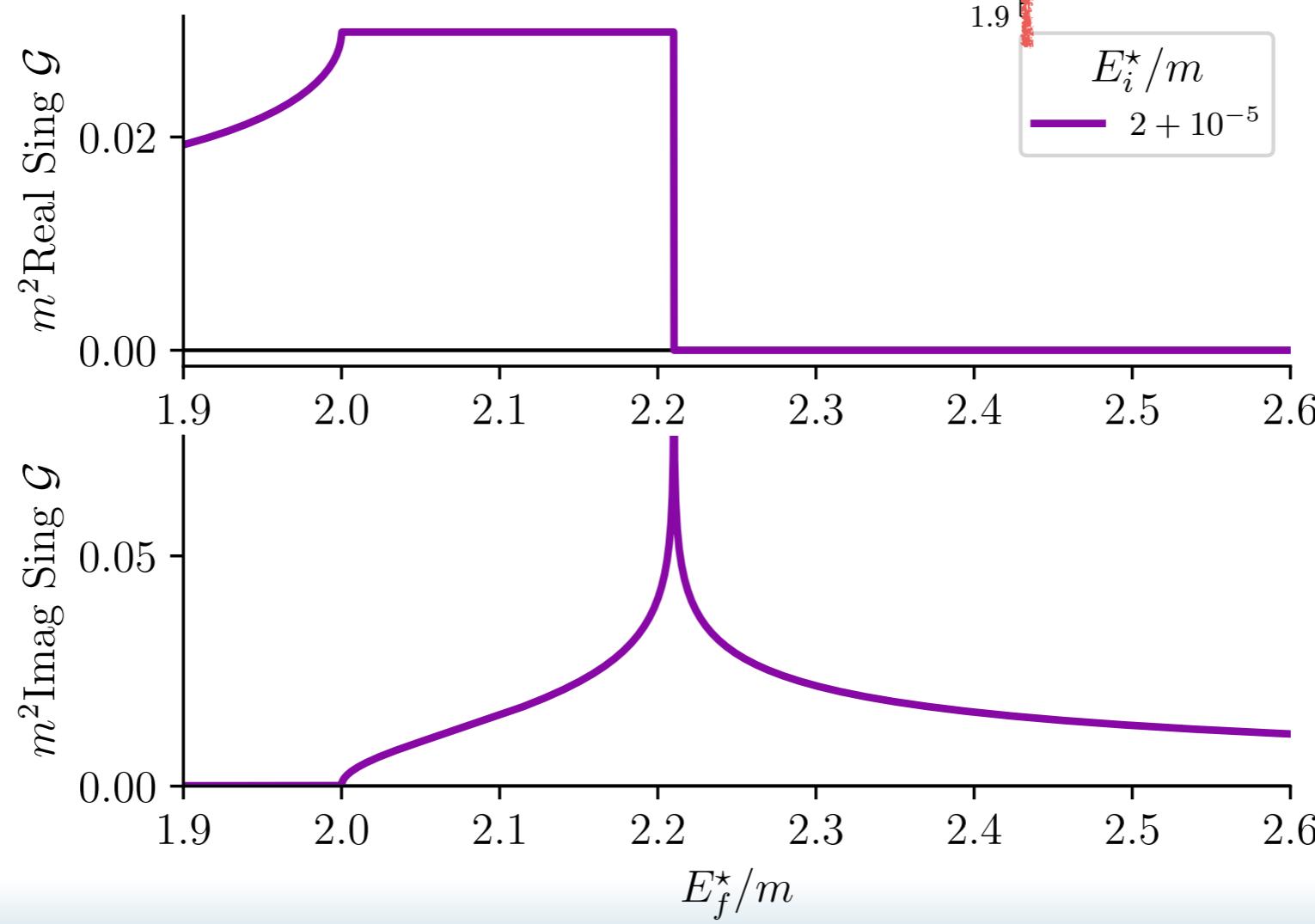
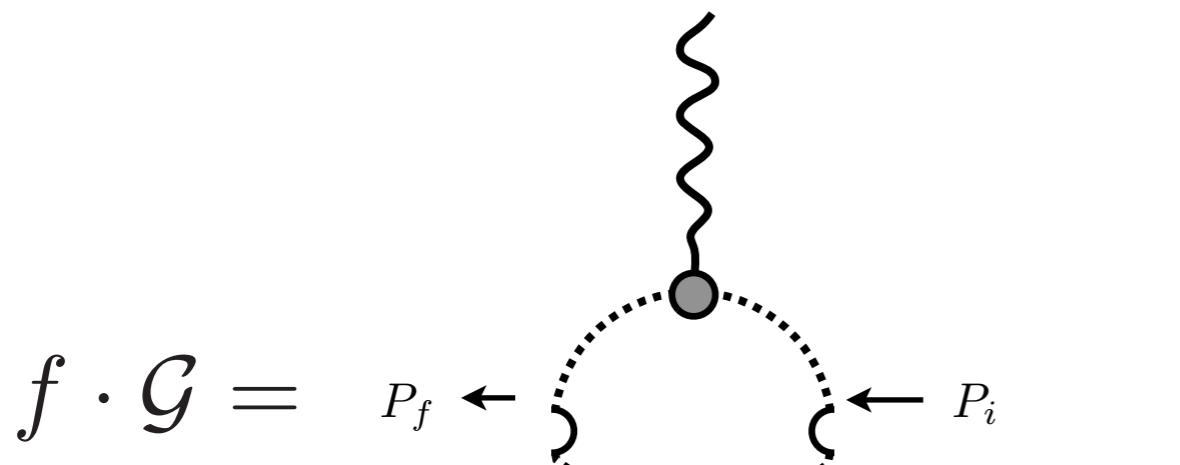
Triangle singularity analytic structure



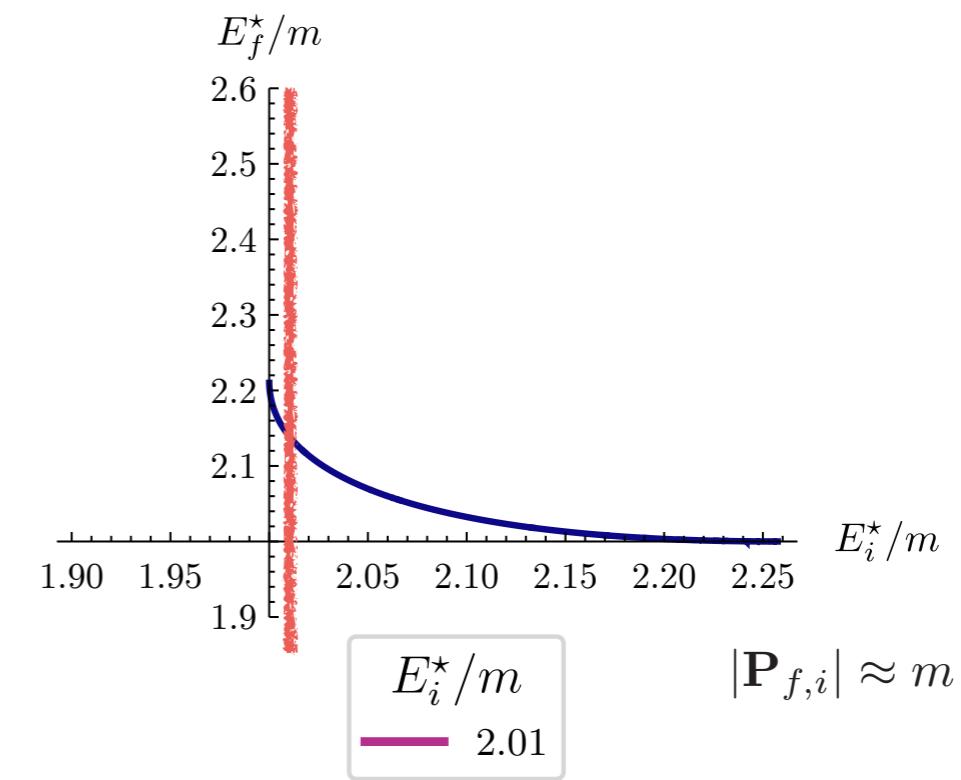
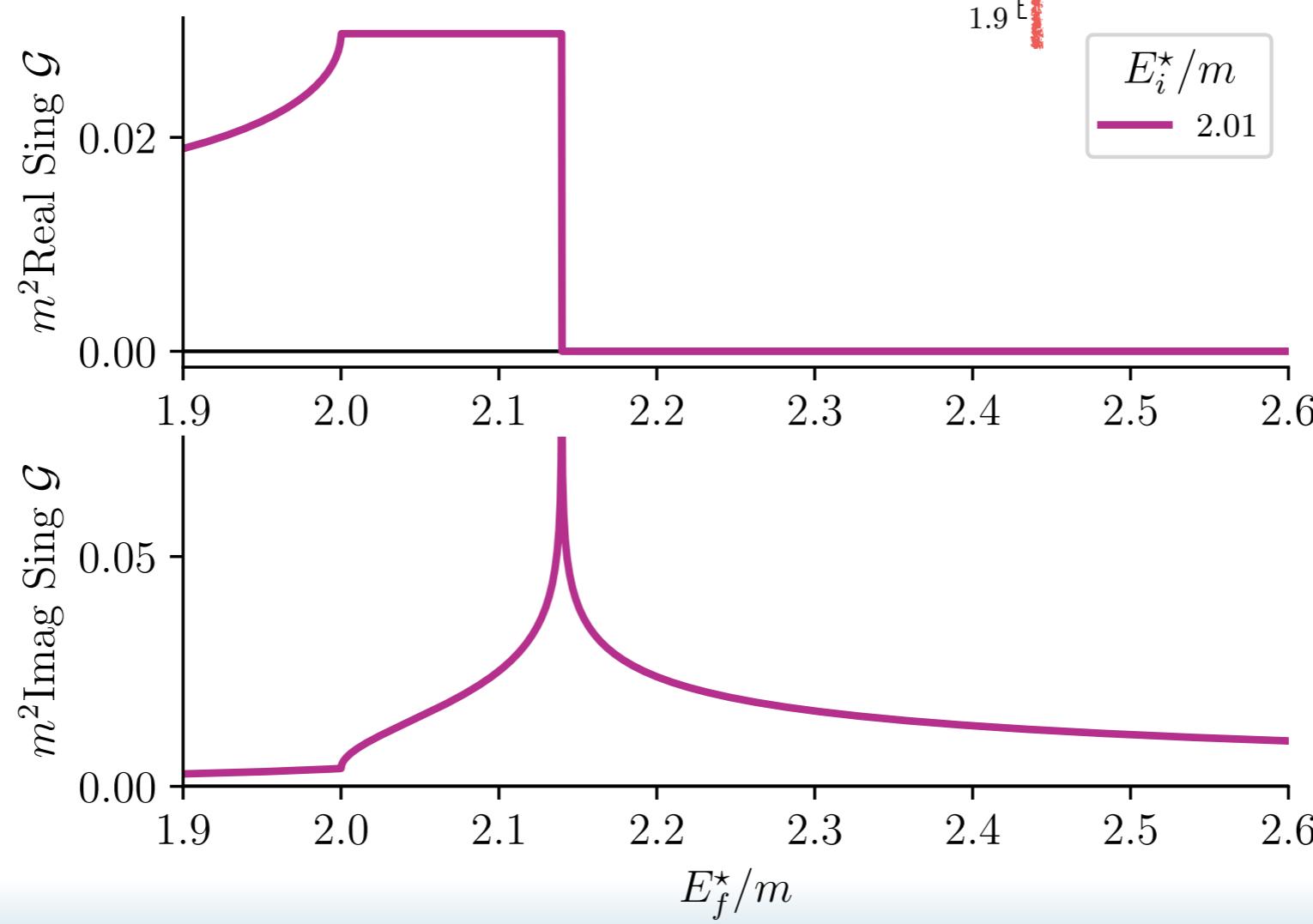
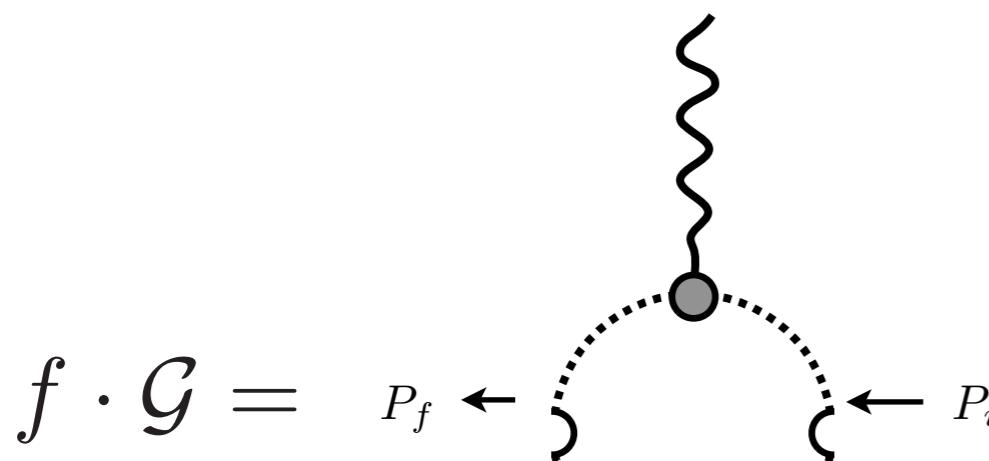
Triangle singularity analytic structure



Triangle singularity analytic structure



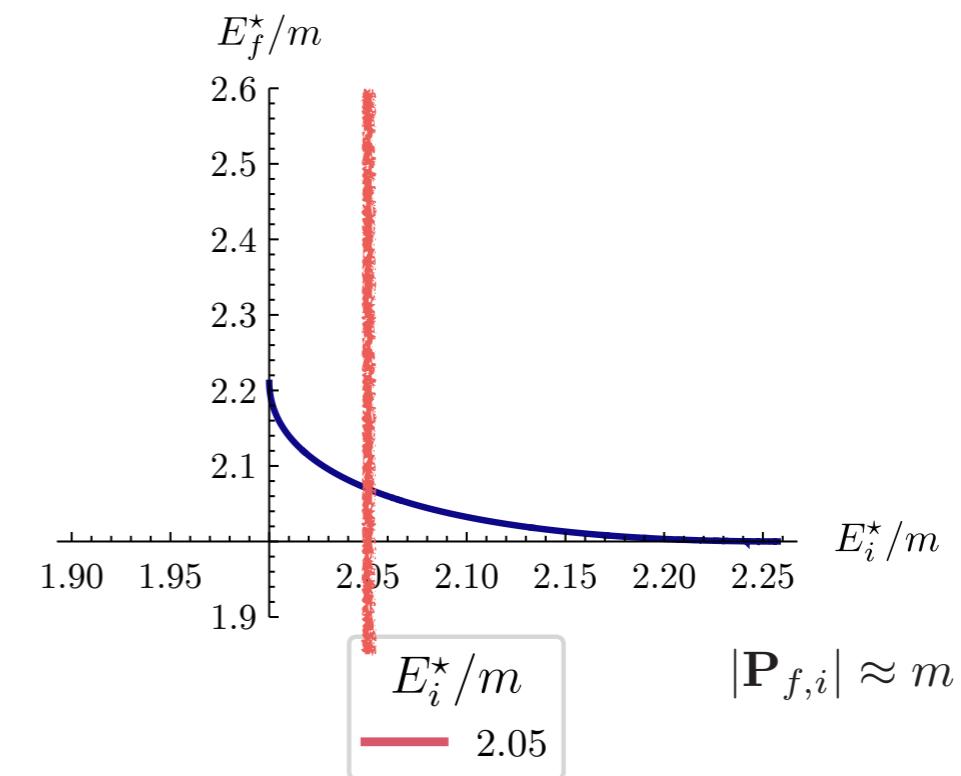
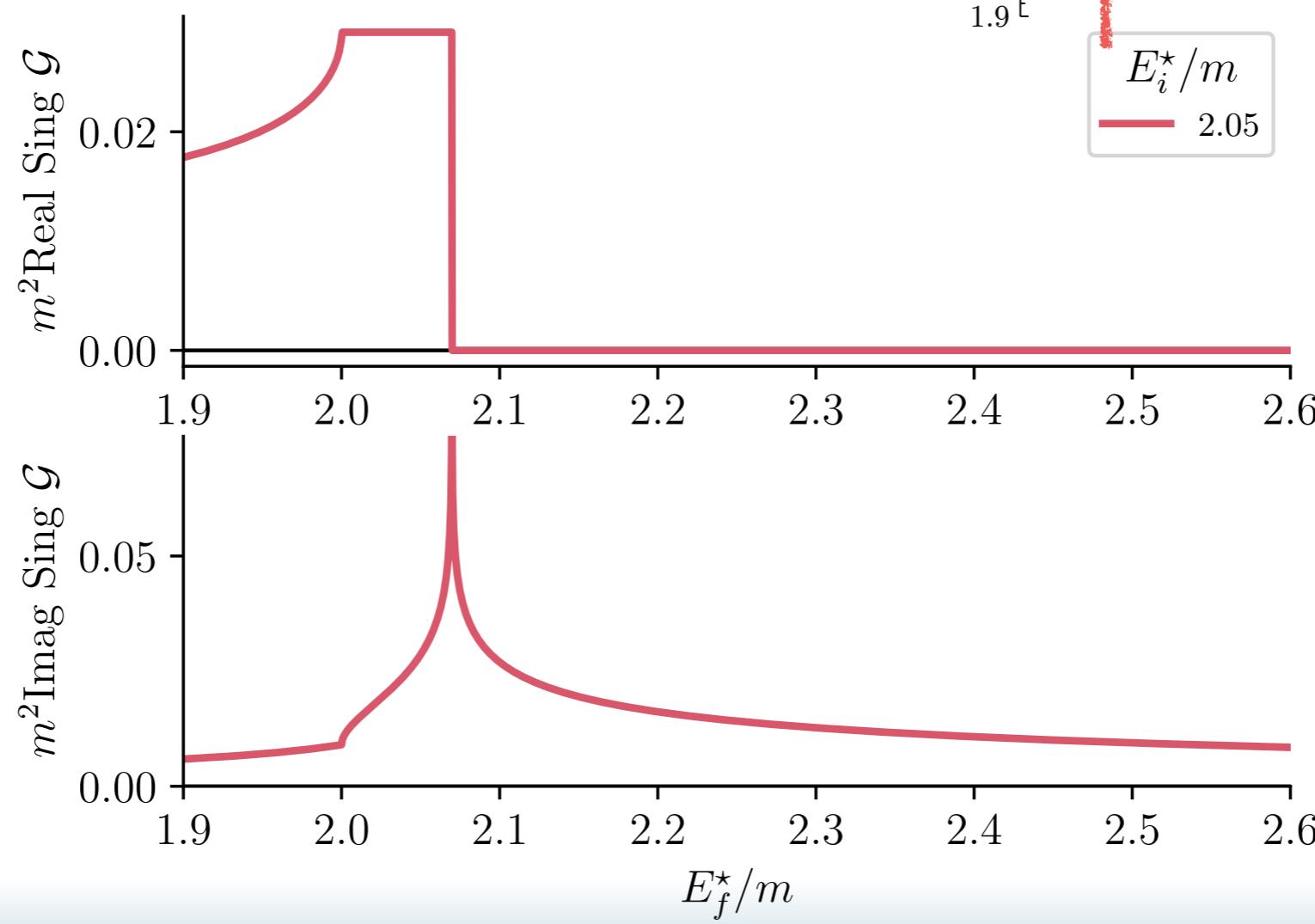
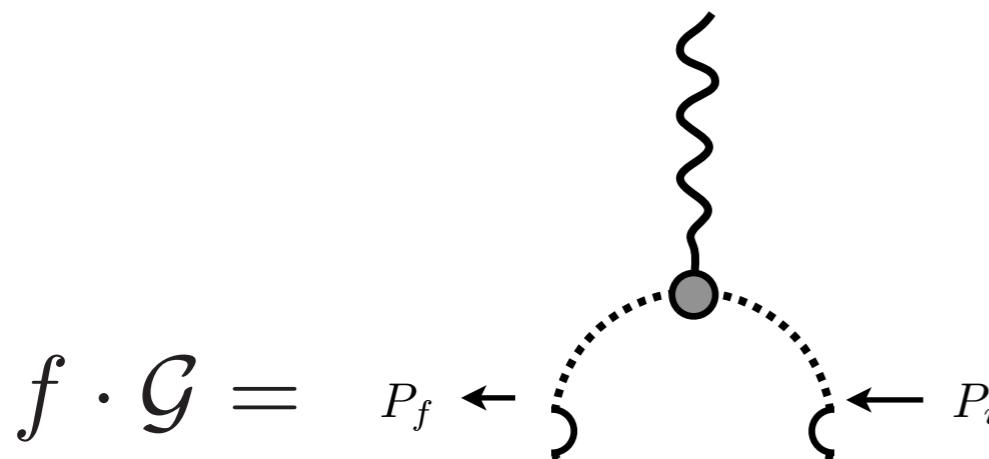
Triangle singularity analytic structure



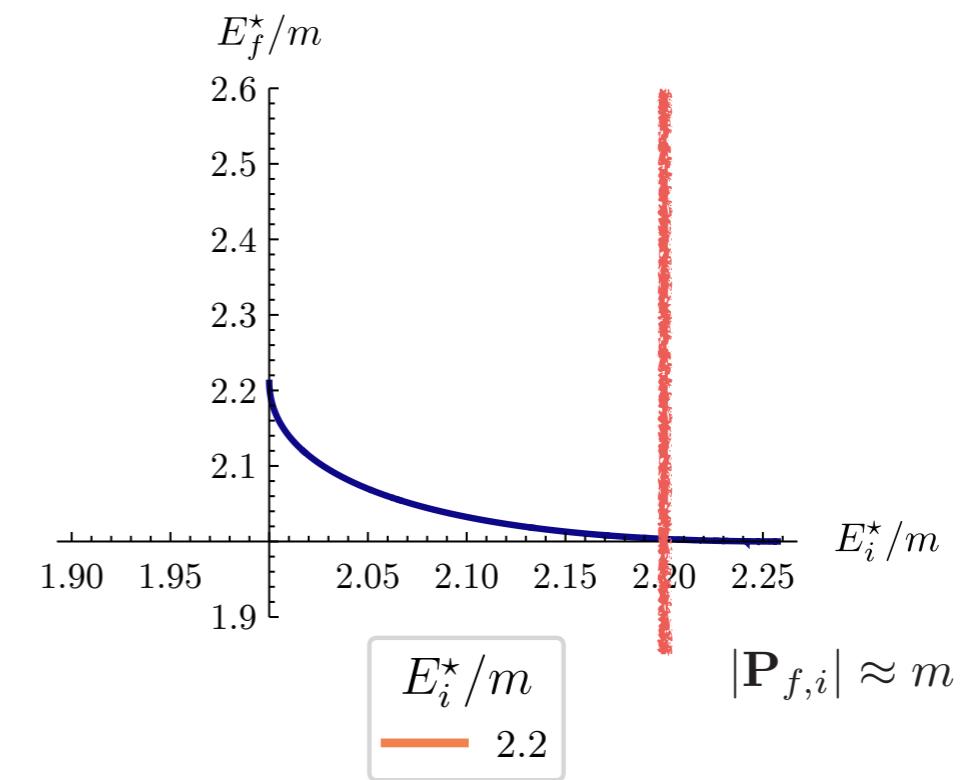
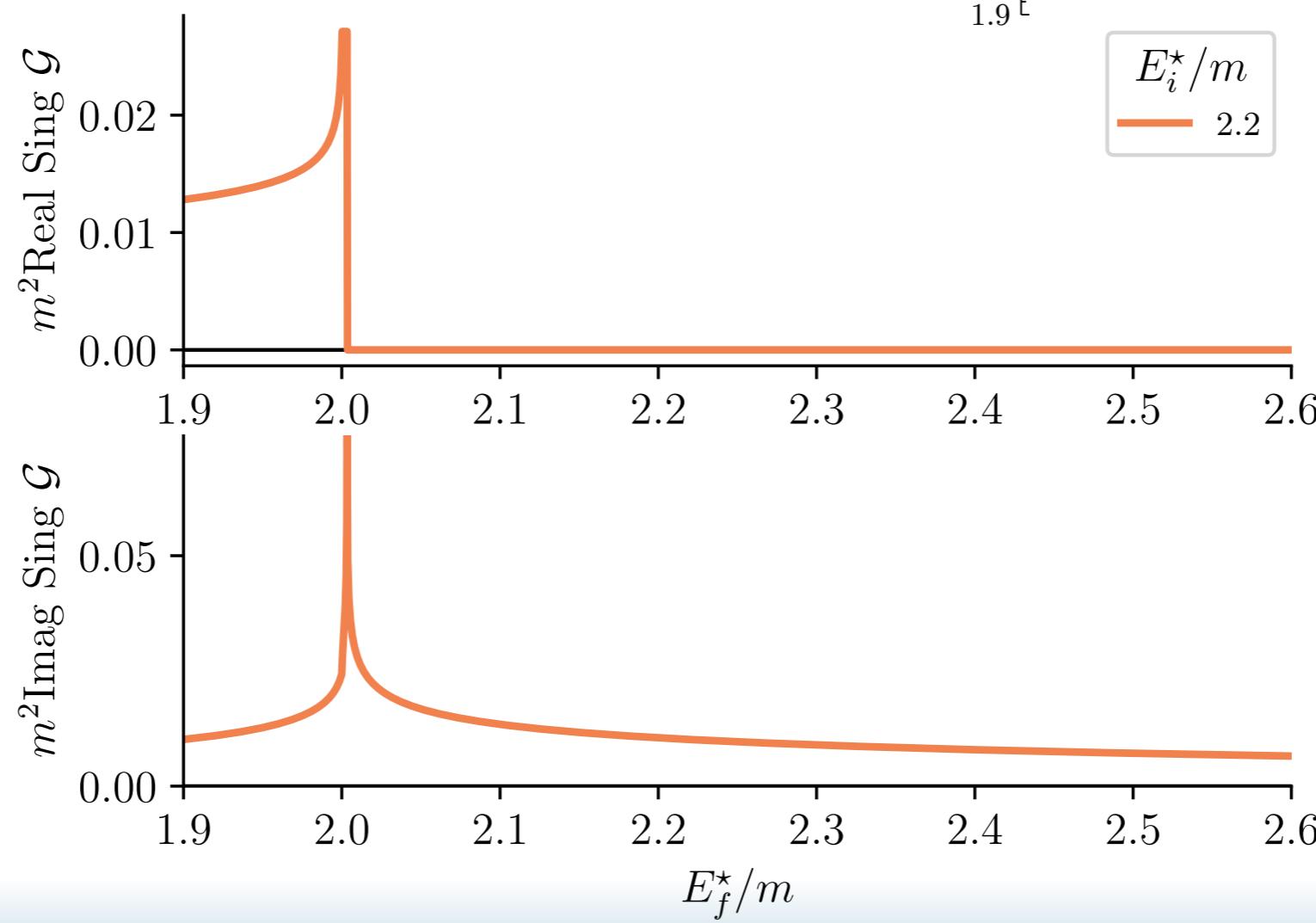
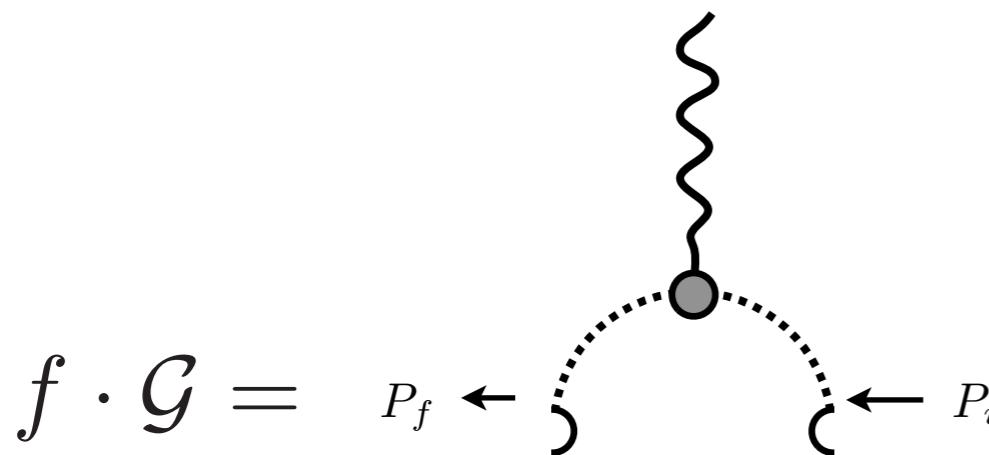
E_i^*/m
2.01

$|\mathbf{P}_{f,i}| \approx m$

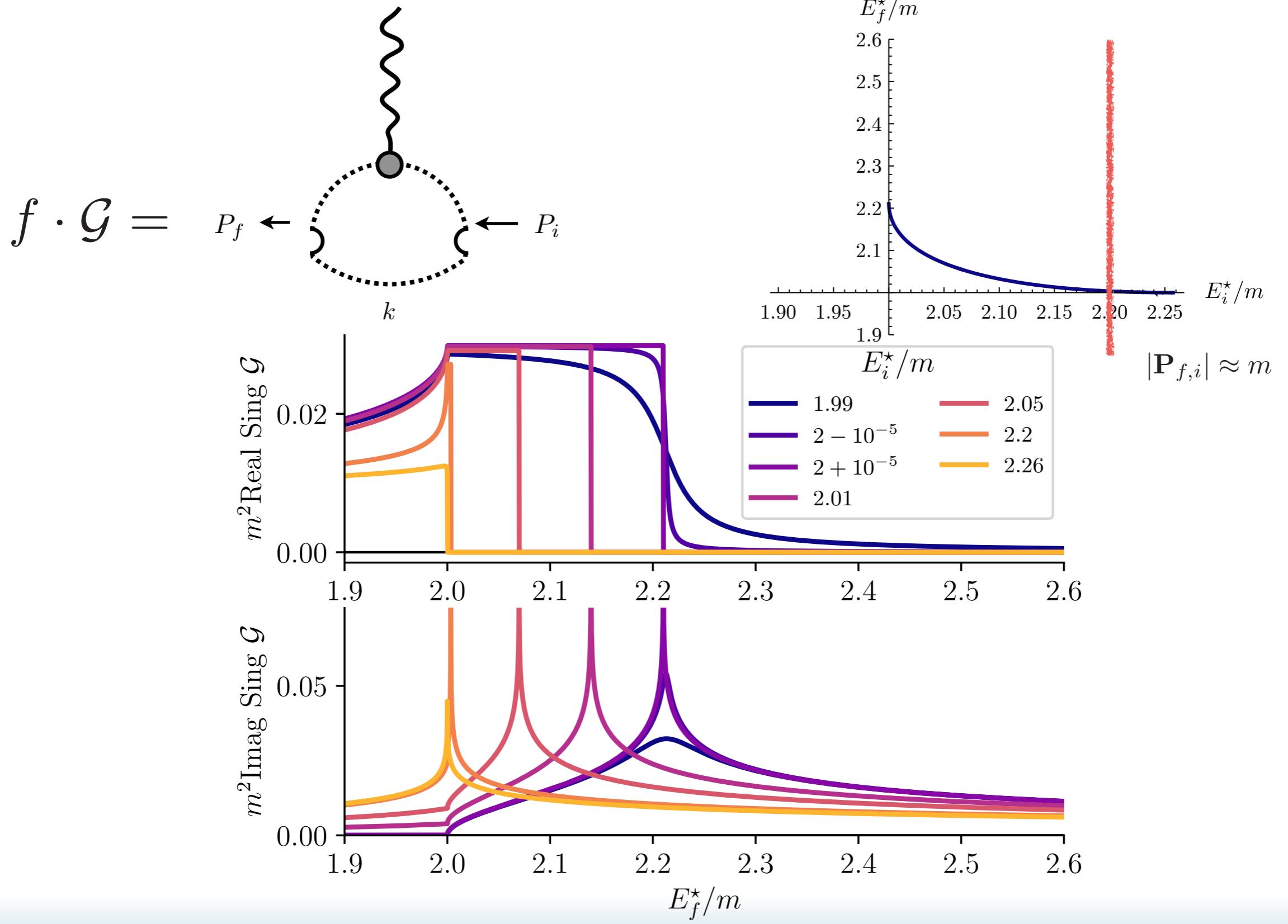
Triangle singularity analytic structure

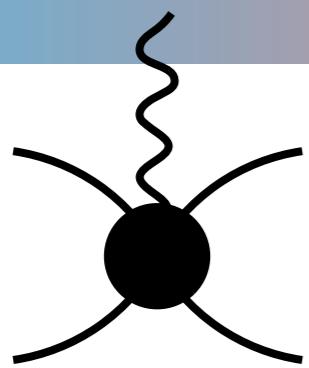


Triangle singularity analytic structure



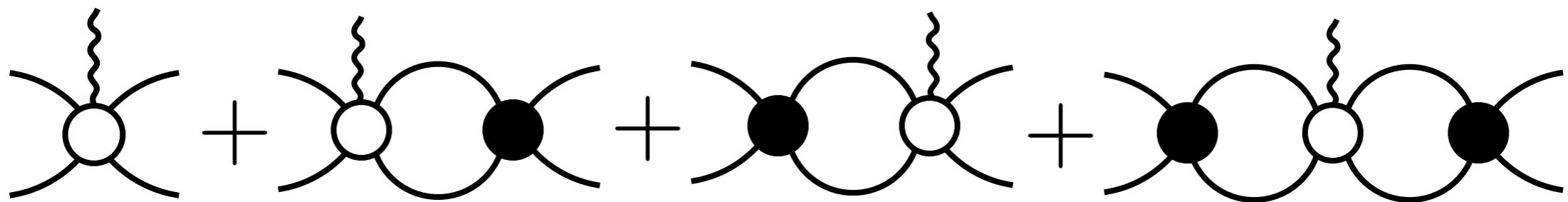
Triangle singularity analytic structure



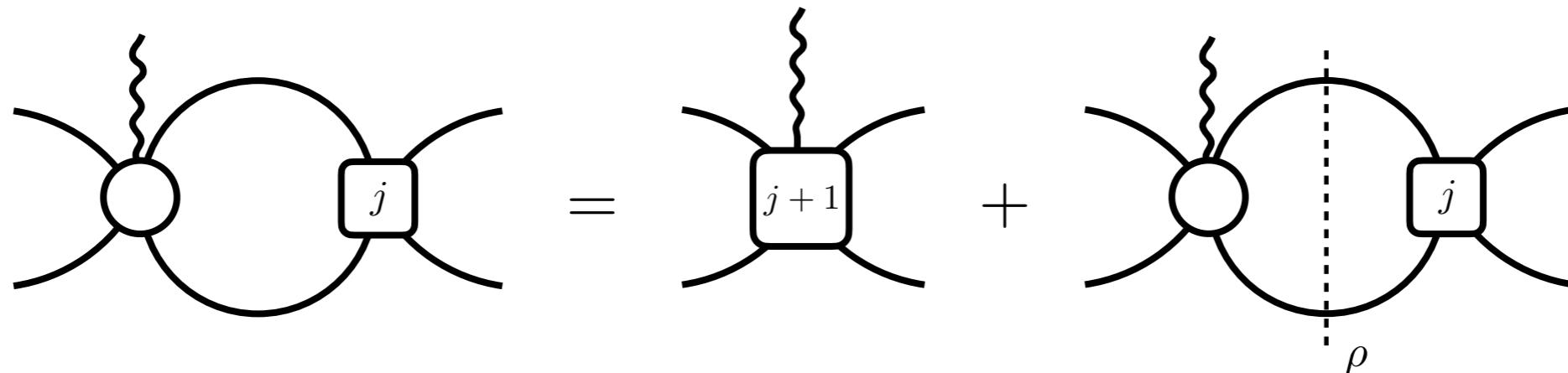


On-shell decomposition
 $2 + \mathcal{J} \rightarrow 2$

Short range contributions



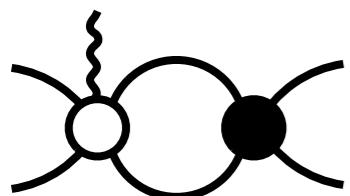
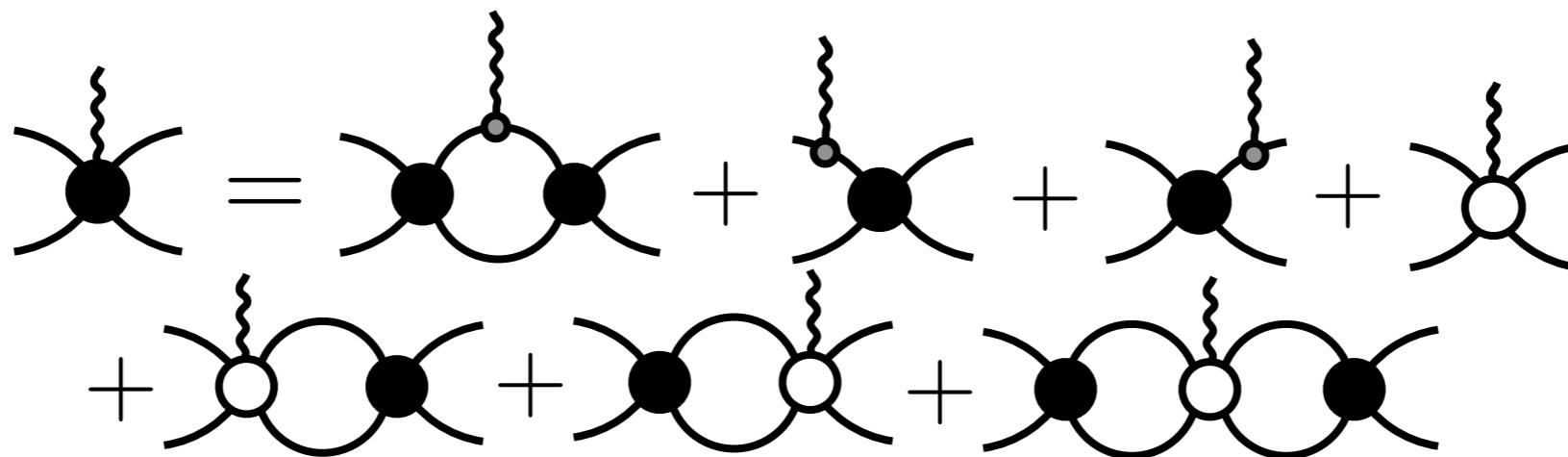
Non-analytic contributions:



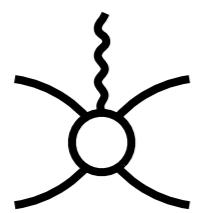
$$\rho \propto \sqrt{s - s_{\text{th}}}$$

On-shell decomposition

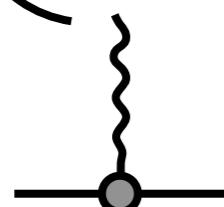
$$2 + \mathcal{J} \rightarrow 2$$



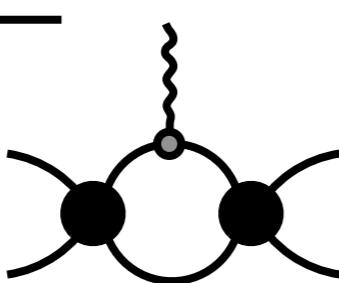
- Phase space singularities.



- Short range current insertions: smooth kernel.



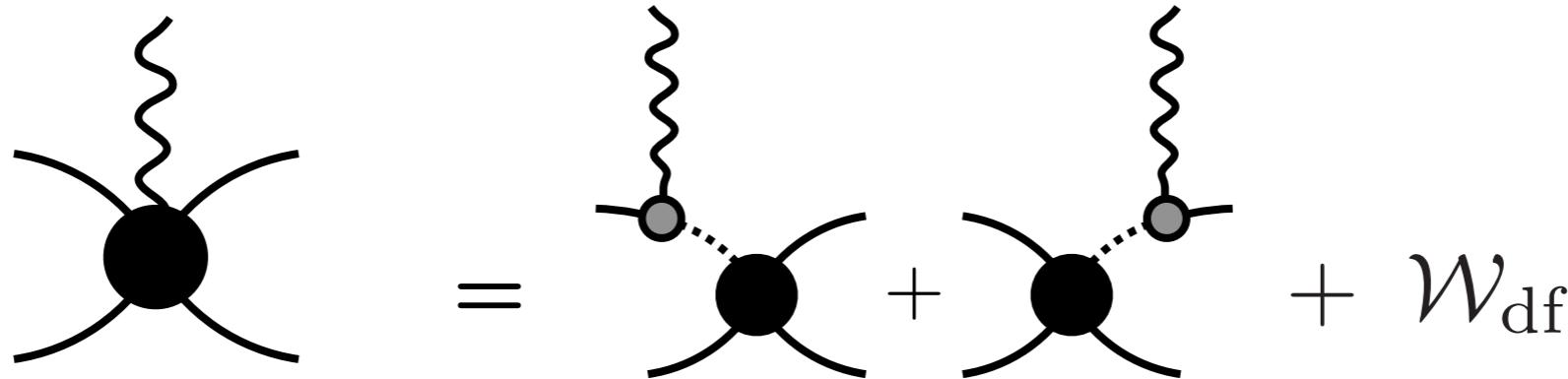
- Single-hadron matrix element.



- Triangle singularities.

On-shell decomposition

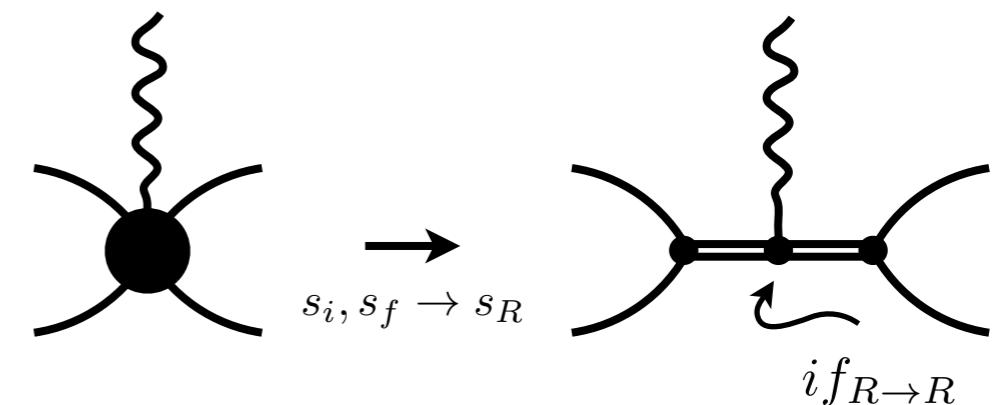
$$2 + \mathcal{J} \rightarrow 2$$



$$\mathcal{W}_{\text{df}} = \mathcal{M} (\mathcal{A}_{22} + \mathbf{f} \cdot \mathcal{G}) \mathcal{M}$$

Resonance structure:

$$\mathcal{M}^{\text{II}}(s) \sim -\frac{c^2}{s - s_R}$$

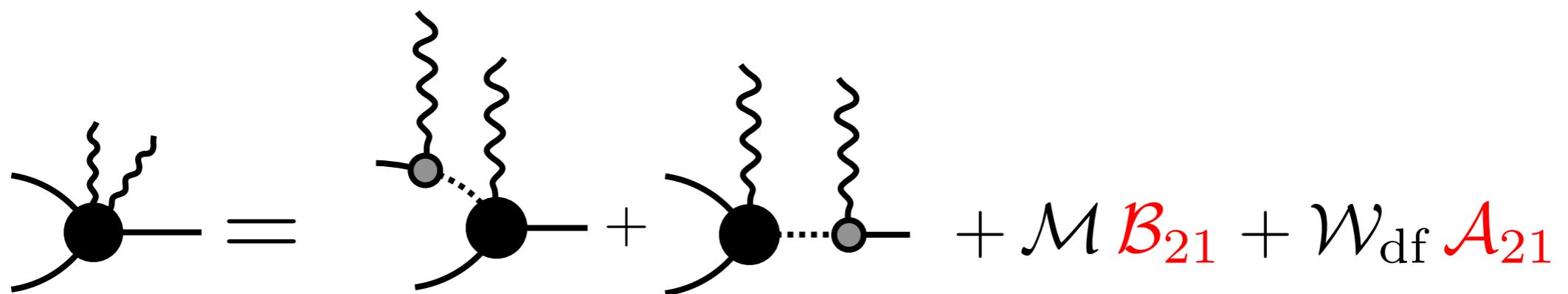
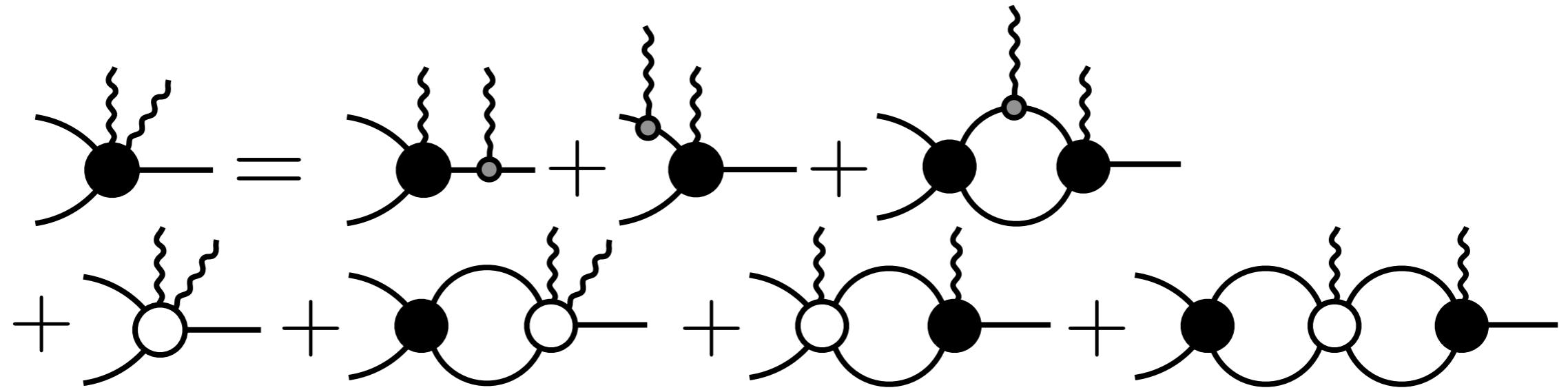


$$f_{R \rightarrow R}(Q^2) = c^2 [\mathcal{A}_{22}(s_R, Q^2, s_R) + f(Q^2) \mathcal{G}^{\text{II}, \text{II}}(s_R, Q^2, s_R)]$$

On-shell decomposition

$$1 + \mathcal{J} \rightarrow 2 + \mathcal{J}$$

$$(\mathcal{J} + \mathcal{J} \rightarrow 2)$$



$$i\mathcal{M} = i\kappa \frac{1}{1 - i\rho\kappa}$$

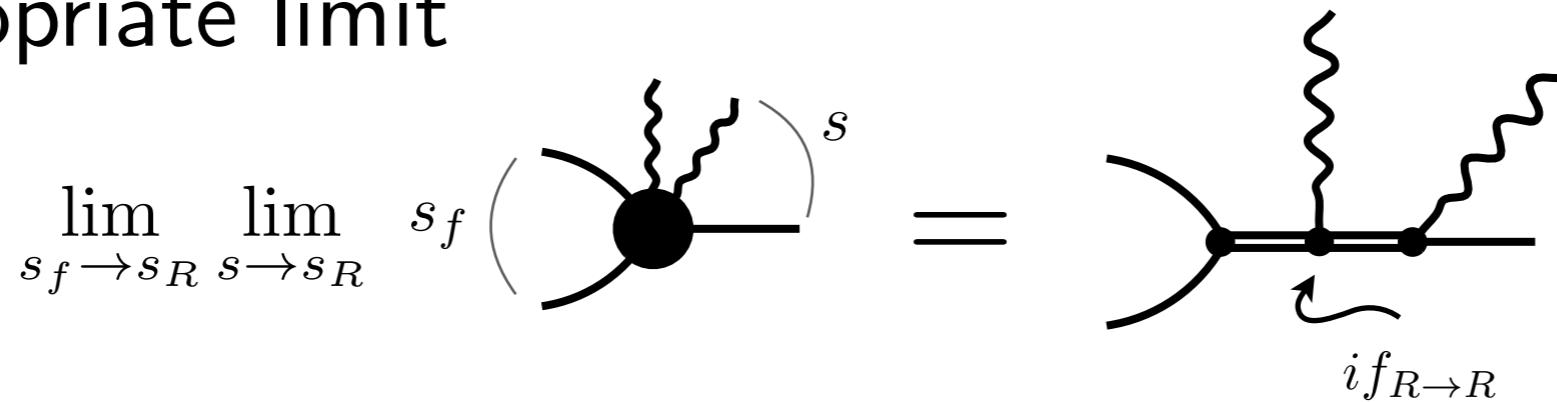
$$\mathcal{W}_{\text{df}} = \mathcal{M}(\mathcal{A}_{22} + \mathbf{f} \cdot \mathcal{G})\mathcal{M}$$

$$\text{1-point function with wavy line} = \mathcal{M} \mathcal{A}_{21}$$

On-shell decomposition

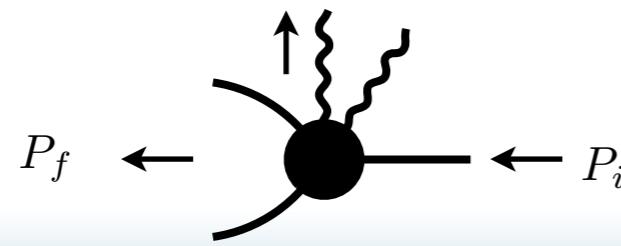
$$1 + \mathcal{J} \rightarrow 2 + \mathcal{J}$$

- The elastic resonance previously derived is recovered in the appropriate limit



- Ward identities sets constraints on the forward limit of different kernels.

$$\lim_{q_f \rightarrow 0} \mathcal{B}_{21}(P_f, P_i, q_f) = Q_0 \left(\frac{\partial}{\partial P_f^\mu} + \frac{\partial}{\partial P_i^\mu} \right) \mathcal{A}_{21}^\nu(P_f, P_i)$$

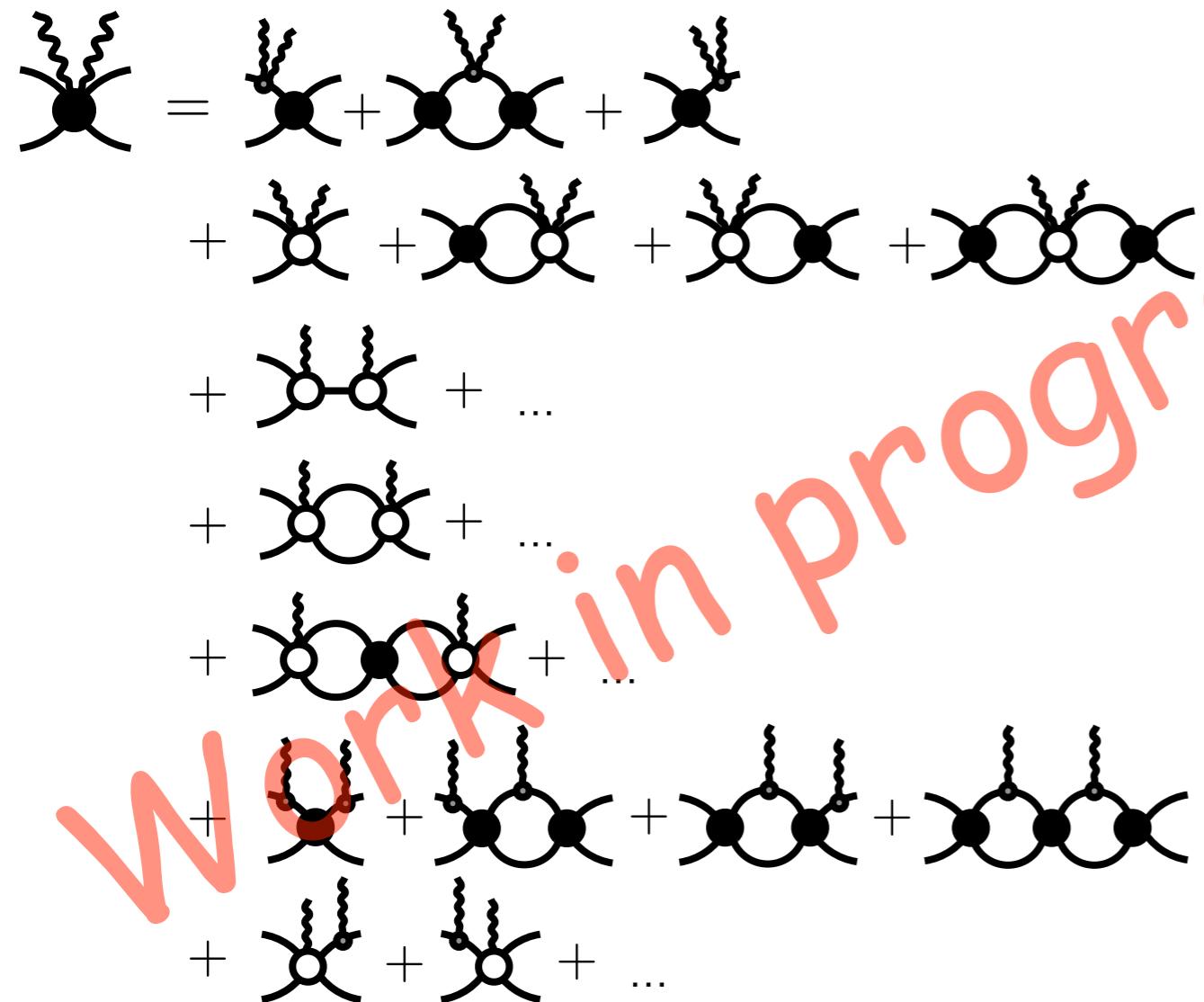


$$= \mathcal{M} \mathcal{A}_{21}$$

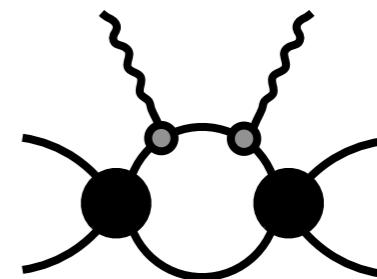
On-shell decomposition

$$2 + \mathcal{J} \rightarrow 2 + \mathcal{J}$$

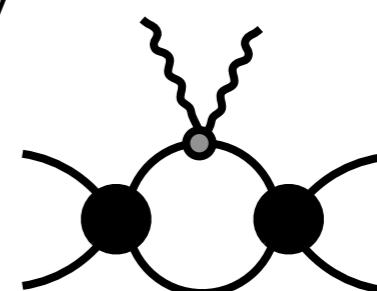
e.g. double beta decay



- Square diagram singularities



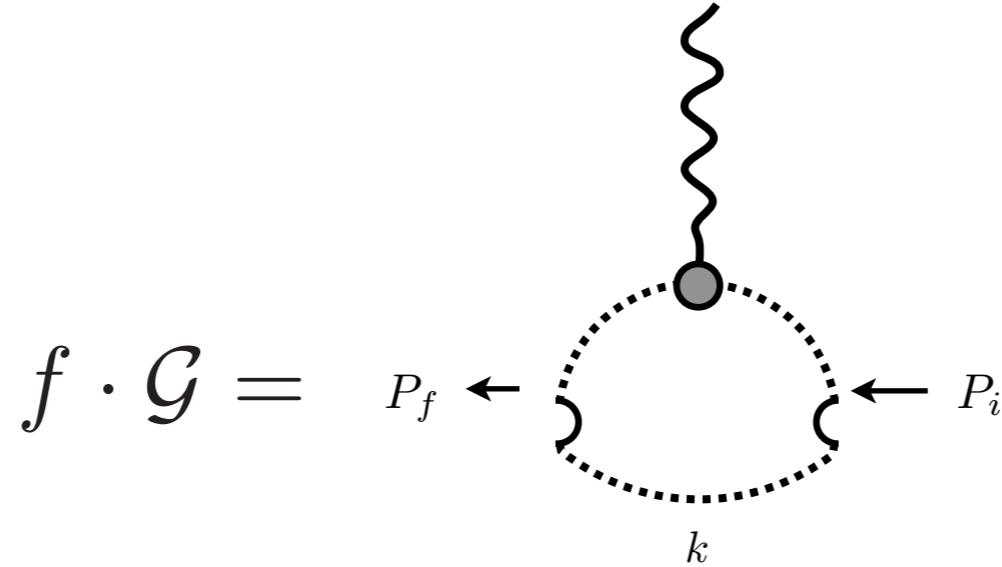
- Single hadron Compton transition in the triangle diagram has to be handled carefully



Summary and outlook

- We presented the analytic decomposition of a set of transitions amplitudes with multi-hadron states.
- The analytic continuation to the complex energy poles of resonances gives access to their form factors.
- These decompositions can also guide the derivation of the finite volume formalism to extract them from lattice QCD.
- Future work on $2 + \mathcal{J} \rightarrow 2 + \mathcal{J}$ and the inclusion of external states with spin.

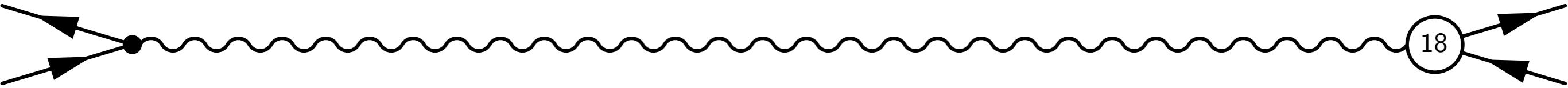
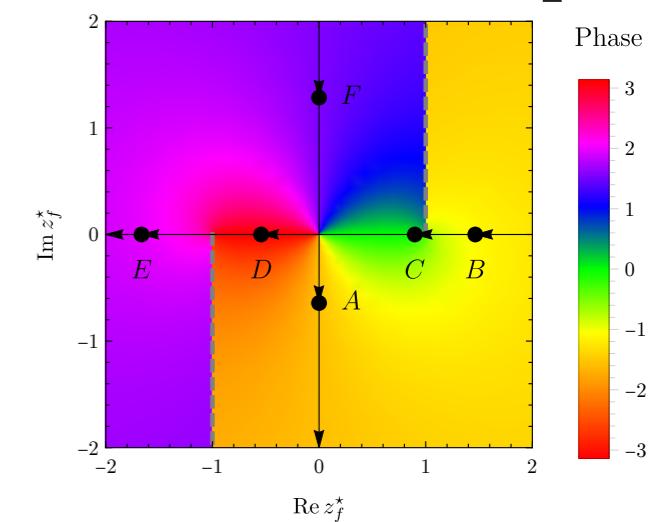
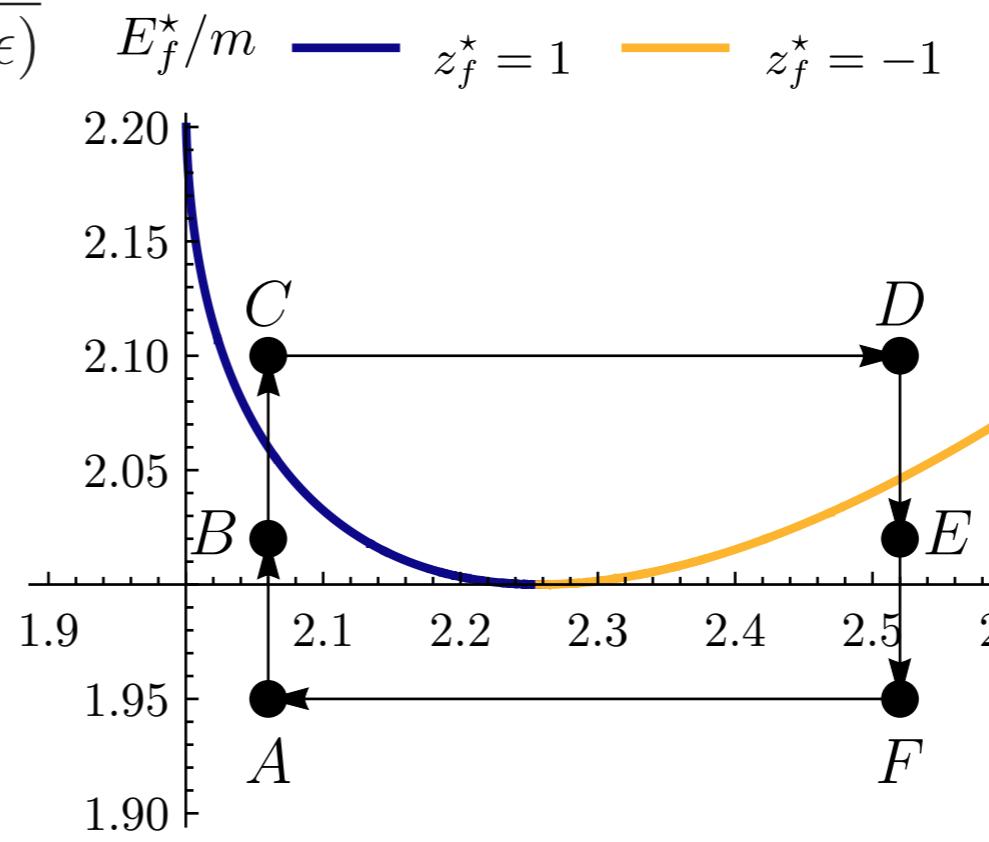
Back up slides



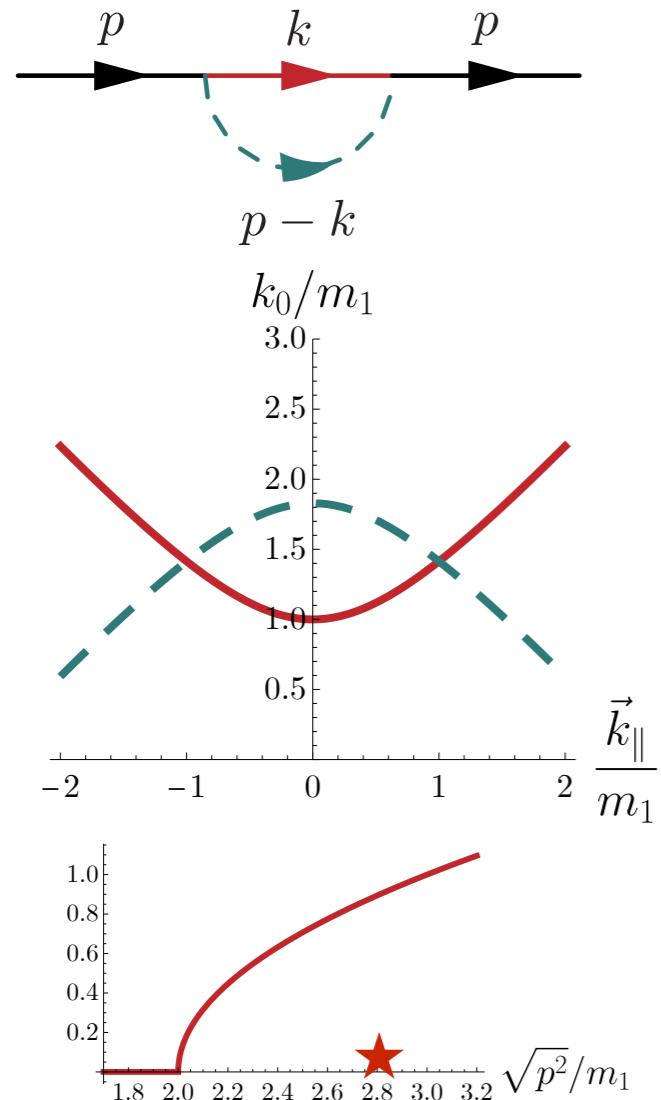
$$\text{Sing } \mathcal{G}(P_f, P_i) = \frac{i}{32\pi \sqrt{(P_f \cdot P_i)^2 - P_i^2 P_f^2}} \left[\log \left(\frac{1 + z_f^* + i\epsilon}{1 - (z_f^* + i\epsilon)} \right) + \log \left(\frac{1 + z_i^* + i\epsilon}{1 - (z_i^* + i\epsilon)} \right) \right]$$

$$D_2(k_i) = \frac{1}{2|\mathbf{P}_{i,f}^*|q_f^*(\cos\theta_f^* - z_f^* + i\epsilon)}$$

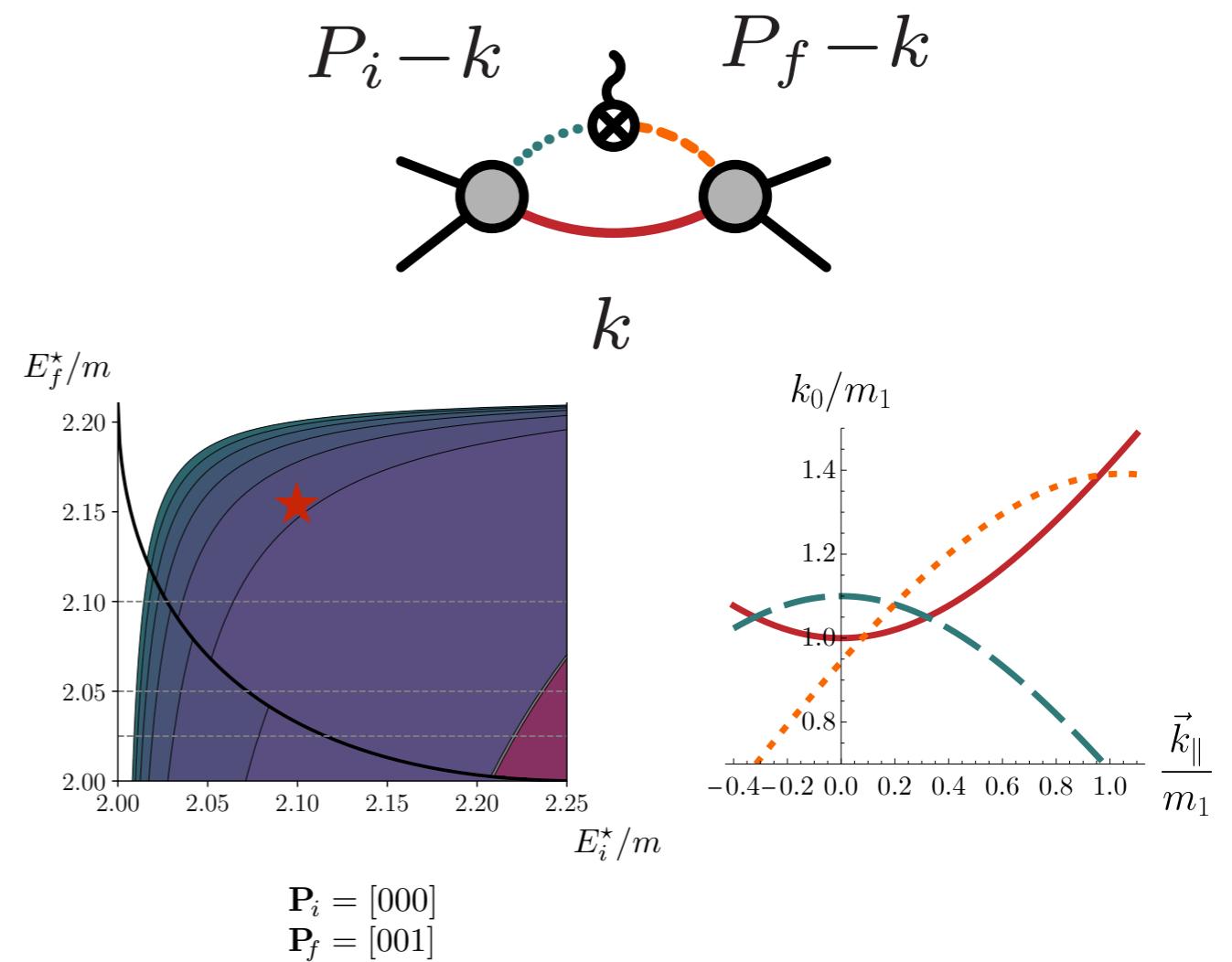
$$z_f^* = \frac{P_i \cdot P_f - s_i}{\frac{2q_f^*}{\sqrt{s_f}} \sqrt{(P_f \cdot P_i)^2 - P_i^2 P_f^2}}$$



Singularities of loop diagrams

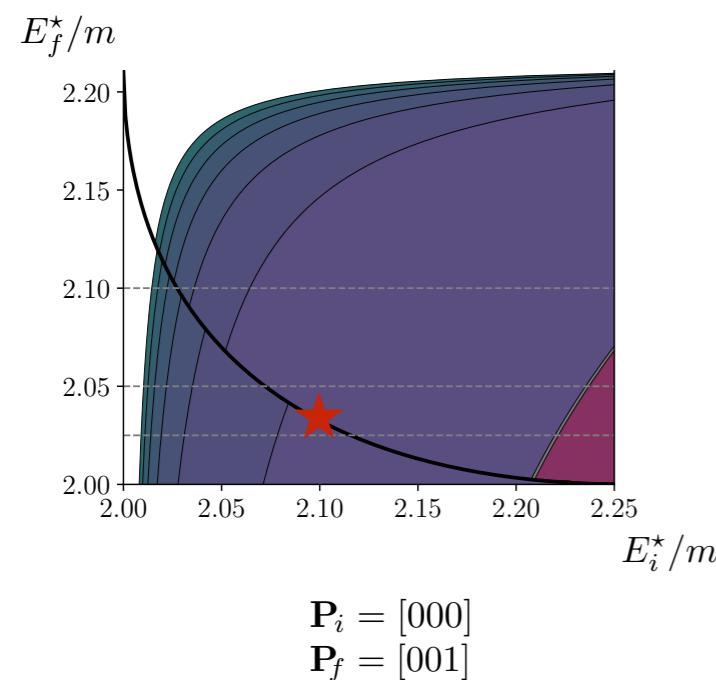


Branch cut: square root singularity

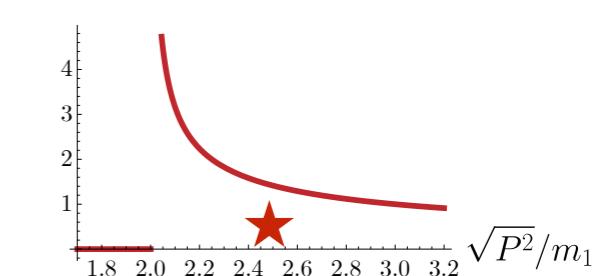
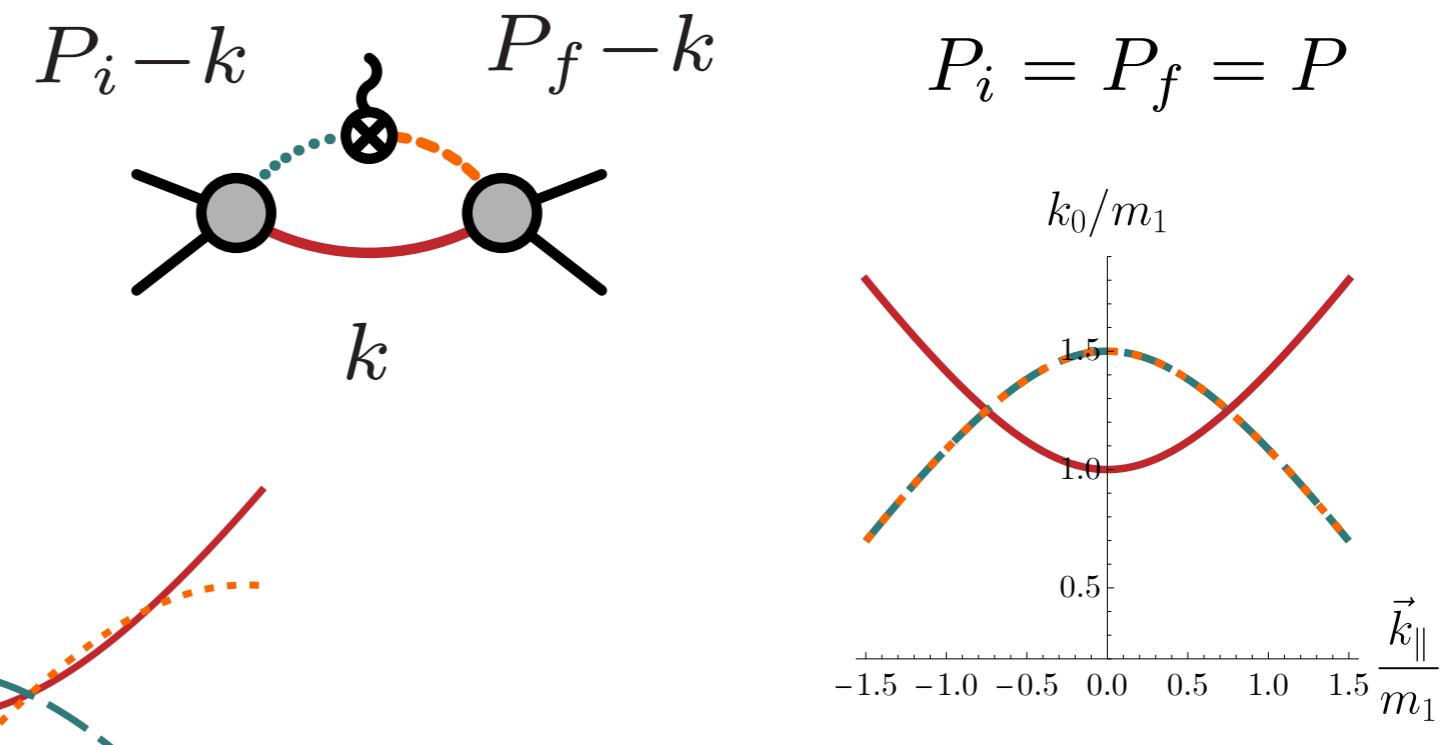
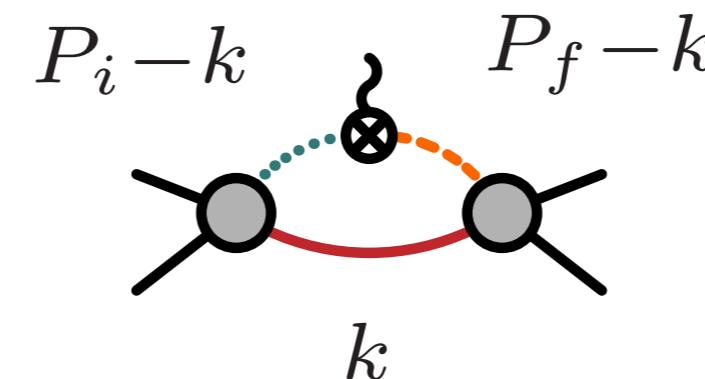
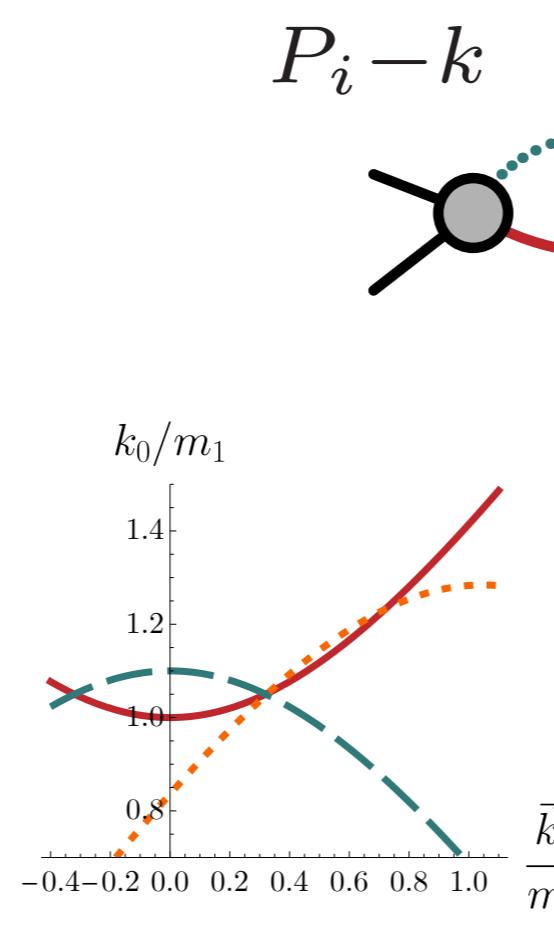


Triangle diagram: poles in k space

Triangle diagram singularities



Logarithmic singularity



Inverse square root singularity