



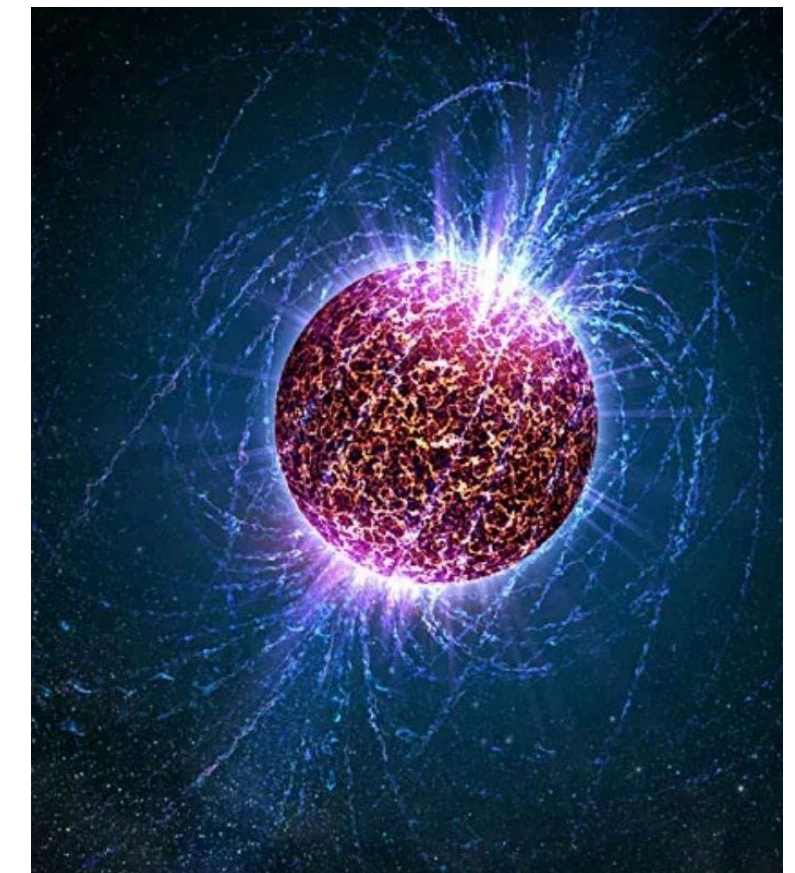
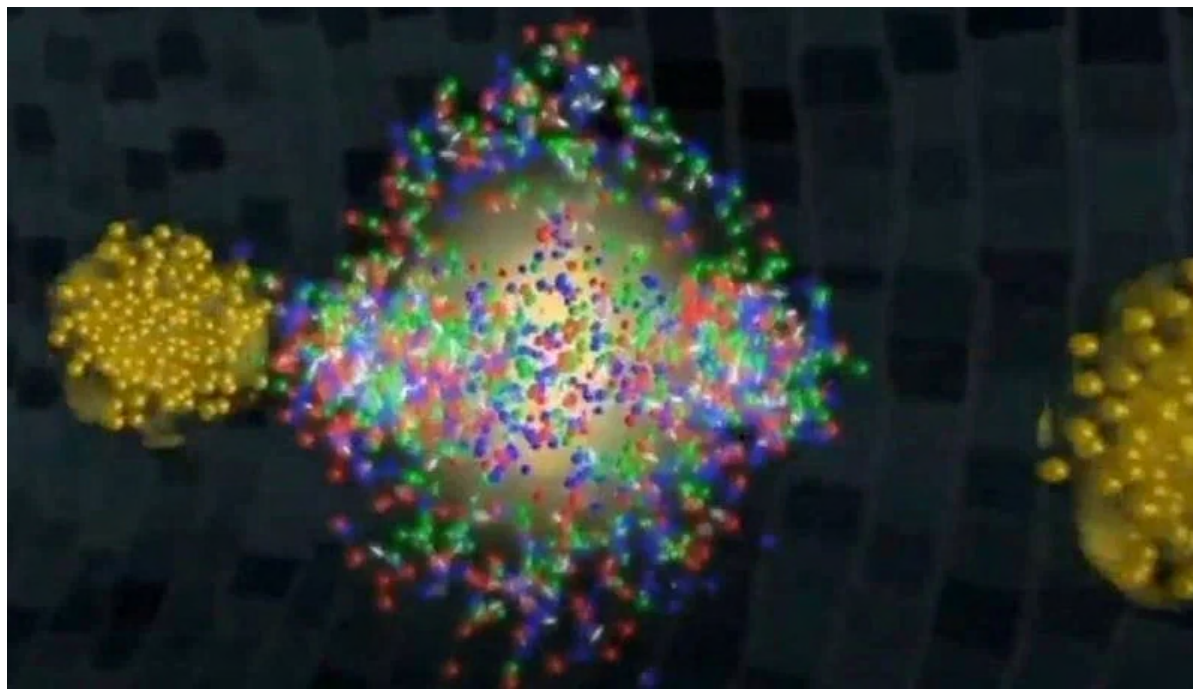
Illinois Center for Advanced Studies of the Universe



How do we connect the dense matter EOS to heavy-ion data?

Jacquelyn Noronha-Hostler

INT 25-94W Sept, 2025



Main takeaway: heavy-ion collisions are complicated.

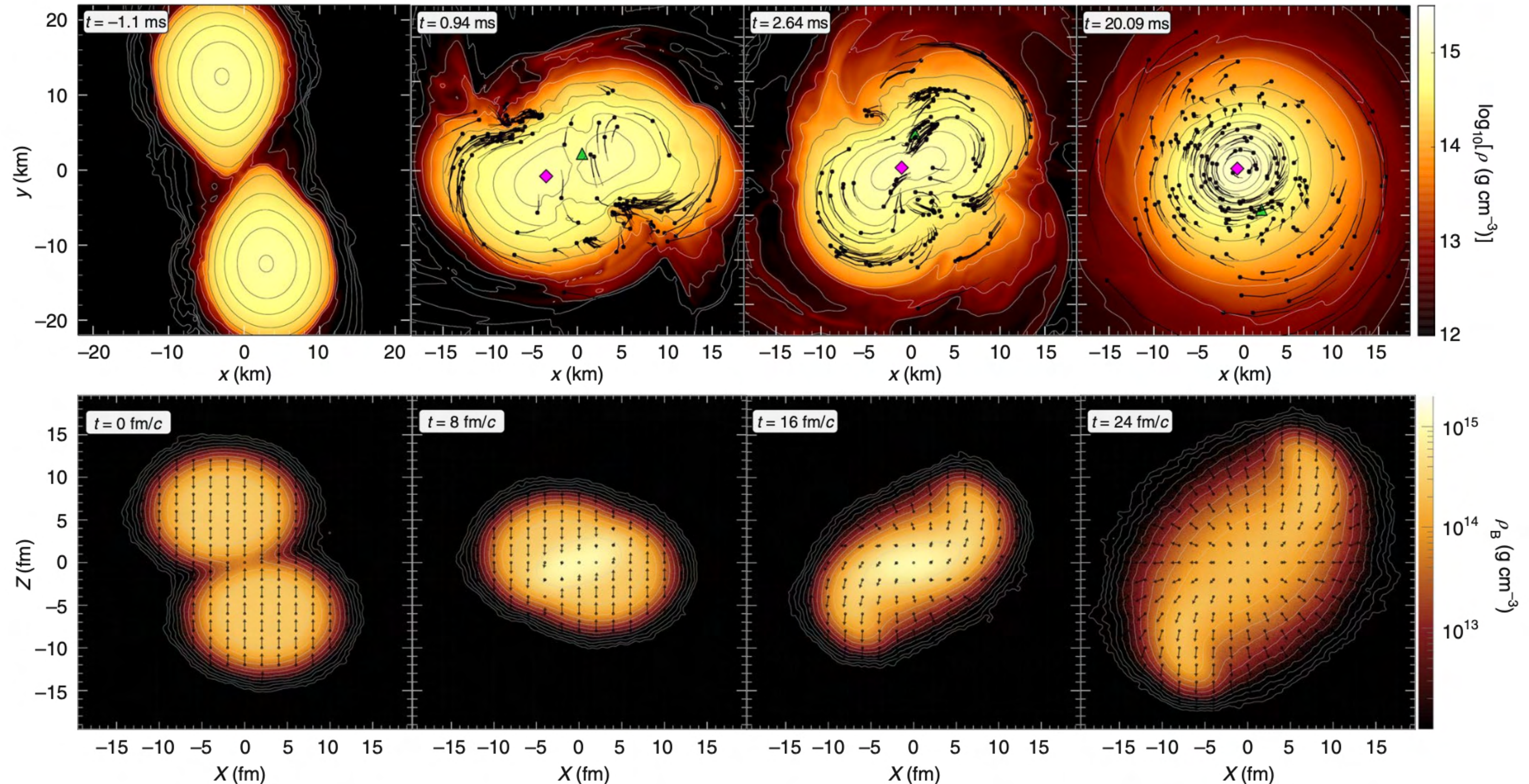
Don't trust anyone who tells you they know the
large n_B EOS.

But there's hope we will with enough time and effort!

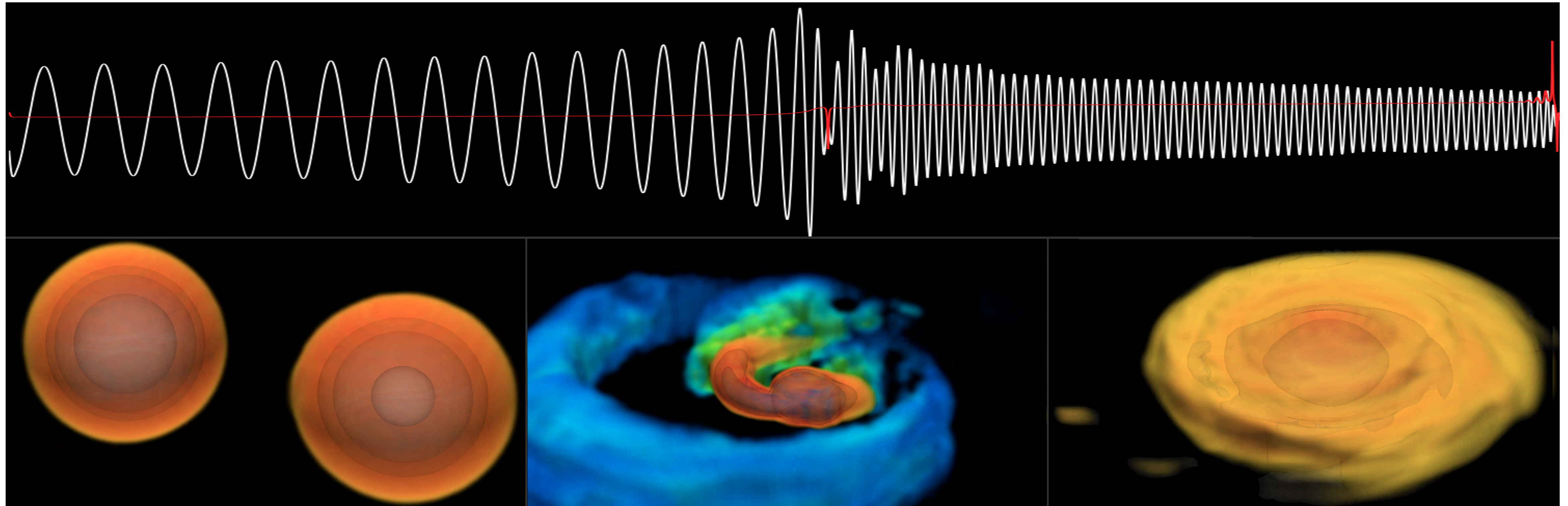
Heavy-ion collisions (HIC) vs Binary neutron star (NS) mergers

Neutron Star Mergers (Numerical Relativity)

[HADES] Nature Phys. 15 (2019) 10, 1040-1045



Stages of a binary neutron star (NS) merger



Cold (inspiral)

$$T \sim 10^{-5} \text{ MeV}$$

Hot (merger)

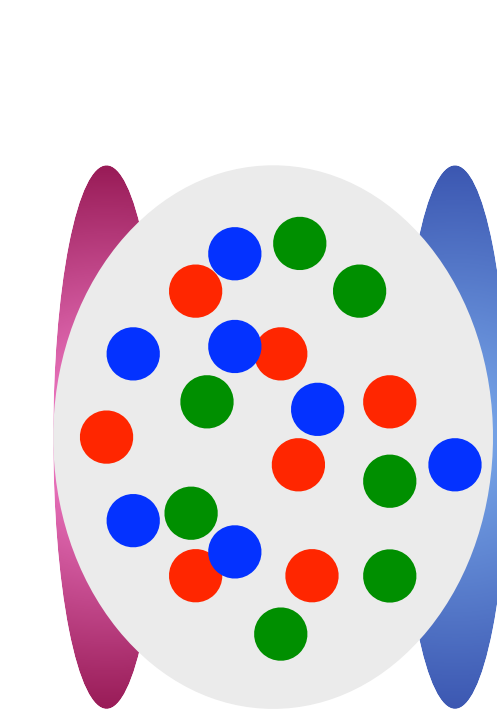
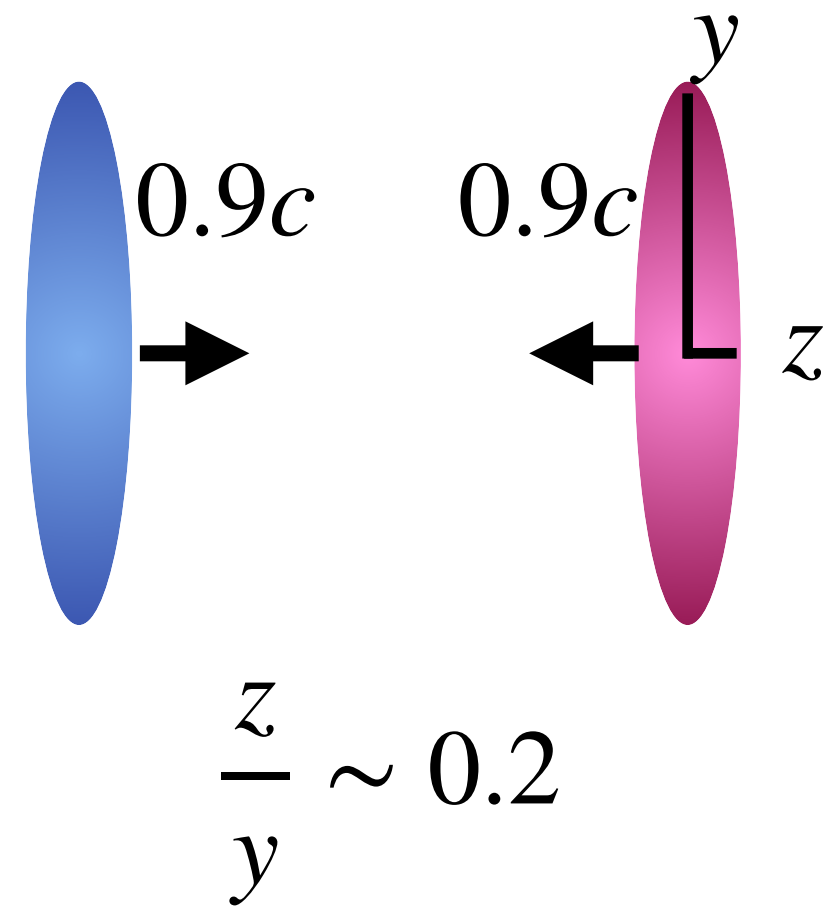
$$T \sim 100 \text{ MeV}$$

Observations here

Not yet measured

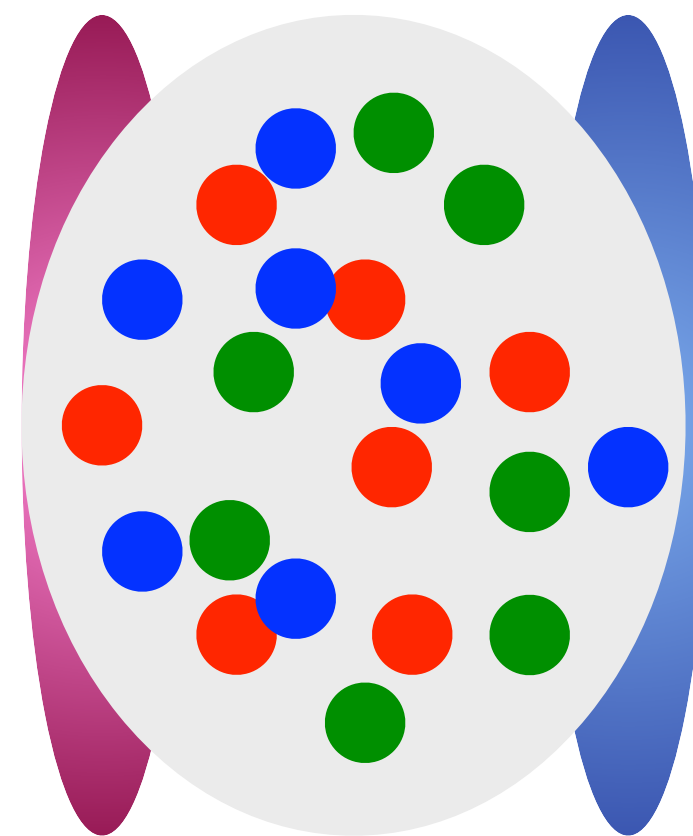
Stages of a heavy-ion collision (HIC)

Example of $\sqrt{s_{NN}} \sim 10 \text{ GeV}$ collision



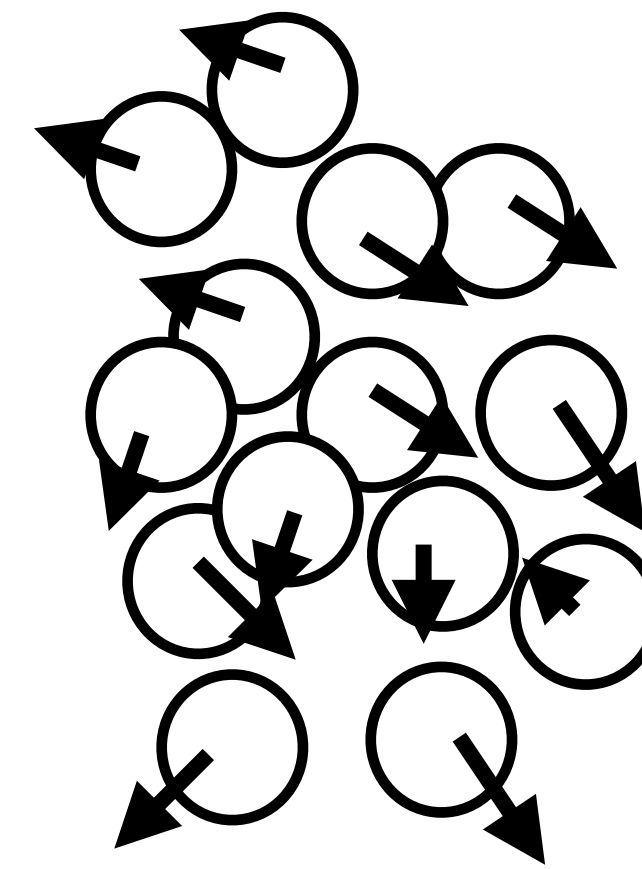
Initial State

$$T \sim 0 \text{ MeV}, \quad n_B \sim n_{sat}$$



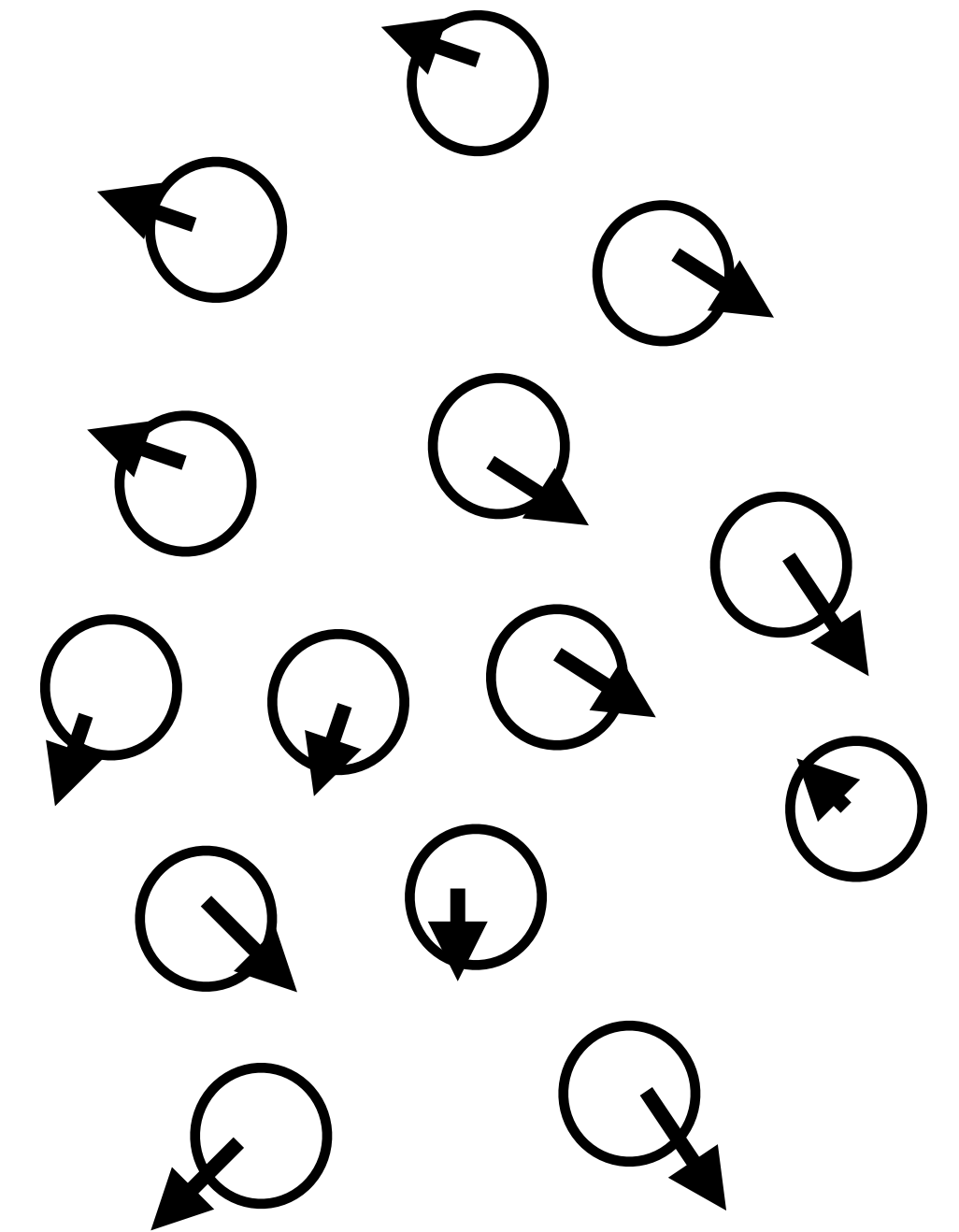
Hydrodynamics

$$T_{max} \sim 200 \text{ MeV} \quad \mu_B^{max} \sim 300 \text{ MeV}$$



Hadron gas

$$T_{FO} \sim 140 \text{ MeV}, \quad \mu_B \sim 200 \text{ MeV}$$



Freeze-out

“Archeology”

Data obtained here

System size BNS vs HIC

Difference $L_{NS}/L_{HIC} \sim 10^{18}$

$$L_{NS} \sim 10^4 m$$

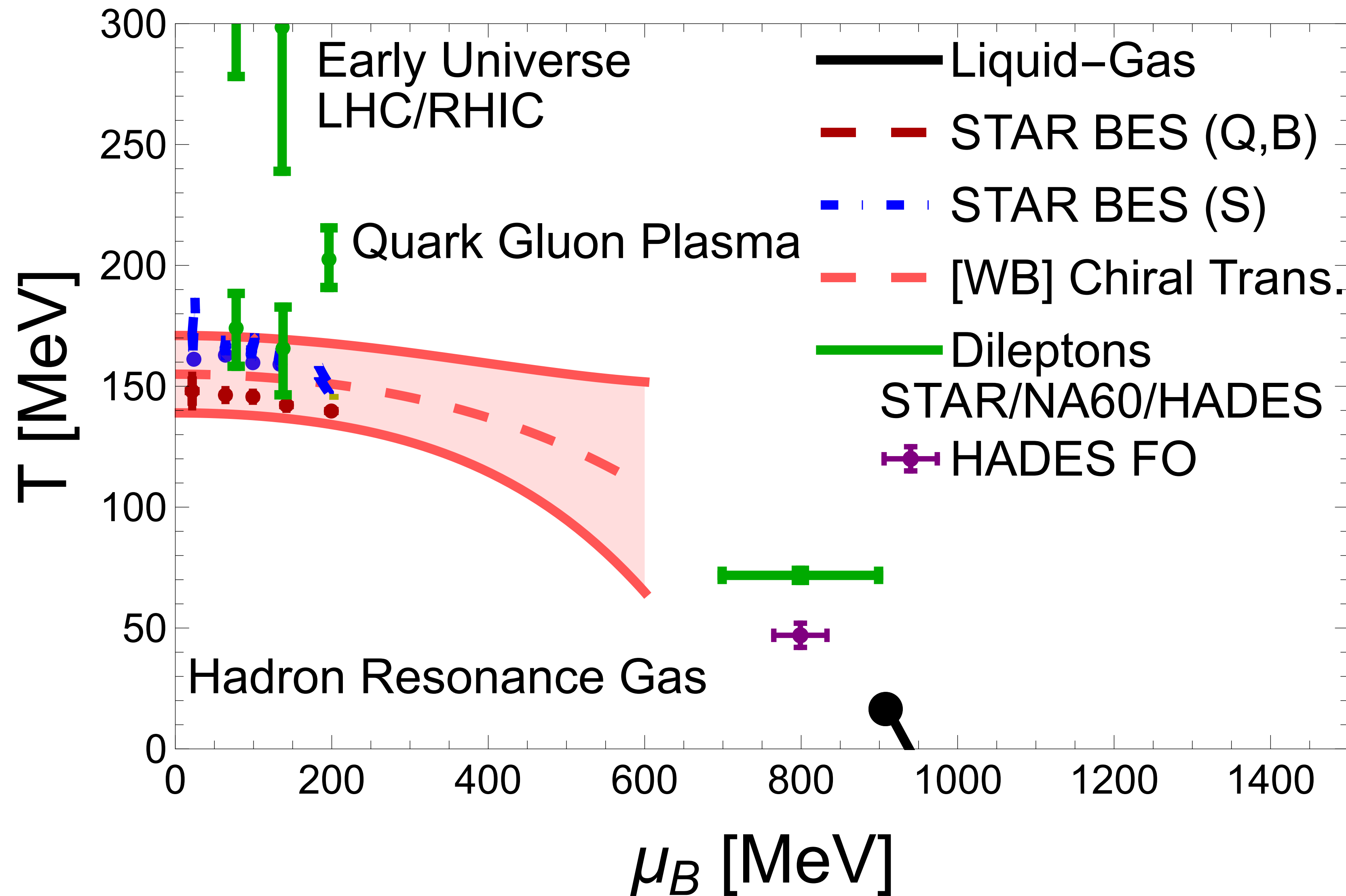
- Neutrinos and leptons matter
- Magnetic fields $\gg$ viscosity
- Convergence issues (Nils talk)
- Time scales milliseconds (10^{-3} sec)
- Causality?
- EOS has constraints at $T = 0$
- EOS ***unknown*** at finite T ??
- $n_B \lesssim 6 n_{sat}$, $T \lesssim 50 MeV$, $Y_Q \lesssim 0.2$
- $\mu_S = 0$
- Ignore anti-matter

$$L_{HIC} \sim 10^{-14} m$$

- QCD (hadrons/quarks) matters
- Viscosity $\gg$ magnetic fields
- Converges
- Time scales femtometer (10^{-23} sec)
- Causality challenges with viscosity
- EOS is ***known*** nearly exactly at high energies
- EOS unknown-***ish*** at finite n_B, n_S, n_Q
- $n_B \lesssim 2 n_{sat}$, $T \gtrsim 50 MeV$, $Y_Q \gtrsim 0.38$
- $n_S = 0$
- We have a TON of anti-matter

Jamie's
talk

HIC: what we know, what we don't



Hydro simulations from $\sqrt{s} = [3, 7.7, 27] \text{ GeV}$
Shen&Schenke, Phys. Rev. C 105, 064905 (2022)

(T, μ_B) extracted from STAR net-(p, π ,K), net-p, net-K
fluctuations
Alba, et al, Phys. Rev. C 101, 054905 (2020)

Chiral transition from lattice QCD
[WB] Phys. Rev. Lett. 125, 052001 (2020)

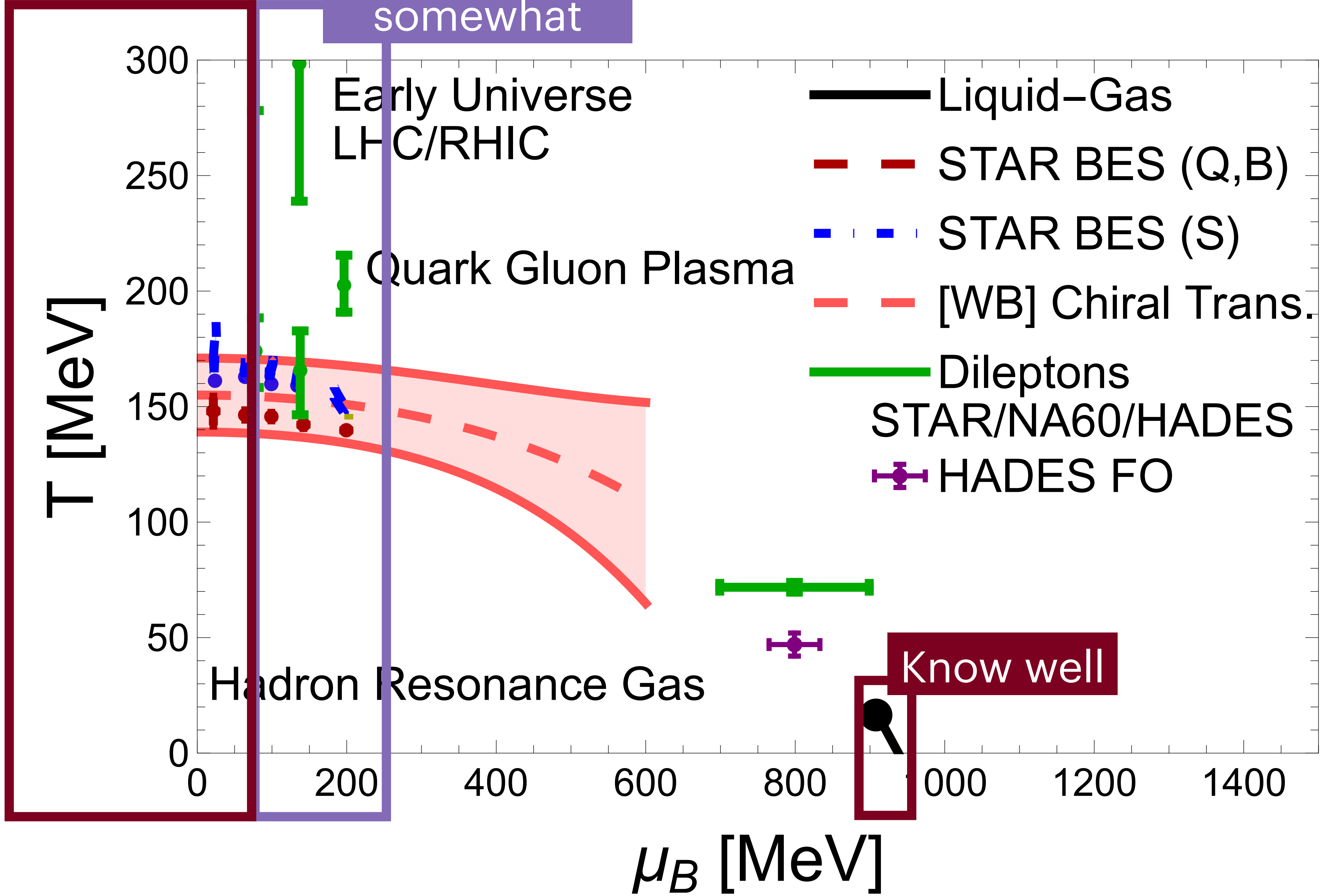
Dilepton measurements from
[STAR] 2402.01998 [nucl-ex]
[HADES] Nature Phys. 15, 1040 (2019)
[NA60] Eur.Phys.J.C59:607-623,2009

Statistical Hadronization Model
[HADES] Phys. Rev. C 102, 054903 (2020)

Liquid-gas phase transition location
Elliott, et al, Phys. Rev. C 87, 054622 (2013)
 μ_B estimate Vovchenko, et al, Phys. Rev. Lett. 118, 182301
(2017)

Hints of a QCD critical point
beginning to appear...

What we know, what we don't



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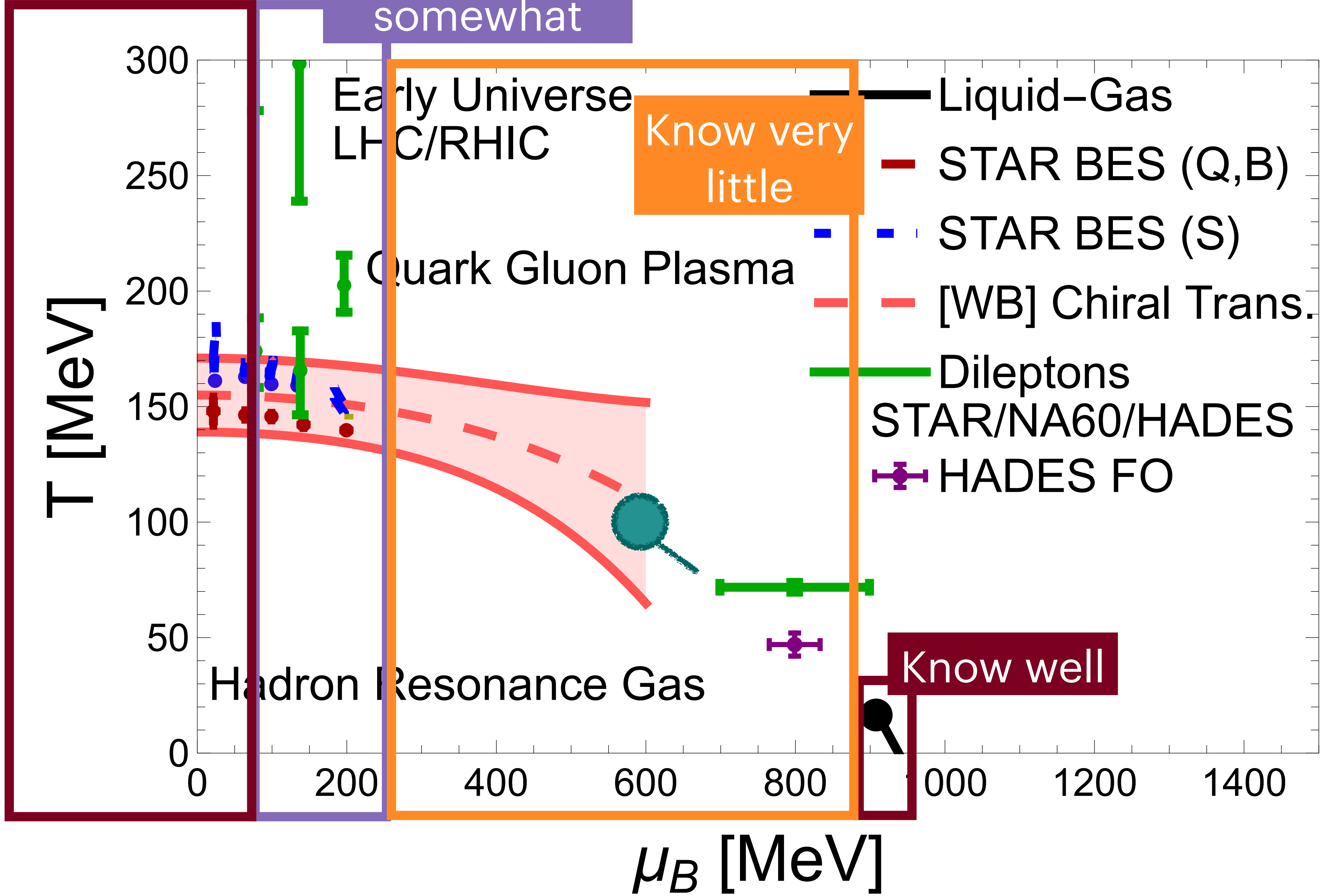
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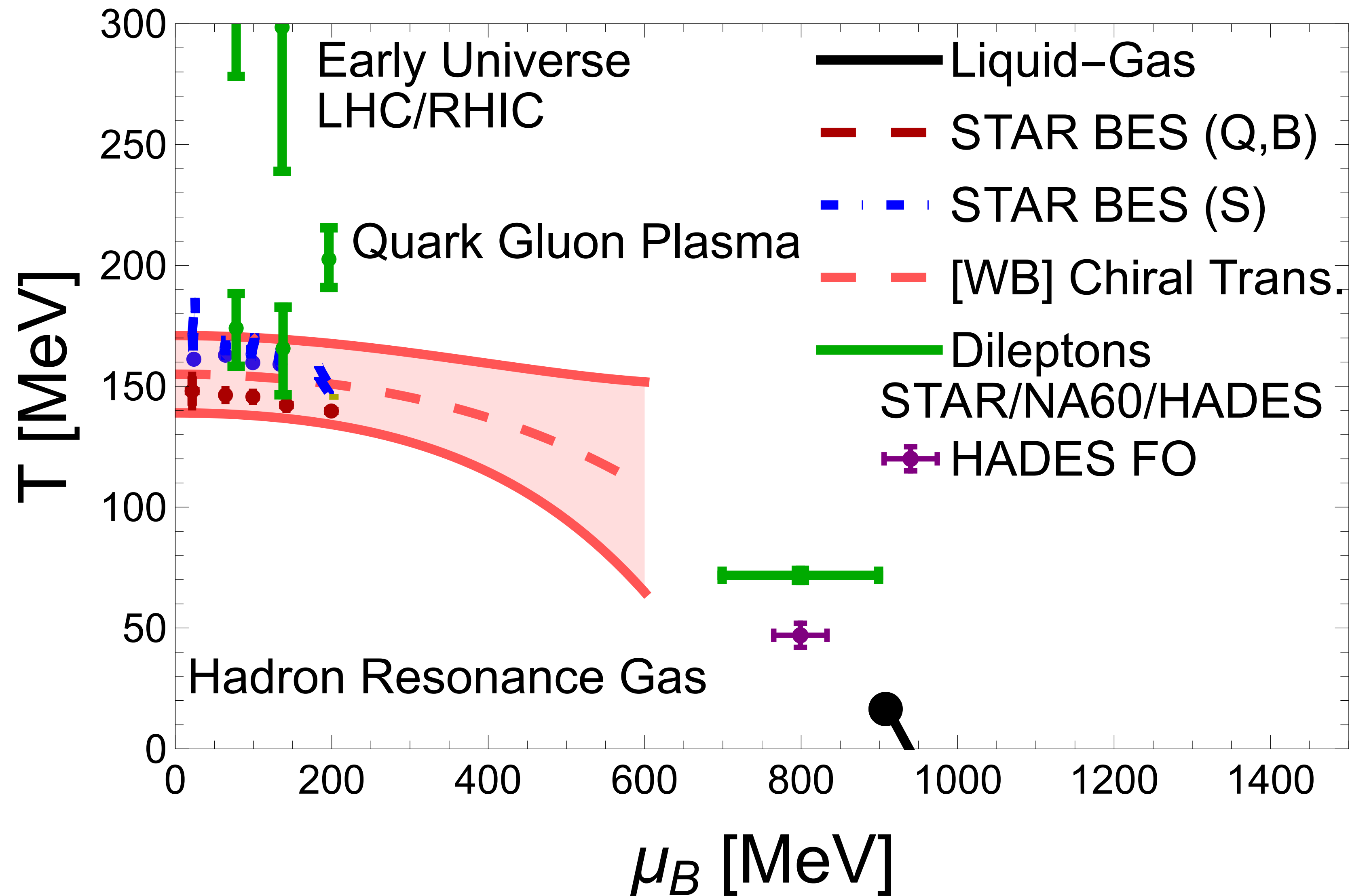
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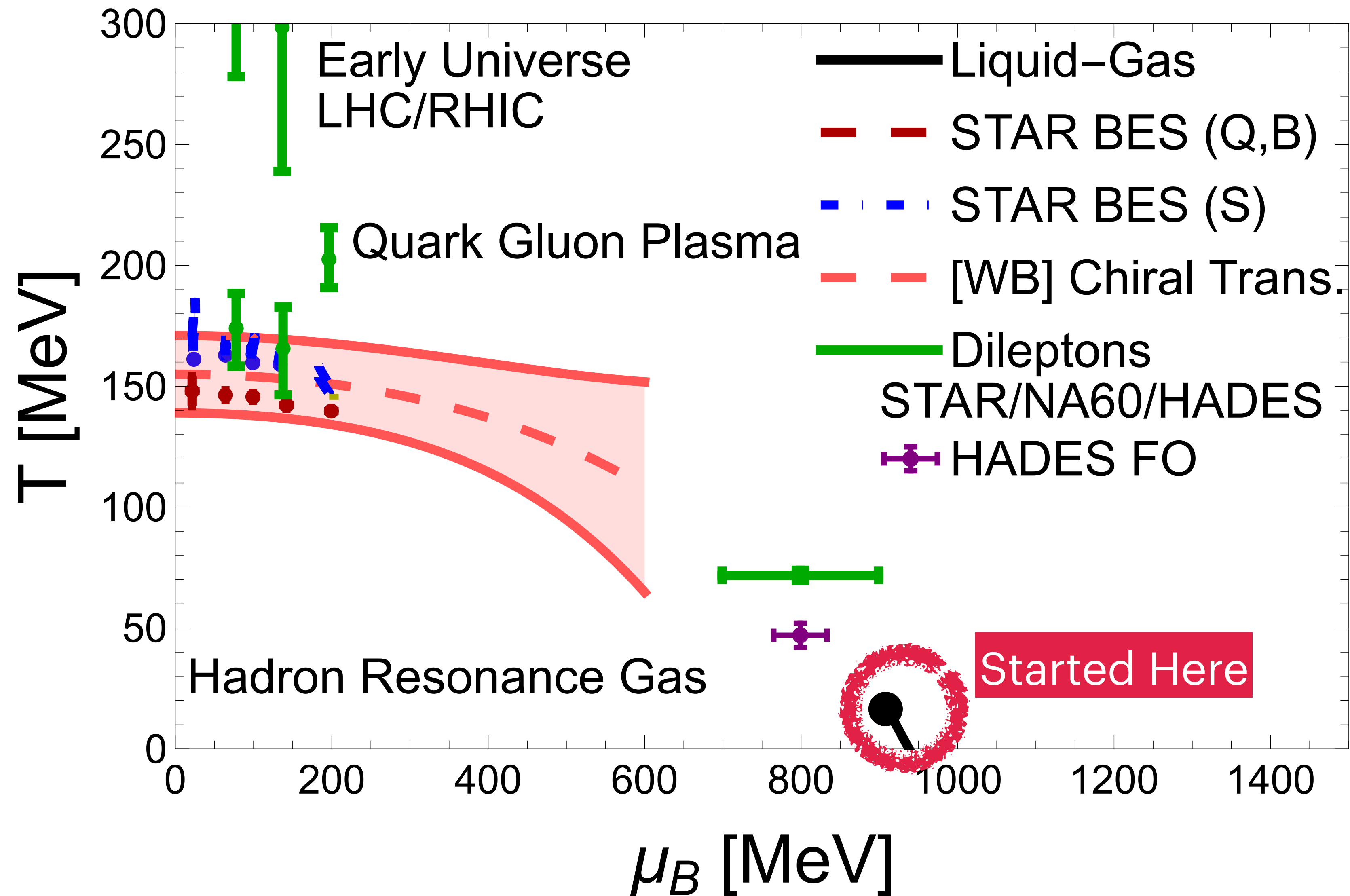
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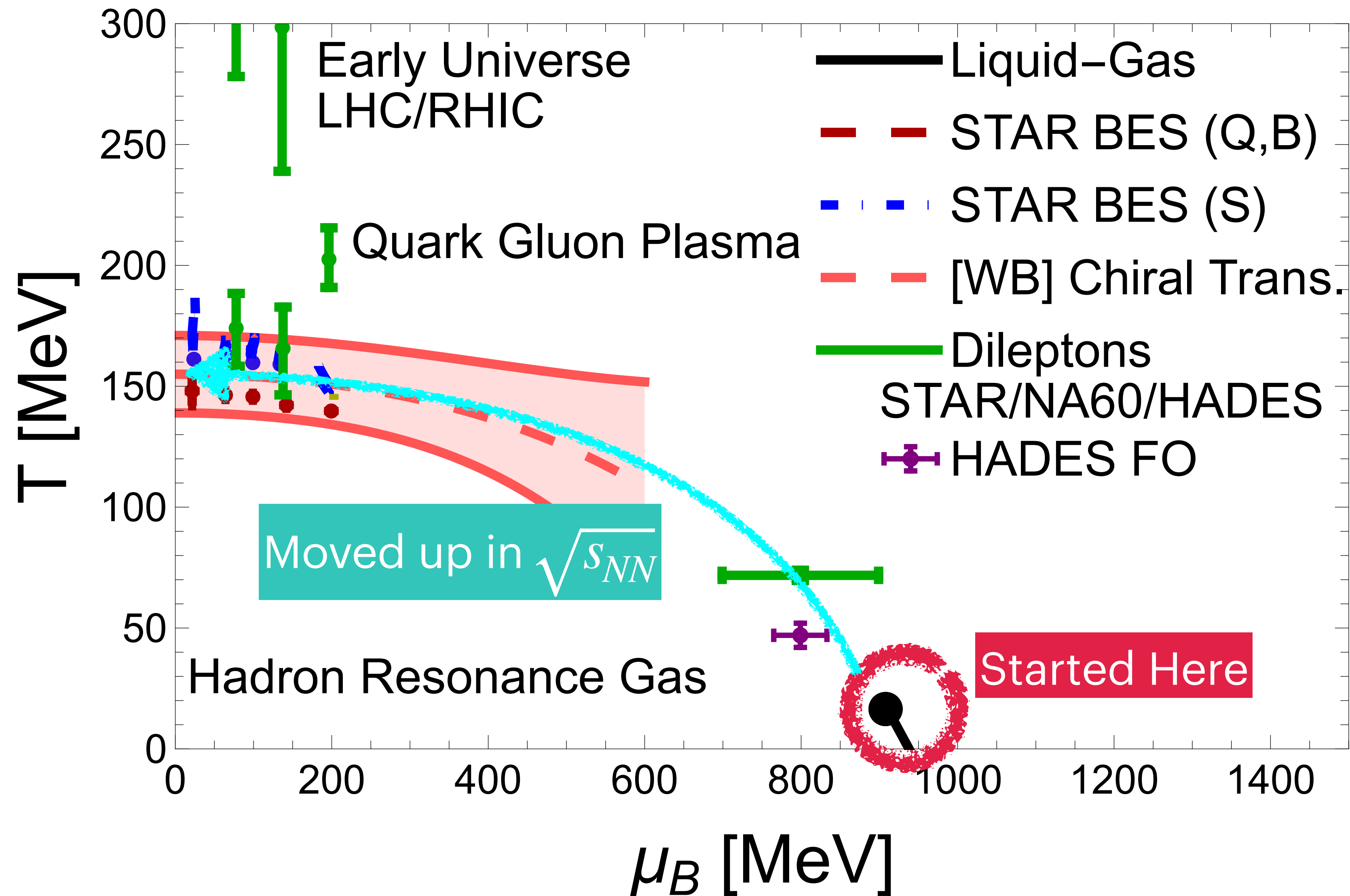
History of HIC: why don't we understand large n_B ?



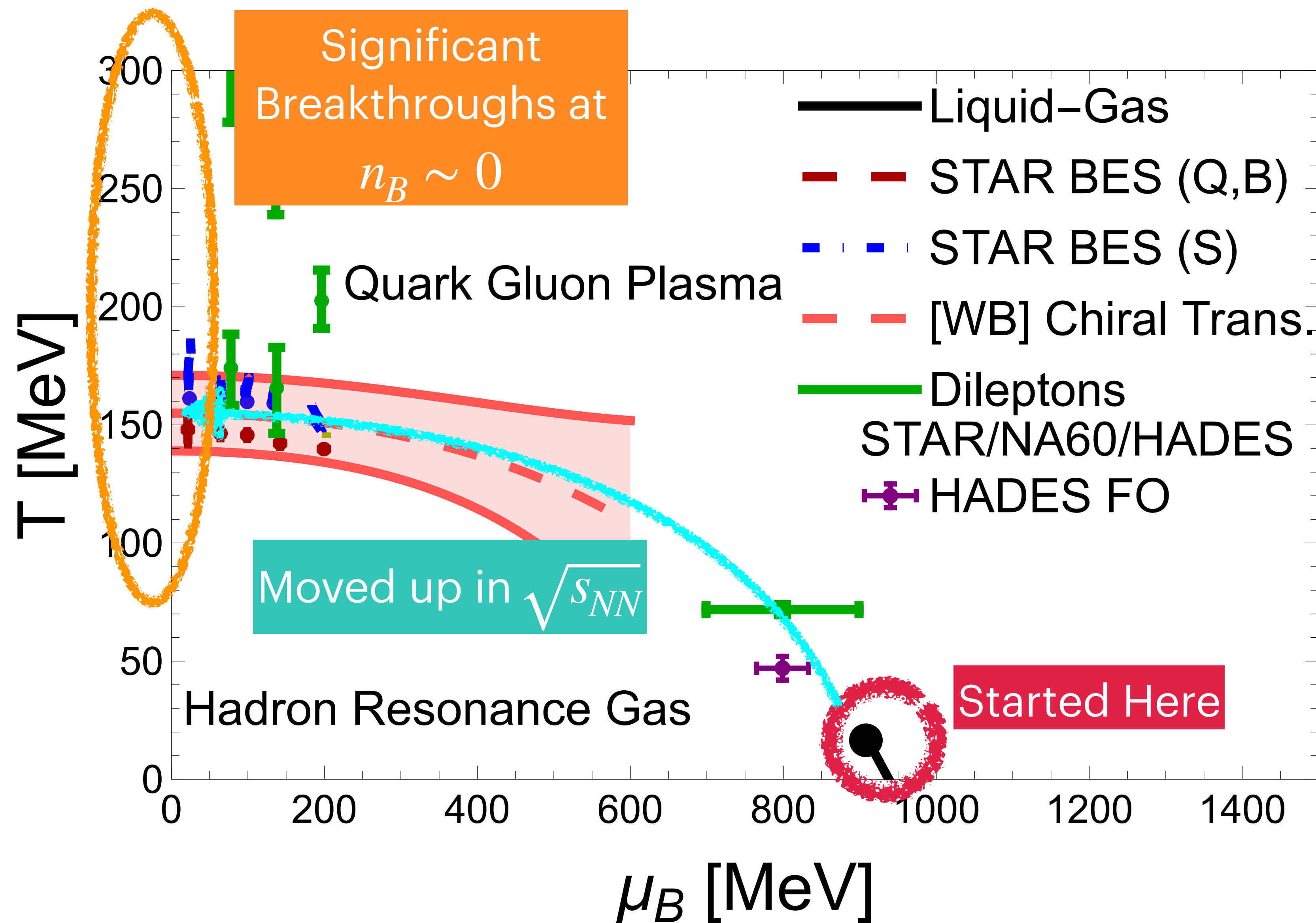
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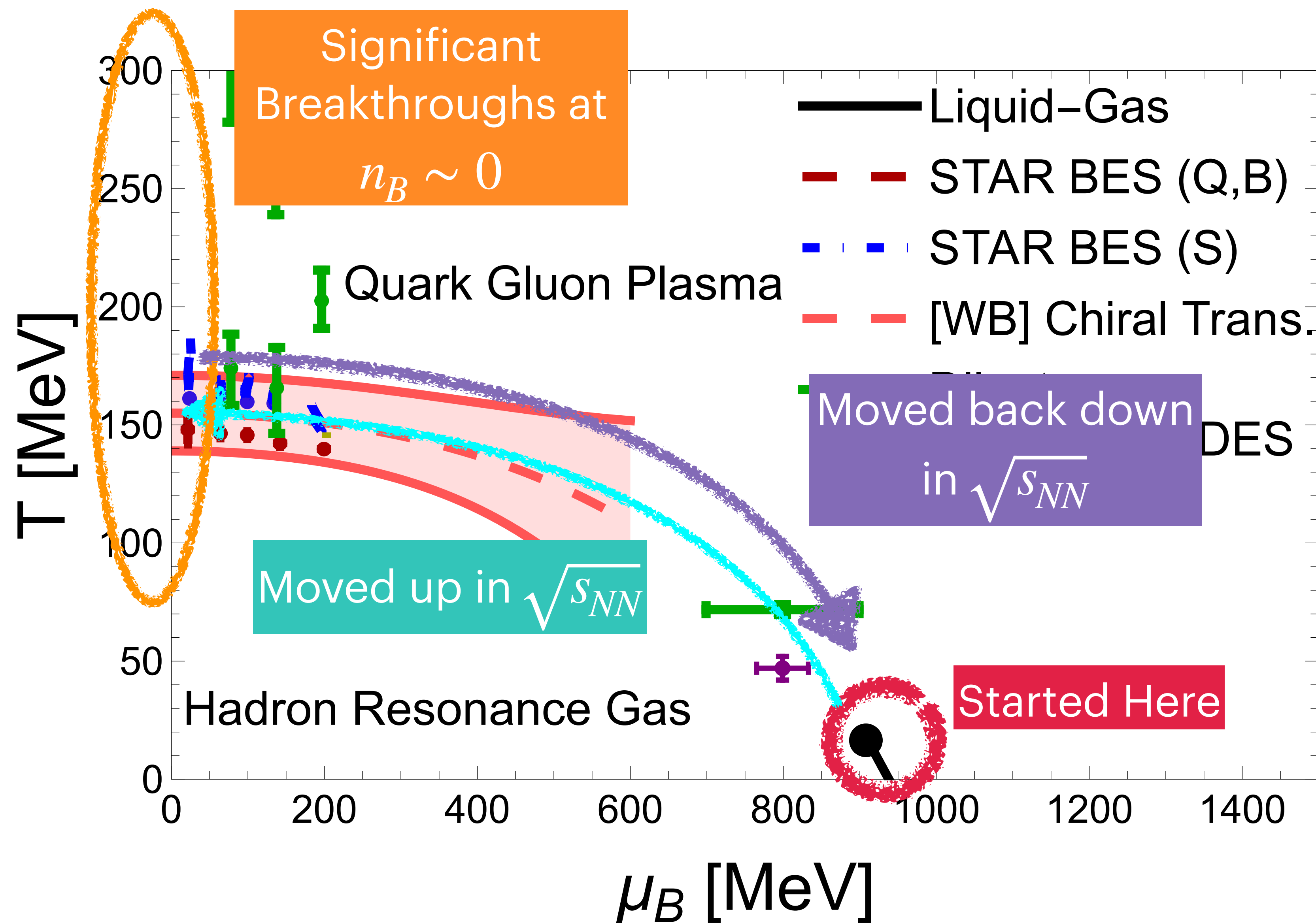


History of HIC: why don't we understand large n_B ?



- Breakthroughs:**
- Event-by-event
 - Relativistic **viscous** hydro
 - Lattice QCD EOS
 - Cross-over EOS
 - Hadron improvements
 - Constrained initial state
 - Nuclear structure
 - Open-source frameworks
 - Bayesian Analyses
 - Quantitative Predictions

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**Beam energy Scan/
HADES/SPS**
Rest of this talks

**Before proceeding , need to
understand how we connect
the EOS (equilibrium) to data**

Heavy-Ion Collisions: **Equilibrium** vs **Out-of-Equilibrium**

Energy-Momentum Tensor

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi + \pi^{\mu\nu}$$

Equation of
State (EOS)

Local rest
frame

$$T_0^{\mu\nu} = \begin{bmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

Bulk Pressure $\Pi = \text{Tr} \left[T^{\mu\nu} - T_0^{\mu\nu} \right]$

Shear stress tensor $\pi^{\mu\nu} = T^{\mu\nu} - T_0^{\mu\nu} - \Pi$

Conserved Currents

Baryon Current $N_B^\mu = n_B u^\mu + q_B^\mu$

Strangeness Current $N_S^\mu = n_S u^\mu + q_S^\mu$ **Diffusion**

Electric Charge Current $N_Q^\mu = n_Q u^\mu + q_Q^\mu$

Contains diffusion matrix and gradients of μ/T

$$\kappa = \begin{bmatrix} \kappa_{BB} & \kappa_{BS} & \kappa_{BQ} \\ \kappa_{SB} & \kappa_{SS} & \kappa_{SQ} \\ \kappa_{QB} & \kappa_{QS} & \kappa_{QQ} \end{bmatrix}$$

Ignoring magnetic fields and rotation

Heavy-Ion Collisions: Equilibrium vs Out-of-Equilibrium

Energy-Momentum Tensor

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi + \pi^{\mu\nu}$$

Local rest frame

$$\begin{bmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

Conserved Currents

Baryon Current $N_B^\mu = n_B u^\mu + q_B^\mu$

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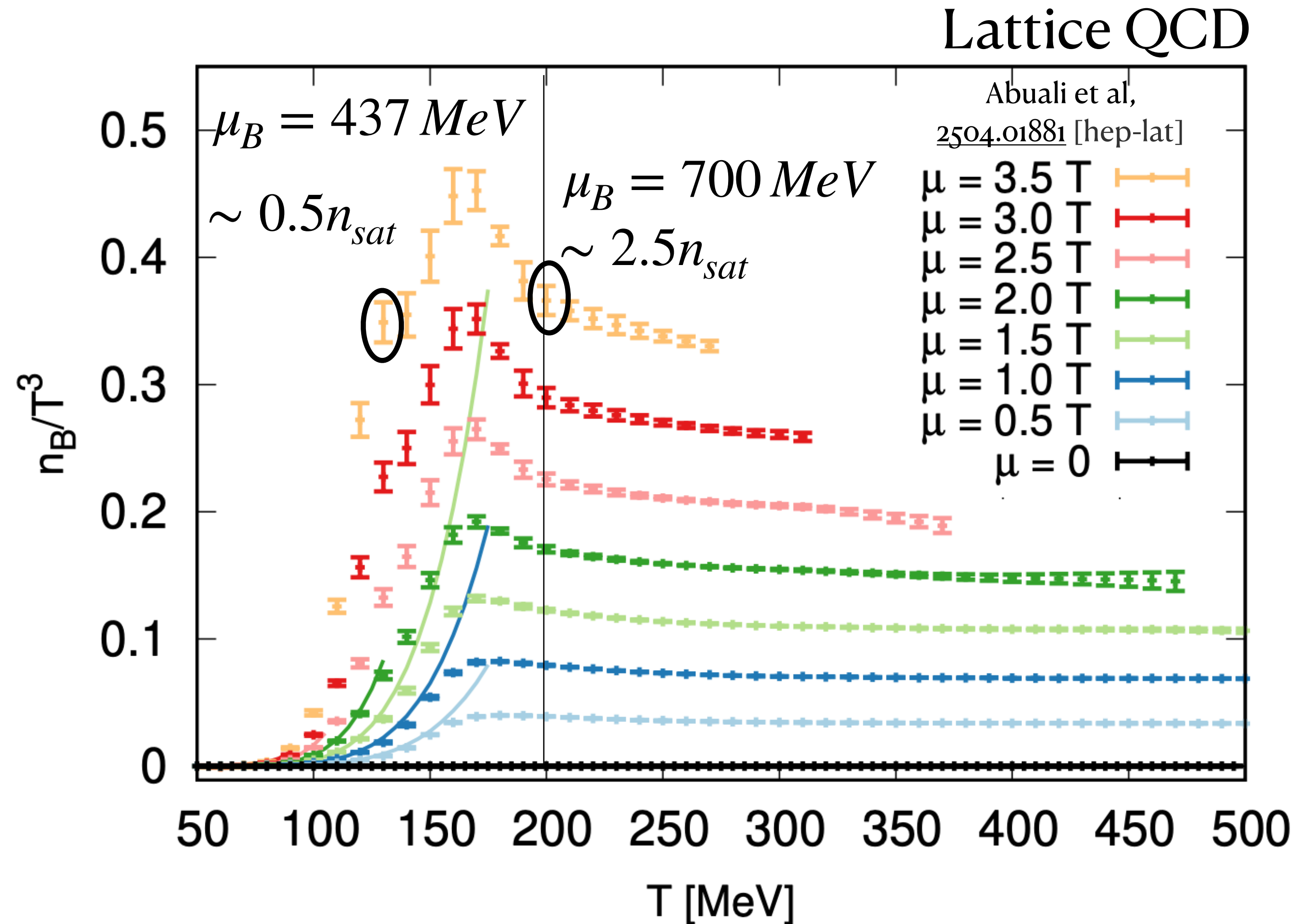
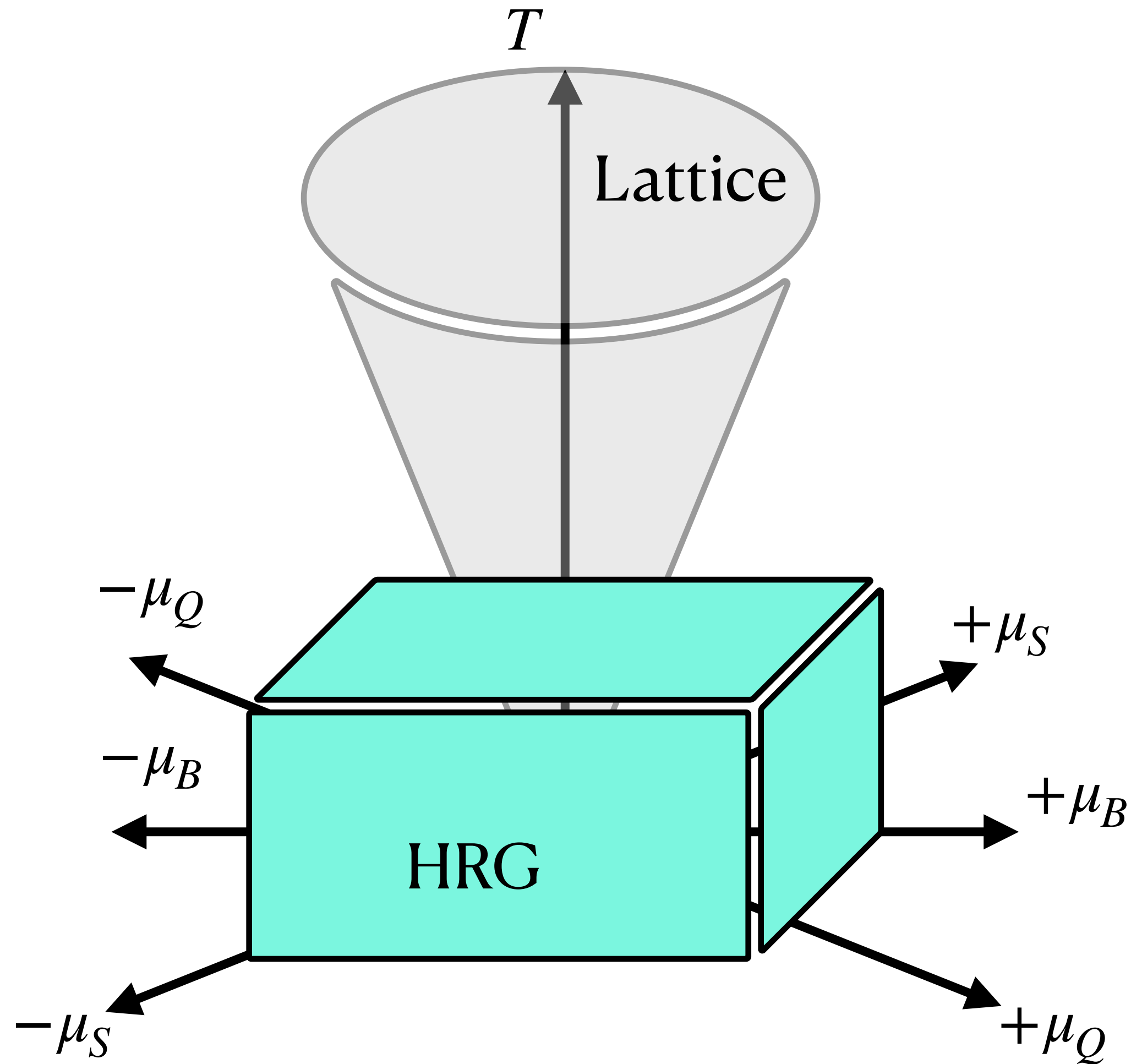
Heavy-ion collisions are NOT in equilibrium, out-of-equilibrium effects must be properly accounted for to extract the EOS!

Shear stress tensor $\pi^{\mu\nu} = T^{\mu\nu} - T_0^{\mu\nu} - \Pi$

$$\kappa = \begin{bmatrix} \overset{BB}{\kappa_{SB}} & \overset{BS}{\kappa_{SS}} & \overset{BQ}{\kappa_{SQ}} \\ \kappa_{QB} & \kappa_{QS} & \kappa_{QQ} \end{bmatrix}$$

Ignoring magnetic fields and rotation

4d EOS of heavy-ions T, n_B, n_S, n_Q



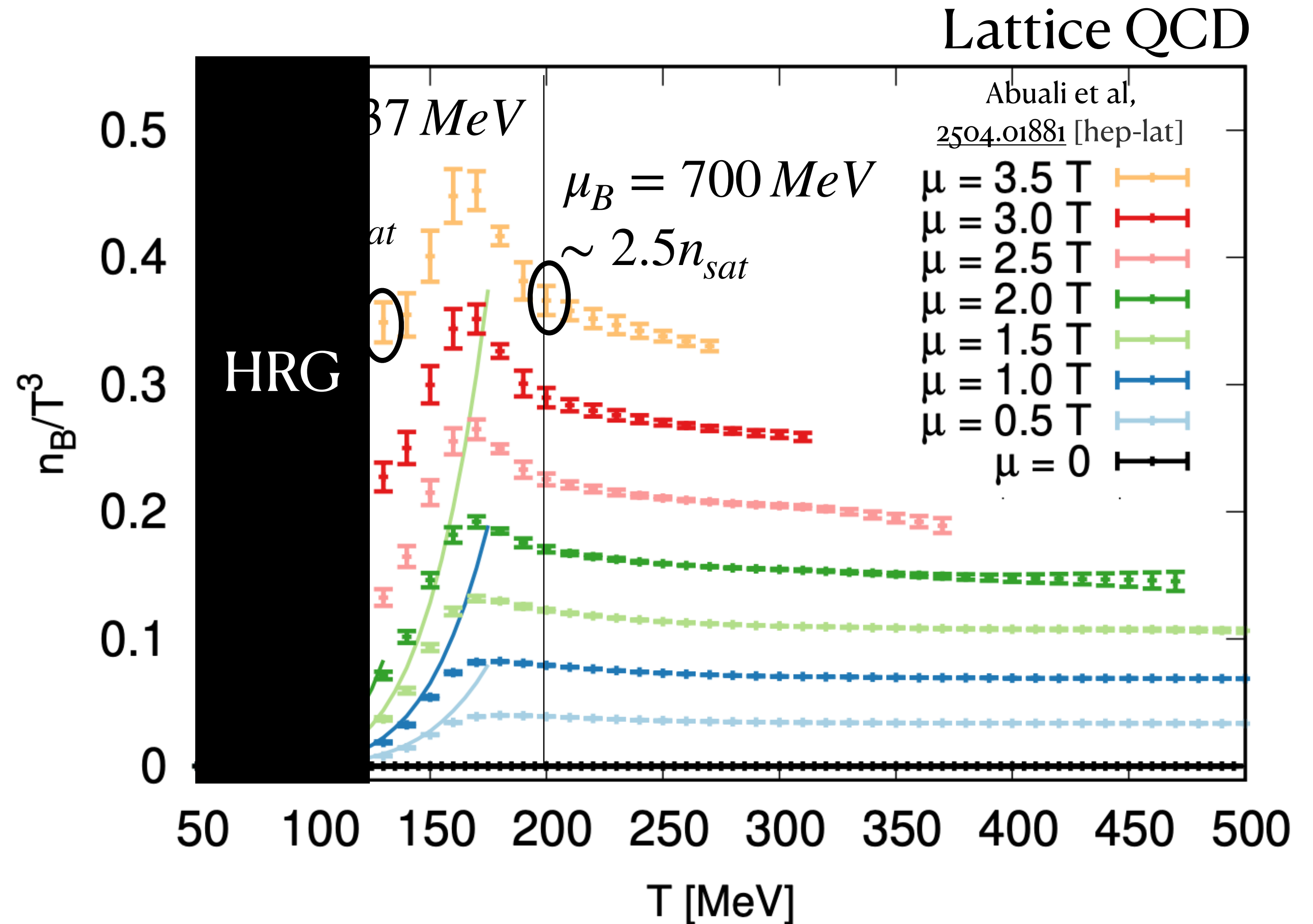
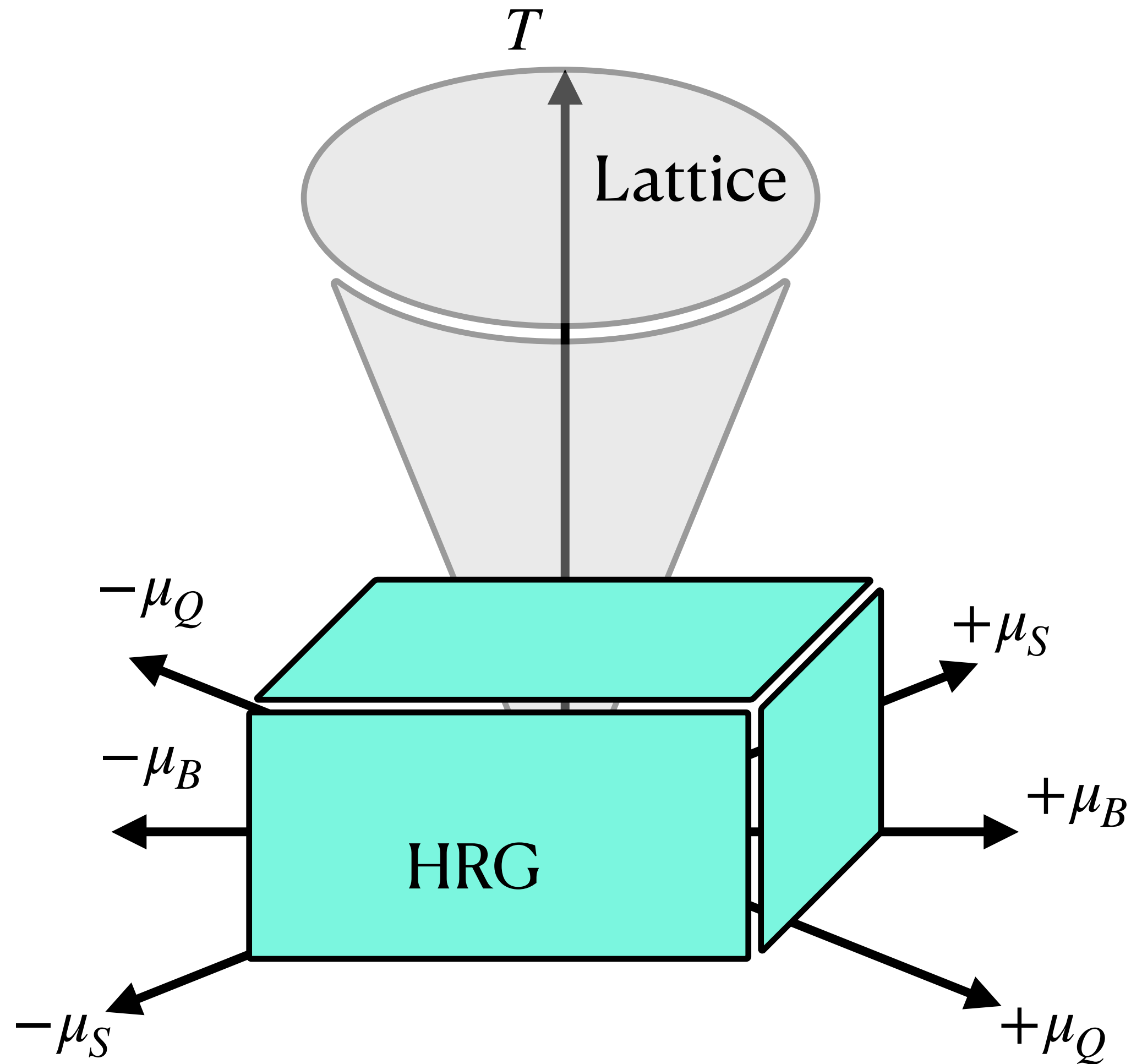
Reasonable estimates: $\mu_S/\mu_B \sim 1/3$
 $\mu_Q/\mu_B \sim -0.1$

But these are fields,
with large fluctuations!

Lines are ideal hadron
resonance gas (HRG)

Data bars come from lattice QCD

4d EOS of heavy-ions T, n_B, n_S, n_Q



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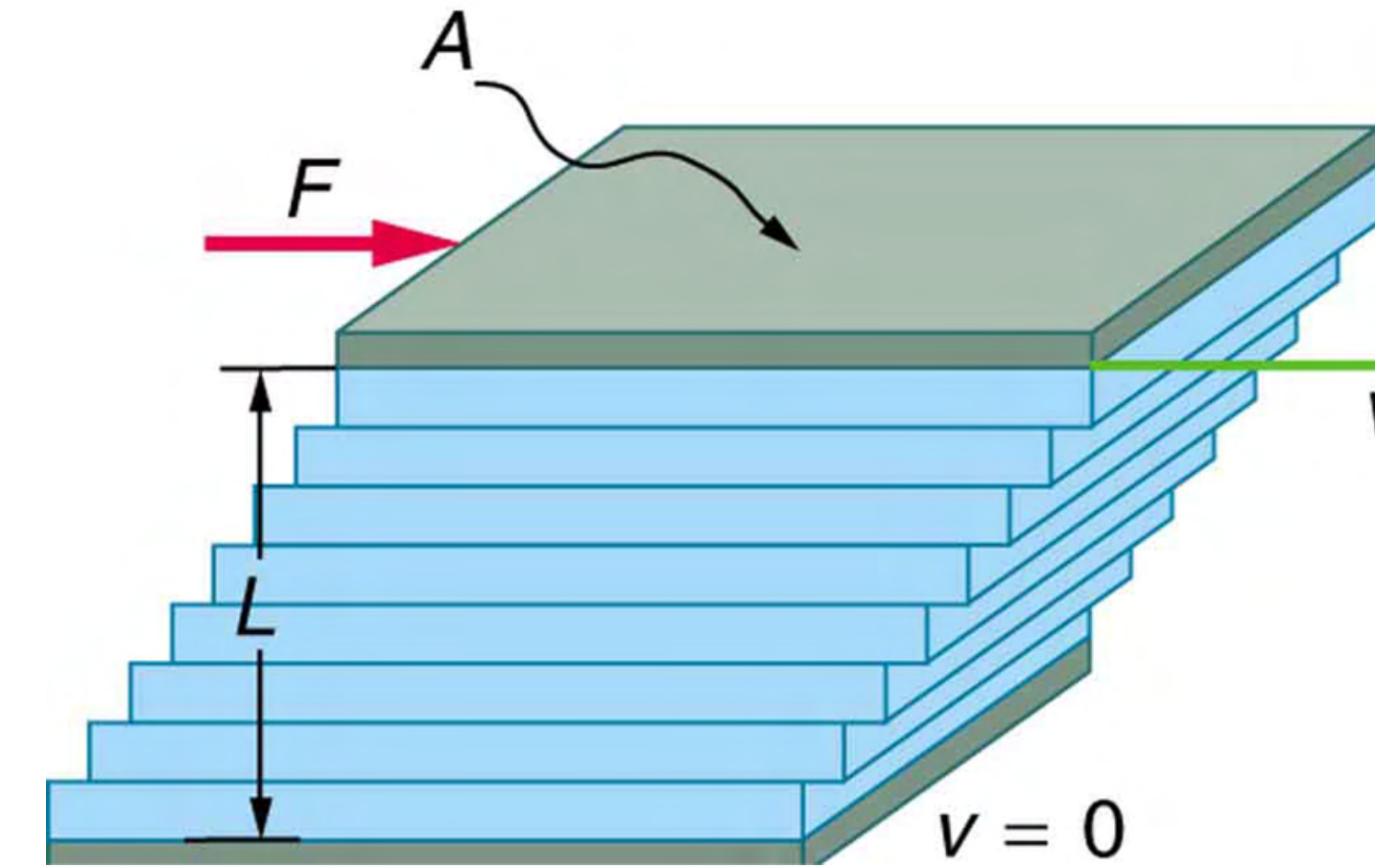
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Equations of motion for out-of-equilibrium pieces

Almaalol, Dore, JNH *Phys.Rev.D* 111 (2025) 1, 014020



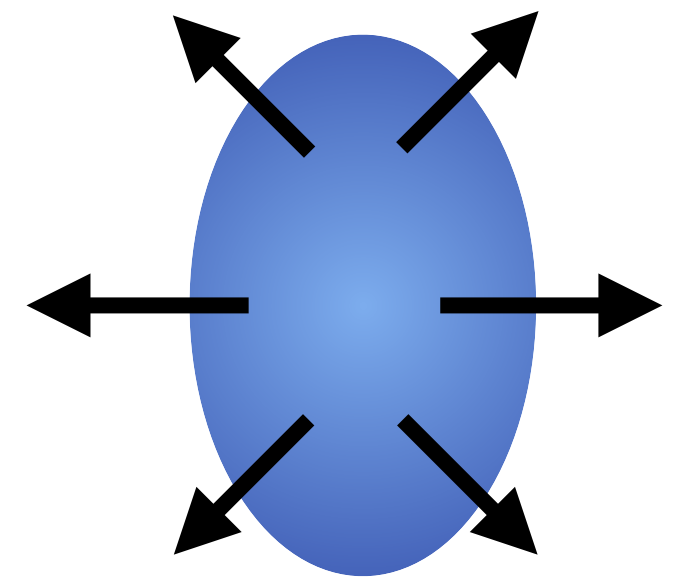
Equations NOT unique: Derived from enforcing only positive entropy production



$$\tau_{\pi} \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_{\pi}\pi^{\mu\nu}}{2}\theta - \frac{\tau_{\pi}}{2\beta_{\pi}}\dot{\beta}_{\pi}\pi^{\mu\nu} - \frac{2\eta}{\beta} \left(\gamma_1^q \nabla^{\langle\mu} n_q^{\nu\rangle} + \frac{1}{2} n_q^{\langle\mu} \nabla^{\nu\rangle} \gamma_1^q \right)$$



$$\tau_{\Pi} \dot{\Pi} + \Pi = -(\zeta + \frac{\tau_{\Pi}}{2}\Pi)\theta - \frac{\tau_{\Pi}}{2\beta_{\Pi}}\dot{\beta}_{\Pi}\Pi - \frac{\zeta}{\beta} \left(\gamma_0^q D_{\mu} n_q^{\mu} + \frac{1}{2} n_q^{\mu} \nabla_{\mu} \gamma_0^q \right)$$

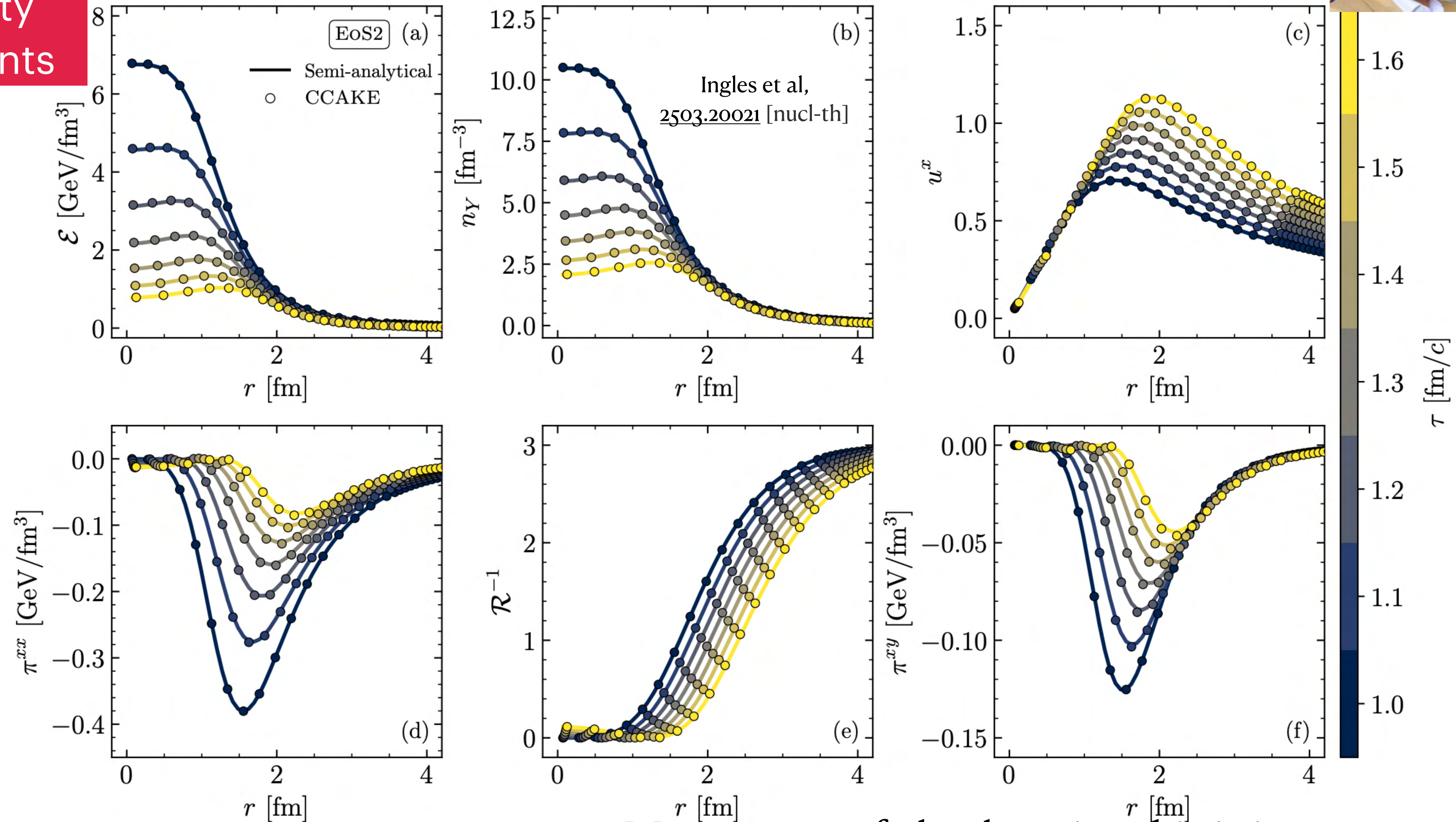


$$\tau_{qq'} \dot{n}_{q'}^{\mu} + n_q^{\mu} = -\kappa_{qq'} \nabla^{\mu} \alpha_{q'} + \frac{\tau_{qq'} n_{q'}^{\mu}}{2} \theta - \frac{\tau_{qq'}}{2\beta_{qq'}} \dot{\beta}_{qq'} n_{qq'}^{\mu} - \frac{\kappa_{qq'}}{\beta} \left(\gamma_0^{qq'} \nabla^{\mu} \Pi - \frac{\Pi}{2} \nabla^{\mu} \gamma_0^{qq'} \right) - \frac{\kappa_{qq'}}{\beta} \left(\gamma_1^{qq'} \nabla_{\nu} \pi^{\mu\nu} + \frac{\pi^{\mu\nu}}{2} \nabla_{\nu} \gamma_1^{qq'} \right)$$

Ask about
Causality
+stability
constraints

Does this actually work? Analytical checks

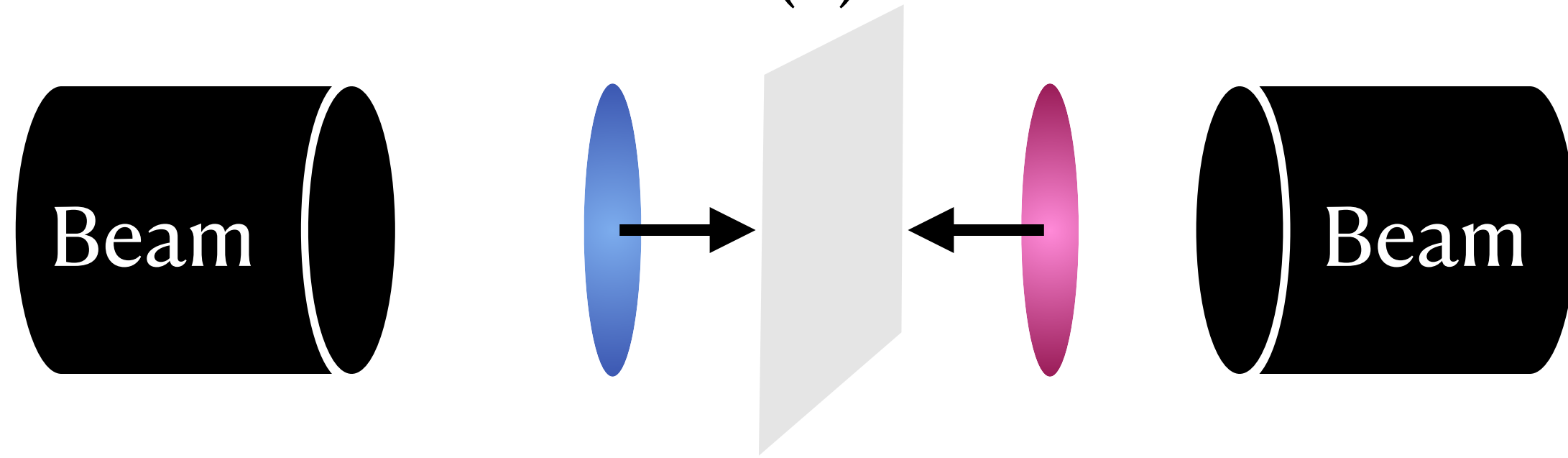
With viscosity, at finite densities



Comparing to data: dynamics of HIC

What do pressure gradients do?

Beam direction (z)

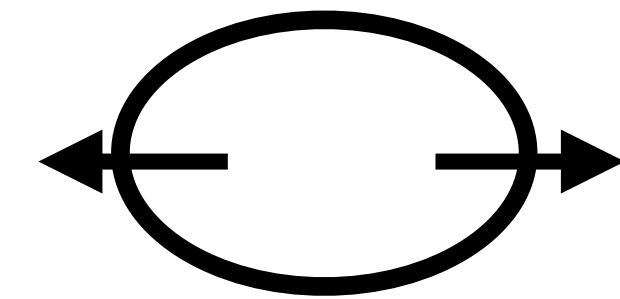
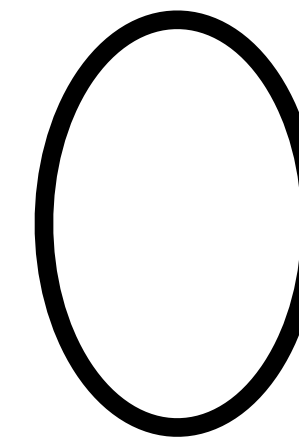


Transverse plane (x,y)

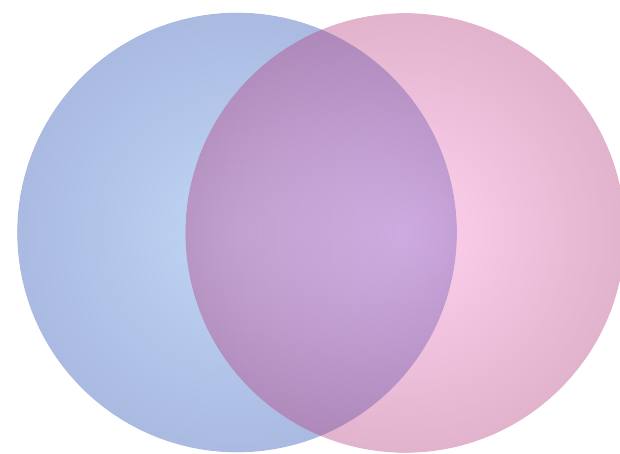
Geometric shape or “eccentricity”, propagated to final state in momentum space
(∇p drive hydro response)

Initial State

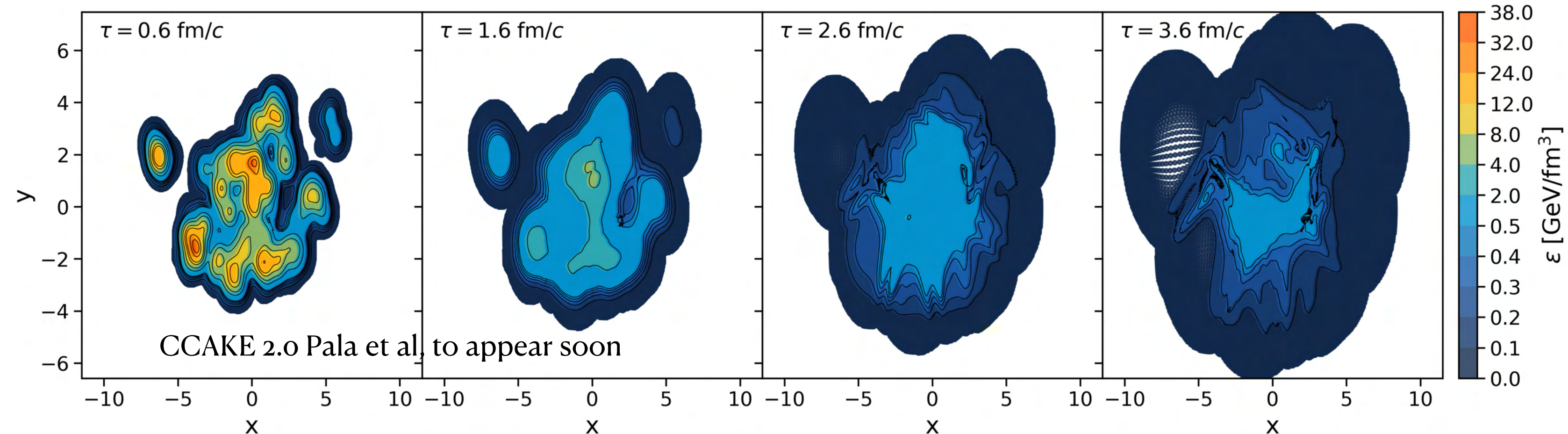
Final State



“Spectators”



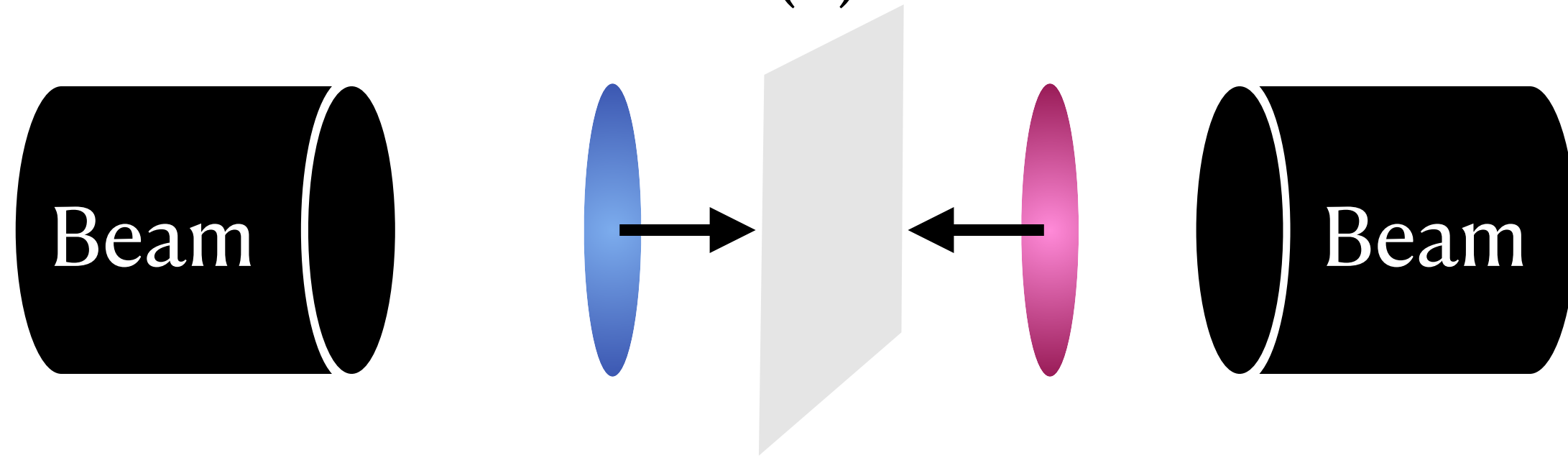
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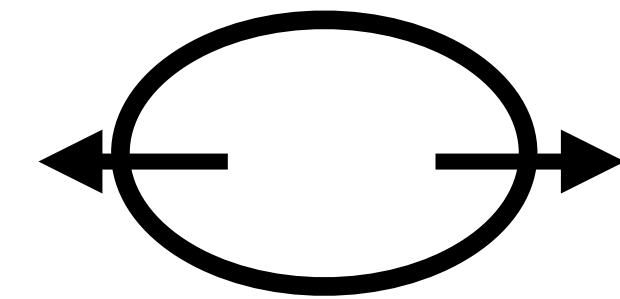
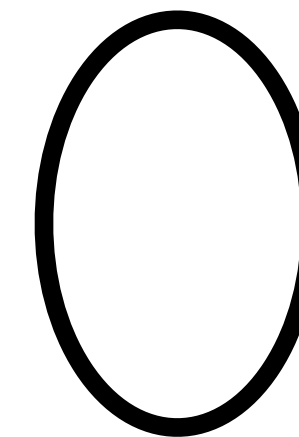


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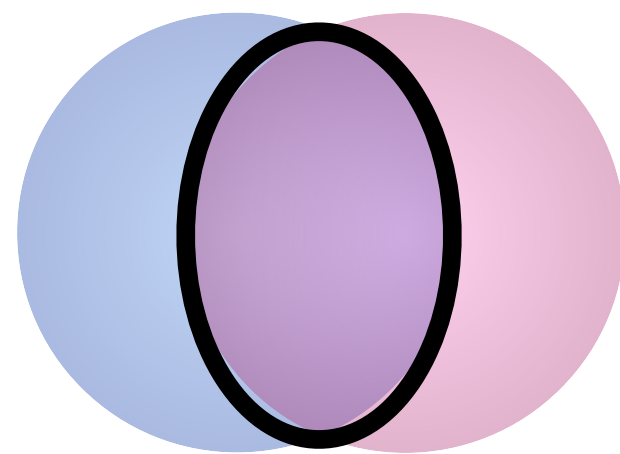
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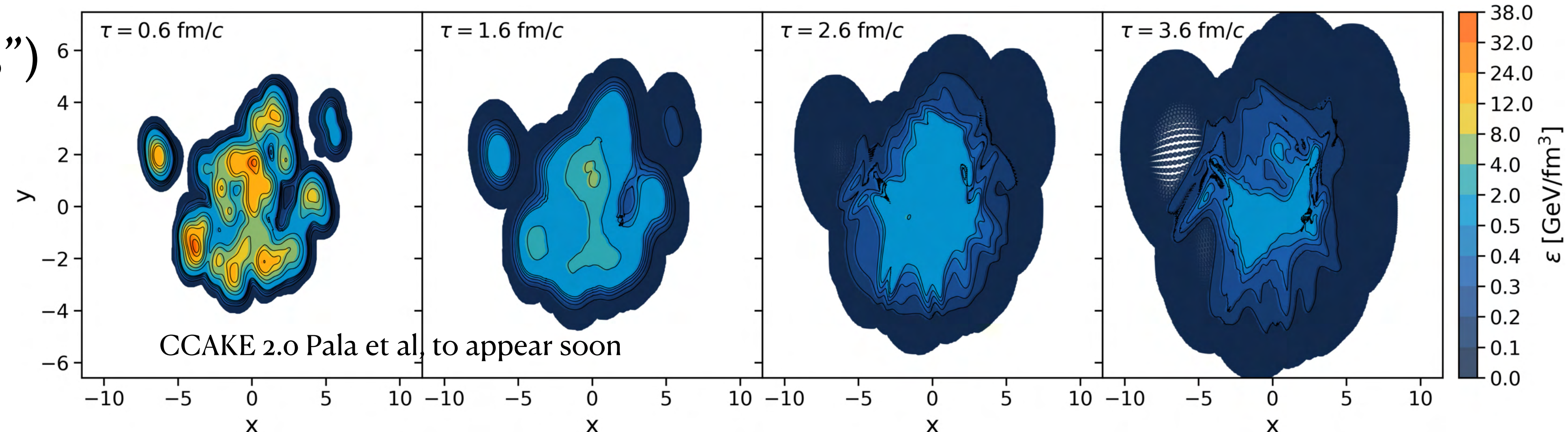


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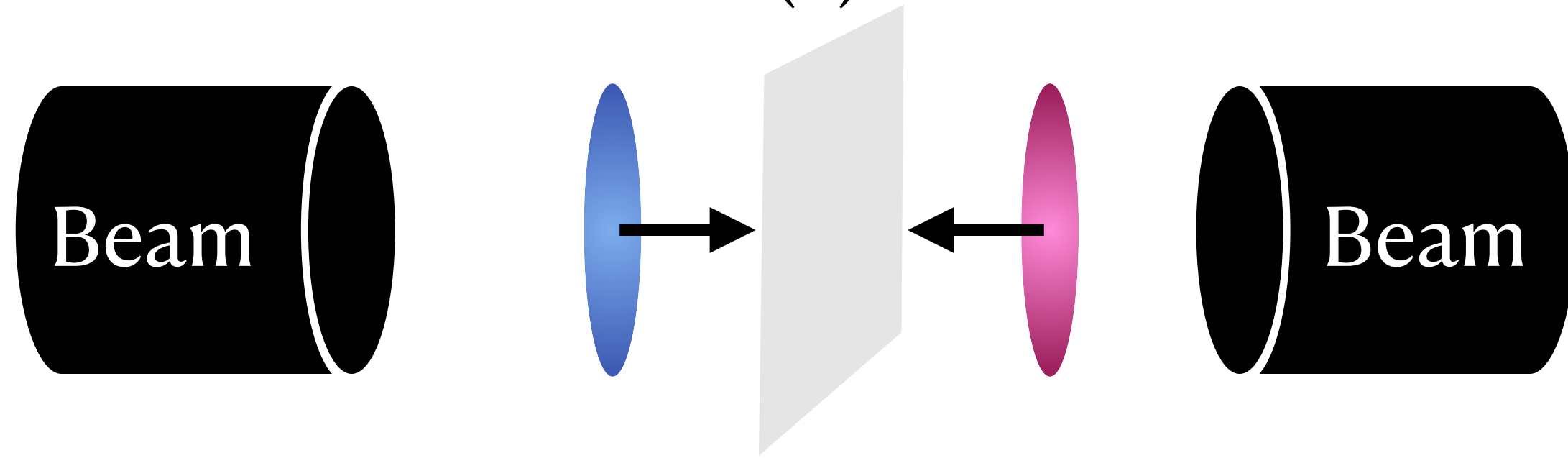
Overlap region (“participants”)
This is the initial condition!



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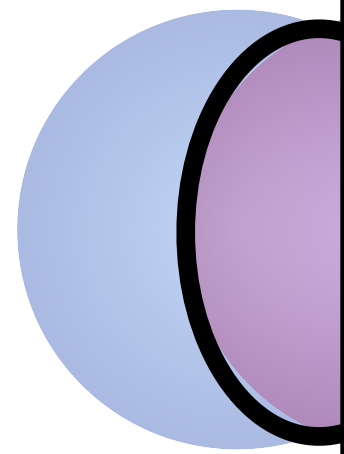
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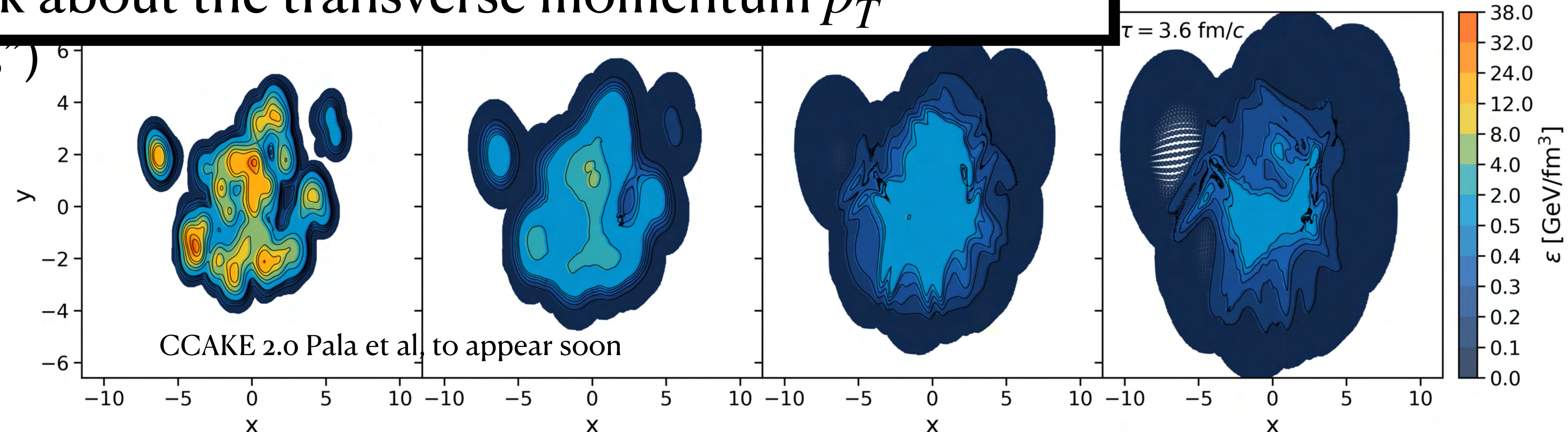
“Spectators”



Since final state is measured in momentum, and the transverse plane is where most of the “action” occurs, we talk about the transverse momentum p_T

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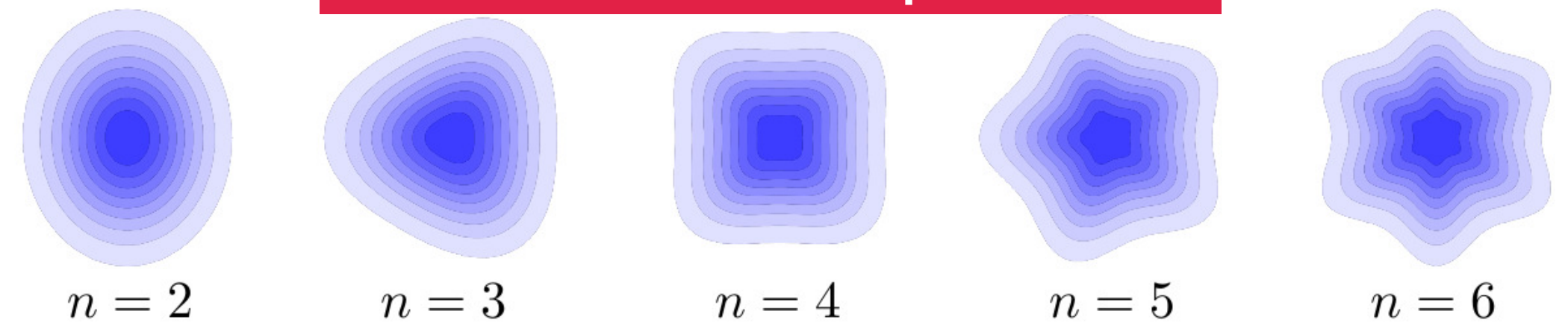


Multiplicity and collective flow

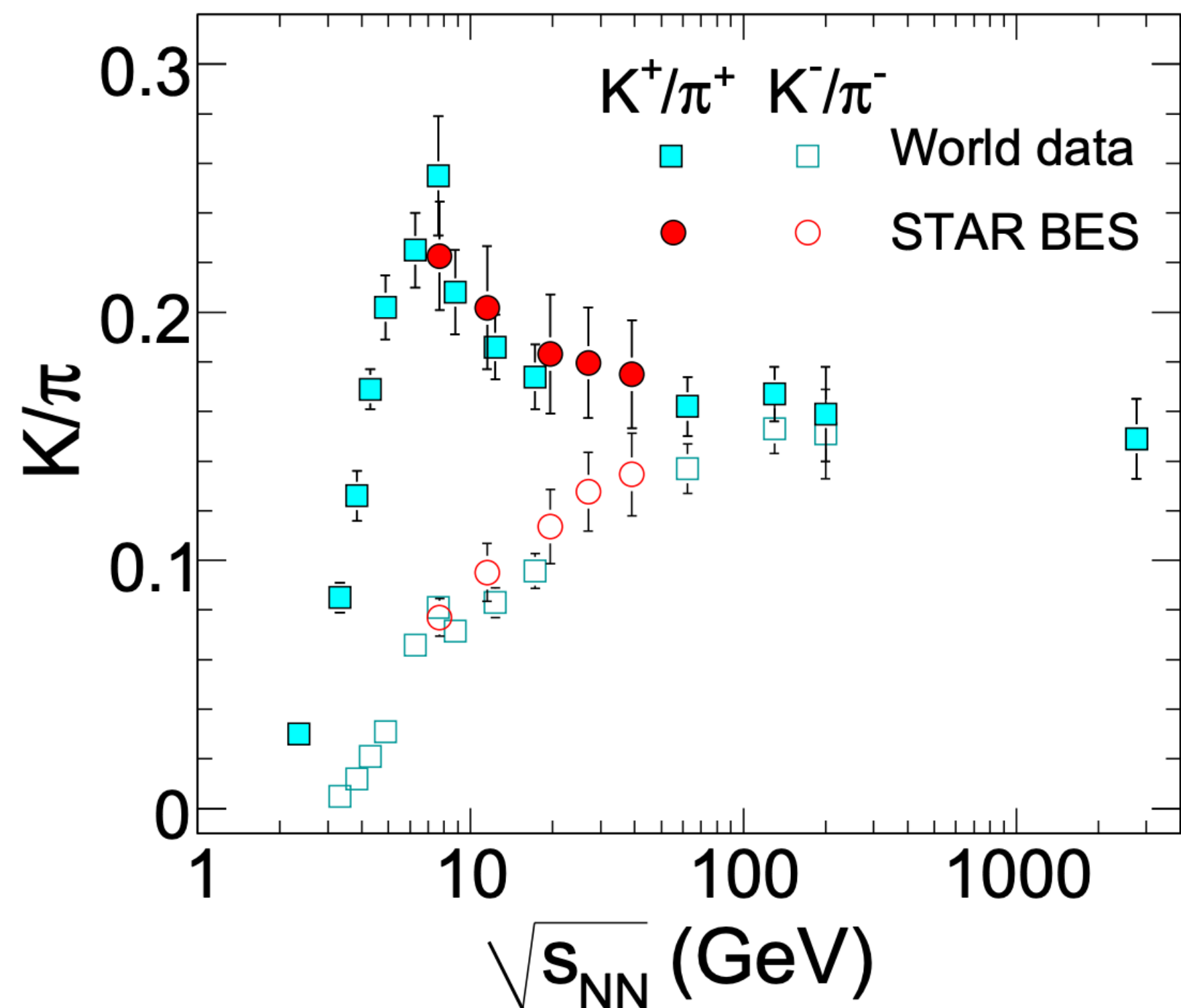
Counting of particles, Fourier analyses: oh my!

Count up particles

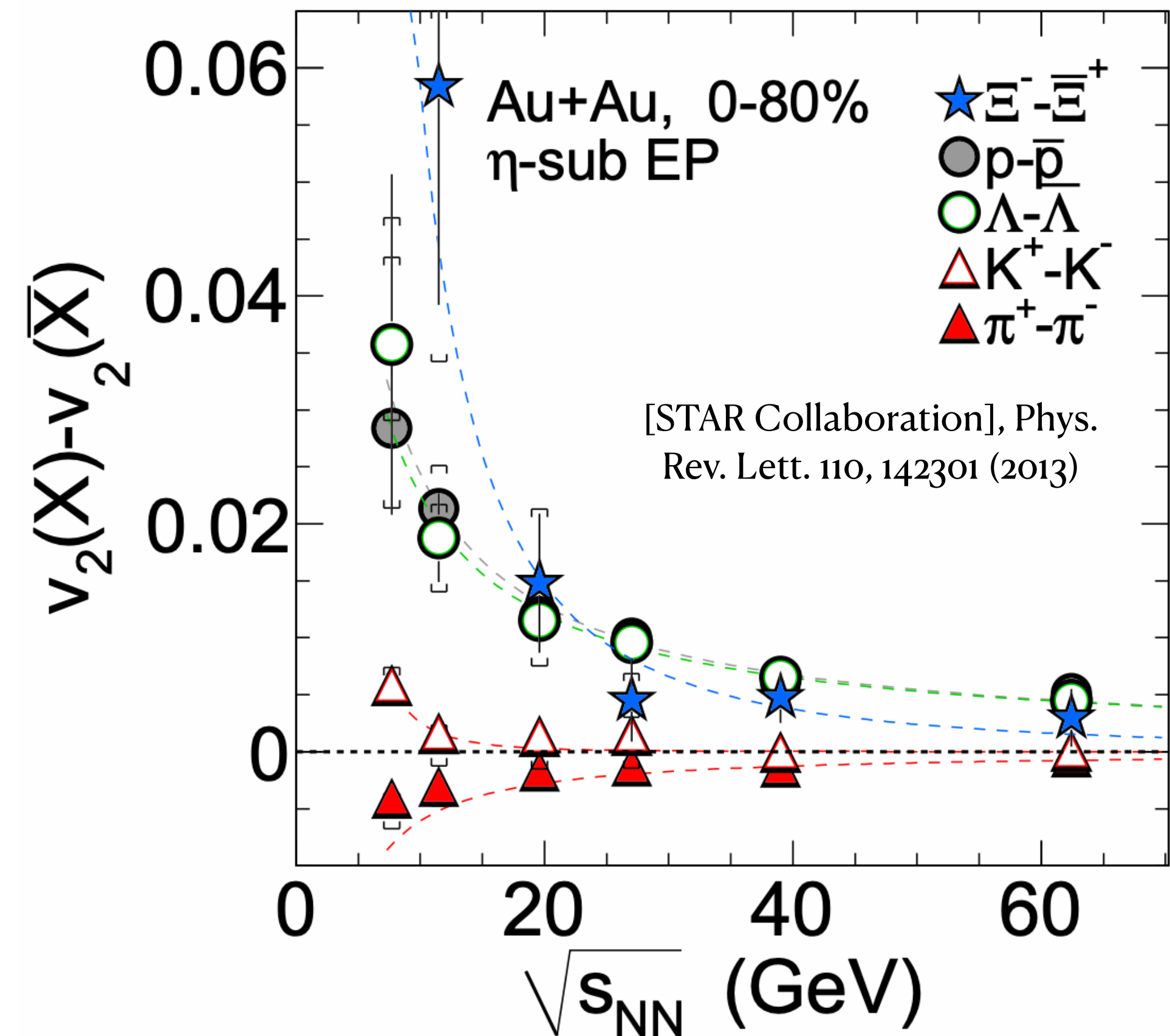
Fourier decomposition



Can compare across p_T or rapidity (more well-defined relativistically “z” direction)



n_B ← ————— → T

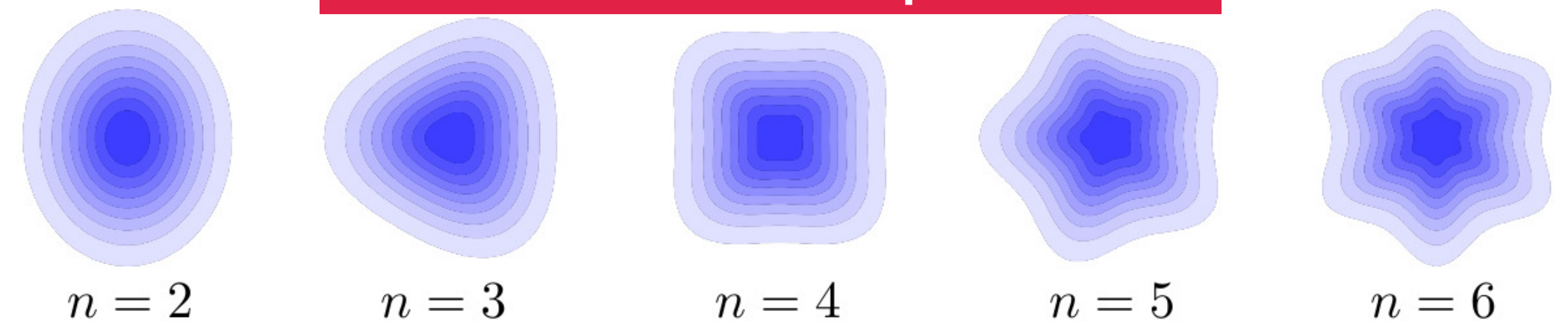


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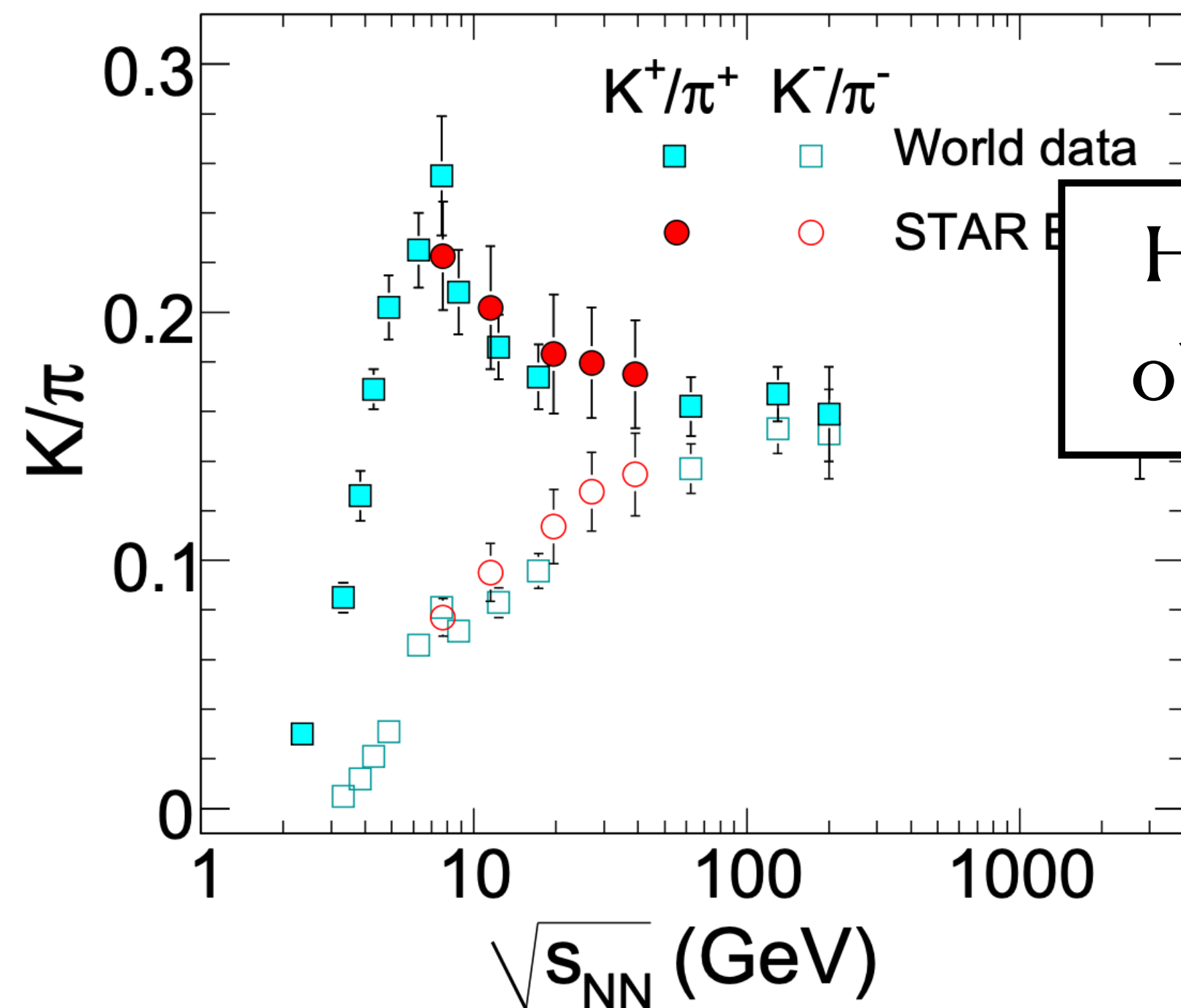
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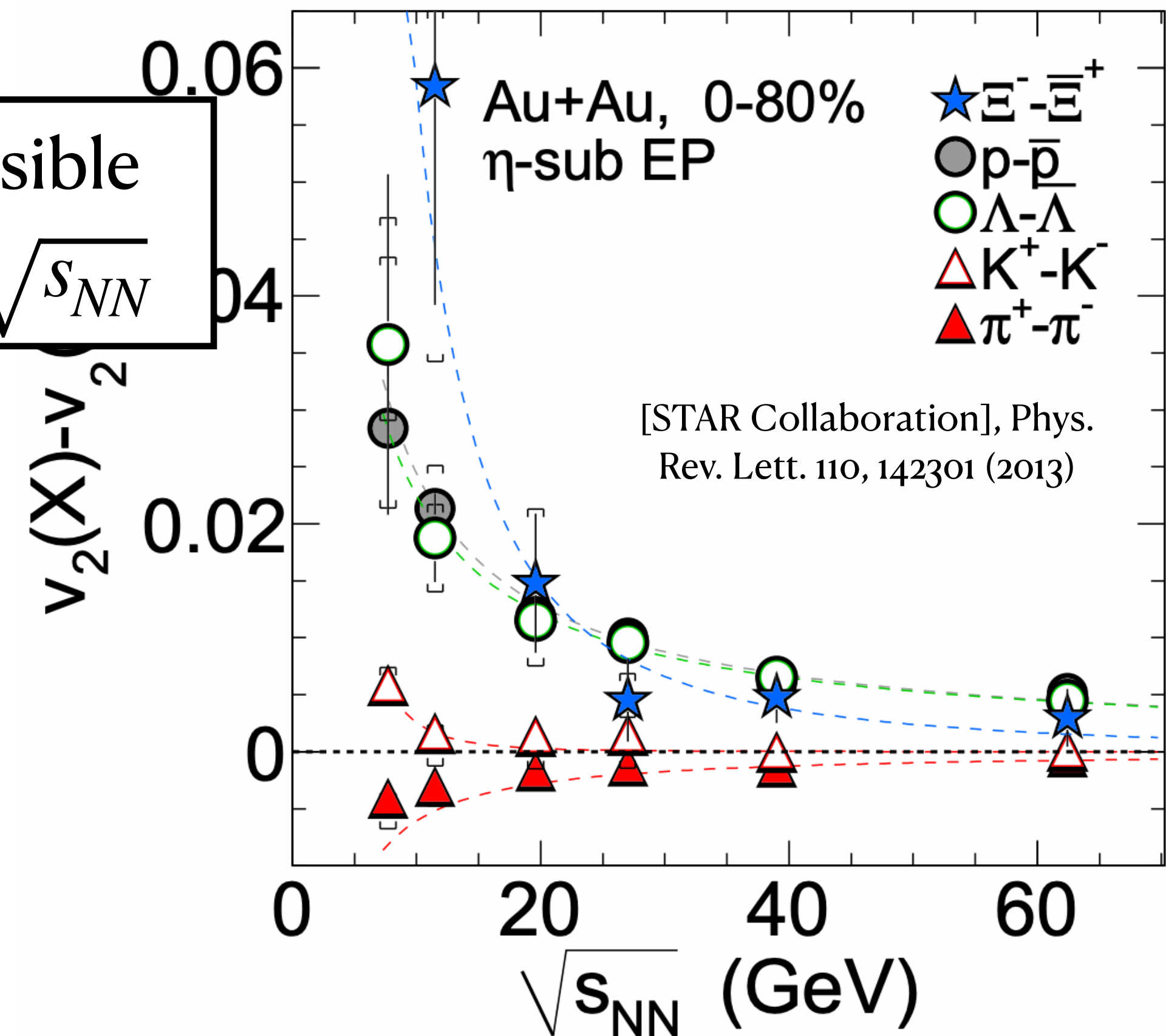
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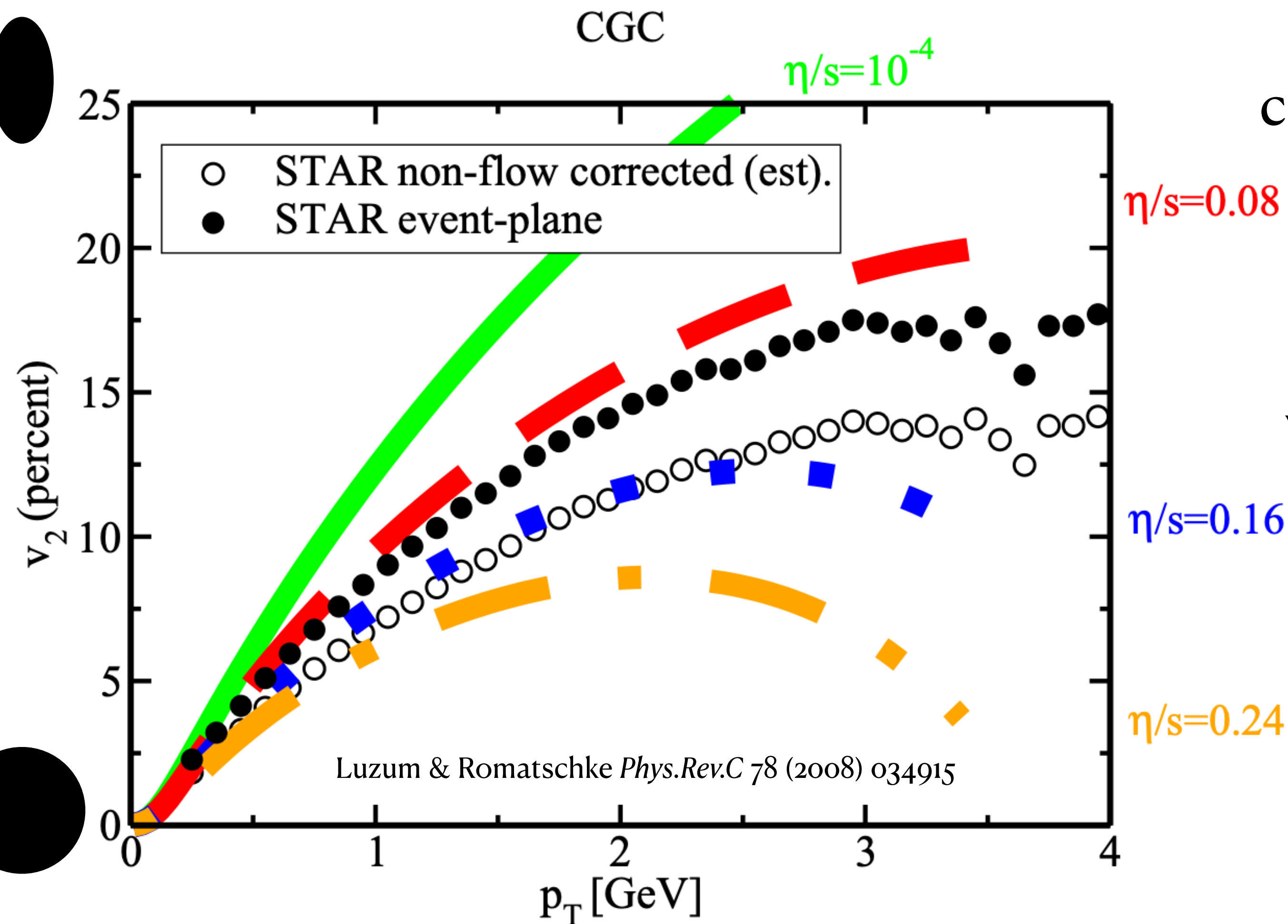
Hundreds of possible observables per $\sqrt{s_{NN}}$



n_B ← ————— → T

Collective flow and viscosity

Out-of-equilibrium and EOS intertwined!



Shear viscosity vital to reproducing collective flow observables at $n_B = 0$ bulk and diffusion are sub-dominate there.

At finite n_B , systematic studies of EOS variations+ all transport coefficients for realistic hydro ***DO NOT EXIST***.

Simpler studies of fixing EOS varying $\eta/s = \text{const}$, sometimes ζ/s do

Simpler studies of varying EOS with transport coefficients in 0+1D or 1+1D exist

How well to HIC models reproduce data?

$$\sqrt{s_{NN}} \sim [200 \text{ GeV}, 6 \text{ TeV}] \text{ RHIC/LHC}$$

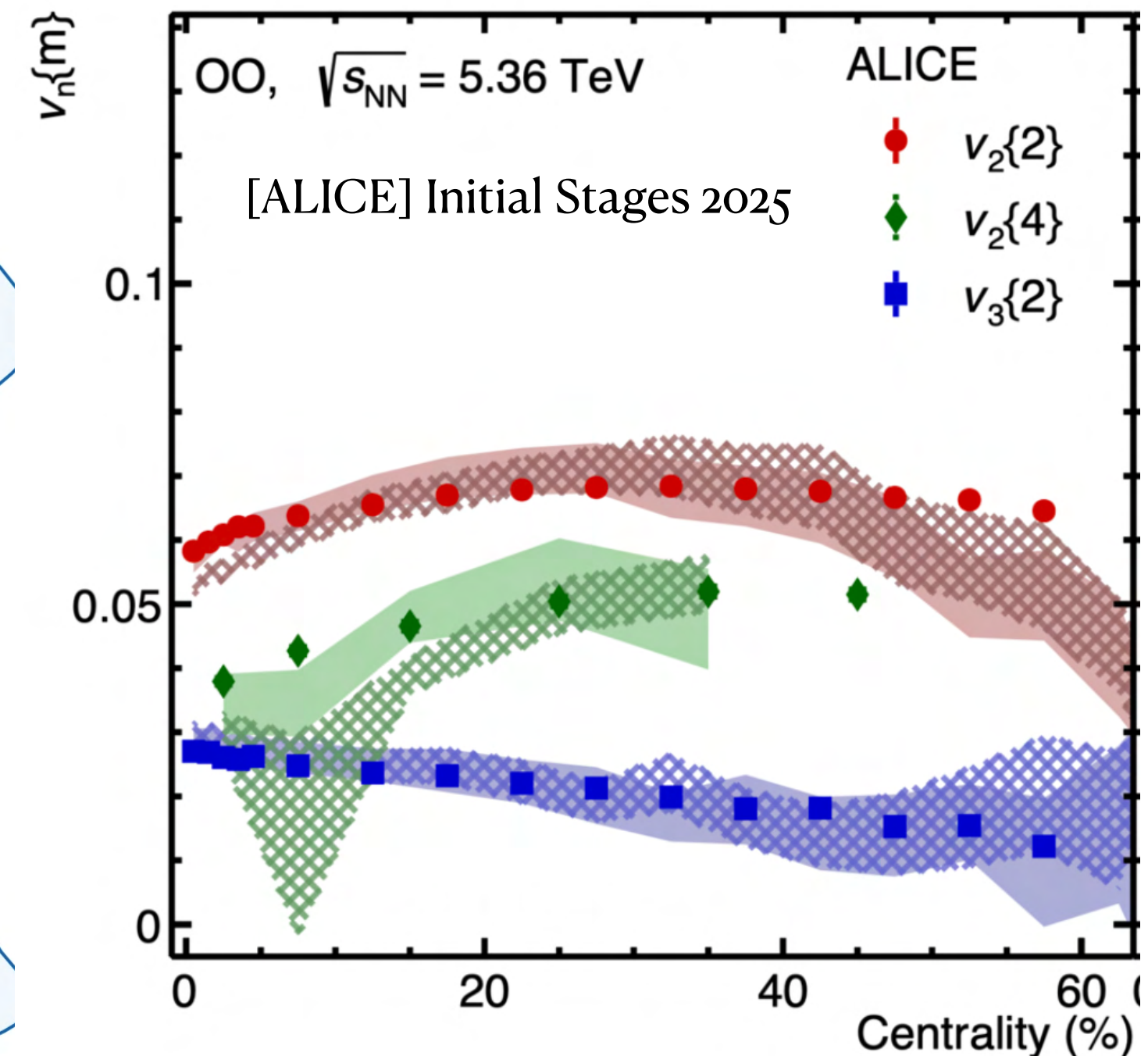
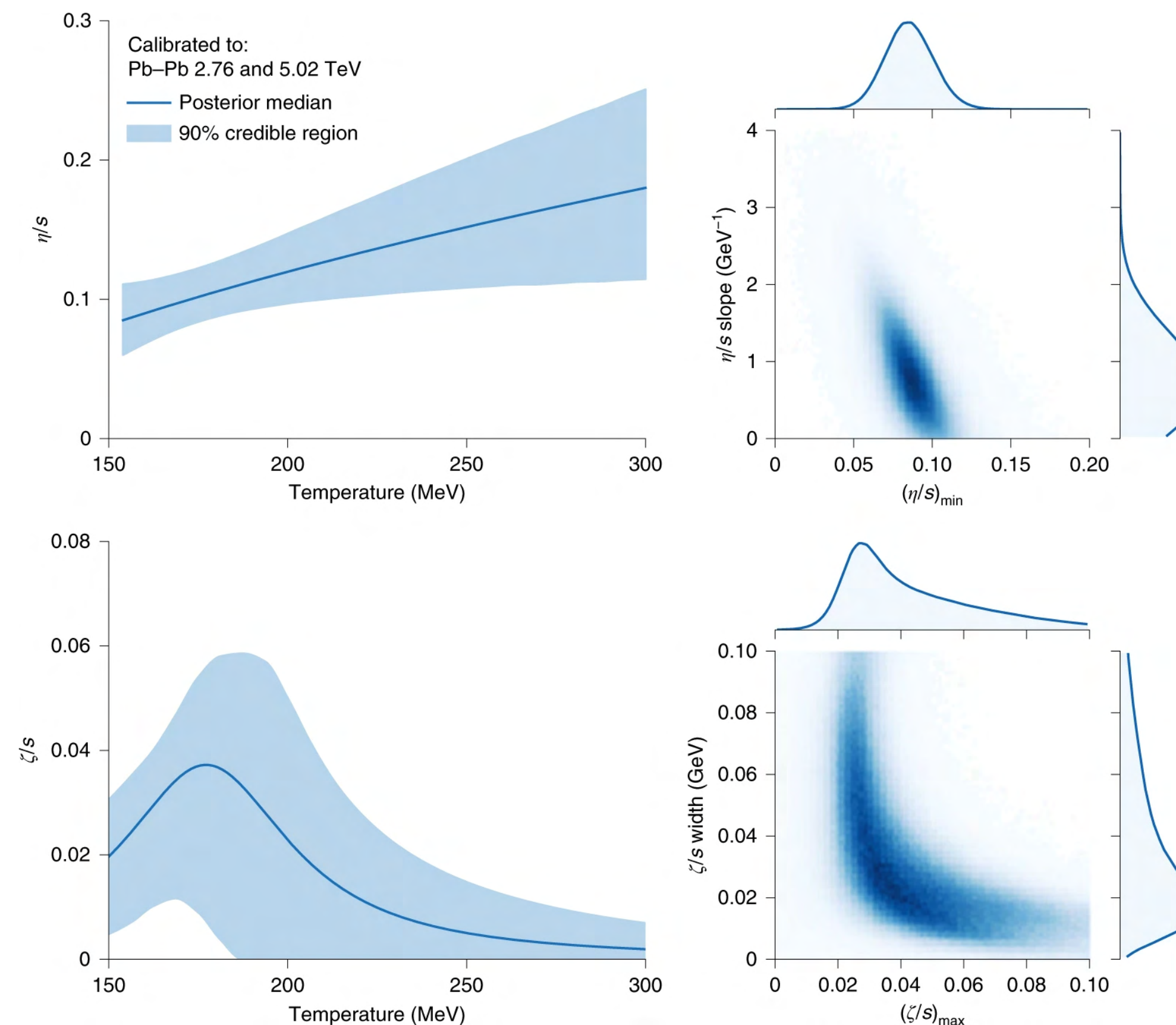
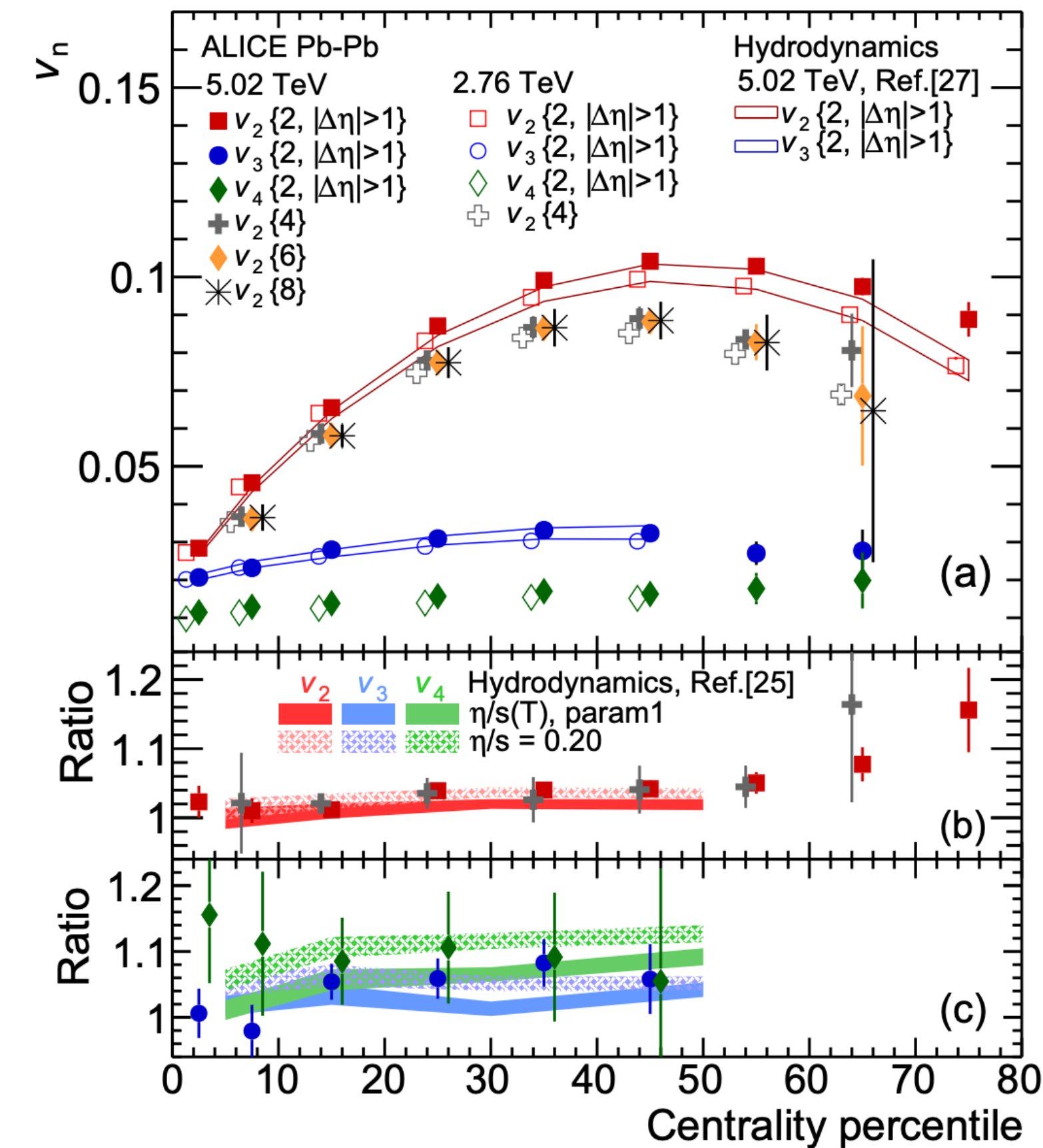
2016 Theory predictions confirmed at the % level.

[ALICE] *Phys.Rev.Lett.* 116 (2016) 13, 132302

2016-2020 Bayesian parameter estimates begin

Bernhard et al, *Nature Phys.* 15 (2019) 11, 1113-1117

2025 Bayesian posteriors accurately predict OO data!



<https://alice-collaboration.web.cern.ch/>
2025-ALICE-Initial-Stages

How well to HIC models reproduce data?

$$\sqrt{s_{NN}} \sim [200 \text{ GeV}, 6 \text{ TeV}] \text{ RHIC/LHC}$$

Know
extremely well

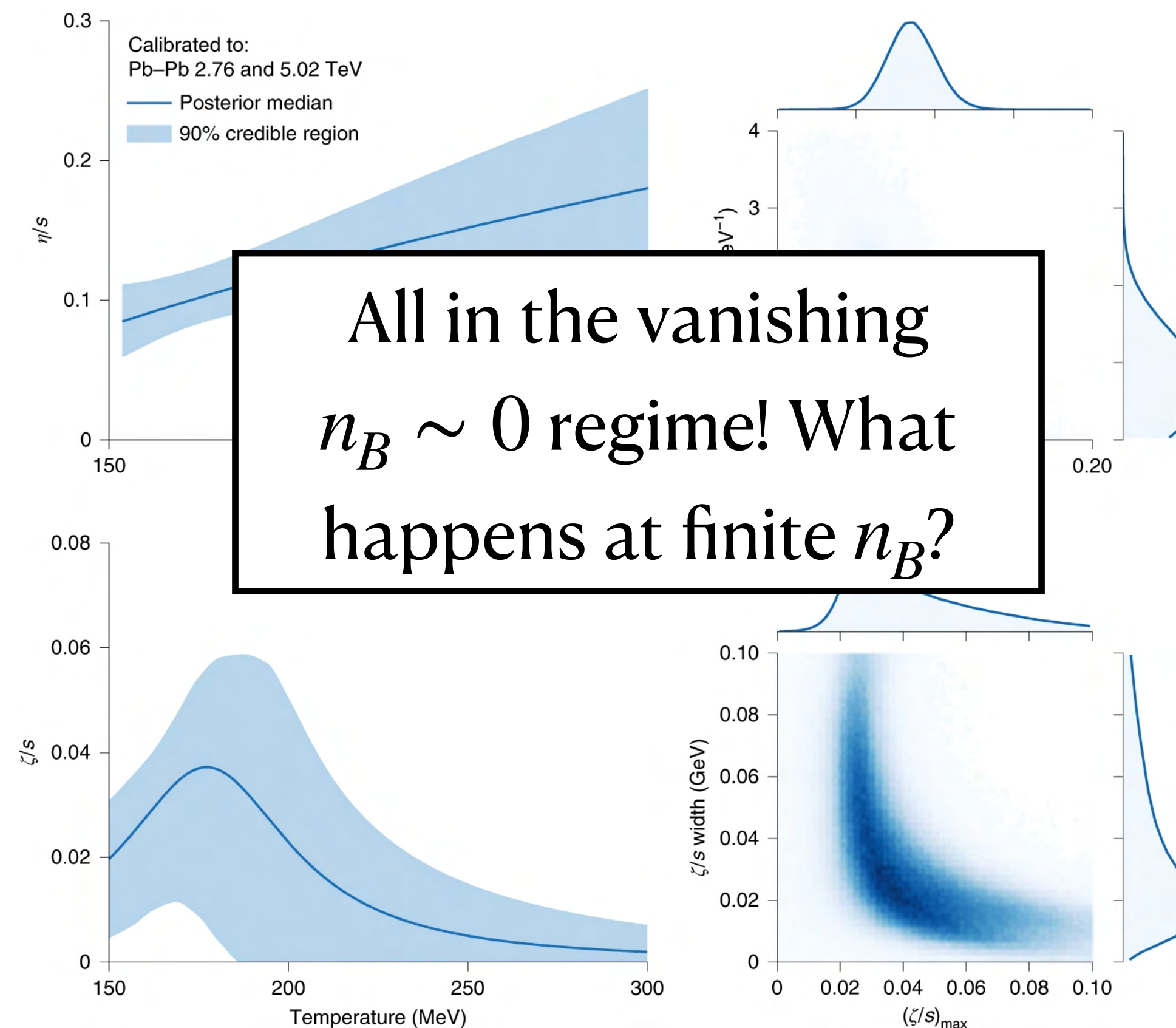
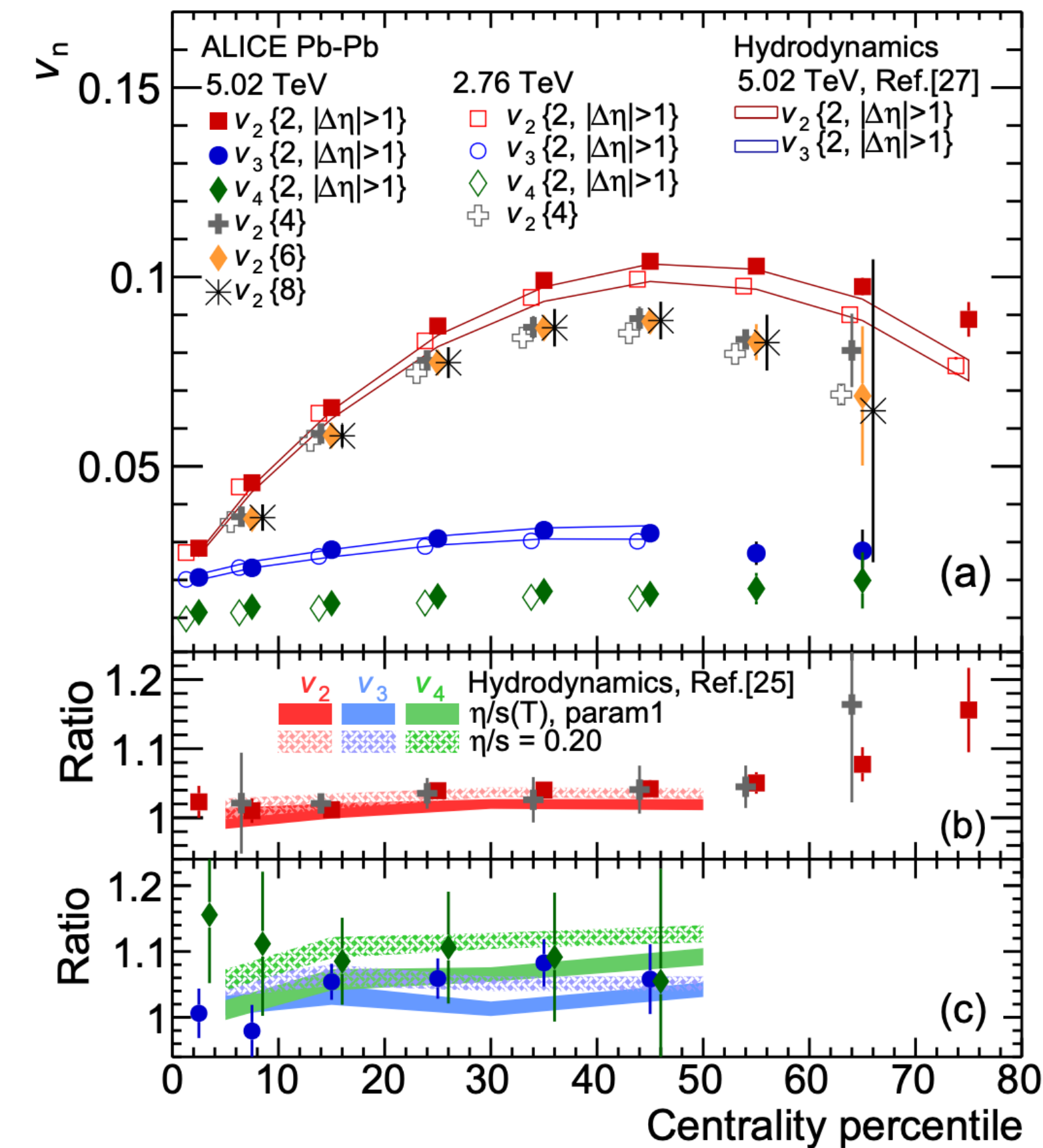
2016 Theory predictions
confirmed at the % level.

[ALICE] *Phys.Rev.Lett.* 116 (2016) 13, 132302

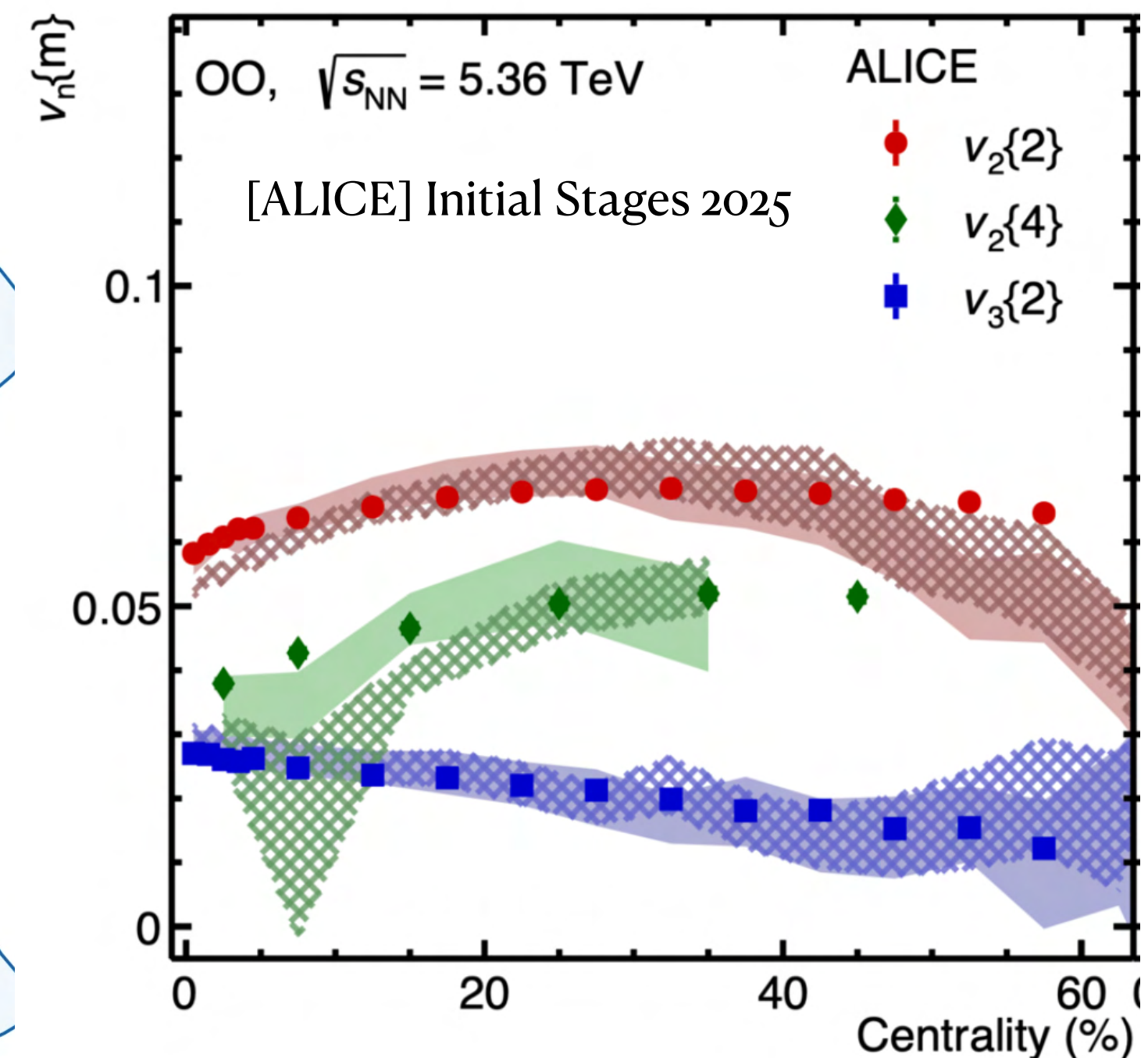
2016-2020 Bayesian parameter
estimates begin

Bernhard et al, *Nature Phys.* 15 (2019) 11, 1113-1117

2025 Bayesian posteriors
accurately predict OO data!



All in the vanishing
 $n_B \sim 0$ regime! What
happens at finite n_B ?

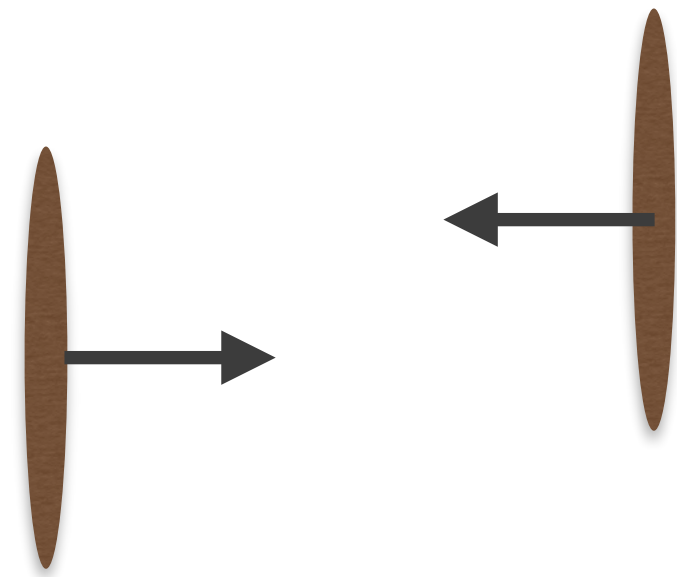


<https://alice-collaboration.web.cern.ch/>
 2025-ALICE-Initial-Stages

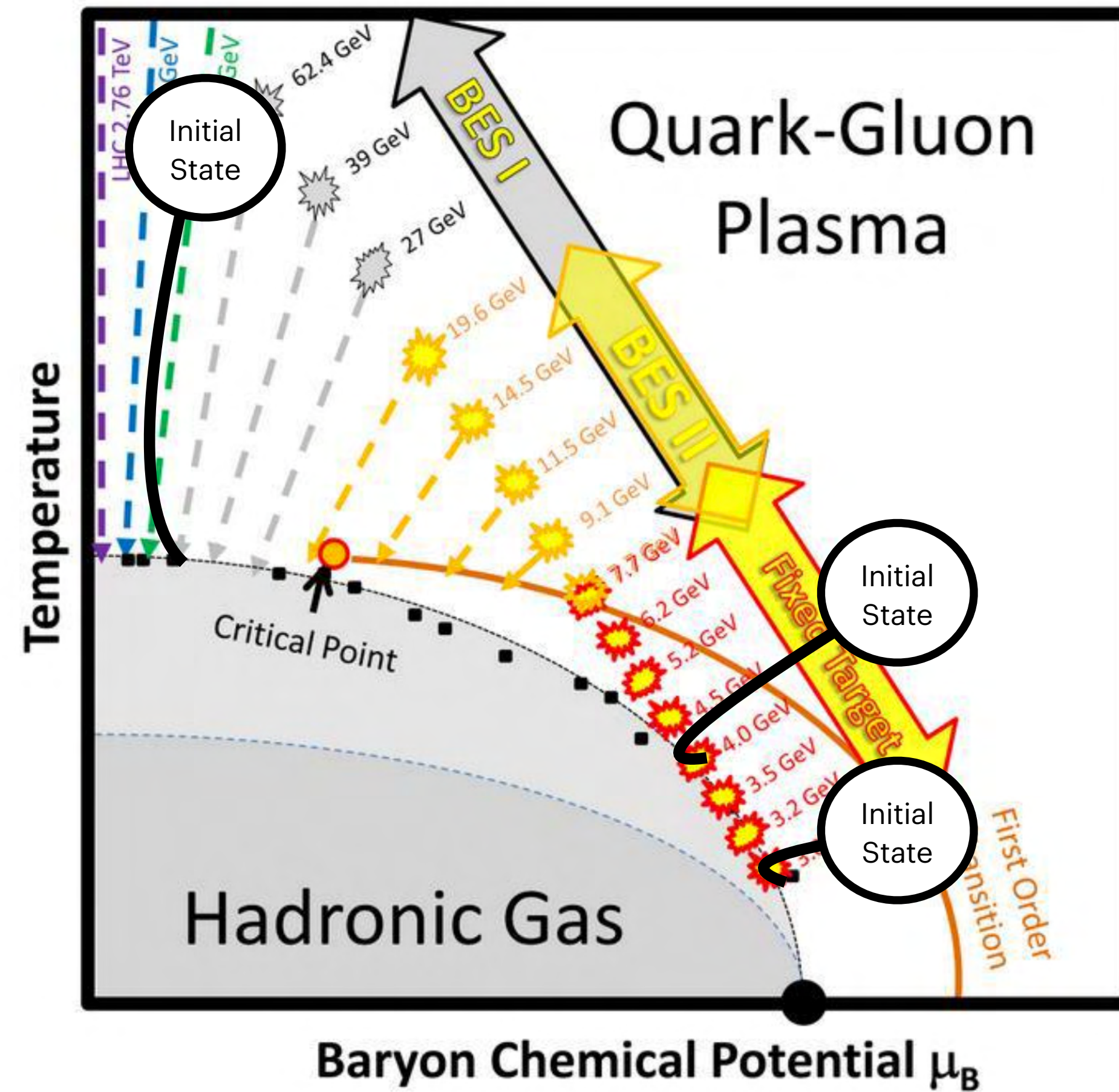
Differences as you lower $\sqrt{s}$ in HIC

Quark-gluon degrees of freedom may matter significantly less

Large $\sqrt{s_{NN}}$

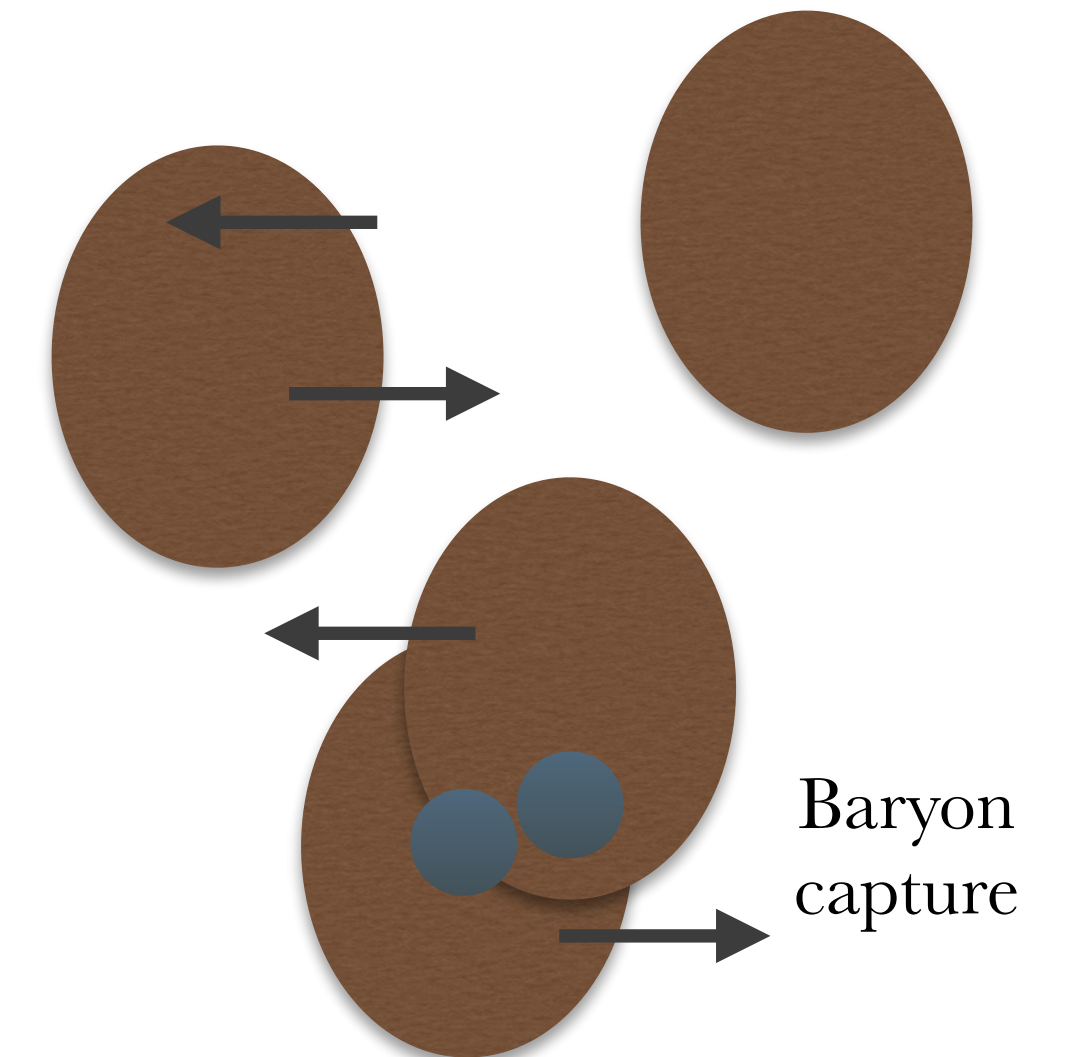


- Lorentz contracted (2D)
- Nuclei pass through instantaneously
- Too quick to capture baryons
- Simulations best understood



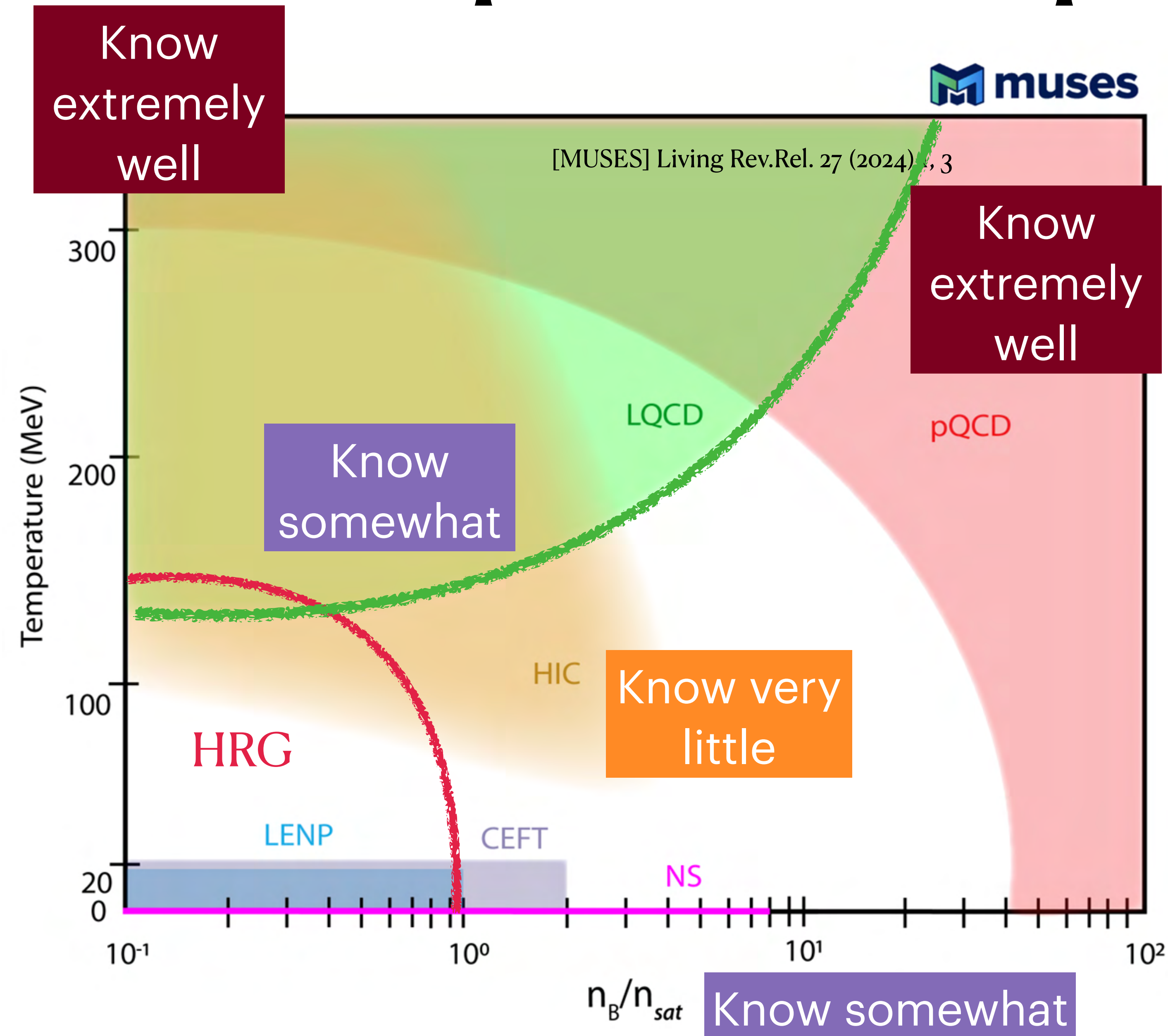
With $\sqrt{s_{NN}}$, different time spent in quark/gluon phase vs hadron phase

Small $\sqrt{s_{NN}}$



- 3D nuclei pass slowly
- Time to capture baryons
- Diffusion, phase transitions matter

Equilibrium: Complexity at low $\sqrt{s_{NN}}$



Large parts of heavy-ion collisions outside of the regime of validity for lattice QCD and the HRG

Jamie's talk

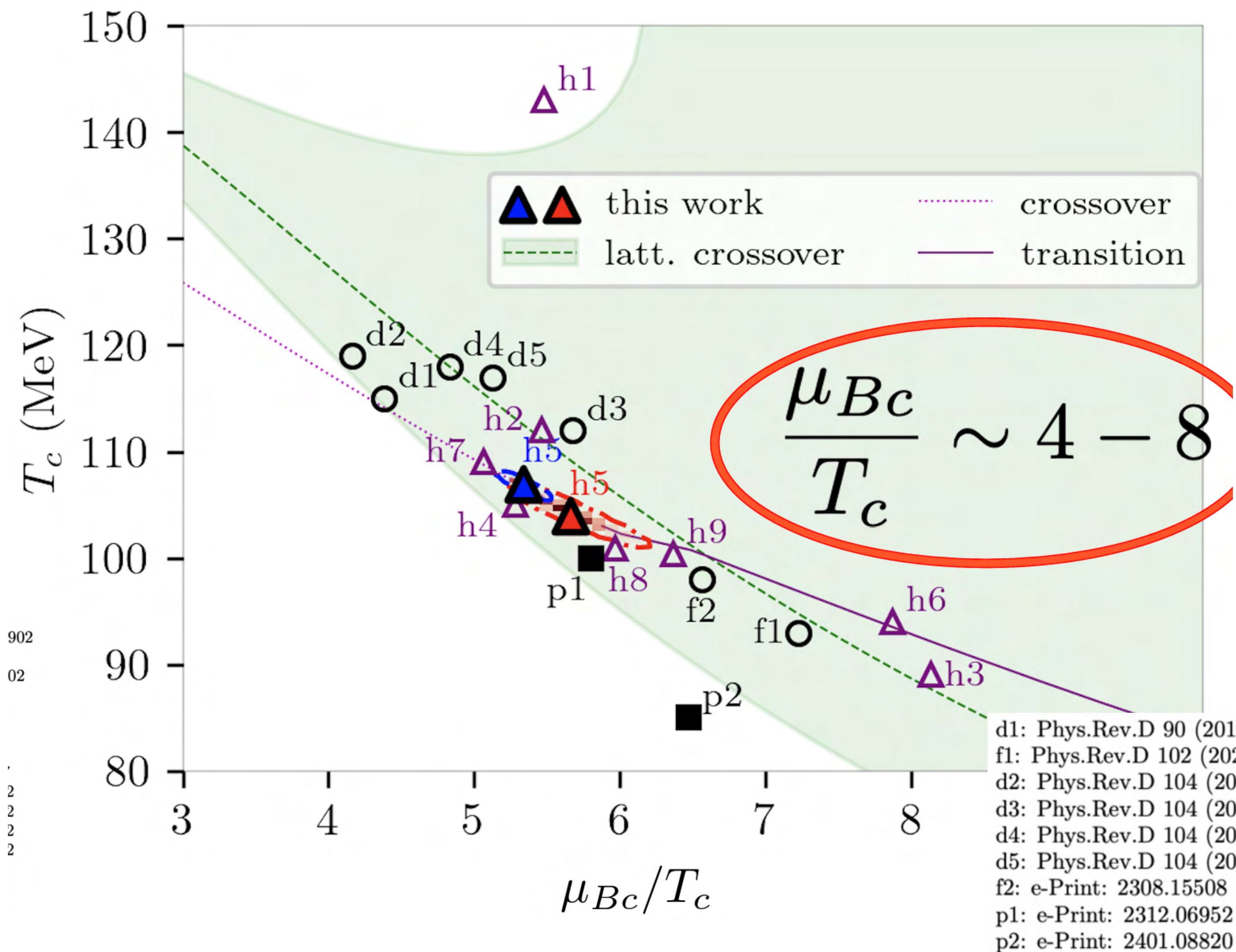
Need to smoothly match to lattice QCD/HRG, include search for critical point/first-order line, match to NS, requires 4D EOS as well!



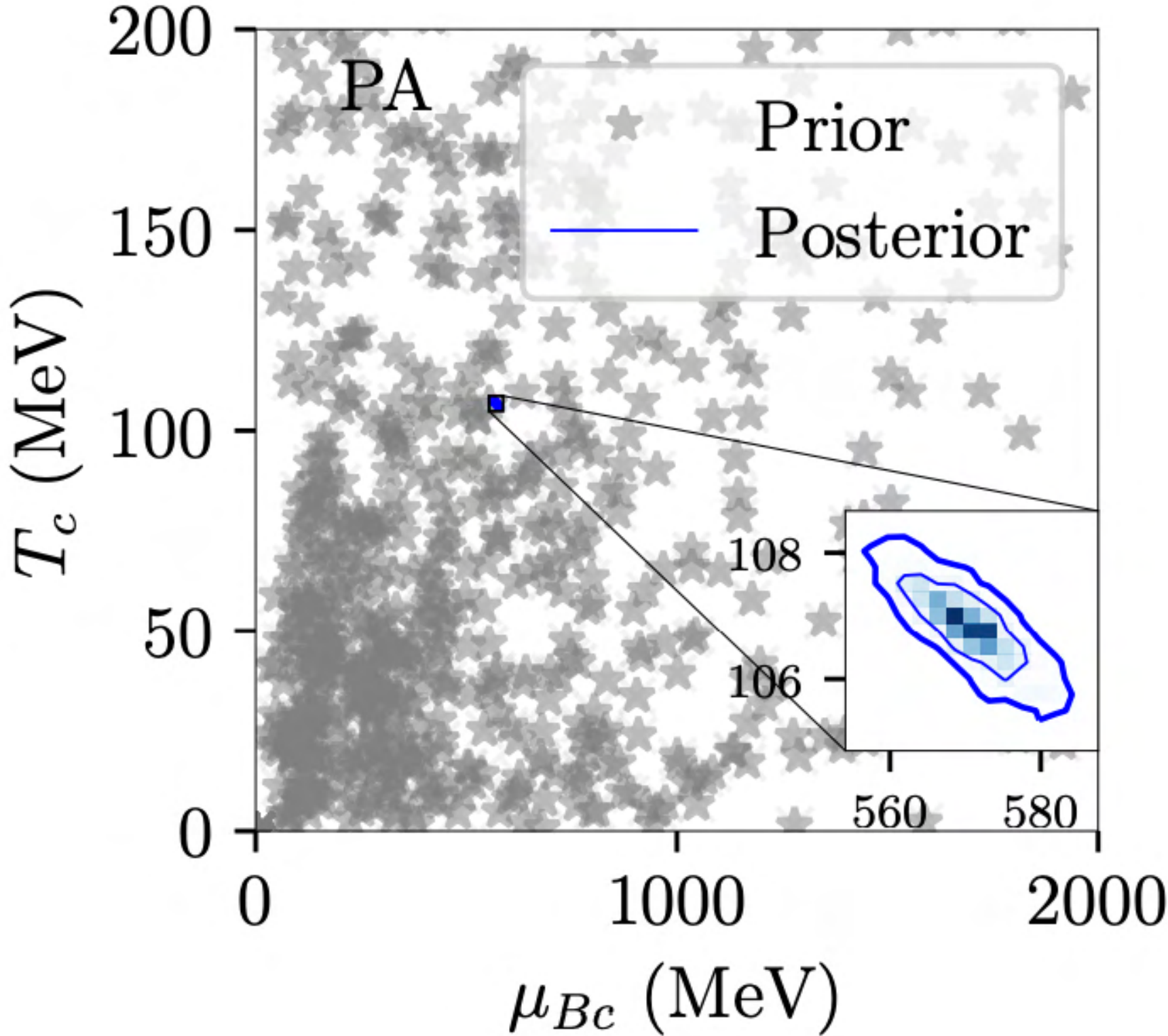
Hints of a critical point

Know
somewhat

Using lattice QCD+models (holography, functional renormalization group, data etc)



Hippert et al, *Phys.Rev.D* 110 (2024) 9, 094006

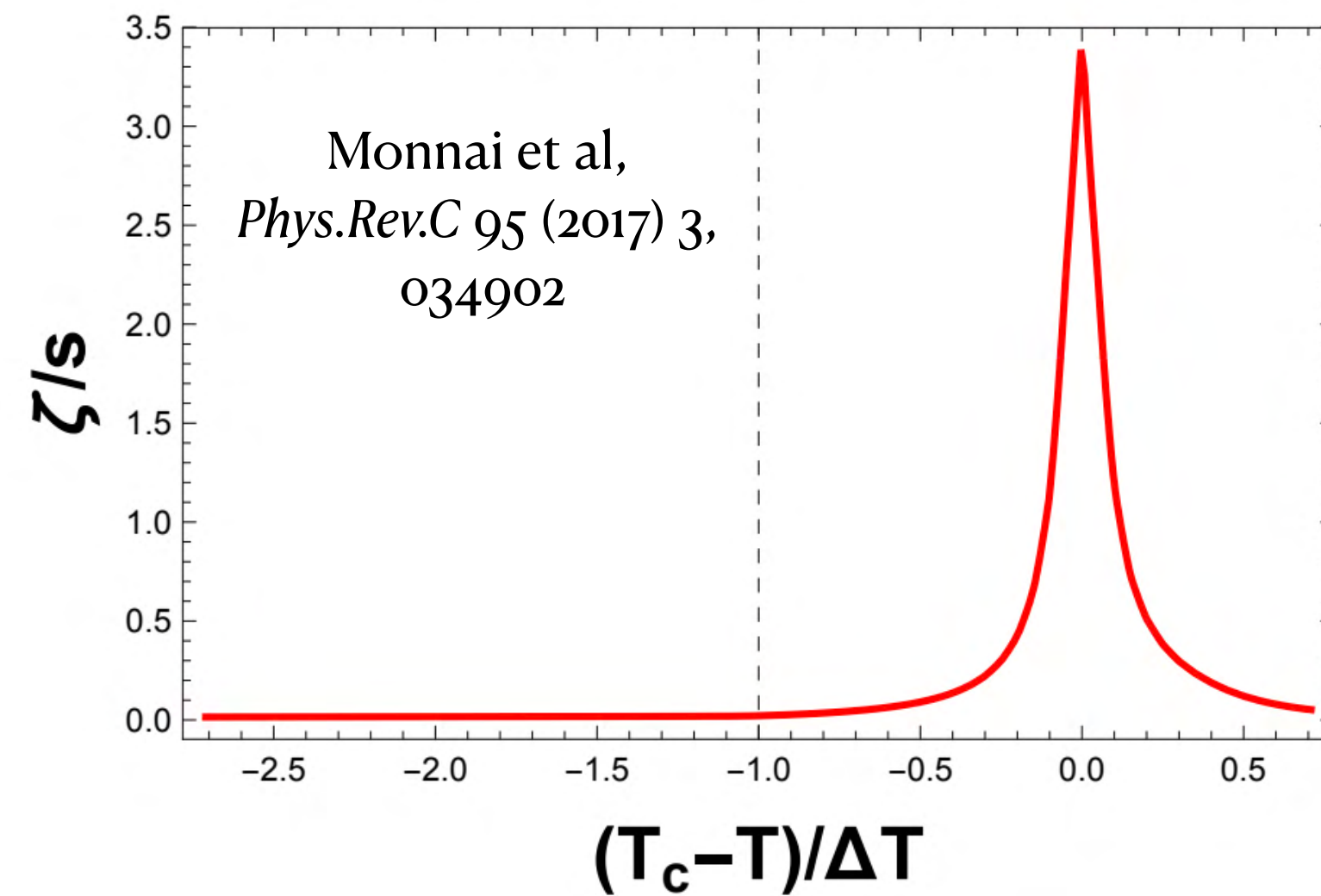
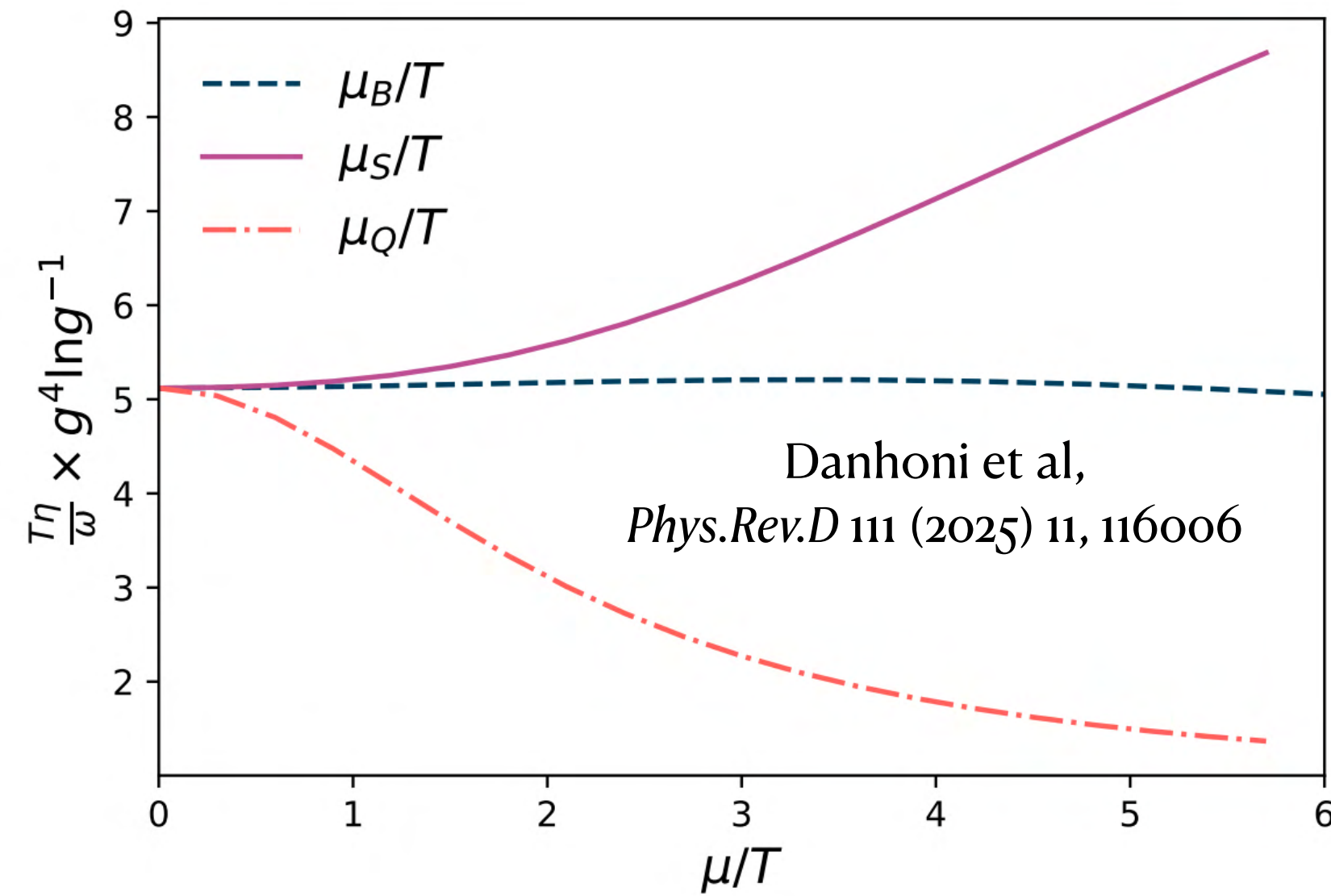


Bayesian analysis to
reproduce lattice QCD,
finds highly constrained
critical point!

Out-of-Equilibrium: Complexity at low $\sqrt{s_{NN}}$

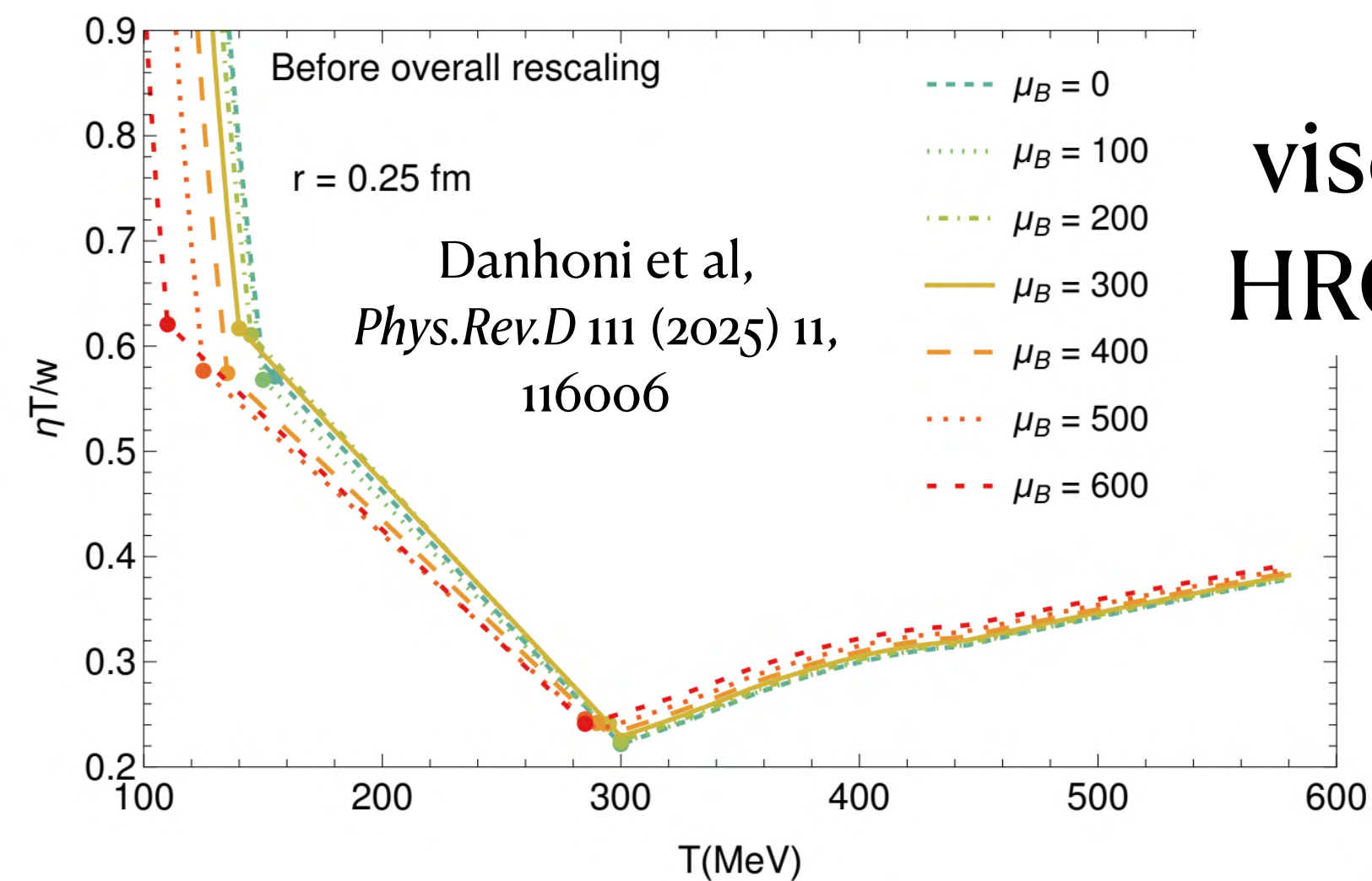
Shear viscosity of pQCD

Bulk viscosity at a critical point

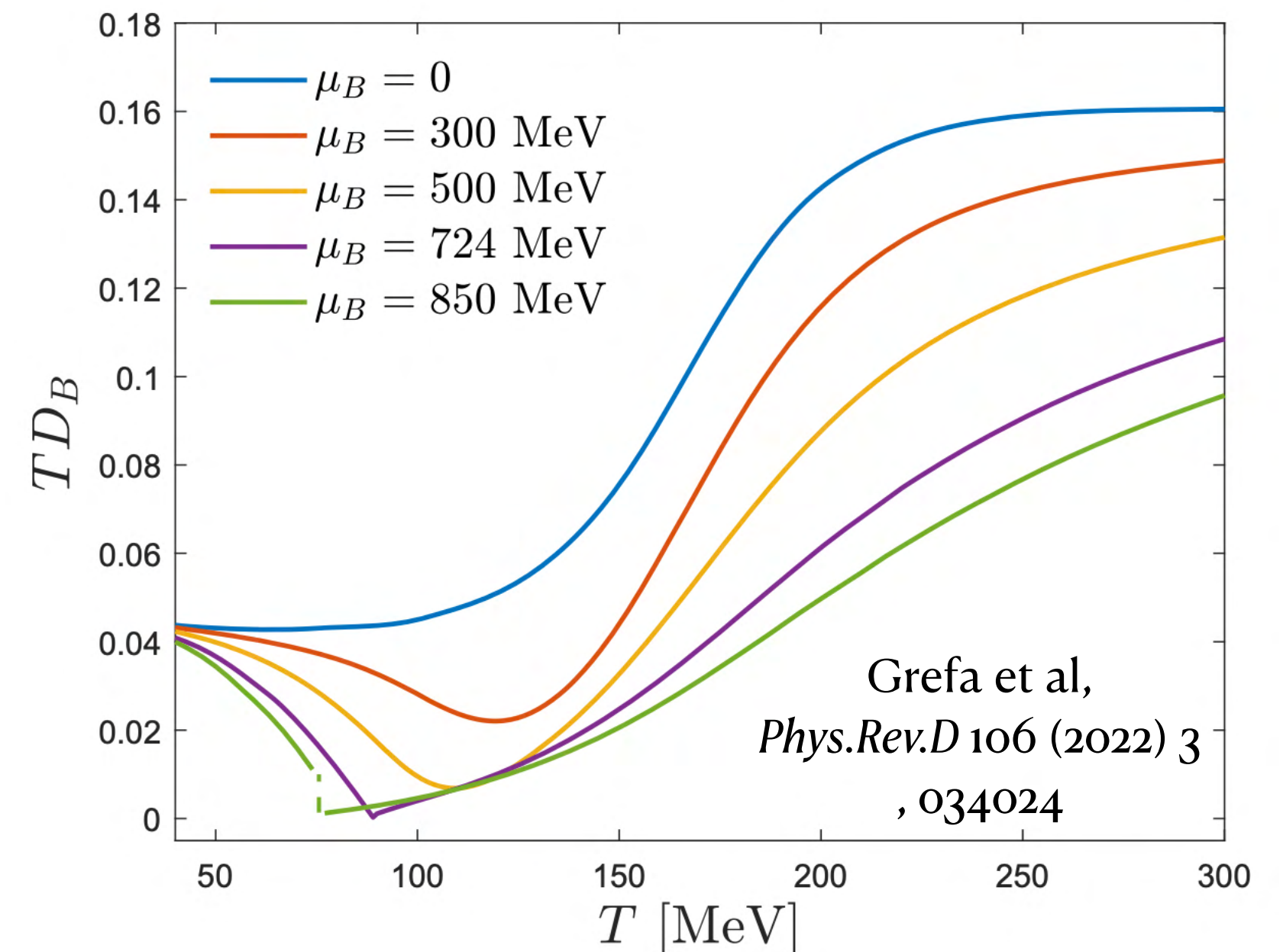


BSQ diffusion matrix ???

Baryon diffusion at a critical point



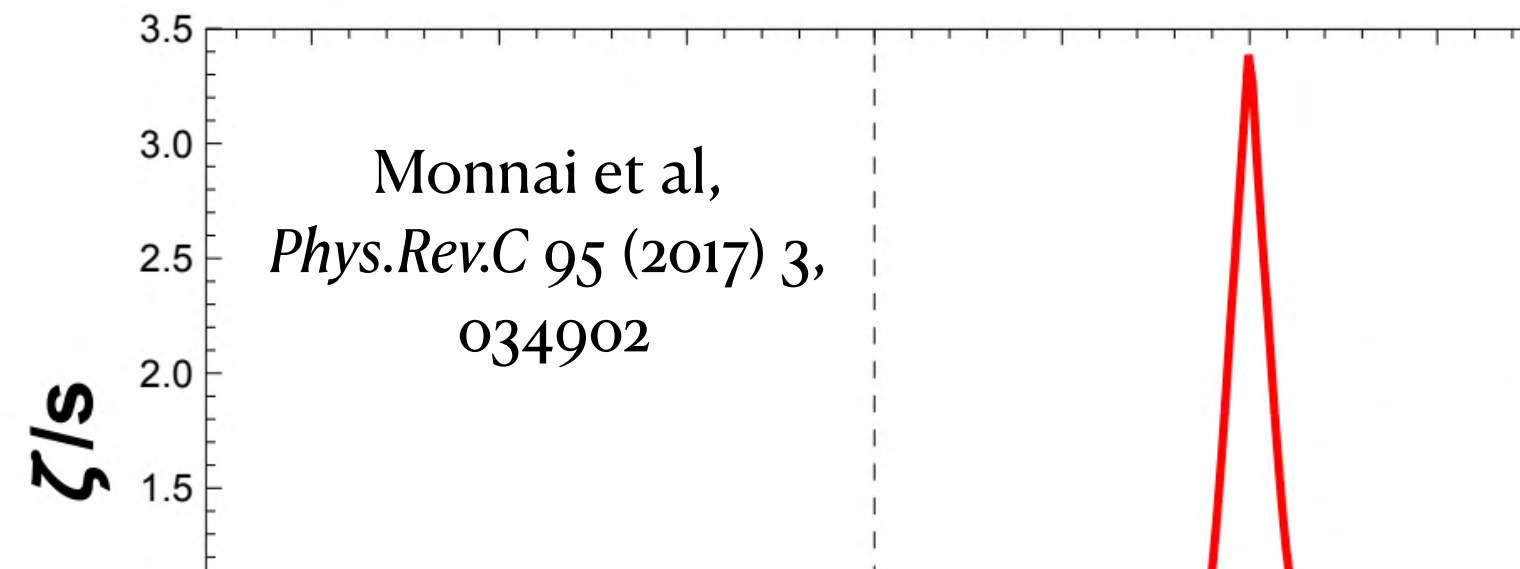
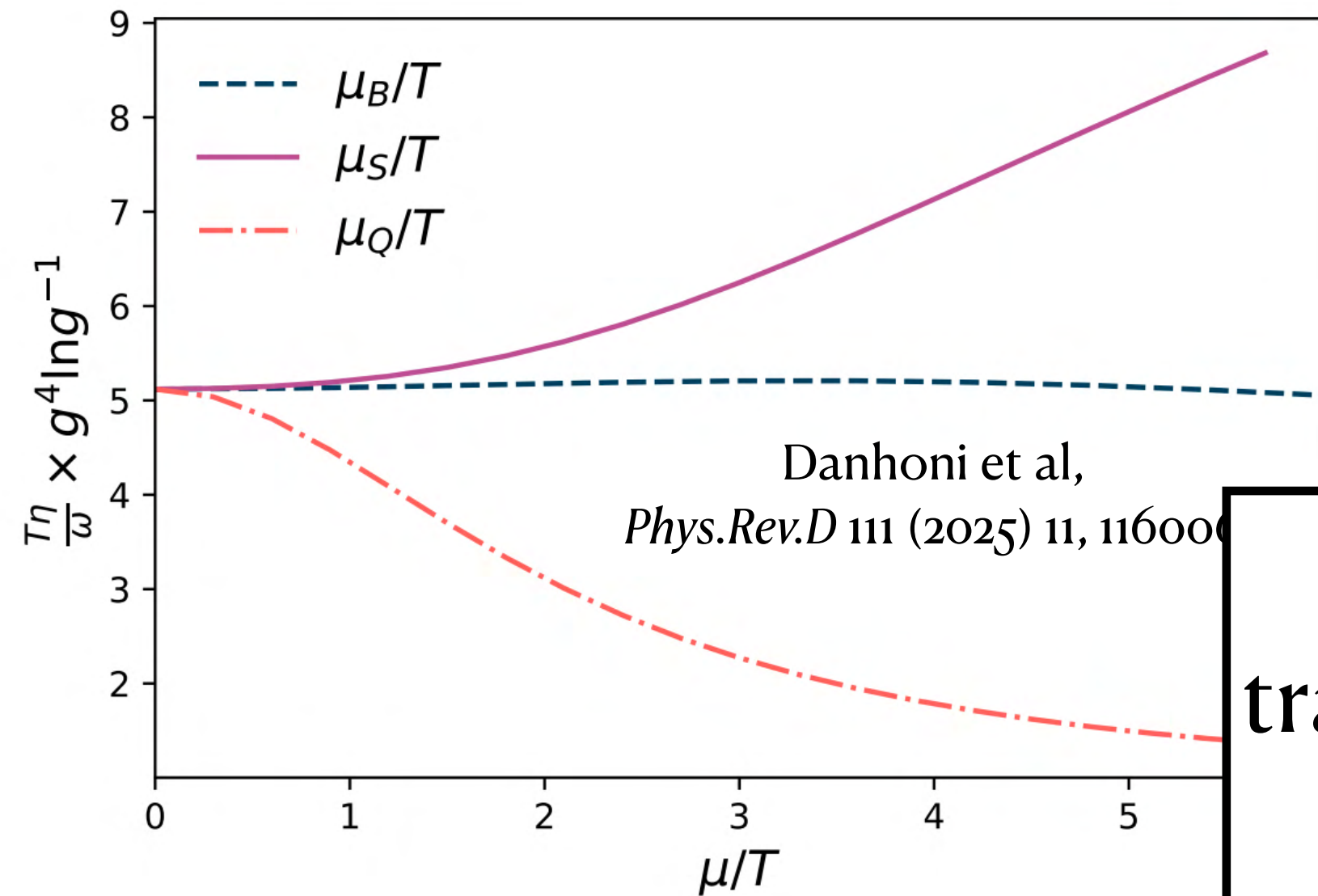
Shear viscosity of HRG+pQCD



Out-of-Equilibrium: Complexity at low $\sqrt{s_{NN}}$

Shear viscosity of pQCD

Bulk viscosity at a critical point



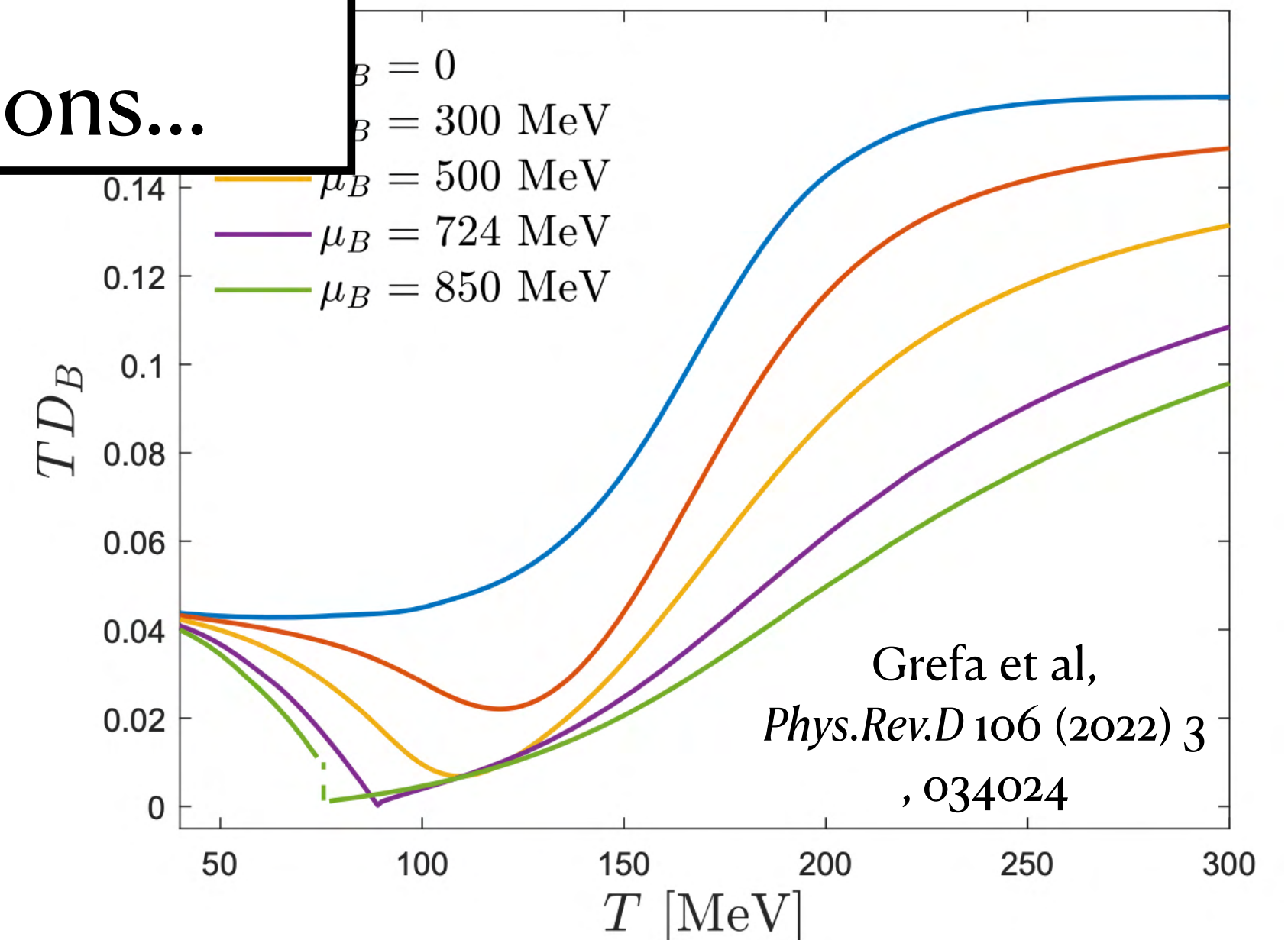
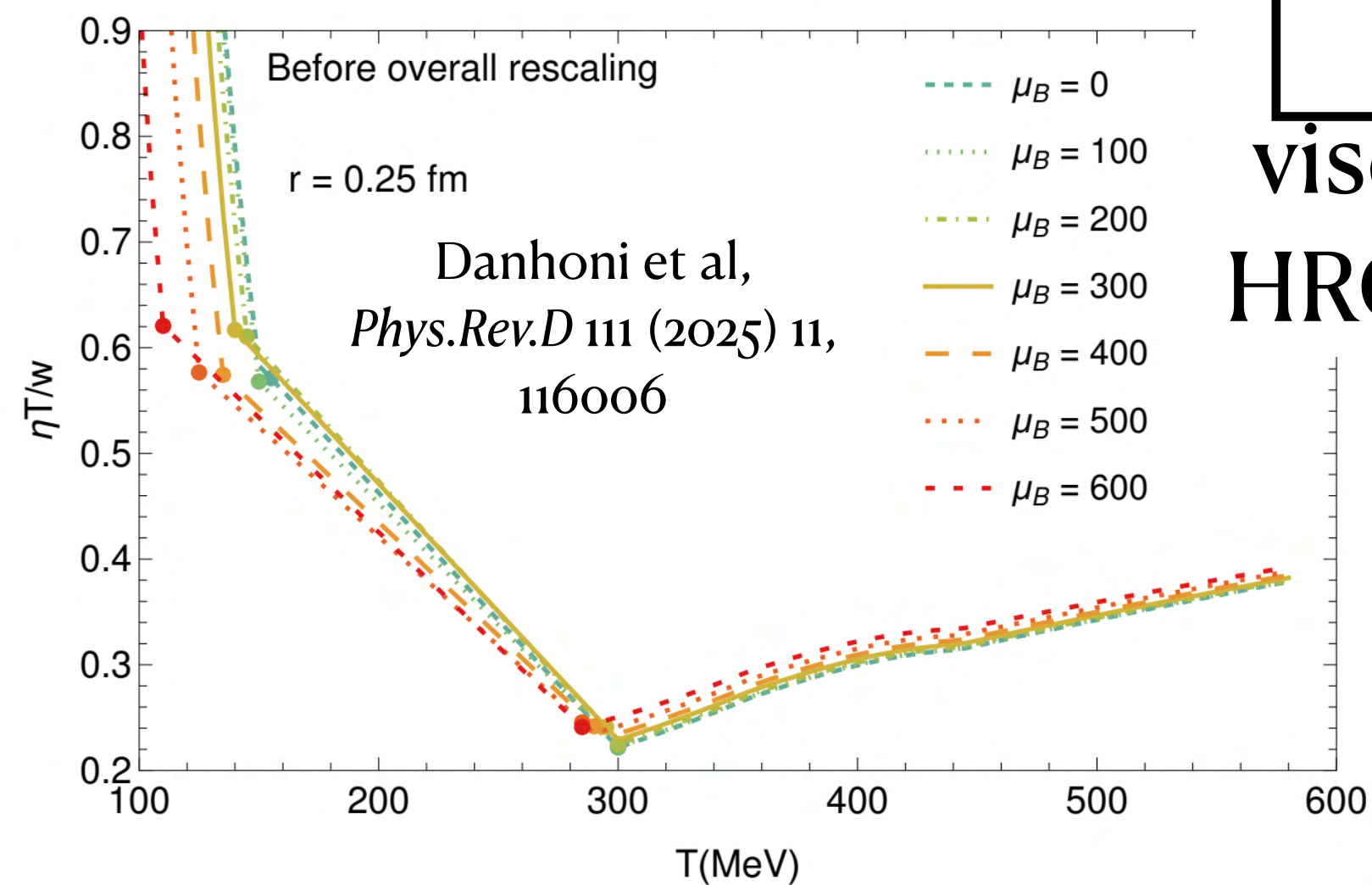
BSQ diffusion matrix ???

Recall, cross-correlations between transport coefficients, this is all very non-trivial!

Also, no lattice QCD calculations...

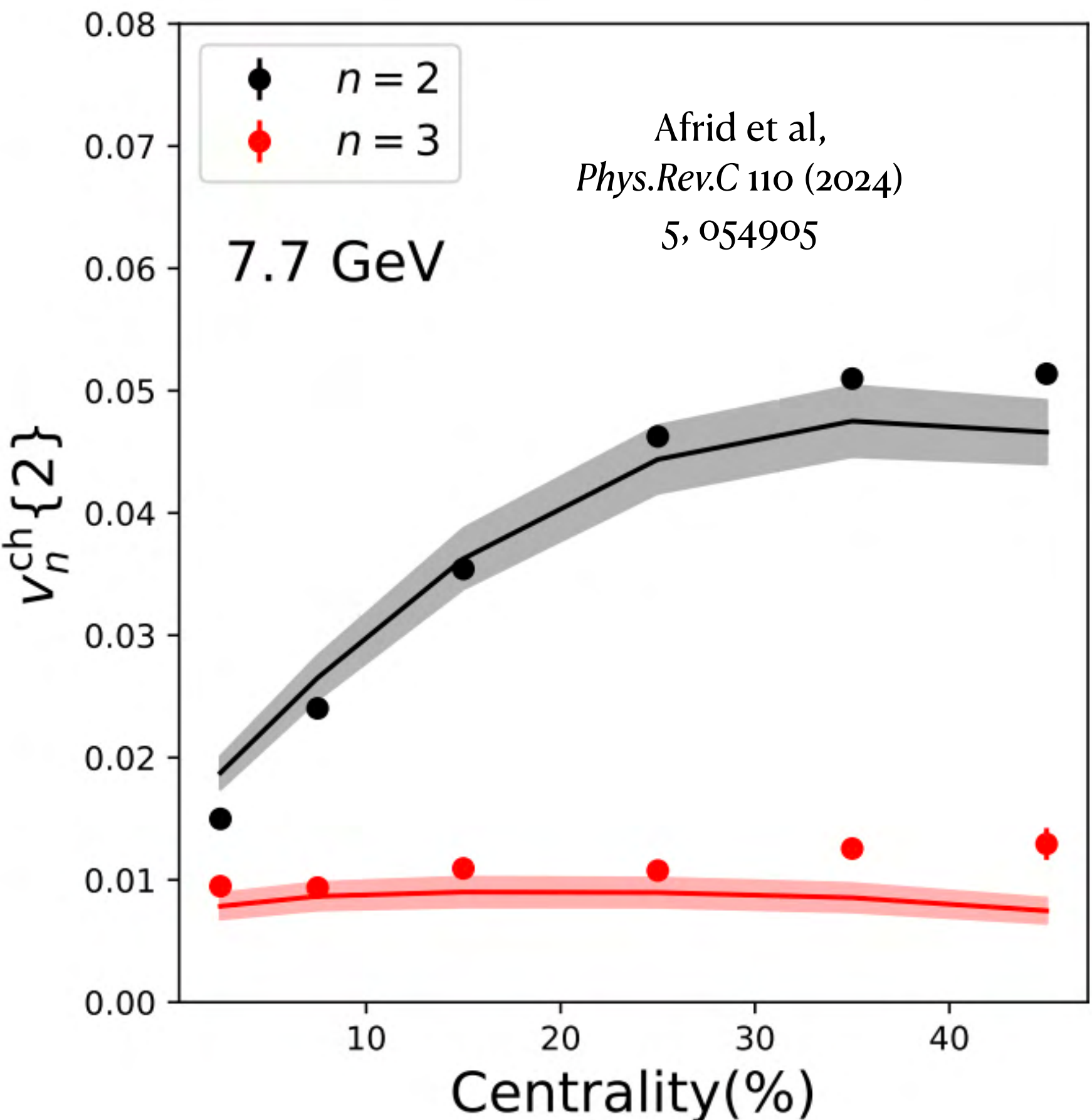
quarkon diffusion at a critical point

viscosity of HRG+pQCD



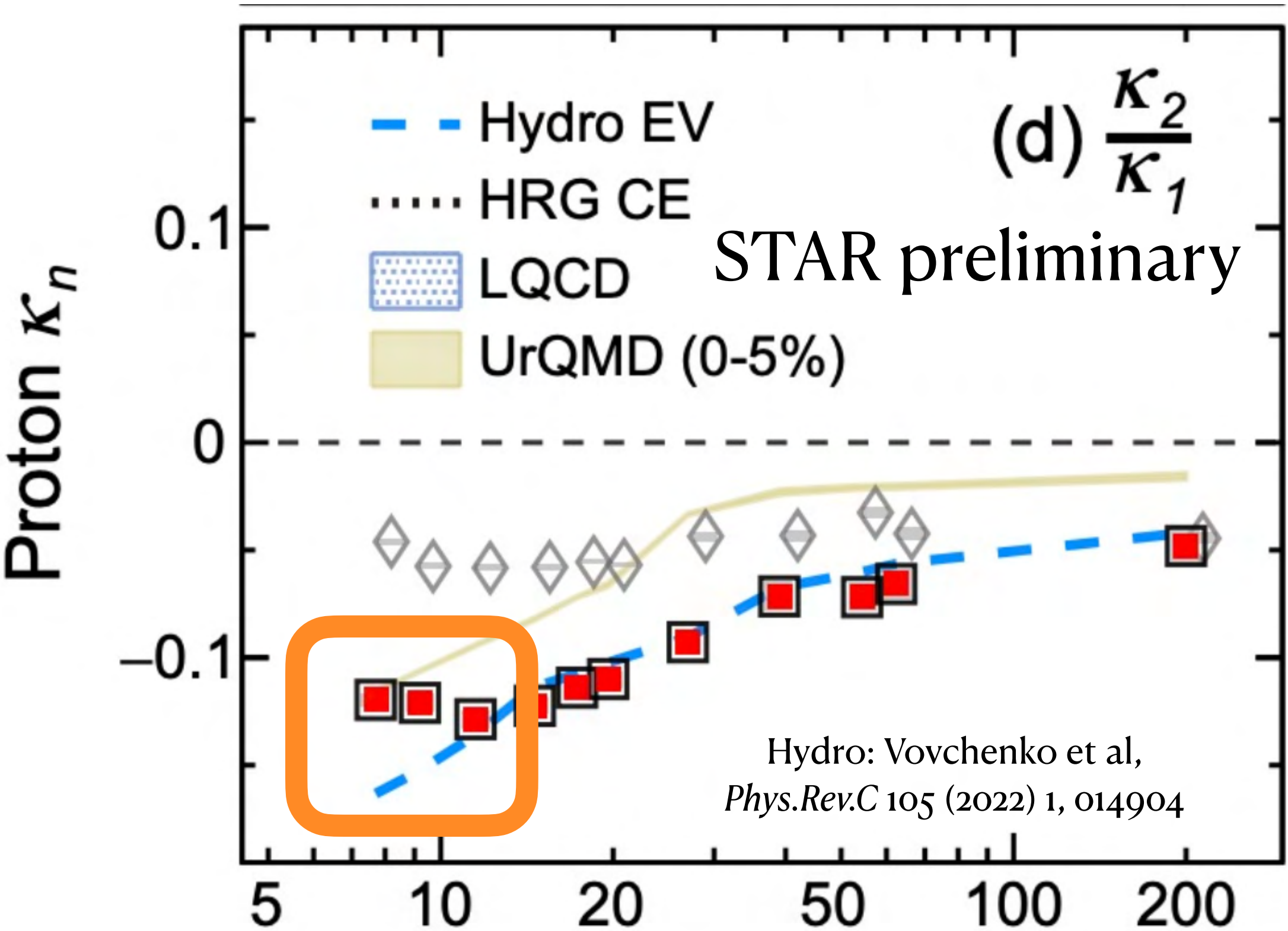
Baseline calculations (cross-over all the way down)

Most recent Bayesian analysis with fixed (lattice QCD+HRG) EOS, vary (simplified) transport coefficients



J. Noronha-Hostler UIUC

Search for the QCD critical point

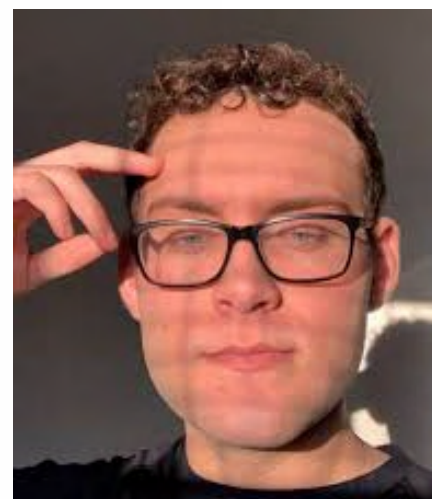
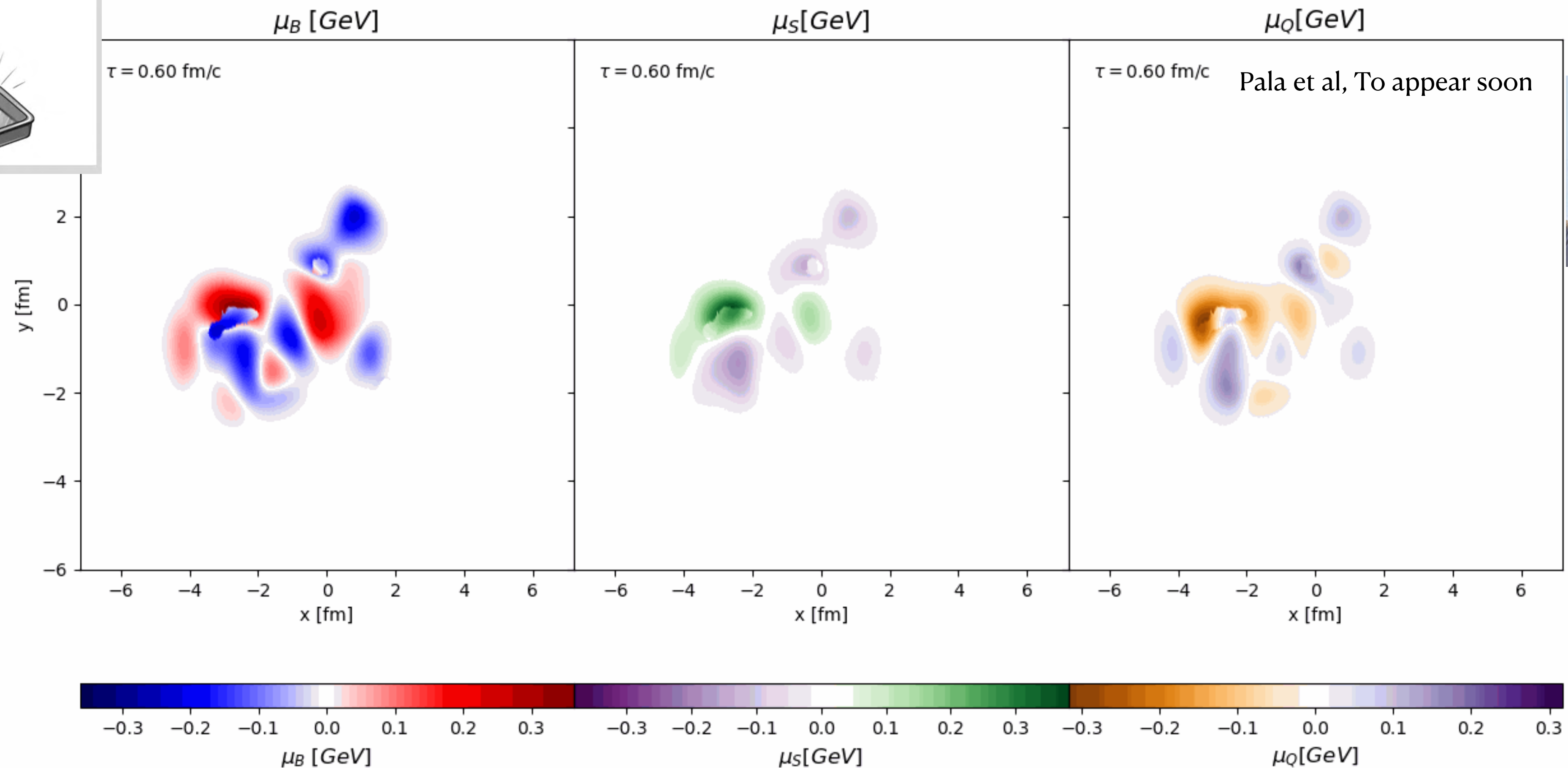


Deviations exactly at $\sqrt{s_{NN}} = 7.7$ GeV, hydro still works. Sign of a critical point?

Coming soon: CCAKE 2.0

3+1D simulations with BSQ diffusion, on CPUs/GPUs, generalized coordinates

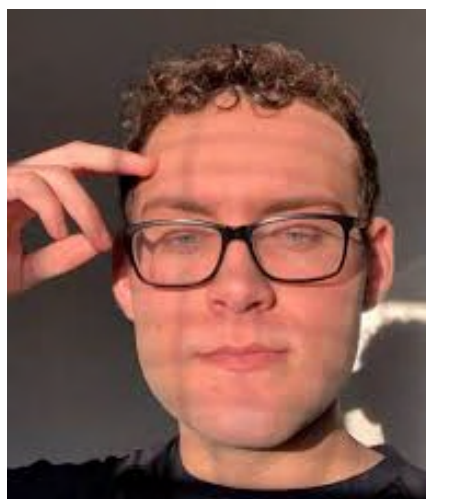
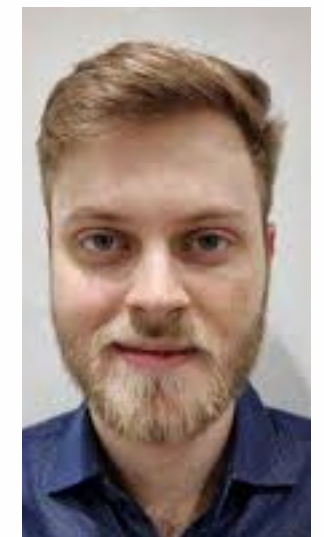
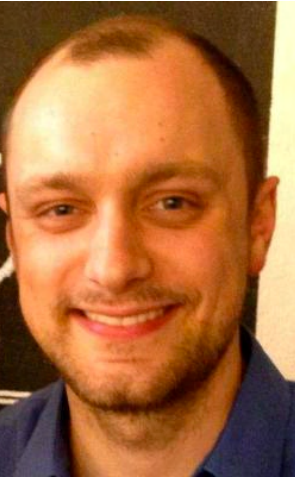
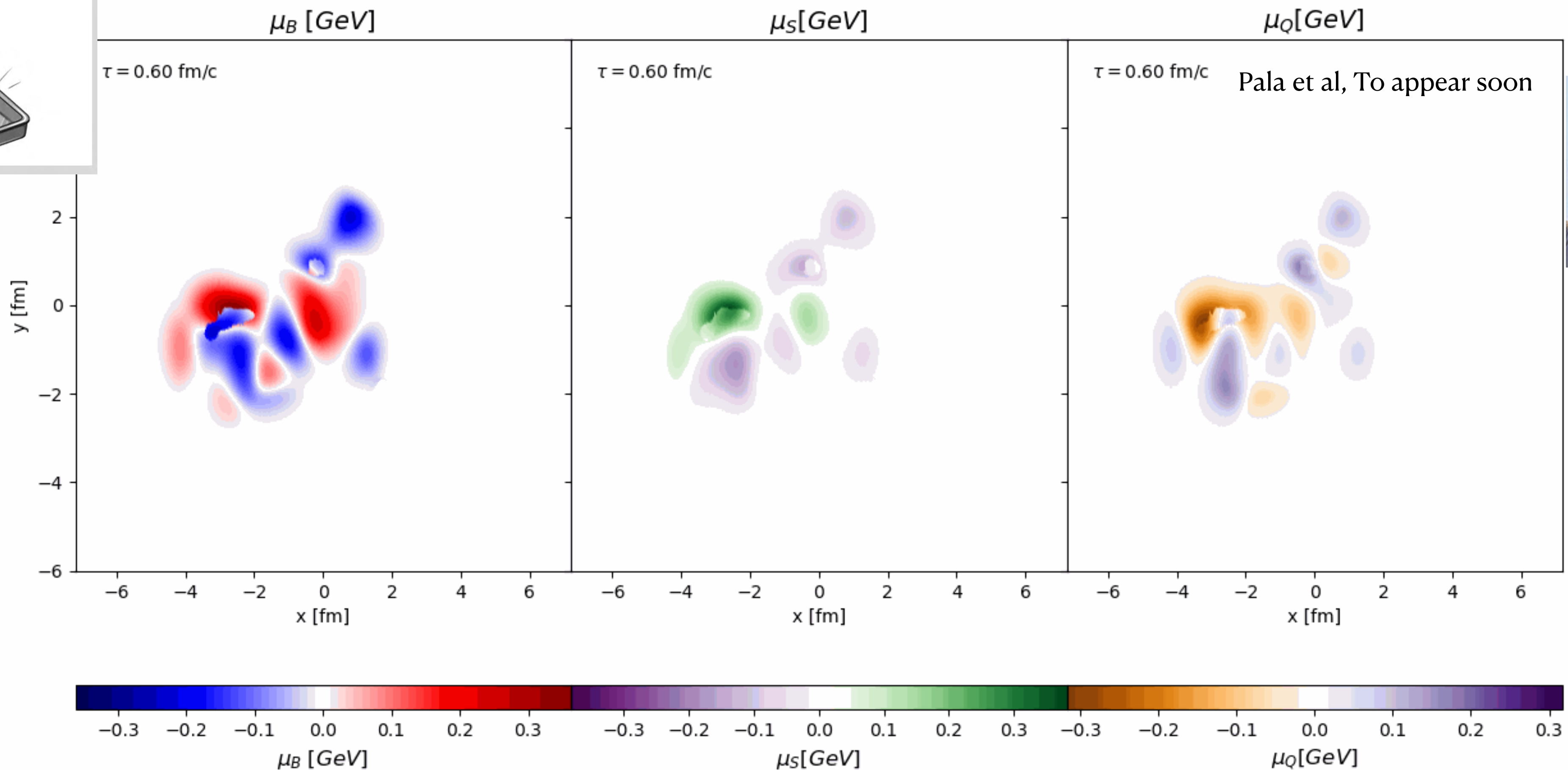
everything but the kitchen sink



Coming soon: CCAKE 2.0

3+1D simulations with BSQ diffusion, on CPUs/GPUs, generalized coordinates

everything but the kitchen sink



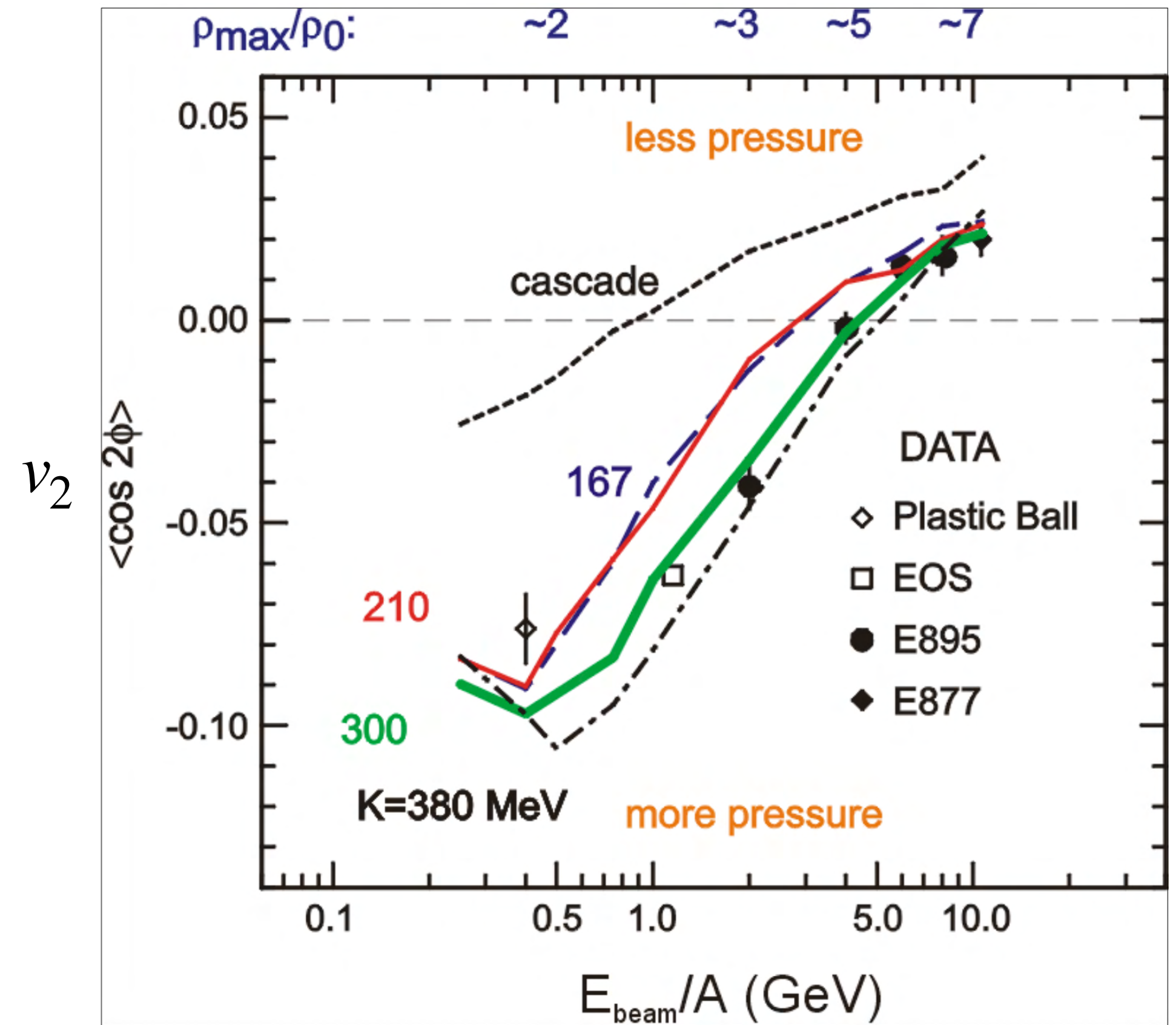
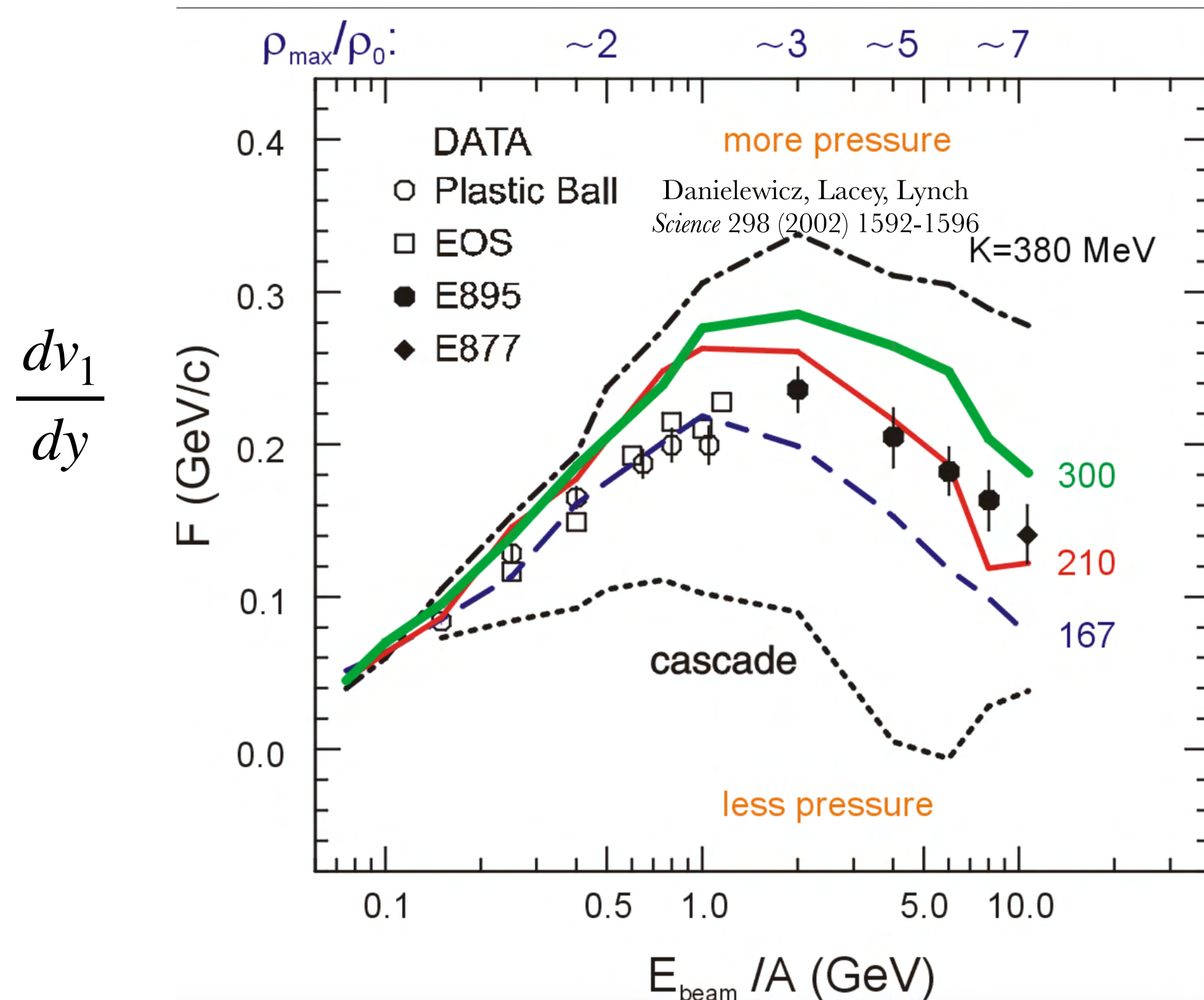
What do we do in the meantime?

Need to work out the math to connect HIC to NS, this isn't very trivial...

2002 Collective flow & extracting the heavy-ion EOS

Varied Symmetry energy properties, extrapolated EOS to $T = 0$

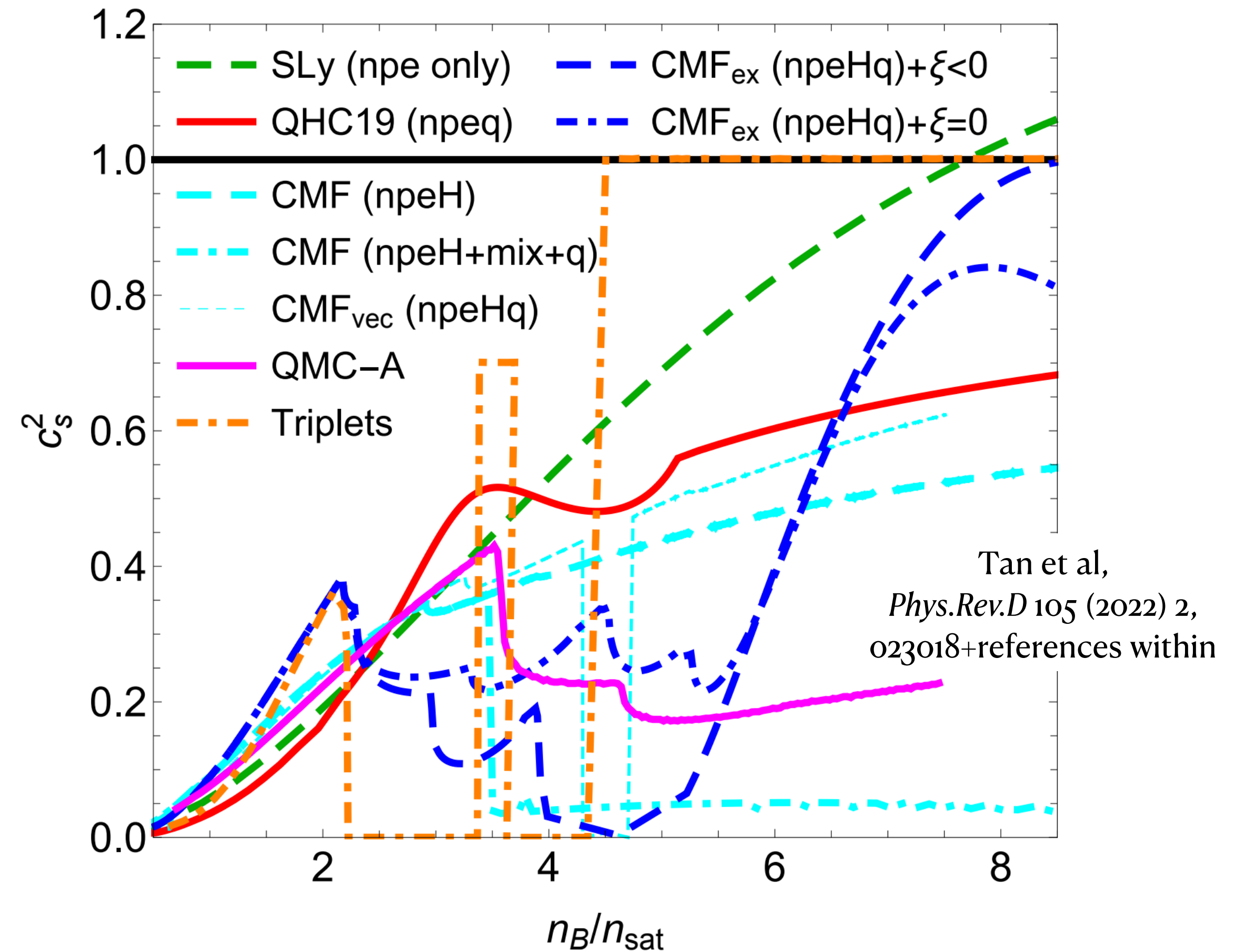
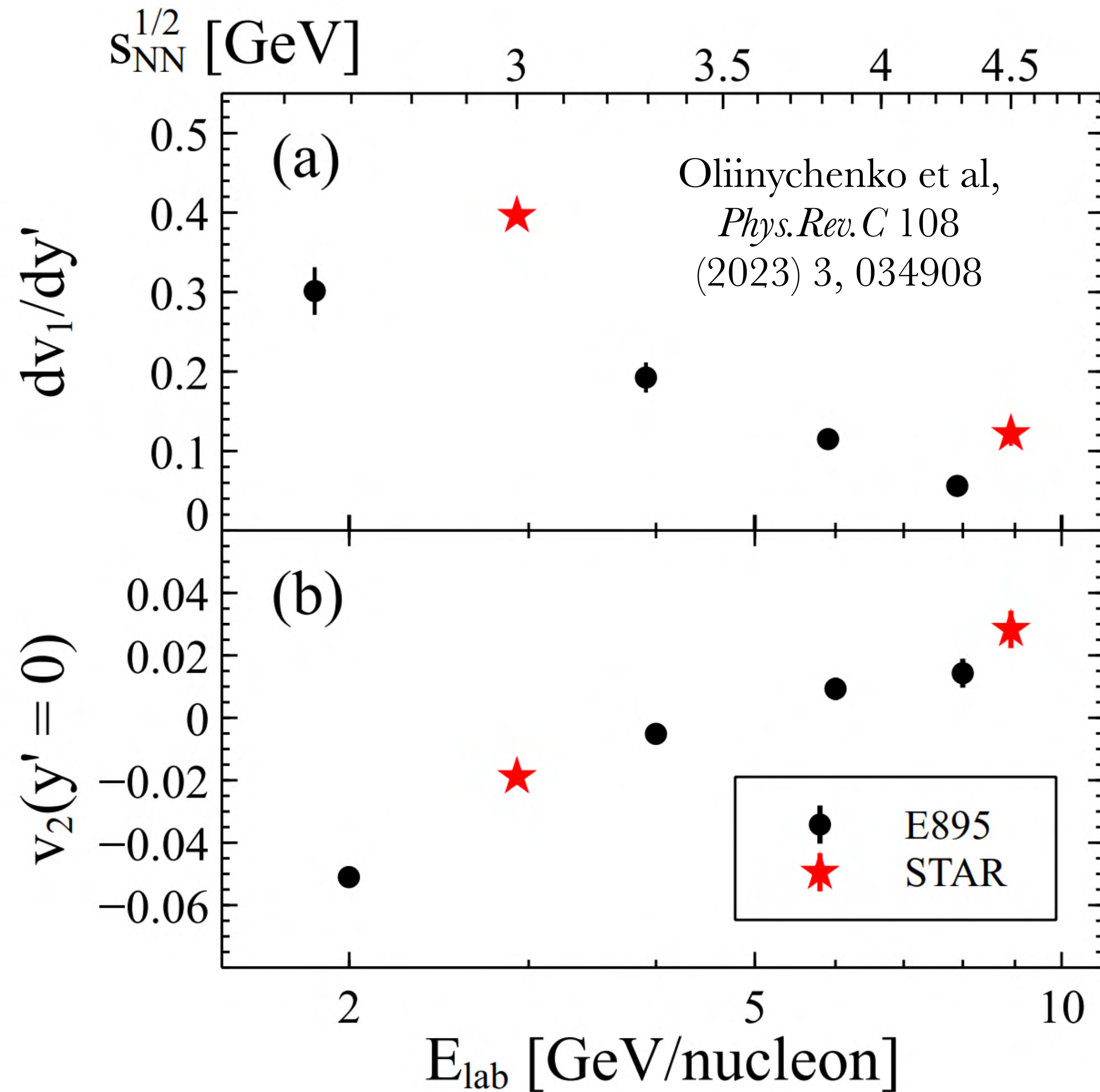
Extremely hard to fit both dv_1/dy and v_2



Fermi-liquid/ π , p, n transport assumed
What does modern HIC find?

A lot has happen in 20+ years

New data, insights from neutron stars and high-energy HIC



New STAR data not identical to old HIC data, bumps in c_s^2 , but 20 years of knowledge from HIC...

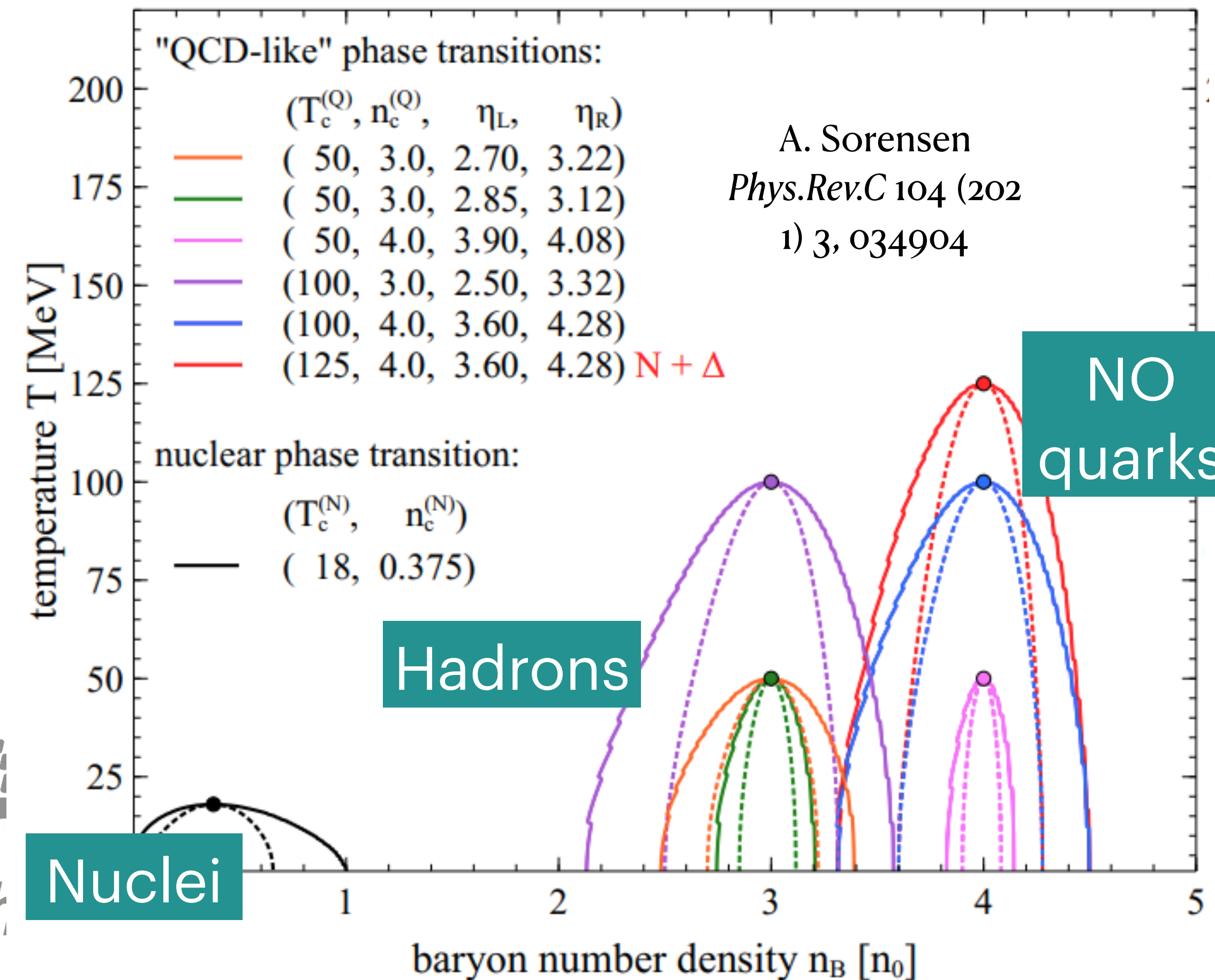
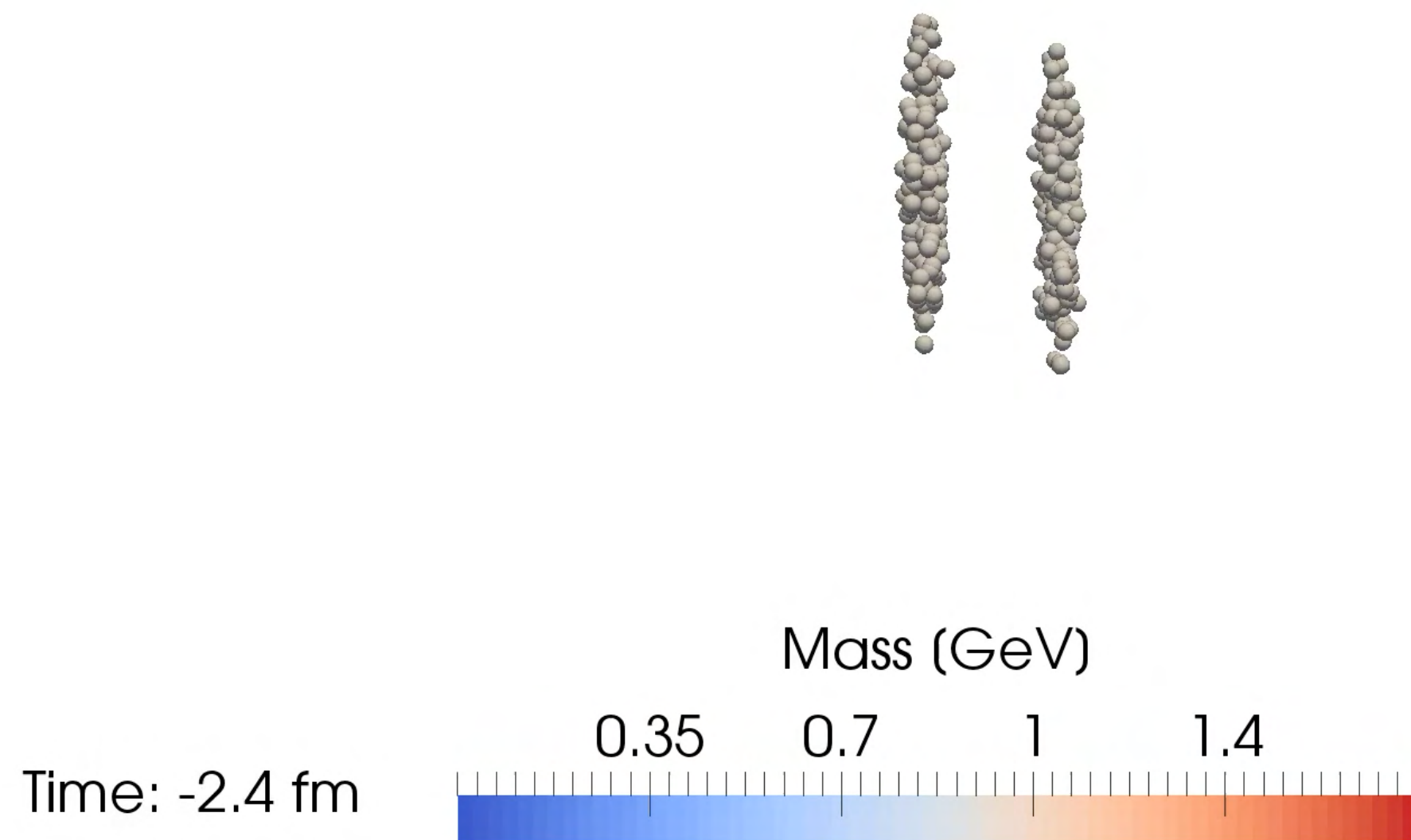
2020's Low-energy $\sqrt{s} \leq 7.7 \text{ GeV}$ HICs

New STAR data, new interpretations

Do quark-gluons d.o.f. matter?

Build phase transitions into hadron transport

Flexible potentials $\rightarrow$ phase transitions



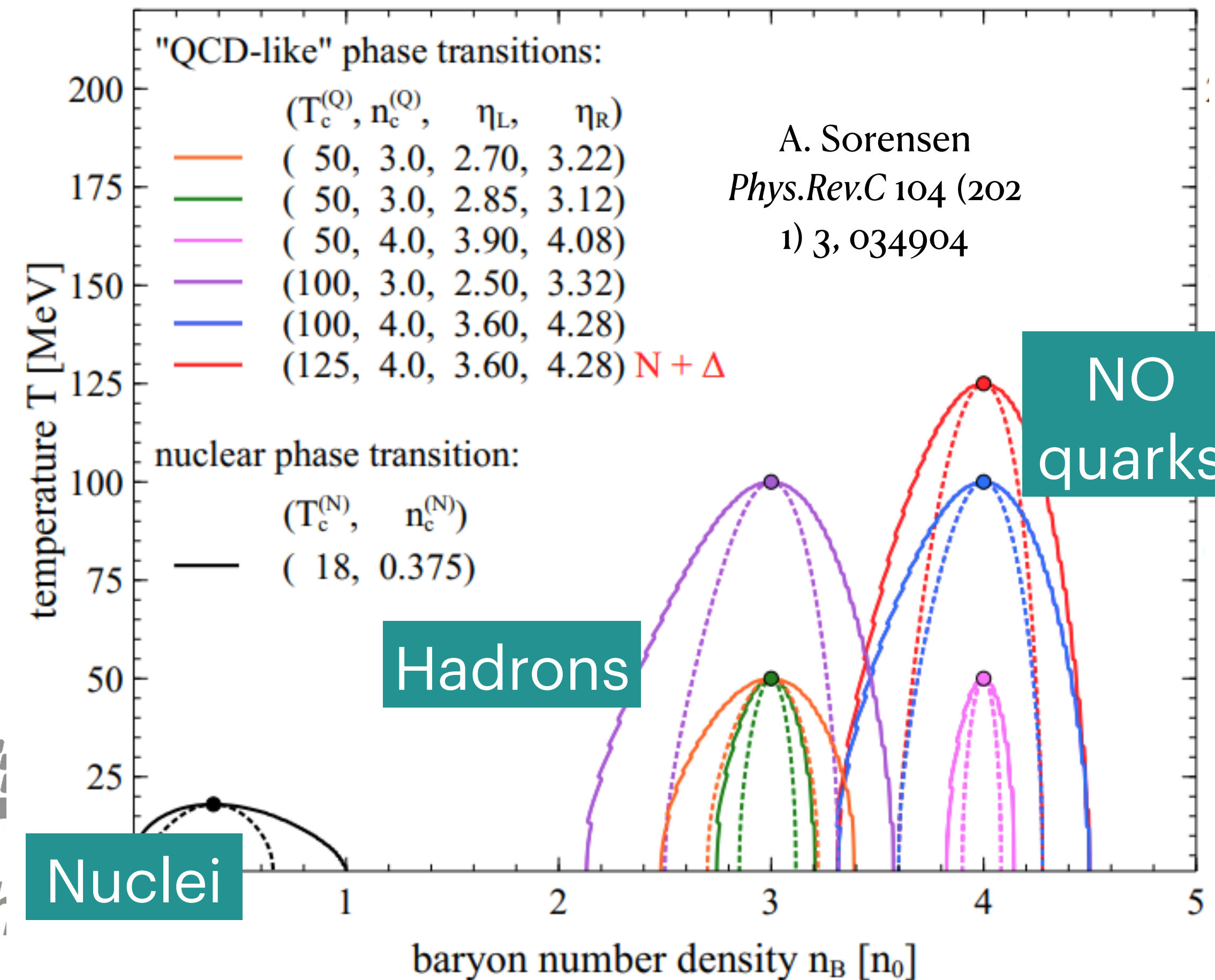
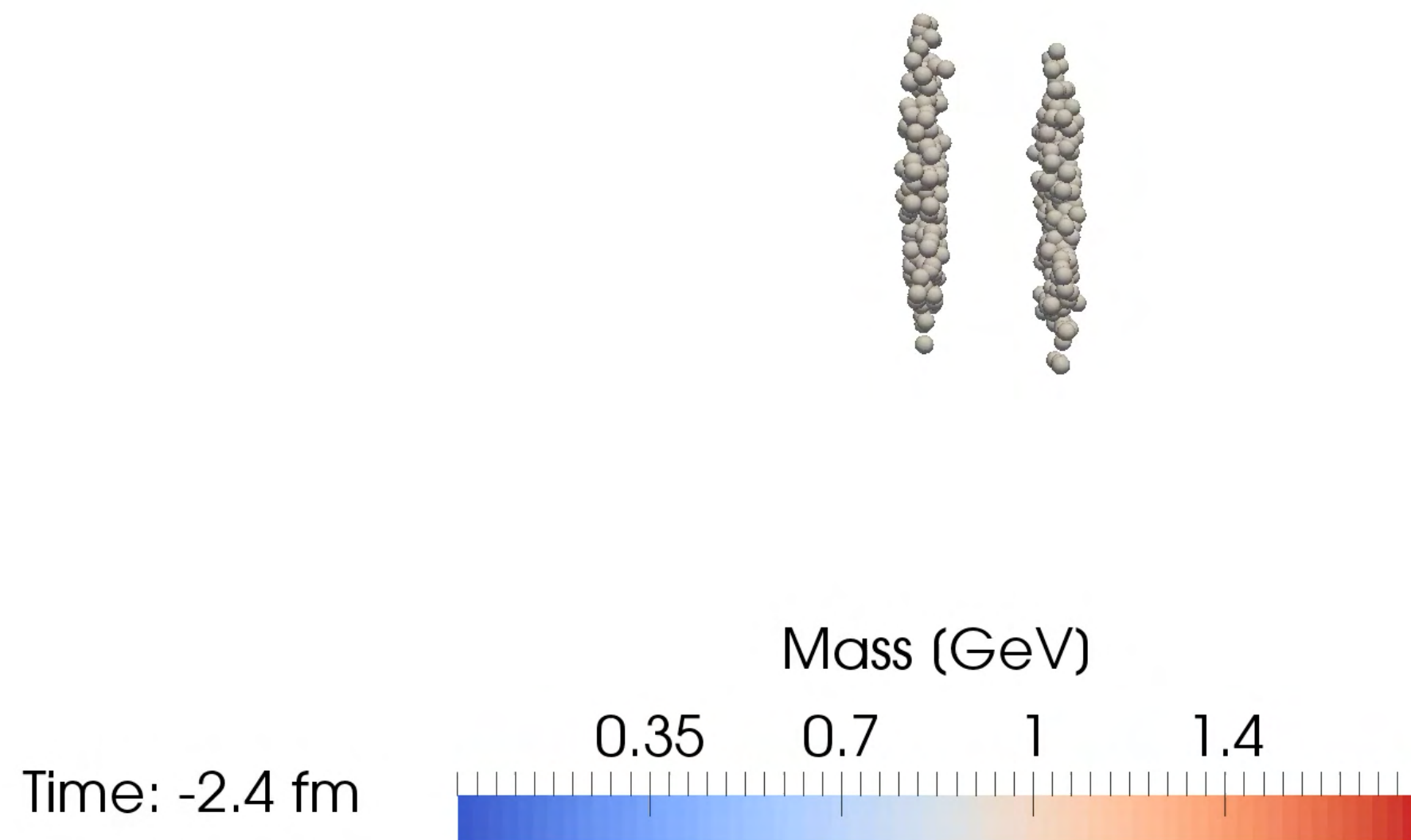
2020's Low-energy $\sqrt{s} \leq 7.7 \text{ GeV}$ HICs

New STAR data, new interpretations

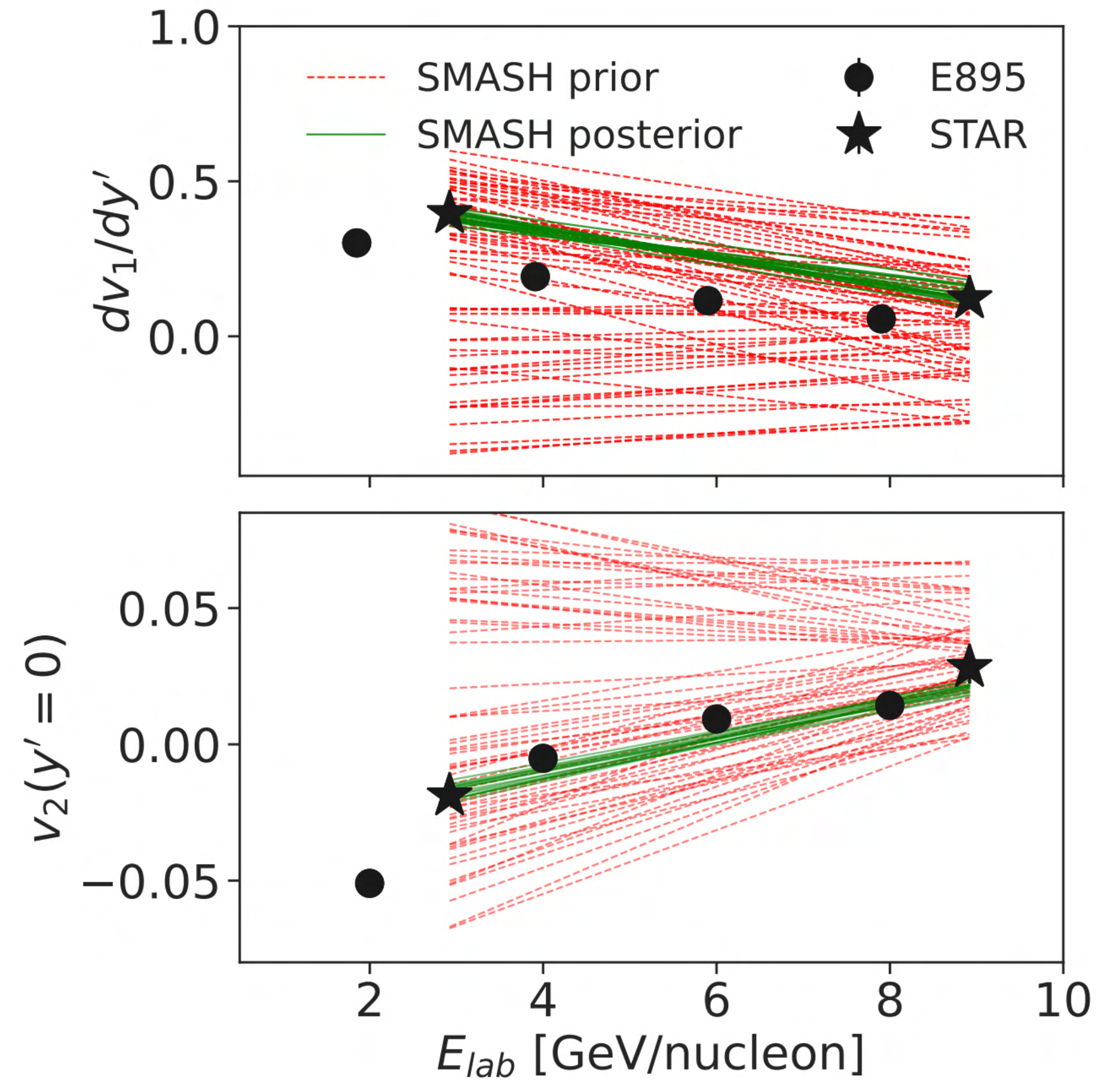
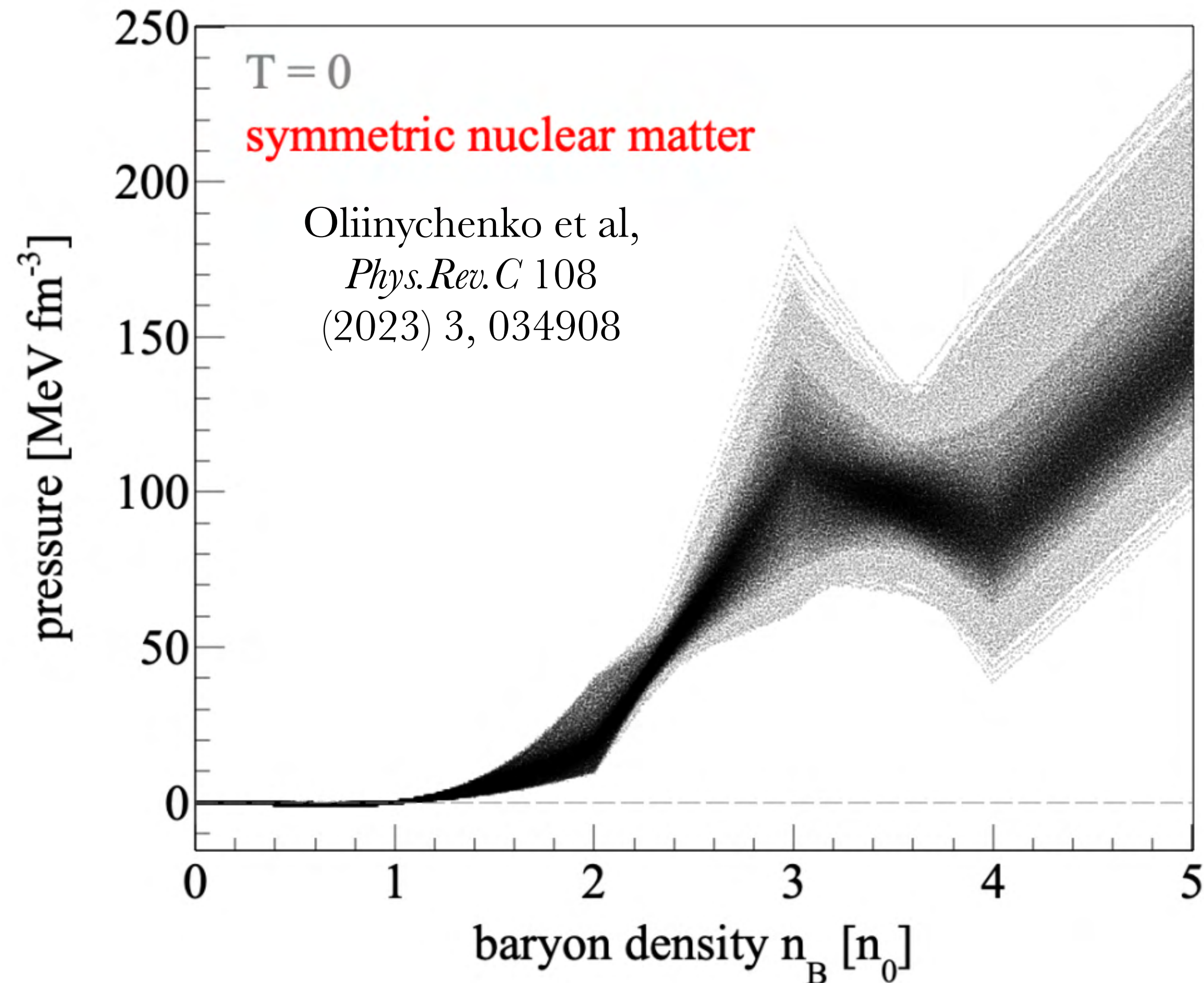
Do quark-gluons d.o.f. matter?

Build phase transitions into hadron transport

Flexible potentials $\rightarrow$ phase transitions



Peak in c_s^2 at $n_B^{peak} = [2,3] n_{sat}$ for HIC



See also *Phys.Rev.Lett.* 131 (2023) 20, 202303



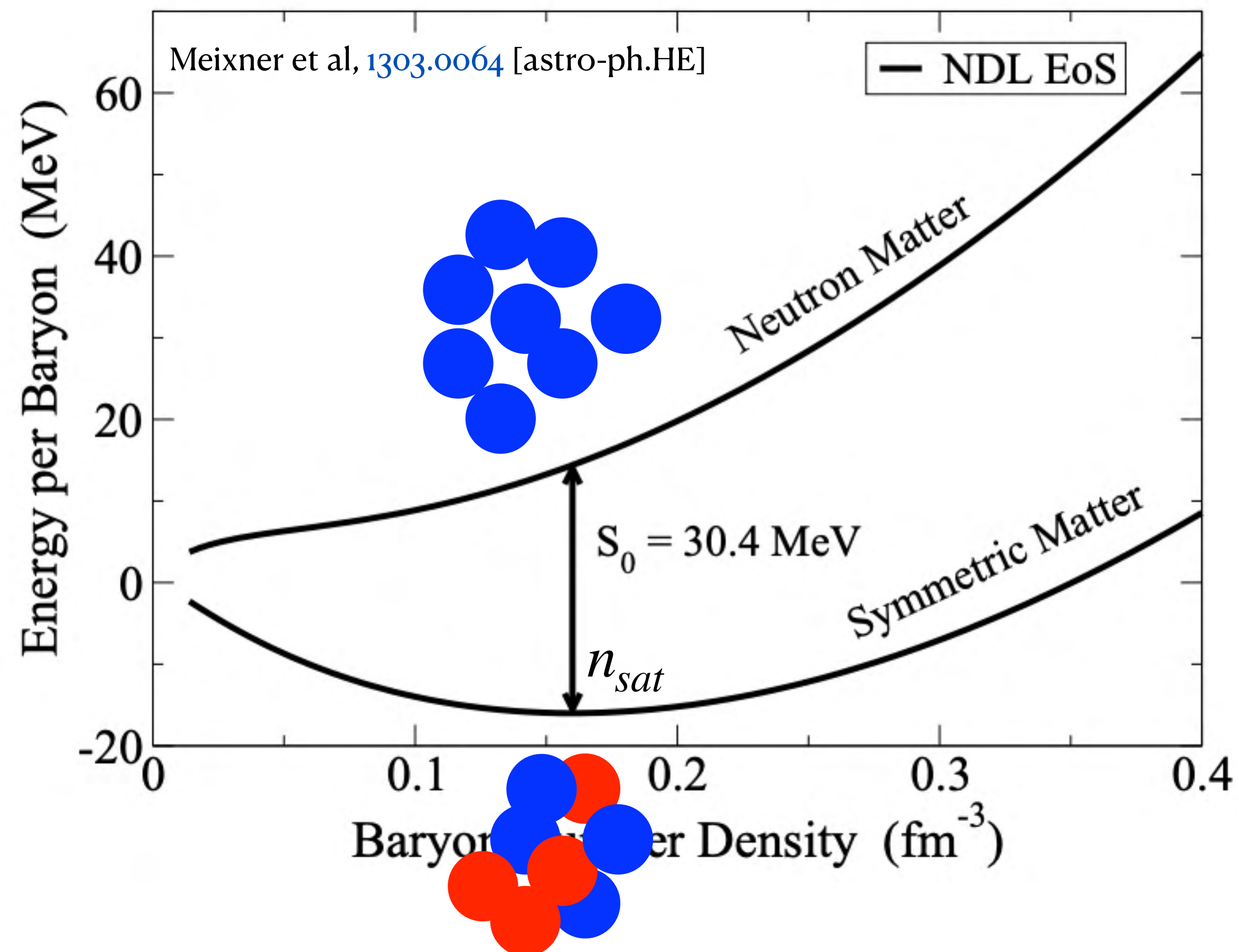
J. Noronha-Hostler UIUC

Systematic Hydro studies still needed

How do we connect HIC to NS?

Symmetric nuclear matter vs Pure neutron Matter

“Symmetry Energy”



Symmetric nuclear matter

$$\mu_Q = 0$$

$$\text{Isospin asymmetry } \delta = 1 - 2Y_Q = 0$$

$$Y_Q = 0.5$$

ONLY true for neutrons and protons!

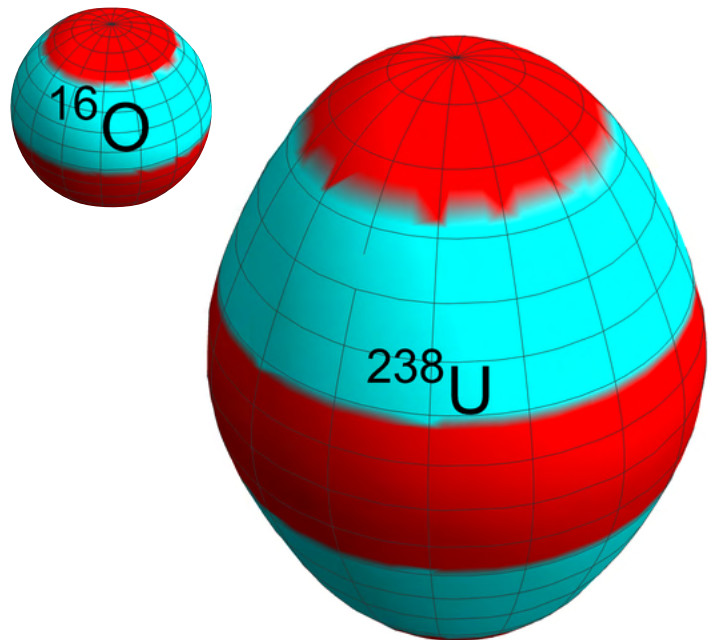
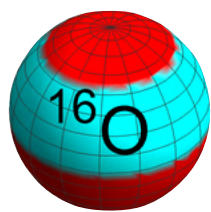
Symmetry energy

$$S(n_B) = \frac{\epsilon_{PNM}}{n_B} - \frac{\epsilon_{SNM}}{n_B}$$

Charge fraction of ions

Isospin asymmetry

$$Y_Q = \frac{Z}{A} = \frac{n_Q}{n_B}$$



System	Z	A	Y_Q	Data?
O+O	8	16	0.5	some
Ne+Ne	10	20	0.5	no
Mg+Mg	12	24	0.5	no
Ca+Ca	20	40	0.5	no
Cu+Cu	29	63	0.46	yes
Ru+Ru	44	96	0.458	some
Ar+Ar	18	40	0.45	no
Xe+Xe	54	128	0.419	yes
Zr+Zr	40	96	0.417	some
Au+Au	79	198	0.399	yes
U+U	92	238	0.387	yes

$$Y_Q \lesssim 0.2$$

Heavy-Ions and Neutron Stars are all on the same phase diagram, but very different Y_Q

Isospin asymmetry

n_Q

Isospin-
Asymmetric
Matter (IAM)
 $Y_Q = \infty$

Heavy-Ion Collisions (HIC)

$Y_Q = [0.38, 0.5]$

$$Y_Q \lesssim 0.2$$
$$n_B$$

31

IGNORE Strange: Symmetry Energy Expansion

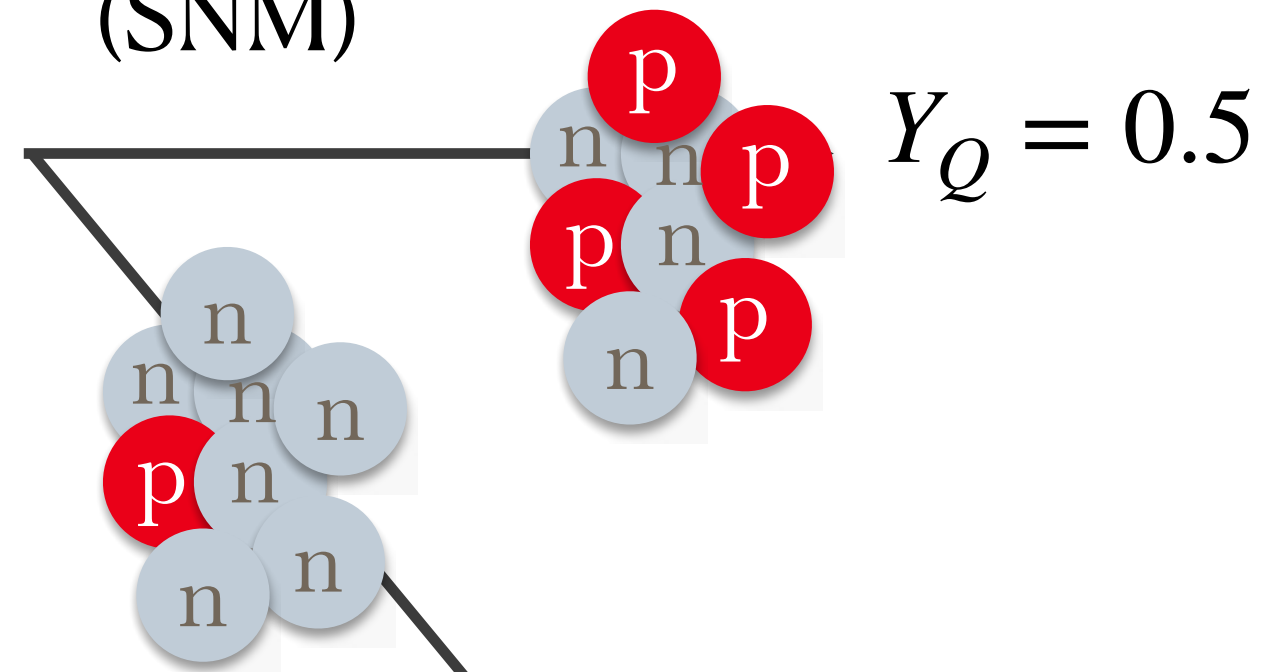
Connecting NS to HIC across Y_Q

Original symmetry energy expansion from binding energies

Bombaci & Lombardo *Phys.Rev.C* 44 (1991) 1892-1900

Symmetric matter

(SNM)



Isospin asymmetry $\delta = 1 - 2Y_Q$

where $\delta = 0$ for SNM and $\delta = 1$ for PNM

$$\frac{E_{ANM}}{N_B} = \frac{E_{SNM}}{N_B} + E_{sym,2}\delta^2 + \mathcal{O}(\delta^4)$$

Asymmetric
matter (ANM)

$Y_Q \lesssim 0.1$

Expand in δ where odd terms drop due to isospin symmetry

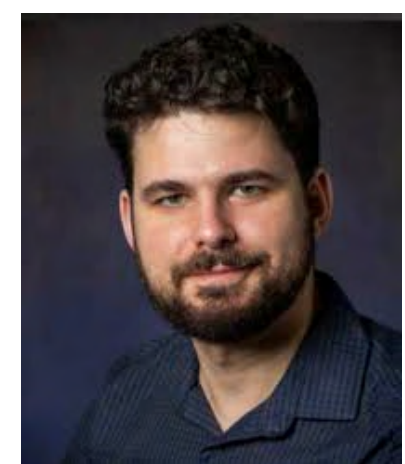
Convert EOS from NS to HIC, expand around n_{sat}

$$\epsilon_{HIC} = \epsilon_{NS} - 4n_B \left[E_{sym,sat} + \frac{L}{3} \left(\frac{n_B}{n_{sat}} - 1 \right) + \frac{K}{18} \left(\frac{n_B}{n_{sat}} - 1 \right)^2 + \frac{J}{162} \left(\frac{n_B}{n_{sat}} - 1 \right)^3 \right] \left[\left(Y_Q^{HIC} - Y_{Q,NS} \right) + \left(Y_{Q,NS}^2 - \left(Y_Q^{HIC} \right)^2 \right) \right]$$

$E_{sym,2}(n_B)$ 4 unknowns

To varying Y_Q^{HIC}

Yao et al, *Phys.Rev.C* 109 (2024) 6, 065803



What about hyperons?

Symmetry energy expansion with strangeness

Yang et al, 2504.18764 [nucl-th]

Original symmetry energy expansion considered only protons, neutrons

isospin asymmetry: $\delta_Q \equiv 1 - 2 Y_Q$

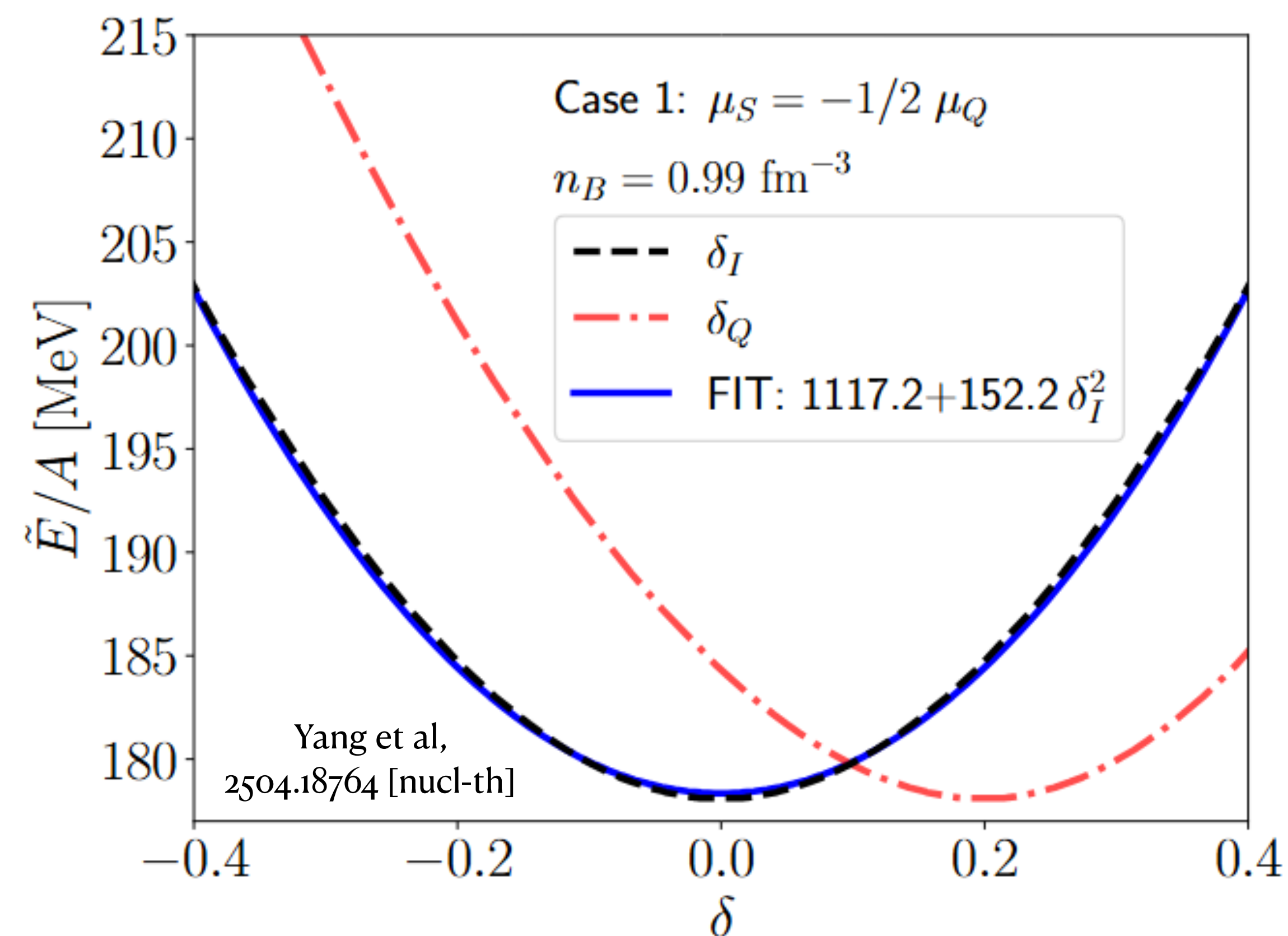
Gell-Mann-Nishijima formula

$$Q = I_z + \frac{1}{2} (B + \mathcal{S})$$

Corrected isospin asymmetry

$$\delta_I \equiv 1 + Y_S - 2 Y_Q$$

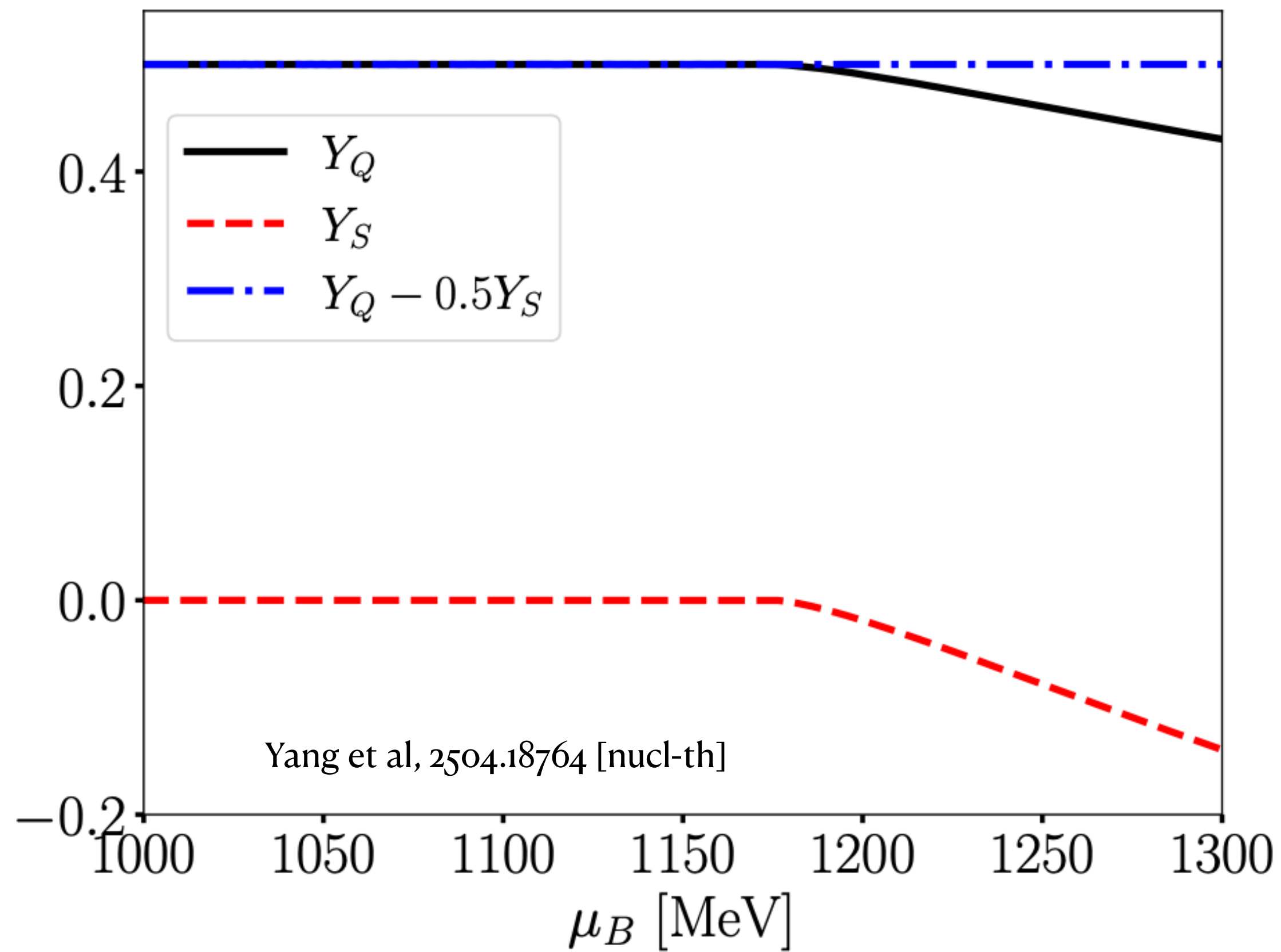
Original symmetry energy expansion misses the ground state of nuclear matter when hyperons are present



Strangeness contribution to SNM

You really **CANNOT** ignore it

For SNM, $Y_Q \neq 0.5$



Wrong isospin
asymmetry

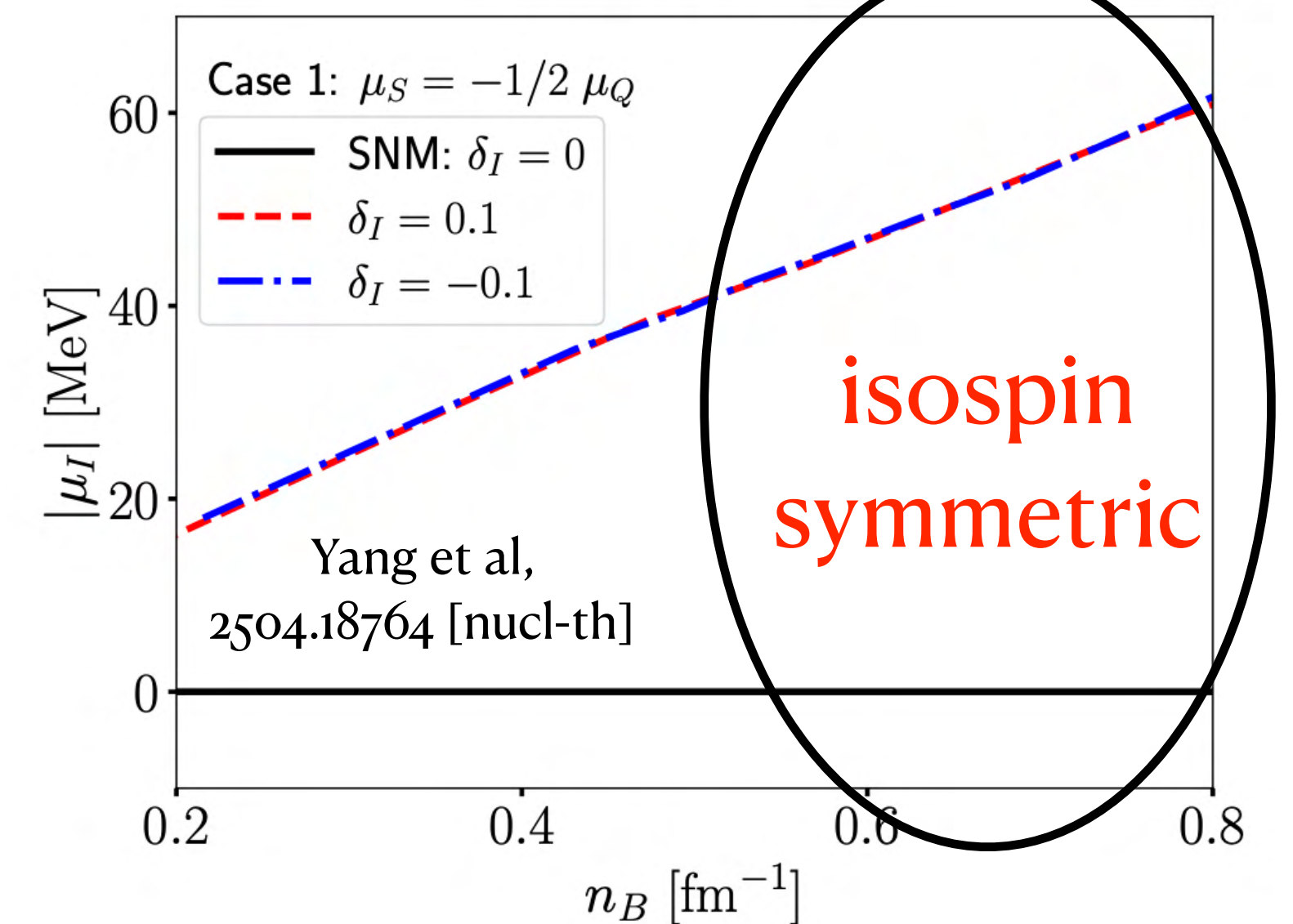
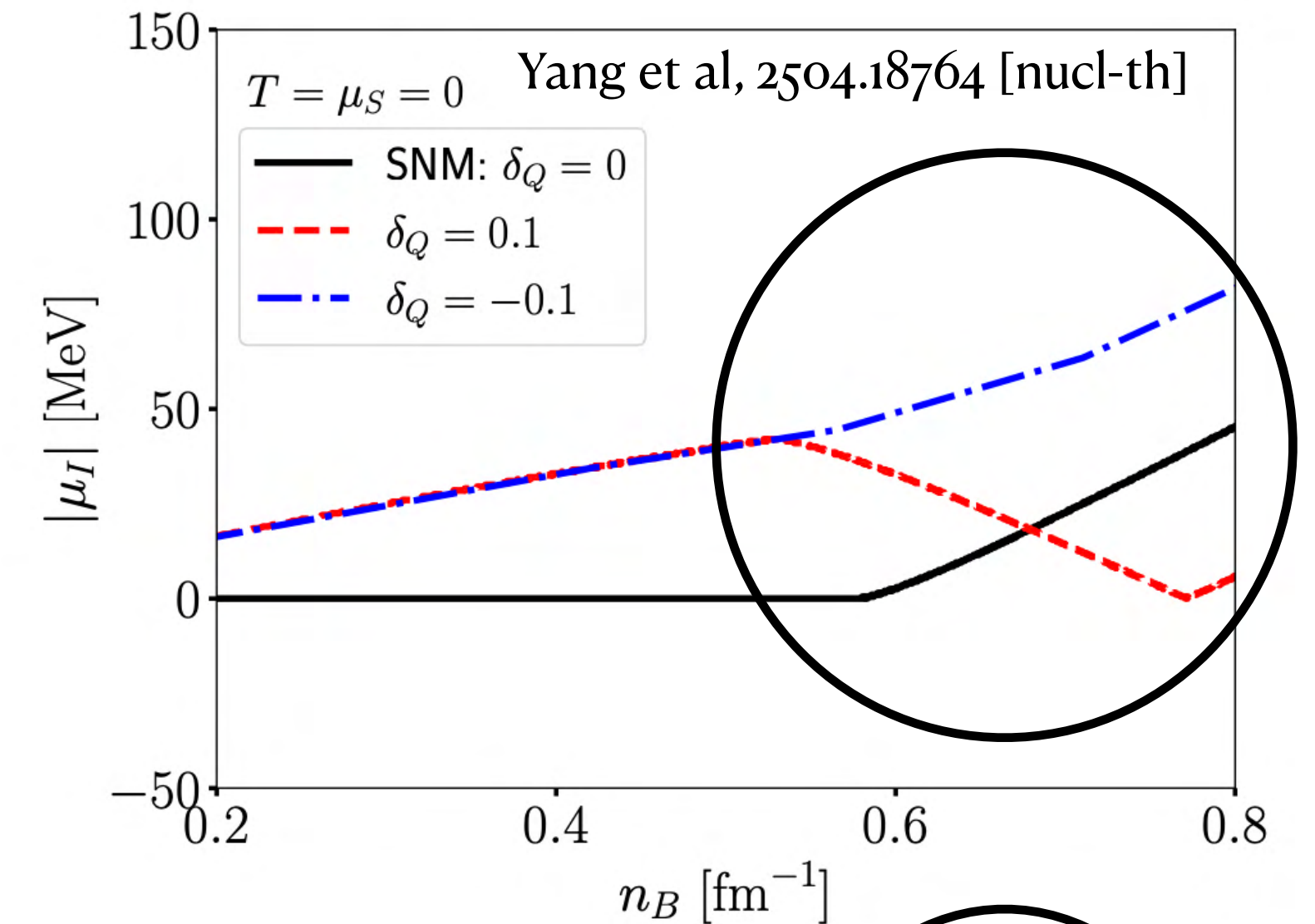
$$\delta_Q = 1 - 2Y_Q$$

Correct isospin
asymmetry

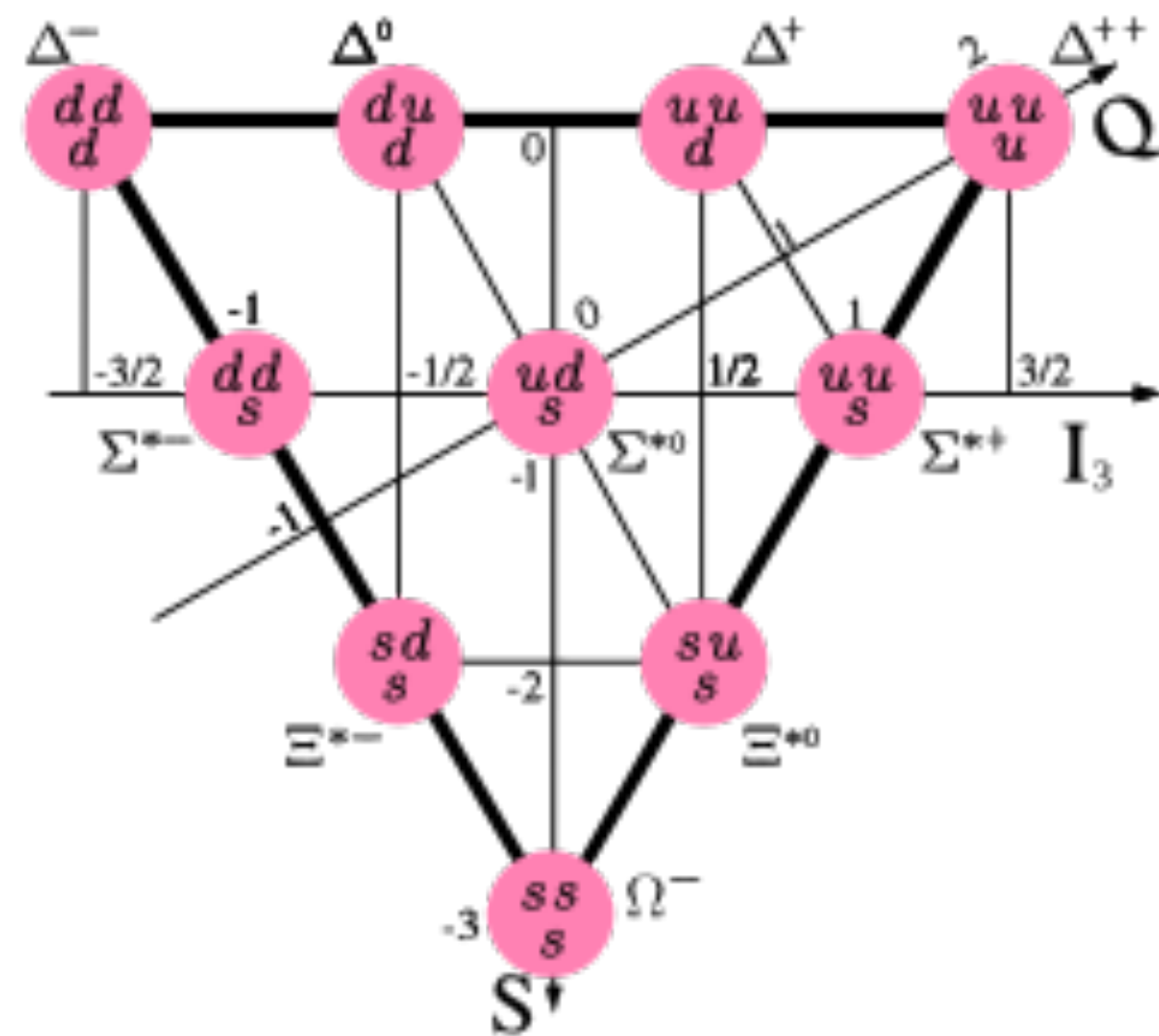
$$\delta_I = 1 - 2Y_Q + Y_S$$

PNM is meaningless for $\delta_I = 1$, then $Y_S = 2Y_Q$

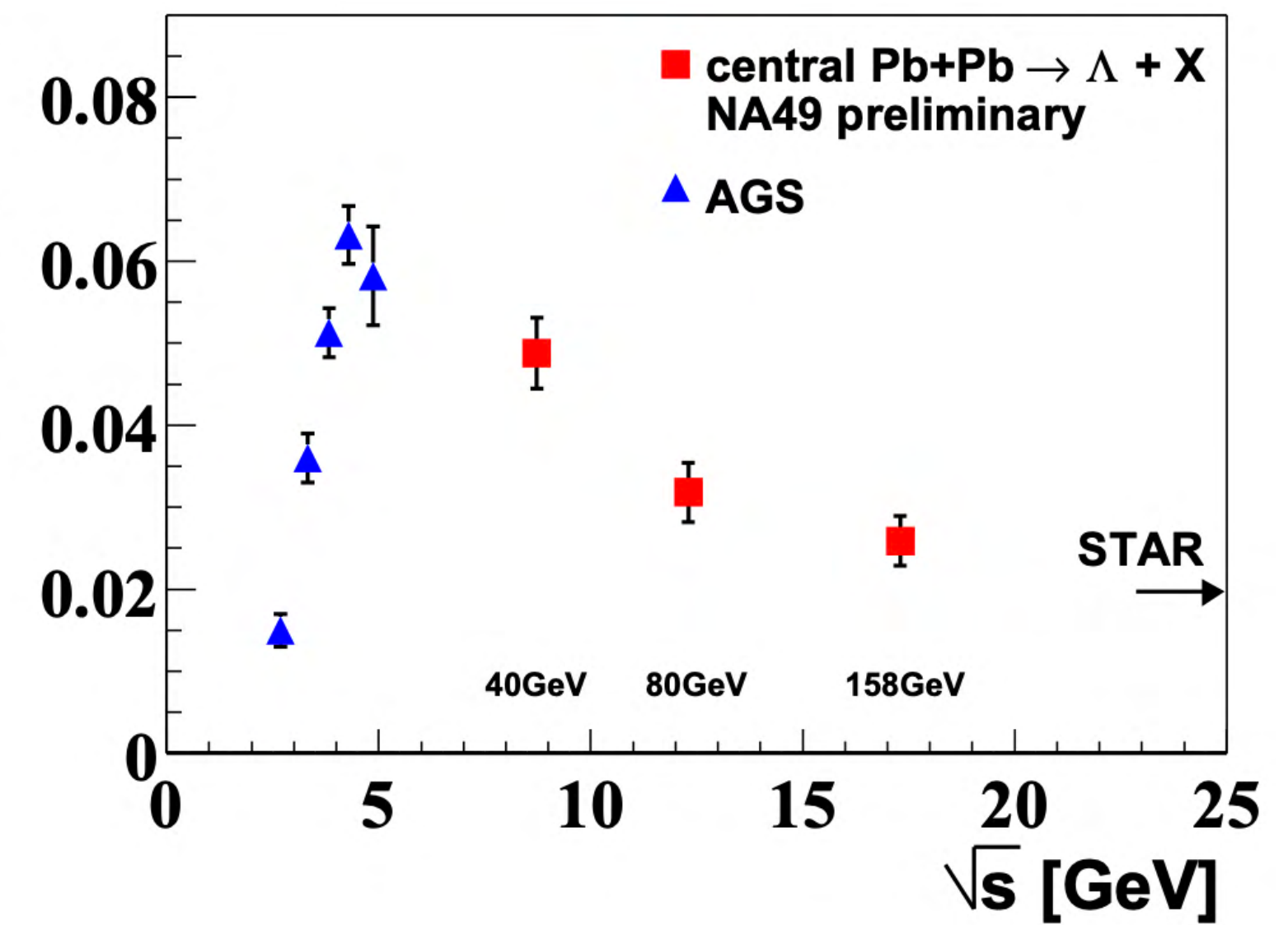
NOT isospin symmetric



Ignoring strangeness



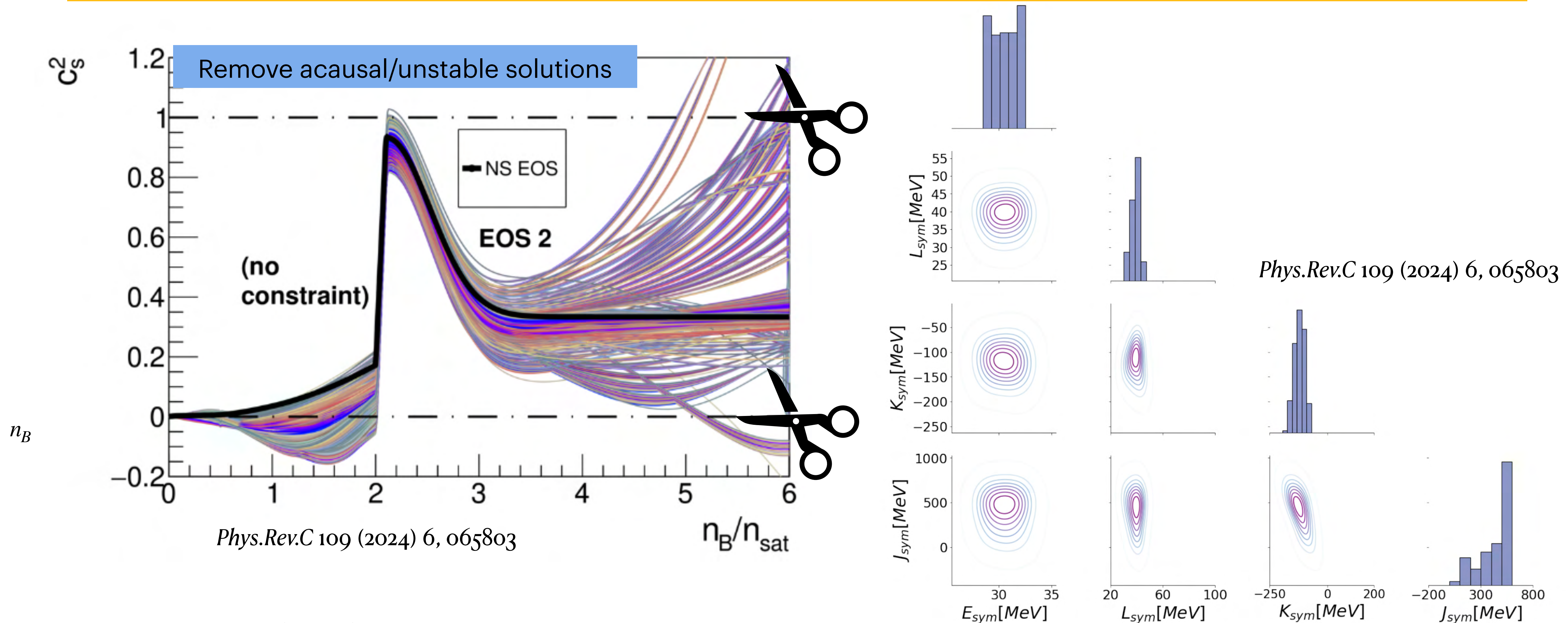
$\Lambda / \pi \ (y=0)$



How do low-energy heavy-ion collisions at $T = 0$ connect to neutron stars?

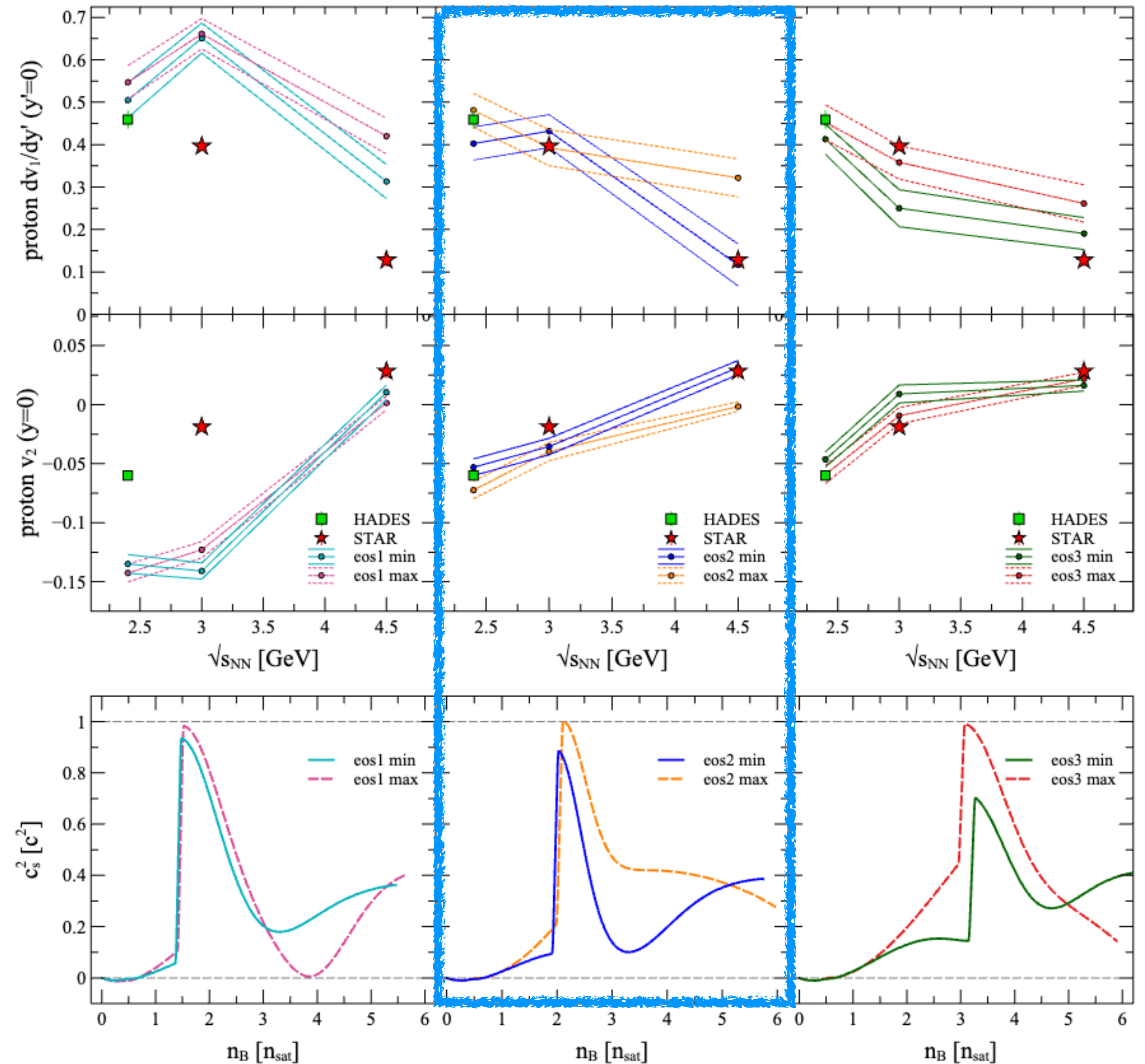
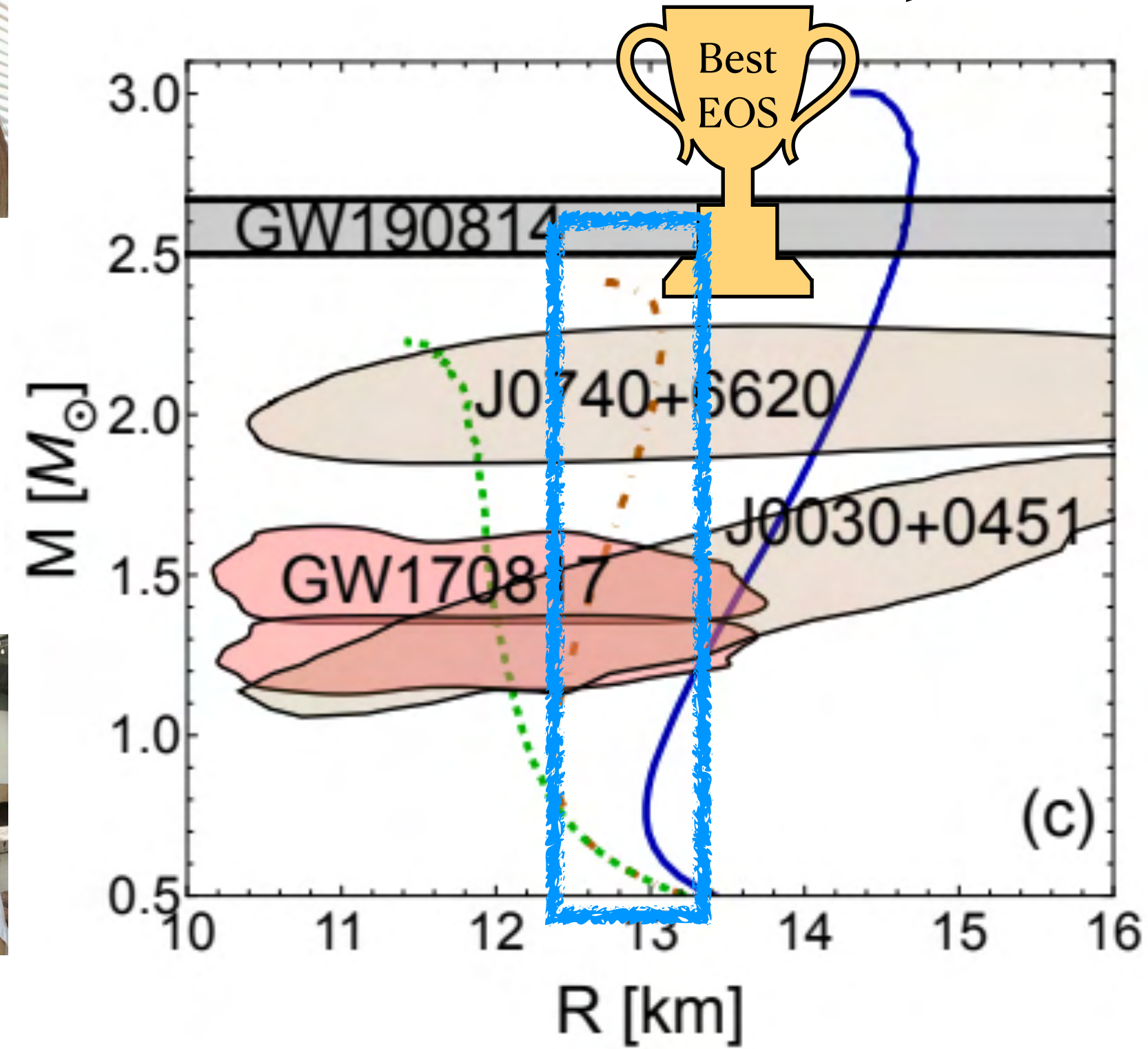
Symmetry energy expansion

Given neutron star equation of state $\rightarrow$ convert to HIC and can constrain by $0 \leq c_s^2 \leq 1$ and saturation properties.



Ultra heavy neutron star EOS works HIC

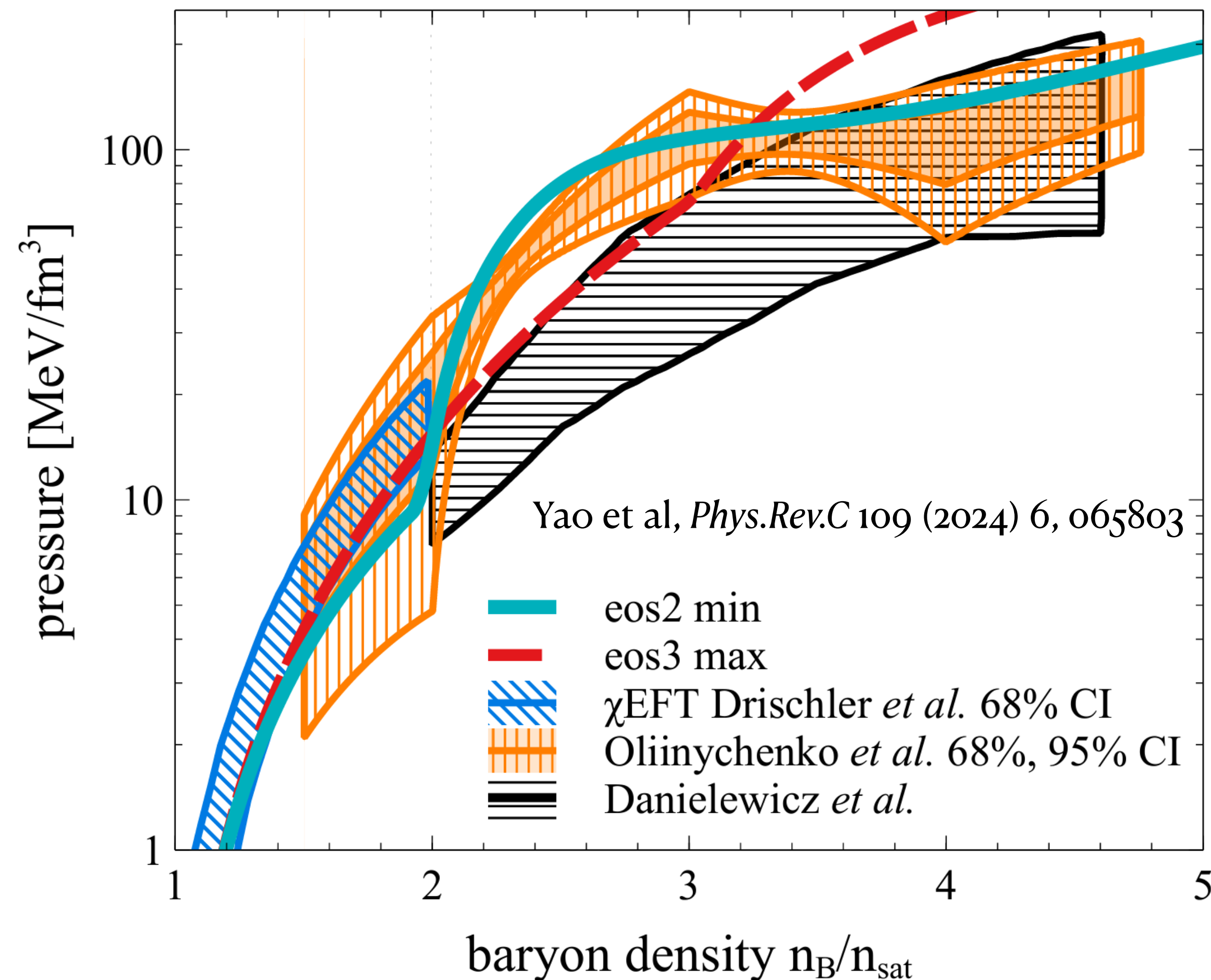
Phys.Rev.C 109 (2024) 6, 065803



Take extremes of the EOS band derived for heavy-ion collisions, run in hadron transport, and compare to heavy-ion collision flow data

Ultra heavy neutron star EOS works HIC

But, this does NOT match hadron only transport models



But, this is NOT the end of the story..

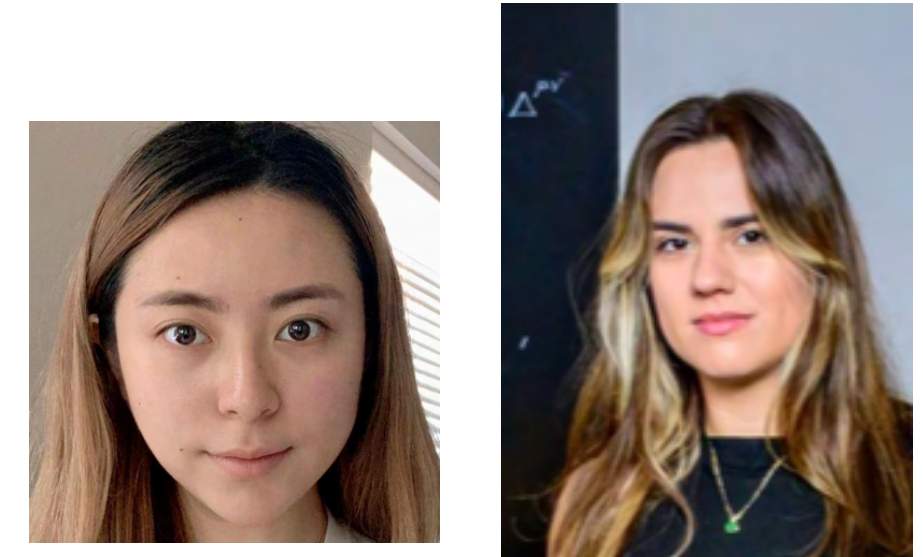
- Strangeness symmetry energy expansion
- Redo with hydrodynamics
- Properly include quarks
- Bayesian analysis BOTH with NS and HIC + symmetry energy coefficients
- Momentum dependence potentials
- A lot of unused HIC data still...

Proposal: isospin HIC scan

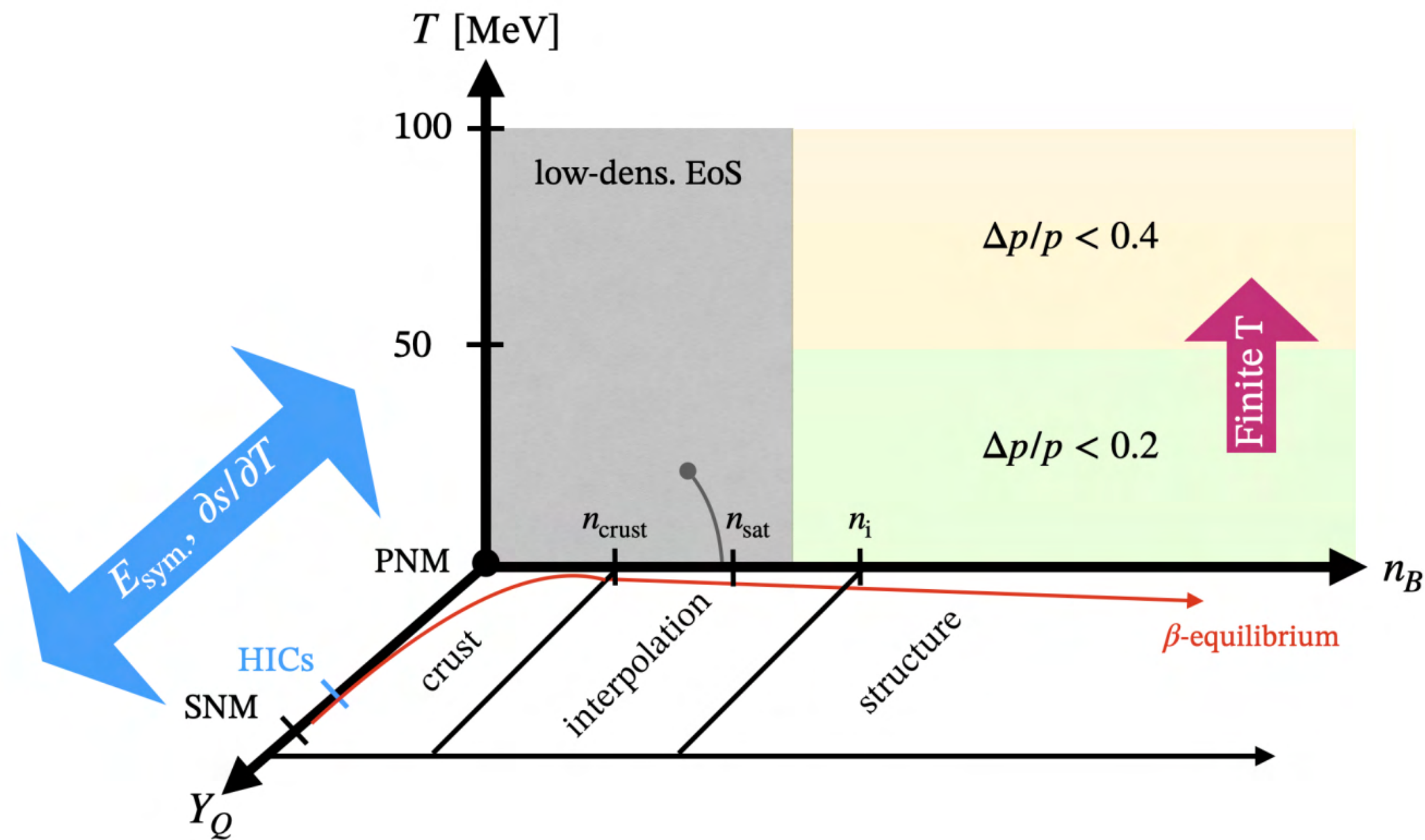
New finite T expansion

Mroczek, Yao et al, 2404.01658 [astro-ph.HE]

1D EOS $\rightarrow$ 3D EOS

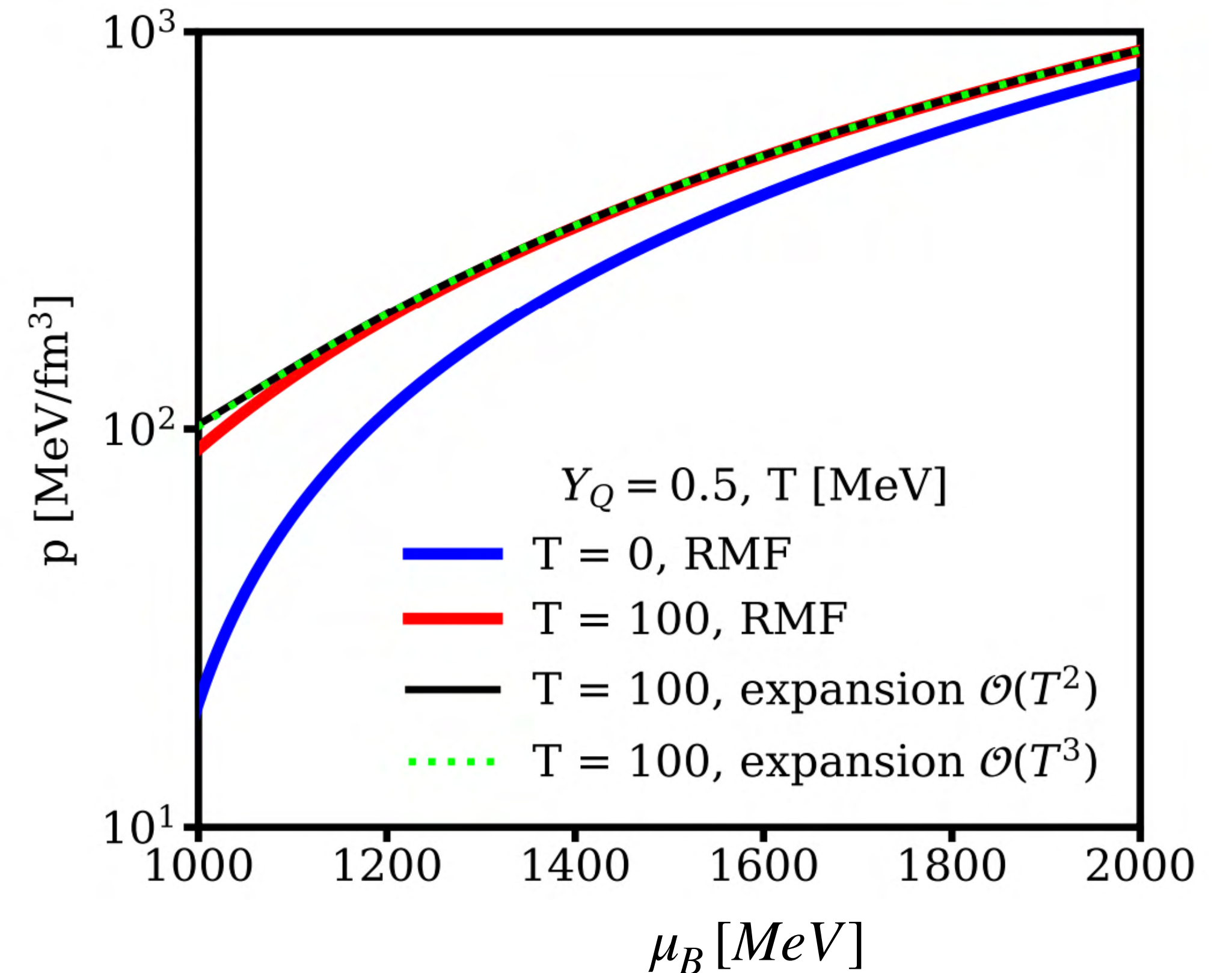


+Liam, Alex, Elias
(in this room!)



Expand from a cold, neutron star EOS

$$p(T, \vec{\mu}) = p_{T=0} + \frac{1}{2} \frac{\partial s}{\partial T} \bigg|_{T=0, \vec{\mu}} T^2 + \mathcal{O}(T^3)$$



Yields across $\frac{Z}{A}$ and the neutron star equation of state

Repurposing isobars

Nana, Salinas san Martin, JNH 2411.03705 [nucl-th]

Need information about how $\partial s/\partial T$ varies with Y_Q

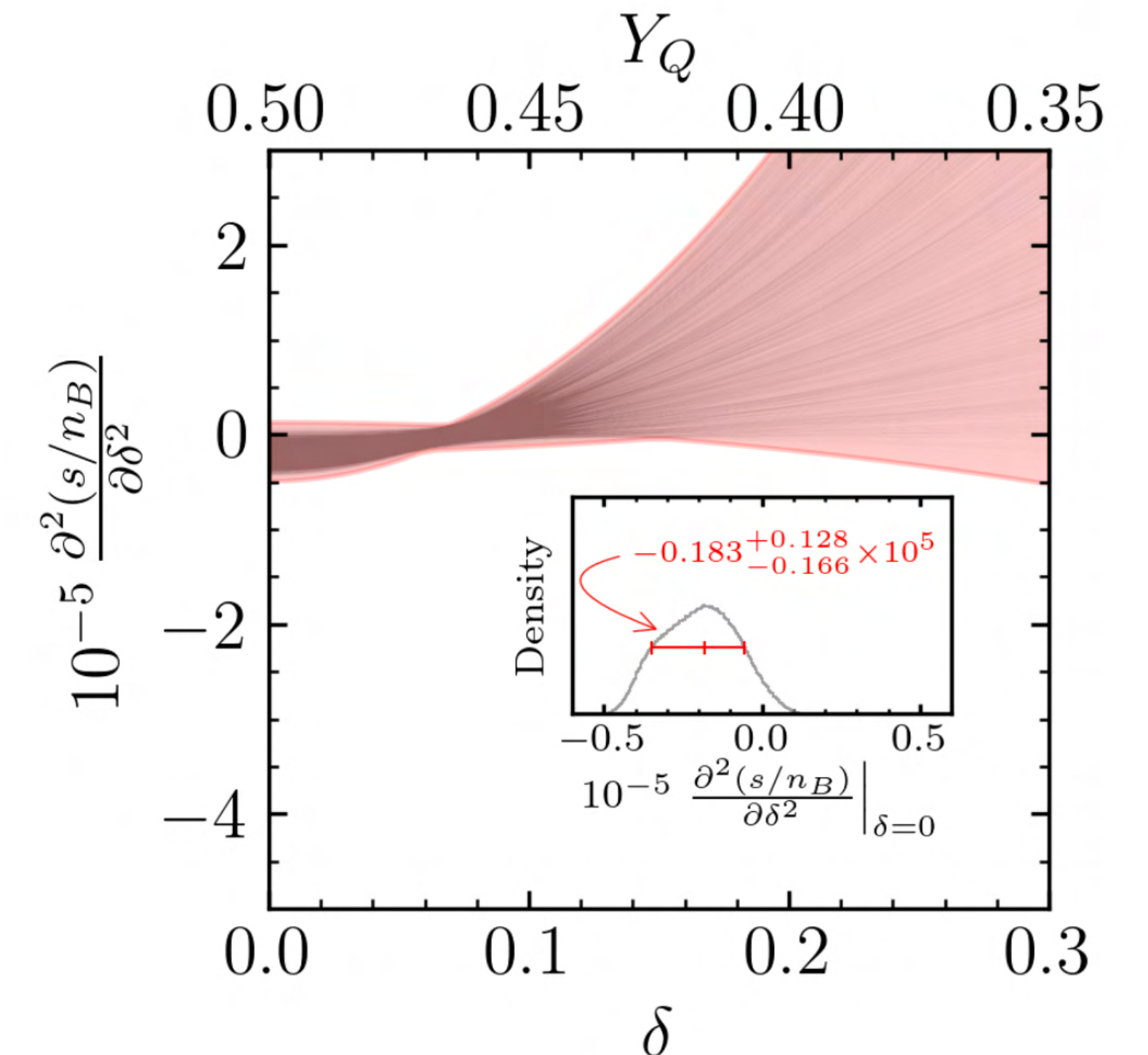
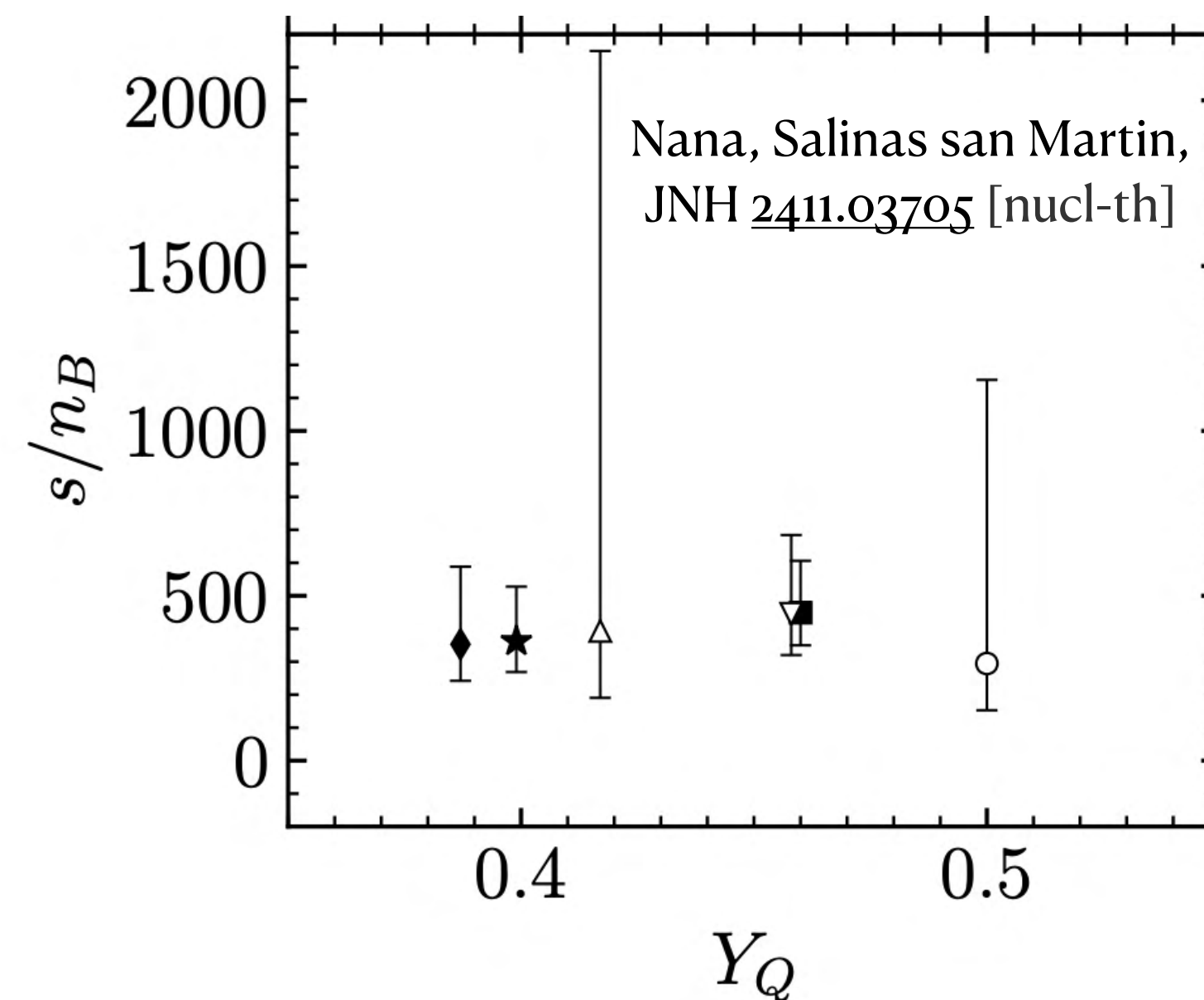
$$\left. \frac{\partial s/n_B(T, n_B, Y_Q)}{\partial T} \right|_{T=0} = \frac{1}{n_B} \left. \frac{\partial s_{\text{HIC}}(T, n_B, Y_Q)}{\partial T} \right|_{T=\delta_{\text{HIC}}=0} + \frac{1}{2} \left(1 - \frac{Y_Q}{Y_Q^{\text{HIC}}} \right)^2 \left. \frac{\partial^3 (s/n_B)_{\text{HIC},2}(T, n_B, \delta_{\text{HIC}})}{\partial T \partial \delta_{\text{HIC}}^2} \right|_{T=\delta_{\text{HIC}}=0}$$

How do we get this information?

Thermal models from heavy-ion collisions and varying Y_Q !

Proof-of-principle from RHIC

$\sqrt{s} = 200 \text{ GeV}$



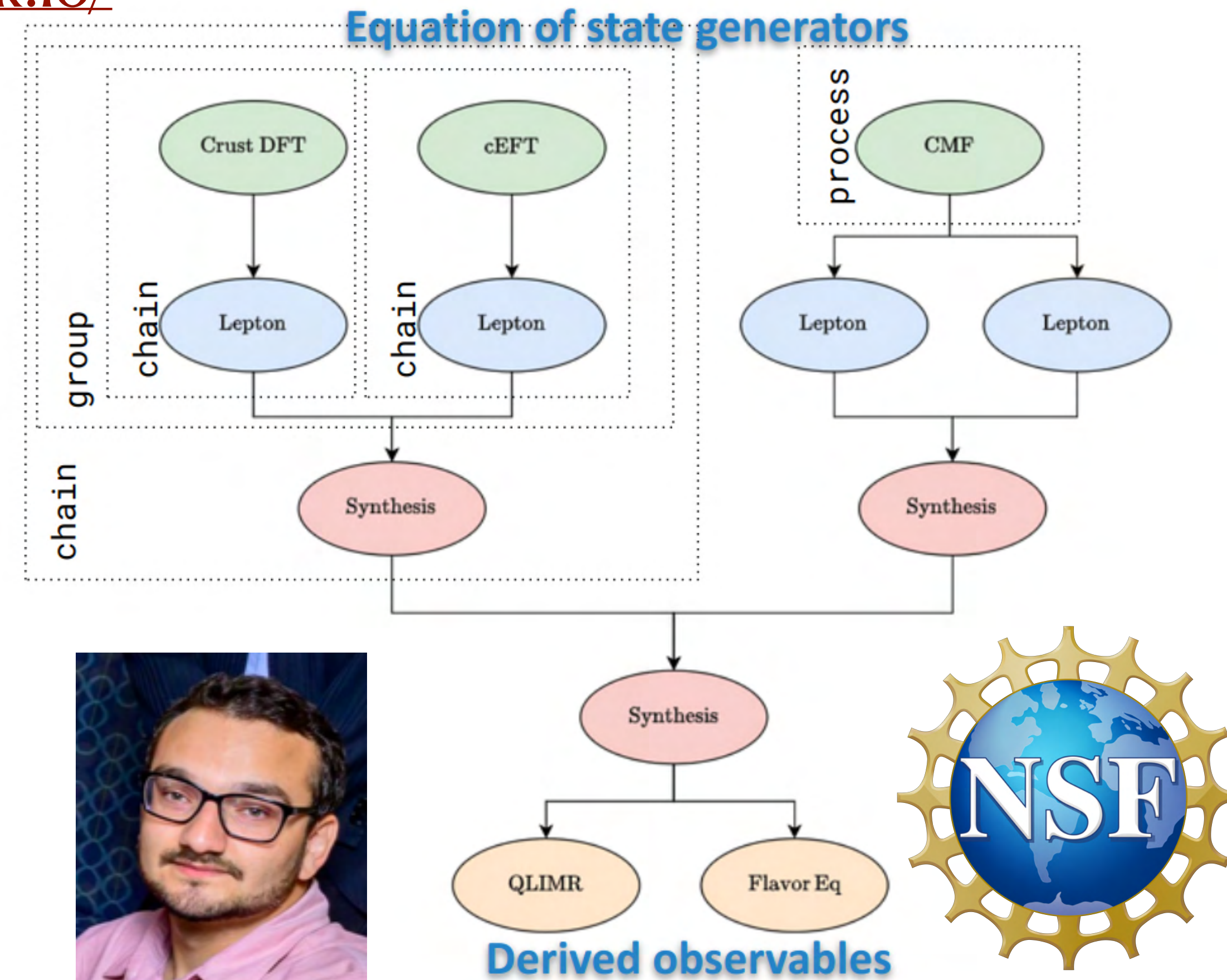
Open-source tools for more cross-disciplinary connections!



<https://ce.musesframework.io/>

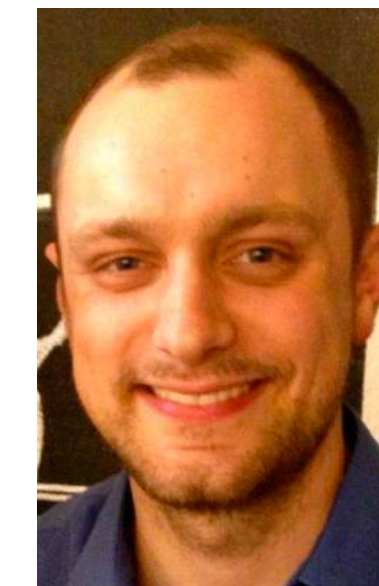
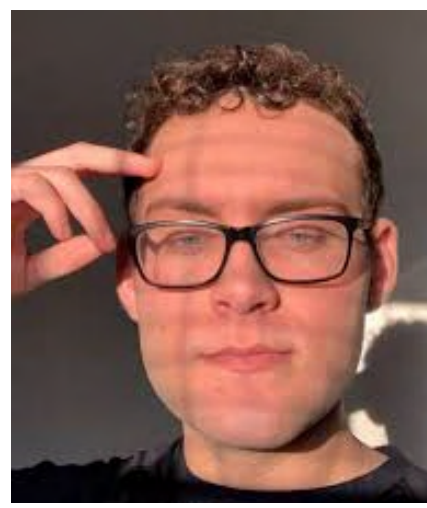
- 7 available equation of state (EOS) modules
- 6 new EOS modules in next year+2D synthesis
- 3 available observable modules (2+ coming soon)
- Both heavy-ion and neutron star EOS available
- β -release out and available! Possible to run crust to core of a neutron star+calculate mass, radius, tidal deformability etc
- Looking for new collaborators!

Software across the QCD phase diagram



Later releases will connect heavy-ion and neutron star EOS across the entire phase diagram!

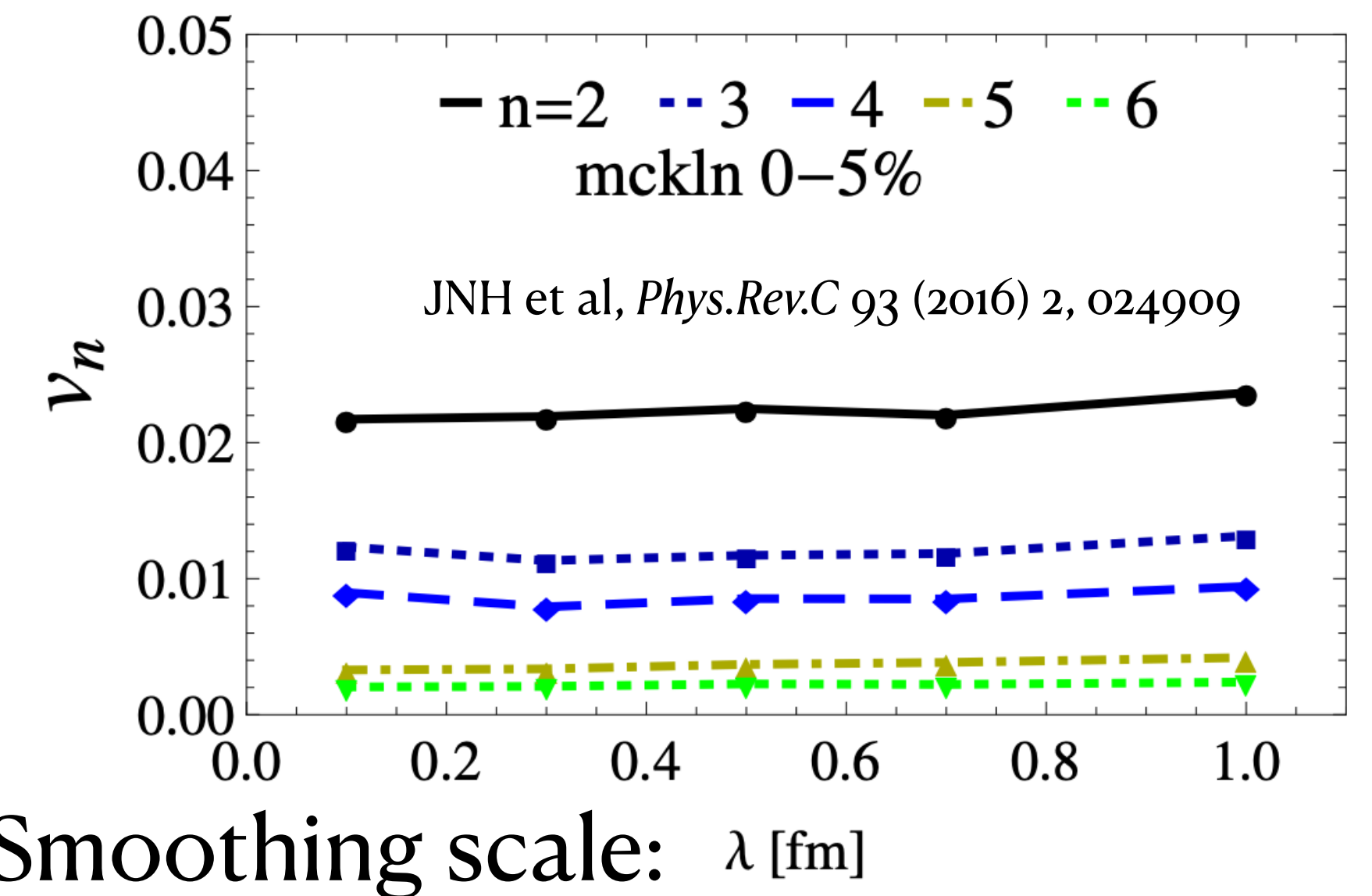
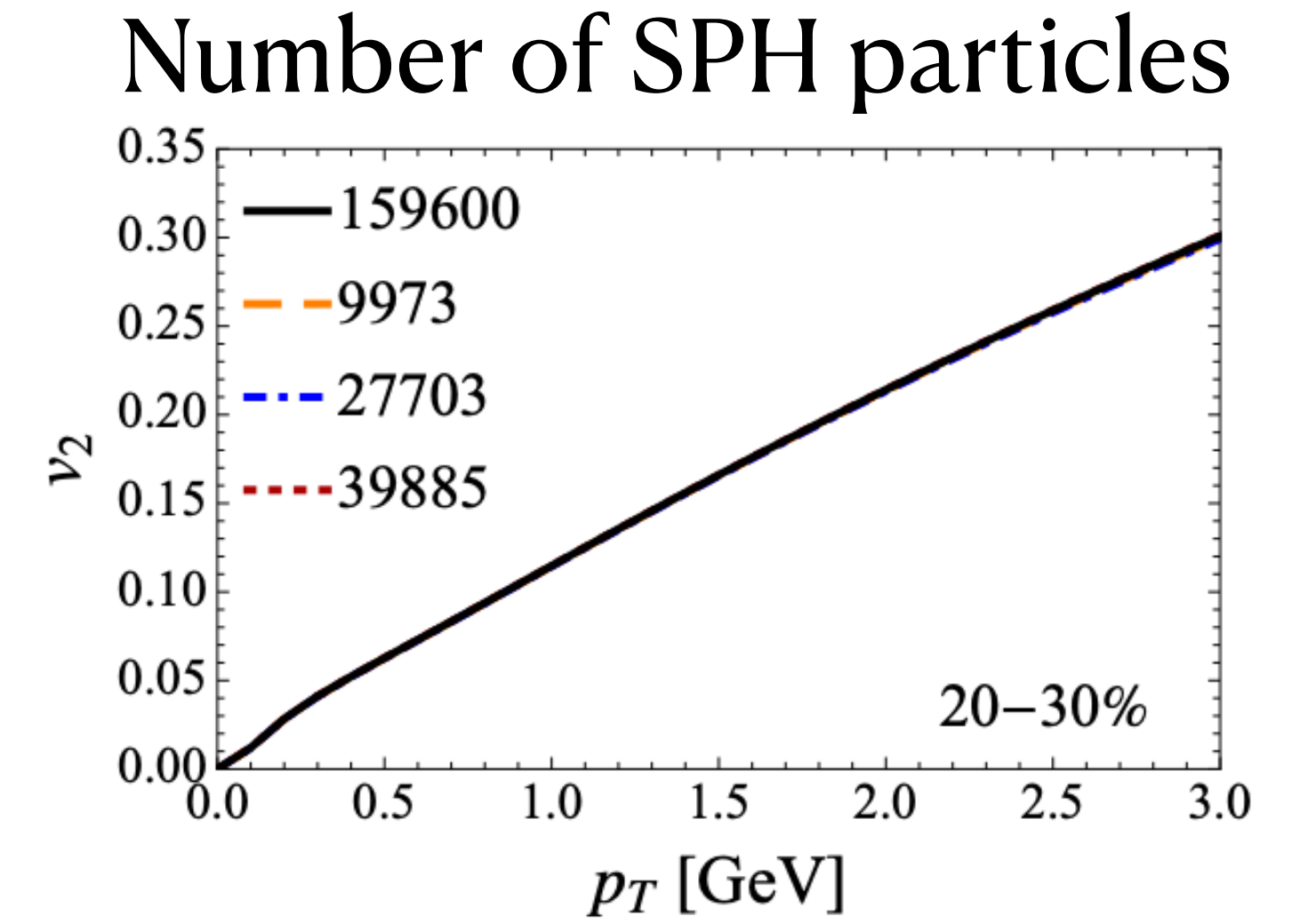
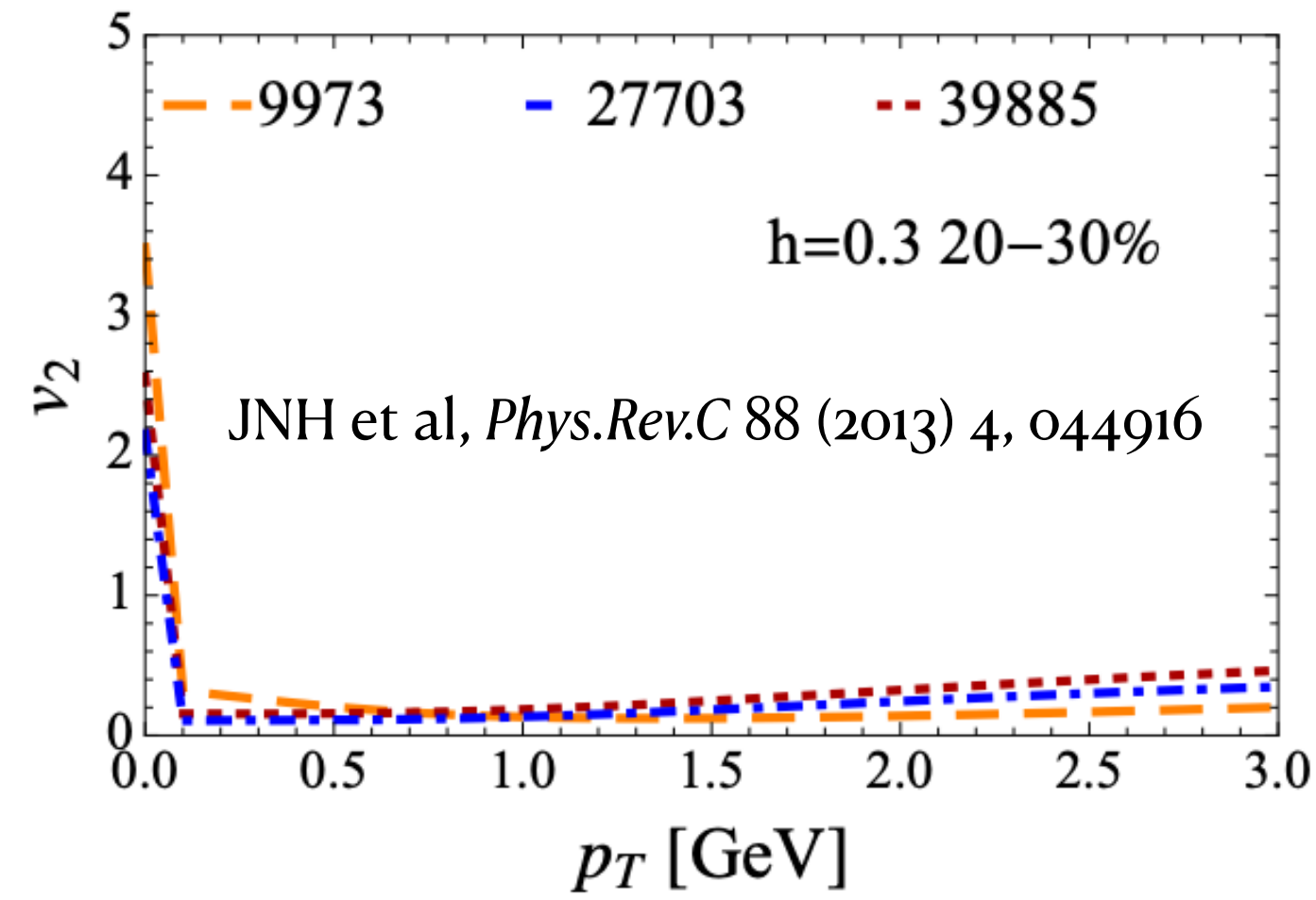
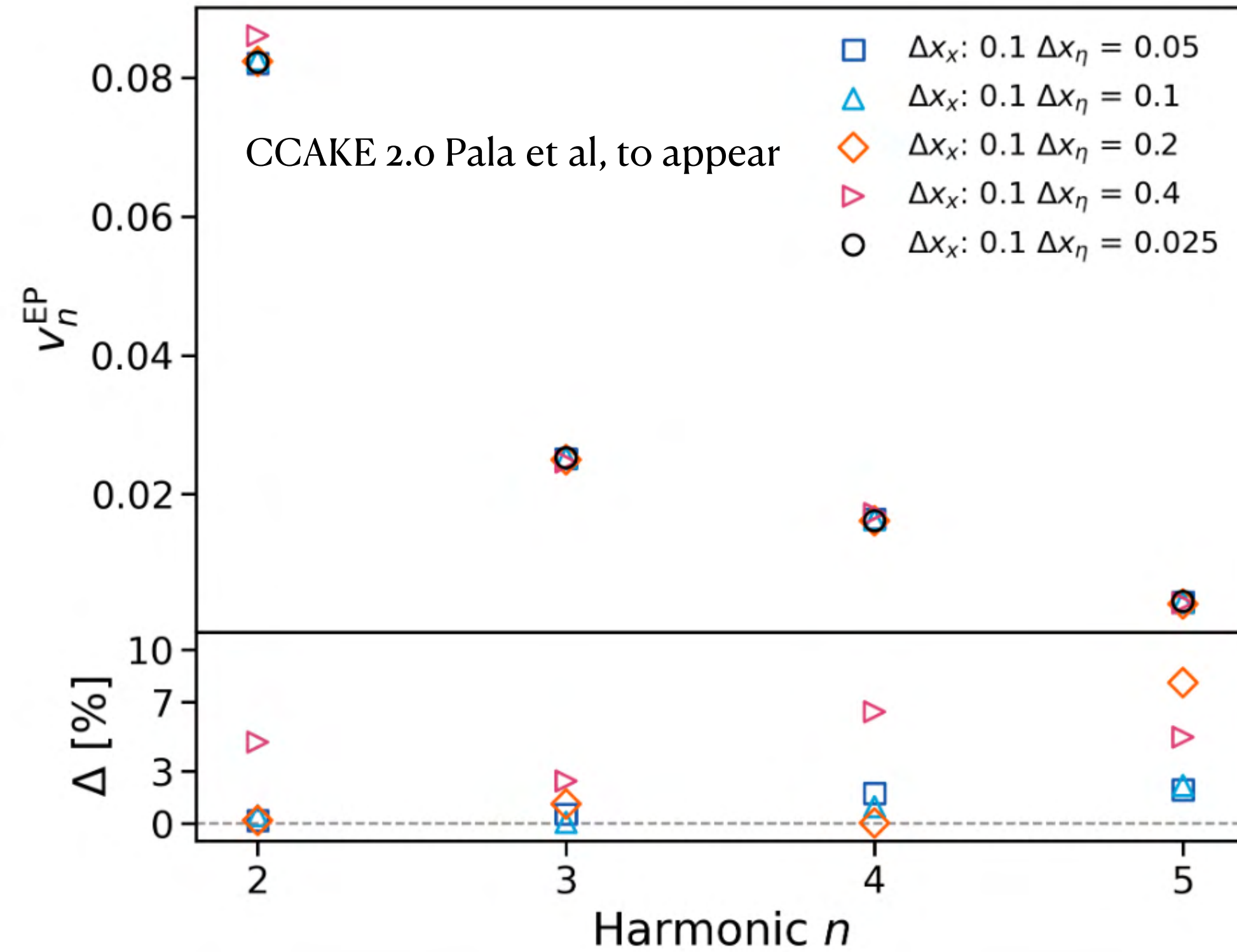
Thank you to my group + collaborators!



Summary and Outlook

- We do NOT know the large n_B EOS probed in heavy-ion collisions, but there's hope
- We DO understand high-energies ($n_B \sim 0$) HIC extremely well: precision studies, Bayesian analyses, predictions possible
- Large n_B has many moving parts (EOS + transport coefficients), slowly working our way towards better models.
 - Biggest limitations: time, computational resources, personnel, \$\$
 - Joint Bayesian analyses in HIC and NS desperately needed, but it's **really** hard
- Need to understand HIC (assumptions, limitations) before applying to astrophysical data
- CMB at FAIR in Germany is being built to study the regime between heavy-ions and neutron stars $\rightarrow$ scans of Z/A
- Possibilities at other fixed target experiments? Here at CERN? AGS? FRIB400?

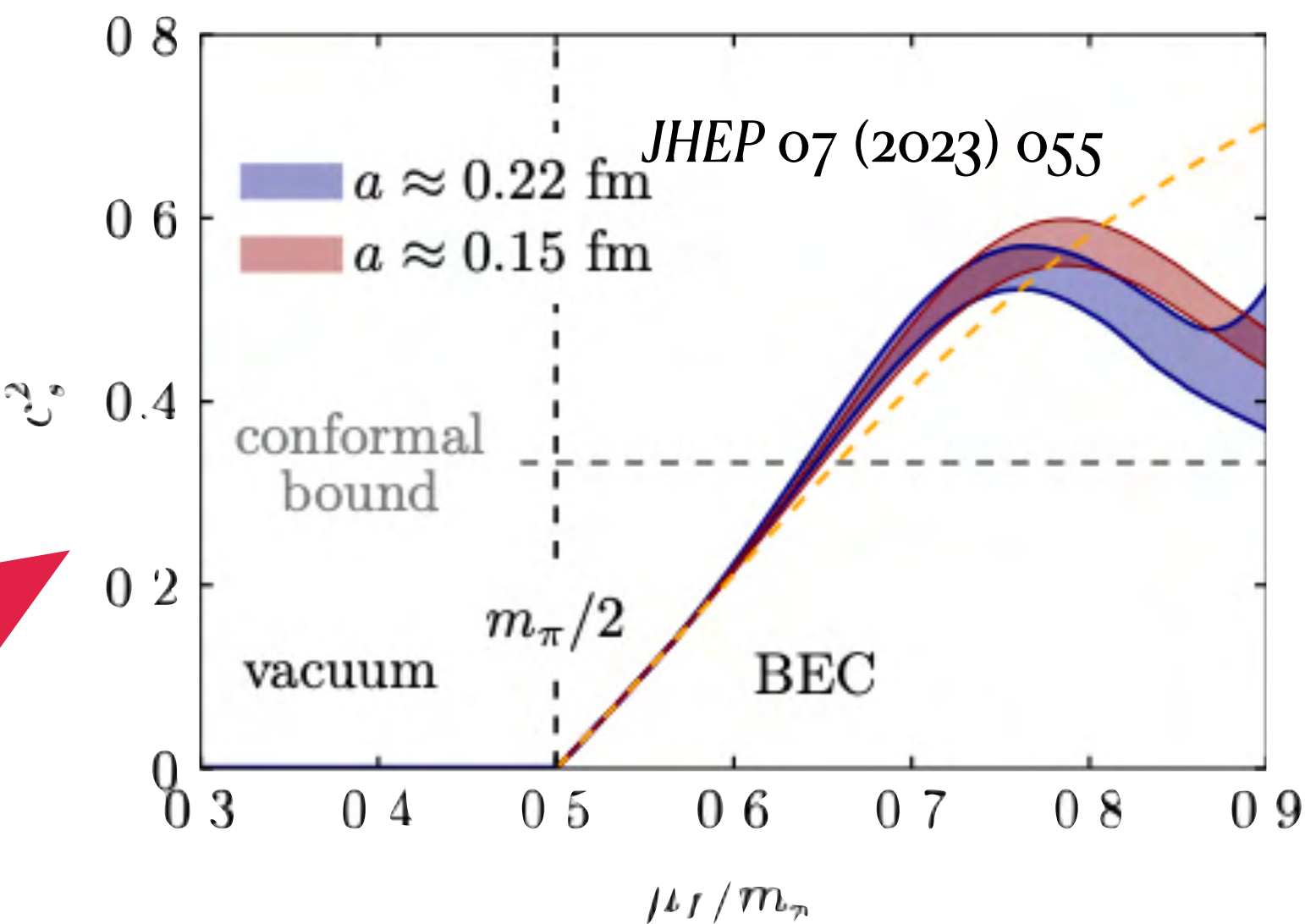
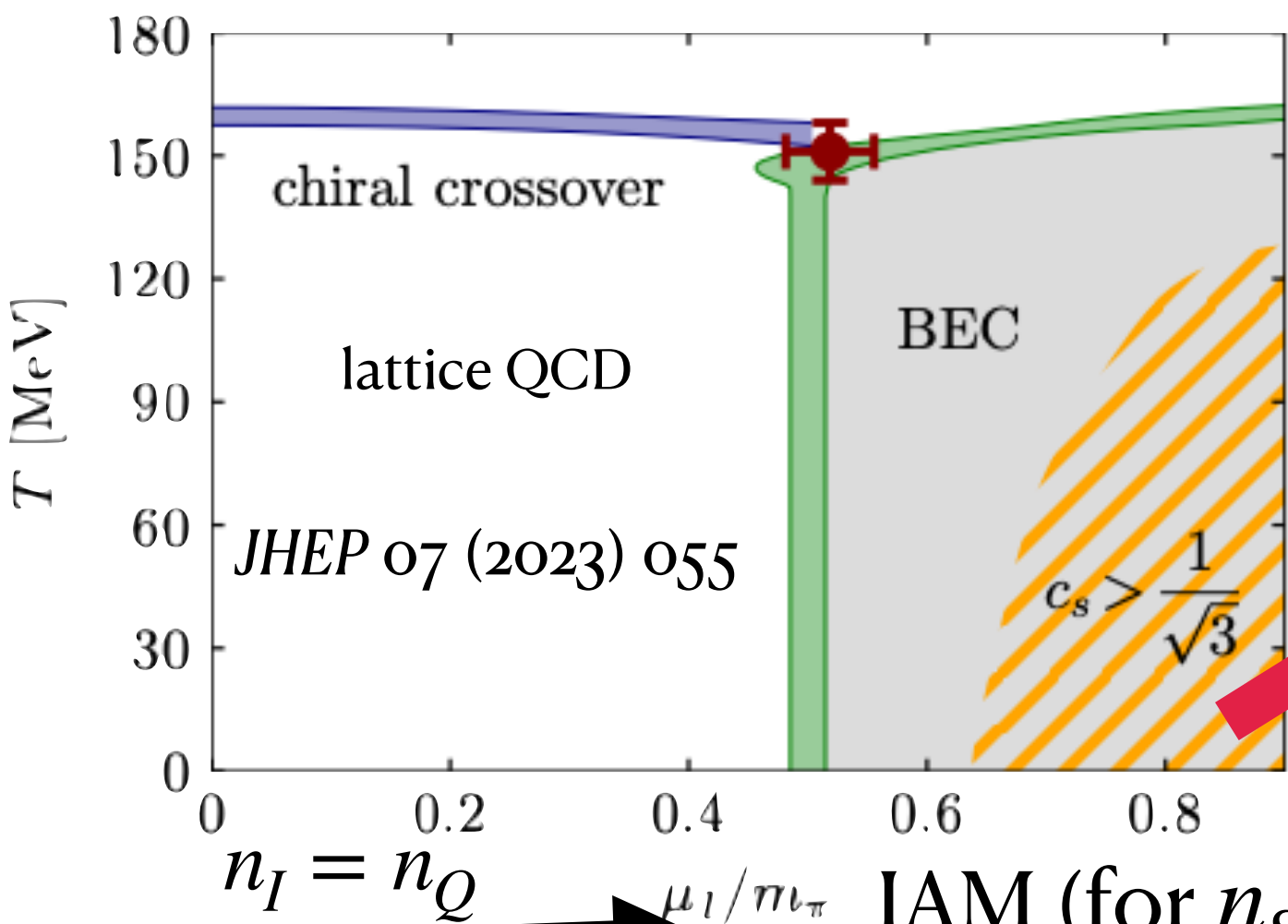
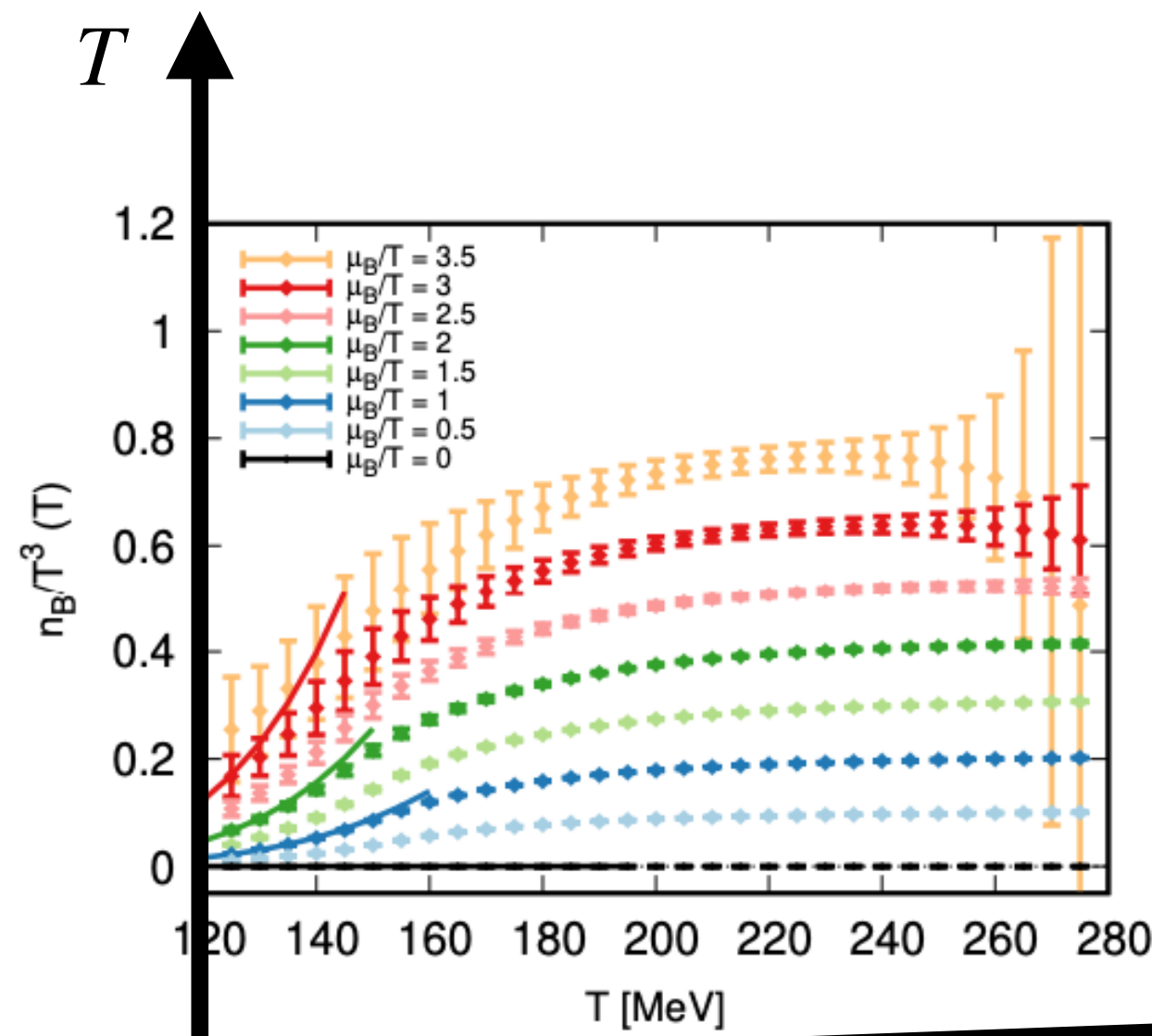
Robustness to grid-size changes, smoothing scale h



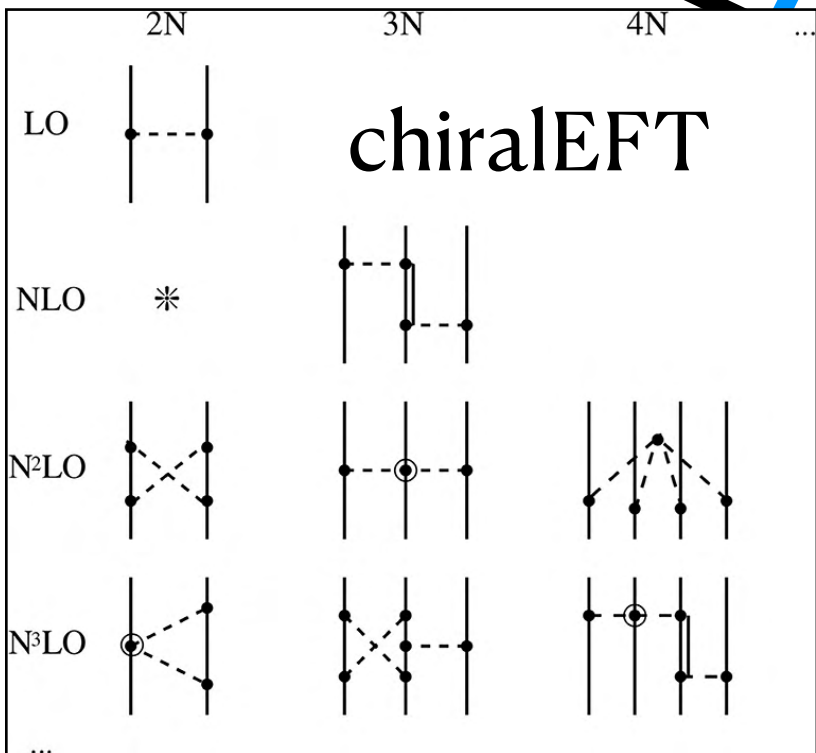
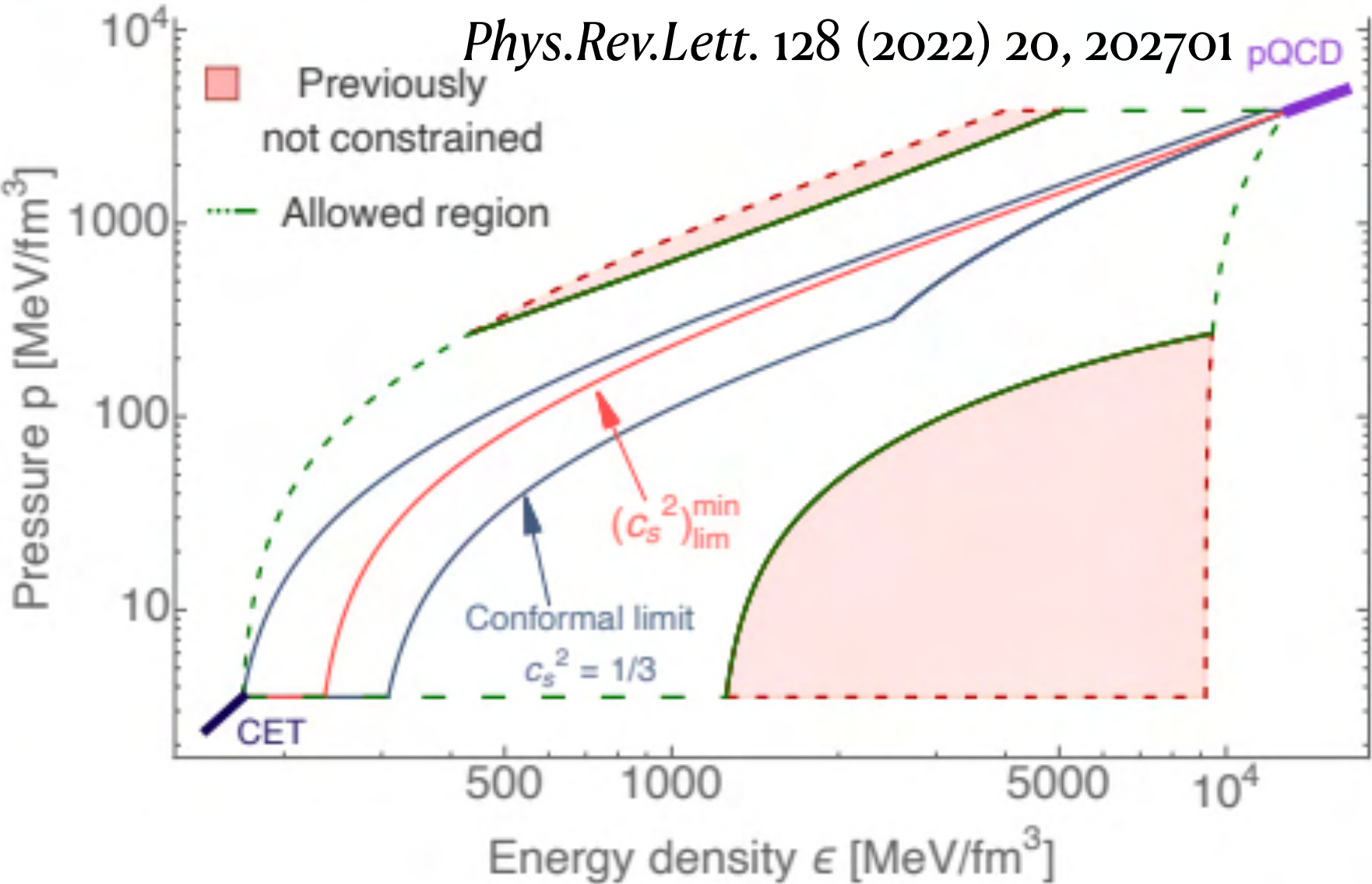
First principle QCD constraints

Equation of State (EOS)

Resumed lattice QCD Lattice *Phys.Rev.Lett.* 126 (2021) 23, 232001; *Phys.Rev.D* 105 (2022) 11, 114504
Open-source code *Phys.Rev.D* 109 (2024) 9, 094046



Perturbative QCD $n_B \gtrsim 40 n_{sat}$



Review: *Phys.Rept.* 503 (2011) 1-75
+ many, many works

n_B SNM $Y_Q = 0.5$

n_B PNM $Y_Q = 0$

Filling gaps in our knowledge

Expansions: symmetry energy, finite T or μ , etc

$$\text{Lattice QCD } \mu_B = 0, \text{ then } p(T, \hat{\mu} = \vec{\mu}/T) = T^4 \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \hat{\mu}_B^i \hat{\mu}_S^j \hat{\mu}_Q^k$$

$$\text{Cold neutron stars } T = 0, \text{ then } p(T, \vec{\mu}) = p_{T=0} + \frac{1}{2} \frac{\partial s}{\partial T} \Big|_{T=0, \vec{\mu}} T^2 + \mathcal{O}(T^3) \quad 2404.01658 [\text{astro-ph.HE}]$$

$$\text{Connect heavy-ions to neutron stars: } \frac{E_{ANM}}{N_B} = \frac{E_{SNM}}{N_B} + E_{sym} \delta^2 + \mathcal{O}(\delta^4) \quad \& \text{ expansion around } n_{sat}$$

Caveat: breaks down outside of regime of validity, struggles with phase transitions

Effective Models: relativistic mean field, NJL, quarkyonic, etc

Minimalist models based on nuclear parameters

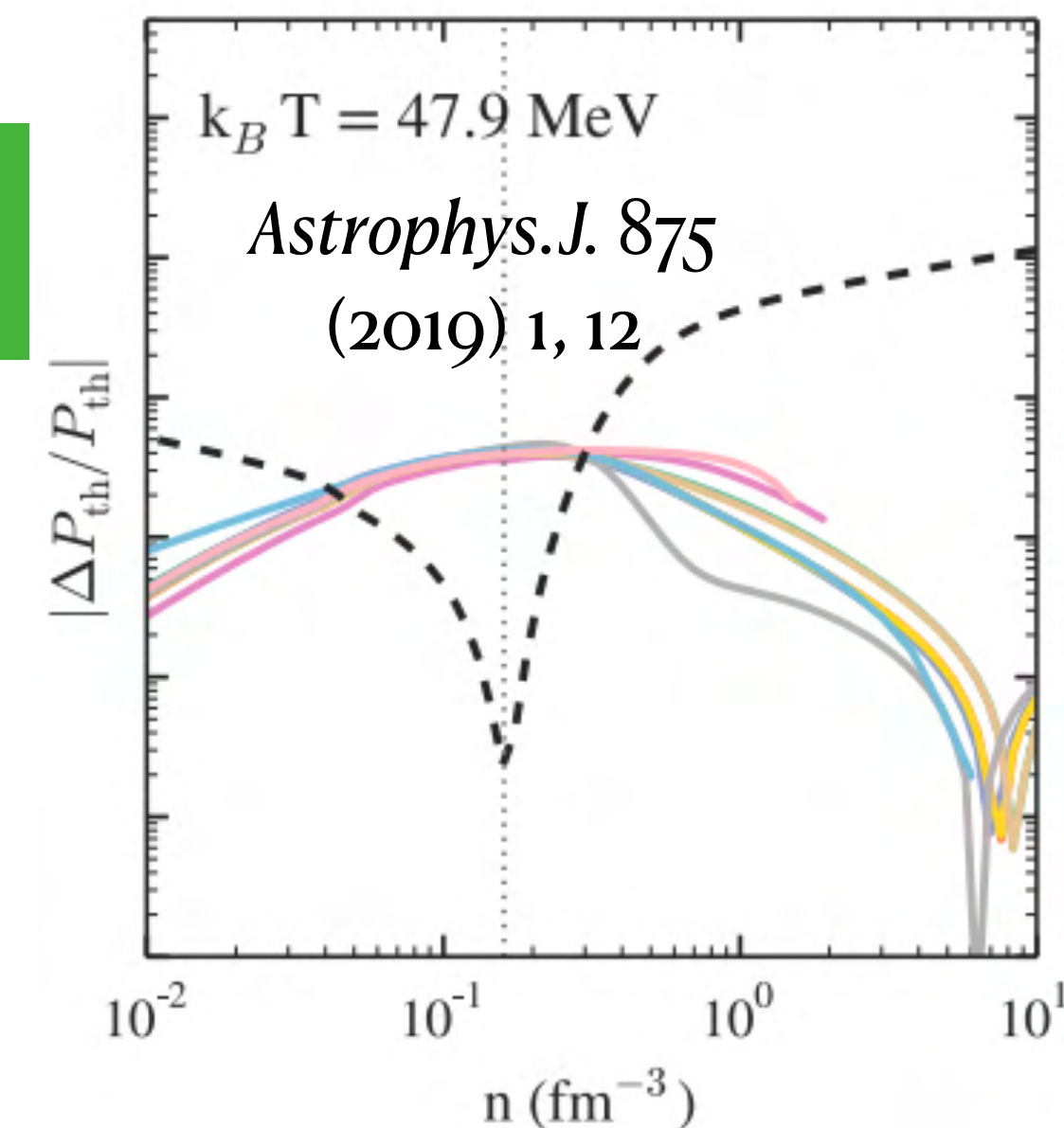
Qualitative features: bump in c_s^2 , phase transitions, ...

Constrain theory vs data; vary free parameters

Caveat: model assumptions and degrees of freedom

Effective Mass approach for neutron, proton, electron matter

Caveat: breaks down outside of regime of validity, no phase transitions, hyperons etc



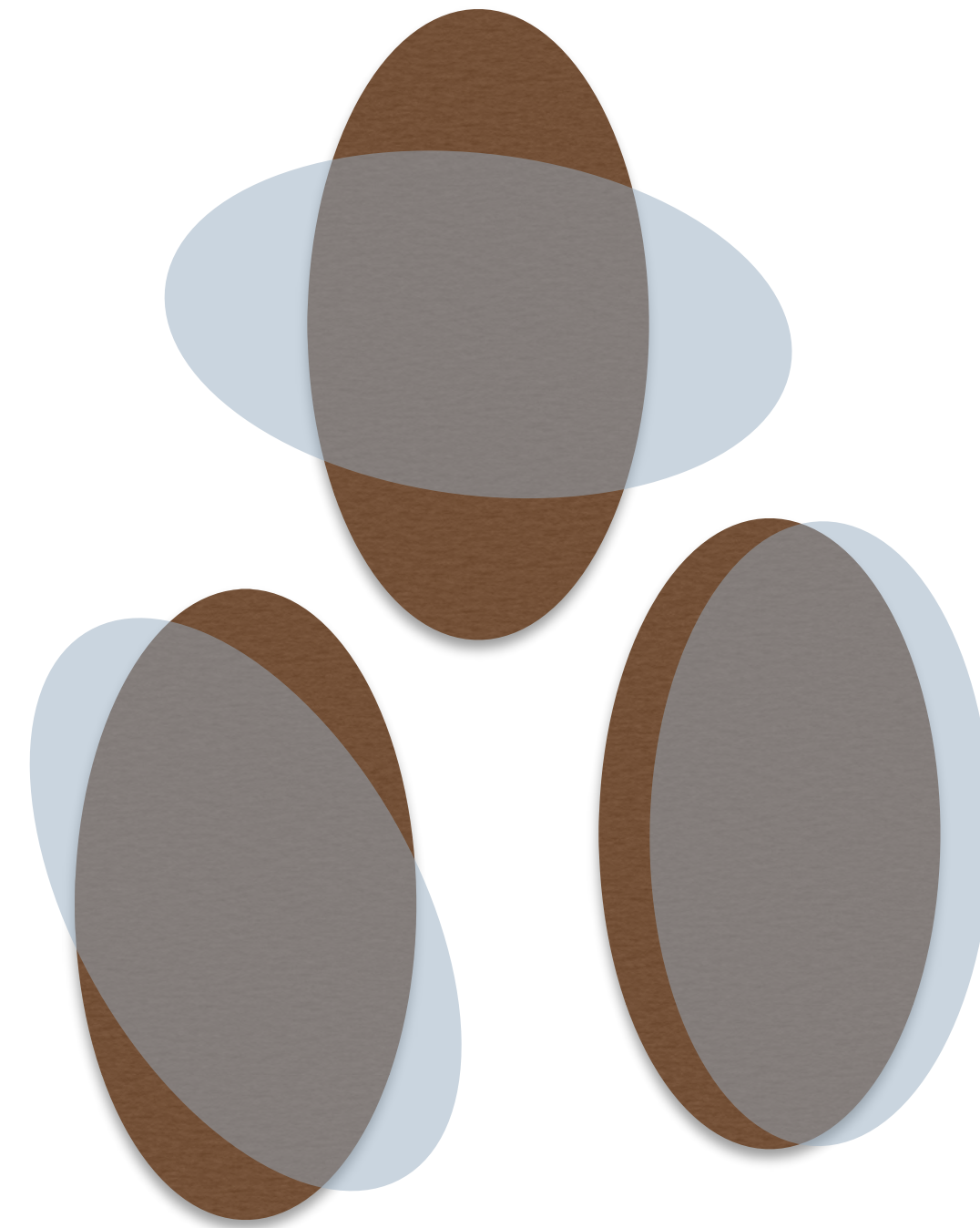
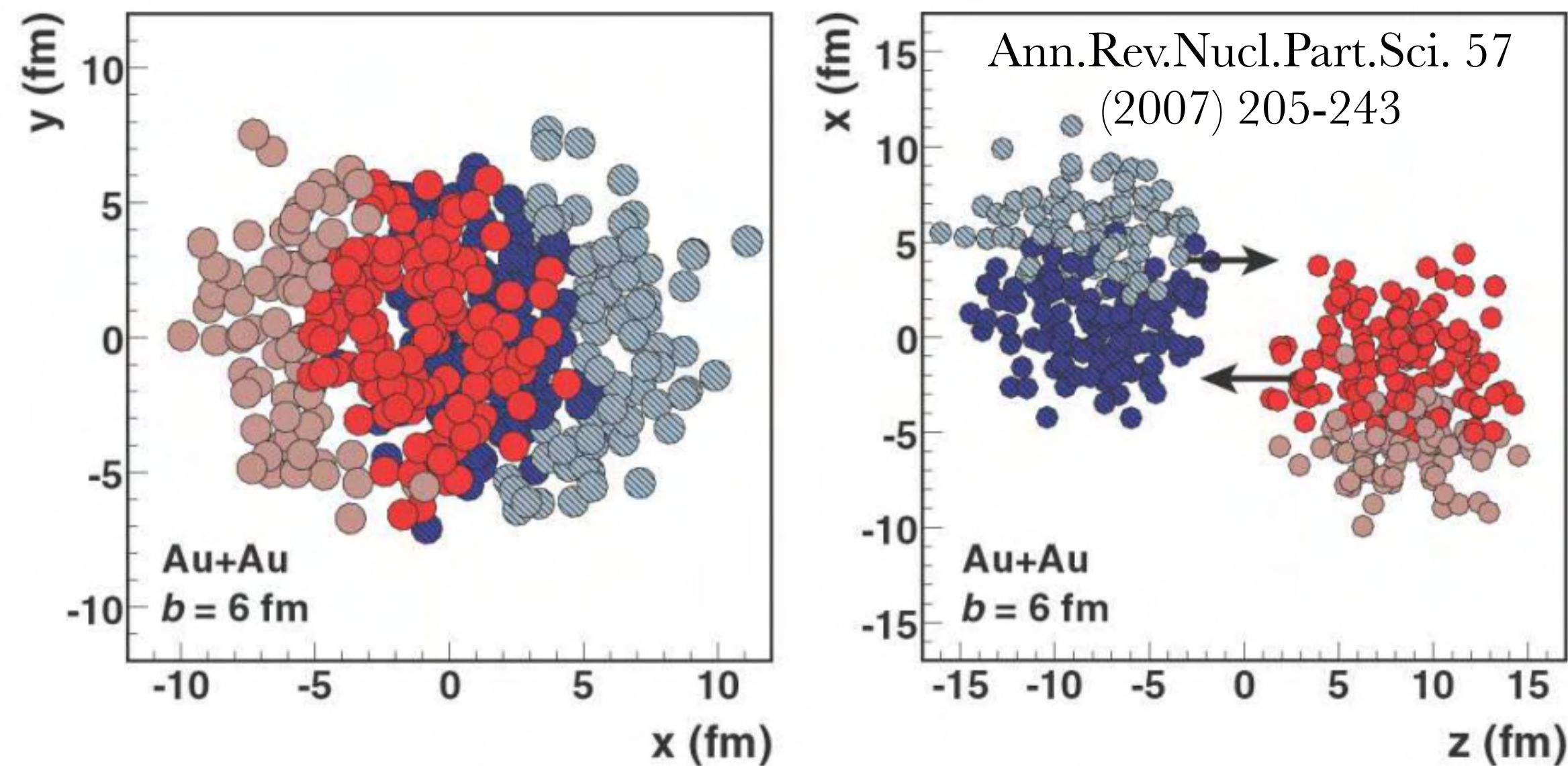
Head on collisions and deformations

Centrality % = 0 for head on collisions

Terminology:

- participants (colliding nucleons)
- spectators (fly off to the detector)

All $b = 0$ impact parameters,
very different shapes

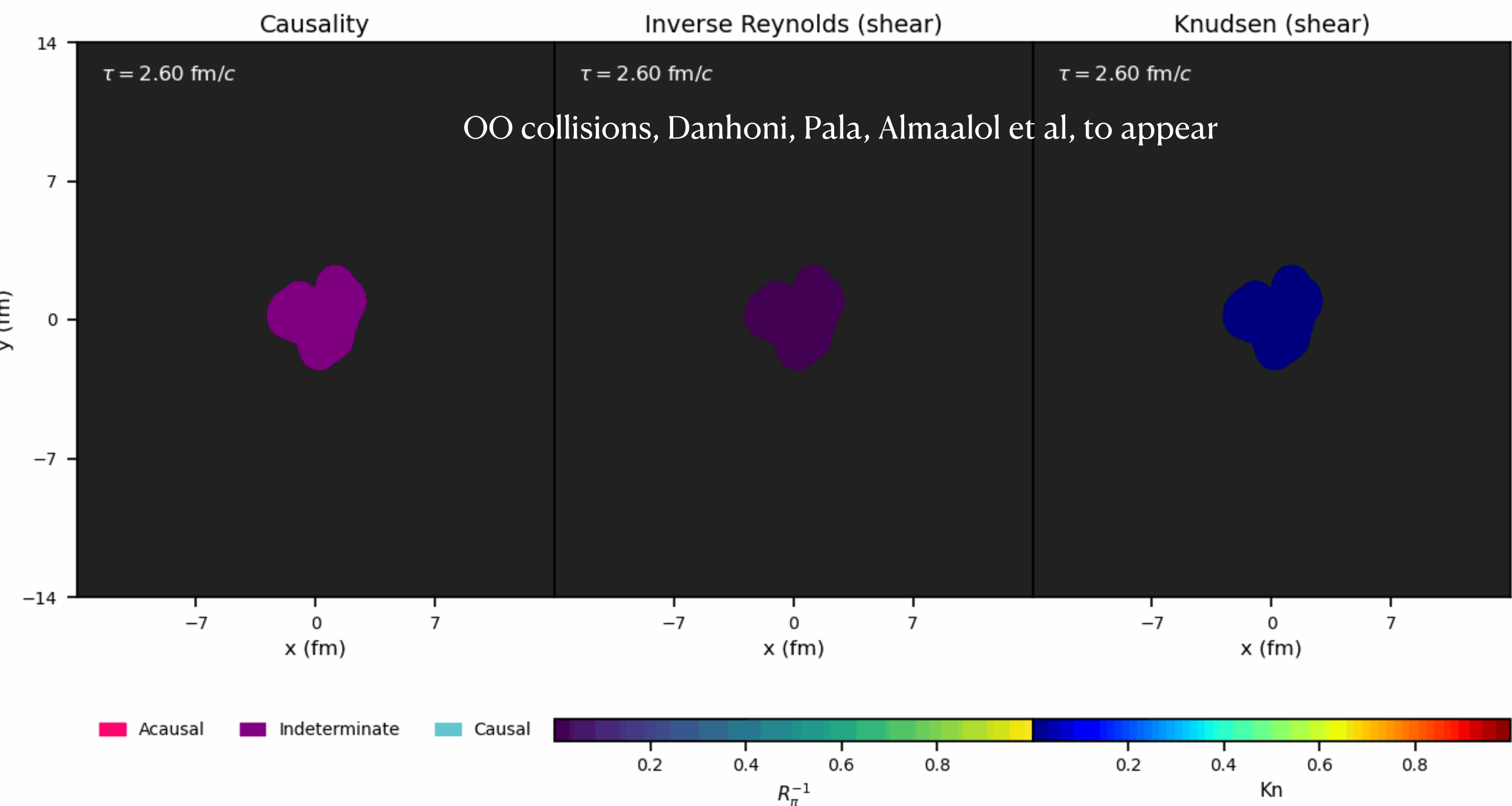


Head-on collisions most sensitive to structure, but $b = 0$ does not necessarily mean ultra central!

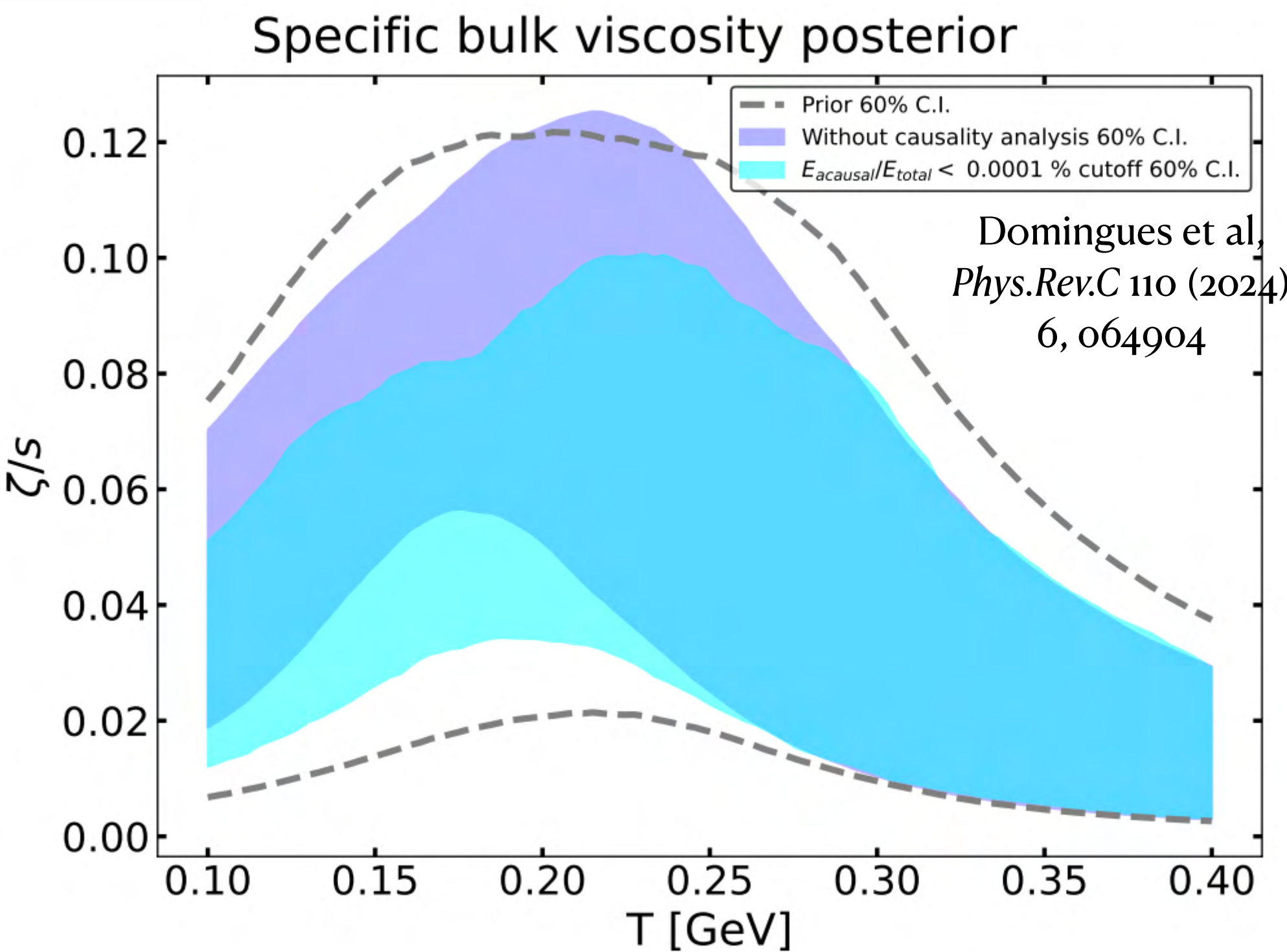
Causality and Stability: math-physics collaborations

Mathematical constraints from nonlinear causality and stability

Derivation: Bemfica et al, *Phys.Rev.Lett.* 126 (2021) 22, 222301 ;



Effects on transport coefficient extraction



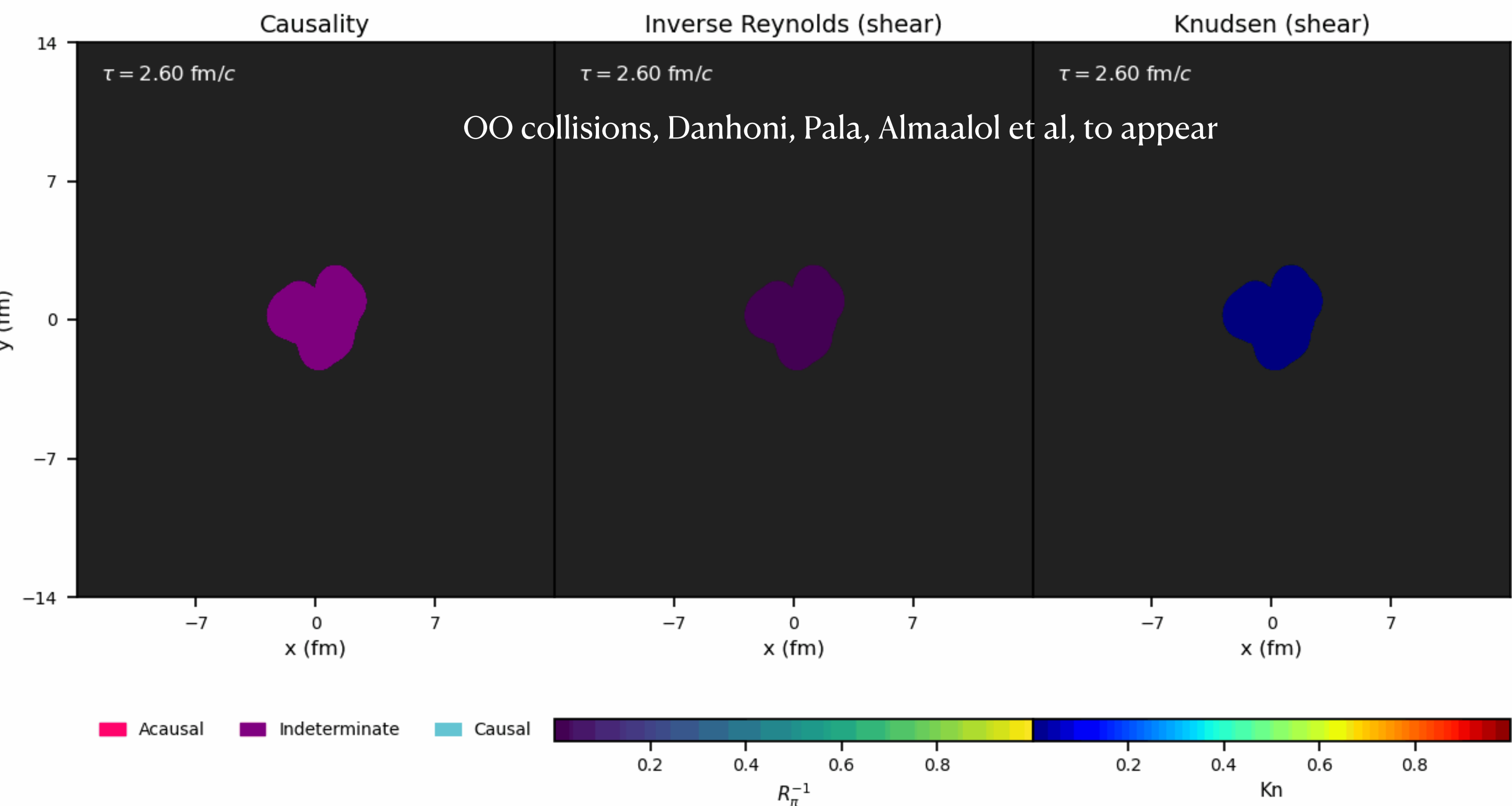
BUT, this goes WAY beyond just heavy-ion collisions!

Other related works in HIC: Plumberg et al, *Phys.Rev.C* 105 (2022) 6, L061901; Chiu et al, *Phys.Rev.C* 103 (2021) 6, 064901, Gavassino QM25, Cordeiro QM25

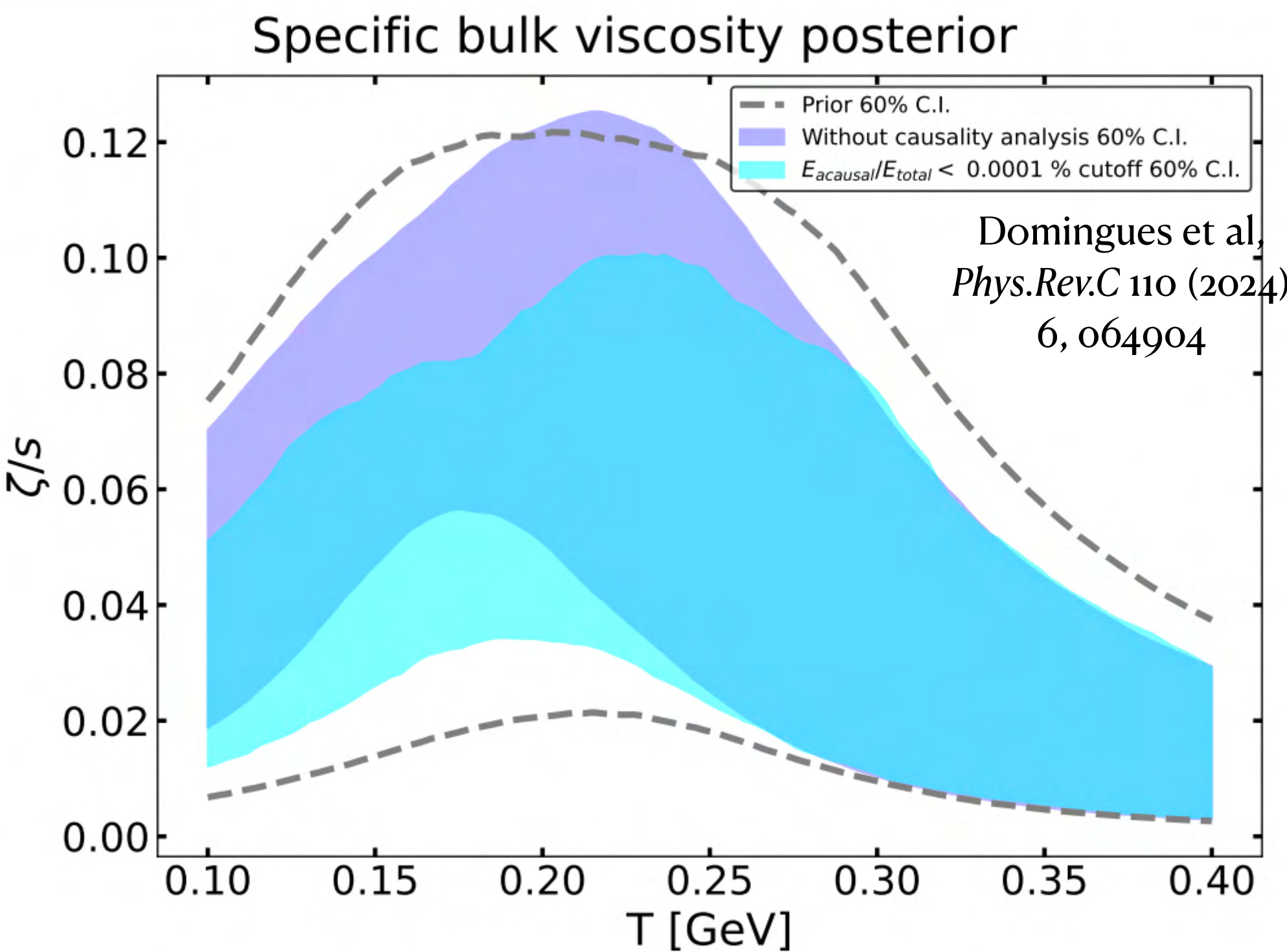
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