Deblurring Source Function from Particle-correlations In Heavy-Ion Collision

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INT 22-84W workshop: **Dense Nuclear Matter Equation of State from Heavy-Ion Collisions**

(Seattle, Washington)

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FRIB

Outlines

- Two–particle correlation function (CF)
- Source function via deblurring: case d-alpha CF
- Transport model for source
- Summary and outlook

Particle Correlation Functions (CF)

Two particle CF

$$R(\boldsymbol{q})+1=\frac{P_{12}(\boldsymbol{p_1},\boldsymbol{p_2})}{P_1(\boldsymbol{p_1})P_2(\boldsymbol{p_2})}$$

Numerator- probability of coincident particle emission

Denomator- product of the probability of single emission

-Two-particle relative momentum ($\boldsymbol{q}$)

Identical particle

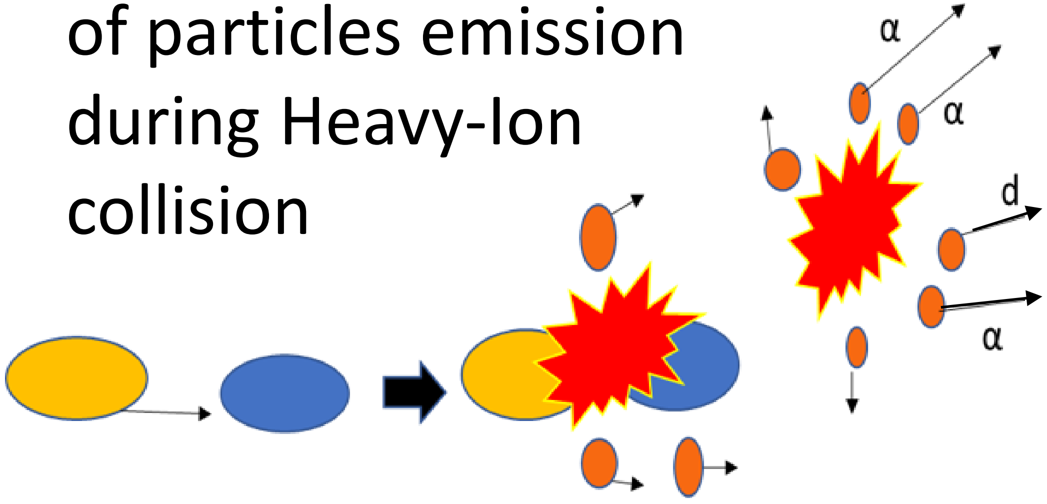
$\boldsymbol{q} = 0.5(\boldsymbol{p_1} - \boldsymbol{p_2})$

$\boldsymbol{p_{1,2}}$ momentum of 1, 2

Non-identical particle

$\boldsymbol{q} = 0.5\,\mu\,(\boldsymbol{v_1} - \boldsymbol{v_2})$ -μ is the reduced mass and velocity -$\boldsymbol{v_{1,2}}$

Pictorial representation of particles emission during Heavy-Ion collision



Pre-equilibrium fast emission

-Particles may be emitted during collision time

-light particles may decay from unstable intermediate nuclei

- Charged particle are more likely to be measured

CF Model

Theoretical definition

$$C(\boldsymbol{q}) = R(\boldsymbol{q}) + 1 = \int d^3r \; \underbrace{S(\boldsymbol{r})} \; \underbrace{|\Psi(\boldsymbol{q},\boldsymbol{r})|^2}$$

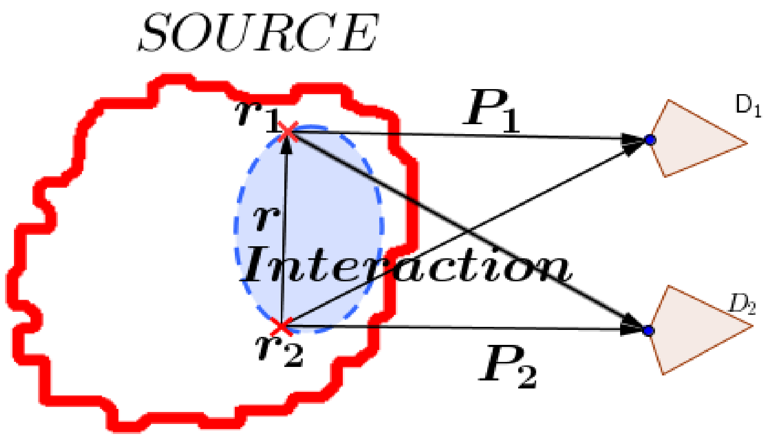
Source function and 2-particle wave function

$$R(\boldsymbol{q}) = \int d^3r \, S(\boldsymbol{r}) \left(|\Psi(\boldsymbol{q},\boldsymbol{r})|^2 - 1\right)$$

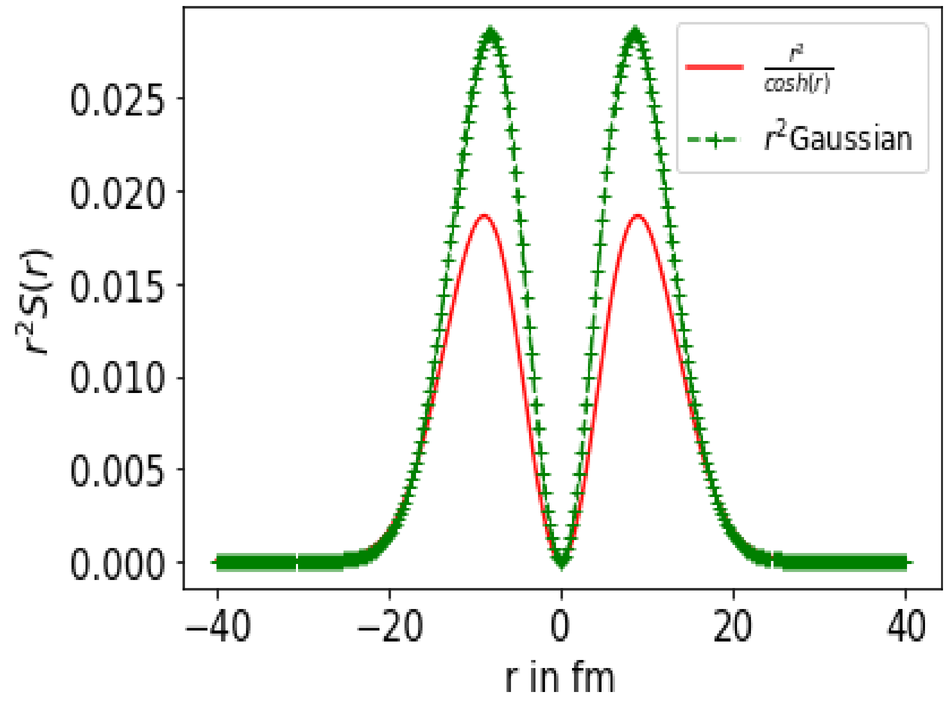
$\boldsymbol{\Psi}(\mathbf{r},\mathbf{q})$, **is relative wave function,** Encodes particle interactions (**short-range and coulomb interaction**.

- Study correlation between particles to learn about nuclear systems: one needs to solve Schrödinger Equation to obtain $\Psi(\boldsymbol{q},\boldsymbol{r})$
- To extract a reliable and realistic S(r) that characterizes nuclear reaction system.

Source Function(SF)



- Interactions and symmetrization, affecting the relative state of two particles on their way to the detectors, and encoded in the relative wave function, allow inferring characteristics of particle emission from correlation functions
- Studying relative distribution of two-particle emission gives access to space-time characteristics of the emitting system



Examples of SF: normalized Gaussian and $\propto \frac{1}{Cosh(r)}$

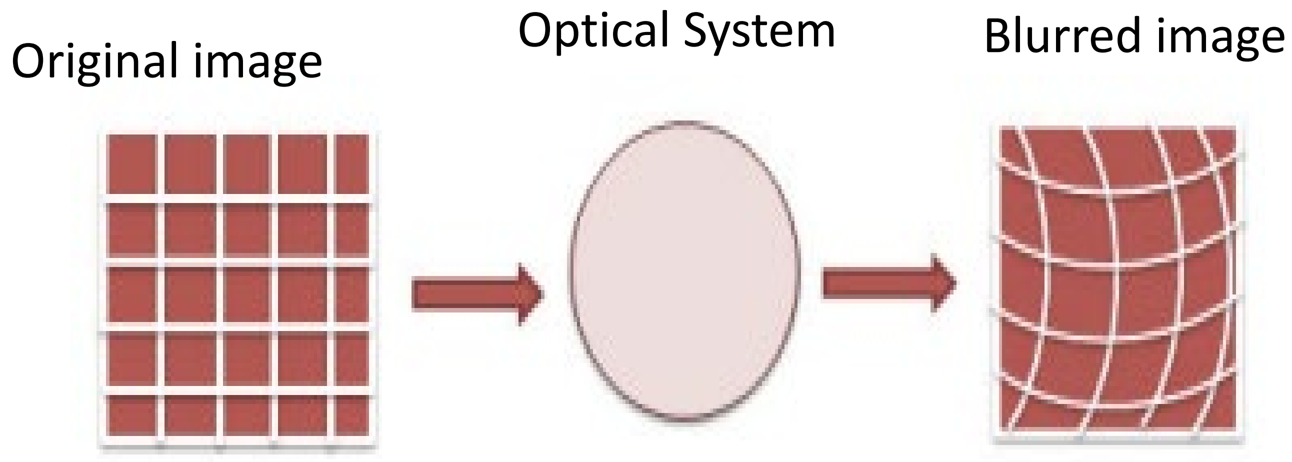
Method to estimate S(r):

-Comparing measured correlation C_{Exp} to C_{TH} one can extract $S(r)$

- Or Imaging Restoration of SF (**D.A. Brown and P. Danielewicz (1997)**)

-Restoration of SF by Deblurring Method

Deblurring overview



DOI: 10.1109/ELINFOCOM.2016.7562935

Deblurring: - Technique to restore the original image
- Originally developed in Optics

Deblurring

1. Deblurring in optical image processing

(a)Sharp (b) Blurred and noisy (c) Deblurred

Before and after deblurring of image of lunar crater Copernicus https://en.wikipedia.org/wiki/Deconvolution

Source: https://doi.org/10.1016/j.matpr.2020.11.076

Detector efficiency ε, n measured particle number, N actual number

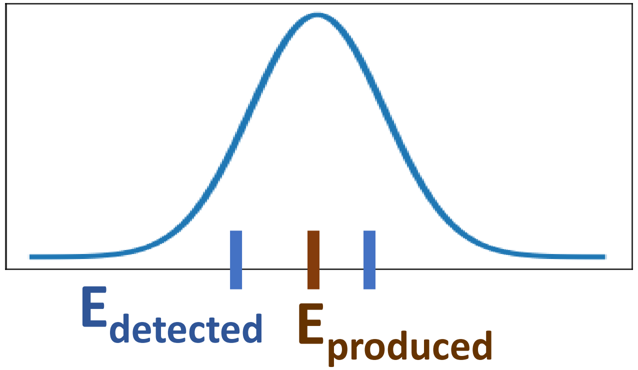
$\mathrm{n} \simeq \varepsilon N$

probabilistically

$$n(E_d) = \int dE_d^{'} P(E_d|E_d^{'}) N(E_d^{'}).$$

with $P(E_d|E_d^{'})$ – probability to measure particle characteristic to be E_d when it is actually $E_d^{'}$

Optical terminology: P - blurring or transfer function.



Schematic description of prob. density of energy measurements

Richardson-Lucy (RL) Algorithm for deblurring

Blurred/Measured function

$f_i = \sum_j H_{ij} F_j$

RL-algorithm solves F (true function) iteratively

- $f_j^n(E) = \sum_j H_{ji} F_i^n$

$$\boldsymbol{F_i^{(n+1)}} = \sum_j \frac{\boldsymbol{f_j}}{\boldsymbol{f_j^n}} \boldsymbol{T_{ji} F_i^n}$$

-H is a transfer matrix (TM) (Kernel Here)

$T_{ji} = \frac{W_j H_{ji}}{\sum_{j'} \mathrm{W_{j'} H_{j'i}}}$, normalized TM

$$\sum_j T_{ji} = 1$$

- n = number of iterations
- $F^0 > 0$, guess i.e., $\boldsymbol{F^0}$=1
- F^n, T_{ji}, f_i, and H_{ji} are kept positive

Add a noise term on the blurring model

$\hat{f}_i = \sum_j H_{ij} F_j + N_j \sim \mathcal{P}\left(\sum_j H_{ij} F_j\right)$

$\mathcal{P}$: statistical noise model

RL algorithm becomes

$$F_i^{(n+1)} = \sum_j \frac{\hat{f}_j}{\hat{f}_j^n} T_{ji} F_i^n$$

W. H. Richardson, JoSA 62, 55 (1972)

L. B. Lucy, the Astronomical journal 79, 745 (1974)

P. Danielewicz and M. Kurata-Nishimura, Physical Review C 105, 034608 (2022)

RL algorithm for source function

- Recall
- $R(\boldsymbol{q}) = \int d^3r\, S(\boldsymbol{r})\, (|\Psi(\boldsymbol{q},\boldsymbol{r})|^2 - 1)$
- $C(\boldsymbol{q}) = \int d^3r S(\boldsymbol{r}) |\Psi(\boldsymbol{q},\boldsymbol{r})|^2$
- Write CF in discretized form
- $R_i = \sum_{j=1}^{N} K_{ij} S_j$
- where $\boldsymbol{K(r,q) = |\Psi(r,q)|^2 - 1}$ or

$\boldsymbol{K(r,q) = |\Psi(r,q)|^2}$ for $R(\boldsymbol{q})$+1.

- Source function: $S \cong \sum_{j=1}^{N} S_j(\boldsymbol{r}) g_j(\boldsymbol{r})$
- $g_j(\mathrm{r}) = \begin{cases} 1 & r_{j-1} < r < r_j \\ 0 & otherwise \end{cases}$

$K_{ij} = 4\pi \int_{r_{j-1}}^{r_j} dr\, r^2 K(q_i, r)$: -Kernel Matrix

(Transfer matrix in the deblurring terminology)

RL algorithm for SF

R_i has some negative values but $C_i = R_i + 1$ is positive

$$S_i^{n+1} = S_i^n \sum_j \frac{C_i}{C_i^n} \frac{|W_j K_{ji}|}{\sum_{j'} |W_{j'} K_{j'i}|}$$

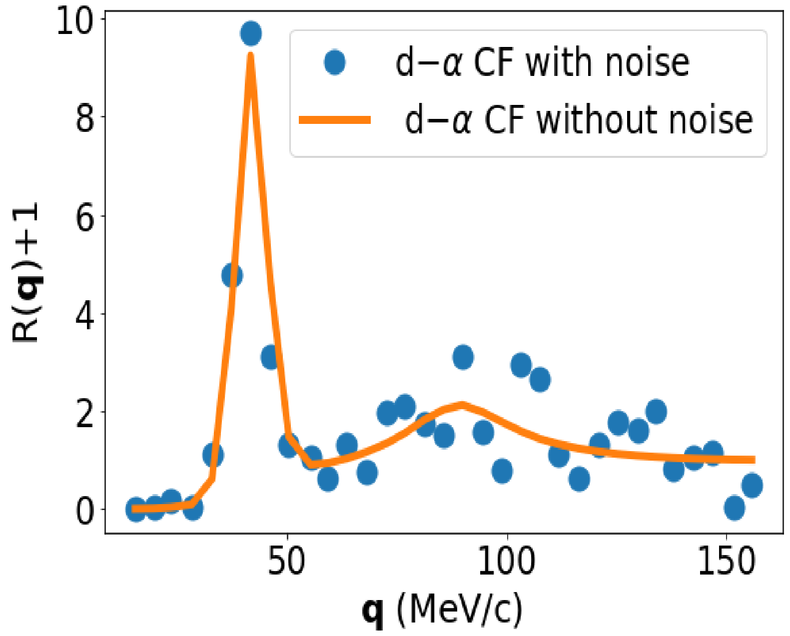
where $C_i^n = R_i^n + 1$/, the absolute value prevent K to be negative, at n^th^ iteration

D.A. Brown and P. Danielewicz, Phys. Lett. B 398, 252 (1997)

D.A. Brown and P. Danielewicz, Phys. Rev. C 64, 014902 (2001)

*The ATLAS collaboration, Aad G, Abajyan T. et al., J. High Energ. Phys. **2013,** 183 (2013)*

$d-\alpha$ source function recovering: Simulated Correlations

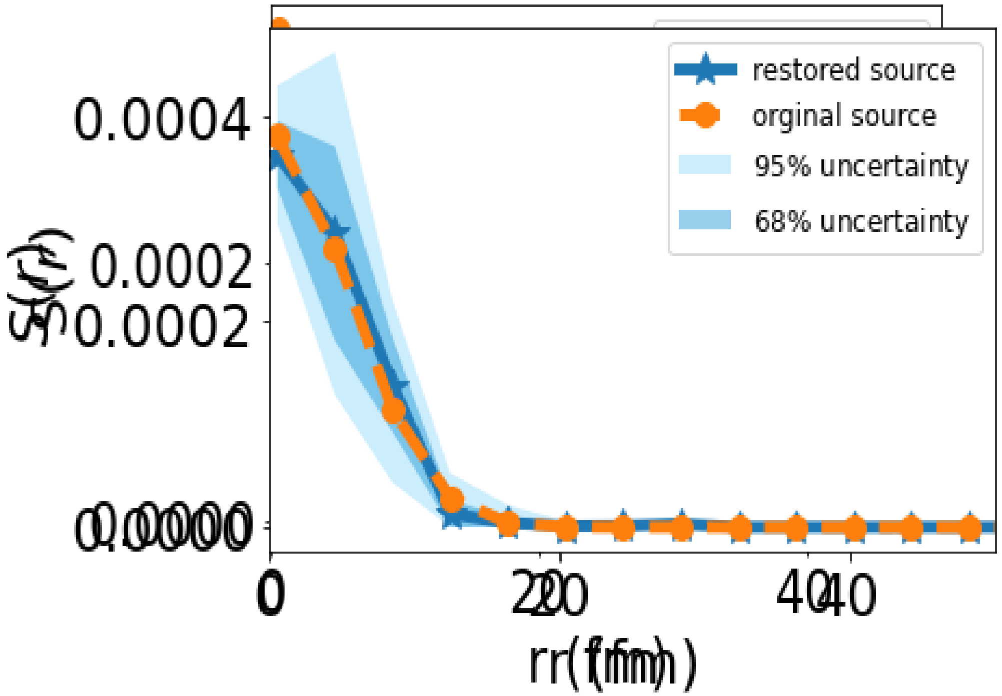


This is the angle average SF

Simulated $d-\alpha$ CF assuming Gaussian source function with radius, $R_0 = 5.5$ fm

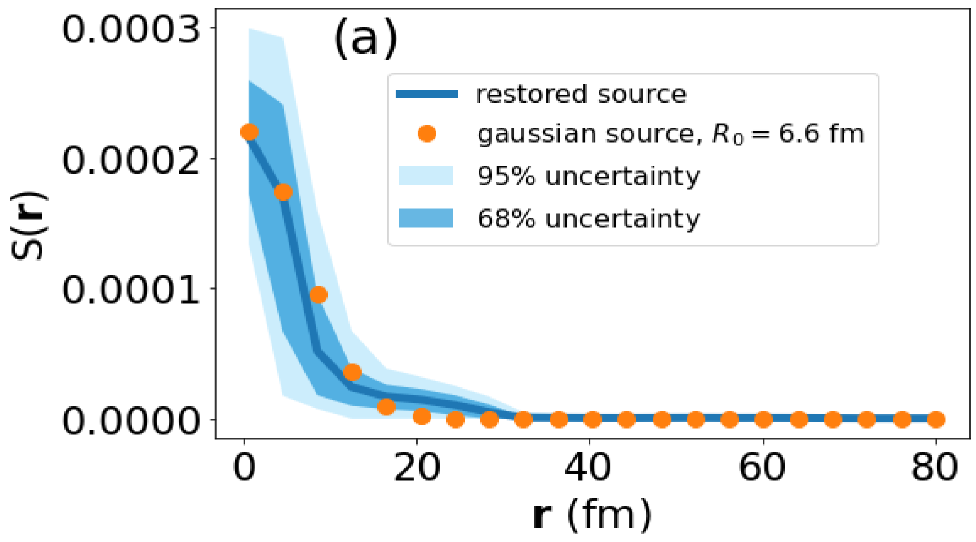
Deblurring S(r) for Correlation function in presence of noise.

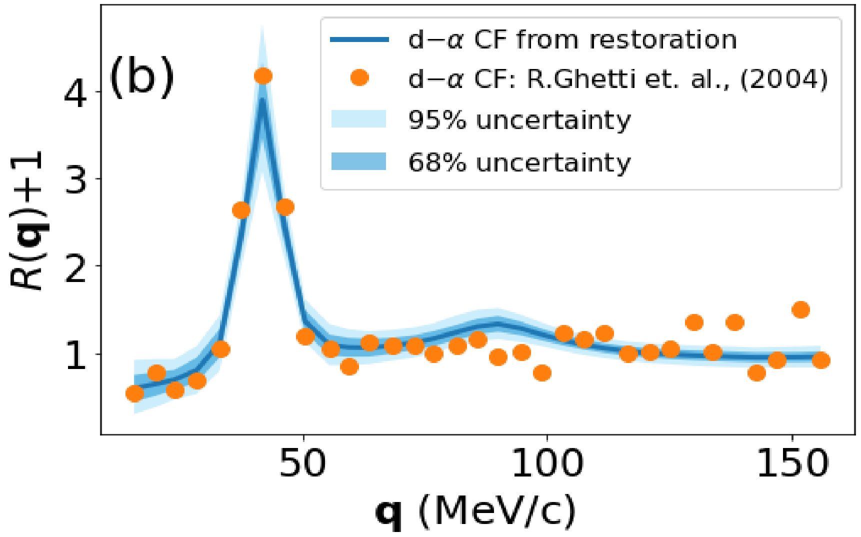
For smooth Correlation function



The blue bands are uncertainties resulting from SF restoration for noisy CF.

$d - \alpha$ source function recovering: Measured correlations





Experimental data

d-α CF with data(stars) from ^{40}Ar+^{27}Al reaction at E=40MeV/U **(R. Ghetti, et al., 2004)**

- Restored solid line (blue line) and (orange dashed) SF for $d - \alpha$ system
- The structure at low **r** corresponds to physics
- Peaks at 42 and 82 MeV/c correspond to the resonance decay of ^{6}Li into $d - \alpha$ at - 2.26 MeV and 4.4 MeV

The blue bands are uncertainties associated with resampling point of measured CF .

R. Ghetti, et al., Physical Review C 70, 027601 (2004).

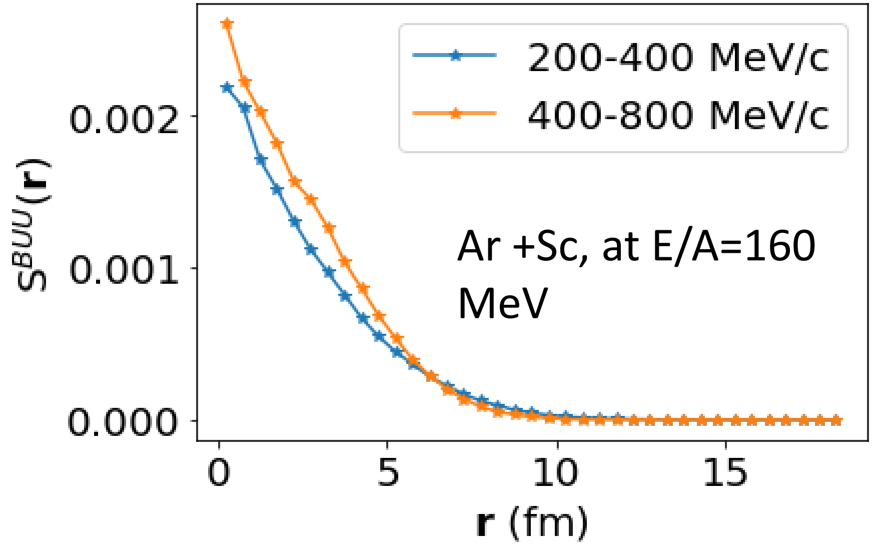
Comparison of deblurring and transport model (BUU) source function to infer EOS

BUU model:

$$\left(\frac{\partial}{\partial t}+\frac{\boldsymbol{P}}{m}.\boldsymbol{\nabla}_{\boldsymbol{r}}-\boldsymbol{\nabla}_{\boldsymbol{r}}U(\boldsymbol{r}).\boldsymbol{\nabla}_{\boldsymbol{p}}\right)f=\boldsymbol{I}_{\boldsymbol{col}}(\boldsymbol{\sigma}_{\boldsymbol{in}},\boldsymbol{f})$$

Collision term: $\boldsymbol{I}_{\boldsymbol{col}}(\boldsymbol{\sigma}_{\boldsymbol{in}},\boldsymbol{f})$

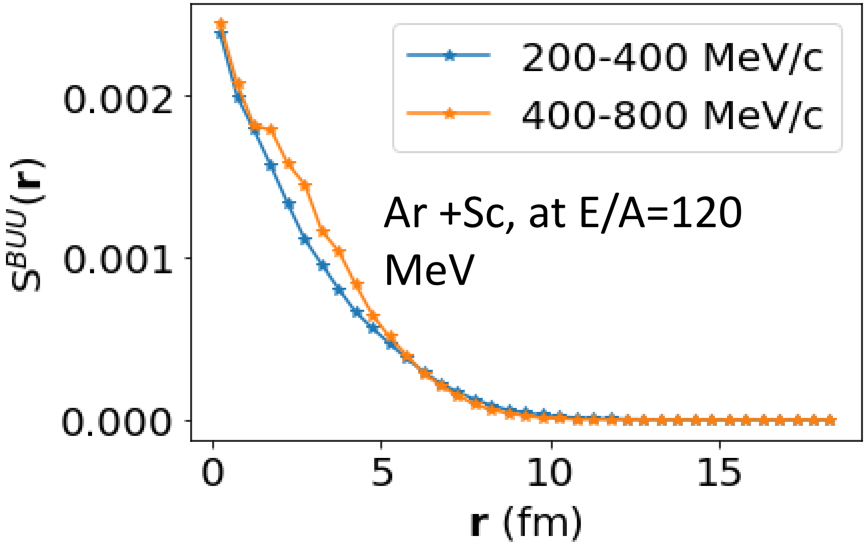
Preliminary results Proton- Proton source function from BUU ($S_p^{buu}(r)$)



The source function from BUU:

$$S^{buu}(r)=\frac{\int d^3R\, f(\frac{\boldsymbol{P}}{2},\boldsymbol{R}+\frac{\boldsymbol{r}}{2},t_>)f(\frac{\boldsymbol{P}}{2},\boldsymbol{R}-\frac{\boldsymbol{r}}{2},t_>)}{\left|\int d^3r\, f\right|^2}$$

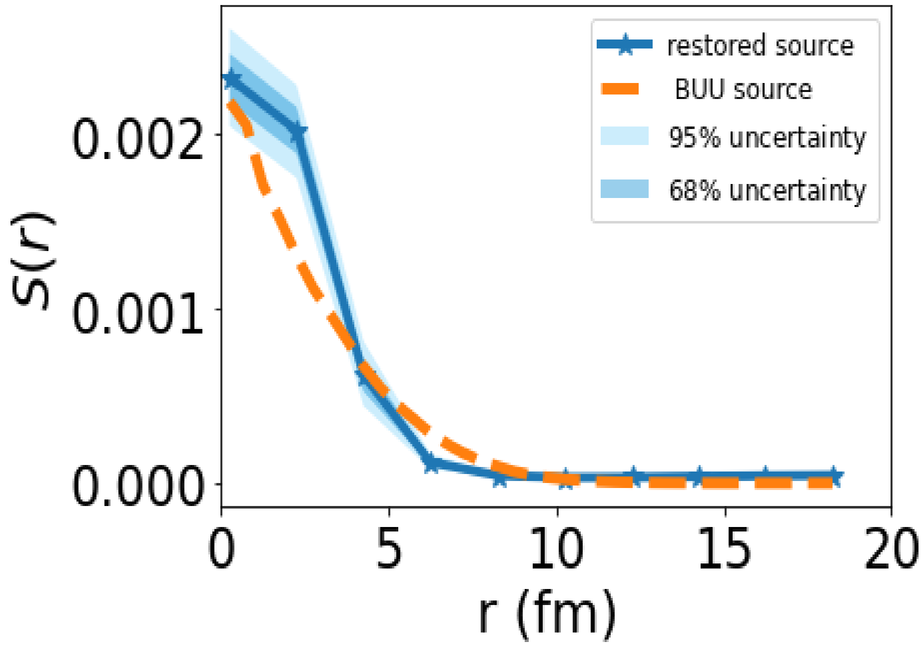
where $\boldsymbol{R}=0.5(\boldsymbol{r_1}+\boldsymbol{r_2}),\ \boldsymbol{r}=\boldsymbol{r_2}-\boldsymbol{r_1}$, -$f$ particle phase-space distribution



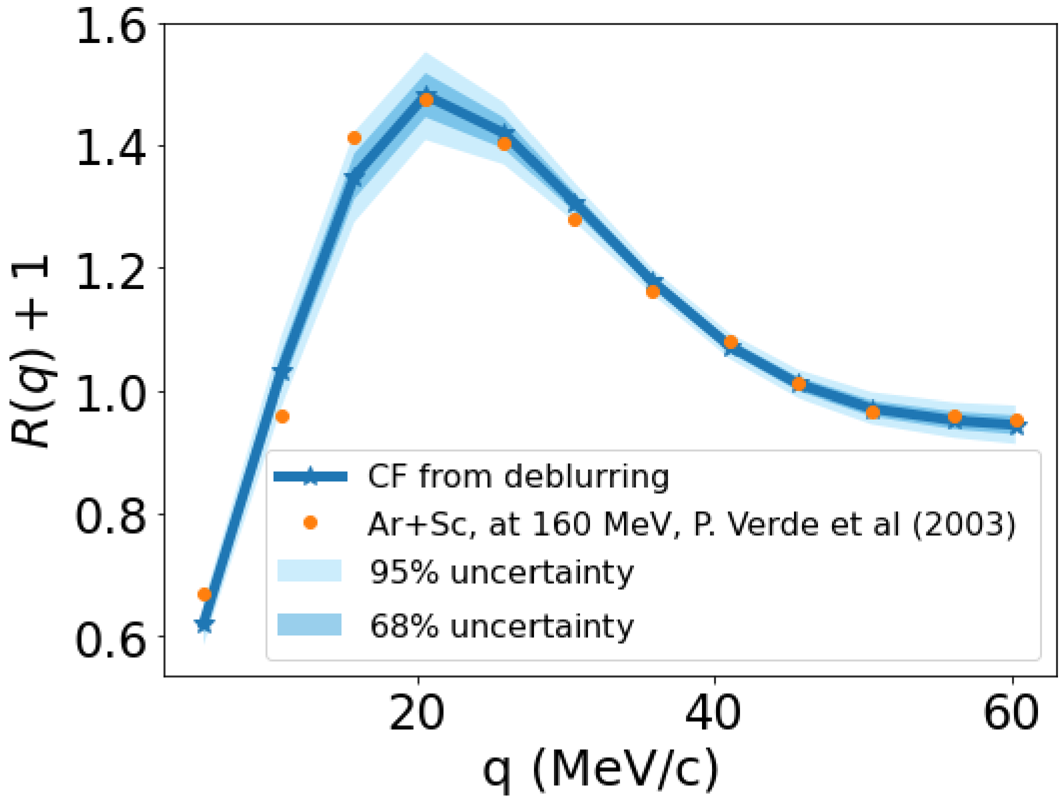
P. Verde et al 2003, PhysRevC.67.034606

Comparison cont….

Preliminary results : Work in progress



P-P correlation, 200-400 MeV/c



- Fit $S^{\mathrm{BUU}}(r)$ to S(r) help to assess the sensitivity of S(r) on EOS :
  - Isospin dependent
  - Nuclear symmetry energy
  - In medium cross section dependence

P. Verde et al 2003, PhysRevC.67.034606

Summary and outlooks

- Discuss deblurring for the source function
- The TMs are very different from the optics one

    - For CF the TM is due to physics and is very different from that in other cases

- RL algorithm gives good results in estimating SF in two-particle correlations (d-alpha system)

Outlook

-Extract source function to infer EoS.

    -symmetry energy

    -isospin dependent, ….

THANK YOU FOR YOUR ATTENTION!

Thanks to the funding agency:

US DOE, DE-SC001920

Deblurring algorithm with noise

Add a noise term on the blurring model

$$\hat{f}_i = \sum_j H_{ij} F_j + N_j \sim \mathcal{P}\left(\sum_j H_{ij} F_j\right),$$

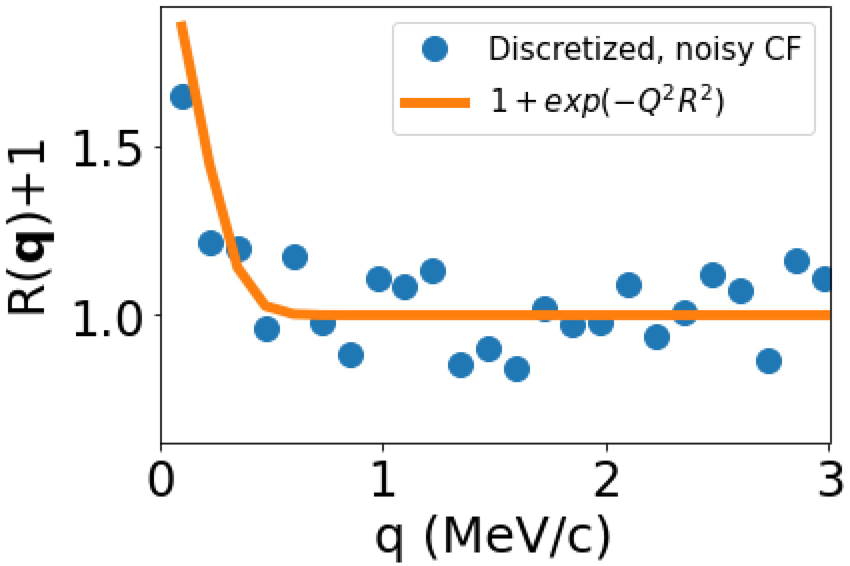
$\mathcal{P}$: statistical noise model as Poisson distribution

RL algorithm becomes: $F_i^{(n+1)} = \sum_j \frac{\hat{f}_j}{\hat{f}_j^n} T_{ji} F_i^n$

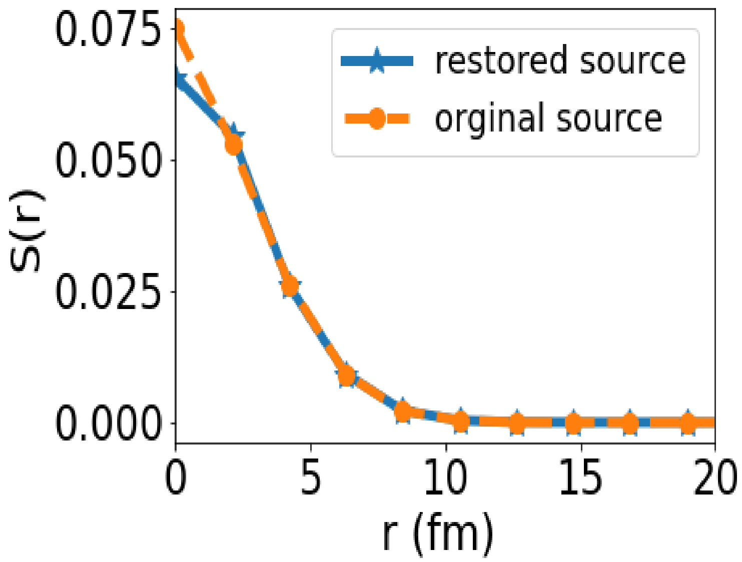
- Regularization to stabilize the solution: $I_i^n = \begin{cases} \frac{1}{1-\lambda} & F_i^n < F_{\{i-1,i+1\}}^n \\ \frac{1}{1+\lambda} & F_i^n > F_{\{i-1,i+1\}}^n \\ 1 & otherwise \end{cases}$
- $F_i^{(n+1)} = \sum_j \frac{\hat{f}_j}{\hat{f}_j^n} T_{ji} I_i^n F_i^n,$
- λ is adjusted parameters

Testing: Deblurring SF for $\pi^0 - \pi^0$ system

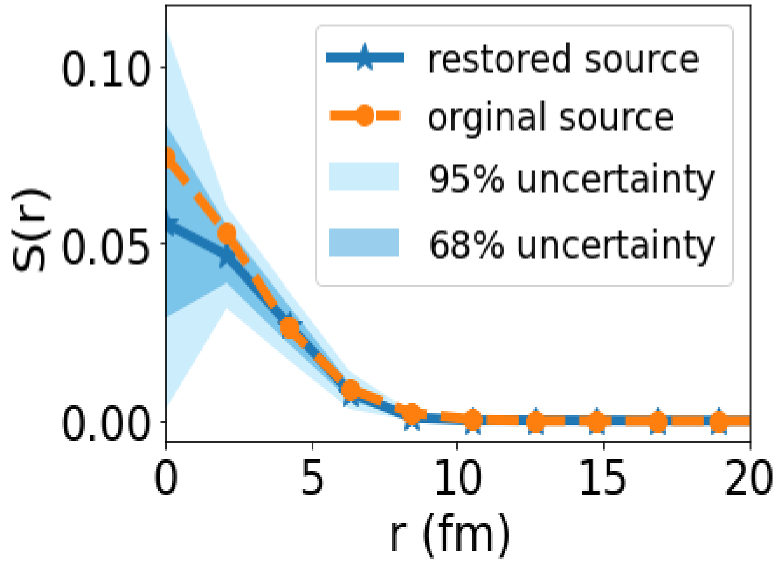
- $K(r,q) \approx \frac{\sin(rq)}{2rq}$ and $S(\boldsymbol{r}) \sim \exp\left(-\frac{\boldsymbol{r}^2}{2R_0^2}\right)$, Gaussian source function of radius R_0
- $\mathrm{R}(q) + 1 \approx 1 + \exp(-q^2 R_0^2)$ (***D.A. Brown and P. Danielewicz, 1997***)



Uncharged $\pi^0 - \pi^0$ Correlation function, $R_0 = 4$ fm



For smooth Correlation function



Deblurring S(r) for Noisy Correlation function

Wave function and Phase shift

Wave function ,

$$\Psi(\boldsymbol{q},\boldsymbol{r}) = \sum_l \frac{2}{\boldsymbol{qr}}(2l+1)i^l\, U_{l,\boldsymbol{q}}(\boldsymbol{r}) P_l(\cos\theta)$$

Radial wave function, $U_{l,\boldsymbol{q}}(\boldsymbol{r})$ is a solution to the Radial Schrödinger Equation (SE):

$$\frac{d^2}{dr^2}U_{l,\boldsymbol{q}}(\boldsymbol{r}) = \left(\frac{2\mu}{\hbar^2}\left(V_f(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} - E\right)\right)U_{l,\boldsymbol{q}}(\boldsymbol{r})$$

$V_f(r)$: Coulomb potential + Nuclear interaction

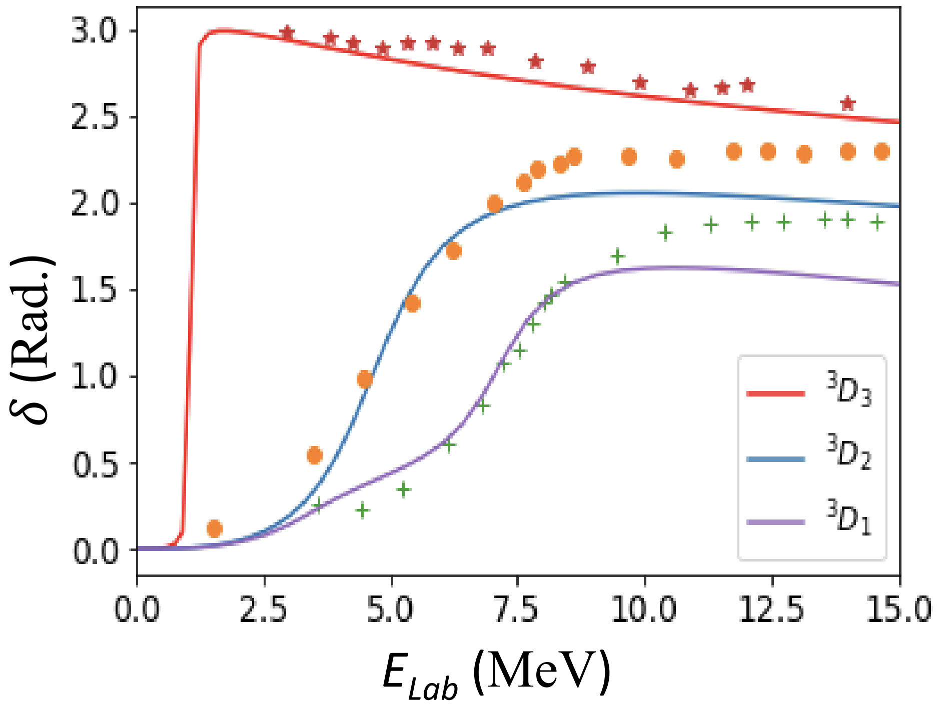
Then phase shifts are obtained by matching $U_{l,\boldsymbol{q}}(\boldsymbol{r})$ numerical and $U_{l,\boldsymbol{q}}(\boldsymbol{r})$ asymptotical solutions (i.e., logarithmic derivative)

We extract nuclear potential (Woods-Saxon form) by comparing theoretical phase shift to phase fit data,

D-alpha phase shifts;

d-wave (**$^3D_3$,$^3D_2$,$^3D_1$ states are responsible of low energy region resonance in ^{6}Li).**

-Data from **(P. E. Shanley, 1969)**



Baye's theory for deblurring

Measured/blurred observable is defined as

$f(E_d) = \int H(E_d|E'_d)F(E'_d)dE'_d$

-F : quantity we want to find

-H: Transfer matrix

In discretized form

$f_i = \sum_j H_{ij}F_j$

-We shall use discrete form in the derivation

-$\sum_j H_{ij} = 1$, implies $\sum_j f_j = \sum_j F_j = N$ all produced particle were detected

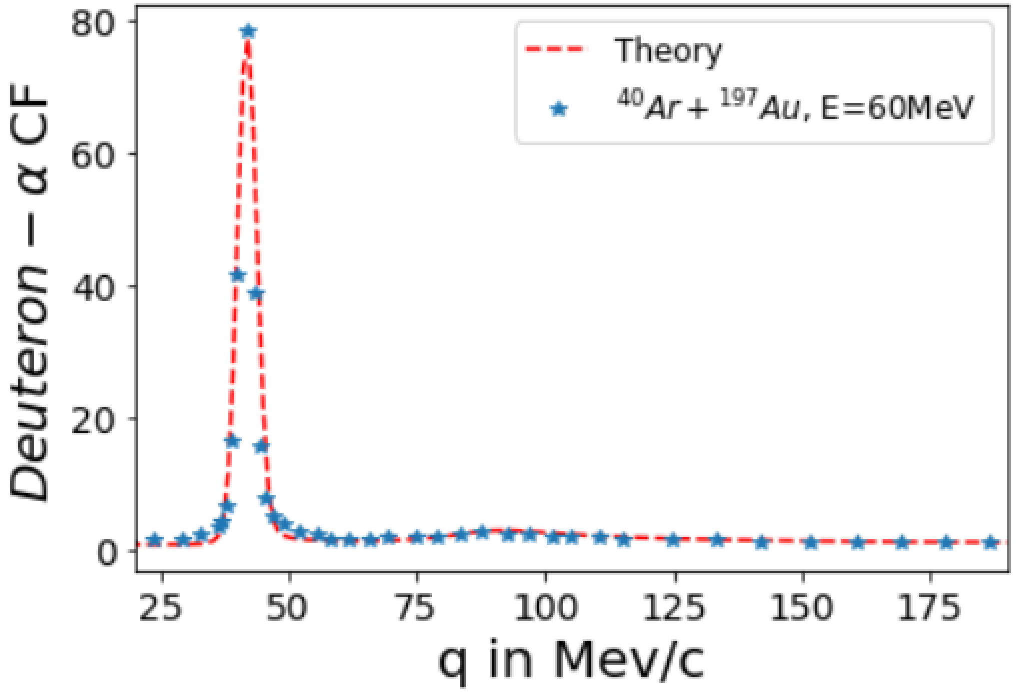
-f, H are know except F

- Assume
- $p(F_i) \sim \frac{F_i}{N}$ -probability that F_i occurs and $p(f_i) \sim \frac{f_i}{N}$ -probability that F_i occurs
- Then Baye's theory
- $H_{ki} = p(F_k|f_i) = \frac{p(f_k|F_i)p(F_i)}{\sum_j p(f_k|F_j)\, p(F_j)}$
- $p(F_k|f_i)$ - probability that F_k occurs given f_i and is a complement of $p(f_k|F_i)$

Pawel Danielewicz and Mizuki Kurata-Nishimura
Phys. Rev. C 105, 034608 (2022)

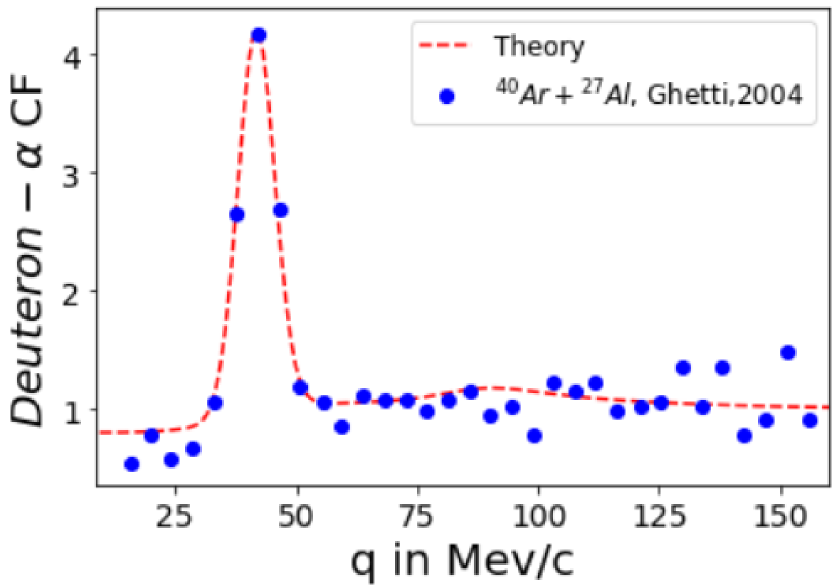
G. D'Agostini, Nucl. Instrum. Methods. Phys. Res. A 362, 487 (1995).

Deuteron-Alpha CF



α-d CF with data(stars) from ^{40}Ar+^{197}Au reaction at E=60MeV/U (**Z. Fan Phd thesis, 1992).**

The estimated Gaussian source par: R=2.3 fm and λ=.32



α-d CF with data(stars) from ^{40}Ar+^{27}Al reaction at E=40MeV/U **(R. Ghetti, et al., 2004).**

The estimated Gaussian source par:

R=6.3 fm and λ=.3

The peak at q=42 MeV/C corresponds to: J=3^+ state of ^{6}Li at E=2.186 decay.

The peak between 80-100 MeV/C is due to overlap of ^{6}Li state at E=4.31MeV ($^3d_1$) and E=5.6MeV ($^3d_2$)