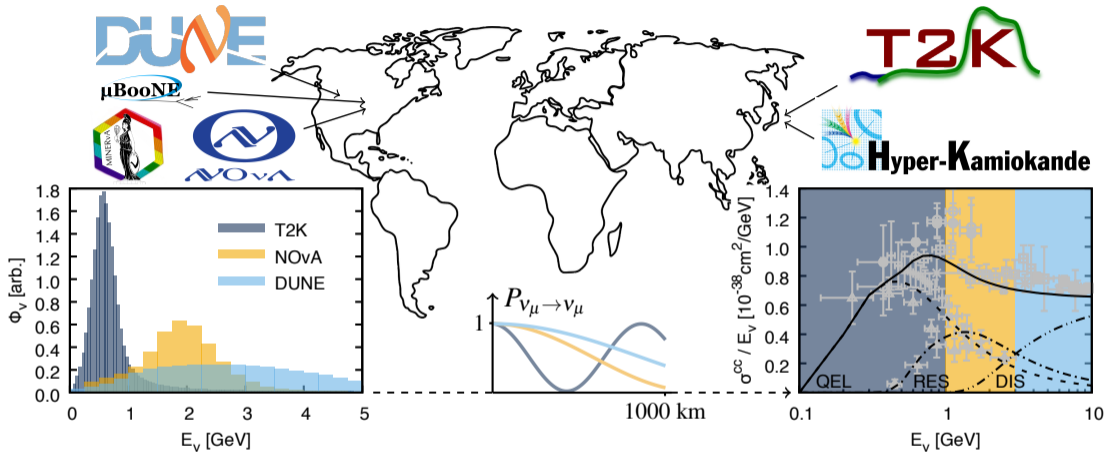


Current Capabilities and Future Plans for Lepton Scattering Uncertainties in NuWro and their Implications for Global Neutrino Oscillation Experiments

Kajetan Niewczas





$$P(\nu_{\mu} \rightarrow \nu_e) \simeq \sin^2(2\theta) \sin^2 \left(1.27 \frac{\Delta m^2 L}{E_{\nu}} \right)$$

↑
oscillation

↑
amplitude

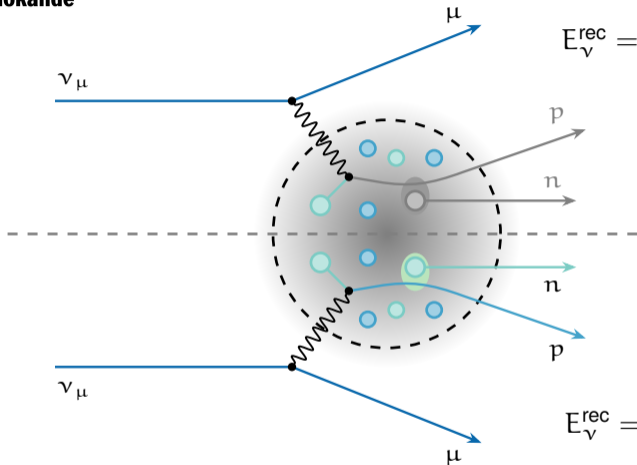
↑
frequency

$$A_{CP} = \frac{P(\nu_{\mu} \rightarrow \nu_e) - P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)}{P(\nu_{\mu} \rightarrow \nu_e) + P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)}$$

↑
asymmetry

↑
oscillation ratio

Kinematical energy reconstruction



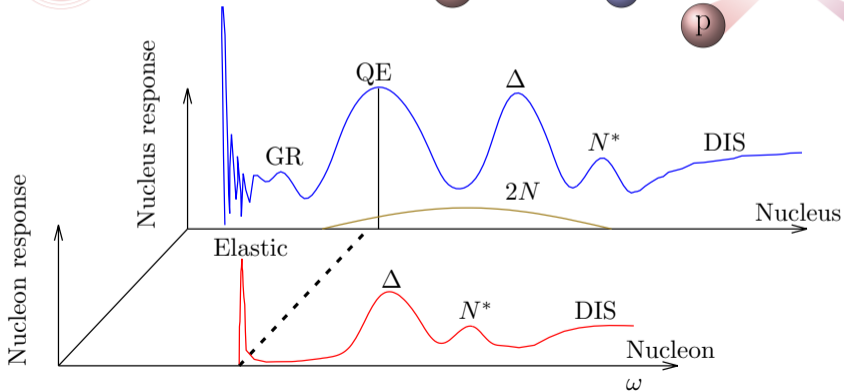
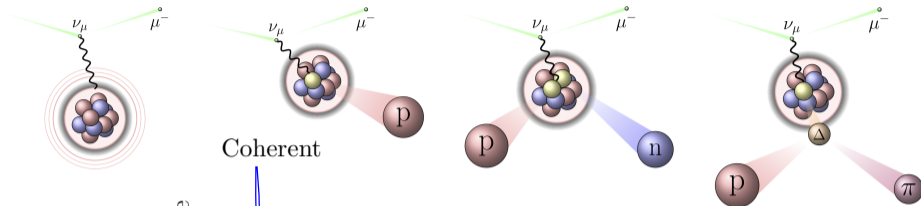
$$E_{\nu}^{\text{rec}} = \frac{2M_N E_{\mu} - m_{\mu}^2 + M_{N'}^2 - M_N^2}{2(M_N - E_{\mu} + p_{\mu} \cos \theta)}$$

$$E_{\nu}^{\text{rec}} = E_{\mu} - E_B + \sum_{\text{nucl.}} T_i + \sum_{\text{mes.}} E_j$$



Calorimetric energy reconstruction

Nuclear response



Outline

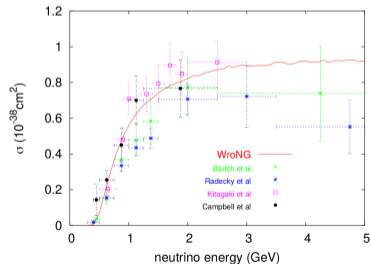
- (6) Brief characteristics of NuWro
- (2) Challenges in modeling quasielastic scattering
- (4) Understanding the current meson-exchange currents implementations
- (3) Enhancing sensitivity to angular distributions in single-pion production
- (5) Testing the cascade model against nuclear transparency
- (1) **The future beyond franken-models**

Standard NuWro assumptions

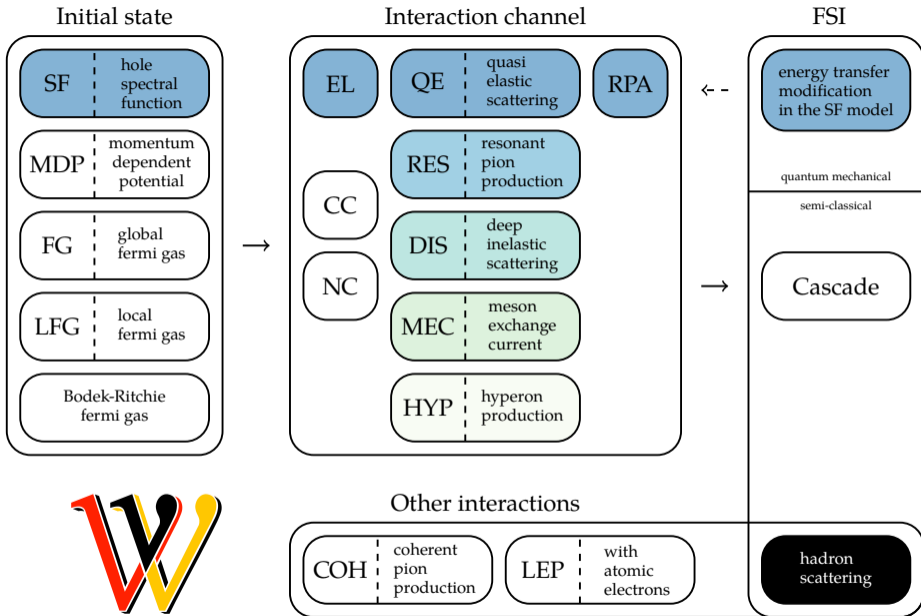
- Nuclei are composed of nucleons → **Nucleon** degrees of freedom
- One-photon exchange (BA, IA) → $(L_{\mu\nu}W^{\mu\nu})$ separation
- Plane-wave impulse approximation → $(\sigma_{\nu A} \propto P(E, p)\sigma_{\nu N})$ **factorization**
- In-medium propagation ($\bar{\lambda} \ll d < \lambda < R$) → **Cascade model** for inelastic FSI

More *WroNG* assumptions

- Not fully relativistic, no distorted waves
- Factorization of the inclusive cross section
- *Frankenstein; or, The Modern Prometheus*



J.T. Sobczyk et al., Nucl.Phys.B Proc.Suppl. 139 (2005) 266-271



Intranuclear cascade

- **Propagates particles** through the nuclear medium
- **Probability** of passing a distance λ :

$$P(\lambda) = e^{-\lambda/\tilde{\lambda}}$$

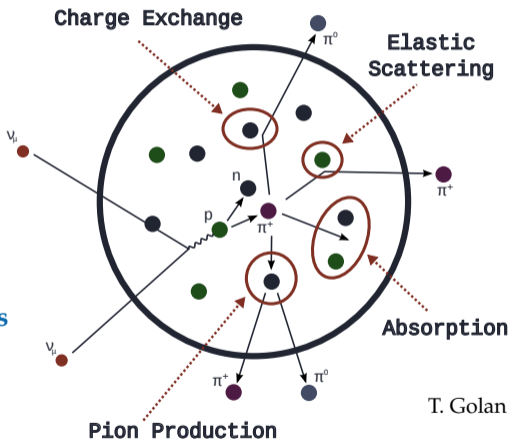
where $\tilde{\lambda} \equiv (\rho\sigma)^{-1}$ and

ρ - local density

σ - cross section

→ Implemented for **nucleons**, **pions** and **kaons**

T. Golan, C. Juszczak, J.T. Sobczyk,
Phys.Rev. C 86 (2012) 015505



Uncertainties of concern

Coming from **models**:

- Form factors, nuclear dynamics, and in-medium effects ...
- Model validity, meaningful degrees of freedom ...

Coming from **event generators**:

- Model implementations, simplifications ...
- Double counting of physical effects and dynamics ...

Factorization of the cross section calculation

Plane-wave impulse approximation

$$\frac{d^2\sigma}{d\omega d|\vec{q}|} = K \int dE d^3\vec{p} S(E, |\vec{p}|) L_{\mu\nu} H^{\mu\nu}$$

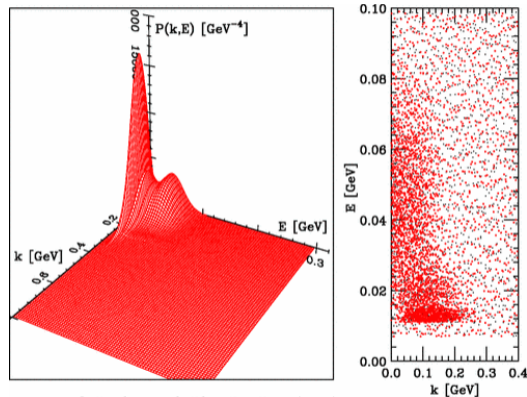
→ effective optical potential prescription

or the **Llewellyn-Smith** formula

$$\frac{d\sigma}{dQ^2} = K \left[A(Q^2) - B(Q^2) \left(\frac{s-u}{M^2} \right) + C(Q^2) \left(\frac{s-u}{M^2} \right)^2 \right]$$

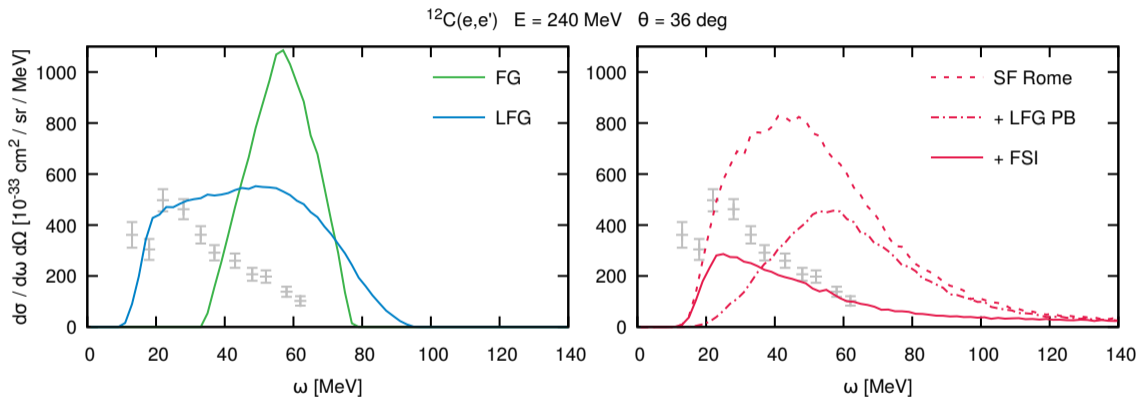
→ after boosting to the N-rest frame

→ folded with nuclear model distributions



O. Benhar et al., Phys.Rev.D 72 (2005) 053005

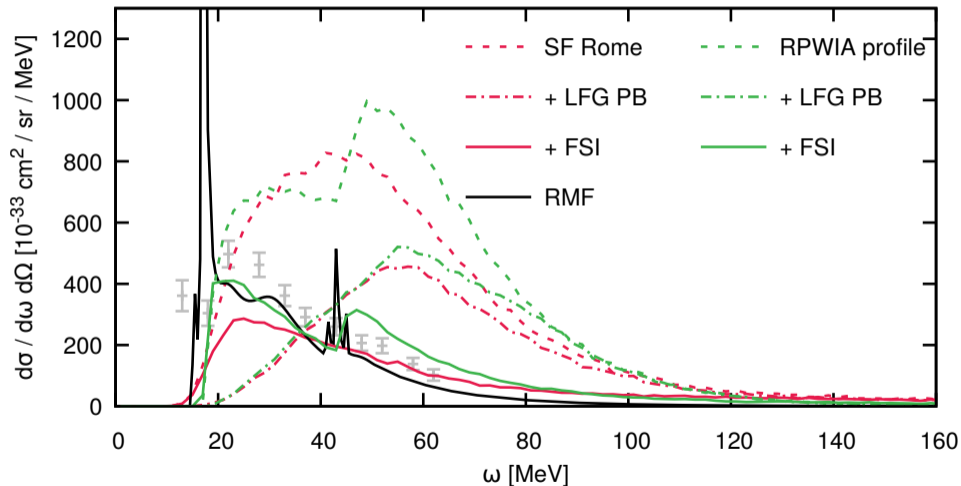
Factorization of the cross section calculation



→ **FG** and **LFG** do not reproduce the inclusive electron results

Factorization of the cross section calculation

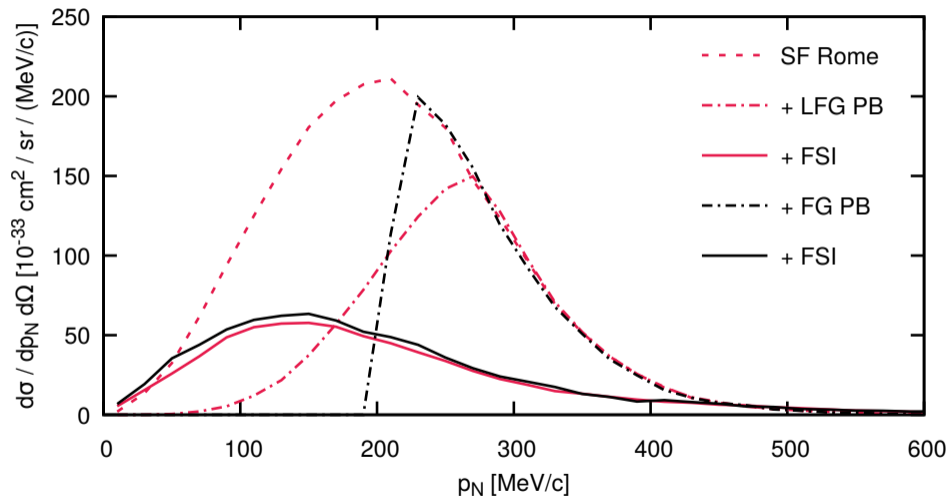
$^{12}\text{C}(e,e')$ $E = 240 \text{ MeV}$ $\theta = 36 \text{ deg}$



→ For inclusive cross sections, the **correction is fine**

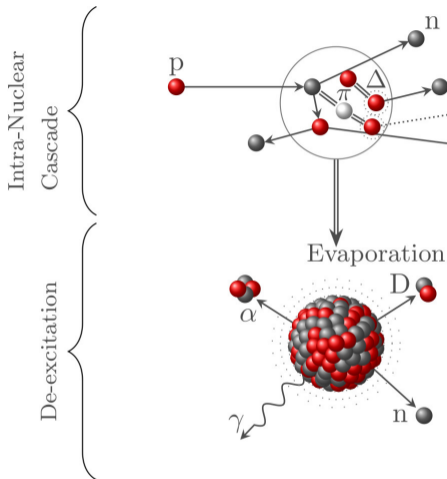
Factorization of the cross section calculation


$^{12}\text{C}(e,e')$ $E = 240 \text{ MeV}$ $\theta = 36 \text{ deg}$



→ The procedure is **inconsistent for exclusive observables**

NuWro + INCL + ABLA



Projectiles: baryons (nucleons, Λ , Σ), mesons (pions and Kaons) or light nuclei ($A \leq 18$). **No neutrinos yet!** We use neutrino vertex from  **NuWro** (widely used ν -nucleus MC generator).

Flexible tool: has been implemented in GEANT4 and GENIE

De-excitation: ABLA, SMM, GEMINI

We will use **ABLA**, since it proved to work for the **light nuclei** (Phys. J. Plus 130, 153 (2015))

First neutrino simulation results:
Phys.Rev.D 106, 3 (2022)

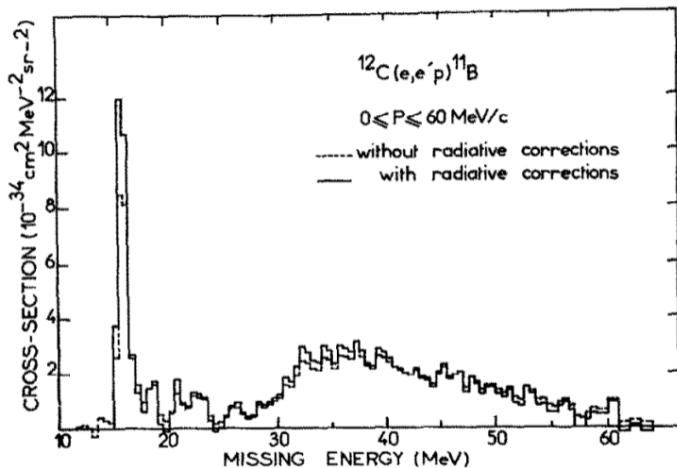
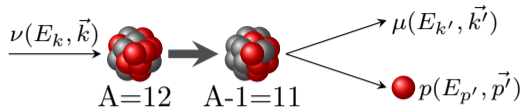


Fig. 7. Energy spectrum of the $^{12}\text{C}(e, e'p)^{11}\text{B}$ reaction before and after the radiative corrections.

NuWro + INCL + ABLA



Experimental definition:

$$E_x^{\text{exp}} = E_{\text{missing}} - (M_A - M_{A-1} - M)$$

- A constant shift of missing energy by ~ 15.4 MeV leads to **non-physical, negative values**
- We use experimental data (J. Phys. G: Nucl. Part. Phys. 16 507 (1999)) to simulate discrete levels
- We assume all strength below the peak comes from the symmetric **$1p_{3/2}$ shell**

M_{A-1} is the rest mass of the $A - 1$ nucleus

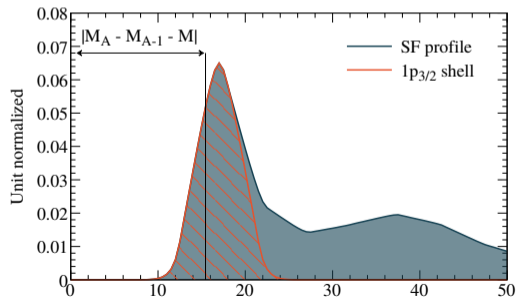
M_A is the rest mass of the initial A nucleus

M is the rest mass of the target nucleon

E_{missing} is the missing energy

For interaction on carbon,

$$M_A - M_{A-1} - M = 15.4 \text{ MeV}$$



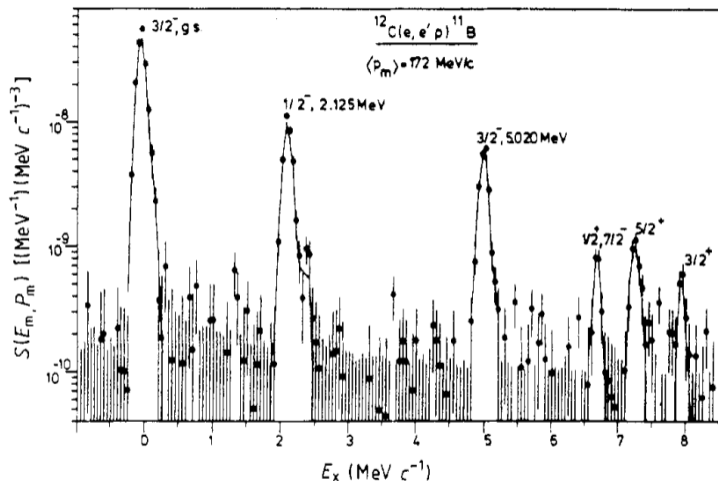
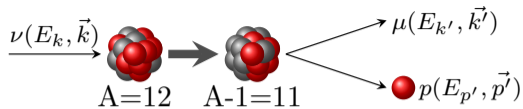


Figure 22. Excitation-energy spectrum of ^{11}B observed in the reaction $^{12}\text{C}(e, e'p)$. Both negative and positive-parity final states are shown.

NuWro + INCL + ABLA



For the continuous spectrum part,
we can calculate excitation energy as:

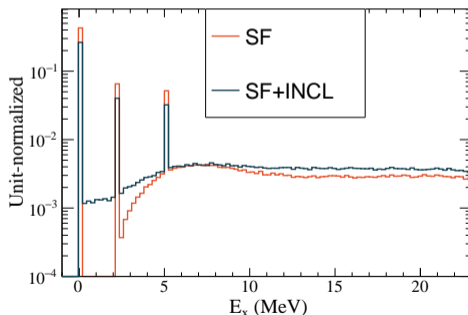
$$E_x = M_R^* - M_R, \text{ where:}$$

$$M_R^* = \sqrt{(E_k + M_A - E_{k'} - E_{p'})^2 - |\vec{p}_{missing}|^2}$$

Otherwise, we model **3 discrete peaks** with
strength of 79%, 12%, and 9% (**p-shell**)

M_R^* is the mass of the excited remnant
 M_R is the rest mass of the remnant
 T_R is the kinetic energy of the excited remnant

$p_{missing}$ is the missing momentum



NuWro + INCL + ABLA

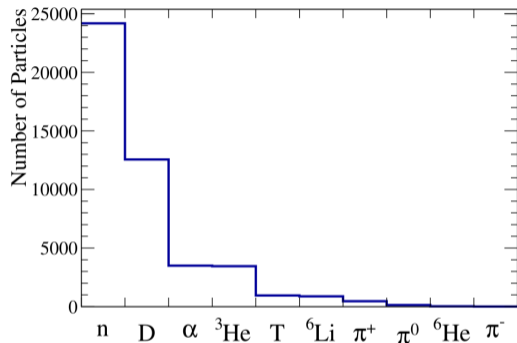


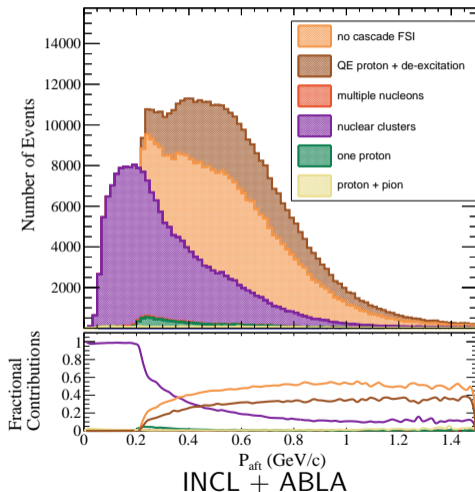
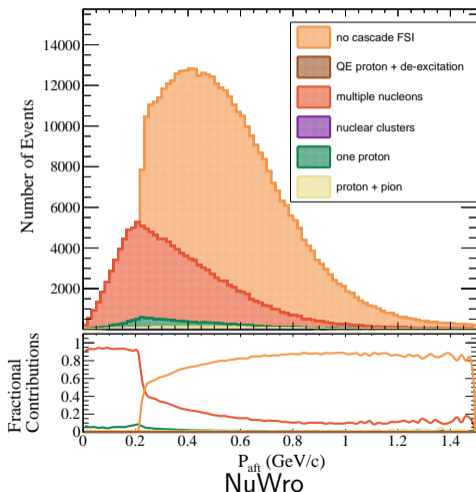
FIG. 11: Particles leaving the nucleus in events without proton in the final state in INCL.

In the last paper: [Phys.Rev.D 106, 3 \(2022\)](#) we show the **nuclear cluster production for the first time** in FSI.

Now we study the impact of the subsequent **de-excitation modelling**, that predicts **more nuclear clusters**.

NuWro + INCL + ABLA

INCL+ABLA simulation features **massive difference** in nucleon kinematics in comparison to NuWro



Anna Ershova, NuFACT 2023

Using $\mu + p$ is **better** than using muon only, but here we show that we gain even **higher precision** by using all subleading particles

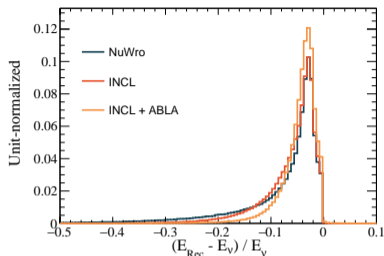
proton only:

$$E_{rec} = E_{\mu} + T_p$$

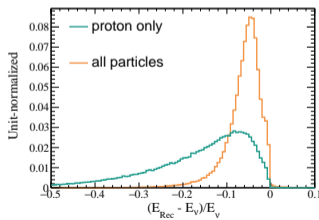


all particles (including clusters)

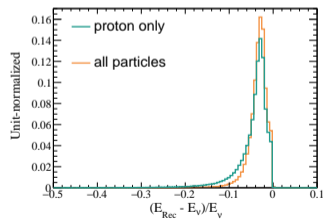
$$E_{rec} = E_{\mu} + \sum_i T_i$$



"all particles" reconstruction



INCL+ABLA cascade FSI



INCL+ABLA no cascade FSI

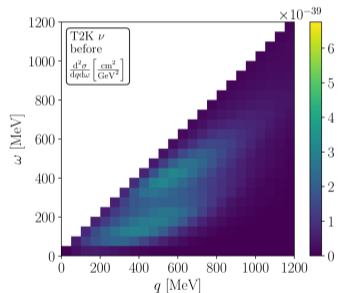
Phenomenological 2p2h model

A simultaneous fit to the T2K and MINERvA CC0 π data

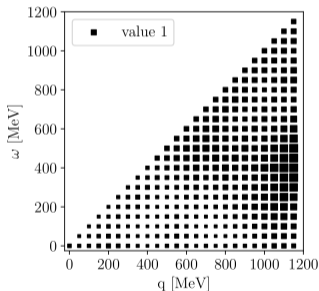
→ ansatz: the **whole error comes from 2p2h**

→ **Valencia 2p2h** model as the **prior** ($|\vec{q}| \leq 1.2 \text{ GeV}/c$)

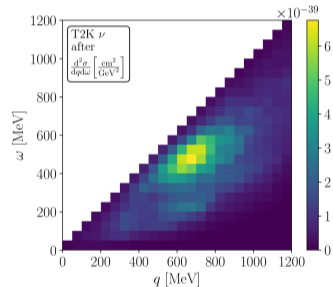
Experiment	D.O.F.	Non-scaled	Scaled
MINERvA ν_μ	156	462.8	358.2
MINERvA $\bar{\nu}_\mu$	60	65.1	62.2
T2K ν_μ	58	143.7	83.9
T2K $\bar{\nu}_\mu$	58	101.2	98.0
Sum	332	772.8	619.6



→



→



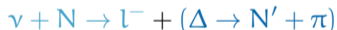
→ the **data** favor contributions from **higher momentum transfers**

T. Bonus, J.T. Sobczyk, M. Siemaszko, and C. Juszczak, Phys.Rev. C 102 (2020) 015502

Single-pion production



- **Single-pion production** (SPP) is an essential dynamics for accelerator-based experiments
- There many measurements sensitive to **pion angular distributions** ($\cos \theta_\pi$)



- **NuWro** models the **Δ -resonance** excitation
→ it decays according to the ANL/BNL angular fits

$$\frac{d^2\sigma_\Delta}{dQ^2 dW} \rightarrow \frac{d^4\sigma_\pi}{dQ^2 dW} \times \frac{df_\Delta(Q^2)}{d\Omega_\pi^*}$$

- The nonresonant background is extrapolated from the DIS formalism into the lower regions of W , Q^2

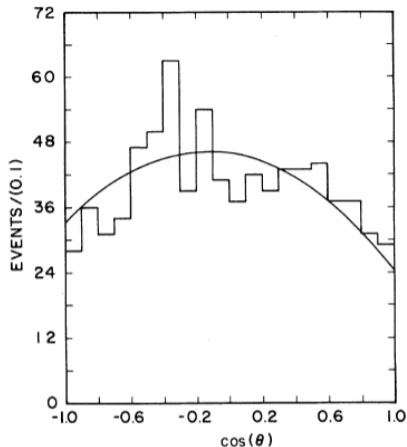


FIG. 15. Distribution of events in the pion polar angle $\cos\theta$ for the final state $\mu^-p\pi^+$, with $M(p\pi^+) < 1.4$ GeV. The curve is the area-normalized prediction of the Adler model.

Radecky et al. [ANL Collaboration], *Phys.Rev. D* 25 (1982) 1161

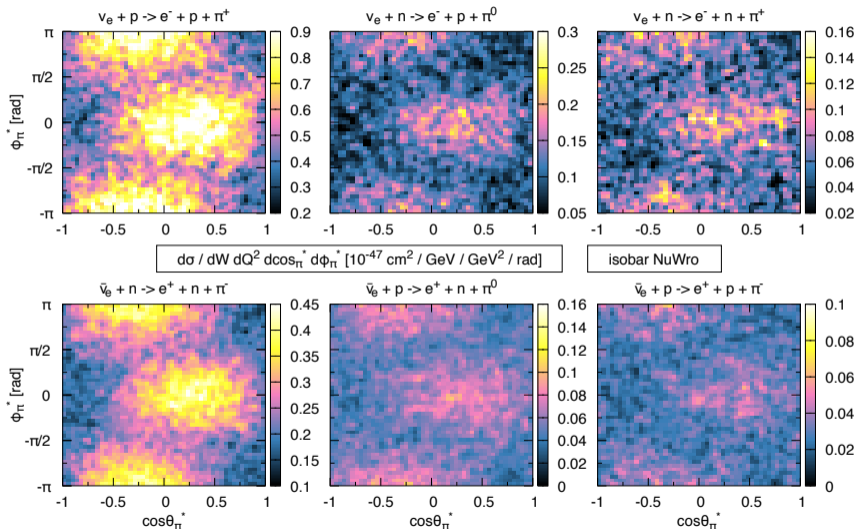
Pion angular distributions

- Default NuWro
- Free nucleon
- Fixed kinematics:

$$E = 1 \text{ GeV}$$

$$Q^2 = 0.1 \text{ GeV}^2$$

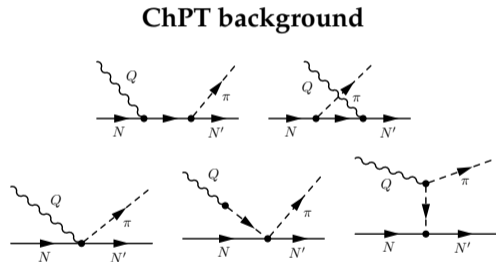
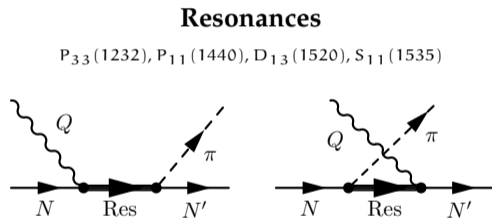
$$W = 1230 \text{ MeV}$$



(total number of 10^7 events over the whole phase space)

Ghent low energy model of SPP

- The model of Ref. [[R. González-Jiménez et al., Phys.Rev. D 95 \(2017\) 113007](#)]
- The **low-energy** part based on the **Valencia model**



- **Bottleneck** for the implementation is the code **execution time**
- Adding a **nuclear model** will further increase the complexity of the implementation

Implementation

- Working in the **Adler frame**, generating an event requires the value of

$$\frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_\pi^*}{k_1^2} [A + B \cos(\phi_\pi^*) + C \cos(2\phi_\pi^*) + D \sin(\phi_\pi^*) + E \sin(2\phi_\pi^*)]$$

→ that is **time consuming** and the MC sampling has an **efficiency** of 10 – 15 %

- Sampling Q^2, W from **precomputed arrays** allows to build the **muon kinematics**
- Then, $\cos \theta_\pi^*$ is given by the A function that is mostly **parabolic** (fit using 3-7 points)
- Finally, for other variables fixed, ϕ_π^* is given by an **analytical expression**

$$\frac{d^2\sigma}{dQ^2 dW} \xrightarrow[\text{numerical}]{\text{fix } Q^2, W} \frac{d^3\sigma}{dQ^2 dW d\cos \theta_\pi^*} \xrightarrow[\text{numerical / analytical}]{\text{fix } \cos \theta_\pi^*} \frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*} \xrightarrow[\text{analytical}]{\text{fix } \phi_\pi^*} \text{event...}$$

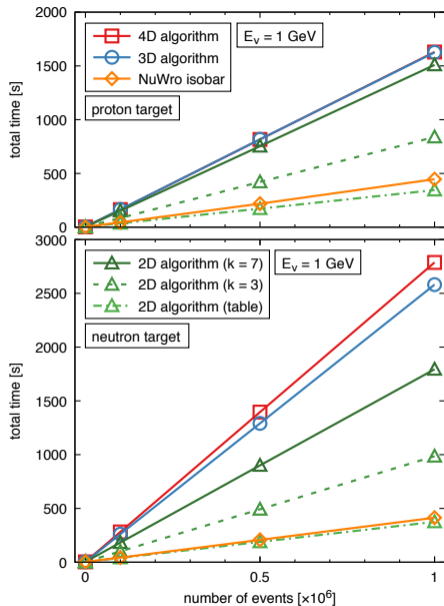
K.N. et al., Phys.Rev.D 103 (2021) 053003

Performance

We propose:

- **4D algorithm:** sampling $(Q^2, W, \cos \theta_\pi^*, \phi_\pi^*)$ together (1 cross section calculation per accepted event)
- **3D algorithm:** sampling $(Q^2, W, \cos \theta_\pi^*)$ together + ϕ_π^* analytical (2 cross section calculation per accepted event)
- **2D algorithm:** sampling (Q^2, W) from tables + $\cos \theta_\pi^*$ from k points or from tables + ϕ_π^* analytical ($k + 1$ cross section calculation per accepted event)

→ $\nu - n$ scattering requires one more code evaluation because it has two channels ($p + \pi^0, n + \pi^+$)



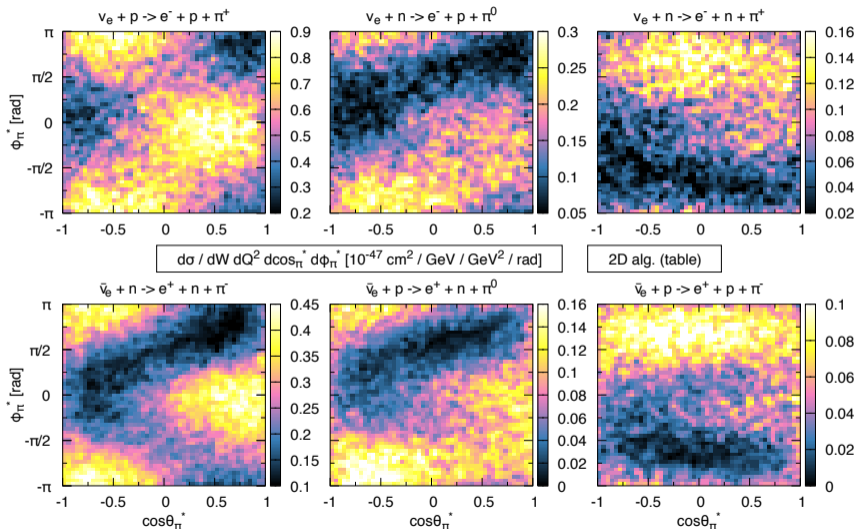
Pion angular distributions

- Ghent LEM
- Free nucleon
- Fixed kinematics:

$$E = 1 \text{ GeV}$$

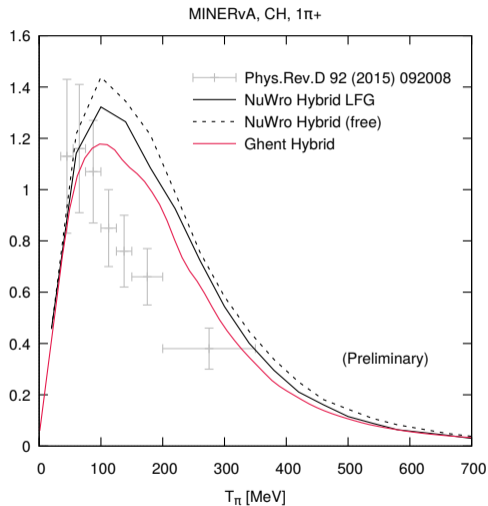
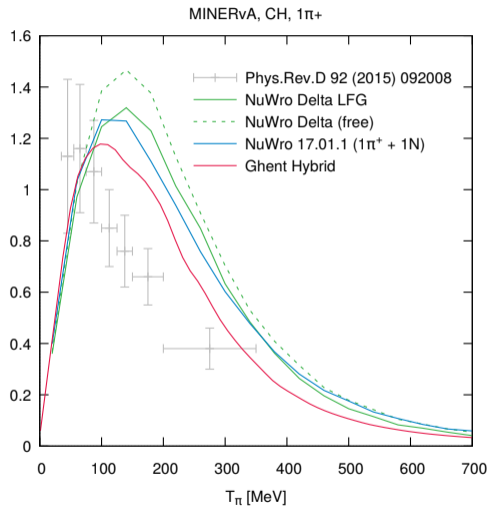
$$Q^2 = 0.1 \text{ GeV}^2$$

$$W = 1230 \text{ MeV}$$



(total number of 10^7 events over the whole phase space)

Hybrid model on the nucleus



R. González-Jiménez et al., Phys.Rev.D 97 (2018) 013004; Q. Yan et al., in preparation

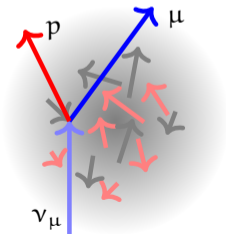
Nuclear transparency

Definition

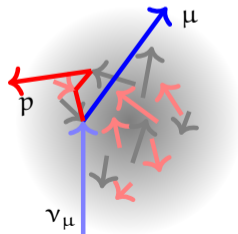
Nuclear transparency is the average **probability** for a knocked-out **proton** to **escape** the nucleus **without significant reinteraction**.

e.g. measured for Carbon: $T \simeq 0.60$ [D. Abbott *et al.*, PRL 80 (1998), 5072]

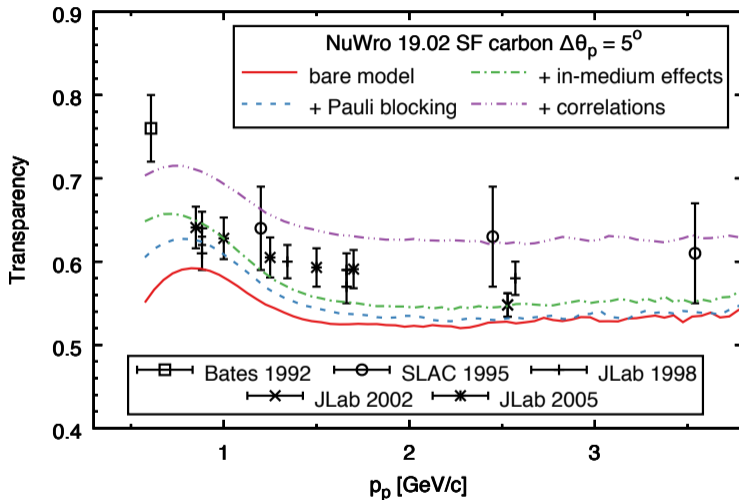
~ 60% without FSI



~ 40% with FSI

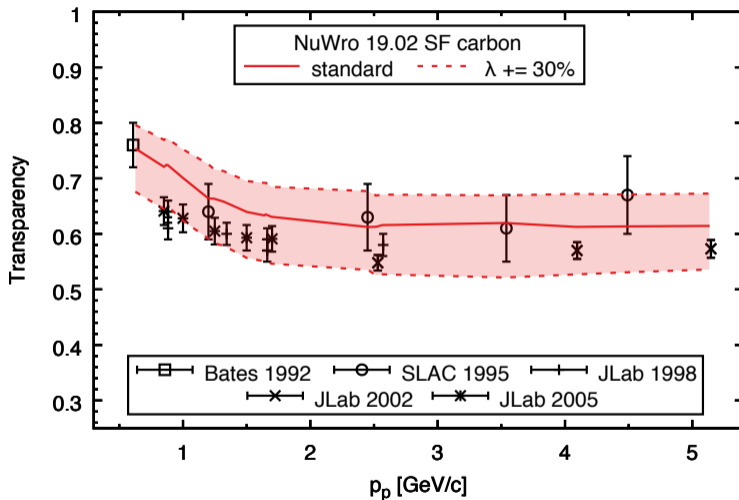


Nuclear transparency



K. Niewczas, J. Sobczyk, Phys.Rev. C 100 (2019) 015505

Nuclear transparency



K. Niewczas, J. Sobczyk, Phys.Rev. C 100 (2019) 015505

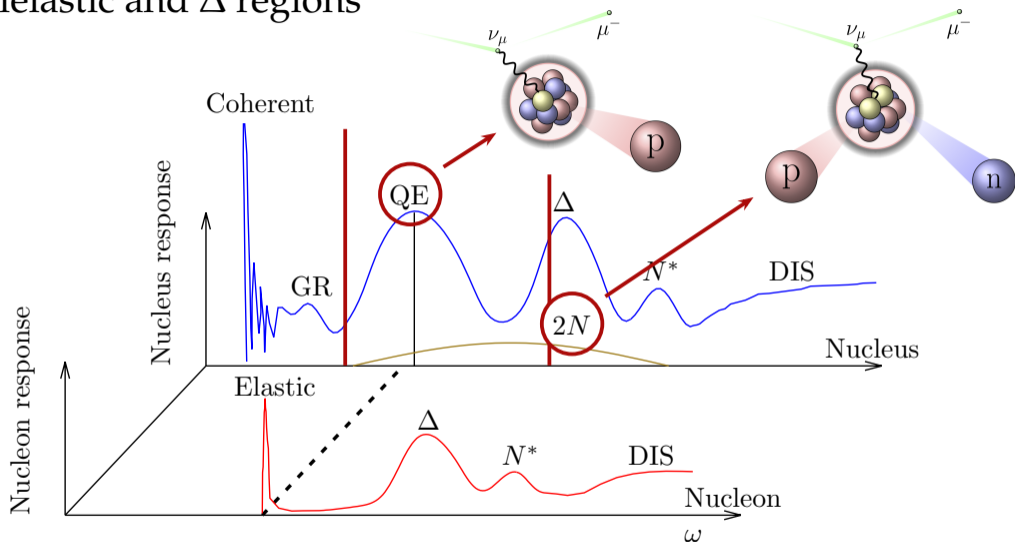
The future beyond franken-models...

with



**UNIVERSITEIT
GENT**

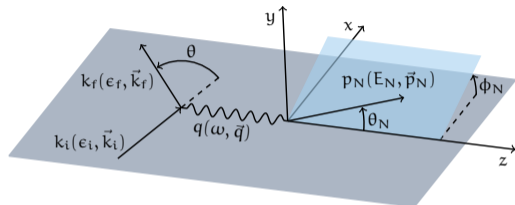
Quasielastic and Δ regions



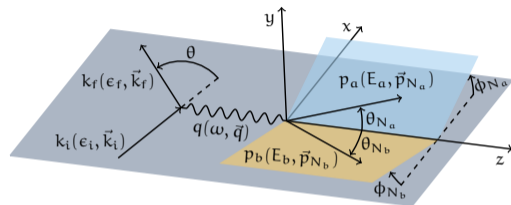
→ Mostly influenced by **one- and two-body physics** at nucleon and Δ levels

Kinematics

One-nucleon knock-out (1p1h)



Two-nucleon knock-out (2p2h)



Inclusive cross section

Electron scattering

$$\frac{d\sigma^\gamma}{d\epsilon_f d\Omega_f} = 4\pi\sigma^{\text{Mott}} [\mathcal{V}_L^e \mathcal{W}_L + \mathcal{V}_T^e \mathcal{W}_T]$$

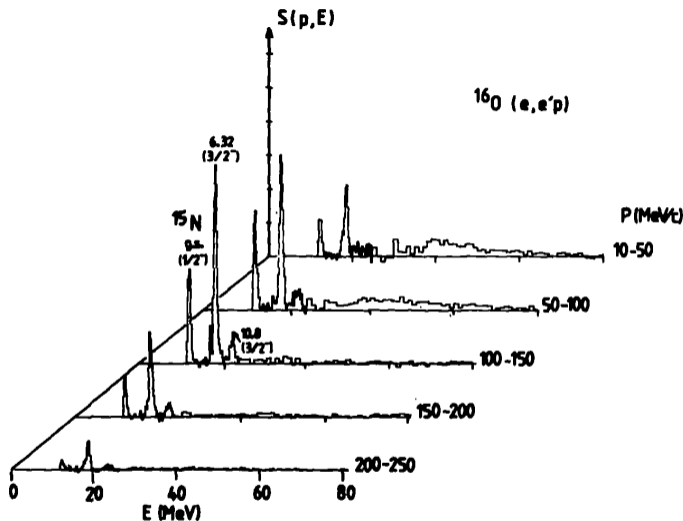
Neutrino scattering

$$\frac{d\sigma^W}{d\epsilon_f d\Omega_f} = 4\pi\sigma^W \zeta [\mathcal{V}_{CC} \mathcal{W}_{CC} + \mathcal{V}_{CL} \mathcal{W}_{CL} + \mathcal{V}_{LL} \mathcal{W}_{LL} + \mathcal{V}_T \mathcal{W}_T + \text{h}\mathcal{V}_{T'} \mathcal{W}_{T'}]$$

\mathcal{V}_x - leptonic factors; \mathcal{W}_x - hadronic responses; L/T - longitudinal/transverse relative to \vec{q}

Nuclear mean-field model

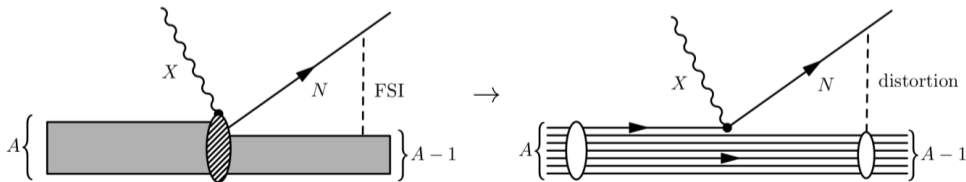
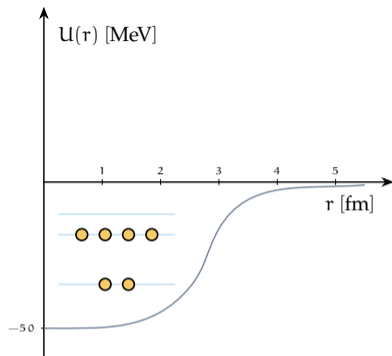
- Nucleons exhibit discrete energy states characteristic of the **mean-field potential** picture
- The redistribution of shell strength is caused by the **nucleon-nucleon correlations**
- Residual nuclei can be excited above the **two-nucleon knock-out** threshold



J. Mougey, Nucl. Phys. A 335 (1980) 35

Our nuclear framework

- Nucleons are solutions to the Schrödinger equation in a **mean-field potential**
- We calculate single-particle states with the **Hartree-Fock** procedure and SkE2 NN force
- We describe outgoing nucleons as **continuum states** of the nuclear potential



Impulse approximation

→ We evaluate the following **hadronic transition currents**

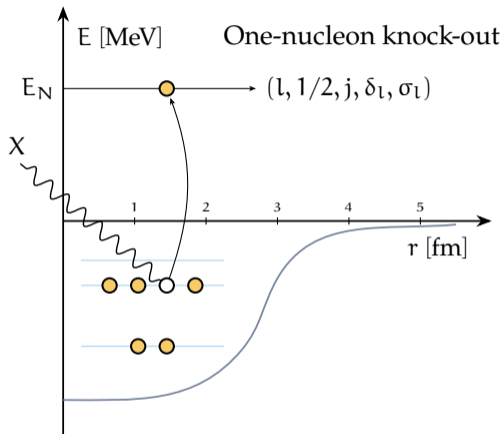
$$\hat{j}(\vec{r})_{\nu}^{\text{had}} = \langle \Psi_f | \hat{j}(\vec{r})_{\nu}^{\text{had}} | \Psi_i \rangle$$

→ The nuclear many-body current is a sum of **one-body operators**

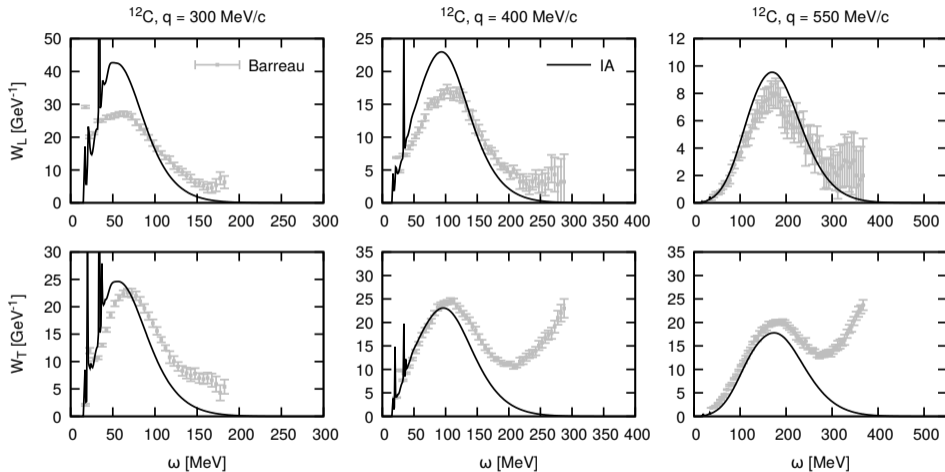
$$\hat{j}(\vec{r})_{\nu}^{\text{had}} \simeq \hat{j}(\vec{r})_{\nu}^{\text{IA}} = \sum_{j=1}^A \hat{j}(\vec{r}_j)_{\nu}^{[1]} \delta^{(3)}(\vec{r} - \vec{r}_j)$$

→ We control numerical precision using a **multipole decomposition**

→ Comparing to **inclusive electron scattering data** allows for benchmarking of the model

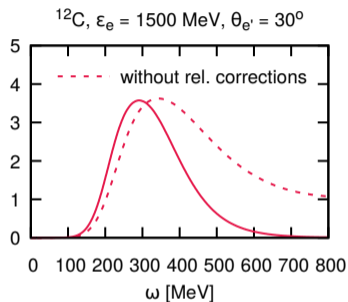
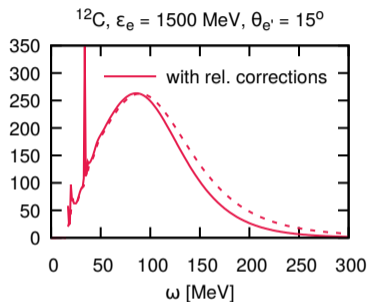
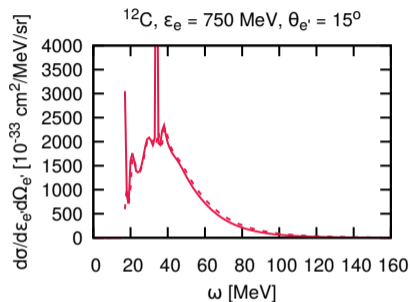


Impulse approximation: electron scattering



→ Calculation using **one-body currents** exhibits typical properties

Relativistic corrections



Fixing the **relativistic** position of the **quasielastic peak**

$$\omega \rightarrow \omega \left(1 + \frac{\omega}{2M_N}\right), \quad \text{then} \quad \omega_{\text{QE}} = \frac{|\vec{q}|^2}{2M_N} \rightarrow \frac{Q^2}{2M_N}$$

Short-range correlations

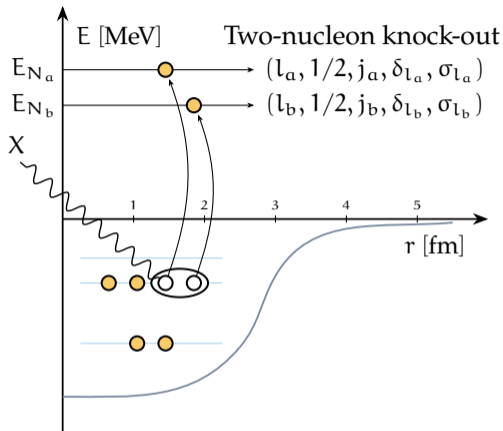
→ Nucleons with strongly **overlapping wave functions** for a short period of time

$$\hat{\rho}_v^{\text{eff}} \simeq \sum_{i=1}^A \hat{\rho}_v^{[1]}(i) + \sum_{i<j}^A \hat{\rho}_v^{[1],\text{SRC}}(i,j) + \left[\sum_{i<j}^A \hat{\rho}_v^{[1],\text{SRC}}(i,j) \right]^\dagger$$

with

$$\hat{\rho}_v^{[1],\text{SRC}}(i,j) = \left[\hat{\rho}_v^{[1]}(i) + \hat{\rho}_v^{[1]}(j) \right] \hat{l}(i,j)$$

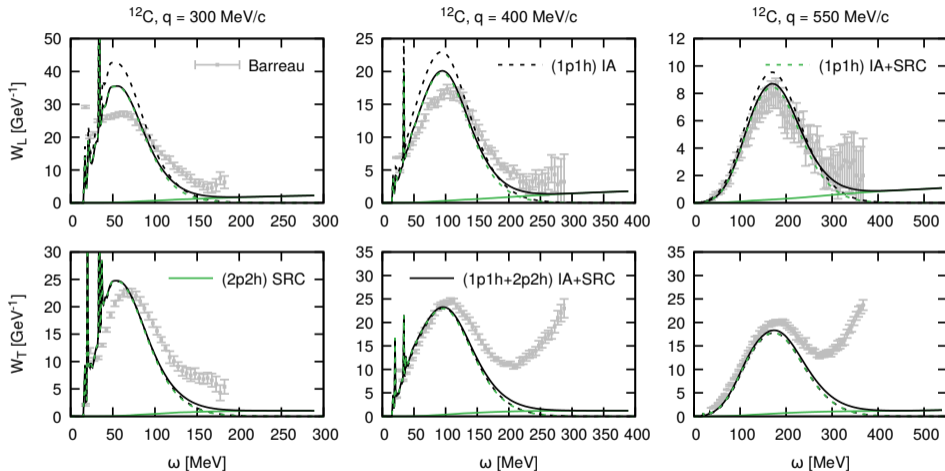
→ The correlation operator $\hat{l}(i,j)$ includes **central**, **tensor**, and **spin-isospin correlations**



→ First corrections to the **independent-particle model** picture for 1p1h

→ **Two-body currents** also leading to **two-nucleon knock-out** reactions

Short-range correlations: electron scattering

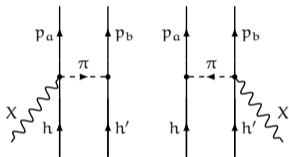


→ Significant reduction of the **longitudinal 1p1h strength** and a minor 2p2h contribution

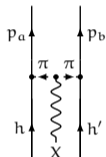
Meson-exchange currents

Explicit **two-body currents** contributing to both **1p1h** and **2p2h** final-states:

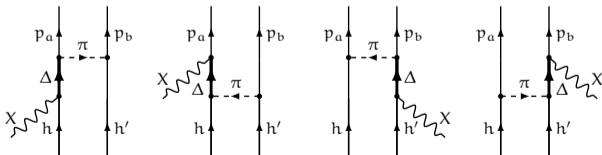
→ **Seagull** currents



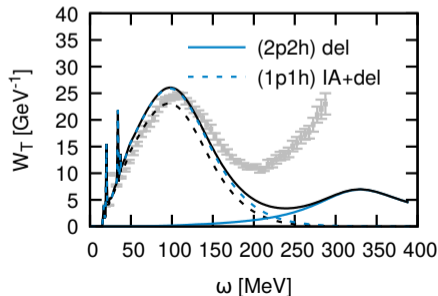
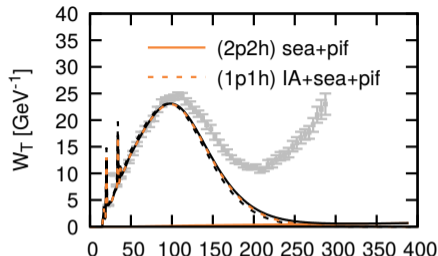
→ **Pion-in-flight** current



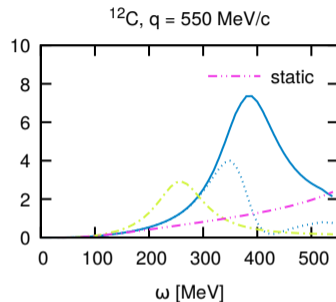
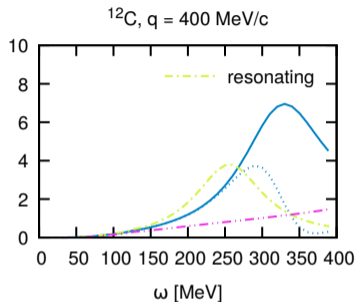
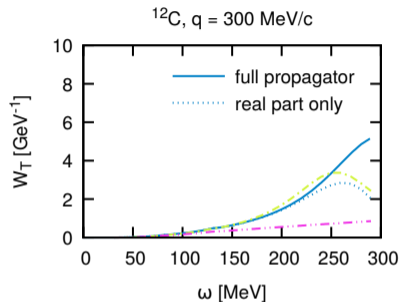
→ **Δ-isobar** degrees of freedom



^{12}C , $q = 400 \text{ MeV}/c$



Delta currents



Full propagators

$$G_{\Delta}^{\text{res}} = \frac{2M_{\Delta}}{M_{\Delta}^2 - s - iM_{\Delta}\Gamma_{\Delta}^{\text{res}} + 2M_{\Delta}V_{\Delta}}$$

$$G_{\Delta}^{\text{nres}} = \frac{2M_{\Delta}}{M_{\Delta}^2 - u}$$

Static approximation

$$G_{\Delta}^{\text{res}} = \frac{1}{M_{\Delta} - M_N}$$

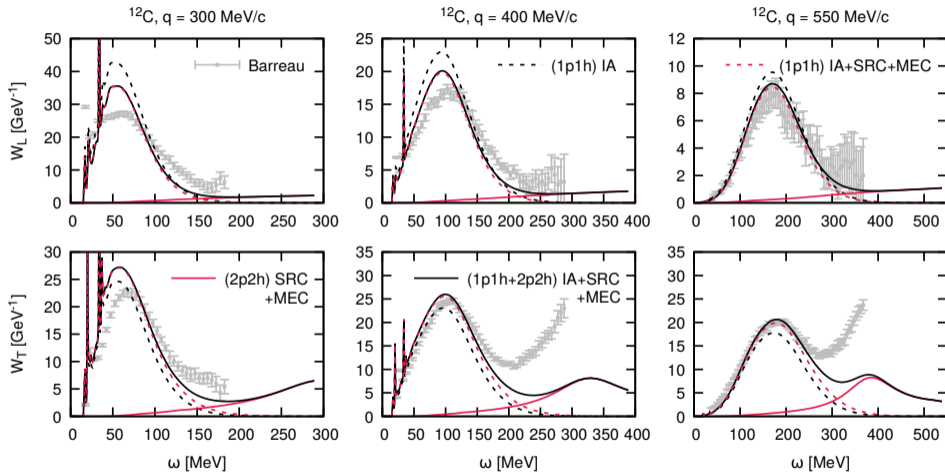
$$G_{\Delta}^{\text{nres}} = 0$$

Resonating approximation

$$G_{\Delta}^{\text{res}} + G_{\Delta}^{\text{nres}} = \frac{1}{M_{\Delta} - M_N - \omega - \frac{i}{2}\Gamma_{\Delta}^{\text{res}}}$$

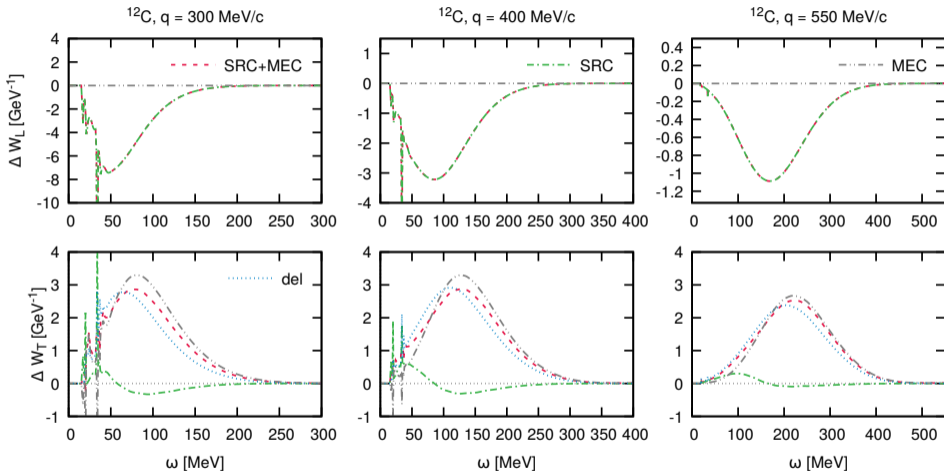
$$G_{\Delta}^{\text{res}} - G_{\Delta}^{\text{nres}} = 0 \quad + \frac{1}{M_{\Delta} - M_N + \omega}$$

Consistent modeling of two-body currents: electron scattering



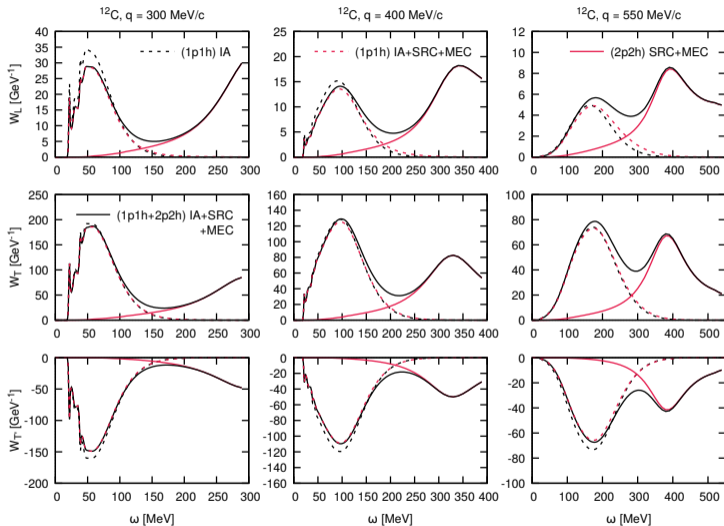
→ **Coherent sum of SRC and MEC** enhances our predictions

Consistent modeling of two-body currents: electron scattering



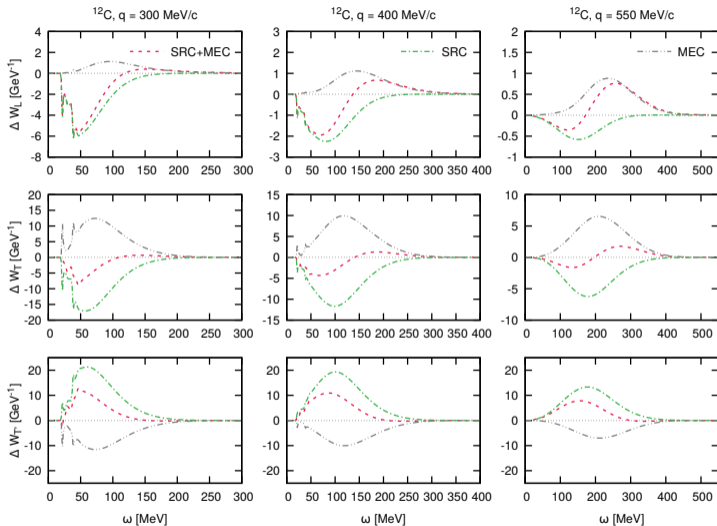
→ **Two-body currents** modify the one-nucleon knock-out responses

Consistent modeling of two-body currents: neutrino scattering



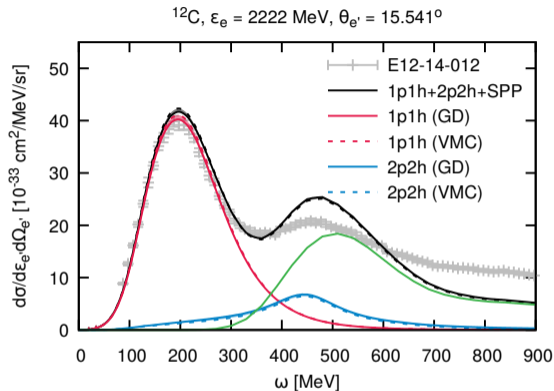
→ **Pronounced Δ peaks** for both longitudinal and transverse responses

Consistent modeling of two-body currents: neutrino scattering

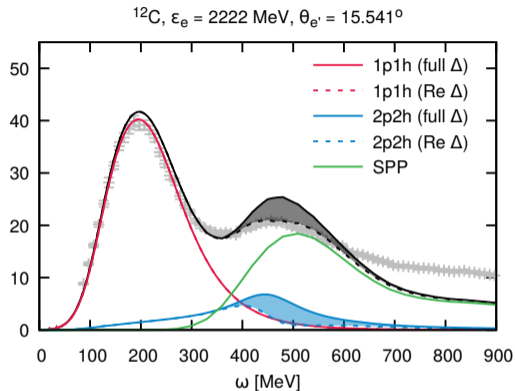


→ **SRC** provides quenching in the longitudinal and transverse responses

JLab Hall A data



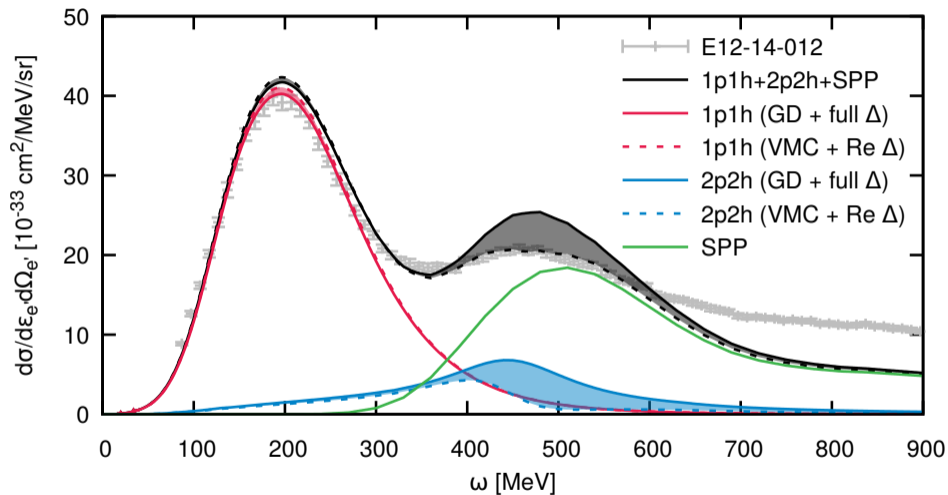
→ The choice of the different **central correlation functions** modifies the **QE peak strength** (GD–stronger, VMC–weaker)



→ Modifying the Δ -propagator governs the **overlap between MEC and SPP** around the Δ peak (Re Δ –only the real part)

JLab Hall A data

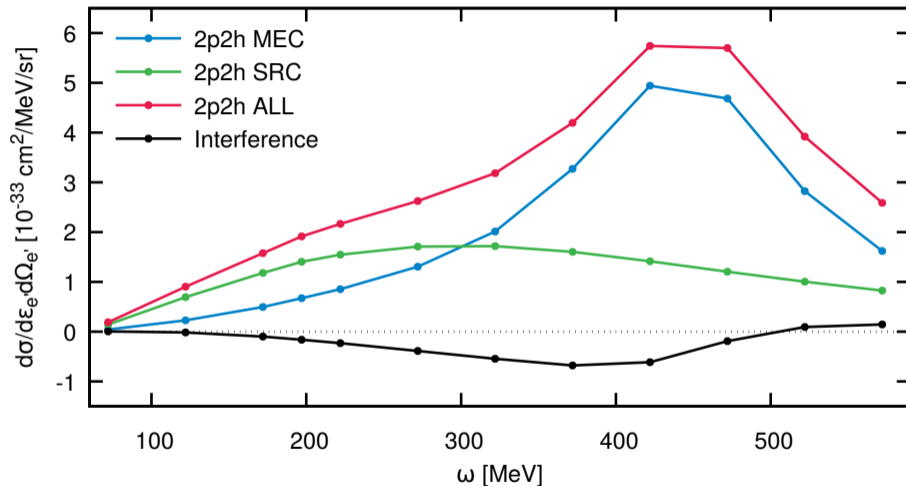
^{12}C , $\epsilon_e = 2222 \text{ MeV}$, $\theta_{e'} = 15.541^\circ$



→ Combining variation in given d.f. provides **flexibility in describing QE and Δ peaks**

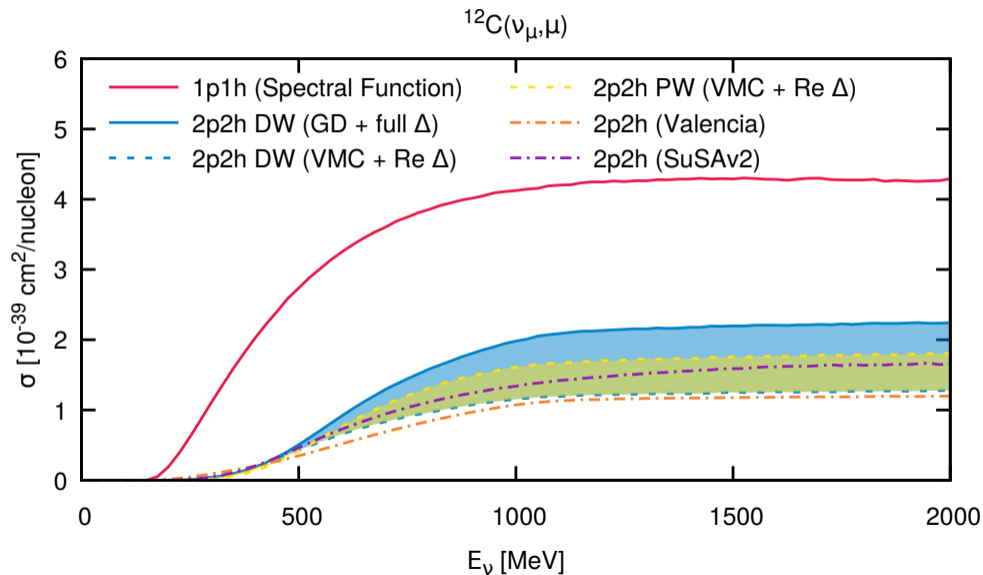
JLab Hall A data

^{12}C , $\epsilon_e = 2222 \text{ MeV}$, $\theta_{e'} = 15.541^\circ$

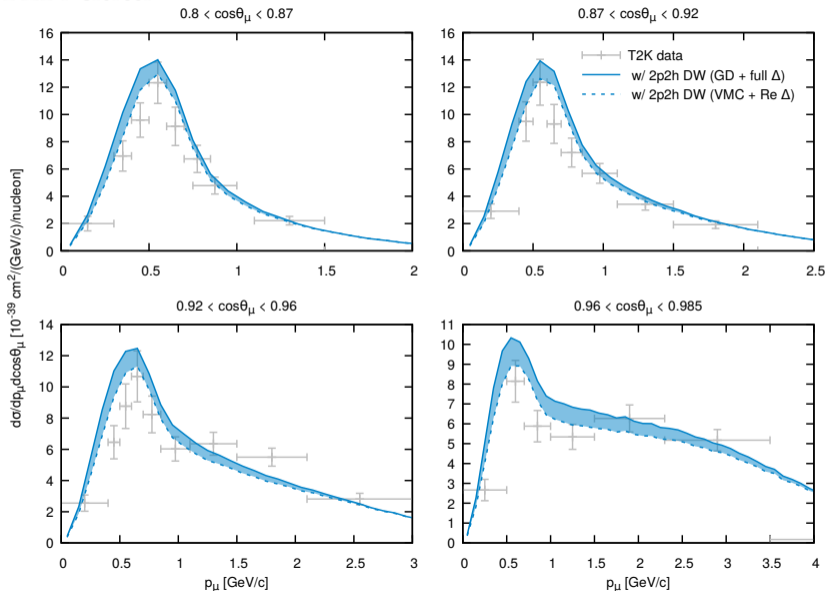


→ We see a significant **negative interference** between the SRC and MEC contributions

Inclusive NuWro implementation

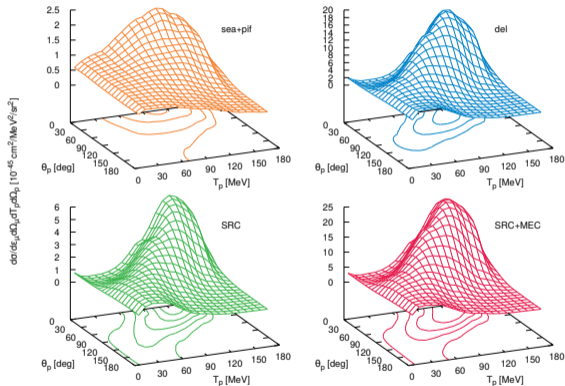


Inclusive T2K data



Going more exclusive... in neutrino scattering

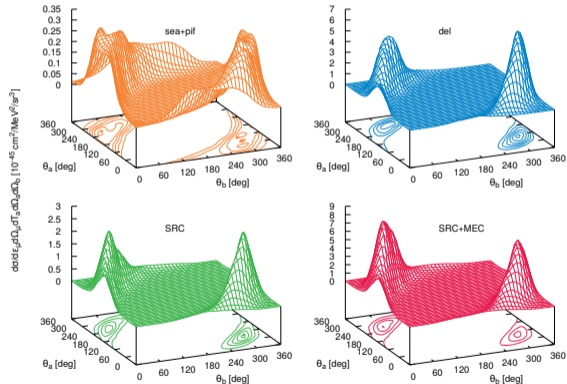
^{12}C , $\epsilon_{\nu\mu} = 750 \text{ MeV}$, $\epsilon_{\mu} = 550 \text{ MeV}$, $\theta_{\mu} = 15^\circ$, $\phi_p = 0^\circ$



Semi-inclusive two-nucleon knock-out

Exclusive two-nucleon knock-out

^{12}C , $\epsilon_{\nu\mu} = 750 \text{ MeV}$, $\epsilon_{\mu} = 550 \text{ MeV}$, $\theta_{\mu} = 15^\circ$



Conclusions

- The current generator methods face **significant challenges**
- We are moving towards precision **exclusive processes modeling**
- More **refined implementation methods** become available
- We are **moving forward**, leaving franken-models behind

You, theoreticians, want consistency. We, experimentalists, want flexibility.

Stephen Dolan, NuXTract 2023