Neutrino-induced pion production on nucleons: data comparisons

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INT workshop 23 - 86W

31 October 2023

Suggested title
“Testing resonance models against constraining variables of interest”

- Will focus on single-pion production
- Try to avoid talking about ‘models’ and focus on data comparisons

→ Heavily ‘inspired’ by recent work: [K. Niewczas PRD 103, 053003 (2021)]
[R.G.J PRD 95, 113007 (2017)]
[A. N. PRD 107 053007 (2023)]
Based on work by Valencia group

Outline

- Electromagnetic pion production
- Angular distributions of pions
- Isovector contribution to charged-current pion production
- Axial currents & Bubble chamber data in the Δ region
- Higher mass resonances and quark-hadron duality

Electro and photoproduction

Many approaches available in the literature e.g.:
-MAID07, CLAS analyses ('unitary isobar model')
- Julich-Bonn, ANL-Osaka, ... (Dynamical models)
- Effective Lagrangian models, SAID, dispersive approaches ...

Ingredients:

- -Nucleon resonances
- -Background terms : Born term, Vector meson exchanges
- -Meson-baryon interactions

Analyses of N(e,e'π)N extract resonance amplitudes/properties
Supported by a large amount of data!

MAID07
~18 000 points for photon induced processes

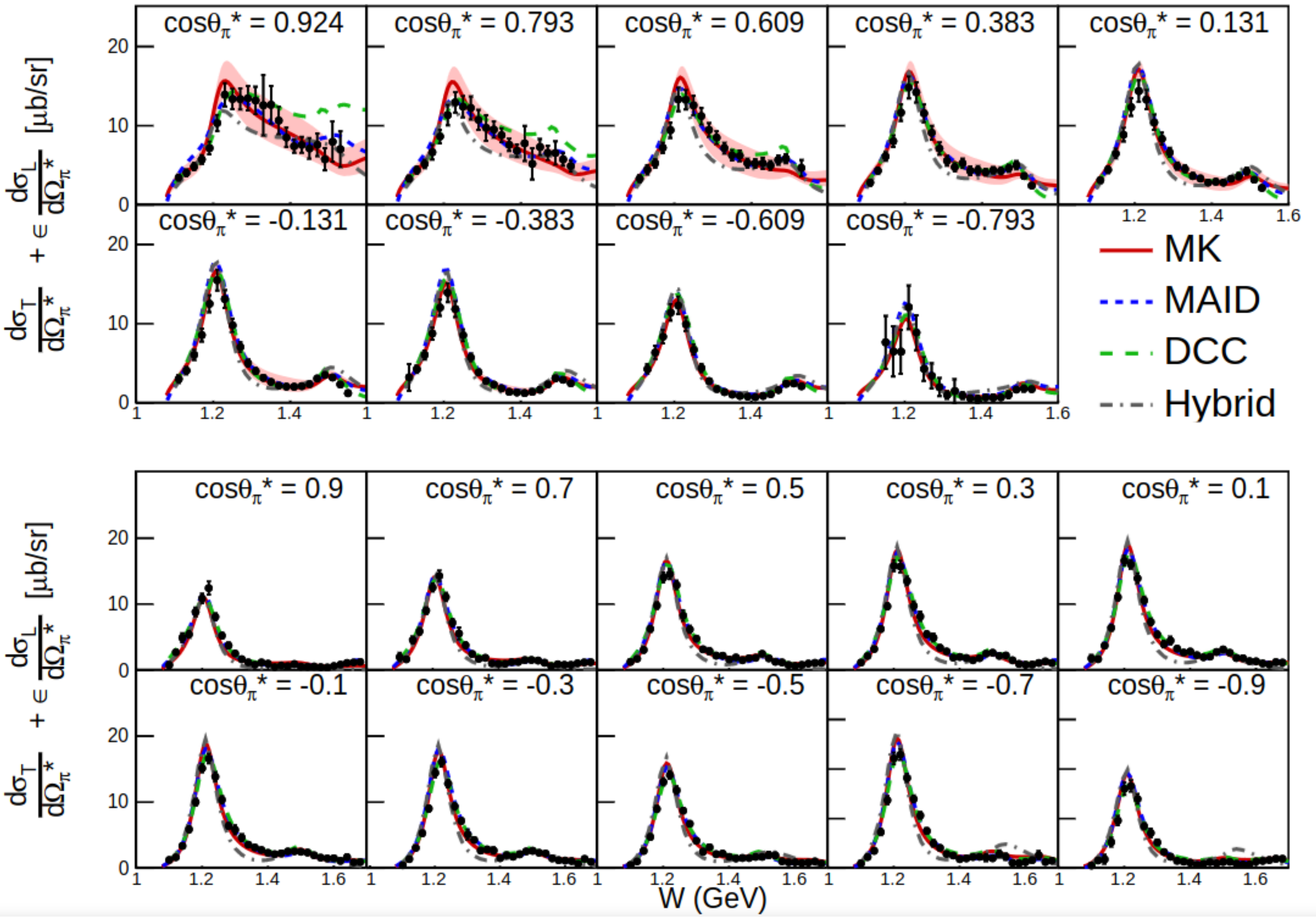
[illegible]	[illegible]	[illegible]
$n\pi^+$	0.4-0.65	$d\sigma_{LT'}$
CLAS06 [46]	1110-1570	4179
$n\pi^+$	0.3-0.60	$d\sigma$
CLAS06 [13]	1110-1390	8491
$p\pi^0$	3.0-6.0	$d\sigma$
total	1074-1975	68457
$p\pi^0$, $n\pi^+$	0.1-6.0	$d\sigma$, ...

Electron-proton datapoints in MAID07

Fermilab

Electron-induced SPP: high quality proton target data

Figures from M. Kabirnezhad [arxiv:2203.15594]

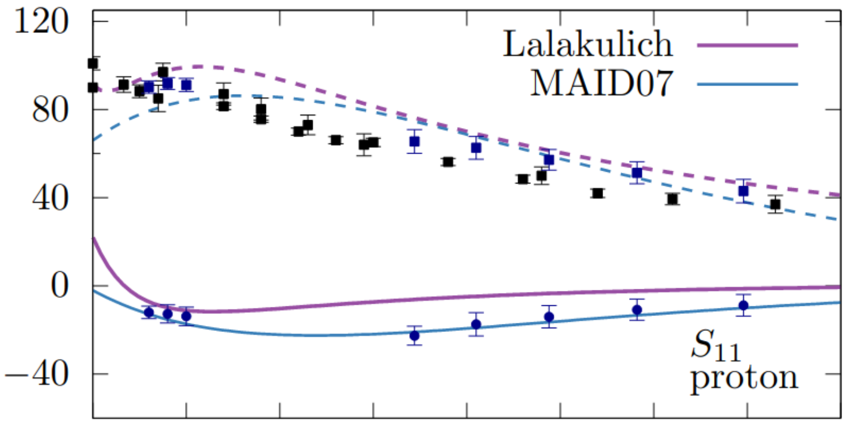


, CLAS data

Differential cross sections for exclusive 1π data is abundant for large Q^2 and W-range

Fermilab

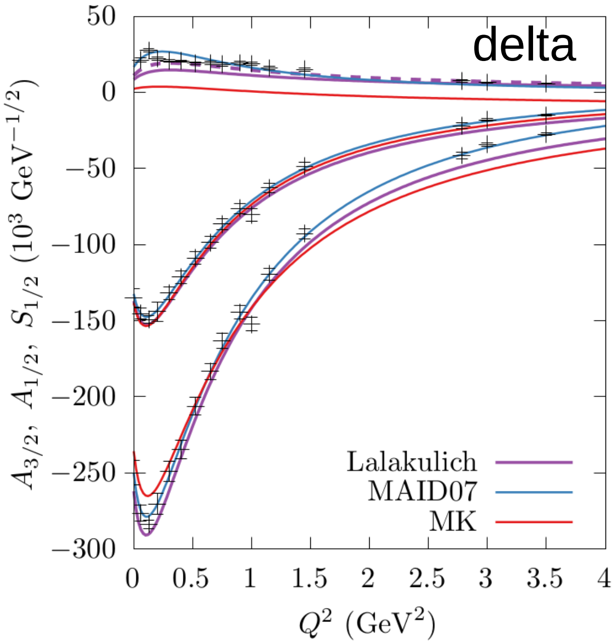
Resonance contributions: proton form factors from helicity amplitudes



$$\mathcal{A}_{1/2} = \sqrt{\frac{2\pi\alpha}{K}}\frac{1}{e}\langle S_{z,R} = \frac{1}{2}|\epsilon^{(+)}_{\mu}(q)J^{\mu}|S_{z,N} = -\frac{1}{2}\rangle$$

$$\mathcal{A}_{3/2} = \sqrt{\frac{2\pi\alpha}{K}}\frac{1}{e}\langle S_{z,R} = \frac{3}{2}|\epsilon^{(+)}_{\mu}(q)J^{\mu}|S_{z,N} = \frac{1}{2}\rangle$$

$$\mathcal{S}_{1/2} = \sqrt{\frac{2\pi\alpha}{K}}\frac{1}{e}\langle S_{z,R} = \frac{1}{2}|\frac{|\vec{q}|}{\sqrt{Q^2}}\epsilon^{(0)}_{\mu}(q)J^{\mu}|S_{z,N} = \frac{1}{2}\rangle$$



Helicity amplitudes at W = M_R can be used to determine γ*p→ R form-factors

There is some model-dependence in RES-BG separation: watch out!

See [userweb.jlab.org/~mokeev/resonance_electrocouplings]

Helicity amplitudes from CLAS and MAID07 analyses used in:

- Lalakulich et al [Phys. Rev. D74, 014009 (2006)]
- Hernandez et al. [Phys Rev D 77 053009 (2008)]
- Nikolakopoulos et al. [arxiv:2210.12144 (2022)]

Electron-induced SPP: high quality proton data & Analyses

In short: If you do electron interactions → e.g. e4nu
No need to reinvent the wheel!

- **MAID** [https://maid.kph.uni-mainz.de/]

- **CLAS resonances** [userweb.jlab.org/~mokeev/resonance_electrocouplings]

- **Julich-Bonn** [http://collaborations.fz-juelich.de/ikp/meson-baryon/]

- **ANL-Osaka** [www.phy.anl.gov/theory/research/anl-osaka-pwa]

- **SAID** [gwdac.phys.gwu.edu]

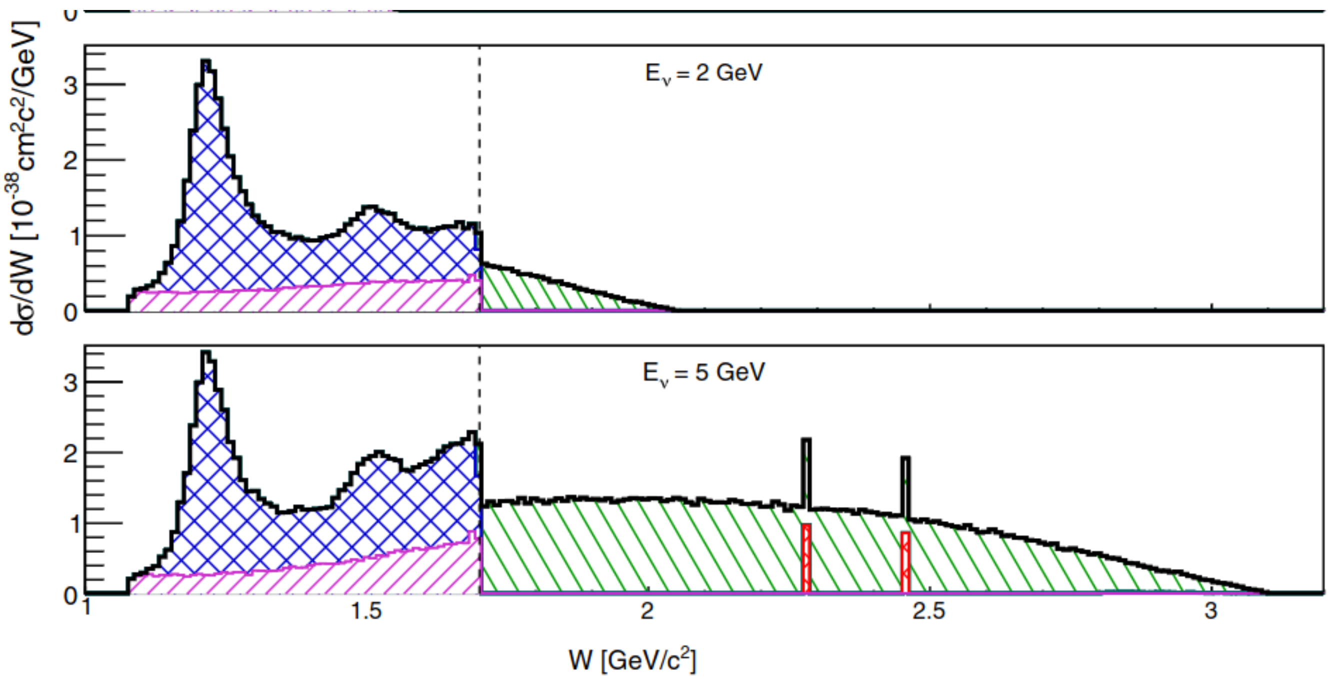
- **CLAS structure functions** [clas.sinp.msu.ru/strfun]

- and many more ...

Cross sections, amplitudes, resonance couplings, data, …

Resonance models in generators

- Incoherent sum of resonance contributions
- Angular distributions πN isotropic in CMS
- Non-resonant background extrapolated from DIS



Resonance models in generators

- **Incoherent sum of resonance contributions**

Can decompose the pion production amplitude in s-channel angular momenta

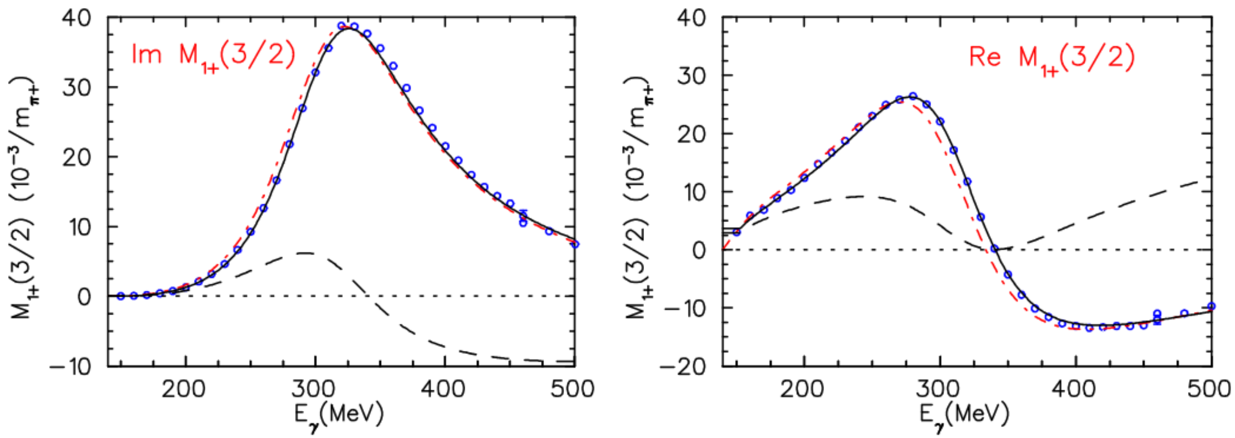
$$H^{r}_{\lambda',\lambda}(\Omega^*) = \sum_{J=\frac{1}{2}}^{3/2} \left(J+\frac{1}{2}\right) \langle \lambda' | T^J | r, \lambda \rangle D^J_{M,\lambda'}(\Omega^*) \qquad (M = \lambda - r)$$

And definite parity:

$$T^{J,P=\pm} = \langle r-\lambda | T^j | \lambda' \rangle \pm \langle r - \lambda | T^J | -\lambda' \rangle$$

(For EM interactions → 6 independent amplitudes $M_{l\pm}, E_{l\pm}, S_{l\pm} \quad (l = J \pm 1/2)$

A resonant structure is found in specific πN partial waves: (I, J, P) e.g. Δ = P_{33}



[MAID07] photoproduction

Resonance models in generators

- **Incoherent sum of resonance contributions**

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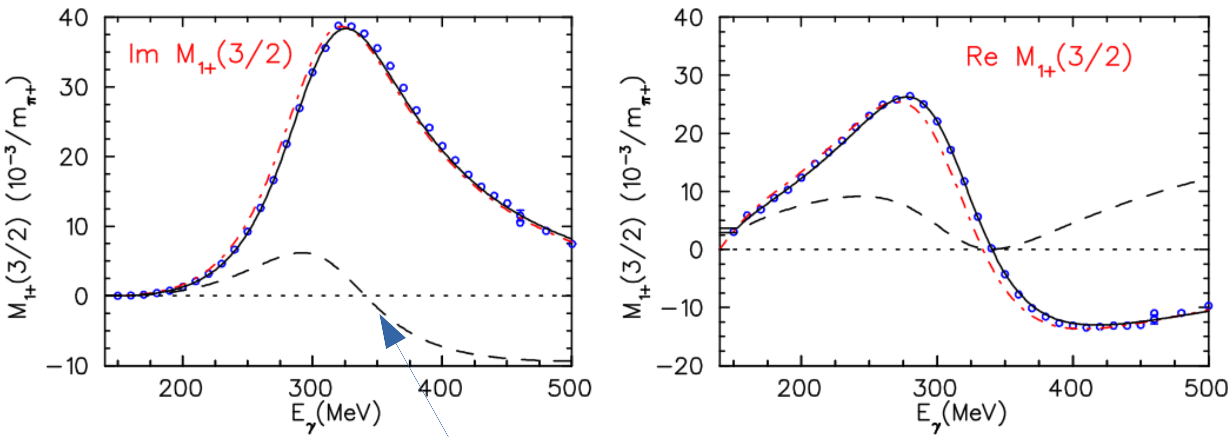


Inclusive cross sections are incoherent sums of angular momentum states

→ Can get away with incoherent sums of resonance contributions

→ Can lead to fair description of inclusive cross section

→ Cannot incoherently add ‘Background’ contribution

Resonance models in generators

- Incoherent sum of resonance contributions
- **Angular distributions πN** isotropic in CMS

→ Sensitive to Spin-parity of resonance

→ Sensitive to interference between multipoles

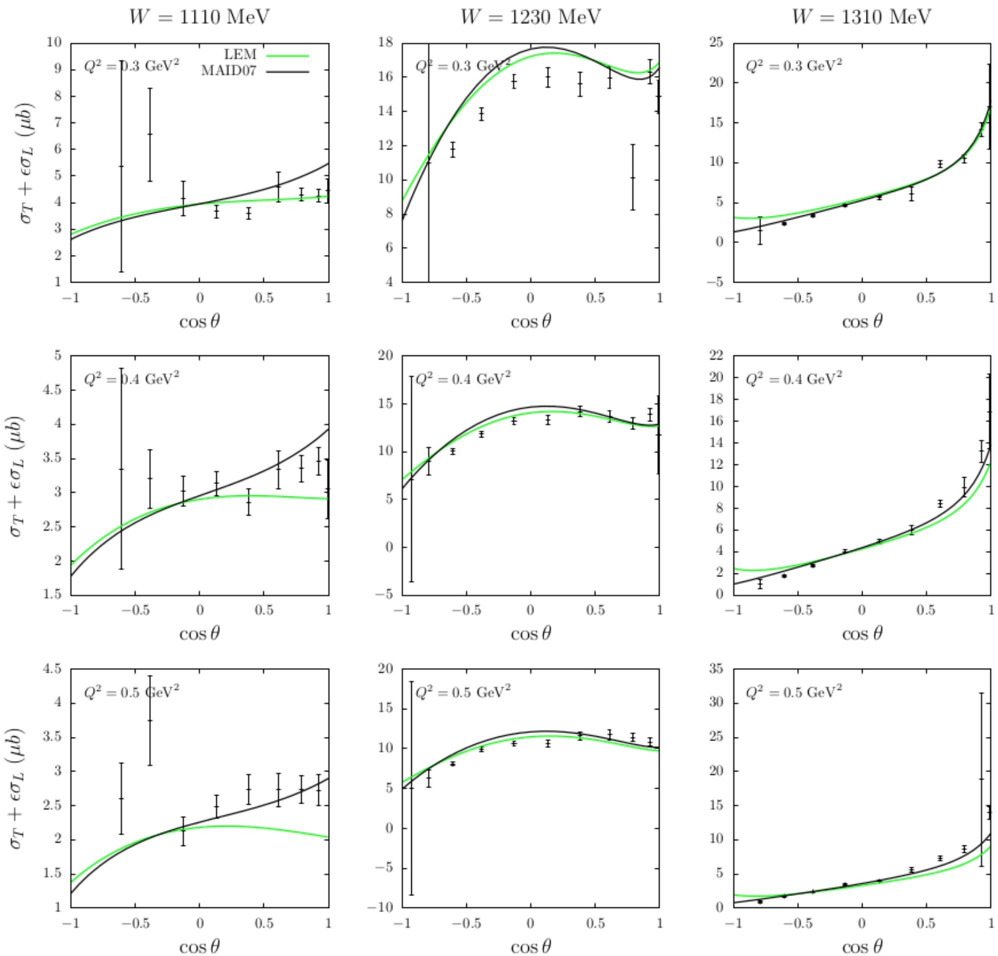
$$\frac{d\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_\pi^*}{k_l^2} \times [A + B\cos(\phi^*)\, C\cos(2\phi^*) + D\sin(\phi^*) + E\sin(2\phi^*)]$$

ϕ-dependence factorizes, A,B,C,D,E functions of (Q^2, W, θ_π)
See e.g. [Sobczyk et al. Phys. Rev. D 98, 073001 (2018)]

Fermilab

Electron-induced SPP: angular-dependence structure functions

$$\frac{d\sigma_e}{d\Omega^*} = \boxed{\sigma_T + \epsilon\sigma_L} + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\left(\phi^*\right) + \epsilon\sigma TT\cos\left(2\phi^*\right) + h\sqrt{2\epsilon\left(1-\epsilon\right)}\sigma_{LT'}\sin\phi^*$$



– LEM from
[R.G.J et al. PRD 95, 113007 (2017)]
(based on
[HNV PRD76, 033005, 2007])

– MAID07

CLAS data

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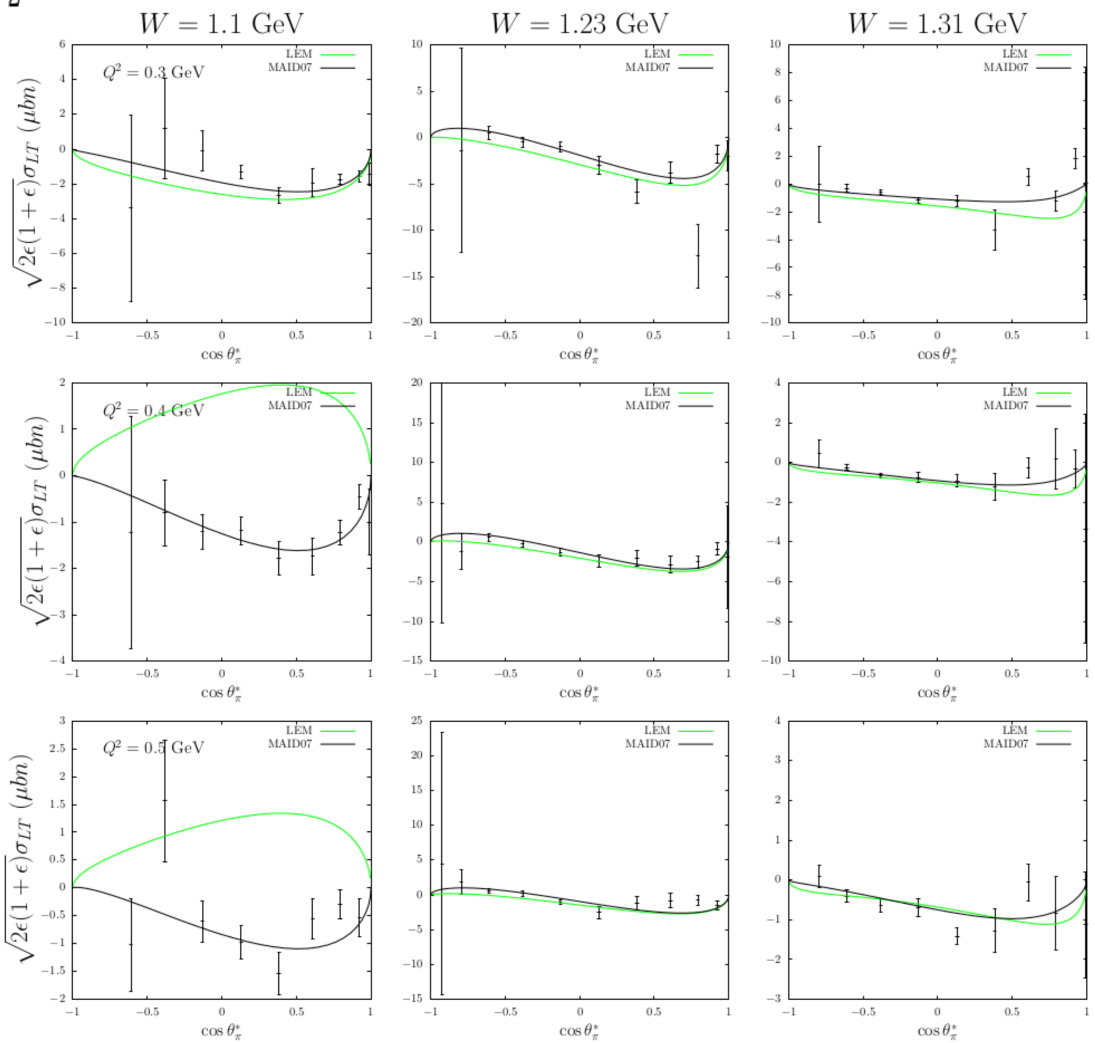

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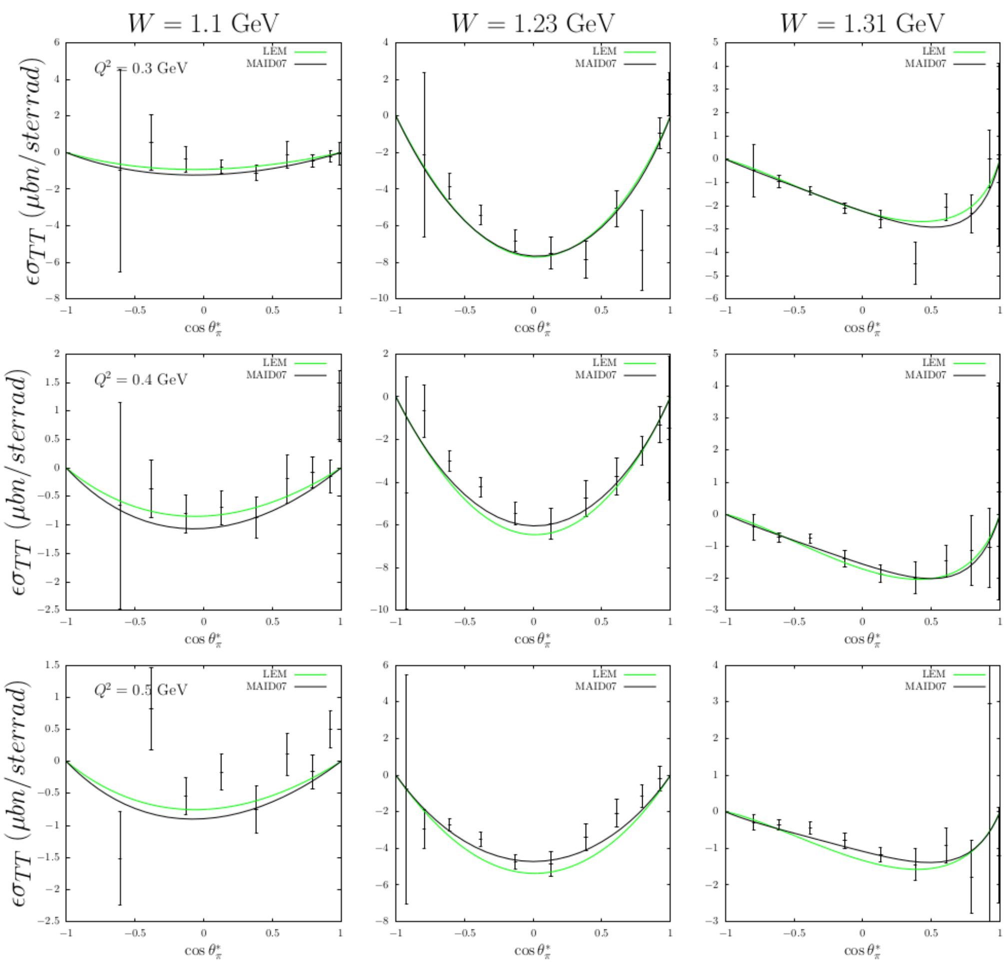

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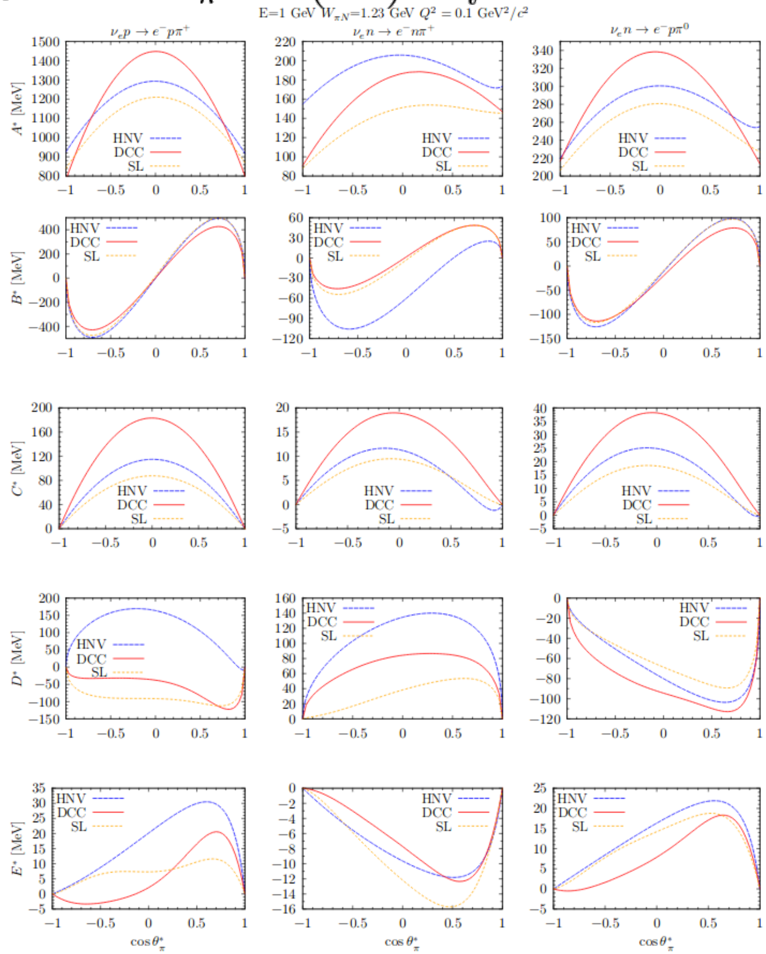
– MAID07

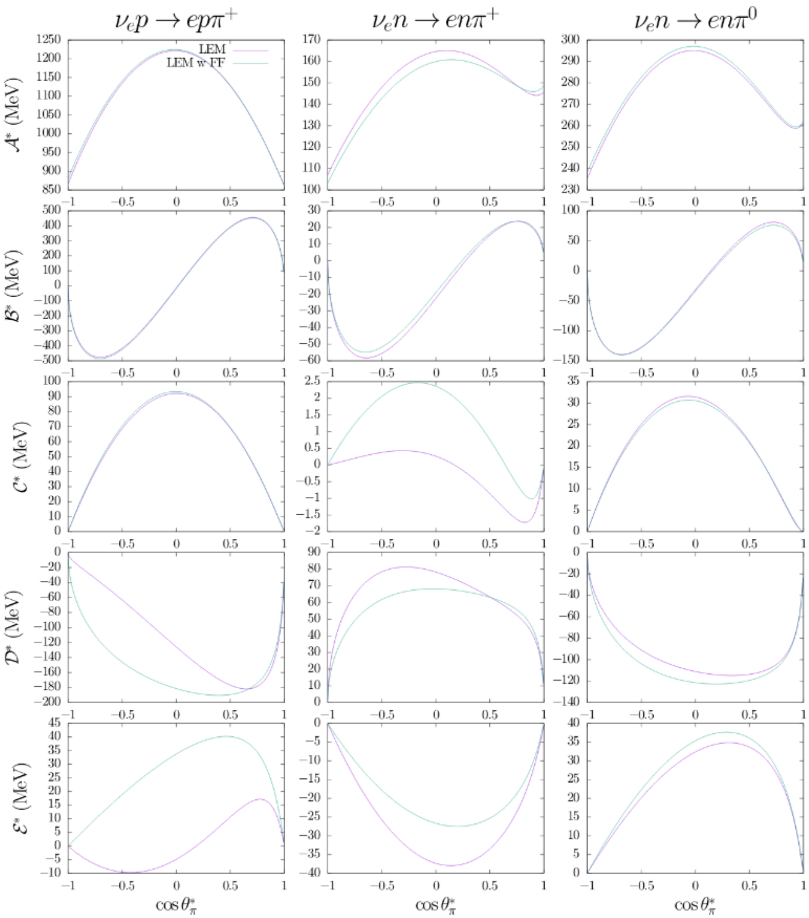
CLAS data

Fermilab

Structure functions for neutrinos: non-trivial angular dependence

$$\frac{d\sigma}{dQ^2 dW\, d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k_\pi^*}{k_l^2}\times\left[A + B\cos\left(\phi^*\right) C\cos\left(2\phi^*\right) + D\sin\left(\phi^*\right) + E\sin\left(2\phi^*\right)\right]$$

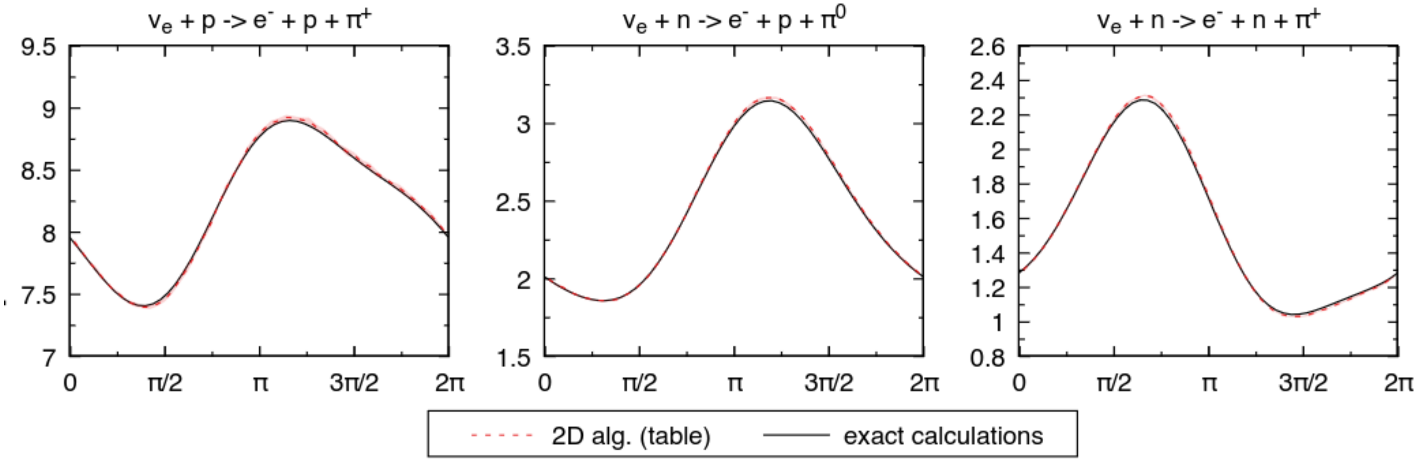




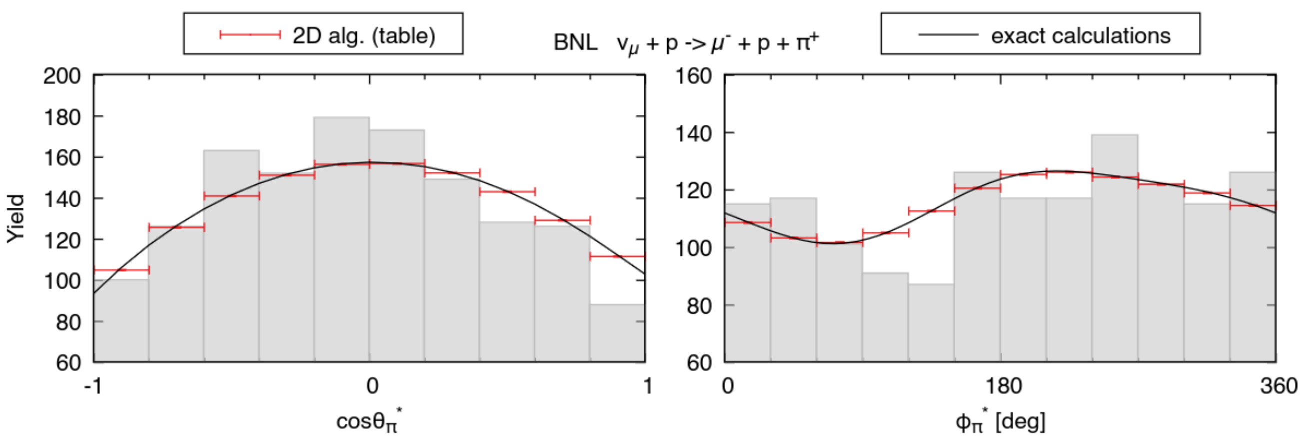
[Sobczyk et al. Phys. Rev. D 98, 073001 (2018)]

Structure functions for neutrinos in NuWro

Full implementation of cross section with all interference in NuWro : [Niewczas, PRD 103, 053003 (2021)]
Completely general: works for every model



Flux-averaged neutrino-nucleus data not sensitive to ϕ
Hydrogen/deuteron bubble chambers:



Will see results for flux-averaged data in near future

Fermilab

Neutrino-induced SPP: isovector form factors

$$J^\mu_{EM} = V^\mu_s + \mathbf{V}^\mu \qquad J^\mu_{CC\pm} = \mathbf{V}^\mu - \mathbf{A}^\mu$$

$$\langle \pi^+ n | J^\mu_{EM} | p \rangle = V^\mu_{3/2} - \sqrt{2}\left(V^\mu_{1/2} + S^\mu\right)$$
$$\langle \pi^- p | J^\mu_{EM} | n \rangle = V^\mu_{3/2} - \sqrt{2}\left(V^\mu_{1/2} - S^\mu\right)$$
$$\langle \pi^0 p | J^\mu_{EM} | p \rangle = \sqrt{2}V^\mu_{3/2} + \boxed{\left(V^\mu_{1/2} + S^\mu\right)} \quad F_p$$
$$\langle \pi^0 n | J^\mu_{EM} | n \rangle = \sqrt{2}V^\mu_{3/2} + \boxed{\left(V^\mu_{1/2} - S^\mu\right)} \quad F_n$$

$$\langle \pi^+ p | V^\mu_+ | p \rangle = 3V^\mu_{3/2},$$
$$\langle \pi^+ n | V^\mu_+ | n \rangle = V^\mu_{3/2} + 2\sqrt{2}V^\mu_{1/2},$$
$$\langle \pi^0 p | V^\mu_+ | n \rangle = -\sqrt{2}V^\mu_{3/2} + 2V^\mu_{1/2}.$$

Determining the isovector couplings requires analysis of proton and neutron target !

Ref. channel	W (MeV) Q^2 (GeV^2)	N_{data} observables	χ^2/N_{data} (2003) χ^2/N_{data} (2007)
total $p\pi^0$, $n\pi^+$	1074-1975 0.1-6.0	68457 dσ, ...	2.724 2.437
SAID00 $p\pi^-$	1253-1976 0.54-1.36	799 dσ	2.100 2.264

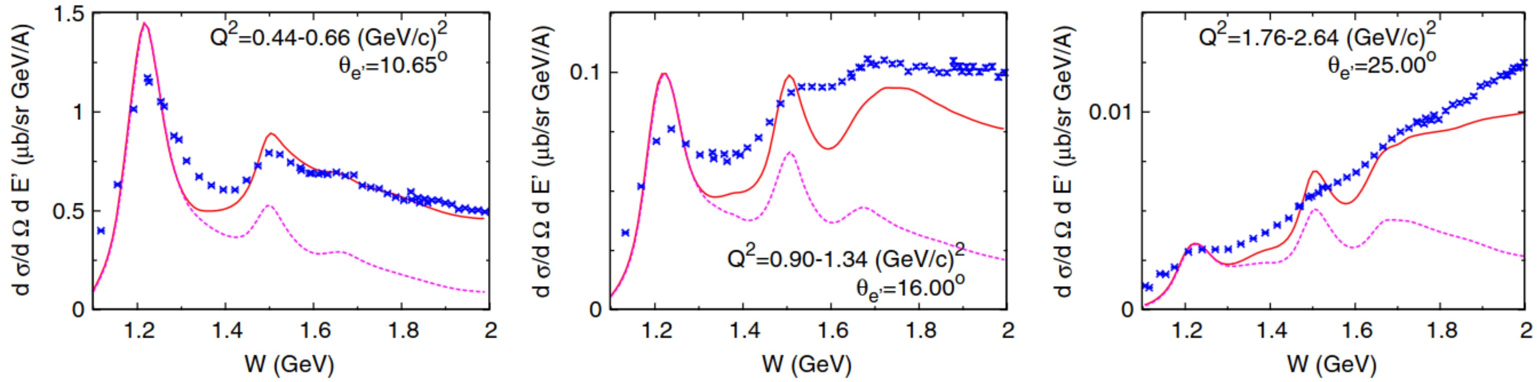
Way less abundant!

Deuteron targets

Analyses of electropion production on deuteron
(only the ones we use for comparison)

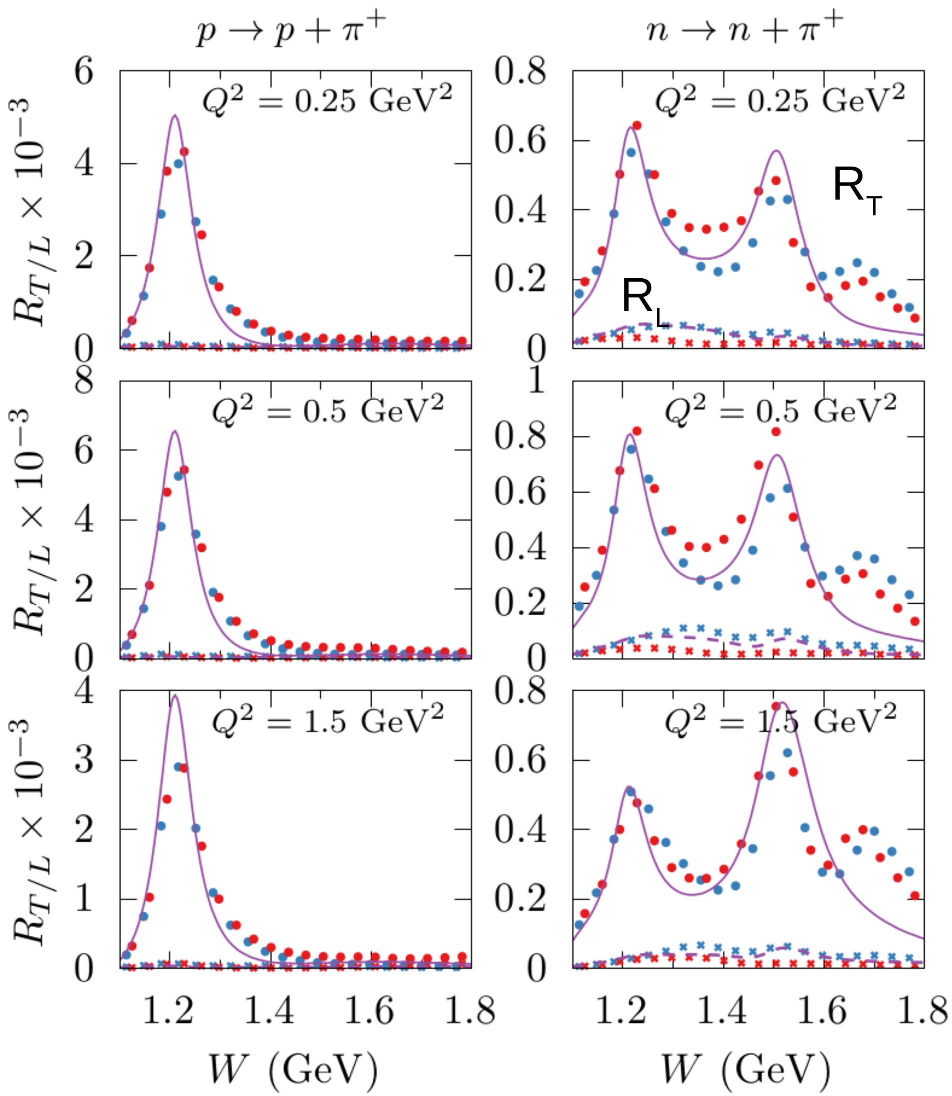
MAID07 [Eur.Phys.J.A34:69-97,2007]
Unitarized background: includes πN scattering phases and inelasticity
S-channel resonances: Breit-Wigner parametrization of helicity amplitudes
→ Fit 'neutron' target exclusive data (deuteron)

ANL-Osaka Dynamic Coupled Channels (DCC) [PRD 92, 074024 (2015)]
Coupled-channels unitarity with consistent meson-baryon amplitudes [PRC94 015201]
→ Fit 'neutron' target exclusive data (deuteron)
→ Fit inclusive structure function on deuteron



Calculation: sum of free nucleon CS, no smearing from deuteron
→ Progress on deuteron FSI in [PRD 99 031301]
+ New CLAS data [Phys. Rev. C 107, 015201 (2023)] over large W-region

Isovector contribution to charged pion production



For high-E, the VV cross section is

$$\frac{d^2\sigma^{VV}}{dW\,dQ^2} = \frac{G_F^2 \cos\theta_c}{2E^2\,(2\pi)^3}\frac{k_W}{1-\epsilon}\left(R_T^{VV} + \epsilon R_L^{VV}\right)$$

● ANL-Osaka DCC model

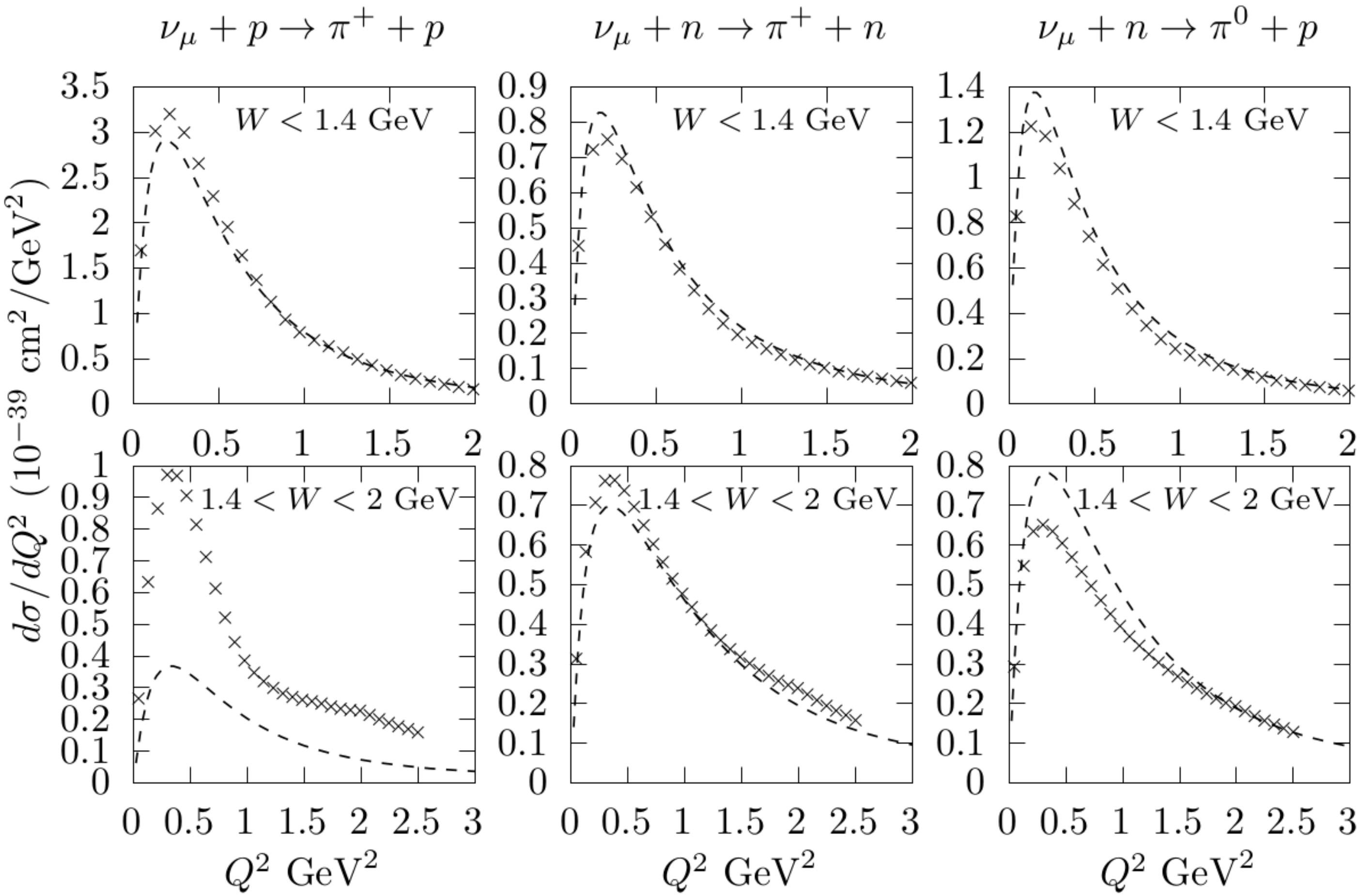
Nakamura, Kamano and Sato,
Phys. Rev. D92, 074024 (2015)

● MAID07

Drechsel, Kamalov and Tiator,
Eur. Phys. J. A34, 69-97 (2007)

The hybrid model is based (partly) on MAID07 results

Isovector contribution to neutrino pion production: flux-averaged



BEBC flux-folded

$$\int dE_\nu \Phi(E_\nu) \frac{d\sigma(E_\nu)}{dQ^2}$$

$\langle E_\nu \rangle \approx 20$ GeV

- - = The hybrid model

XX = ANL-Osaka DCC model

Nakamura, Kamano and Sato,
[Phys. Rev. D92, 074024 (2015)]

How to assign uncertainty to isovector ?

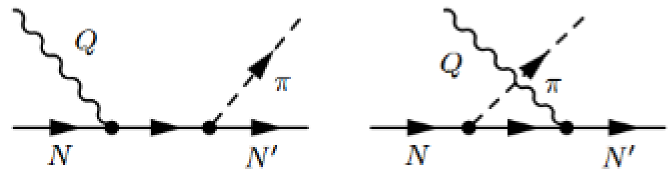
Electroweak single pion production: some modeling

Ingredients

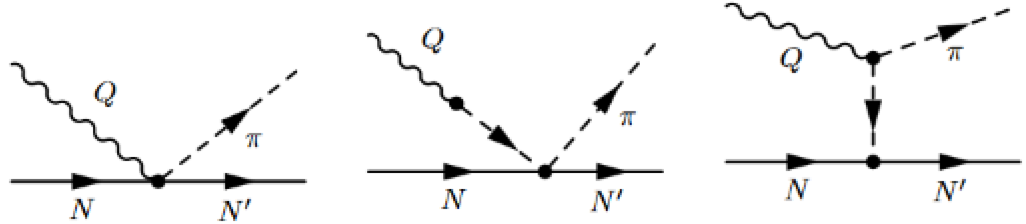
Background in non-linear σ model
Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]

Regge model for BG at high W
R. Gonzalez-Jimenez et al.
[Phys. Rev. D 95, 113007 (2017)]

Partial unitarization in Delta region
Fit of Delta axial coupling
L. Alvarez-Ruso et al.
[Physical Review D93, 014016 (2016)]



Nucleon pole (NP) Crossed nucleon pole (CNP)



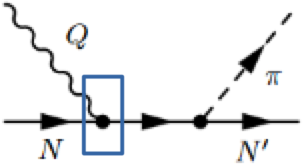
Contact-term (CT) Pion-pole (PP) Pion-in-flight (PF)

Tree-level 'background'
- Dressed with form-factors
- Added coherently to resonances (see extra slides)

Future work → (M. Hooft Ugent)

Includes πN rescattering
Through K-Matrix

Resonance axial couplings



Spin ½ : $\Gamma^{\mu}_{QRN,A} = G_A \gamma^\mu \gamma^5 + \frac{G_P}{M_N} Q^\mu \gamma^5$

Naive PCAC and pion-pole dominance inform both couplings at low-Q^2

$$F_A(0) = f_\pi \frac{\sqrt{2} f_{\pi NR}}{m_\pi} \qquad G_P = 2M_N (M_R \pm M_N) \frac{F_A(Q^2)}{Q^2 + m_\pi^2}$$

Spin 3/2 :

$$\Gamma^{\beta\mu}_A = \frac{C_3^A}{M}\left(g^{\beta\mu} \not{Q} - Q^\beta \gamma^\mu\right) + \frac{C_4^A}{M^2}\left(g^{\beta\mu} Q \cdot k_R - Q^\beta k_R^\mu\right) + C_5^A g^{\beta\mu} + \frac{C_6^A}{M^2} Q^\beta Q^\mu,$$

PCAC and π-pole dominance:

$$C_A^5(0) = f_\pi I_{iso} \frac{\sqrt{2} f_{\pi NR}}{m_\pi \sqrt{3}}$$

$$C_A^6(Q^2) = -M_N^2 \frac{C_A^5(Q^2)}{Q^2 + m_\pi}$$

No constraints for Q^2 dependence

No constraint on C_3 or C_4

Fit bubble chamber data in the delta region: *partial* unitarity

L. Alvarez-Ruso, E. Hernández, J. Nieves, M.J. Vicente Vacas
[Phys. Rev. D93, 014016 (2016)]

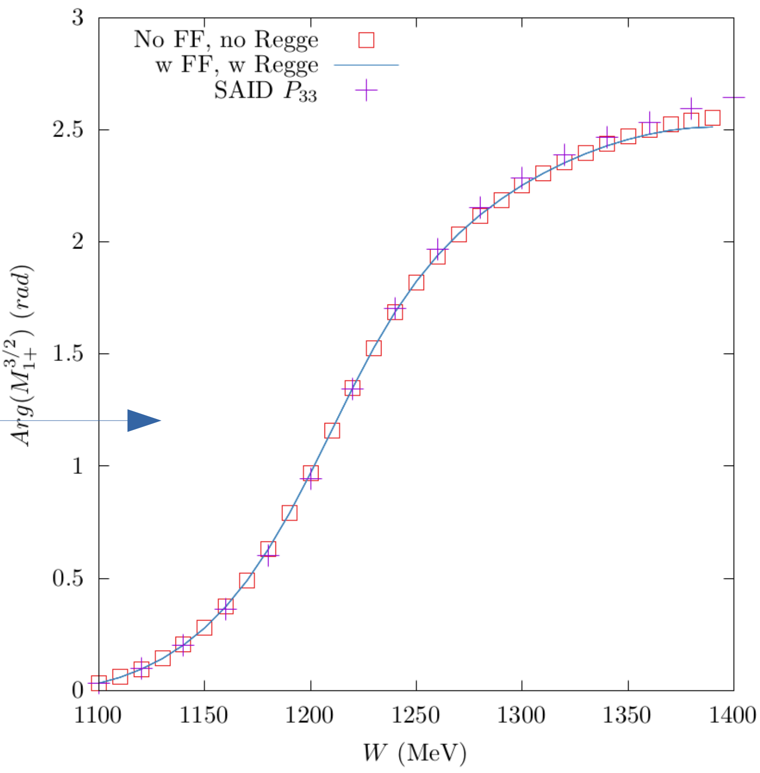
Fit of axial Delta form factor(s)

- ANL data
- Adler model for Delta
  → $C_4 = -C_5/4$, $C_3 = 0$
  → C_4 and C_3 contributions small

- **Watson's theorem in *Δ* dominated Vector and Axial multipoles**

$$J^\mu = J^\mu_V(s,t,Q^2)\Phi_V(Q^2,s) - J^\mu_A(s,t,Q^2)\Phi_A(Q^2,s)$$

The Olsson phases are fixed by
Requiring that the dominant P_{33} amplitudes
Have the πN scattering phase-shift



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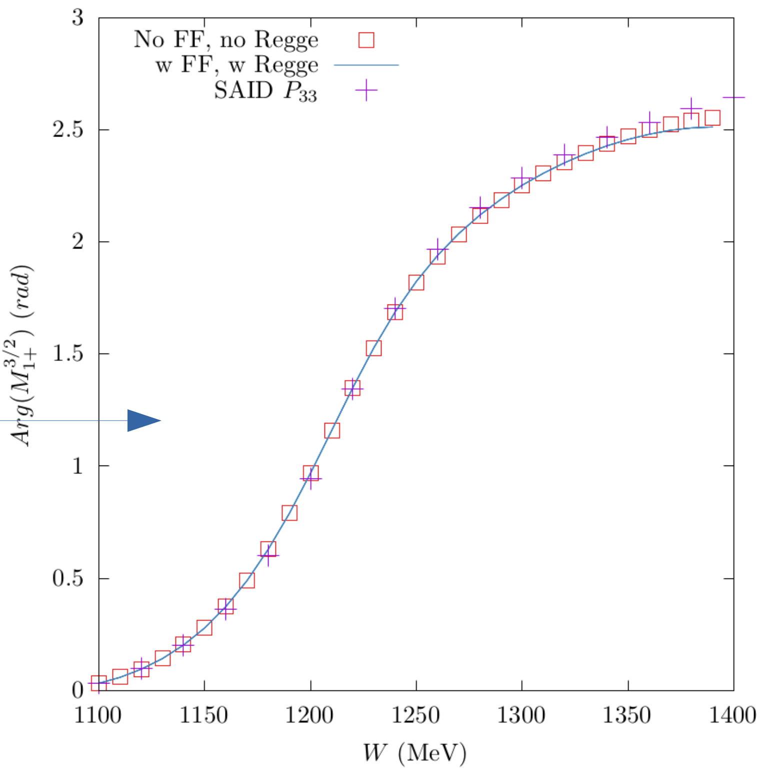
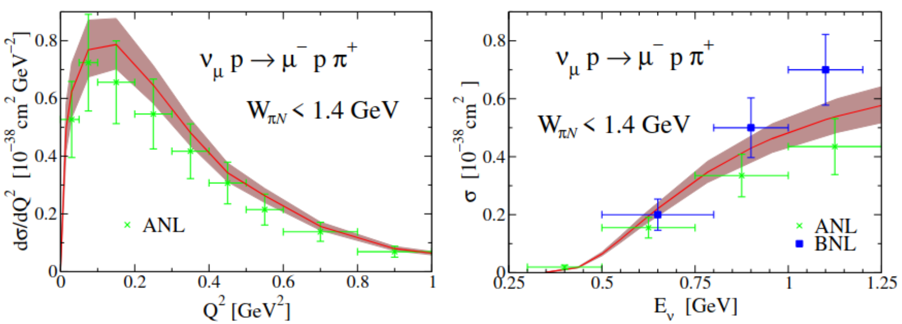
- **Watson's theorem in *Δ* dominated Vector and Axial multipoles!**



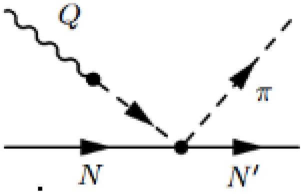
$$J^\mu = J^\mu_V(s,t,Q^2)\Phi_V(Q^2,s) - J^\mu_A(s,t,Q^2)\Phi_A(Q^2,s)$$



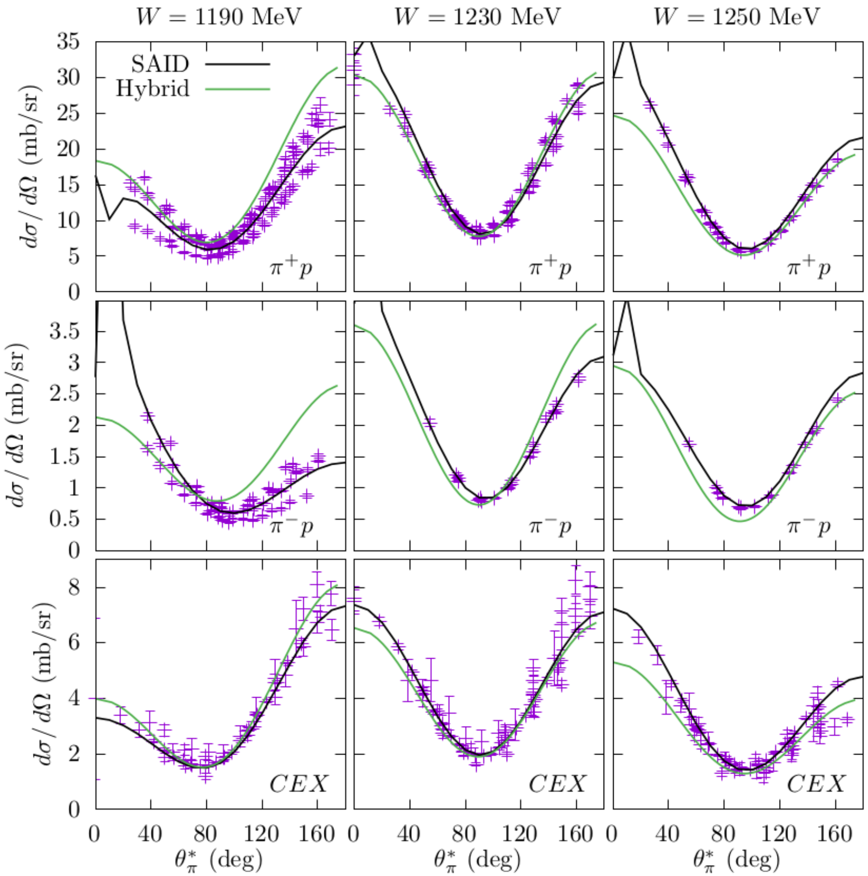
Find $C_5(0)$ consistent with PCAC
parametrize $C_5(Q^2)$

Pion-nucleon scattering as limiting case

$$\vec{A}^{\mu} = f_{\pi}\partial^{\mu}\vec{\phi} + g_A\bar{\Psi}\gamma^{\mu}\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{1}{2f_{\pi}}\bar{\Psi}\gamma^{\mu}(\vec{\phi}\times\vec{\tau})\Psi$$



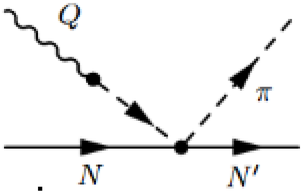
Axial-current has a pion-pole → the same dynamics for πN scattering



- Can gauge model with πN angular distributions

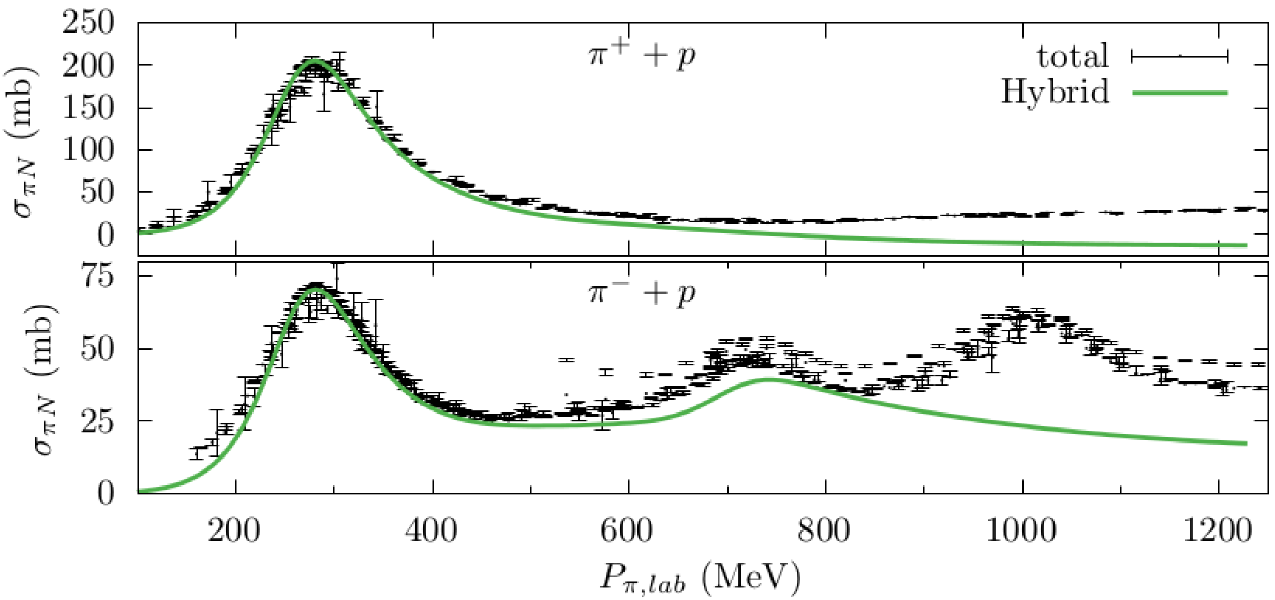
Pion-nucleon scattering

$$\vec{A}^\mu = f_\pi \partial^\mu \vec{\phi} + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{1}{2 f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi$$



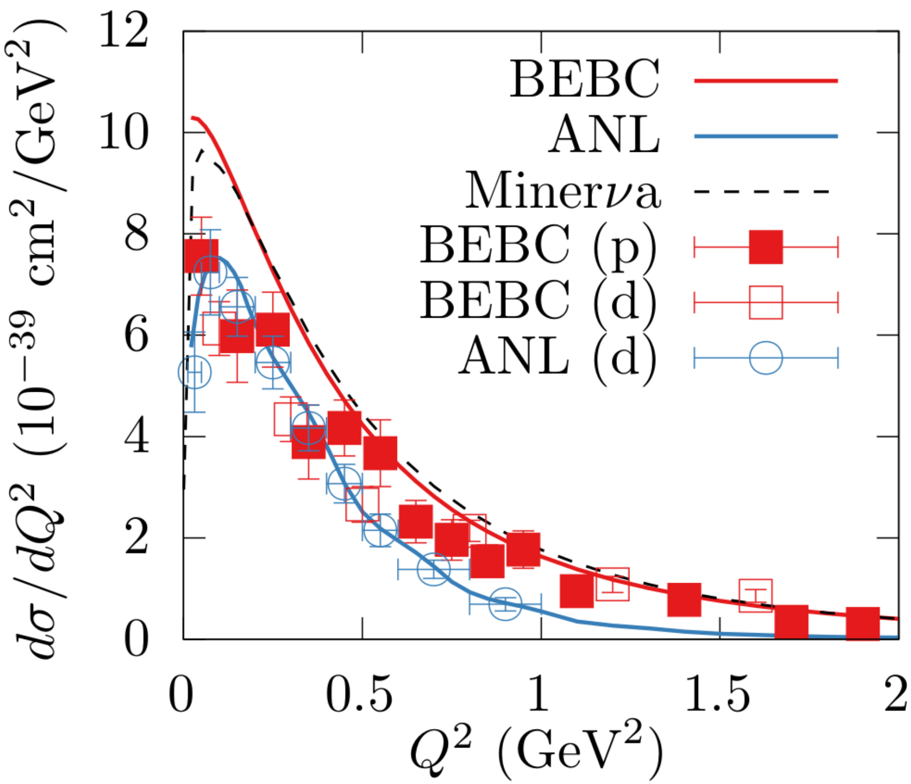
Axial-current has a pion-pole → the same dynamics for πN scattering

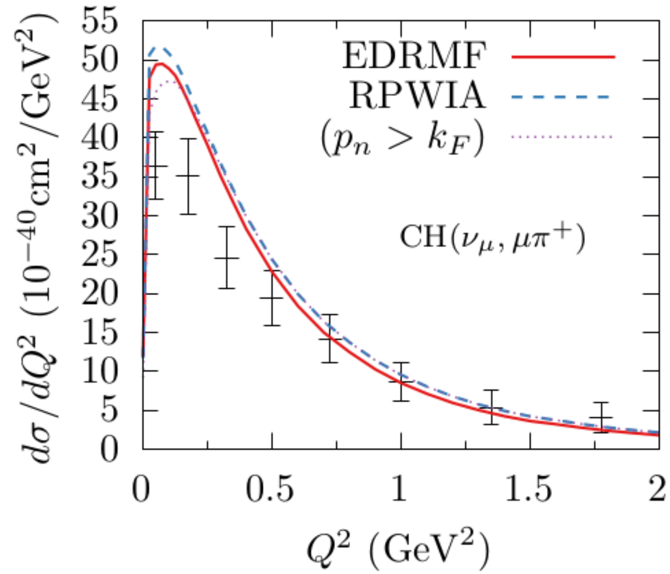
- Can gauge model with πN angular distributions
- Total CS sensitive to Im(A)



Comparison to bubble chamber data & MINERvA

[A.N. et al. PRD 107 (2023) 5, 053007], see also : [J. Garcia-Marcos et al. Arxiv:2310.18056] (new!)

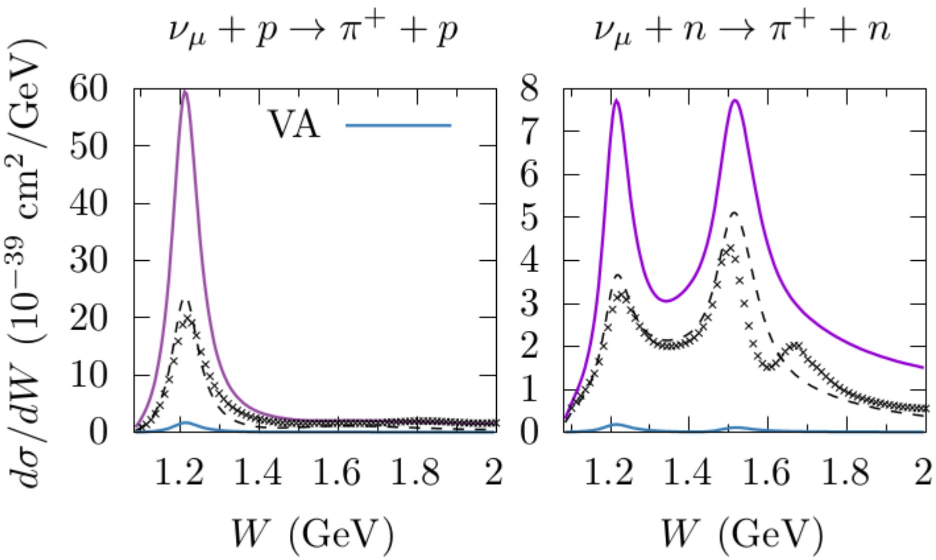


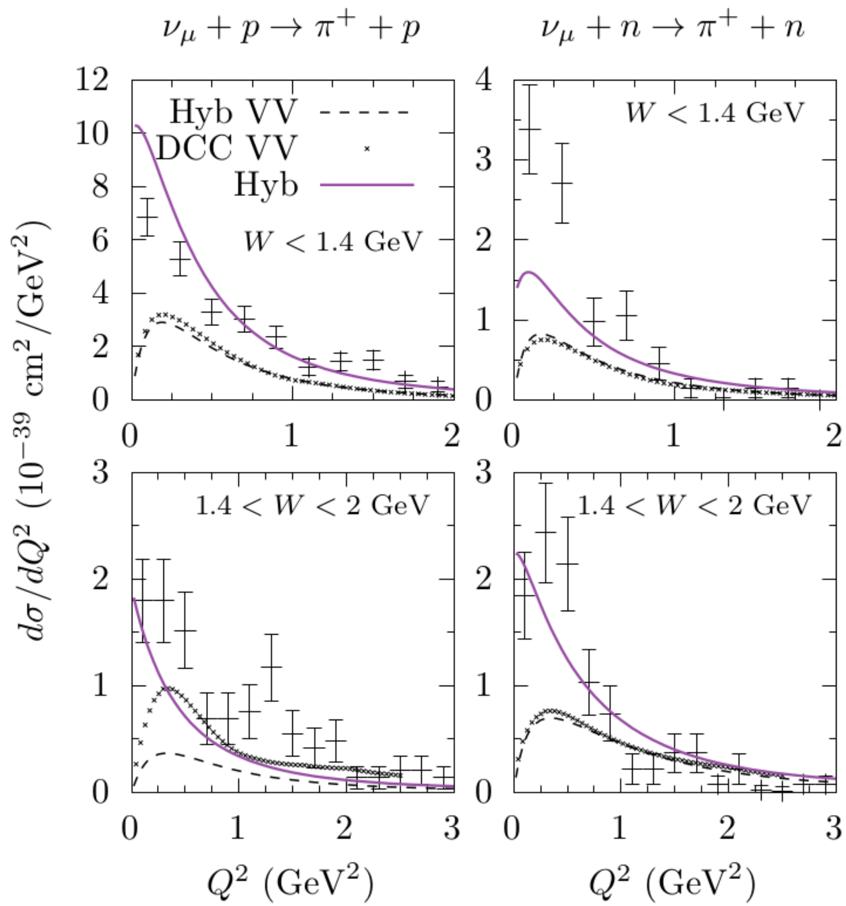


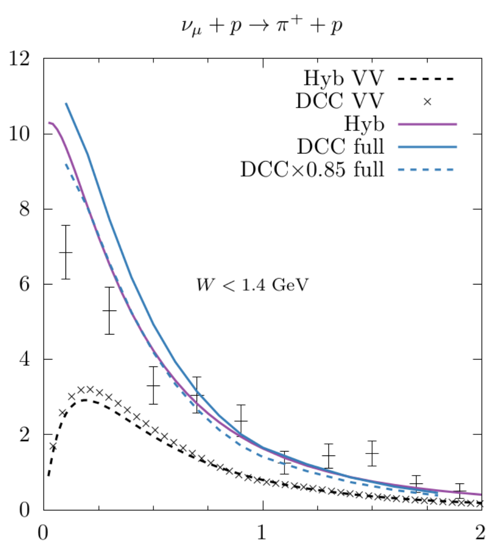
Data and calculations for W < 1.4 GeV ~ Δ dominated

The fit to ANL data does not reproduce the BEBC data
Even in the π^+ channel

BEBC flux-folded = VV + AA

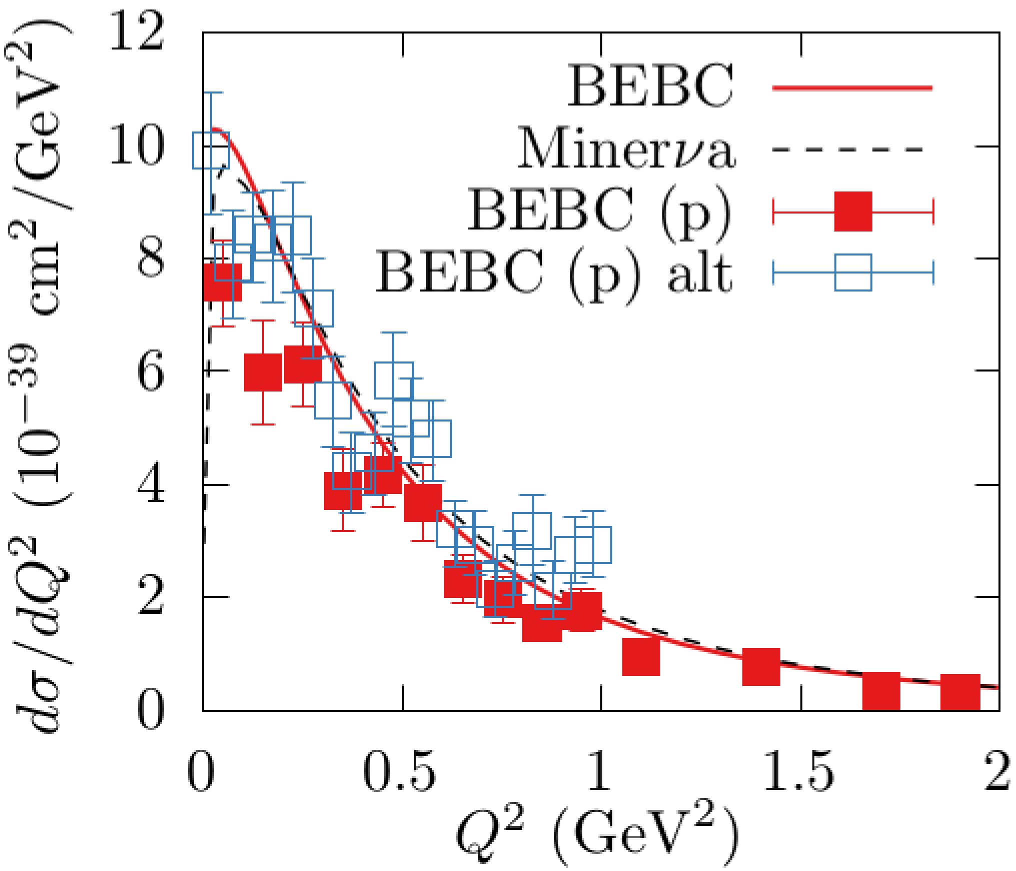






At high energy VA terms are negligible
→ Can isolate AA contribution

BEBC flux-folded: alternative dataset



Data from BEBC:

[Z. Phys. C 43, 527–540 (1989)]

[Nucl.Phys.B 176 (1980) 269]

- related to low-p_N efficiency
- Unclear how correction is done
- Errors likely underestimated?

Will revisit data with unitary model
(M. Hooft, UGent)

Need new data on proton (deuteron)!

Fermilab

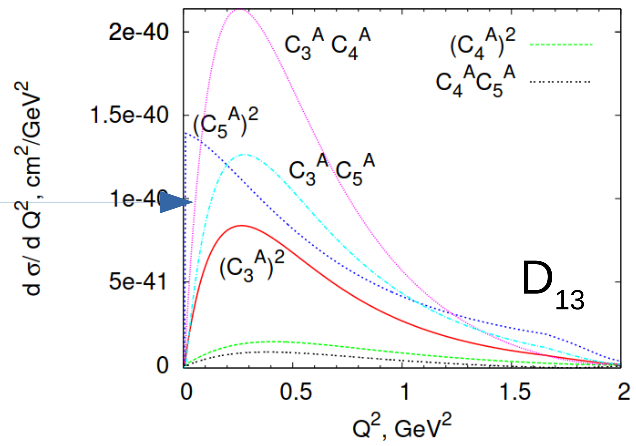
Axial couplings to higher-mass resonances

For Δ we're relatively 'safe':

C_3 and C_4 give small contribution

Bubble chamber data to constrain Q^2-dependence

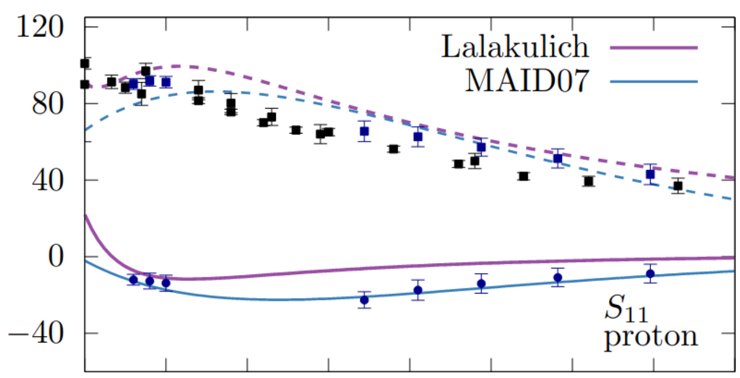
Lalakulich et al.
[PRD 74, 014009 (2006)]



Higher mass resonances:

- The contribution from C_3 and C_4 can be large!
  set $C_i = 1$ → 100 % uncertainty

- Form factors can be far from dipoles

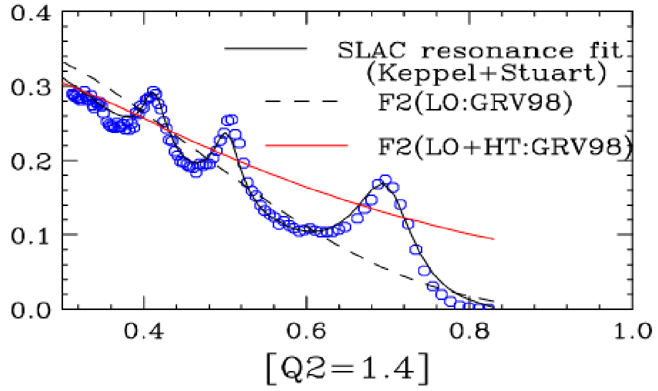


S_{11} amplitude drops off remarkably slowly With Q^2 in interactions with the proton

Pion production in the GENIE-based analysis: higher mass resonances
(based on description in [Phys. Rev. D 100, 072005 (2019)])

Add incoherently DIS contribution using Bodek&Yang pdf [J. Phys. G: Nucl. Part. Phys. 29 1899] with a hadronization model

But hadronic & DIS descriptions
Should be ‘dual’ ?



Pion production in the GENIE-based analysis: electron scattering
(based on description in [Phys. Rev. D 100, 072005 (2019)])

But hadronic & DIS descriptions
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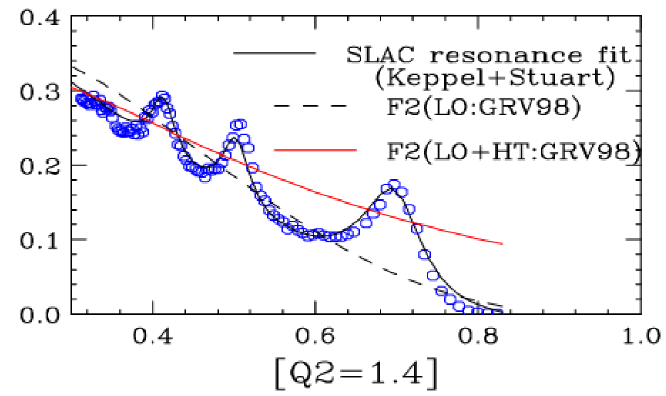
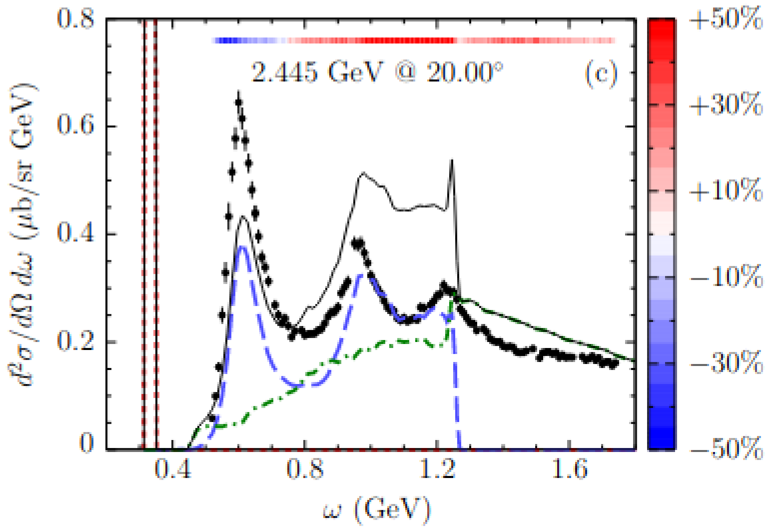
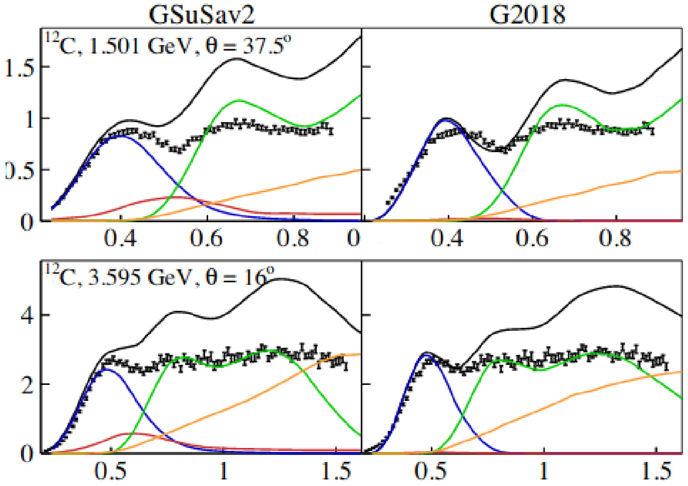


This addition of DIS/HAD in GENIE has been shown explicitly to lead to double-counting



Ankowski & Friedland
[PRD 102, 053001 (2020)]



E4nu collaboration
[PRD 103, 113003 (2021)]

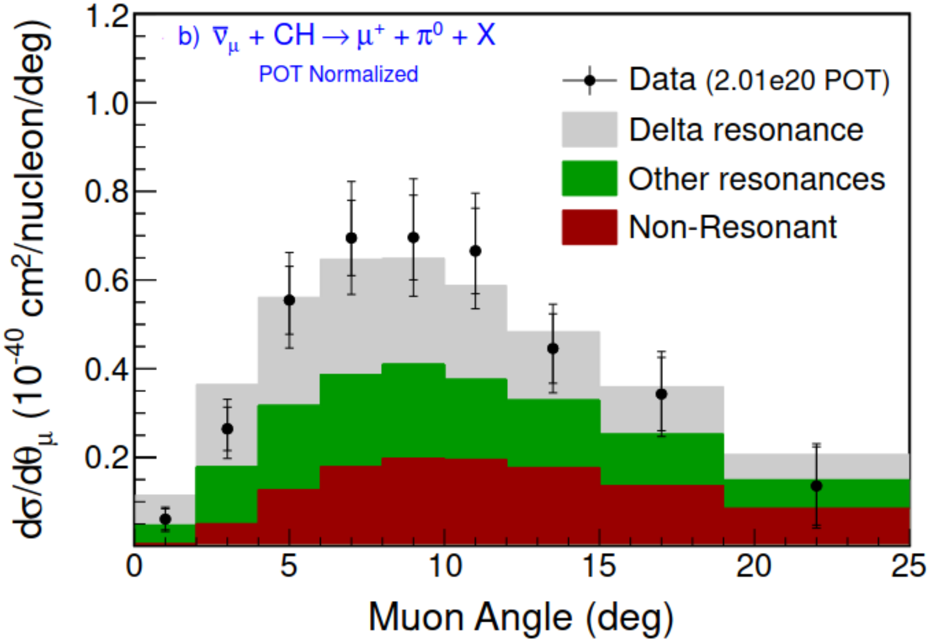
Fermilab

Pion production in the GENIE-based analysis: electron & neutrino

(based on description in [Phys. Rev. D 100, 072005 (2019)])

This addition of DIS/HAD in GENIE has been shown explicitly to lead to double-counting

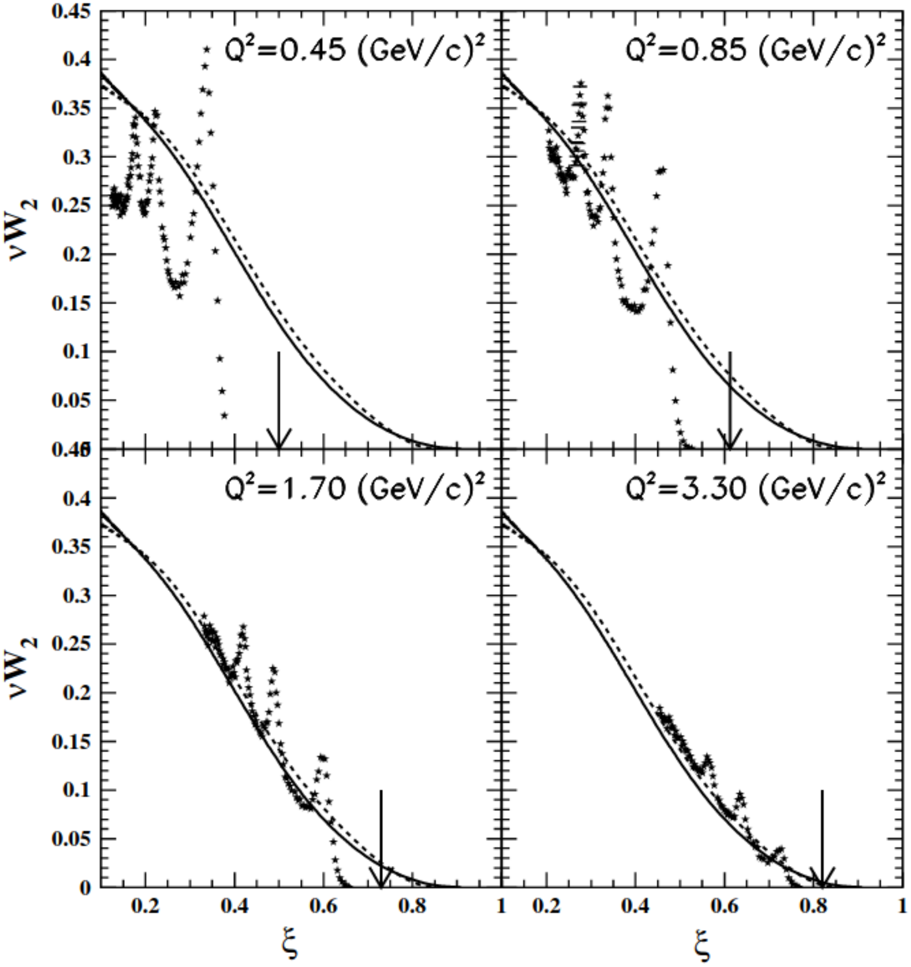
But the large 'non-resonant' contribution gives reasonable magnitude



Quark-hadron duality in neutrino interactions
[T. Sato EPJ:ST (2021) 230:4409-4418 (2022)]!!

Bloom-Gilman duality in (e,e') experiments on proton

- At large Q^2 the F_2 structure function in the resonance region oscillates around and approaches the DIS structure function



[Niculescu et al. PRL85, 1186 (2000)]

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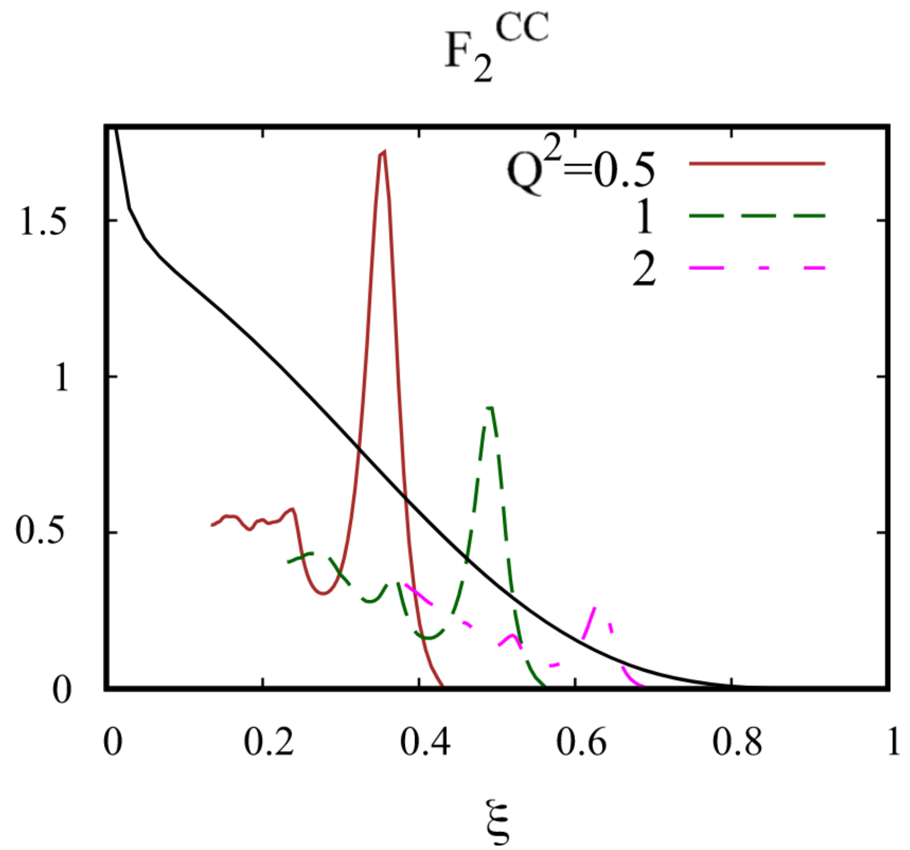
For CC ν scattering

- ANL-Osaka DCC model underestimates F_2 from DIS → Unconstrained FF = 0



[T. Sato EPJ:ST (2021) 230:4409-4418 (2022)]

see also:
[O. Lalakulich et al. PRC79 015206 (2009)]

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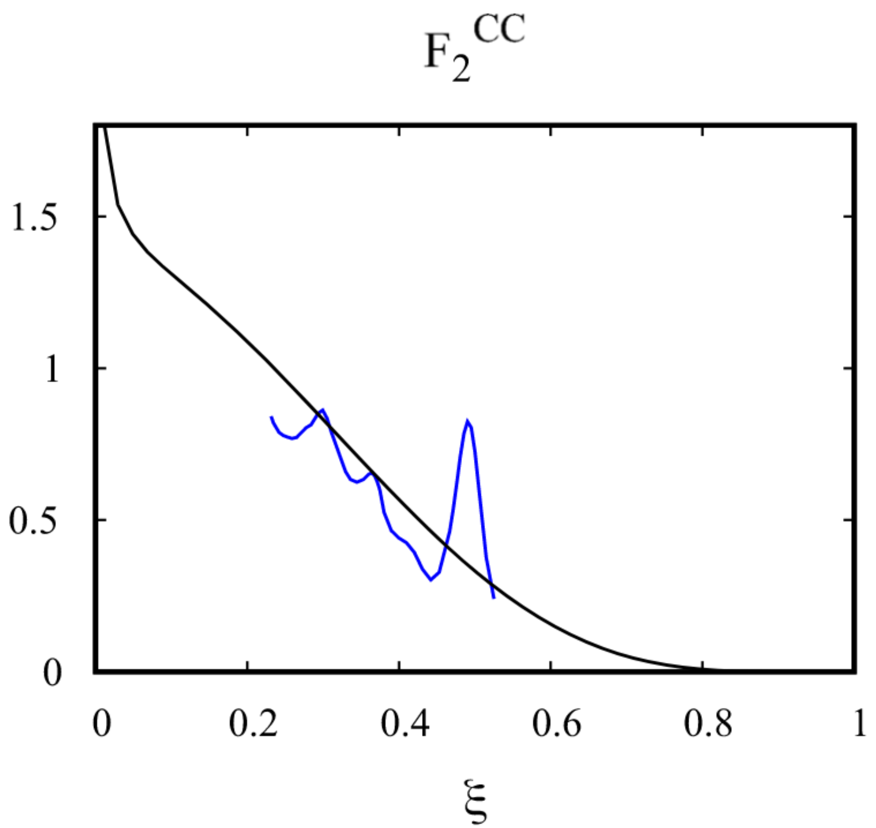

For CC ν scattering

- ANL-Osaka DCC model underestimates F_2 from DIS

- Upon modifying the Q^2-dependence:

$$A_\lambda^A(Q^2) = A_\lambda^A(0)\frac{A_\lambda^V(Q^2)}{A_\lambda^V(0)}$$

→ Improved agreement with DIS

Quark-hadron duality in neutrino interactions
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For CC ν scattering

- Upon modifying the Q^2-dependence:

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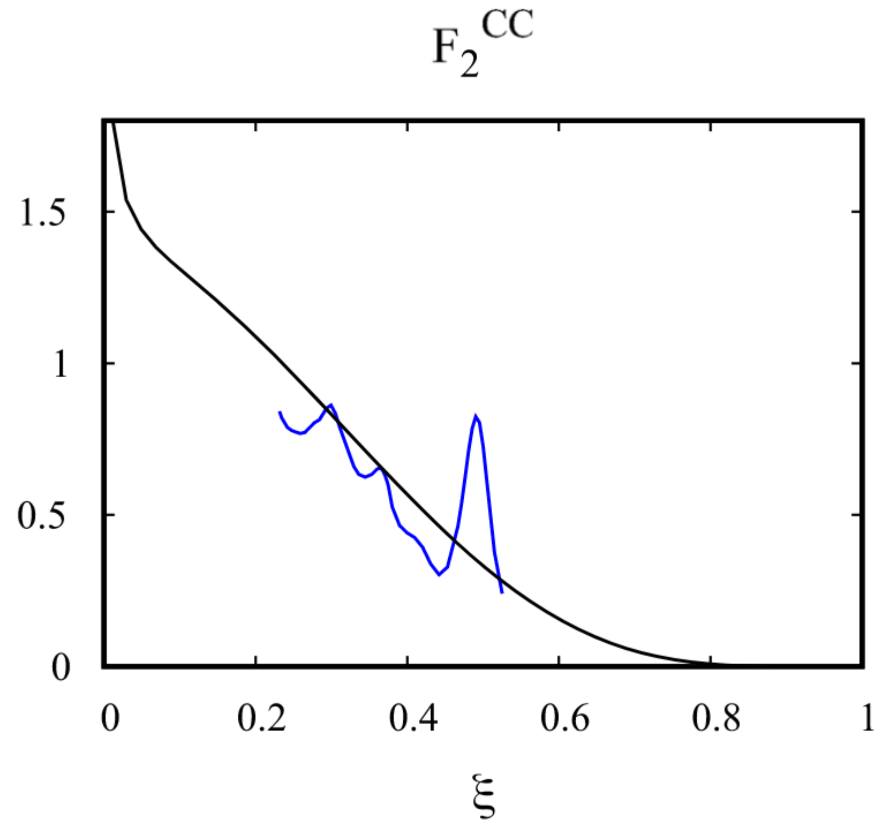
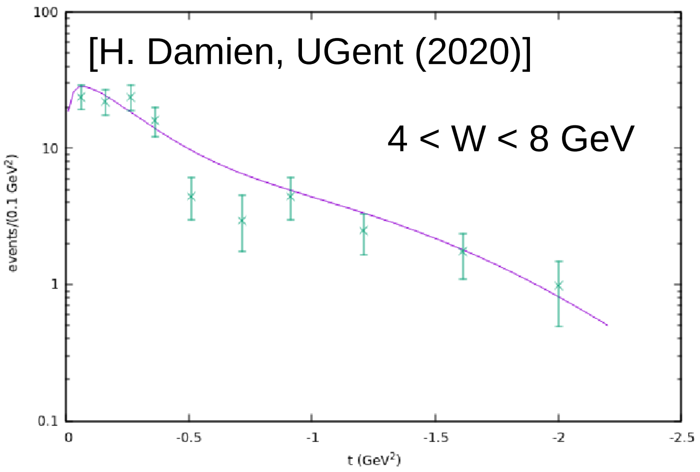
Improved agreement with DIS

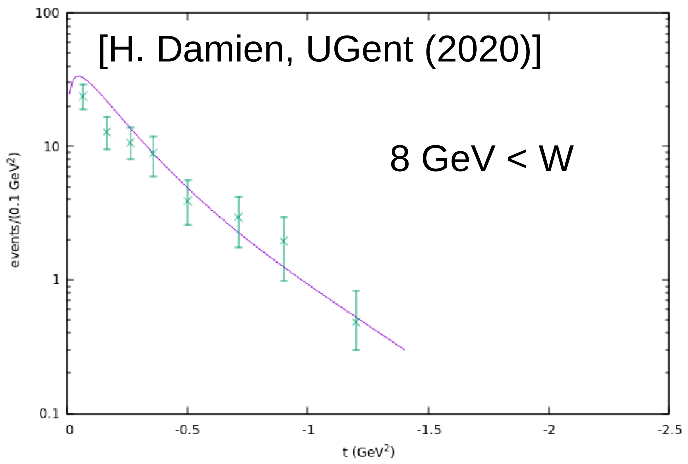
Can duality be used as a constraint ?
= work in progress



Fermilab

Regge approach for high-energy pion production
(& ‘Regge-Resonance duality’)





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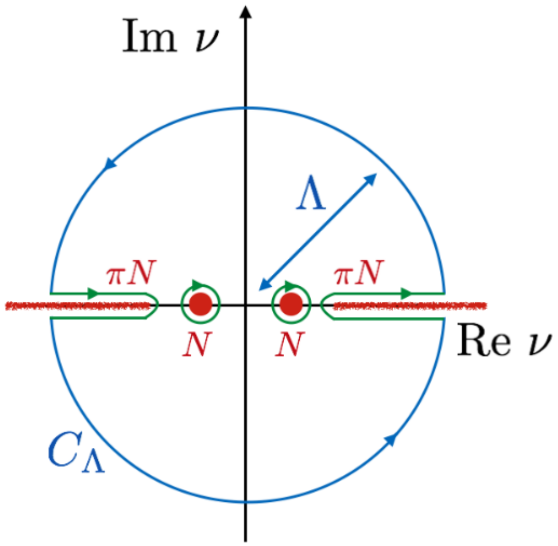


From [PRD 98, 014041 (2018)]

Analiticity connects the high-energy amplitude to the resonance region
See work by JPAC:

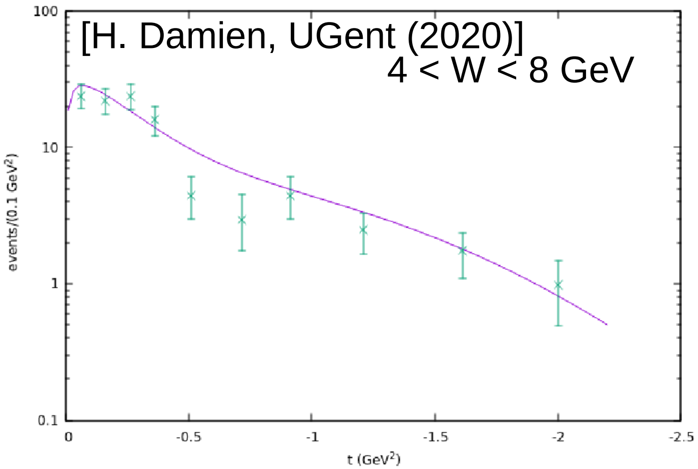
[PRD 98, 014041 (2018)]
[PRD 95, 034014 (2017)]
[PRD 92, 074004 (2015)]

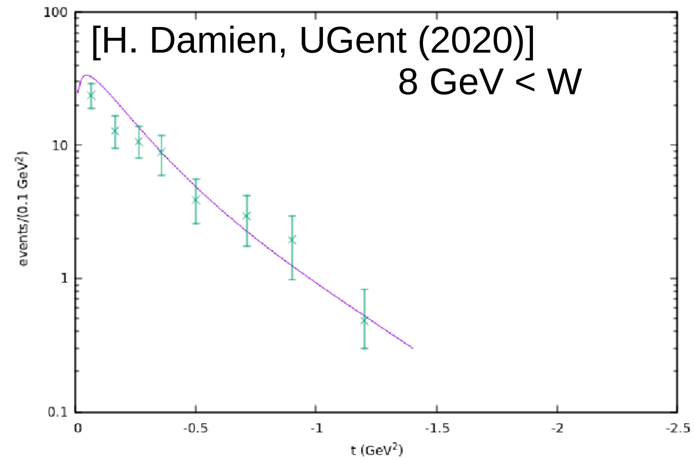
(Details in extra slides)

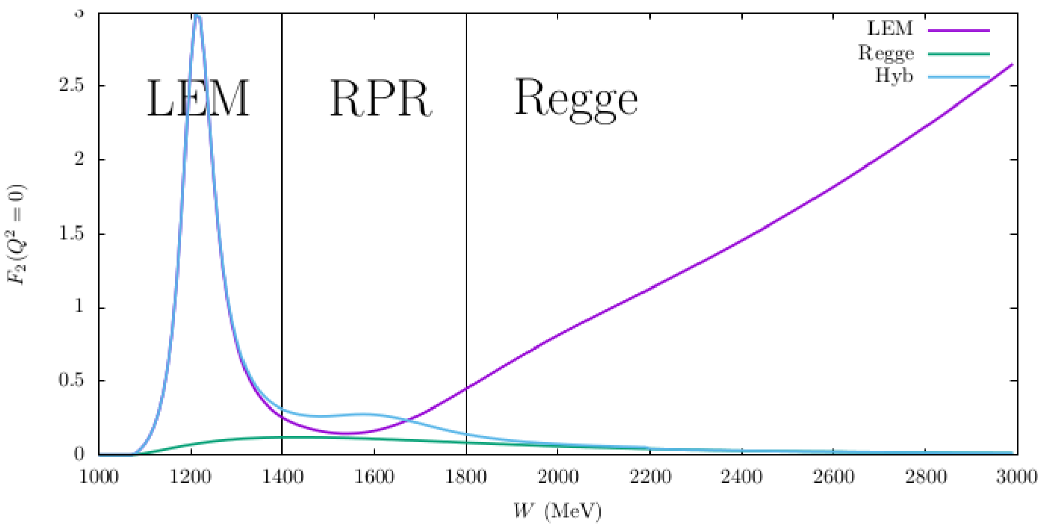
Fermilab

Hybrid Regge-plus-resonance description

R. Gonzalez-Jimenez et al.
[Phys. Rev. D 95, 113007 (2017)]







The high-energy behaviour can be built in naturally

Conclusions

- Electron and photoproduction data on proton is plentiful
  → Many theoretical approaches and analyses available

- Require neutron measurements for isovector-isoscalar separation
  → Deuteron target data: need to describe FSI
  → New experimental efforts : [CLAS, PRC 107, 015201]
  → How to assess uncertainty due to isovector current ?

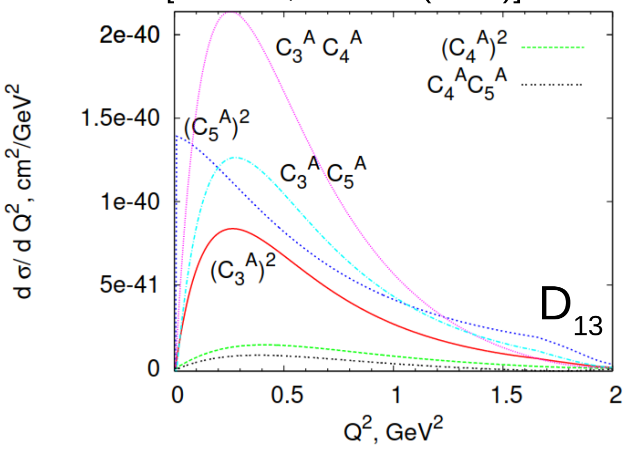
- Delta resonance mostly dominated by 1 axial FF
  → Can constrain with bubble chamber data
  → Inconsistencies between datasets, need deuteron

- Higher mass resonances not well constrained

Where to go from here ? : nucleons

Difficulties in describing data in the delta region
→ This is more severe for higher-mass resonances

Lalakulich et al.
[PRD 74, 014009 (2006)]



Constraints could come from:

- Progress in (l)QCD for axial form-factors
  [L. Barca et al. PoS LATTICE2021, 359 (2022)]
- ChPT calculations with delta d.o.f
  [Yao et al. Phys. Rev. D 98, 076004 (2018)]
- Quark-Hadron duality
  [T. Sato, Eur. Phys. J. ST 230, 4409 (2021)]
- Analyticity & Duality with the Regge regime → FESR
  [V. Mathieu et al. (JPAC) Phys. Rev. D 98, 014041 (2018)]
- **Modern experiments on hydrogen & deuterium**
  L. Alvarez-Ruso et al., (2022), arXiv:2203.11298 [hep-ex]

Fermilab

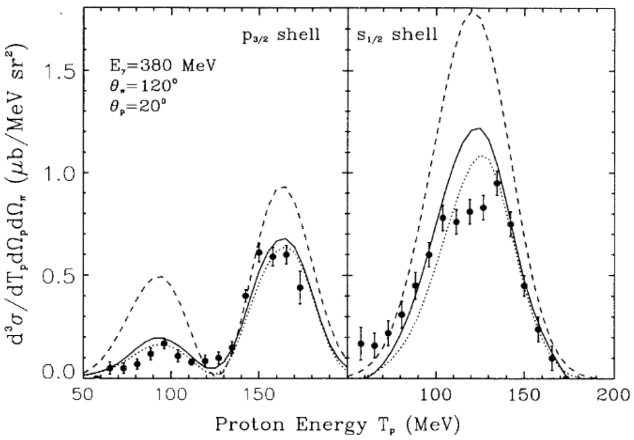
Where to go from here ? : nuclei

Neutrino experiments with broad fluxes
→ Easy because details are smeared out
→ Hard because **exclusive** signals are impossible

In exclusive conditions one can cut away everything that is hard to describe
→ 'Absorb' it in an optical potential

For neutrino experiments rescattering has to be modeled explicitly! → Cascade models/transport

Exclusive γ pion production
[Li et al. PRC 48 816]



exclusive
(e,e'p)

Severely restrict E_m and lepton kinematics

Fixed-E experiment
(ν_l,l'p) or (e,e'p)
with large acceptance

Combining E_m cuts with broad lepton kinematics to study rescattering

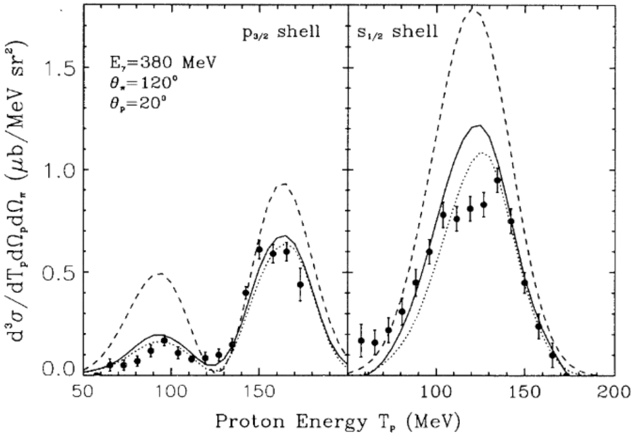
Accelerator
(ν_l,l'p)

No E_m restriction possible

Fermilab

Where to go from here ? : nuclei

Exclusive γ pion production
[Li et al. PRC 48 816]



Neutrino experiments with broad fluxes
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→ 'Absorb' it in an optical potential

For neutrino experiments rescattering has to be modeled explicitly! → Cascade models/transport

Electron scattering experiments Positioned here can provide significant Information for ν-A interactions!

e.g. e4nu collaboration with CLAS data

exclusive
(e,e'p)

Severely restrict E_m and lepton kinematics

Accelerator
(ν_l,l'p)

No E_m restriction possible

Other stuff

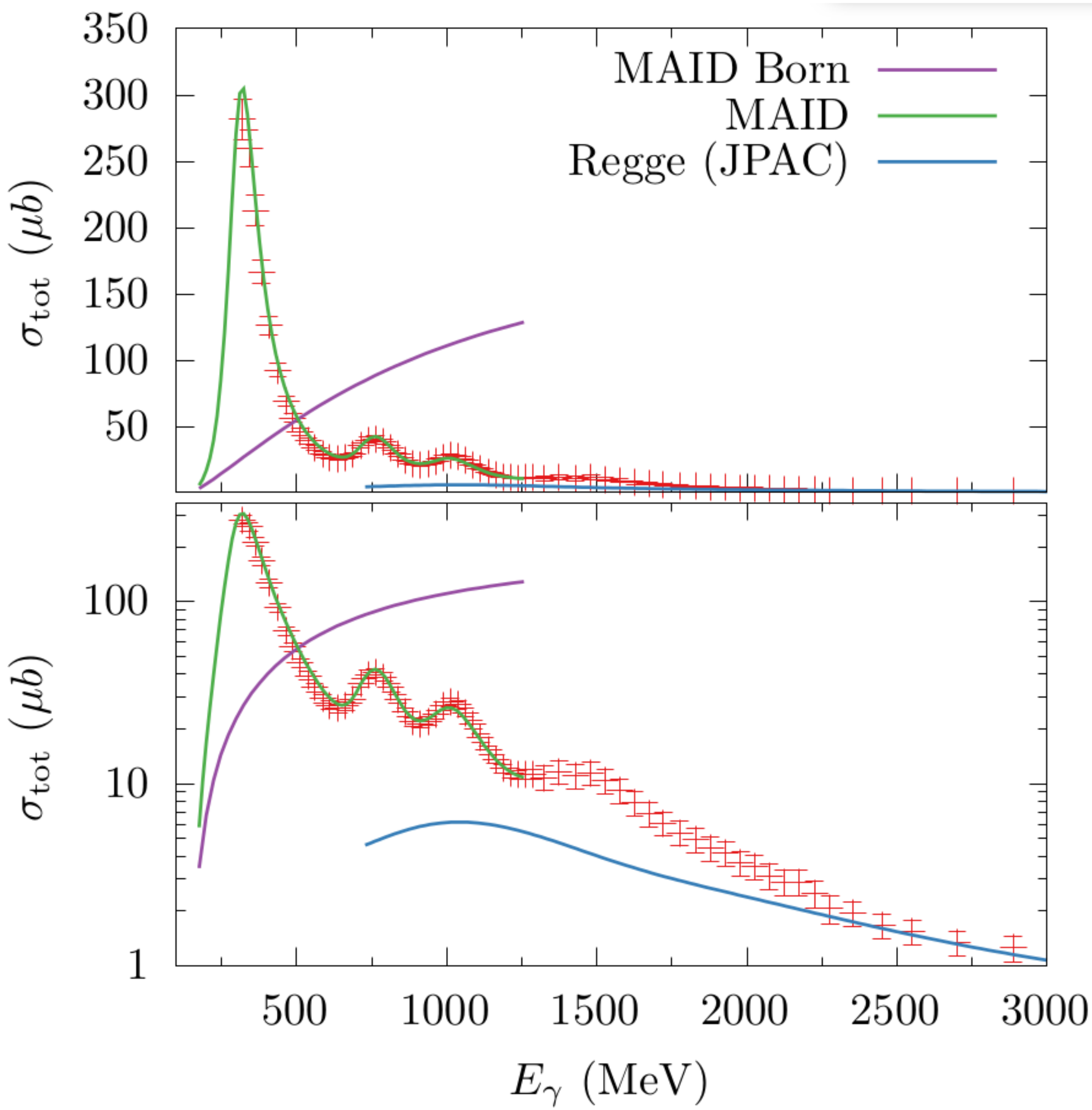
Regge model for the high-W background

Background in non-linear σ model
Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]

The effective tree-level terms are suitable at low-W

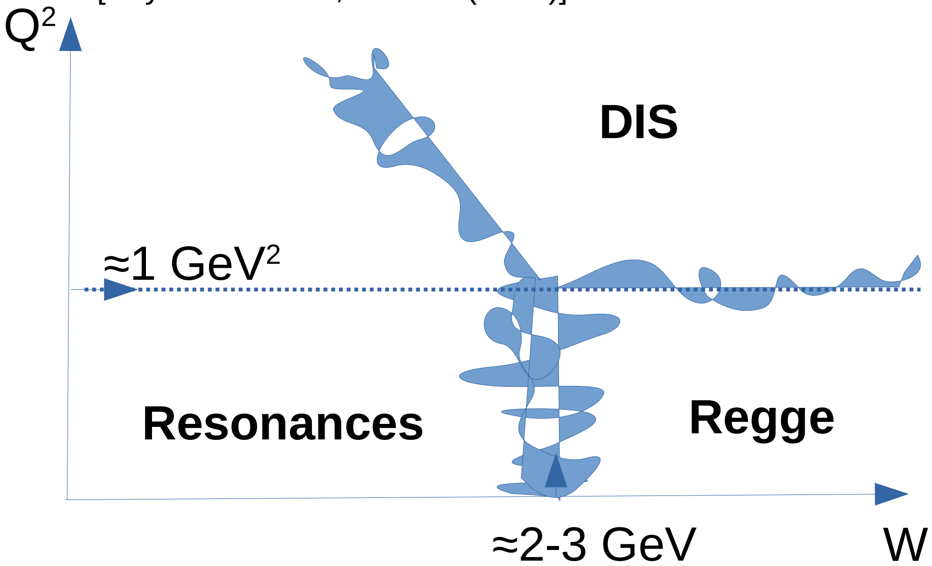
For intermediate W adjusting the phases of resonant contributions is necessary e.g. in MAID

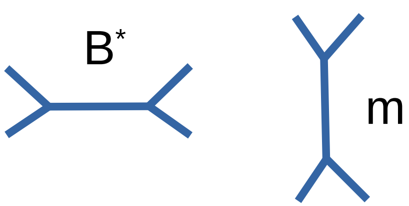
At high-W (low Q^2) a Regge approach describes the amplitude



Regge model (briefly)

R. Gonzalez-Jimenez et al.
[Phys. Rev. D 95, 113007 (2017)]



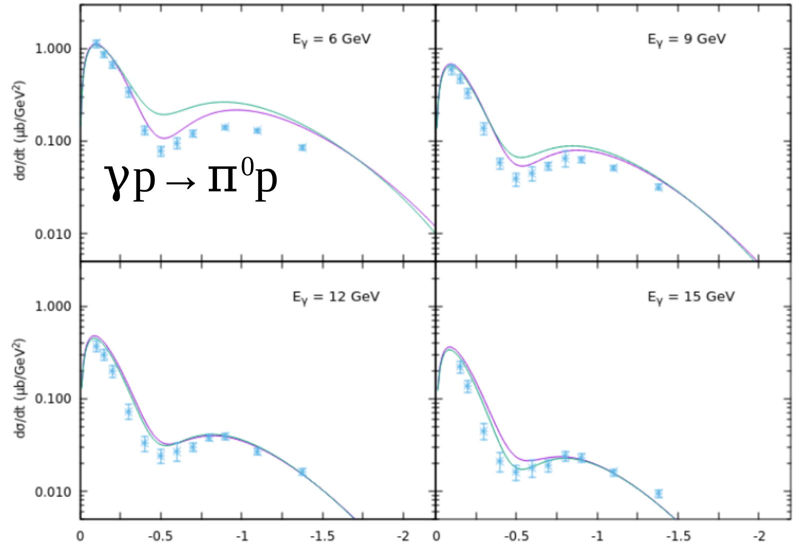


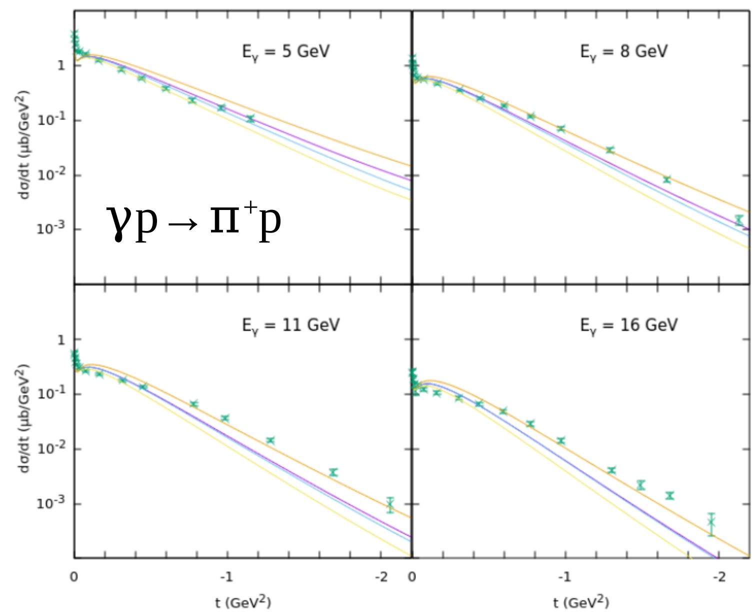
For high-s
Small -t
↓
t-channel meson d.o.f

$s >> M_{B^*}^2 \approx (1\text{-}2\ \text{GeV})^2$

$-t \approx M_m^2 \approx (0.1\text{-}1\ \text{GeV})^2$

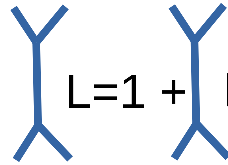
[H. Damien, Masters thesis, UGent (2020)]

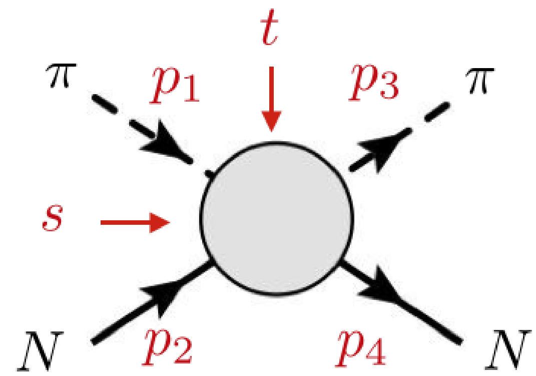




Regge poles (briefly)

$$A(s,t) = \sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$





Partial wave series is a natural description in t-channel does not converge in physical region of s-channel

$$z_t \equiv \cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$

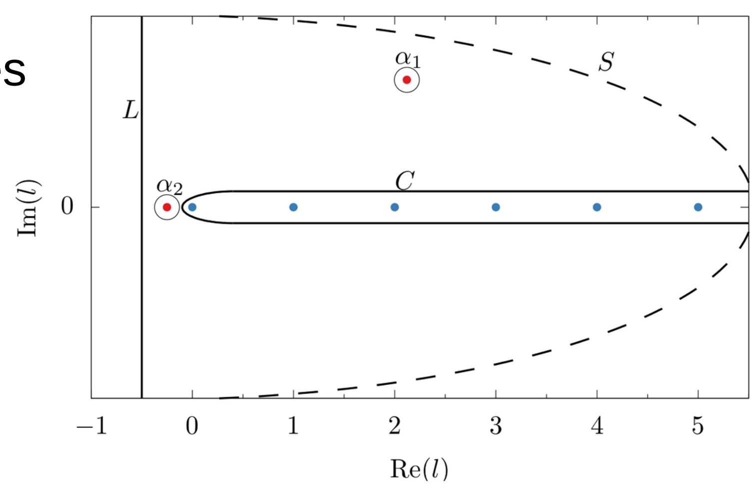
Analytic continuation A(l,t):

$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t) = \frac{-1}{2i} \oint_C A(l,t) \frac{P_l(-z_t)}{\sin(l\pi)}\, dl$$

Assume A(l,t) has isolated singularities

$$A(l,t) \to \frac{\beta(t)}{l - \alpha(t)} \quad \text{for } l \to \alpha(t)$$

α(t) is the position of a
Regge pole
With residue β(t)



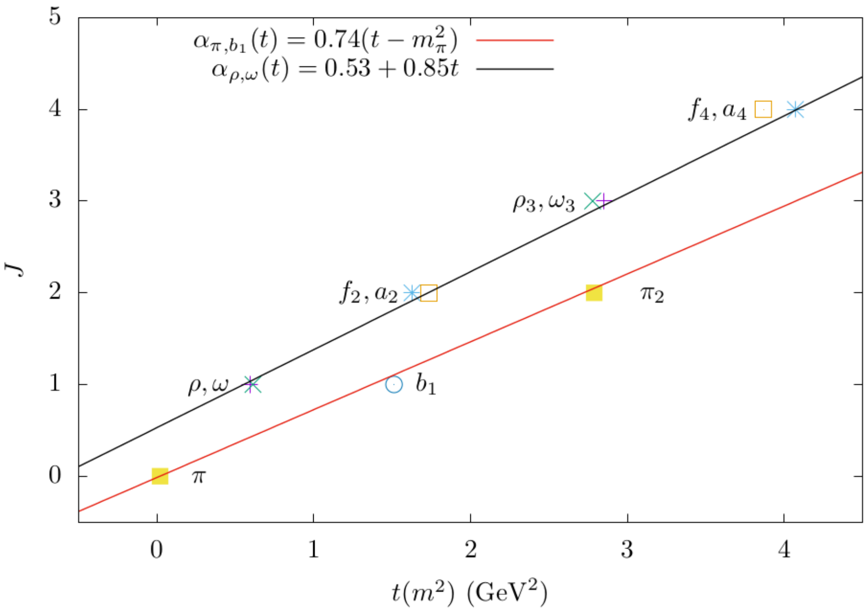
Fermilab

Regge poles (briefly)

$$A(l,t) \to \frac{\beta(t)}{l-\alpha(t)} \text{ for } l \to \alpha(t)$$

$$A(s,t) = \frac{-1}{2i} \oint_{L+S} A(l,t) \frac{P_l(-z_t)}{\sin l\pi} \, dl - \sum_{i=0}^{n} \pi \frac{\beta_i(t) P_{\alpha_i(t)}(-z_t)}{\sin(\pi\alpha_i(t))}.$$

$$A_{l=n}(t) = A(l=n,t) \quad \alpha(t_0) = n$$



Trajectories $\alpha(t) = \alpha' t + \alpha_0$

For t > 0 $\alpha(t)$ describes the spin-mass relation of exchanged particles

For large s: $$A_{pole}(s,t) \to \beta(t)\alpha'\Gamma\left[-\alpha(t)\right](\alpha' s)^{\alpha(t)}$$

$$A_{pole}(s,t) \to \frac{\beta(t)}{t-m^2} \qquad \text{for } t \to m^2$$

Regge poles (briefly)

$$\beta(t)\alpha'\Gamma\left[-\alpha(t)\right](\alpha' s)^{\alpha(t)}$$

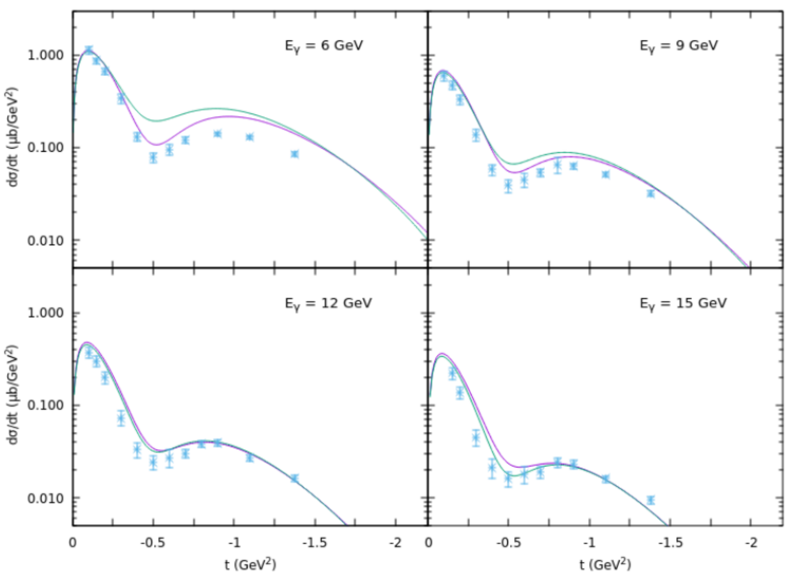
for $t \to m^2$

$$A_{pole}(s,t) \to \frac{\beta(t)}{t-m^2}$$

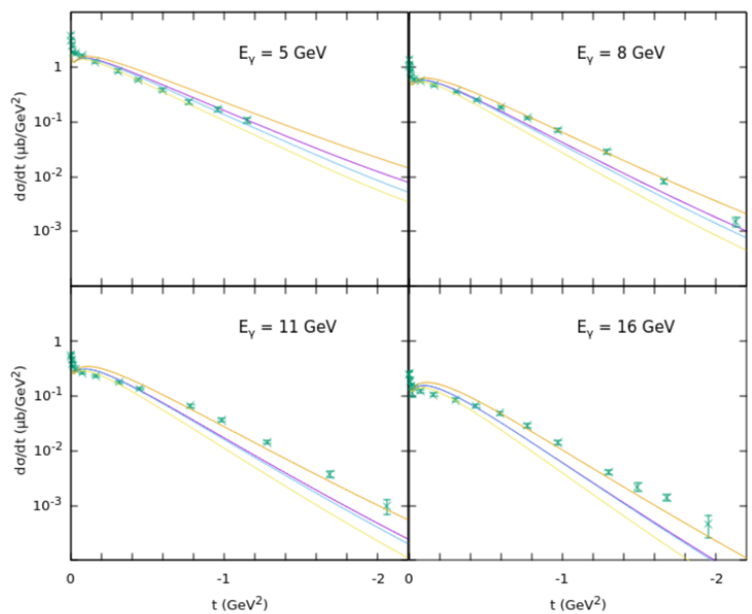
For small -t, β(t) is approximated as

$$A_{Regge}(s,t,Q^2) \approx A_{tree}(t,Q^2) \times \left(t^2 - m^2\right)\left\{\alpha'\Gamma\left[-\alpha(t)\right](\alpha' s)^{\alpha(t)}\right\}$$

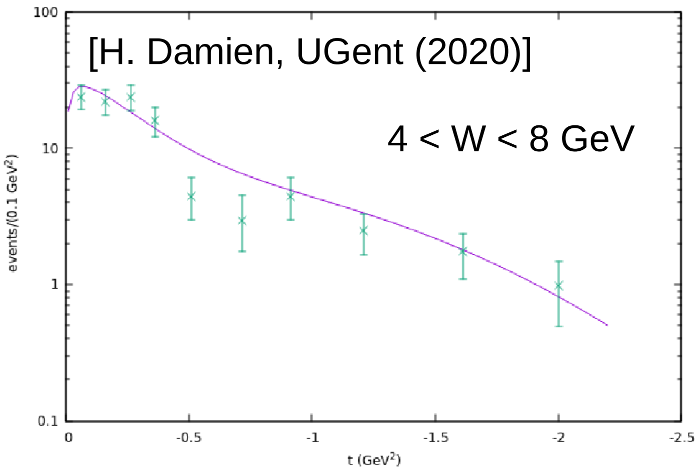
$\gamma p \to \pi^0 p$ dominated by ω exchange

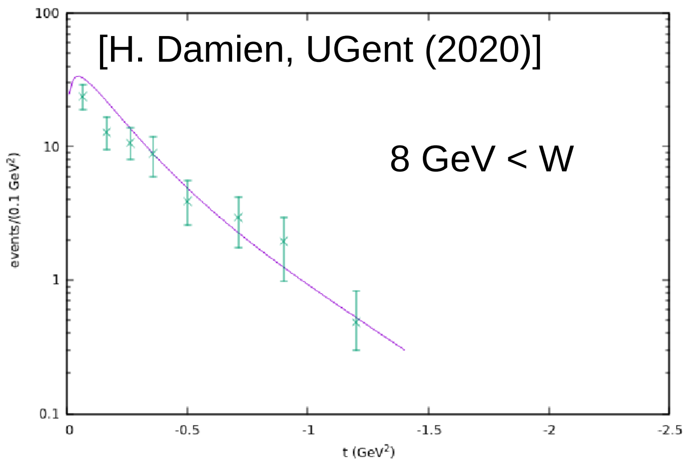


$\gamma p \to \pi^+ p$ from pion-exchange (GLV perscription)



Regge approach for high-energy neutrino-pion production





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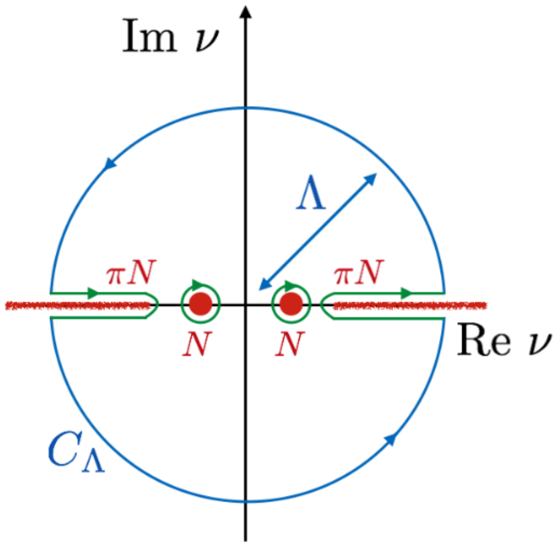

From [PRD 98, 014041 (2018)]

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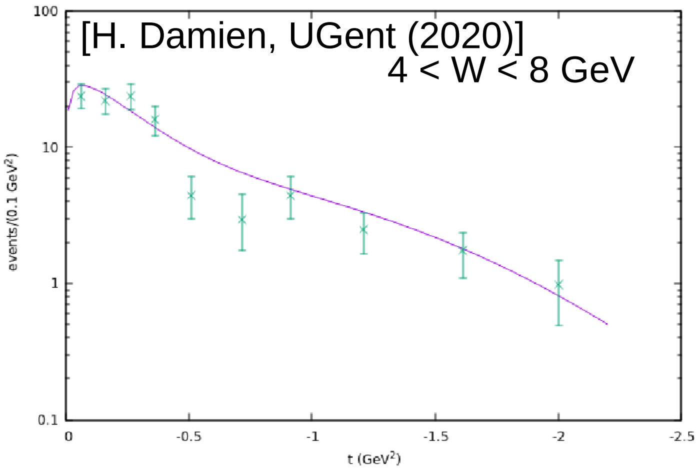
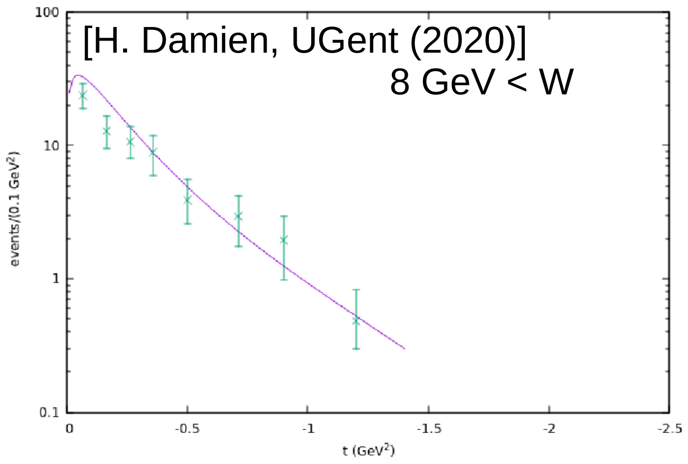
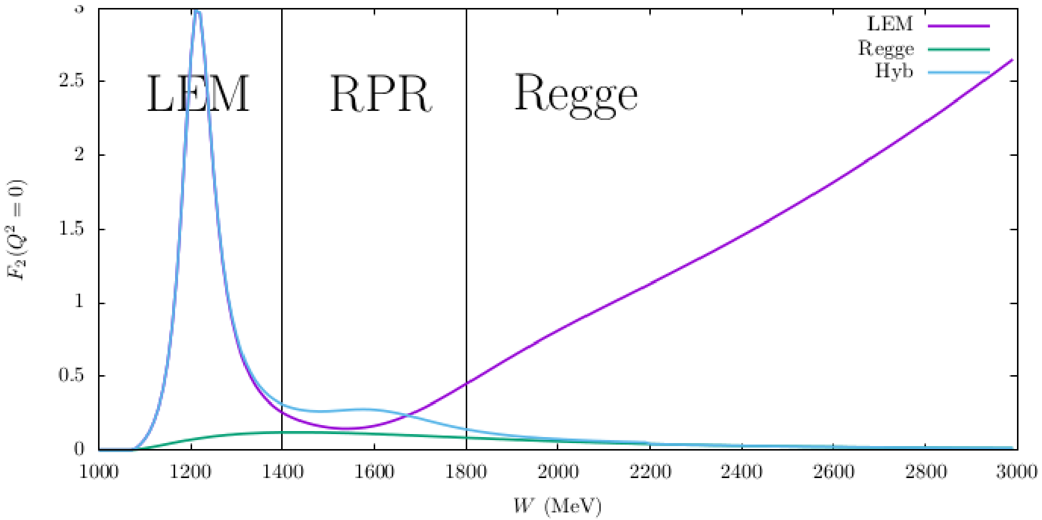
R. Gonzalez-Jimenez et al.
[Phys. Rev. D 95, 113007 (2017)]



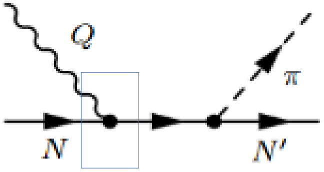




$p \to p + \pi^+$

Background contributions: nucleon form-factors

Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]



$$\mathcal{L}_{\rm int}^{\sigma} = \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi \qquad \vec{V}^\mu = \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi \qquad \vec{A}^\mu = f_\pi\partial^\mu\vec{\phi} + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi$$

$$\overline{u}\left[\Gamma_V^\mu - \Gamma_A^\mu\right]u = \overline{u}\left[\gamma^\mu - g_A\left(\gamma^\mu + q^\mu\frac{\not{q}}{m_\pi - q^2}\right)\gamma^5\right]u$$

$$\overline{u}\left[\tilde{\Gamma}_V^\mu(q^2) - \tilde{\Gamma}_A^\mu(q^2)\right]u$$

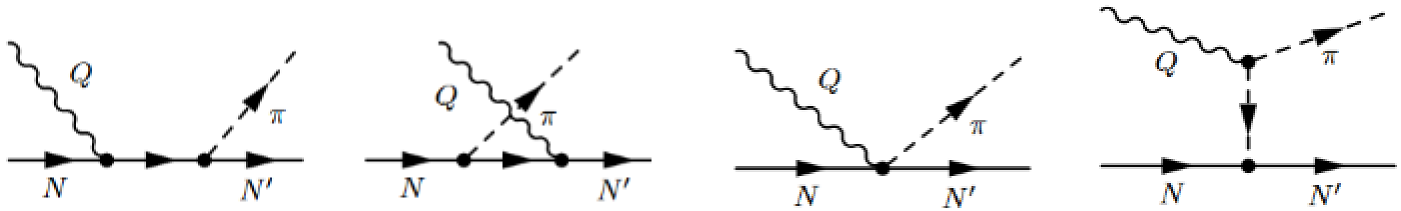
$$\tilde{\Gamma}_V^\mu(q^2) = F_1(q^2)\gamma^\mu + i\frac{F_2(q^2)}{2M_N}\sigma^{\mu\nu}q_\nu$$

$$\tilde{\Gamma}_A^\mu(q^2) : \; g_A \to G_A(q^2) = \frac{g_A}{(1 - q^2/M_A^2)^2}$$

Background contributions: vector current

Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]

$$J^\mu_V = \overline{u}\left[\mathcal{O}^\mu_{NP,V} + \mathcal{O}^\mu_{CNP,V} + \mathcal{O}^\mu_{CT,V} + \mathcal{O}^\mu_{PIF,V}\right] u$$



Nucleon pole (NP) Crossed nucleon pole (CNP) Contact-term (CT) Pion-in-flight (PIF)

For $F_1 = 1$ $\quad Q_\mu J^\mu_V = 0 \text{ (CVC)}$

Introduction of $F_1(Q^2)$ in NP and CNP breaks conservation for $Q^2 > 0$

$$\mathcal{O}^\mu_{CT,V} = \frac{g_A}{\sqrt{2}f_\pi} F_{CT}(q^2)\gamma^\mu\gamma^5 \rightarrow \boxed{F_{CT}(q^2) = F_1(q^2)}$$

$$\mathcal{O}^\mu_{PF,V} = \frac{g_A}{\sqrt{2}f_\pi} F_{PF}(q^2)\frac{(2k_\pi - q)^\mu}{t^2 - m_\pi^2} 2M\gamma^5 \rightarrow \boxed{F_{PF}(q^2) = F_1(q^2)}$$

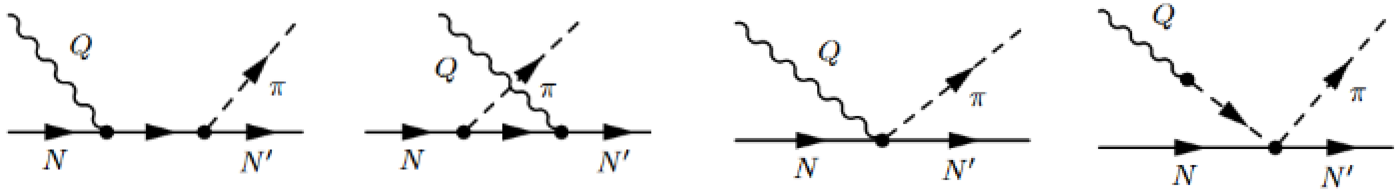
Introduce of $F_1(Q^2)$ in CT and PF to recover CVC

Fermilab

Background contributions: axial current

Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]

$$J^\mu_A = \overline{u}\left[\mathcal{O}^\mu_{NP,A} + \mathcal{O}^\mu_{CNP,A} + \mathcal{O}^\mu_{CT,A} + \mathcal{O}^\mu_{PP}\right] u$$



Nucleon pole (NP) | Crossed nucleon pole (CNP) | Contact-term (CT) | Pion-pole (PP)

$$\mathcal{O}^\mu_{CT,A} = \frac{1}{\sqrt{2} f_\pi}\gamma^\mu \to \frac{F_\rho(t)}{\sqrt{2} f_\pi}\gamma^\mu$$

Rho-meson propagator to regularize CT,A
"rho dominance of the ππNN coupling"

→ Need to include it in the PP term

$$\mathcal{O}^\mu_{PP} = \frac{1}{\sqrt{2} f_\pi}\frac{q^\mu}{q^2 - m_\pi^2}\not{q} \to \frac{F_\rho(t)}{\sqrt{2} f_\pi}\frac{q^\mu}{q^2 - m_\pi^2}\not{q}$$

$$\mathcal{O}^\mu_{PP} + \mathcal{O}^\mu_{CT,A} =$$

$$\mathcal{O}^\mu_\rho = \frac{g_{\rho NN} g_{W\rho\pi}}{t - m_\rho^2} F_A(Q^2)\{g^{\mu\alpha} + \frac{Q^\mu Q^\alpha}{Q^2 + m_\pi^2}\}\left(\gamma_\alpha + i\frac{k_\rho}{2M_N}\sigma_{\alpha\nu}K^\nu_\rho\right)$$

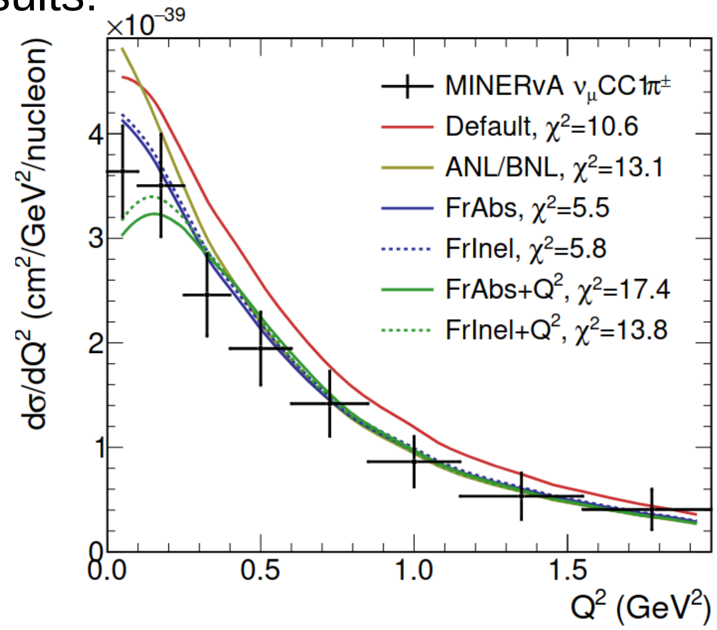
$F_A(Q^2) \to 1$

$\kappa_\rho = 0$

Tensions in the resonance region ?

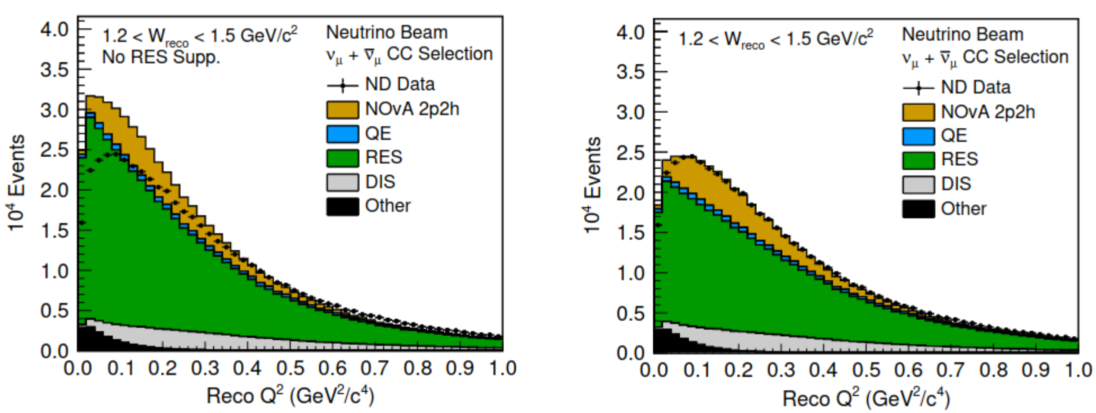
To resolve tension between deuteron / carbon results:

> "An additional *ad hoc* correction for
> the low-Q2 region,
> where collective nuclear effects are expected
> to be large"



[MINERvA PRD100, 072005 (2019)]

Similar correction introduced by NOvA



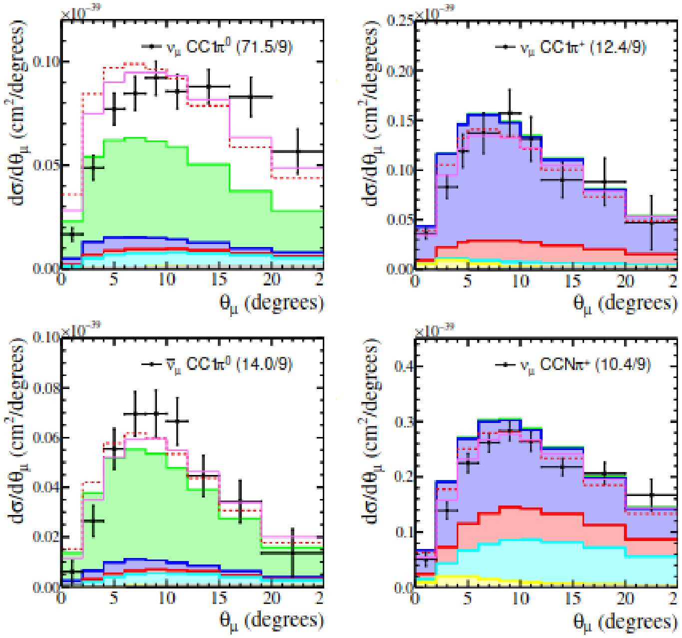
[NOvA Eur. Phys. J. C 80, 1119 (2020)]

Tensions in the resonance region ?

In [PRD 100, 072005 (2019)] a simultaneous fit of

1. ANL/BNL bubble-chamber data for
   pion production on deuteron

2. MINERvA pion production data on carbon



Conclusion ?

“the Monte Carlo models which are currently widely used in the field are unable to explain multiple data sets, even when they are from a single Experiment.”

Local RDWIA for pion production on the nucleus: nucleon FSI

Full expression of single-nucleon current in IA:

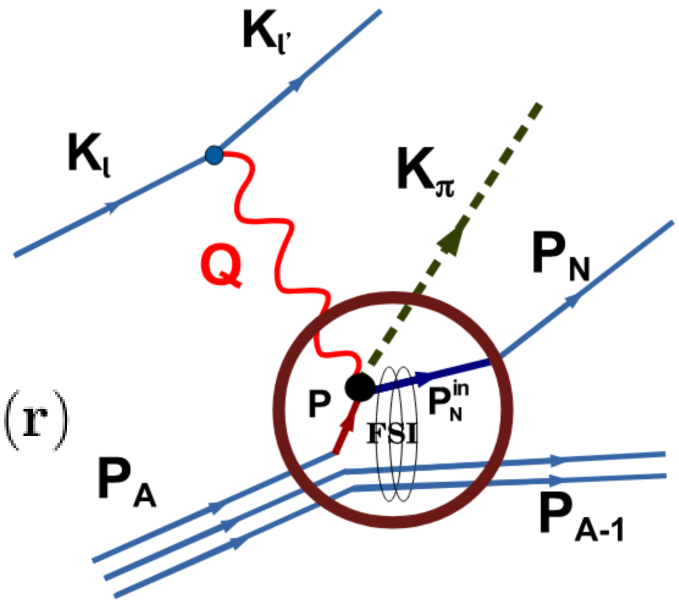
$$J^{\nu} = \frac{1}{(2\pi)^{3/2}} \int d\mathbf{p}'_N \int d\mathbf{p}'_\pi \overline{\psi}^{s_N}(\mathbf{p}'_N, \mathbf{k}_N)\, \phi^*(\mathbf{p}'_\pi, \mathbf{k}_\pi)$$
$$\mathcal{O}^{\nu}(q^\mu, p'_N, p'_\pi, p'_m)\, \psi_\kappa^{m_j}(\mathbf{p}'_m = \mathbf{p}'_N + \mathbf{p}'_\pi - \mathbf{q}). \quad (13)$$

Local/asymptotic approximation

$$\mathcal{O}^{\mu}(q, p'_m, p'_N, p'_\pi) \rightarrow \mathcal{O}^{\mu}(q, p_m, k_N, k_\pi)$$

$$= \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \phi^*(\mathbf{r}, \mathbf{k}_\pi)\, \overline{\psi}^{s_N}(\mathbf{r}, \mathbf{k}_N)\, \mathcal{O}^{\nu} \psi_\kappa^{m_j}(\mathbf{r})$$

PW | **ED-RMF** | **RMF**



Local RDWIA for pion production on the nucleus: nucleon FSI

Full expression of sing

$$J^{\nu} = \frac{1}{(2\pi)^{3/2}} \int d\mathbf{p}'_N$$

$$\mathcal{O}^{\nu}\left(q^{\mu}, p'_N, p'_{\pi}, p'_m\right)$$

Local/asympt

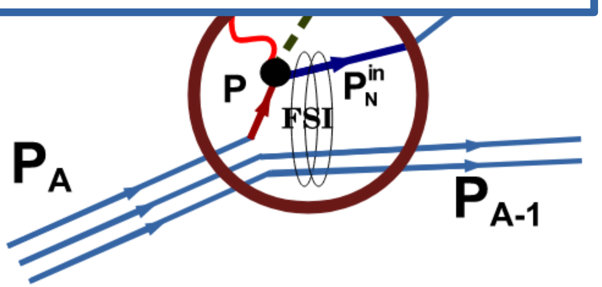
$$\mathcal{O}^{\mu}\left(q, p'_m, p'_N, p'_{\pi}\right.$$

How to deal with the pion in ν experiment?

- **Charge exchange → redistribution**
- **Absorption**
- **'elastic' rescattering**

affect the signal differently!
The conditions are not exclusive!
→ Optical potentials won't work

$$= \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \phi^{*}(\mathbf{r}, \mathbf{k}_{\pi})\, \overline{\psi}^{s_N}(\mathbf{r}, \mathbf{k}_N)\, \mathcal{O}^{\nu} \psi_{\kappa}^{m_j}(\mathbf{r})$$



PW | **ED-RMF** | **RMF**

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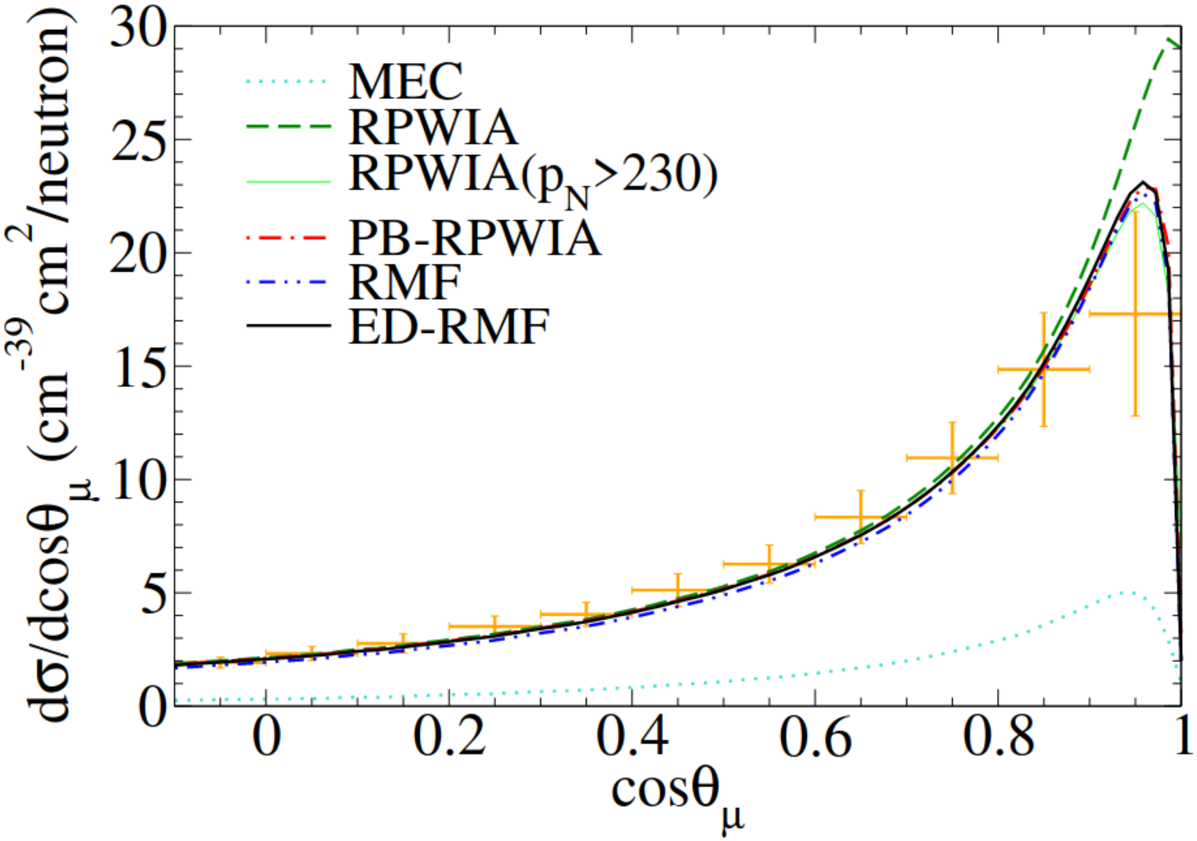
Low-Q^2 suppression in the CCQE region: RDWIA

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]

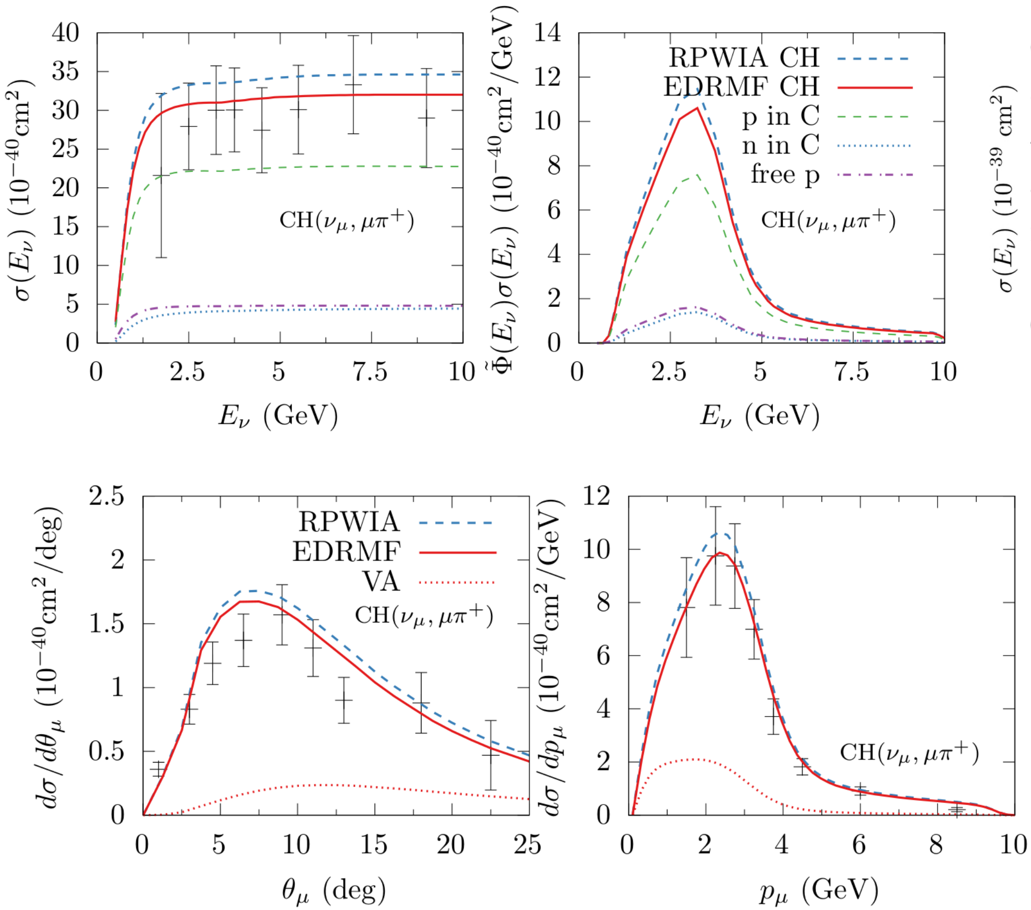


The RDWIA leads to a suppression at small angles (≈low Q^2) compared to RPWIA
Seen to be mostly due to Pauli-Blocking
→ What is the effect in single-pion production ?

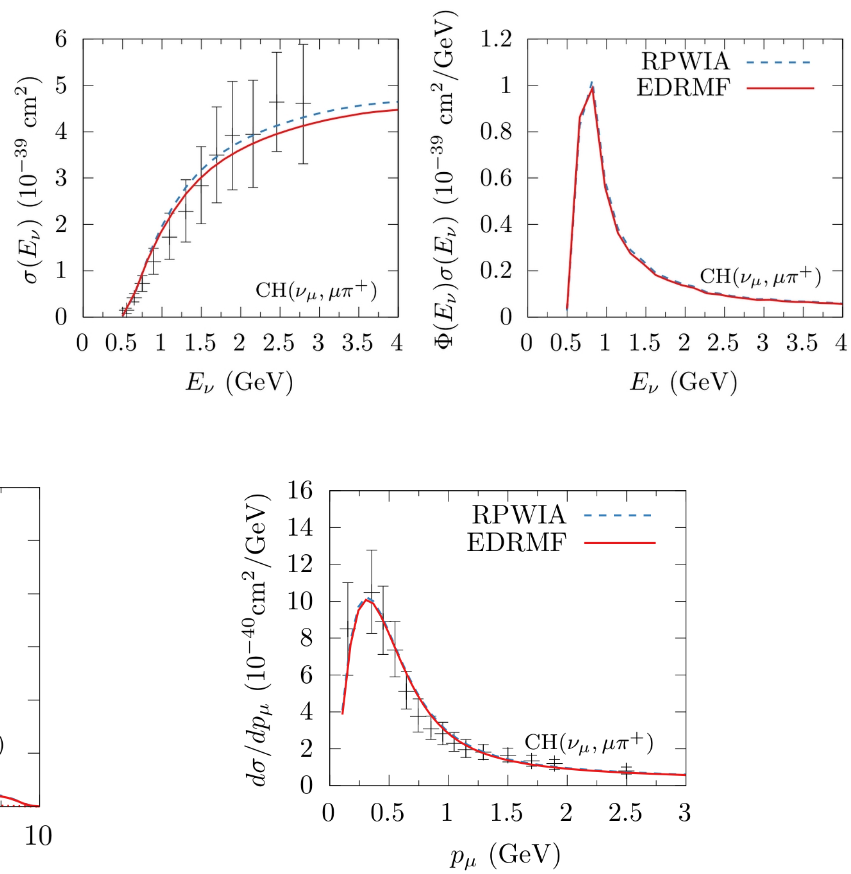
Neutrinoproduction of charged pions on CH

MINERνA experiment $1\pi^+$

$W_{(exp)} < 1.4$ GeV

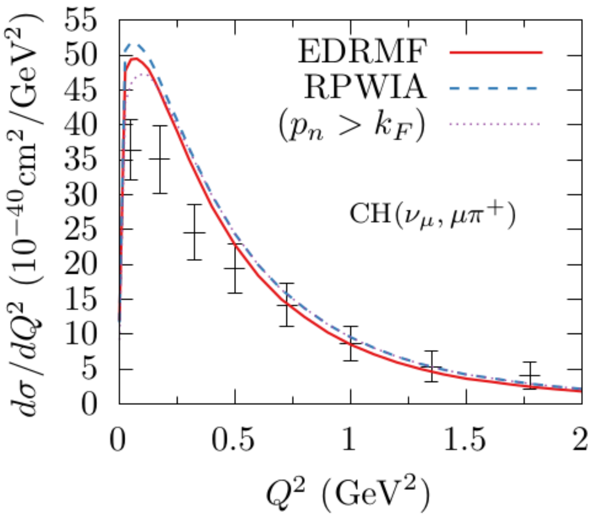


T2K experiment $1\pi^+$

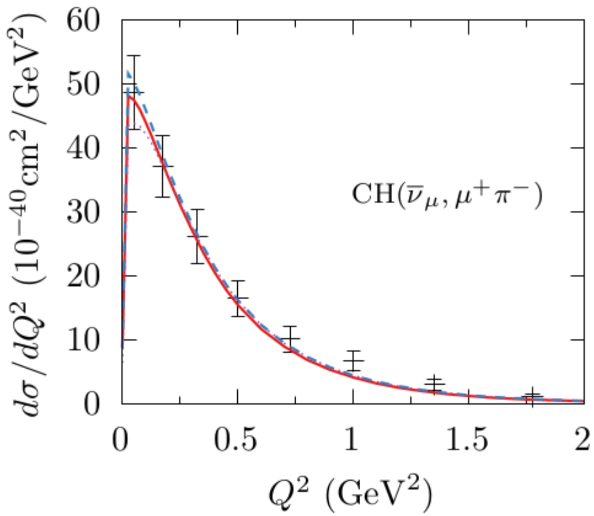


Neutrinoproduction of charged pions on CH: Q^2 distributions

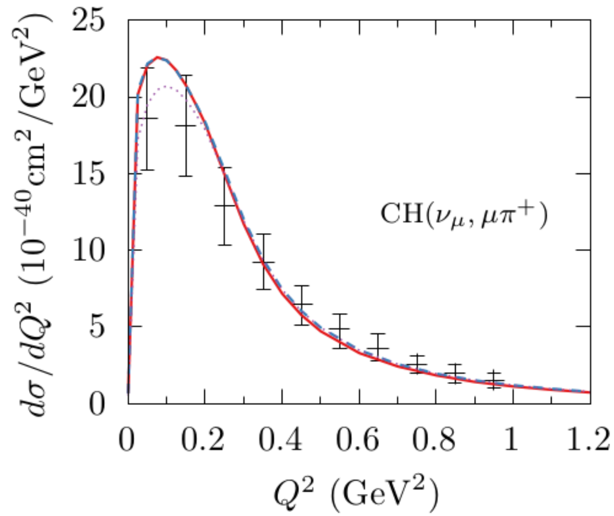
MINERνA $1\pi^+$
$W_{(exp)} < 1.4$ GeV



MINERνA $1\pi^-$
$W_{(exp)} < 1.8$ GeV



T2K $1\pi^+$



Similar overprediction of low-Q^2 region in EDRMF and RPWIA
- Many caveats in interpretation of data-theory comparison!

But certainly:
Nucleon FSI does not produce a significant reduction in the low-Q^2 region!
See also : [J. Garcia-Marcos Arxiv:2310.18056]