Differential observables for $0\nu\beta\beta$ decay to resolve mechanism degeneracies and interpret NMEs

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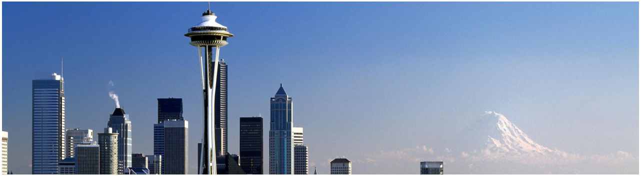
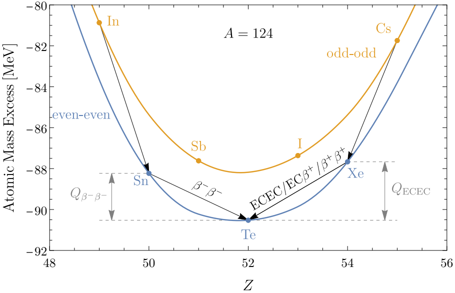


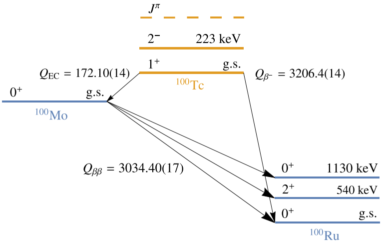
Outlook

Double-beta decay

Two-neutrino DBD is the rarest observed decay with $T_{1/2}^{2\nu} > 10^{18}$ yr.

$$2\nu\beta^-\beta^- : (A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$





F. Šimkovic, O. Nițescu and J. D. Vergados, Double-beta decay, Book Chapter (2026)

There is an increasing interest in the process due to the possible observation of $0\nu\beta^-\beta^-$ decay.

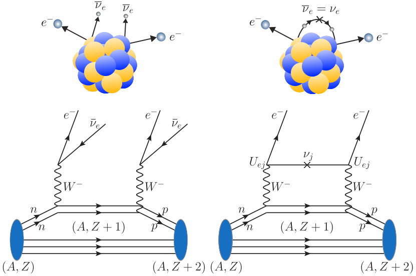
$$0\nu\beta^-\beta^- : (A,Z) \rightarrow (A,Z+2) + 2e^-$$

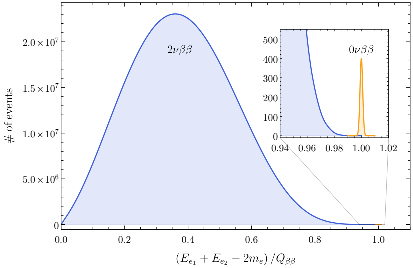
$\nu_e = \overline{\nu}_e$ Majorana particle

GERDA, CUPID-0, CUPID-Mo, NEMO, CUORE, EXO-200 , KamLAND-ZEN
nEXO, LEGEND-200(×5), CUPID, SuperNEMO, CROSS, AMoRE, SNO+, PandaX
COBRA, CROSS, ACCESS, MONUMENT, BINGO, ...

Finding the needle in the haystack ($2\nu\beta\beta$ vs $0\nu\beta\beta$)

F. Šimkovic, O. Nițescu and J. D. Vergados, Double-beta decay, Book Chapter (2026)



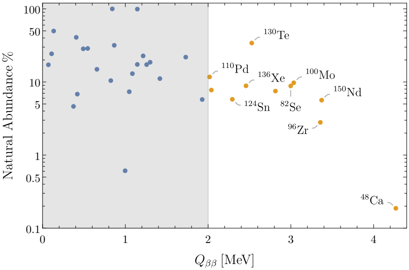


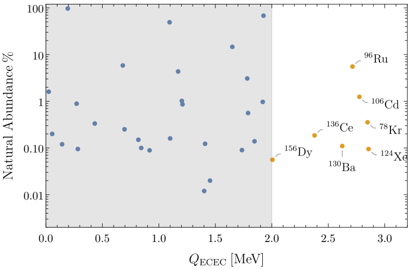
$$\left[T_{1/2}^{2\nu}\right]^{-1} = G^{2\nu} g_A^4 \left|M^{2\nu}\right|^2$$

$$\left[T_{1/2}^{0\nu}\right]^{-1} = \sum_i G_i^{0\nu} g_A^4 \left|M_i^{0\nu}\right|^2 \frac{\eta_i^2}{m_e^2}$$

Finding the needle in the haystack ($2\nu\beta\beta$ vs $0\nu\beta\beta$)

F. Šimkovic, O. Nițescu and J. D. Vergados, Double-beta decay, Book Chapter (2026)



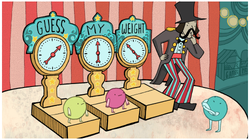


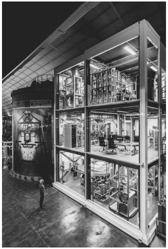
Nucleus	Experiment	Mass [kg]	$T^{0\nu}_{1/2}$ [yr]	$m_{\beta\beta}$ [meV]	Status
^{76}Ge	LEGEND	200	$\sim 10^{27}$	$\sim 34 - 80$	Current
		1000	1.7×10^{28}	$8.5 - 19.4$	R&D
^{100}Mo	CUPID	250	1.4×10^{27}	$10 - 17$	R&D
		1000	9.2×10^{28}	$4.1 - 6.8$	R&D
	AMoRE	100	$\sim (5 - 8) \times 10^{26}$	$13 - 28$	R&D
^{130}Te	SNO+	1300	$21. \times 10^{26}$	$37 - 89$	In progress
		$\sim$ 8000	$\sim 10^{27}$	$\sim 20 - 40$	R&D
^{136}Xe	nEXO	5000	1.35×10^{28}	$4.7 - 20.3$	R&D
	KamLAND2-Zen	1000	$\sim 2 \times 10^{27}$	$12 - 52$	R&D

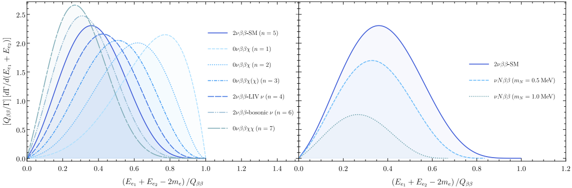
Why precise predictions for DBD?

The theoretical predictions are required in:

- neutrino mass scale determination from $0\nu\beta\beta$ decay and from (ultra-)low Q value β and EC transitions (KATRIN, Project-8, ECHo, HOLMES, PTOLEMY, NuMECS)
- dark matter searches as they are unavoidable background sources in liquid xenon experiments (XENON, LUX-ZEPLIN, PandaX, XMASS, DarkSide)
- constraining BSM parameters from the shape of β and $\beta\beta$ spectra (most DBD experiments)







DHFS self-consistent method

The relativistic wave function of the bound electron

$$\psi_{n,\kappa}(\boldsymbol{r}) = \sum_{\kappa m} \begin{pmatrix} g_{n,\kappa}(r)\Omega_{\kappa,m}(\hat{\boldsymbol{r}}) \\ i f_{n,\kappa}(r)\Omega_{-\kappa,m}(\hat{\boldsymbol{r}}) \end{pmatrix},$$

where the large- and small-component radial functions obey the radial Dirac equation

$$\left(\frac{d}{dr} + \frac{\kappa+1}{r}\right) g_{n\kappa} - (E_{n\kappa} - V(r) + m_e) f_{n\kappa} = 0,$$

$$\left(\frac{d}{dr} - \frac{\kappa-1}{r}\right) f_{n\kappa} + (E_{n\kappa} - V(r) - m_e) g_{n\kappa} = 0.$$

Bound states ($E_e < m_e$): discrete energy levels (n, κ, $t_{n\kappa}$, $E_{n\kappa} = m_e - |t_{n\kappa}|$),

$\langle \psi_{n\kappa} | \psi_{n'\kappa'} \rangle = \delta_{nn'}\delta_{\kappa\kappa'}$ and

$$V(r) \equiv V_{\mathrm{DHFS}}(r) = V_{\mathrm{nuc}}(r) + V_{\mathrm{el}}(r) + V_{\mathrm{ex}}(r)$$

The nuclear potential is obtained from a Fermi distribution for the proton density

$$\rho_p(r) = \frac{\rho_0}{1 + e^{(r - c_{\mathrm{rms}})/a}}, \qquad V_{\mathrm{nuc}}(r) = -\alpha \int \frac{\rho_p(r')}{|\boldsymbol{r} - \boldsymbol{r}'|} d\boldsymbol{r}'.$$

DHFS self-consistent method

The electronic potential is

$$V_{\text{el}}(r) = \alpha \int \frac{\rho(r')}{|\boldsymbol{r} - \boldsymbol{r}'|} d\boldsymbol{r}'.$$

The local exchange potential with the correct the asymptotic behavior is

$$V_{\text{ex}}(r) = \begin{cases} V_{\text{ex}}^{\text{Slater}}(r) = -\frac{3}{2}\alpha\left(\frac{3}{\pi}\right)^{1/3}[\rho(r)]^{1/3} & r < r_{\text{Latter}}, \\ -\frac{\alpha(Z-N+1)}{r} - V_{\text{nuc}}(r) - V_{\text{el}}(r) & r \geq r_{\text{Latter}}. \end{cases}$$

- start from an approximate electron density (Molière parametrization of the Thomas-Fermi potential)
- solve the Dirac equation
- update the electron density from the obtained wave functions

$$\rho(r) = \sum_{n\kappa} \psi_{n\kappa}^{\dagger}(r)\psi_{n\kappa}(r)$$

- repeat and stop when the atomic binding energy is not changing (ϵ tolerance)

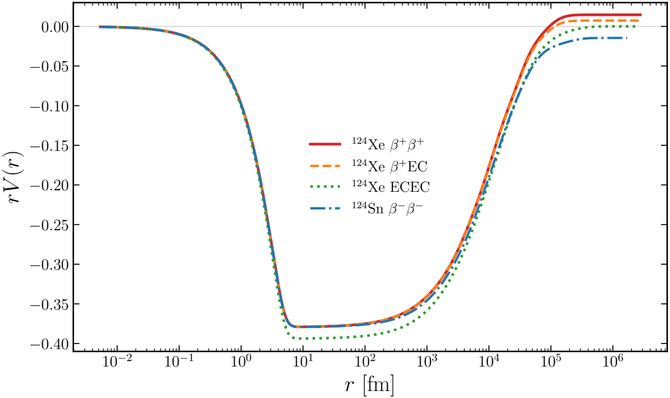
Self-consistent potentials

$$V^{2+}(Z,r) = V_{\mathrm{nuc}}(r) + V_{\mathrm{el}}^{2+}(Z,r) + V_{\mathrm{ex}}^{2+}(Z,r), \qquad (\beta^-\beta^-),$$

$$-V^{2-}(Z,r) = V_{\mathrm{nuc}}(r) + 2V_{\mathrm{el}}(Z,r) - V_{\mathrm{el}}^{2+}(Z,r), \qquad (\beta^+\beta^+),$$

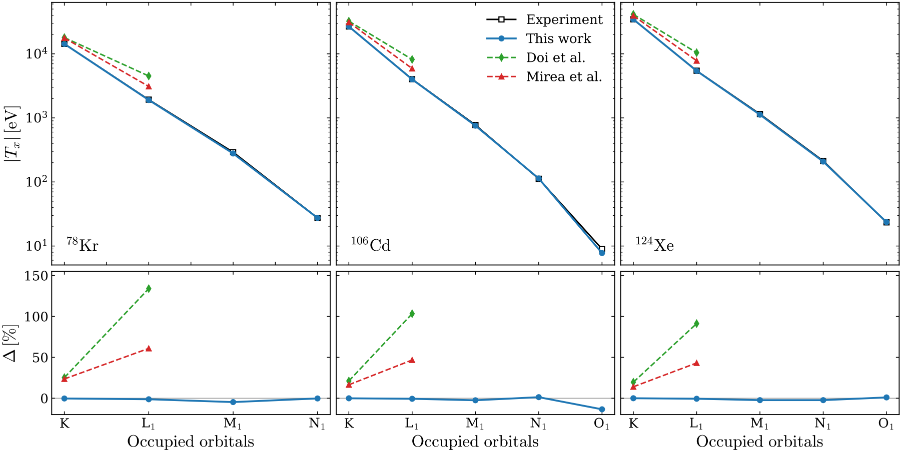
$$-V^{1-}(Z,r) = V_{\mathrm{nuc}}(r) + 2V_{\mathrm{el}}(Z,r) - V_{\mathrm{el}}^{1+}(Z,r), \qquad (\beta^+\mathrm{EC}),$$

$$V(Z+2,r) = V_{\mathrm{nuc}}(r) + V_{\mathrm{el}}(Z+2,r) + V_{\mathrm{ex}}(Z+2,r), \qquad (\mathrm{ECEC}).$$



Bound states for neutrino-emitting modes

*O. Nițescu, S. Ghinescu, S. Stoica and F. Šimkovic, Universe **10** (2), 98, (2024)*

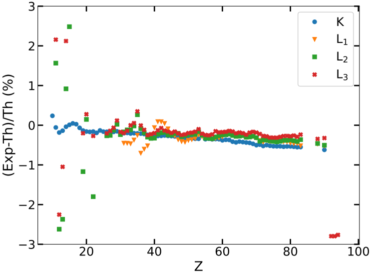


Total biding energy and atomic relaxation energy

$$\mathscr{H} = \sum_{i=1}^{Z} \left[\boldsymbol{\alpha}_i \cdot \boldsymbol{p}_i + (\beta - 1)\, m_e\right] + \sum_{i=1}^{Z} V_{\text{nuc}}(\boldsymbol{r}_i) + \sum_{i<j=1}^{Z} \frac{\alpha}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$$

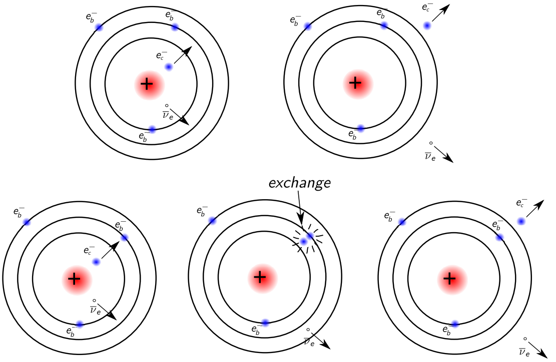
$$\Psi = \frac{1}{\sqrt{Z!}} \begin{vmatrix} \psi_1(\boldsymbol{r}_1) & \dots & \psi_1(\boldsymbol{r}_Z) \\ \vdots & \ddots & \vdots \\ \psi_Z(\boldsymbol{r}_1) & \dots & \psi_Z(\boldsymbol{r}_Z) \end{vmatrix} \qquad \langle \psi_i | \psi_j \rangle = \int \psi_i^\dagger(\boldsymbol{r}) \psi_j(\boldsymbol{r}) d\boldsymbol{r} = \delta_{ij}$$

$$R_{\text{x}} = B_{\text{gs}}(Z) - B_{\text{x}}(Z) = \langle \Psi_{\text{gs}} | \mathscr{H} | \Psi_{\text{gs}} \rangle - \langle \Psi_{\text{x}} | \mathscr{H} | \Psi_{\text{x}} \rangle$$



V. A. Sevestrean, O. Nițescu, S. Ghinescu and S. Stoica, Phys. Rev. A ***108****, 012810 (2023)*

Atomic exchange correction in β^- decay



$$\frac{d\Gamma}{dE_e} \Rightarrow \frac{d\Gamma}{dE_e} \times \left[1 + \eta^T(E_e)\right]$$

$$\eta^T(E_e) = f_s(2T_s + T_s^2) + (1 - f_s)(2T_{\bar{p}} + T_{\bar{p}}^2) = \eta_s(E_e) + \eta_{\bar{p}}(E_e)$$

$$T_s = \sum_{(ns)} T_{ns} = -\sum_{(ns)} \frac{\langle \psi_{E_e s} | \psi'_{ns} \rangle}{\langle \psi_{ns} | \psi'_{ns} \rangle} \frac{g_{n,-1}(R)}{g_{-1}(E_e, R)}, \quad f_s = \frac{g_{-1}^2(E_e, R)}{g_{-1}^2(E_e, R) + f_{+1}^2(E_e, R)}.$$

Exchange correction calculation: current status

The overlap between final continuum and initial bound states,

$$\left\langle \psi_{E_e \kappa} \middle| \psi'_{n\kappa} \right\rangle = \int_0^\infty r^2 \left[g_\kappa(E_e, r) g'_{n,\kappa}(r) + f_\kappa(E_e, r) f'_{n,\kappa}(r) \right] dr$$

- good knowledge of the continuum w.f. over large distances
- integration method
- orthogonal continuum and bound w.f. for the same atomic system

Phys. Rev. A ***45****, 6282 (1992)*

$$\left\langle \psi_{E_e \kappa} \middle| \psi_{n\kappa} \right\rangle = 0$$

Standard calculations (true DHFS)

$|\psi_{n\kappa}\rangle$: $V_{\mathrm{nuc}}(r) + V_{\mathrm{el}}(r) + V_{\mathrm{ex}}(r)$

$|\psi_{E_e s}\rangle$: $V_{\mathrm{nuc}}(r) + V_{\mathrm{el}}(r)$

Phys. Rev. A ***90****, 012501 (2014)*
Rev. Mod. Phys. ***90****, 015008 (2018)*
Phys. Rev. C ***102****, 065501 (2020)*
Phys. Rev. D ***102****, 072004 (2020)*
Appl. Radiat. Isot. ***185****, 110237 (2022)*

Our calculations (modified DHFS)

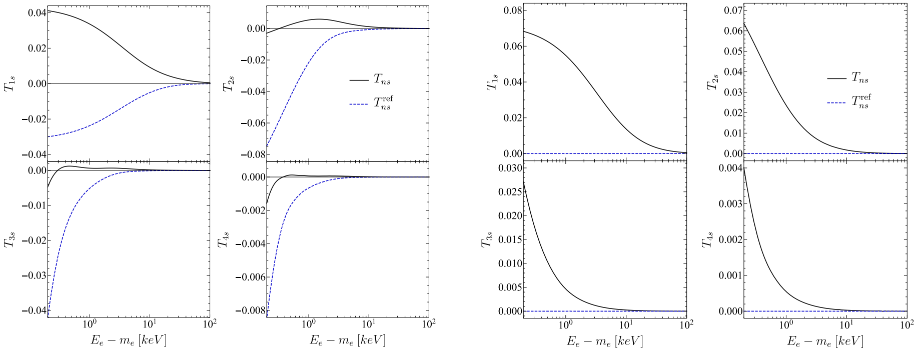
$|\psi_{n\kappa}\rangle$ and $|\psi_{E_e s}\rangle$:

$V_{\mathrm{nuc}}(r) + V_{\mathrm{el}}(r) + V_{\mathrm{ex}}^{\mathrm{Slater}}(r)$

O. Nițescu, S. Stoica and F. Šimkovic Phys. Rev. C ***107****, 025501 (2023)*

The β-decay of ^{45}Ca $\rightarrow$ ^{45}Sc^{+}+e^{-} + $\overline{\nu}_e$

$$T_{ns} = -\frac{\langle \psi_{E_e s} | \psi'_{ns} \rangle}{\langle \psi_{ns} | \psi'_{ns} \rangle} \frac{g_{n,-1}(R)}{g_{-1}(E_e, R)} \qquad T^{\mathrm{ref}}_{ns} = -\frac{\langle \psi_{E_e s} | \psi_{ns} \rangle}{\langle \psi_{ns} | \psi'_{ns} \rangle} \frac{g_{n,-1}(R)}{g_{-1}(E_e, R)}$$

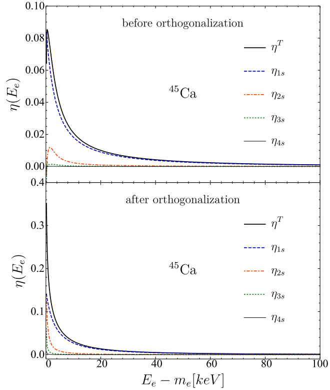


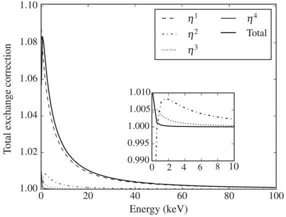
ψ'_{ns}, ψ_{ns}: true DHFS
$\psi_{E_e s}$: $V_{\mathrm{nuc}}(r) + V_{\mathrm{el}}(r)$

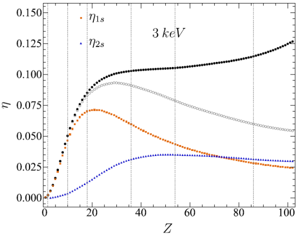
ψ'_{ns}, ψ_{ns}: modified DHFS
$\psi_{E_e s}$: modified DHFS

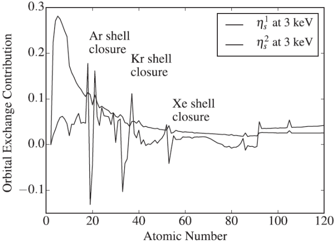
our result: *Phys. Rev. C* ***107***, *025501 (2023)*

Rev. Mod. Phys. ***90***, *015008 (2018)*



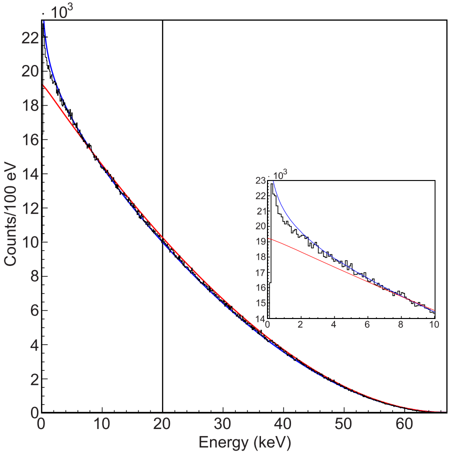




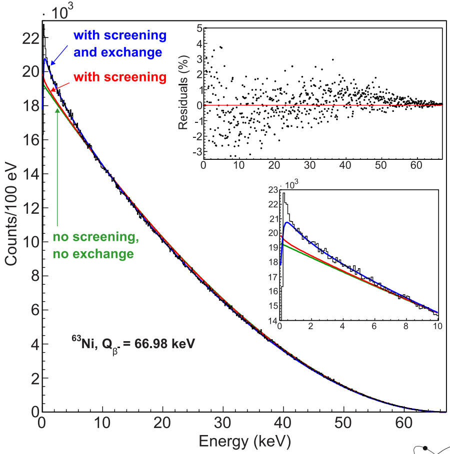


$$^{63}\text{Ni} \longrightarrow {}^{63}\text{Cu}^{+} + e^{-} + \overline{\nu}_e \qquad Q = 66.945 \text{ keV}$$

*our result: Phys. Rev. C **107**, 025501 (2023)*

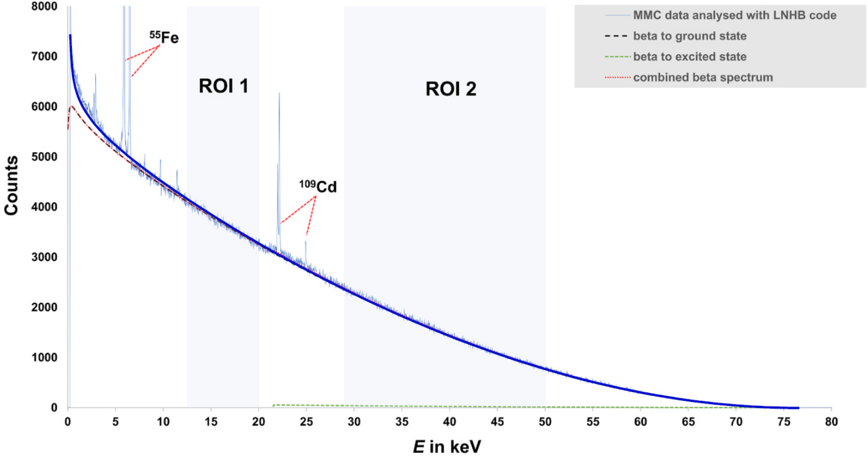


*Phys. Rev. A **90**, 012501 (2014)*



Experiment vs Theory

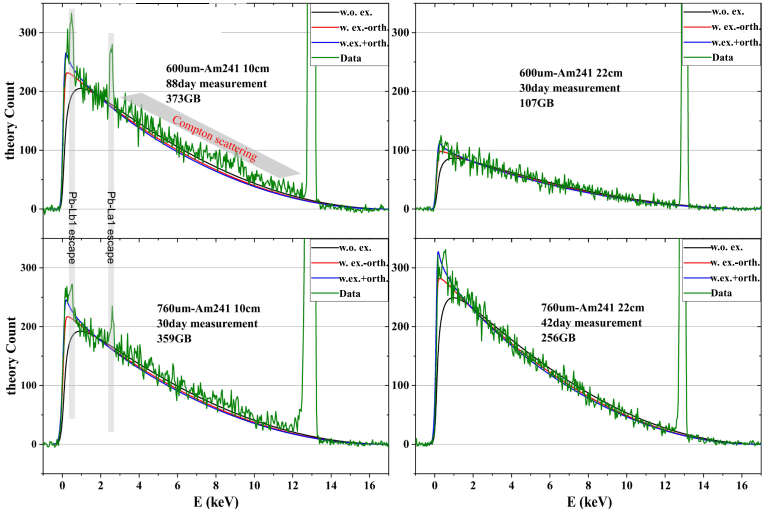
$$^{151}\mathrm{Sm} \rightarrow {}^{151}\mathrm{Eu}^{+} + e^{-} + \overline{\nu}_e \qquad Q = 76.6\ \mathrm{keV}$$



Appl. Radiat. Isot. ***185***, *110237 (2022)*

Experiment vs Theory*

$$^{210}\text{Pb} \longrightarrow {}^{210}\text{Bi}^{*} + e^{-} + \overline{\nu}_e \qquad Q = 16.96\ \text{keV}$$



arXiv:2307.16276 (2023)

$2\nu\beta\beta$ decay rate via Taylor expansion

We get a more accurate expression of the $2\nu\beta\beta$ decay rate

$$\left[T_{1/2}^{2\nu}(\xi_{31},\xi_{51})\right]^{-1} = G_0^{2\nu}\left(g_A^{\text{eff}}\right)^4\left|M_{GT}^{2\nu}\right|^2 \times\left\{1+\xi_{31}\frac{G_2^{2\nu}}{G_0^{2\nu}}+\frac{1}{3}\xi_{31}^2\frac{G_{22}^{2\nu}}{G_0^{2\nu}}+\left(\frac{1}{3}\xi_{31}^2+\xi_{51}\right)\frac{G_4^{2\nu}}{G_0^{2\nu}}\right\},$$

by performing in the matrix elements the following Taylor expansion

$$\begin{aligned} M_{F,GT}^{K,L} &= m_e\sum_n M_{F,GT}(n)\frac{E_n-(E_i-E_f)/2}{\left[E_n-(E_i-E_f)/2\right]^2-\epsilon_{K,L}^2} \\ &= m_e\sum_n M_{F,GT}(n)\frac{1}{E_n-(E_i-E_f)/2} \\ &\times\left\{1+\left(\frac{\epsilon_{K,L}}{E_n-(E_i-E_f)/2}\right)^2+\left(\frac{\epsilon_{K,L}}{E_n-(E_i-E_f)/2}\right)^4+\ldots\right\} \end{aligned}$$

F.Šimkovic, R. Dvornický, D. Štefánik, and A. Faessler, Phys. Rev. C 97, 034315 (2018)

*O. Nițescu, S. Stoica, R. Dvornický and F.Šimkovic, Univers **7** (5), 147 (2021)*

$2\nu\beta\beta$ decay rate via Taylor expansion

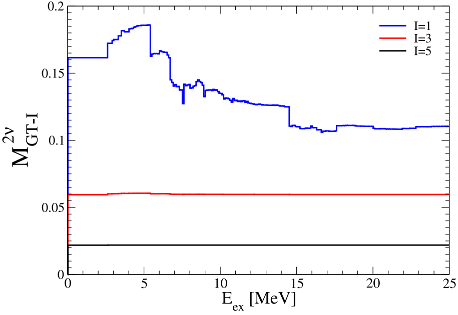
$$\xi_{31} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}},$$

$$\xi_{51} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}.$$

$$M^{2\nu}_{GT-1} \equiv M^{2\nu}_{GT} = \sum_n M_{GT}(n) \frac{m_e}{E_n(1^+) - (E_i + E_f)/2},$$

$$M^{2\nu}_{GT-3} = \sum_n M_{GT}(n) \frac{4\ m_e^3}{\left[E_n(1^+) - (E_i + E_f)/2\right]^3},$$

$$M^{2\nu}_{GT-5} = \sum_n M_{GT}(n) \frac{16\ m_e^5}{\left[E_n(1^+) - (E_i + E_f)/2\right]^5},$$



$$M_{GT}(n) = \langle 0^+_f \| \sum_m \tau^+_m \sigma_m \| 1^+_n \rangle \langle 1^+_n \| \sum_m \tau^+_m \sigma_m \| 0^+_i \rangle,$$

Adding exchange and radiative corrections to $2\nu\beta\beta$ decay

O. Nițescu and F. Šimkovic Phys. Rev. C 111, 035501 (2025)

$$G_N^{2\nu} = \frac{m_e (G_\beta m_e^2)^4}{8\pi^7 \ln 2} \frac{1}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} dE_{e_1} \int_{m_e}^{E_i - E_f - E_{e_1}} dE_{e_2}$$
$$\times p_{e_1} E_{e_1} \left[1 + \eta^T(E_{e_1})\right] R(E_{e_1}, E_i - E_f - m_e) p_{e_2} E_{e_2} \left[1 + \eta^T(E_{e_2})\right]$$
$$\times R(E_{e_2}, E_i - E_f - m_e) F_{ss}(E_{e_1}) F_{ss}(E_{e_2}) \mathscr{I}_N$$

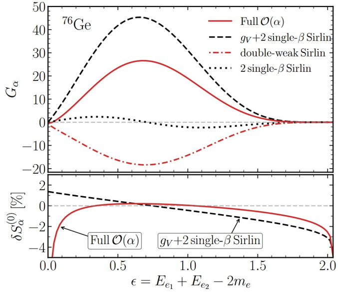
with $N = \{0, 2, 22, 4\}$.

$$R(E_e, E_e^{\mathsf{max}}) = 1 + \frac{\alpha}{2\pi} g(E_e, E_e^{\mathsf{max}})$$

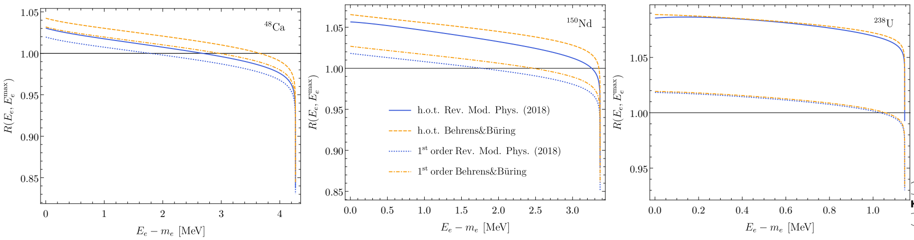
$$g(E_e, E_e^{\mathsf{max}}) = 3\ln(m_p) - \frac{3}{4} - \frac{4}{\beta}\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right)$$
$$+ \frac{\tanh^{-1}\beta}{\beta}\left[2\left(1+\beta^2\right) + \frac{\left(E_e^{\mathsf{max}} - E_e\right)^2}{6E_e^2} - 4\tanh^{-1}\beta\right]$$
$$+4\left(\frac{\tanh^{-1}\beta}{\beta} - 1\right)\left\{\frac{E_e^{\mathsf{max}} - E_e}{3E_e} - \frac{3}{2} + \ln\left[2\left(E_e^{\mathsf{max}} - E_e\right)\right]\right\}$$

where $\beta = p_e/E_e$.

Radiative corrections to $2\nu\beta\beta$ decay

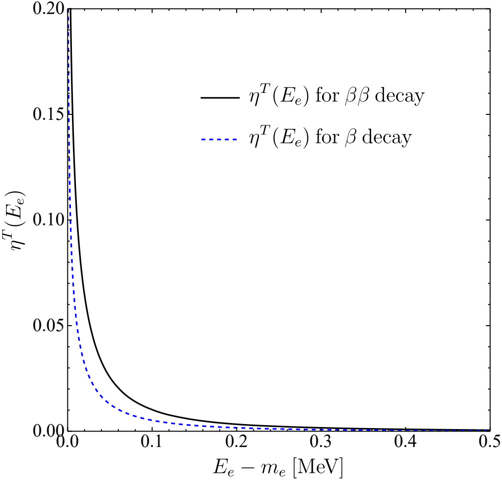


J. de Vries, E. Mereghetti, S. el Morabit and S. Sandner arXiv:2603.23592 (2026)



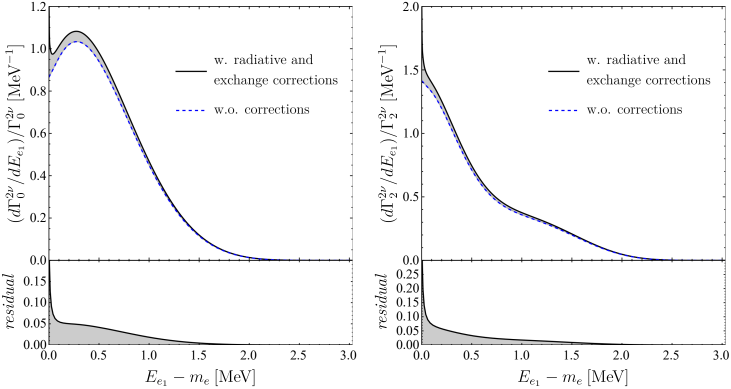
Exchange correction for β and $\beta\beta$ decay

O. Nițescu and F. Šimkovic Phys. Rev. C 111, 035501 (2025)



Single electron distributions and decay rates for ^{100}Mo

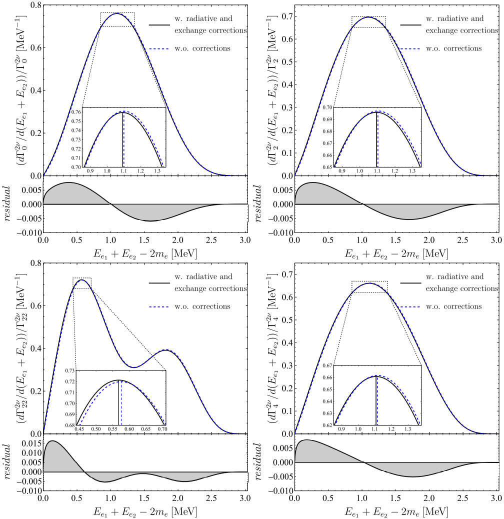
O. Nițescu and F. Šimkovic *Phys. Rev. C 111, 035501 (2025)*



Nucleus	Correction(s)	$G_0^{2\nu}$	$G_2^{2\nu}$	$G_{22}^{2\nu}$	$G_4^{2\nu}$
^{100}Mo	DHFS	3.307×10^{-18}	1.511×10^{-18}	1.989×10^{-19}	8.652×10^{-19}
	Exchange	3.343×10^{-18}	1.536×10^{-18}	2.031×10^{-19}	8.835×10^{-19}
	Radiative	3.432×10^{-18}	1.568×10^{-18}	2.066×10^{-19}	8.974×10^{-19}
	Radiative and Exchange	3.470×10^{-18}	1.593×10^{-18}	2.109×10^{-19}	9.164×10^{-19}
	δ	4.91%	5.42%	5.97%	5.92%

Summed electron distributions*

O. Nițescu and F. Šimkovic *Phys. Rev. C 111, 035501 (2025)*



Stella laboratory + D-L. Fang, O. Nițescu and F. Šimkovic *Eur. Phys. J. C 85:174 (2025)*
CUORE collaboration + D. Castillo, J. Kotila, J. Menèndez. O. Nițescu, F. Šimkovic *Phys. Rev. Lett. 135, 082501 (2025)*

Angular correlations in DBD

The differential DBD rate for a $0^+ \rightarrow 0^+$ nuclear transition with respect to the angle $0 \leq \theta \leq \pi$ between the emitted electrons can be written as,

$$\frac{d\Gamma}{d\left(\cos\theta\right)} = \frac{\Gamma}{2}\left(1 + K\cos\theta\right), \quad \text{where} \quad K = -\frac{\Lambda}{\Gamma}.$$

*O. Nițescu, S. Ghinescu and F. Šimkovic, Universe **10** (12), 442, (2024)*

For $2\nu\beta\beta$ decay we have,

$$\begin{Bmatrix} \Gamma^{2\nu} \\ \Lambda^{2\nu} \end{Bmatrix} = (g_A^{\text{eff}})^4 \left|M_{GT}^{2\nu}\right|^2 \ln(2) \begin{Bmatrix} G_0^{2\nu} + \xi_{31} G_2^{2\nu} + \frac{1}{3}\xi_{31}^2 G_{22}^{2\nu} + \left(\frac{1}{3}\xi_{31}^2 + \xi_{51}\right) G_4^{2\nu} \\ H_0^{2\nu} + \xi_{31} H_2^{2\nu} + \frac{5}{9}\xi_{31}^2 H_{22}^{2\nu} + \left(\frac{2}{9}\xi_{31}^2 + \xi_{51}\right) H_4^{2\nu} \end{Bmatrix},$$

$$\begin{Bmatrix} G_N^{2\nu} \\ H_N^{2\nu} \end{Bmatrix} = \frac{m_e \left(G_F \left|V_{ud}\right| m_e^2\right)^4}{8\pi^7 \ln(2)} \frac{1}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} p_{e_1} E_{e_1} \int_{m_e}^{E_i - E_f - E_{e_1}} p_{e_2} E_{e_2} \mathscr{I}_N \begin{Bmatrix} F_{ss}(E_{e_1}) F_{ss}(E_{e_2}) \\ E_{ss}(E_{e_1}) E_{ss}(E_{e_2}) \end{Bmatrix} dE_{e_2} dE_{e_1}.$$

For $0\nu\beta\beta$ decay we have,

$$\begin{Bmatrix} \Gamma^{0\nu} \\ \Lambda^{0\nu} \end{Bmatrix} = \frac{\left(G_F \left|V_{ud}\right|\right)^4 m_e^2}{32\pi^5 R^2} \left(g_A\right)^4 \left|M^{0\nu}\right|^2 \frac{\left|m_{\beta\beta}\right|^2}{m_e^2} \int_{m_e}^{E_i - E_f - m_e} p_{e_1} E_{e_1} p_{e_2} E_{e_2} \begin{Bmatrix} F_{ss}(E_{e_1}) F_{ss}(E_{e_2}) \\ E_{ss}(E_{e_1}) E_{ss}(E_{e_2}) \end{Bmatrix} dE_{e_1}.$$

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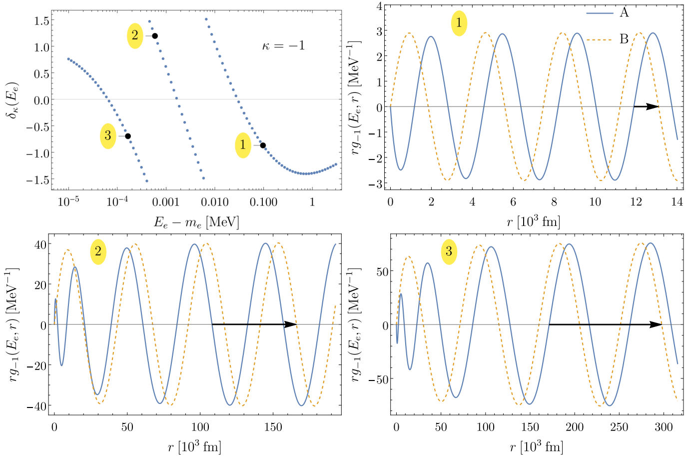
Electron phase shifts

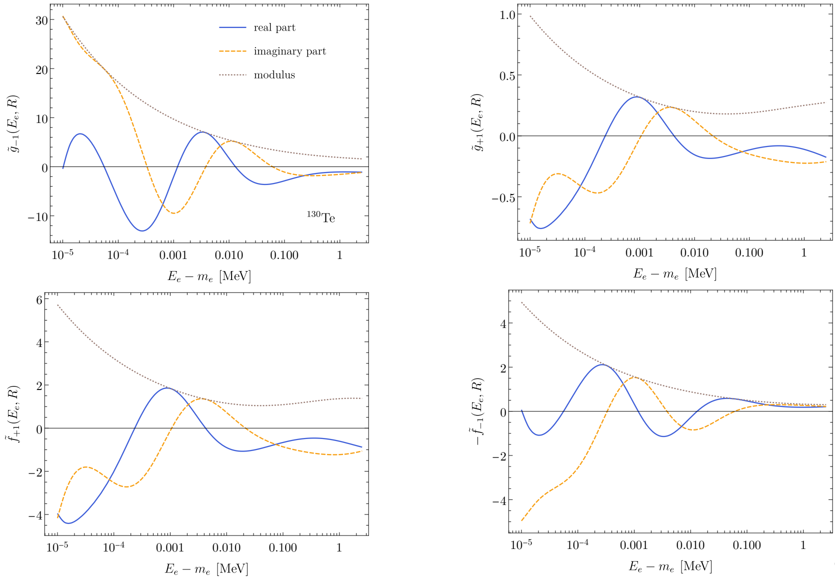
$$\begin{Bmatrix} \tilde{g}_\kappa(E_e,r) \\ \tilde{f}_\kappa(E_e,r) \end{Bmatrix} \xrightarrow[r\to\infty]{} \frac{\exp(-i\bar{\Delta}_\kappa)}{p_e r} \begin{Bmatrix} \sqrt{\frac{E_e+m_e}{2E_e}} \sin\left(p_e r - \ell_\kappa \frac{\pi}{2} + \eta \ln(2p_e r) + \bar{\Delta}_\kappa\right) \\ \sqrt{\frac{E_e-m_e}{2E_e}} \cos\left(p_e r - \ell_\kappa \frac{\pi}{2} + \eta \ln(2p_e r) + \bar{\Delta}_\kappa\right) \end{Bmatrix},$$

$$\bar{\Delta}_\kappa = \Delta_\kappa + \delta_\kappa .$$

*O. Nițescu, S. Ghinescu and F. Šimkovic, Universe **10** (12), 442, (2024)*



Electron phase shifts



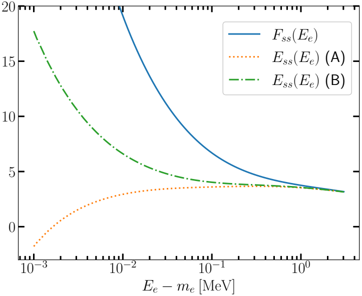
Angular distributions for $2\nu\beta\beta$ decay

$$F_{ss}(E_e) = |\tilde{g}_{-1}(E_e,R)|^2 + \left|\tilde{f}_1(E_e,R)\right|^2$$

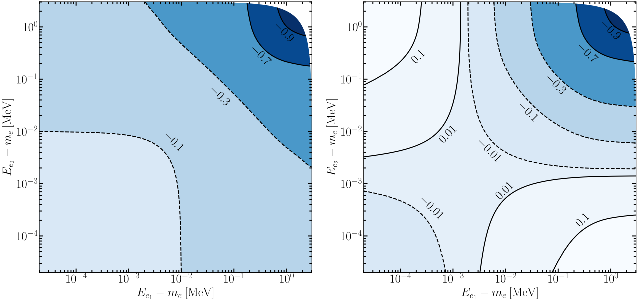
$$E_{ss}(E_e) = 2\,\mathrm{Re}\{\tilde{g}_{-1}(E_e,R)\tilde{f}_1^*(E_e,R)\}$$

$$\simeq 2\,|\tilde{g}_{-1}(E_e,R)|\left|\tilde{f}_1(E_e,R)\right|.$$

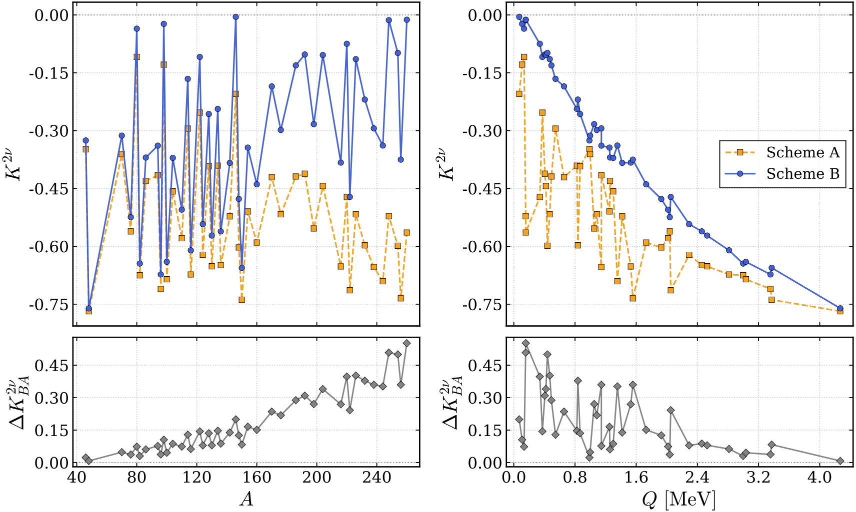
$$\kappa^{2\nu}(E_{e_1},E_{e_2}) = -\frac{d^2\Lambda^{2\nu}/(dE_{e_1}dE_{e_2})}{d^2\Gamma^{2\nu}/(dE_{e_1}dE_{e_2})}$$



O. Nițescu, S. Ghinescu and F. Šimkovic, *Universe* **10** (12), 442, (2024)



Angular correlation factors for $2\nu\beta\beta$ decay



Phase shifts in $0\nu\beta\beta$ decay within LRSM

$$f_{11}^{(0)} = \left|{}^{-1}f^{-1}\right|^2 + \left|{}_1f_1\right|^2 + \left|{}^{-1}f_1\right|^2 + \left|{}_1f^{-1}\right|^2$$

$$f_{11}^{(1)} = -2\,\mathrm{Re}\left[{}^{-1}f^{-1}\left({}_1f_1\right)^* + {}^{-1}f_1\left({}_1f^{-1}\right)^*\right]$$

$$f_{33}^{(0)} = \left(\frac{E_{e_1}-E_{e_2}}{m_e}\right)^2\left[\left|{}^{-1}f_1\right|^2 + \left|{}_1f^{-1}\right|^2\right]$$

$$f_{33}^{(1)} = 2\left(\frac{E_{e_1}-E_{e_2}}{m_e}\right)^2\mathrm{Re}\left[{}^{-1}f_1\left({}_1f^{-1}\right)^*\right]$$

$$f_{44}^{(0)} = \left(\frac{1}{m_eR}\right)^2\left[\left|{}^{-1}f^1 + {}_{-1}f_1\right|^2 + \left|{}_1f_{-1} + {}^1f^{-1}\right|^2\right]$$

$$f_{44}^{(1)} = -2\left(\frac{1}{m_eR}\right)^2\mathrm{Re}\left[\left({}^{-1}f^1 + {}_{-1}f_1\right)\left({}_1f_{-1} + {}^1f^{-1}\right)^*\right]$$

$$f_{55}^{(0)} = 4\left(\frac{1}{m_eR}\right)^2\left[\left|{}^{-1}f_{-1} + {}_{-1}f^{-1}\right|^2 + \left|{}_1f^1 + {}^1f_1\right|^2\right]$$

$$f_{55}^{(1)} = -8\left(\frac{1}{m_eR}\right)^2\mathrm{Re}\left[\left({}^{-1}f_{-1} + {}_{-1}f^{-1}\right)\left({}_1f^1 + {}^1f_1\right)^*\right]$$

$$f_{66}^{(0)} = 16\left[\left|{}^{-1}f^{-1}\right|^2 + \left|{}_1f_1\right|^2\right]$$

$$f_{66}^{(1)} = 32\,\mathrm{Re}\left[{}^{-1}f^{-1}\left({}_1f_1\right)^*\right]$$

$$f_{13}^{(0)} = \frac{E_{e_1}-E_{e_2}}{m_e}\left[\left|{}^{-1}f_1\right|^2 - \left|{}_1f^{-1}\right|^2\right]$$

$$f_{13}^{(1)} = 0$$

$$f_{16}^{(0)} = -4\left[\left|{}^{-1}f^{-1}\right|^2 - \left|{}_1f_1\right|^2\right]$$

$$f_{16}^{(1)} = 0$$

$$f_{14}^{(0)} = \frac{1}{m_eR}\left[{}^{-1}f_1\left({}^{-1}f^1 + {}_{-1}f_1\right)^* + {}_1f^{-1}\left({}_1f_{-1} + {}^1f^{-1}\right)^*\right]$$

$$f_{14}^{(1)} = -\frac{1}{m_eR}\mathrm{Re}\left[{}^{-1}f_1\left({}_1f_{-1} + {}^1f^{-1}\right)^* + {}_1f^{-1}\left({}^{-1}f^1 + {}_{-1}f_1\right)^*\right]$$

$$f_{34}^{(0)} = \frac{E_{e_1}-E_{e_2}}{m_e^2R}\left[{}^{-1}f_1\left({}^{-1}f^1 + {}_{-1}f_1\right)^* - {}_1f^{-1}\left({}_1f_{-1} + {}^1f^{-1}\right)^*\right]$$

$$f_{34}^{(1)} = -\frac{E_{e_1}-E_{e_2}}{m_e^2R}\mathrm{Re}\left[{}^{-1}f_1\left({}_1f_{-1} + {}^1f^{-1}\right)^* - {}_1f^{-1}\left({}^{-1}f^1 + {}_{-1}f_1\right)^*\right]$$

$$f_{15}^{(0)} = -\frac{2}{m_eR}\left[{}_1f_1\left({}_1f^1 + {}^1f_1\right)^* + {}^{-1}f^{-1}\left({}^{-1}f_{-1} + {}_{-1}f^{-1}\right)^*\right]$$

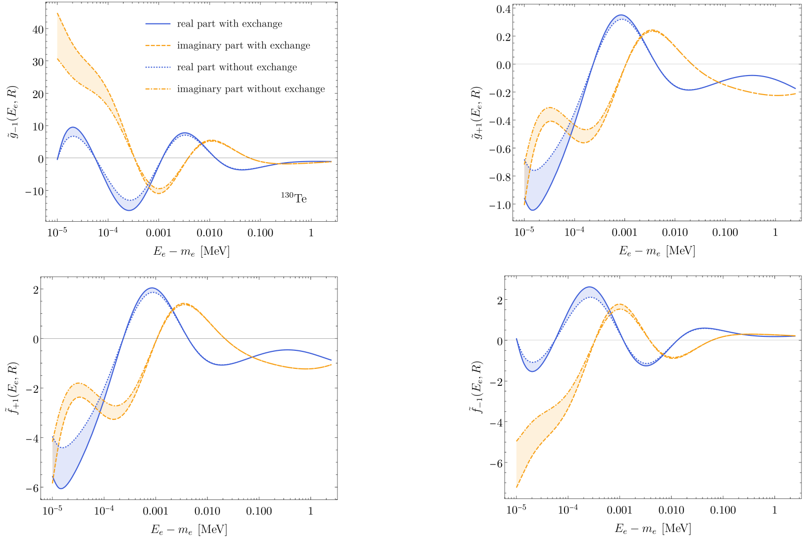
$$f_{15}^{(1)} = \frac{2}{m_eR}\mathrm{Re}\left[{}_1f_1\left({}^{-1}f_{-1} + {}_{-1}f^{-1}\right)^* + {}^{-1}f^{-1}\left({}_1f^1 + {}^1f_1\right)^*\right]$$

$$f_{56}^{(0)} = -\frac{8}{m_eR}\left[{}_1f_1\left({}_1f^1 + {}^1f_1\right)^* - {}^{-1}f^{-1}\left({}^{-1}f_{-1} + {}_{-1}f^{-1}\right)^*\right]$$

$$f_{56}^{(1)} = \frac{8}{m_eR}\mathrm{Re}\left[{}_1f_1\left({}^{-1}f_{-1} + {}_{-1}f^{-1}\right)^* - {}^{-1}f^{-1}\left({}_1f^1 + {}^1f_1\right)^*\right]$$

Atomic exchange correction for $0\nu\beta\beta$ decay within LRSM

$$\tilde{g}_\kappa(E_e, r) \rightarrow \tilde{g}_\kappa(E_e, r) - \sum_n \frac{\langle \psi_{E_e\kappa} | \psi'_{n\kappa} \rangle}{\langle \psi_{n\kappa} | \psi'_{n\kappa} \rangle} g_{n\kappa}(r), \qquad \tilde{f}_\kappa(E_e, r) \rightarrow \tilde{f}_\kappa(E_e, r) - \sum_n \frac{\langle \psi_{E_e\kappa} | \psi'_{n\kappa} \rangle}{\langle \psi_{n\kappa} | \psi'_{n\kappa} \rangle} f_{n\kappa}(r).$$



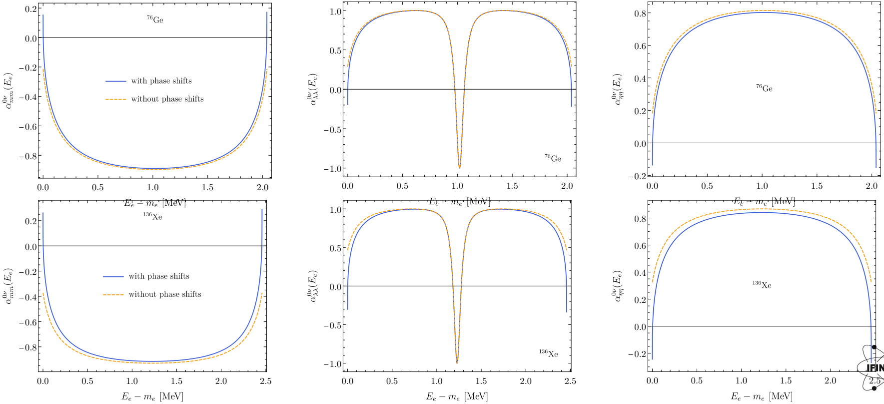
Angular distributions for $0\nu\beta\beta$ decay within LRSM

$$dC_{mm} = (1-\chi_F)^2 d\mathcal{G}_{01},$$

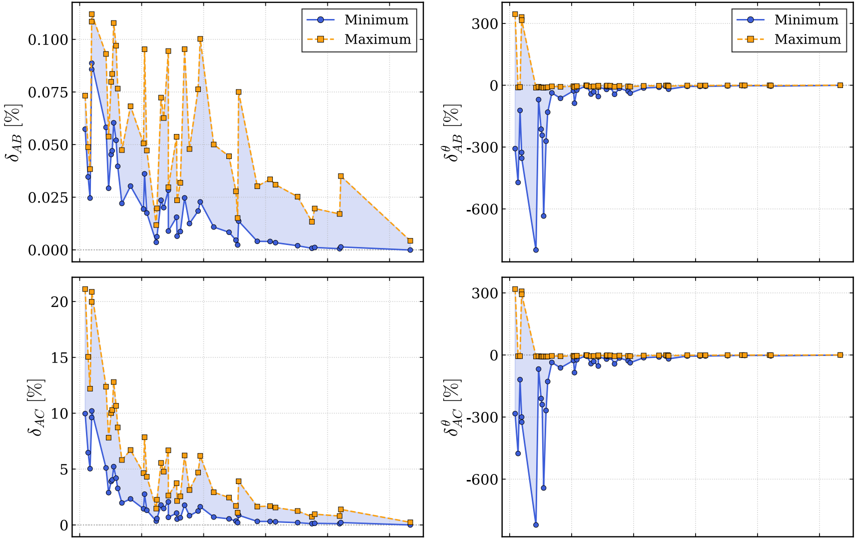
$$dC_{\lambda\lambda} = \chi_{2-}^2 d\mathcal{G}_{02} + \frac{1}{9}\chi_{1+}^2 d\mathcal{G}_{011} - \frac{2}{9}\chi_{1+}\chi_{2-} d\mathcal{G}_{010},$$

$$dC_{\eta\eta} = \chi_{2+}^2 d\mathcal{G}_{02} + \frac{1}{9}\chi_{1-}^2 d\mathcal{G}_{011} - \frac{2}{9}\chi_{1-}\chi_{2+} d\mathcal{G}_{010} + \chi_P^2 d\mathcal{G}_{08} - \chi_P\chi_R d\mathcal{G}_{07} + \chi_R^2 d\mathcal{G}_{09}.$$

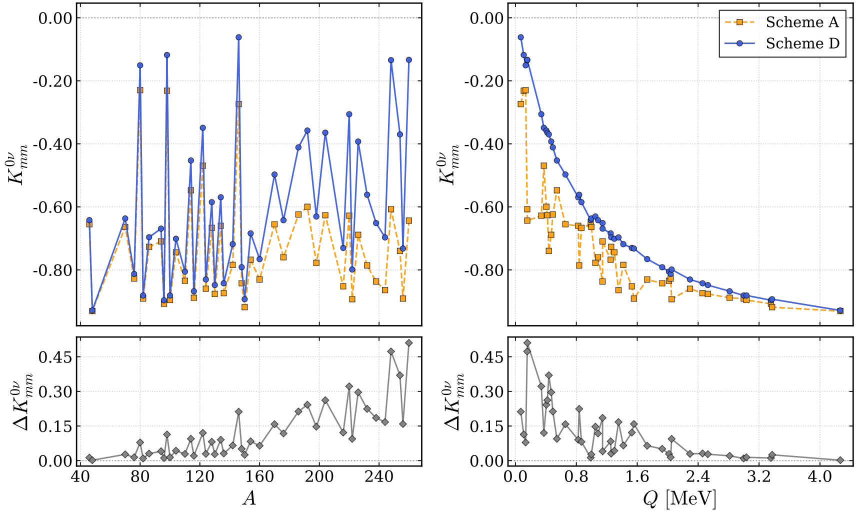
$$d\mathcal{G}_{0k} = d\cos\theta\, dE_{e_2} \frac{G_\beta^4 m_e^2 Q_{\beta\beta}}{64\pi^5 R^2}\left[h_{0k}(E_{e_1}, E_{e_2}, R)\cos\theta + g_{0k}(E_{e_1}, E_{e_2}, R)\right] p_1 p_2 E_{e_1} E_{e_2}$$



Deviations in the decay rates for $0\nu\beta\beta$ decay



Angular correlation factors for $0\nu\beta\beta$ decay



Conclusions and Outlook

Interaction of charged leptons with atomic systems:

- precise model based on self-consistent DHFS method for emitted/captured charged leptons; atomic relaxation energies for EC/ECEC processes;

Exchange correction for β^- decay:

- the orthogonality condition, i.e. $\left\langle \psi_{E_e\kappa} \middle| \psi_{n\kappa} \right\rangle = 0$, is the most important ingredient in the atomic exchange correction; our model is in agreement with the experimental β decay spectra;

Phase shifts in the angular distributions for DBD:

- electrons are most likely emitted in the same direction when one of them is below 2 keV;

Phase shifts in the angular correlation factors for DBD:

- large deviations in K for both $2\nu\beta\beta$ decay and $0\nu\beta\beta$ decay; smooth trend in K vs Q with electron phase shifts; NMEs constrains from a K fit.