Polytropes, polytropes and more polytropes

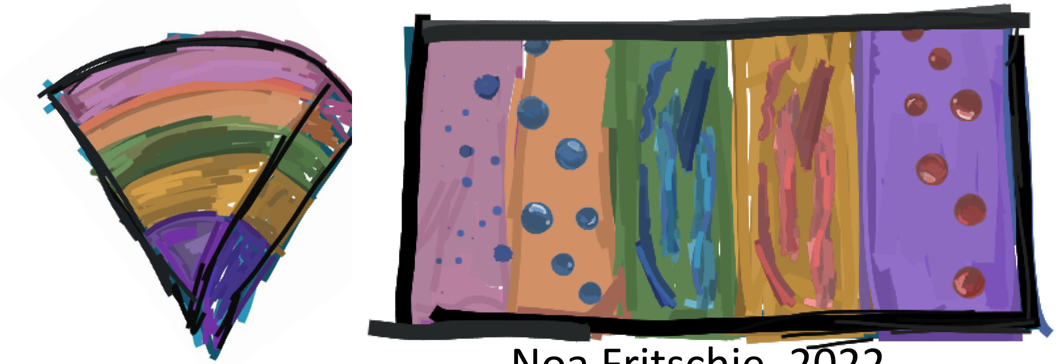
William G. Newton

The work presented in this talk would not be possible without an amazing team of undergraduates and Master’s students, including

Rebecca Preston, Lauren Balliet, Michael Ross Amber Stinson, August Doss, Gabriel Crocombe, Josh Belieu, Savannah Wright, Parker Reeves

Texas A&M University-Commerce

Duncan Neill, David Tsang – University of Bath



Noa Fritschie, 2022

Crust to core: different nuclear and astrophysical observables for different densities, and systematic errors

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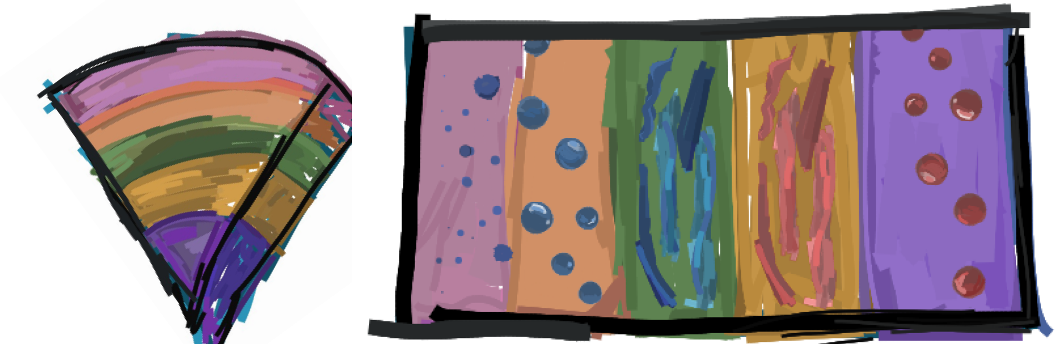
EAST TEXAS A&M UNIVERSITY



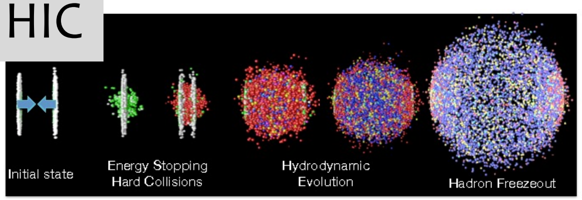
Noa Fritschie, 2022

Nuclear structure/ dynamics

Multi-messenger Nuclear & Astro Physics

Neutron star structure/ dynamics

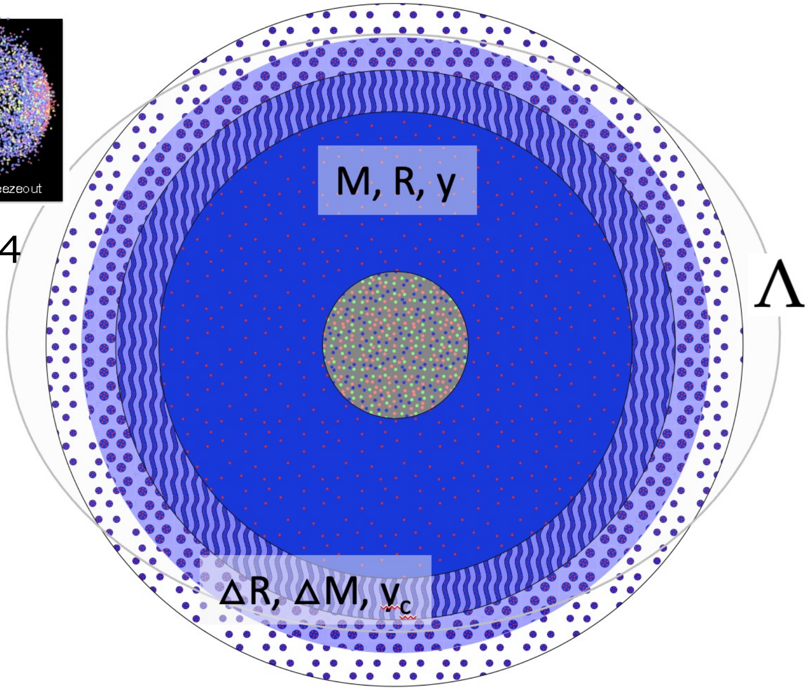
HIC



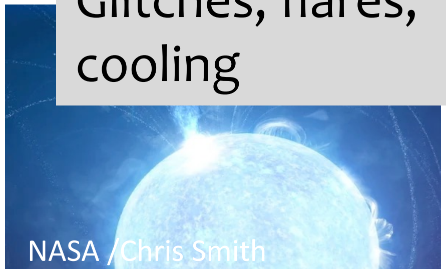
T.K.Nayak, arxiv:1201.4264

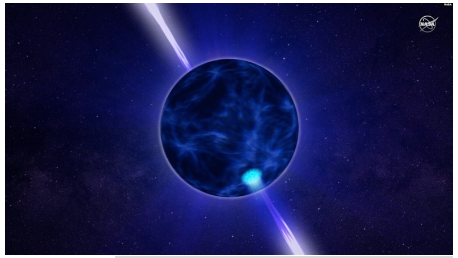
neutron skins

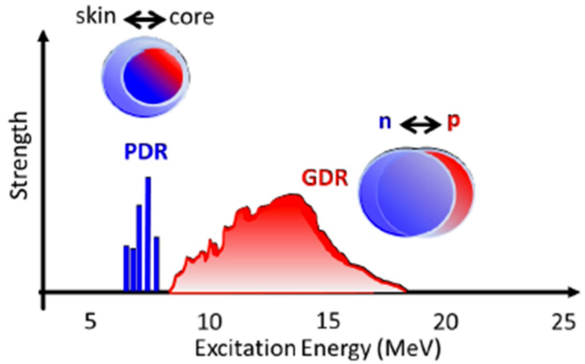
Abrahamyan+,
PRL 108, 112592 (2012)



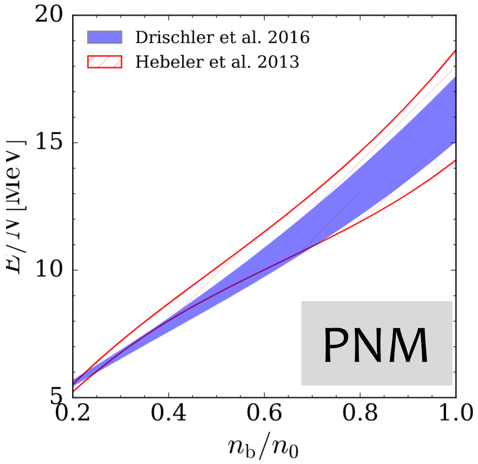
Glitches, flares, cooling





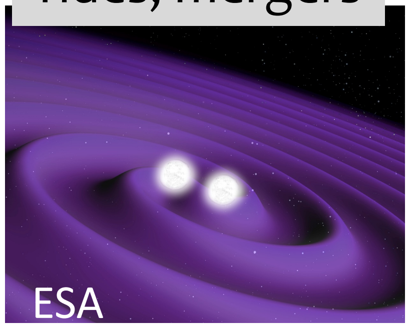


Bracco, Lanza, Tamii,
PPNP 106, 360 (2019)



Haensel, Fortin JPhysG 2017

Tides, mergers



Hot spots
Oscillations,
Crust cooling

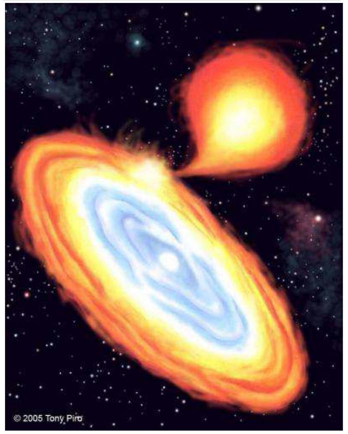


Figure: Artist's impression of a LMXB - credit Tony Piro, 2005.

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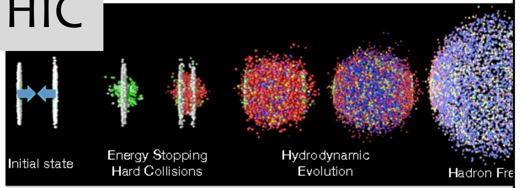
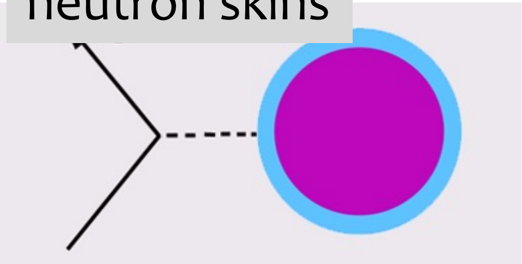


T.K.Nayak, arxiv:1201.426

neutron skins



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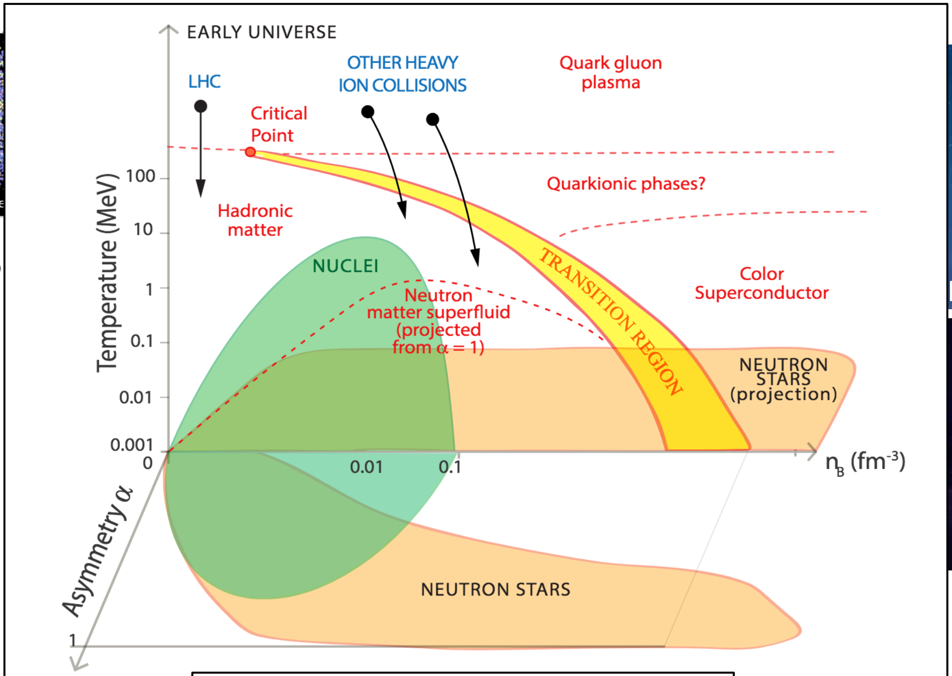
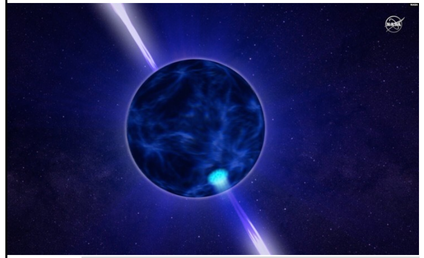


Watts et al arxiv:1501.00042

Glitches, flares, cooling



Hot spots
Oscillations,
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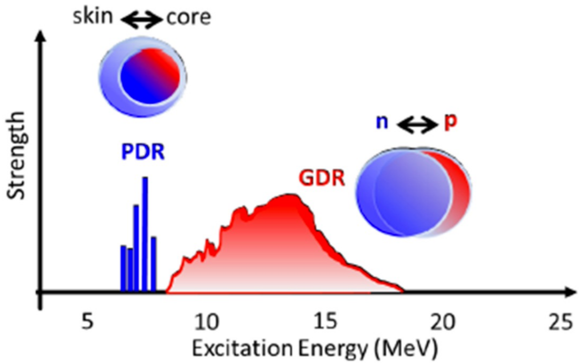
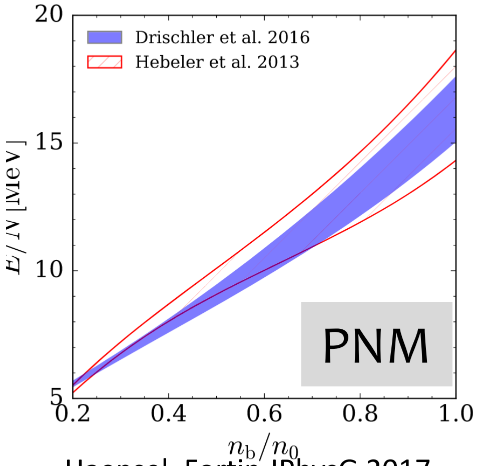
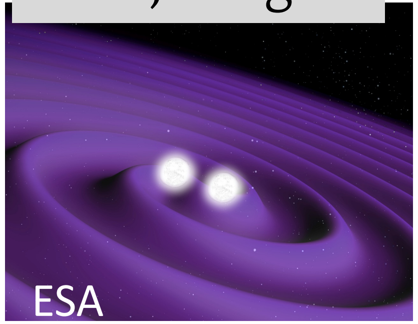


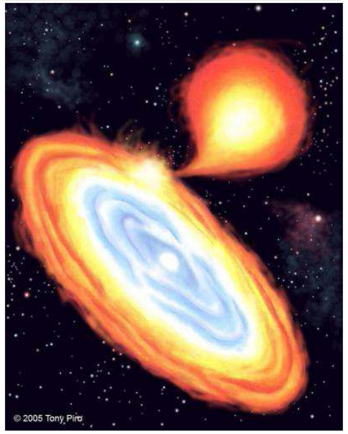
Bracco, Lanza, Tamii,
PPNP 106, 360 (2019)



Haensel, Fortin JPhysG 2017

Tides, mergers





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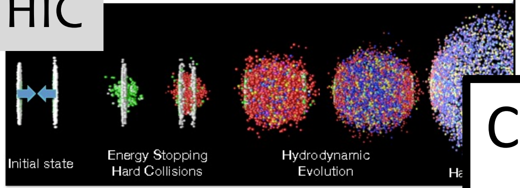
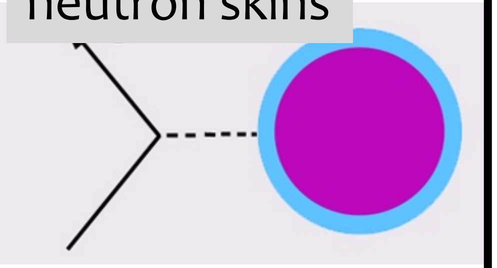


T.K.Nayak, arxiv:1201.4

neutron skins



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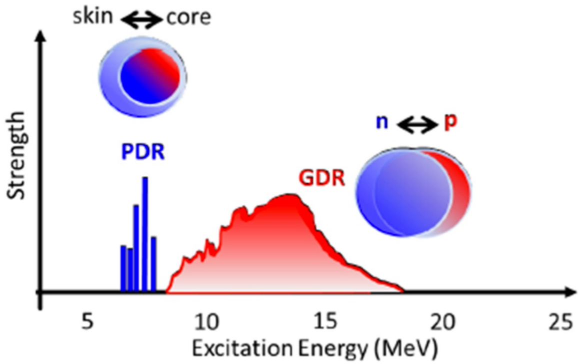
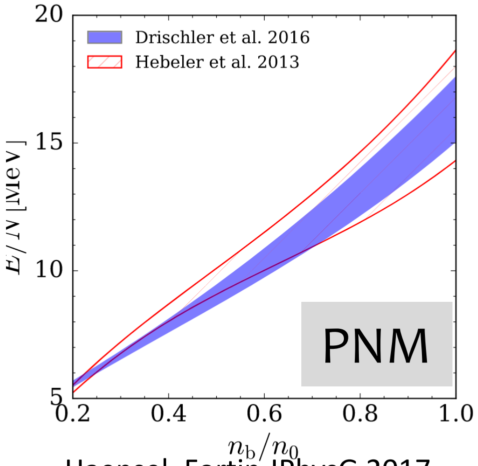


Bracco, Lanza, Tamii, PPNP 106, 360 (2019)

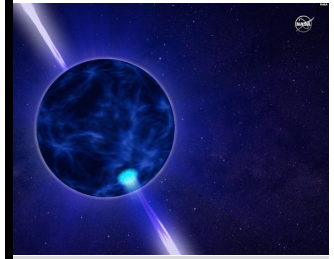
Communication between communities:

- What are we interested in?
- What do we need?
- How are we defining quantities?
- What are the limits of models?
- How are the uncertainties quantified?
- What are the model dependencies?



Haensel, Fortin JPhysG 2017

Glitches, flares, cooling



Hot spots
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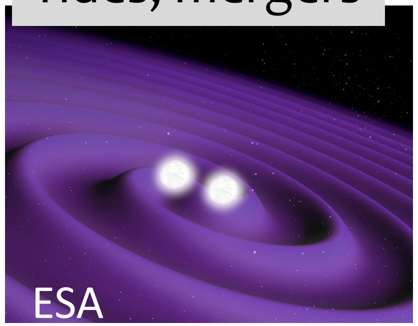


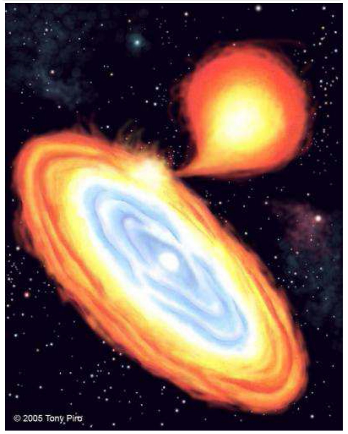


Figure: Artist's impression of a LMXB - credit Tony Piro, 2005.

Some questions

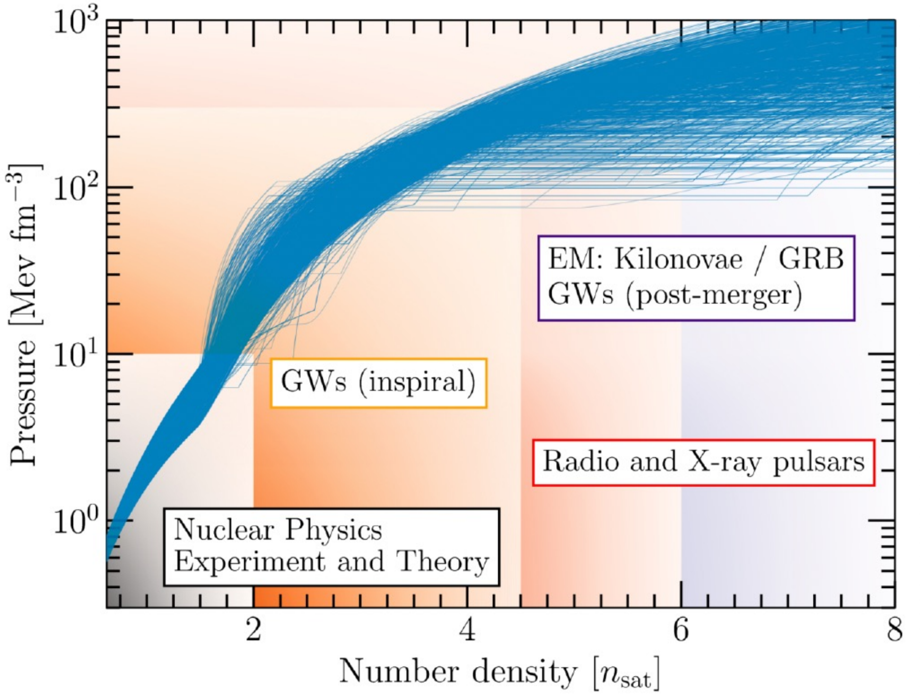
How to combine many nuclear and astro observables minimizing systematic model uncertainties

- Modeling observables as directly as possible
- Different density dependence of models when extrapolating constraints up/down in density
- Which densities are best probed by which observables?
- Including the crust consistently to bring other observables into play

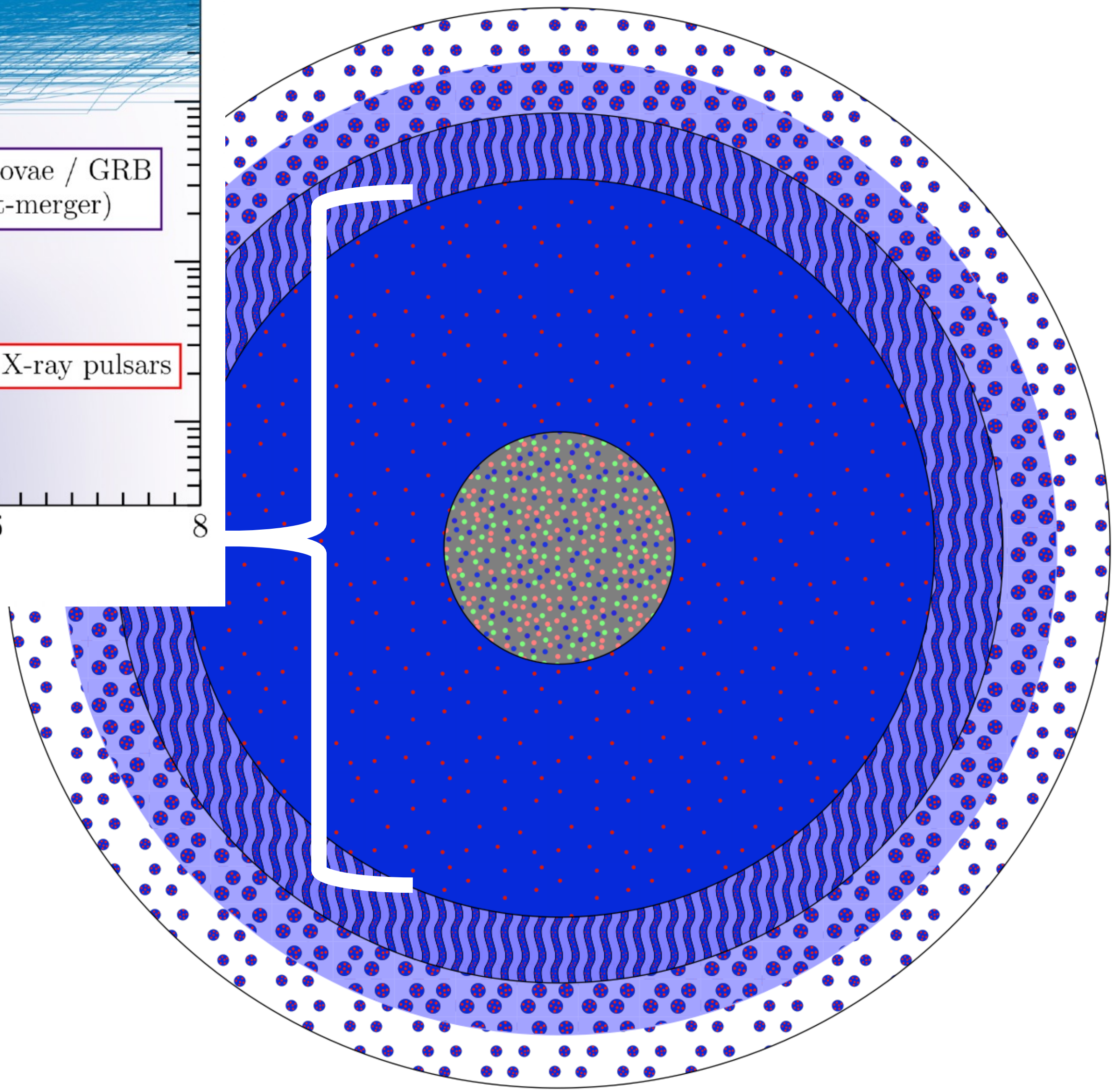
We'll illustrate these considerations schematically, take a look at some examples from the literature and then delve deeper using our own EoS models

I'm picking out systematic modeling uncertainties in a number of really excellent studies – they are not a slight on the studies!

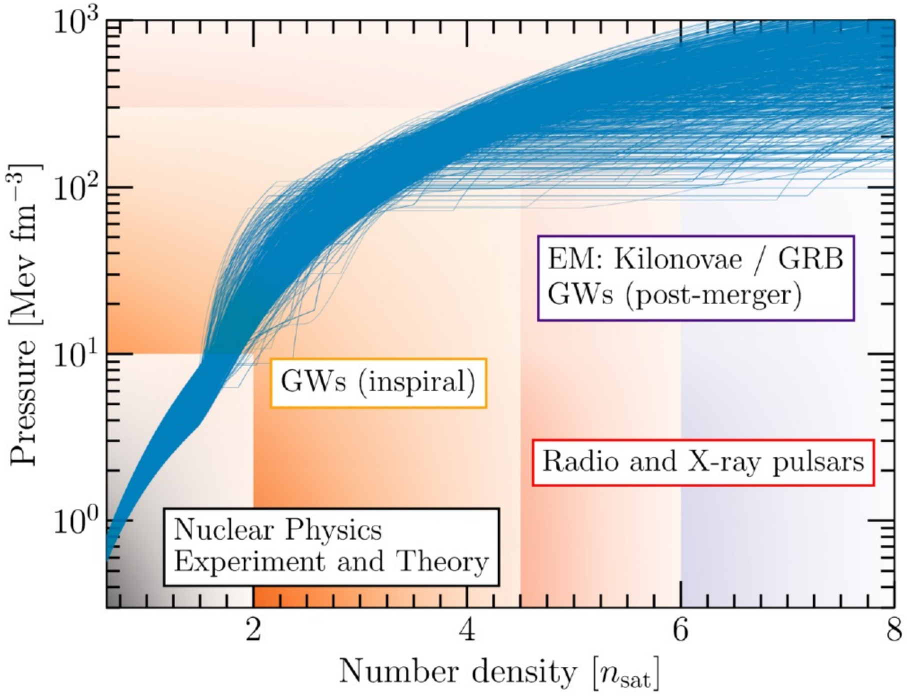
Modern approach: create ensembles of EOSs/neutron star models for statistical inference



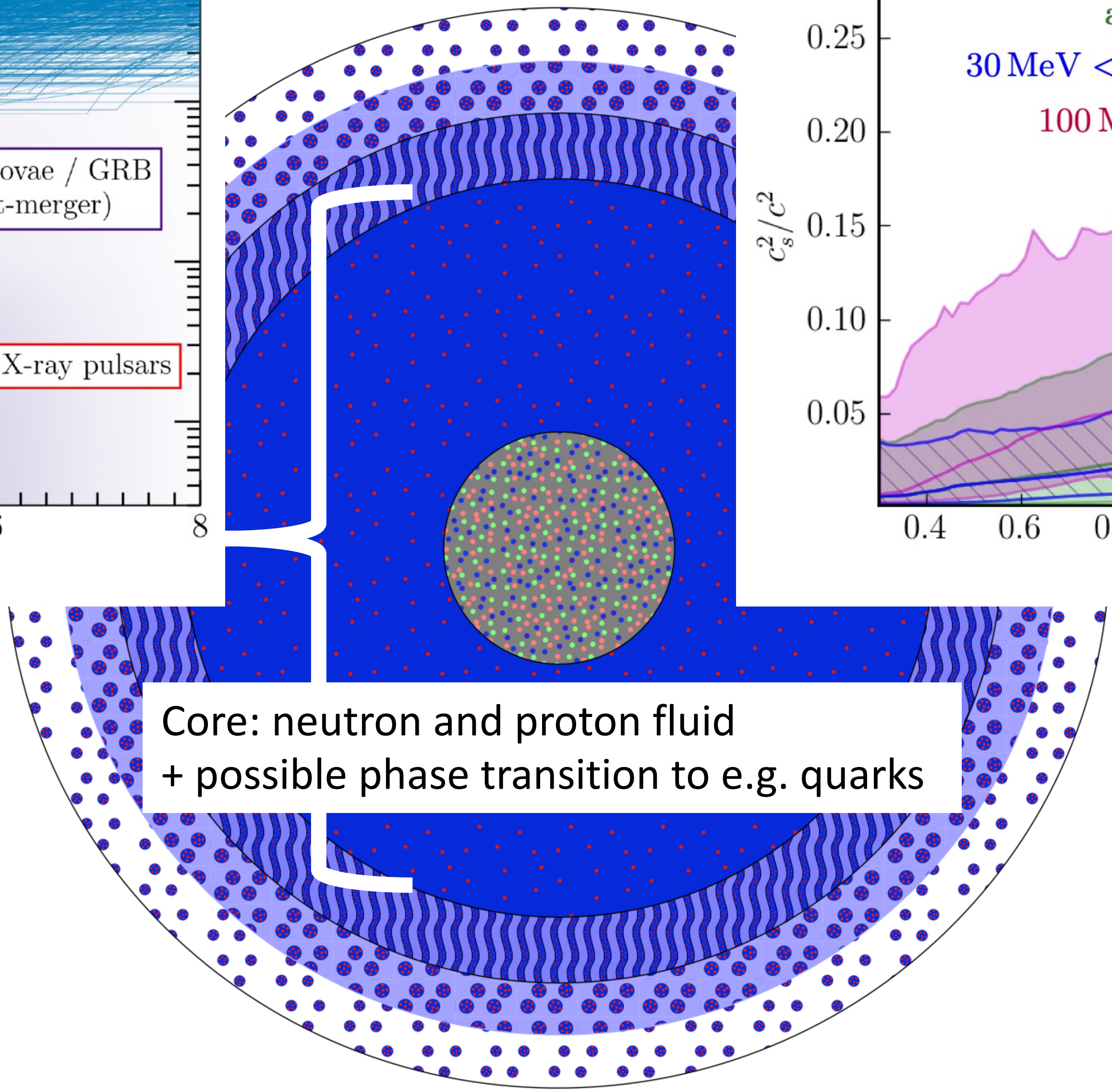
Pang et al, arxiv:2205.08513



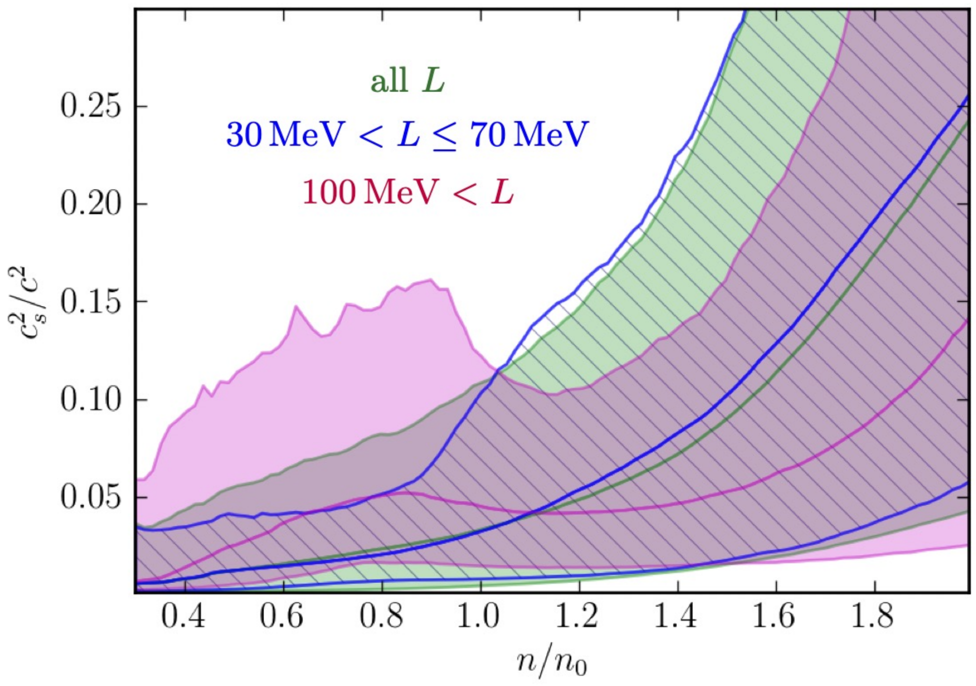
And go ahead and infer! To date, emphasis has been on the EOS of the core



Pang et al, arxiv:2205.08513



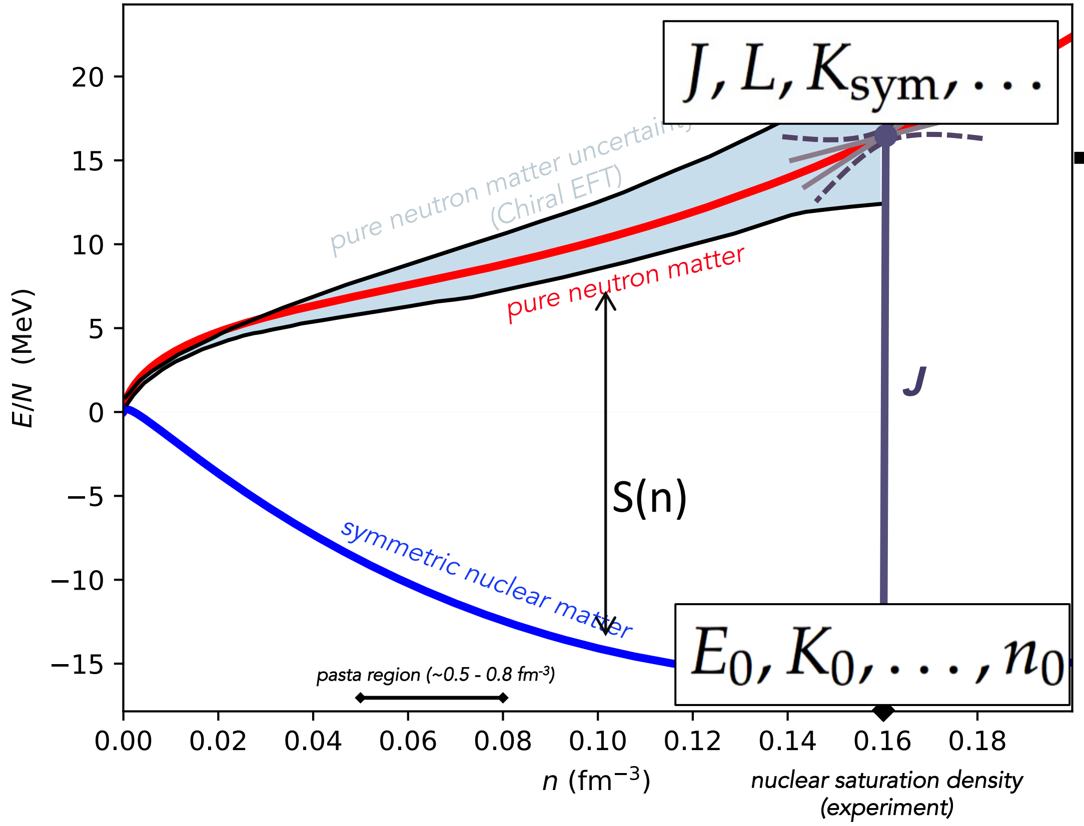
Core: neutron and proton fluid
+ possible phase transition to e.g. quarks



Essick+ arXiv 2102.10074

The Nuclear Matter Equation of State

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L(\frac{\rho - \rho_0}{3\rho_0}) + \frac{K_{sym}}{2}(\frac{\rho - \rho_0}{3\rho_0})^2 + \frac{Q_{sym}}{6}(\frac{\rho - \rho_0}{3\rho_0})^3 \ldots$$



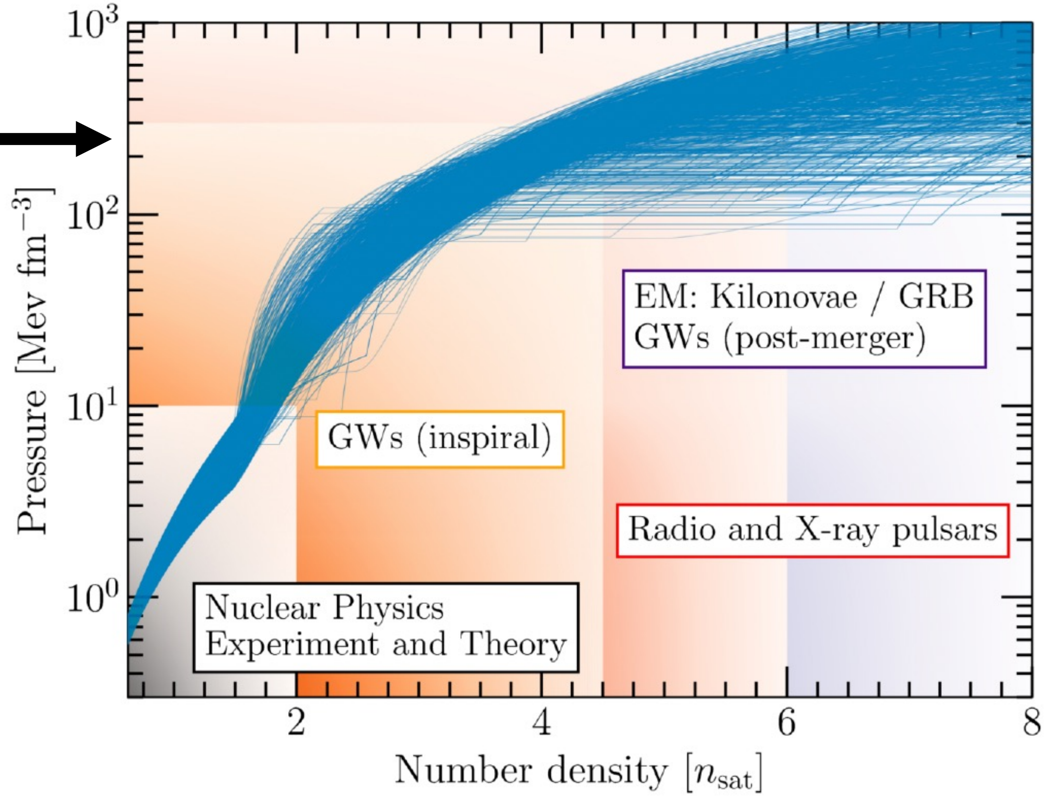


Figure: Lauren Balliet

Pang et al, arxiv:2205.08513

Li, arxiv:2105.04629

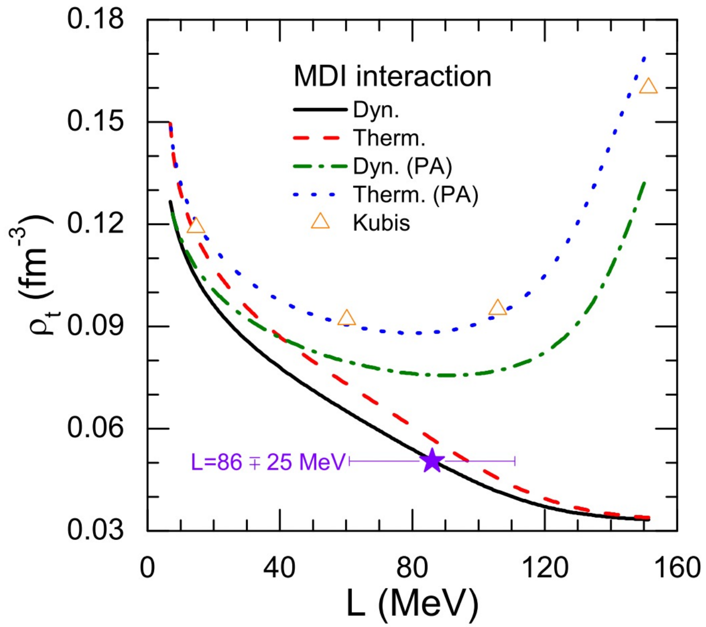
$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}(\frac{\rho - \rho_0}{3\rho_0})^2 + \frac{Q_0}{6}(\frac{\rho - \rho_0}{3\rho_0})^3 \ldots$$

Symmetry energy: some communication problems

1. Proliferation of nomenclature

E_{sym}, S, a_{sym}, c_{sym},

2. 2^{nd} order or all orders



$$E_{\text{sym},2}(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial \delta^2}\bigg|_{\delta=0}$$

$$E_{\text{sym}}(\rho) = E(\rho,\delta=1) - E(\rho,\delta=0) = E_{\text{PNM}}(\rho) - E_{\text{SNM}}(\rho)$$

Difference < 3 MeV at saturation density
In neutron-rich matter $\delta \cong 1$ so higher order terms become important

Xu et al, arxiv:0807.4477

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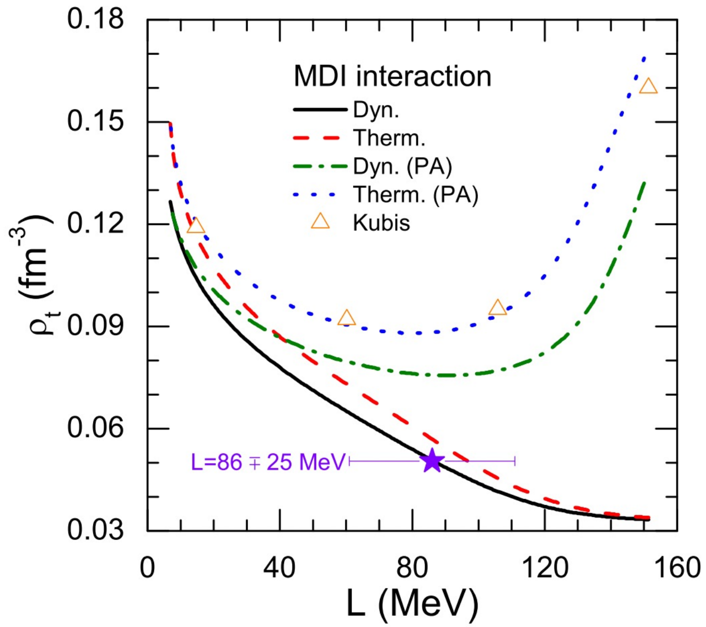

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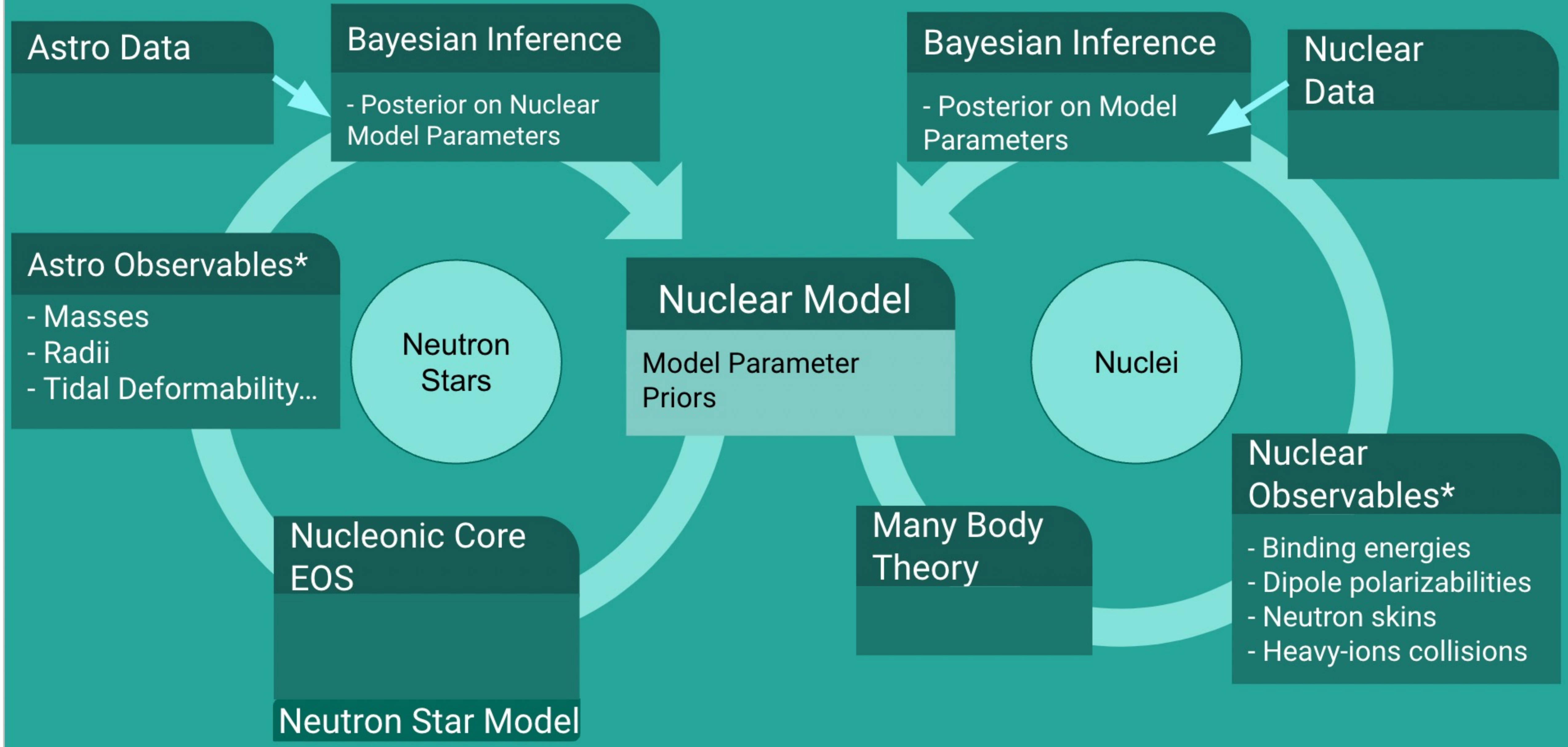
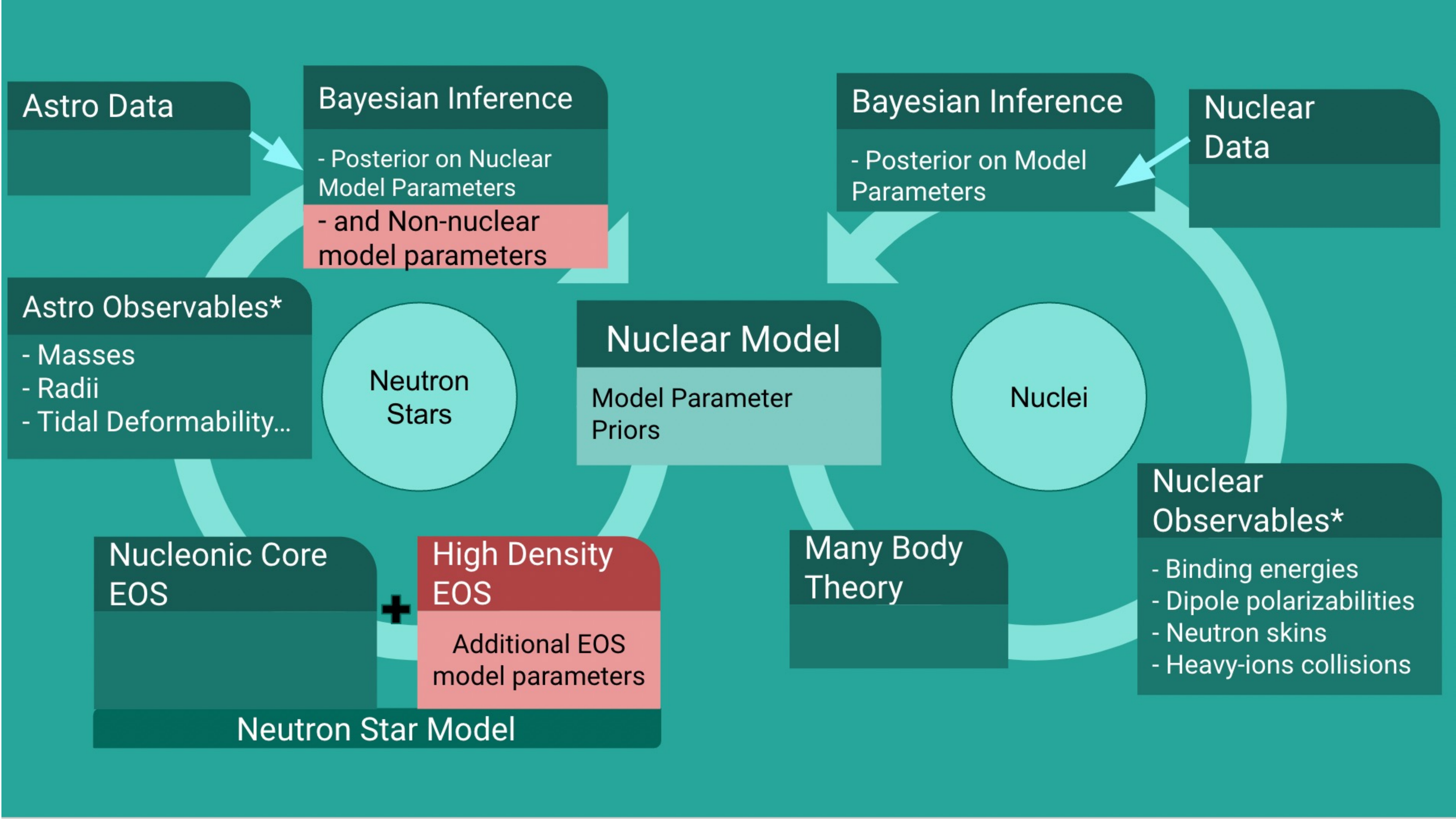
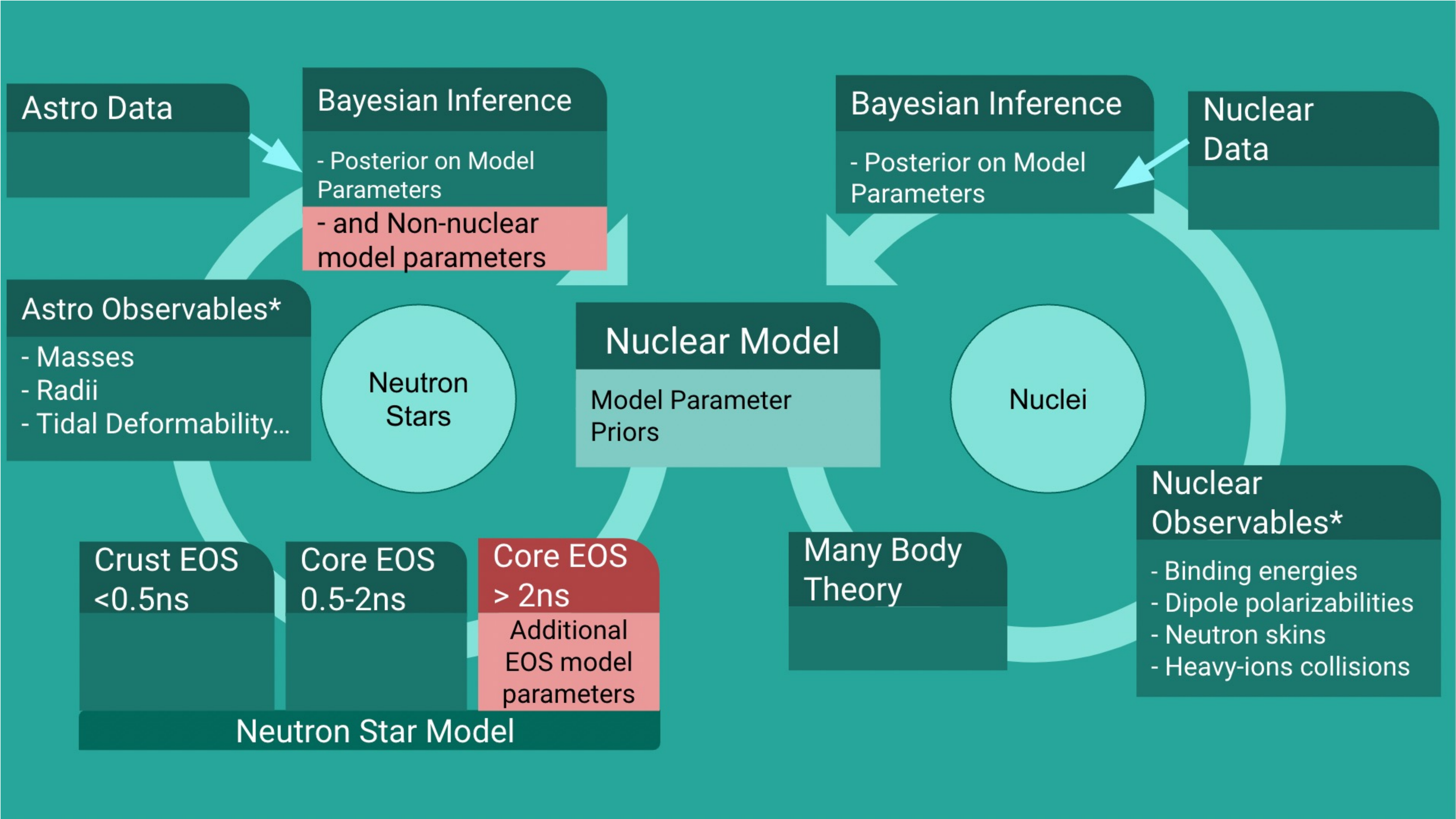
An idealized scheme




Astro Data
Bayesian Inference
- Posterior on Nuclear Model Parameters
- and Non-nuclear model parameters
Bayesian Inference
- Posterior on Model Parameters
Nuclear Data
Astro Observables*
- Masses
- Radii
- Tidal Deformability...
Neutron Stars
Nuclear Model
Model Parameter Priors
Nuclei
Nuclear Observables*
- Binding energies
- Dipole polarizabilities
- Neutron skins
- Heavy-ions collisions
Nucleonic Core EOS
+
High Density EOS
Additional EOS model parameters
Many Body Theory
Neutron Star Model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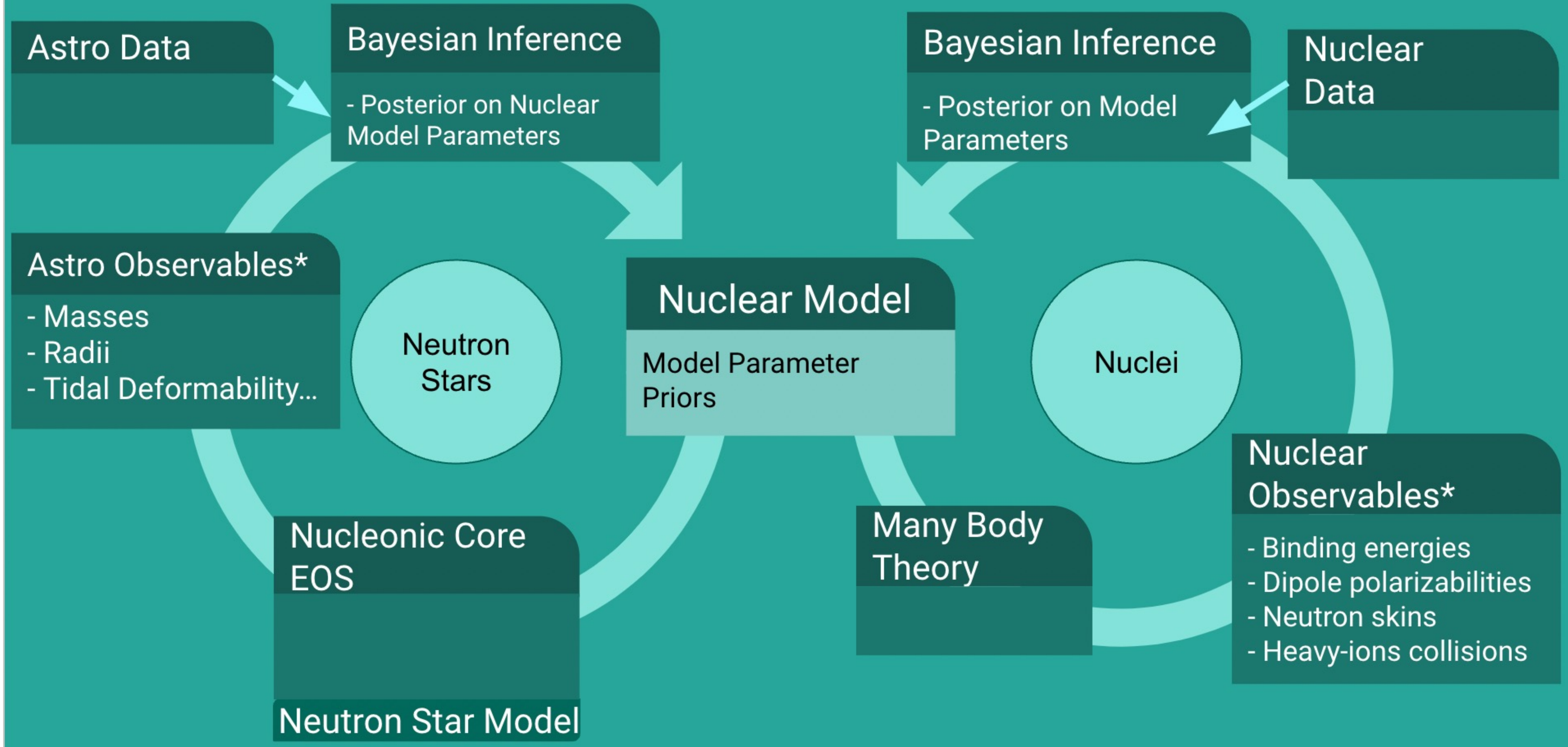
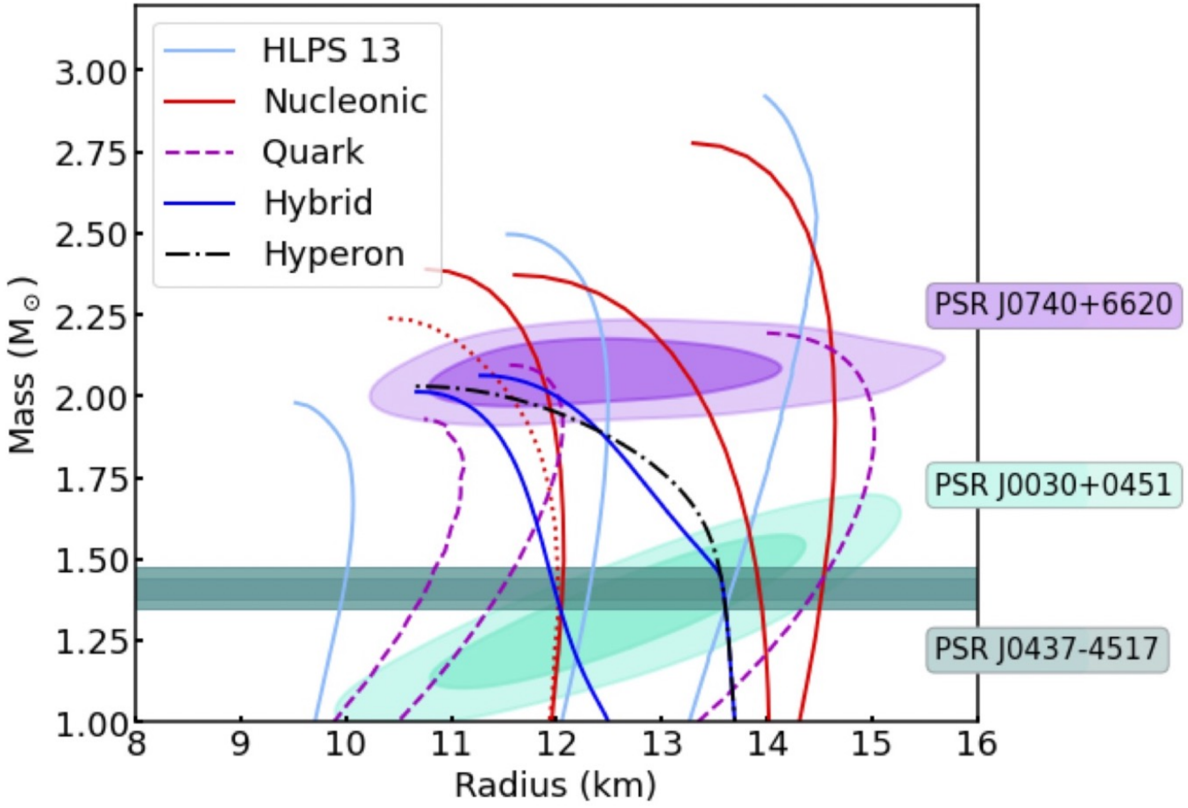
Crust EOS <0.5ns
Core EOS 0.5-2ns
Core EOS > 2ns
Additional EOS model parameters
Many Body Theory
Neutron Star Model

An idealized scheme

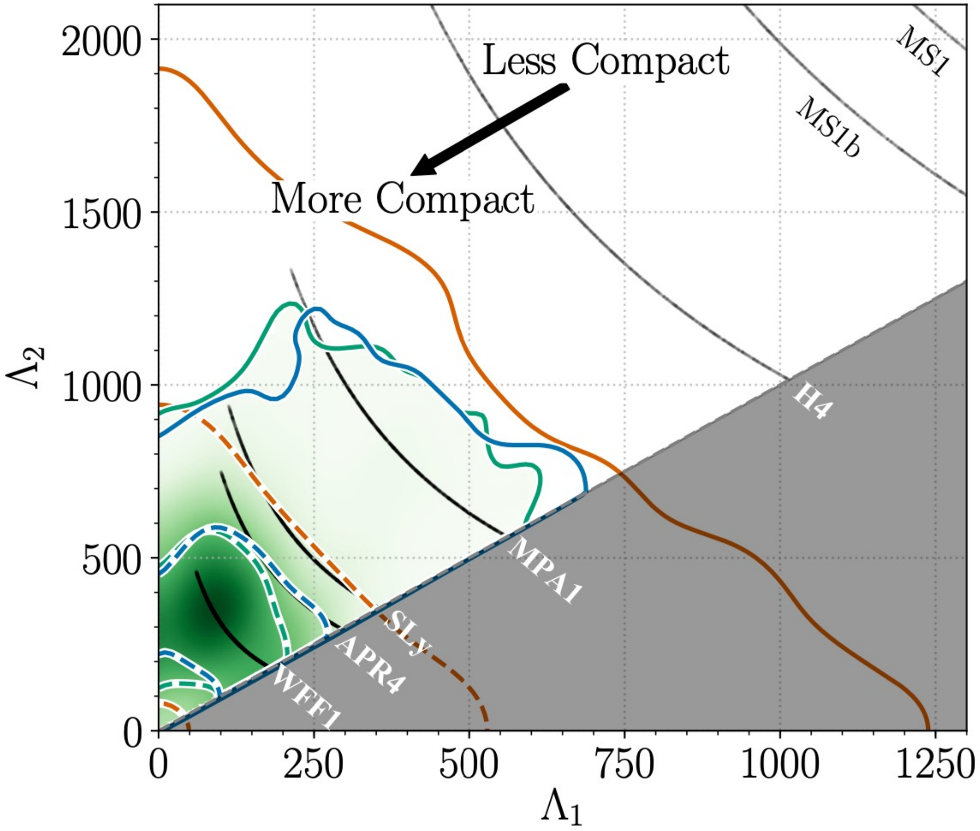


Data: Neutron star mass/radii (e.g. NICER)

Data: Tidal Deformability



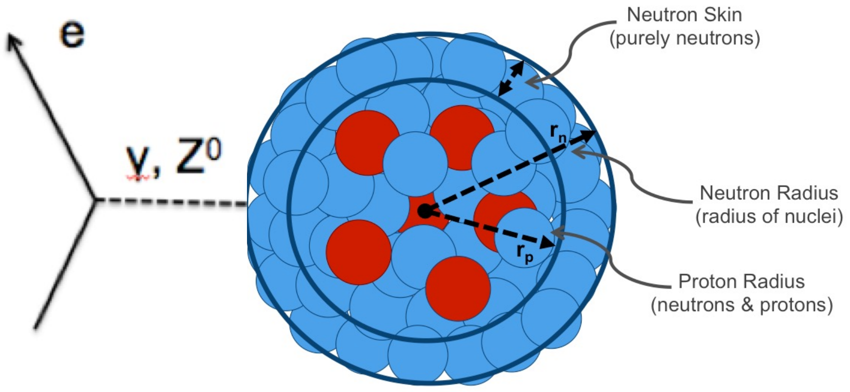
Riley arxiv:1912.05702, arxiv:2105.06980
Miller et al arxiv:2105.06979, arxiv:1912.05705
Raajimakers et al arxiv: 1912.05703, 2105.06981



LIGO/Virgo arxiv:1805.11581

Data: Neutron Skins

PREX, CREX



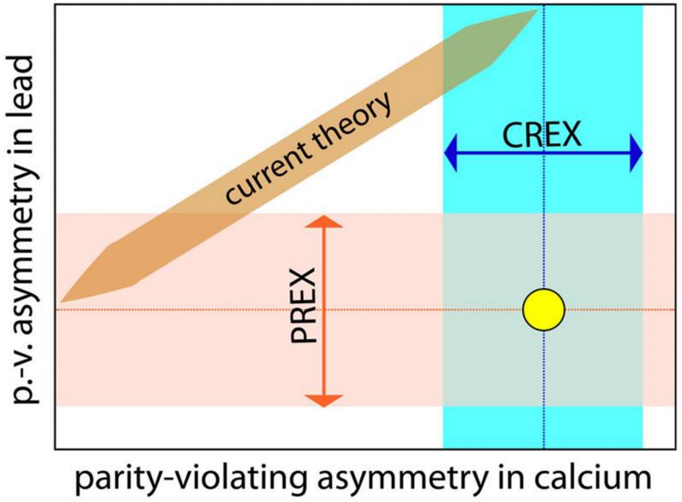
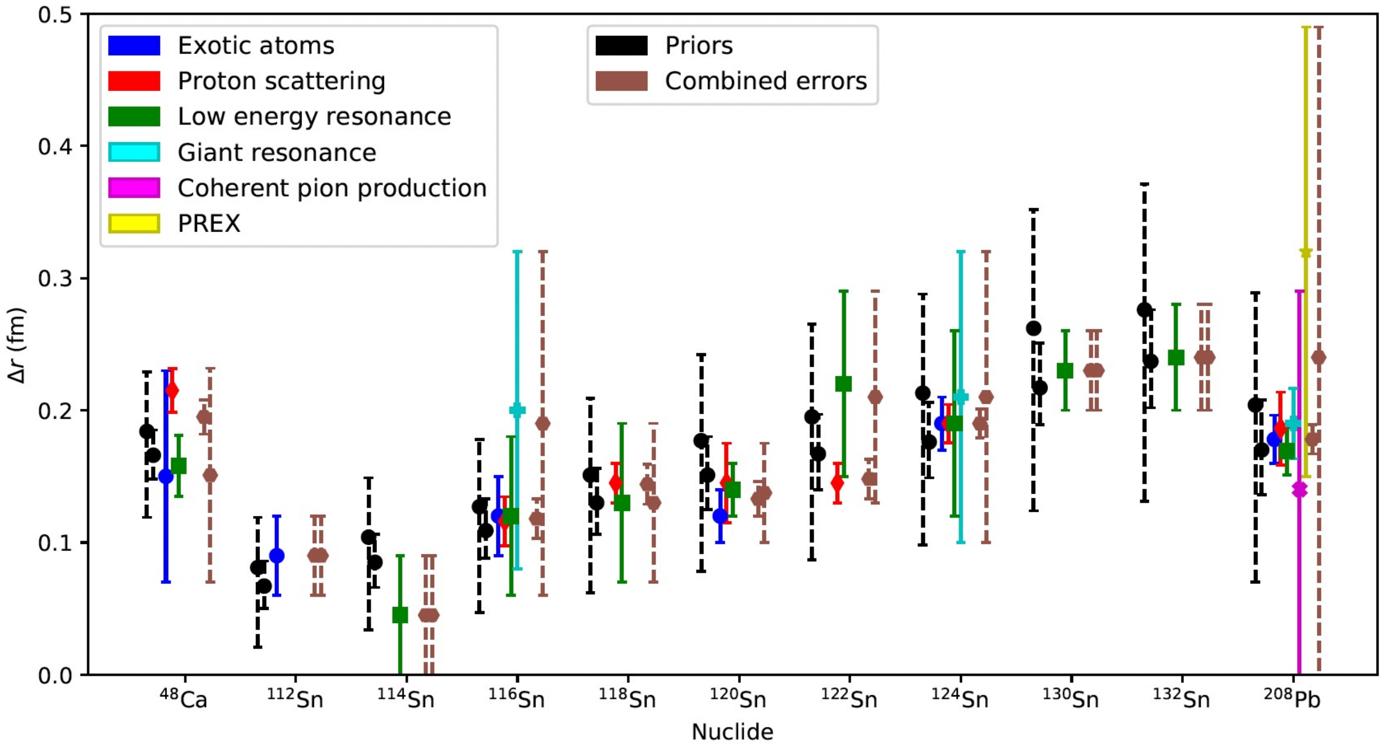


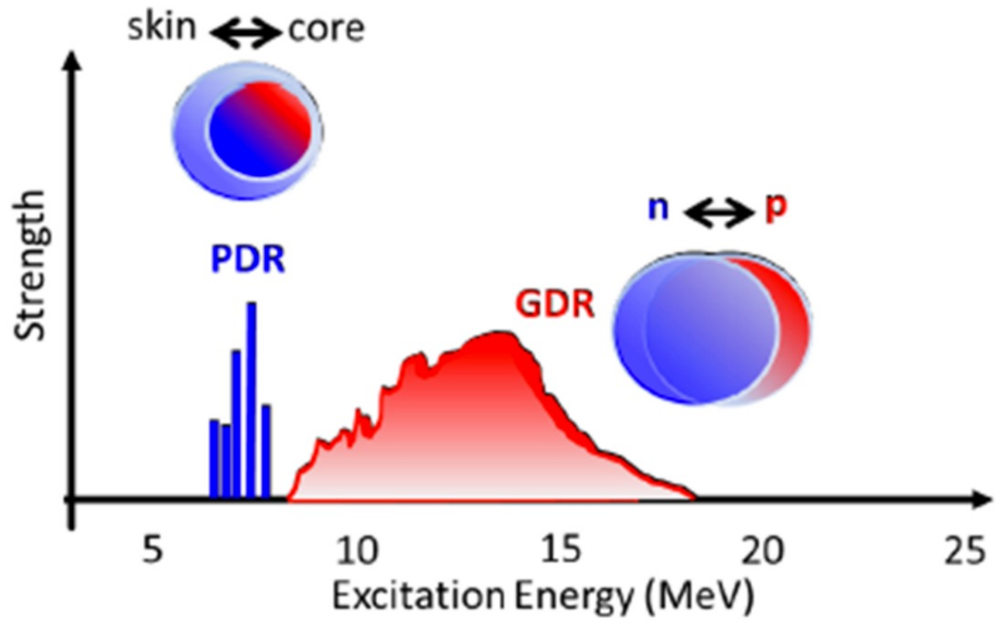
Image: Witold Nazarewicz

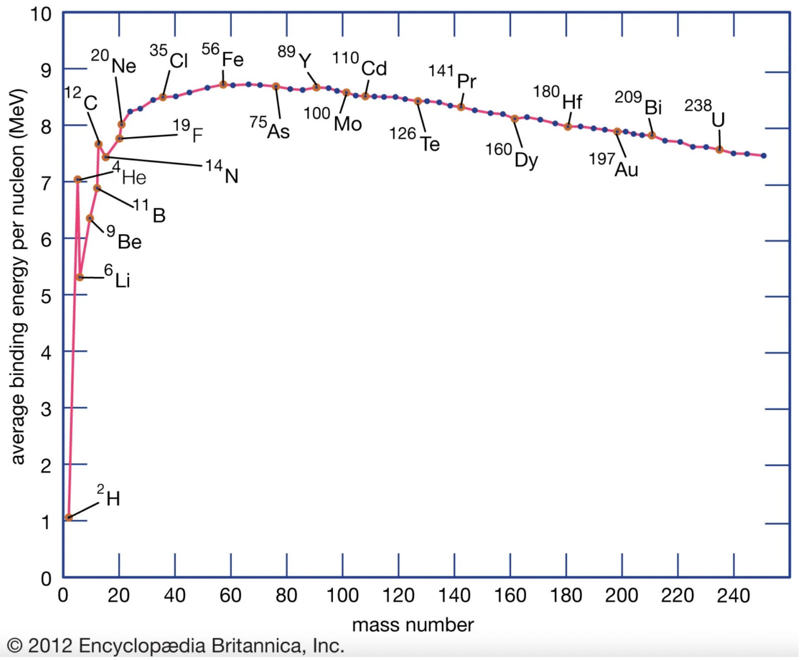
Other Probes



Newton, Crocombe arxiv:2008.00042

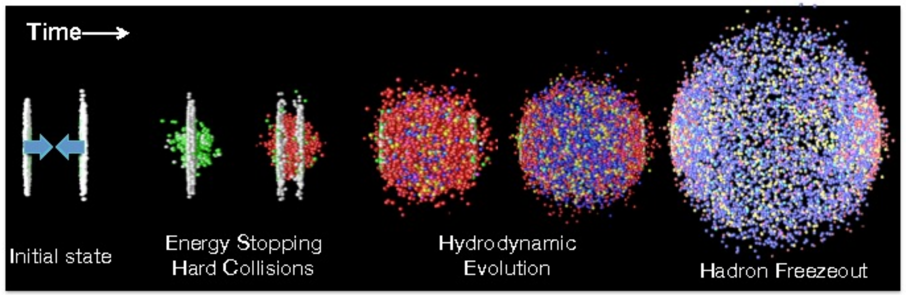
Data: Dipole Polarizability, Nuclear Masses, HIC

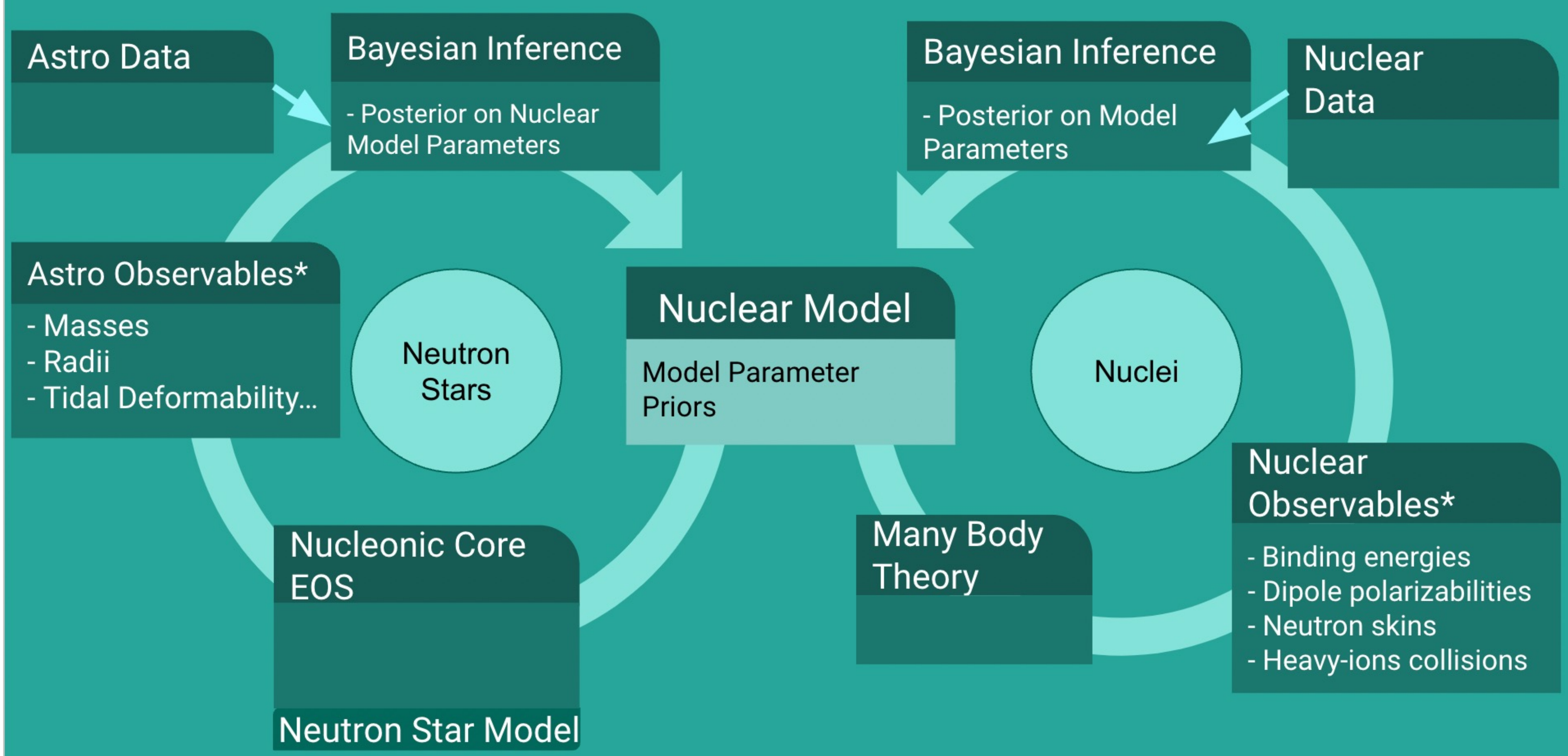




Bracco, Lanza, Tamii,
PPNP 106, 360 (2019)

e.g. proton scattering

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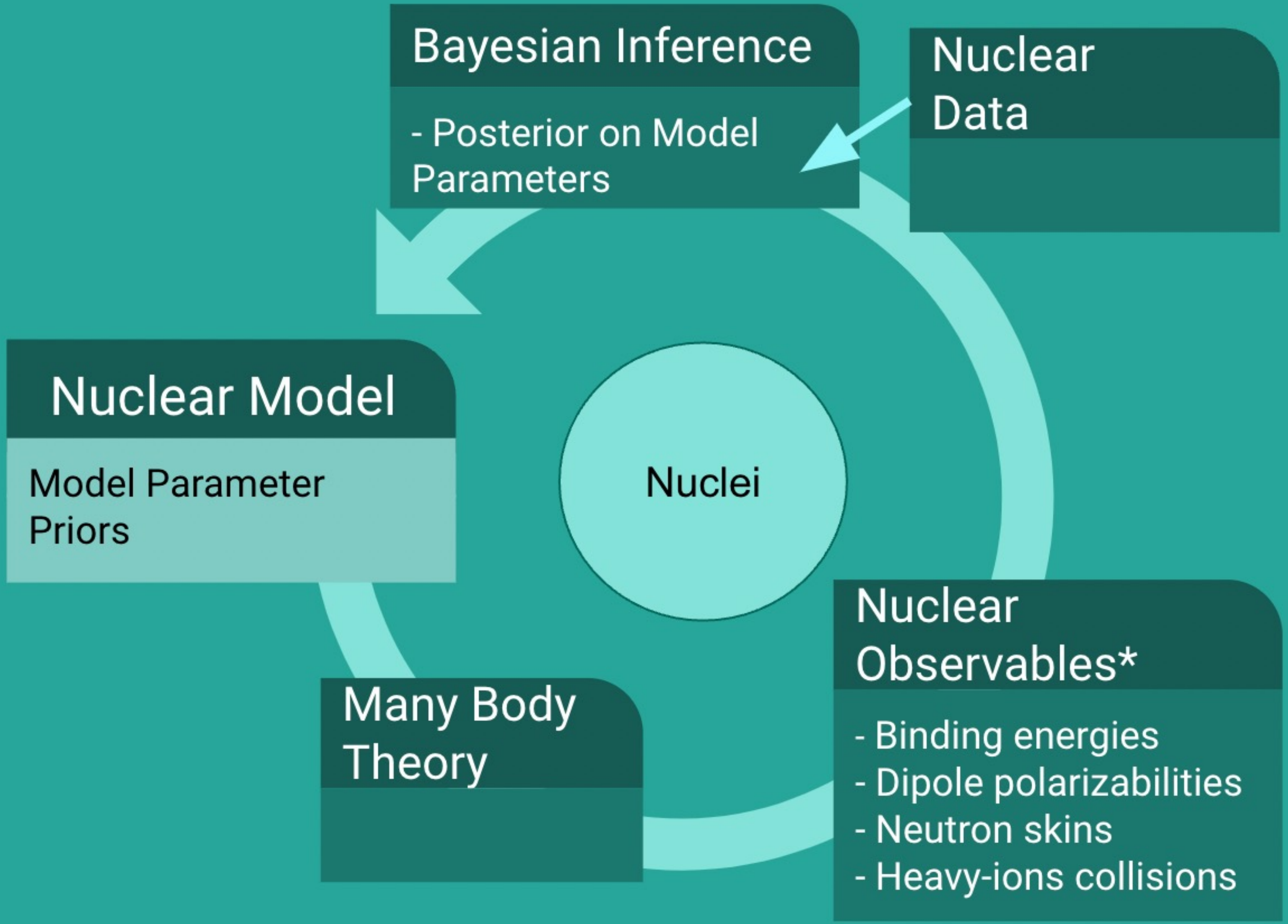
Nucleonic Core EOS
Neutron Star Model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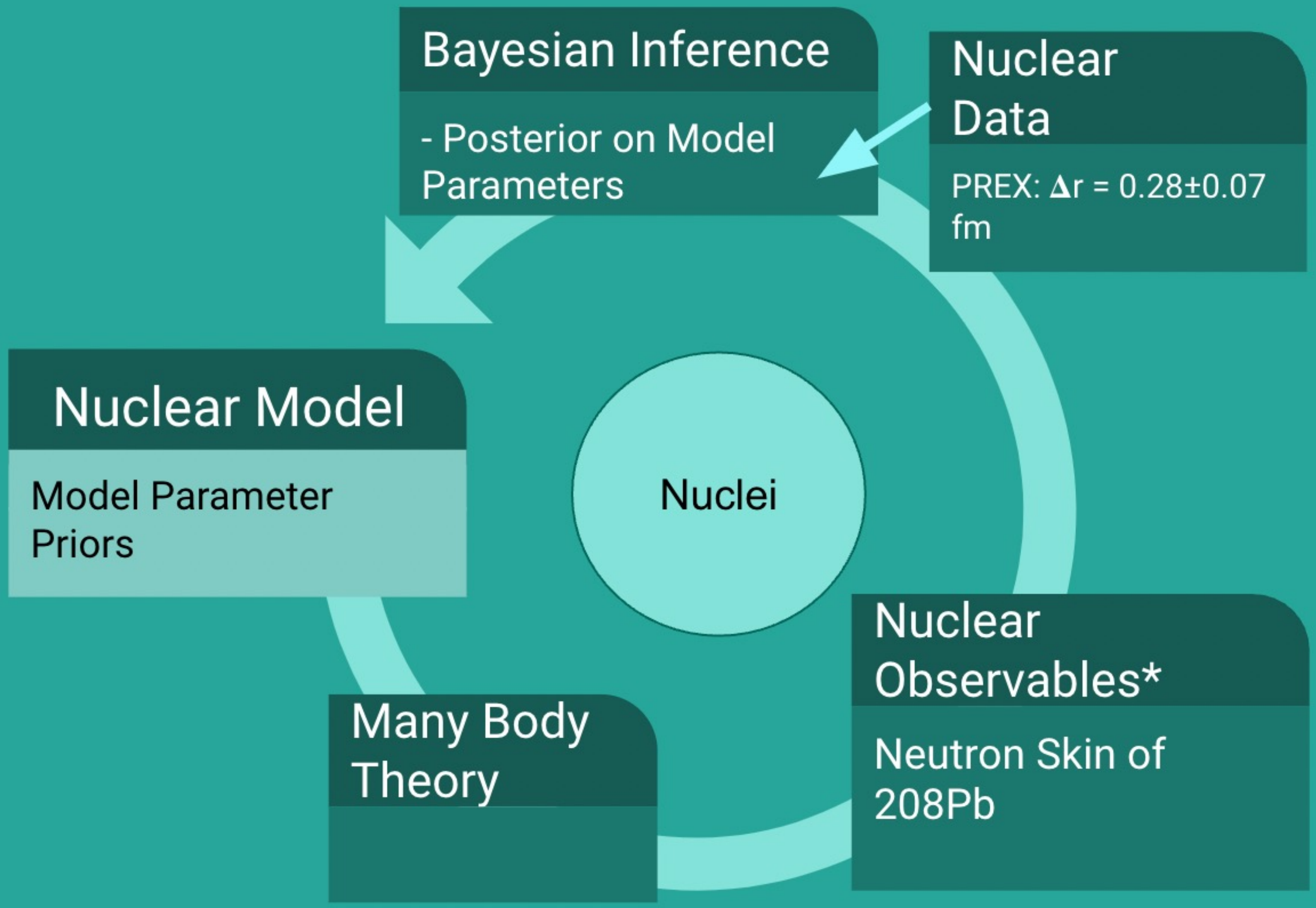
Bayesian Inference
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Nuclear Data
PREX: Δr = 0.28±0.07 fm
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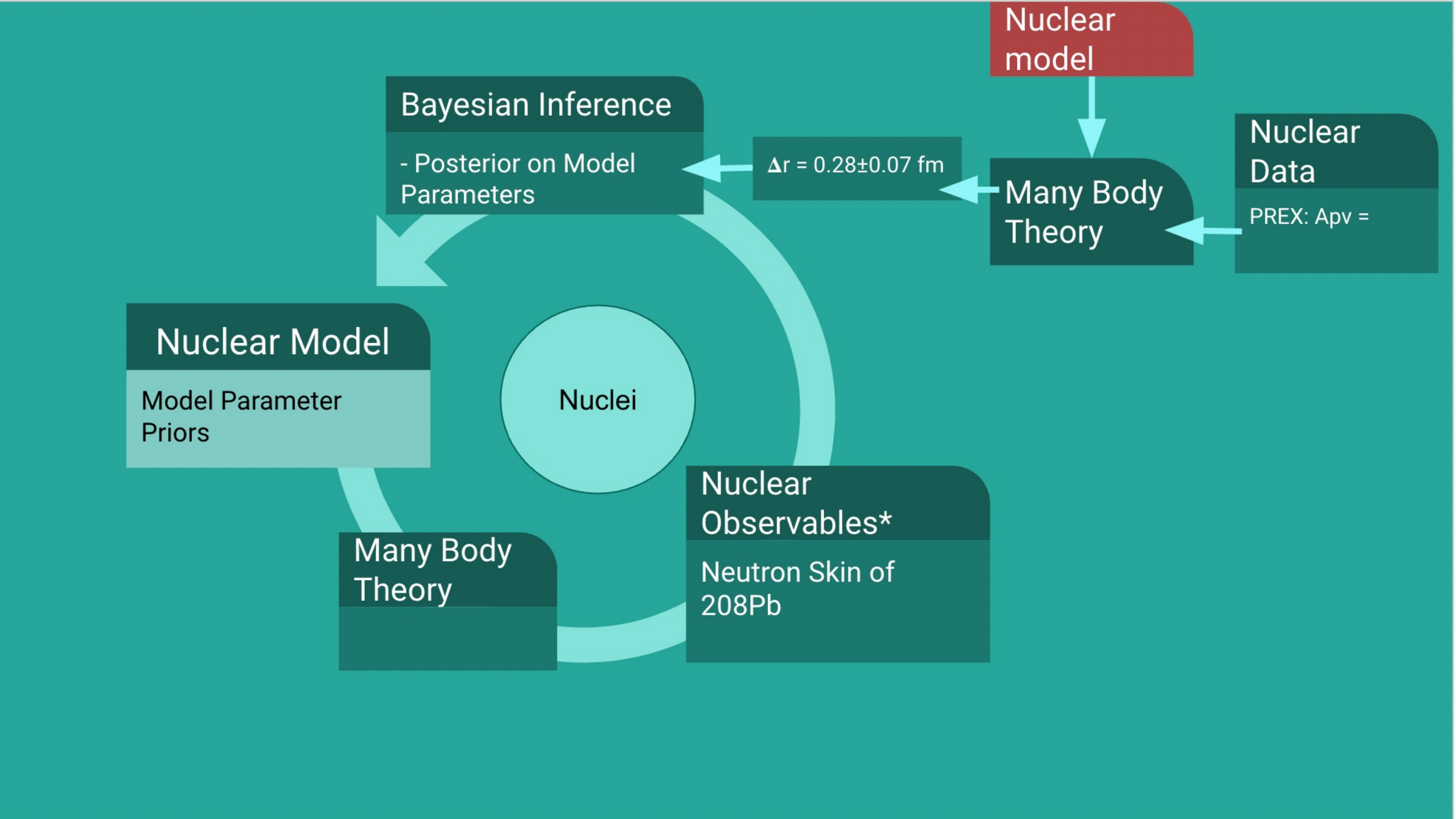
Neutron Skin of 208Pb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Bayesian Inference
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Many Body Theory
Nuclear Data
PREX: Apv =
Nuclear Model
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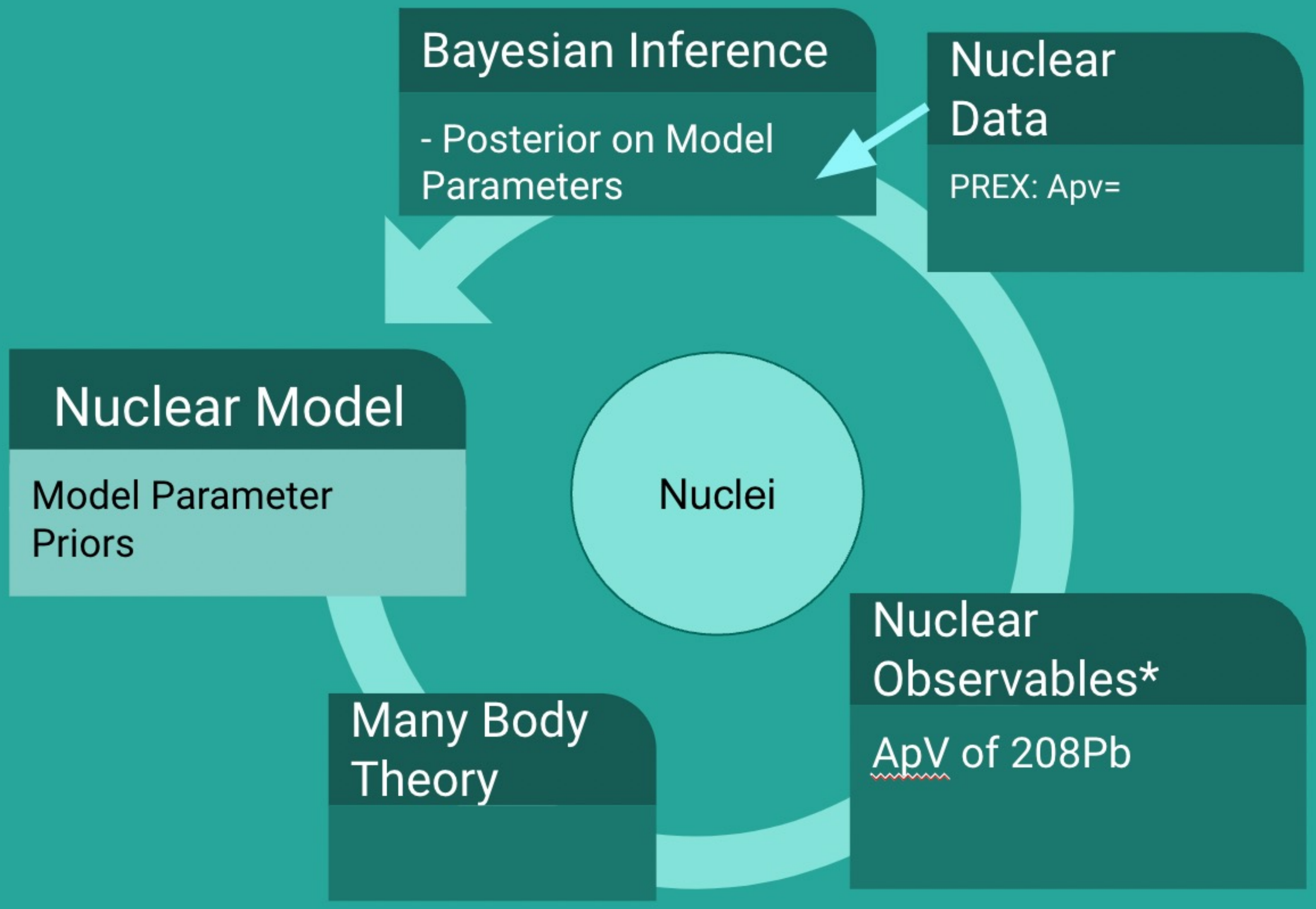
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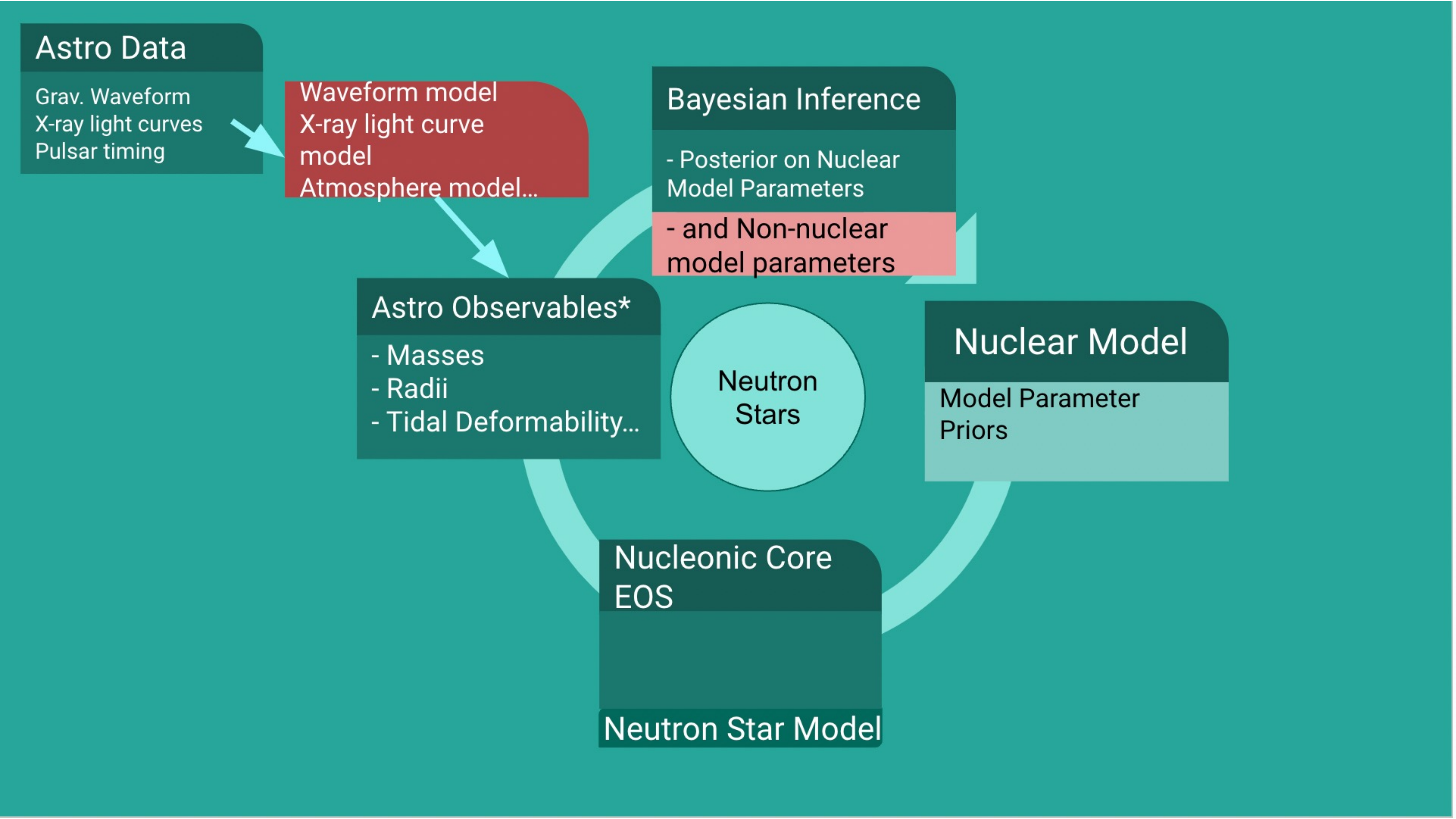
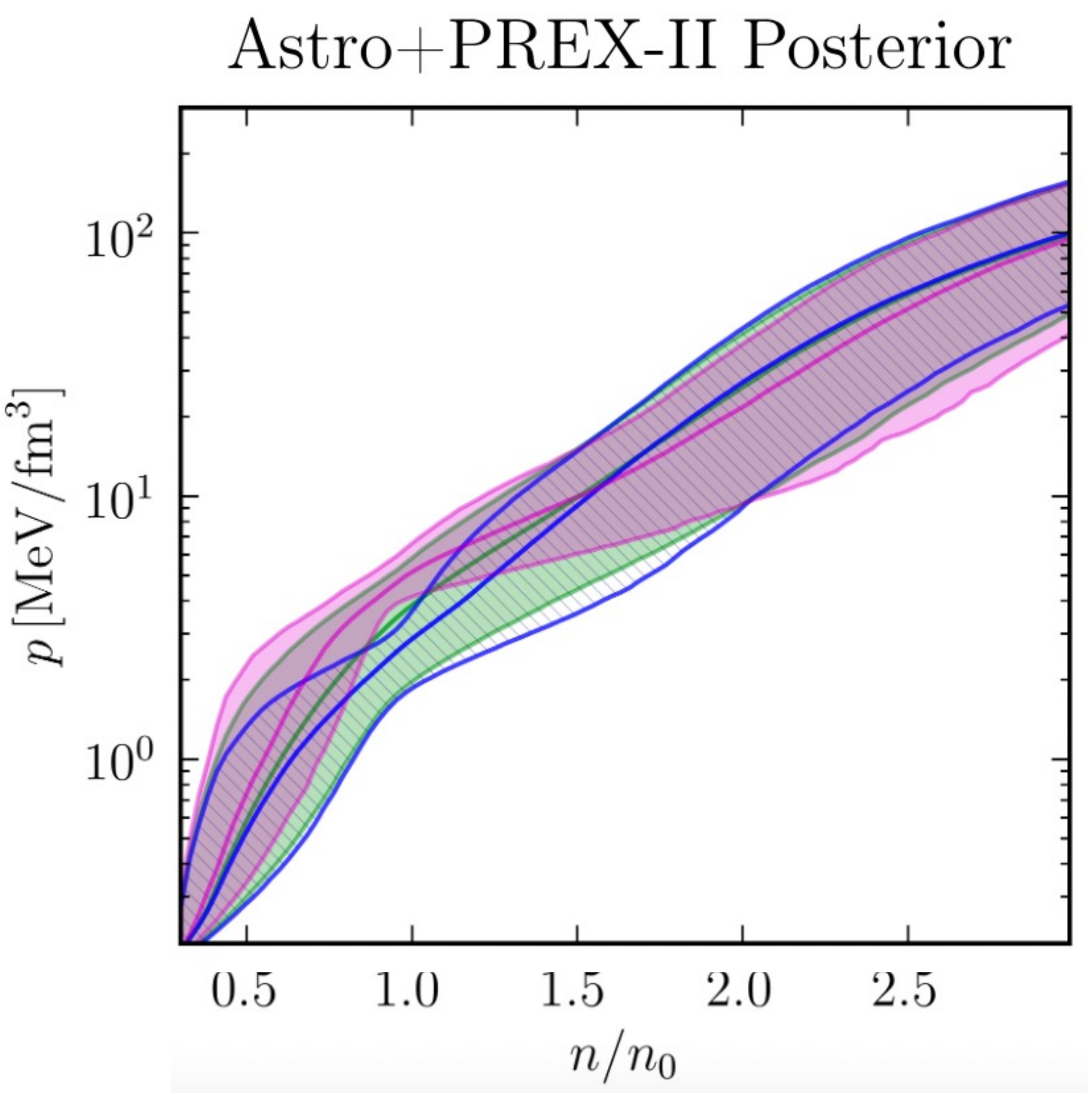
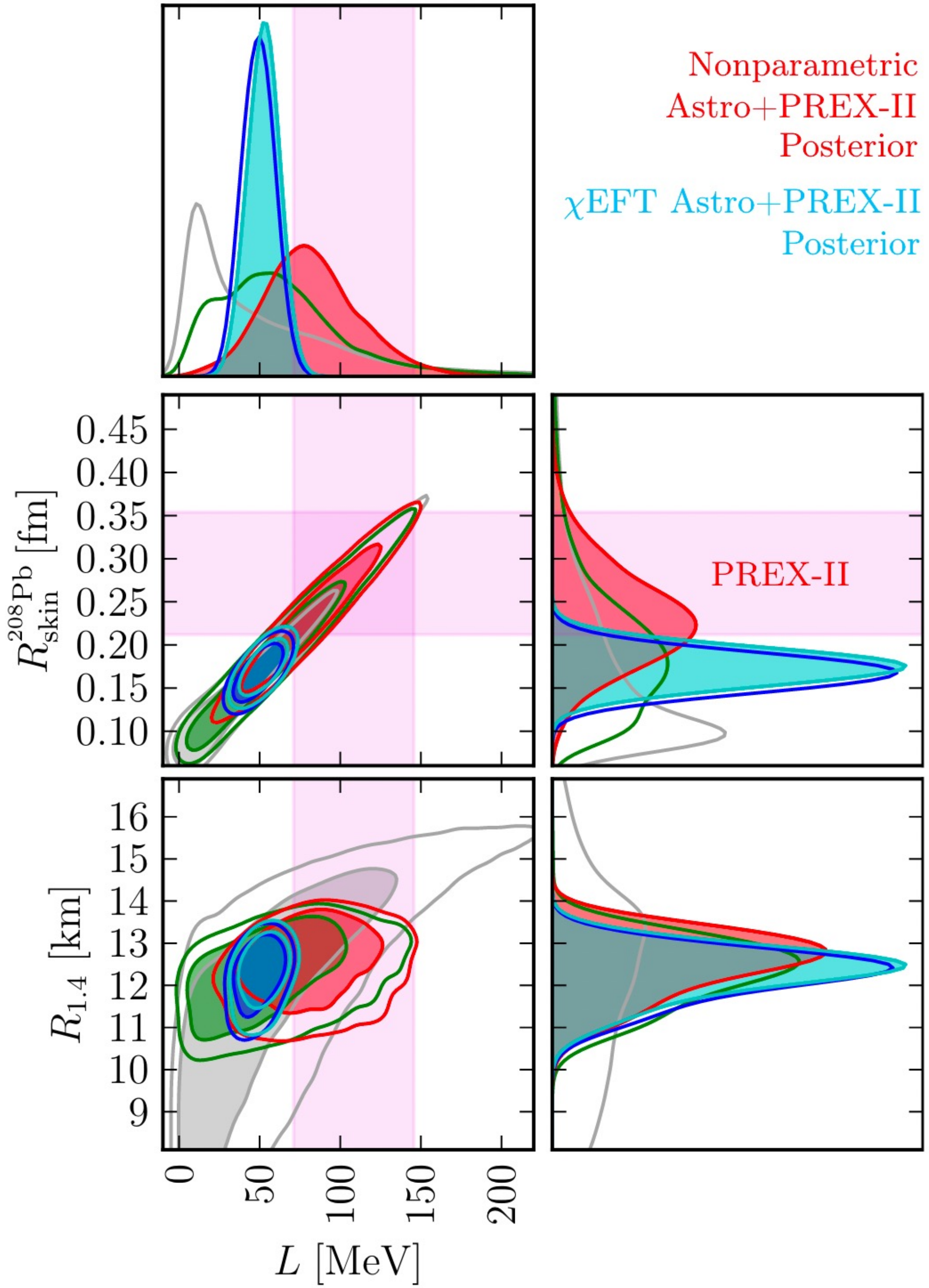
Nuclear Observables*
ApV of 208Pb
Many Body Theory

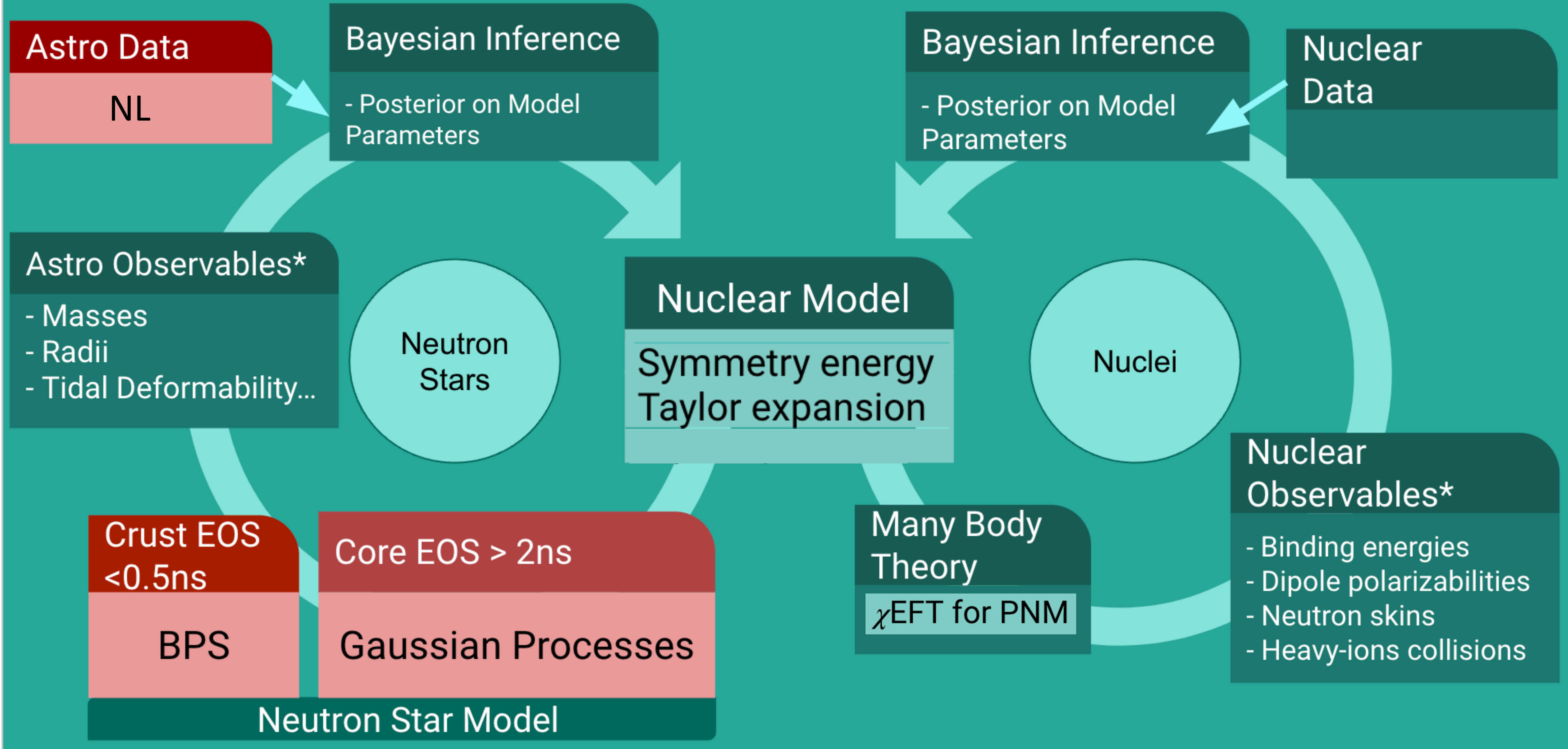

Astro Data
Grav. Waveform
X-ray light curves
Pulsar timing
Waveform model
X-ray light curve model
Atmosphere model...
Bayesian Inference
- Posterior on Nuclear Model Parameters
- and Non-nuclear model parameters
Astro Observables*
- Masses
- Radii
- Tidal Deformability...
Neutron Stars
Nuclear Model
Model Parameter Priors
Nucleonic Core EOS
Neutron Star Model

Example 1: Essick+ arXiv 2102.10074

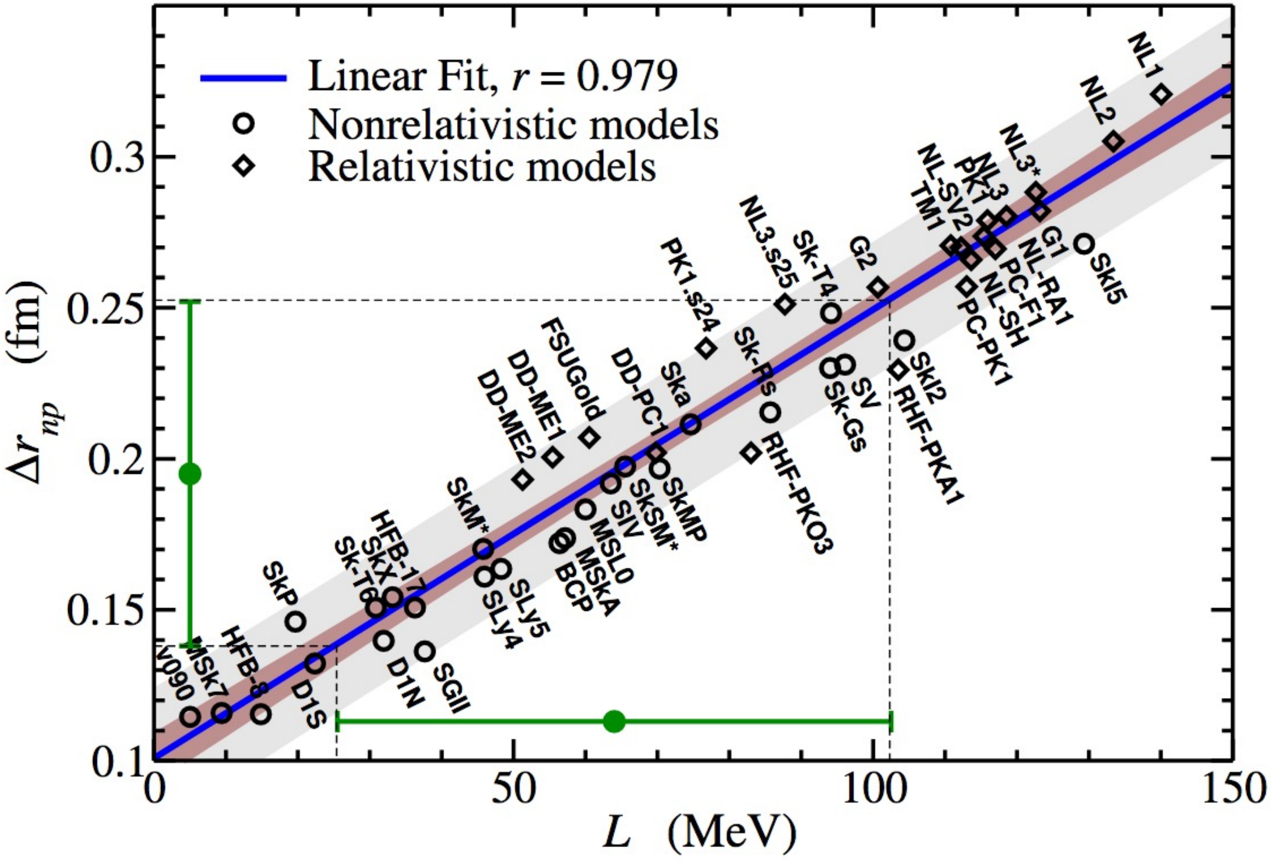




Essick+ arXiv 2102.10074



Connecting L to neutron skin: Existing DFTs predict neutron skin-L relation



Roca-Maza et al, arxiv:1103.1762

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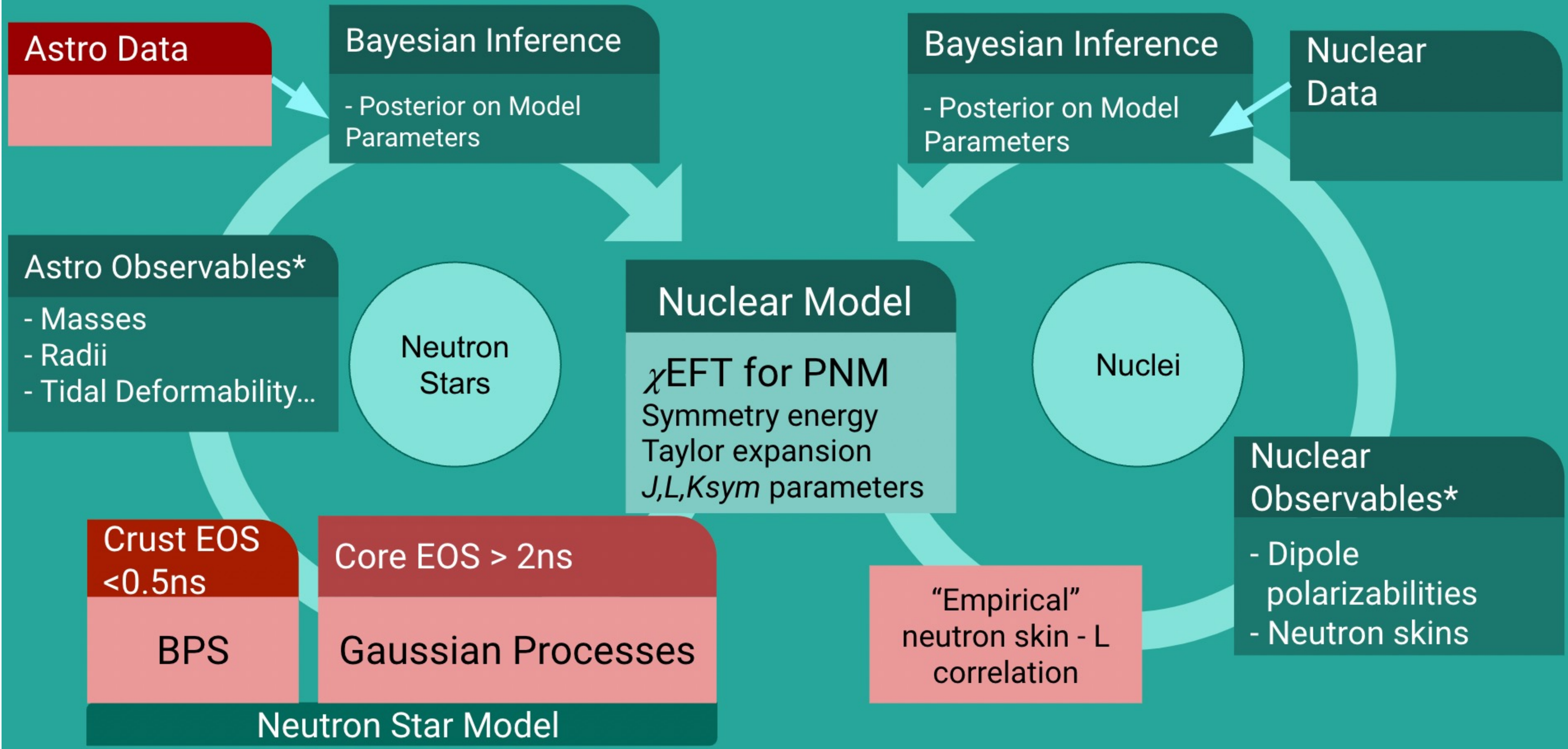

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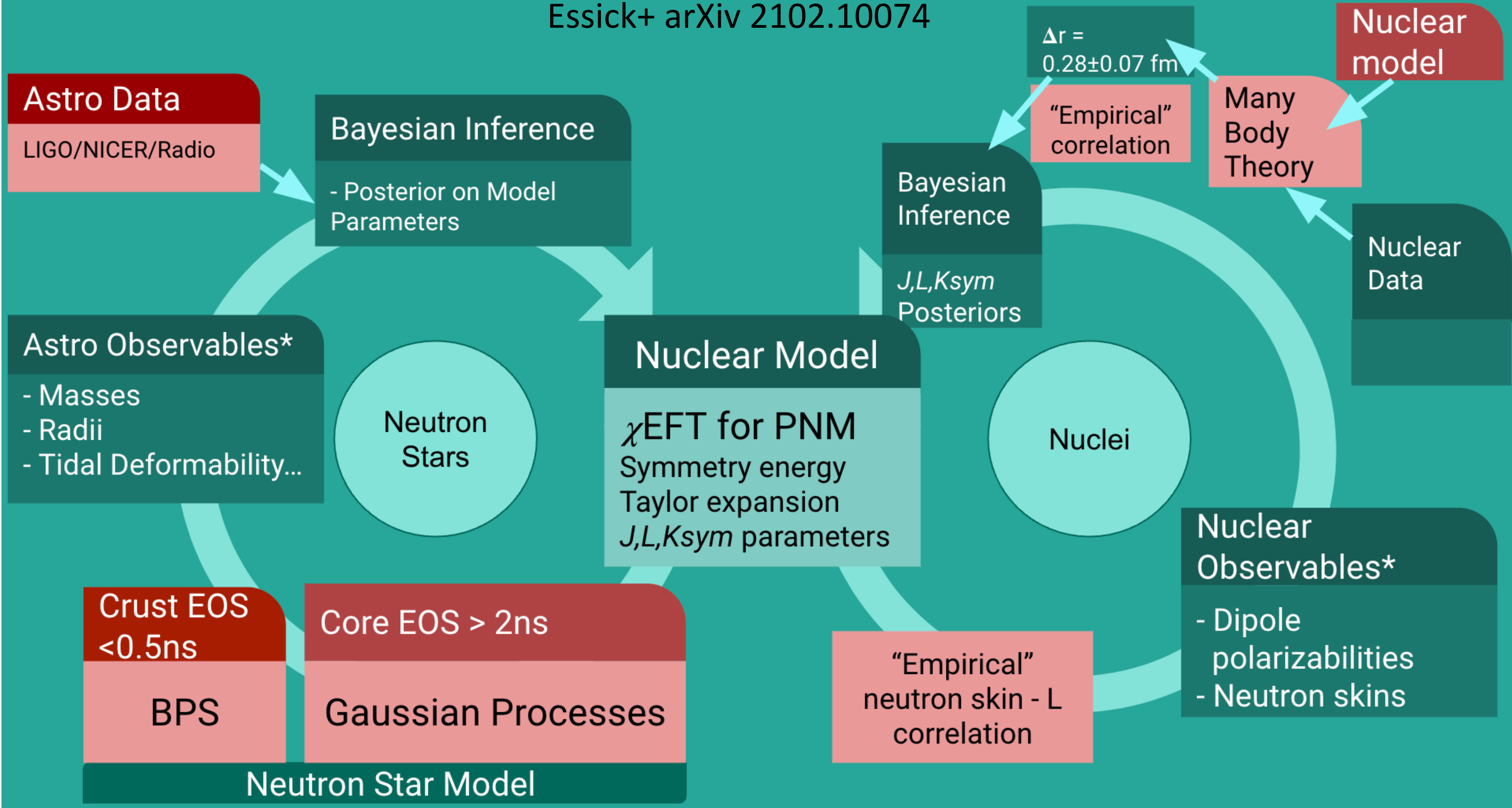
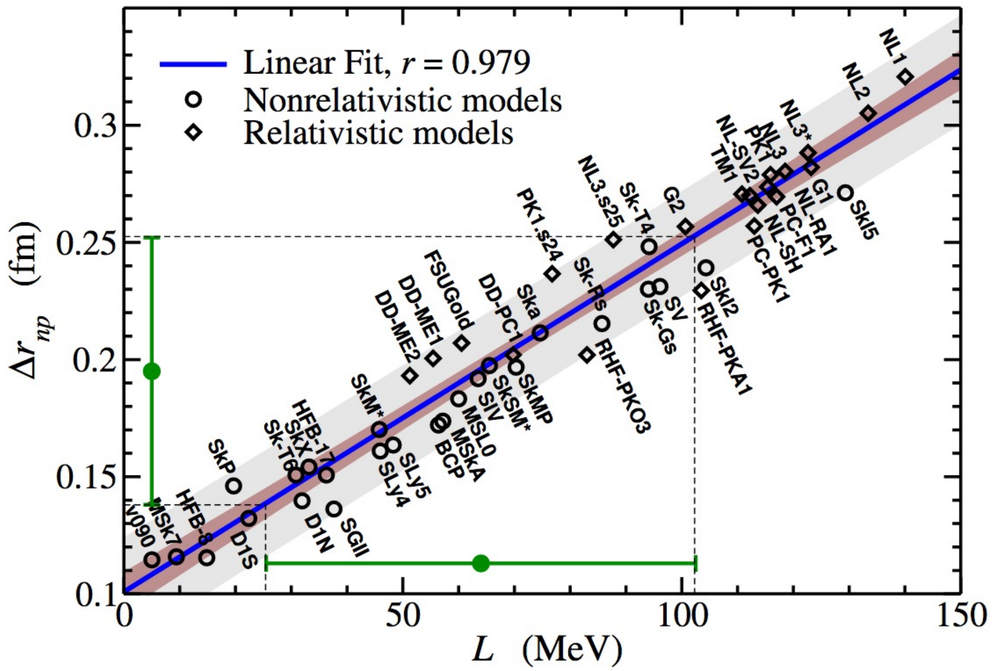
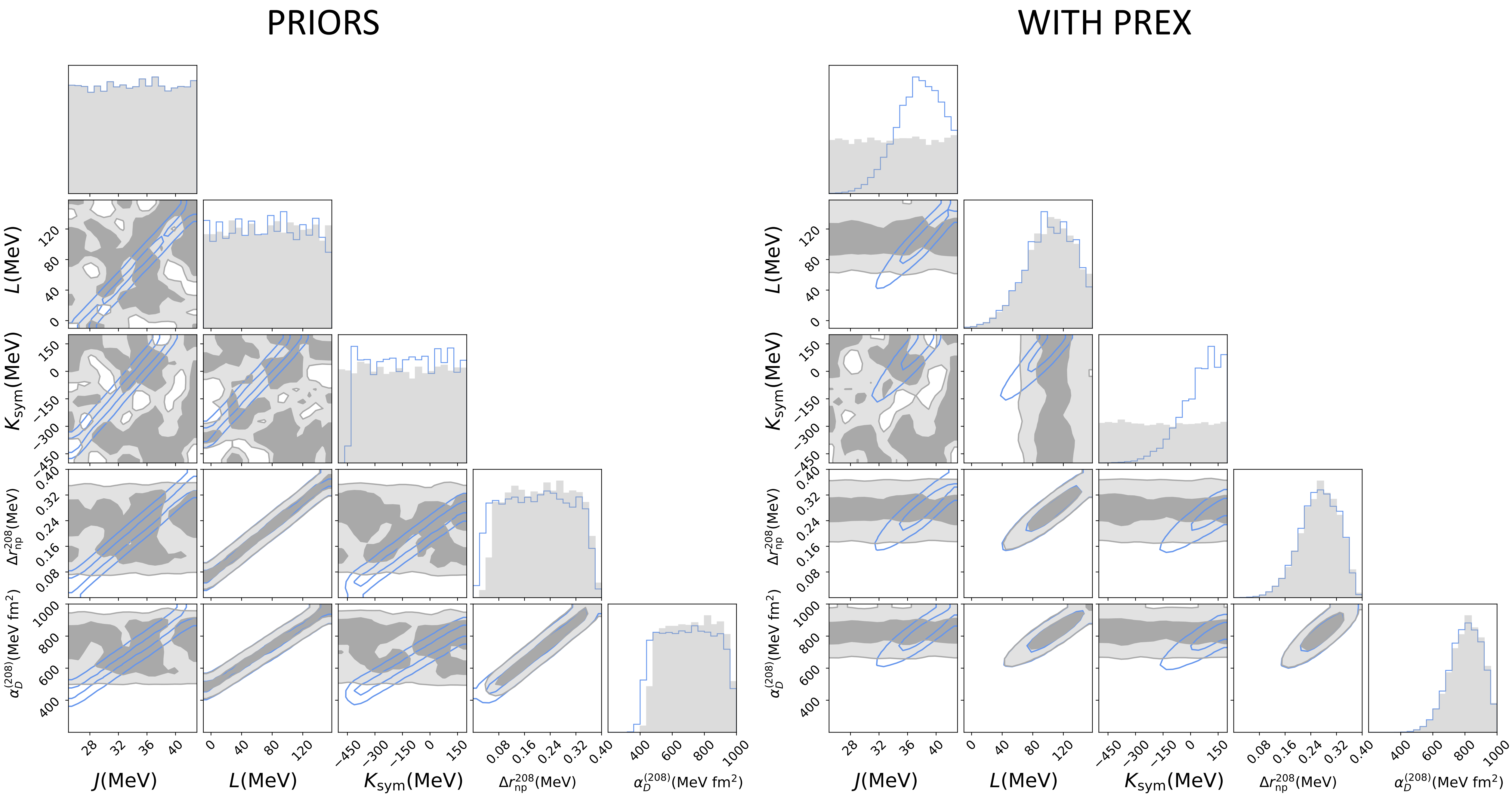


Connecting L to neutron skin: Existing DFTs predict neutron skin-L relation



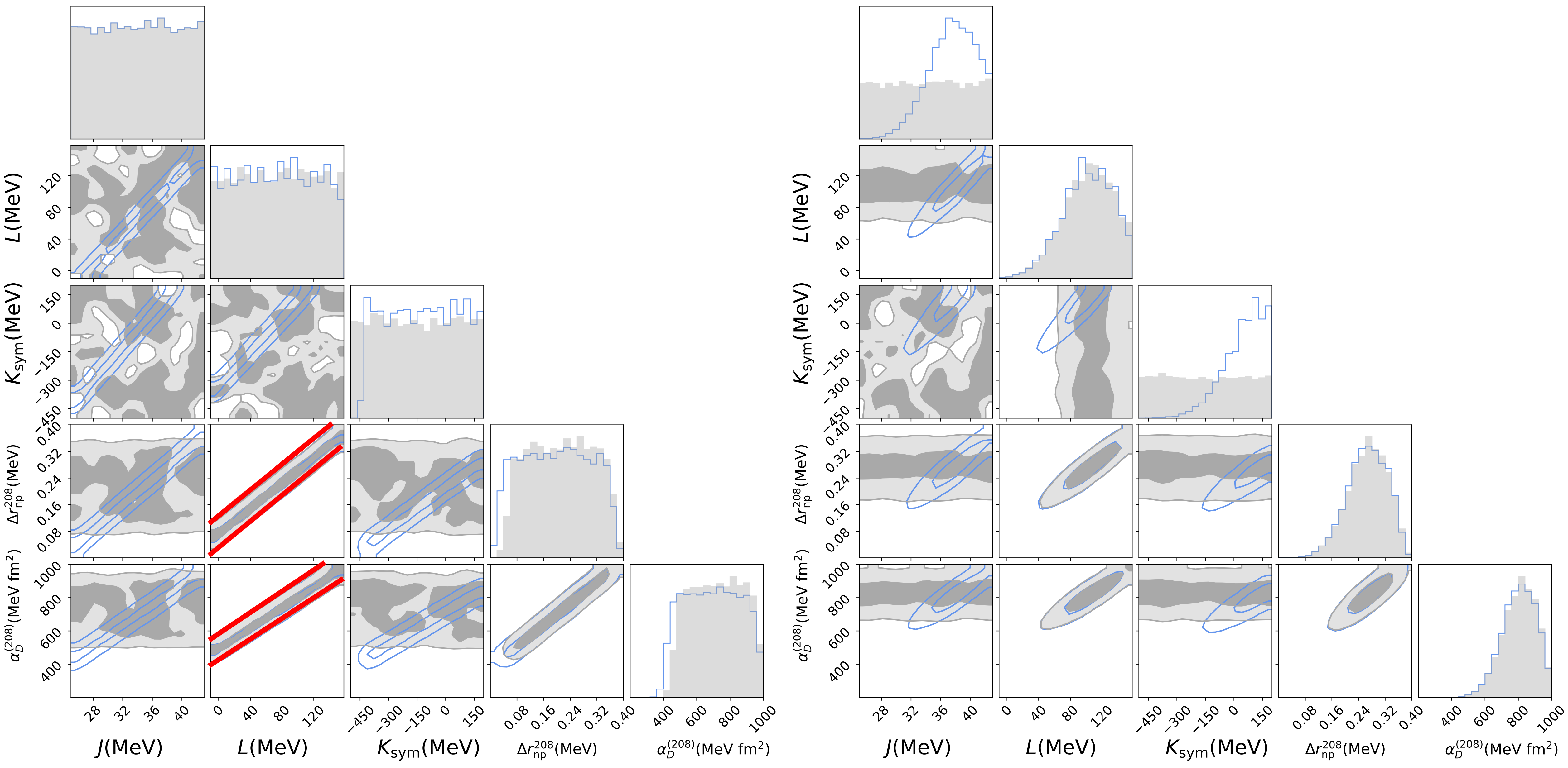
Roca-Maza et al, arxiv:1103.1762

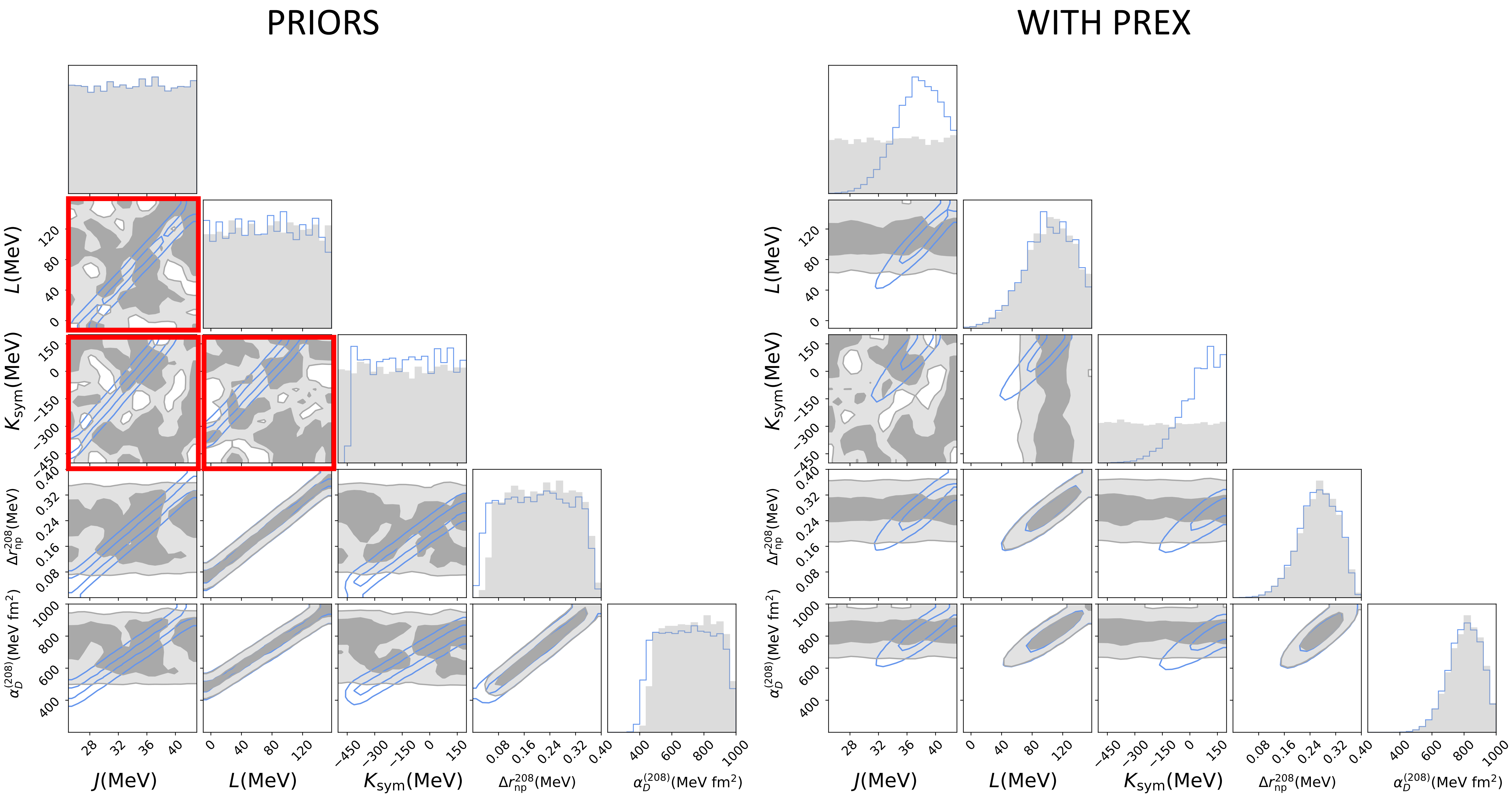
- Models already fit to different datasets which induce additional correlations between symmetry energy parameters
- This relation *includes* nuclear binding energy data, something that can be obscured
- Induces correlations between J,L, K_{sym},L


PRIORS
WITH PREX
J(MeV)
L(MeV)
K_sym(MeV)
Δr_np^208(MeV)
α_D^(208)(MeV fm^2)
28 32 36 40
0 40 80 120
−450 −300 −150 0 150
0.08 0.16 0.24 0.32 0.40
400 600 800 1000

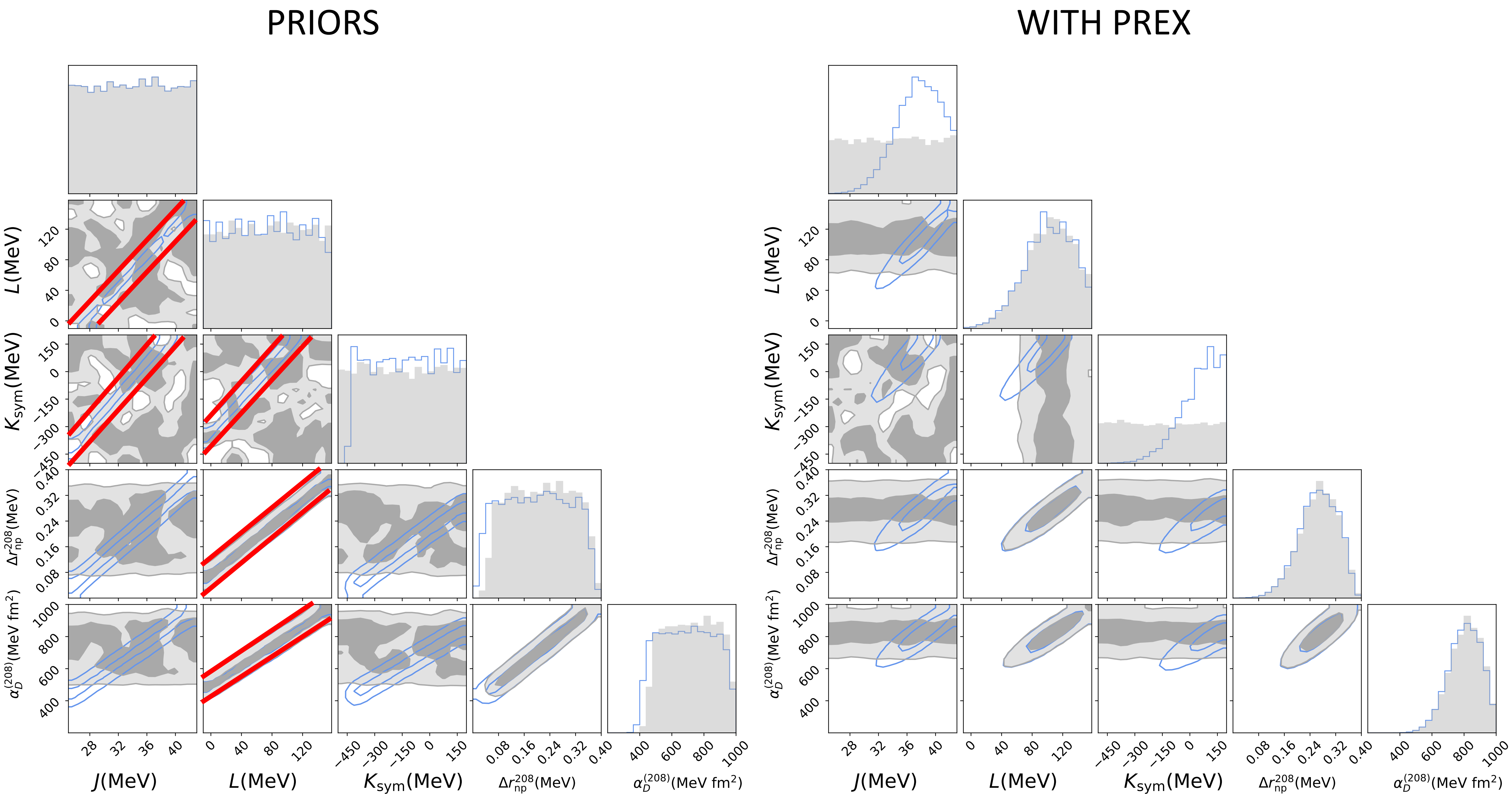
PRIORS

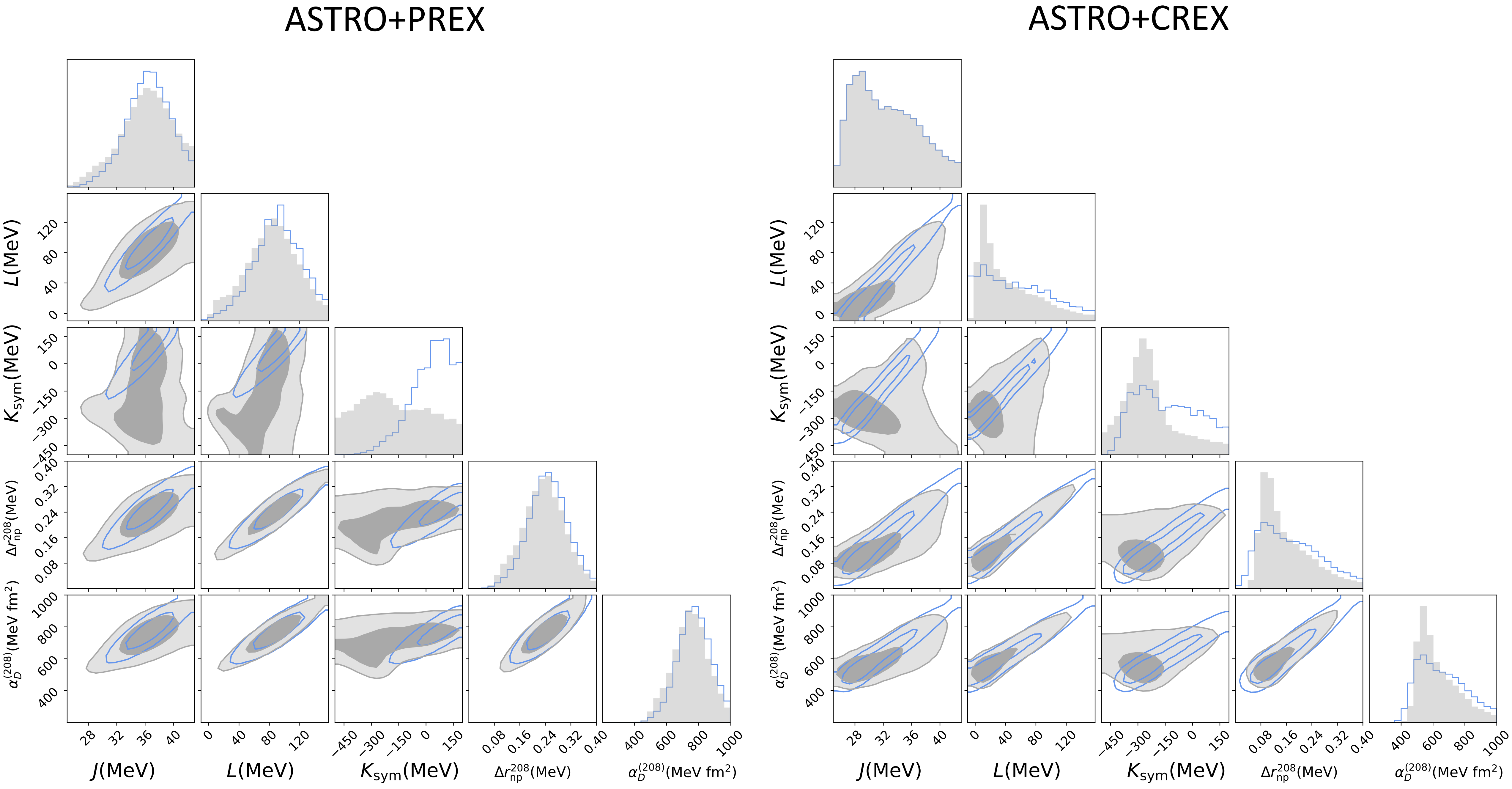
WITH PREX





PRIORS
WITH PREX
J(MeV)
L(MeV)
K_sym(MeV)
Δr_np^208(MeV)
α_D^(208)(MeV fm²)


PRIORS
WITH PREX
J(MeV)
L(MeV)
K_{sym}(MeV)
Δr_{np}^{208}(MeV)
$\alpha_D^{(208)}$(MeV fm^2)


ASTRO+PREX
ASTRO+CREX
J(MeV)
L(MeV)
Ksym(MeV)
Δr_np^208(MeV)
α_D^(208)(MeV fm^2)
28 32 36 40
0 40 80 120
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0.08 0.16 0.24 0.32 0.40
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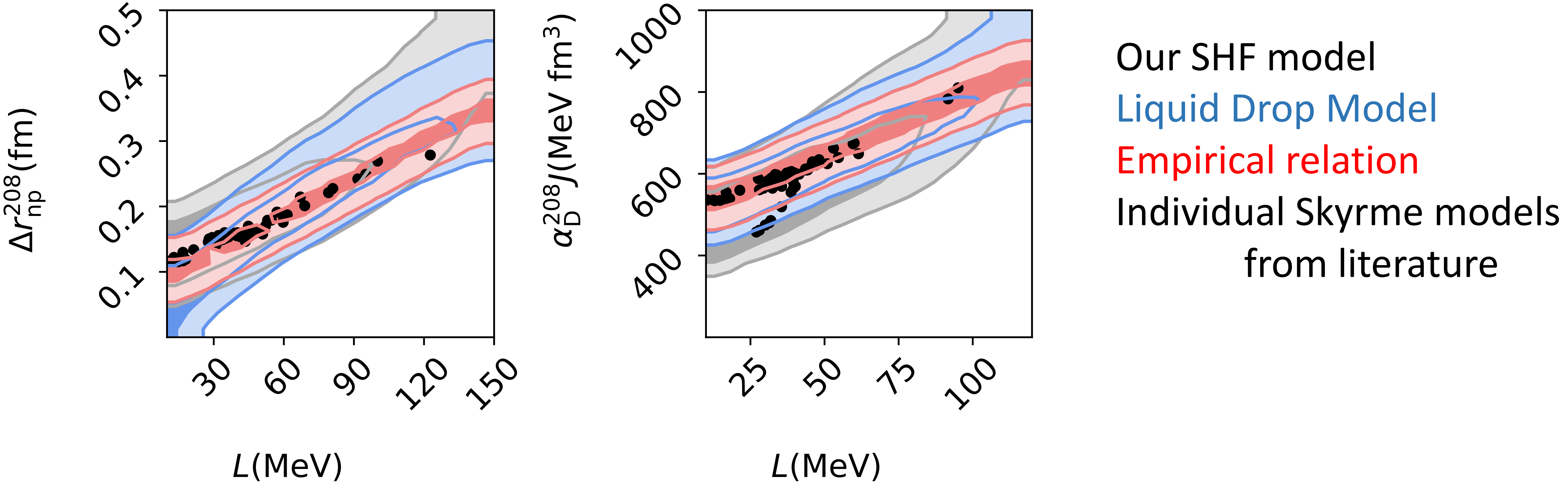
If you don't want to use those J vs L, Ksym vs L relations as priors in your Astro inference (or want to combine a previous Astro inference with skins) – you CANNOT use that empirical relation.



Liquid drop model:

$$\Delta r_{\rm np} = \sqrt{\frac{3}{5}}\left[\frac{3}{2}A^{1/3}\left(\frac{J}{Q}\frac{\delta}{r_{\rm sym}} - \frac{J}{Q}\frac{1}{J r_{\rm sym}}\frac{e^2 Z}{20}A^{-1/3}\right) - \frac{1}{J}\frac{e^2 Z}{70}\right] + a\frac{J}{Q}\delta + b\delta.$$

$$\frac{J}{Q} = \frac{4}{9}(r_{\rm sym} - 1)A^{-1/3} \qquad r_{\rm sym}(\rho_{\rm A}) = J/S(\rho_{\rm A})$$

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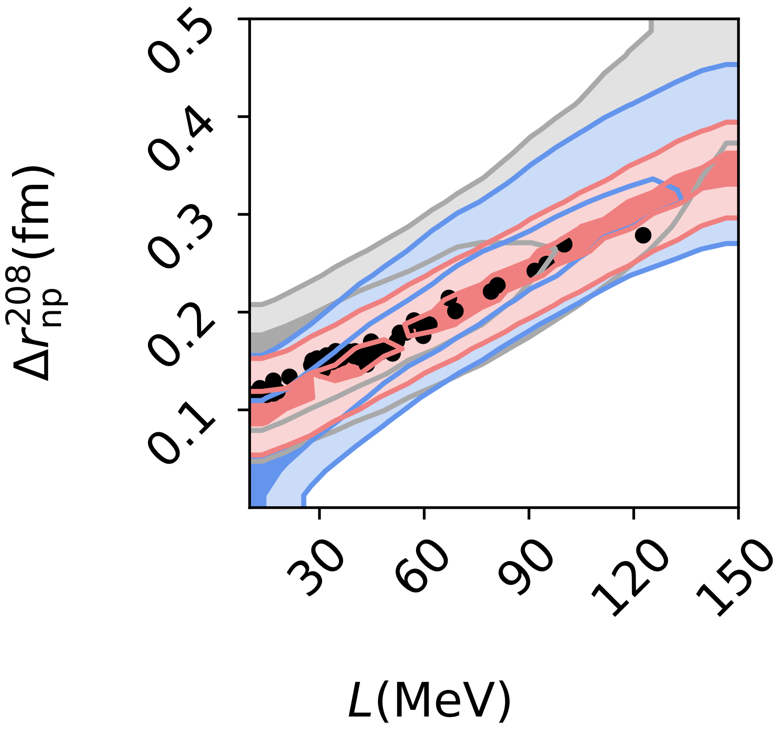

Our SHF model
Liquid Drop Model
Empirical relation
Individual Skyrme models from literature

L(MeV):

Astro+PREX-II using empirical: $66.1^{+35.4}_{-33.7}$

Astro+PREX-II using SHF: $83.5^{+27.6}_{-54.9}$

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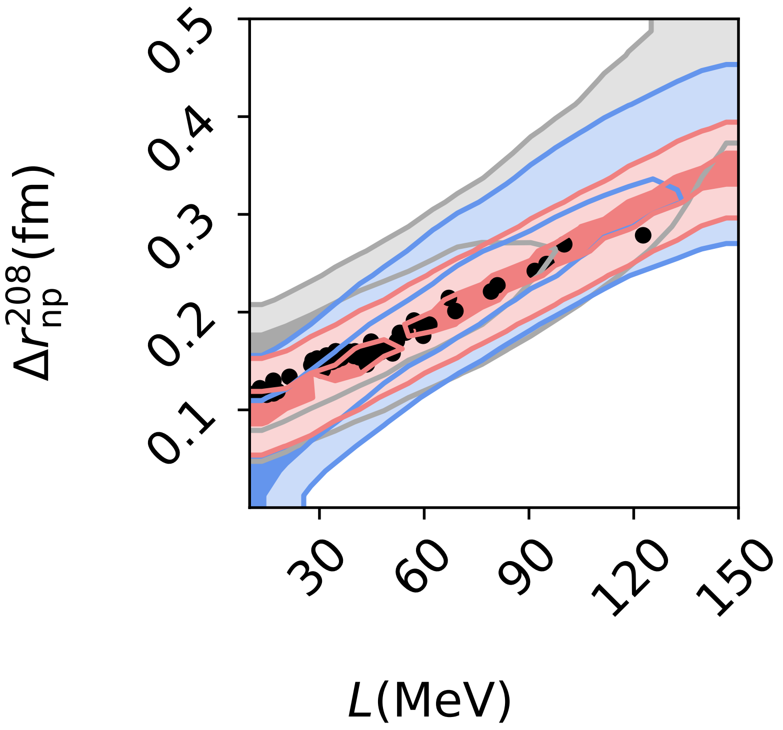


Our SHF model

Liquid Drop Model

Empirical relation

Individual Skyrme models from literature

Systematic Errors

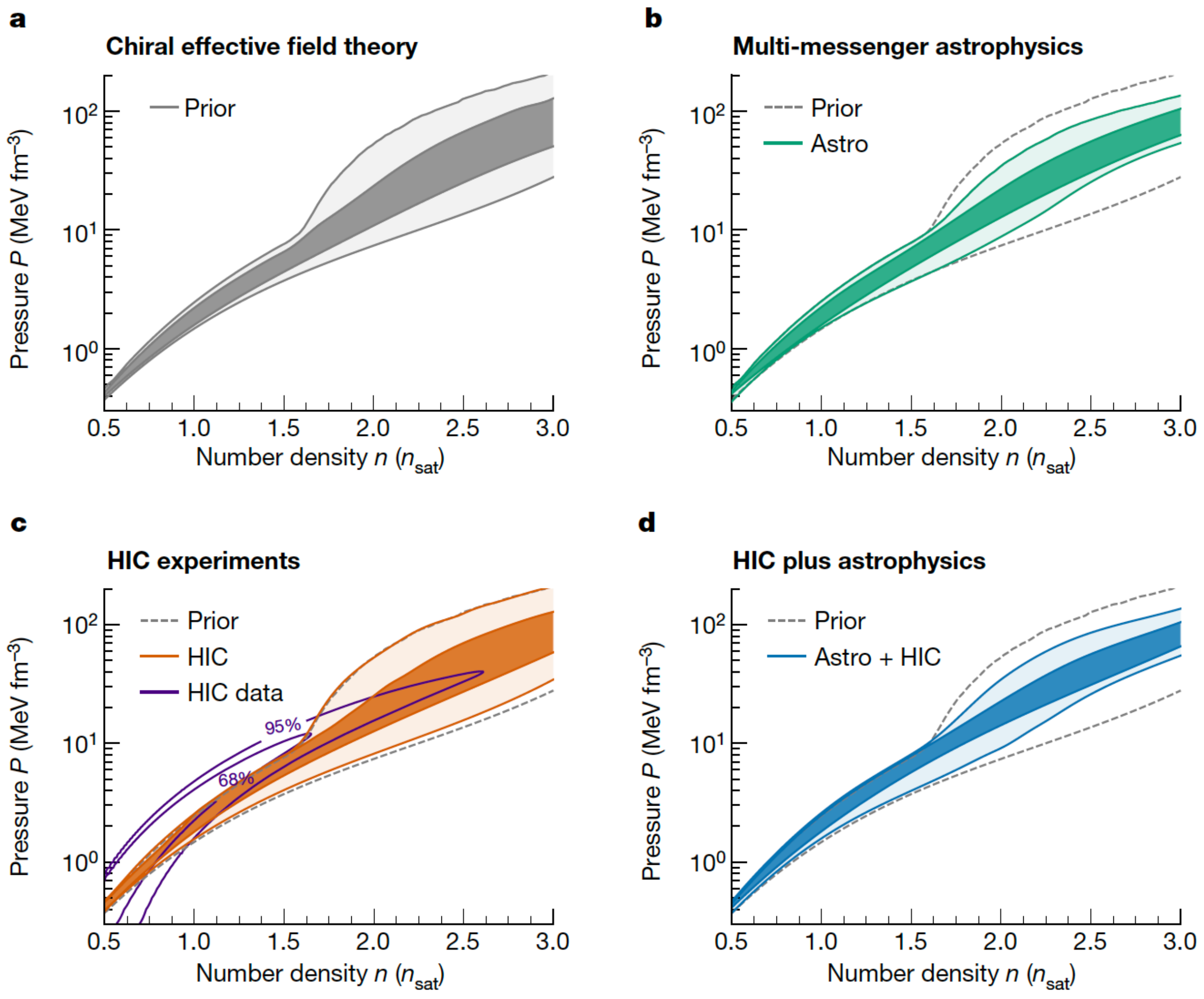
Low Accuracy
High Precision

L(MeV):

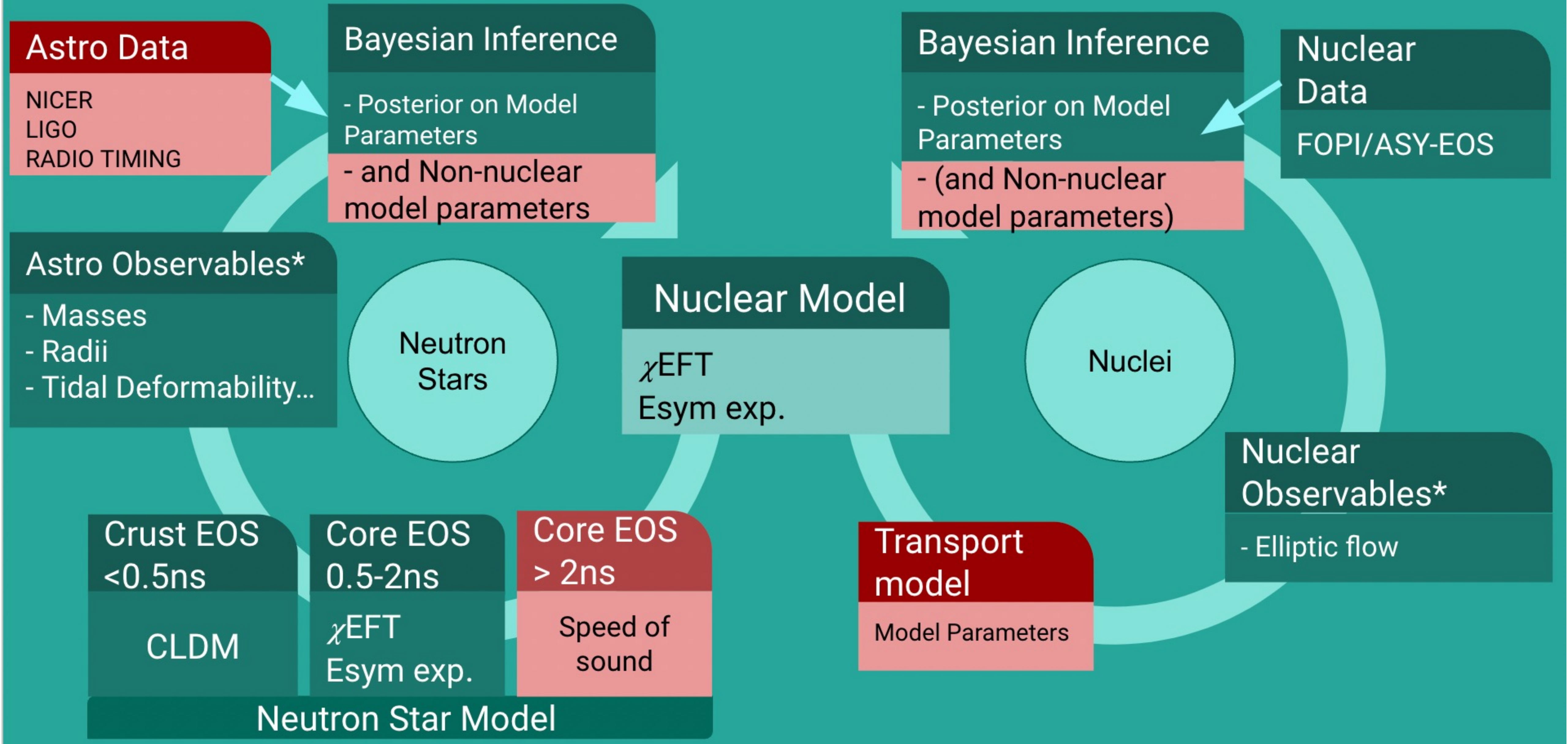
Astro+PREX-II using empirical: $66.1^{+35.4}_{-33.7}$

Astro+PREX-II using SHF: $83.5^{+27.6}_{-54.9}$

Example 2: Huth+, Nature 276,606 (2022)

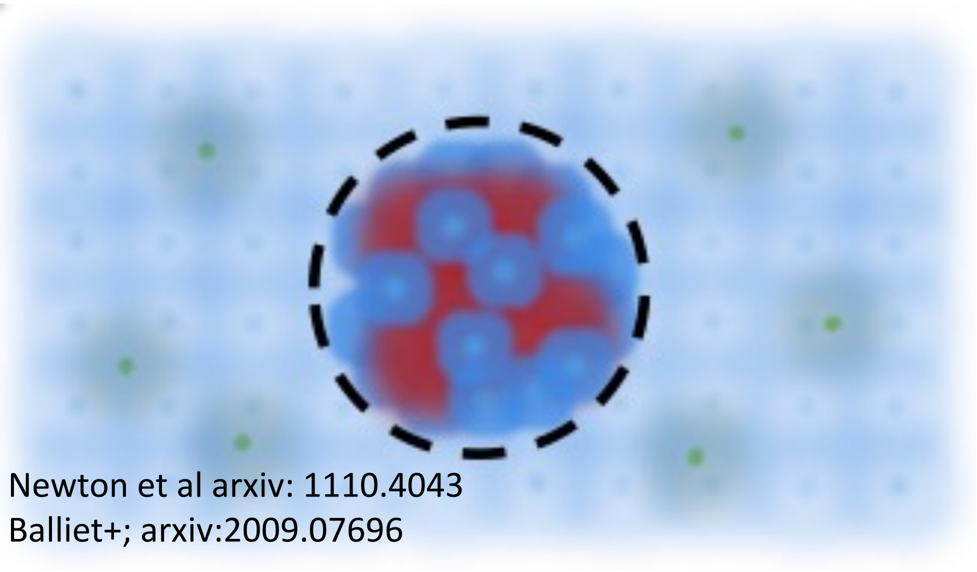


Huth+, Nature 276,606 (2022)



Modeling the crust

CLDM:Bulk fluid and surface degrees of freedom



Newton et al arxiv: 1110.4043
Balliet+; arxiv:2009.07696

Bulk nuclear matter (Skyrme-type)

$$\frac{E^{\rm nuc}}{A}(n,x) = T_0\left(\frac{3}{5}\left(x^{\frac{5}{3}}+(1-x)^{\frac{5}{3}}\right)\left(\frac{2n}{n_0}\right)^{\frac{2}{3}} - [(2\alpha-4\alpha_L)x(1-x)+\alpha_L]\frac{n}{n_0} + [(2\eta-4\eta_L)x(1-x)+\eta_L]\left(\frac{n}{n_0}\right)^{\gamma}\right)$$

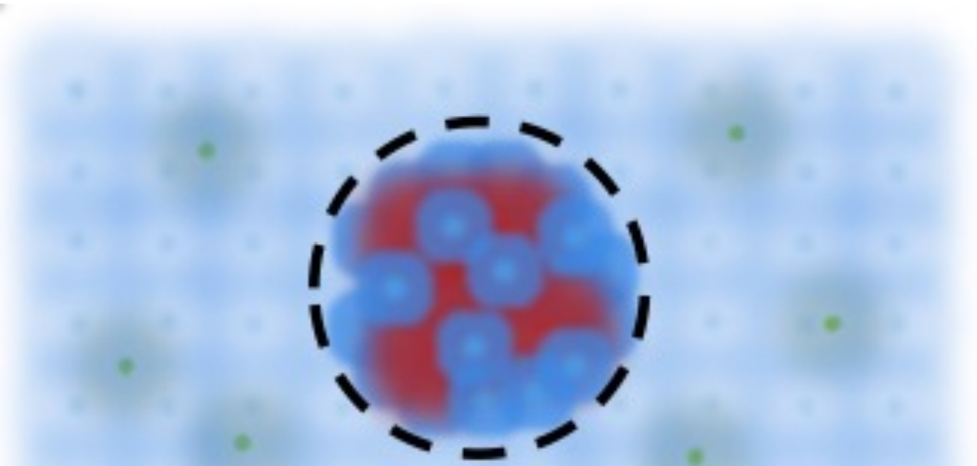
Tews, 2017 arxiv:1607.06998

Surface energy

$$\sigma_s(y_p) = \sigma_0 \frac{2^{p+1}+b}{\frac{1}{y_p^p}+b+\frac{1}{(1-y_p)^p}}$$

Modeling the crust

CLDM:Bulk fluid and surface degrees of freedom



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$$- \left[(2\alpha - 4\alpha_L)x(1-x) + \alpha_L\right]\frac{n}{n_0}$$
$$\left. + \left[(2\eta - 4\eta_L)x(1-x) + \eta_L\right]\left(\frac{n}{n_0}\right)^{\gamma}\right)$$

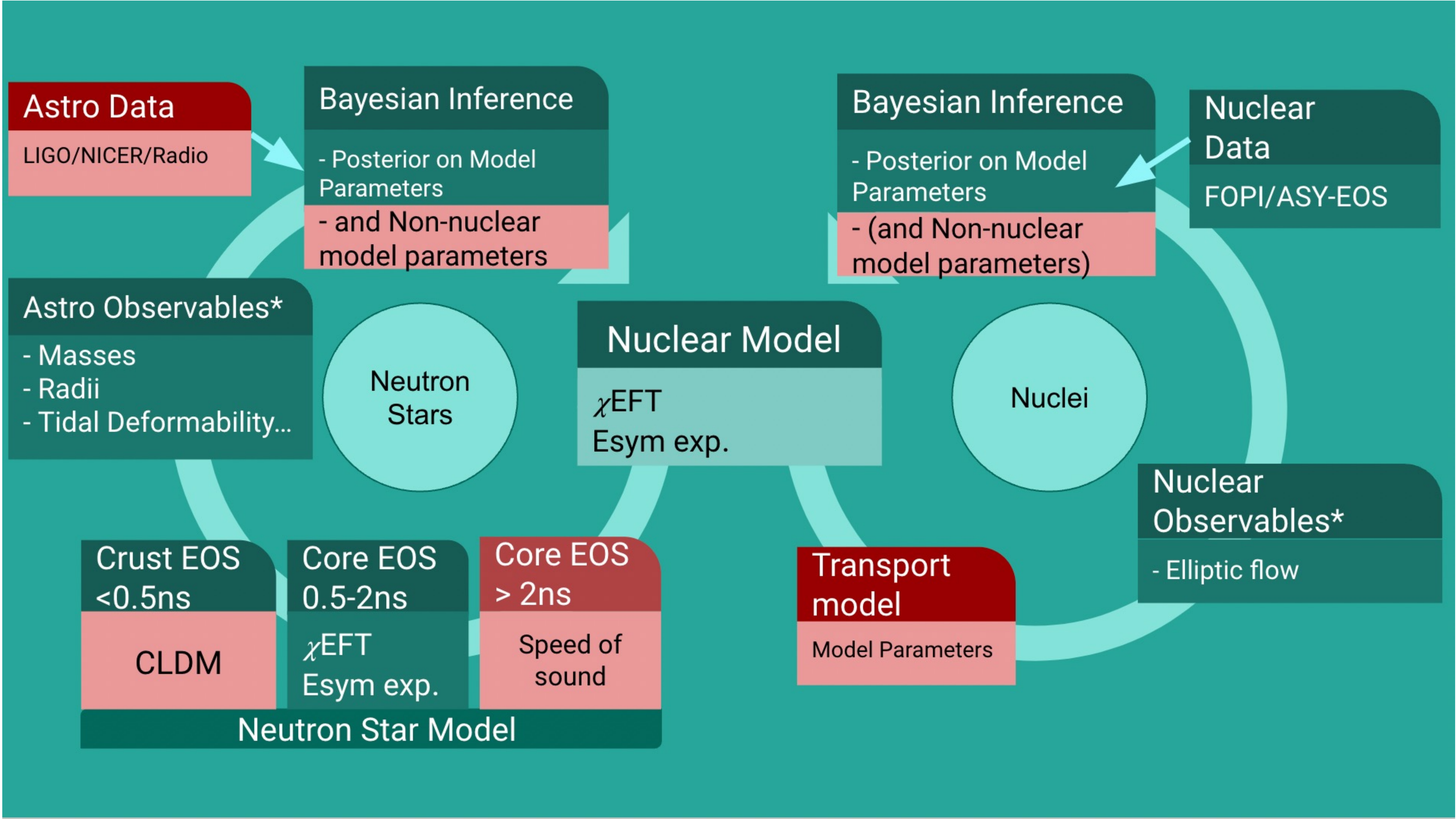
Tews, 2017 arxiv:1607.06998

Surface energy

$$\sigma_s(y_p) = \sigma_0 \frac{2^{p+1} + b}{\frac{1}{y_p^p} + b + \frac{1}{(1-y_p)^p}}$$

Systematic Errors

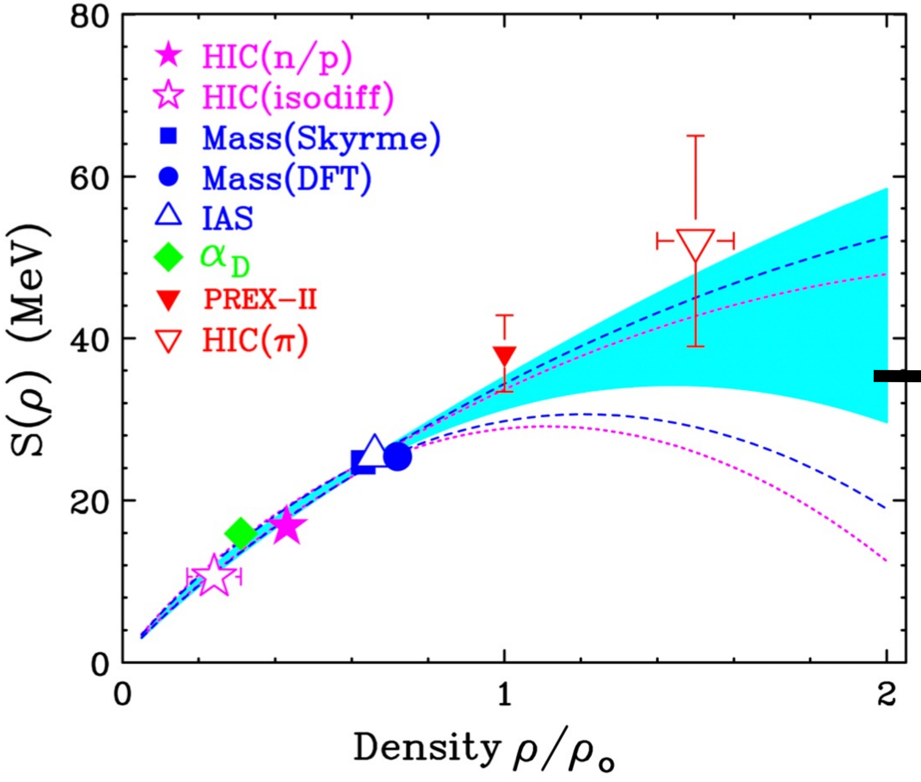
Low Accuracy
High Precision



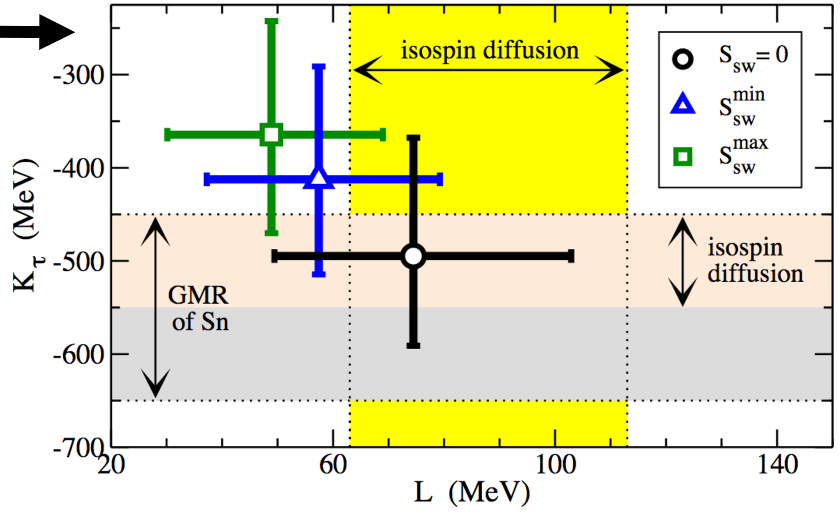
Astro Data
LIGO/NICER/Radio
Bayesian Inference
- Posterior on Model Parameters
- and Non-nuclear model parameters
Bayesian Inference
- Posterior on Model Parameters
- (and Non-nuclear model parameters)
Nuclear Data
FOPI/ASY-EOS
Astro Observables*
- Masses
- Radii
- Tidal Deformability...
Neutron Stars
Nuclear Model
χEFT
Esym exp.
Nuclei
Nuclear Observables*
- Elliptic flow
Crust EOS <0.5ns
CLDM
Core EOS 0.5-2ns
χEFT
Esym exp.
Core EOS > 2ns
Speed of sound
Neutron Star Model
Transport model
Model Parameters

Different observables constrain at different densities…

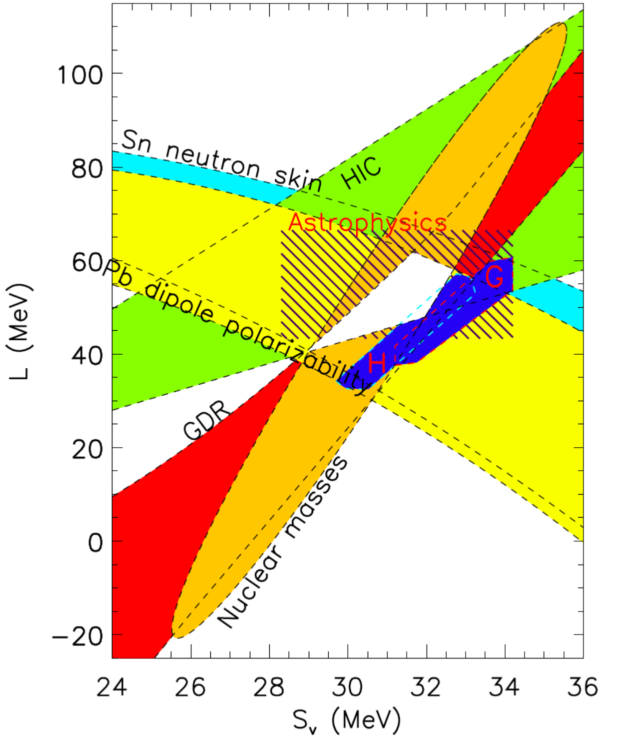
… so resulting constraints on nuclear matter parameters at saturation density involve model-dependent extrapolation



Tsang and Lynch, arxiv:2106.10119



Centelles et al, arxiv:0806.2886



Lattimer, Lim ApJ771(2013)
Lattimer, Steiner EPJA50 (2013)

Different observables constrain at different densities…

… so resulting constraints on nuclear matter parameters at saturation density involve model-dependent extrapolation

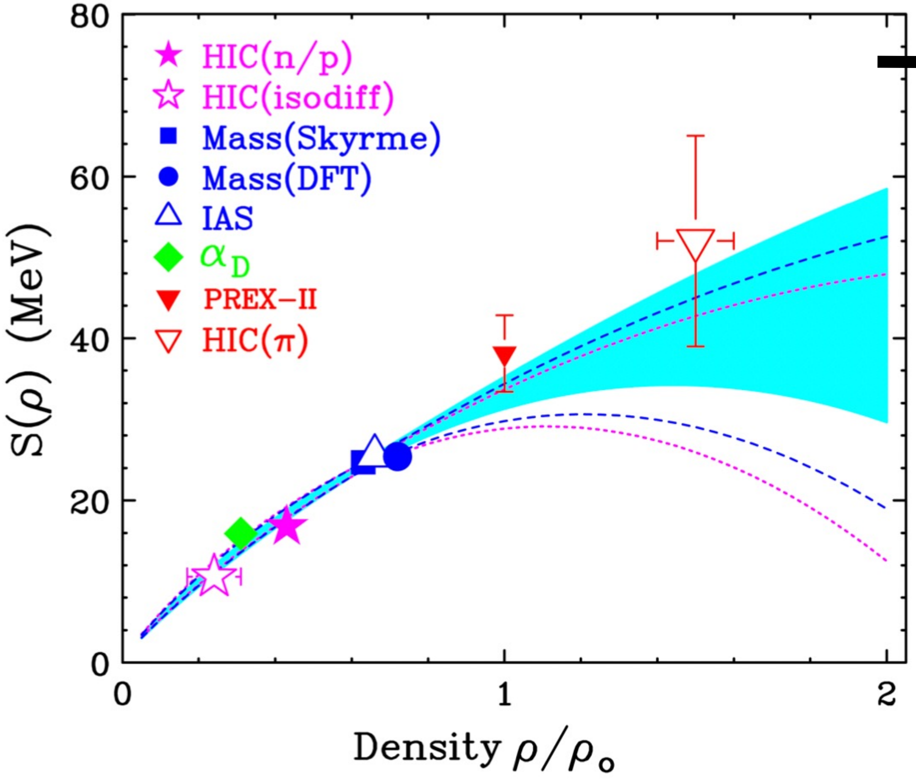
$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L(\frac{\rho-\rho_0}{3\rho_0}) + \frac{K_{\rm sym}}{2}(\frac{\rho-\rho_0}{3\rho_0})^2 + \frac{J_{\rm sym}}{6}(\frac{\rho-\rho_0}{3\rho_0})^3$$

$$S_{\rm RMF}(\rho) = A(\rho)\rho^{2/3} + B(\rho)\rho\ ,$$

$$S_{\rm SHF}(\rho) = a\rho^{2/3} - b\rho - c\rho^{5/3} - d\rho^{\sigma+1}$$

$$S({\rm SHF,ext}) = {\rm a}\rho + {\rm b}\rho^{4/3} + {\rm c}\rho^{5/3} + {\rm d}\rho^2 + ...$$

$$S(n) = E_{{\rm kin},0}\left(\frac{n}{n_{\rm sat}}\right)^{2/3} + E_{{\rm pot},0}\left(\frac{n}{n_{\rm sat}}\right)^{\gamma_{\rm asy}}$$



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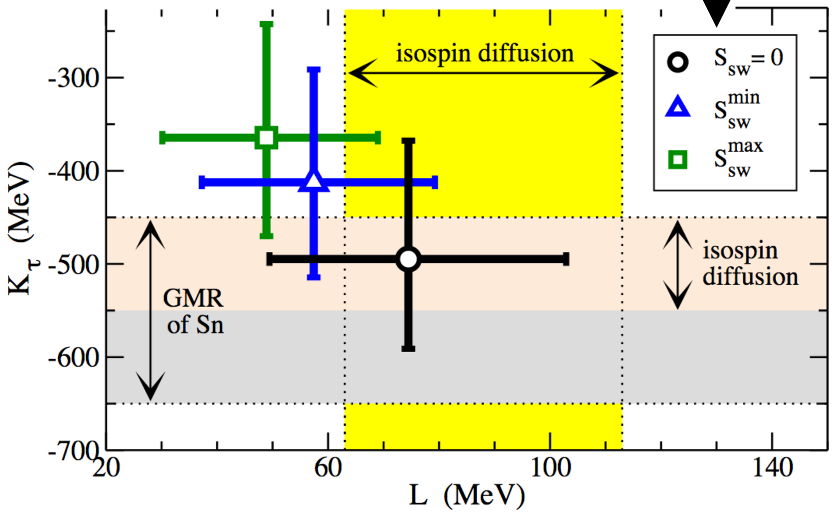

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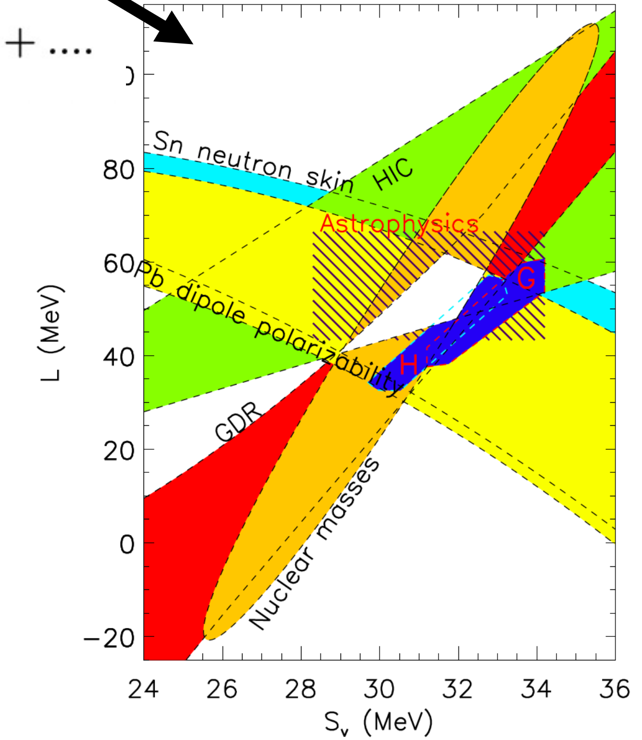

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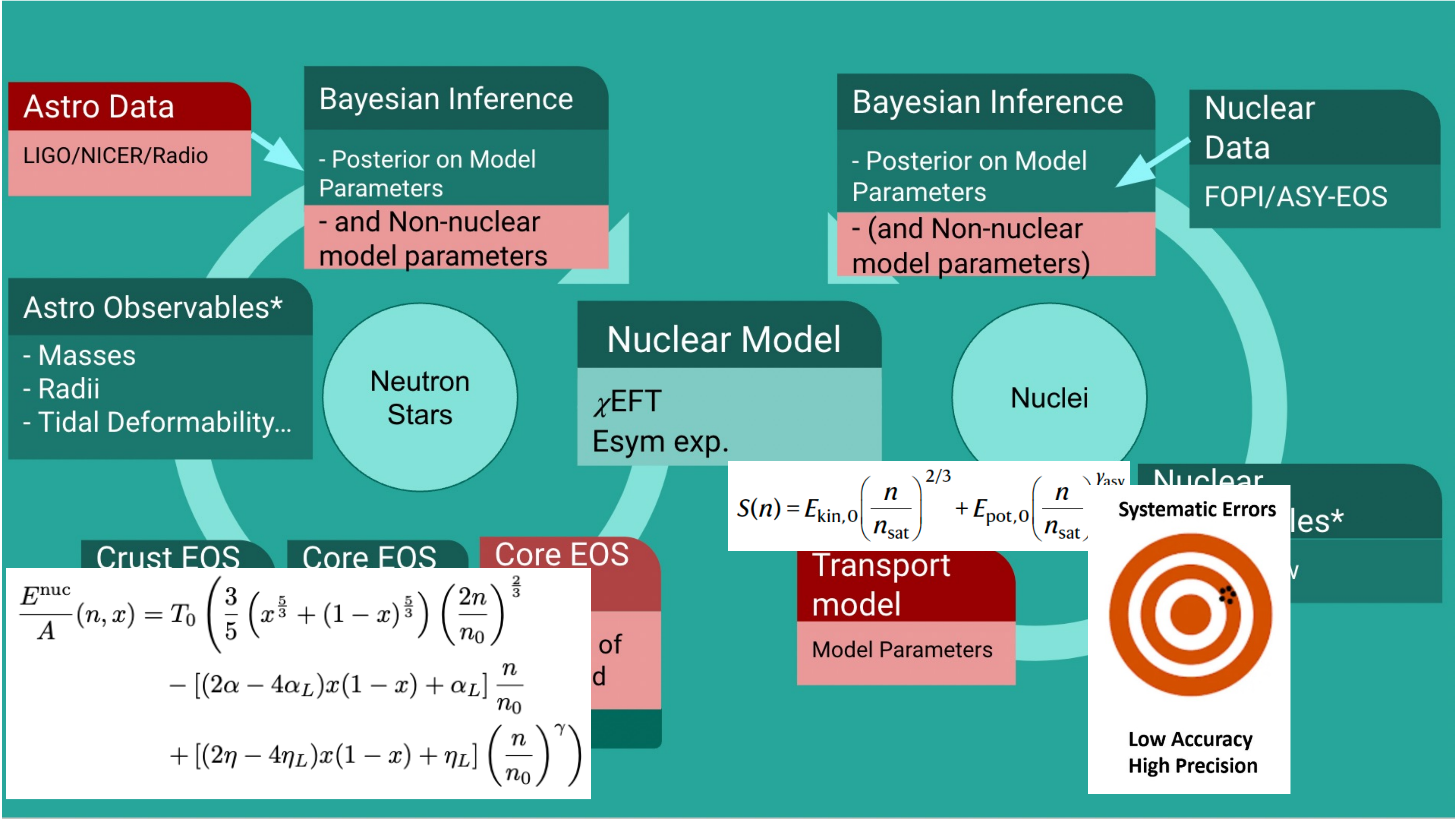
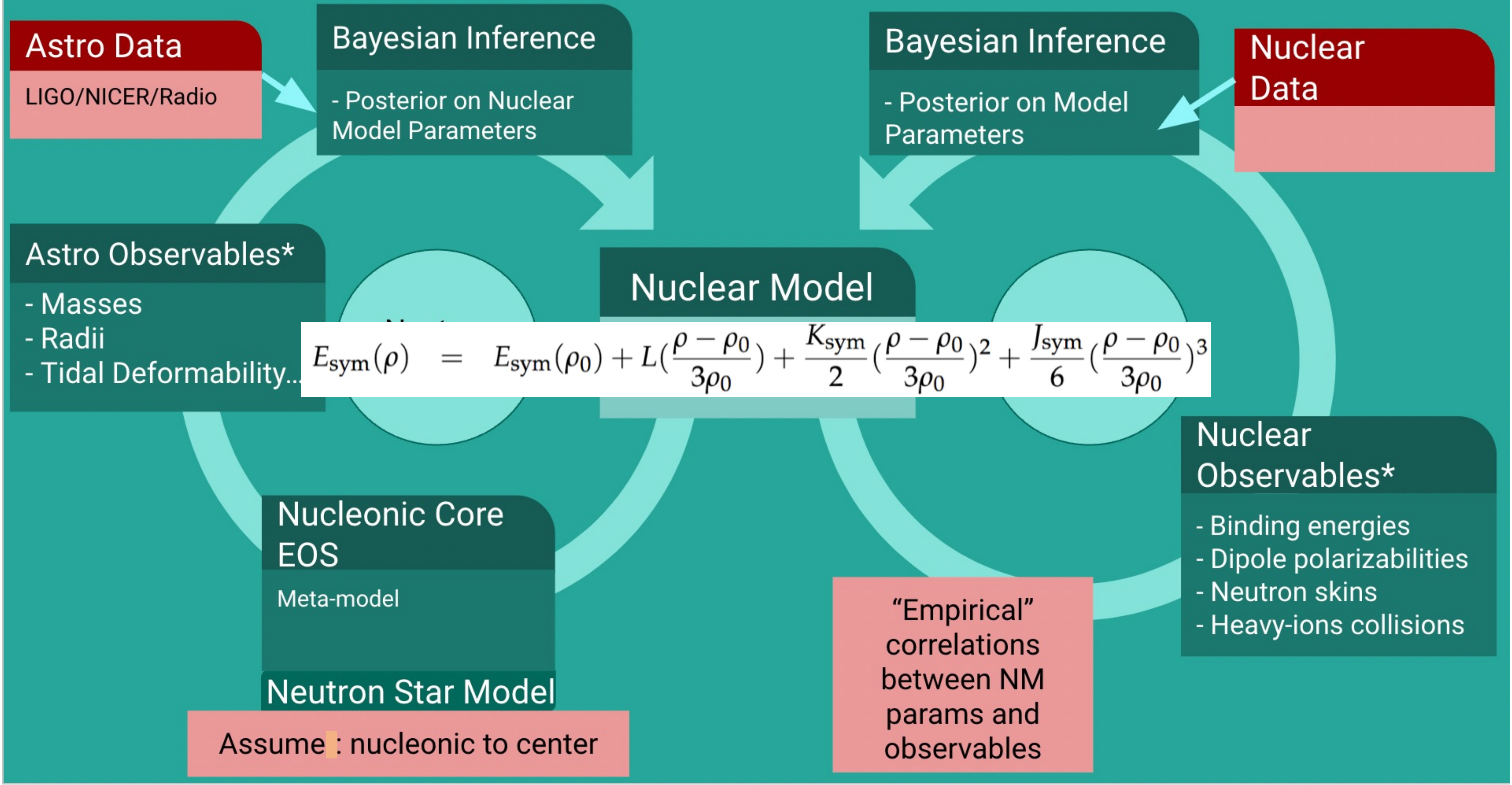


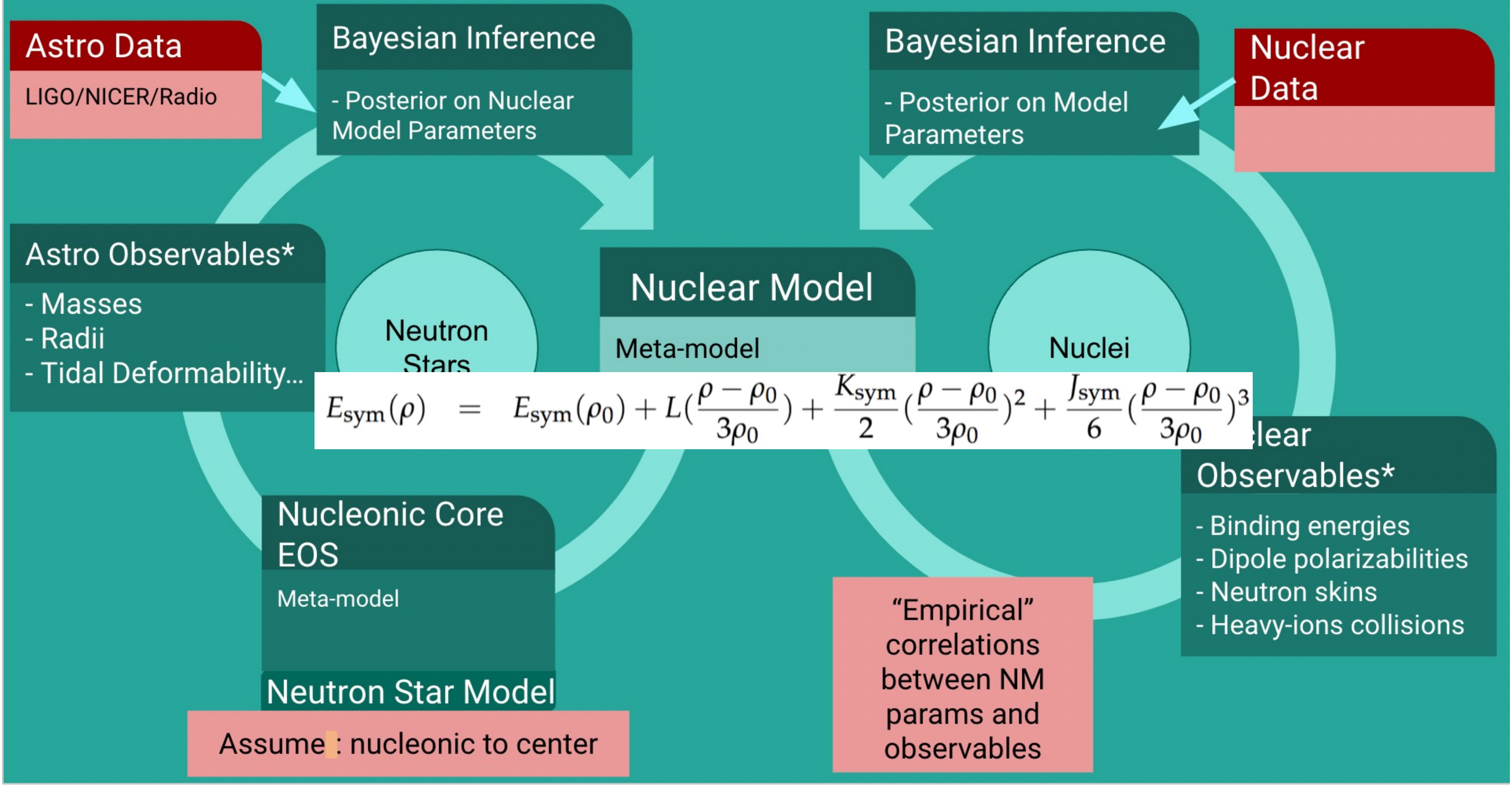
$$S(n) = E_{\mathrm{kin},0}\left(\frac{n}{n_{\mathrm{sat}}}\right)^{2/3} + E_{\mathrm{pot},0}\left(\frac{n}{n_{\mathrm{sat}}}\right)^{\gamma_{\mathrm{asy}}}$$

$$\frac{E^{\mathrm{nuc}}}{A}(n,x) = T_0\left(\frac{3}{5}\left(x^{\frac{5}{3}}+(1-x)^{\frac{5}{3}}\right)\left(\frac{2n}{n_0}\right)^{\frac{2}{3}} - [(2\alpha-4\alpha_L)x(1-x)+\alpha_L]\frac{n}{n_0} + [(2\eta-4\eta_L)x(1-x)+\eta_L]\left(\frac{n}{n_0}\right)^{\gamma}\right)$$

Meta-models, e.g. Margueron+ 1708.08694, Li+ 1905.13175

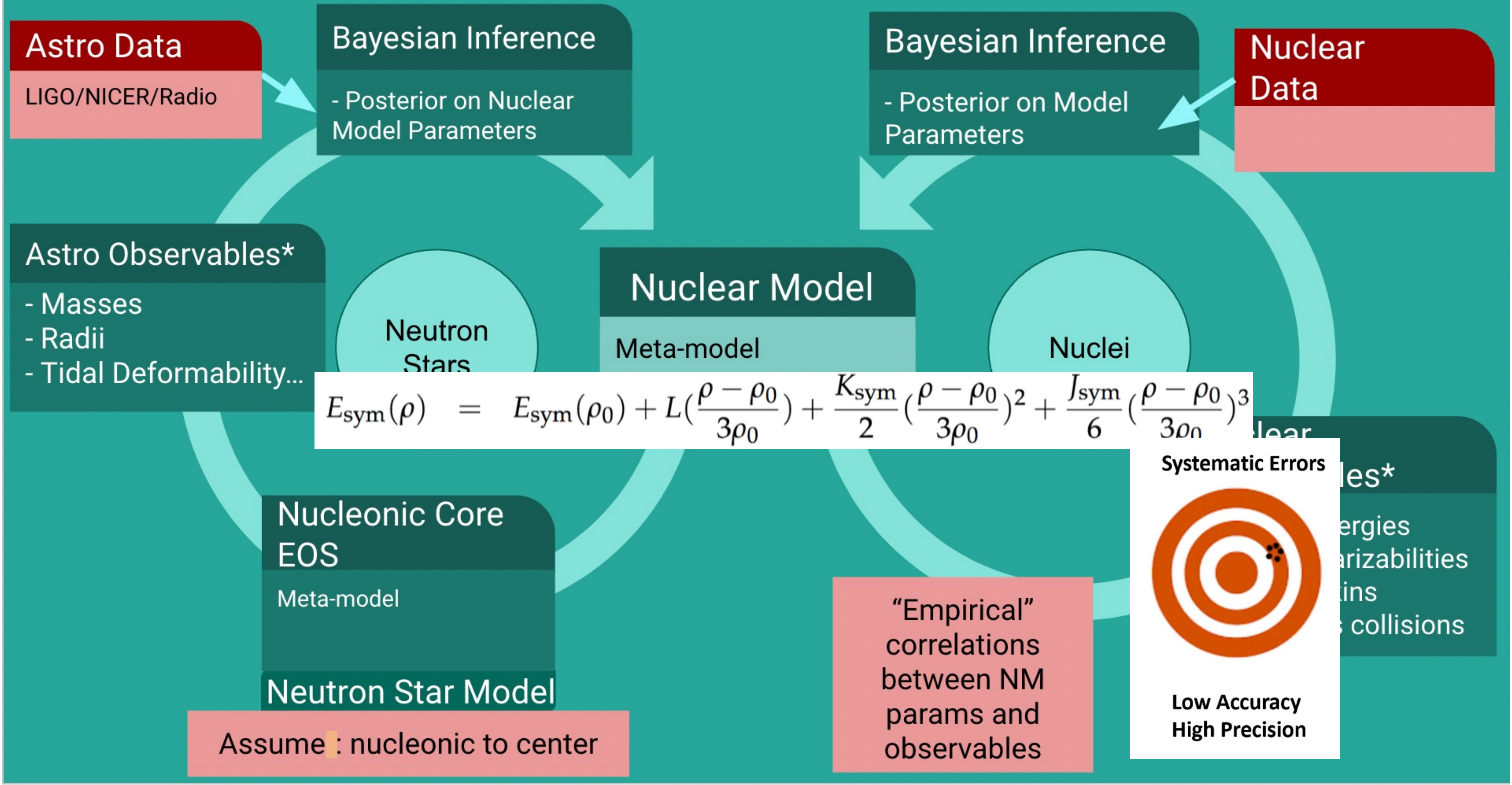


Example 3: Meta-models, e.g. Margueron+ 1708.08694, Li+ 1905.13175



$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\rm sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{\rm sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3$$

Example 3: Meta-models, e.g. Margueron+ 1708.08694, Li+ 1905.13175



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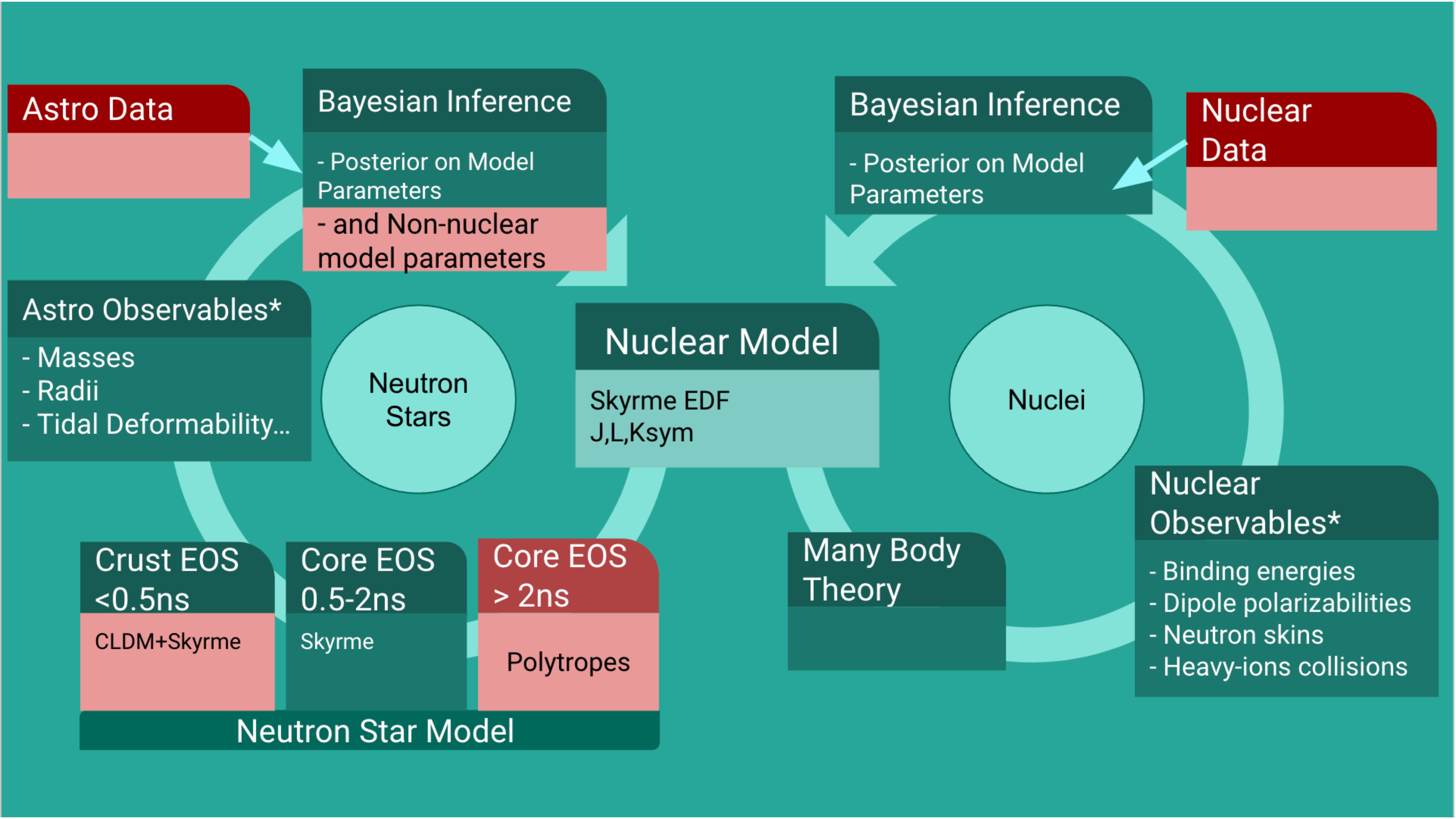
An example of an inference of the symmetry energy parameters
Using both neutron star crust and core data and nuclear properties.



Astro Data
Bayesian Inference
- Posterior on Model Parameters
- and Non-nuclear model parameters
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Nuclear Data
Astro Observables*
- Masses
- Radii
- Tidal Deformability...
Neutron Stars
Nuclear Model
Skyrme EDF
J,L,Ksym
Nuclei
Nuclear Observables*
- Binding energies
- Dipole polarizabilities
- Neutron skins
- Heavy-ions collisions
Crust EOS <0.5ns
CLDM+Skyrme
Core EOS 0.5-2ns
Skyrme
Core EOS > 2ns
Polytropes
Many Body Theory
Neutron Star Model

Our choice of model: Skyrme-Hartree-Fock

Density Functional Theory (e.g. Skyrme)

$$\mathcal{H}_\delta = \frac{1}{4} t_0 \rho^2 [(2 + x_0) - (2x_0 + 1)(y_p^2 + y_n^2)]$$

Local interaction

$$\mathcal{H}_\rho = \frac{1}{4} t_3 \rho^{2+\alpha_3} [(2 + x_3) - (2x_3 + 1)(y_p^2 + y_n^2)] + \frac{1}{4} t_4 \rho^{2+\alpha_4} [(2 + x_4) - (2x_4 + 1)(y_p^2 + y_n^2)]$$

Density dependent

$$\mathcal{H}_{\text{eff}} = \frac{1}{8} \rho [t_1(2 + x_1) + t_2(2 + x_2)] \tau + \frac{1}{8} \rho [t_1(2x_1 + 1) + t_2(2x_2 + 1)](\tau_p y_p + \tau_n y_n)$$

3 body

$$\mathcal{H}_{\text{grad}} = \frac{1}{32} (\nabla \rho)^2 [3t_1(2 + x_1) - t_2(2 + x_2)] - \frac{1}{32} [3t_1(2x_1 + 1) + t_2(2x_2 + 1)][(\nabla \rho_p)^2 + (\nabla \rho_n)^2)$$

Gradient…

Used in a variational principle on total energy leads to coupled Schrödinger-like equations for the wavefunctions.
Solutions converge to ground state (Hohenberg-Kohn theorem)

Map nuclear matter parameters to model parameters and systematically generate models

$$
\begin{bmatrix} t_0 \\ \vdots \\ t_4 \\ x_0 \\ \vdots \\ x_4 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}
\leftarrow
\begin{bmatrix} E_0 \\ K_0 \\ n_0 \\ m_{\mathrm{p}}^* \\ m_{\mathrm{n}}^* \\ G_{\mathrm{s}} \\ G_{\mathrm{v}} \\ J \\ L \\ K_{\mathrm{sym}} \end{bmatrix}
$$

SNM

Dynamics of n,p

Surface energy

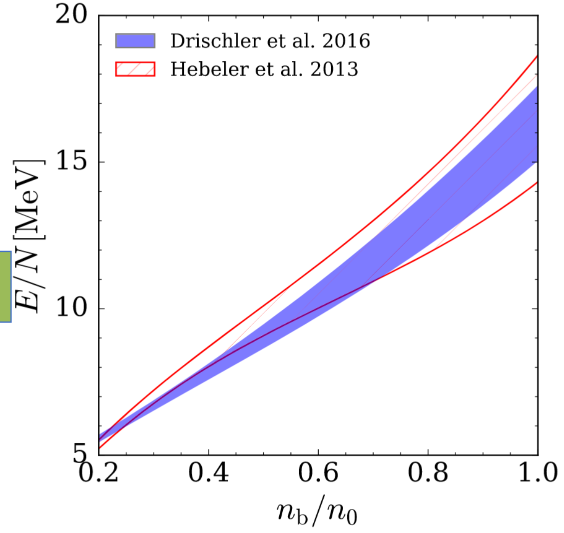
Symmetry Energy

Map nuclear matter parameters to model parameters and systematically generate models

$$\begin{bmatrix} x_0 \\ x_3 \\ x_4 \end{bmatrix} \longleftarrow \begin{bmatrix} J \\ L \\ K_{\mathrm{sym}} \end{bmatrix}$$

$$\begin{bmatrix} t_0 \\ \vdots \\ t_4 \\ x_1 \\ x_2 \end{bmatrix} \longleftarrow \begin{bmatrix} E_0 \\ K_0 \\ n_0 \\ m_{\mathrm{p}}^* \\ m_{\mathrm{n}}^* \\ G_{\mathrm{s}} \\ G_{\mathrm{v}} \end{bmatrix} \begin{bmatrix} x_0 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_3 \\ \alpha_4 \end{bmatrix} \longleftarrow$$



Haensel, Fortin JPhysG 2017
Lim, Holt arXiv:1702.02898

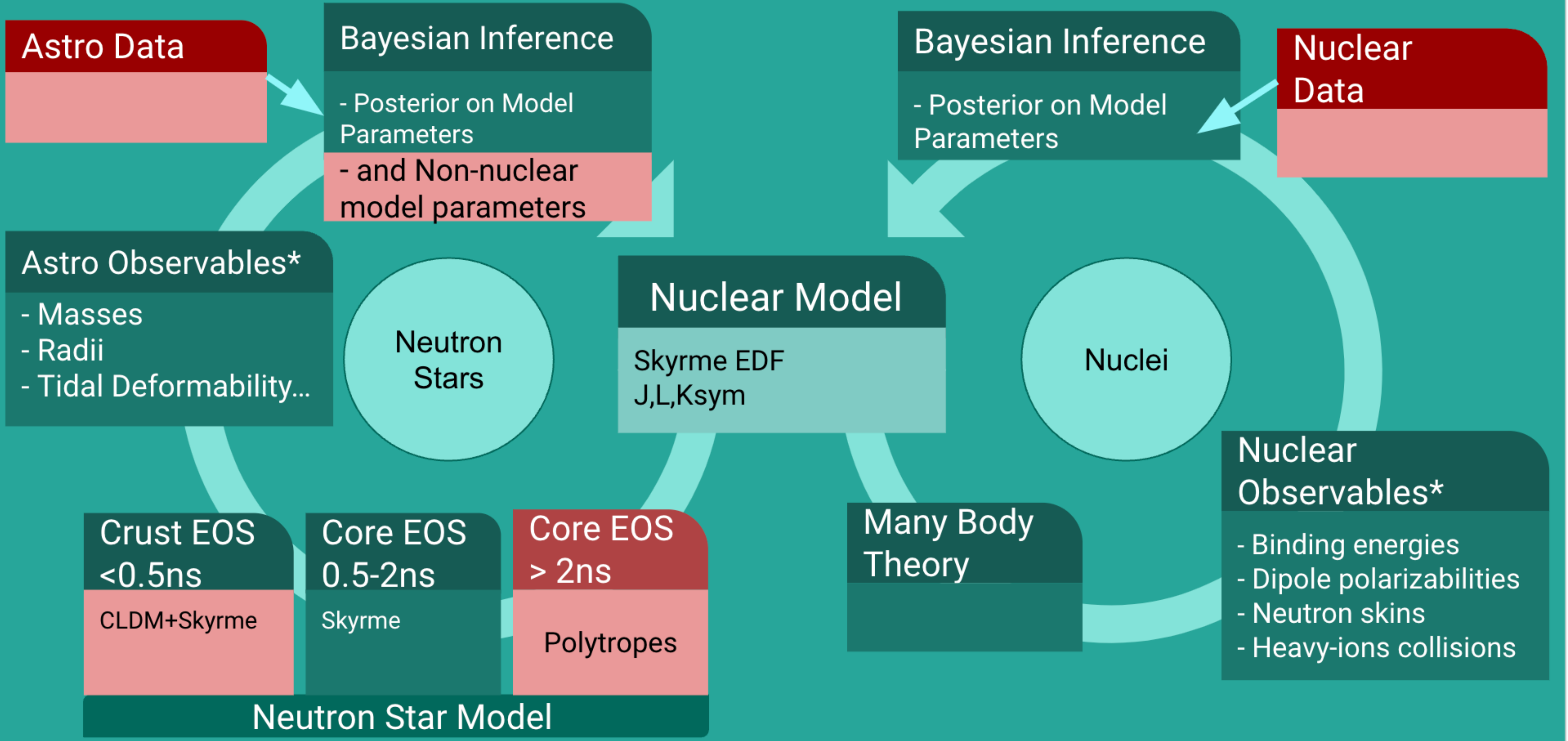
Nuclear masses, giant resonances

ρ_0 (fm^{-3})	0.160 ± 0.005
B (MeV)	16.0 ± 0.5
K (MeV)	230 ± 30

Lim, Holt arXiv:1702.02898

Fixed: potential source of systematic model error

Neutron skins, dipole polarizability, binding energy: SkyrmeRPA Comp Phys Comms, 184, (2013)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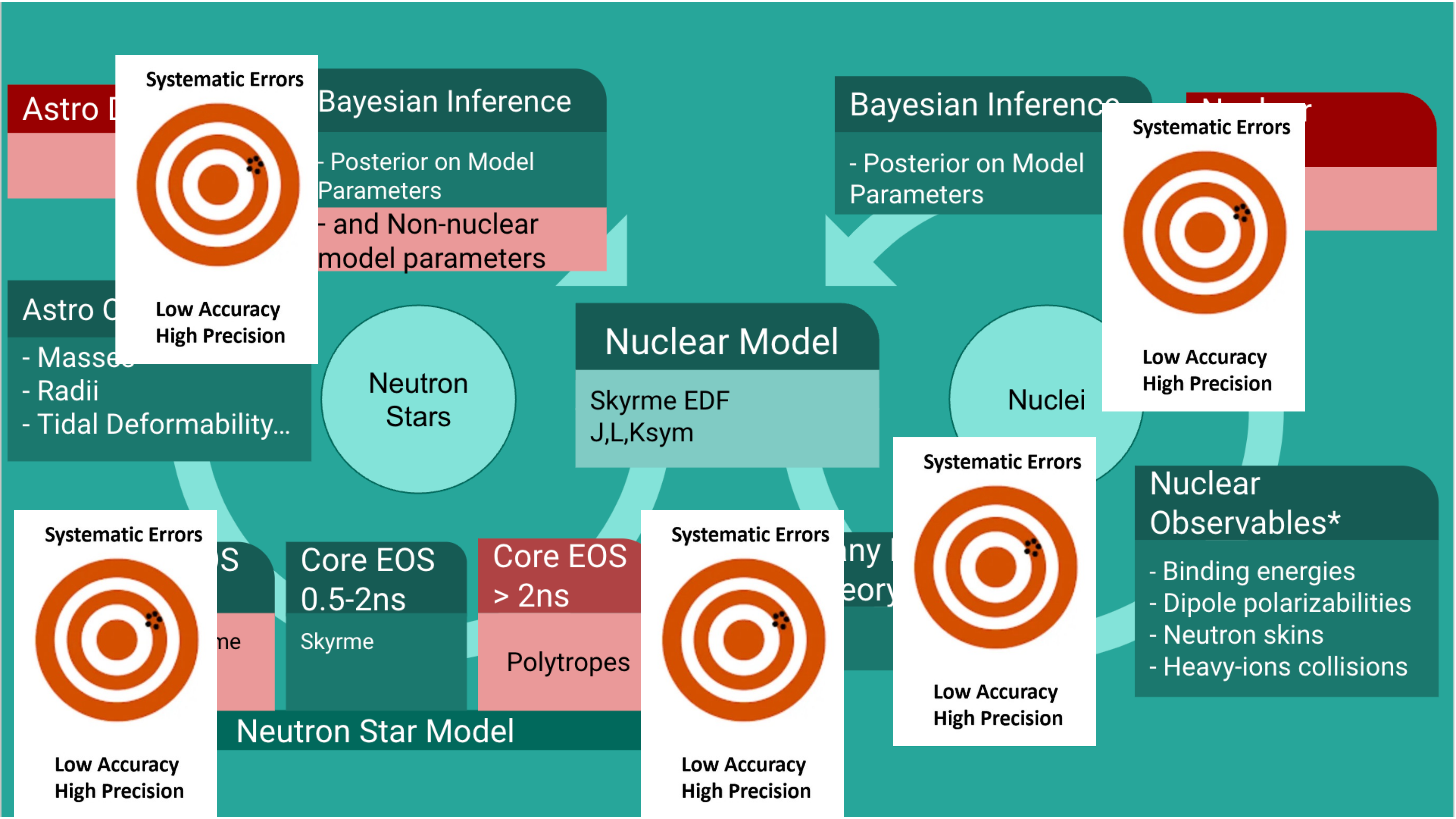

Astro D
Systematic Errors
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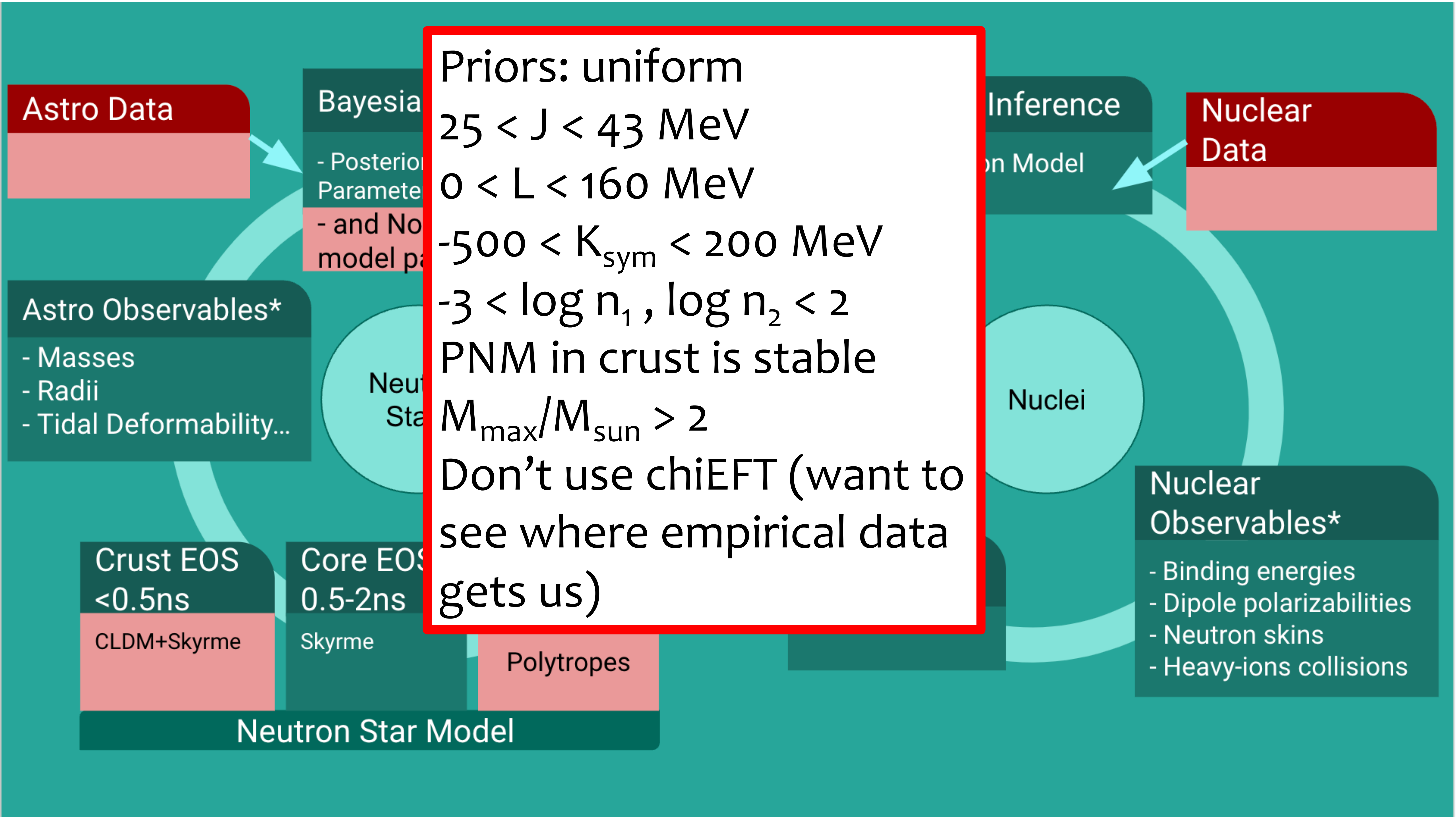


Priors: uniform

$25 < J < 43$ MeV

$0 < L < 160$ MeV

$-500 < K_{sym} < 200$ MeV

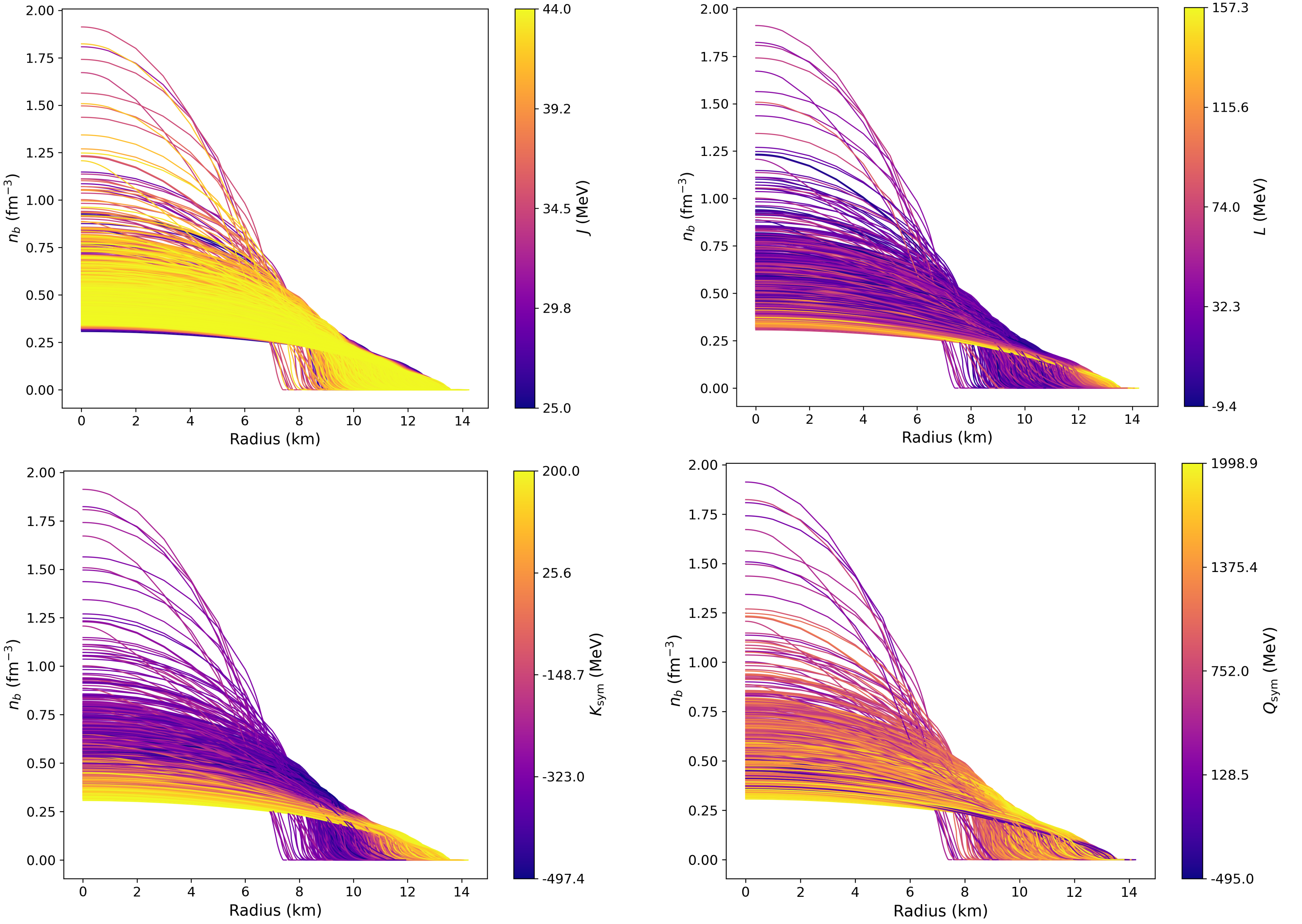
$-3 < \log n_1 , \log n_2 < 2$

PNM in crust is stable

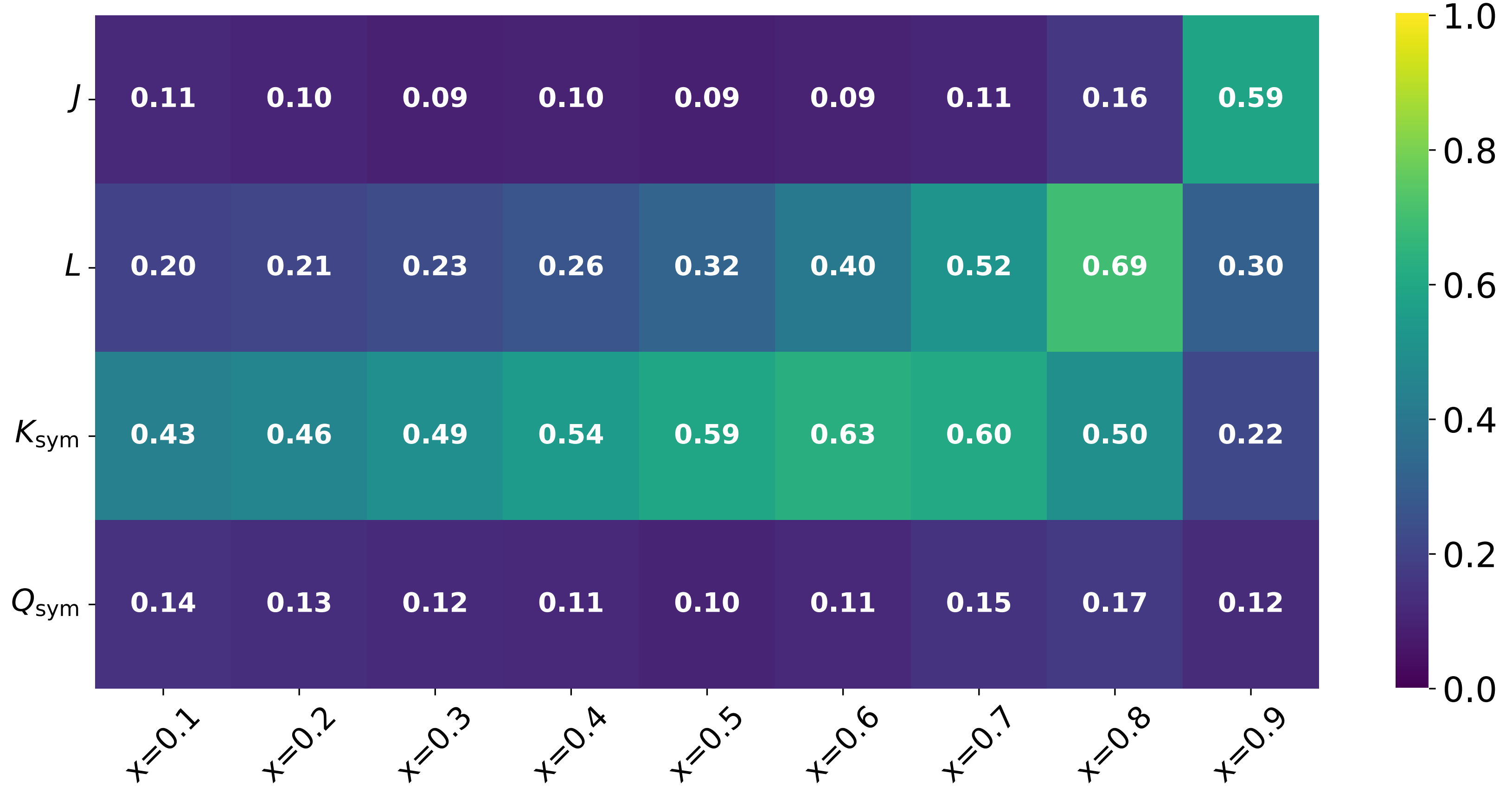
$M_{max}/M_{sun} > 2$

Don't use chiEFT (want to see where empirical data gets us)

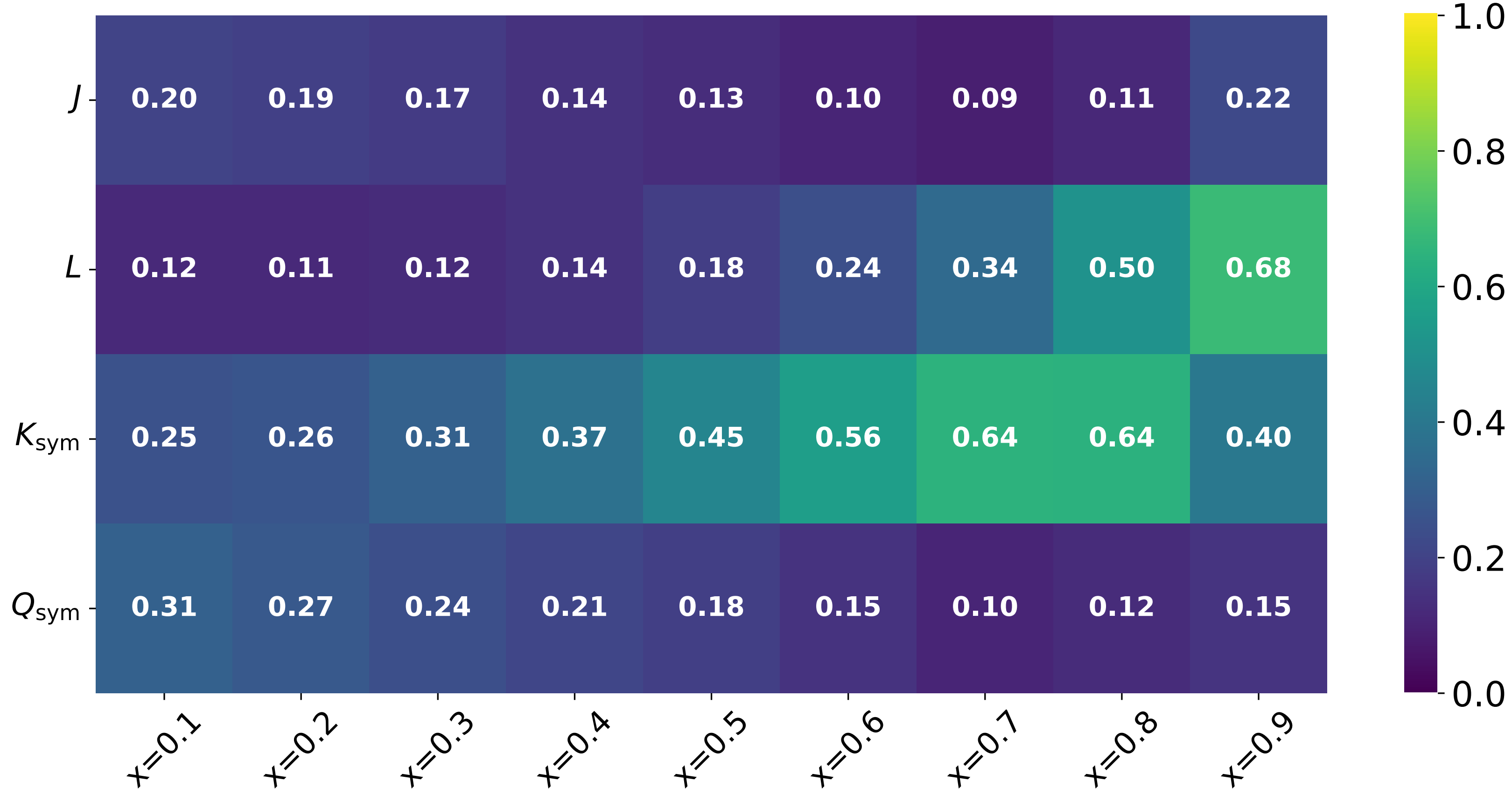
Density profiles of a 1.4M$_{sun}$ star (priors)



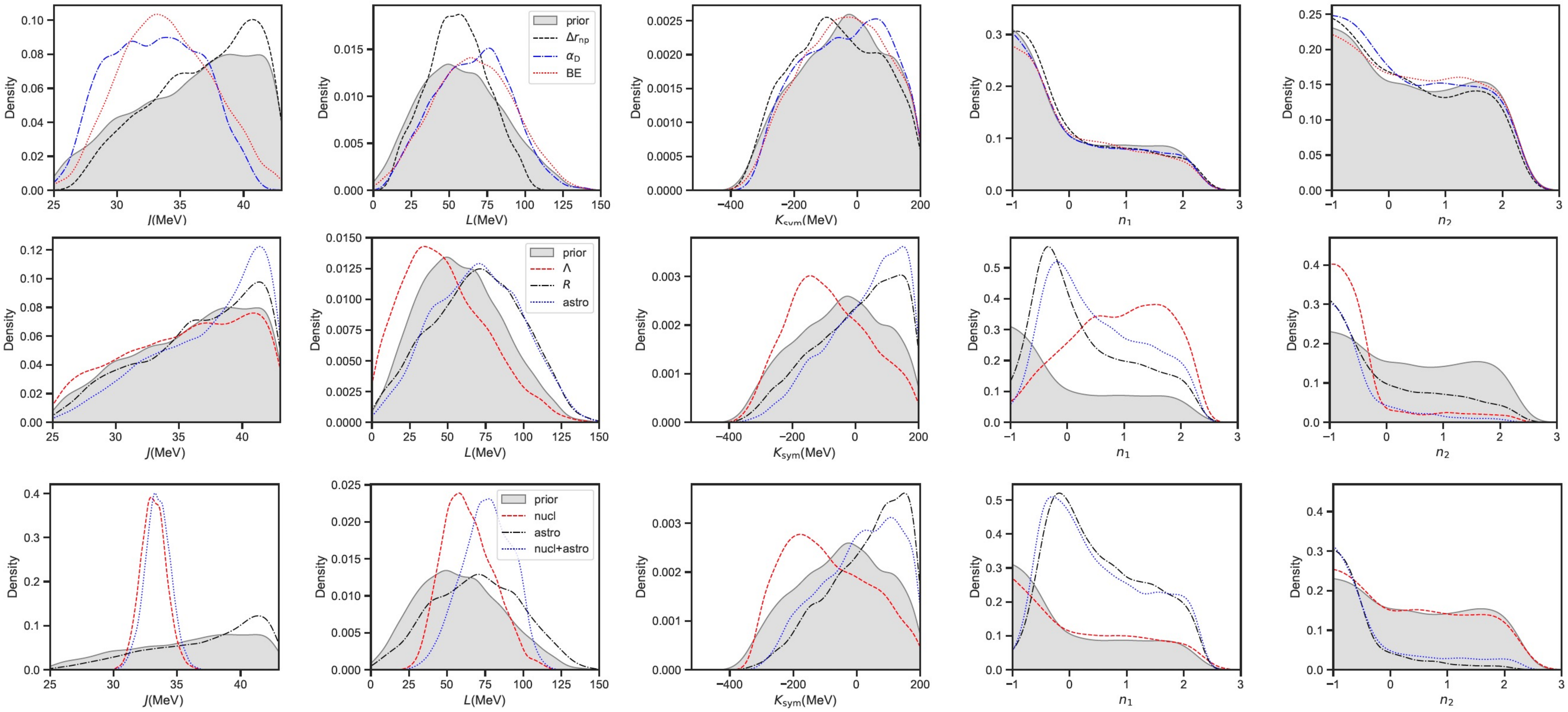
Maximal information coefficients between symmetry energy parameters and density at different radial co-ordinates



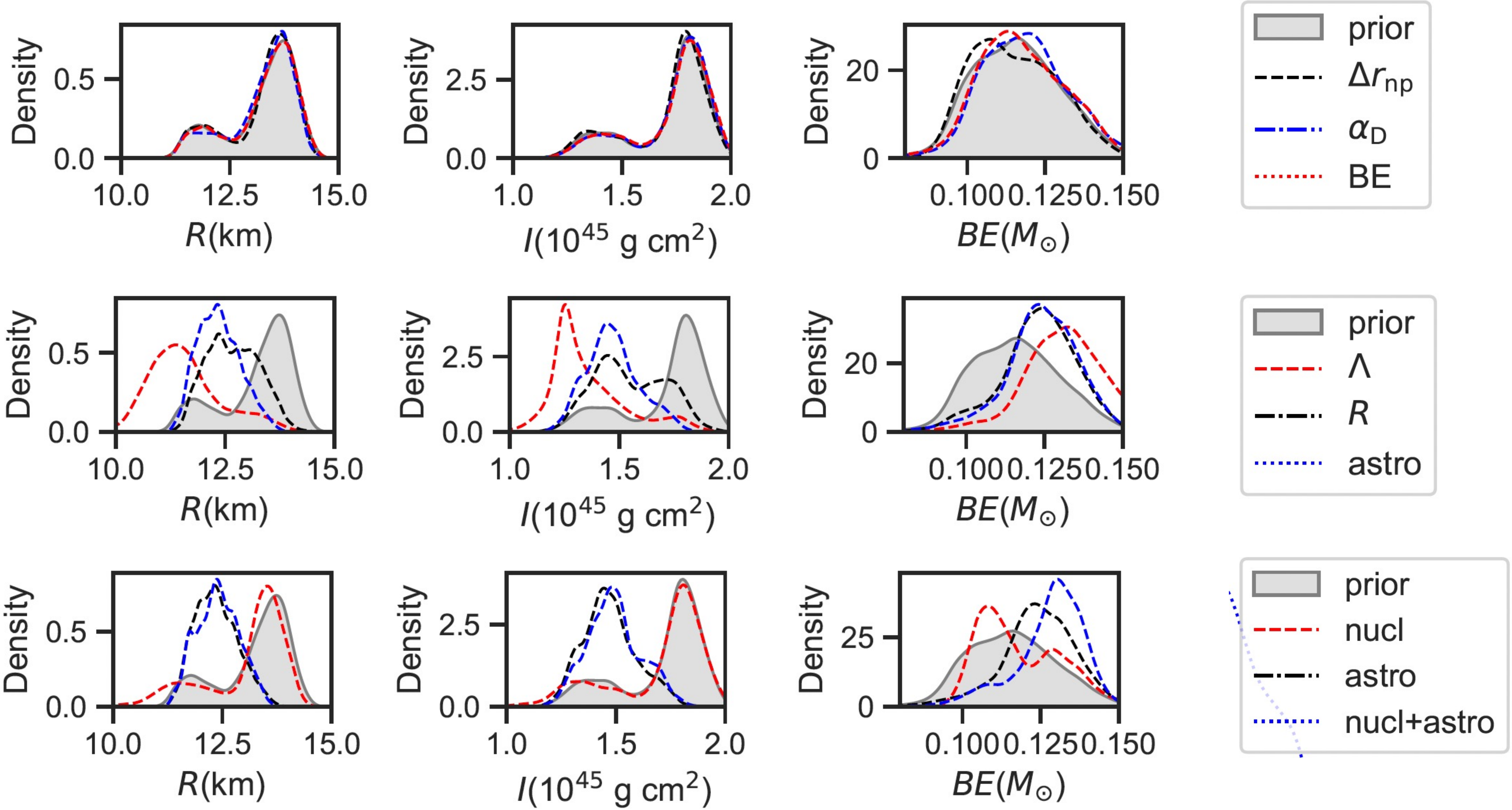
Maximal information coefficients between symmetry energy parameters and density at different radial co-ordinates



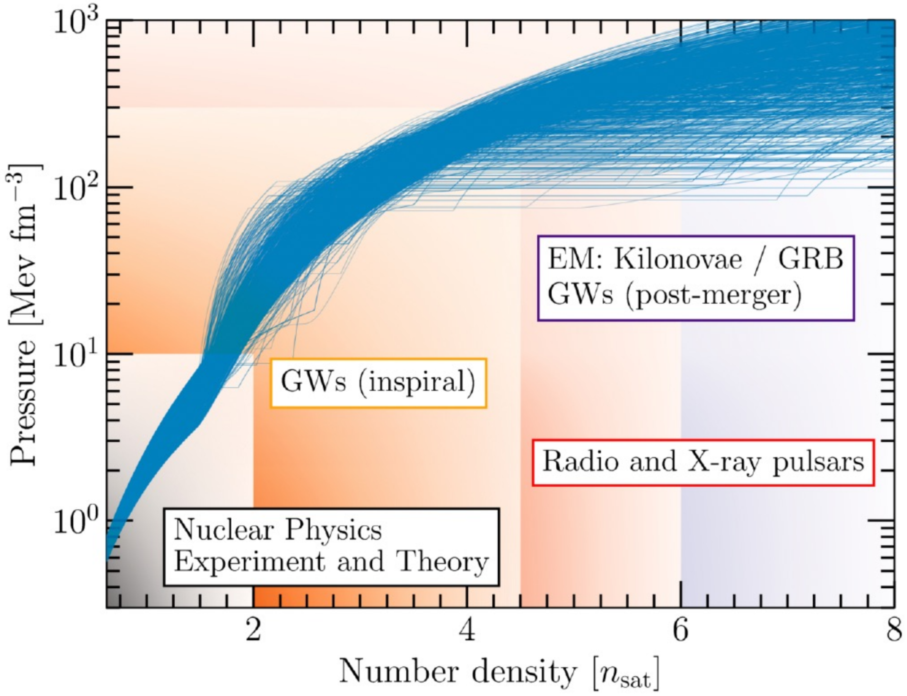
Posteriors on EOS model parameters



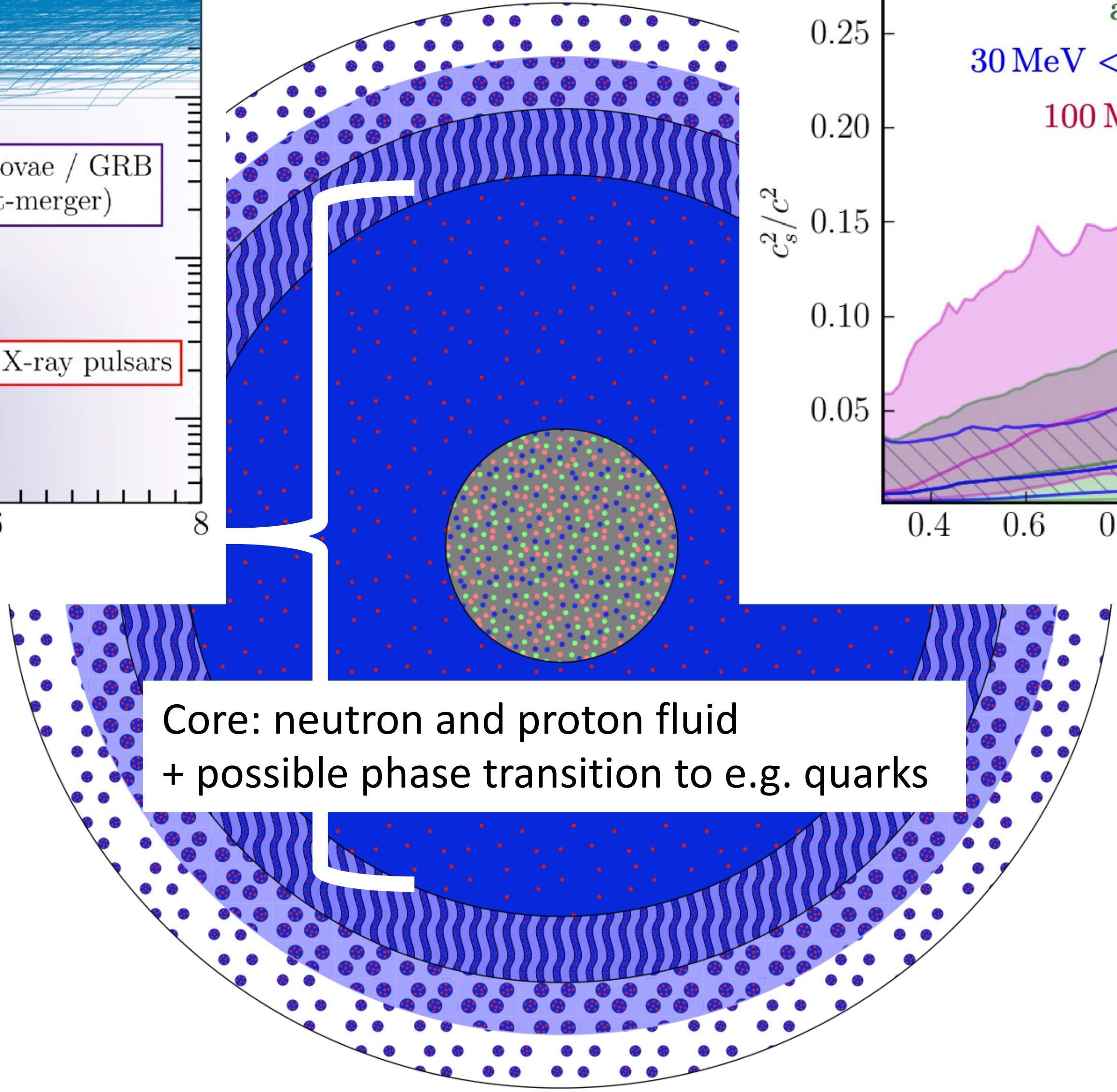
Posteriors on neutron star structure



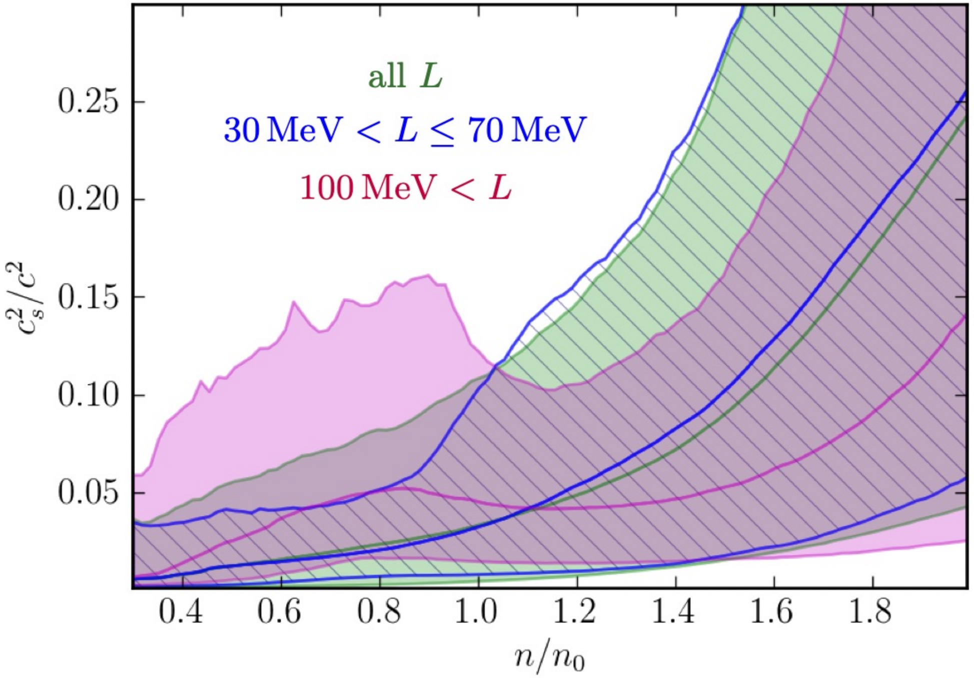
And go ahead and infer! To date, emphasis has been on the EOS of the core



Pang et al, arxiv:2205.08513

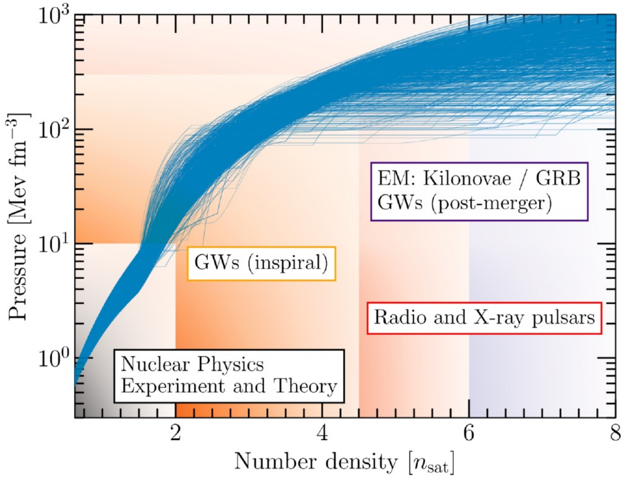


Core: neutron and proton fluid
+ possible phase transition to e.g. quarks



Essick+ arXiv 2102.10074

But the crust is there too, and several observables are sensitive to it



Pang et al, arxiv:2205.08513

Core: neutron and proton fluid
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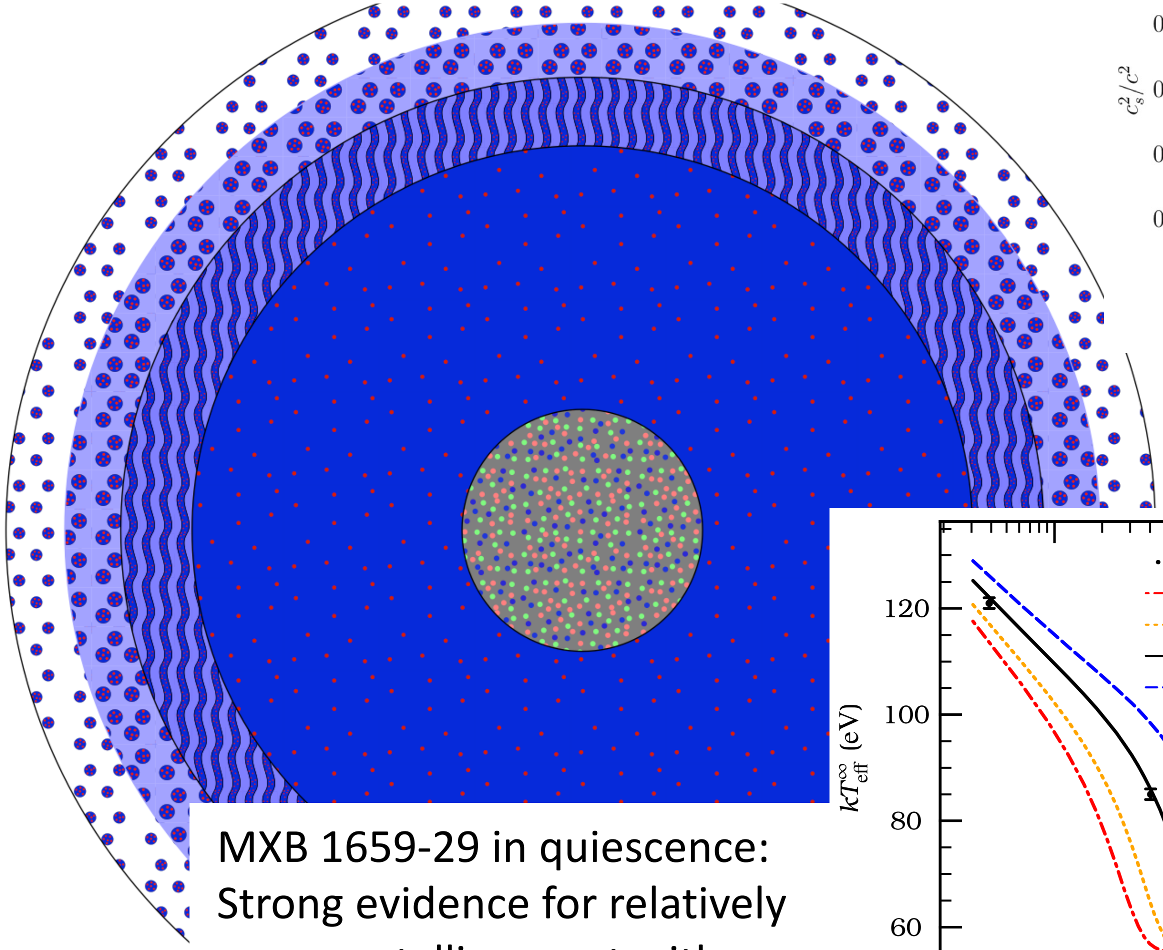
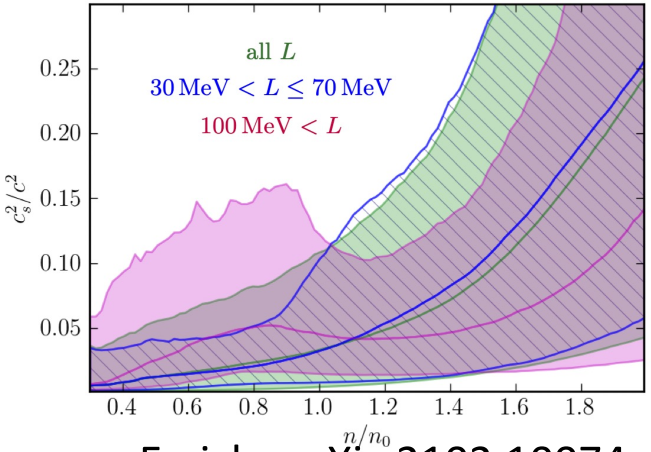
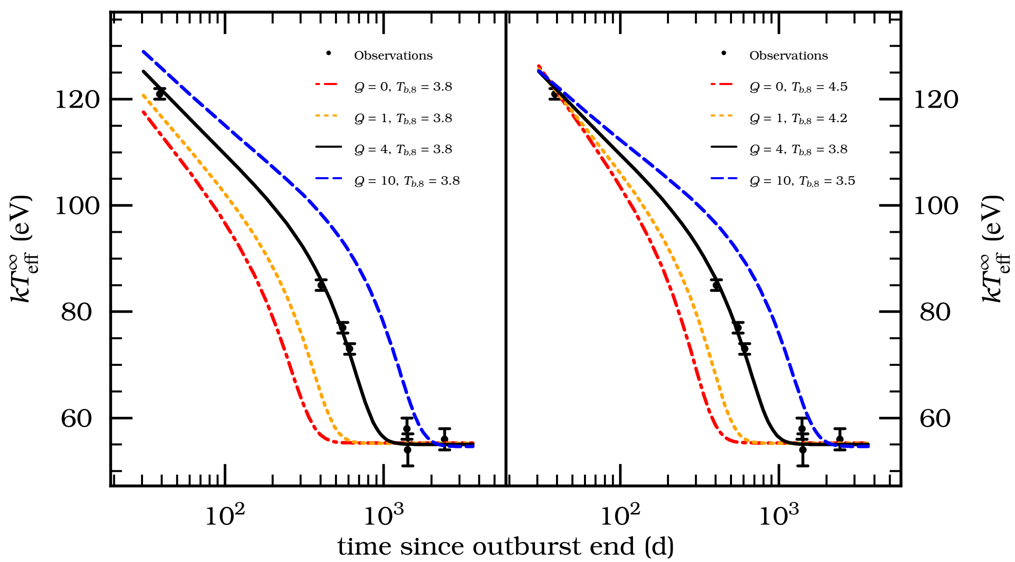


MXB 1659-29 in quiescence: Strong evidence for relatively pure crystalline crust with superfluid neutrons in the inner layer

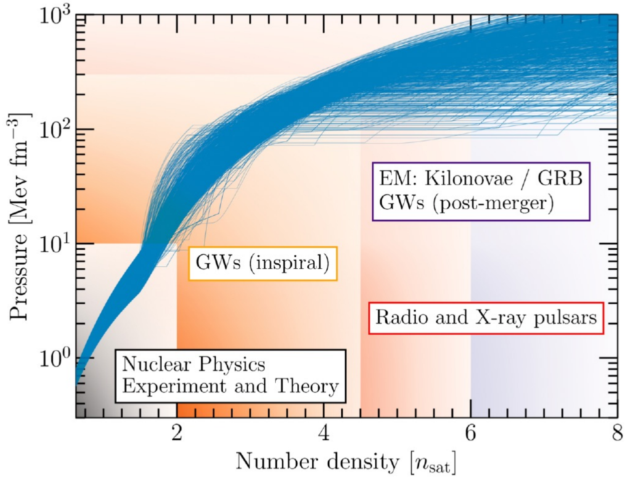


Essick+ arXiv 2102.10074



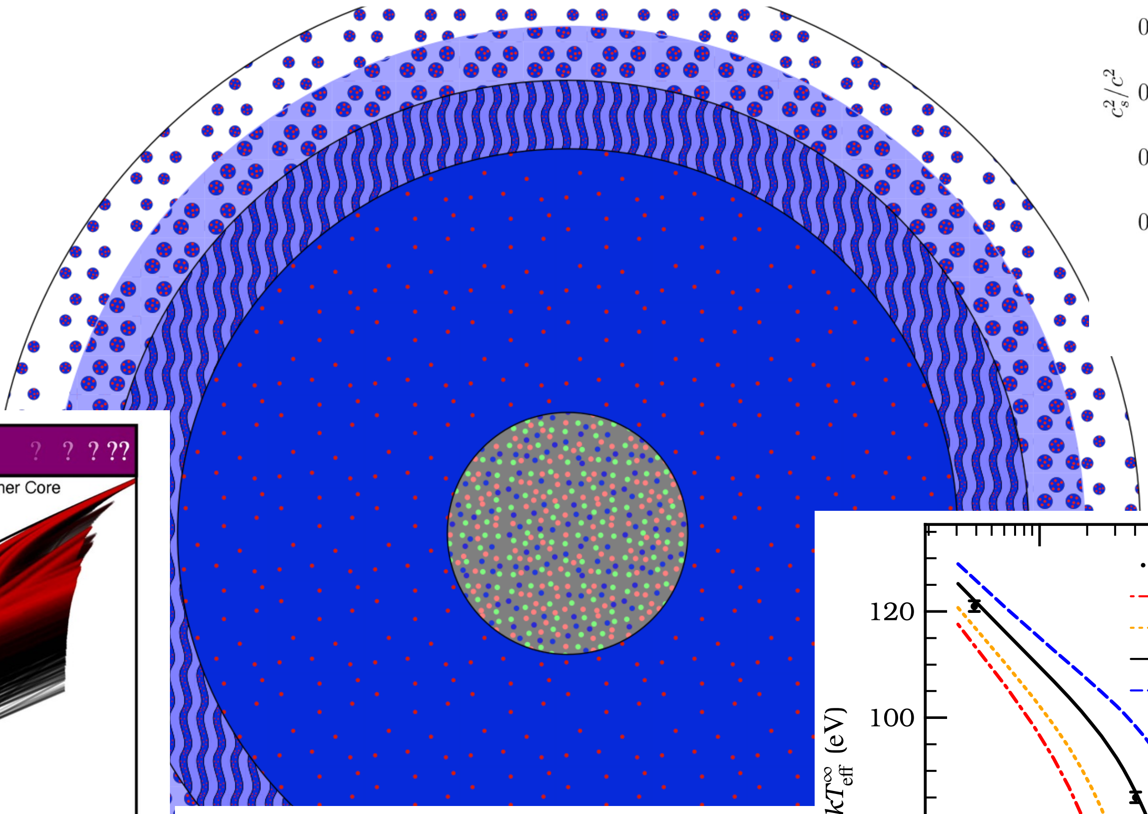
Brown and Cumming, ApJ 2009

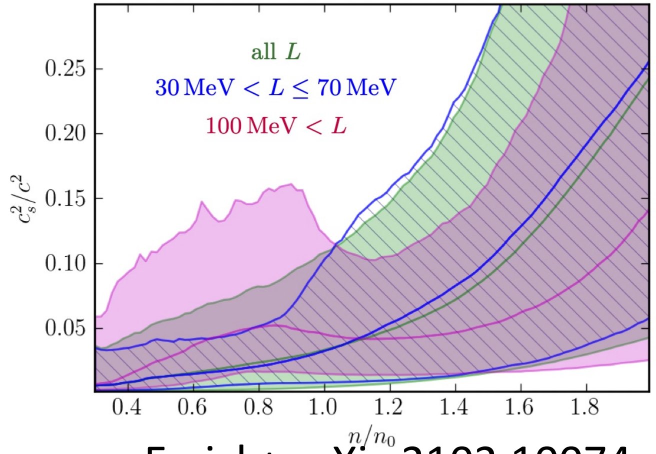
So let's include the crust when we build our ensembles



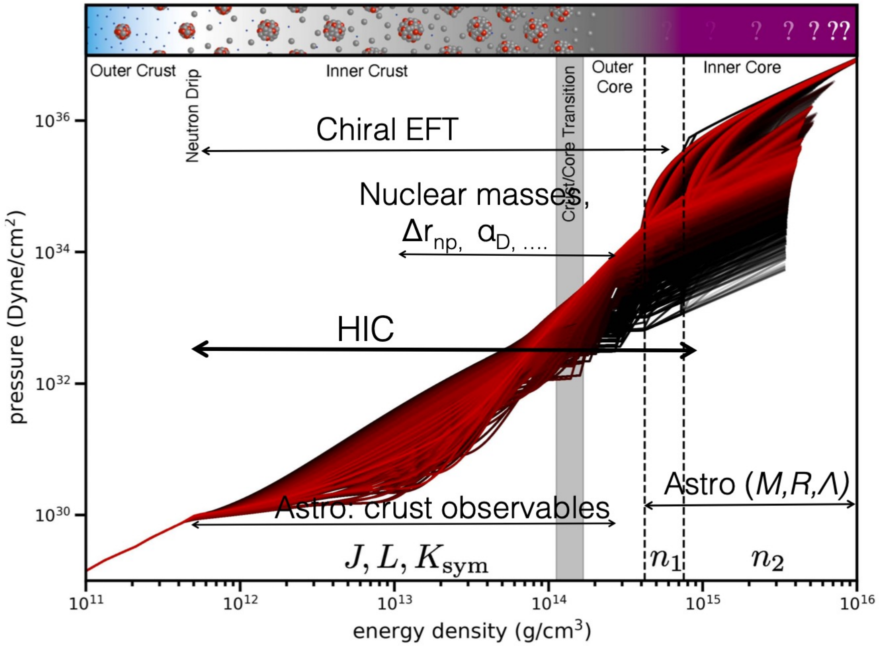
Pang et al, arxiv:2205.08513

Core: neutron and proton fluid
+ possible phase transition to e.g. quarks



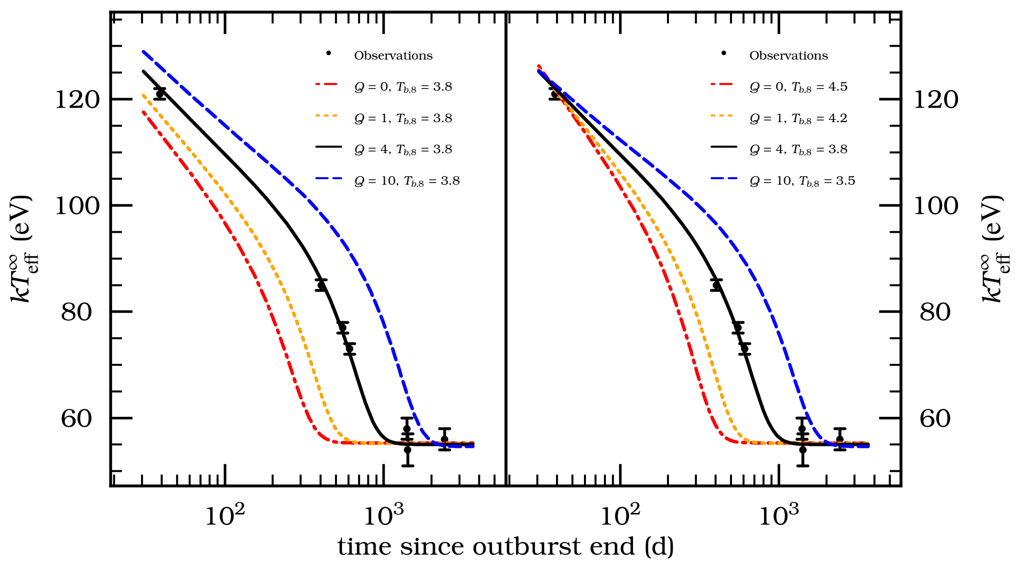


Essick+ arXiv 2102.10074



Neill+ 2208.00994; Sorenson+ 2301.13253

MXB 1659-29 in quiescence:
Strong evidence for relatively pure crystalline crust with superfluid neutrons in the inner layer

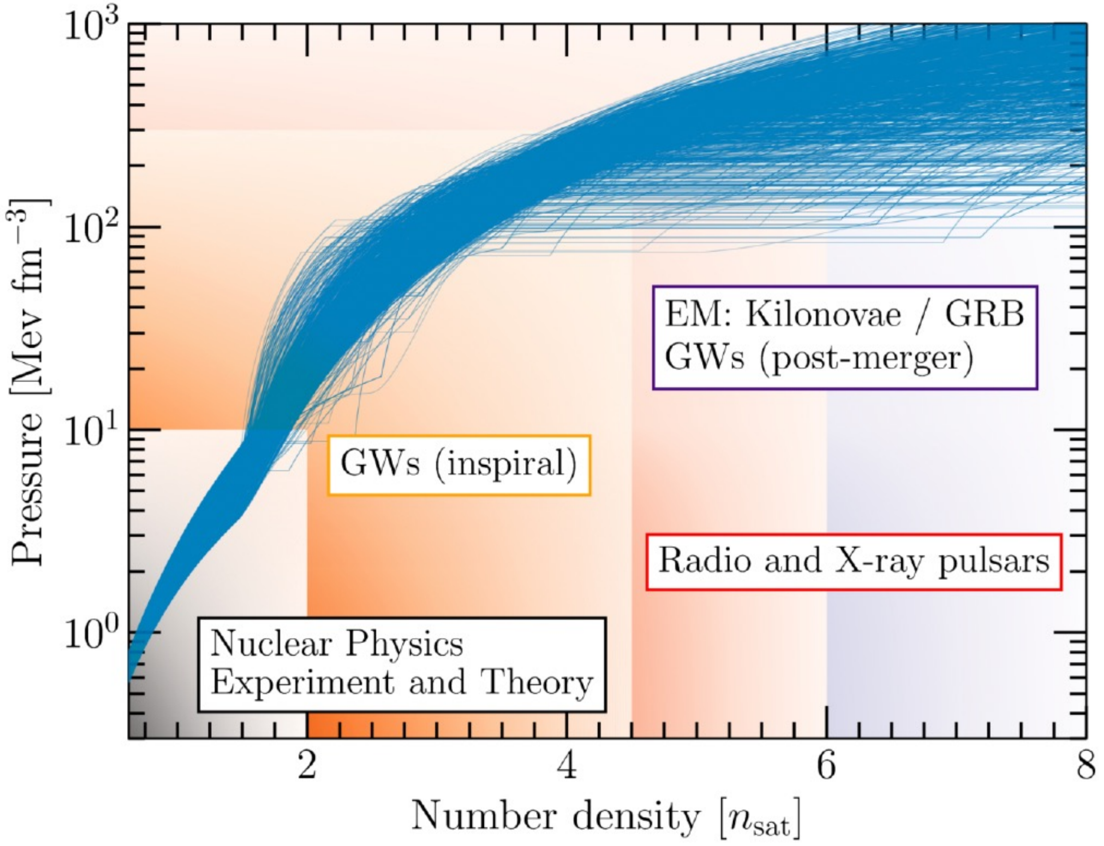


Brown and Cumming, ApJ 2009

Experiments such as neutron skin and dipole polarizability probes EOS below mostly saturation density, how relevant are those in determine neutron star properties?

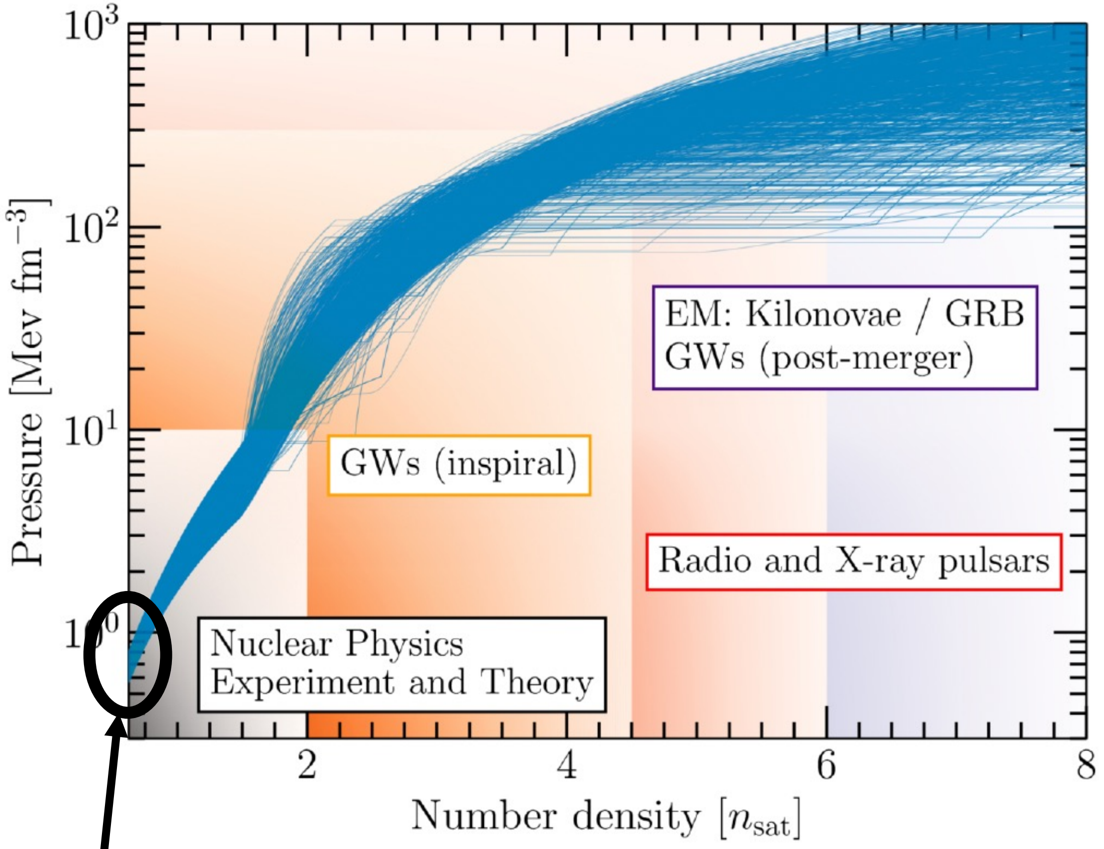
(with apologies to Matt Groening)

Crust usually de-emphasized in EOS plots



Pang et al, arxiv:2205.08513

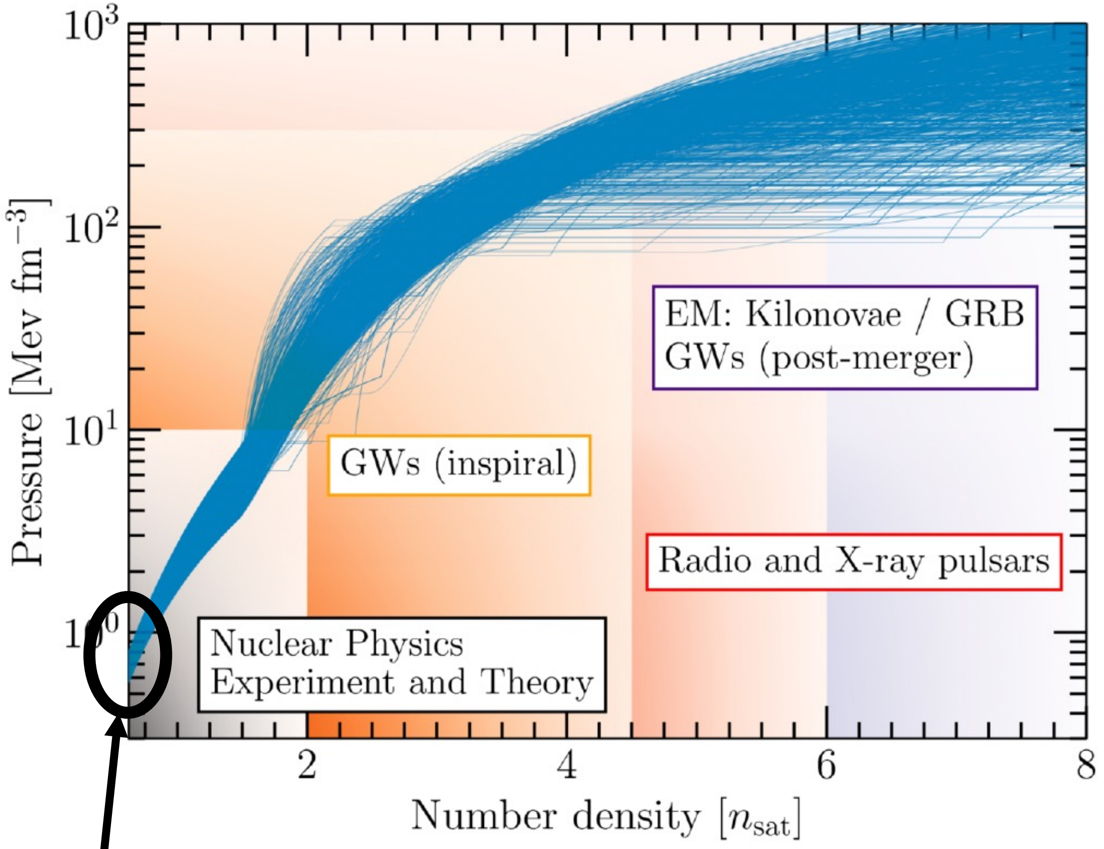
Crust usually de-emphasized in EOS plots



Pang et al, arxiv:2205.08513

There's the crust!

Crust usually de-emphasized in EOS plots



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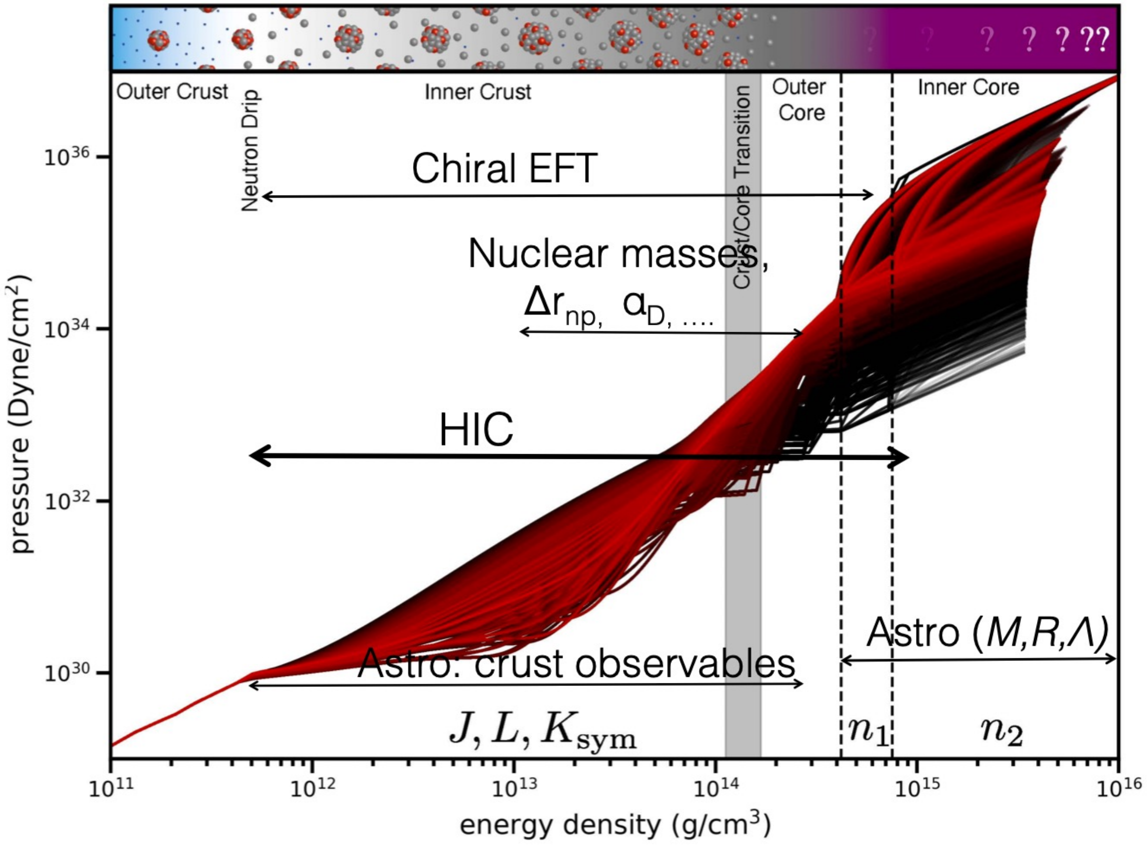
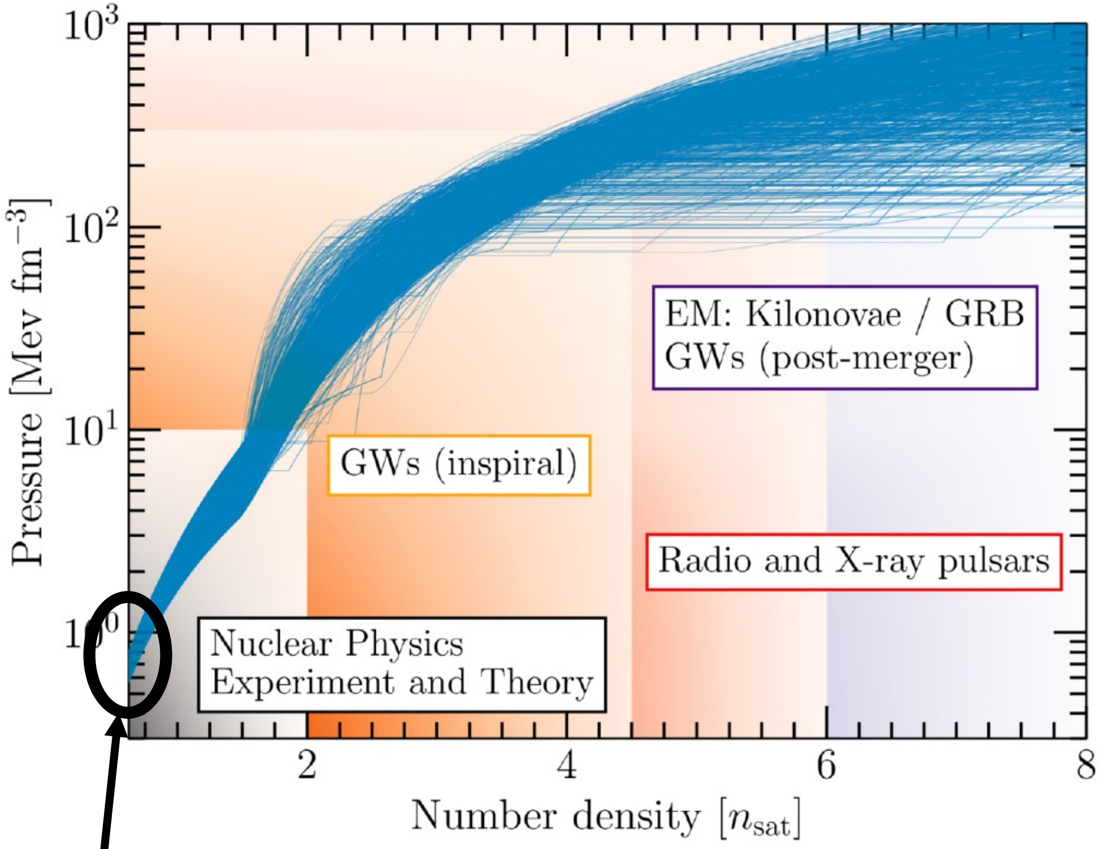


Neill+ 2208.00994; Sorenson+ 2301.13253

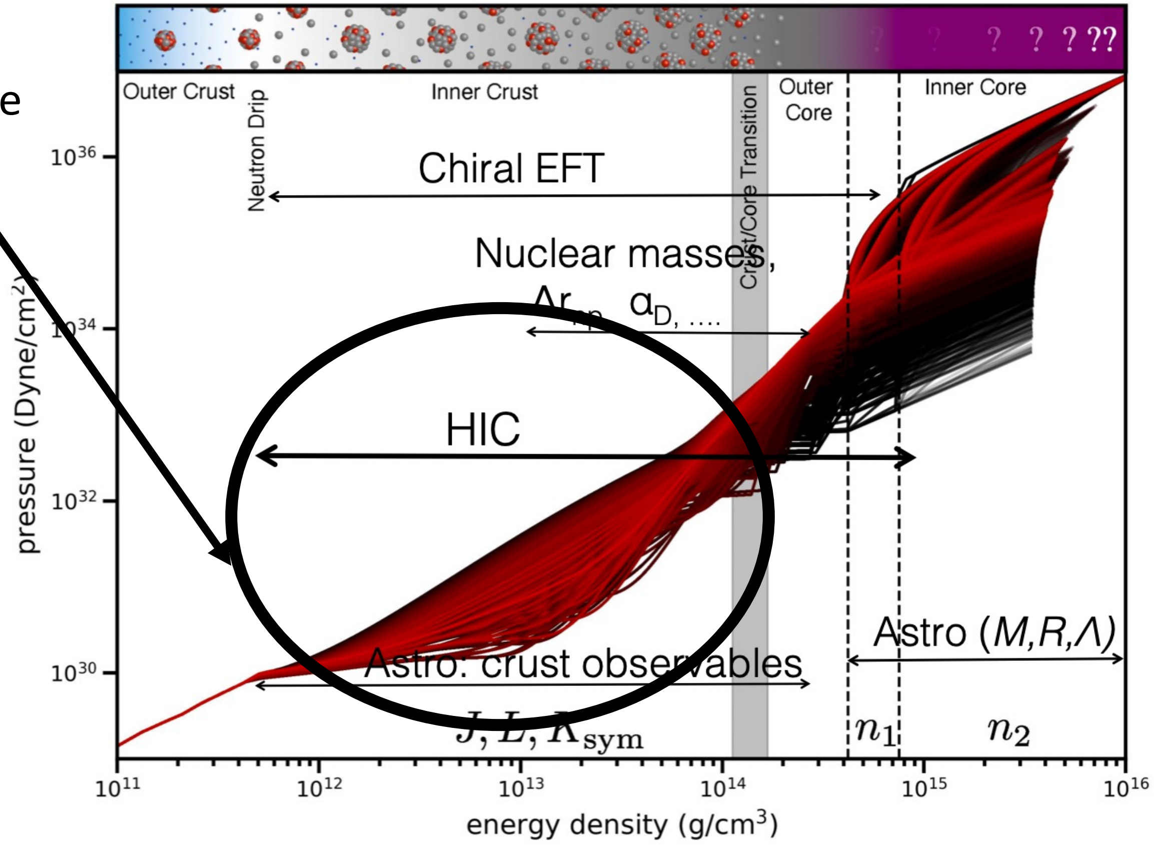
Crust usually de-emphasized in EOS plots



Pang et al, arxiv:2205.08513

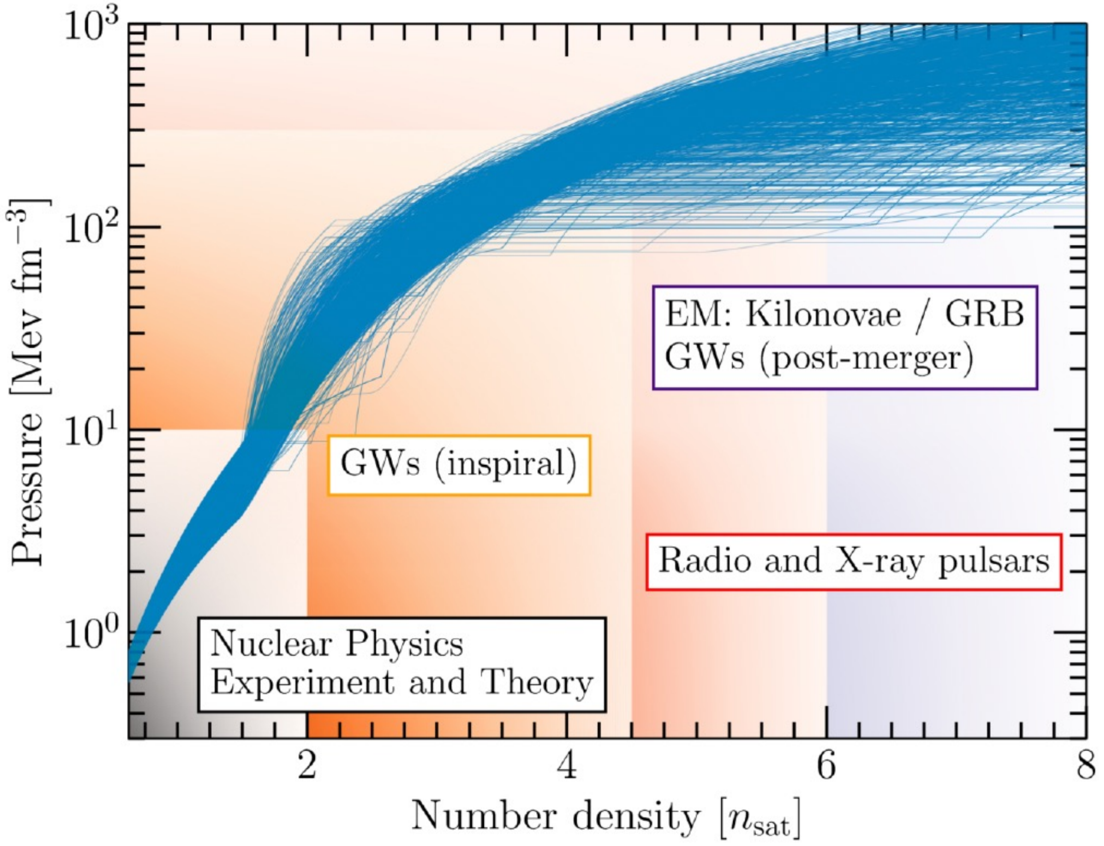
There's the crust!

There's the crust!

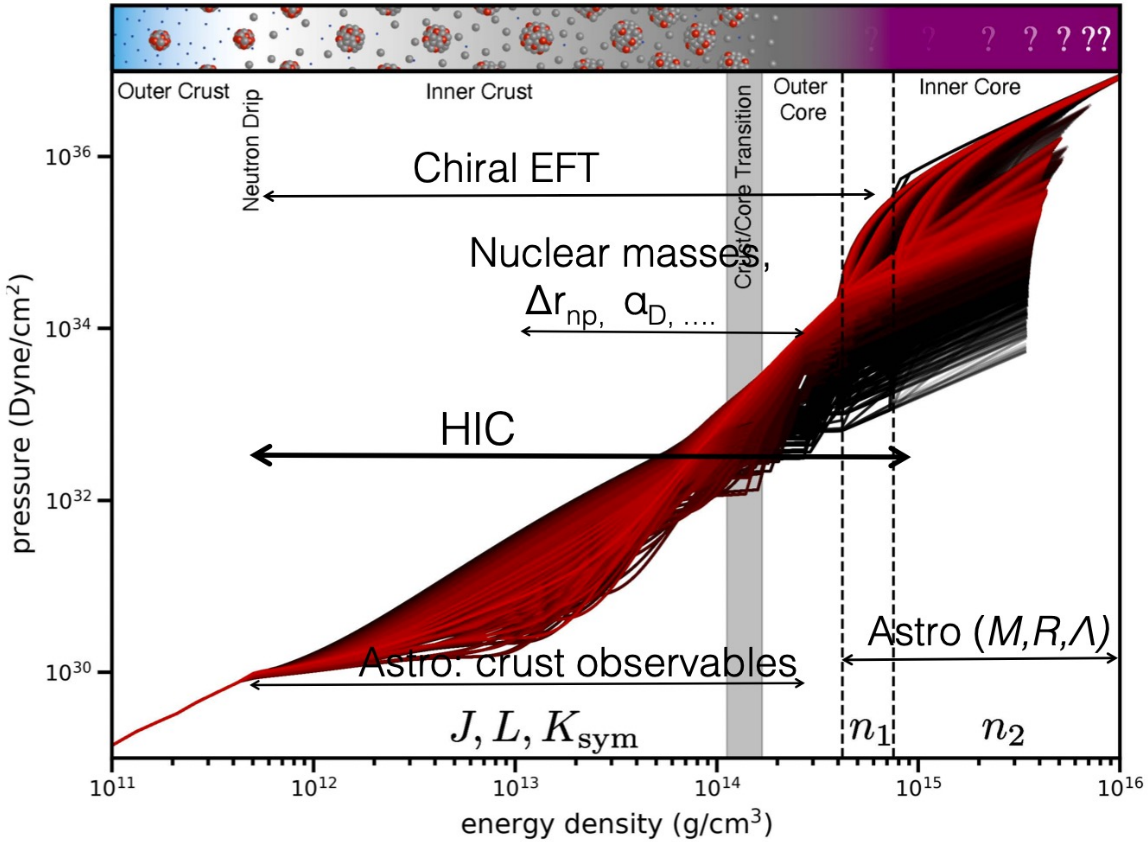


Neill+ 2208.00994; Sorenson+ 2301.13253

Core consistent with crust needed for inference of bulk properties



Pang et al, arxiv:2205.08513



Neill+ 2208.00994; Sorenson+ 2301.13253

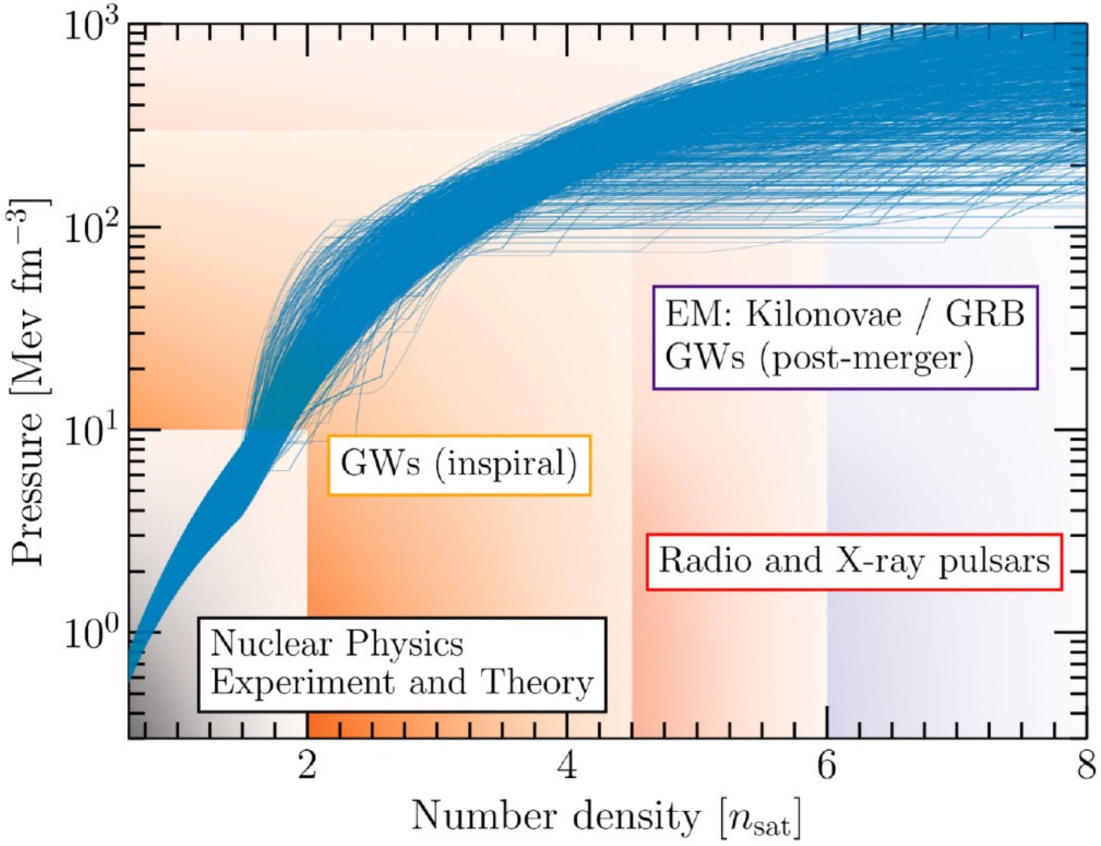
Lami Suleiman + *Phys.Rev.C* 104 (2021) 1, 015801

Crust inconsistent with core EOS leads to **errors up to 5% in radius inference – that's 0.5km**

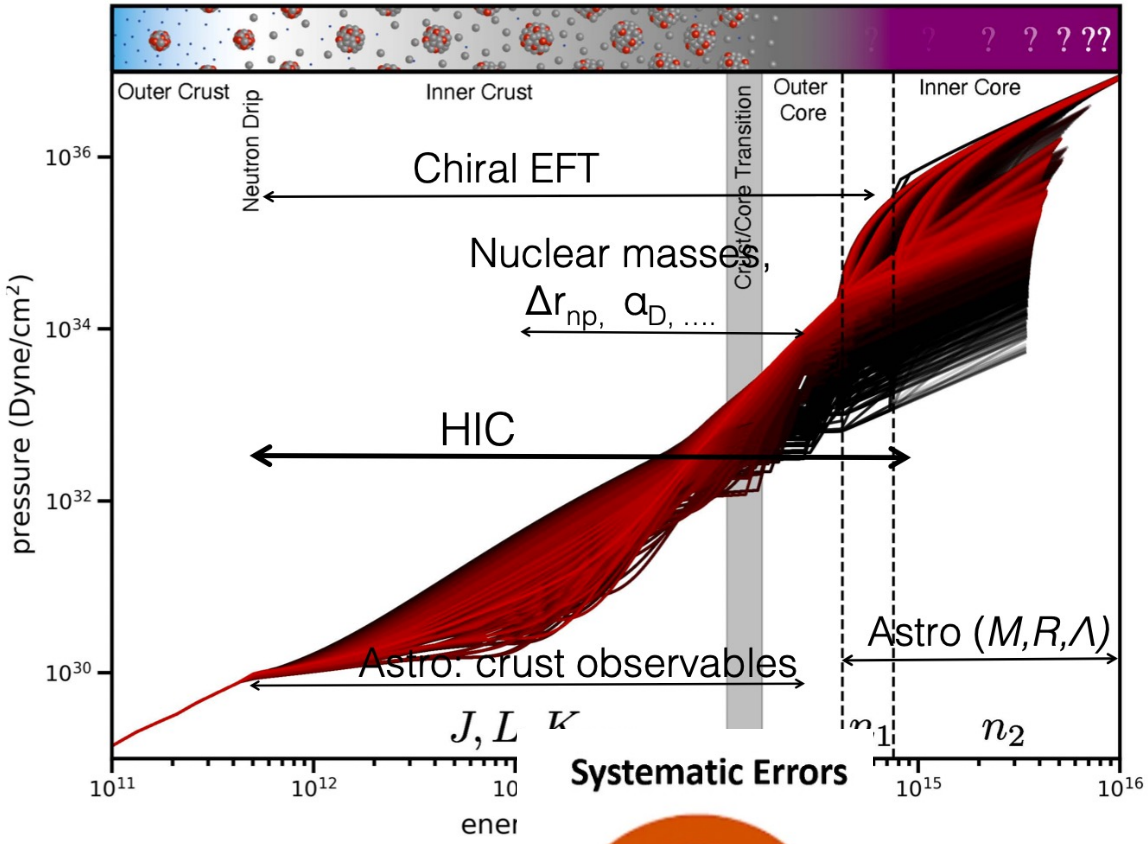
Precision of universal relations underestimated

NEXT GENERATION X-RAY/GW MEASUREMEMENTS WILL NEED BETTER CRUST MODELING

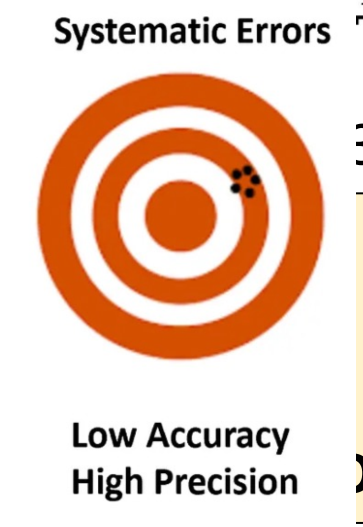
Core consistent with crust needed for inference of bulk properties



Pang et al, arxiv:2205.08513



Neill+ 2208.00994, 301.13253



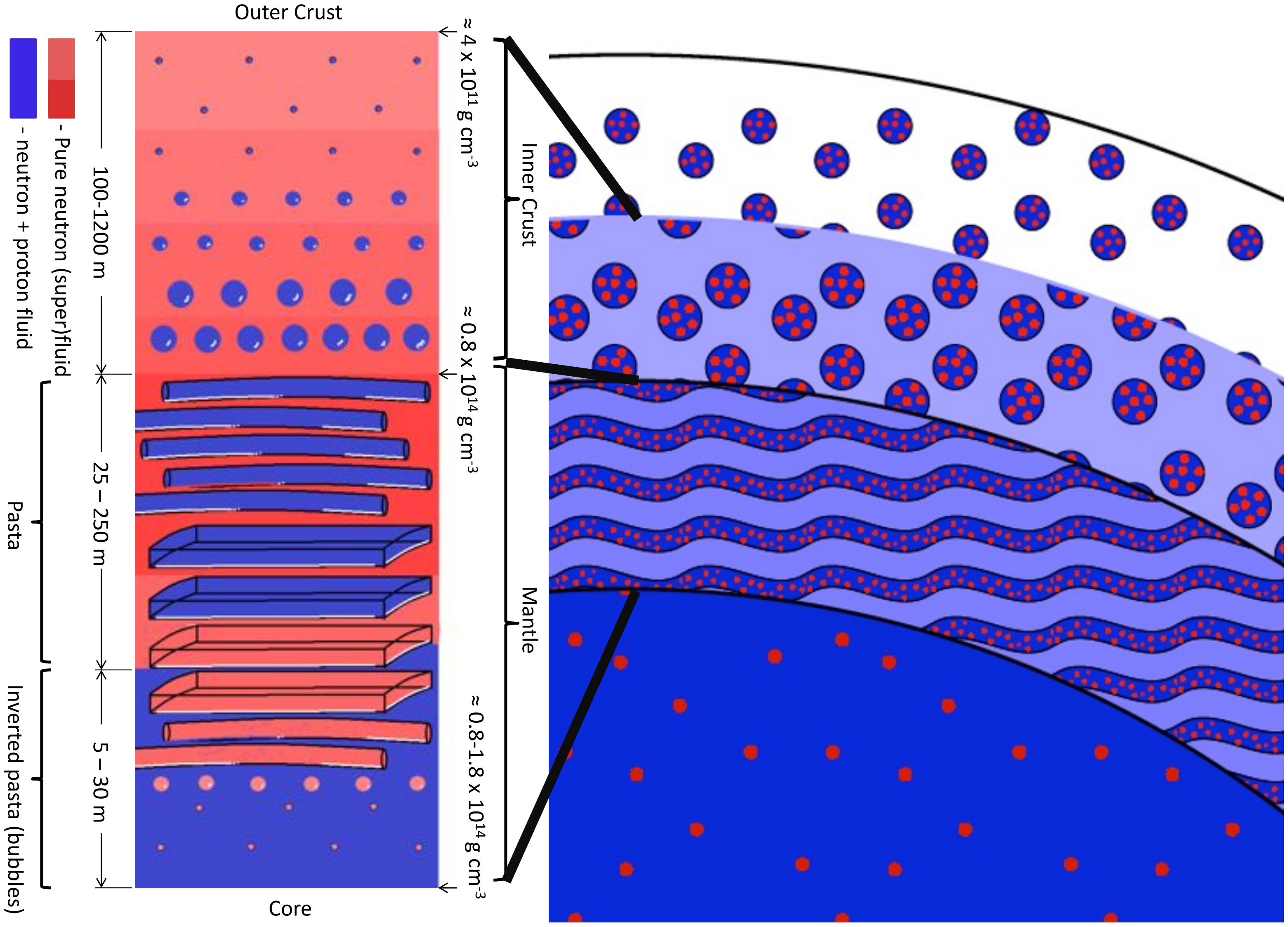
Lami Suleiman + *Phys.Rev.C* 104 (2021) 1, 015801

Crust inconsistent with core EOS leads to **errors up to 5% in radius infe** **0.5km**

Precision of universal relations underestimated

NEXT GENERATION X-RAY/GW MEASUREMEMENTS WILL NEED BETTE **DELING**

Crust structure



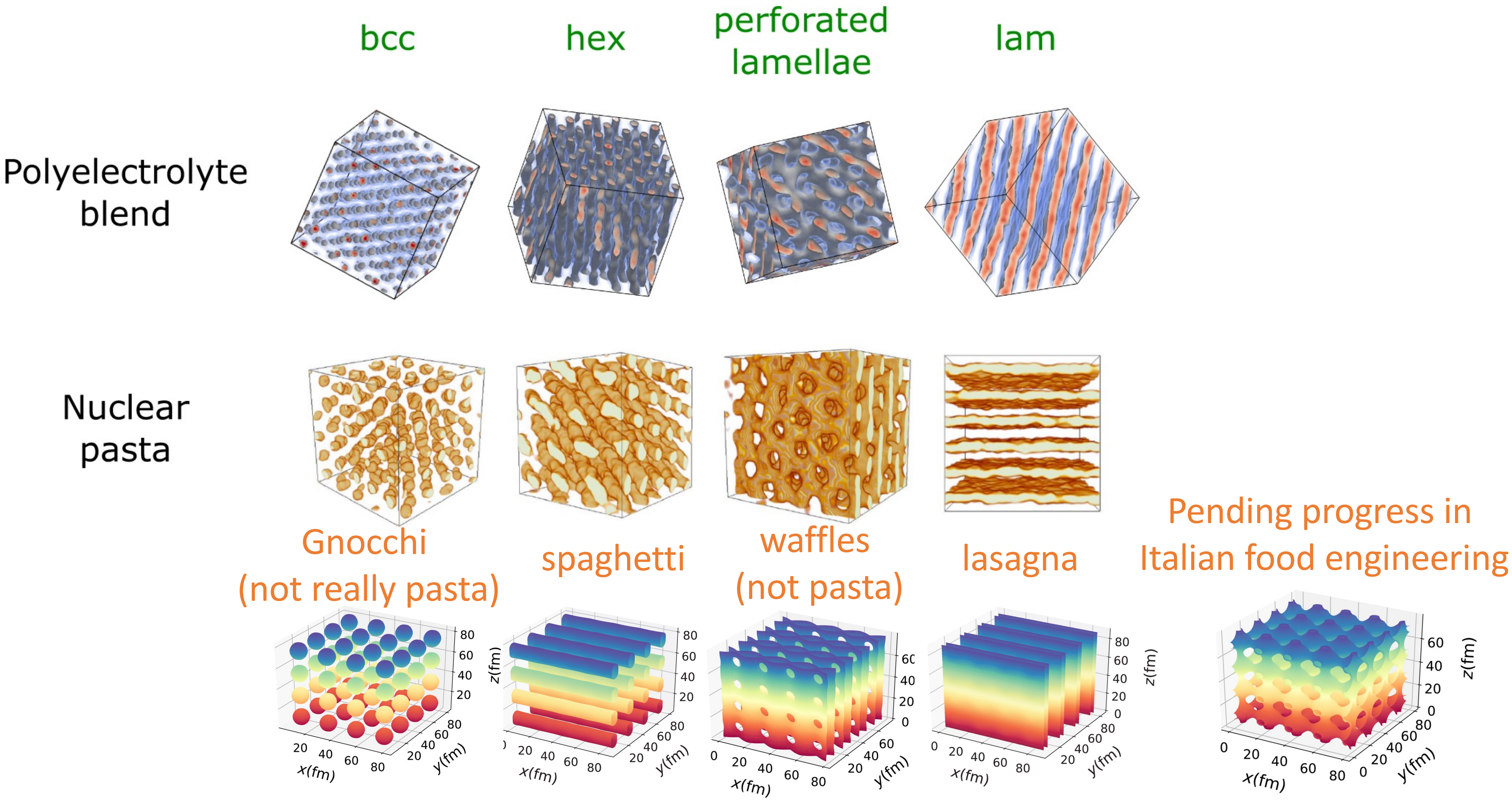
Outer crust: nuclei, e^-, elastic solid

Inner crust: nuclei, e^-, n; two components (elastic solid, neutron superfluid)

Mantle: crust-core interface region; deformed, continuous nuclear clusters, e^-, n; soft condensed matter

Crust breaking, mountains, crust modes… originate at the bottom of the crust (e.g. Morales, Horowitz 2409.14482)

Driven by competition between short range attractive and long-range repulsive interactions - a generic feature of soft-condensed matter systems



Rumyantsev, dePablo: Macromolecules 53, 2020
Molecular dynamics simulations: Caplan, Horowitz, Rev. Mod. Phys. 89, 041002 (2017)
Quantum simulations: Newton et al, arxiv:2104.11835

Structure of Matter below Nuclear Saturation Density

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and
C. J. Pethick
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and
J. R. Wilson
Lawrence Livermore National Laboratory, Livermore, California 94550

(Received 5 May 1983)

It will be interesting to explore the consequences of these spaghettilike and lasagnalike phases of dense matter. Their physical properties will have to reflect the great departure from isotropy that these phases possess. Neutrino scattering

properties. After all, the cooking of spaghetti, while it spoils the perfect straightness of the strands, does not destroy the characteristic short-range order.

NUCLEAR PASTA RECIPE: ANGEL HAIR WITH CARROTS

crammed into a 20km-wide sphere... Because of the immense gravity, the outer layers of neutron stars freeze solid to form a crust that surrounds a liquid core. Below the crust, protons and neutrons compete and end up forming long cylindrical shapes or flat planes. These have become known as 'spaghetti' and 'lasagna'—or nuclear pasta."

Given this exciting discovery, Barilla Executive Chef Lorenzo Boni decided to get creative and make his own version of nuclear pasta using Barilla Angel Hair, carrots, red bell peppers and Romano cheese. A few pieces of Barilla Collezione Orecchiette and some sprinkles of Barilla Pastina make the perfect garnish for the plate. Try it for dinner tonight—it's out of this world!

https://www.barilla.com/en-us/posts/2018/10/22/nuclear-pasta-recipe-angel-hair-with-carrots


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Why care about the crust?

Pulsar glitches
- Link, Lattimer, Epstein PRL 1999

Magnetic field evolution
- Pons, Vigano, Rea, Nature Physics 2013

Crust cooling
- Newton, Murphy, Li ApJL 2013
- Brown and Cumming, ApJ 2009
- Horowitz+, PRL 2015

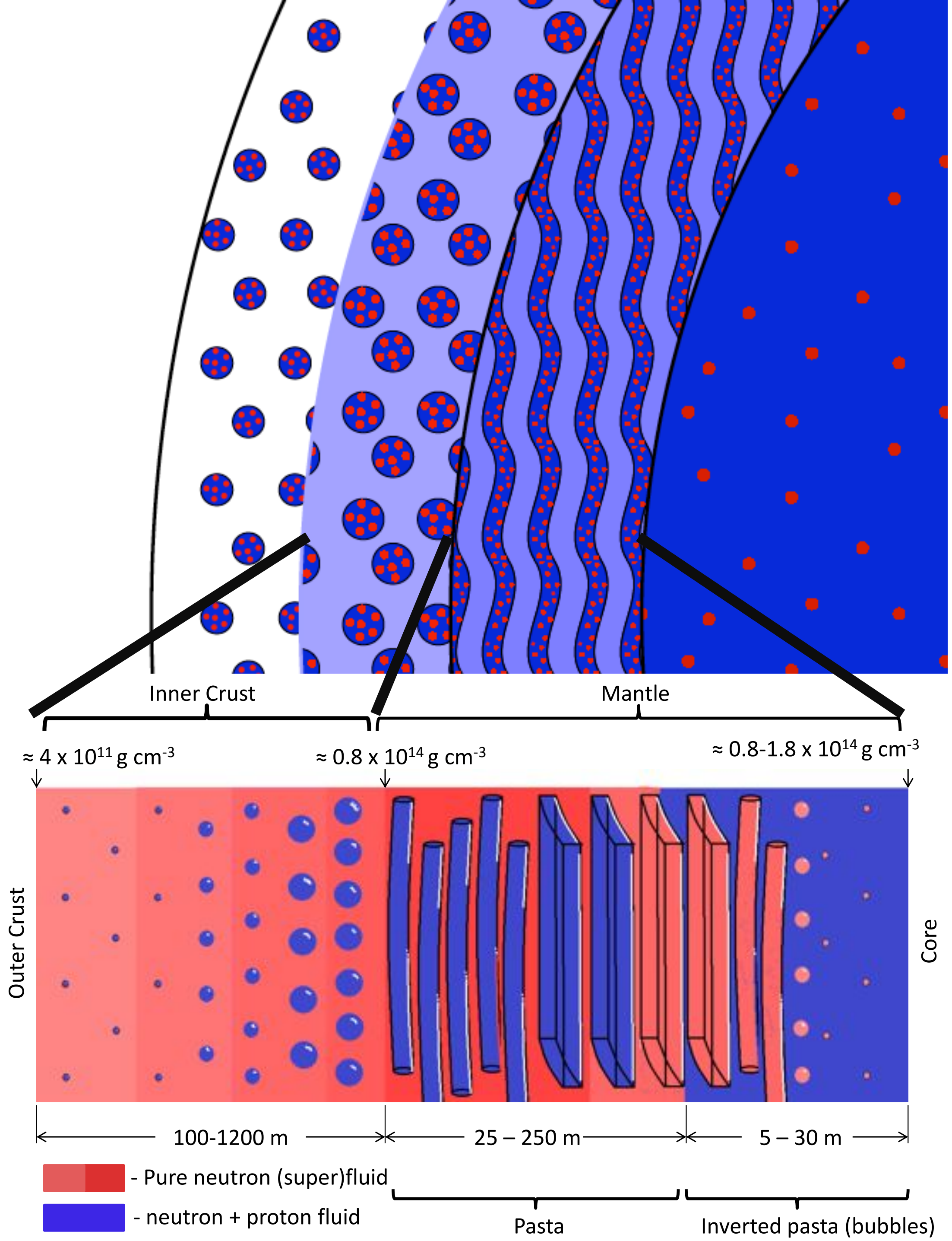
GWs from mountains
- Caplan,Horowitz,Schneider, PRL 2018

Spin evolution, r-modes
- Fattoyev, Newton, Li PRC 2014

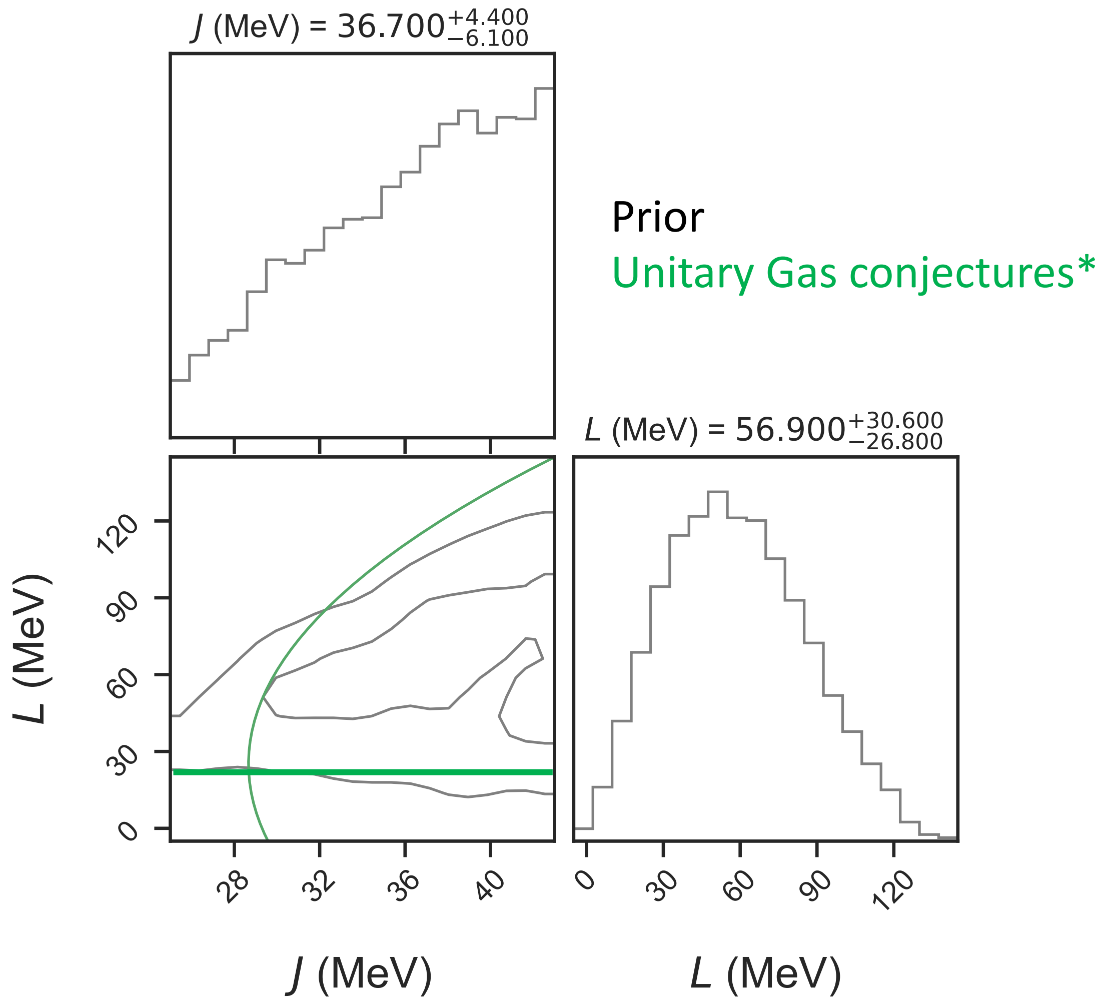
Crust shattering flares
- Tsang et al PRL108, 2012

Chamel, Haensel,
- Living Reviews in Relativity 2008

Constraining the symmetry energy:
- Newton+ EPJA 2014

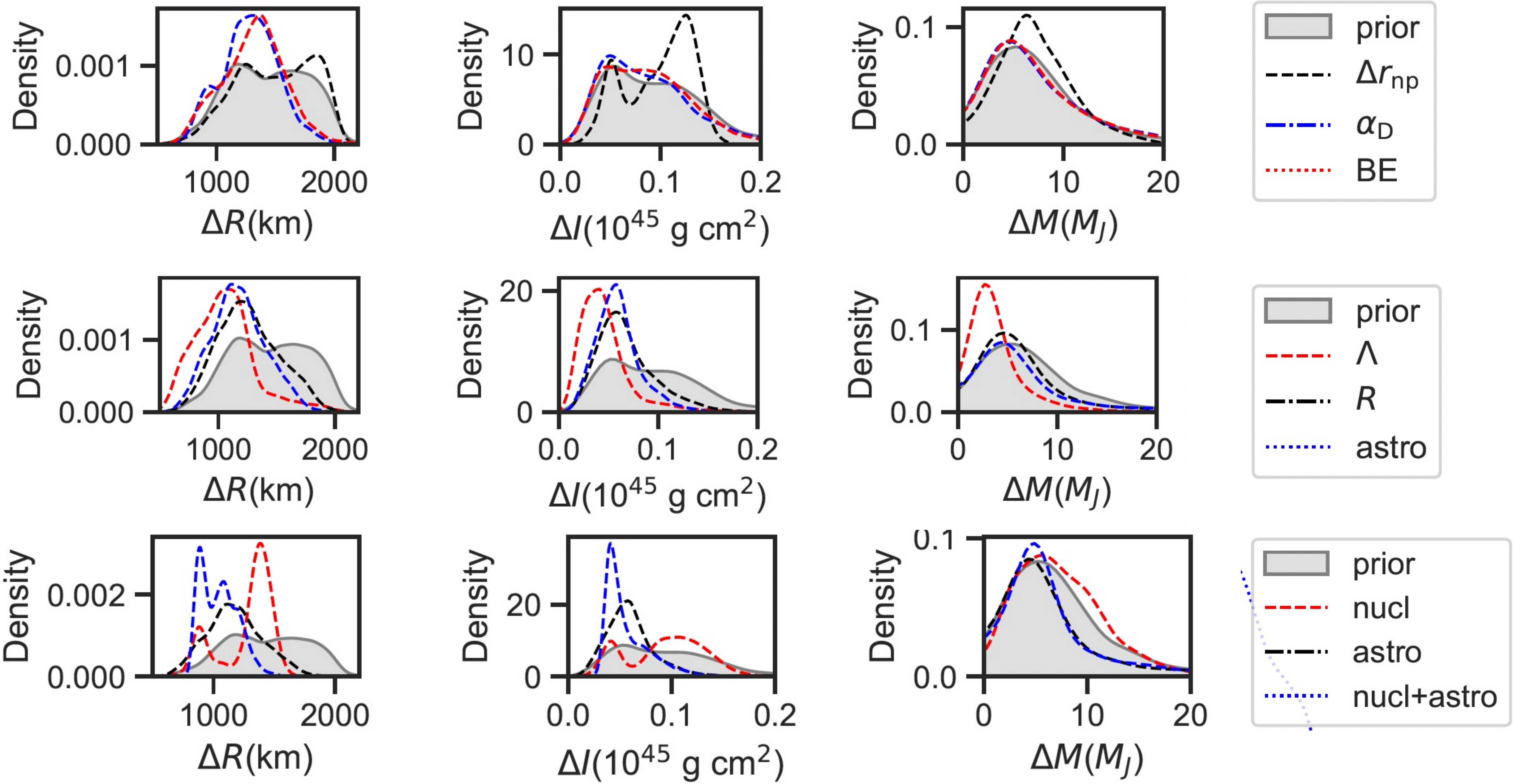


Unitary Gas Constraint equivalent-ish to "The crust exists!"

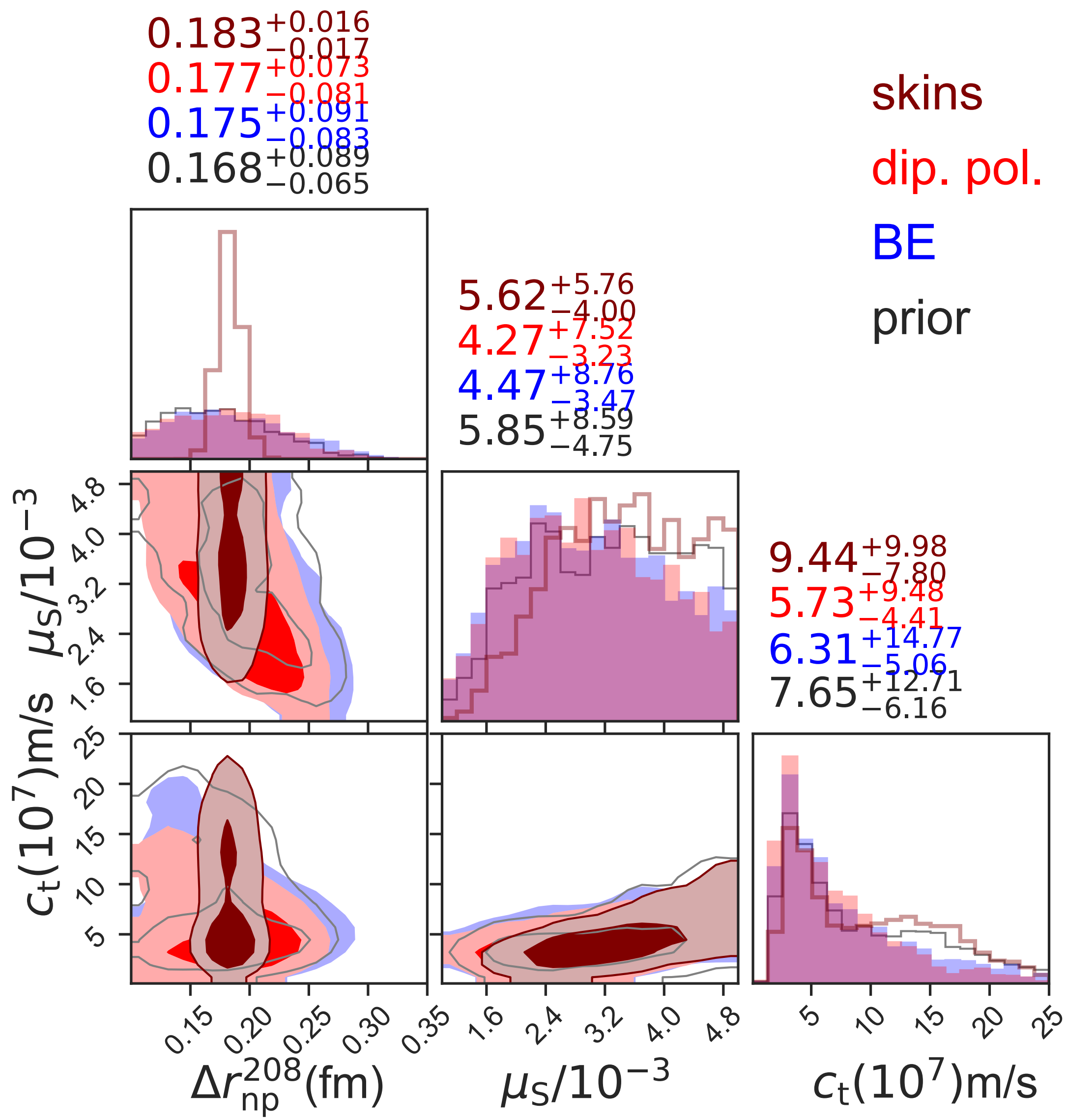


*The energy and pressure of pure neutron matter is higher at all densities than those of a unitary gas (Tews+ arxiv:1611.07133)

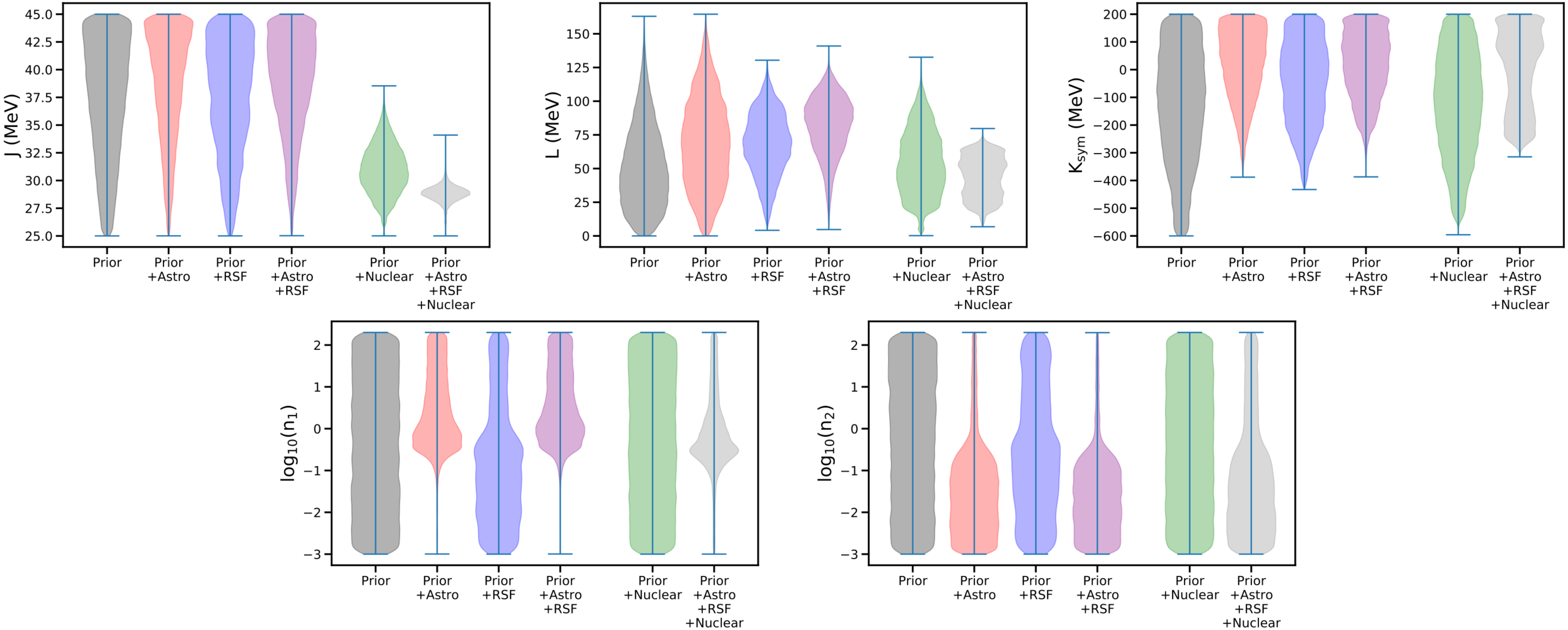
Posteriors on neutron star crust



Shear modulus and speed at base of crust



Inference using a synthetic detection of an RSF at a frequency of 250 Hz, comparison with Nicer-Ligo and nuclear binding energy data



- *J* not constrained by astro
- *L* constrained by nuclear, RSF
- K_{sym} constrained by RSF/NL
- Polytrope parameters constrained by NL

Elastic properties of pasta

Point ions

$$\mu = \frac{0.1194}{r_c^3}\frac{(Ze)^2}{r_c}$$

Ogata, Ichimura Phys. Rev. A, 42, 4867

$$\mu = \frac{0.1106}{r_c^3}\frac{(Ze)^2}{r_c}$$

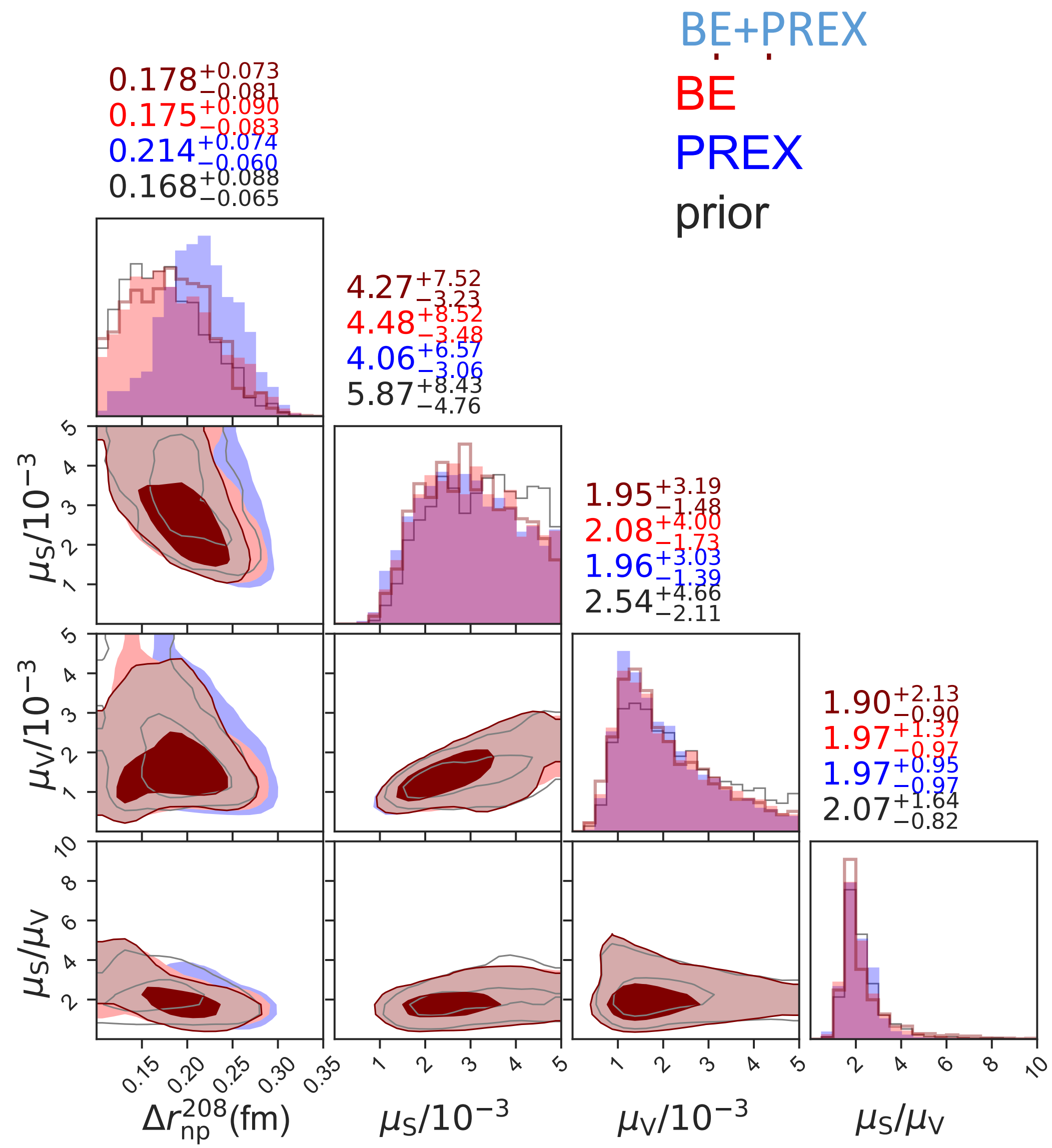
Horowitz, Hughto arxiv:0812.2650

Nuclear surface tension

Uniformly oriented lasagna

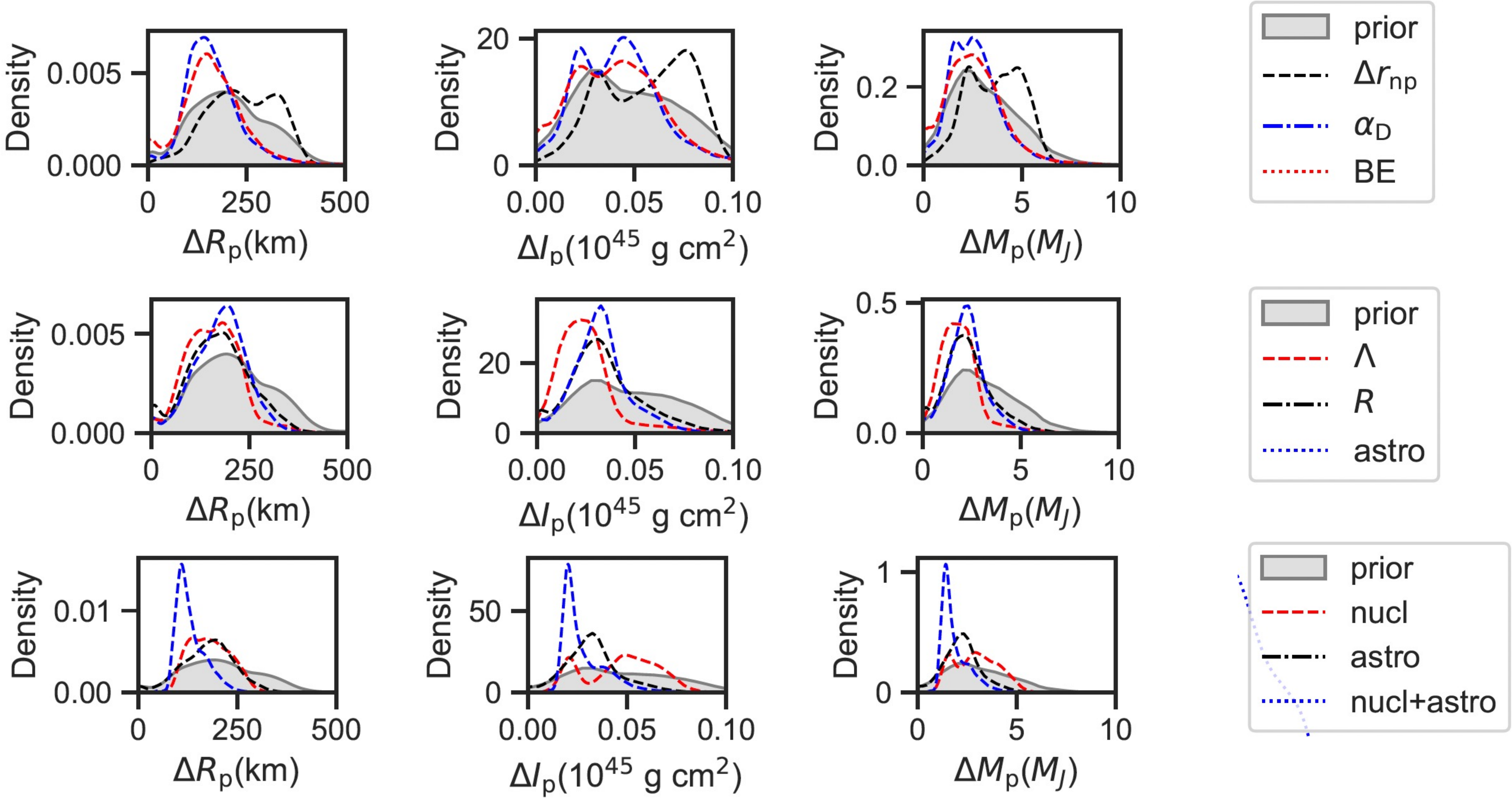
$$\mu_{\rm Las} \leq 0.32\left[\frac{1}{r_c^3}\,\frac{(Ze)^2}{r_c}\,\left(\frac{\sigma}{r_c}\right)^2\,(1-u)^2\right]^{1/3}$$

Pethick, Zheng, Kobyakov arxiv:2003.13430

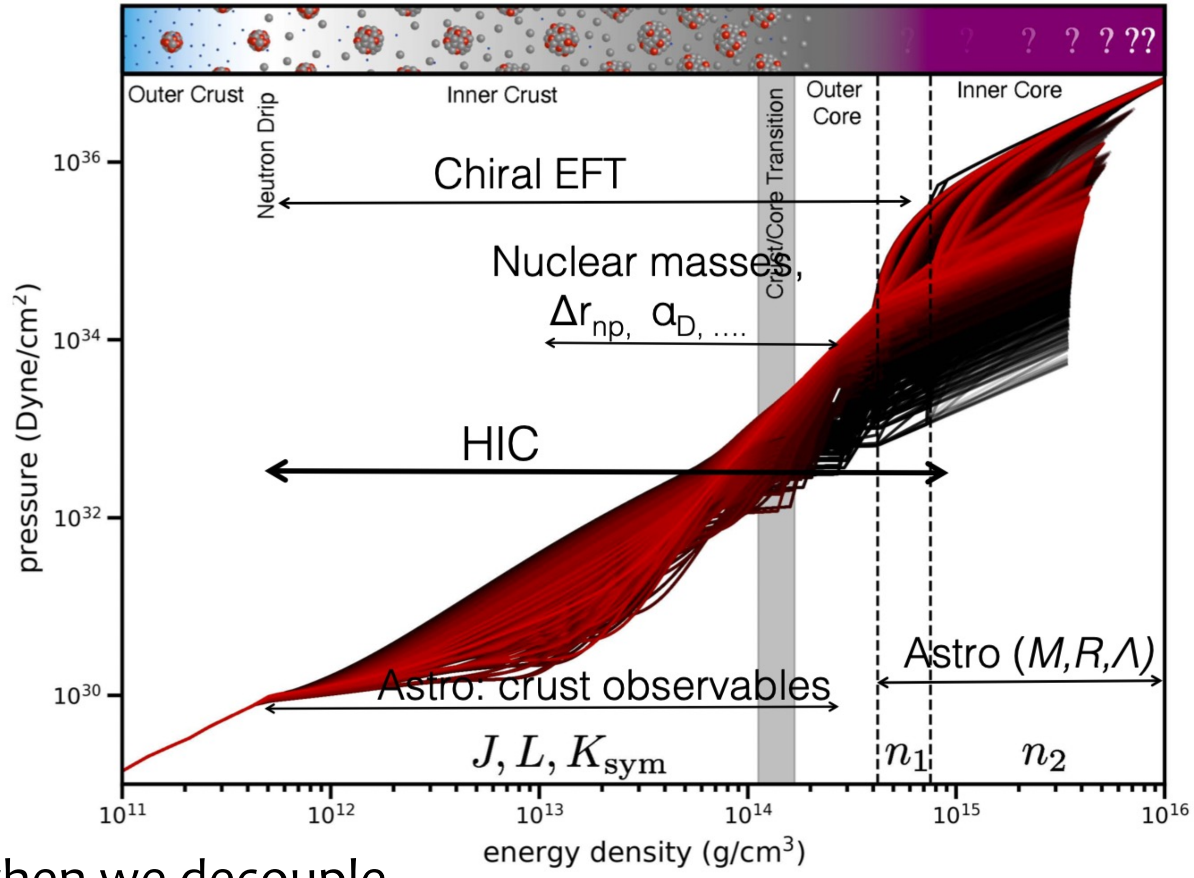


Danger: overestimate by factor of 2

Posteriors on nuclear pasta



Take-aways



Communication!

- Make explicit
  - model dependencies and assumptions
  - Nuclear matter extrapolations to different densities
- In MMNA analysis, work towards getting at the things we measure consistently
- Don't forget the crust! Many observables are sensitive to the layers of the neutron star around the crust-core transition, *exactly where much of our experimental measurements probe*
- L, K_{sym} are sensitive to astrophysical observables, even when we decouple higher densities.

Density functional theory allows access to NS EOS and nuclear observables consistently
Proof of concept with Resonant Shattering Flares: there are astrophysical observables that directly probe the symmetry energy