# FACTORIZATION & RESUMMATION FOR LHC JET PROCESSES

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# ON THE MEANING OF "FACTORIZATION"

Most fundamental: **Separation of energy/distance scales**

- Without this principle, physics would not exist
- Effective Field Theories (EFTs) describe phenomena using only the relevant degrees of freedom, quantum effects from shorter distances are "integrated out" and included in the couplings of the EFT
- EFT for collider physics: Soft-Collinear Effective Theory

[Bauer, Fleming, Pirjol, Stewart (2000)
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Beneke, Chapovsky, Diehl, Feldmann (2002)]

# ON THE MEANING OF "FACTORIZATION"

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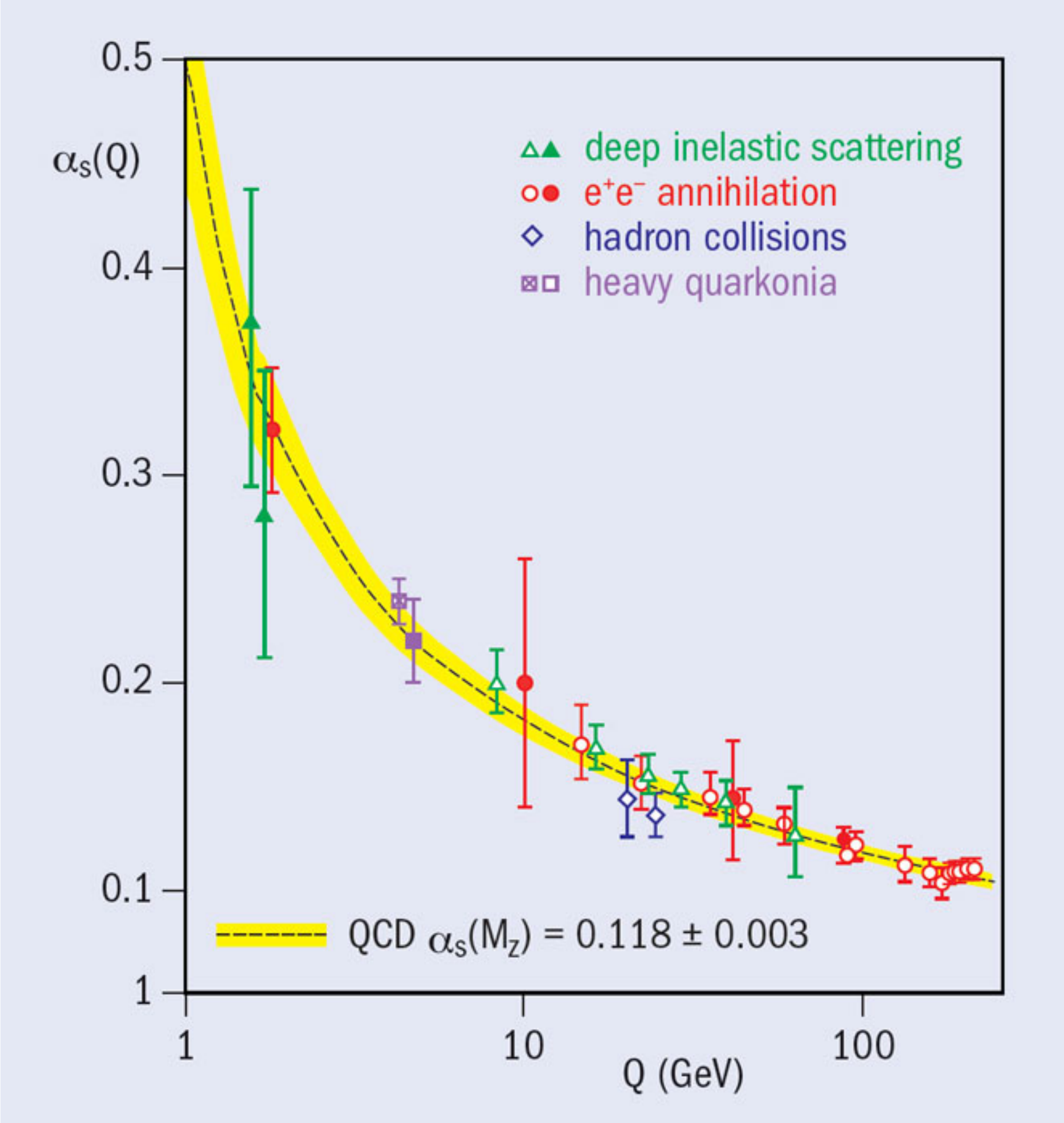
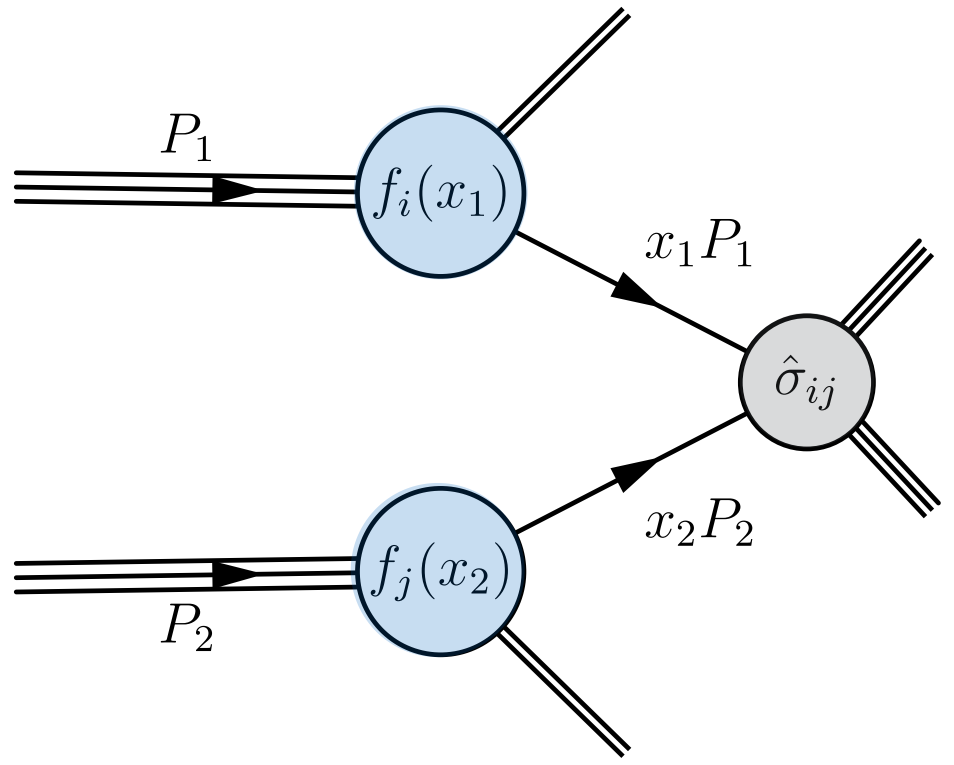
- EFT for collider physics: Soft-Collinear Effective Theory
- Relevant in QCD: separation of perturbative partonic (short-distance) from non-perturbative hadronic (long-distance) effects



# "PDF FACTORIZATION"



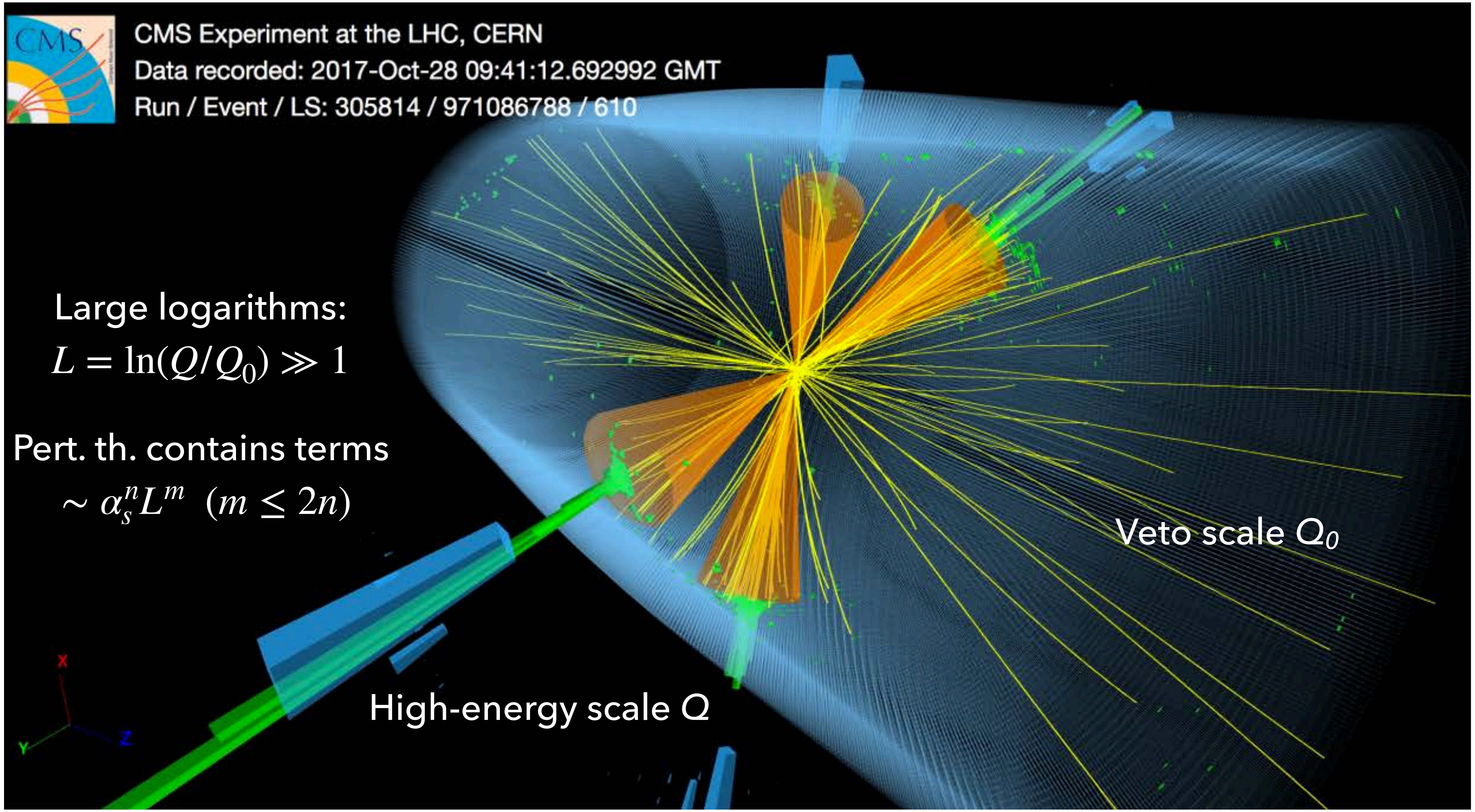

## Stronger assumption:

- Up to power corrections, all long-distance effects in hadron collider scattering are contained in universal parton distribution functions (PDFs) of the nucleon:

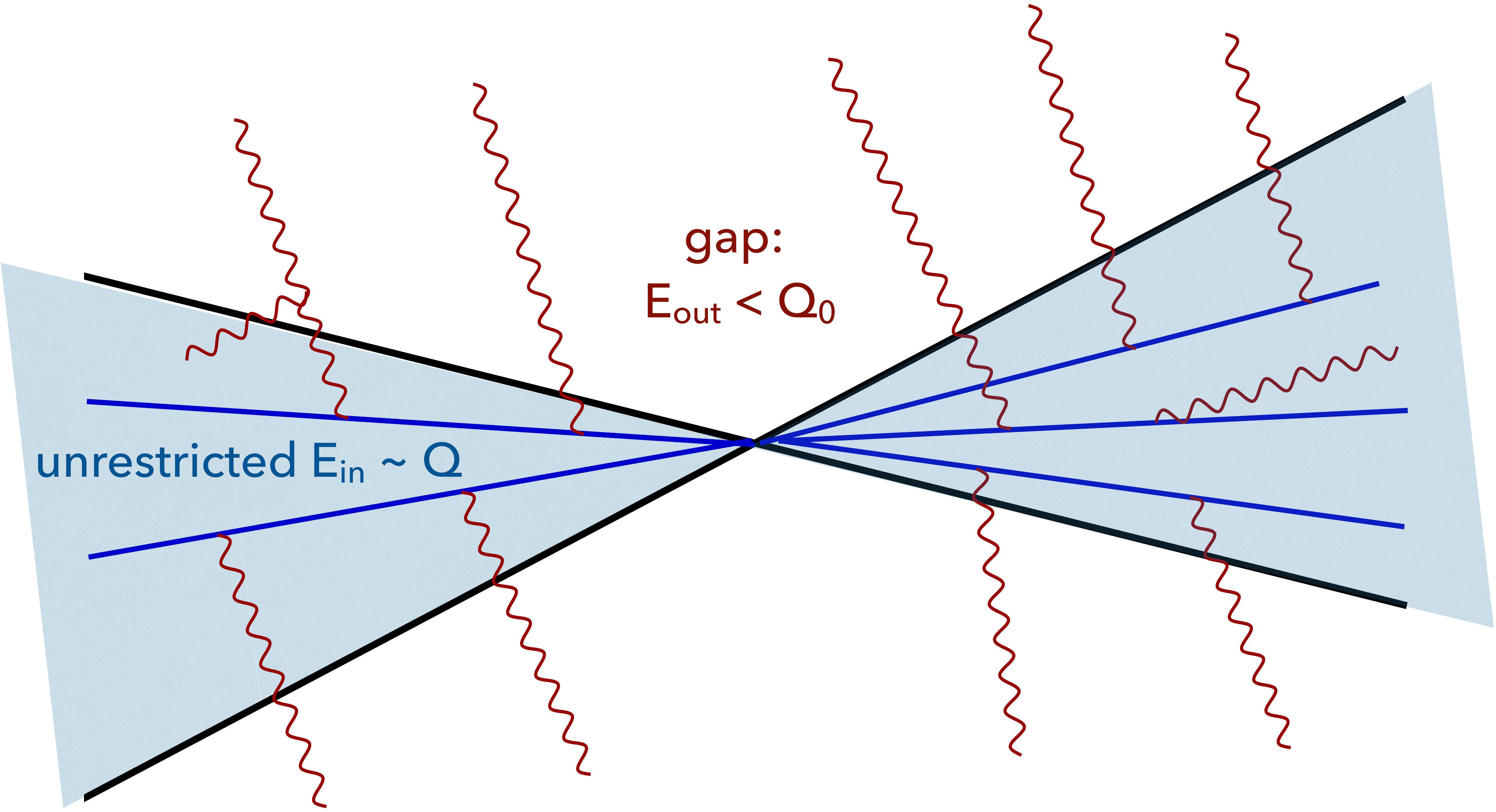
$$d\sigma_{pp\to f}(s) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2\, f_{a/p}(x_1,\mu)\, f_{b/p}(x_2,\mu)\, d\sigma_{ab\to f}(\hat{s}=x_1x_2s,\mu)$$

- Used in all calculations of LHC processes, but proved only for Drell-Yan processes: $pp \to$ color-neutral state $(\gamma^*, W, Z, H)$ [Collins, Soper, Sterman (1985)]
- Entails the absence of low-energy interactions between the colliding hadrons
- What about processes with colored particles (jets) in the final state?

# JET PROCESSES AT HADRON COLLIDERS



# LARGE LOGARITHMS IN JET PROCESSES

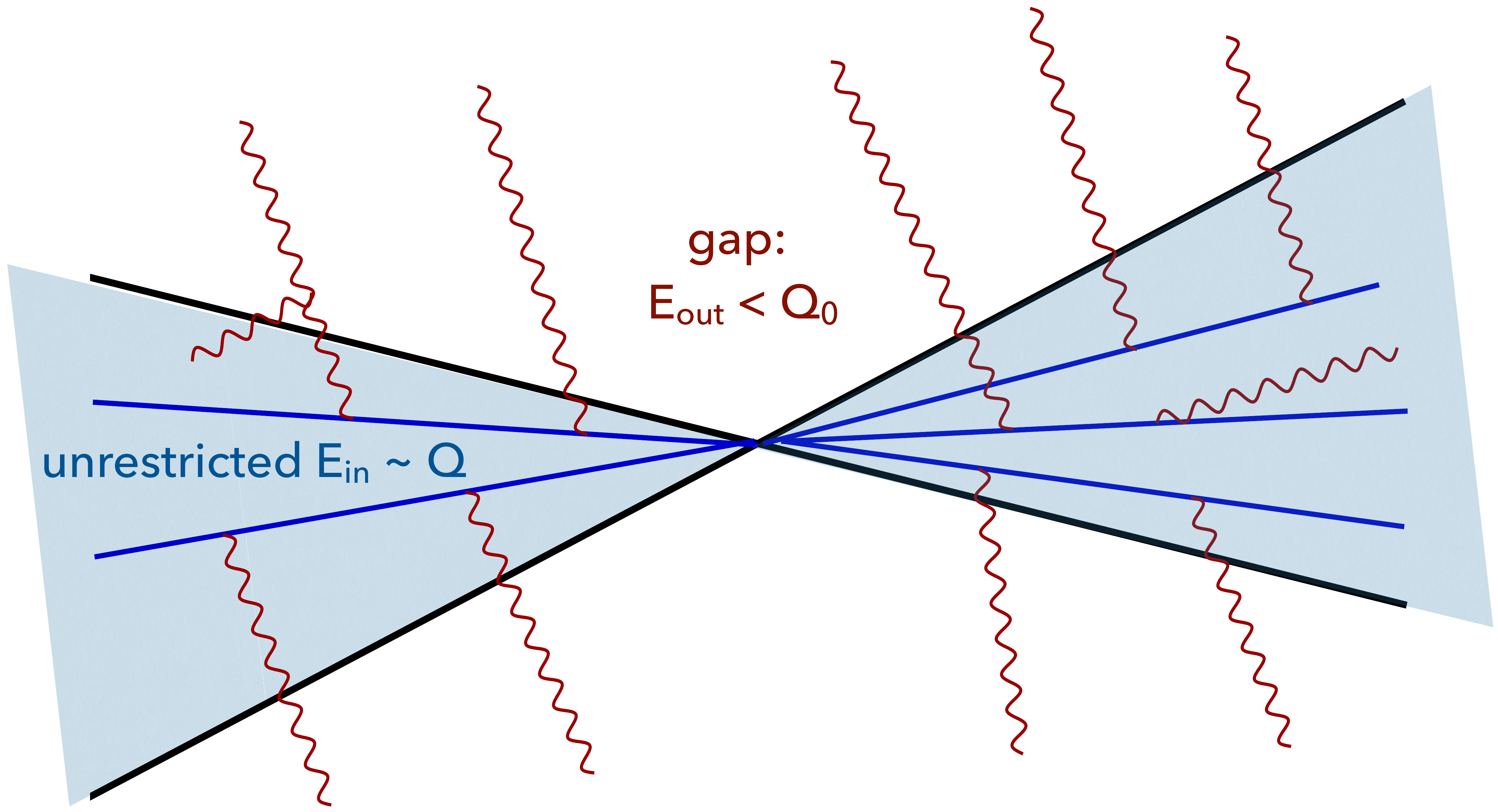



Perturbative expansion includes "super-leading" logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \dots\}$$

state-of-the-art

# LARGE LOGARITHMS IN JET PROCESSES

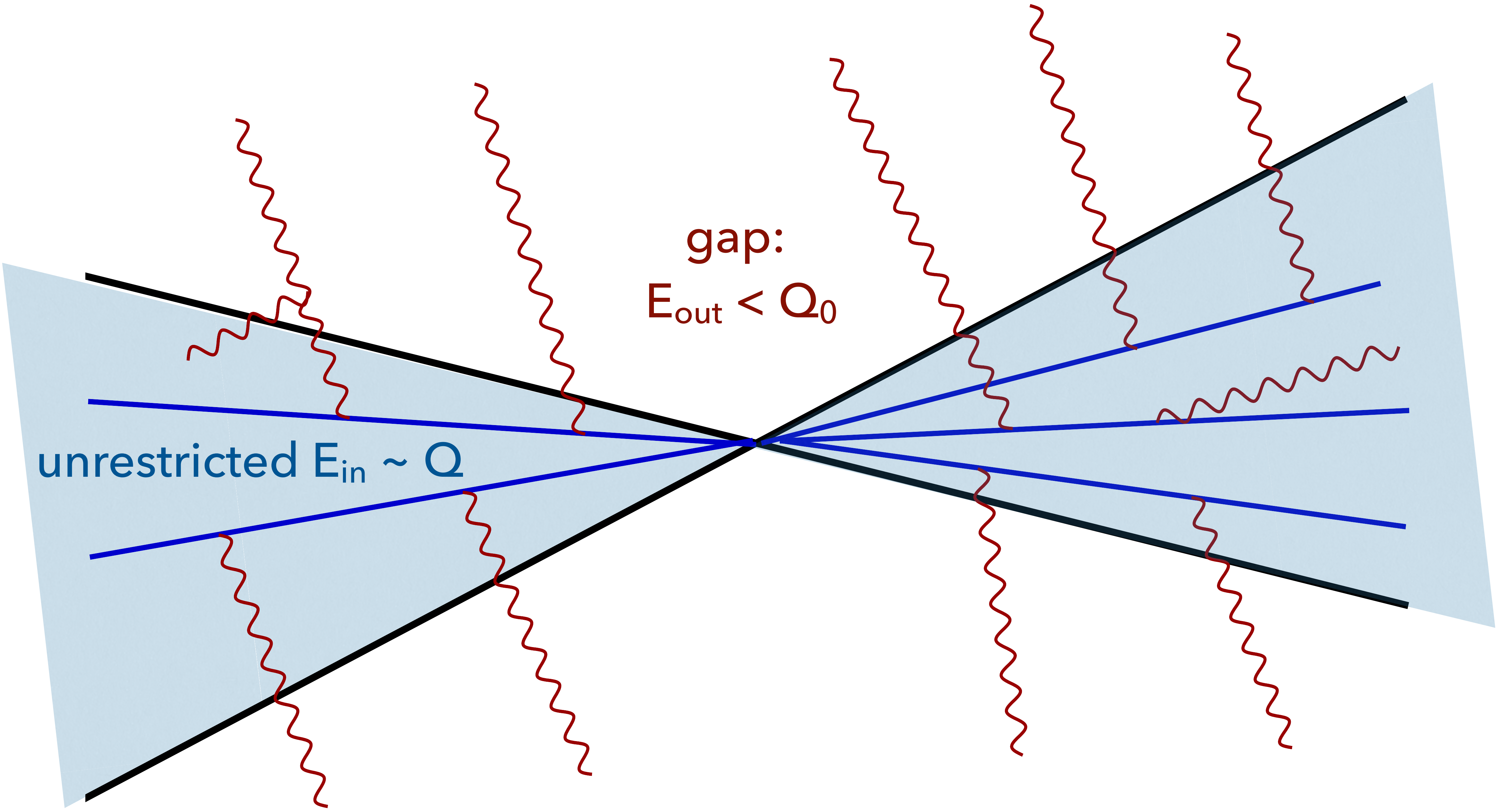



Perturbative expansion includes “super-leading” logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \underset{\uparrow\ \text{state-of-the-art}}{\alpha_s^2 L^2} + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 L^5 + \alpha_s^5 L^7}_{\text{formally larger than } O(1)} + \dots\}$$

[Forshaw, Kyrieleis, Seymour (2006)]

# LARGE LOGARITHMS IN JET PROCESSES




Really, a double-logarithmic series starting at 3-loop order:

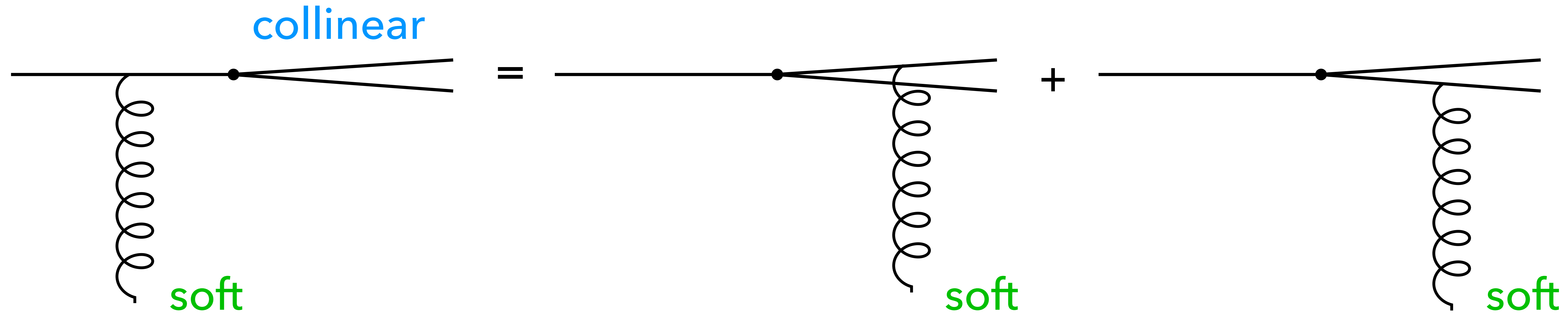
$$\sigma \sim \sigma_{\text{Born}} \times \Big\{ 1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \pi^2) \underbrace{\left[ \alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots \right]}_{\text{formally larger than } O(1)} \Big\}$$

$$\pi^2 \leftarrow (\Im\mathrm{m}\, L)^2$$

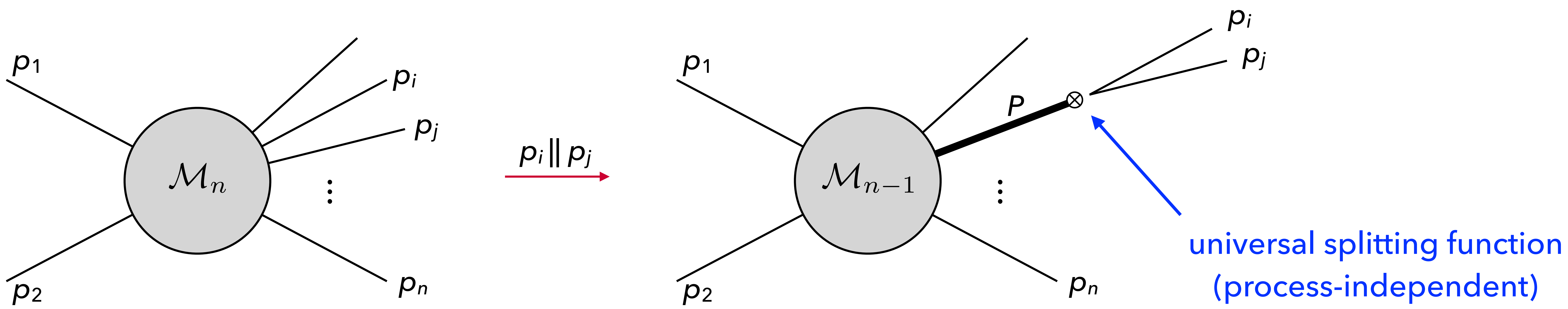
[Forshaw, Kyrieleis, Seymour (2006)]

# IMPORTANCE OF COLOR COHERENCE

- Color coherence (familiar from Low's theorem) holds if all three particles are in the final state of a scattering process (time-like splitting):

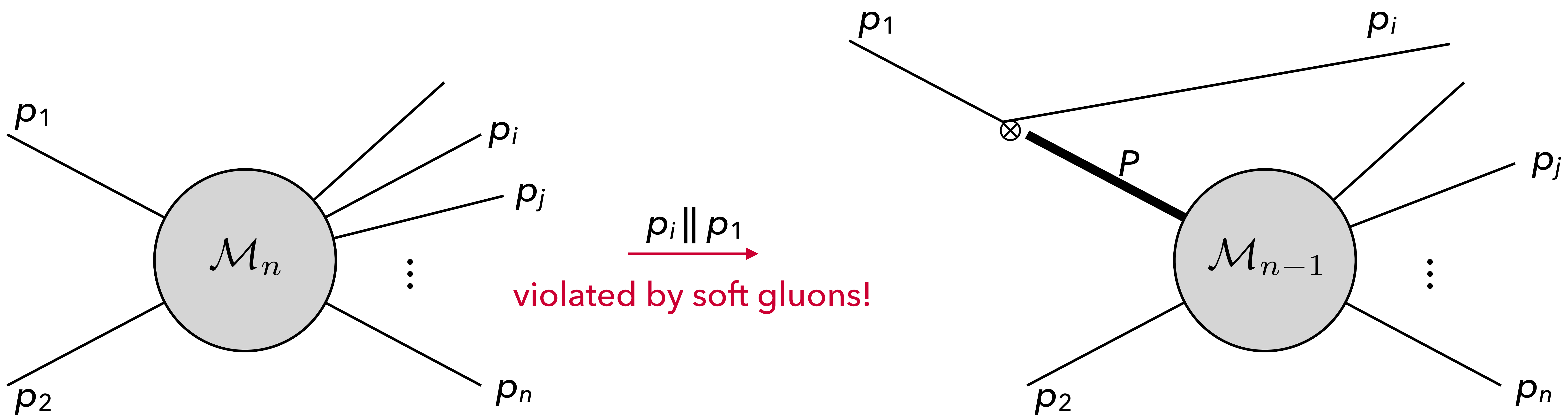



- Then also collinear factorization holds:

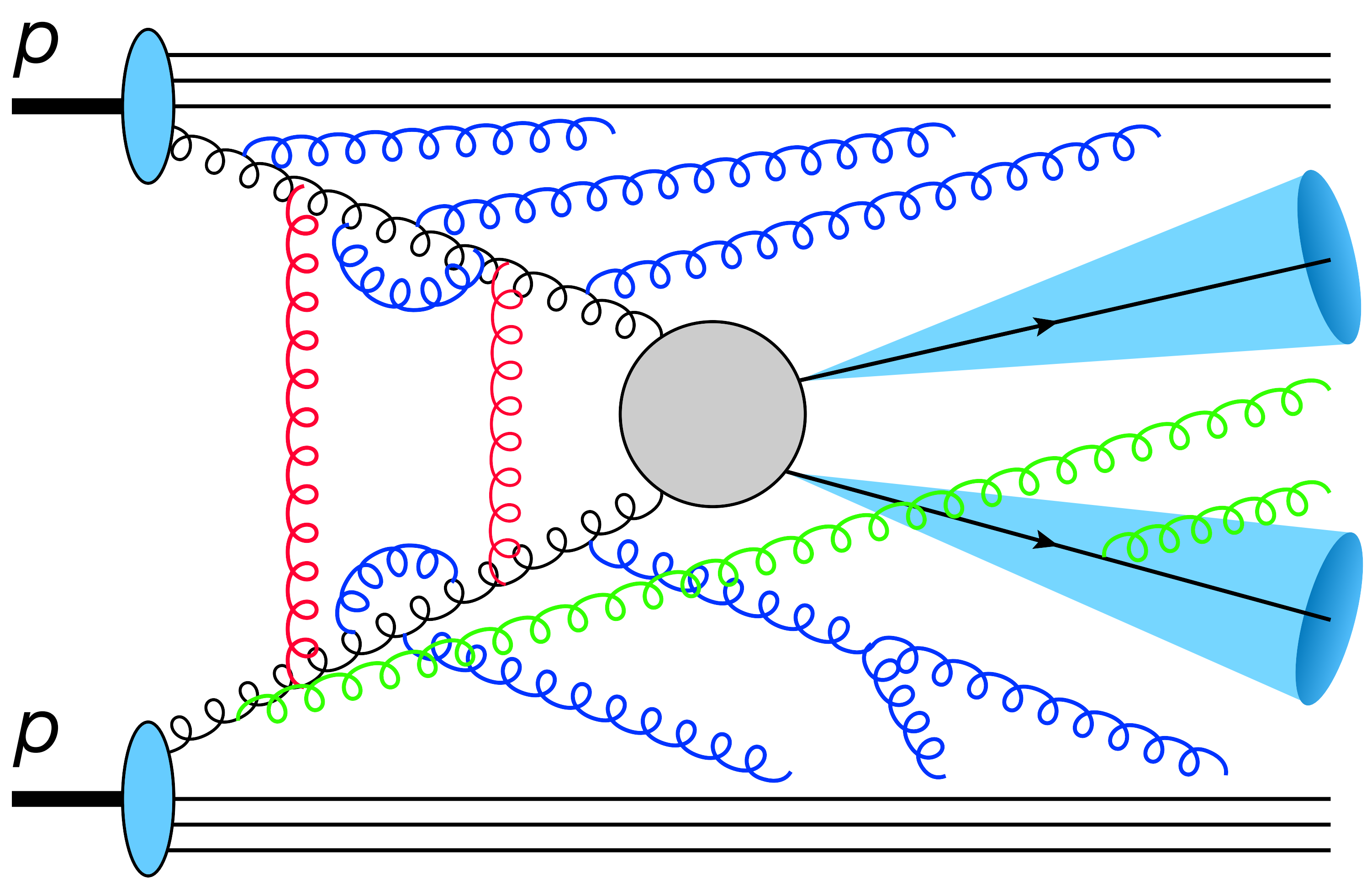


# BREAKING OF COLOR COHERENCE

- Color coherence is broken if not all particles are outgoing (space-like splitting), since then both sides receive **different phase factors** at higher orders:
- Collinear factorization is violated:




[Catani, de Florian, Rodrigo (2011); Forshaw, Seymour, Siodmok (2012)
see also: Henn, Ma, Xu, Yan, Zhang, Zhu (2024)]




# BREAKING OF COLOR COHERENCE

- Origin lies in **Glauber phases** from initial-state soft gluon exchange
- **Soft anomalous dimension:**

$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(ij)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\,\gamma_{\text{cusp}}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

[Bern, Carrasco, Dixon, Johansson, Roiban (2008) Becher, MN (2009); Gardi, Magnea (2009)]

where $s_{ij} > 0$ if particles $i$ and $j$ are both in the initial or final state

- Imaginary part (only at hadron colliders):

$$\boxed{\mathrm{Im}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = -2\pi\,\gamma_{\text{cusp}}(\alpha_s)\;\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + (\dots)\,\mathbf{1}}$$

irrelevant

# GAP-BETWEEN-JETS OBSERVABLES

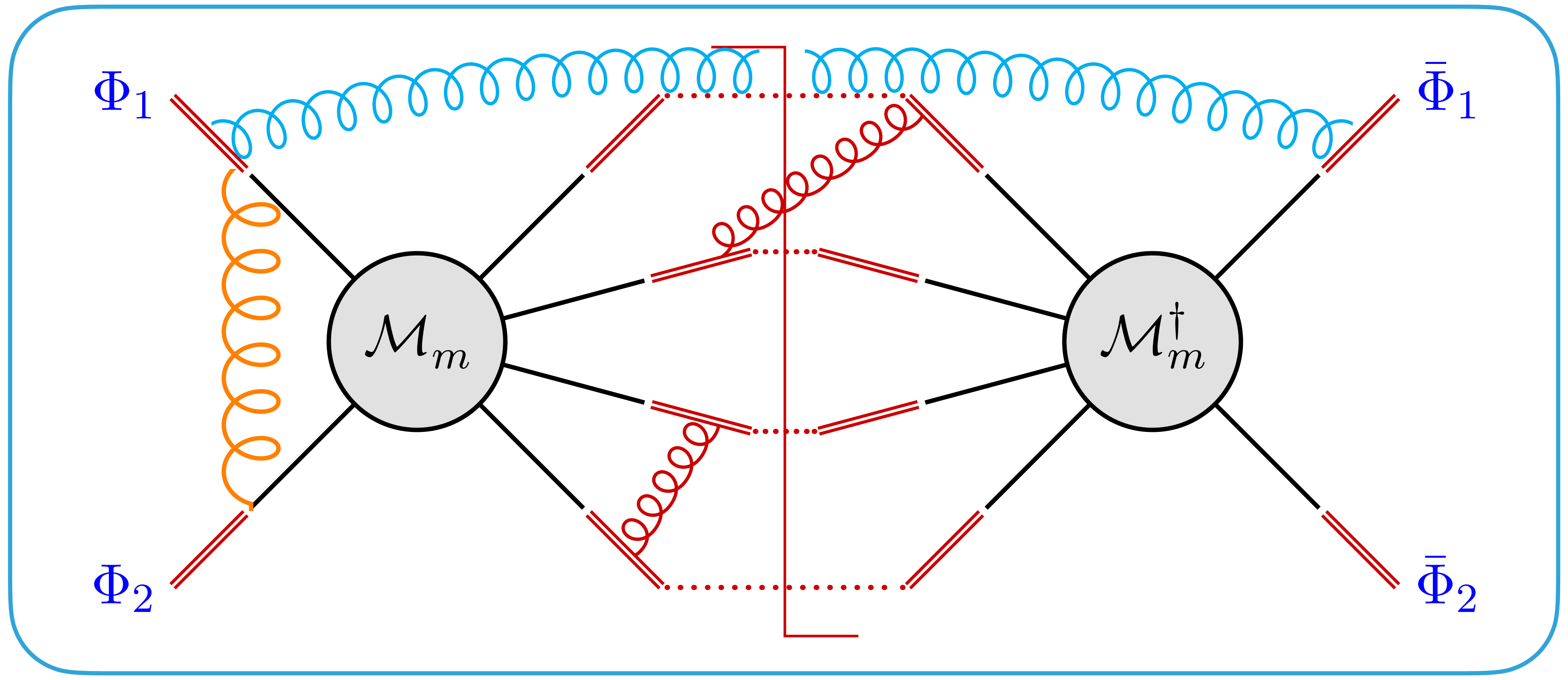
## SCET factorization theorem for *M*-jet production at the LHC

$$\sigma(Q_0) = \sum_{m=m_0}^{\infty} \int d\xi_1 d\xi_2 \, \langle \mathcal{H}_m(\{\underline{n}\}, Q, \xi_1, \xi_2, \mu) \otimes \mathcal{W}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu) \rangle$$

[Becher, MN, Shao (2021)
Becher, MN, Shao, Stillger (2023)]

high scale

low scales $Q_0$ and $\Lambda_{QCD}$




- **hard functions $\mathcal{H}_m$** consisting of squared hard-scattering amplitudes at fixed parton directions $\{n_i\}$, integrated over energy
- **low-energy functions** containing soft Wilson lines for all particles and collinear fields for initial-state parsons

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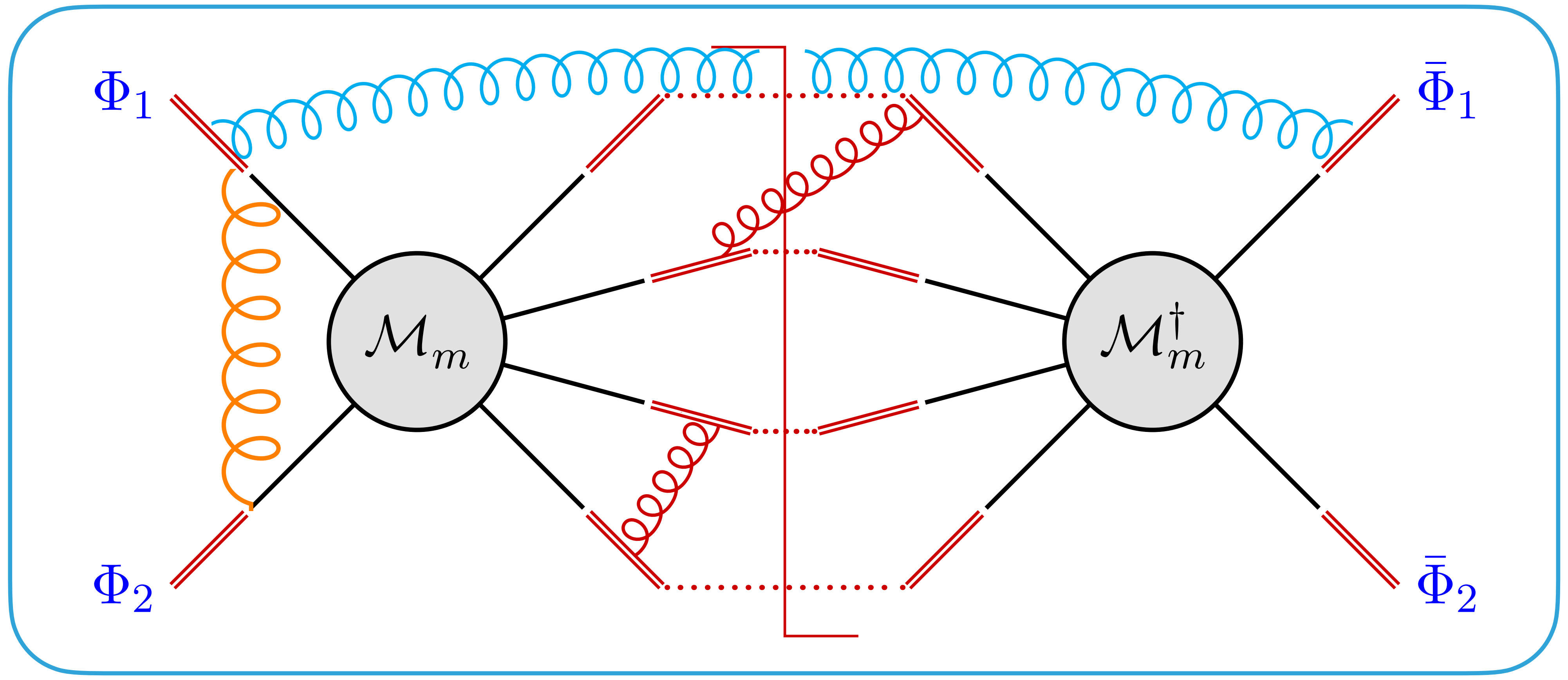


- new perspective to think about non-global observables
- large logs can be resummed using RGEs
- all-order understanding of super-leading logarithms for arbitrary processes

# GAP-BETWEEN-JETS OBSERVABLES

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[Becher, MN, Shao (2021)
Becher, MN, Shao, Stillger (2023)]

high scale

low scales $Q_0$ and $\Lambda_{QCD}$

- Renormalization-group evolution equation:

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_l \, (\{\underline{n}\}, Q, \mu) = -\sum_{m \le l} \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \, \boldsymbol{\Gamma}^H_{ml}(\{\underline{n}\}, Q, \mu)$$

operator in color space and in the infinite space of parton multiplicities

- All-order summation of large logarithmic corrections!

# RESUMMATION OF SUPER-LEADING LOGARITHMS

**Evaluate factorization theorem at a low scale $\mu_s \sim Q_0$**

- Low-energy functions:

$$\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1)\, f_{b/p}(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_s)$$

- Hard functions:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\}, Q, \mu_s) = \sum_{l \le m} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\}, Q, Q)\, \mathbf{P} \exp\left[\int_{\mu_s}^{Q} \frac{d\mu}{\mu}\, \boldsymbol{\Gamma}^H(\{\underline{n}\}, Q, \mu)\right]_{lm}$$

- Super-leading logs correspond to the leading logarithmic approximation!

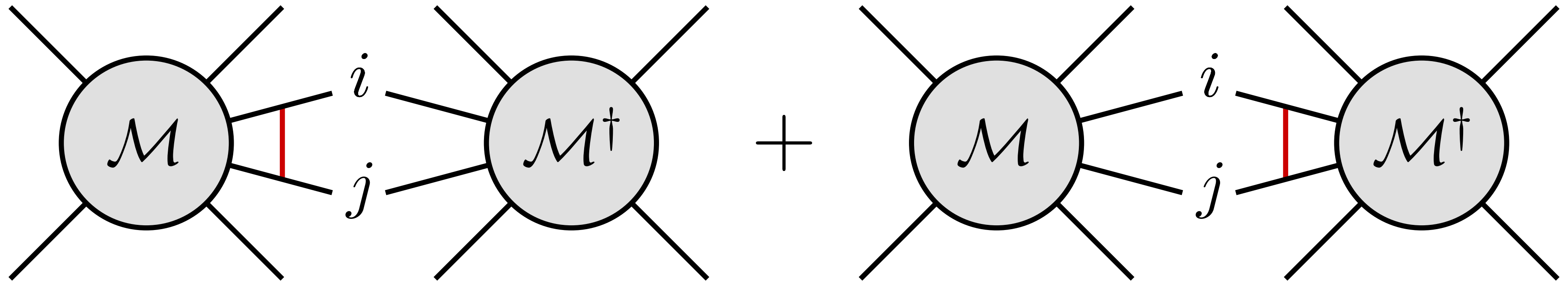
# RESUMMATION OF SUPER-LEADING LOGARITHMS

## Anomalous dimension matrix:

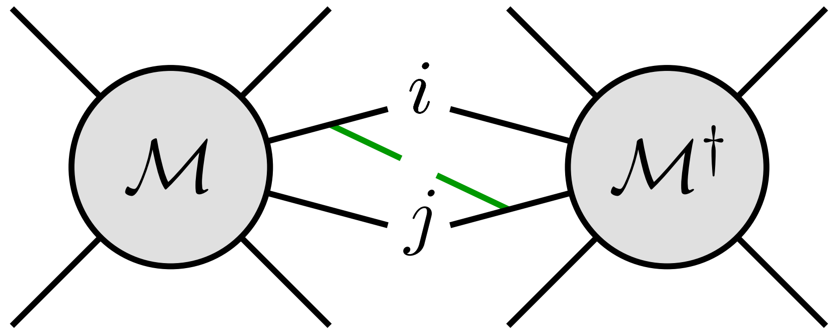
$$\mathbf{\Gamma}^H = \frac{\alpha_s}{4\pi}\begin{pmatrix} \boldsymbol{V}_{2+M} & \boldsymbol{R}_{2+M} & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{2+M+1} & \boldsymbol{R}_{2+M+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{2+M+2} & \boldsymbol{R}_{2+M+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{2+M+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- Action on hard functions:

$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}_m = \sum_{(ij)}$




$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{R}_m = \sum_{(ij)}$




exponentiation generates arbitrarily high parton multiplicities starting from the $2 \to M$ Born process

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- Virtual and real contributions contain collinear singularities, which must be regularized and subtracted:

$$\boldsymbol{\Gamma}^H(\xi_1,\xi_2) = \delta(1-\xi_1)\,\delta(1-\xi_2)\,\boldsymbol{\Gamma}^S + \boldsymbol{\Gamma}_1^C(\xi_1)\,\delta(1-\xi_2) + \delta(1-\xi_1)\,\boldsymbol{\Gamma}_2^C(\xi_2)$$

soft/soft-collinear part

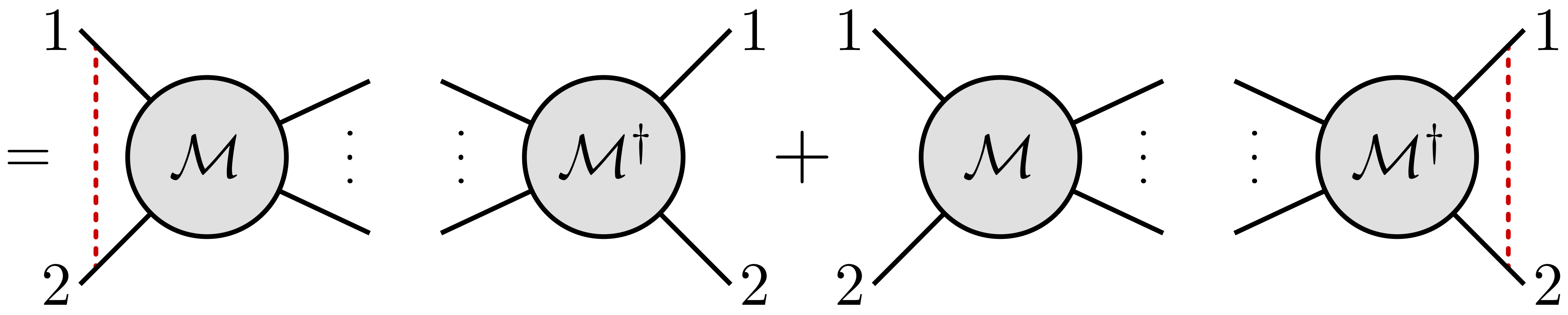
collinear parts

# SOFT ANOMALOUS DIMENSION

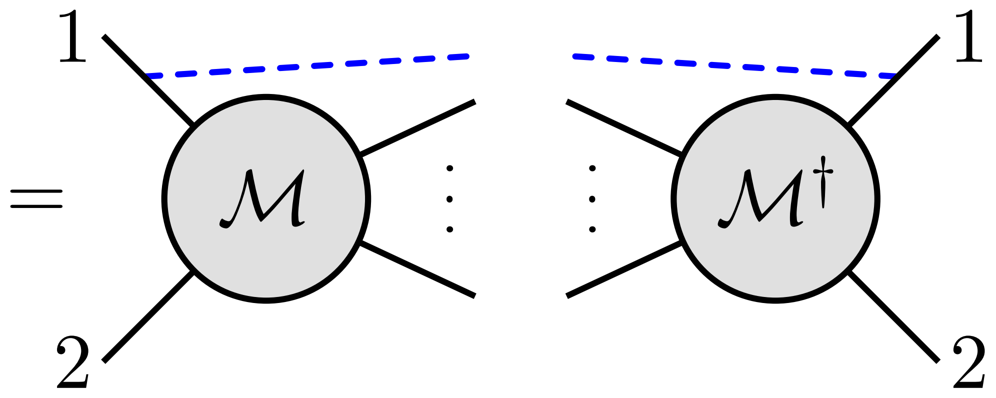
Glauber phase

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma}^S = \overline{\boldsymbol{\Gamma}} + \boxed{\boldsymbol{V}^G + \boldsymbol{\Gamma}^c} \ln\frac{\mu^2}{\hat{s}}$$

soft emission (collinear div. subtracted) collinear emission

$\mathcal{H}_m\, \boldsymbol{V}^G =$ 

$$\boldsymbol{V}^G = -2i\pi \left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

$\mathcal{H}_m\, \boldsymbol{R}_1^c =$ 

new color space of emitted gluon

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2} \left[C_i\,\boldsymbol{1} - \boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{i,R}\,\delta(n_k - n_i)\right]$$

# SOFT ANOMALOUS DIMENSION

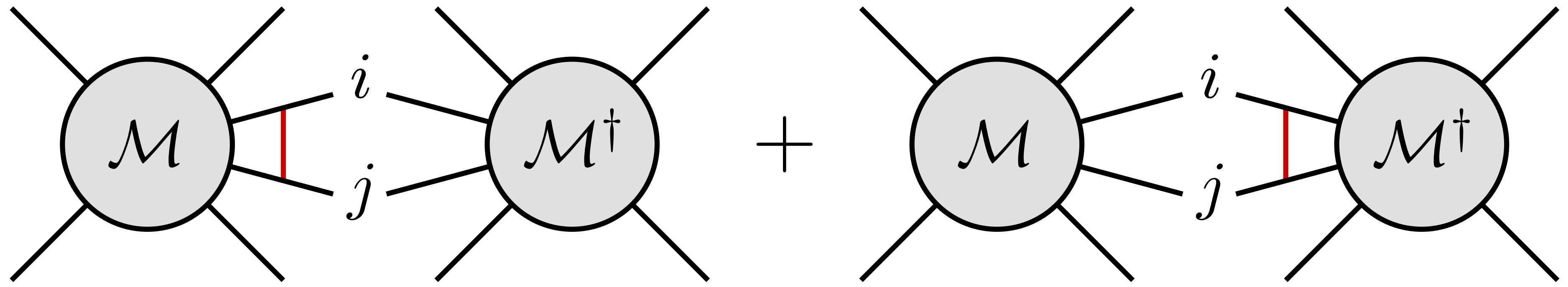
$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \boxed{\overline{\boldsymbol{\Gamma}}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

Glauber phase

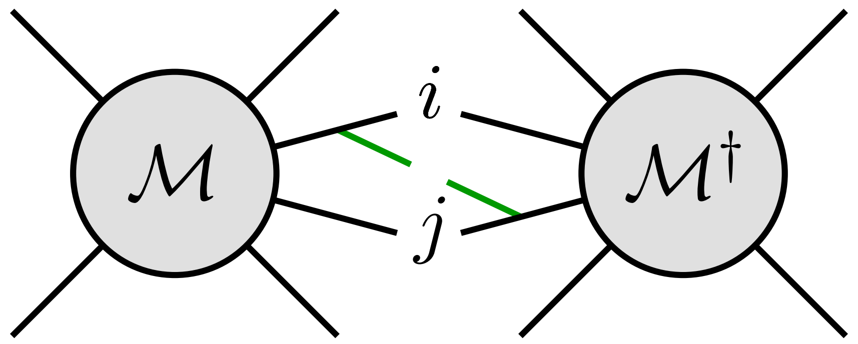
soft emission
(collinear div. subtracted)

collinear emission

$$\mathcal{H}_m \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$




$$\mathcal{H}_m \overline{\boldsymbol{R}}_m = \sum_{(ij)}$$




$$\overline{\boldsymbol{\Gamma}} = 2\sum_{(ij)} \left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right) \int \frac{d\Omega(n_k)}{4\pi}\, \overline{W}_{ij}^k$$

$$- 4\sum_{(ij)} \boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{j,R}\, \overline{W}_{ij}^k\, \Theta_{\text{hard}}(n_k)$$

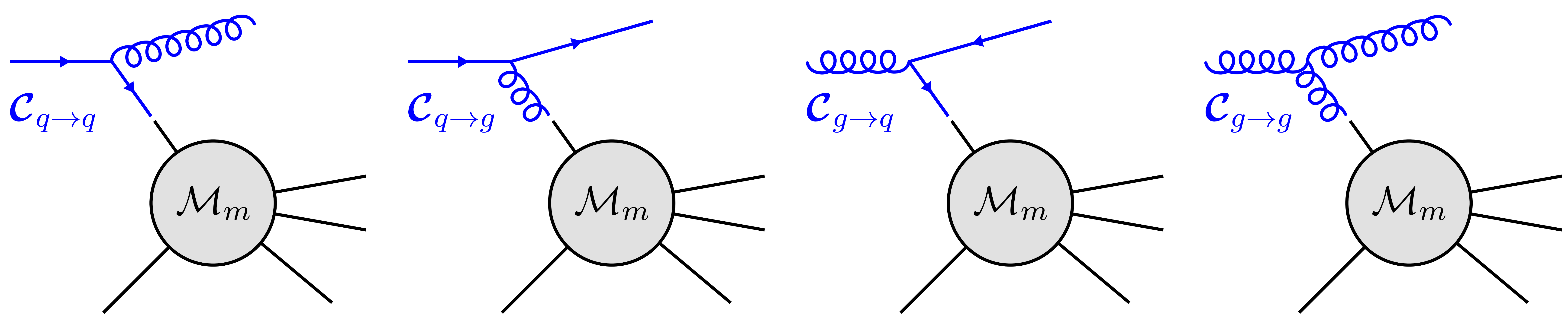
subtracted dipole emitter

# COLLINEAR ANOMALOUS DIMENSION

non-trivial color structures

splitting functions

$$\Gamma_i^C(\xi_i) = \frac{\alpha_s}{4\pi}\left[2\left(2\overline{\mathcal{P}}_{i\to P}(\xi_i) - C_i\gamma_0^{\rm cusp}\ln\frac{\mu_h}{2E_i}\delta(1-\xi_i)\,\delta_{iP}\right)\delta(n_k-n_i)\,\boldsymbol{\mathcal{C}}_{i\to P}\,\boldsymbol{\mathcal{C}}_{i\to P}^\dagger \right.$$
$$\left. -2\left(\gamma_0^i - C_i\gamma_0^{\rm cusp}\ln\frac{\mu_h}{2E_i}\right)\delta(1-\xi_i)\,\delta_{iP}\right]$$



# RESUMMATION OF SUPER-LEADING LOGARITHMS

recall: $\boldsymbol{\Gamma}^S = \overline{\boldsymbol{\Gamma}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$

**SLLs arise from the terms in** $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of $\boldsymbol{\Gamma}^c$**

- Four algebraic identities simplify the calculation:
  - color coherence without Glauber phases: $\boldsymbol{\mathcal{H}}_m\,\boldsymbol{\Gamma}^c\,\overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m\,\overline{\boldsymbol{\Gamma}}\,\boldsymbol{\Gamma}^c$, also for $\boldsymbol{\Gamma}_i^C(\xi_i)$
  - collinear safety: $\langle \boldsymbol{\mathcal{H}}_m\,\boldsymbol{\Gamma}^c \otimes \mathbf{1}\rangle = 0$
  - cyclicality of trace: $\langle \boldsymbol{\mathcal{H}}_m\,\boldsymbol{V}^G \otimes \mathbf{1}\rangle = 0$
  - DGLAP evolution without Glauber phases: $\langle \boldsymbol{\mathcal{H}}\,\boldsymbol{\Gamma}_i^C(\xi_i) \otimes \mathbf{1}\rangle = \langle \boldsymbol{\mathcal{H}} \otimes \mathbf{1}\rangle\,\frac{\alpha_s}{\pi}\,\mathcal{P}_{i\to P}(\xi_i)$

DGLAP splitting functions

# RESUMMATION OF SUPER-LEADING LOGARITHMS

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- Relevant color traces at $\mathcal{O}(\alpha_s^{n+3}L^{2n+3})$:

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\,(\mathbf{\Gamma}^c)^r\,\boldsymbol{V}^G\,(\mathbf{\Gamma}^c)^{n-r}\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}\otimes\mathbf{1}\right\rangle$$

- Kinematic information contained in $(M+1)$ angular integrals from $\overline{\Gamma}$:

$$J_j = \int\frac{d\Omega(n_k)}{4\pi}\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\text{veto}}(n_k)\,;\quad \text{with}\quad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\,n_j\cdot n_k}$$

# RESUMMATION OF SUPER-LEADING LOGARITHMS

**SLLs arise from the terms in** $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of** $\mathbf{\Gamma}^{\mathrm{c}}$

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- Series of SLLs, starting at 3-loop order:

$$\sigma_{\mathrm{SLL}} = \sigma_{\mathrm{Born}}\sum_{n=0}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^{n+3} L^{2n+3}\,\frac{(-4)^n\,n!}{(2n+3)!}\sum_{r=0}^{n}\frac{(2r)!}{4^r\,(r!)^2}\,C_{rn}$$

from scale integrals (at fixed coupling)

# RESUMMATION OF SUPER-LEADING LOGARITHMS

**SLLs arise from the terms in** $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of** $\boldsymbol{\Gamma}^c$

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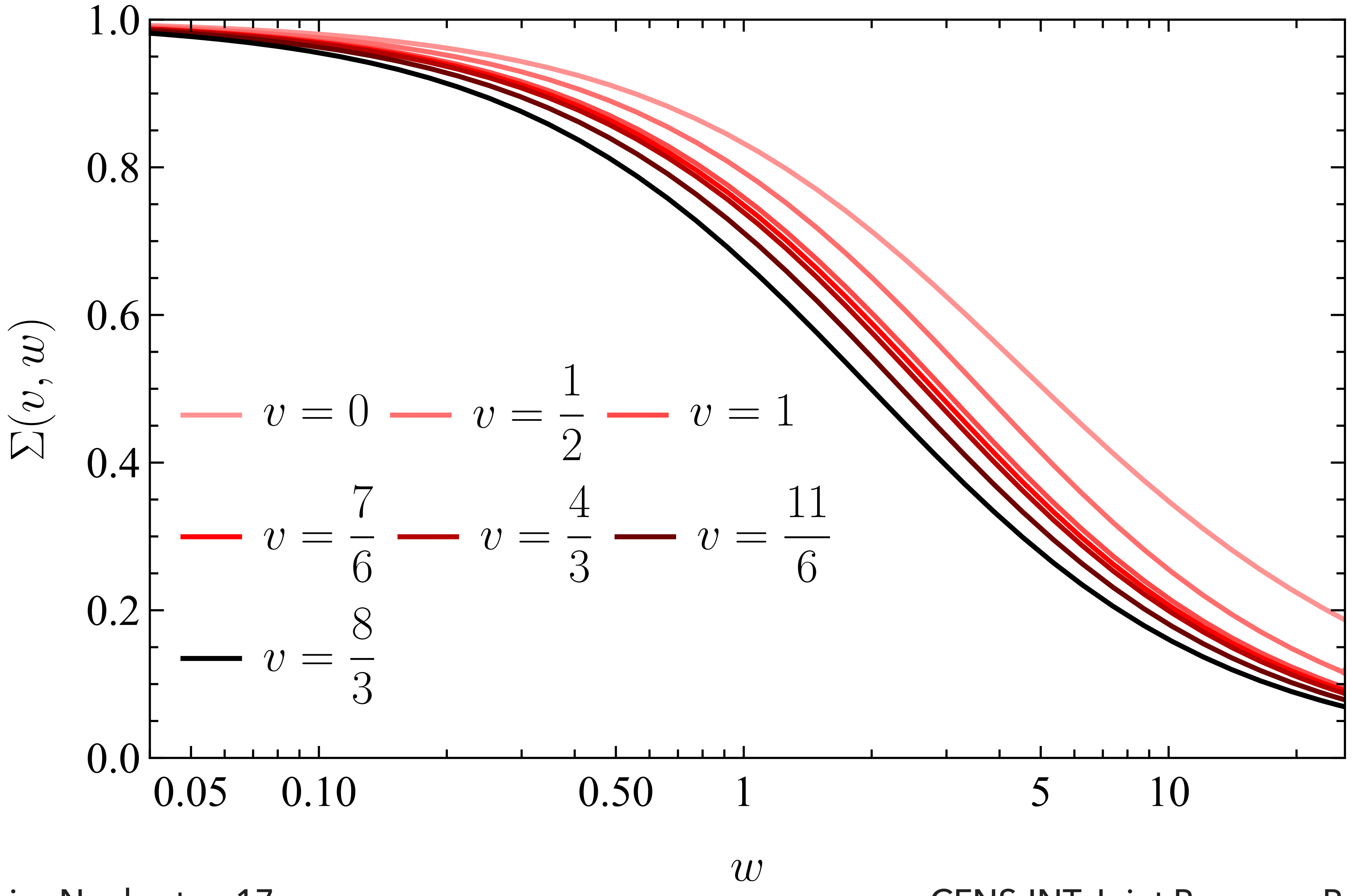
- Series of SLLs, starting at 3-loop order:

$$\sigma_{\text{SLL}} = \sigma_{\text{Born}}\,\frac{\alpha_s L}{\pi N_c}\underbrace{\left(\frac{N_c\,\alpha_s}{\pi}\,\pi^2\right)\sum_{n=0}^{\infty}c_n\left(\frac{N_c\,\alpha_s}{\pi}\,L^2\right)^{n+1}}_{O(1)}$$

# RESUMMATION OF SUPER-LEADING LOGARITHMS

- Infinite series can be expressed in closed form in terms of a prefactor times Kampé de Fériet functions $\Sigma(v_i, w)$, with $w = \dfrac{N_c \alpha_s}{\pi} L^2$ and:

$$v_0 = 0\,, \qquad v_1 = \frac{1}{2}\,, \qquad v_2 = 1\,, \qquad v_{3,4} = \frac{3N_c \pm 2}{2N_c}\,, \qquad v_{5,6} = \frac{2\,(N_c \pm 1)}{N_c}$$




**Asymptotic behavior for $w \gg 1$:**

$$\Sigma_0(w) = \frac{3}{2w}\Big(\ln(4w) + \gamma_E - 2\Big) + \frac{3}{4w^2} + \mathcal{O}(w^{-3})$$

$$\Sigma(v,w) = \frac{3\arctan\left(\sqrt{v-1}\right)}{\sqrt{v-1}\,w} - \frac{3\sqrt{\pi}}{2\sqrt{v}\,w^{3/2}} + \mathcal{O}(w^{-2})$$

$\Rightarrow$ much slower fall-off than Sudakov form factors ~$e^{-cw}$

# RG-IMPROVED RESUMMATION OF SLLS

## Multiple insertions of $\boldsymbol{\Gamma}^c$ exponentiate

- Expand out all terms except the log-enhanced soft-collinear piece:

$$\boldsymbol{U}_{\mathrm{SLL}}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

easy to include running-coupling effects

with the Sudakov operator:

resums all double logs

$$\boldsymbol{U}_c(\mu_i,\mu_j) = \exp\left[\boldsymbol{\Gamma}^c\int_{\mu_j}^{\mu_i}\frac{d\mu}{\mu}\,\gamma_{\mathrm{cusp}}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right]$$

$$\mu_h \simeq Q$$
$$\mu_s \simeq Q_0$$

# RG-IMPROVED RESUMMATION OF SLLS

## Introduce a color basis

- Simplest case of (anti-)quark-initiated scattering processes:

$$\boldsymbol{X}_1 = \sum_{j>2} J_j \, i f^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_4 = \frac{1}{N_c} \, J_{12} \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \,,$$

$$\boldsymbol{X}_2 = \sum_{j>2} J_j \, (\sigma_1 - \sigma_2) \, d^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_5 = J_{12} \, \mathbf{1} \,,$$

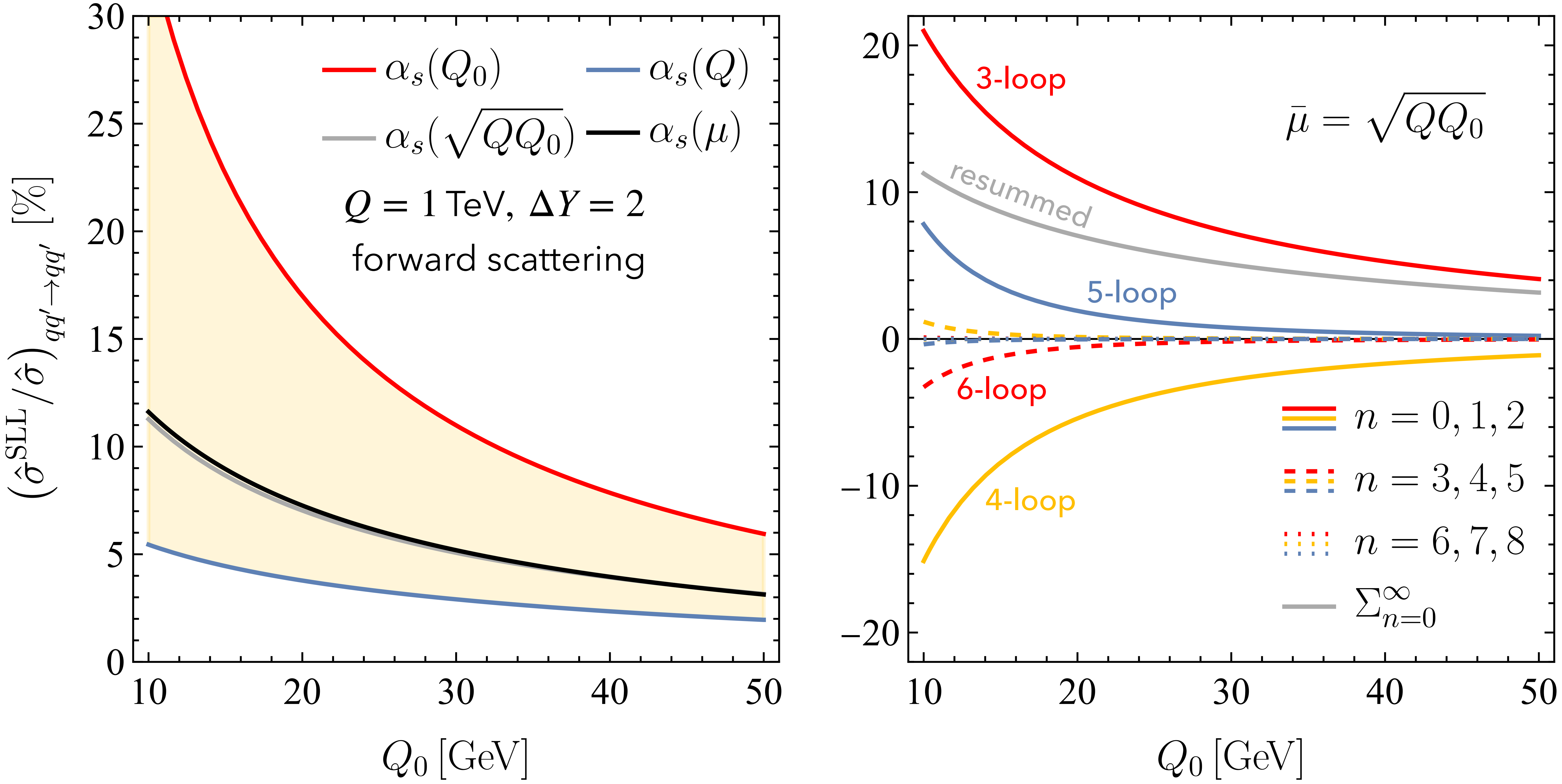
$$\boldsymbol{X}_3 = \frac{1}{N_c} \sum_{j>2} J_j \, (\boldsymbol{T}_1 - \boldsymbol{T}_2) \cdot \boldsymbol{T}_j \,,$$

where $\sigma_i = -1 \, (+1)$ for an initial-state quark (anti-quark)

- In the general case, the **basis contains 11 operators**

# GAP-BETWEEN-JETS OBSERVABLES

Based on this approach, we have performed the **first all-order resummation of super-leading logarithms** for jet processes [Becher, MN, Shao, Stillger (2023)]



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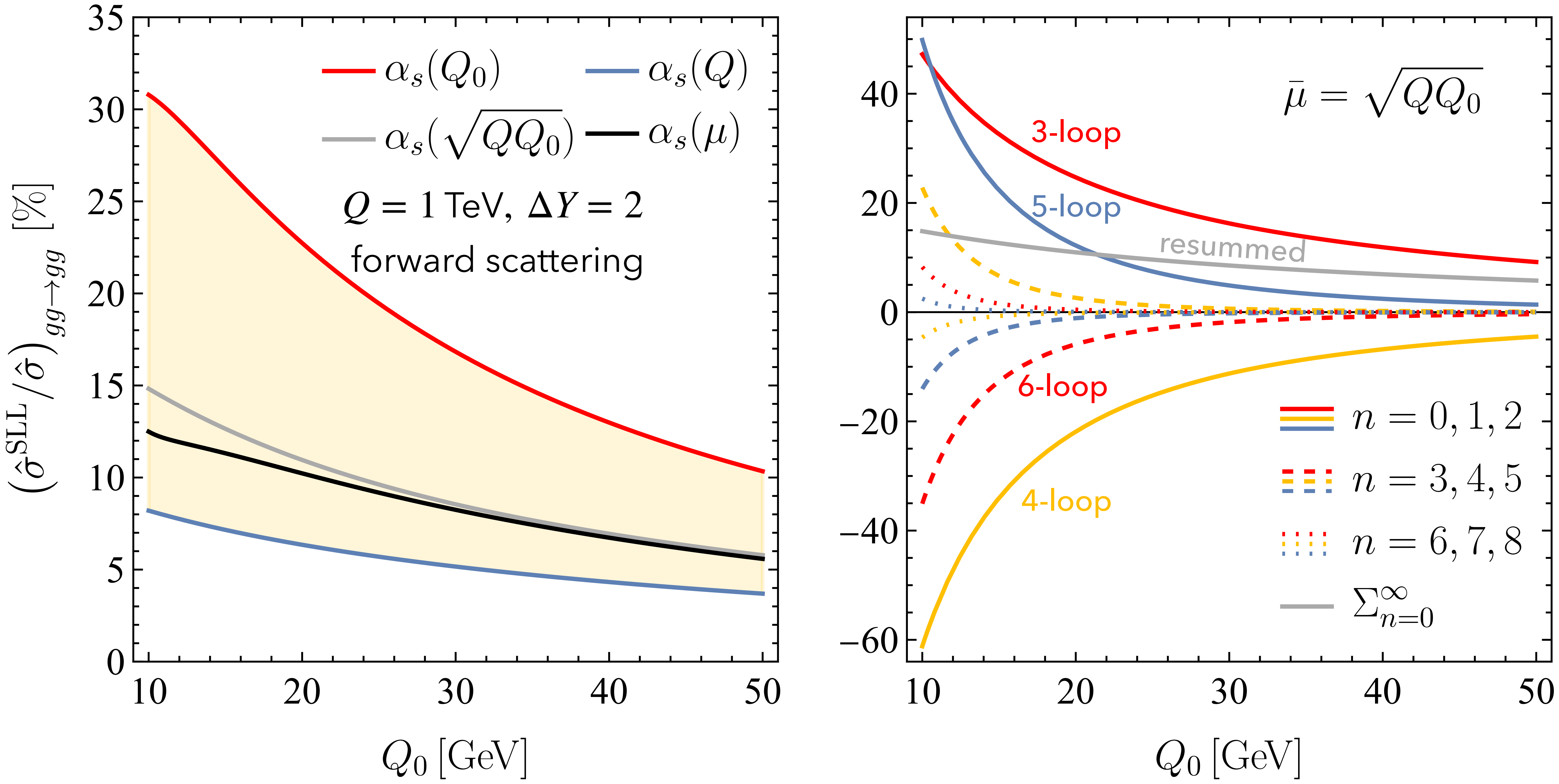

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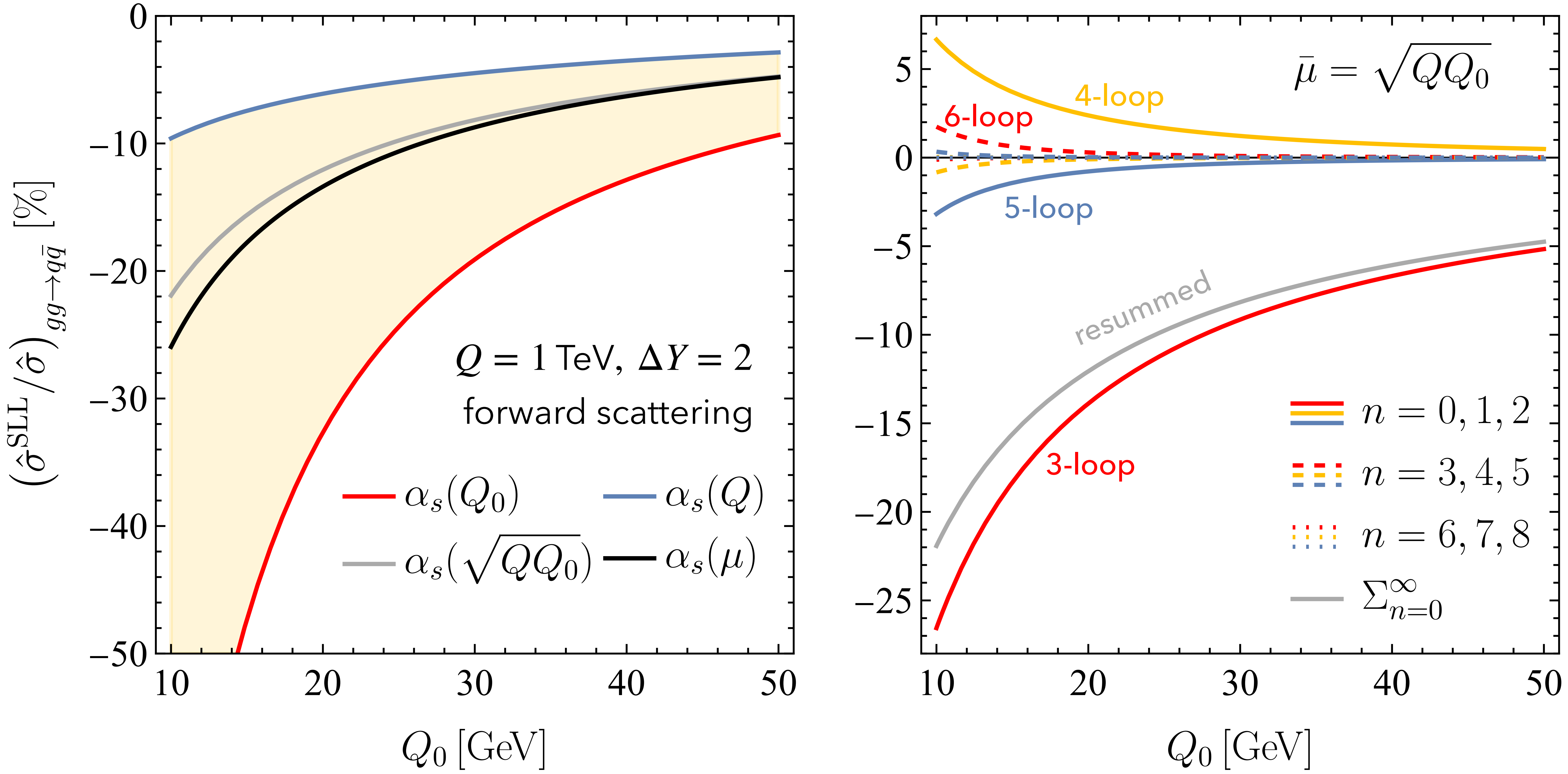


# PHYSICAL GAP-BETWEEN-JETS CROSS SECTION

[Becher, Hager, Martinelli, MN, Schwienbacher, Stillger (2024)]

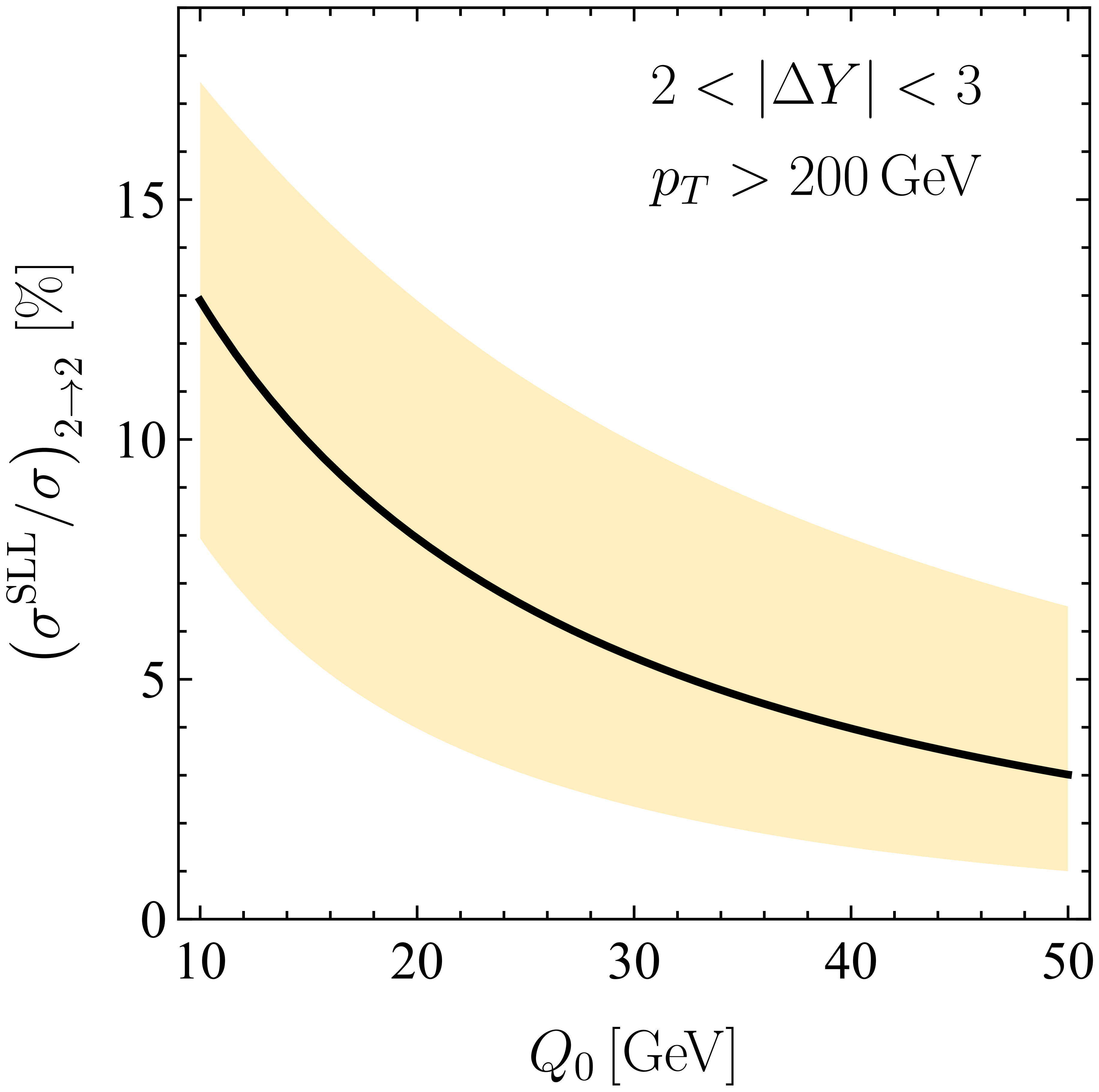
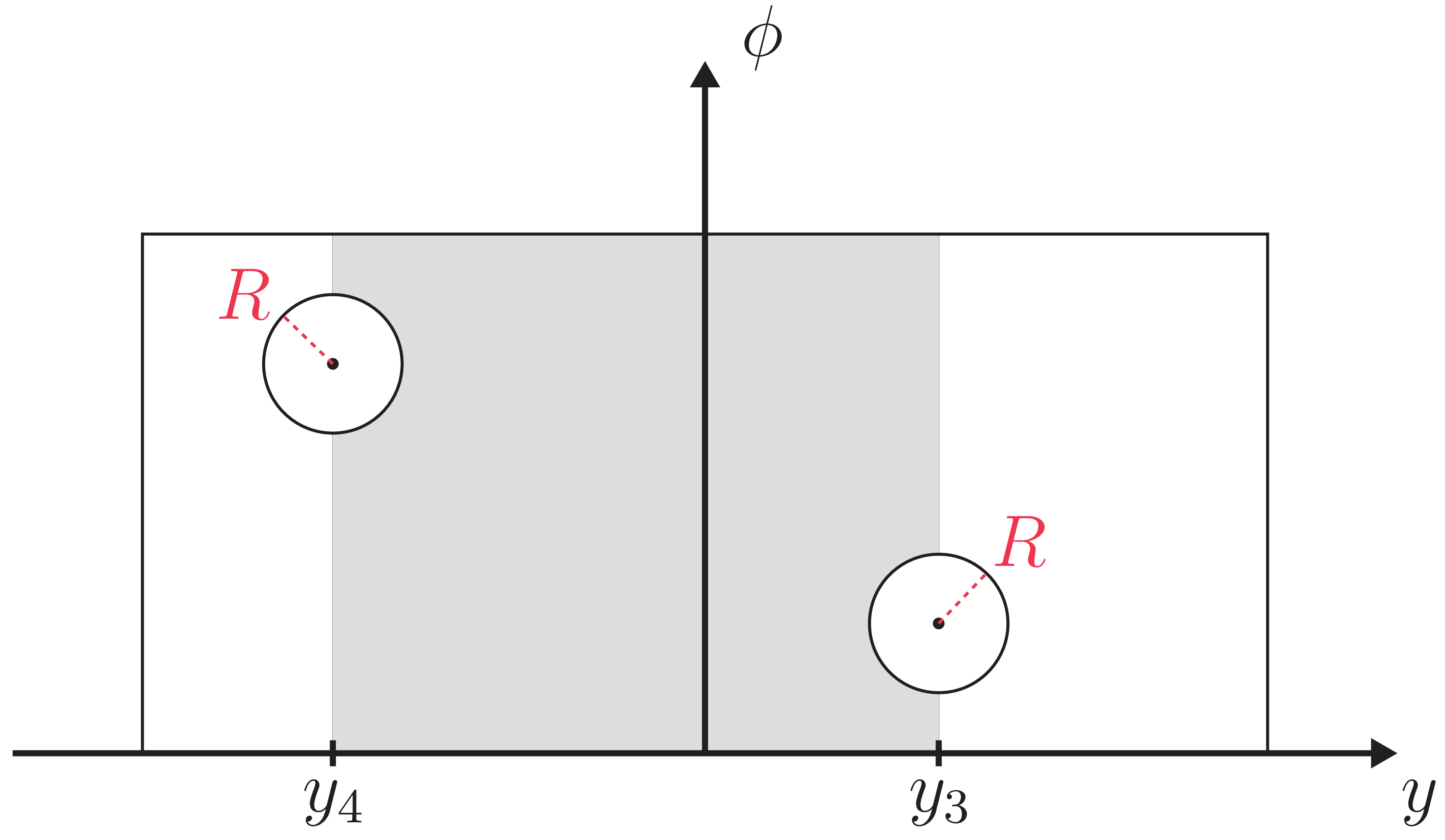



**Figure 2:** SLL contribution to the $pp \to 2\,\text{jets}$ cross section at the LHC as a function of the veto scale $Q_0$, for a center-of-mass energy $\sqrt{s} = 13\,\text{TeV}$ and jet radius $R = 0.6$.

| process | $\sigma_{2\to 2}$ [pb] | $\sigma^{\text{SLL}}_{2\to 2}$ [pb] | process | $\sigma_{2\to 2}$ [pb] | $\sigma^{\text{SLL}}_{2\to 2}$ [pb] |
|---|---|---|---|---|---|
| $qq \to qq$ | 231.5 | 12.0 | $q\bar{q} \to gg$ | 12.4 | −0.9 |
| $qq' \to qq'$ | 454.4 | 22.2 | $qg \to qg$ | 4104.6 | 403.3 |
| $q\bar{q} \to q\bar{q}$ | 142.0 | 7.4 | $gg \to q\bar{q}$ | 57.5 | −4.4 |
| $q\bar{q}' \to q\bar{q}'$ | 372.9 | 18.0 | $gg \to gg$ | 2281.1 | 150.6 |
| $q\bar{q} \to q'\bar{q}'$ | 3.6 | <0.1 | | | |
| $\sum$ | 1204.4 | 59.6 | $\sum$ | 6455.6 | 548.6 |
| $\sum_{\text{all channels}}$ | | 7660.0 | | 608.2 | |



# PDF FACTORIZATION ?

Several authors have expressed doubts that PDF factorization will be valid in general

[e.g.: Collins, Qiu (2007); Gaunt (2014); Zeng (2015)]

Observed breakdown of collinear factorization in space-like splittings was taken as indication that PDF factorization may also be violated

[Catani, de Florian, Rodrigo (2011)
Forshaw, Seymour, Siodmok (2012); Schwartz, Yan, Zhu (2017)
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## Soft gluon emission at two loops in full color

**Lance J. Dixon,[a] Enrico Herrmann,[a] Kai Yan[b] and Hua Xing Zhu[c]**

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In the limit where the outgoing soft gluon is also collinear with an incoming hard parton, potentially dangerous factorization-violating terms can arise.

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In the limit where the outgoing soft gluon is also collinear with an incoming hard parton, potentially dangerous factorization-violating terms can arise.

We speculate that at next-to-next-to-next-to-leading order (NNNLO) in QCD, integrating over the phase space of the collinear splitting can give rise to soft-collinear poles which depend on the color charge of non-collinear partons entering the process. Such poles cannot be canceled by the conventional counterterms associated with renormalization of the parton distribution functions (PDFs), which by definition are process independent.

# STRUCTURE OF THE FACTORIZATION THEOREM ?

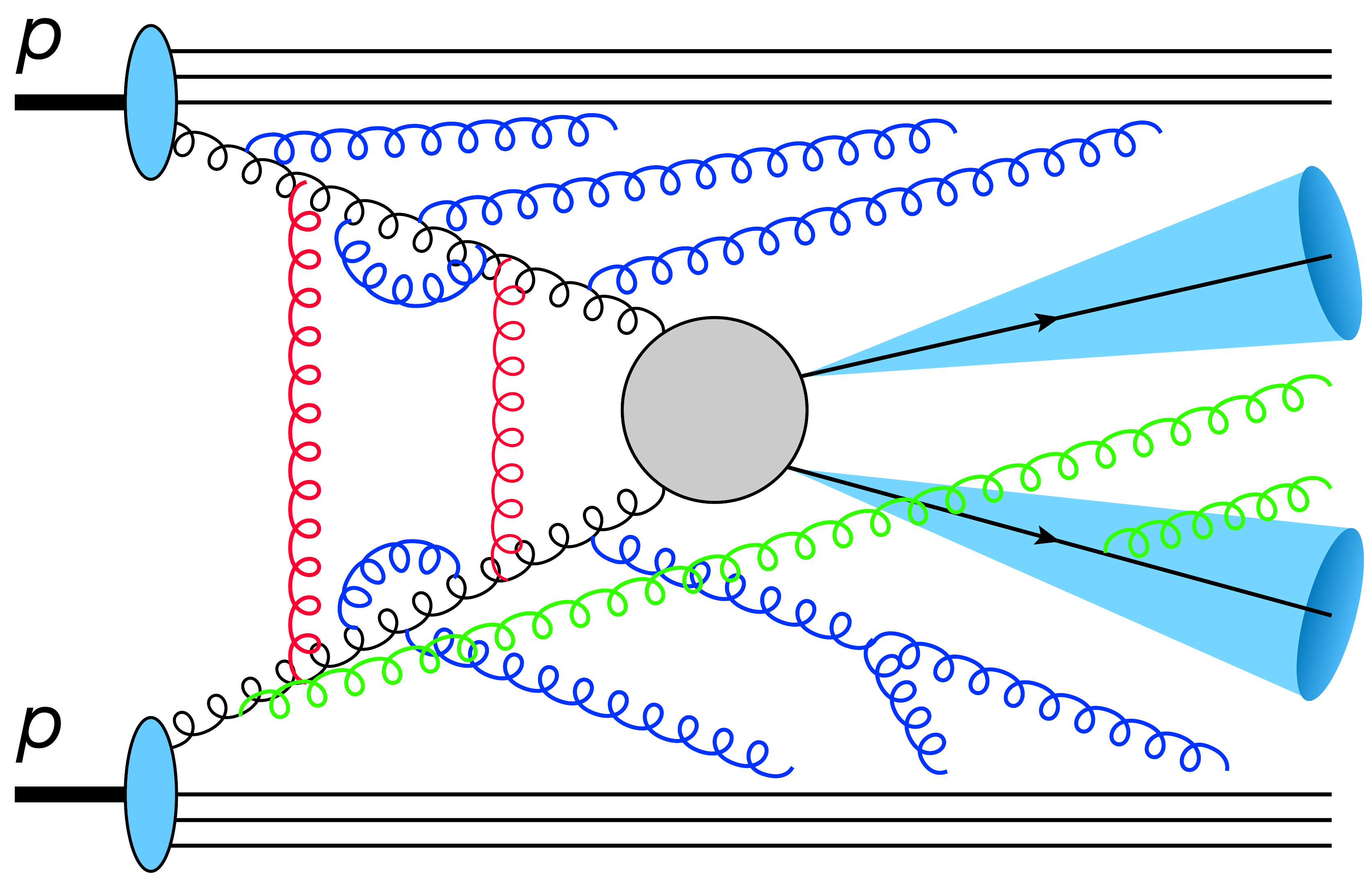
hard scale $Q \sim \sqrt{\hat{s}}$ — $\mathcal{H}_m$

double-logarithmic evolution (super-leading logs) & non-DGLAP *) collinear evolution

jet-veto scale $Q_0$ — $\mathcal{W}_m^{\text{pert}}$

single-logarithmic PDF evolution

hadronic scale $\Lambda_{\text{QCD}}$ — $f_{a/p}(\xi_1)\, f_{b/p}(\xi_2)$




$$\sigma \overset{?}{\sim} \sum_m \mathcal{H}_m \otimes \mathcal{W}_m^{\text{pert}} \otimes f_{a/p} \otimes f_{b/p}$$

*) Dokshitzer–Gribov–Lipatov–Altarelli–Parisi

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double-logarithmic evolution (super-leading logs) & non-DGLAP *) collinear evolution

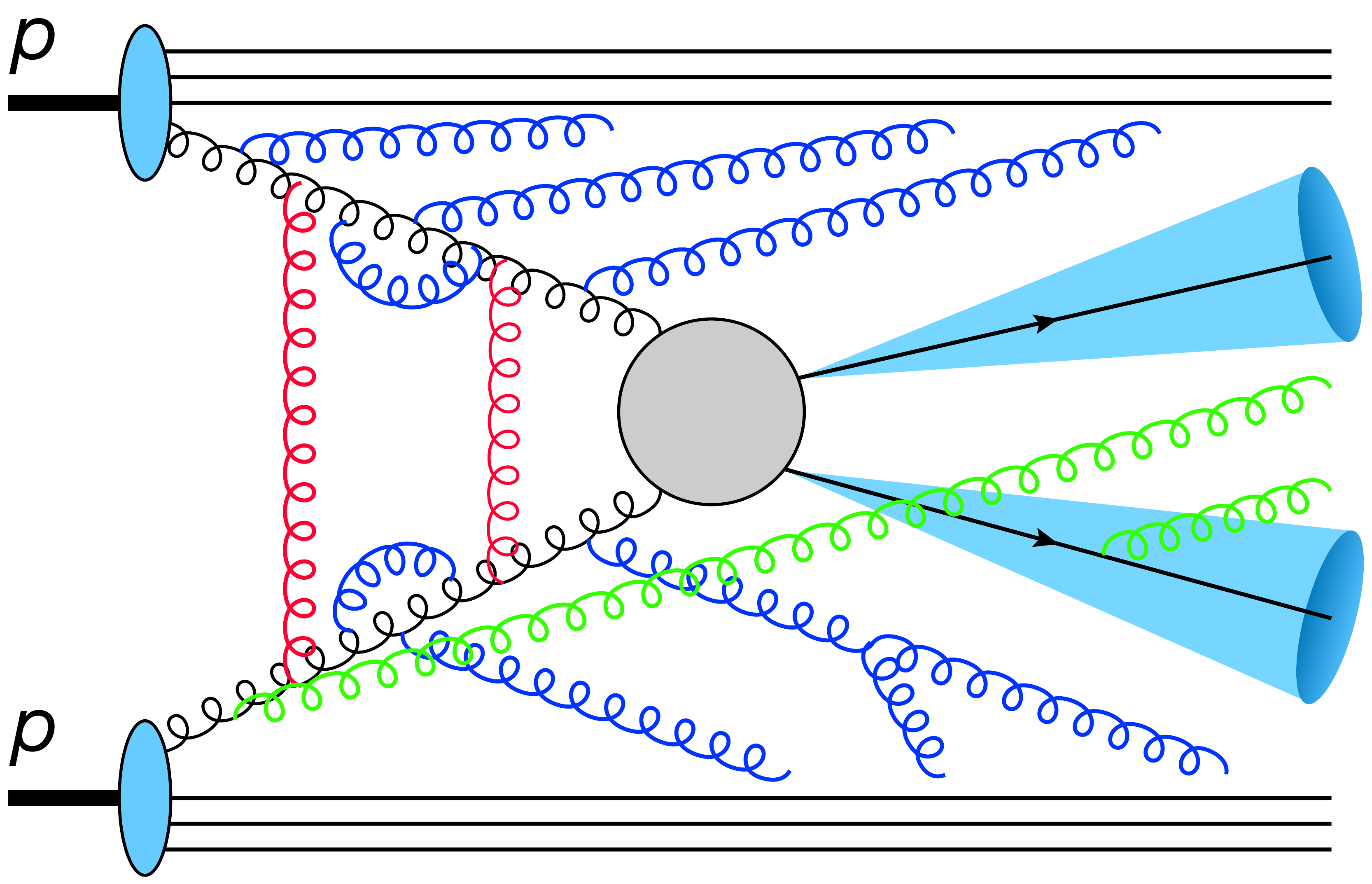
jet-veto scale $Q_0$ — $\mathcal{W}_m^{\rm pert}$

single-logarithmic PDF evolution

**OR:** *double-logarithmic evolution below $Q_0$ and breaking of PDF factorization?*

hadronic scale $\Lambda_{\rm QCD}$ — $f_{a/p}(\xi_1)\, f_{b/p}(\xi_2)$

**OR:** *something more complicated, e.g. a combined two-proton distribution?*




$$\sigma \sim \sum_m \mathcal{H}_m \otimes \mathcal{W}_m^{\rm pert} \otimes \cancel{f_{a/p} \otimes f_{b/p}}$$

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# STRUCTURE OF THE FACTORIZATION THEOREM ?

To settle the question, we calculate the **perturbative $\mu$ dependence** of $\mathcal{W}_m^{\rm pert}$ ($\leftrightarrow 1/\varepsilon^n$ poles in dim. reg.) associated with the veto scale $Q_0$, and check whether the remaining scale dependence is that of the PDFs

- **Assuming PDF factorization**, we predict:

$$\boldsymbol{\mathcal{W}}_m^{\rm bare} = \mathbf{1} + \frac{\alpha_s}{4\pi}\,\frac{\overline{\boldsymbol{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{2\varepsilon^2} + \dots\right)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left[\frac{\boldsymbol{\Gamma}^c\,\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3}\left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2}\ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{1}{12\varepsilon^3}\left[\boldsymbol{\Gamma}^C, [\boldsymbol{V}^G, \overline{\boldsymbol{\Gamma}}]\right] + \frac{\boldsymbol{V}^G\,\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} + \ \dots\right] + \mathcal{O}(\alpha_s^4)$$

where:

$\overline{\boldsymbol{\Gamma}}$ : soft emission operator

$\boldsymbol{\Gamma}^c$ : soft-collinear emission operator

$\boldsymbol{V}^G$: Glauber operator

$\boldsymbol{\Gamma}^C$: collinear emission operator

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To settle the question, we calculate the perturbative $\mu$ dependence of $\mathcal{W}_m^{\text{pert}}$ ($\leftrightarrow 1/\varepsilon^n$ poles in dim. reg.) associated with the veto scale $Q_0$, and check whether the remaining scale dependence is that of the PDFs

- Assuming PDF factorization, we predict:

$$\mathcal{W}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi}\frac{\overline{\mathbf{\Gamma}}}{2\varepsilon} ✓ + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{2\varepsilon^2} ✓ + \dots\right)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \left[\frac{\mathbf{\Gamma}^c\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3}\left(\frac{11}{6\varepsilon} + \ln\frac{\mu_s^2}{Q^2} + \frac{9}{2}\ln\frac{\mu_s^2}{Q_0^2}\right) + \frac{1}{12\varepsilon^3}\left[\mathbf{\Gamma}^C, [\boldsymbol{V}^G, \overline{\mathbf{\Gamma}}]\right] + \frac{\boldsymbol{V}^G\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3} ✓ + \dots\right] + \mathcal{O}(\alpha_s^4)$$

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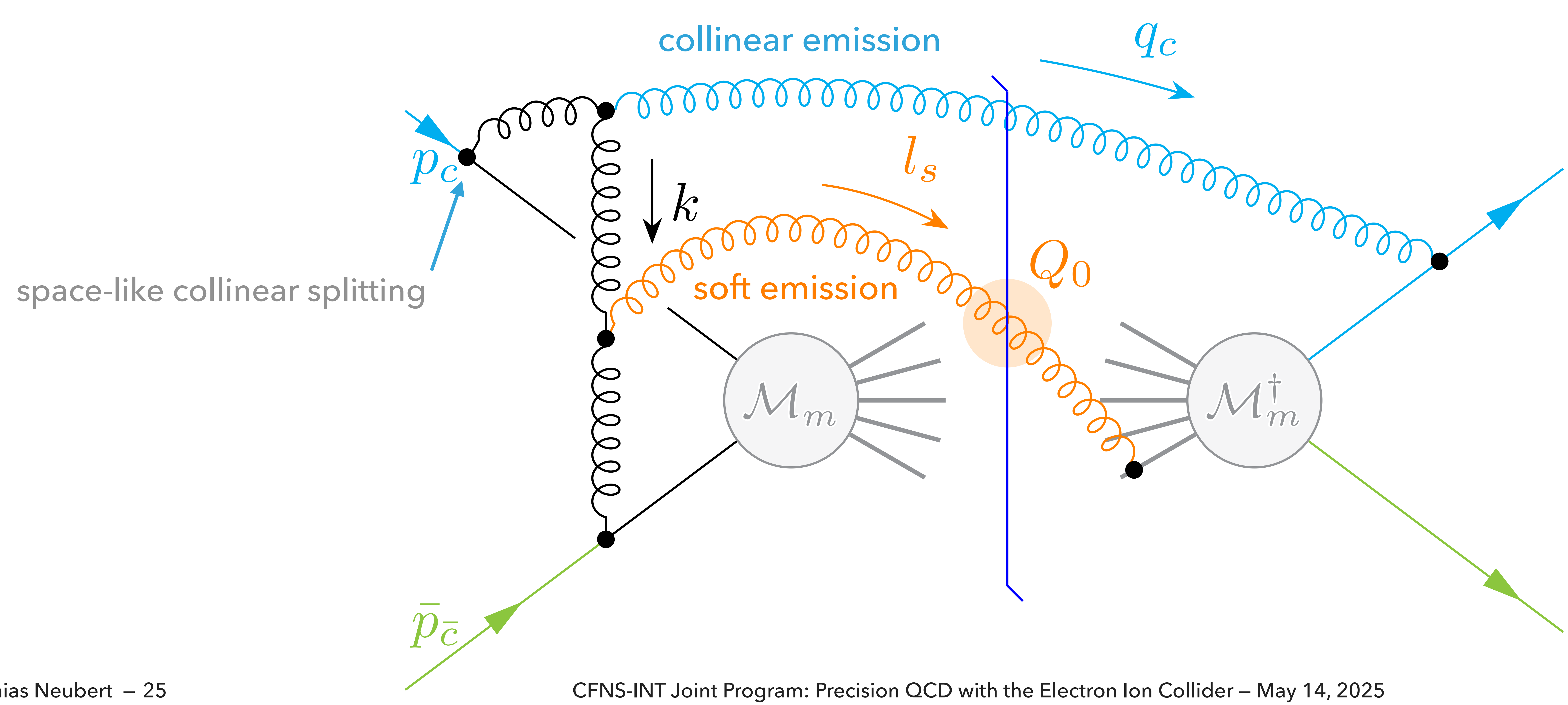
# REGION ANALYSIS OF 3-LOOP DIAGRAMS

[Becher, Hager, Jaskiewicz, MN, Schwienbacher (2024)
Phys. Rev. Lett. **134** (2025) 6, 061901]

- Relevant graphs feature a soft gluon emission into the gap, a space-like collinear splitting, and a virtual gluon exchange:



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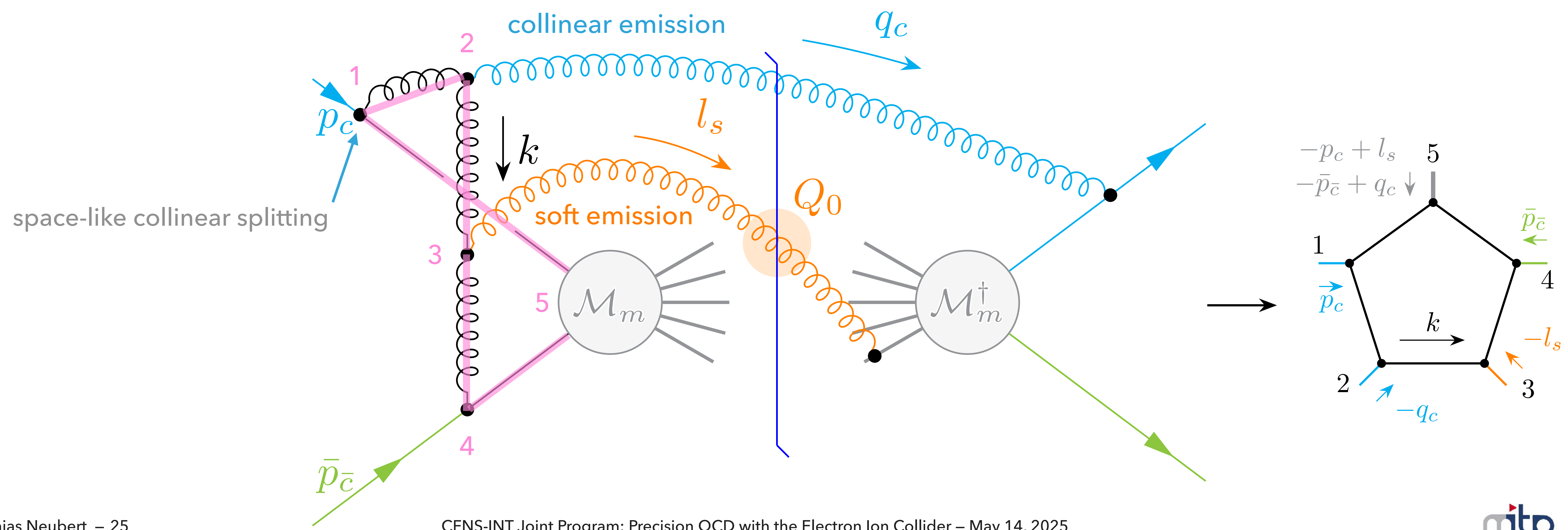
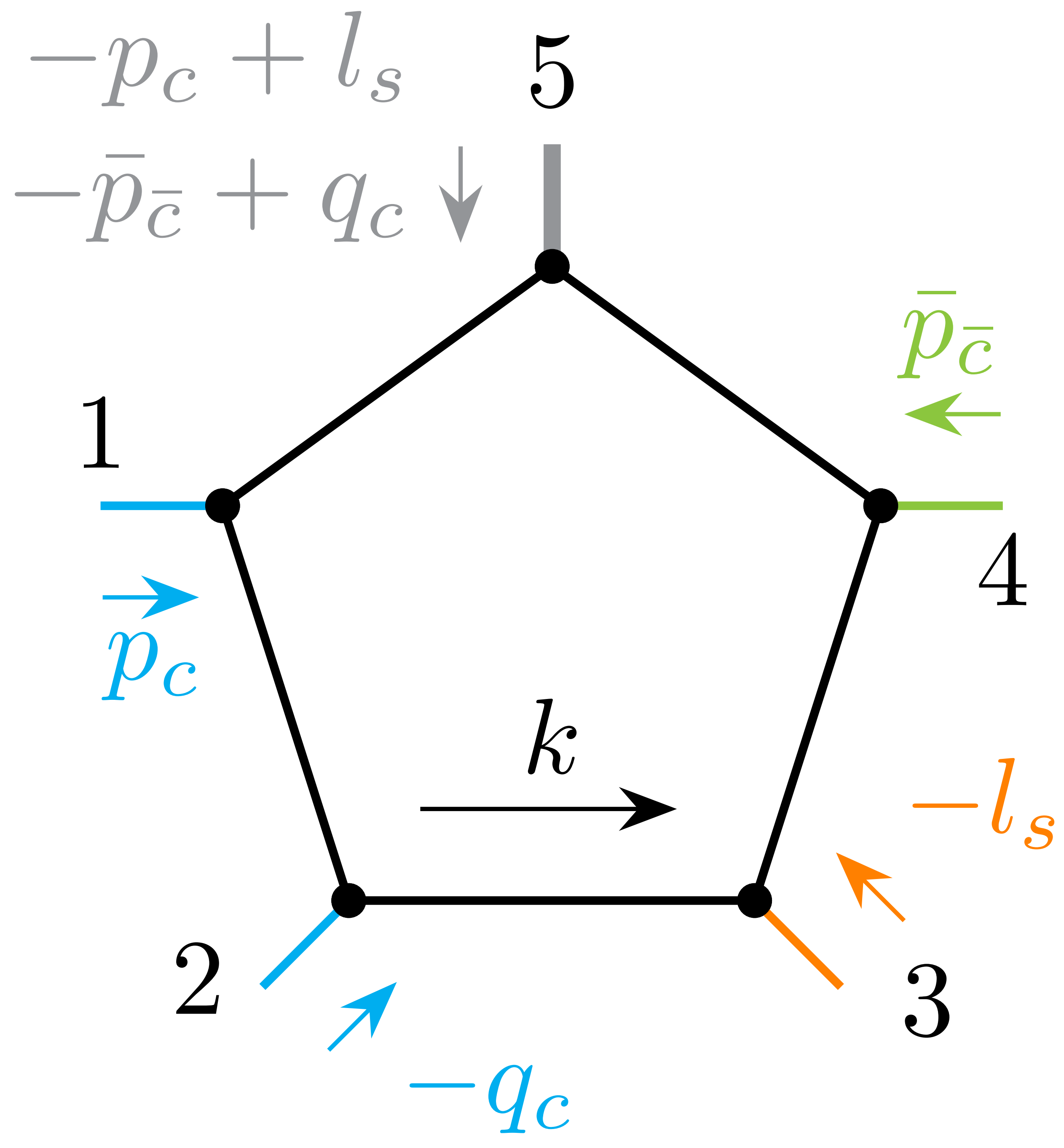


# REGION ANALYSIS OF 3-LOOP DIAGRAMS

[Becher, Hager, Jaskiewicz, MN, Schwienbacher (2024)
Phys. Rev. Lett. **134** (2025) 6, 061901]

- Region analysis of the pentagon integral (exact expression known):

[Bern, Dixon, Kosower (1993)]




- **Euclidean kinematics:** only a single region (soft-collinear) contributes

- **Physical kinematics:** appearance of non-trivial phase factors $e^{2\pi i\varepsilon}$ introduces a new Glauber region, which we were unable to find using existing region-finder codes such as Asy2.1

[Pak, Smirnov (2011); Jantzen, Smirnov, Smirnov (2012)]

# REGION ANALYSIS OF 3-LOOP DIAGRAMS

[Becher, Hager, Jaskiewicz, MN, Schwienbacher (2024)
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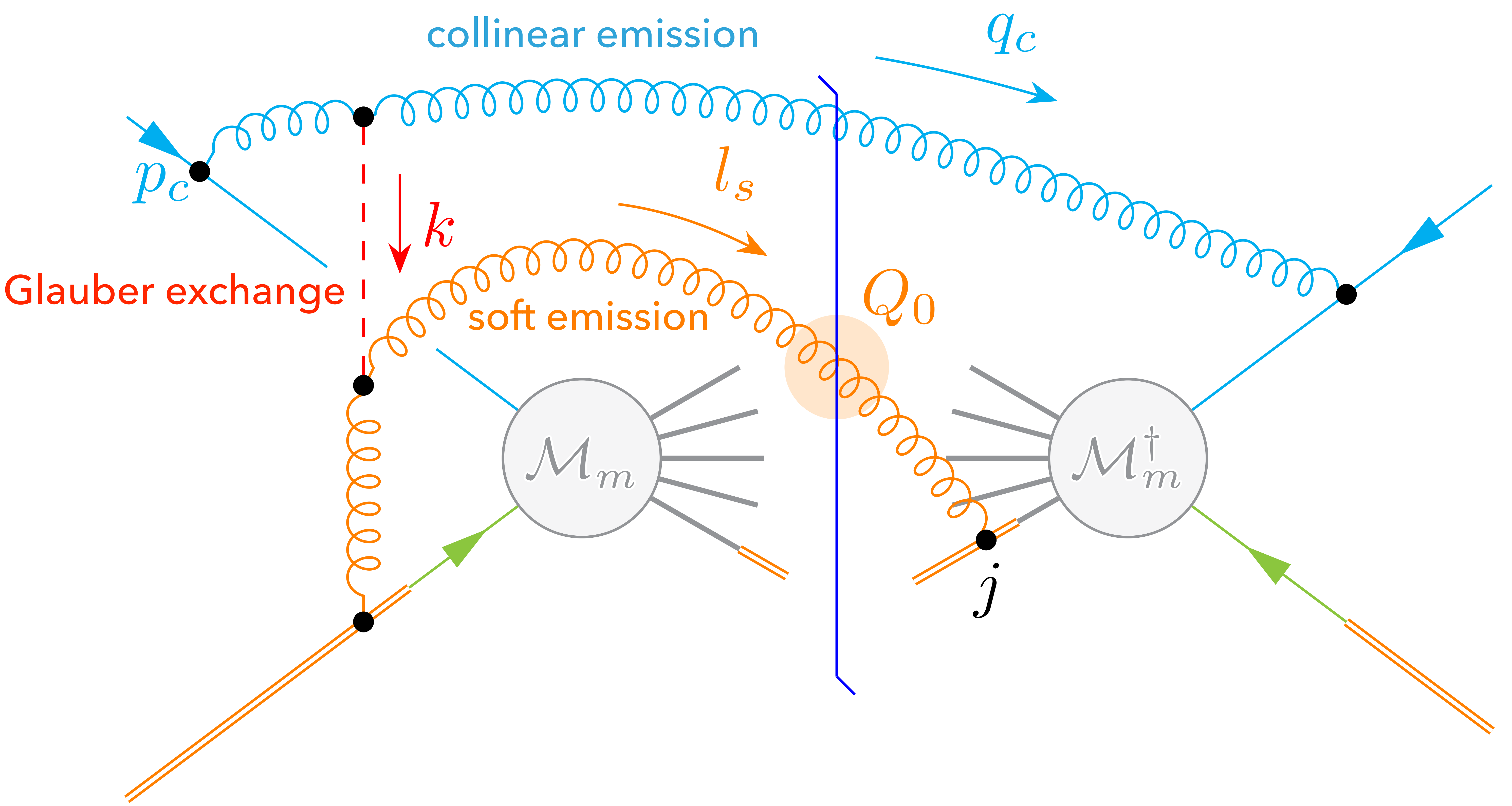
- Relevant graphs in SCET with Glauber gluons: [Rothstein, Stewart (2016)]



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double-logarithmic evolution above $Q_0$

non-DGLAP collinear evolution above $Q_0$ (to appear)

where:

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# STRUCTURE OF THE FACTORIZATION THEOREM ?

$$\in 128\pi\, \mathcal{P}_{q\to g}(\xi_1)\, \delta(1-\xi_2)\, f_{abc}\, \mathrm{Tr}(t^a\, t^A\, t^B)$$

$$\times \textstyle\sum_{j=3}^m J_j\, \mathbf{T}_2^b\, \mathbf{T}_j^c + (1 \leftrightarrow 2)$$

- **Assuming PDF factorization**, we predict:

$$\mathcal{W}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi} \frac{\overline{\mathbf{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\, \overline{\mathbf{\Gamma}}}{2\varepsilon^2} + \dots\right)$$
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# "FACTORIZATION RESTORATION" THROUGH GLAUBER GLUONS

Collinear factorization violation at $\mu \sim Q$

x

Soft-collinear factorization violation by Glauber gluons at $\mu \sim Q_0$

=

PDF factorization restored for $\mu < Q_0$

- We have **proved this to 3-loop order** and **conjecture** that it holds in general !

# SUMMARY

- Based on a SCET factorization theorem, we have performed the **first all-order resummation of super-leading logarithms**
- The main open challenge is to **combine this with the resummation of non-global single logarithms**
- We have uncovered a **new mechanism** reconciling the breaking of collinear factorization with **unbroken PDF factorization:** in an interplay of space-like collinear splittings and soft emissions, **perturbative Glauber gluons restore the factorization** of the cross section
- Understanding the all-order structure of these effects would pave the way to a **proof of PDF factorization** for a much wider class of observables!