

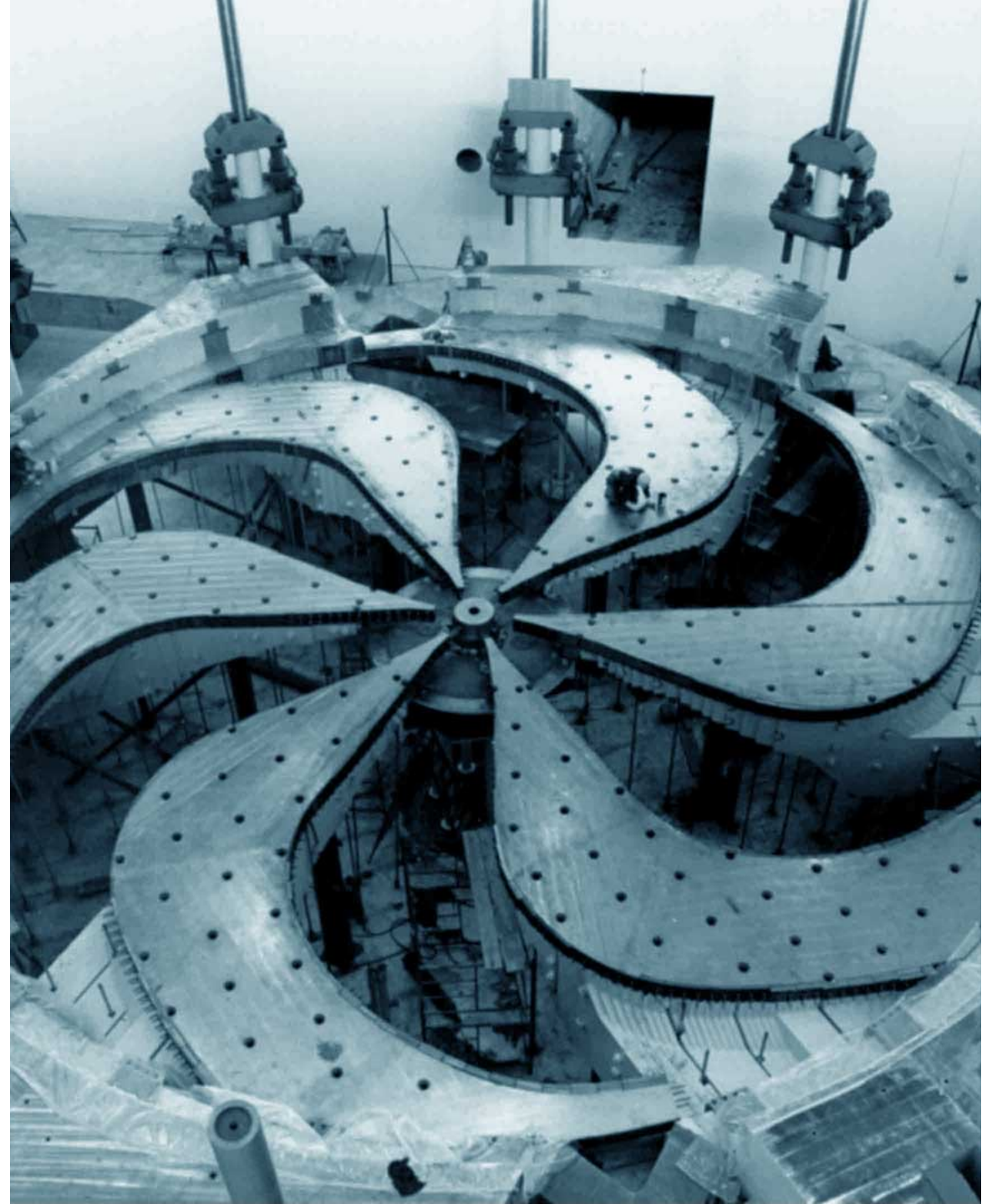
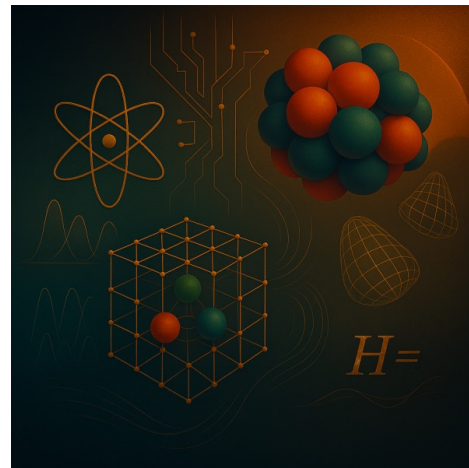


Digital Research Alliance of Canada

Ab initio calculations with sub-leading three-nucleon interaction

INT Program 26-1
Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond
April 29, 2026

Petr Navratil
TRIUMF



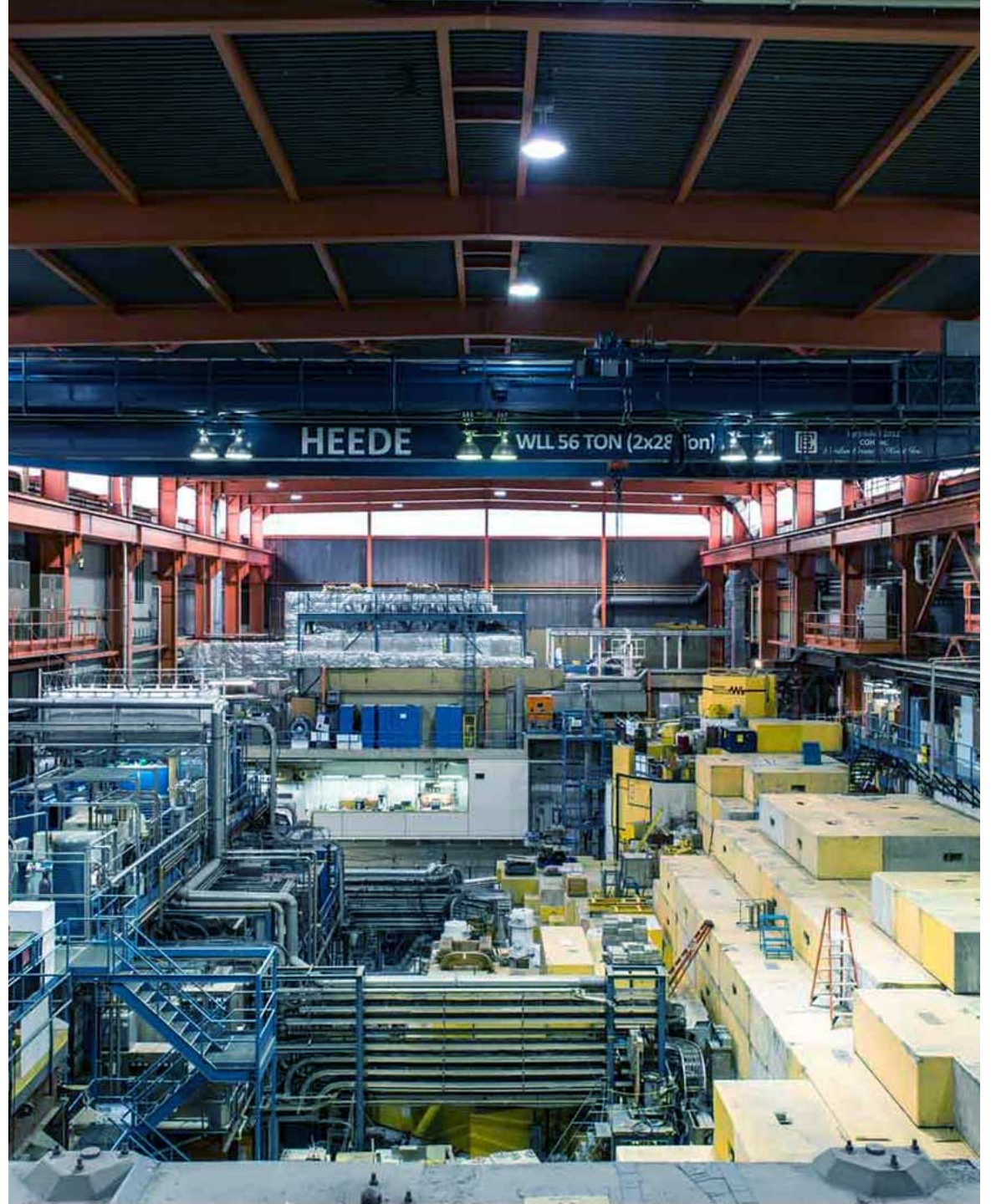
Discovery, accelerated

Outline

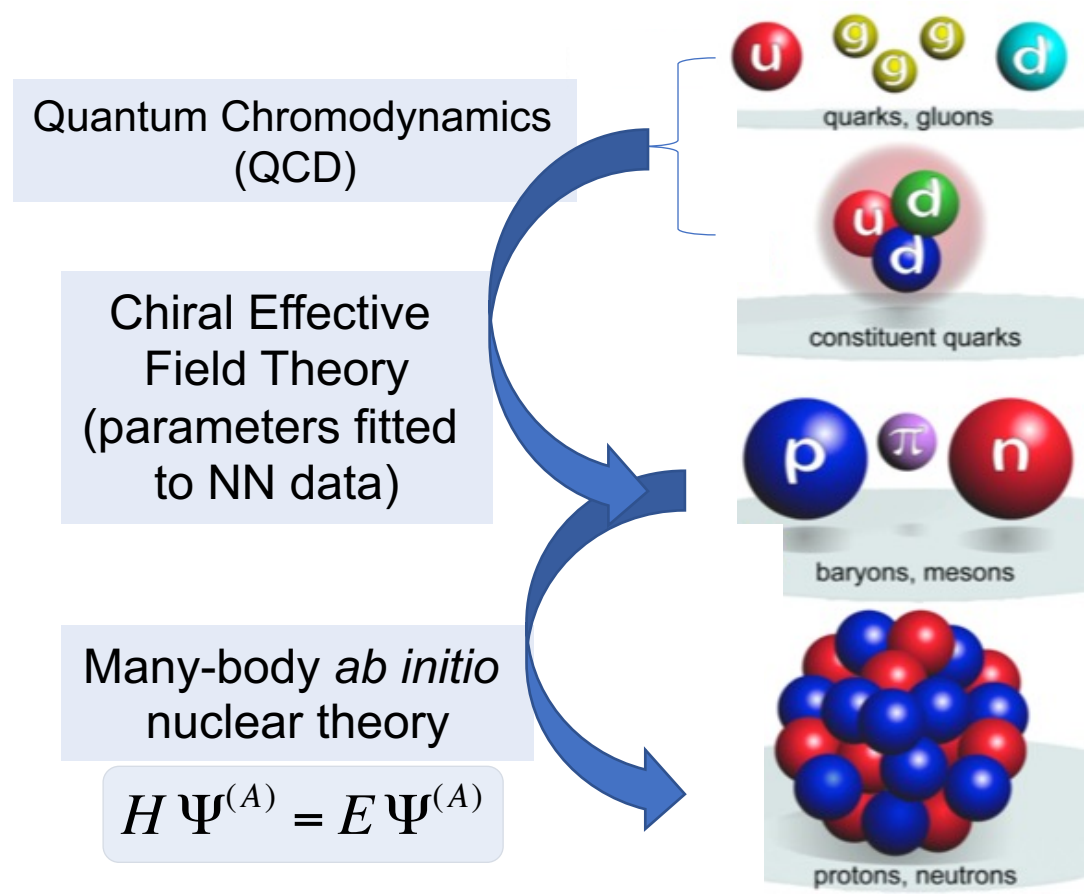
- *Ab initio* nuclear theory – **no-core shell model with continuum (NCSMC)**
- Precision chiral EFT NN N³LO + 3N_{int} interaction
- Precision chiral EFT NN Hamiltonian - order by order convergence
- Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term
 - Energies of light nuclei
 - n+⁴He scattering
 - ⁷Be(p,γ)⁸B radiative capture
 - p+¹²C scattering and capture, n+¹²C cross section
 - Ground-state energies of medium mass nuclei
- Enhanced short-range 3N interaction with two-pion exchange
 - Results for 3H
- Conclusions

Ab initio nuclear theory -
no-core shell model with
continuum (NCSMC)

2026-04-29



First principles or *ab initio* nuclear theory



	NN force	NNN force	NNNN force
Q^0 LO			
Q^2 NLO			
Q^3 N ² LO			
Q^4 N ³ LO			



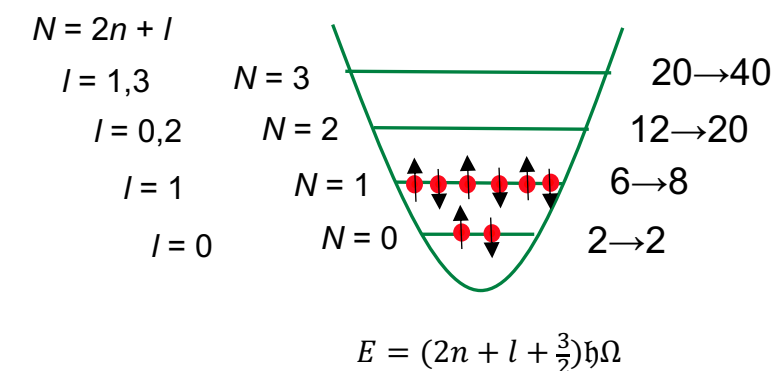
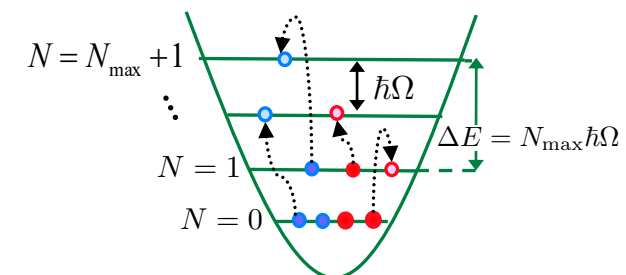
Review

Ab initio no core shell modelBruce R. Barrett^a, Petr Navrátil^b, James P. Vary^{c,*}

5

Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

- Basis expansion method (CI)
 - Harmonic oscillator (HO) basis truncated in a particular way (N_{\max})
 - Why HO basis?
 - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^4He , ^{16}O , ^{40}Ca)
 - Equivalent description in relative(Jacobi)-coordinate and Slater determinant basis – nuclei self-bound, $[\text{H}, \text{P}_{\text{CM}}]=0$
 - Exact factorization of CM and intrinsic eigenfunctions at each N_{\max}
 - Good for well-bound states, approximation of weakly-bound states and narrow resonances



Ab initio calculations of structure including weakly bound halo states, scattering, reactions

Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} (A) \\ \text{Nucleus} \\ \lambda \end{array} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} (A-a) \quad (a) \\ \text{Nucleus} \quad \text{Nucleus} \\ \nu \end{array} \right\rangle$$

Invited Comment

Unified *ab initio* approaches to nuclear structure and reactions

Petr Navrátil¹, Sofia Quaglioni², Guillaume Hupin^{3,4},
Carolina Romero-Redondo² and Angelo Calci¹

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

Ab initio calculations of structure including weakly bound halo states, scattering, reactions

Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \underbrace{\sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \\ \lambda \end{matrix} \right\rangle}_{\text{static solutions}} + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & \vec{r} & (a) \\ \nu & & \nu \end{matrix} \right\rangle$$

Static solutions for aggregate system, describe all nucleons close together

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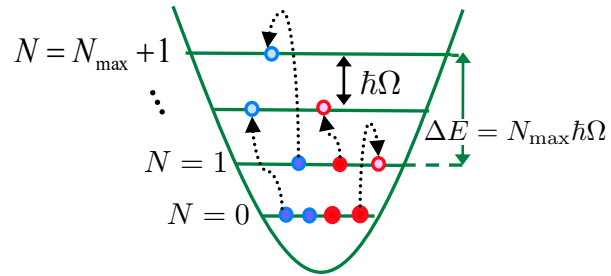
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$$\Psi^{(A)} = \underbrace{\sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \\ \lambda \end{matrix} \right\rangle}_{\text{Static solutions for aggregate system}} + \underbrace{\sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & \vec{r} & (a) \\ \nu & & \nu \end{matrix} \right\rangle}_{\text{Continuous microscopic cluster states}}$$



Continuous microscopic cluster states, describe long-range projectile-target

Static solutions for aggregate system, describe all nucleons close together

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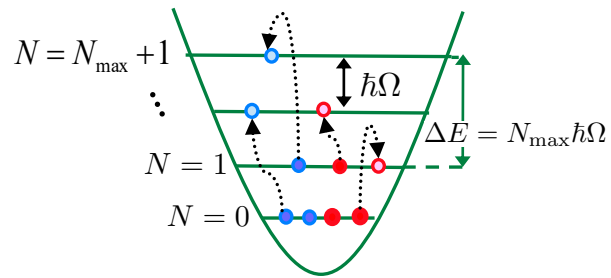
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$$\Psi^{(A)} = \underbrace{\sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{NCSM} \\ \lambda \end{matrix} \right\rangle}_{\text{Static solutions for aggregate system, describe all nucleons close together}} + \underbrace{\sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{NCSM} \\ \nu \end{matrix} \right\rangle}_{\text{Continuous microscopic cluster states, describe long-range projectile-target}}$$



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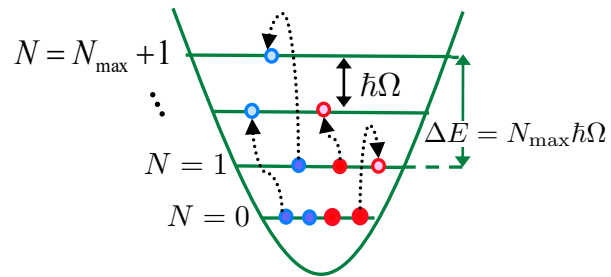
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$$\Psi^{(A)} = \underbrace{\sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \\ \lambda \end{matrix} \right\rangle}_{\text{Unknowns}} + \underbrace{\sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & \vec{r} & (a) \\ \text{cluster} & & \text{cluster} \\ \nu & & \nu \end{matrix} \right\rangle}_{\text{Unknowns}}$$



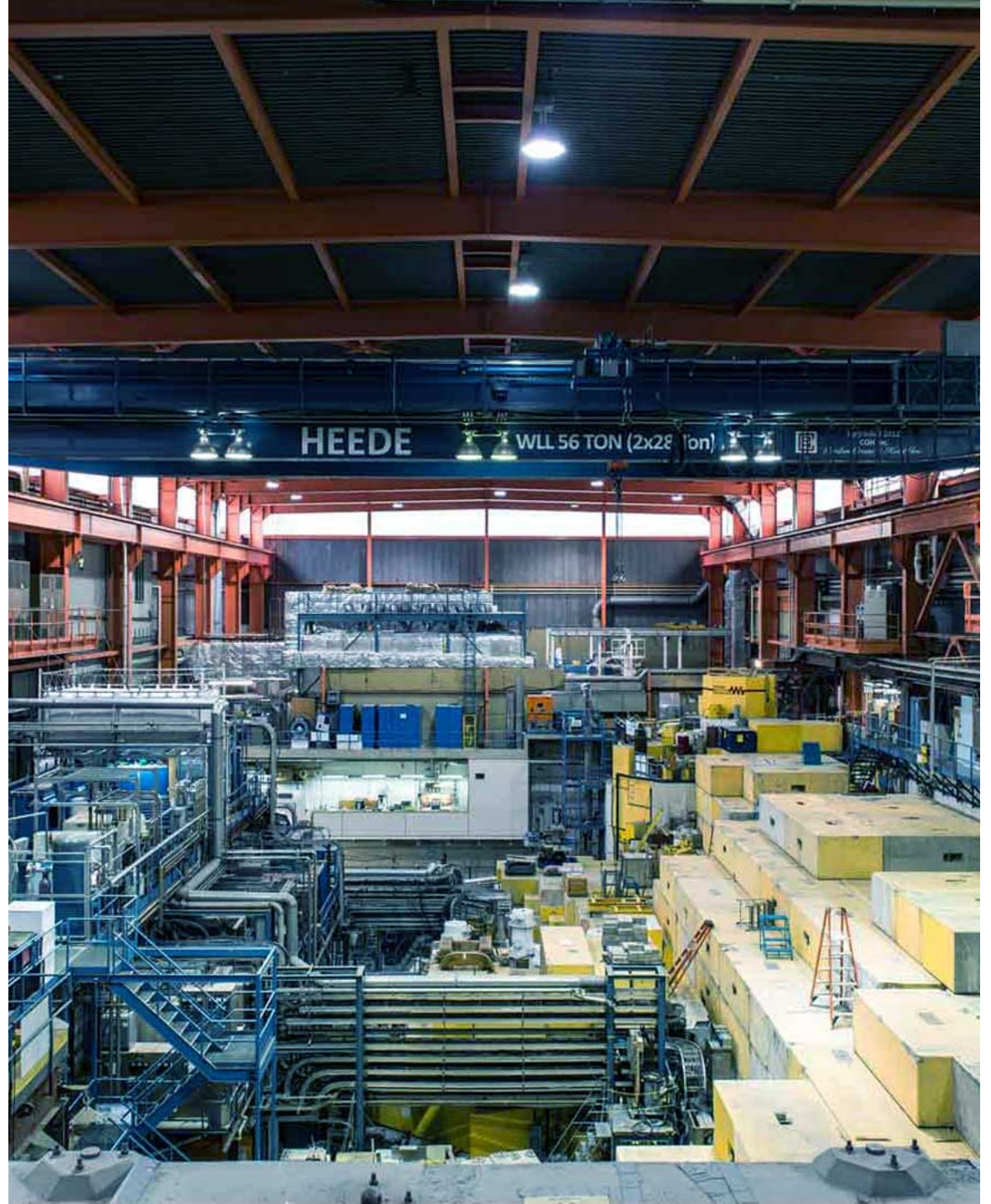
Continuous microscopic cluster states, describe long-range projectile-target

Static solutions for aggregate system, describe all nucleons close together

Precision chiral EFT

NN N^3LO + $3N_{|n|}$ interaction

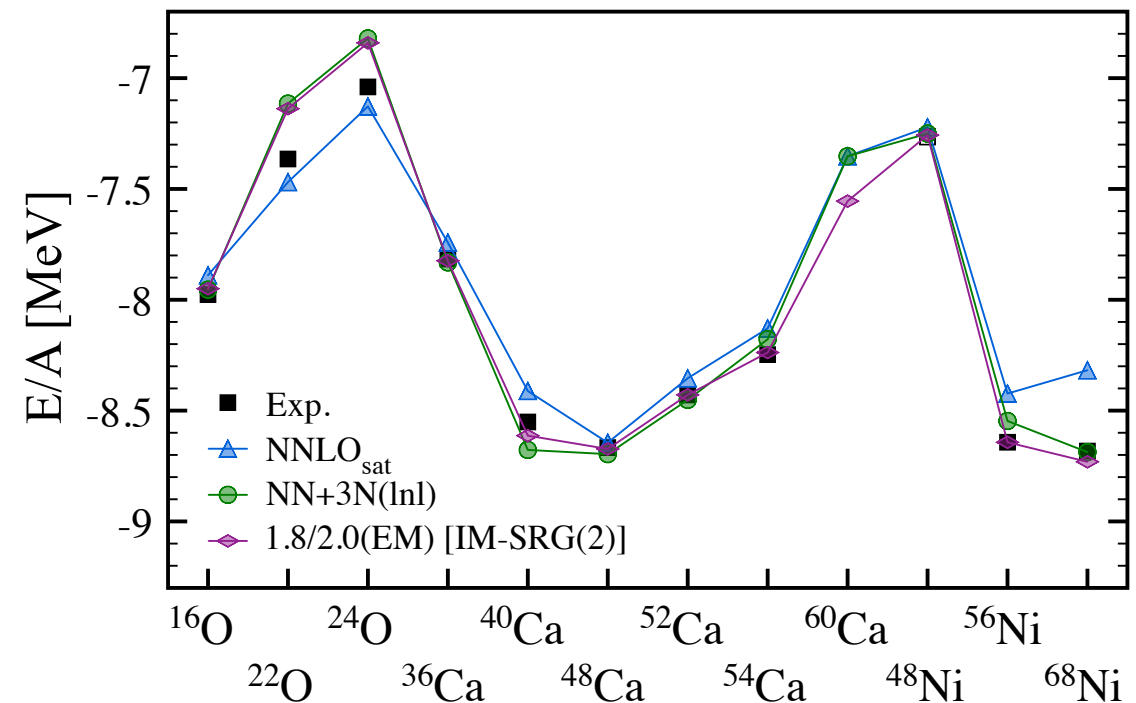
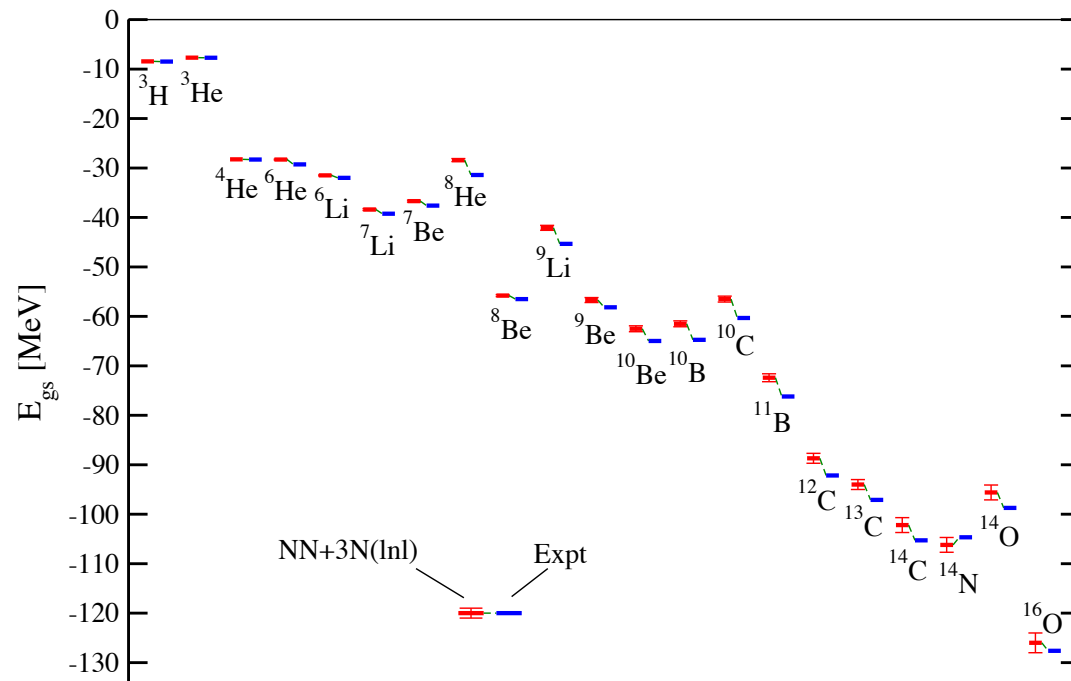
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Binding energies of atomic nuclei from nuclear forces from chiral Effective Field Theory

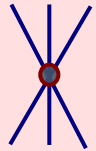
- Quite reasonable description of binding energies across the nuclear charts becomes feasible
 - **The Hamiltonian fully determined in $A=2$ and $A=3,4$ systems**
 - Nucleon–nucleon scattering, deuteron properties, ^3H and ^4He binding energy, ^3H half life
 - Light nuclei – NCSM
 - Medium mass nuclei – Self-Consistent Green’s Function method

NN N³LO (Entem-Machleidt 2003)
3N N²LO w local/non-local regulator



Local vs. non-local chiral 3N interaction

- Regulator depending on momentum transfer \Rightarrow local NNN interaction in coordinate space
 - Different space-tensor structure (compared to regulation with nucleon momenta)
 - Example:** Contact term



$$W_1^{\text{cont}} = E \vec{\tau}_2 \cdot \vec{\tau}_3 \delta(\vec{r}_1 - \vec{r}_2) \delta(\vec{r}_3 - \vec{r}_1)$$

$$= E \vec{\tau}_2 \cdot \vec{\tau}_3 \frac{1}{(2\pi)^6} \frac{1}{(\sqrt{3})^3} \int d\vec{\pi}_1 d\vec{\pi}_2 d\vec{\pi}'_1 d\vec{\pi}'_2 |\vec{\pi}_1 \vec{\pi}_2\rangle \langle \vec{\pi}'_1 \vec{\pi}'_2|$$

- Local

$$W_1^{\text{cont},Q} = E \vec{\tau}_2 \cdot \vec{\tau}_3 \frac{1}{(2\pi)^6} \frac{1}{(\sqrt{3})^3} \int d\vec{\pi}_1 d\vec{\pi}_2 d\vec{\pi}'_1 d\vec{\pi}'_2 |\vec{\pi}_1 \vec{\pi}_2\rangle F(\vec{Q}^2; \Lambda) F(\vec{Q}'^2; \Lambda) \langle \vec{\pi}'_1 \vec{\pi}'_2|$$

$$= E \vec{\tau}_2 \cdot \vec{\tau}_3 \int d\vec{\xi}_1 d\vec{\xi}_2 |\vec{\xi}_1 \vec{\xi}_2\rangle Z_0(\sqrt{2}\xi_1; \Lambda) Z_0(|\frac{1}{\sqrt{2}}\vec{\xi}_1 + \sqrt{\frac{3}{2}}\vec{\xi}_2|; \Lambda) \langle \vec{\xi}_1 \vec{\xi}_2|$$

- Non-local

$$W_1^{\text{cont},\text{ENGKMW}} = E \vec{\tau}_2 \cdot \vec{\tau}_3 \frac{1}{(2\pi)^6} \frac{1}{(\sqrt{3})^3} \int d\vec{\pi}_1 d\vec{\pi}_2 d\vec{\pi}'_1 d\vec{\pi}'_2 |\vec{\pi}_1 \vec{\pi}_2\rangle$$

$$\times F\left(\frac{1}{2}(\pi_1^2 + \pi_2^2); A\right) F\left(\frac{1}{2}(\pi_1'^2 + \pi_2'^2); A\right) \langle \vec{\pi}'_1 \vec{\pi}'_2|$$

- Local/Non-local

$$F\left(\frac{1}{2}(\pi_1^2 + \pi_2^2); \Lambda_{\text{nonloc}}\right) W_1^Q(\Lambda_{\text{loc}}) F\left(\frac{1}{2}(\pi_1'^2 + \pi_2'^2); \Lambda_{\text{nonloc}}\right) \leftarrow$$

Use completeness
in HO basis to calculate
products of $F W F$

$$\vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{\xi}_2 = \sqrt{\frac{2}{3}} \left(\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3 \right)$$

$$\vec{\pi}_1 = \frac{1}{\sqrt{2}}(\vec{p}_1 - \vec{p}_2)$$

$$\vec{\pi}_2 = \sqrt{\frac{2}{3}} \left(\frac{1}{2}(\vec{p}_1 + \vec{p}_2) - \vec{p}_3 \right)$$

$$\vec{Q} = \vec{p}'_2 - \vec{p}_2 = -\frac{1}{\sqrt{2}}(\vec{\pi}'_1 - \vec{\pi}_1) + \frac{1}{\sqrt{6}}(\vec{\pi}'_2 - \vec{\pi}_2)$$

$$\vec{Q}' = \vec{p}'_3 - \vec{p}_3 = \sqrt{\frac{2}{3}}(\vec{\pi}_2 - \vec{\pi}'_2)$$

$$Z_0(r; \Lambda) = \frac{1}{2\pi^2} \int dq q^2 j_0(qr) F(q^2; \Lambda)$$

$$F(q^2; \Lambda) = \exp[-q^4/\Lambda^4]$$

Few Body Syst (2007) 41: 117-140
DOI 10.1007/s00601-007-0193-3
Printed in The Netherlands

Few
Body
Systems

Local three-nucleon interaction from chiral
effective field theory

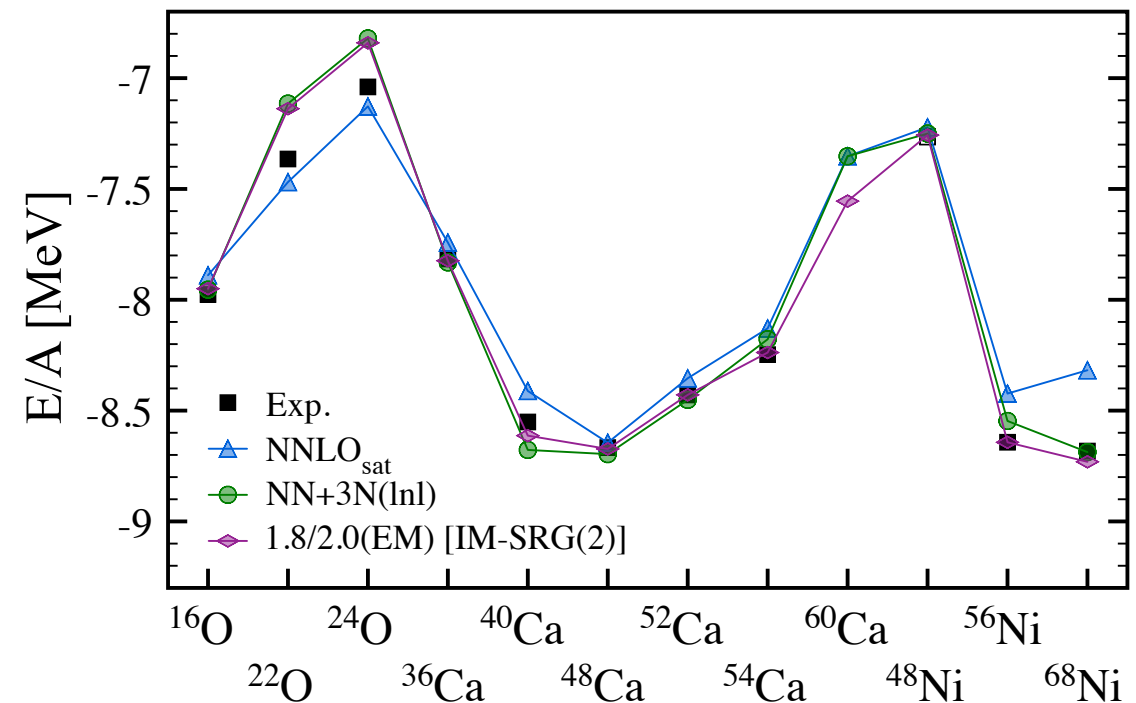
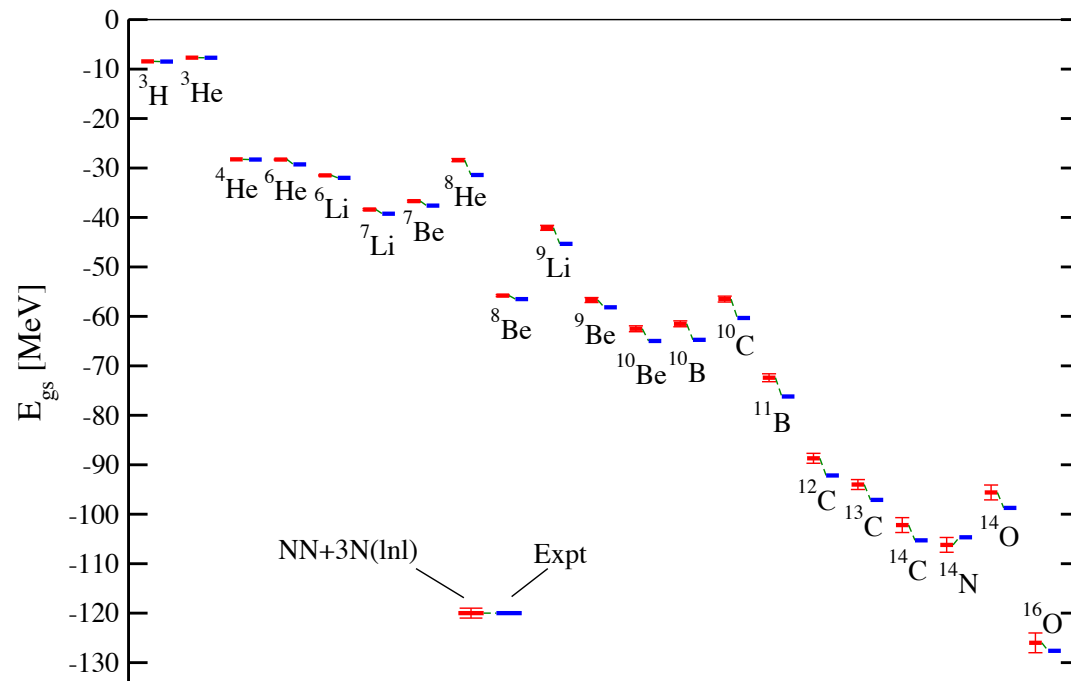
P. Navrátil*

Lawrence Livermore National Laboratory, Livermore, CA, USA

Binding energies of atomic nuclei from nuclear forces from chiral Effective Field Theory

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
 - **The Hamiltonian fully determined in $A=2$ and $A=3,4$ systems**
 - Nucleon–nucleon scattering, deuteron properties, ^3H and ^4He binding energy, ^3H half life
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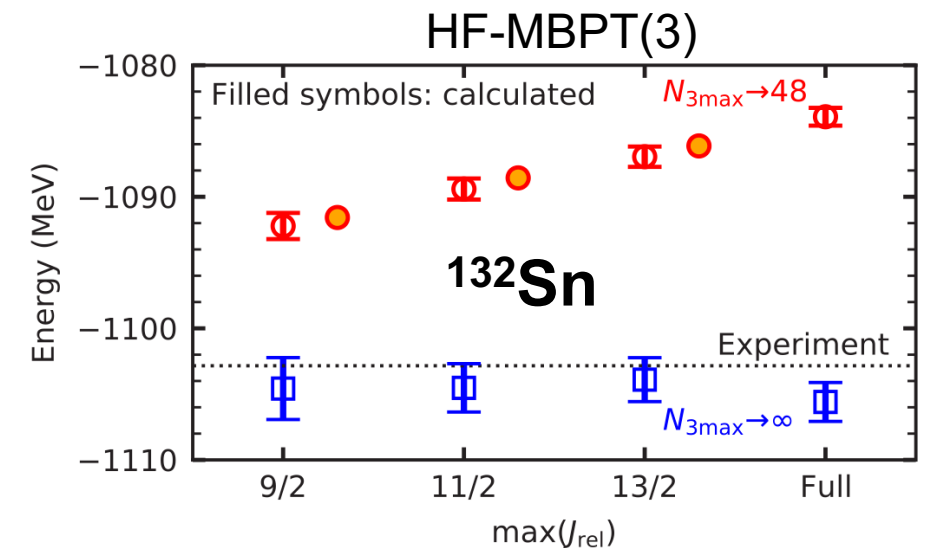
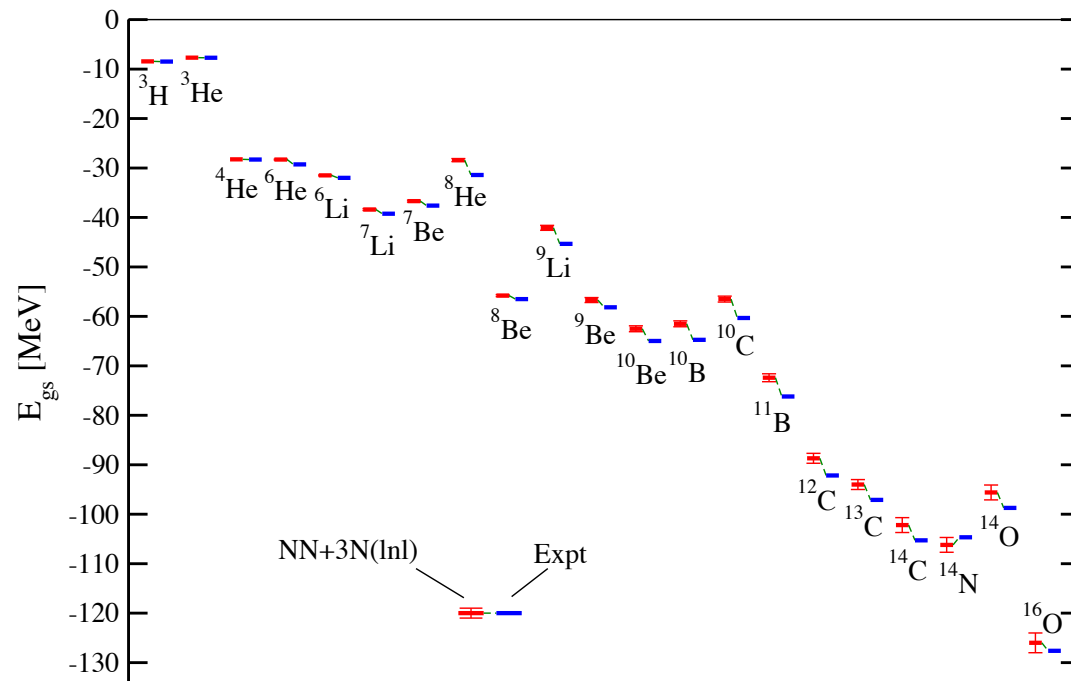
NN N³LO (Entem-Machleidt 2003)
3N N²LO w local/non-local regulator



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 - Heavy nuclei – HF-MBPT(3)

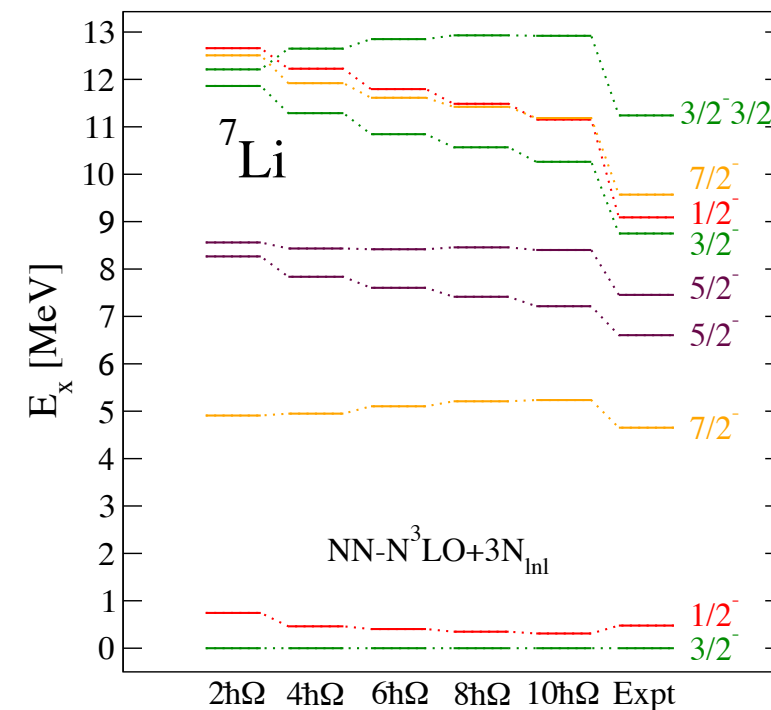
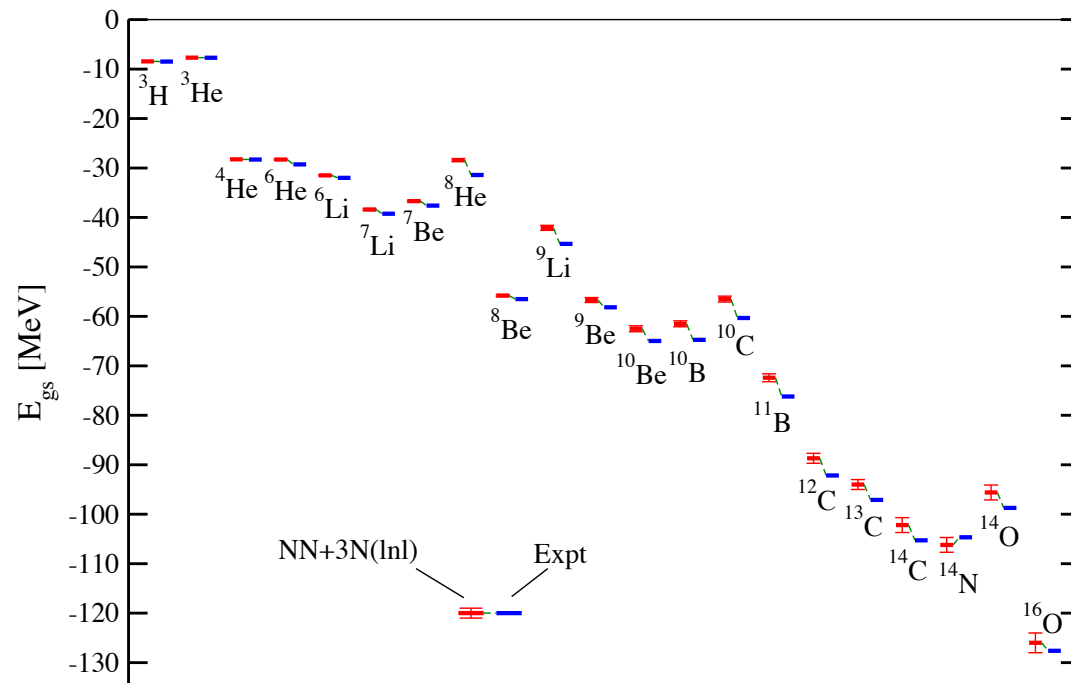
NN N³LO (Entem-Machleidt 2003)
3N N²LO w local/non-local regulator



Binding and excitation energies of atomic nuclei from nuclear forces from chiral Effective Field Theory

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 - Light nuclei – NCSM
 - Denoted as NN $N^3\text{LO} + 3N_{\text{Inl}}$

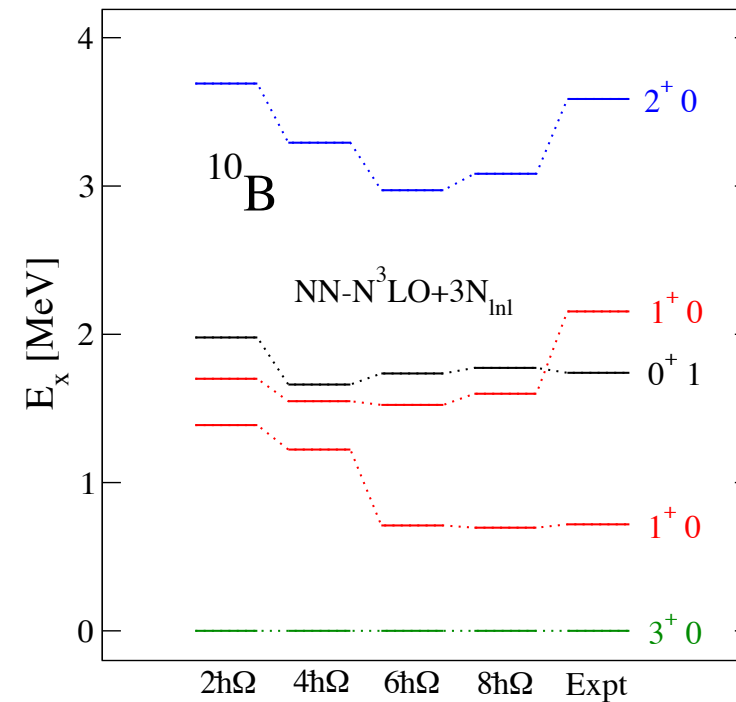
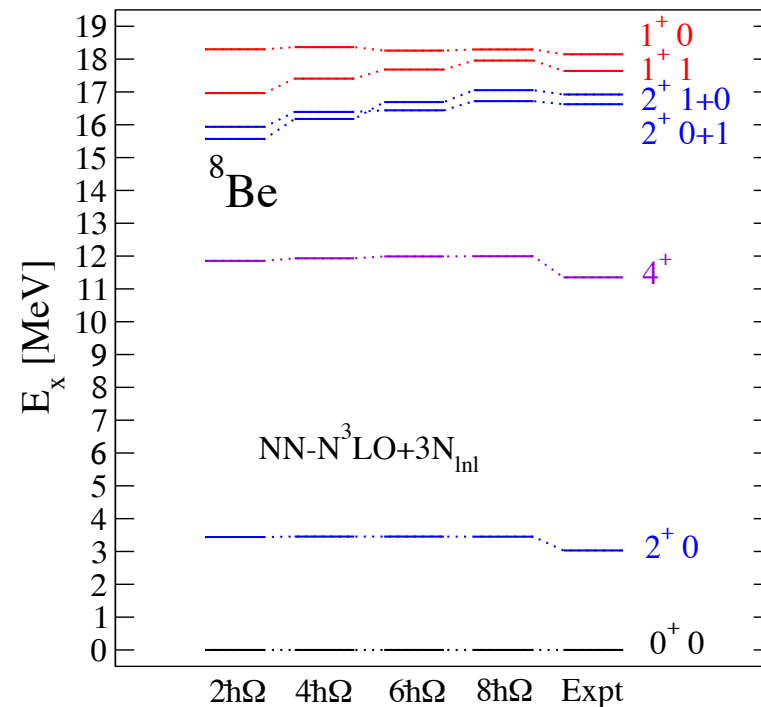
NN $N^3\text{LO}$ (Entem-Machleidt 2003)
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Excitation energies of atomic nuclei from nuclear forces from chiral Effective Field Theory

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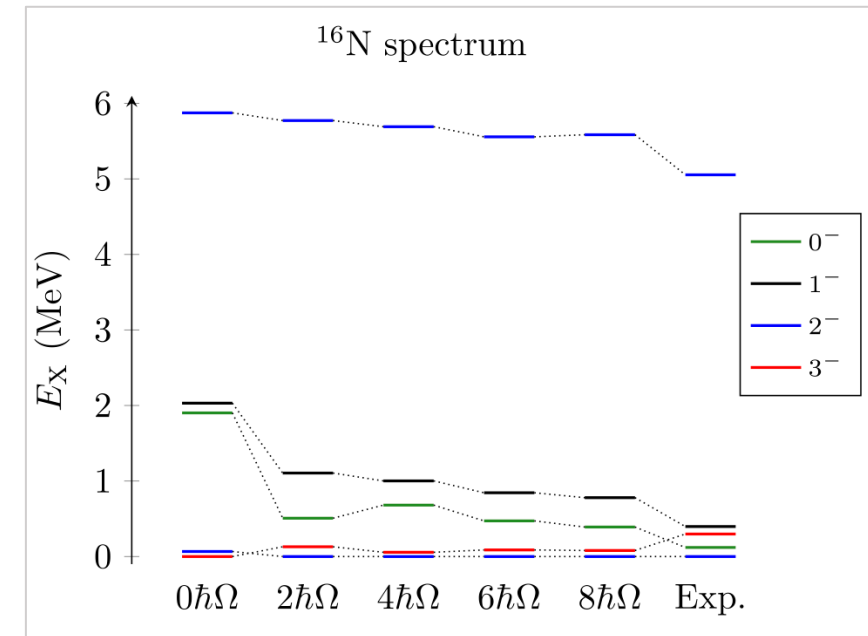
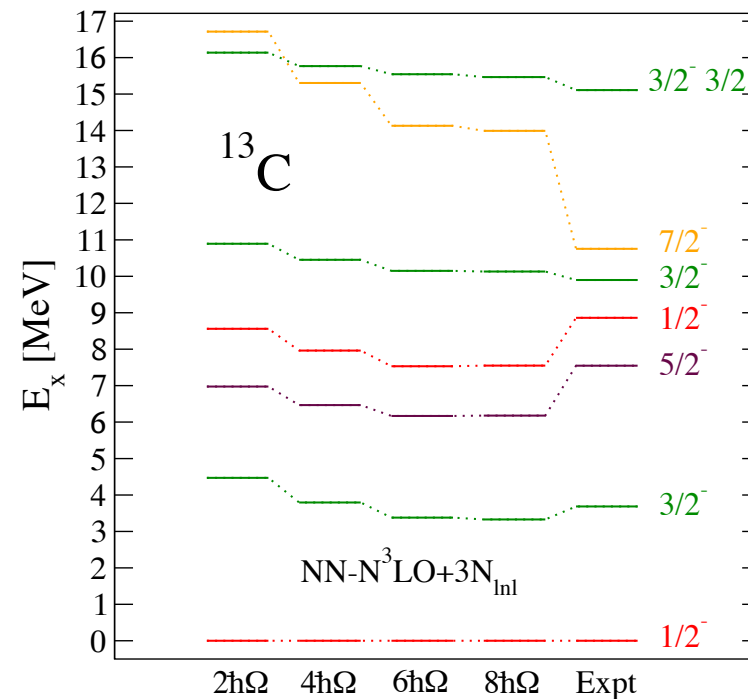
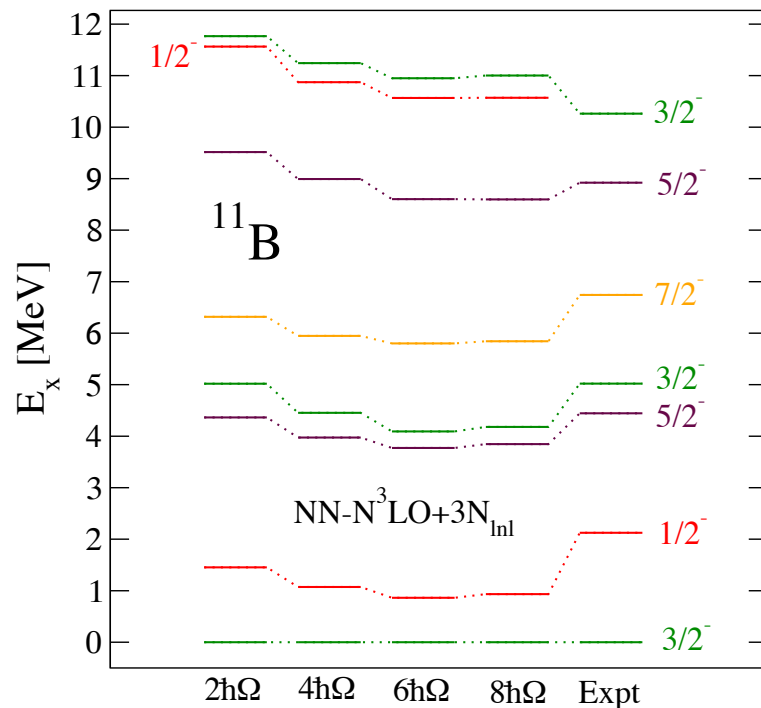
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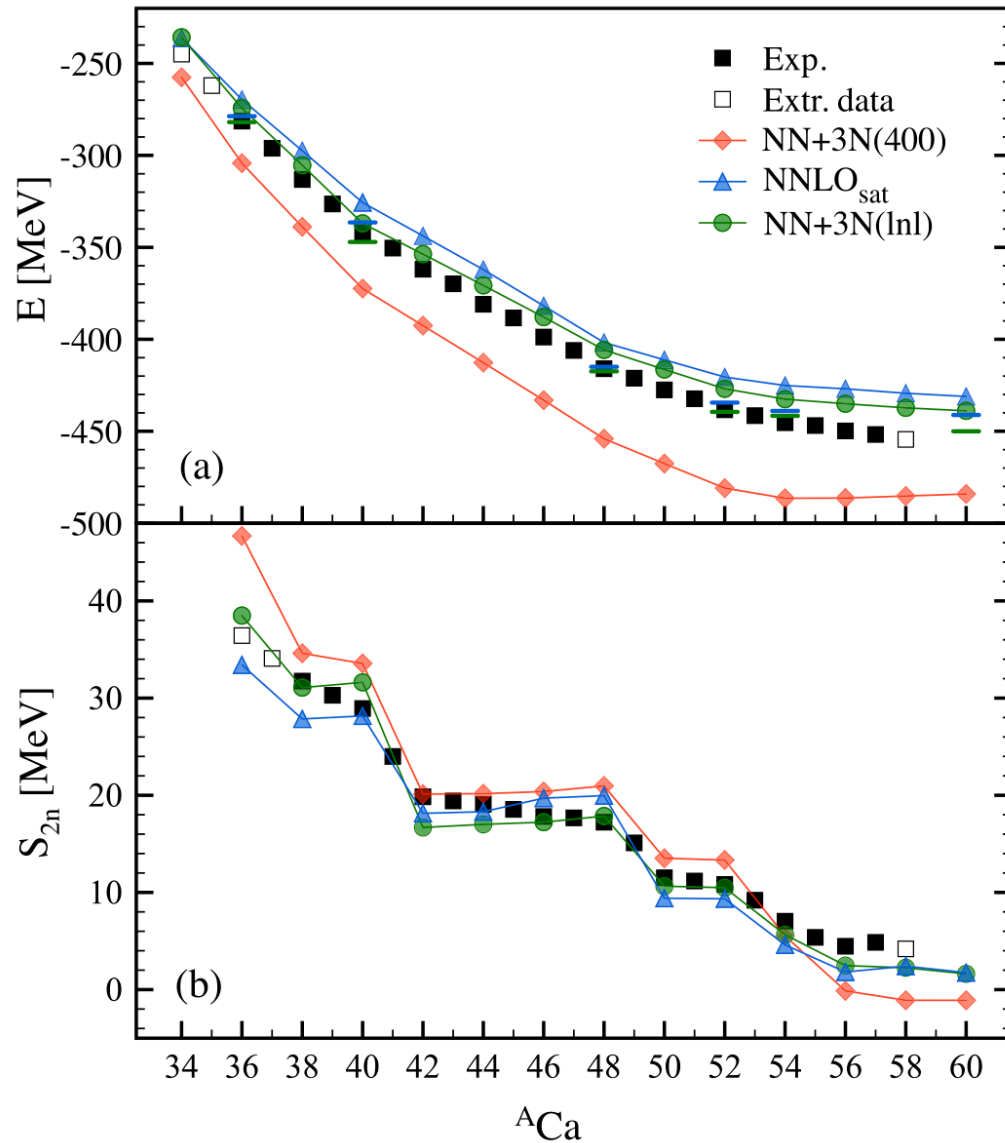


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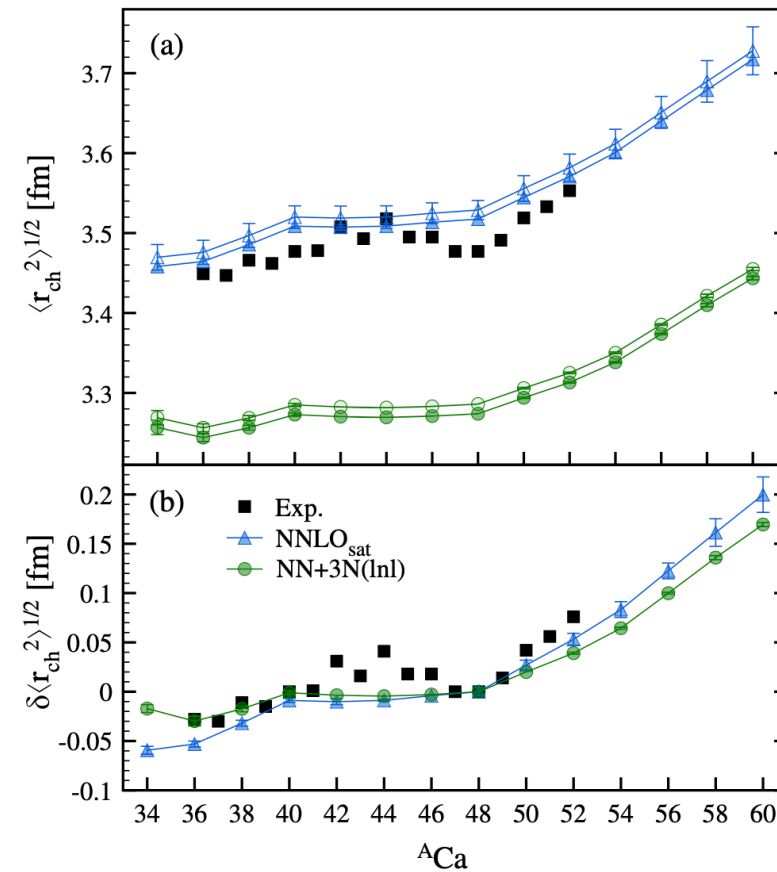
Performance of NN N³LO + 3N_{lnl} in medium-mass nuclei

Novel chiral Hamiltonian and observables in light and medium-mass nuclei

V. Somà,^{1,*} P. Navrátil^{2,†} F. Raimondi,^{3,4,‡} C. Barbieri^{4,§} and T. Duguet^{1,5,||}Eur. Phys. J. A (2021) 57:135
https://doi.org/10.1140/epja/s10050-021-00437-4THE EUROPEAN
PHYSICAL JOURNAL A

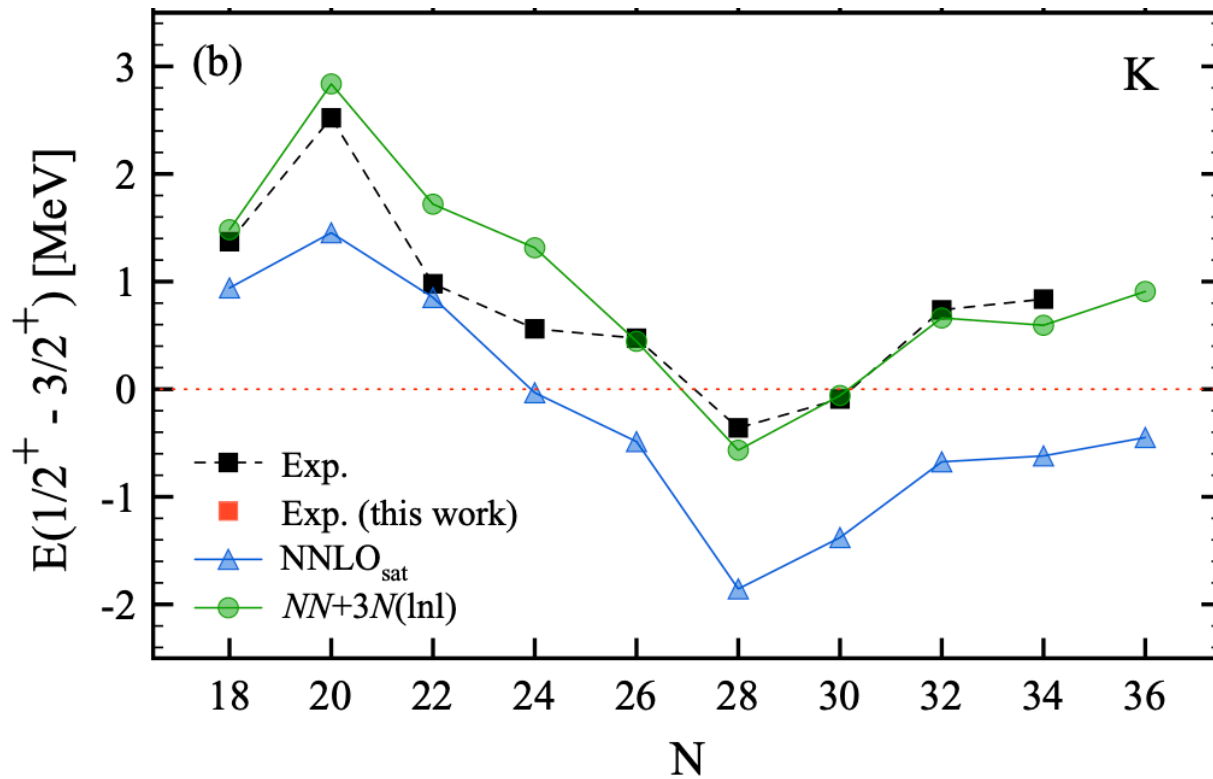
Regular Article - Theoretical Physics

Moving away from singly-magic nuclei with Gorkov Green's function theory

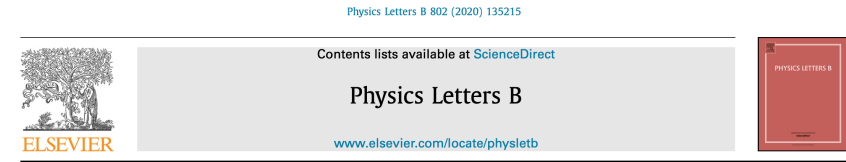
V. Somà^{1,4}, C. Barbieri^{2,3,4}, T. Duguet^{1,5}, P. Navrátil⁶

Performance of NN N³LO + 3N_{lnl} in medium-mass nuclei

- RIKEN experiment on neutron rich odd K isotopes
 - Restoration of the Natural E(1/2⁺) - E(3/2⁺) Energy Splitting towards N=40



NN N³LO+3N_{lnl} performs quite well



Restoration of the natural E(1/2⁺) - E(3/2⁺) energy splitting in odd-K isotopes towards N = 40

Y.L. Sun^{a,b,*}, A. Obertelli^{a,b,c}, P. Doornenbal^c, C. Barbieri^d, Y. Chazono^e, T. Duguet^{b,f}, H.N. Liu^{a,b,g}, P. Navrátil^h, F. Nowacki^{i,j}, K. Ogata^{e,k}, T. Otsuka^{l,m}, F. Raimondi^{d,n}, V. Somà^b, Y. Utsuno^o, K. Yoshida^o, N. Achouri^p, H. Baba^q, F. Browne^c, D. Calvet^b, F. Château^b, S. Chen^{q,c,r}, N. Chiga^c, A. Corsi^b, M.L. Cortés^c, A. Delbart^b, J.-M. Gheller^b, A. Giganon^b, A. Gillibert^b, C. Hilaire^b, T. Isobe^c, T. Kobayashi^q, Y. Kubota^{s,m}, V. Lapoux^b, T. Motobayashi^c, I. Murray^c, H. Otsu^c, V. Panin^c, N. Paul^b, W. Rodriguez^{t,c}, H. Sakurai^{c,l}, M. Sasano^c, D. Steppenbeck^c, L. Stuhl^m, Y. Togano^u, T. Uesaka^c, K. Wimmer^l, K. Yoneda^c, O. Aktas^g, T. Aumann^{a,v}, L.X. Chung^w, F. Flavigny^p, S. Franchoo^p, I. Gašparić^{x,c}, R.-B. Gerst^v, J. Gibelin^z, K.I. Hahn^{aa}, D. Kim^{aa,c}, T. Koiwai¹, Y. Kondo^{ab}, P. Koseoglou^{a,v}, J. Lee^r, C. Lehr^a, B.D. Linh^w, T. Lokotko^r, M. MacCormick^p, K. Moschner^y, T. Nakamura^{ab}, S.Y. Park^{aa,c}, D. Rossi^a, E. Sahin^{ac}, D. Sohler^{ad}, P.-A. Söderström^a, S. Takeuchi^{ab}, H. Törnqvist^{a,v}, V. Vaquero^{ac}, V. Wagner^a, S. Wang^{af}, V. Werner^a, X. Xu¹, H. Yamada^{ab}, D. Yan^{af}, Z. Yang^c, M. Yasuda^{ab}, L. Zanetti^a

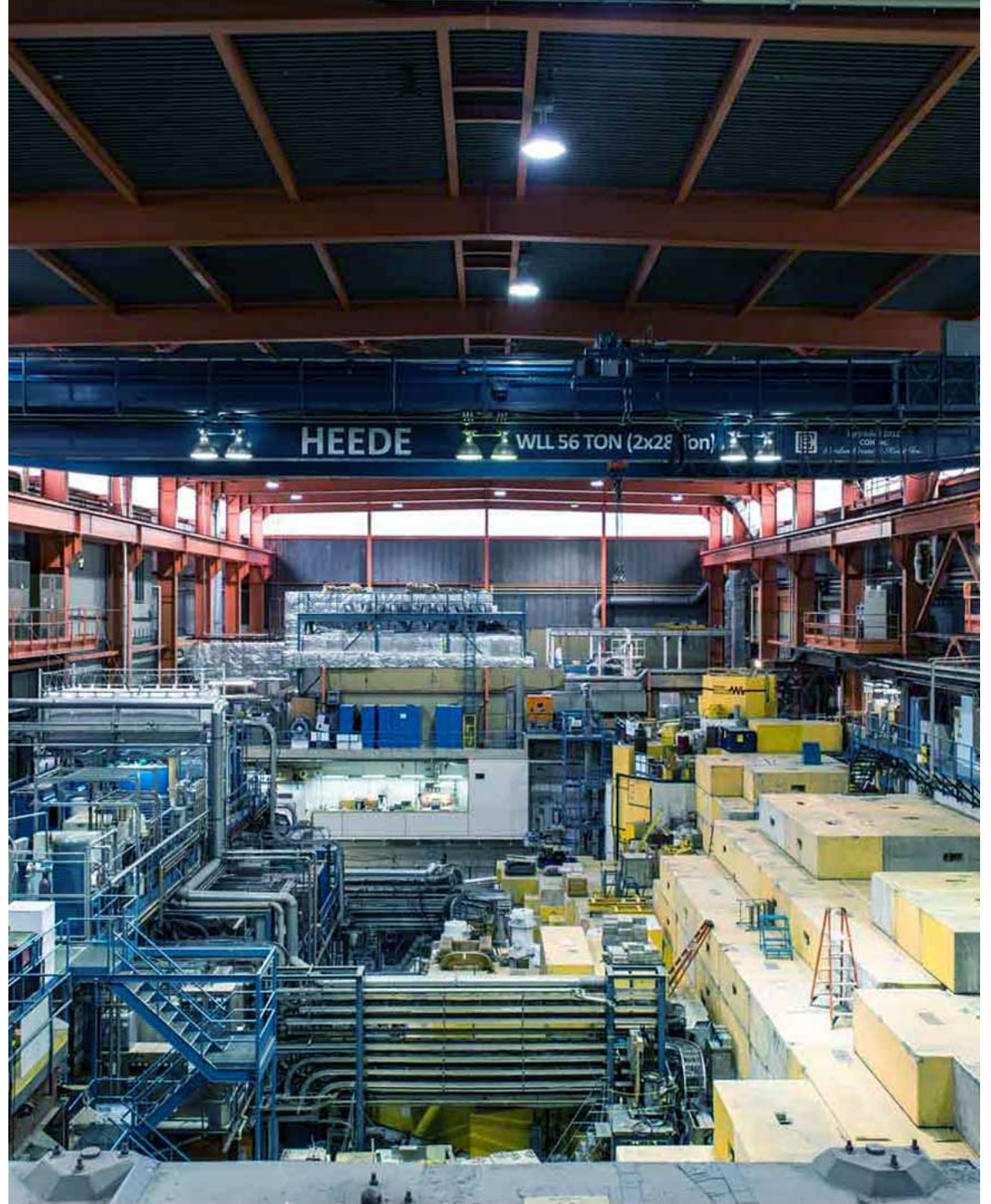
PHYSICAL REVIEW C **104**, 044331 (2021)

Investigation of the ground-state spin inversion in the neutron-rich ^{47,49}Cl isotopes

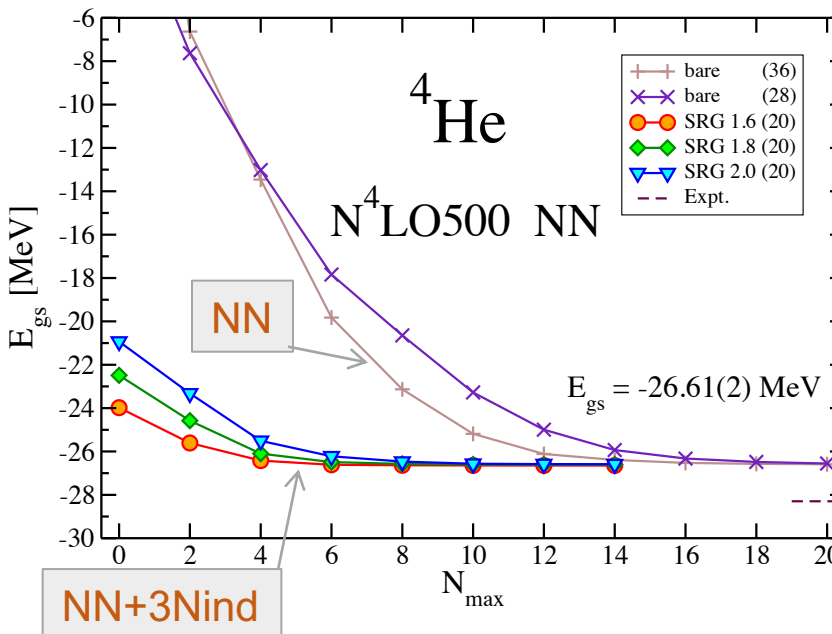
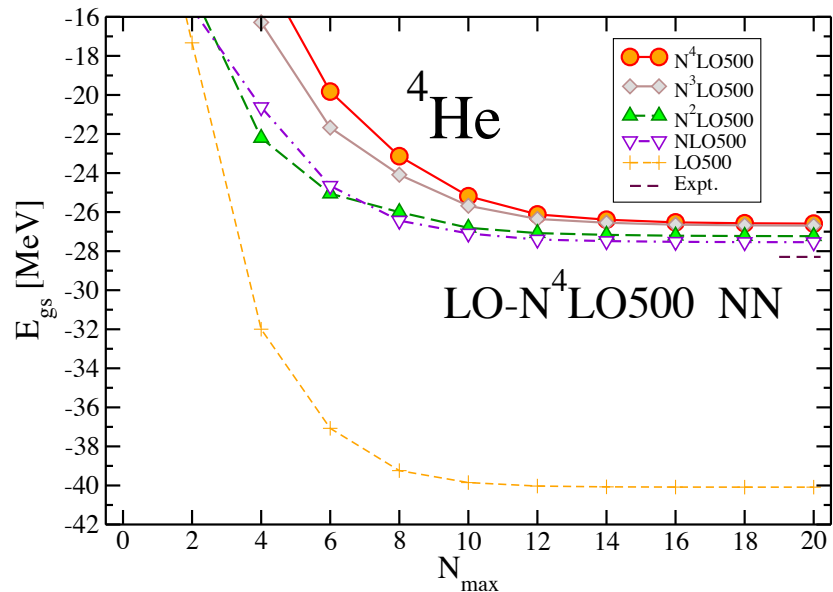
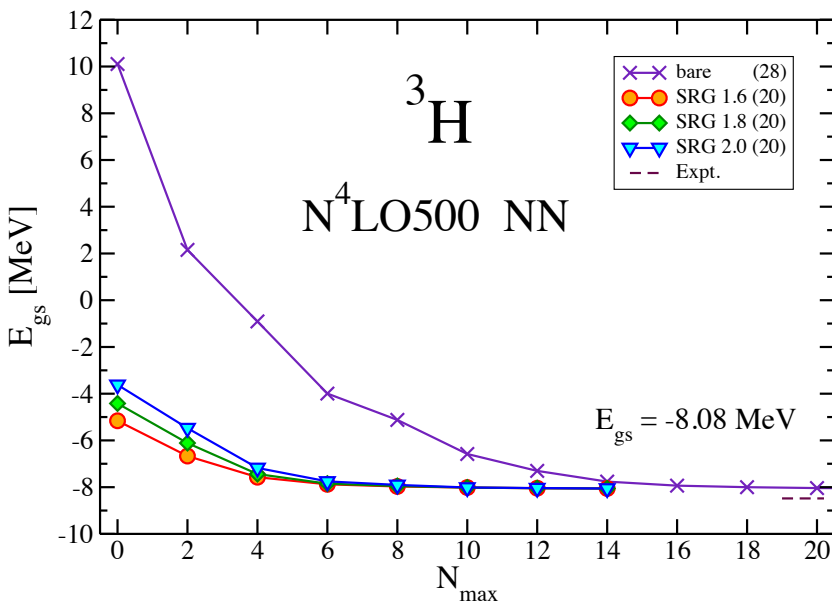
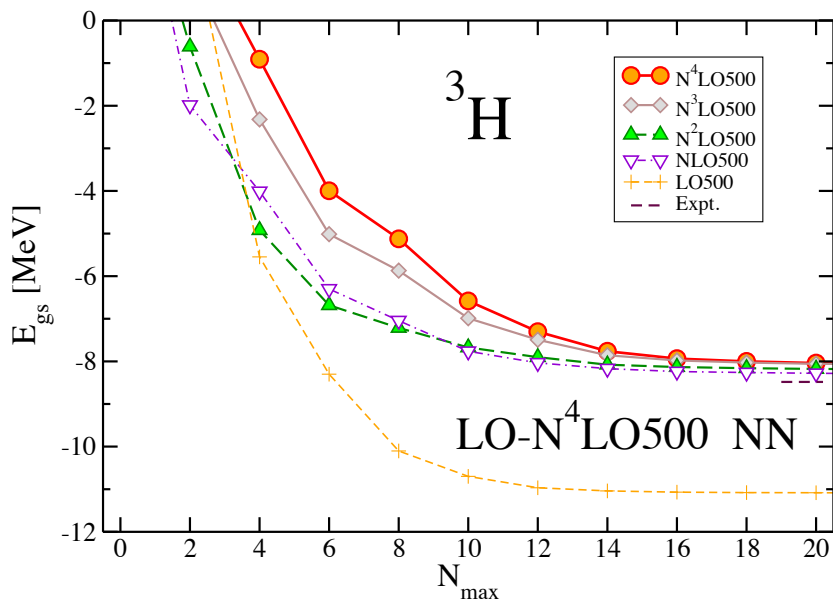
B. D. Linh,¹ A. Corsi,² A. Gillibert,^{2,*} A. Obertelli,^{2,3,4} P. Doornenbal,³ C. Barbieri,^{5,6,7} S. Chen,^{8,3,9} L. X. Chung,¹ T. Duguet,^{2,10} M. Gómez-Ramos,^{4,11} J. D. Holt,^{12,13} A. Moro,¹¹ P. Navrátil,¹² K. Ogata,^{14,15} N. T. T. Phuc,^{16,17} N. Shimizu,¹⁸ V. Somà,² Y. Utsuno,^{18,19} N. L. Achouri,²⁰ H. Baba,³ F. Browne,³ D. Calvet,² F. Château,² N. Chiga,³ M. L. Cortés,³ A. Delbart,² J.-M. Gheller,² A. Giganon,² C. Hilaire,² T. Isobe,³ T. Kobayashi,²¹ Y. Kubota,^{3,18} V. Lapoux,² H. N. Liu,^{2,4,22} T. Motobayashi,³ I. Murray,^{23,3} H. Otsu,³ V. Panin,³ N. Paul,^{2,24} W. Rodriguez,^{3,25,26} H. Sakurai,^{3,27} M. Sasano,³ D. Steppenbeck,³ L. Stuhl,^{18,28,29} Y. L. Sun,^{2,4} Y. Togano,³⁰ T. Uesaka,³ K. Wimmer,^{27,3} K. Yoneda,³ O. Aktas,²² T. Aumann,^{4,31} F. Flavigny,^{23,20} S. Franchoo,²³ I. Gašparić,^{32,4,3} R.-B. Gerst,³³ J. Gibelin,²⁰ K. I. Hahn,^{34,29} N. T. Khai,³⁵ D. Kim,^{34,3,29} T. Koiwai,²⁷ Y. Kondo,³⁶ P. Koseoglou,^{4,31} J. Lee,⁸ C. Lehr,⁸ T. Lokotko,⁸ M. MacCormick,²³ K. Moschner,³³ T. Nakamura,³⁶ S. Y. Park,^{34,29} D. Rossi,⁴ E. Sahin,³⁷ D. Sohler,²⁸ P.-A. Söderström,⁴ S. Takeuchi,³⁶ N. D. Ton,¹ H. Törnqvist,^{4,31} V. Vaquero,³⁸ V. Wagner,⁴ H. Wang,³⁹ V. Werner,⁴ X. Xu,⁸ Y. Yamada,³⁶ D. Yan,³⁹ Z. Yang,³ M. Yasuda,³⁶ and L. Zanetti⁴

Precision chiral EFT NN Hamiltonian
- order by order convergence

2026-04-29

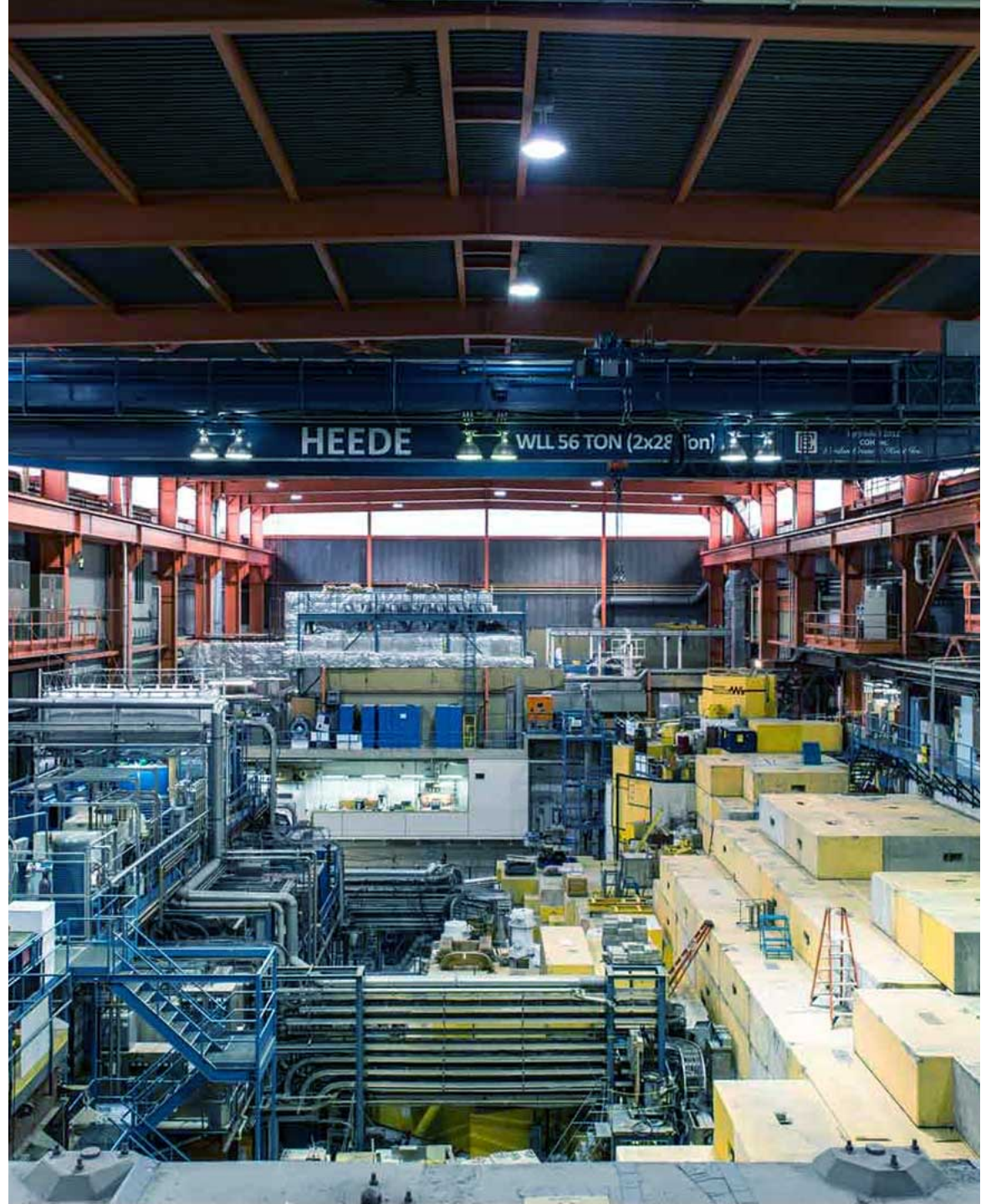


³H and ⁴He with chiral EFT interactions up to N⁴LO



Precision chiral EFT Hamiltonian with a sub-leading $3N$ interaction term

2026-04-29



Precision chiral EFT Hamiltonian with sub-leading 3N interaction terms

- NN N⁴LO 500 interaction by Entem-Machleidt-Nosyk (2017)
- 3N N²LO plus a sub-leading spin-orbit enhancing term with a new LEC (E_7) – Girlanda 2011
 - local/non-local regulator
 - **The Hamiltonian fully determined in $A=2, A=3,4$, and ${}^6\text{Li}$ systems**
 - Nucleon–nucleon scattering, deuteron properties, ${}^3\text{H}$ and ${}^4\text{He}$ binding energy, ${}^3\text{H}$ half life
 - New LEC (E_7) fitted to improve excitation levels in ${}^6\text{Li}$
 - Denoted as NN N⁴LO + 3N_{InLE7}

$$\begin{aligned}
 V = & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
 & + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
 & + [(E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik}] Z_0'(r_{ij}) Z_0'(r_{ik})
 \end{aligned}$$

PHYSICAL REVIEW C, VOLUME 60, 034001

Phenomenological spin-orbit three-body force

A. Kievsky*

$$Z_0(r; \Lambda) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} F(\mathbf{p}^2; \Lambda)$$

PHYSICAL REVIEW C **84**, 014001 (2011)

Subleading contributions to the three-nucleon contact interaction

L. Girlanda,¹ A. Kievsky,² and M. Viviani²

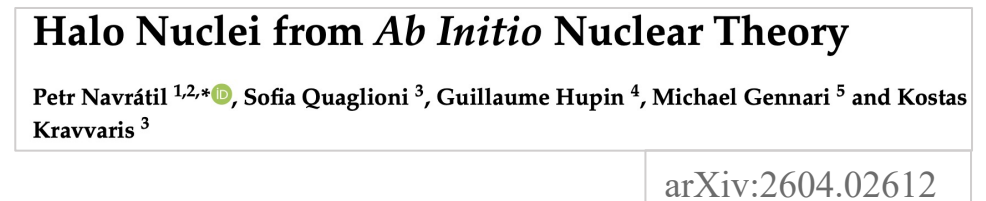
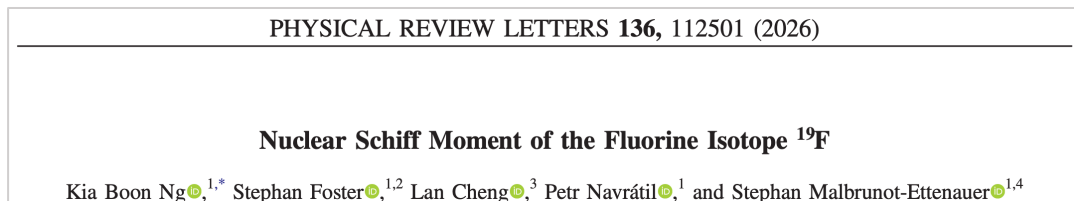
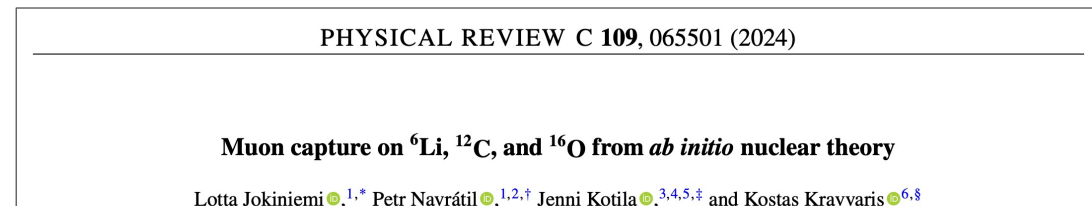
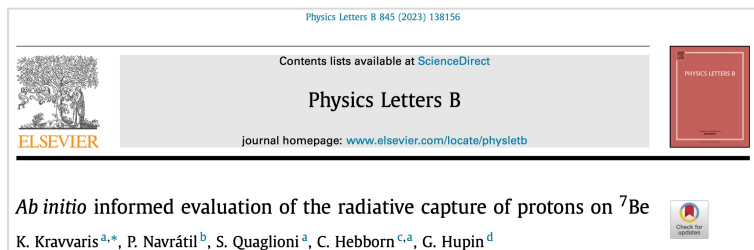
PHYSICAL REVIEW C **102**, 019903(E) (2020)

Erratum: Subleading contributions to the three-nucleon contact interaction
[Phys. Rev. C **84**, 014001 (2011)]

L. Girlanda , A. Kievsky, and M. Viviani

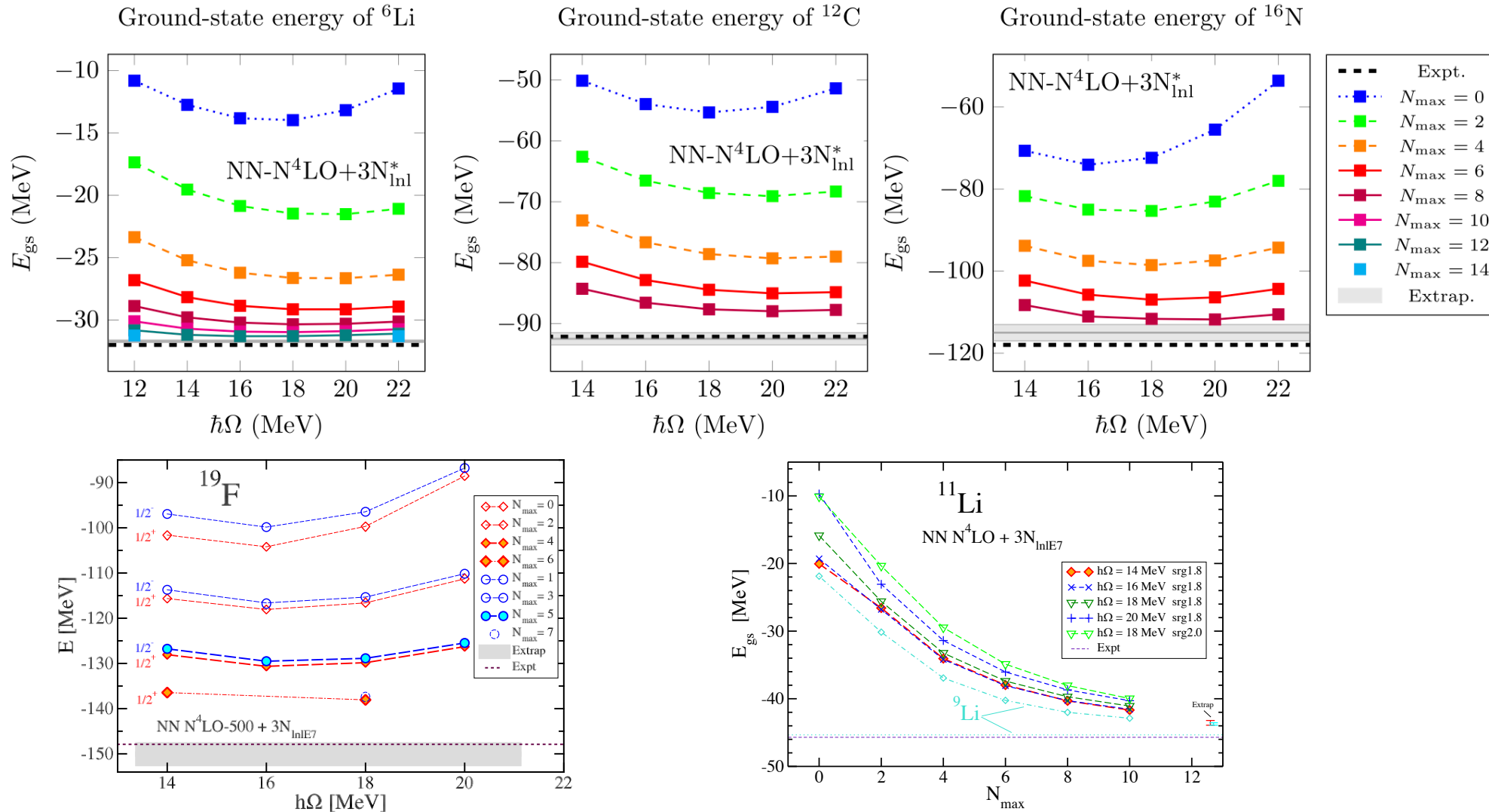
Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- NN N⁴LO 500 interaction by Entem-Machleidt-Nosyk (2017)
- 3N N²LO plus a sub-leading spin-orbit enhancing term with a new LEC (E_7) – Girlanda 2011
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 - New LEC (E_7) fitted to improve excitation levels in ${}^6\text{Li}$
 - Denoted as NN N⁴LO + 3N_{InIE7}
- Successfully applied to ${}^7\text{Be}(p,\gamma){}^8\text{B}$, muon capture on ${}^6\text{Li}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$, ${}^{11}\text{Li}$ structure, ${}^{19}\text{F}$ structure and exotic moments, N=20 isotones



Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- Energies of light nuclei



Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

▪ Energies of light nuclei





TABLE IV. The extrapolated ground-state energies of ${}^6\text{He}$, ${}^6\text{Li}$, ${}^{12}\text{C}$, ${}^{12}\text{B}$, ${}^{16}\text{O}$, and ${}^{16}\text{N}$.

Nucleus	Interaction	$E_{\text{g.s.}}$ (MeV)	Expt. (MeV) [48]
${}^6\text{He}$	NN-N ${}^3\text{LO}+3\text{N}_{\text{Inl}}$	-28.3(1)	-29.27
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}$	-28.3(1)	
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}^*$	-28.6(1)	
${}^6\text{Li}$	NN-N ${}^3\text{LO}+3\text{N}_{\text{Inl}}$	-31.5(1)	-31.99
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}$	-31.4(1)	
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}^*$	-31.7(1)	
${}^{12}\text{C}$	NN-N ${}^3\text{LO}+3\text{N}_{\text{Inl}}$	-88.7(10)	-92.16
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}$	-88.6(10)	
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}^*$	-92.5(10)	
${}^{12}\text{B}$	NN-N ${}^3\text{LO}+3\text{N}_{\text{Inl}}$	-76.1(10)	-79.57
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}$	-76.0(10)	
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}^*$	-79.5(10)	
${}^{16}\text{O}$	NN-N ${}^3\text{LO}+3\text{N}_{\text{Inl}}$	-126(2)	-127.62
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}$	-127(2)	
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}^*$	-127(2)	
${}^{16}\text{N}$	NN-N ${}^3\text{LO}+3\text{N}_{\text{Inl}}$	-114(2)	-116.58
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}$	-114(2)	
	NN-N ${}^4\text{LO}+3\text{N}_{\text{Inl}}^*$	-115(2)	

	NN Bare	N ${}^4\text{LO}+3\text{N}_{\text{Inl}}^{\text{E7}}$ srg2.0	srg1.8	Expt. (MeV)
${}^3\text{H}$	-8.49			-8.481795
${}^3\text{He}$	-7.73			-7.71804
${}^4\text{He}$	-28.23(5)	-28.38	-28.45	-28.29566

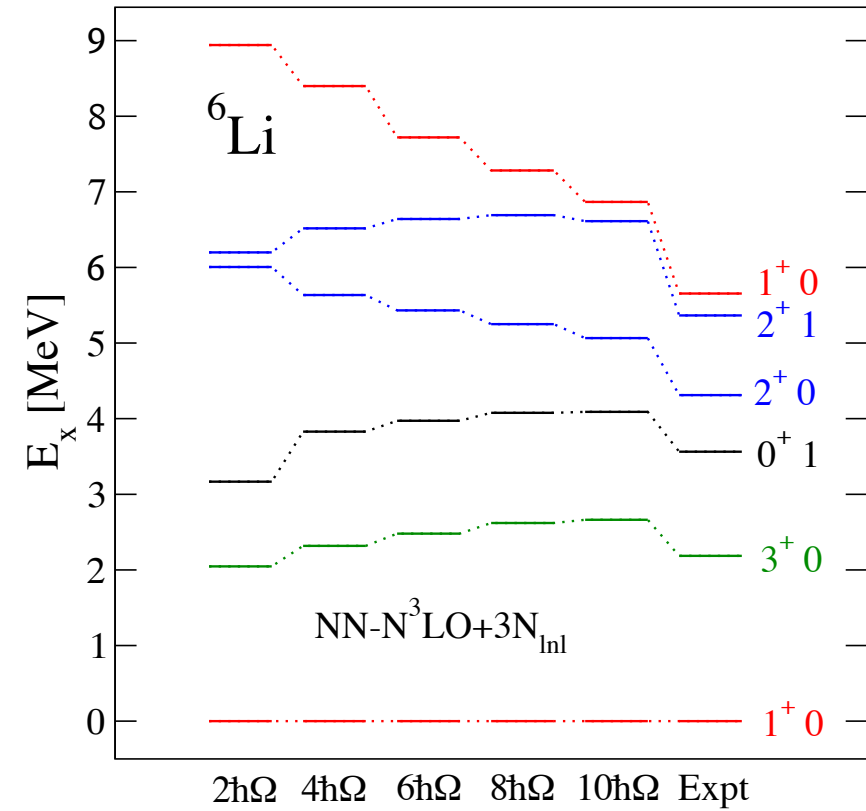
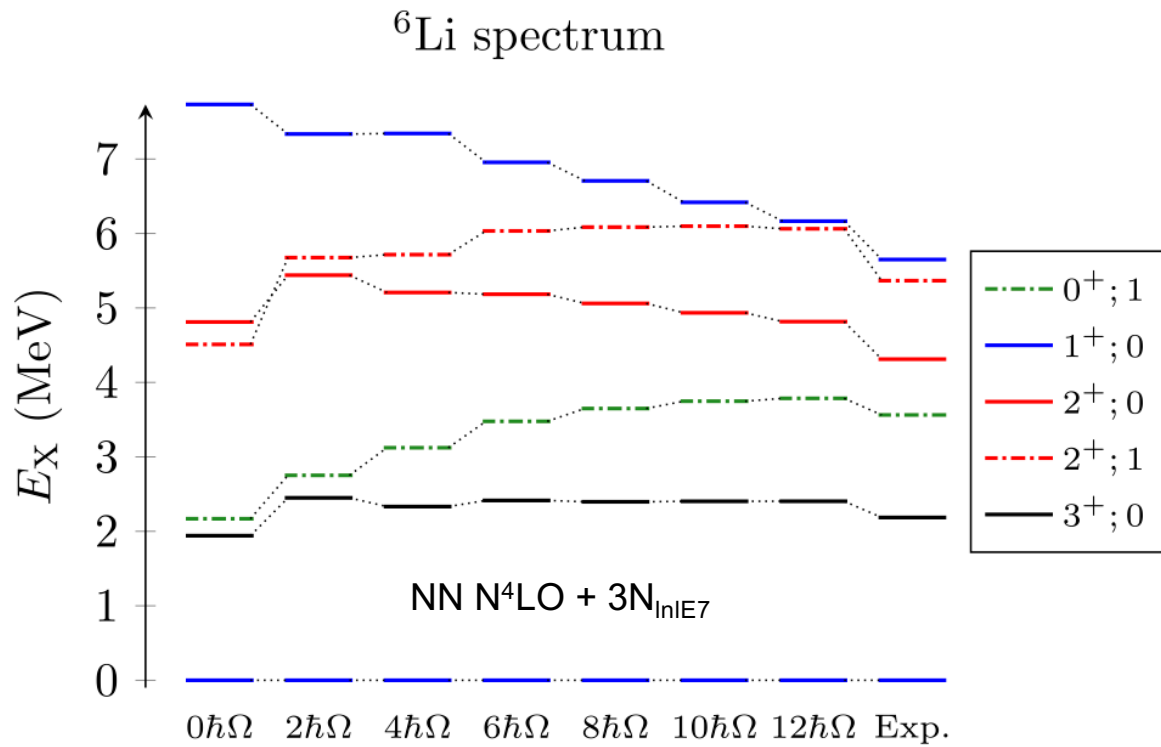
PHYSICAL REVIEW C **109**, 065501 (2024)

Muon capture on ${}^6\text{Li}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$ from *ab initio* nuclear theory

Lotta Jokiniemi ^{1,*} Petr Navrátil ^{1,2,†} Jenni Kotila ^{3,4,5,‡} and Kostas Kravvaris ^{6,§}

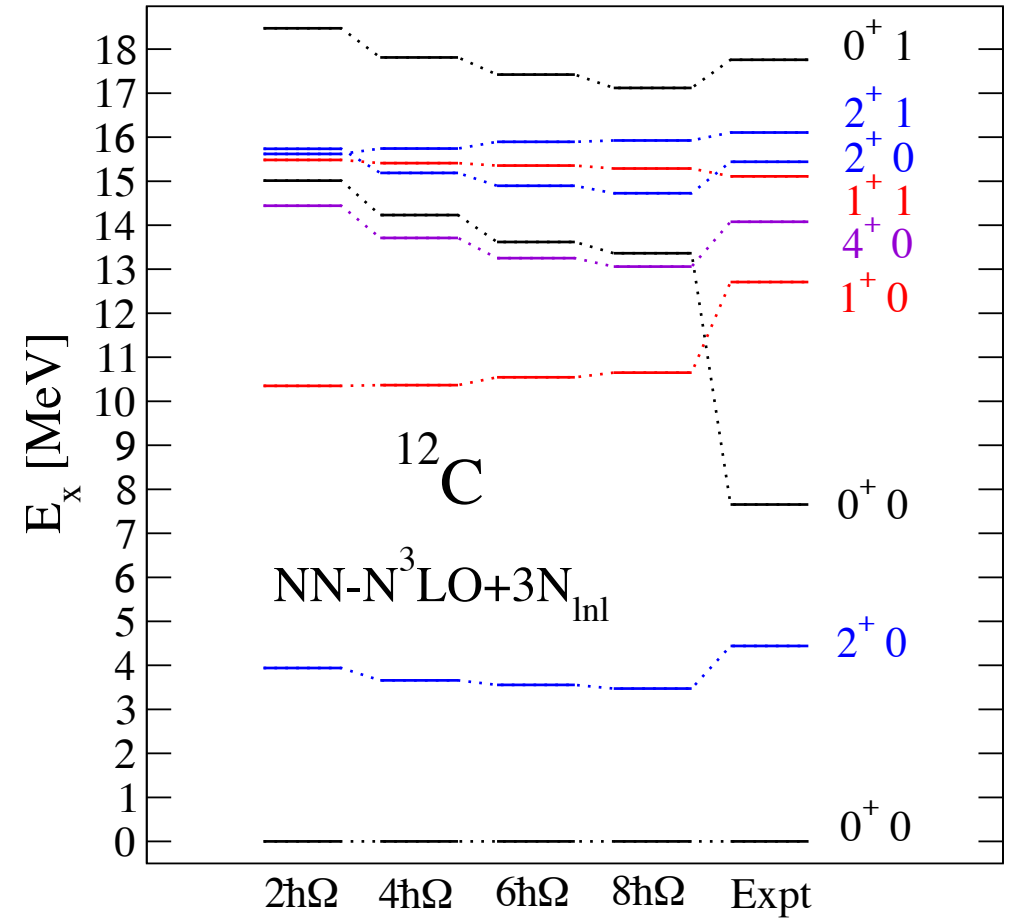
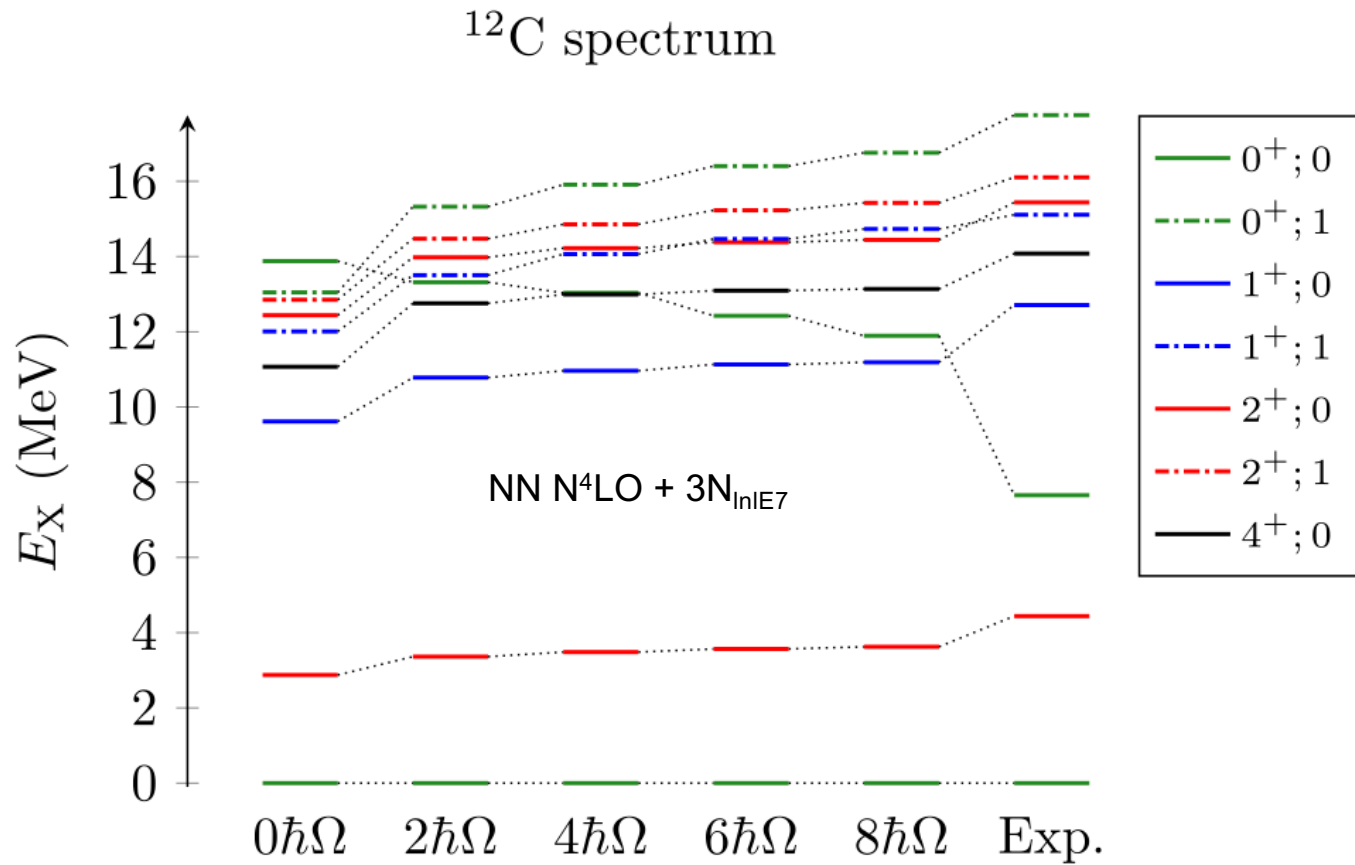
Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- Excitation energies of light nuclei



Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

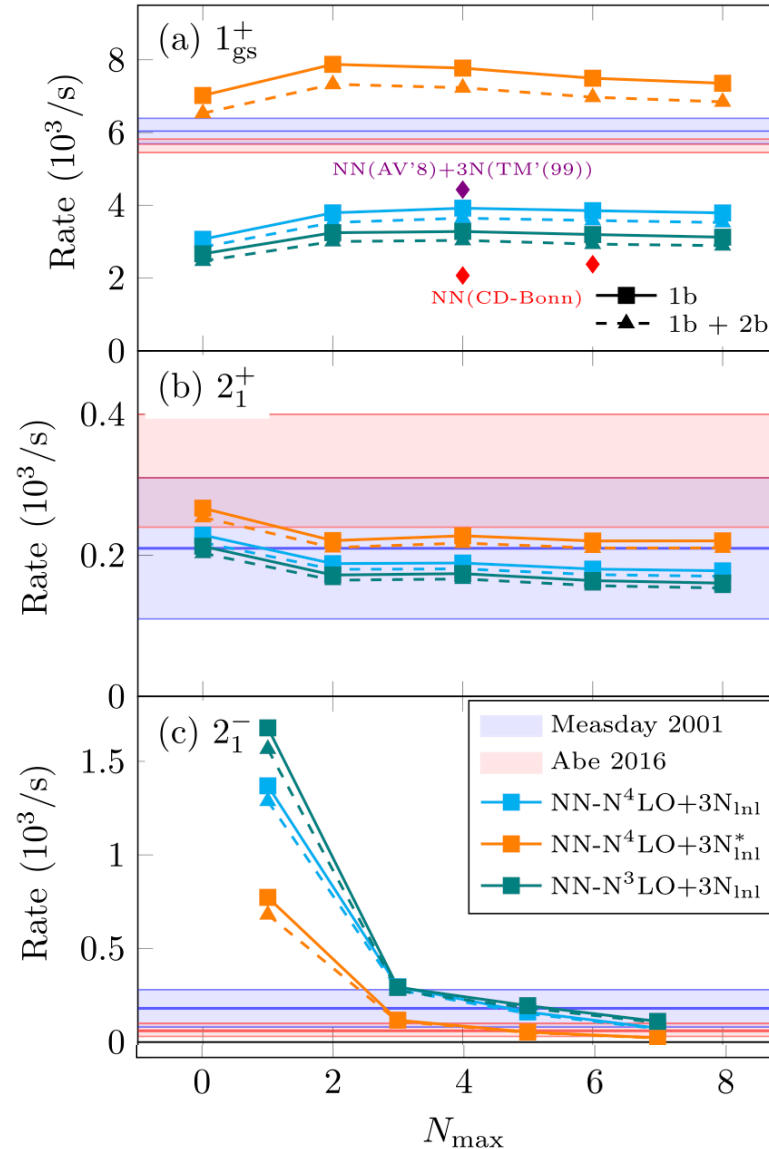
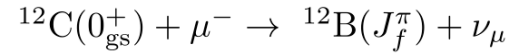
- Excitation energies of light nuclei



Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- Muon capture rate on ^{12}C

Improvement for ^{12}C muon capture rate compared to other interactions



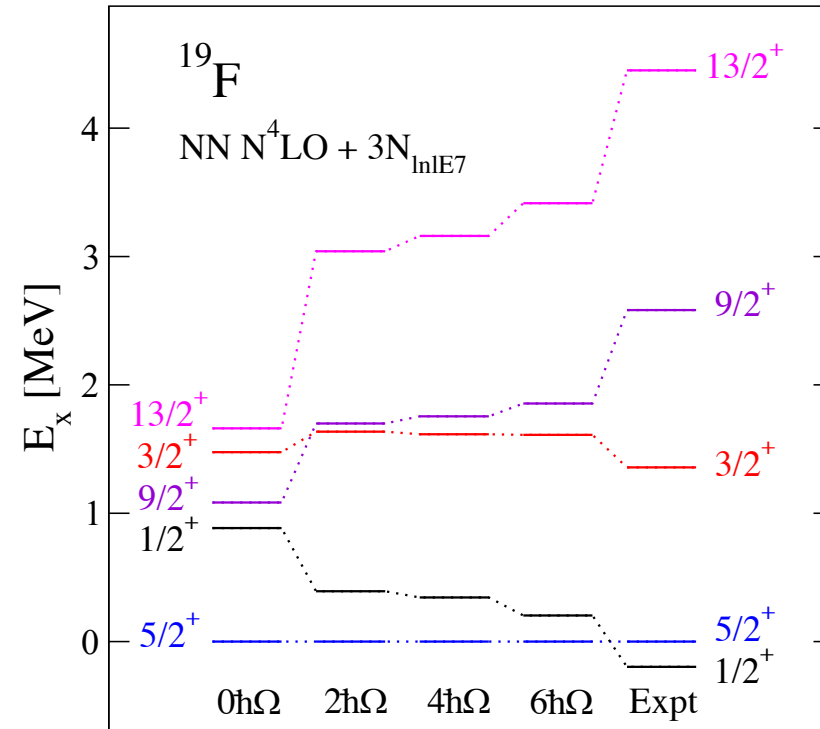
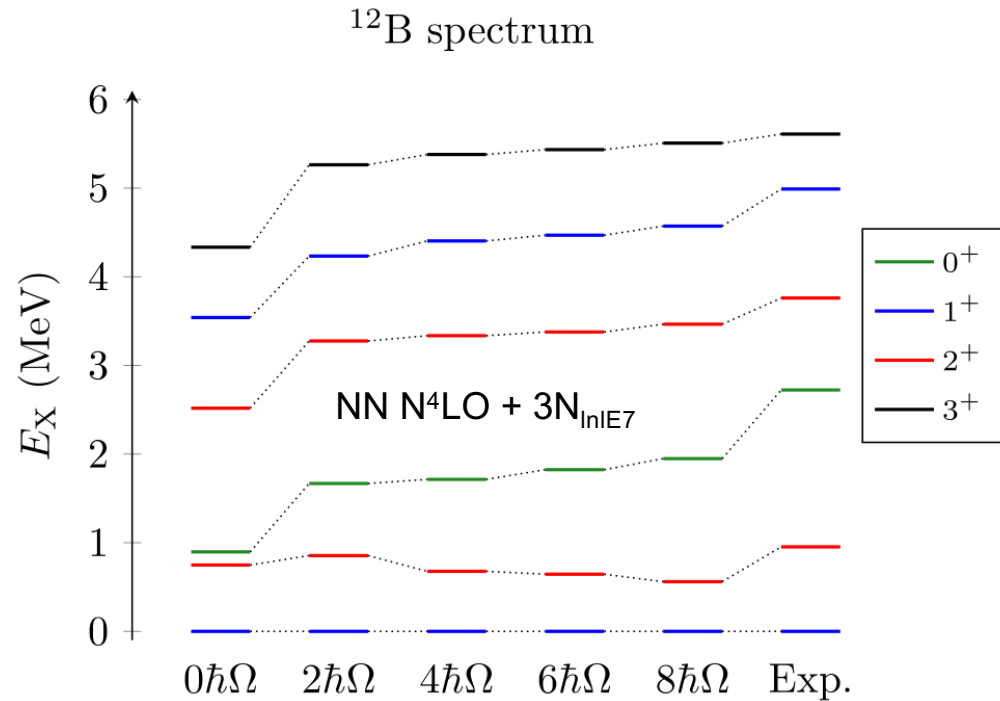
PHYSICAL REVIEW C **109**, 065501 (2024)

Muon capture on ^6Li , ^{12}C , and ^{16}O from *ab initio* nuclear theory

Lotta Jokiniemi ^{1,*}, Petr Navrátil ^{1,2,†}, Jenni Kotila ^{3,4,5,‡} and Kostas Kravvaris ^{6,§}

Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- Excitation energies of light nuclei

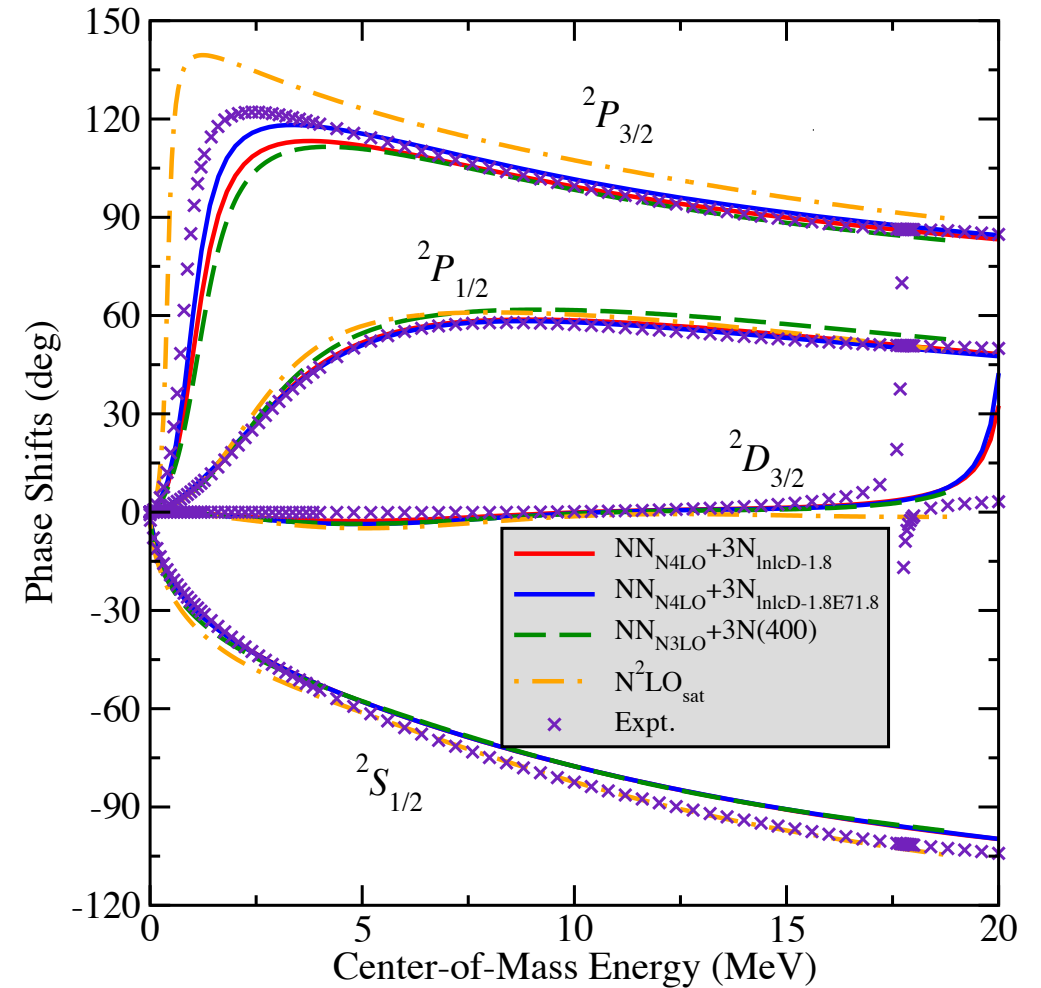
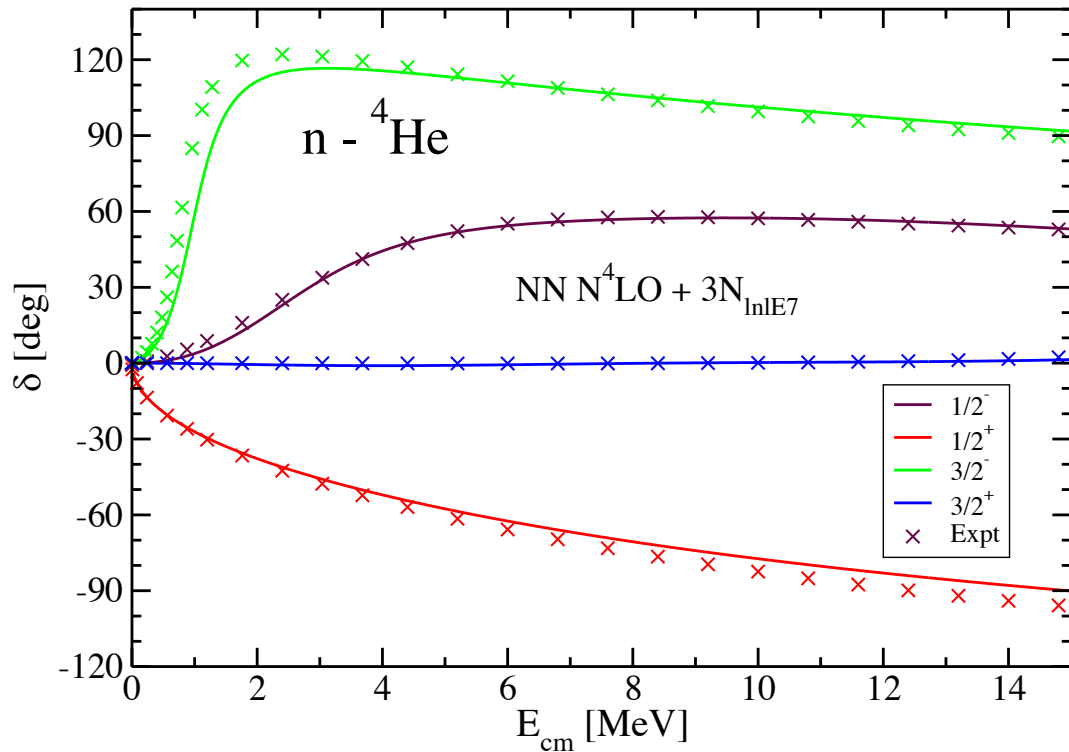


Basis dimension
1.35 billion

Describes well ground-state energies & excitation levels of light nuclei

Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

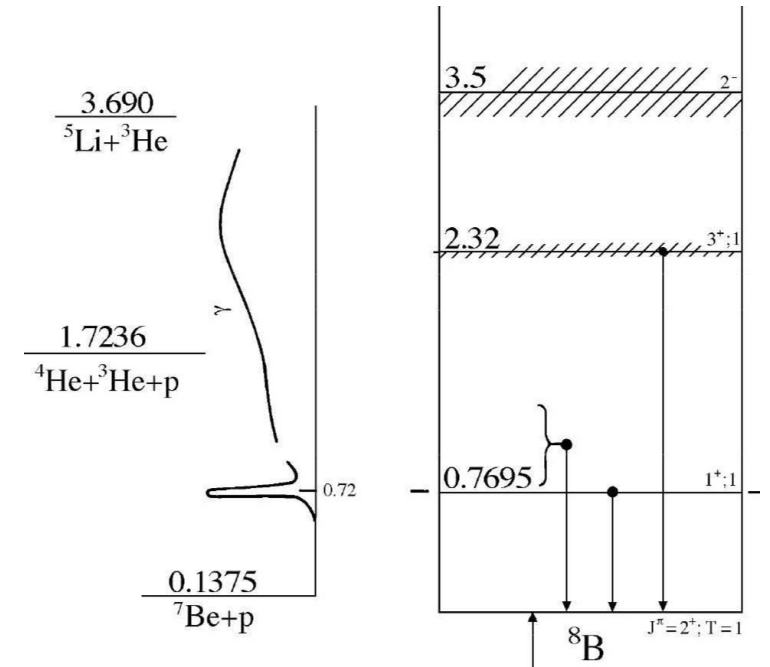
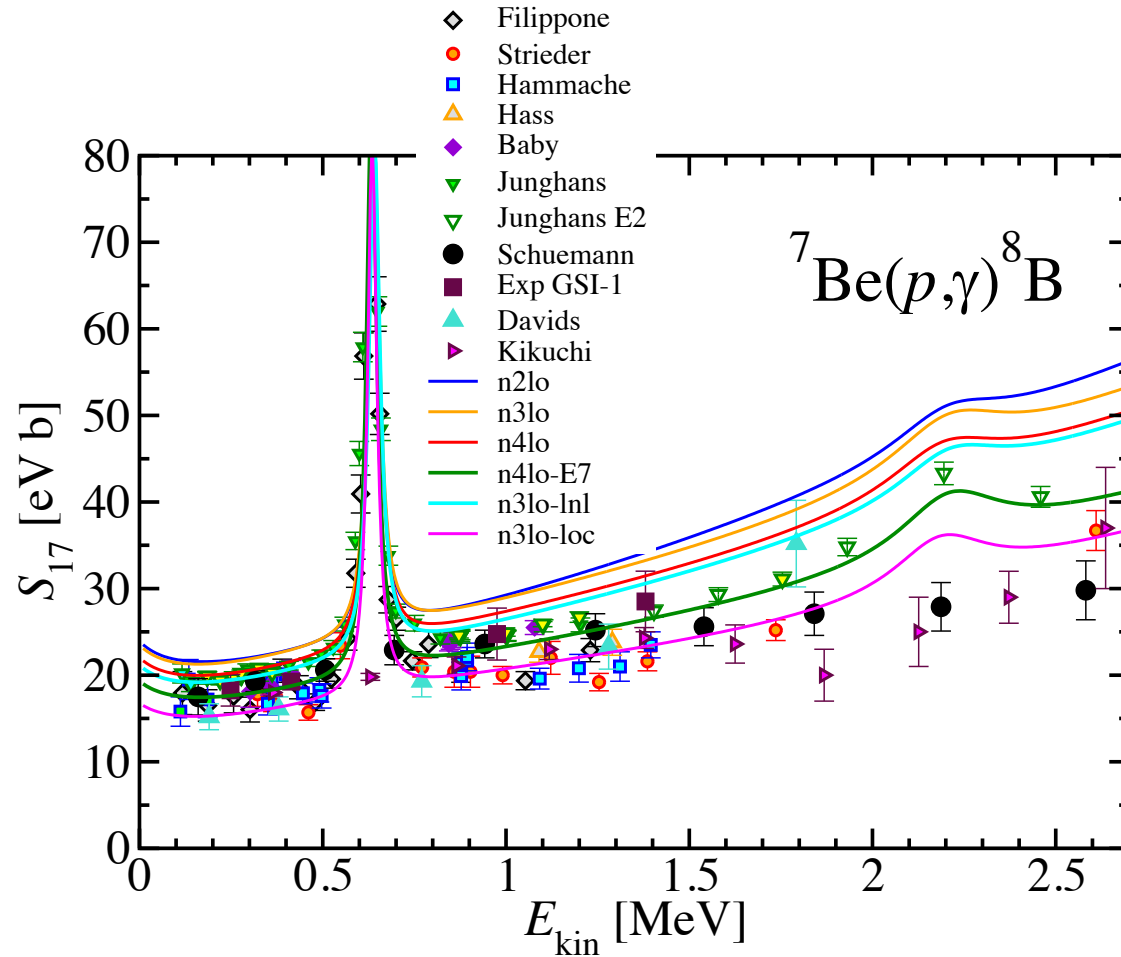
n+⁴He scattering



Improvement for ⁵He P-wave resonances compared to other interactions

Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- ${}^7\text{Be}(p,\gamma){}^8\text{B}$ radiative capture



Improvement for ${}^7\text{Be}(p,\gamma){}^8\text{B}$ S-factor compared to other interactions

Physics Letters B 845 (2023) 138156

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Ab initio informed evaluation of the radiative capture of protons on ${}^7\text{Be}$

K. Kravvaris^{a,*}, P. Navrátil^b, S. Quaglioni^a, C. Hebborn^{c,a}, G. Hupin^d

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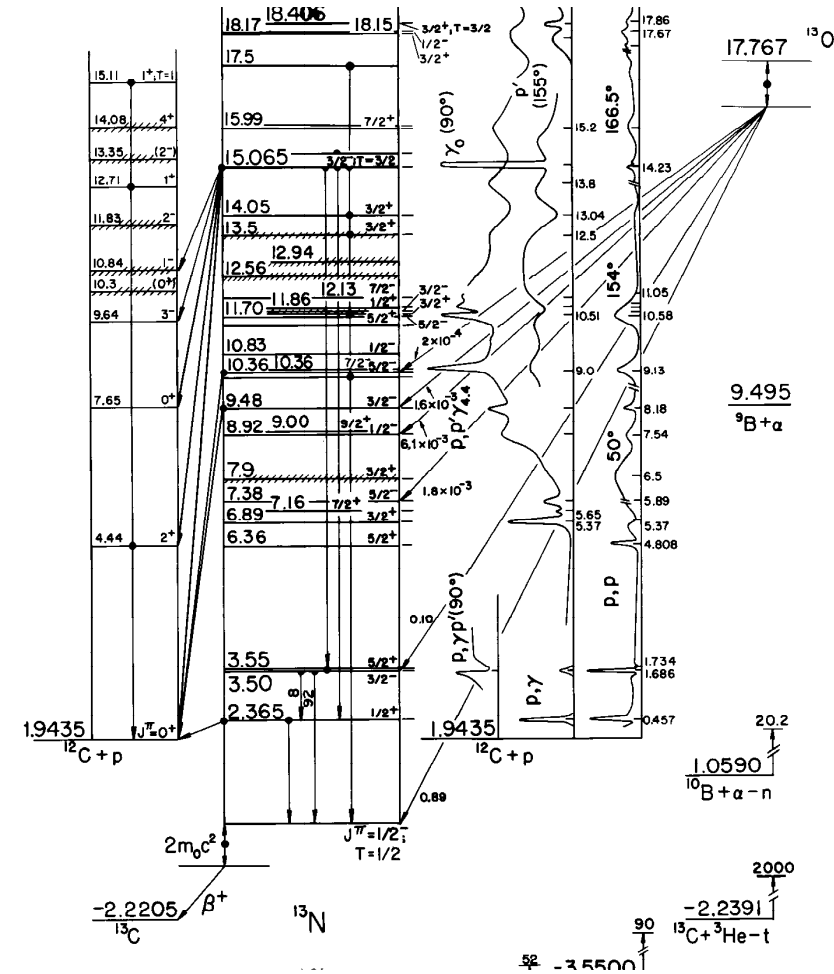
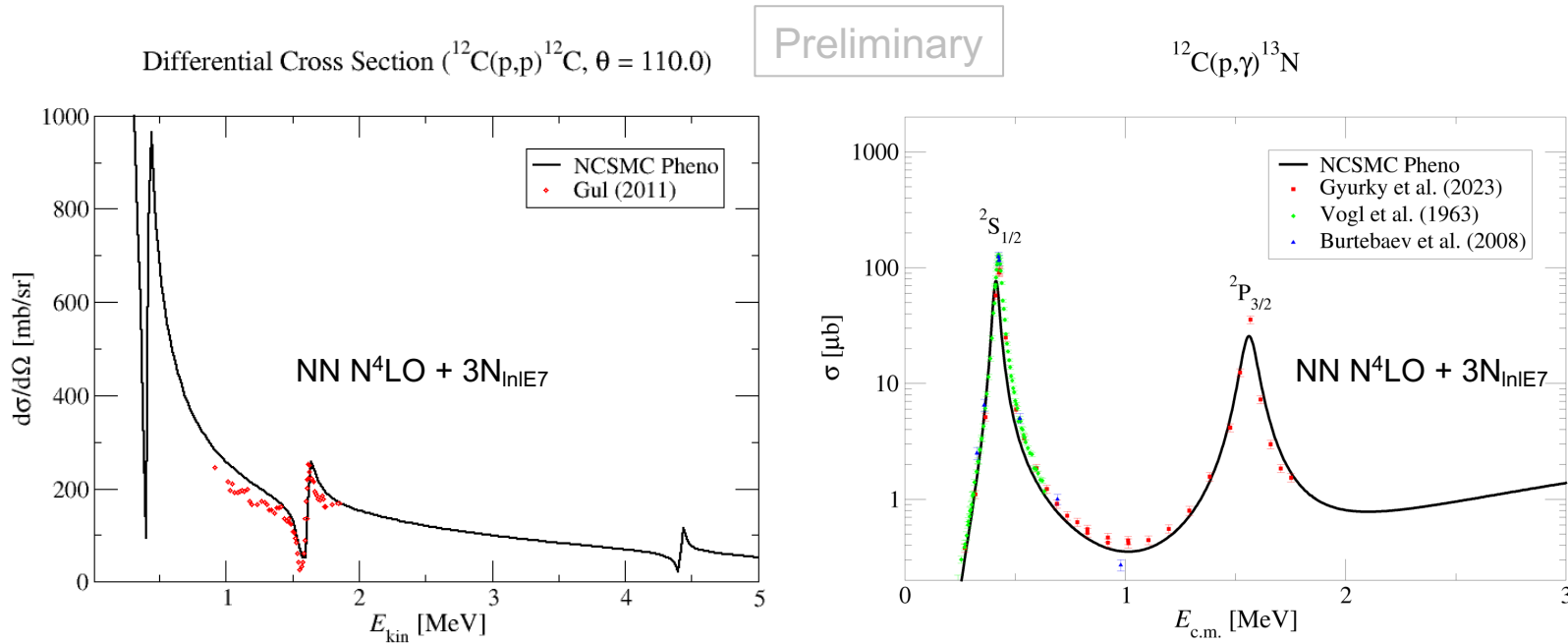
PHYSICS LETTERS B

Check for updates

Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- $p+^{12}\text{C}$ scattering and capture

Calculations in collaboration with Marshall Kodar (UVic undergrad)



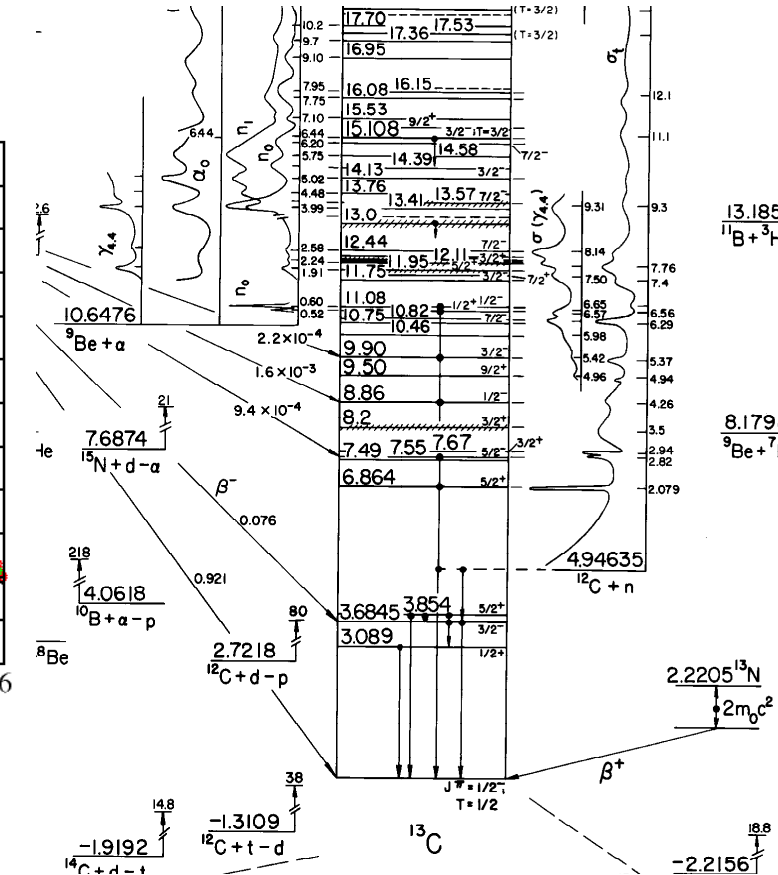
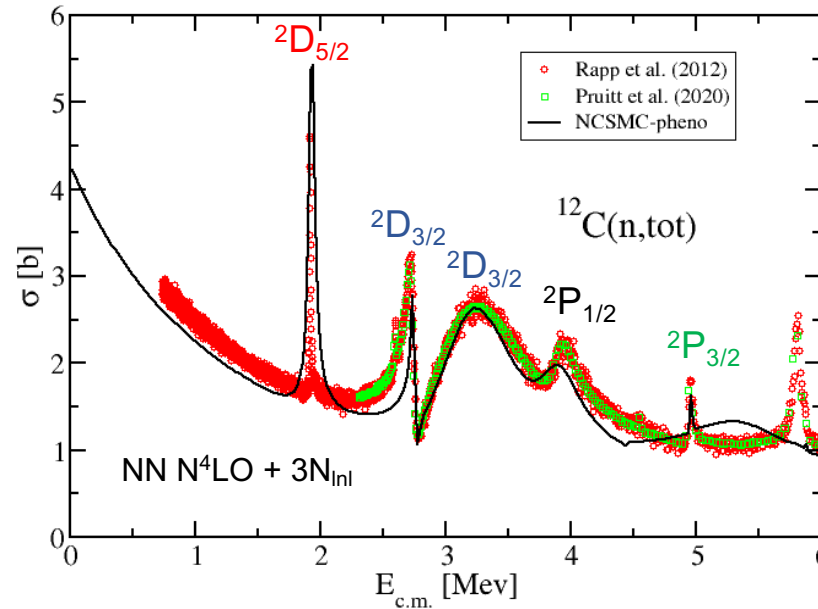
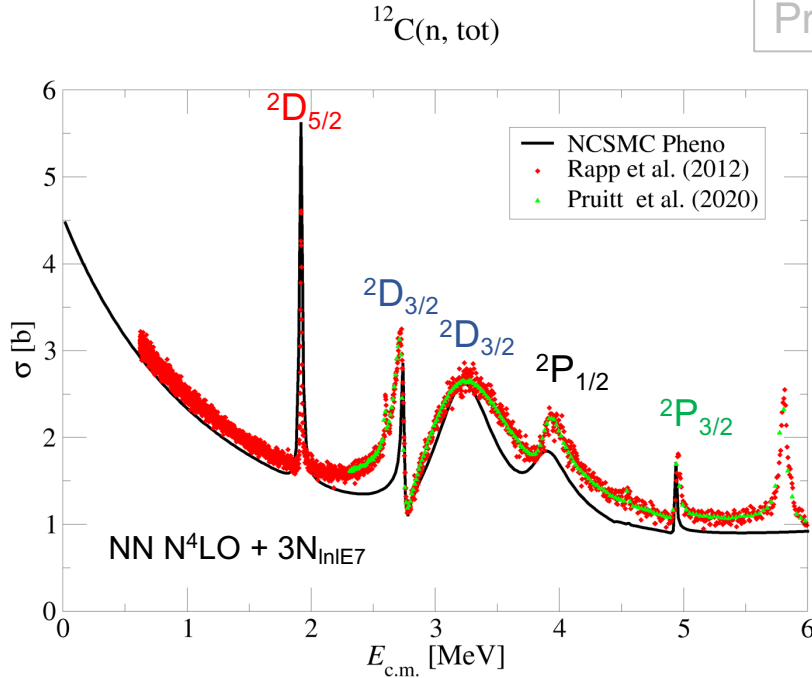
Excellent agreement with experimental data

Precision chiral EFT Hamiltonian with a sub-leading 3N interaction term

- n+¹²C cross section

Calculations in collaboration with Marshall Kodar (UVic undergrad),
Matteo Vorabbi (Surrey)

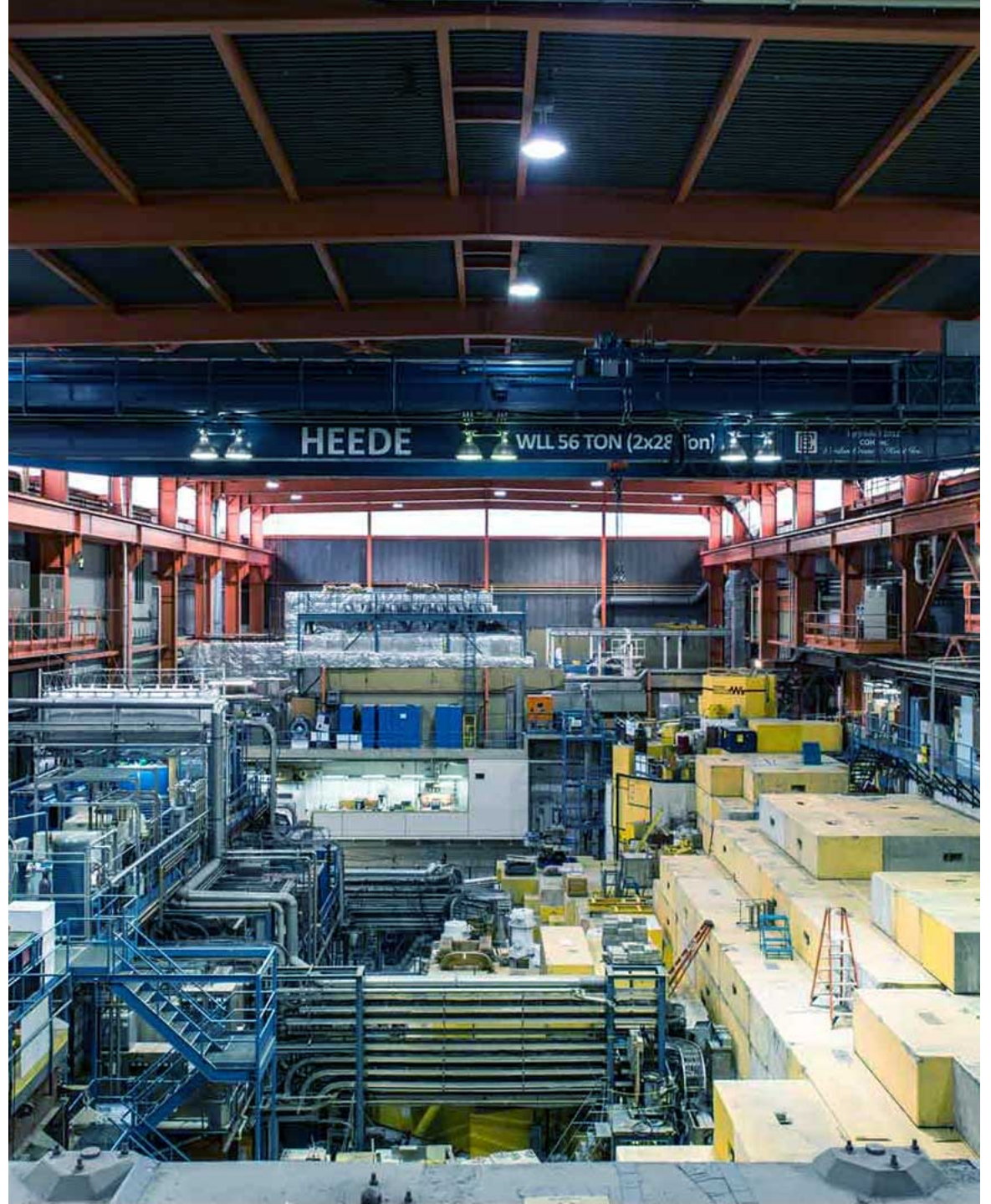
Preliminary



Excellent agreement with experimental data. Impact of E_7 term in progress.

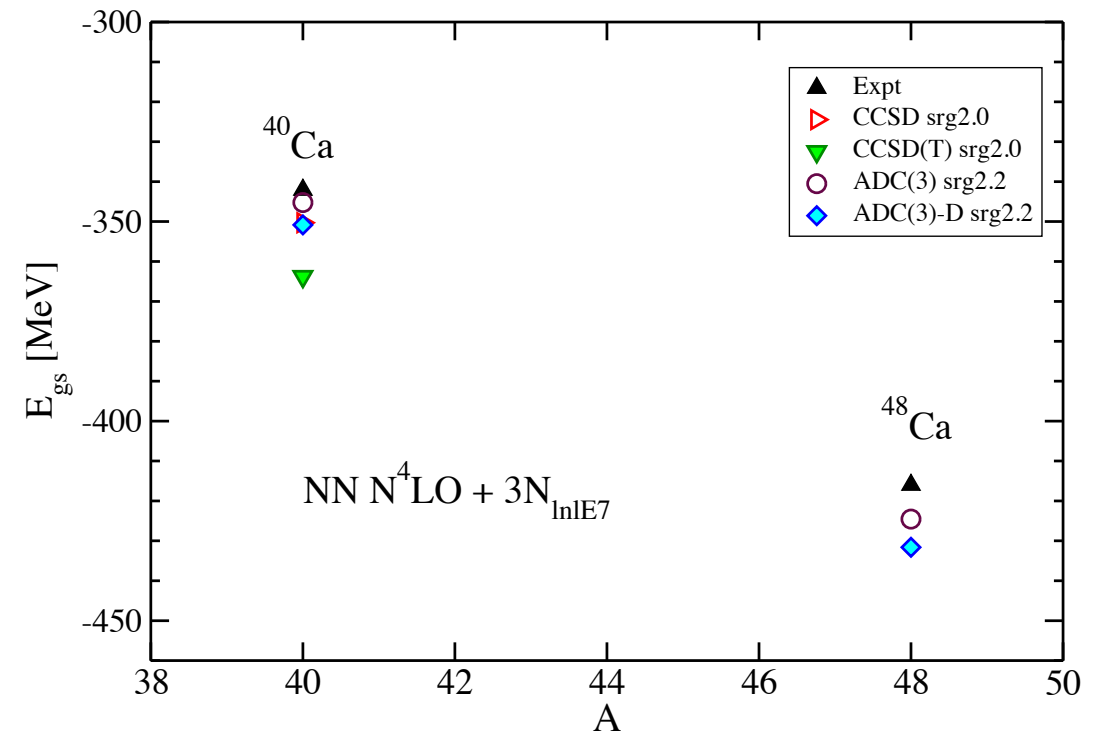
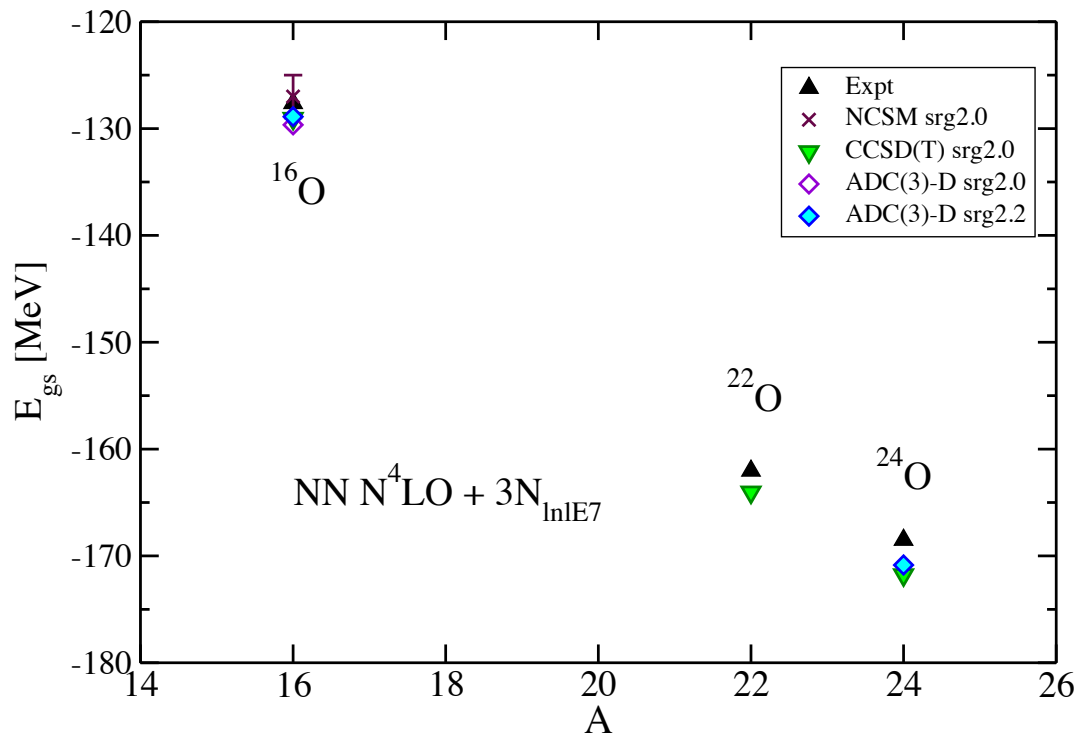
Precision chiral EFT Hamiltonian with
a sub-leading $3N$ interaction term
- medium mass nuclei

2026-04-29



Coupled cluster and Self-Consistent Green's Function calculations for O and Ca isotopes

Calculations by Gaute Hagen and Carlo Barbieri



^{16}O	srg1.8	srg2.0	srg2.2	Expt. (MeV)
CCSD(T)	-130.76	-129.05		
ADC(3)-D		-129.65	-128.89	-127.619

Overall slight overbinding – SRG effect or LECs re-fit needed?

Bogoliubov many-body perturbation theory (BMBPT) calculations for N=20 isotones

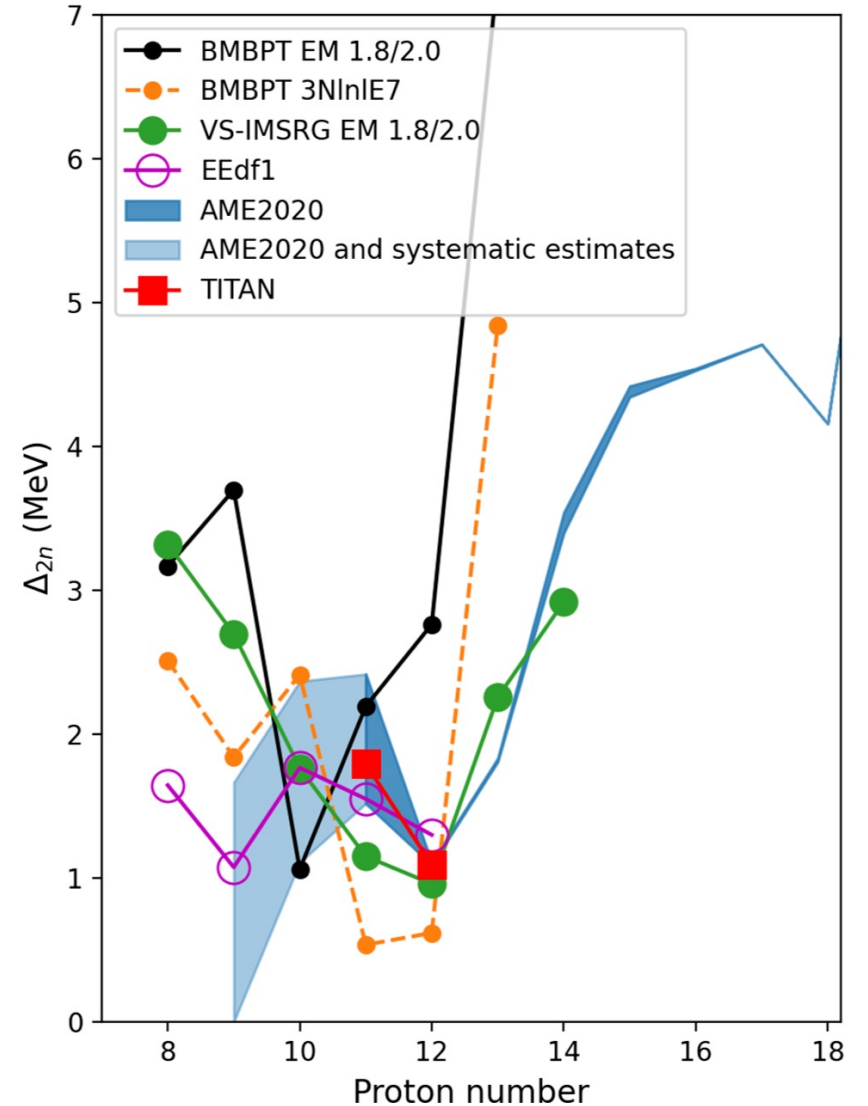
Calculations by Vittorio Soma

PHYSICAL REVIEW LETTERS **134**, 052503 (2025)

Refined Topology of the $N=20$ Island of Inversion with High Precision Mass Measurements of $^{31-33}\text{Na}$ and $^{31-35}\text{Mg}$

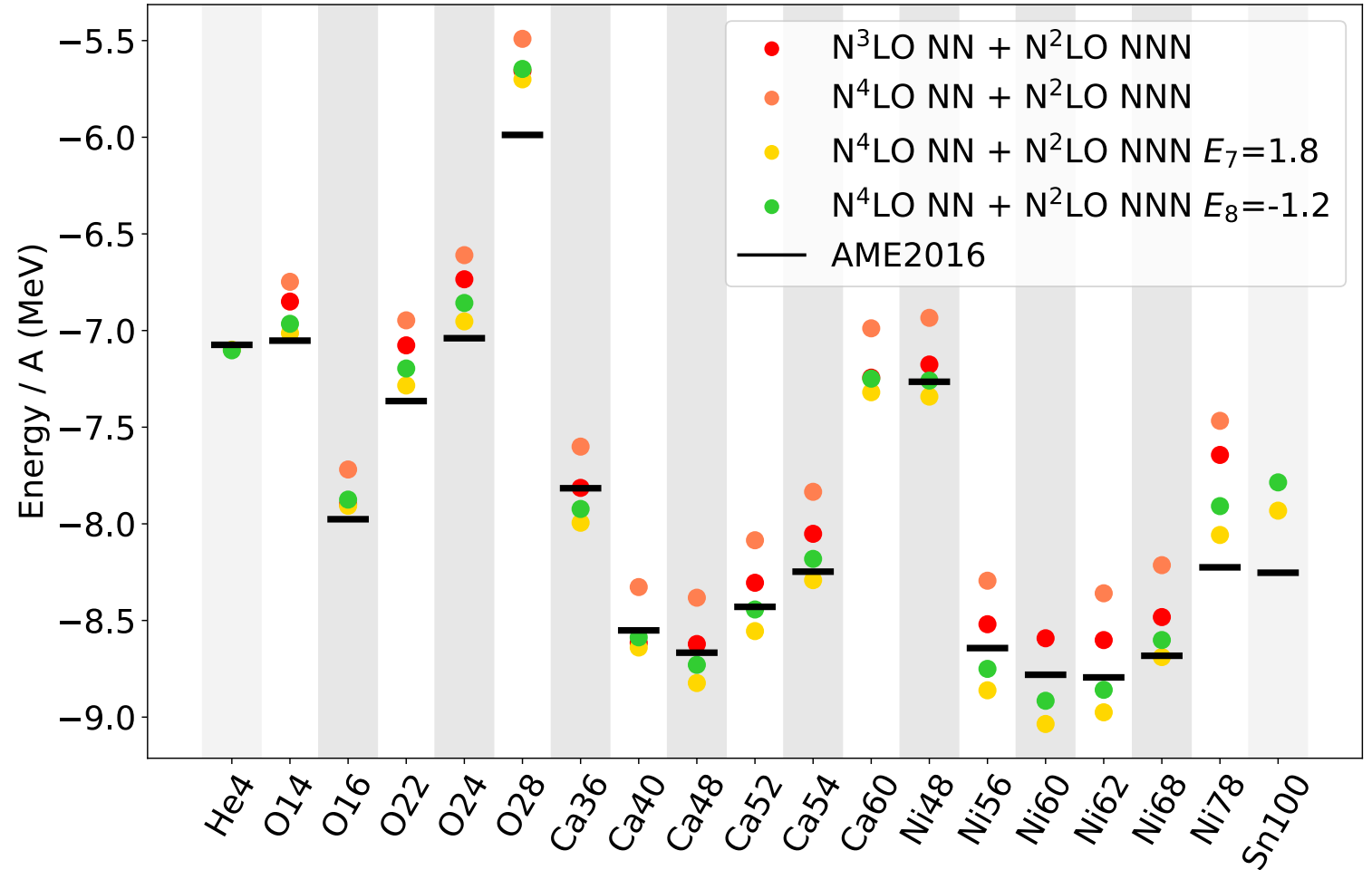
E. M. Lykiardopoulou,^{1,2} C. Walls,^{1,3} J. Bergmann,⁴ M. Brodeur,⁵ C. Brown,⁶ J. Cardona,^{1,3} A. Czihaly,^{1,7} T. Dickel,^{8,4} T. Duguet,^{9,10} J.-P. Ebran,^{11,12} M. Frosini,¹³ Z. Hockenbery,^{1,14} J. D. Holt,^{1,14} A. Jacobs,^{1,2} S. Kakkar,^{1,3} B. Kootte,¹⁵ T. Miyagi,^{1,16} A. Mollaebrahimi,^{1,4} T. Murboeck,⁴ P. Navratil,^{1,7} T. Otsuka,^{17,18} W. R. Plaß,^{4,8} S. Paul,^{1,19} W. S. Porter,⁵ M. P. Reiter,⁶ A. Scalesi,⁹ C. Scheidenberger,^{8,4,20} V. Somà,⁹ N. Shimizu,²¹ Y. Wang,^{1,2} D. Lunney,²² J. Dilling,^{1,2,23,24} and A. A. Kwiatkowski^{1,7}

NN $N^4\text{LO} + 3N_{\text{InIE7}}$ predictions resembling those of the successful phenomenological EEdf1 with the same trend for $Z = 8 - 10$



MBPT calculations up to 3rd order for (semi-)magic medium mass nuclei

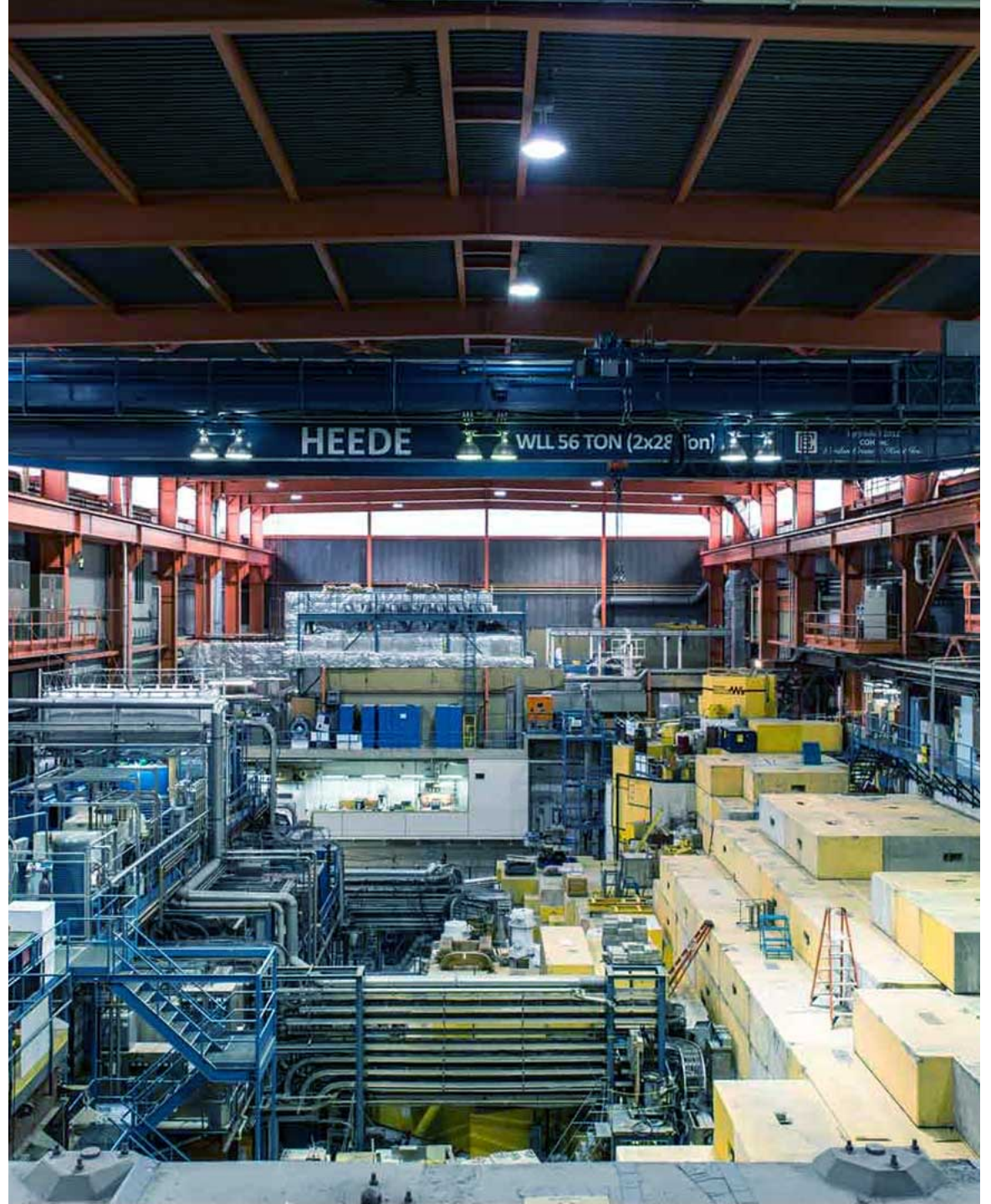
- Calculations by Takayuki Miyagi



N⁴LO 3N contact contributions worth further exploring!

Enhanced short-range 3N interaction with two-pion exchange

Results for ${}^3\text{H}$



A new class of three-nucleon forces – enhanced sub-leading terms?

PHYSICAL REVIEW LETTERS **135**, 022501 (2025)

New Class of Three-Nucleon Forces and Their Implications

Vincenzo Cirigliano^{⊙,*}, Maria Dawid[†], Wouter Dekens^{⊙,‡}, and Sanjay Reddy[§]

Closer look at enhanced three-nucleon forces

E. Epelbaum,¹ A. M. Gasparyan,¹ J. Gegelia,^{1,2} D. Hog,¹ and H. Krebs¹

arXiv:2512.14117

- Enhanced short-range 3N interaction with two-pion exchange

$$W_{D_2} = \sum_{i \neq j \neq k} \frac{9g_A^2 D_2 m_\pi^3}{128\pi f_\pi^4} \frac{(d_2^S + d_2^T \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)}{d_2^S - 3d_2^T} \mathcal{I} \left(\frac{q_k^2}{4m_\pi^2} \right)$$

$$W_{F_2} = - \sum_{i \neq j \neq k} \frac{15g_A^2 m_\pi^3}{16\pi f_\pi^4} (f_2^S + f_2^T \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \mathcal{J} \left(\frac{q_k^2}{4m_\pi^2} \right)$$

$$D_2 = \frac{c_{D_2}}{5F_\pi^4} \quad \leftarrow \quad D_2 = \frac{c_{D_2}}{F_\pi^2 \Lambda_\chi^2}$$

$$F_2 = \frac{c_{F_2}}{5F_\pi^4} \quad \quad \quad F_2 = \frac{c_{F_2}}{F_\pi^2 \Lambda_\chi^2}$$

	N ² LO (Q ³)	N ³ LO (Q ⁴)	N ⁴ LO (Q ⁵)	N ⁵ LO (Q ⁶)
a				unknown
b	—			unknown
c	—			unknown
d				unknown
e	—			
f		—		—

Exploring quark mass dependent three-nucleon forces in medium-mass nuclei

Urban Vernik^{⊙,1,2,*}, Kai Hebeler^{⊙,1,2,3,†} and Achim Schwenk^{⊙,1,2,3,‡}

A new class of three-nucleon forces – enhanced sub-leading terms?

PHYSICAL REVIEW LETTERS **135**, 022501 (2025)

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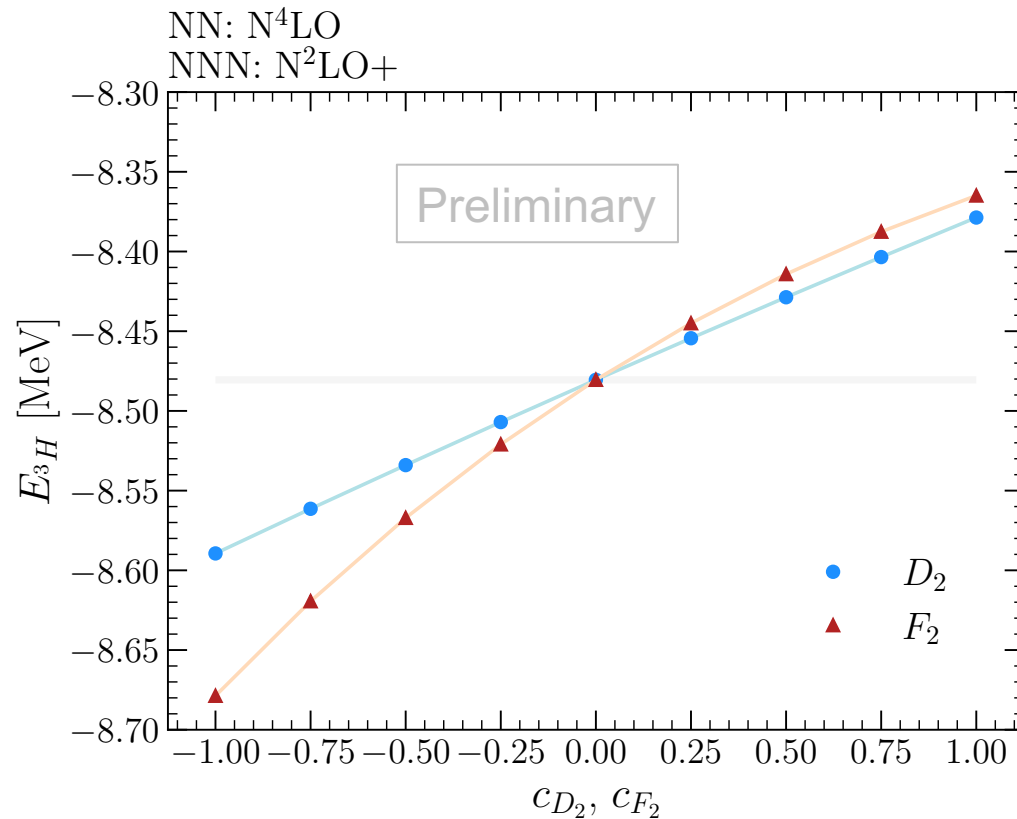
arXiv:2512.14117

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- Application to ${}^3\text{H}$ gs energy – Jacobi NCSM

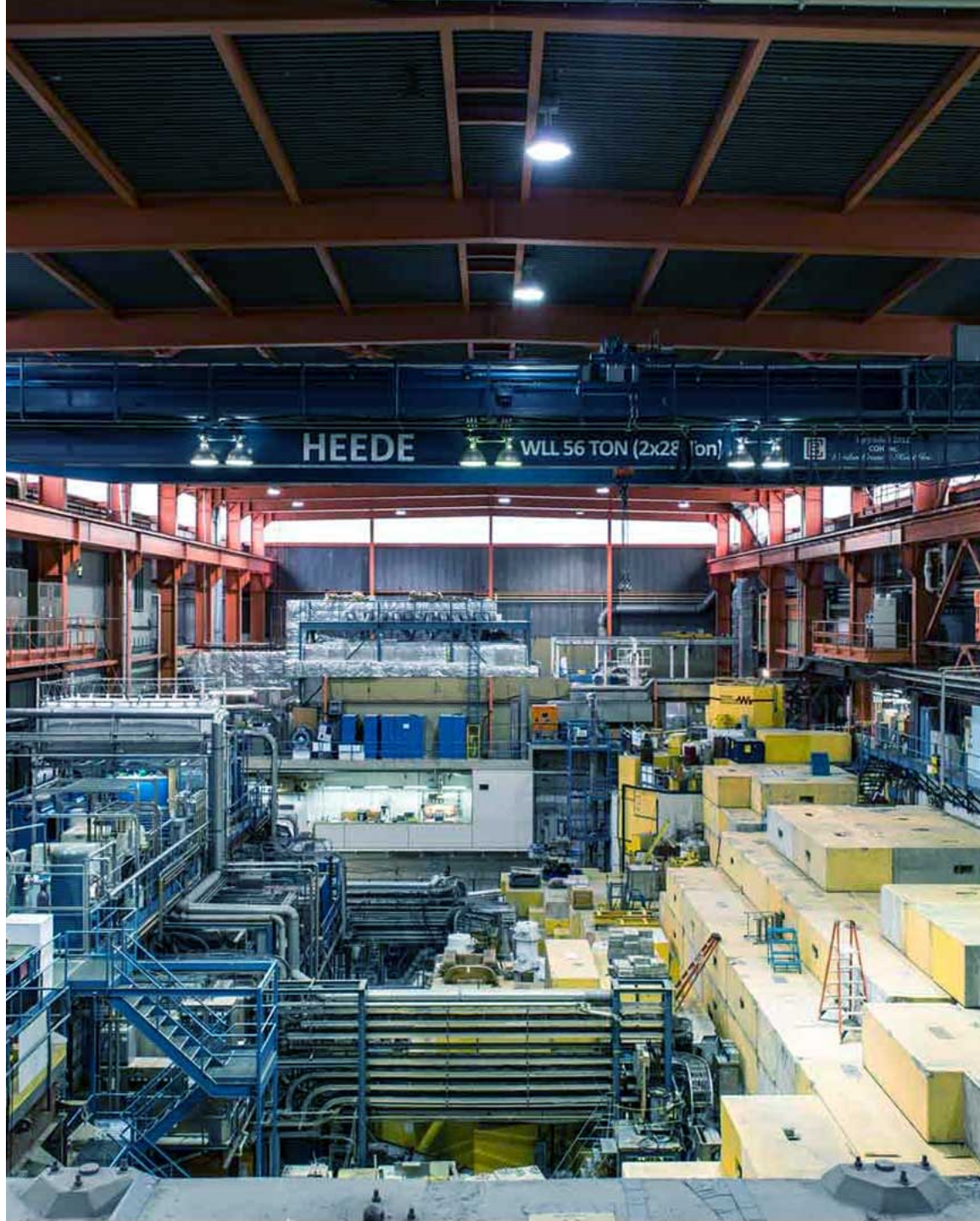
Georgios Palkanoglou (TRIUMF)

Calculations for ${}^4\text{He}$,
p-shell nuclei in progress



Conclusions

2026-04-29



Conclusions

- Big picture
 - High-order (N^3 LO and N^4 LO) chiral NN interactions are OK
 - **3N interactions need to be improved**
 - Including N^4 LO 3N contact terms straightforward and a logical thing to do
 - Spin-orbit enhancing E_7 and/or E_8 terms important, should be included
 - Optimally fitted to $n+^4\text{He}$ phase shifts
 - Good evidence that there is a correlation between ^4He , ^{16}O , ^{40}Ca binding energy
 - Important to get the ^4He binding energy properly
 - Beware that SRG induced 4N can play a role
 - Neutron rich nuclei seems to be underbound – add $T=3/2$ 3N contact(s)?
- More medium-mass and heavy nuclei calculations needed to test Hamiltonians with subleading 3N interactions
- Open question
 - How important is the “enhanced short-range 3N interaction with two-pion exchange”?
 - Applied to ^3H
 - Calculations for ^4He and p-shell nuclei in progress