Azimuthal anisotropy in heavy-ion collisions

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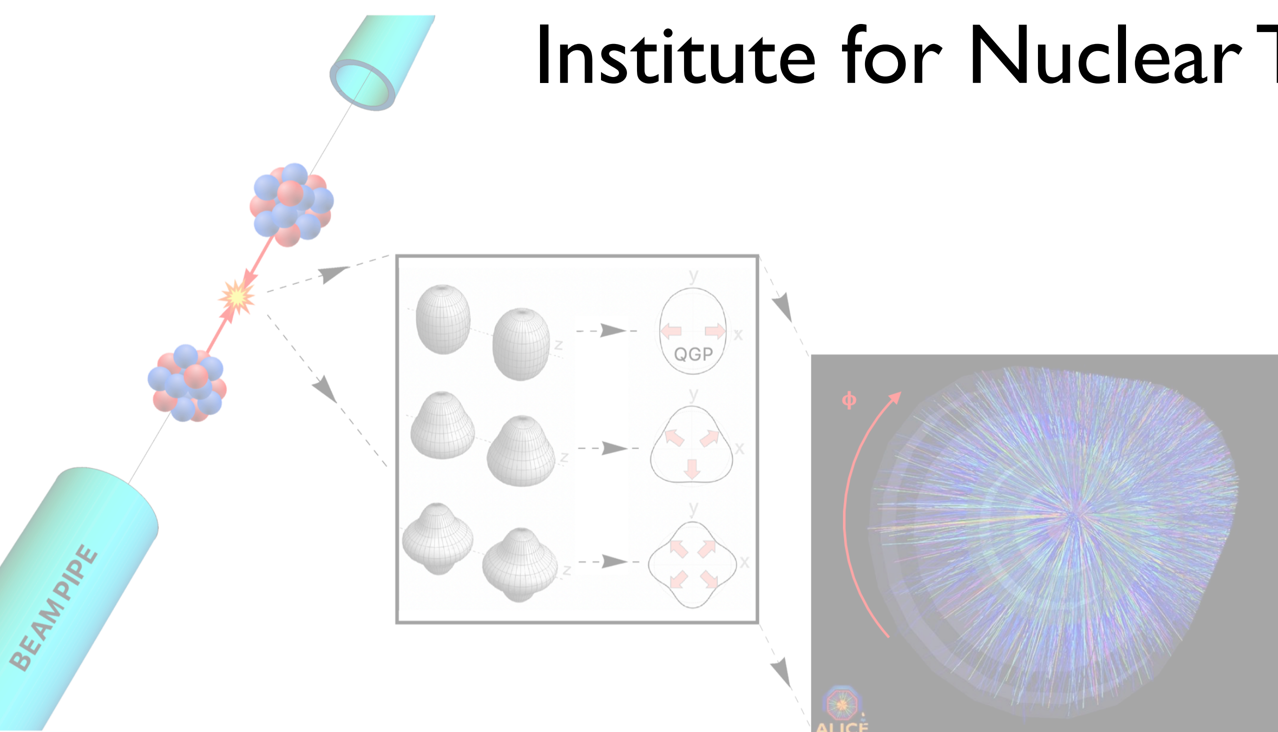
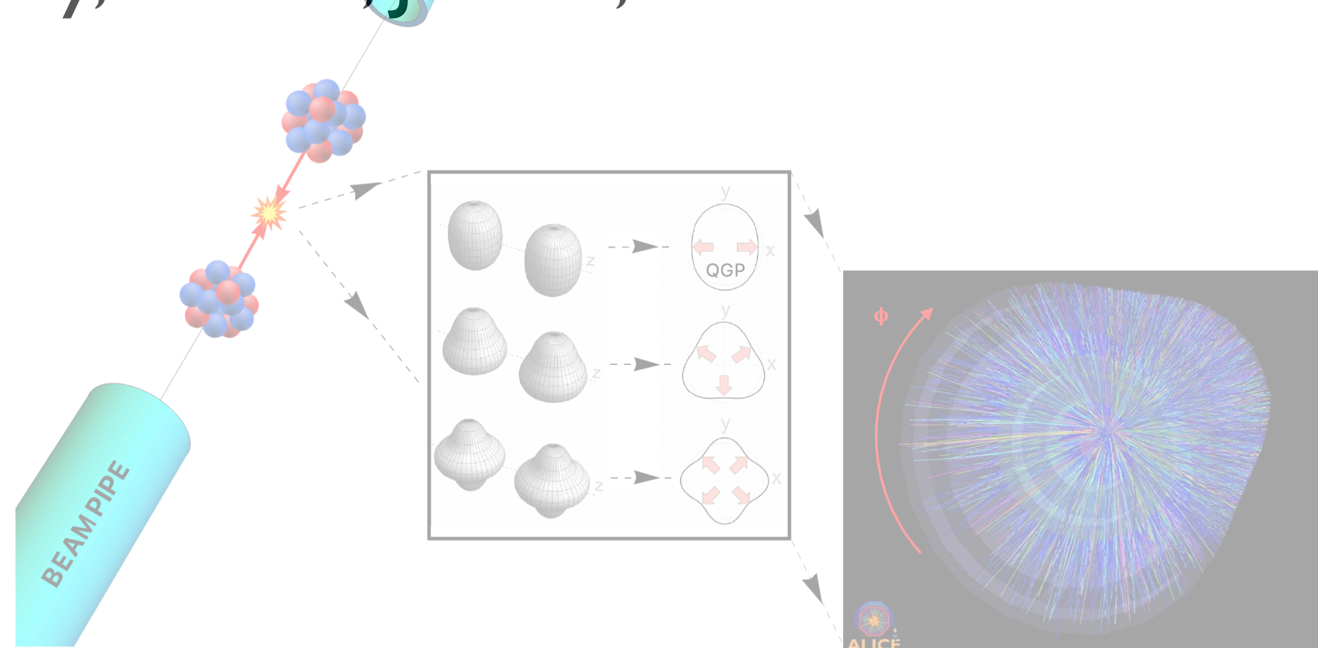
Intersection of nuclear structure
and high−energy nuclear collisions

Institute for Nuclear Theory, Seattle, Jan. 24, 2023





Outline

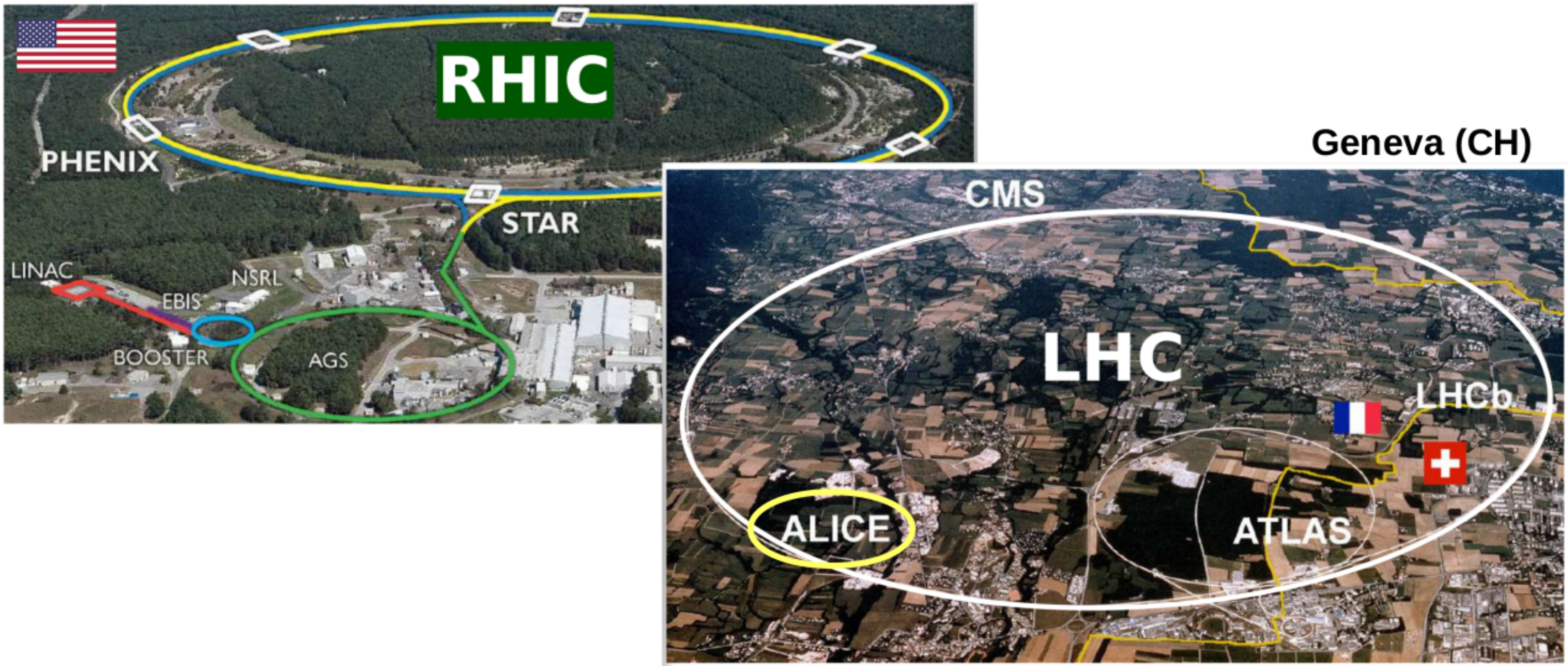
1. Introduction. Why heavy ions.
2. Early days: Elliptic flow in non-central collisions
3. Modern picture of azimuthal anisotropy (2010-)
4. Where to look for influence of nuclear structure
5. Back to early days: Measuring azimuthal anisotropy

Collider experiments with atomic nuclei

(ultra-relativistic heavy-ion collisions)

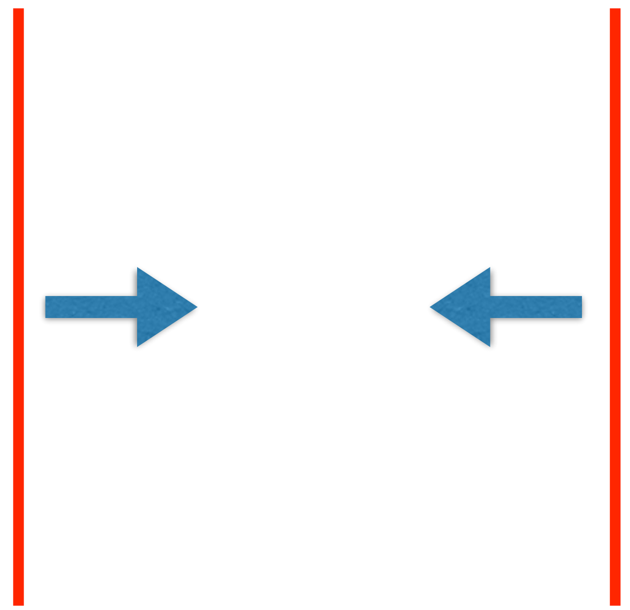
Long Island (NY)

Geneva (CH)



- Program started around year 2000 (RHIC) and 2010 (LHC).
- RHIC dedicated to nuclear collisions.
- Nuclei collided ~1 month/year @ LHC.

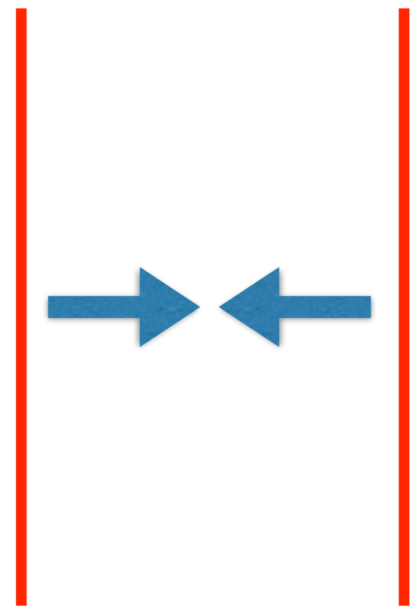
Central Pb+Pb collision at the LHC



Relativistic length contraction in the direction of motion, by a factor ~2700 at LHC

➔ Colliding nuclei appear as disks

Central Pb+Pb collision at the LHC



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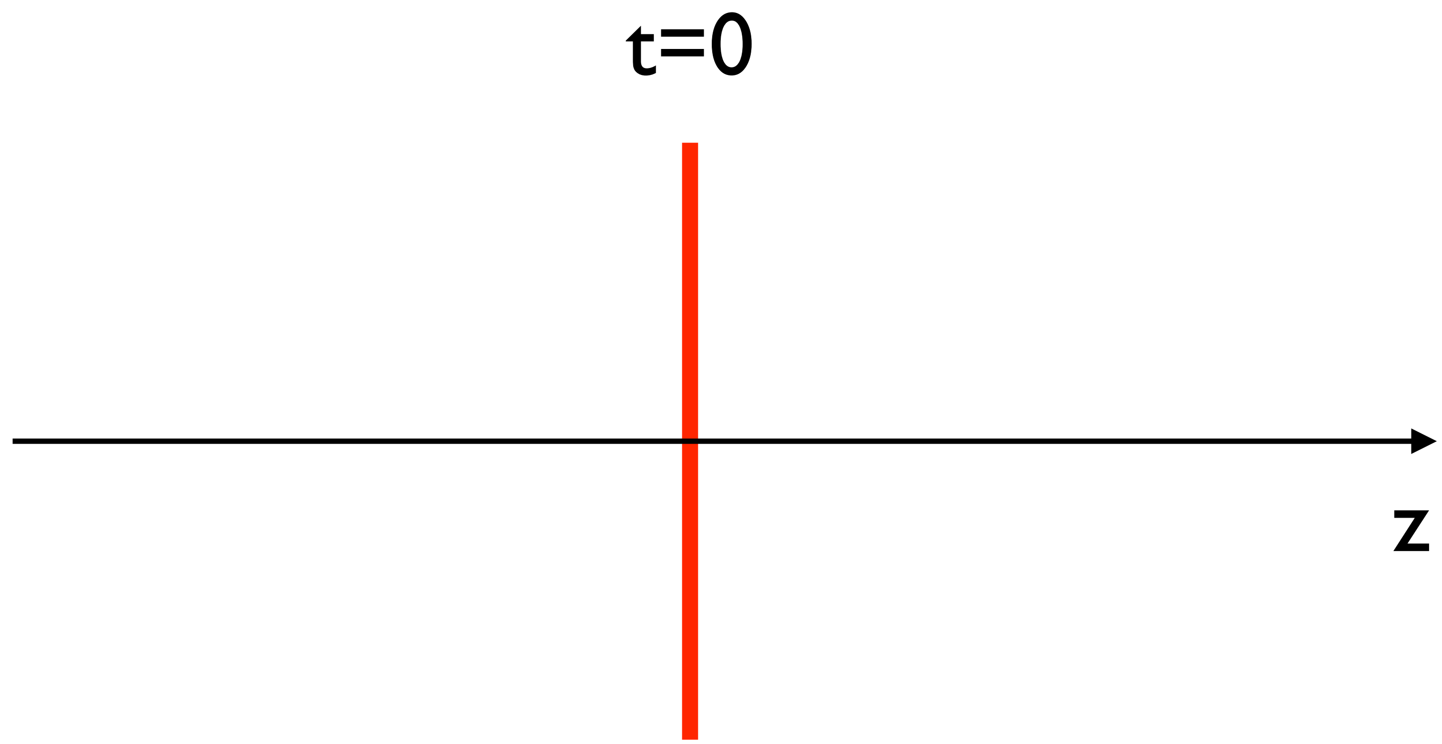
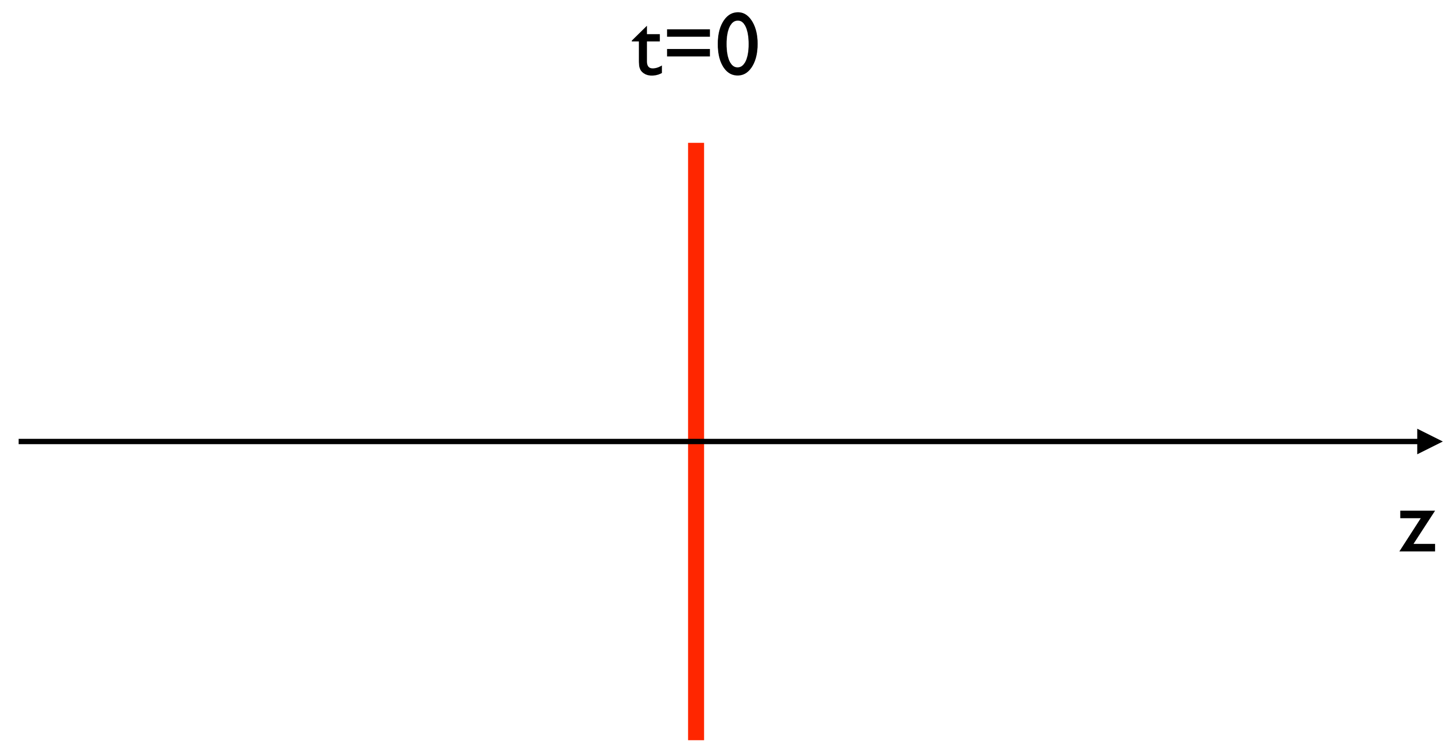


Collision = instantaneous process at z=t=0

How instantaneous ? $t_{coll} = 2 \times 10^{-26}$ s

Strong interactions, local in « transverse » (x,y) plane

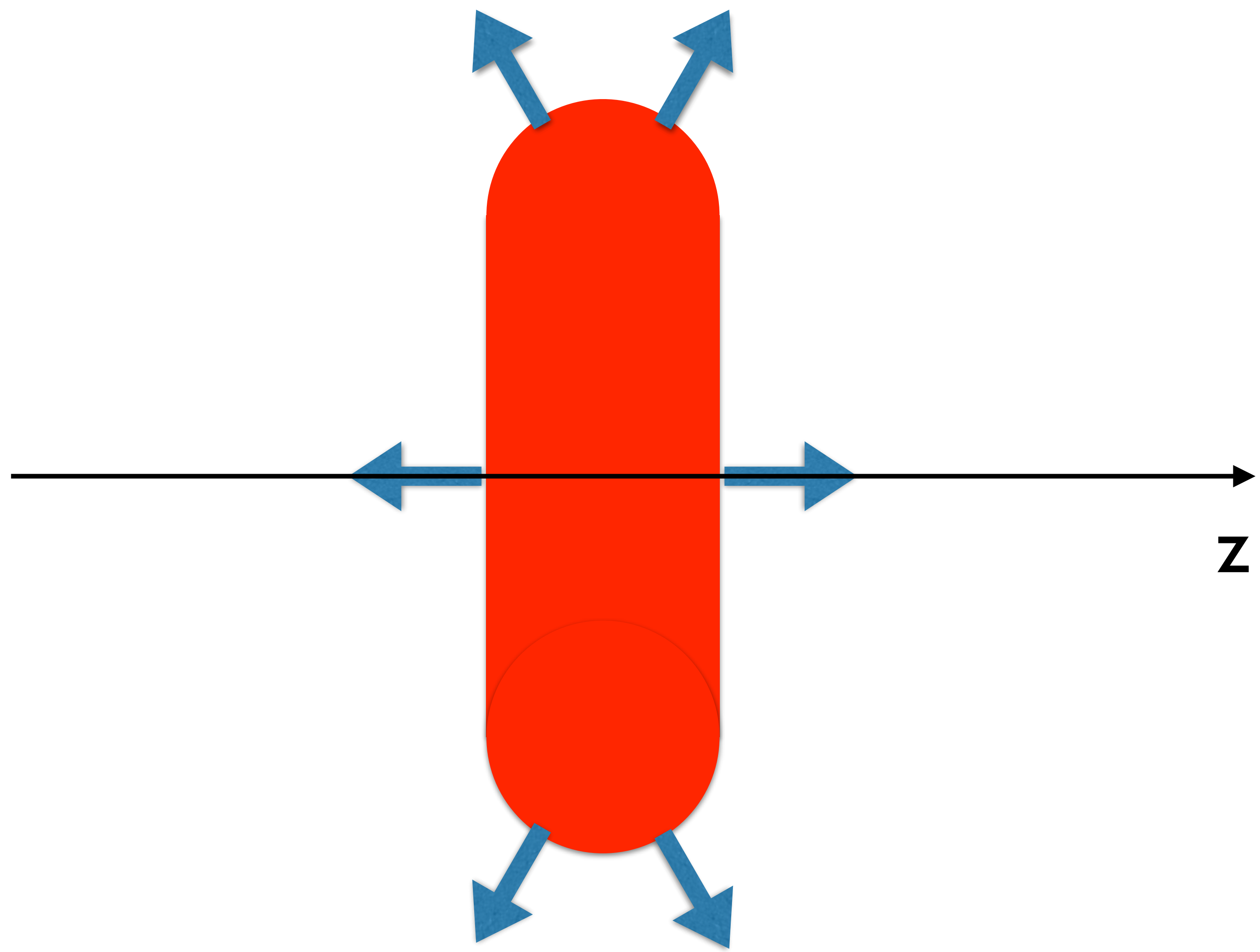
Intersection with nuclear structure?



The collision takes a snapshot of the local density in the 2 nuclei at t=0. Quantum fluctuations = essential

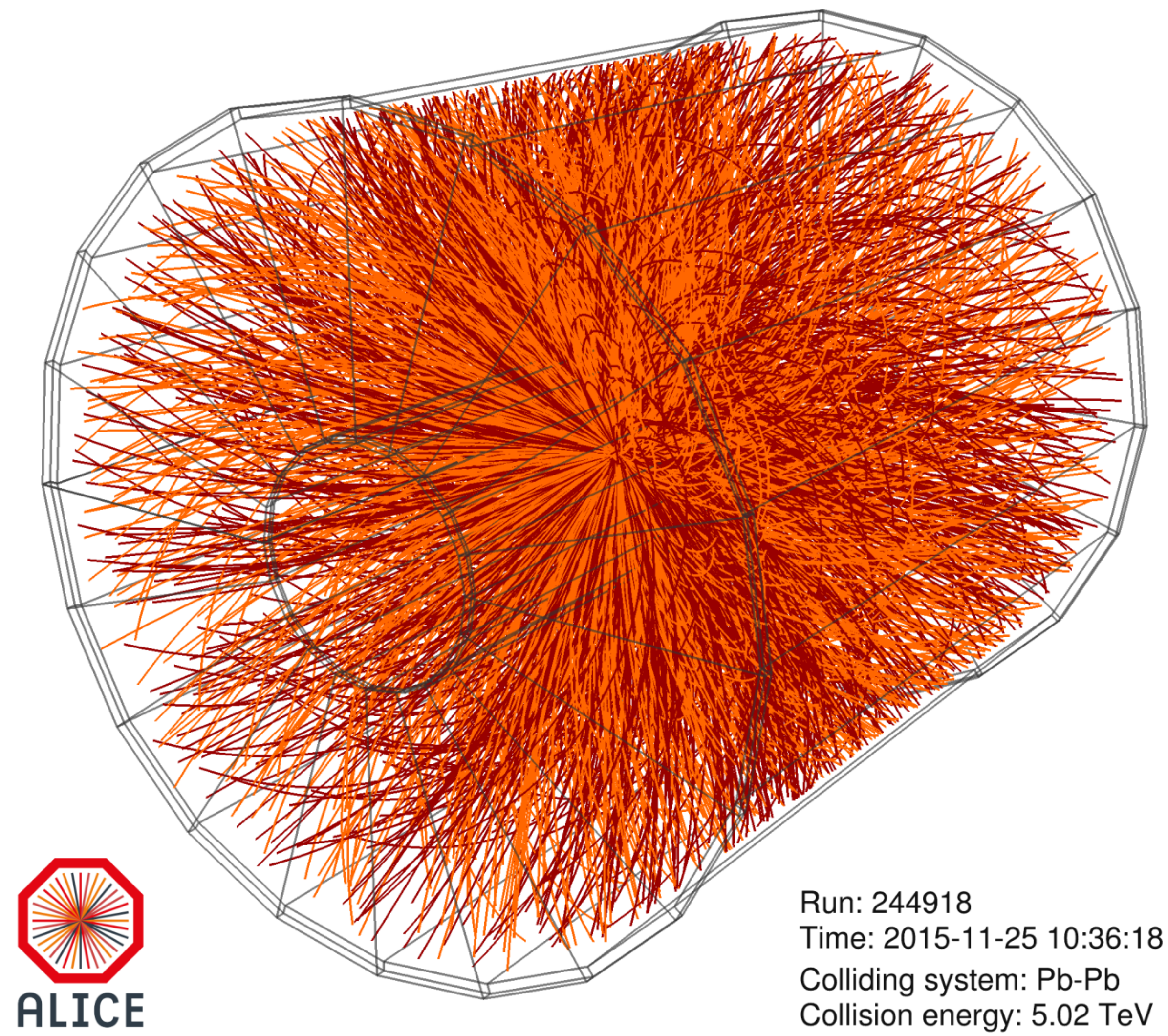
The density is projected onto the transverse plane (integral along z at fixed x,y)

Central Pb+Pb collision at the LHC



- Strongly-coupled quark-gluon matter is created.
- Expands into the vacuum at ~ velocity of light

Central Pb+Pb collision at the LHC



A single Pb+Pb collision *creates* ~35000 particles:

~10000 π^+, ~10000 π^0, ~10000 π^-, and a few thousand heavier hadrons (K, p, n).

Intersection with nuclear structure?

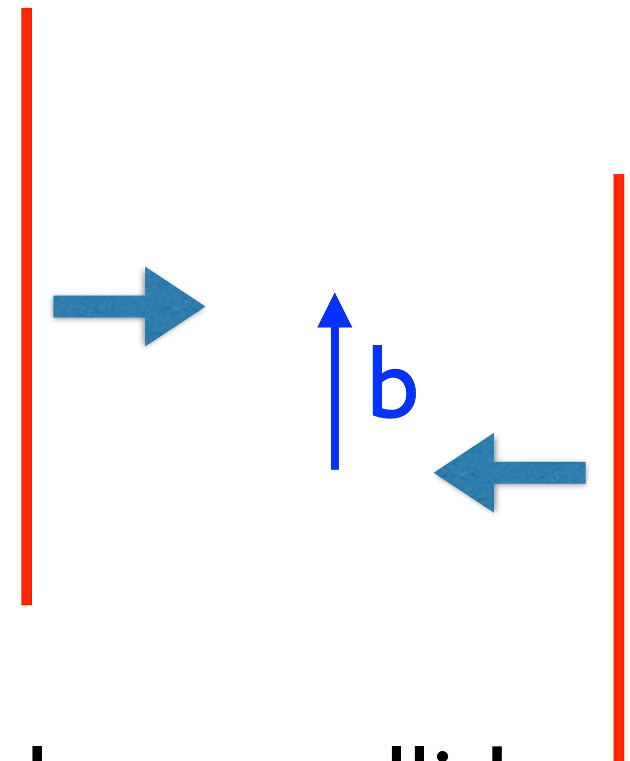
Shape described by similar quantities, but in 2 dimensions

Nuclear structure	Heavy-ion collisions
Multipole moments (θ,φ)	Fourier harmonics (φ)
Quadrupole moment Q_{20}	Elliptic flow v_2
Octupole deformation	Triangular flow v_3
Hexadecapole…	Quadrangular flow v_4 …

Crucial advantage of high-energy collisions over low-energy experiments = huge multiplicity: *More information is available.*

Why high multiplicity (N) matters

Simple example: determination of impact parameter b



Larger b : not all nucleons collide ⇒ smaller N on average.

Sorting collisions according to N is equivalent to sorting them according to b (in opposite order) within ~1% at the LHC. Excellent determination of b thanks to high N.

Note: Better detectors would not improve this, as fluctuations of N at fixed b are dominated by quantum fluctuations.

Why the 1st talk is about azimuthal anisotropy

- Azimuthal anisotropy, a.k.a. anisotropic flow, is the phenomenon through which it was shown, ~20 years ago, that a heavy-ion collision produces a tiny droplet of relativistic fluid expanding into the vacuum, which can be modeled reliably.
- Azimuthal anisotropy is typically enhanced if nuclei are deformed, hence the connection to nuclear structure.
- The information from heavy-ion collisions that is relevant for nuclear structure can likely be encapsulated into a few quantities (my prejudice): azimuthal anisotropies + mean transverse momentum of outgoing particles (see talk by Giuliano Giacalone yesterday).

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A bit of history: Heavy ions in the 1990s.

Fixed-targed relativistic heavy-ion collisions started at Brookhaven (AGS) and CERN (SPS), first with light projectiles, O, S, Si (1986-7) then with heavy projectiles, Au, Pb (1992-4).

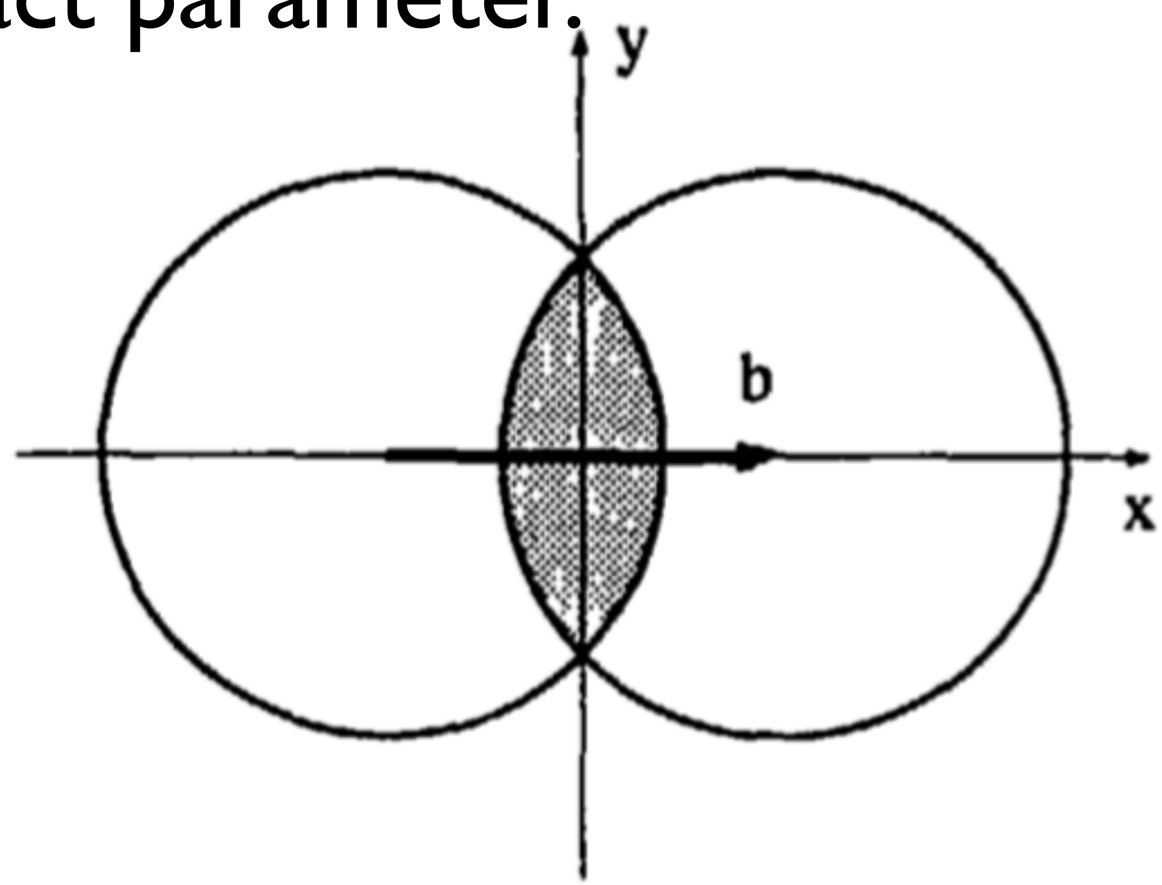
On the theory side, several groups (including myself) had developed hydrodynamic codes to model these collisions. But there was not the slightest hint from experiment that hydrodynamics made any sense in this context.

Hydro codes were typically written for central collisions only (b=0), to make use of azimuthal symmetry, but the excitement on the experimental side was all about how observables depended on b.

Azimuthal anisotropy in non-central collisions

JYO, Phys. Rev. D 46, 229 (1992)

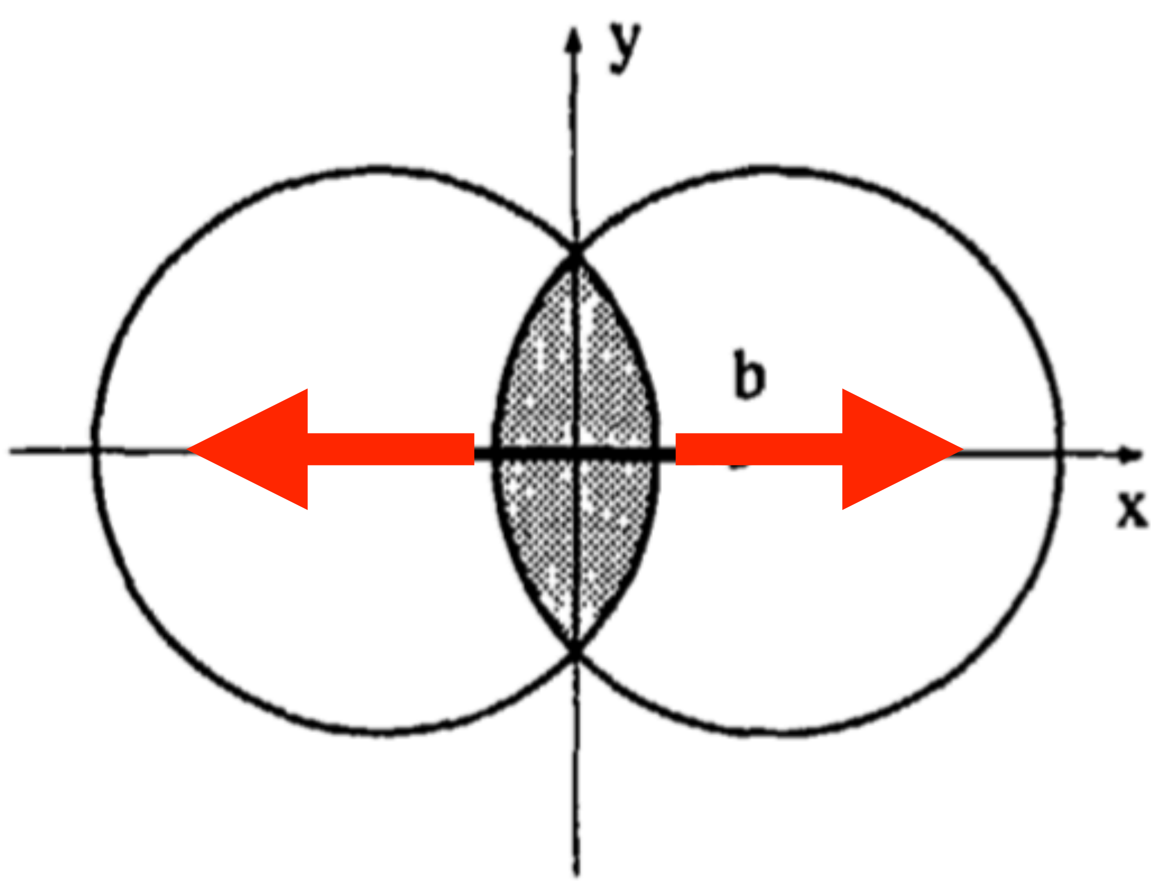
Before writing a hydrodynamic code for non-central collisions, I tried to figure out if it was worth the trouble, and whether one would see something new just by increasing impact parameter.



Azimuthal anisotropy in non-central collisions

JYO, Phys. Rev. D 46, 229 (1992)

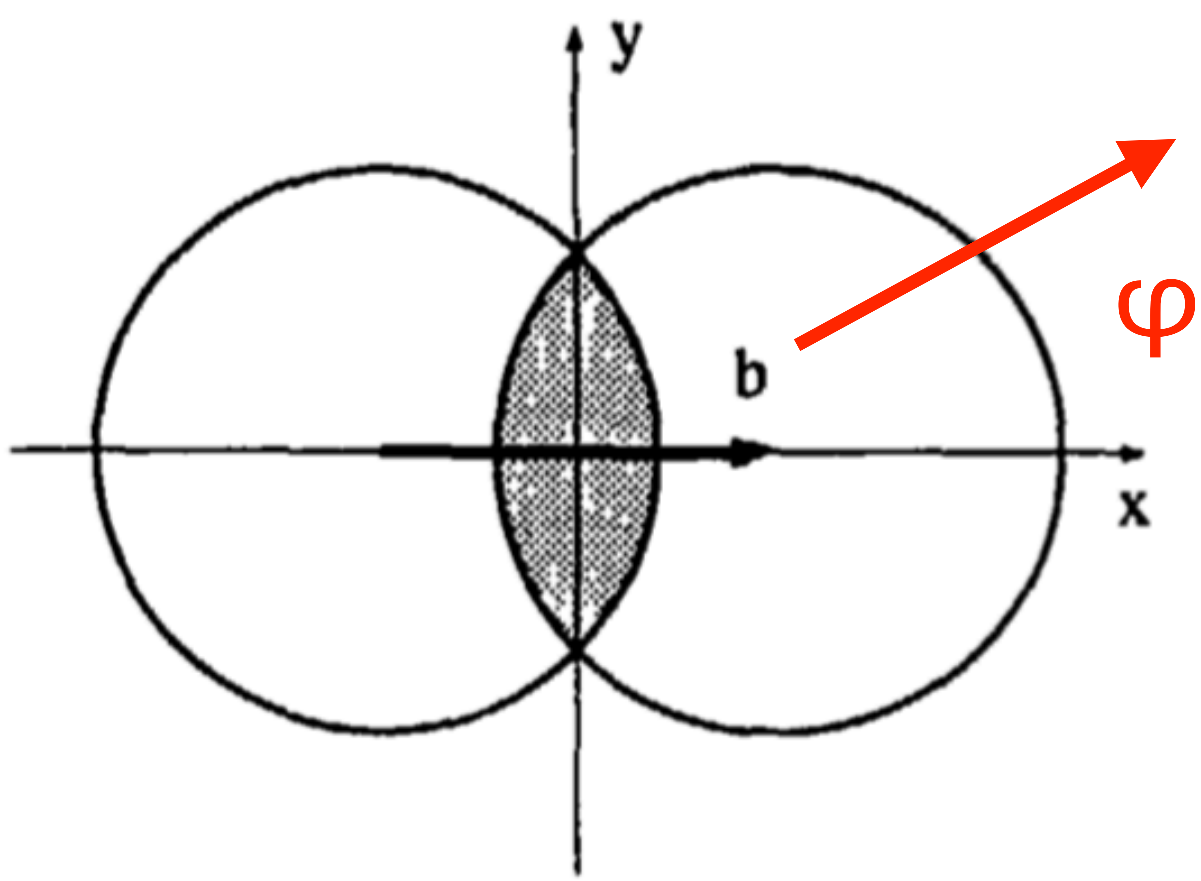
In hydrodynamics, fluid acceleration is proportional to pressure gradient: Larger acceleration along smaller dimension x. Azimuthal anizotropy is generated.



Elliptic flow v_2

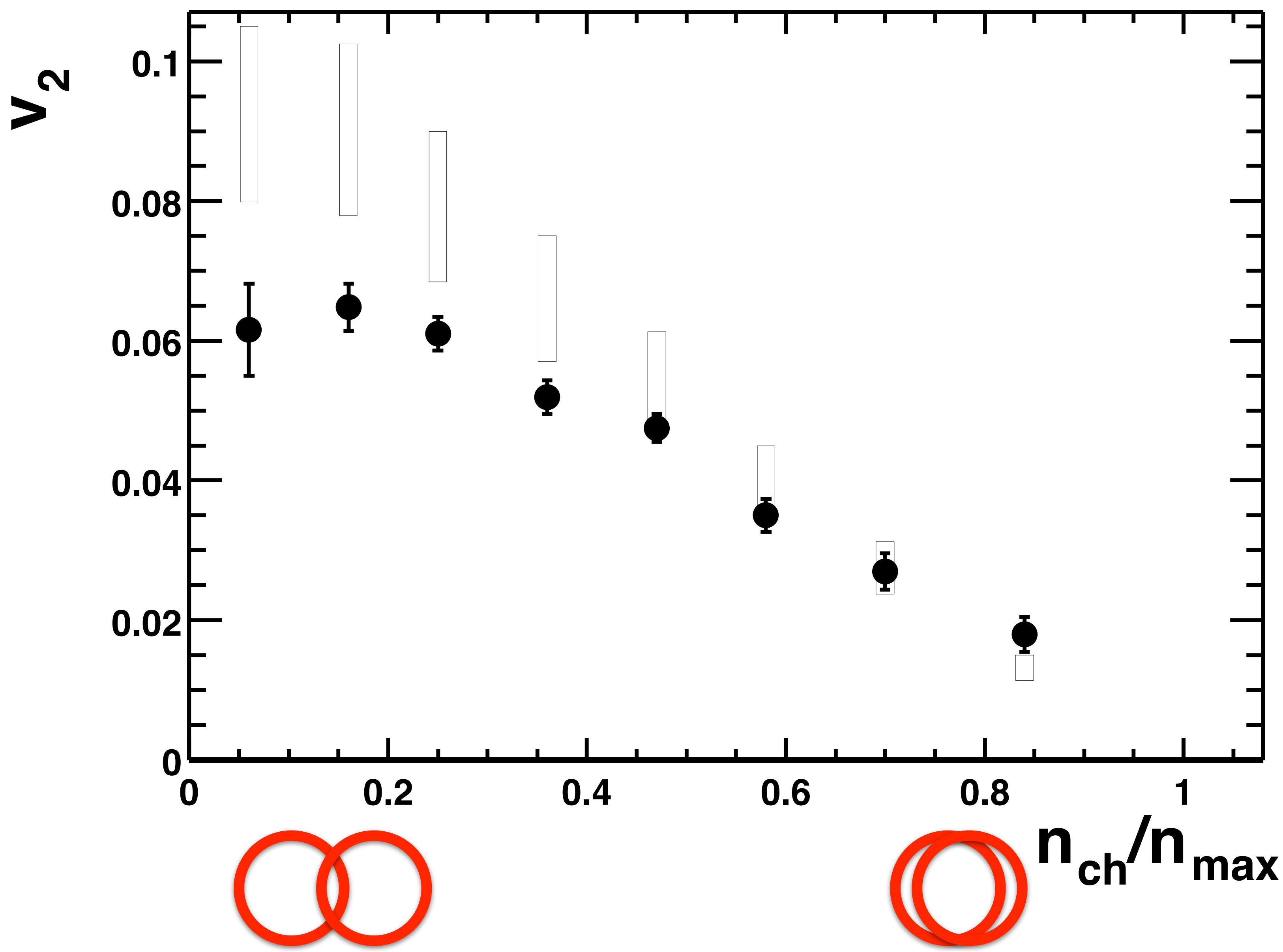
Voloshin Zhang , hep-ph/9407282

The magnitude of this anisotropy can be characterized by v_2 ≡ average over all particles of $\cos 2\varphi$:
>0 along x, <0 along y



Elliptic flow at RHIC

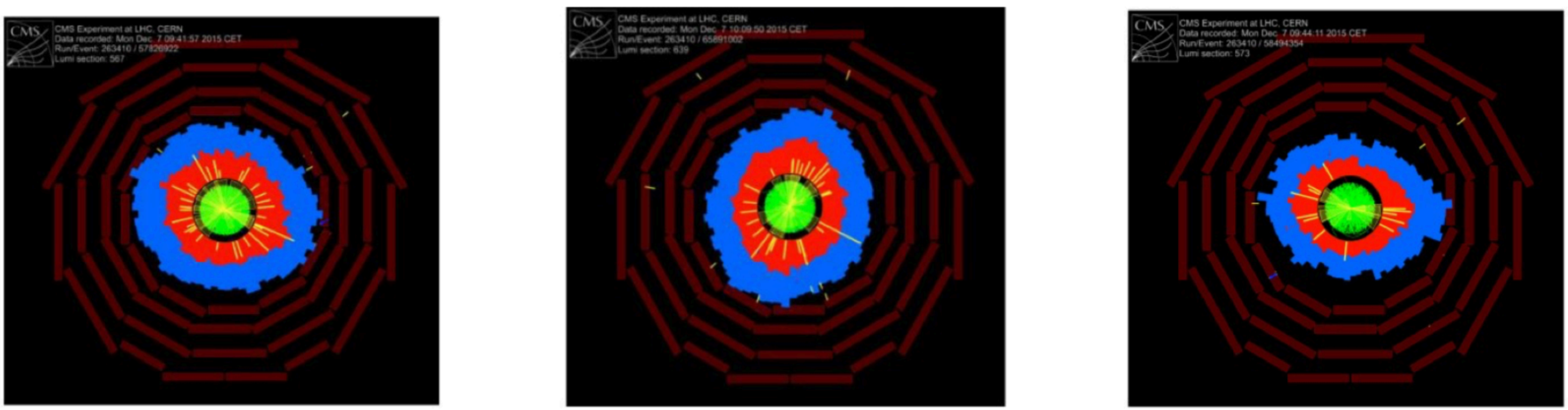
STAR nucl-ex/0009011



The observation of a large v_2, compatible with hydrodynamic predictions, came as a surprise, and soon established hydrodynamics as the only way of modeling the expansion

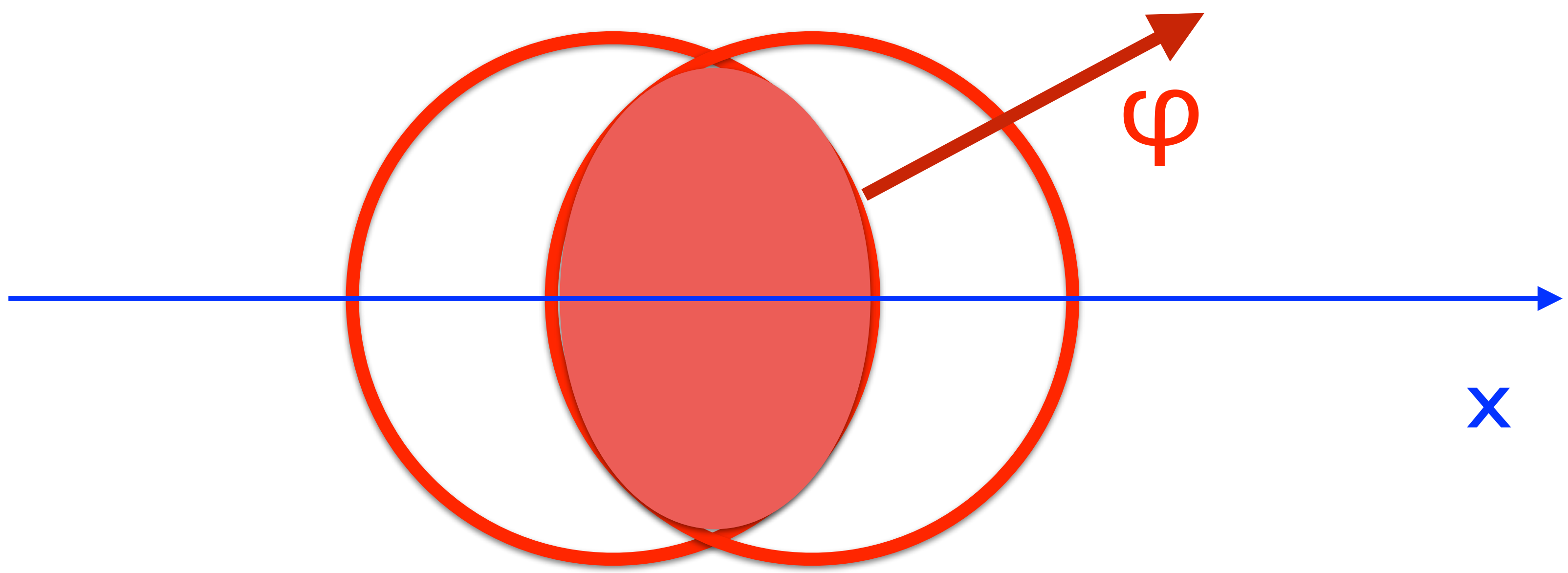
Elliptic flow at the LHC

Not significantly larger than at RHIC, but the detectors see so many particles (due to larger multiplicity at higher energy, and better coverage in polar angle θ) that you see it by eye.



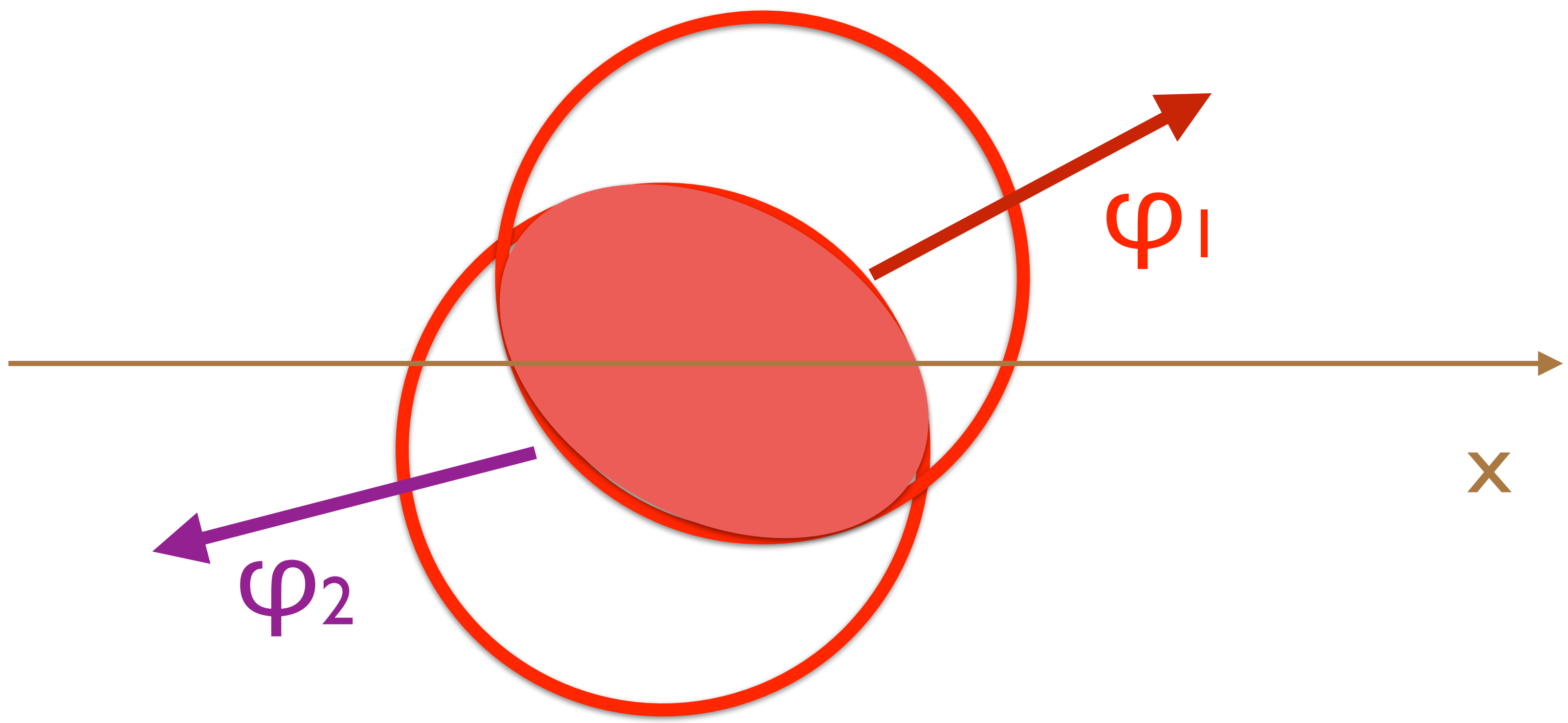
3 Pb+Pb collisions seen in the CMS detector.
(shown by Georgios Konstantinos Krintiras, Moriond 2021)

Measuring elliptic flow



Trivial in intrinsic frame where x is along impact parameter.
Just take average over all particles: $v_2 = \langle \cos 2\varphi \rangle$

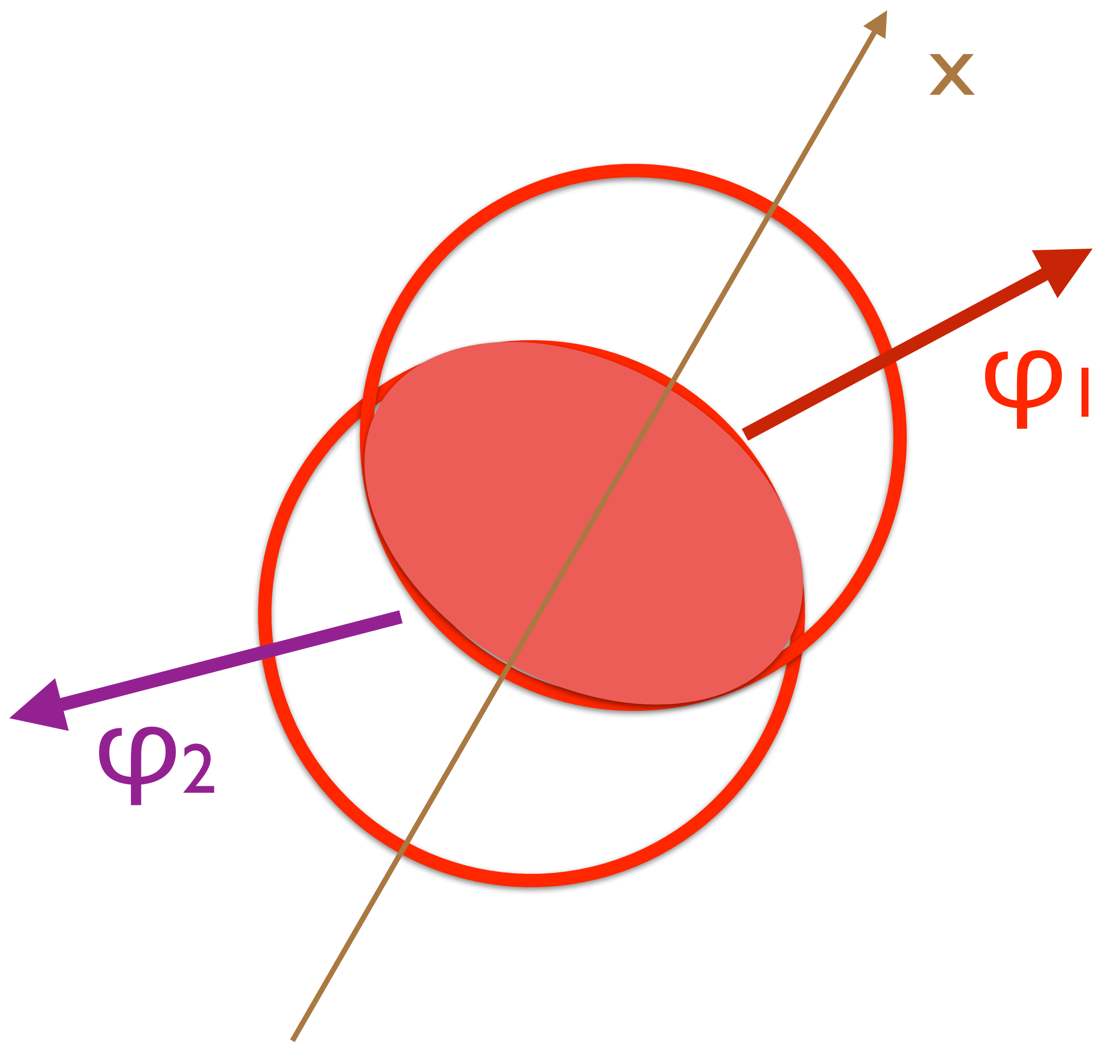
Measuring elliptic flow



In the laboratory frame, orientation of impact parameter is random. One measures instead a pair correlation : average over all pairs of $\cos(2\varphi_1-2\varphi_2)$ or $\exp(2i\varphi_1-2i\varphi_2)$.

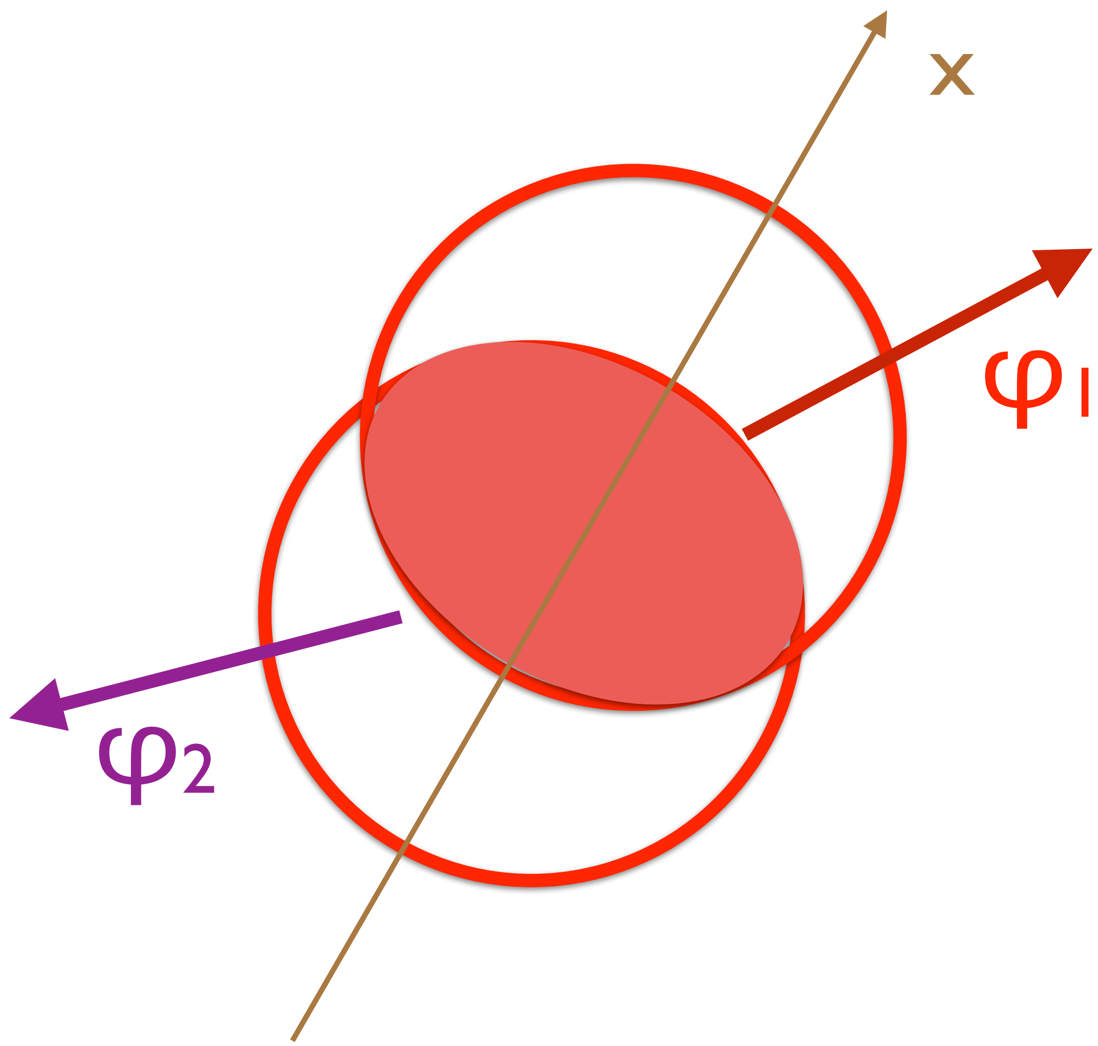
Depends only on difference of angles: frame independent.

Measuring elliptic flow



Evaluate correlation in intrinsic frame.
If particles are independent in the intrinsic frame, then
$<\exp(2i\varphi_1-2i\varphi_2)> = <\exp(2i\varphi_1)><\exp(-2i\varphi_2)> = (v_2)^2$.
This is how v_2 is measured in practice. Note: sign not measured.

Measuring elliptic flow



Two-body distributions are the best way of seeing intrinsic shapes, not only in nuclear structure, but also in heavy-ion collisions!

Evaluate correlation in intrinsic frame.
If particles are independent in the intrinsic frame, then
$\langle \exp(2i\varphi_1 - 2i\varphi_2) \rangle = \langle \exp(2i\varphi_1) \rangle \langle \exp(-2i\varphi_2) \rangle = (v_2)^2$.
This is how v_2 is measured in practice. Note: sign not measured.

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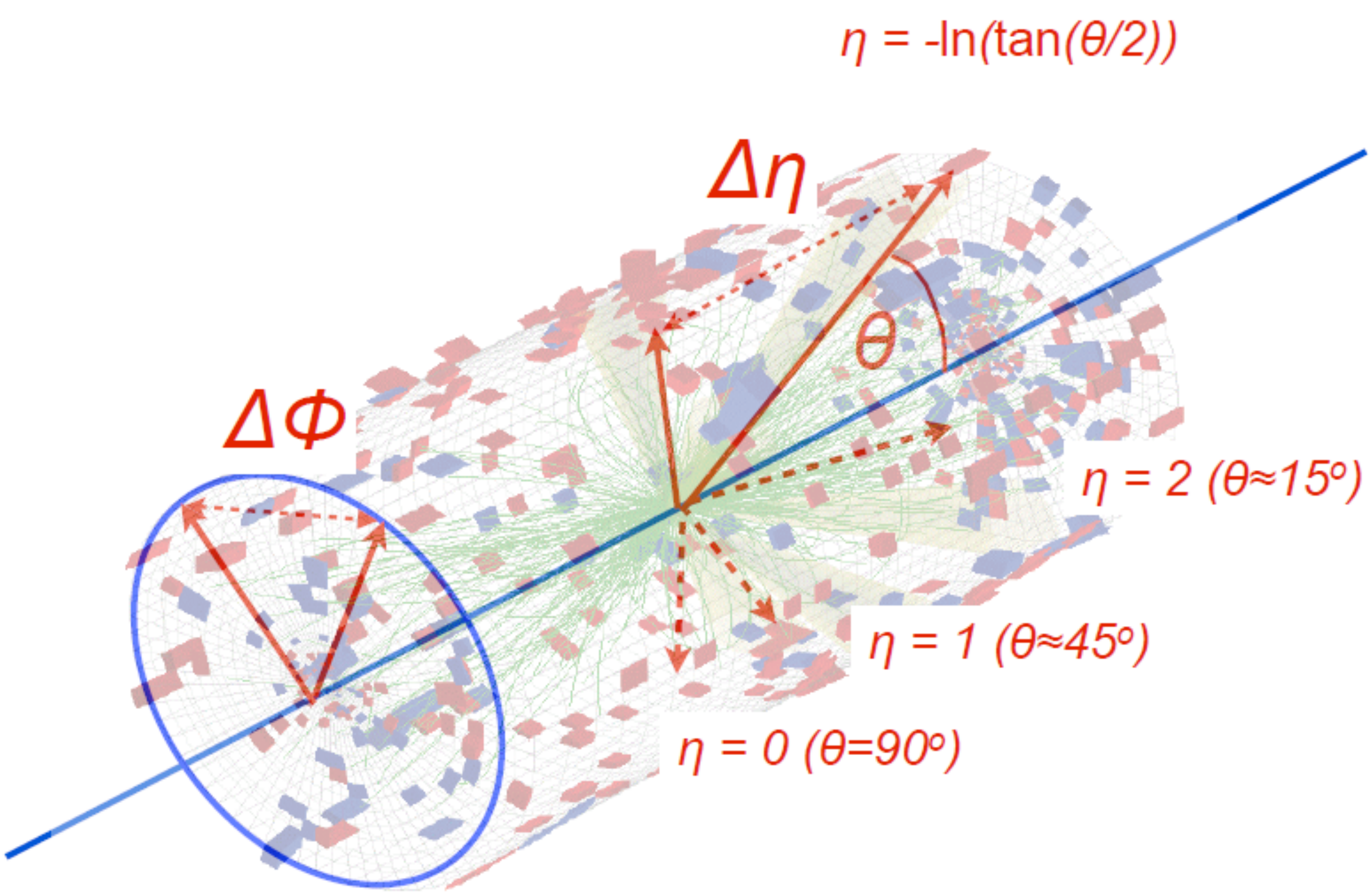
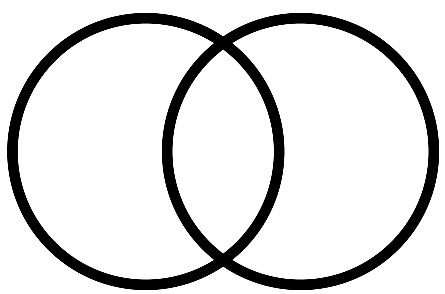
Looking at pair correlations

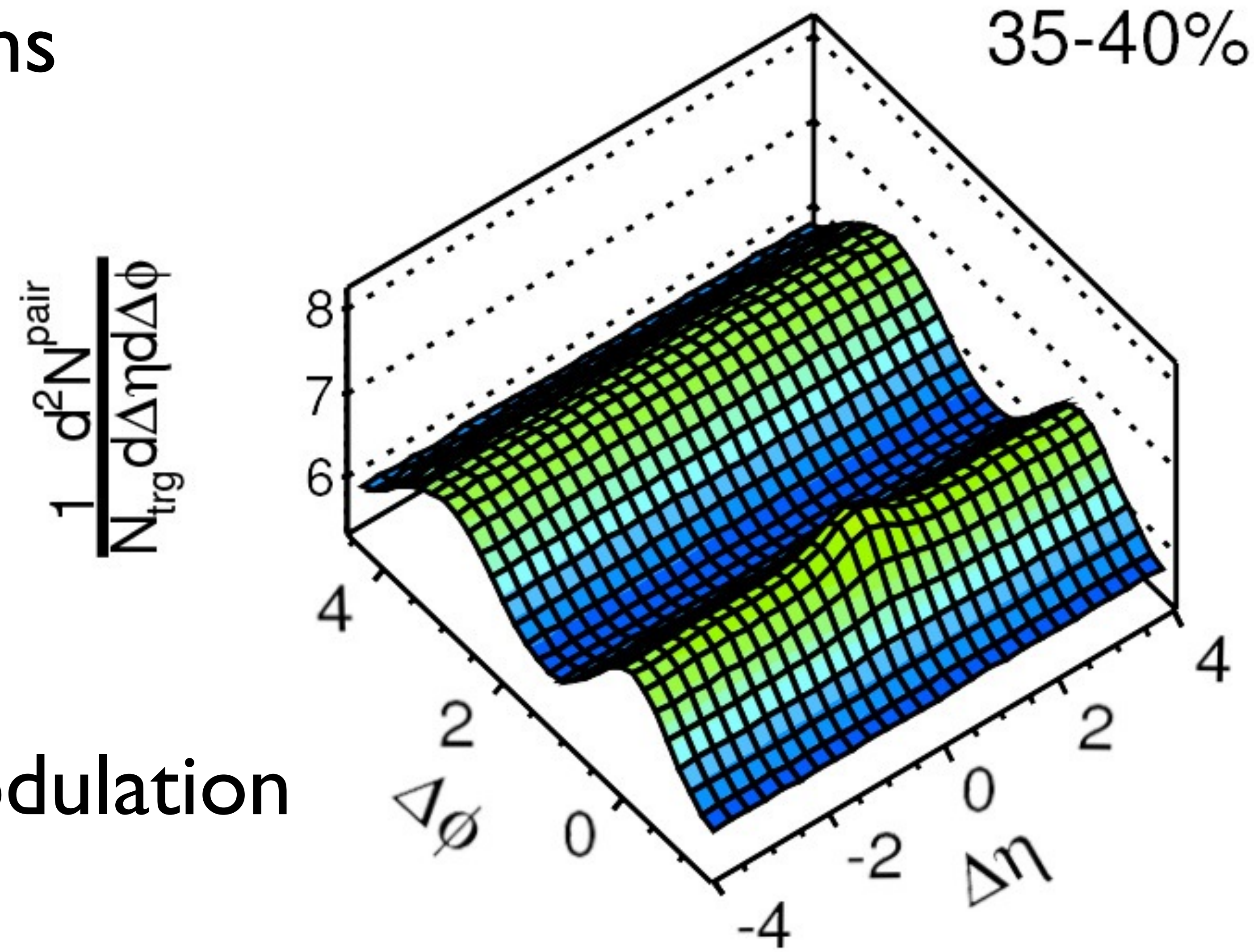


For each pair of particles, one can measure not only the relative azimuthal angle $\Delta\varphi=\varphi_1-\varphi_2$, but also the relative polar angle θ, or pseudorapidity η: measure also $\Delta\eta=\eta_1-\eta_2$

Correlations in Pb+Pb collisions

Semi-central collisions

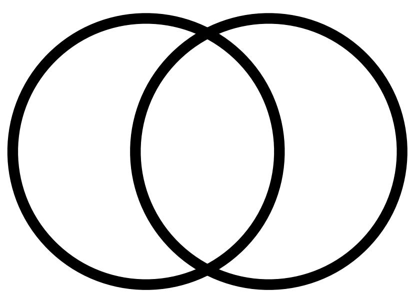


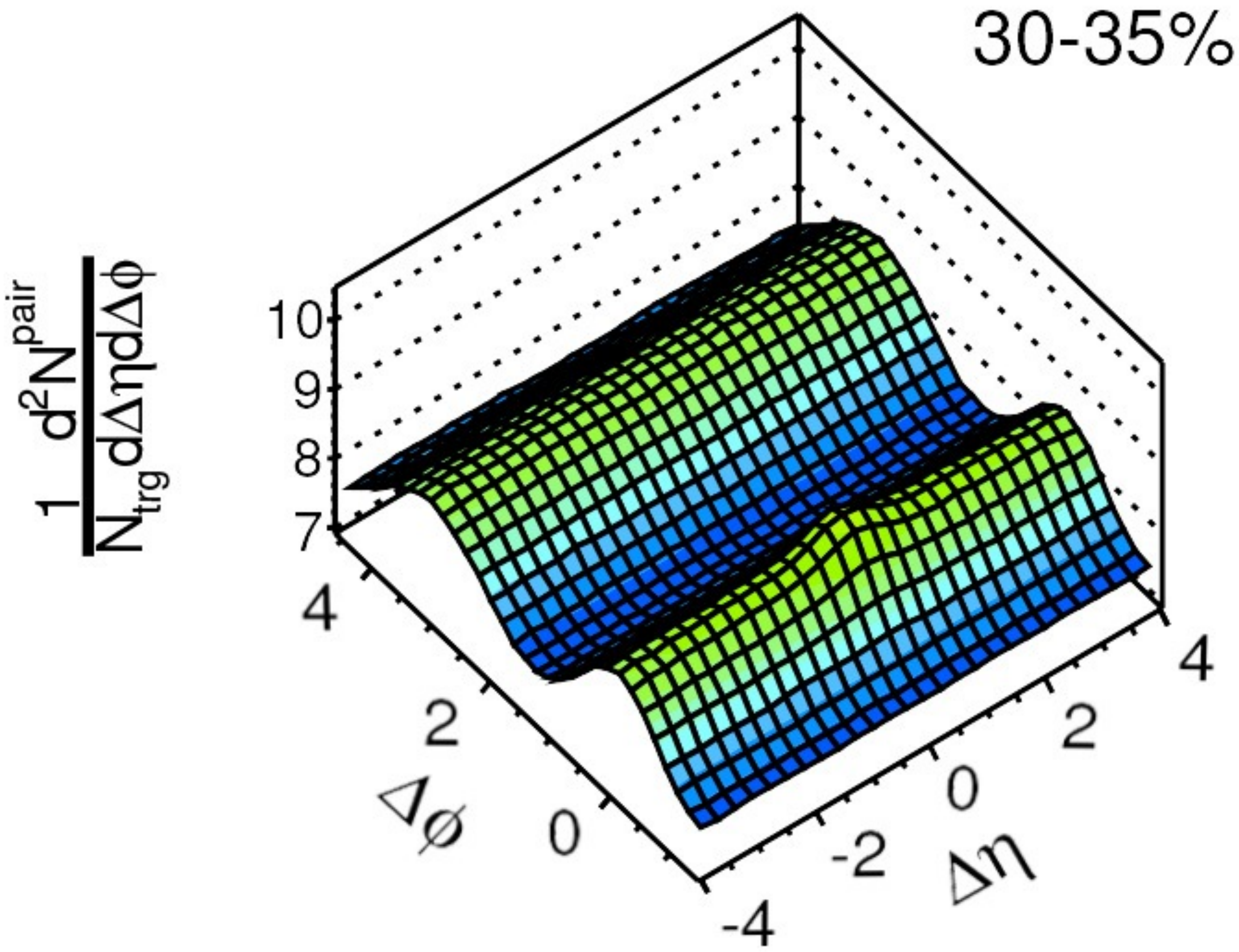


Mostly a cos(2Δφ) modulation due to elliptic flow, independent of Δη: Longitudinal invariance.

CMS 1201.3158

Correlations in Pb+Pb collisions





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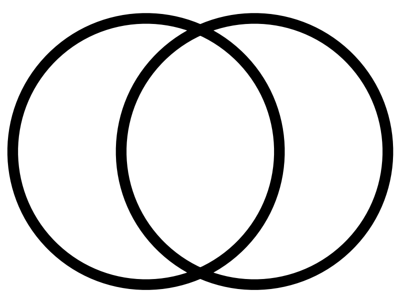

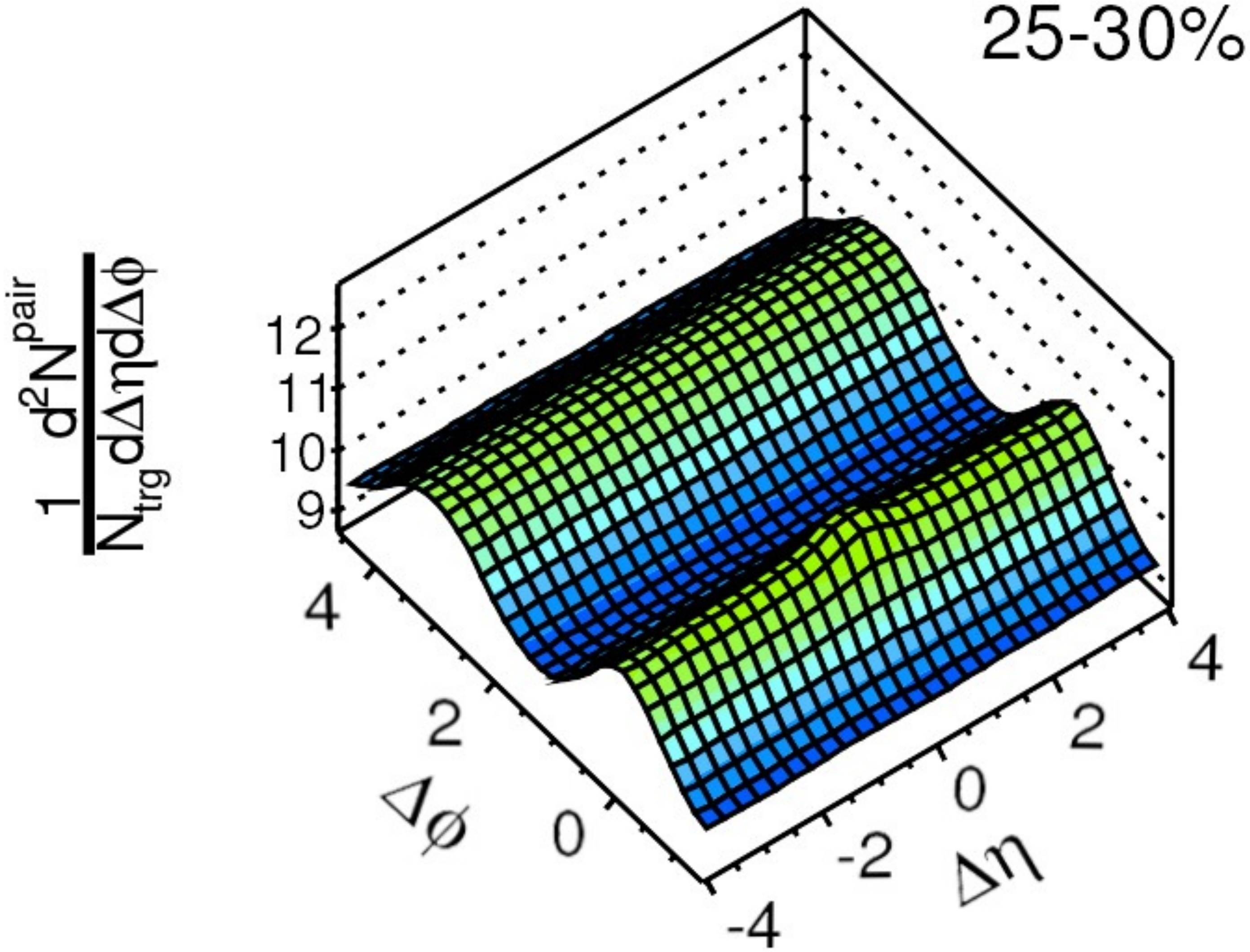

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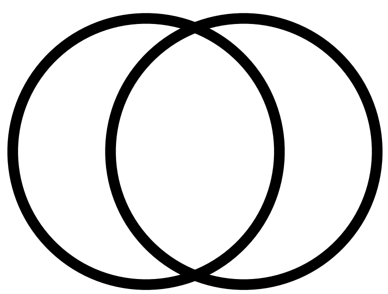

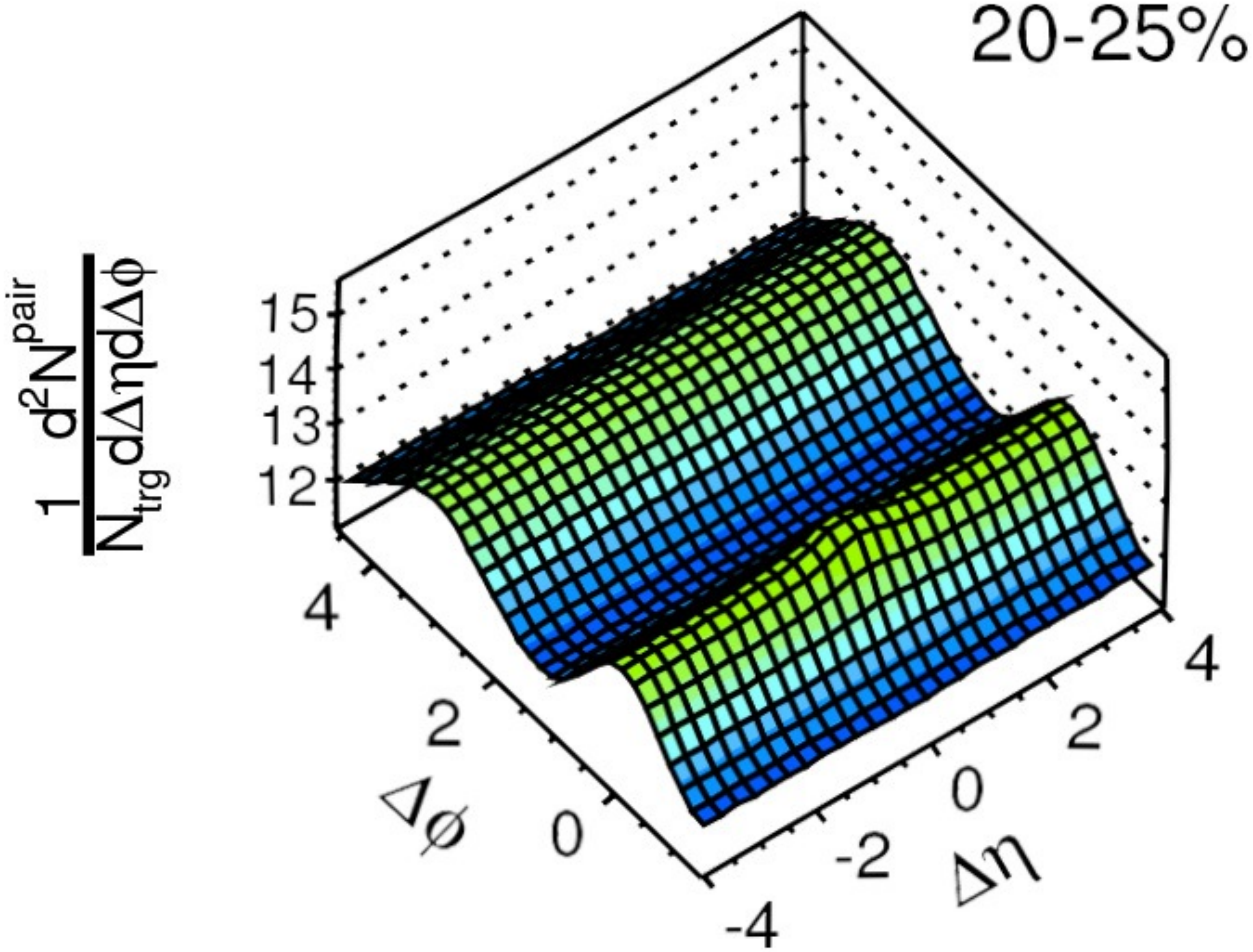

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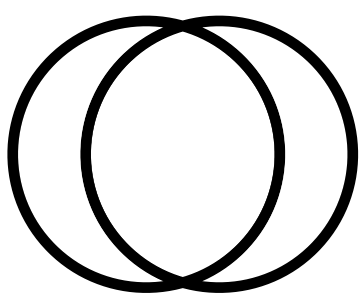

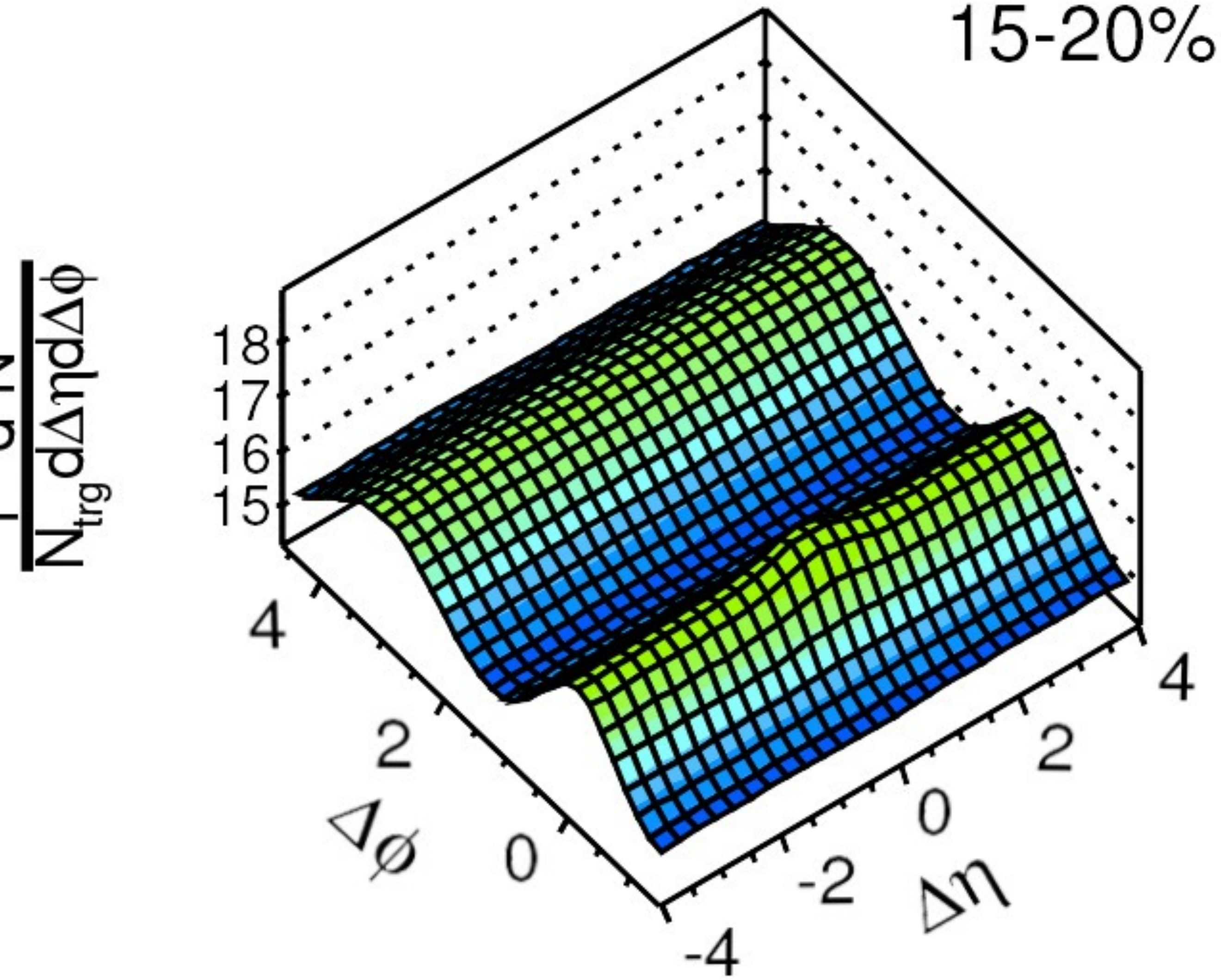

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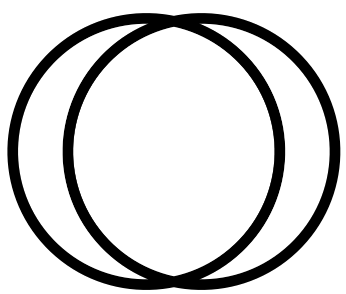

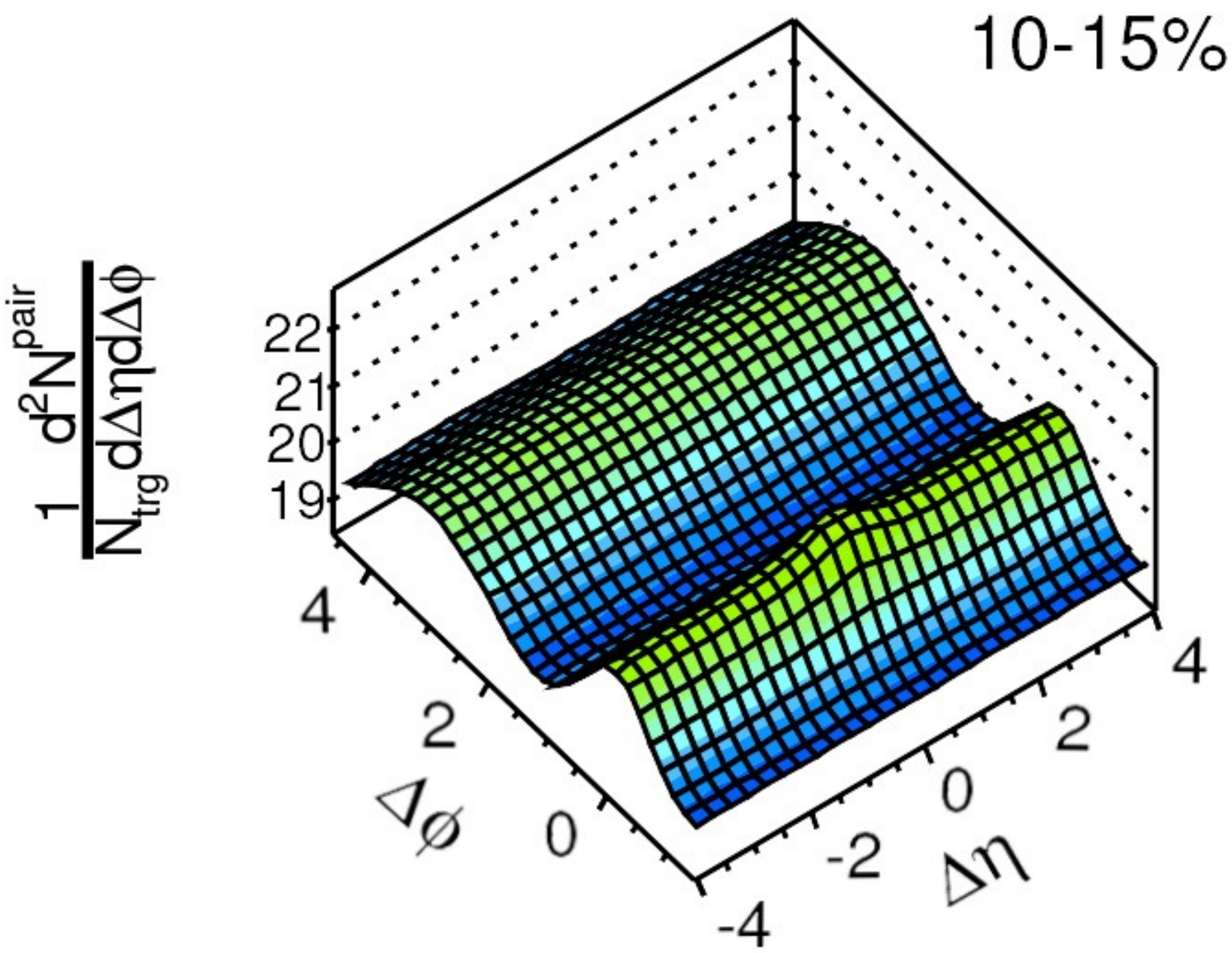

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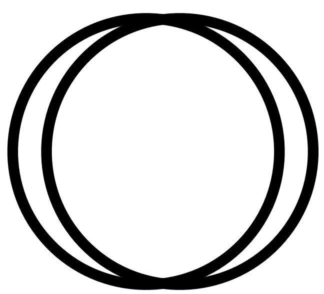
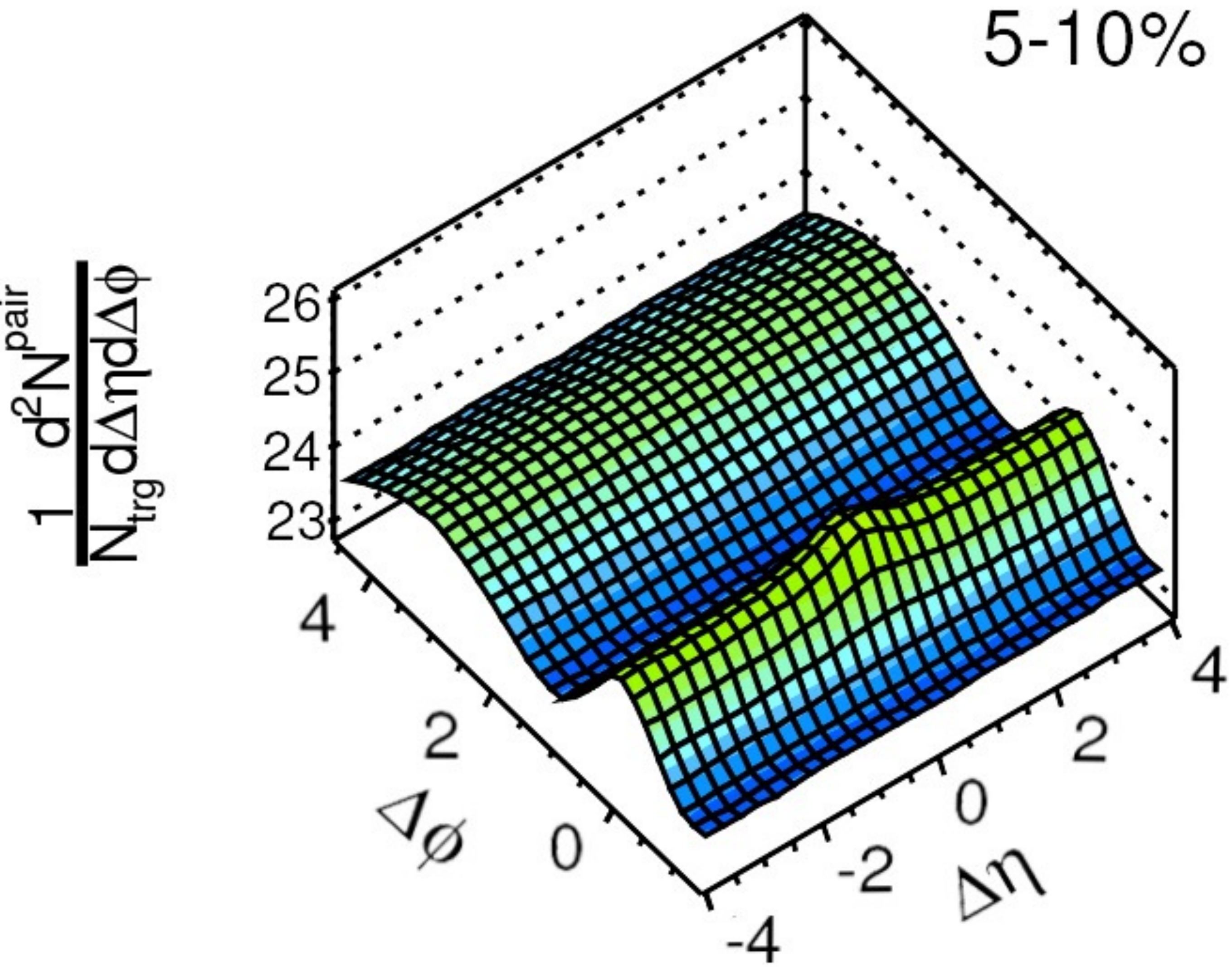
Correlations in Pb+Pb collisions





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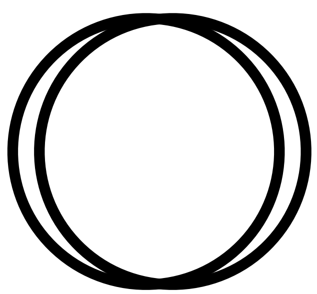
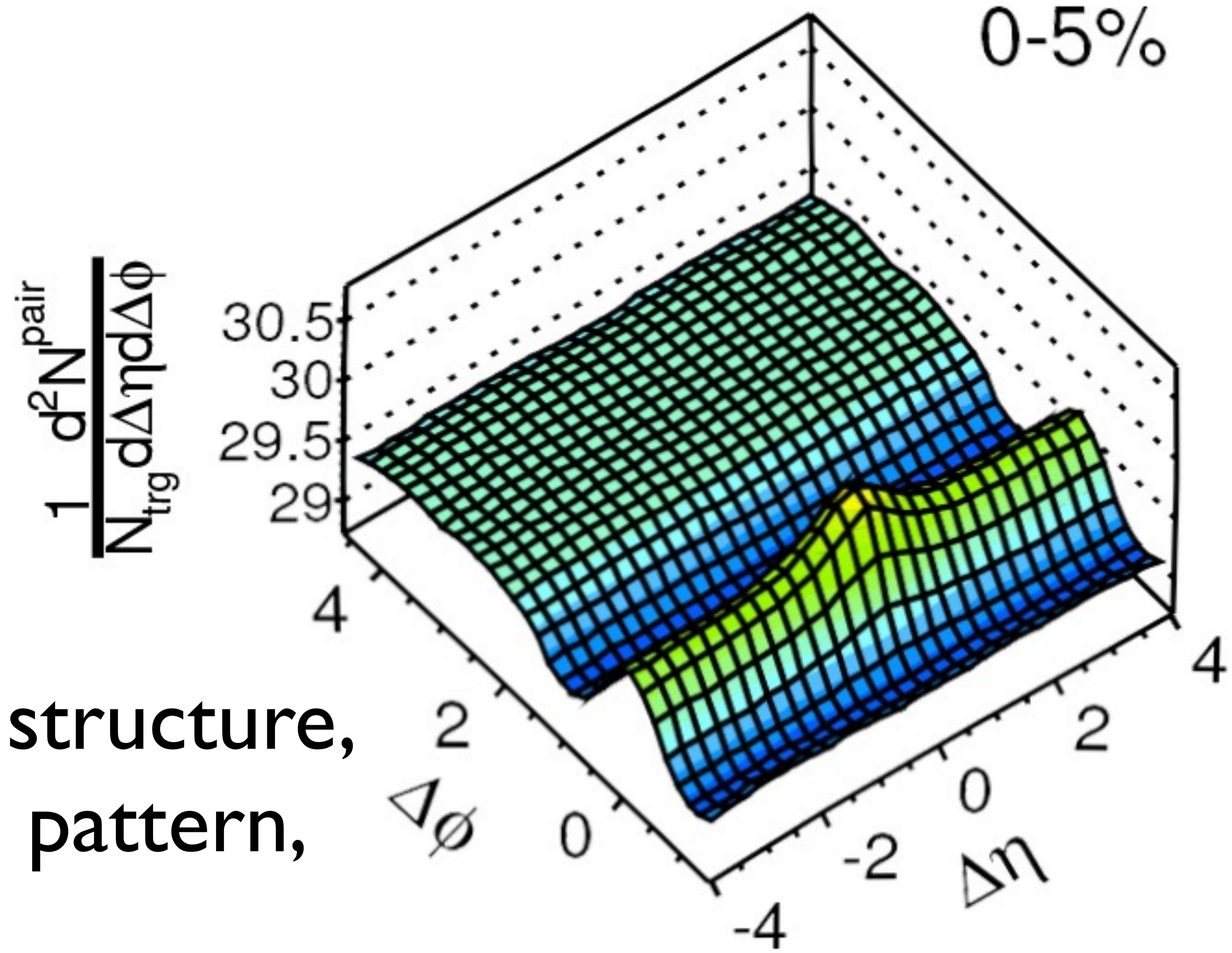
Correlations in Pb+Pb collisions

Central collisions





Not at all a cos(2Δφ) structure, but still a regular ridge pattern, independent of Δη.

Major puzzle for several years! (~2005-2010)

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Independent emission in intrinsic frame

Alver Roland 1003.0194 Luzum 1011.5773

Assume that in a collision, particles are sampled independently from an underlying probability distribution which depends on azimuthal angle φ, not on pseudorapidity, and can be *any function* of φ. Write as Fourier series:

$$dN/d\varphi d\eta = \sum_n V_n \exp(-in\varphi)$$

where $V_{-n}=V_n^*$ because the distribution is real.

The underlying probability distribution is different in every collision event.

Independent emission in intrinsic frame

The pair distribution is

$dN_{pair}/d\varphi_1 d\eta_1 d\varphi_2 d\eta_2 = (dN/d\varphi_1 d\eta_1)(dN/d\varphi_2 d\eta_2)$

$=\sum_{n1,n2} V_{n1} V_{n2} \exp(-in_1\varphi_1 - in_2\varphi_2)$

Write $\varphi_2=\varphi_1+\Delta\varphi$, $\eta_2=\eta_1+\Delta\eta$, integrate over φ_1, η_1:

$\exp(-i(n_1+n_2)\varphi_1)$ gives 0 unless $n_2=-n_1$. Use $V_{-n1} = V_{n1}^*$

$dN_{pair}/d\Delta\varphi d\Delta\eta=\sum_n |V_n|^2 \exp(-in\Delta\varphi)$

$=\sum_n |V_n|^2 \cos(n\Delta\varphi)$

Independent emission in intrinsic frame

Eventually, average over collision events:

$dN_{pair}/d\Delta\varphi d\Delta\eta = \sum_n <|V_n|^2> \cos(n\Delta\varphi)$

Fourier series whose all coefficients are positive

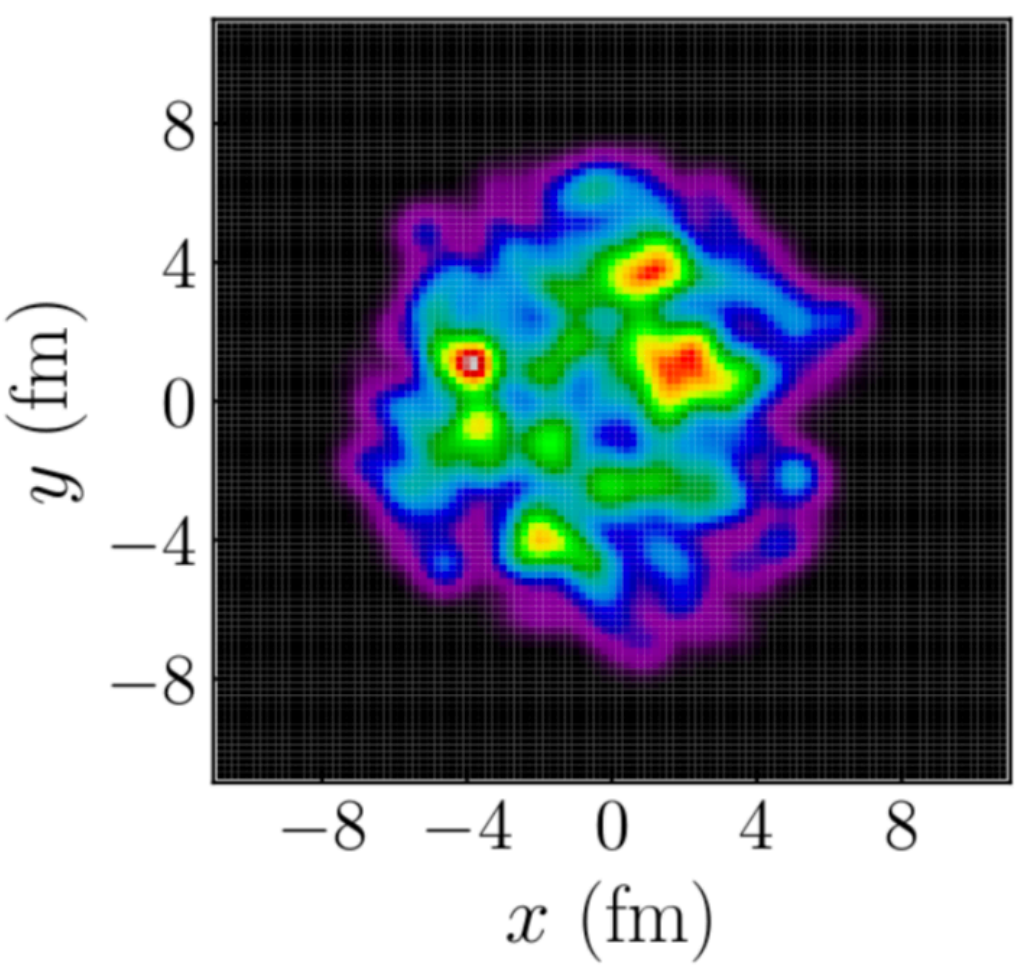
Independent emission in intrinsic frame

This is a very simple yet predictive model.

- Naturally explains the regular structure seen in data.
- Predicts that the absolute maximum of the pair distribution is at $\Delta\varphi=0$. This is the most difficult feature, that other models typically don't reproduce.
- Explains why the peak at $\Delta\varphi=0$ is slightly narrower than the peak at $\Delta\varphi=\pi$. This is mostly due to a small contribution in $\cos(3\Delta\varphi)$

Independent emission in intrinsic frame

Built-in feature of hydrodynamic models, where particles are emitted independently. Due to event-to-event fluctuations in nucleon positions, the initial density profile has no symmetry in (x,y) plane. Therefore:



(borrowed from G. Giacalone)

- 2nd Fourier component, elliptic flow, still exists in central collisions
- Elliptic flow not strictly along impact parameter in non-central collisions
- 3rd Fourier component, triangular flow, allowed even though it breaks parity.

Measuring anisotropic flow

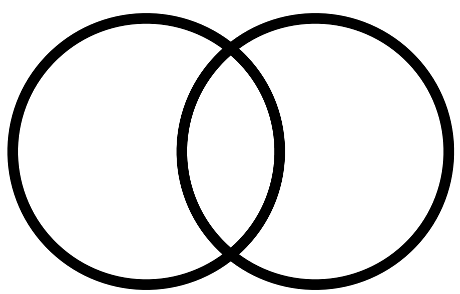
Anisotropic flow, v_n, is defined as the normalized amplitude of the Fourier coefficient: $v_n = |V_n|/V_0$

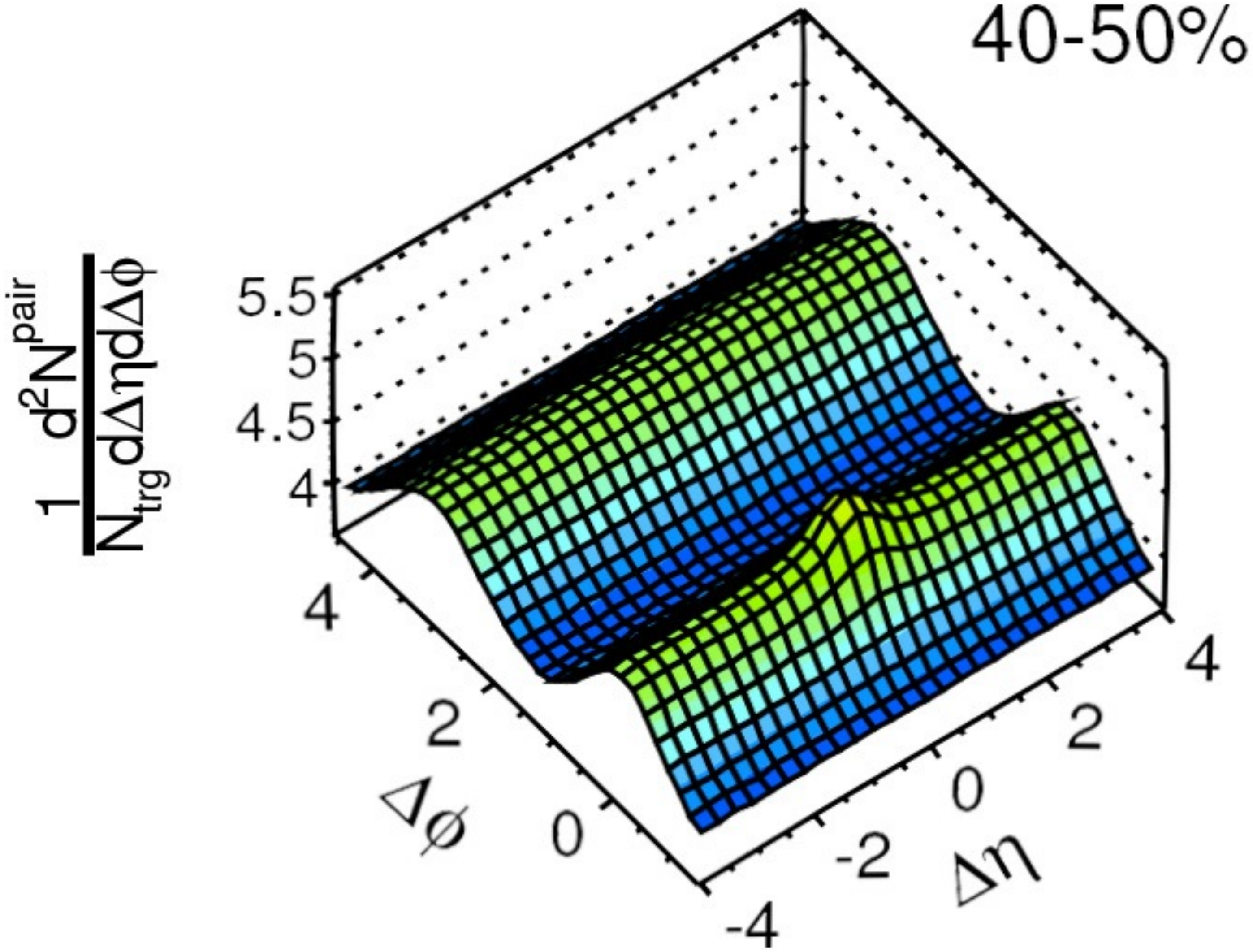
The Fourier coefficient of the pair distribution gives:

$\langle \exp(-in\,\Delta\varphi)\rangle = \langle \cos(n\,\Delta\varphi)\rangle = \langle |V_n|^2\rangle / \langle V_0^2\rangle \approx \langle v_n^2\rangle$

One usually imposes in addition that the particles have a minimum separation in $\Delta\eta$, in order to eliminate short-range correlations, which show up as a small peak referred to as nonflow, which is typically larger for smaller systems.

Correlations in Pb+Pb collisions





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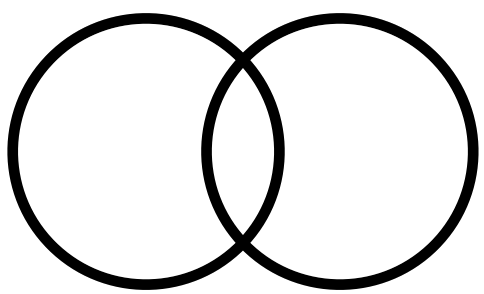

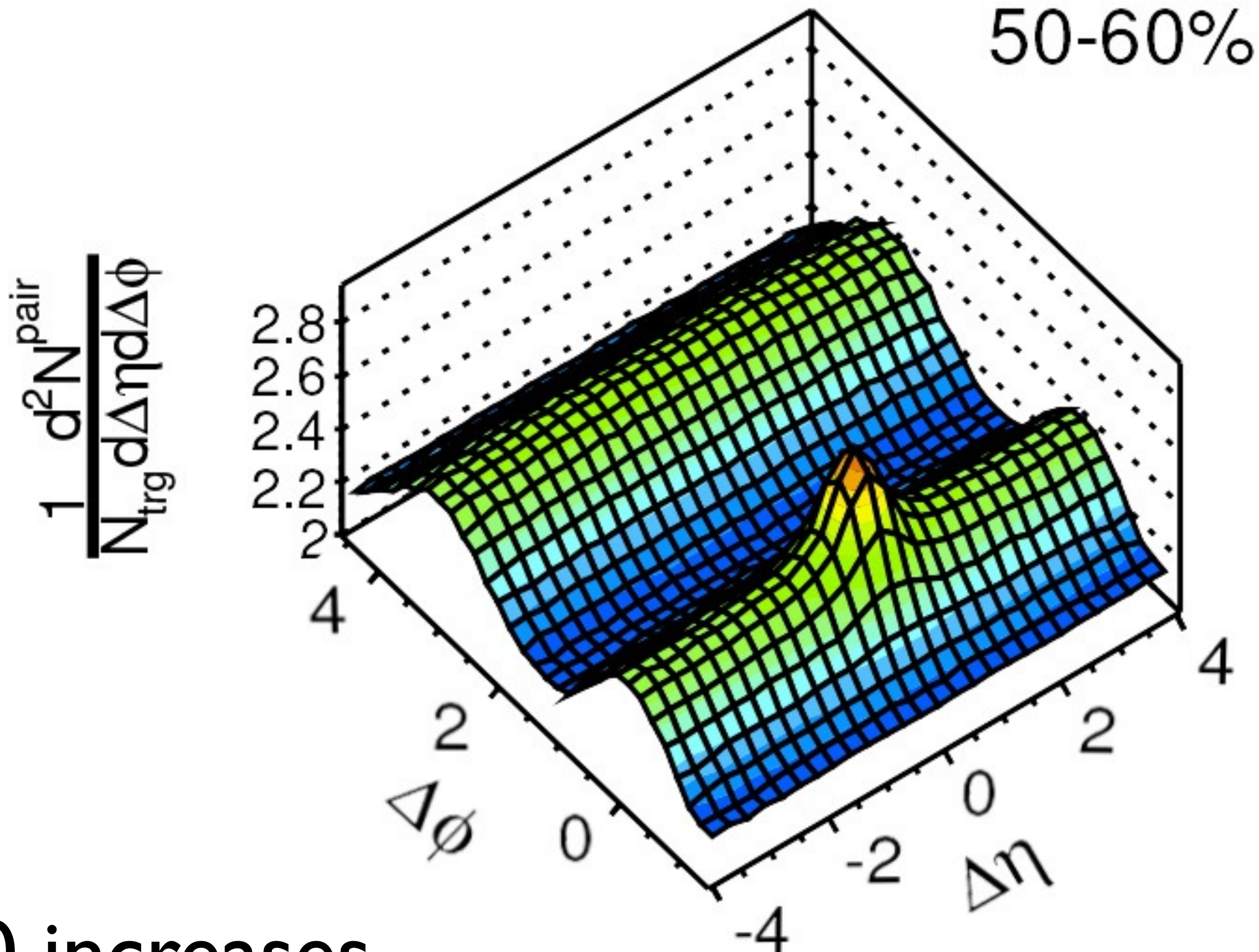

The nonflow peak at 0 increases

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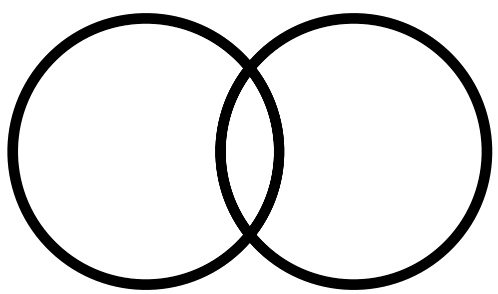
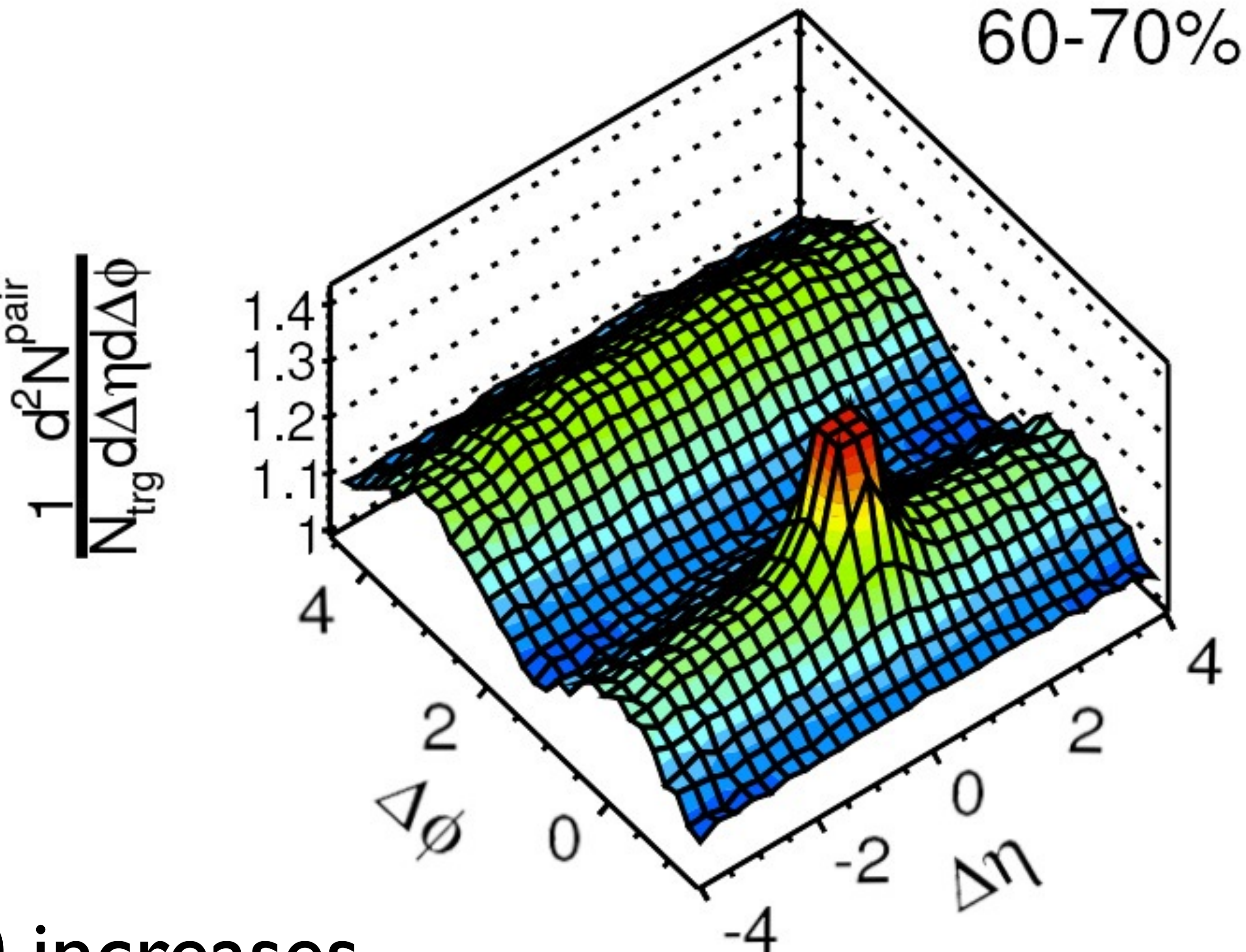
Correlations in Pb+Pb collisions





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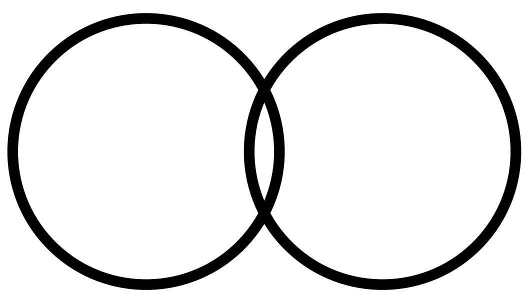

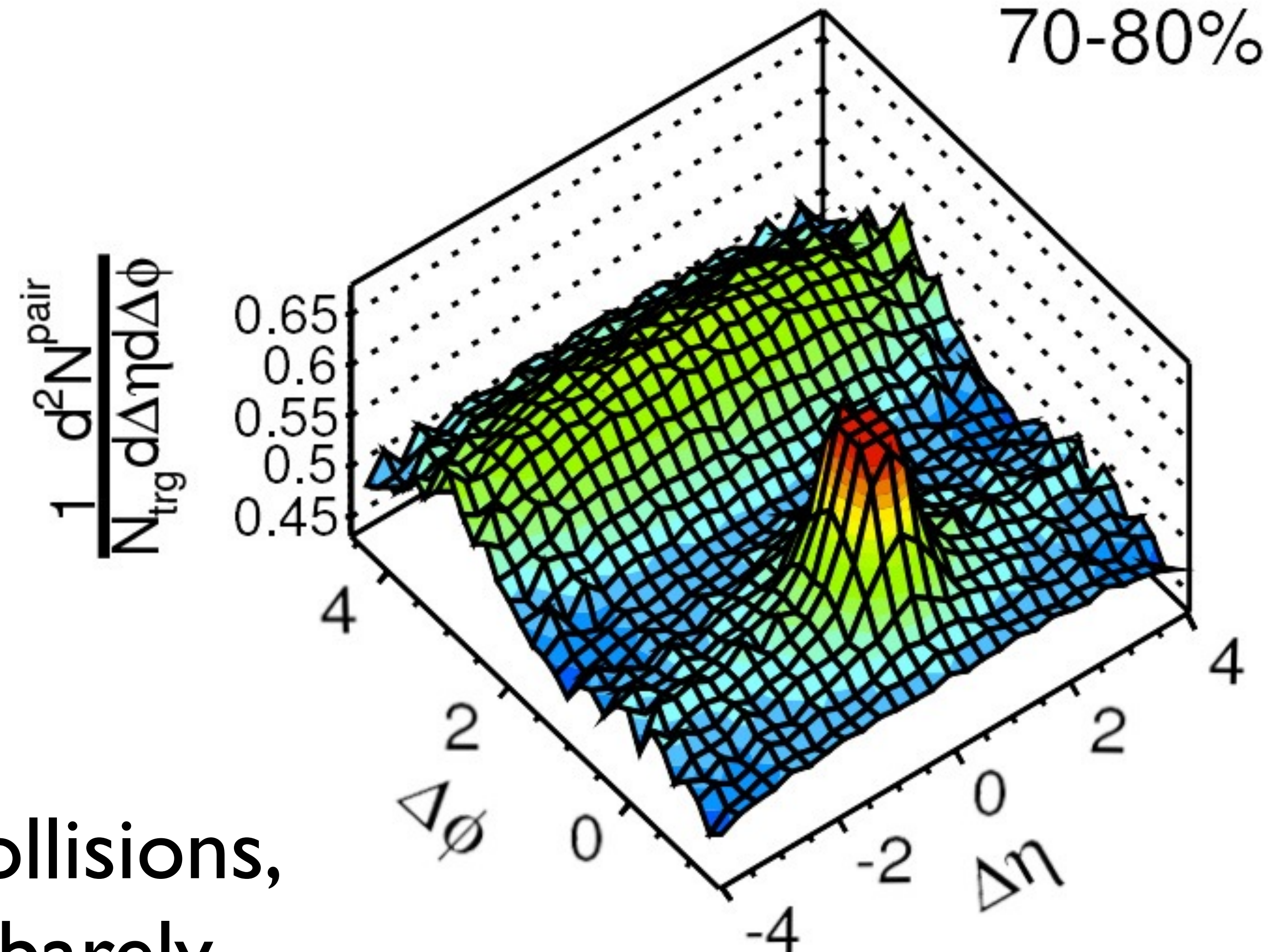

For very peripheral collisions, the ridge structure is barely visible.

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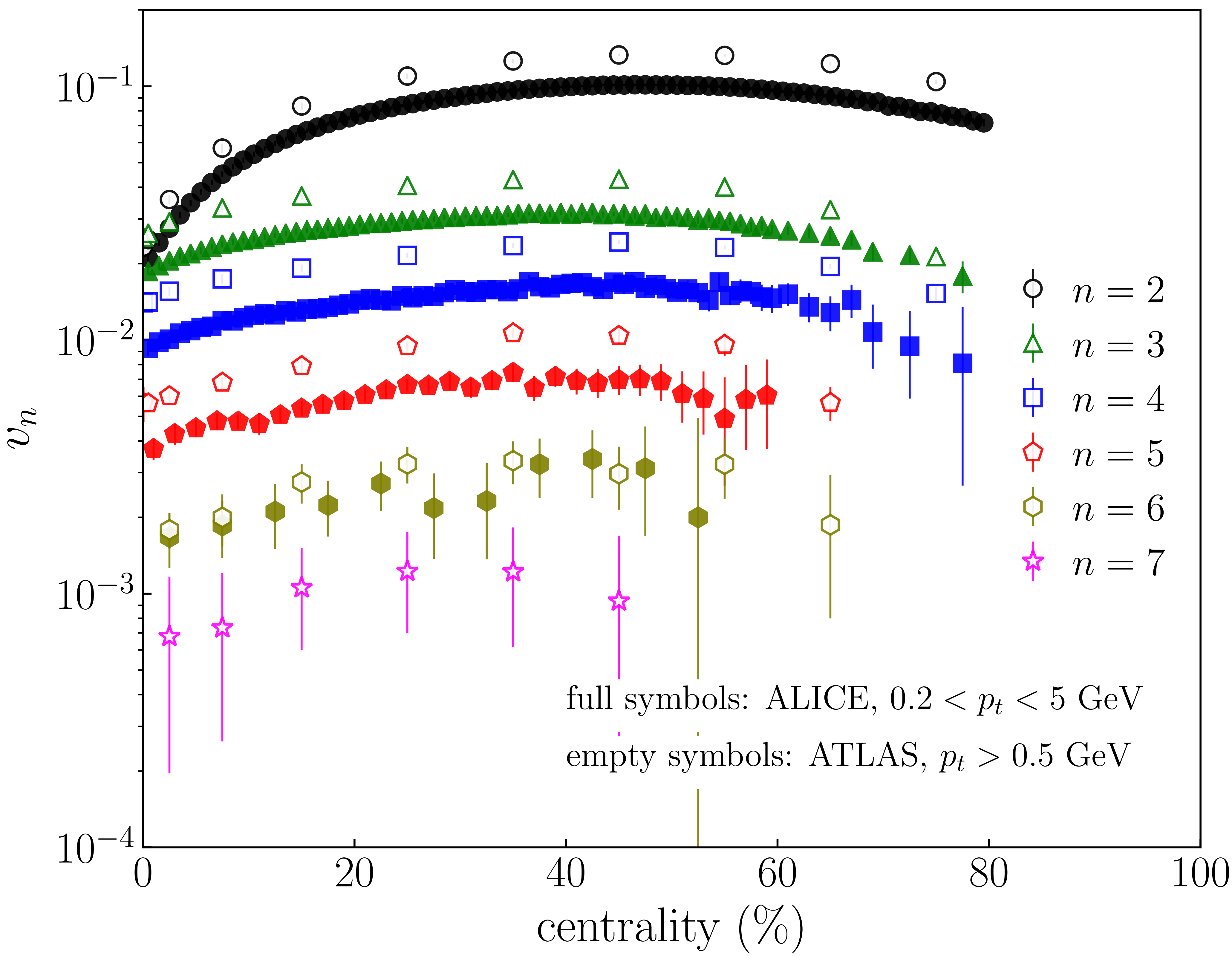
Measuring anisotropic flow

The estimate of v_n from pair correlations with a rapidity gap is the most common measure of anisotropic flow.

It is denoted by $v_n\{2\}$ and corresponds to a rms average over events. $v_n\{2\}^2 \equiv \langle (v_n)^2 \rangle = \langle \cos(n\,\Delta\varphi) \rangle$

It is important to keep in mind that it always comes from a correlation, and may depend on the rapidity gap.

$v_n\{2\}$ in Pb+Pb collisions at the LHC



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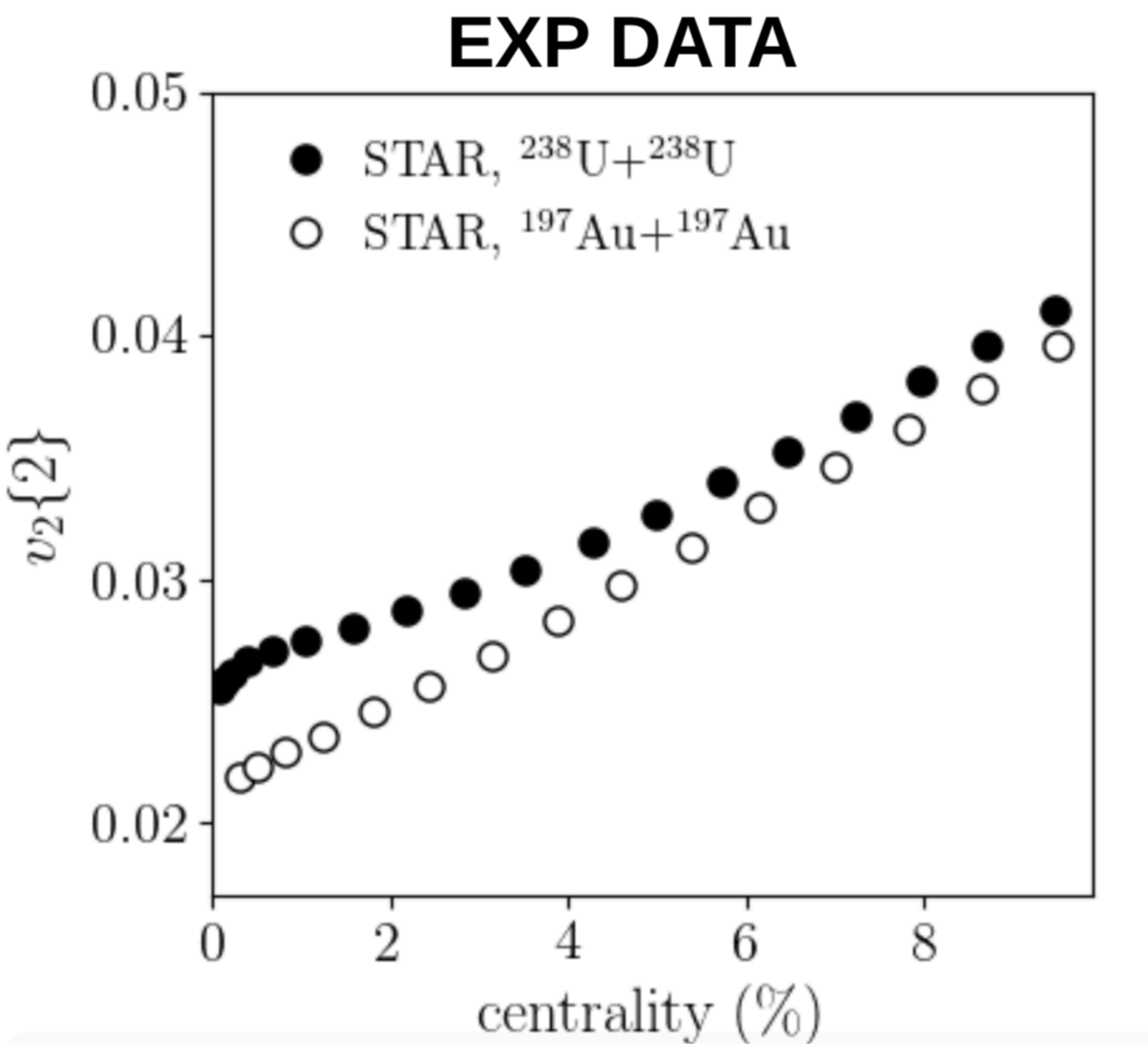
Deformation of ^{238}U seen by STAR

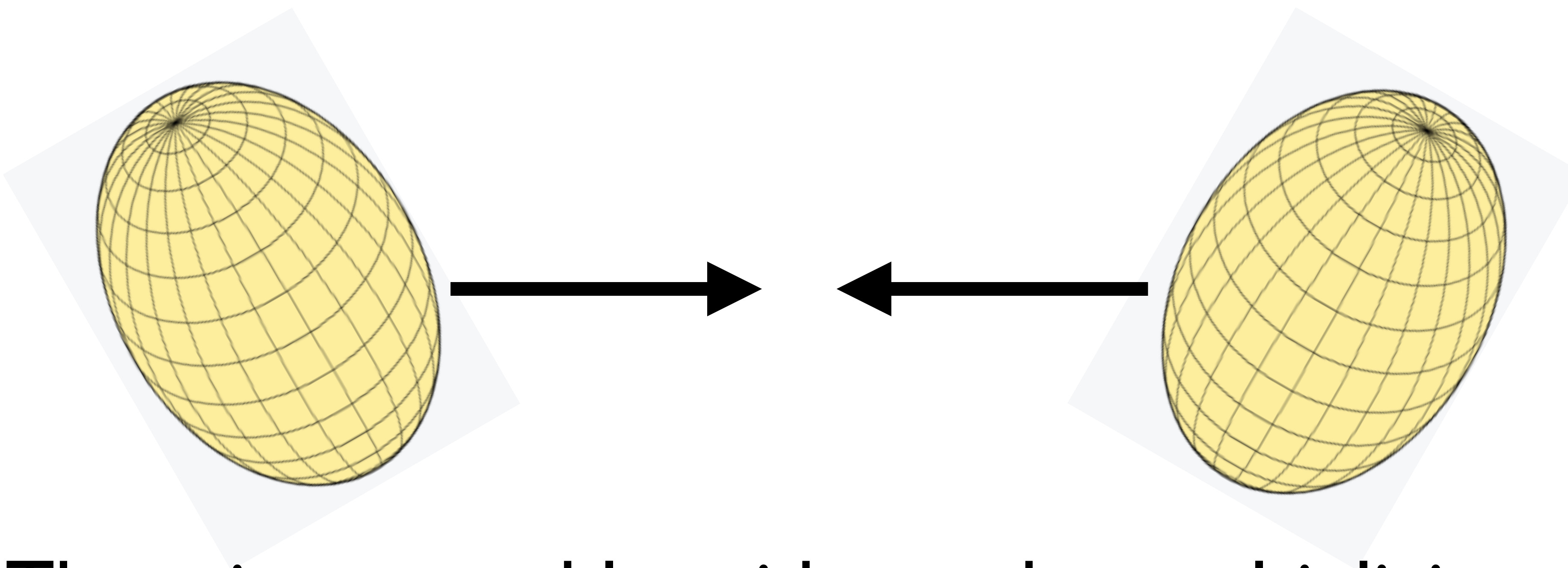
[STAR Collaboration, 1505.07812]



(borrowed from G. Giacalone)

The large quadrupole deformation of ^{238}U results in a larger v_2. The effect is clear only in central collisions.

Central collisions

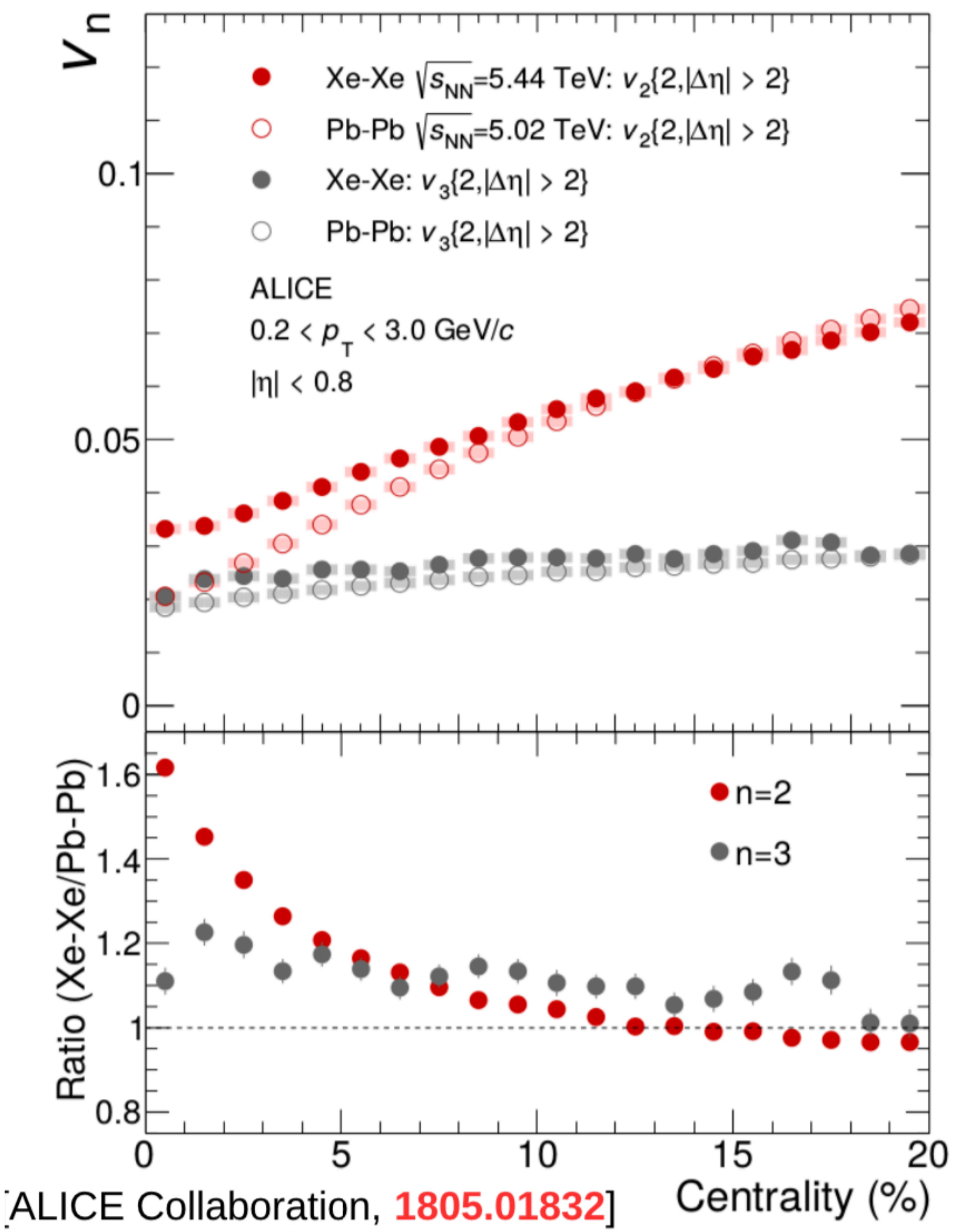


- There is reasonable evidence that multiplicity of produced particles

  $\propto$ Number of nucleons colliding at least once.

- In experiment, central collisions are defined as those producing the largest multiplicity
- Therefore, for identical nuclei, they correspond to collisions where they fully overlap

Deformation of ^{129}Xe seen at LHC



Increase due to quadrupole deformation is again seen in the most central collisions.

(borrowed from G. Giacalone)

1. Introduction. Why heavy ions.
2. Early days: Elliptic flow in non-central collisions
3. Modern picture of azimuthal anisotropy (2010-)
4. Where to look for influence of nuclear structure
5. Back to early days: Measuring azimuthal anisotropy

Higher-order correlations

Due to the large multiplicity in every event, one can measure correlations of arbitrary order with excellent precision.

In a single event, assuming independent particles in intrinsic frame:

$<\exp(2i\varphi_1-2i\varphi_2)> \; = <\exp(2i\varphi_1)> <\exp(-2i\varphi_2)> = |v_2|^2$

Similarly, if one averages over all possible 4-plets

$<\exp(2i\varphi_1-2i\varphi_2+2i\varphi_3-2i\varphi_4)>$
$= <\exp(2i\varphi_1)> <\exp(-2i\varphi_2)> <\exp(2i\varphi_3)> <\exp(-2i\varphi_4)>$
$= |v_2|^4$

Higher-order correlations

The only difference comes upon averaging over events.
The pair correlation gives $<|v_n|^2>$, while the 4-particle correlation gives $<|v_n|^4>$, which is not simply related to $<|v_n|^2>$ if $|v_n|$ differs in every collision.

In the same way as one defines $v_n\{2\}^2 \equiv <(v_n)^2>$ (rms average)
it would be natural to define $v_n\{4\}^4 \equiv <(v_n)^4>$ (4th moment), etc.
For historical reasons, however, this is not how it is done in practice.

Cumulants

Borghini Dinh JYO nucl-th/0105040

4-particle correlations were initially used in combination with 2-particle correlations in order to sutract nonflow correlations and isolate collective flow:

$<\exp(2i\varphi_1-2i\varphi_2+2i\varphi_3-2i\varphi_4)>$ gets contributions from pairwise correlations:

$<\exp(2i\varphi_1-2i\varphi_2)>$ $<\exp(2i\varphi_3-2i\varphi_4)>$ or

$<\exp(2i\varphi_1+2i\varphi_3)>$ $<\exp(-2i\varphi_2-2i\varphi_4)>$ or

$<\exp(2i\varphi_1-2i\varphi_4)>$ $<\exp(-2i\varphi_2+2i\varphi_3)>$

which one subtracts to isolate the genuine 4-particle correlation, or *cumulant.*

Cumulants

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Anisotropic flow
= independent particle emission in intrinsic frame
= produces correlations of arbitrary high order in lab frame

$<\exp(2i\varphi_1-2i\varphi_2+2i\varphi_3-2i\varphi_4)> = <|v_2|^4>$
$<\exp(2i\varphi_1-2i\varphi_2)> <\exp(2i\varphi_3-2i\varphi_4)> = <|v_2|^2>^2$
$<\exp(2i\varphi_1+2i\varphi_3)> <\exp(-2i\varphi_2-2i\varphi_4)> = 0$
$<\exp(2i\varphi_1-2i\varphi_4)> <\exp(-2i\varphi_2+2i\varphi_3)> = <|v_2|^2>^2$

After subtraction, one obtains $<|v_2|^4>-2 <|v_2|^2>^2$
One defines $-v_2\{4\}^4 = <|v_2|^4>-2 <|v_2|^2>^2$

Cumulants

This can be generalized to arbitrarily high orders.

Anisotropic flow is the spontaneous breaking of azimuthal symmetry and can only be seen, as any symmetry breaking, in the limit of an infinitely large system. It is analogous to a phase transition and can be analyzed using the same methods (Lee-Yang zeroes)

Bhalerao Borghini JYO nucl-th/0310016

Conclusion

The concepts and tools that we use to describe azimuthal anisotropy in heavy-ion collisions are surprisingly similar to those used to describe the deformation of atomic nuclei.

What we call collective flow is a deformation of the single-particle probability distribution in some intrinsic frame, in the same way as collective excitations in nuclei are described as independent nucleons in some intrinsic state.