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Ab initio calculations of beta decays of light nuclei

INT Workshop 23-1b
New physics searches at the precision frontier
May 8, 2023

Petr Navratil

TRIUMF

Collaborators:
Peter Gysbers (TRIUMF/UBC), Michael Gennari (UVic/TRIUMF),
Lotta Jokiemi (TRIUMF), Mehdi Drissi (TRIUMF), Ayala Glick-Magid (INT),
Doron Gazit (Hebrew U), Christian Forssen (Chalmers UT),
Daniel Gazda (NPI Rez), Kostas Kravvaris (LLNL), Mack Atkinson (LLNL),
Chien Yeah Seng (INT), Misha Gorshteyn (U Mainz)

Outline

- Calculations of ^{6}He β-decay electron spectrum including nuclear structure and recoil corrections – published in PLB (2022)
- Calculations of ^{16}N β-decay electron spectrum including nuclear structure and recoil corrections – ongoing, related to calculations of the muon capture on ^{16}O
- Ongoing calculations of nuclear structure corrections δ_C and δ_{NS} for the extraction of the V_{ud} matrix element from the ^{10}C→^{10}B superallowed Fermi transition (Michael Gennari on May 1st)
- Investigation of the β-delayed proton emission from ^{11}Be – published in PRC (2022)

Calculations performed within the no-core shell model (NCSM), δ_C and ^{11}Be decay within the NCSM with continuum (NCSMC)

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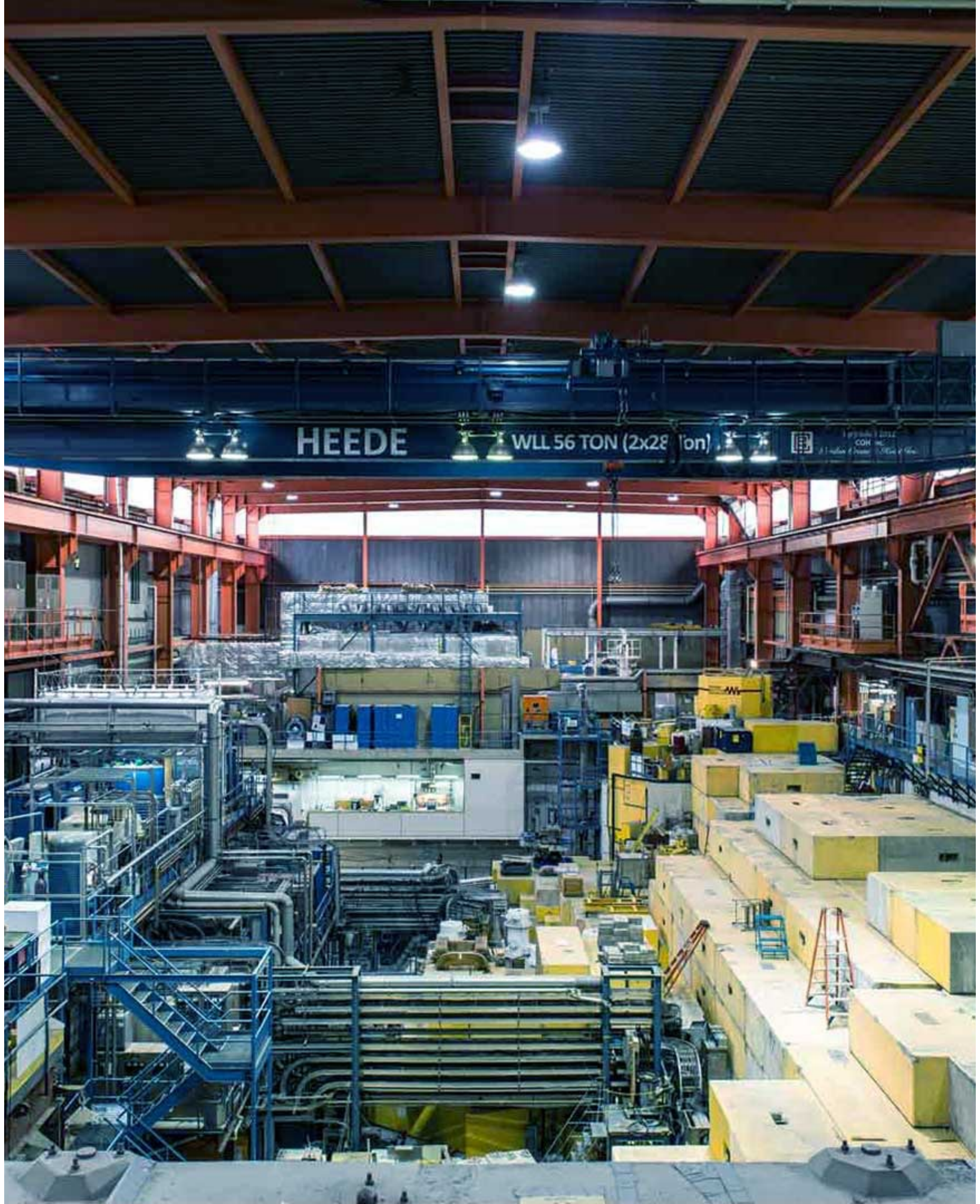
^{6}He β-decay



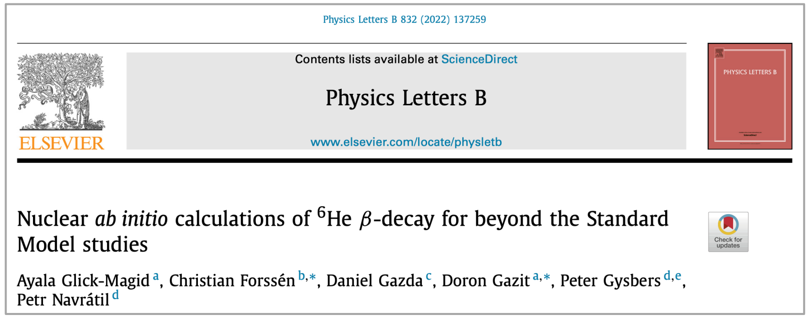
Discovery, accelerated

Precise measurements of β decays to search for Physics Beyond the Standard Model

- Precision measurements of β-decay observables offer the possibility to search for deviations from the Standard Model
  - β-decay observables are sensitive to interference of currents of SM particles and hypothetical BSM physics
  - Such couplings are proportional to v/Λ, with $v \approx 174$ GeV, the SM vacuum expectation value, and Λ the new physics energy scale
    - a $\sim 10^{-4}$ coupling between SM and BSM physics would suggest new physics at a scale that is out of the reach of current particle accelerators
  - Discovering such small deviations from the SM predictions demands also high-precision theoretical calculations
    - ⇒ Nuclear structure calculations with quantified uncertainties

- Theoretical analysis of β-decay observables of the pure Gamow-Teller (GT) transition ^{6}He(0^{+} g.s.) → ^{6}Li(1^{+} g.s.) using *ab initio* nuclear structure calculations in combination with the chiral effective field theory (χEFT)

  - Details published in



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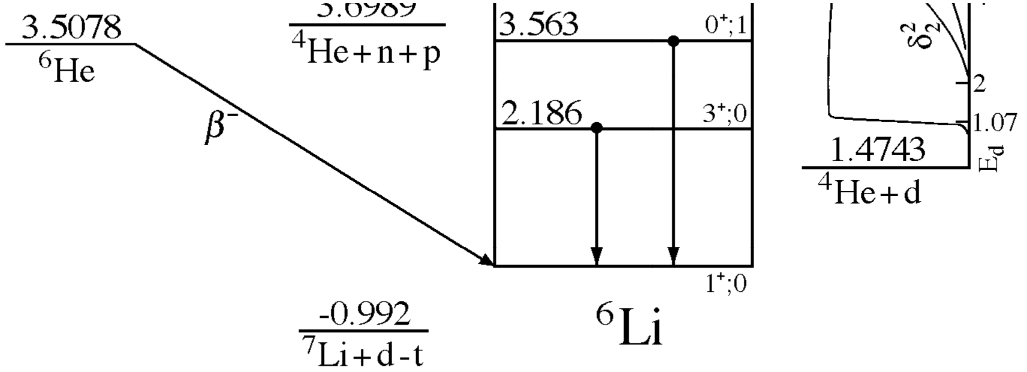
Nuclear *ab initio* calculations of ^{6}He β-decay for beyond the Standard Model studies

Ayala Glick-Magid [a], Christian Forssén [b,*], Daniel Gazda [c], Doron Gazit [a,*], Peter Gysbers [d,e], Petr Navrátil [d]

Precise measurements of β decays to search for Physics Beyond the Standard Model

- Decay rate proportional to

$$d\omega \propto 1 + a_{\beta\nu}\vec{\beta}\cdot\hat{\nu} + b_F\frac{m_e}{E} \qquad \vec{\beta} = \frac{\vec{k}}{E} \qquad \vec{\nu} = \nu\hat{\nu}$$



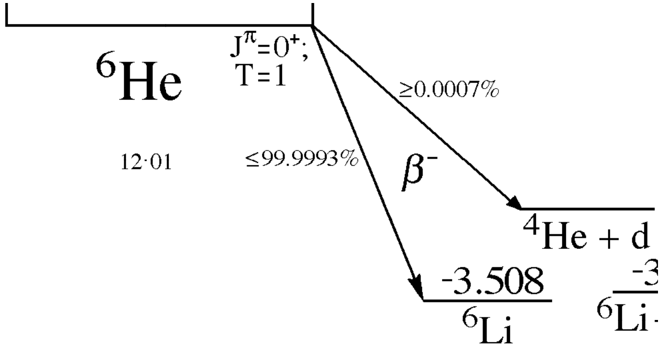
$a_{\beta\nu}$ angular correlation coefficient between the emitted electron and the antineutrino

b_F Fierz interference term that can be extracted from electron energy spectrum measurements

- The *V-A* structure of the weak interaction in the Standard Model implies for a Gamow-Teller transition

$$a_{\beta\nu} = -\frac{1}{3}$$

$$b_F = 0$$

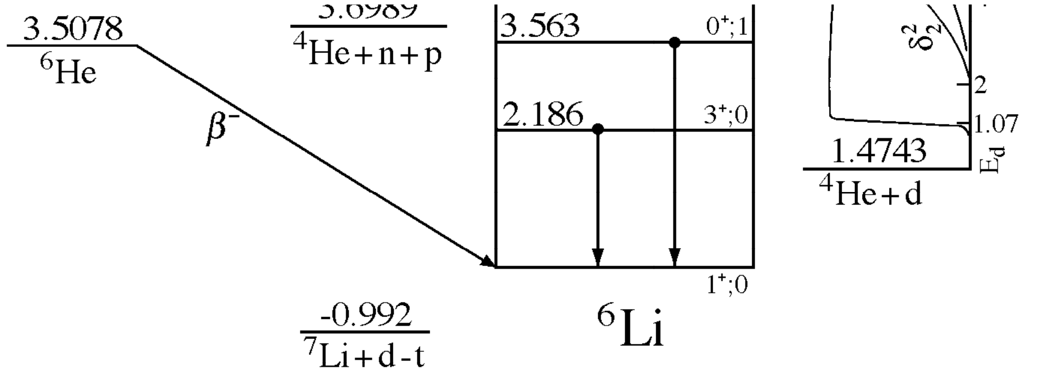


Precise measurements of β decays to search for Physics Beyond the Standard Model

- In the presence of Beyond the Standard Model interactions

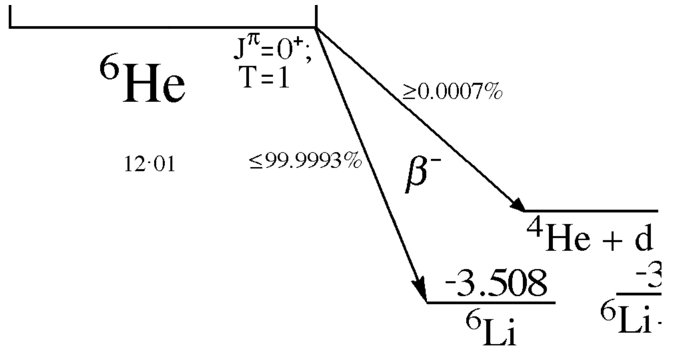
$$a_{\beta\nu}^{\mathrm{BSM}} = -\frac{1}{3}\left(1 - \frac{|C_T|^2 + |C_T'|^2}{2|C_A|^2}\right)$$

$$b_{\mathrm{Fierz}}^{\mathrm{BSM}} = \frac{C_T + C_T'}{C_A}$$



  - with tensor and pseudo-tensor contributions

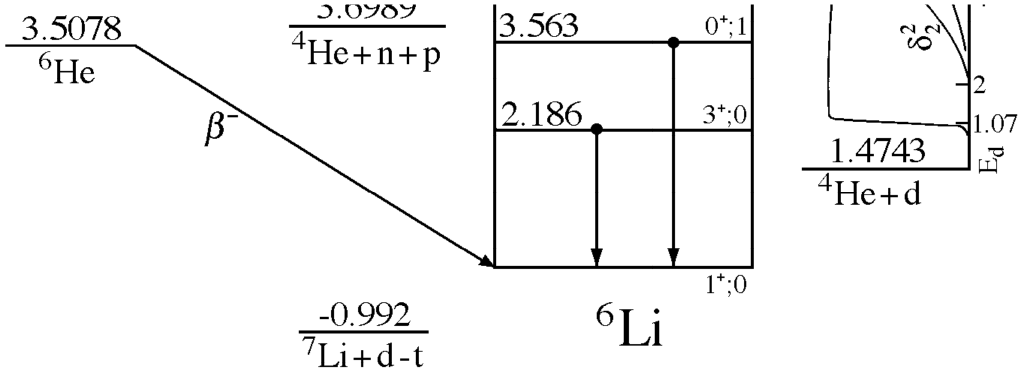
- However, deviations also within the Standard Model caused by the finite momentum transfer, higher-order transition operators, and nuclear structure effects
  - Detailed, accurate, and precise calculations required



Precise measurements of β decays to search for Physics Beyond the Standard Model

- ^{6}He β^--decay differential distribution within the SM—including the leading shape and recoil corrections (NLO in GT)

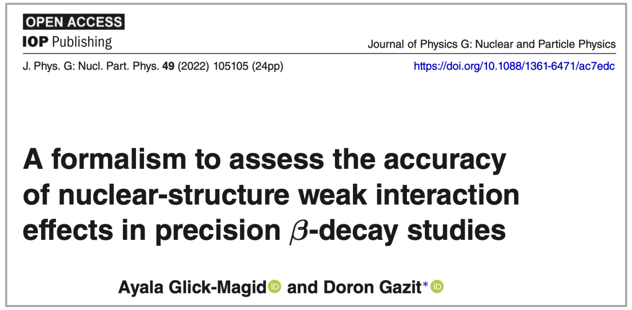
$$\frac{d\omega^{1^+\beta^-}}{dE\,\frac{d\Omega_k}{4\pi}\,\frac{d\Omega_\nu}{4\pi}} = \frac{4}{\pi^2}\,(E_0 - E)^2\,kEF^-(Z_f, E)\,C_{\mathrm{corr}}\left|\langle\|\hat{L}_1^A\|\rangle\right|^2$$
$$\times 3\left(1+\delta_1^{1^+\beta^-}\right)\left[1 + a_{\beta\nu}^{1^+\beta^-}\,\vec{\beta}\cdot\hat{\nu} + b_{\mathrm{F}}^{1^+\beta^-}\,\frac{m_e}{E}\right]$$



$\hat{L}_1^A \propto 1$ … longitudinal operator of the axial current, Gamow-Teller leading order

$F^-(Z_f, E)$ … Fermi function, deformation of the electron wave function due to the EM interaction with the nucleus

C_{corr} … radiative corrections, finite-mass and electrostatic finite-size effects, and atomic effects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A formalism to assess the accuracy of nuclear-structure weak interaction effects in precision β-decay studies

Ayala Glick-Magid and Doron Gazit*

Precise measurements of β decays to search for Physics Beyond the Standard Model

- Higher-order Standard Model recoil and shape corrections

$$a_{\beta\nu}^{1^+\beta^-} = -\frac{1}{3}\left(1 + \tilde{\delta}_a^{1^+\beta^-}\right)$$

$$b_{\mathrm{F}}^{1^+\beta^-} = \delta_b^{1^+\beta^-}$$

$$\delta_1^{1^+\beta^-} \equiv \frac{2}{3}\mathfrak{Re}\left[-E_0\frac{\langle\|\hat{C}_1^A/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle} + \sqrt{2}\,(E_0 - 2E)\,\frac{\langle\|\hat{M}_1^V/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle}\right] - \frac{4}{7}ER\alpha Z_f - \frac{233}{630}\left(\alpha Z_f\right)^2,$$

$$\tilde{\delta}_a^{1^+\beta^-} \equiv \frac{4}{3}\mathfrak{Re}\left[2E_0\frac{\langle\|\hat{C}_1^A/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle} + \sqrt{2}\,(E_0 - 2E)\,\frac{\langle\|\hat{M}_1^V/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle}\right] + \frac{4}{7}ER\alpha Z_f - \frac{2}{5}E_0R\alpha Z_f,$$

$$\delta_b^{1^+\beta^-} \equiv \frac{2}{3}m_e\mathfrak{Re}\left[\frac{\langle\|\hat{C}_1^A/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle} + \sqrt{2}\frac{\langle\|\hat{M}_1^V/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle}\right],$$

$\vec{q} = \vec{k} + \vec{\nu}$ momentum transfer

$\hat{C}_1^A$ axial charge

$\hat{M}_1^V$ vector magnetic or weak magnetism

$\hat{L}_1^A \propto 1$ Gamow-Teller leading order

$\hat{C}_1^A$ $\hat{M}_1^V$ NLO recoil corrections, order q/m_N

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$$\frac{\hat{C}^A_{JM_J}}{q} = \sum_{j=1}^{A} \frac{i}{m_N} \left[g_A \hat{\Omega}'_{JM_J}(q\vec{r}_j) - \frac{1}{2}\frac{\tilde{g}_P}{2m_N}(E_0 + \Delta E_c)\,\hat{\Sigma}''_{JM_J}(q\vec{r}_j) \right] \tau_j^+,$$

$$\hat{L}^A_{JM_J} = \sum_{j=1}^{A} i \left(g_A + \frac{\tilde{g}_P}{(2m_N)^2} q^2 \right) \hat{\Sigma}''_{JM_J}(q\vec{r}_j)\,\tau_j^+,$$

$$\frac{\hat{M}^V_{JM_J}}{q} = \sum_{j=1}^{A} \frac{-i}{m_N} \left[g_V \hat{\Delta}_{JM_J}(q\vec{r}_j) - \frac{1}{2}\mu \hat{\Sigma}'_{JM_J}(q\vec{r}_j) \right] \tau_j^+$$

$$\hat{\Sigma}''_{JM_J}(q\vec{r}_j) = \left[\frac{1}{q} \vec{\nabla}_{\vec{r}_j} M_{JM_J}(q\vec{r}_j) \right] \cdot \vec{\sigma}(j),$$

$$\hat{\Omega}'_{JM_J}(q\vec{r}_j) = M_{JM_J}(q\vec{r}_j)\,\vec{\sigma}(j) \cdot \vec{\nabla}_{\vec{r}_j} + \frac{1}{2}\hat{\Sigma}''_{JM_J}(q\vec{r}_j),$$

$$\hat{\Delta}_{JM_J}(q\vec{r}_j) = \vec{M}_{JJM_J}(q\vec{r}_j) \cdot \frac{1}{q}\vec{\nabla}_{\vec{r}_j},$$

$$\hat{\Sigma}'_{JM_J}(q\vec{r}_j) = -i \left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j} \times \vec{M}_{JJM_J}(q\vec{r}_j) \right] \cdot \vec{\sigma}(j),$$

$$M_{JM_J}(q\vec{r}_j) = j_J(qr_j) Y_{JM_J}(\hat{r}_j)$$

$$\vec{M}_{JLM_J}(q\vec{r}_j) = j_L(qr_j) \vec{Y}_{JLM_J}(\hat{r}_j)$$

Hadronic vector, axial vector and pseudo-scalar charges

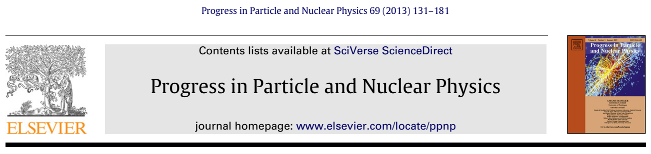
$g_V = 1$ $g_A = -1.2756(13)$ $\tilde{g}_P = -\frac{(2m_N)^2}{m_\pi^2 - q^2} g_A$

$\mu \approx 4.706$ is the nucleon isovector magnetic moment

$$\Delta E_c \equiv \langle {}^6\text{Li}\ 1^+_{gs} | V_c | {}^6\text{Li}\ 1^+_{gs} \rangle - \langle {}^6\text{He}\ 0^+_{gs} | V_c | {}^6\text{He}\ 0^+_{gs} \rangle$$

Ultimately, we need to calculate ^{6}He(0^+ 1) → ^{6}Li(1^+ 0) matrix elements of these "one-body" operators

Apply *ab initio* No-Core Shell Model (NCSM) to calculate the ^{6}Li and ^{6}He wave functions and the operator matrix elements


Progress in Particle and Nuclear Physics 69 (2013) 131–181
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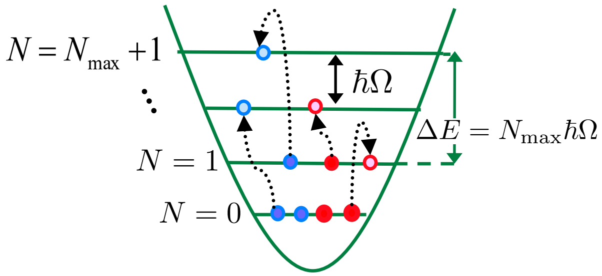
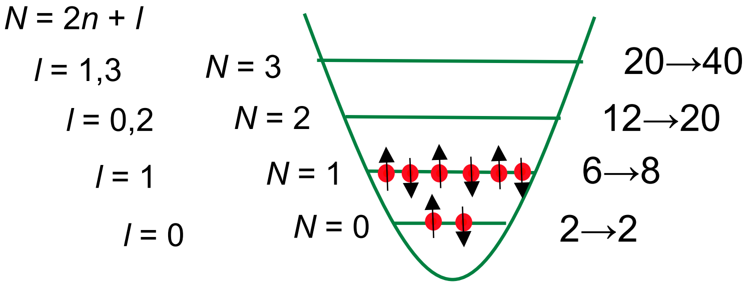
Review
Ab initio no core shell model
Bruce R. Barrett[a], Petr Navrátil[b], James P. Vary[c,*]

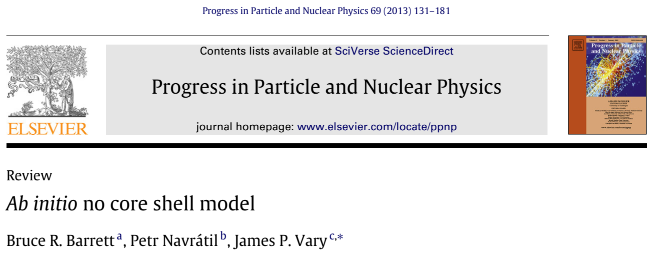
- Basis expansion method
  - Harmonic oscillator (HO) basis truncated in a particular way (N_{max})
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^{4}He, ^{16}O, ^{40}Ca)
    - Equivalent description in relative(Jacobi)-coordinate and Slater determinant (SD) basis
- Short- and medium range correlations
- Bound-states, narrow resonances

NCSM





$E = (2n + l + \frac{3}{2})\hbar\Omega$



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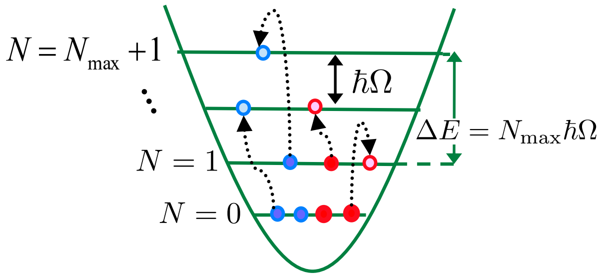
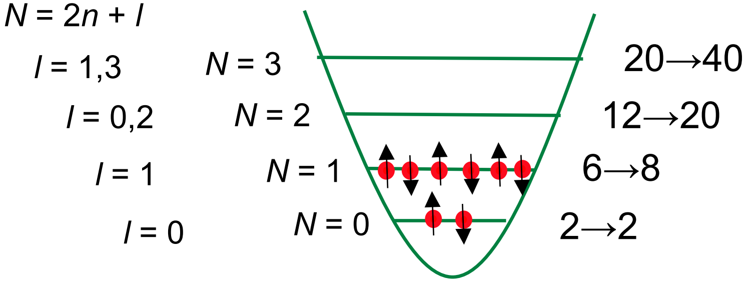
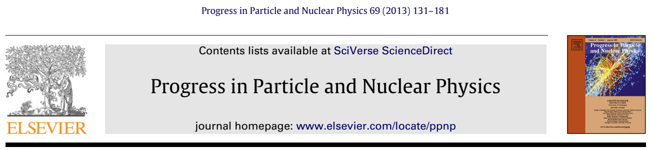




$E = (2n + l + \frac{3}{2})\hbar\Omega$

$$\Psi^A = \sum_{N=0}^{N_{max}} \sum_i c_{Ni} \Phi^{HO}_{Ni}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$

$$\Psi^A_{SD} = \sum_{N=0}^{N_{max}} \sum_j c^{SD}_{Nj} \Phi^{HO}_{SD\,Nj}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$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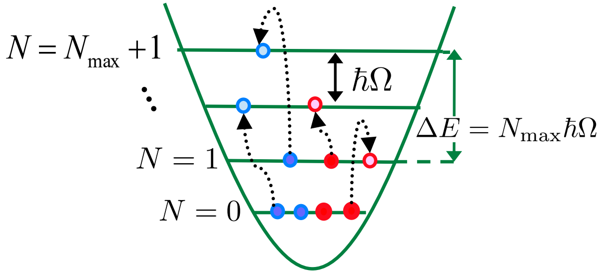
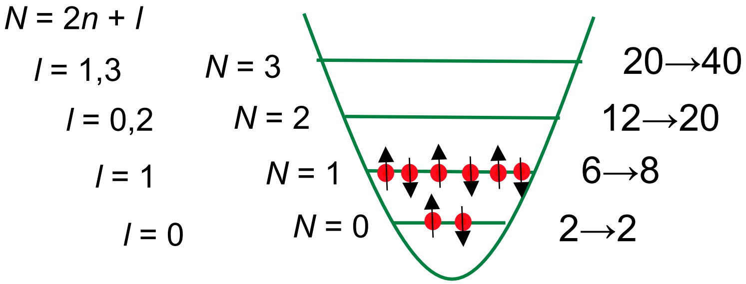


For ^{6}Li, ^{6}He and heavier nuclei we use the SD basis



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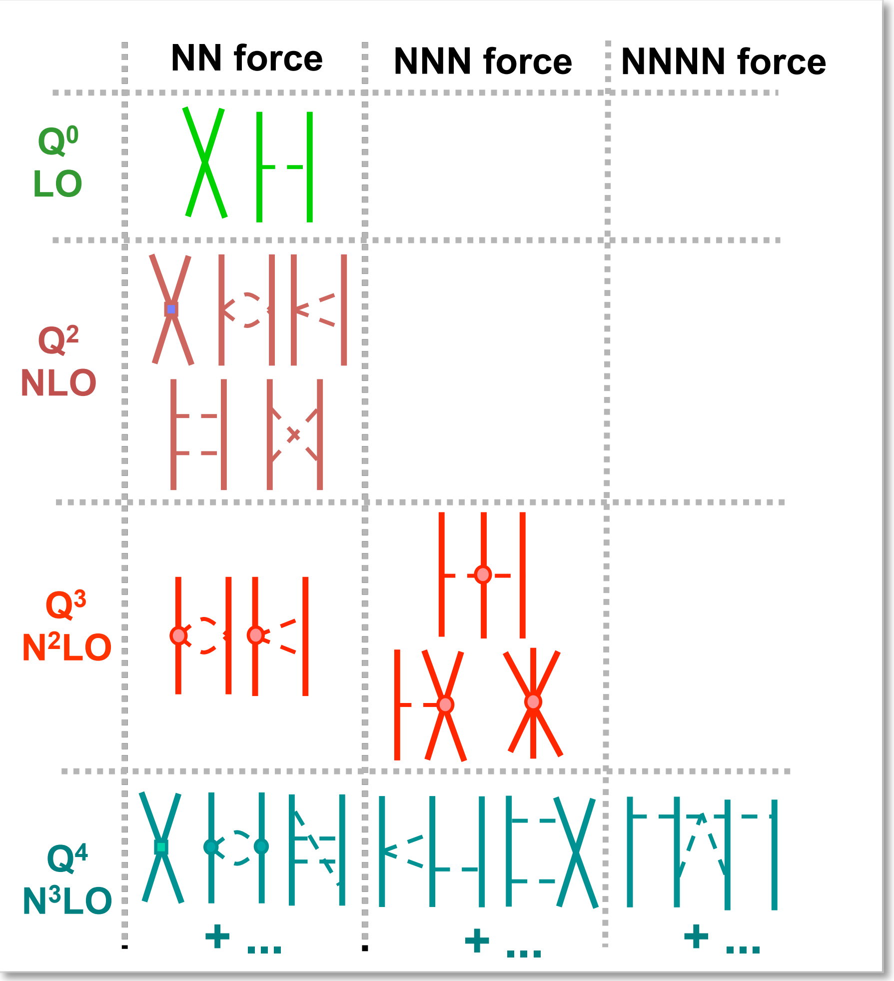
Input for NCSM calculations: Nuclear forces from chiral Effective Field Theory

- Approach taking advantage of the separation of scales
  - Based on the symmetries of QCD
    - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order (Q/Λ_χ)
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD



Λ_χ~1 GeV :
Chiral symmetry breaking scale

Input for NCSM calculations: Nuclear forces from chiral Effective Field Theory

- Interactions used in this study
  - $NNLO_{opt}$
    - NN only
    - Reproduces reasonably well binding energies & radii of A = 3,4 and 6 nuclei

PRL 110, 192502 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Optimized Chiral Nucleon-Nucleon Interaction at Next-to-Next-to-Leading Order

A. Ekström,[1,2] G. Baardsen,[1] C. Forssén,[3] G. Hagen,[4,5] M. Hjorth-Jensen,[1,2,6] G. R. Jansen,[4,5] R. Machleidt,[7] W. Nazarewicz,[5,4,8] T. Papenbrock,[5,4] J. Sarich,[9] and S. M. Wild[9]

  - $NNLO_{sat}$
    - NN+3N
    - More accurate for medium mass nuclei especially for radii

RAPID COMMUNICATIONS

PHYSICAL REVIEW C 91, 051301(R) (2015)

Accurate nuclear radii and binding energies from a chiral interaction

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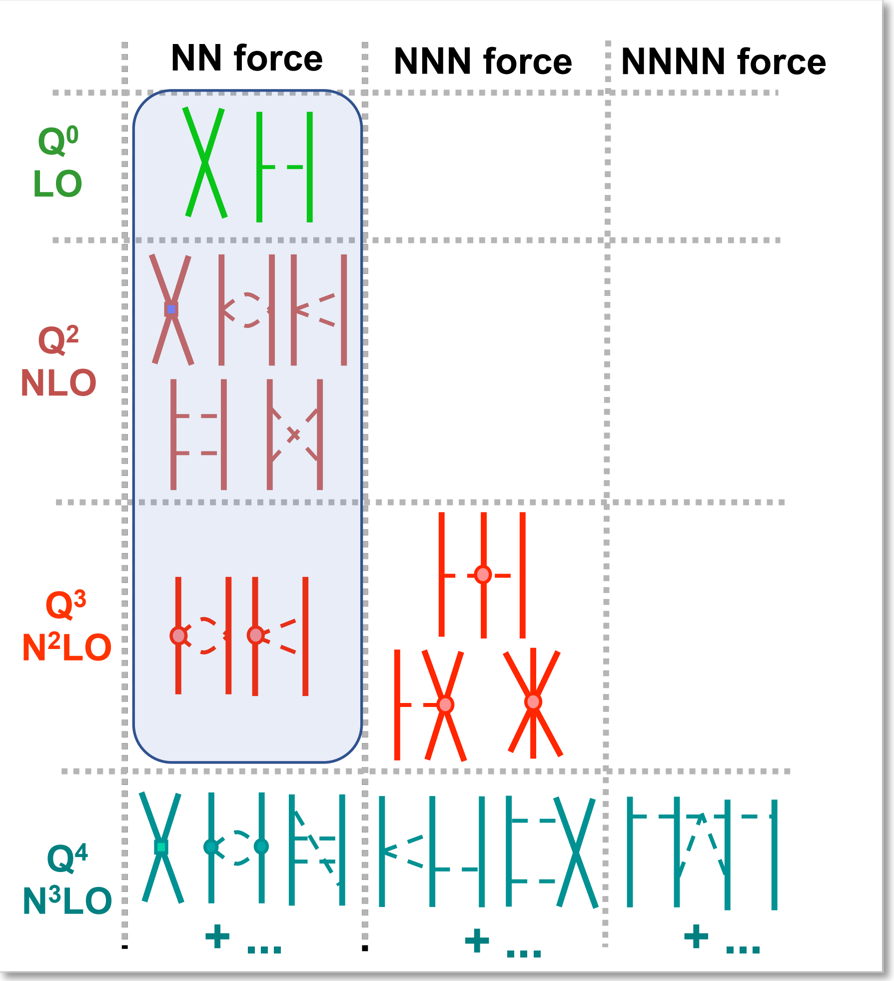
- No further renormalization (no SRG or OLS …)



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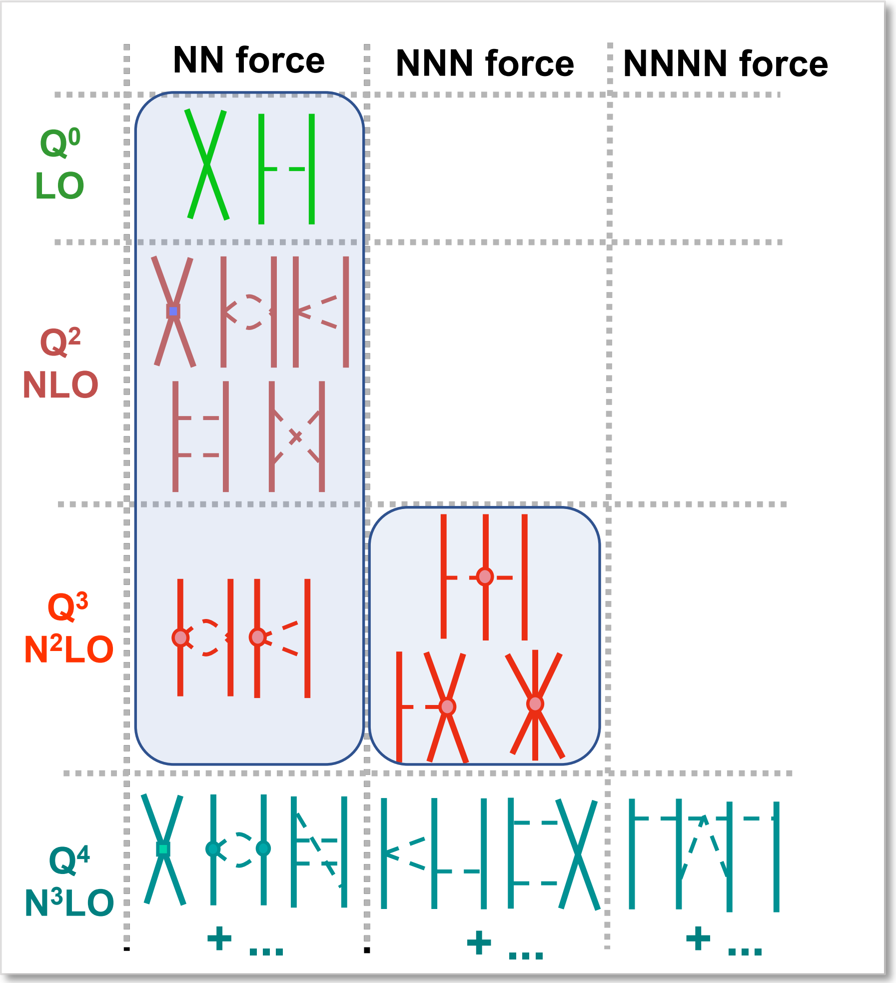

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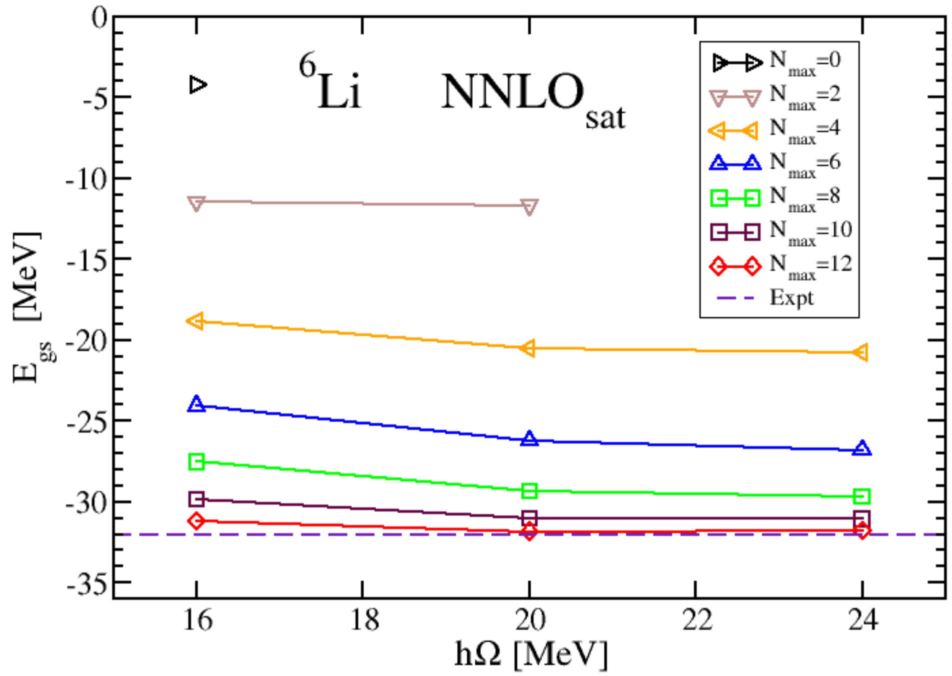

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Apply *ab initio* No-Core Shell Model to calculate the ^{6}Li and ^{6}He wave functions and the operator matrix elements

NCSM

- Straightforward to calculate matrix elements of one-body operators

$$\langle \Psi_f \| \sum_{j=1}^{A} \hat{O}_J(\vec{r}_j) \| \Psi_i \rangle = \frac{-1}{\sqrt{2J+1}} \sum_{|\alpha|,|\beta|} \langle |\alpha| \| \hat{O}_J(\vec{r}) \| |\beta| \rangle \times \langle \Psi_f \| (a^\dagger_{|\alpha|} \tilde{a}_{|\beta|}) J \| \Psi_i \rangle,$$

One-body operator matrix element

One-body density

- In our case $J=1$, $|\Psi_i\rangle = |^6\mathrm{He\,gs}\,0^+1\rangle$, $|\Psi_f\rangle = |^6\mathrm{Li\,gs}\,1^+0\rangle$

However, NCSM wave function include spurious center of mass component and the "one-body" operator depends on coordinates measured from the center of mass of the nucleus: $\vec{r}_i \to \vec{r}_i - \vec{R}_{\mathrm{CM}}$

$$\Psi^A_{\mathrm{SD}} = \sum_{N=0}^{N_{\max}} \sum_j c^{\mathrm{SD}}_{Nj} \Phi^{HO}_{\mathrm{SD}\,Nj}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$

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NCSM

- How to do this right?
  - Introduce Jacobi coordinates, use transformations of HO wave functions
  - Done successfully in the past for radial density

$$\rho_{op}(\vec{r}) = \sum_{i=1}^{A} \delta(\vec{r} - \vec{r}_i)$$

PHYSICAL REVIEW C **70**, 014317 (2004)

Translationally invariant density

Petr Navrátil
Lawrence Livermore National Laboratory, L-414, P.O. Box 808, Livermore, California 94551, USA
(Received 23 May 2004; published 30 July 2004)

PHYSICAL REVIEW C **97**, 034619 (2018)

Microscopic optical potentials derived from *ab initio* translationally invariant nonlocal one-body densities

Michael Gennari[*]
University of Waterloo, 200 University Avenue West Waterloo, Ontario N2L 3G1, Canada and TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

Matteo Vorabbi,[†] Angelo Calci, and Petr Navrátil[‡]
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

PHYSICAL REVIEW C **99**, 024305 (2019)

Nuclear kinetic density from *ab initio* theory

Michael Gennari[*]
University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada and TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

Petr Navrátil[†]
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

PHYSICAL REVIEW LETTERS **124**, 162501 (2020)

Elastic Antiproton-Nucleus Scattering from Chiral Forces

Matteo Vorabbi,[1,2] Michael Gennari,[2,3] Paolo Finelli,[4] Carlotta Giusti,[5] and Petr Navrátil[2]

PHYSICAL REVIEW C **104**, 064322 (2021)

Translationally invariant matrix elements of general one-body operators

Petr Navrátil*

TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

Apply *ab initio* No-Core Shell Model to calculate the ^{6}Li and ^{6}He wave functions and the operator matrix elements

NCSM

- How to do this right?
  - Can this be generalized for an arbitrary operator $\sum_{j=1}^{A} \hat{O}(\vec{r}_j - \vec{R}_{\rm CM})$? Yes!

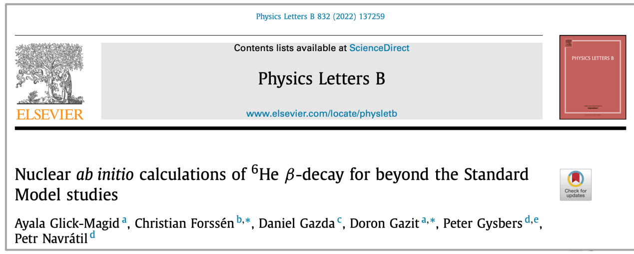
"One-body" operator matrix element

$$\langle \Psi_f || \sum_{j=1}^{A} \hat{O}_J(\vec{r}_j - \vec{R}_{\rm CM}) || \Psi_i \rangle = \frac{-1}{\sqrt{2J+1}} \sum_{|a||b||\alpha||\beta|} \langle |a|| \hat{O}_J(-\sqrt{A-1/A}\vec{\xi}) || |b| \rangle$$

$$\times (M^J)^{-1}_{|a||b|,|\alpha||\beta|} \langle \Psi_f || (a^\dagger_{|\alpha|} \tilde{a}_{|\beta|}) J || \Psi_i \rangle$$

One-body density

$$(M^K)_{n_1 l_1 j_1 n_2 l_2 j_2, nljn'l'j'}$$
$$= \sum_{N_1 L_1} \hat{j}_1 \hat{j}_2 \hat{j} \hat{j}' \hat{l} \hat{l}' (-1)^{K+L_1+l_1+l_2+j'+j_2}$$
$$\times \begin{Bmatrix} j' & L_1 & j_2 \\ l_2 & \frac{1}{2} & l' \end{Bmatrix} \begin{Bmatrix} j_1 & L_1 & j \\ l & \frac{1}{2} & l_1 \end{Bmatrix} \begin{Bmatrix} j_1 & L_1 & j \\ j' & K & j_2 \end{Bmatrix}$$
$$\times \langle nl00l | N_1 L_1 n_1 l_1 l \rangle_{\frac{1}{A-1}} \langle n'l'00l' | N_1 L_1 n_2 l_2 l' \rangle_{\frac{1}{A-1}}.$$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Nuclear *ab initio* calculations of ^{6}He β-decay for beyond the Standard Model studies

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Apply *ab initio* No-Core Shell Model to calculate the ^{6}Li and ^{6}He wave functions and the operator matrix elements

NCSM

- Matrix elements of the relevant operators

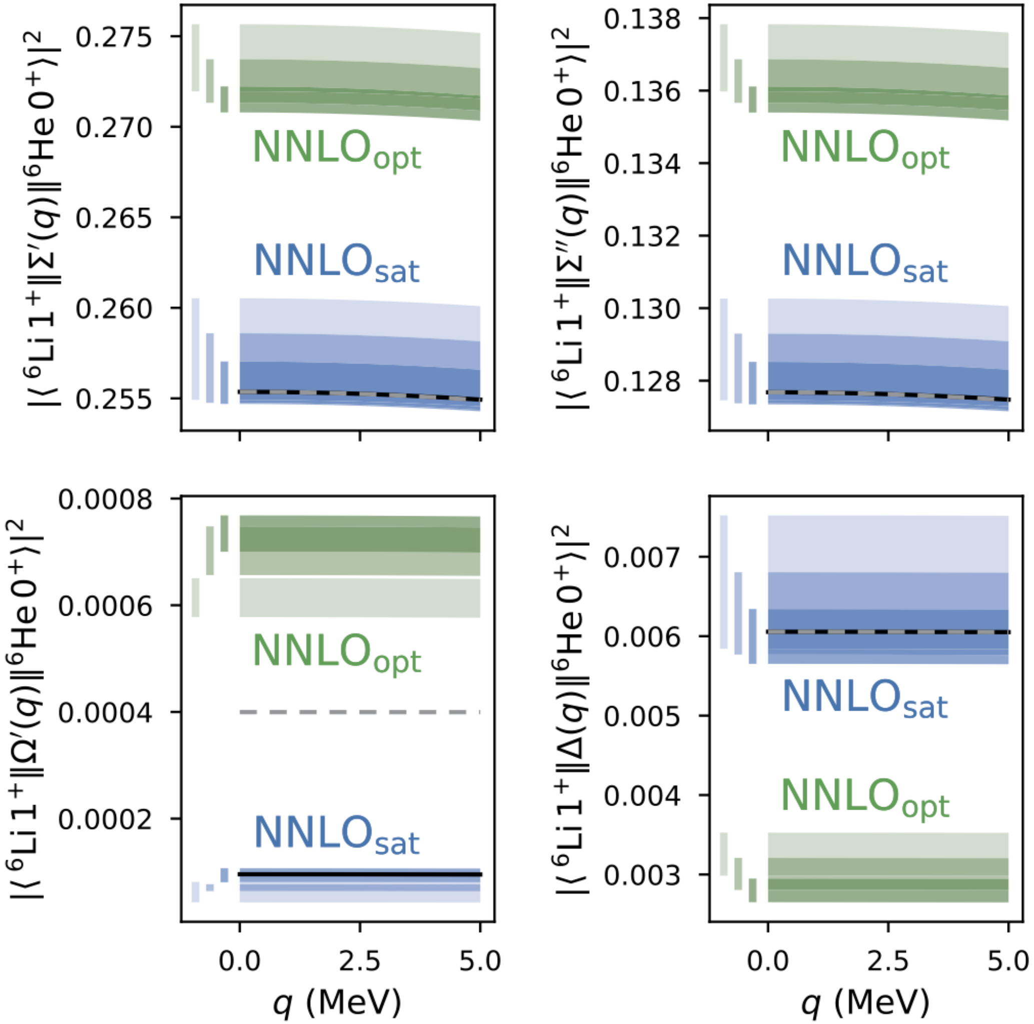
$$\hat{\Sigma}''_{JM_J}(q\vec{r}_j) = \left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j} M_{JM_J}(q\vec{r}_j)\right]\cdot\vec{\sigma}(j),$$

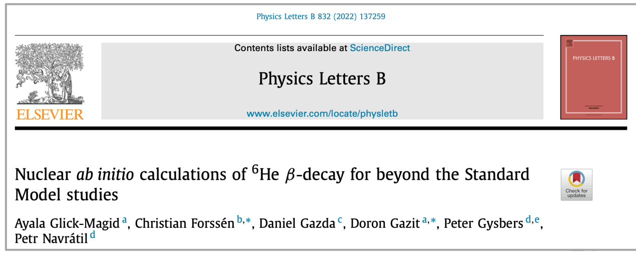
$$\hat{\Omega}'_{JM_J}(q\vec{r}_j) = M_{JM_J}(q\vec{r}_j)\,\vec{\sigma}(j)\cdot\vec{\nabla}_{\vec{r}_j} + \frac{1}{2}\hat{\Sigma}''_{JM_J}(q\vec{r}_j),$$

$$\hat{\Delta}_{JM_J}(q\vec{r}_j) = \vec{M}_{JJM_J}(q\vec{r}_j)\cdot\frac{1}{q}\vec{\nabla}_{\vec{r}_j},$$

$$\hat{\Sigma}'_{JM_J}(q\vec{r}_j) = -i\left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j}\times\vec{M}_{JJM_J}(q\vec{r}_j)\right]\cdot\vec{\sigma}(j),$$

- Convergence investigation
  - Variation of HO frequency
    - $h\Omega$ = 16 - 24 MeV
  - Variation of basis size
    - N_{max}= 0 - 14 for $NNLO_{opt}$
    - N_{max}= 0 - 12 for $NNLO_{sat}$

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$$\hat{\Delta}_{JM_J}(q\vec{r}_j) = \vec{M}_{JJM_J}(q\vec{r}_j)\cdot\frac{1}{q}\vec{\nabla}_{\vec{r}_j},$$

$$\hat{\Sigma}'_{JM_J}(q\vec{r}_j) = -i\left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j}\times\vec{M}_{JJM_J}(q\vec{r}_j)\right]\cdot\vec{\sigma}(j),$$

- Impact of the CM correction

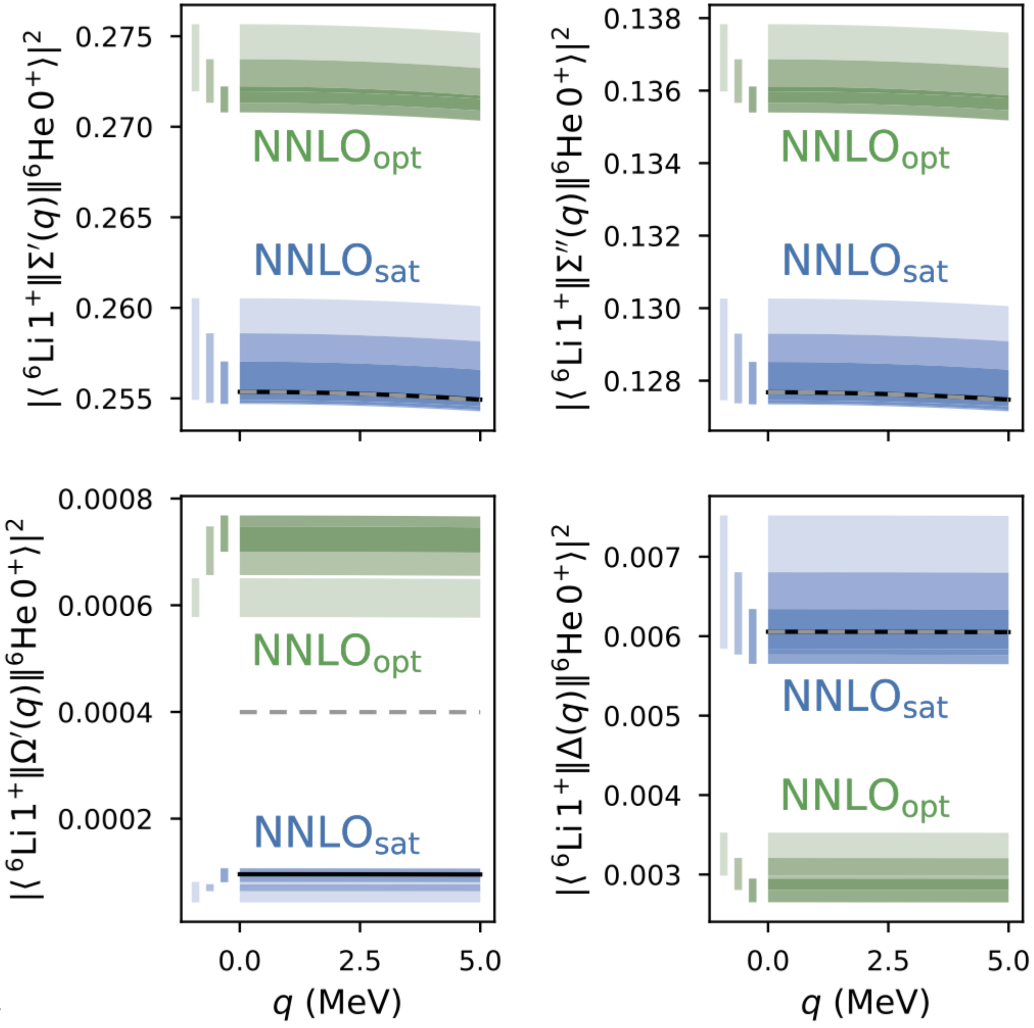
$$\langle\Psi_f\|\sum_{j=1}^{A}\hat{O}_J(\vec{r}_j)\|\Psi_i\rangle \longleftrightarrow \langle\Psi_f\|\sum_{j=1}^{A}\hat{O}_J(\vec{r}_j-\vec{R}_{\rm CM})\|\Psi_i\rangle$$

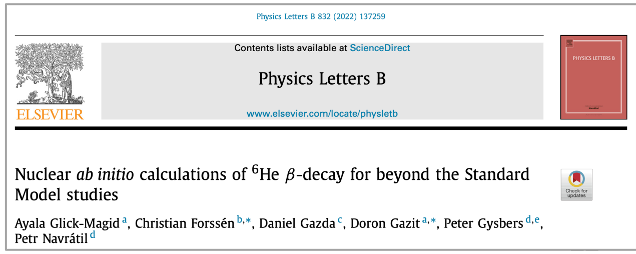
At $q = 0$:

No difference for $\hat{\Sigma}'_{JM_J}(q\vec{r}_j)$, $\hat{\Sigma}''_{JM_J}(q\vec{r}_j)$, and $\hat{\Delta}_{JM_J}(q\vec{r}_j)$

Change by a factor of ~2 for $\hat{\Omega}'_{JM_J}(q\vec{r}_j)$

Increasing deviations for all operators with increase of q

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Apply *ab initio* No-Core Shell Model to calculate the ^{6}Li and ^{6}He wave functions and the operator matrix elements

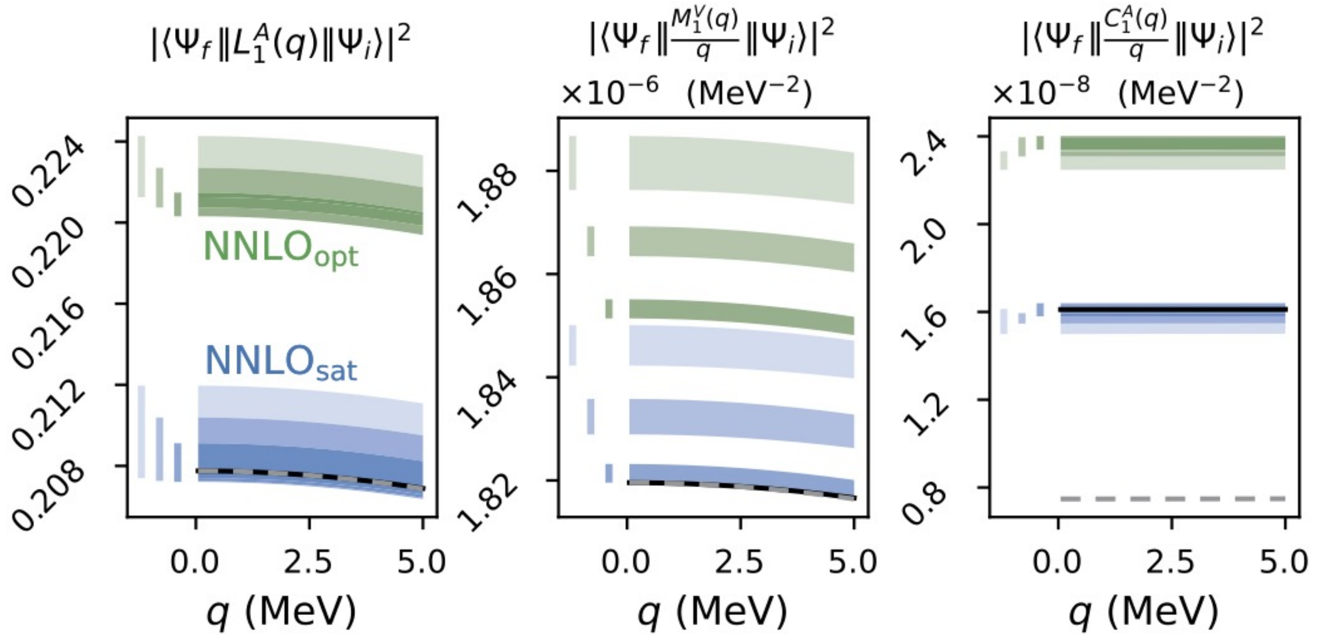
- Matrix elements of the relevant operators

$$\frac{\hat{C}^A_{JM_J}}{q} = \sum_{j=1}^{A} \frac{i}{m_N}\Big[g_A \hat{\Omega}'_{JM_J}(q\vec{r}_j) - \frac{1}{2}\frac{\tilde{g}_P}{2m_N}(E_0+\Delta E_c)\,\hat{\Sigma}''_{JM_J}(q\vec{r}_j)\Big]\tau_j^+,$$

$$\hat{L}^A_{JM_J} = \sum_{j=1}^{A} i\left(g_A + \frac{\tilde{g}_P}{(2m_N)^2}q^2\right)\hat{\Sigma}''_{JM_J}(q\vec{r}_j)\,\tau_j^+,$$

$$\frac{\hat{M}^V_{JM_J}}{q} = \sum_{j=1}^{A} \frac{-i}{m_N}\Big[g_V\,\hat{\Delta}_{JM_J}(q\vec{r}_j) - \frac{1}{2}\mu\hat{\Sigma}'_{JM_J}(q\vec{r}_j)\Big]\tau_j^+$$

- Convergence investigation
  - Variation of HO frequency
    - $h\Omega$ = 16 - 24 MeV
  - Variation of basis size
    - N_{max}= 0 - 14 for $NNLO_{opt}$
    - N_{max}= 0 - 12 for $NNLO_{sat}$



- Impact of the CM correction

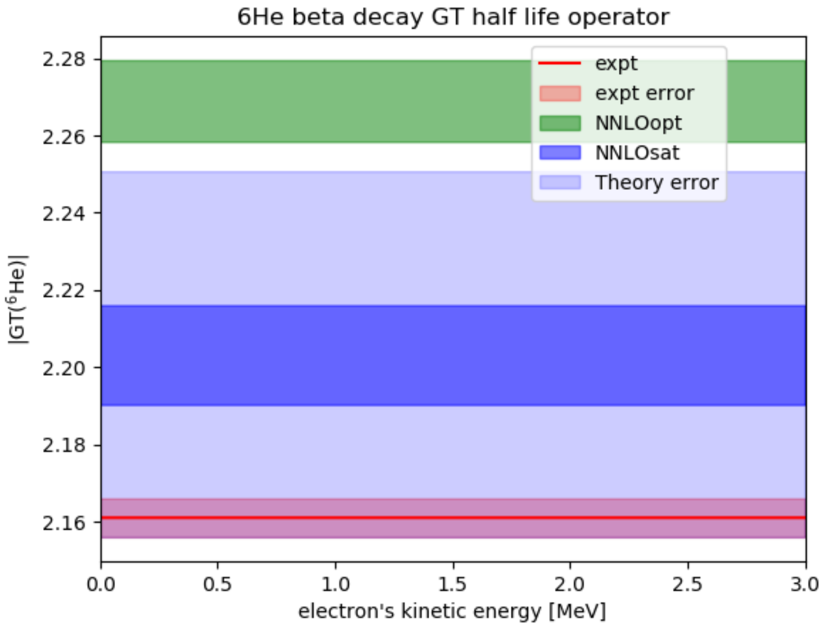
$$\langle\Psi_f\|\sum_{j=1}^{A}\hat{O}_J(\vec{r}_j)\|\Psi_i\rangle \longleftrightarrow \langle\Psi_f\|\sum_{j=1}^{A}\hat{O}_J(\vec{r}_j-\vec{R}_{\rm CM})\|\Psi_i\rangle$$

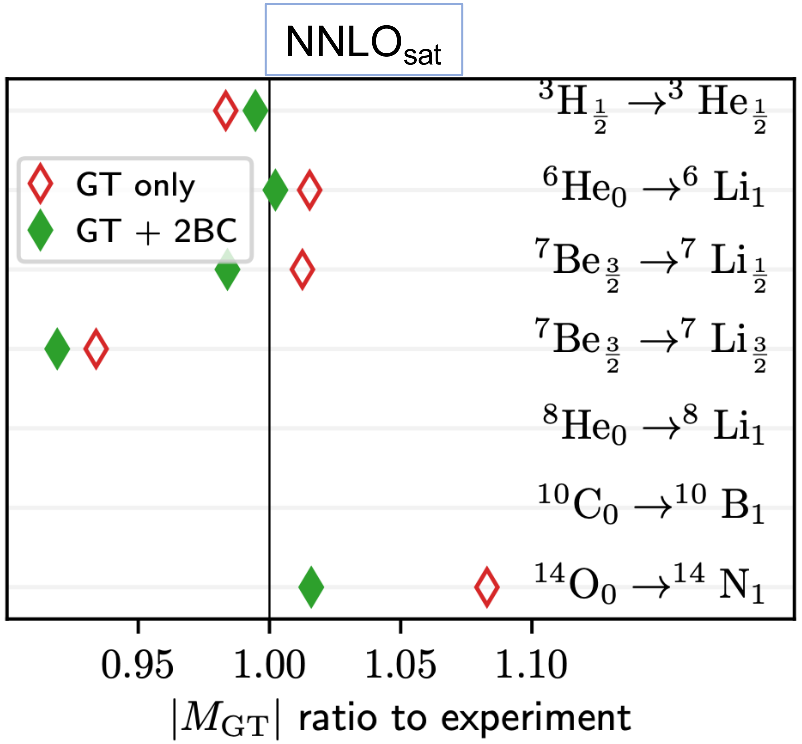
Almost no difference for $\hat{L}^{A\pm}_{JM_J}$ and $\hat{M}^{V\pm}_{JM_J}$

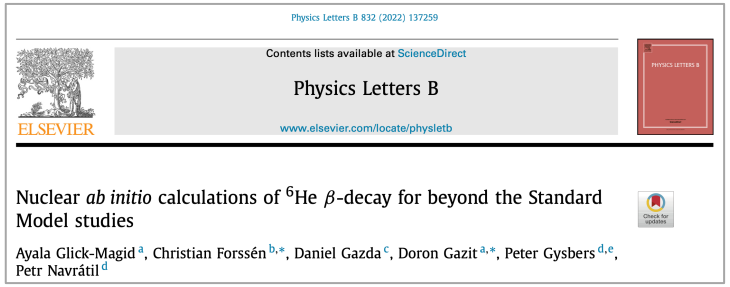
Change of ~40% for $\hat{C}^{A\pm}_{JM_J}$

Overall results for ^{6}He(0^+ 1) → ^{6}Li(1^+ 0) + e^- + $\bar{\nu}$

- Calculations performed in the impulse approximations
  - Weak magnetism M_1^V receives two-body current correction of the order the χEFT expansion parameter ϵ_{EFT}
  - L_1^A and C_1^A two-body current terms are associated with the next order, $\epsilon^2_{\mathrm{EFT}}$
- The effect of two-body currents on the Gamow-Teller matrix element (q=0) quite small, ~2%
- Two-body contribution to the magnetic moment of ^{6}Li negligible, correction to the B(M1; $1^+ \to 0^+$) ~ 10%



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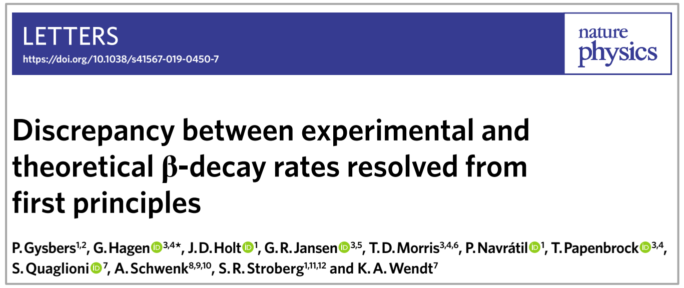
Conservative estimate $\epsilon_{\mathrm{EFT}} \lesssim 0.15$


LETTERS

https://doi.org/10.1038/s41567-019-0450-7

nature physics

Discrepancy between experimental and theoretical β-decay rates resolved from first principles

P. Gysbers[1,2], G. Hagen[3,4*], J. D. Holt[1], G. R. Jansen[3,5], T. D. Morris[3,4,6], P. Navrátil[1], T. Papenbrock[3,4], S. Quaglioni[7], A. Schwenk[8,9,10], S. R. Stroberg[1,11,12] and K. A. Wendt[7]

Overall results for $^{6}He(0^{+}\ 1) \rightarrow {}^{6}Li(1^{+}\ 0) + e^{-} + \overline{\nu}$

- We find up to 1% correction for the β spectrum and up to 2% correction for the angular correlation
- Propagating nuclear structure and χEFT uncertainties results in an overall uncertainty of 10^{-4}
  - Comparable to the precision of current experiments

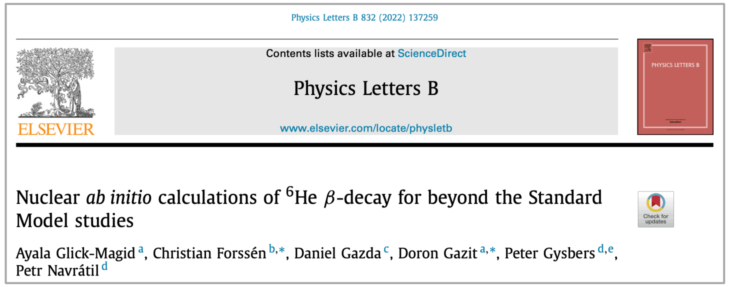
$$b_{\mathrm{F}}^{1^{+}\beta^{-}} = \delta_{b}^{1^{+}\beta^{-}} = -1.52\,(18)\cdot 10^{-3}$$

$$\left\langle \tilde{\delta}_{a}^{1^{+}\beta^{-}} \right\rangle = -2.54\,(68)\cdot 10^{-3}$$

Non-zero Fierz interference term due to nuclear structure corrections

Note that new physics at TeV scale implies

$$b_{\mathrm{Fierz}}^{\mathrm{BSM}} = \frac{C_T + C_T'}{C_A} \sim 10^{-3}$$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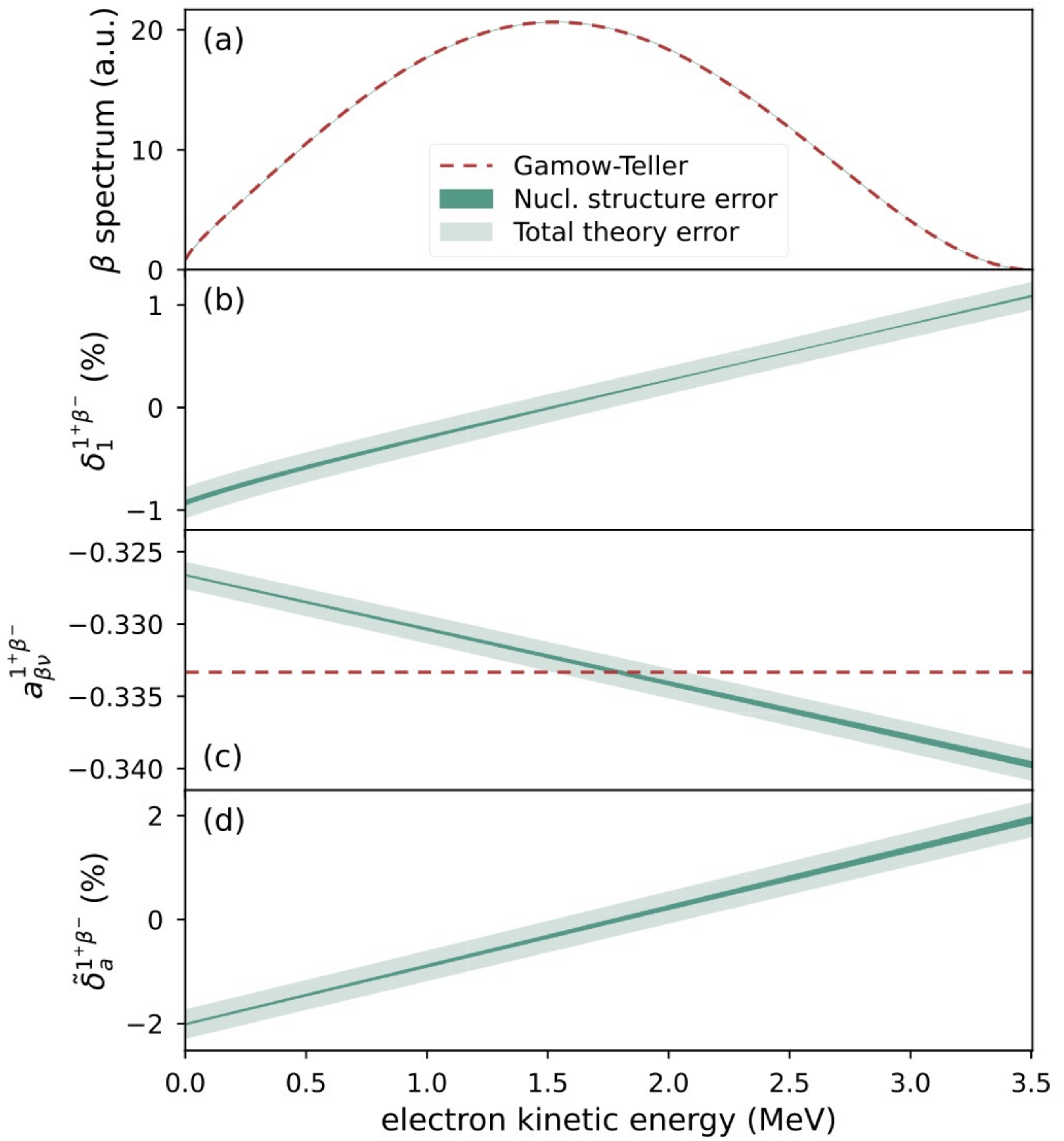
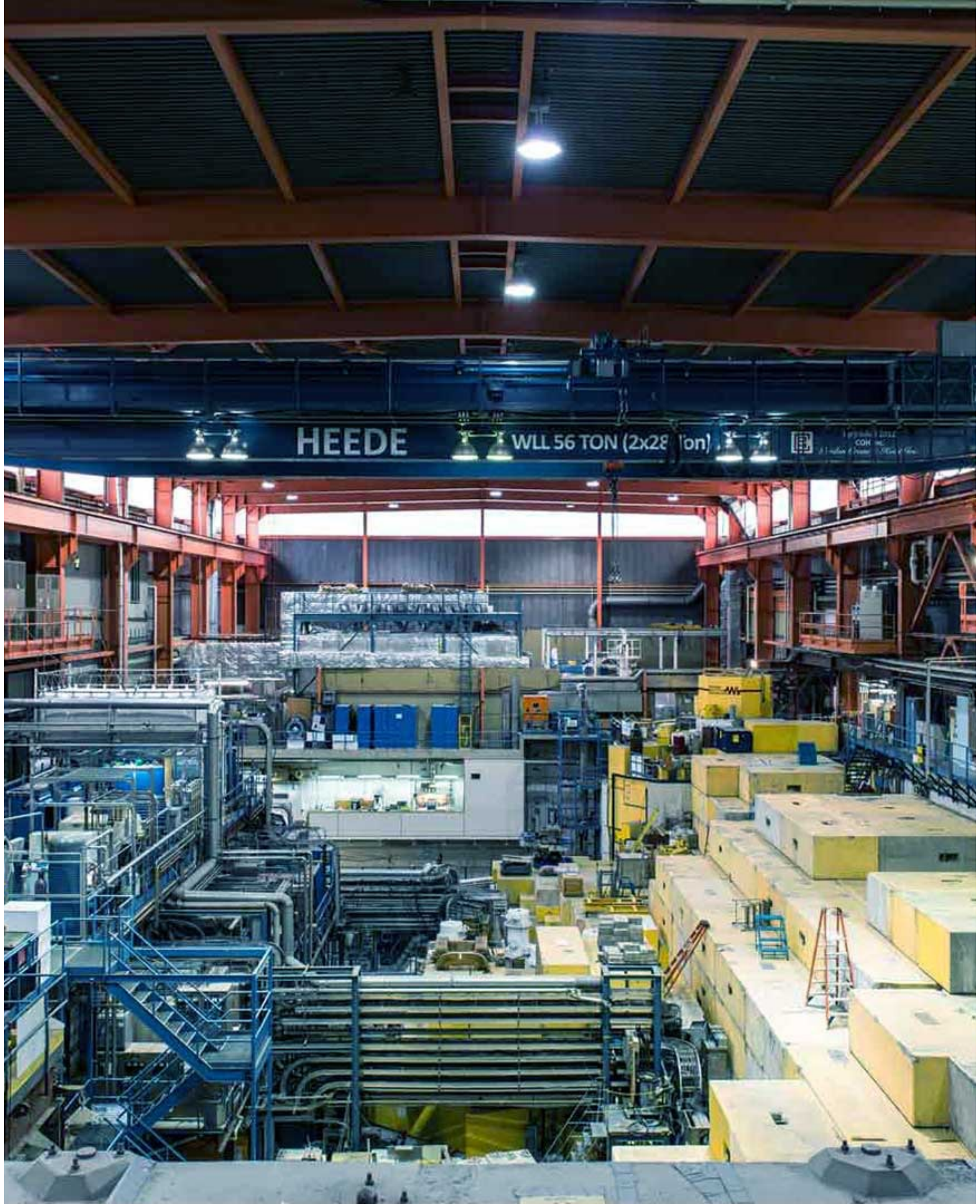


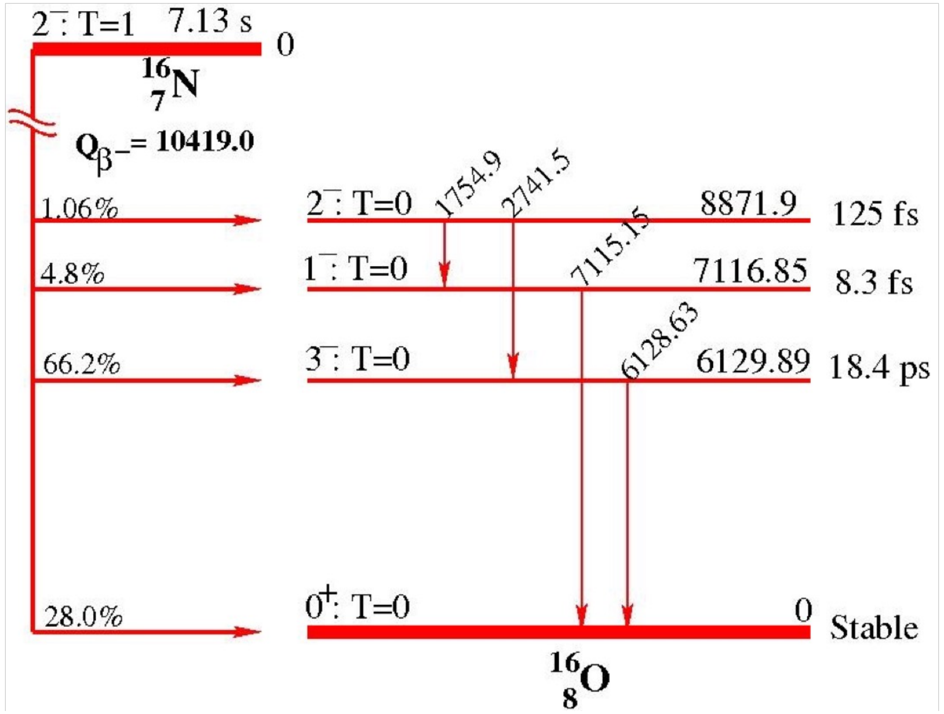
Unique first-forbidden beta decay

$^{16}N(2^-) \rightarrow {}^{16}O(0^+)$

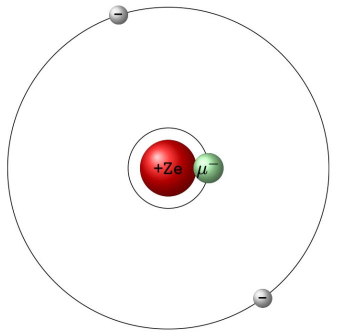


Unique first-forbidden beta decay $^{16}N(2^-) \rightarrow {}^{16}O(0^+)$

- The unique first-forbidden transition, $J^{\Delta\pi} = 2^-$, is of great interest for BSM searches
  - Energy spectrum of emitted electrons sensitive to the symmetries of the weak interaction, gives constraints both in the case of right and left couplings of the new beyond standard model currents
  - Ayala Glick-Magid *et al*., PLB 767 (2017) 285
- Ongoing experiment at SARAF, Israel



Ordinary muon capture on ^{16}O within the NCSM



- Investigated using three sets of chiral EFT NN+3N interactions:
  - NN(N^{4}LO)+3N(N^{2}LO,lnl)

    Entem, Machleidt, Nosyk, Phys. Rev. C 96, 024004 (2017) (NN)

    Gysbers *et al.*, Nature Phys. 15, 428 (2019) (3N)
  - NN(N^{4}LO)+3N(N^{2}LO,lnl,E7)

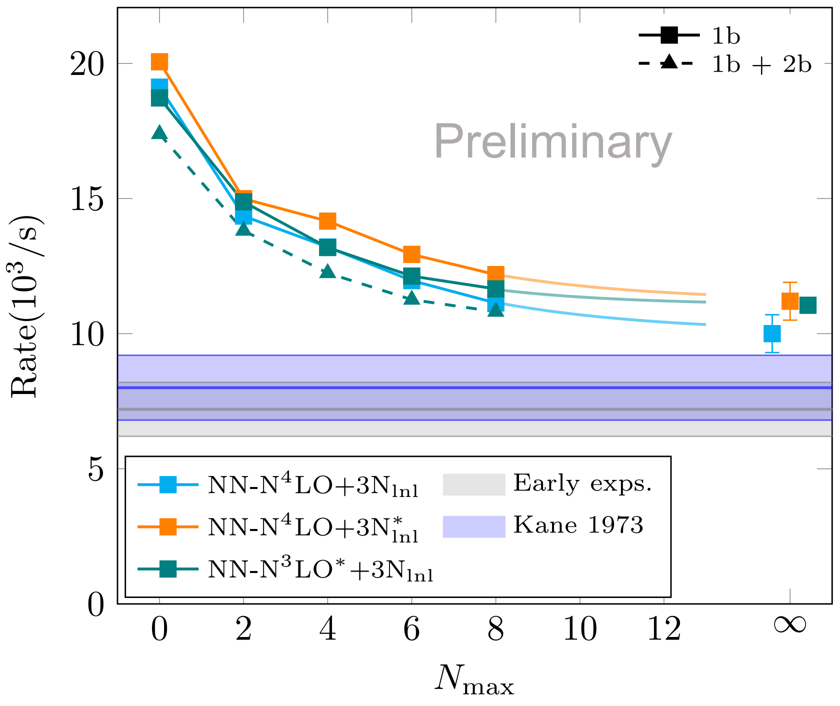
    Girlanda, Kievsky, Viviani, Phys. Rev. C 84, 014001 (2011) (E7)
  - NN(N^{3}LO)+3N(N^{2}LO,lnl)

    Entem, Machleidt, Phys. Rev. C 68, 041001 (2003) (NN)

    Soma, Navratil *et al.*, Phys. Rev. C 101, 014318 (2020) (3N)

- Results quite encouraging
  - NCSM describes well the complex systems ^{16}O and ^{16}N
  - → Feasible to apply NCSM to the ^{16}N beta decay

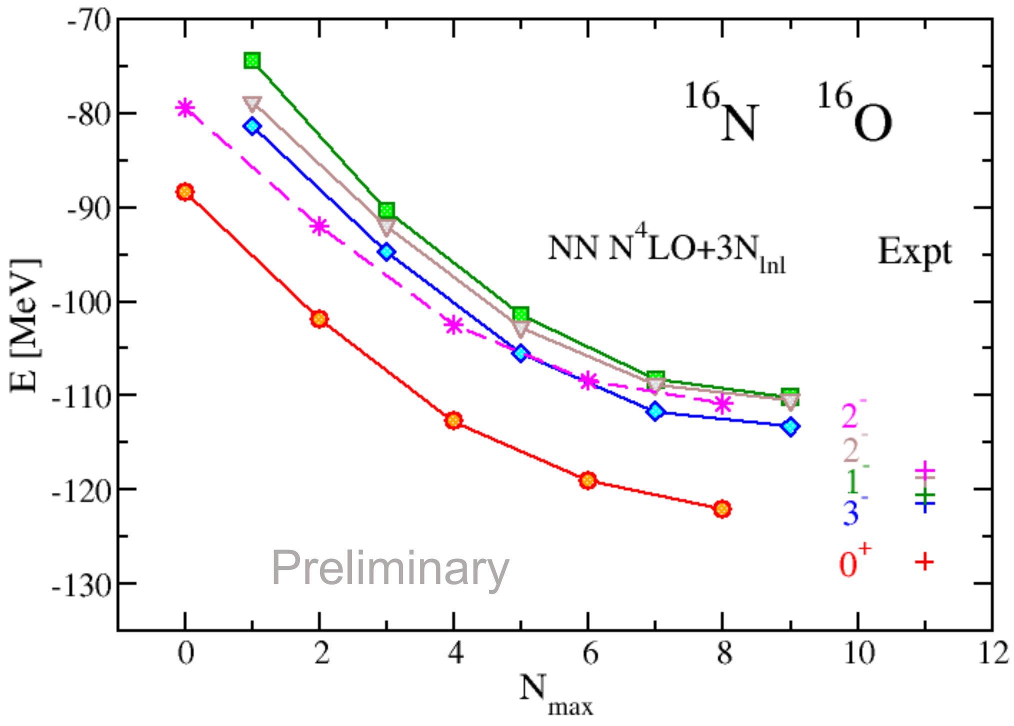
$$^{16}\mathrm{O}(0^+_{\mathrm{gs}}) + \mu^- \rightarrow \; ^{16}\mathrm{N}(2^-_{\mathrm{gs}}) + \nu_\mu$$

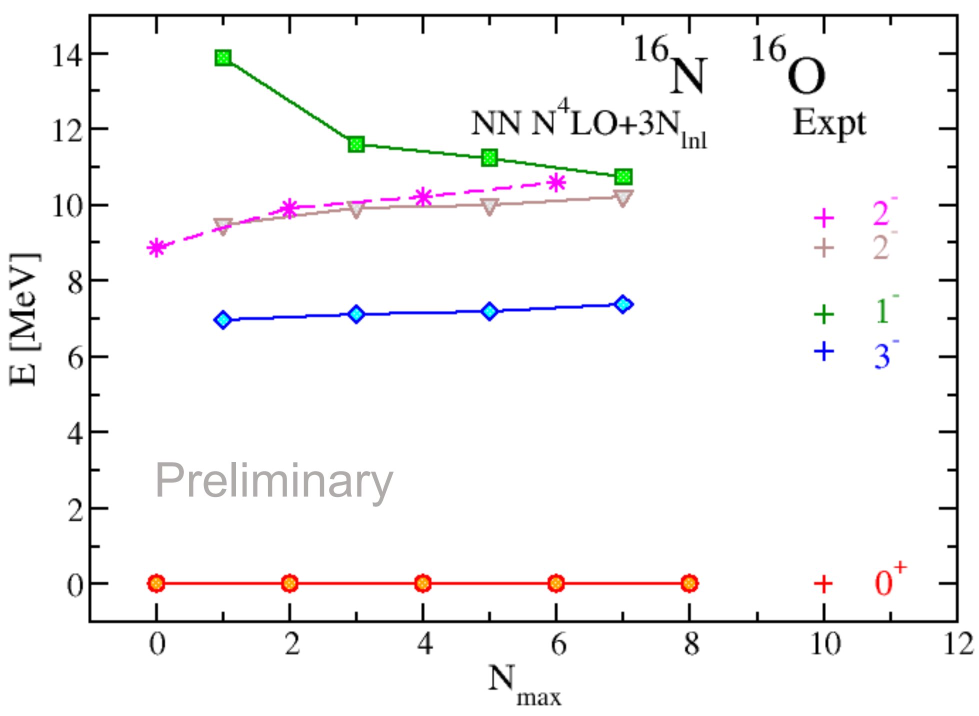


Lotta Jokiniemi, PN, Kotila, and Kravvaris, in progress

^{16}N and ^{16}O energies

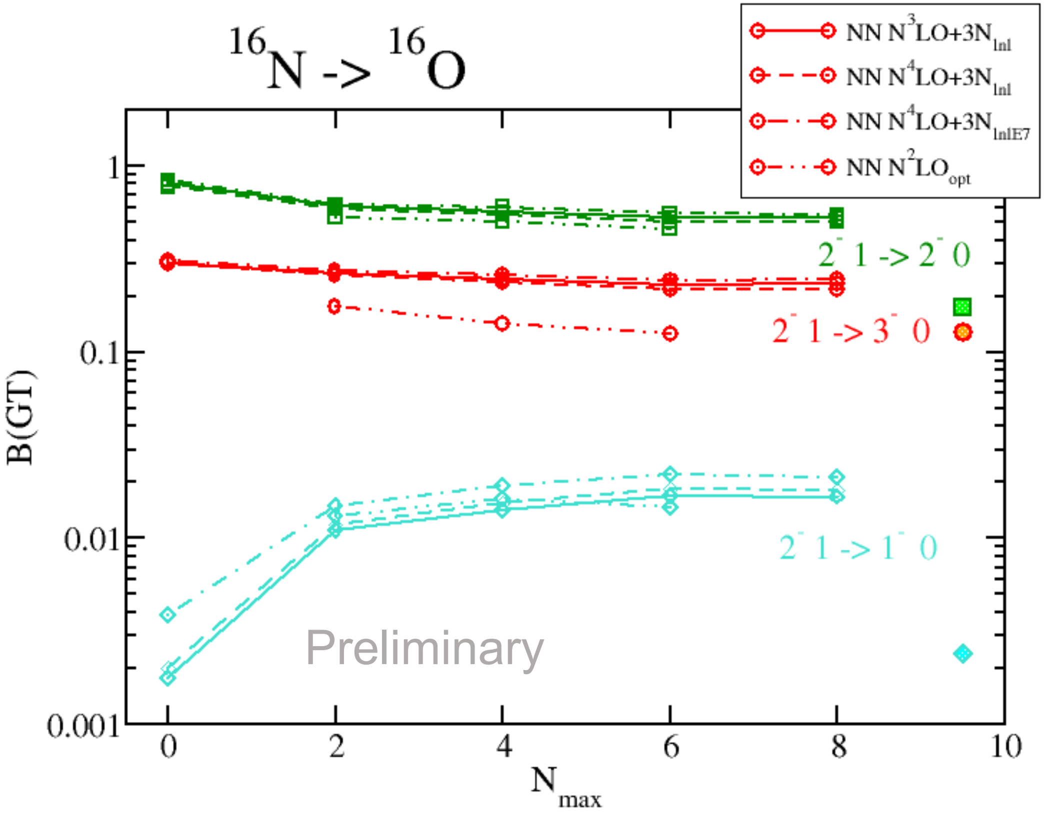
- Chiral NN N^4LO+3N_{lnl} interaction:
  - Binding energies underestimated by ~2 MeV
  - Excitation energies overestimated by ~1 MeV

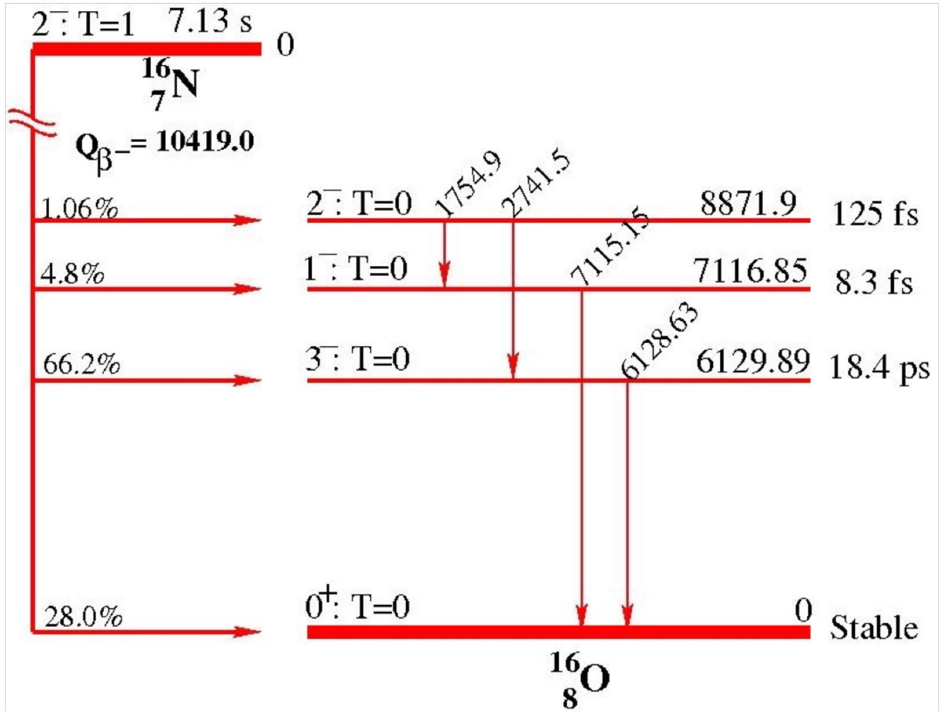




$^{16}N(2^-)$ Gamow-Teller transitions to the negative parity excited states of ^{16}O

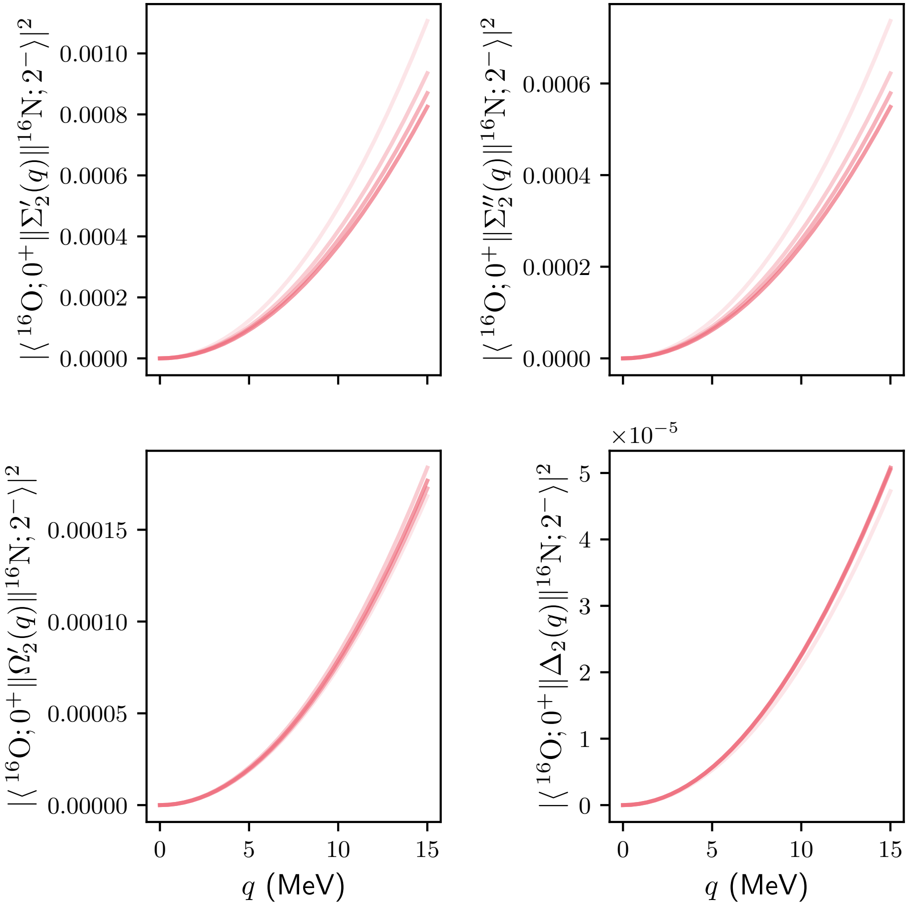
- Tests of NCSM wave functions
  - B(GT)s overestimated – operator SRG, 2BC need to be included
  - Correct hierarchy of transitions



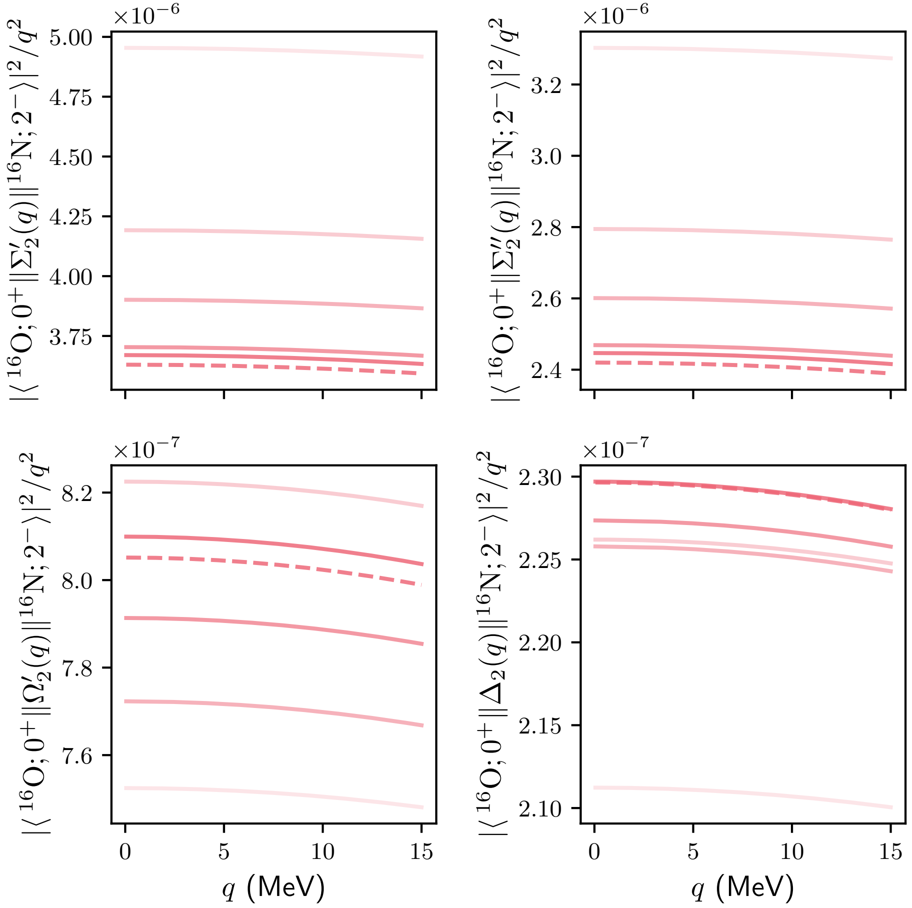


Unique first-forbidden beta decay $^{16}N(2^-) \rightarrow {}^{16}O(0^+)$

- Basic operator matrix elements
  - NN-N^{3}LO+3N$_{\text{lnl}}$ - N_{max} dependence, COM effect

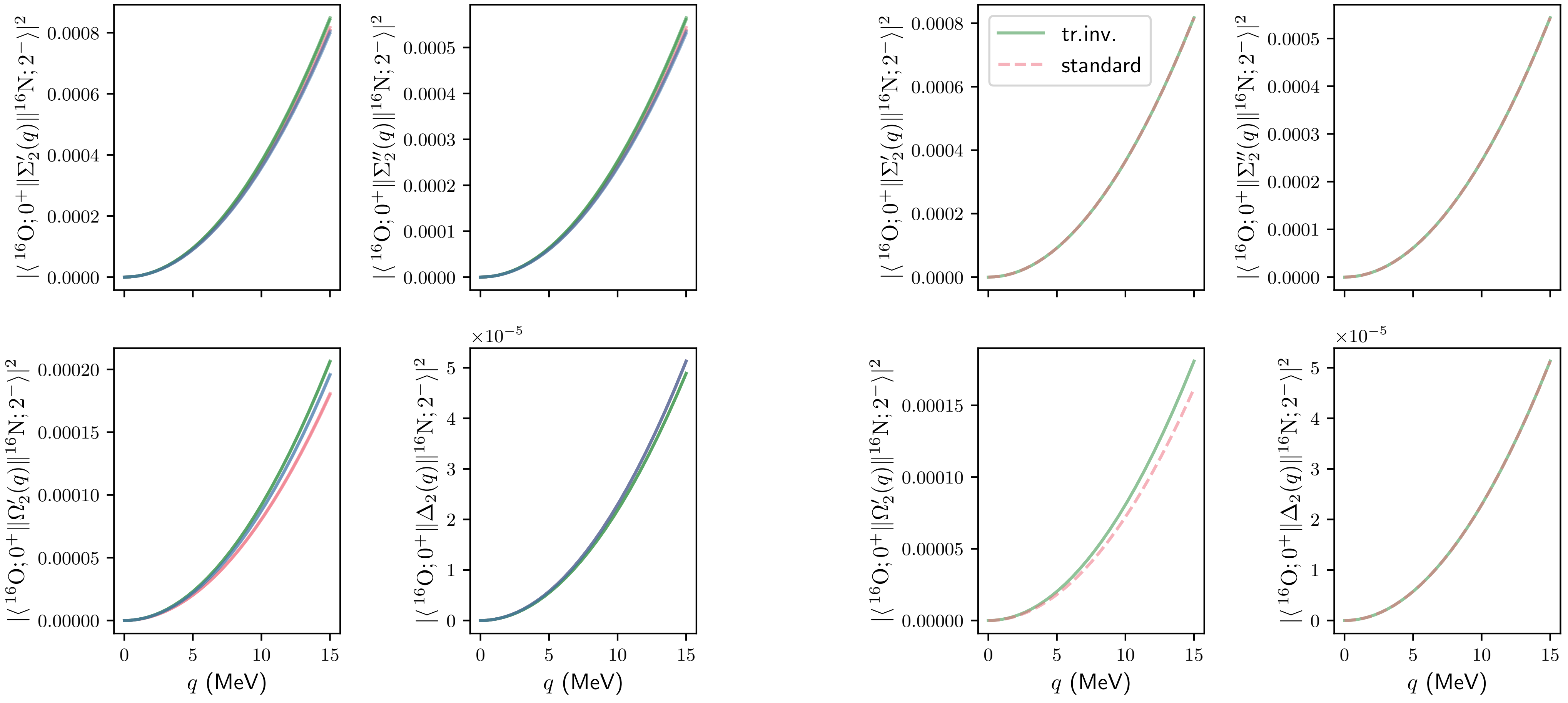


Preliminary

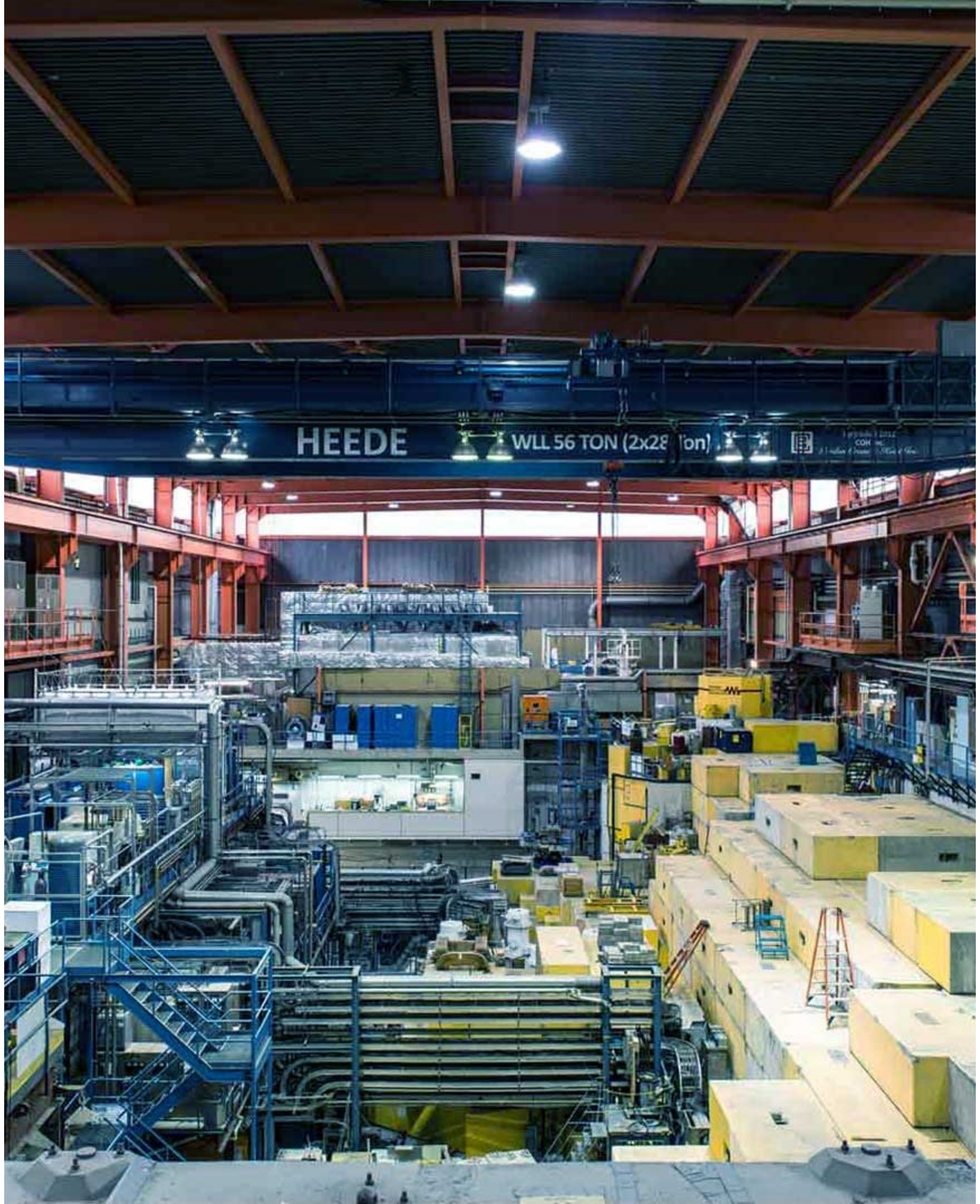


Unique first-forbidden beta decay $^{16}N(2^-) \rightarrow {}^{16}O(0^+)$

- Basic operator matrix elements
  - Interaction dependence, COM effect   Preliminary



Electroweak radiative correction δ_{NS}



V_{ud} element of CKM matrix

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}(\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^{\mu} W_{\mu}^{+} V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

- Precise V_{ud} from superallowed Fermi transitions

$$|V_{ud}|^2 = \frac{\hbar^7}{G_F^2 m_e^5 c^4} \frac{\pi^3 \ln(2)}{\mathcal{F}t(1+\Delta_R^V)}$$

$G_F \equiv$ Fermi coupling constant determined from muon β decay

  - hadronic matrix elements modified by nuclear environment
  - Fermi matrix element renormalized by isospin non–conserving forces

$$\mathcal{F}t = ft(1+\delta_R')\underline{(1-\delta_C+\delta_{NS})}$$

$$\mathcal{F}t = \frac{K}{G_V^2 |M_{F0}|^2 (1+\Delta_R^V)}$$

Δ_R^V and δ_{NS}

- Tree level beta decay amplitude

$$M_{tree} = -\frac{G_F}{\sqrt{2}} L_\lambda F^\lambda(p', p)$$

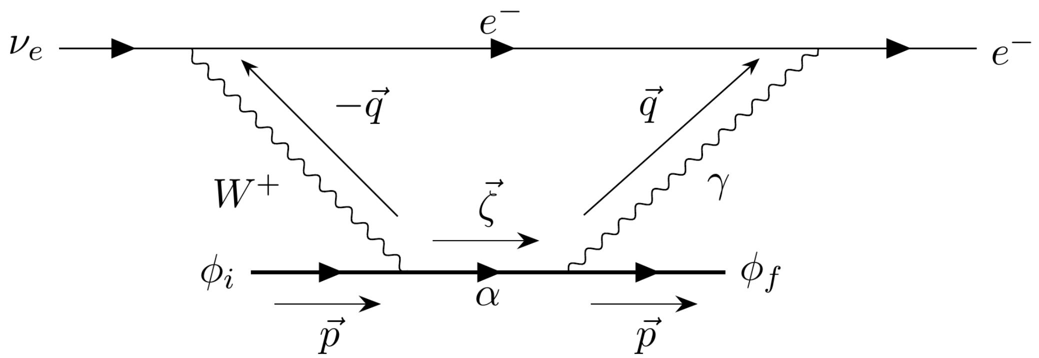
Leptonic current (L_λ)

NME of charged weak current ($F^\lambda(p', p)$)

- Hadronic correction in forward scattering limit

$$\delta M = -i\sqrt{2} G_F e^2 L_\lambda \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q_\alpha}{[(p_e - q)^2 - m_e^2] q^2} \underline{T_{\mu\nu}(p', p, q)}$$

$$\boxed{\delta M = \Box_{\gamma W}(E_e) M_{tree}}$$



[6] Seng et al. (2023)

Δ_R^V and δ_{NS}

Leptonic current

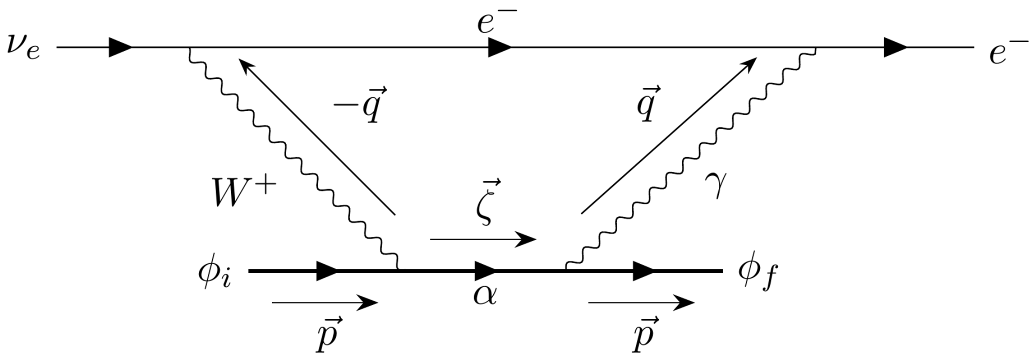
NME of charged weak current

- Tree level beta decay amplitude

$$M_{tree} = -\frac{G_F}{\sqrt{2}} L_\lambda F^\lambda(p', p)$$

- Hadronic correction in forward scattering limit

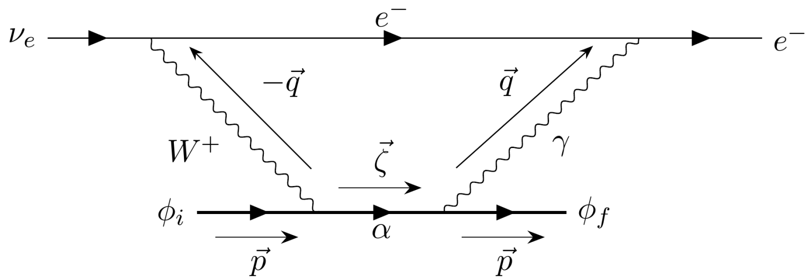
$$\delta M = \Box_{\gamma W}(E_e) M_{tree}$$



$$\Box^b_{\gamma W}(E_e) = \frac{e^2}{M} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{1}{q^2 + i\epsilon} \frac{1}{(p_e - q)^2 + i\epsilon'} \frac{M\nu\left(\frac{p_e \cdot q}{p \cdot p_e}\right) - q^2}{\nu} \frac{T_3(\nu, |\vec{q}|)}{f_+(0)}$$

[6] Seng et al. (2023)

Nonrelativistic Compton amplitude

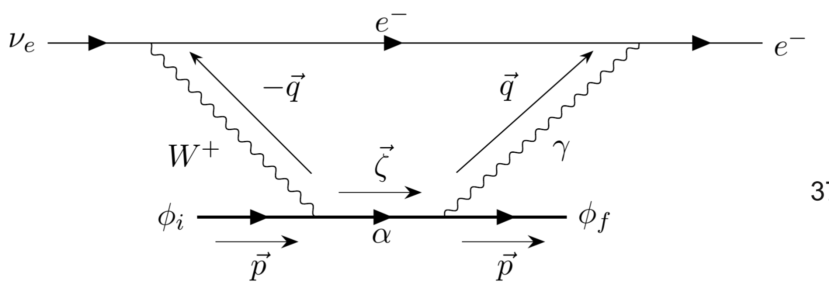


- **Goal**: Non-relativistic currents in momentum space
- Rewrite currents with A-body propagators
- Fourier transform currents into momentum space
- General multipole expansion of currents

$$M_{JM}(q) := \int d^3r\ \mathcal{M}_{JM}(q,\vec{r})\,\rho(\vec{r}) \qquad T^{\mathrm{el}}_{JM}(q) := \int d^3r\ \frac{1}{q}\left(\vec{\nabla} \times \vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r})\right) \cdot \vec{J}(\vec{r})$$

$$L_{JM}(q) := \int d^3r\ \frac{i}{q}\left(\vec{\nabla}\mathcal{M}_{JM}(q,\vec{r})\right) \cdot \vec{J}(\vec{r}) \qquad T^{\mathrm{mag}}_{JM}(q) := \int d^3r\ \vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r}) \cdot \vec{J}(\vec{r})$$

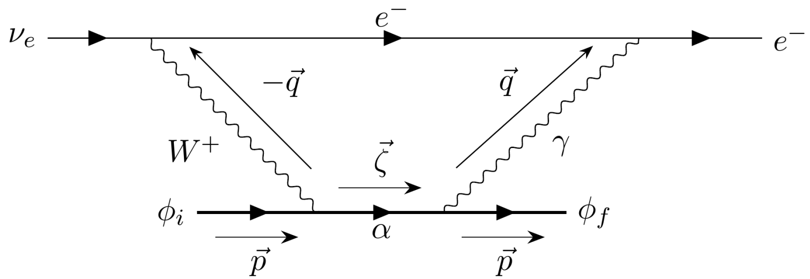
Nonrelativistic Compton amplitude



- **Goal**: Non-relativistic currents in momentum space
- Rewrite currents with A-body propagators
- Fourier transform currents into momentum space
- General multipole expansion of currents

$$T_3(\nu, |\vec{q}\,|) = 4\pi i \frac{\nu}{|\vec{q}\,|} \sqrt{M_i M_f} \sum_{J=1}^{\infty} (2J+1) \langle \Psi_f | \Big\{ T_{J0}^{\text{mag}} \, G(\nu + M_f + i\epsilon) \, T_{J0}^{5,\text{el}} + T_{J0}^{\text{el}} \, G(\nu + M_f + i\epsilon) \, T_{J0}^{5,\text{mag}} + T_{J0}^{5,\text{mag}} \, G(-\nu + M_i + i\epsilon) \, T_{J0}^{\text{el}} + T_{J0}^{5,\text{el}} \, G(-\nu + M_i + i\epsilon) T_{J0}^{\text{mag}} \Big\} (|\vec{q}\,|) | \Psi_i \rangle$$

Nonrelativistic Compton amplitude



- **Goal**: Non-relativistic currents in momentum space
- Rewrite currents with A-body propagators
- Fourier transform currents into momentum space
- General multipole expansion of currents

Lanczos continued fraction method to compute nuclear Green's functions

$$T_3(\nu, |\vec{q}\,|) = 4\pi i \frac{\nu}{|\vec{q}\,|}\sqrt{M_i M_f}\sum_{J=1}^{\infty}(2J+1)\,\langle \Psi_f | \Big\{ T_{J0}^{\text{mag}} G(\nu + M_f + i\epsilon) T_{J0}^{5,\text{el}} + T_{J0}^{\text{el}} G(\nu + M_f + i\epsilon) T_{J0}^{5,\text{mag}} + T_{J0}^{5,\text{mag}} G(-\nu + M_i + i\epsilon) T_{J0}^{\text{el}} + T_{J0}^{5,\text{el}} G(-\nu + M_i + i\epsilon) T_{J0}^{\text{mag}} \Big\} (|\vec{q}\,|) | \Psi_i \rangle$$

The $\nu \equiv q_0$ integration performed using Wick rotation

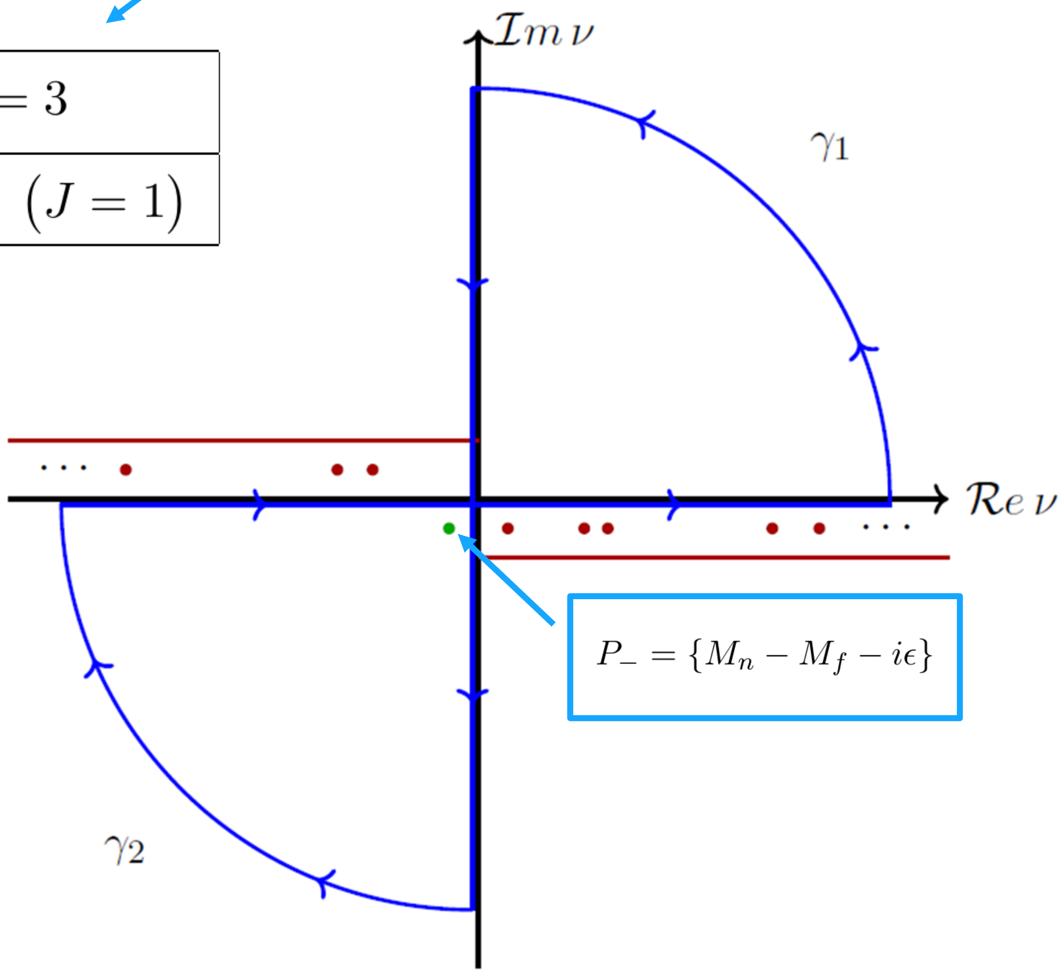
Residues for $^{10}C \rightarrow {}^{10}B$ in NCSM

Second 1^+ below 0^+ sensitive to interaction and N_{max}

Poles	$n = 1$	$n = 2$	$n = 3$
P_- [MeV]	-1.6572 $(J = 3)$	-0.6974 $(J = 1)$	-0.1861 $(J = 1)$

Table 1: Pole locations along ν axis corresponding to n–th excited state in T_3 for $^{10}C \rightarrow {}^{10}B$ transition at $N_{max} = 5$.

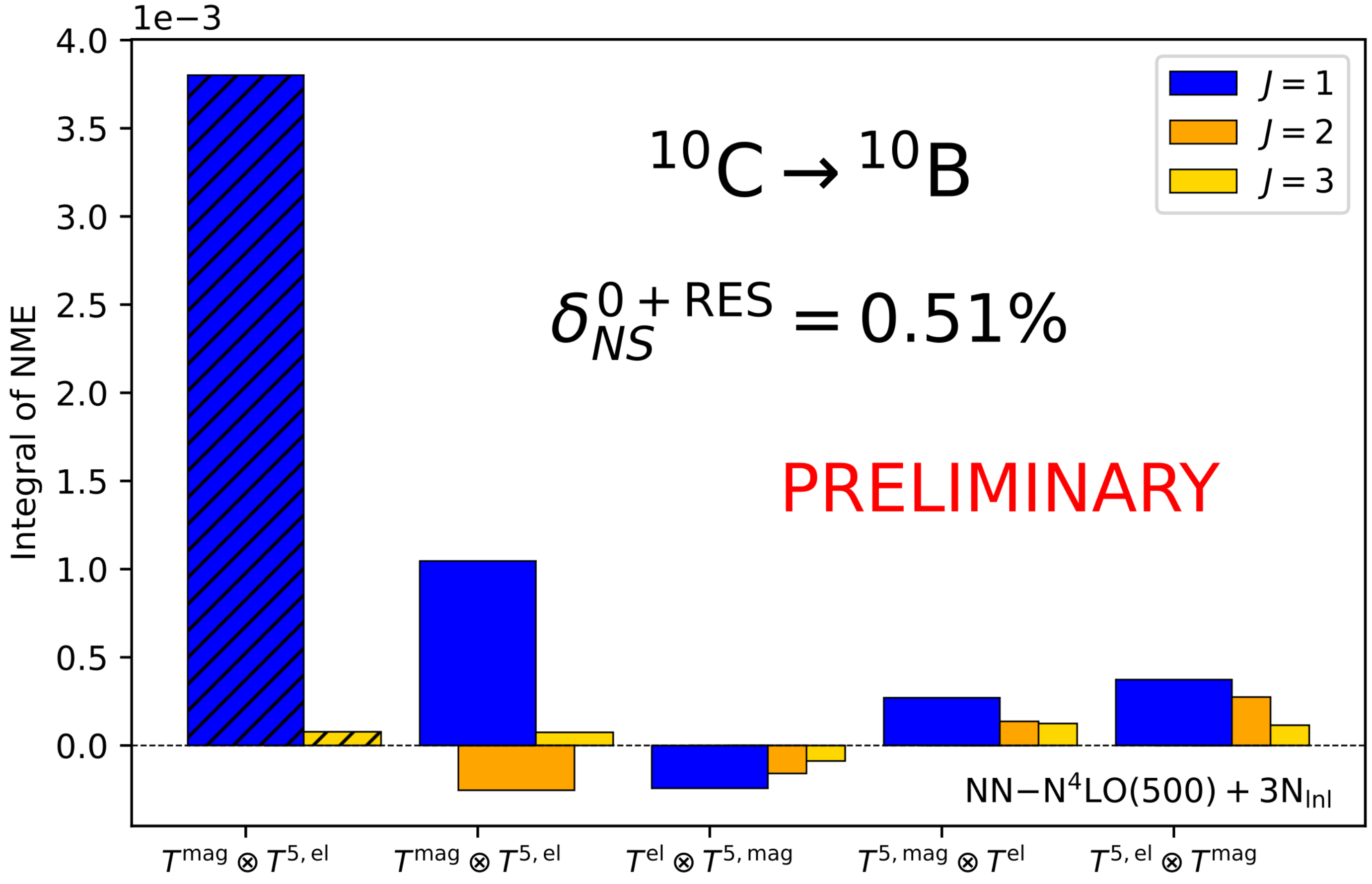
- Ground state 3^+ and low-lying 1^+ incur residues after Wick rotation
- Remaining pole in residue terms must also be treated



Preliminary δ_{NS} result at N_{max}=3 and N_{max}=5 still being double checked

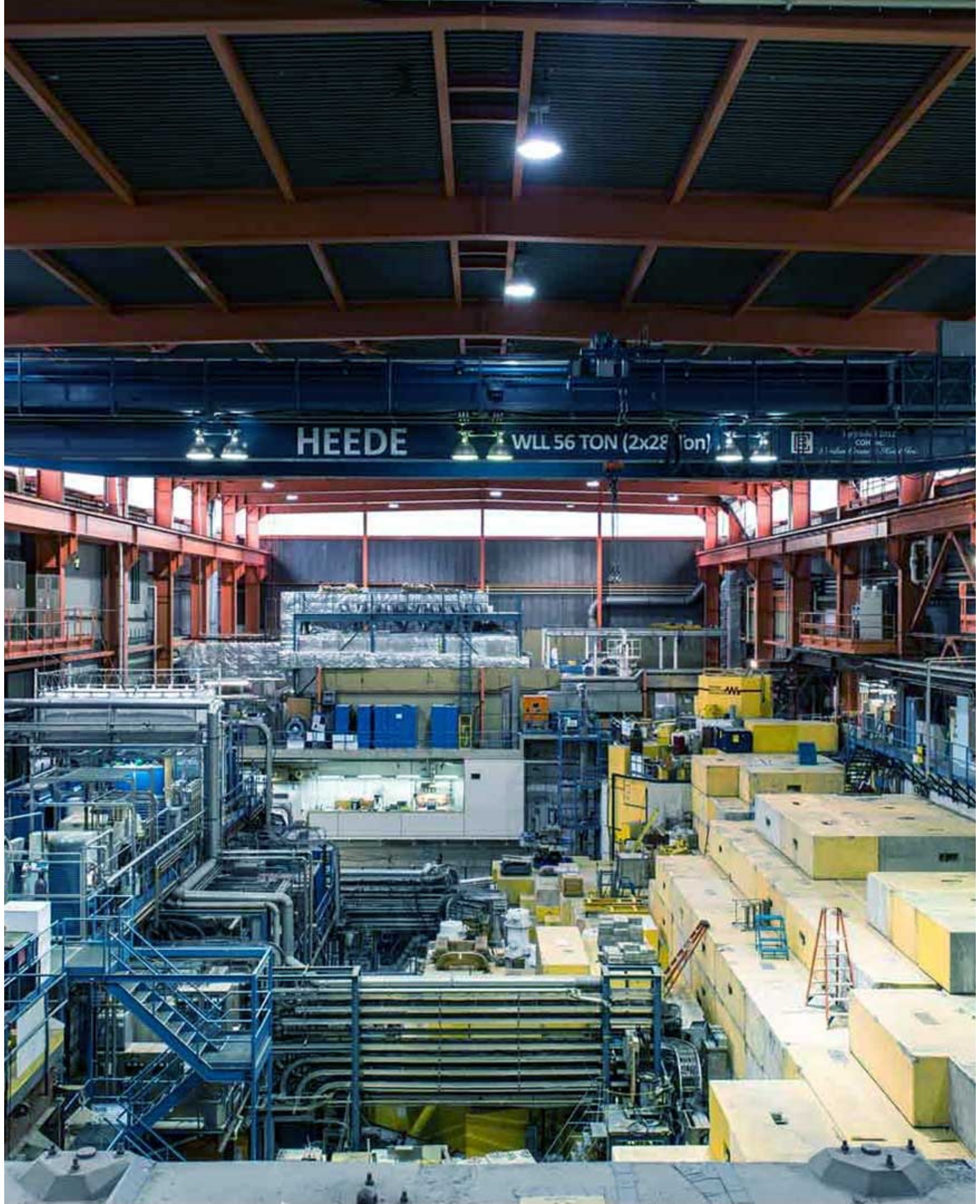
Feasible to reach N_{max}=11

Towner & Hardy used δ_{NS} = -0.4



TRIUMF

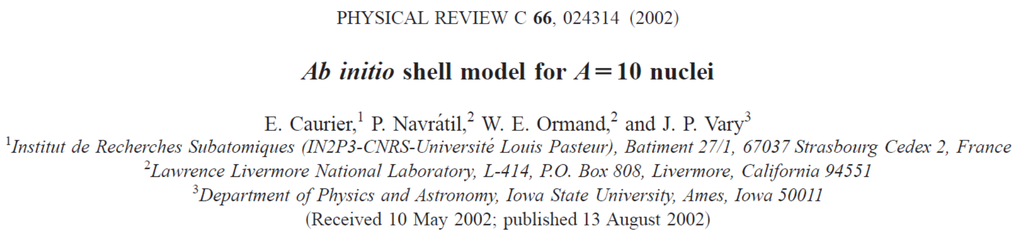
Isospin symmetry breaking correction δ_C



Discovery, accelerated

The pathway to δ_C

- δ_C in *ab initio* NCSM over 20 years ago

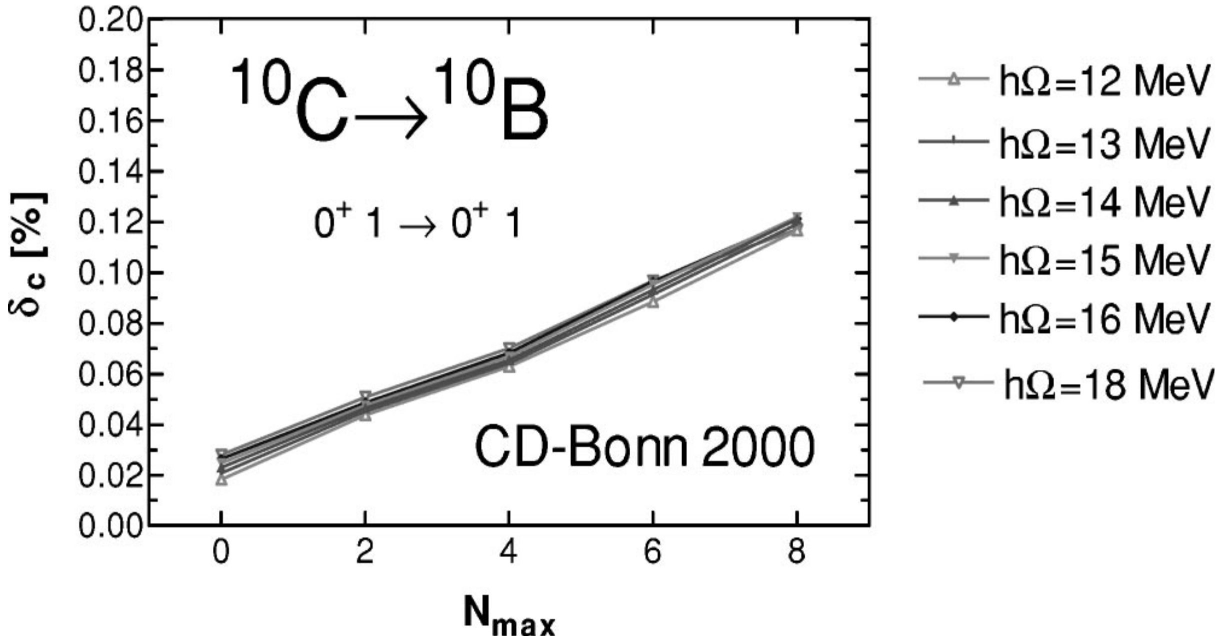

PHYSICAL REVIEW C **66**, 024314 (2002)

***Ab initio* shell model for $A=10$ nuclei**

E. Caurier,[1] P. Navrátil,[2] W. E. Ormand,[2] and J. P. Vary[3]

[1]*Institut de Recherches Subatomiques (IN2P3-CNRS-Université Louis Pasteur), Batiment 27/1, 67037 Strasbourg Cedex 2, France*
[2]*Lawrence Livermore National Laboratory, L-414, P.O. Box 808, Livermore, California 94551*
[3]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011*

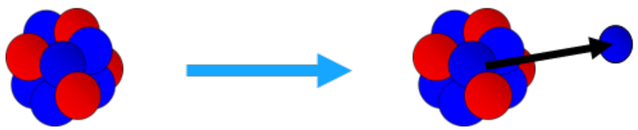
(Received 10 May 2002; published 13 August 2002)



HO expansion incompatible with reaction theory

i. imprecise asymptotics
ii. missing correlations in excited states
iii. description of scattering not feasible

Combine NCSM with resonating group method (RGM)



Ab Initio Calculations of Structure, Scattering, Reactions

Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left|^{(A)} , \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left|_{(A-a)} \xrightarrow{\vec{r}} {}_{(a)} , \nu \right\rangle$$

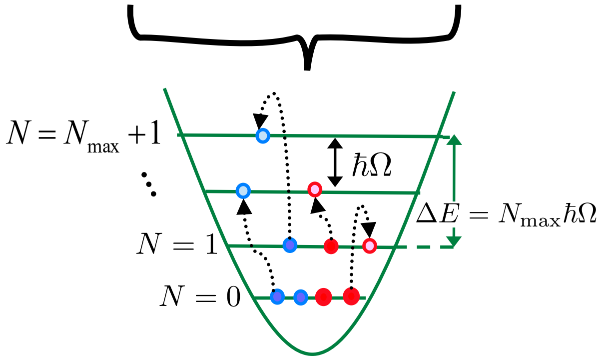
S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

Ab Initio Calculations of Structure, Scattering, Reactions

Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left|^{(A)} \;, \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left|_{(A-a)} \;\; \vec{r} \;\; (a) \;, \nu \right\rangle$$



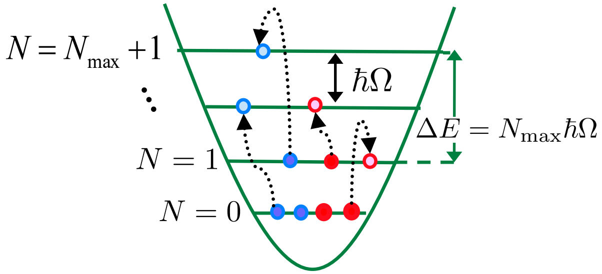
Static solutions for aggregate system,
describe all nucleons close together

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

Ab Initio Calculations of Structure, Scattering, Reactions
Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| {}^{(A)} \, , \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left| {}_{(A-a)} \;\; \vec{r} \;\; {}_{(a)} \, , \nu \right\rangle$$



Static solutions for aggregate system, describe all nucleons close together

Continuous microscopic cluster states, describe long-range projectile-target

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

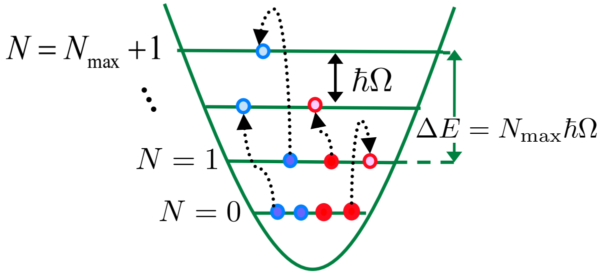
Ab Initio Calculations of Structure, Scattering, Reactions

Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

Unknowns

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| {}^{(A)}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r})\; \hat{A}_{\nu} \left| {}_{(A-a)} \;\vec{r}\; (a), \nu \right\rangle$$



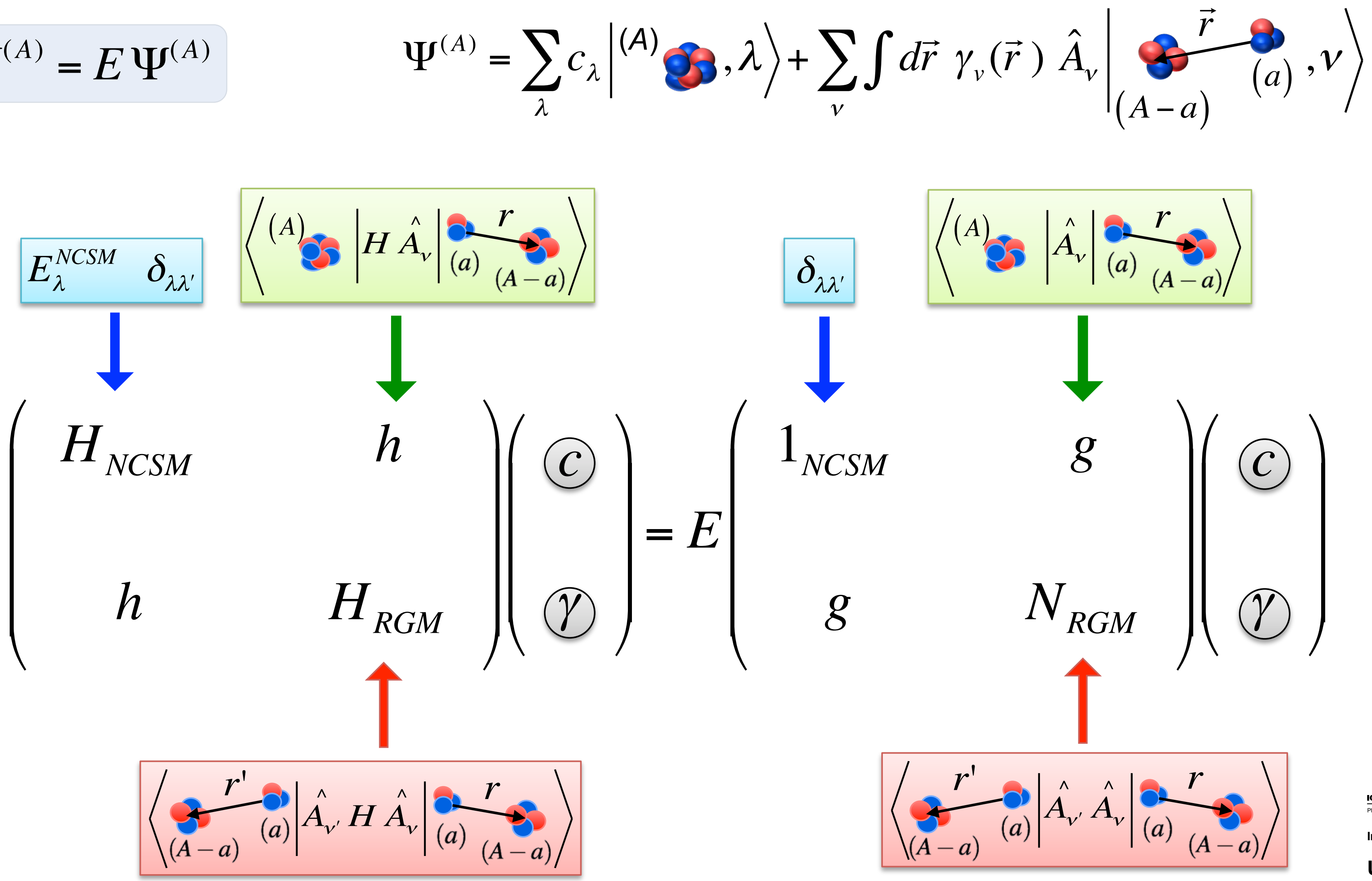
Continuous microscopic cluster states, describe long-range projectile-target

Static solutions for aggregate system, describe all nucleons close together

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

Coupled NCSMC equations

$$H\,\Psi^{(A)} = E\,\Psi^{(A)}$$



δ_C in NCSMC

PHYSICAL REVIEW C **105**, 054316 (2022)

***Ab initio* calculation of the β decay from ^{11}Be to a ^{10}Be + p resonance**

M. C. Atkinson,[1] P. Navrátil,[1] G. Hupin,[2] K. Kravvaris,[3] and S. Quaglioni[3]

- Compute Fermi matrix element in NCSMC

$$M_F = \left\langle \Psi^{J^\pi T_f M_{T_f}} \middle| T_+ \middle| \Psi^{J^\pi T_i M_{T_i}} \right\rangle \longrightarrow |M_F|^2 = |M_{F0}|^2(1-\delta_C)$$

- Total isospin operator $T_+ = T_+^{(1)} + T_+^{(2)}$ for partitioned system

$$M_F \sim \langle A\lambda_f J_f T_f M_{T_f}|T_+|A\lambda J_i T_i M_{T_i}\rangle + \langle A\lambda J_f T_f M_{T_f}|T_+ \mathcal{A}_{\nu i}|\Phi_{\nu r}^{J_i T_i M_{T_i}}\rangle$$
$$+ \langle \Phi_{\nu r}^{J_f T_f M_{T_f}}|\mathcal{A}_{\nu f} T_+|A\lambda_i J_i T_i M_{T_i}\rangle + \langle \Phi_{\nu r}^{J_f T_f M_{T_f}}|\mathcal{A}_{\nu f} T_+ \mathcal{A}_{\nu i}|\Phi_{\nu r}^{J_i T_i M_{T_i}}\rangle$$

NCSM matrix element

NCSM-Cluster matrix elements

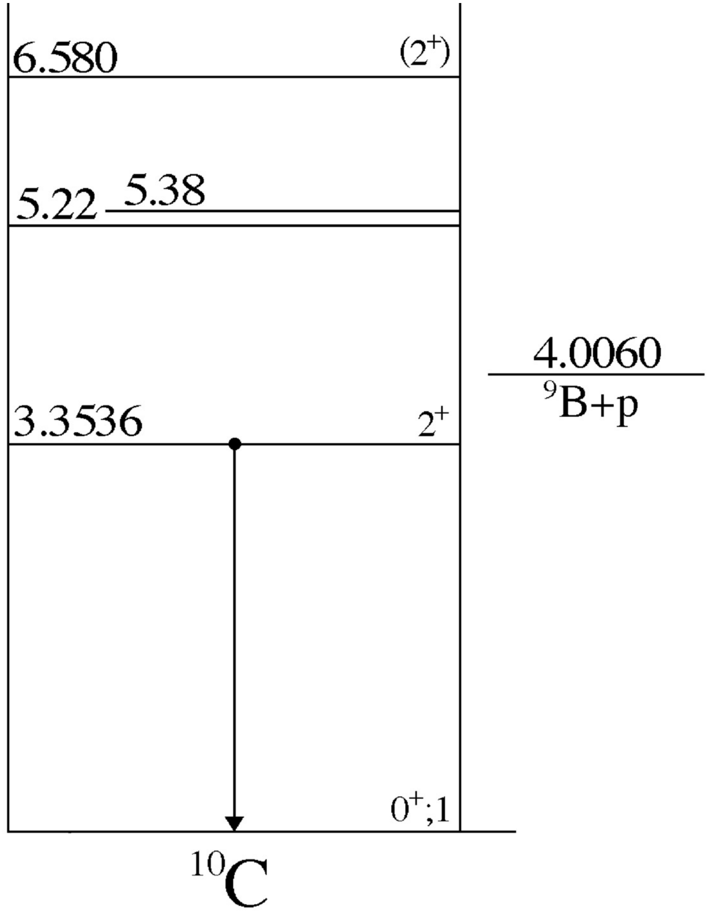
Continuum (cluster) matrix element

^{10}C structure from chiral EFT NN(N^{4}LO)+3N(N^{2}LO,lnl) interaction ($N_{max} = 9$)

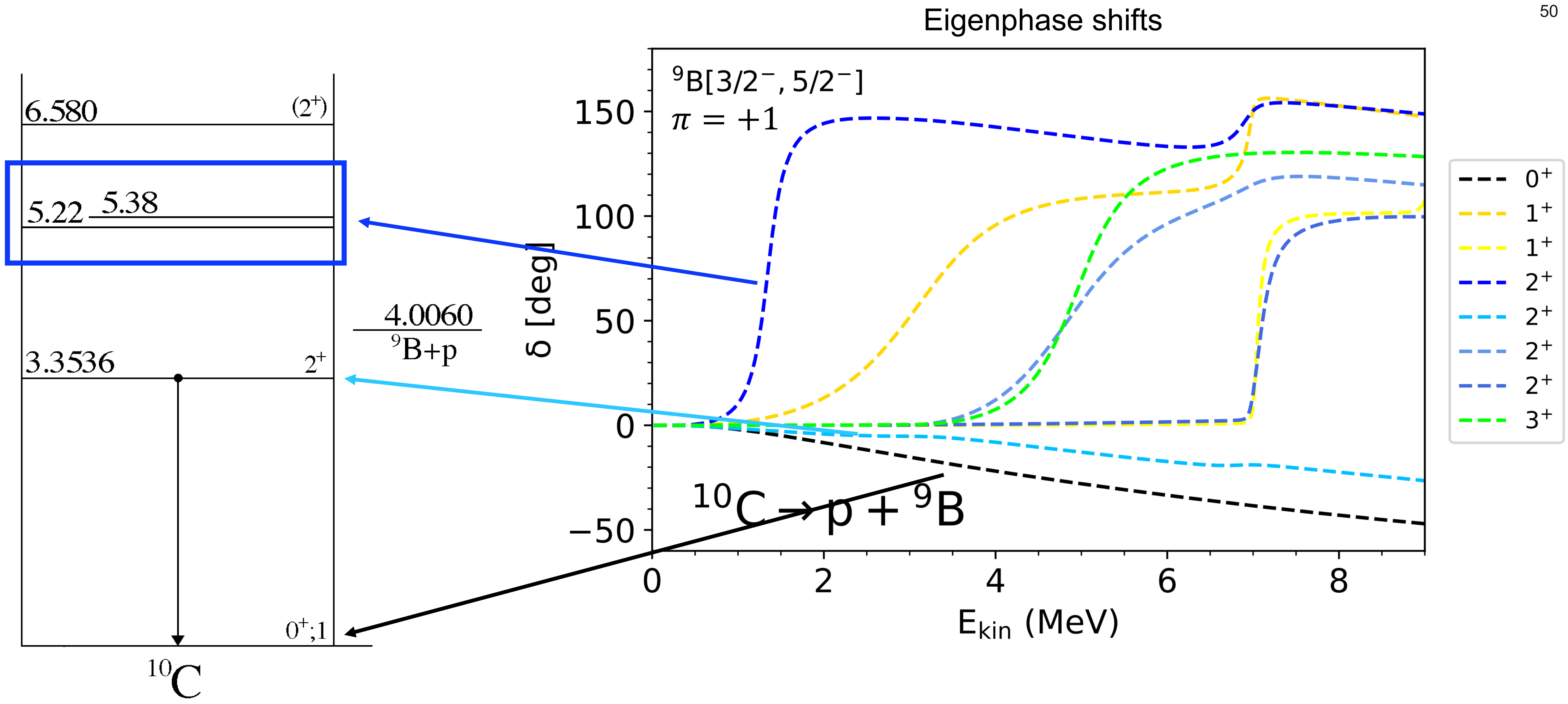
$$|{}^{10}\mathrm{C}\rangle = \sum_\alpha c_\alpha |{}^{10}\mathrm{C},\alpha\rangle_{\mathrm{NCSM}} + \sum_\nu \int dr\, \gamma_\nu^{J^\pi T}(r) \mathcal{A}_\nu |{}^9\mathrm{B}+\mathrm{p},\nu\rangle$$

- Treat as mass partition of proton plus ^{9}B
- Use $3/2^-$ and $5/2^-$ states of ^{9}B
- Known bound states captured by NCSMC

State	E_{NCSM} (MeV)	E (MeV)	E_{exp} (MeV)
0^+	-3.09	-3.46	-4.006
2^+	$+0.40$	-0.03	-0.652

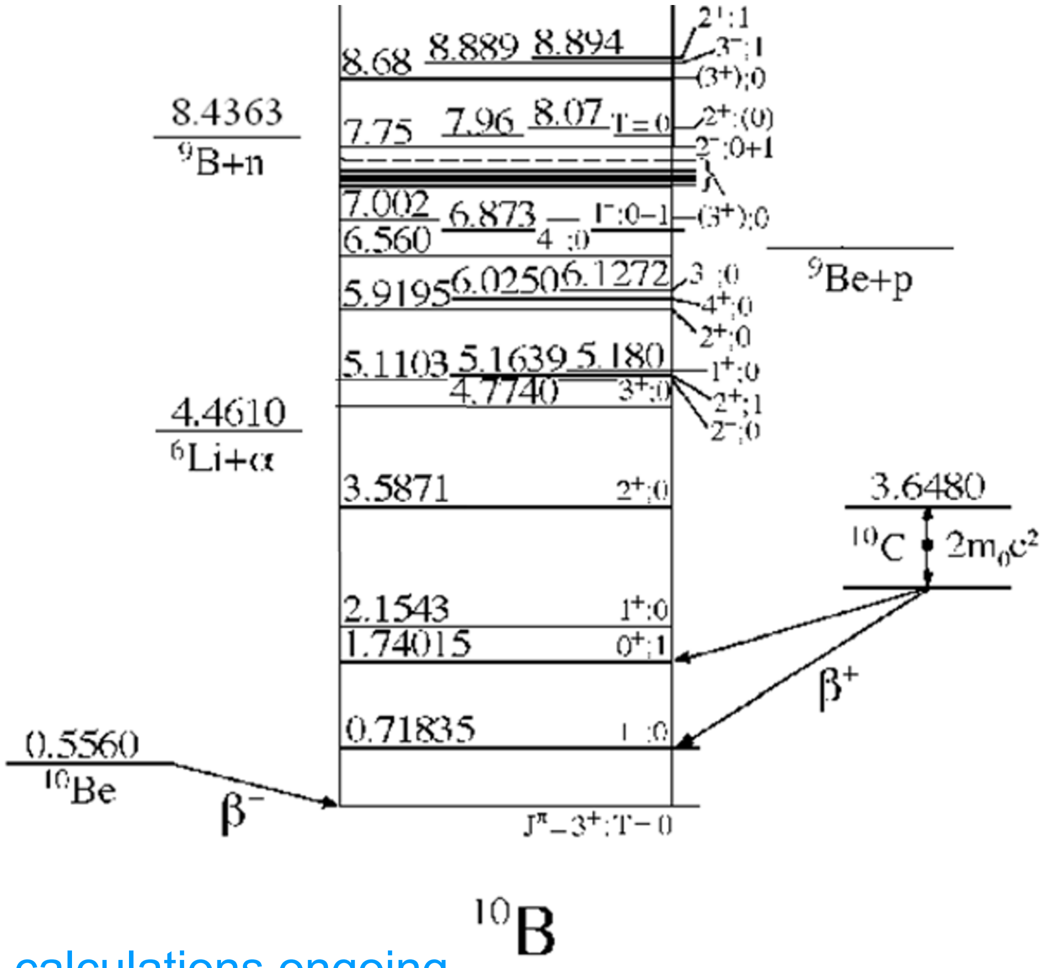


^{10}C structure from chiral EFT NN(N^{4}LO)+3N(N^{2}LO,lnl) interaction ($N_{max} = 9$)



^{10}B structure from chiral EFT NN(N^{4}LO)+3N(N^{2}LO,lnl) interaction ($N_{max} = 9$)

$$|^{10}\mathrm{B}\rangle = \sum_{\alpha} c_\alpha \, |^{10}\mathrm{B}, \alpha\rangle_{\mathrm{NCSM}} + \sum_{\nu} \int dr \, \gamma_\nu(r) \mathcal{A}_\nu \, |^{9}\mathrm{Be} + p, \nu\rangle + \sum_{\mu} \int dr \, \gamma_\mu(r) \mathcal{A}_\mu \, |^{9}\mathrm{B} + n, \mu\rangle$$

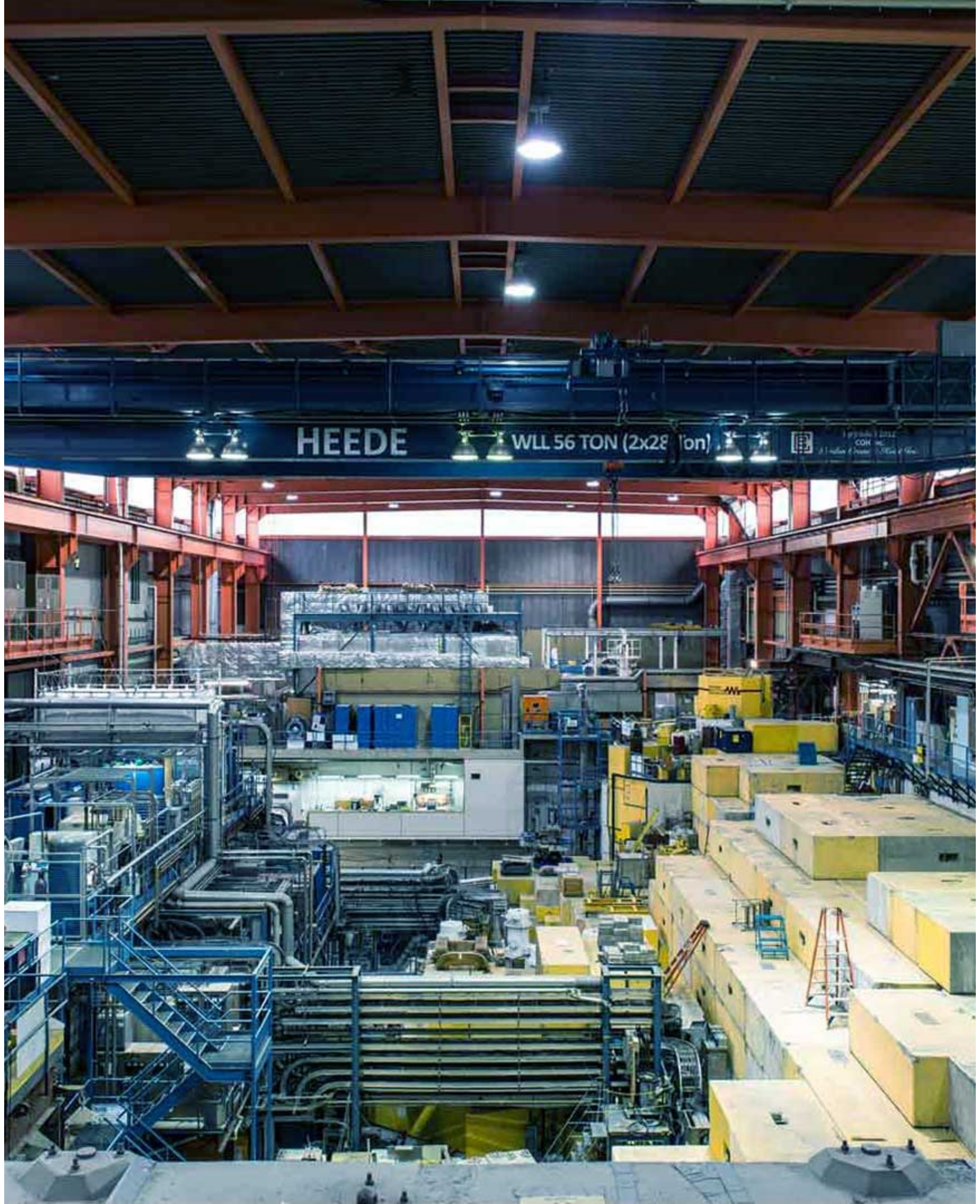


- Use $3/2^-$ and $5/2^-$ states of ^{9}B and ^{9}Be
- Eight of twelve bound states predicted

State	E (MeV)	E_{exp} (MeV)
3^+	-5.75	-6.5859
1^+	-5.33	-5.8676
0^+	-4.30	-4.8458
1^+	-4.26	-4.4316
2^+	-2.69	-2.9988
2^+	-0.93	-1.4220
2^+	-0.70	-0.6664
4^+	-0.19	-0.5609

δ_C calculations ongoing …

β-delayed proton emission in ^{11}Be



Discovery, accelerated

β-delayed proton emission in ^{11}Be

Physics Letters B 732 (2014) 305–308

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Physics Letters B

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^{11}Be(βp), a quasi-free neutron decay?

K. Riisager[a,*], O. Forstner[b,c], M.J.G. Borge[d,e], J.A. Briz[e], M. Carmona-Gallardo[e], L.M. Fraile[f], H.O.U. Fynbo[a], T. Giles[g], A. Gottberg[e,g], A. Heinz[h], J.G. Johansen[a,1], B. Jonson[h], J. Kurcewicz[d], M.V. Lund[a], T. Nilsson[h], G. Nyman[h], E. Rapisarda[d], P. Steier[b], O. Tengblad[e], R. Thies[h], S.R. Winkler[b]

- Indirectly observed $^{11}\mathrm{Be}(\beta p)^{10}\mathrm{Be}$
- Measured an extremely high branching ratio $b_p = 8.3 \pm 0.9 \times 10^{-6}$
  - Orders of magnitude larger than theoretical predictions (*e.g.* 3.0×10^{-8})
- Two proposed explanations: D. Baye and E.M. Tursunov, PLB **696**, 4, 464-467 (2011)
  1. The neutron decays to an unobserved $p+^{10}$Be resonance in ^{11}B
  2. There are unobserved dark decay modes

β-delayed proton emission in ^{11}Be

Eur. Phys. J. A (2020) 56:100
https://doi.org/10.1140/epja/s10050-020-00110-2

THE EUROPEAN PHYSICAL JOURNAL A

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Regular Article - Experimental Physics

Search for beta-delayed proton emission from ^{11}Be

K. Riisager[1,a], M. J. G. Borge[2,3], J. A. Briz[3], M. Carmona-Gallardo[4], O. Forstner[5], L. M. Fraile[4], H. O. U. Fynbo[1], A. Garzon Camacho[3], J. G. Johansen[1], B. Jonson[6], M. V. Lund[1], J. Lachner[5], M. Madurga[2], S. Merchel[7], E. Nacher[3], T. Nilsson[6], P. Steier[5], O. Tengblad[3], V. Vedia[4]

- New Accelerator Mass Spectrometry experiment that supersedes the 2014 measurement
  - Branching ratio $b_p \sim 2.2 \times 10^{-6}$
    - Upper limit, possible contamination by BeH molecular ions

β-delayed proton emission in ^{11}Be

PHYSICAL REVIEW LETTERS **123**, 082501 (2019)

Editors' Suggestion

Direct Observation of Proton Emission in ^{11}Be

Y. Ayyad,[1,2,*] B. Olaizola,[3] W. Mittig,[2,4] G. Potel,[1] V. Zelevinsky,[1,2,4] M. Horoi,[5] S. Beceiro-Novo,[4] M. Alcorta,[3] C. Andreoiu,[6] T. Ahn,[7] M. Anholm,[3,8] L. Atar,[9] A. Babu,[3] D. Bazin,[2,4] N. Bernier,[3,10] S. S. Bhattacharjee,[3] M. Bowry,[3] R. Caballero-Folch,[3] M. Cortesi,[2] C. Dalitz,[11] E. Dunling,[3,12] A. B. Garnsworthy,[3] M. Holl,[3,13] B. Kootte,[3,8] K. G. Leach,[14] J. S. Randhawa,[2] Y. Saito,[3,10] C. Santamaria,[15] P. Šiurytė,[3,16] C. E. Svensson,[9] R. Umashankar,[3] N. Watwood,[2] and D. Yates[3,10]

- Directly observed the protons from $^{11}\text{Be}(\beta p)^{10}\text{Be}$
- Measured consistent branching ratio $b_p = 1.3(3) \times 10^{-5}$
  - Still orders of magnitude larger than theoretical predictions
- Predict the proton resonance at 11.425(20) MeV from the proton energy distribution
  - Predicted to be either $\frac{1}{2}^+$ or $\frac{3}{2}^+$
  - Corresponds to excitation energy of 197 keV

NCSMC extended to describe exotic ^{11}Be βp emission

^{11}Be or ^{11}B

$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_\lambda c_\lambda^{J^\pi T} |A\lambda J^\pi T\rangle + \sum_\nu \int dr r^2 \frac{\gamma_\nu^{J^\pi T}(r)}{r} \hat{\mathcal{A}}_\nu \left|\Phi_{\nu r}^{J^\pi T}\right\rangle$$

$$\left|\Phi_{\nu r}^{J^\pi T}\right\rangle = \left[\left(\left|{}^{10}\mathrm{Be}\,\alpha_1 I_1^{\pi_1} T_1\right\rangle \left|N\,\tfrac{1}{2}^+\tfrac{1}{2}\right\rangle\right)^{(sT)} Y_\ell(\hat{r}_{10,1})\right]^{(J^\pi T)} \times \frac{\delta(r-r_{10,1})}{r r_{10,1}},$$

n for ^{11}Be or *p* for ^{11}B

Input chiral interaction

NN N^{4}LO(500) + 3N(lnl)

Entem-Machleidt-Nosyk 2017

3N N^{2}LO w local/non-local regulator

Including 0^+_{gs} and 2^+_1 states of ^{10}Be

$$B(\mathrm{GT}) = \frac{1}{2}\left|\left\langle \Psi^{\frac{1}{2}^+\frac{1}{2}}_{^{11}B} \right\| \hat{\mathrm{GT}} \left\| \Psi^{\frac{1}{2}^+\frac{3}{2}}_{^{11}\mathrm{Be}} \right\rangle\right|^2$$

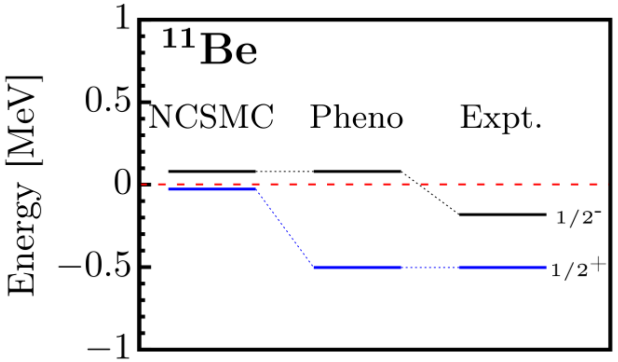
PHYSICAL REVIEW C **105**, 054316 (2022)

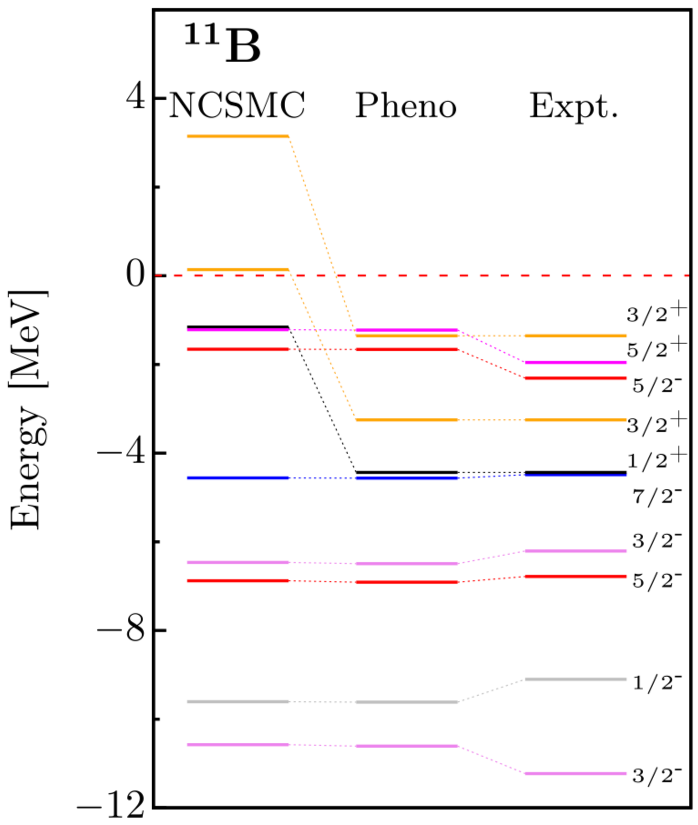
***Ab initio* calculation of the β decay from ^{11}Be to a ^{10}Be + *p* resonance**

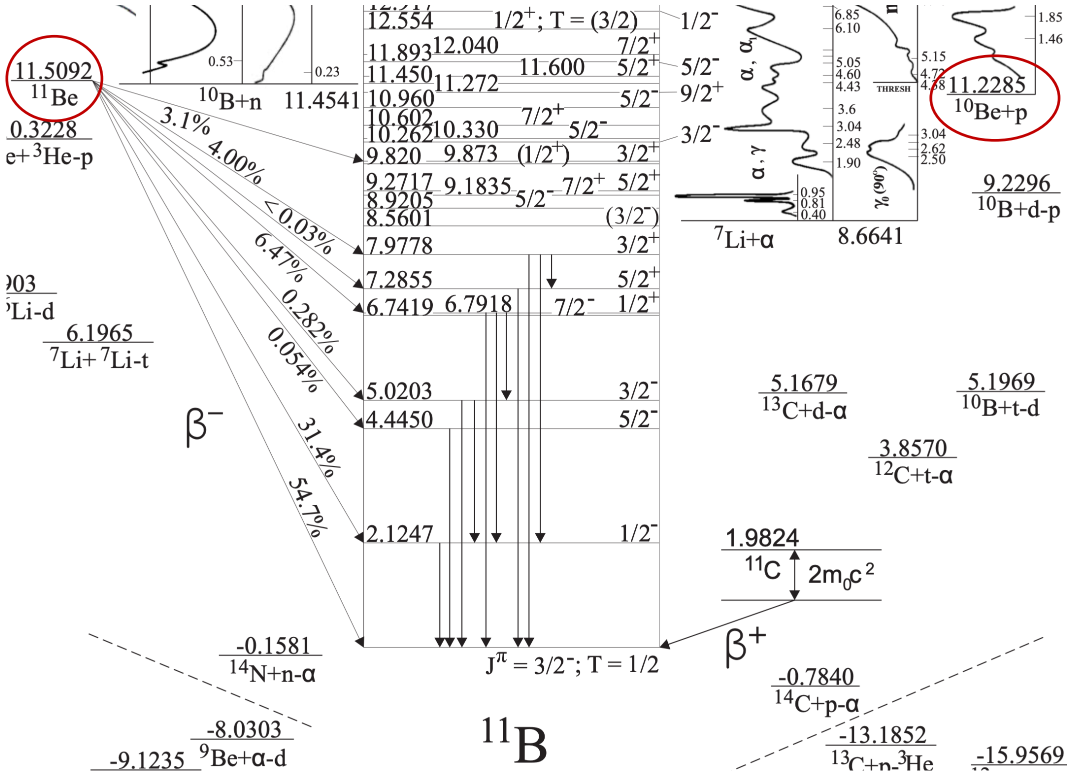
M. C. Atkinson,[1] P. Navrátil,[1] G. Hupin,[2] K. Kravvaris,[3] and S. Quaglioni[3]

^{11}Be and ^{11}B nuclear structure results

- Bound states wrt ^{10}Be+N thresholds



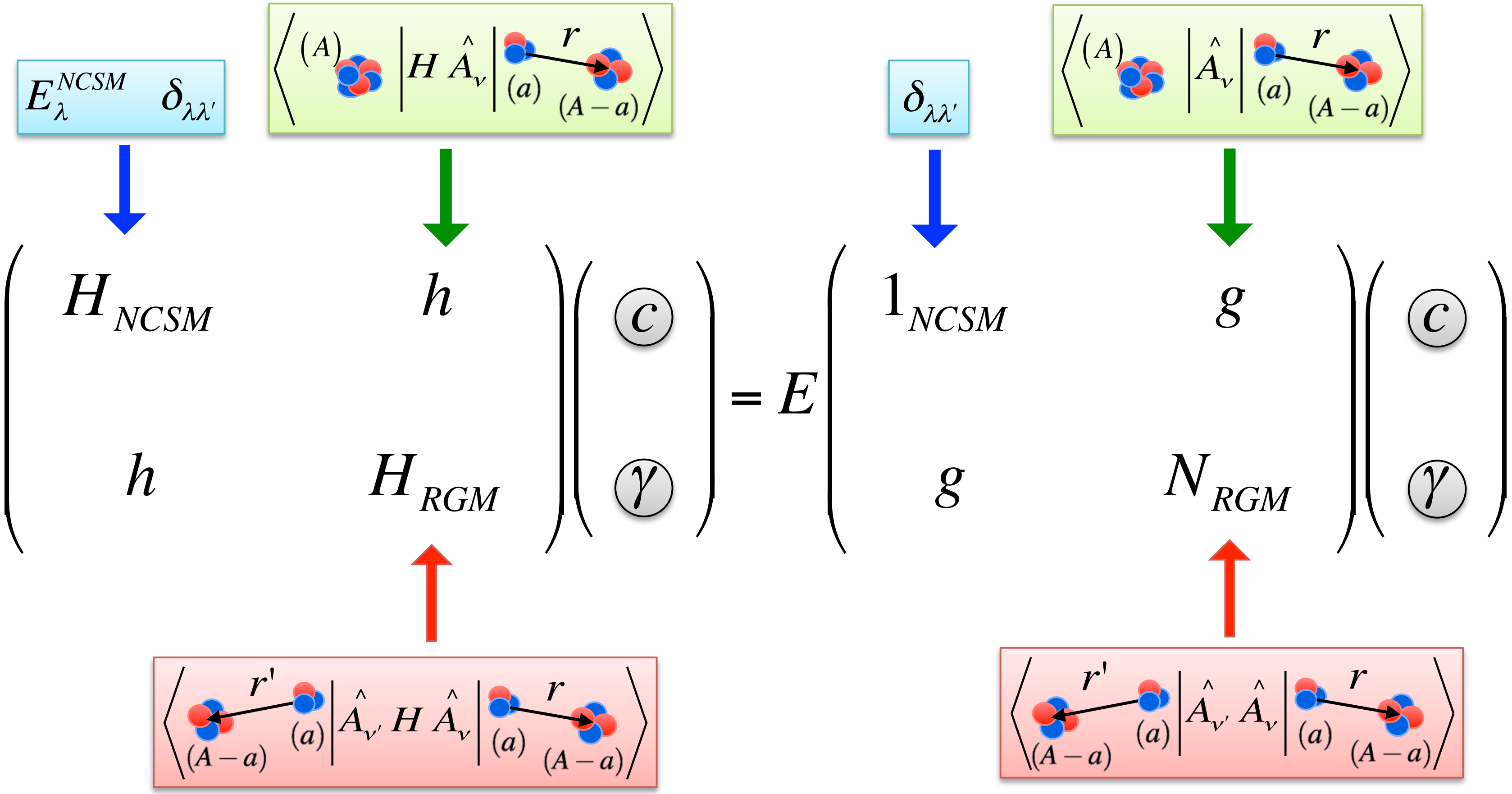




NCSMC phenomenology

$$H\,\Psi^{(A)} = E\,\Psi^{(A)}$$

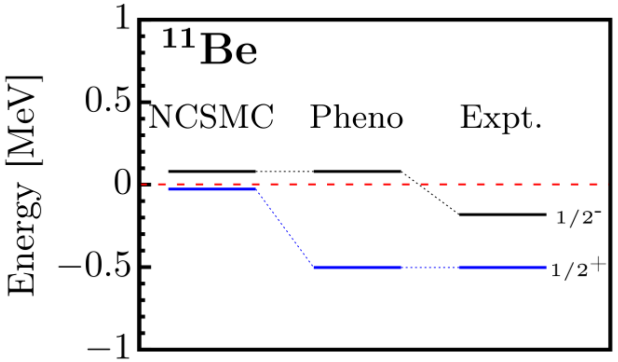
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| (A), \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\, \gamma_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| (A-a)\ \vec{r}\ (a), \nu \right\rangle$$

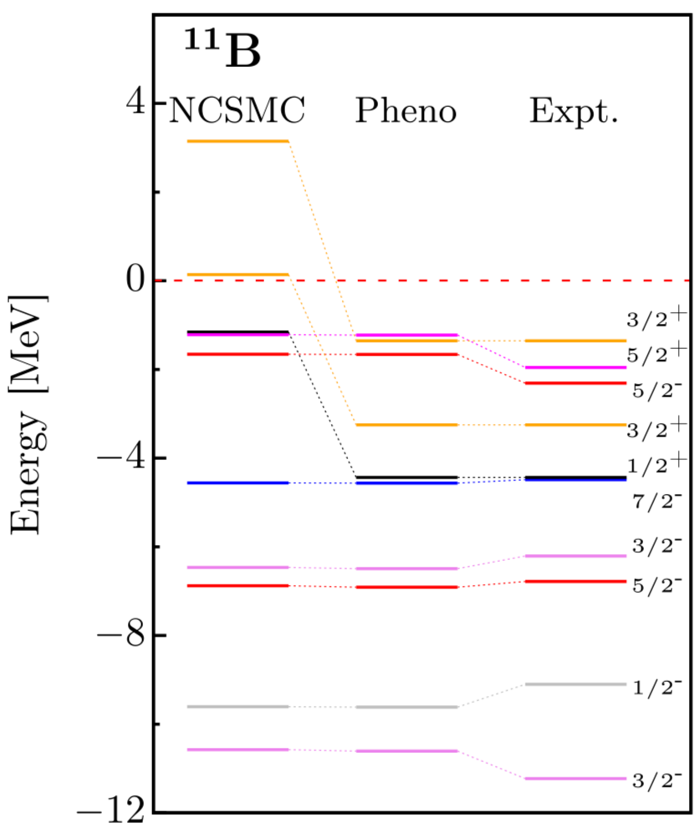


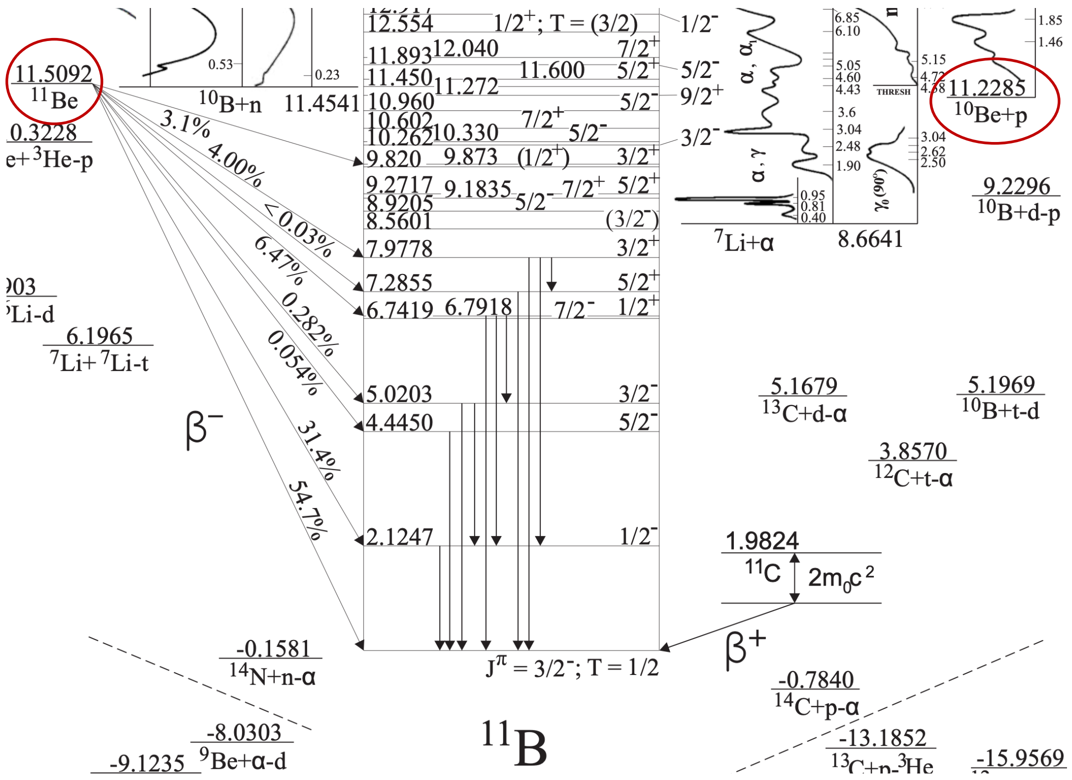
E_{λ}^{NCSM} energies treated as adjustable parameters

^{11}Be and ^{11}B nuclear structure results

- Bound states wrt ^{10}Be+N thresholds

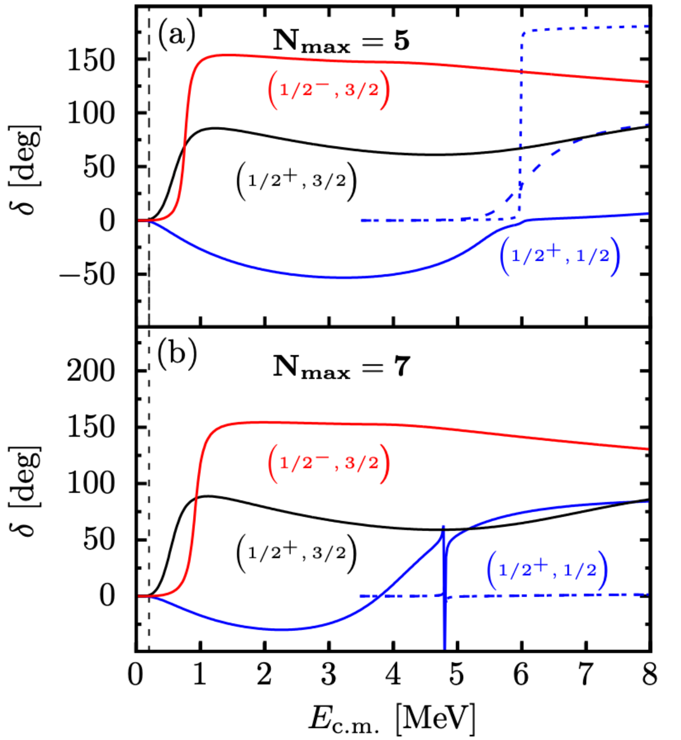




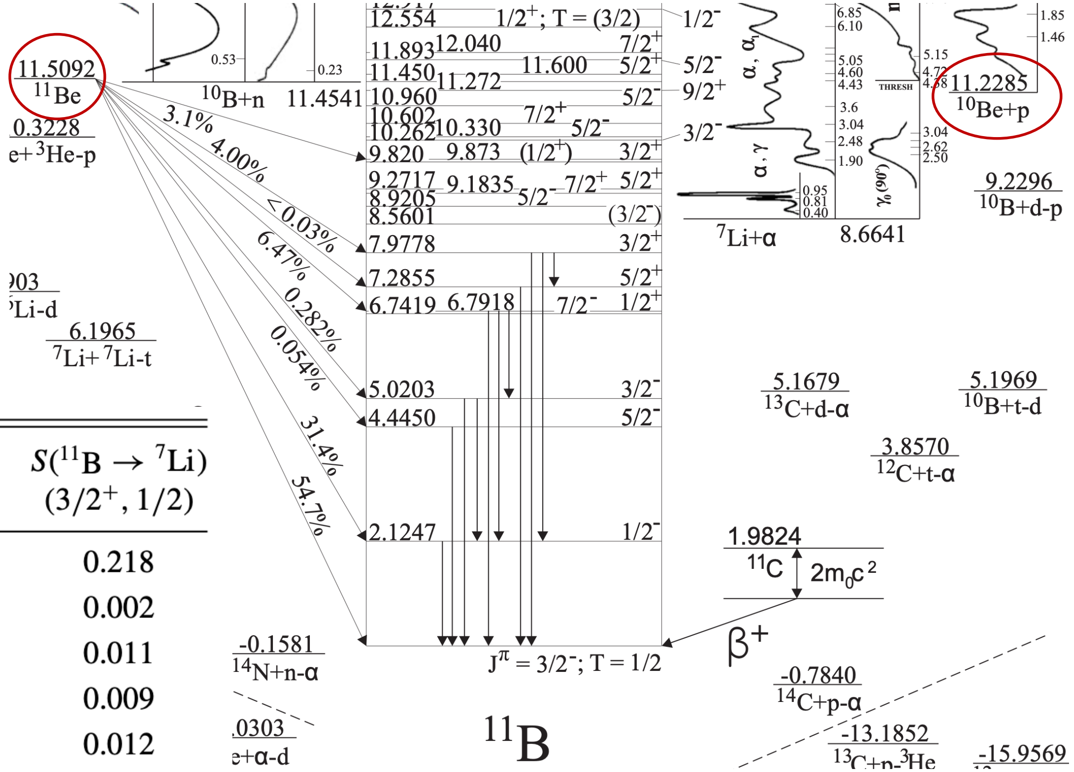


^{11}Be and ^{11}B nuclear structure results

- ^{11}B resonances above ^{10}Be+p threshold



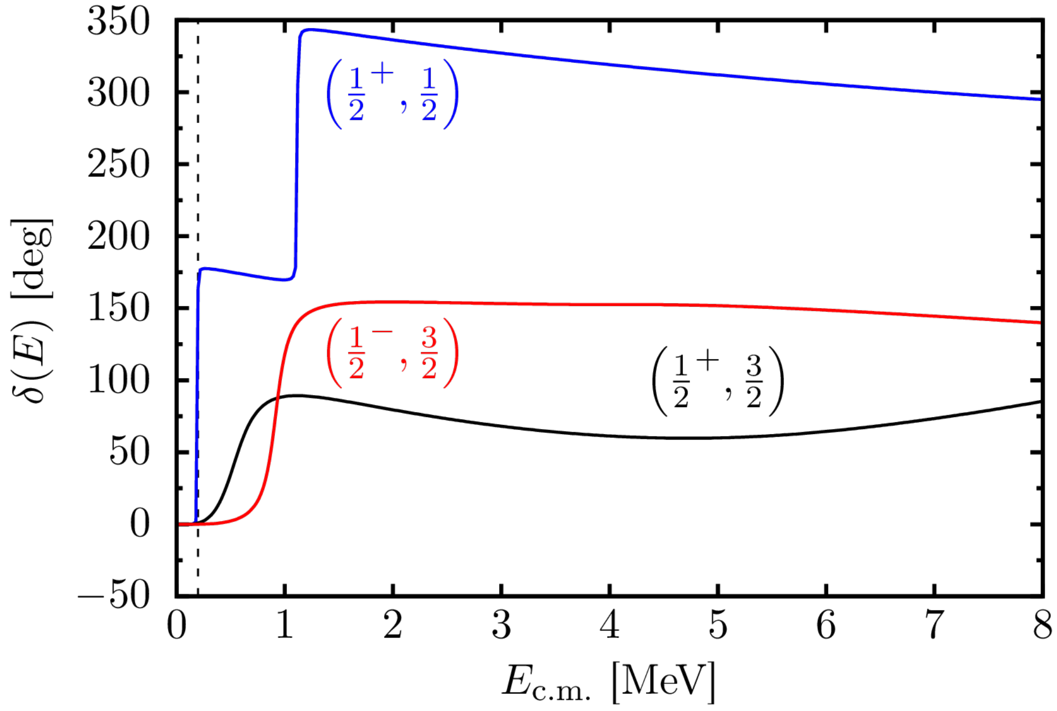
J^π	$S(^{11}B \to {}^{10}Be)$ $(0^+, 1)$	$S(^{11}B \to {}^{10}B)$ $(1_1^+, 0)$	$(1_2^+, 0)$	$S(^{11}B \to {}^{7}Li)$ $(3/2^+, 1/2)$
$1/2_1^+$	0.276	0.250	2×10^{-4}	0.218
$1/2_2^+$	0.0525	0.171	0.562	0.002
$1/2_3^+$	0.067	0.231	0.188	0.011
$3/2_1^+$	0.079	6×10^{-4}	0.215	0.009
$3/2_2^+$	4×10^{-4}	0.581	0.002	0.012
$3/2_3^+$	6×10^{-4}	0.011	0.006	0.021
$3/2_4^+$	0.067	0.034	0.35	0.006

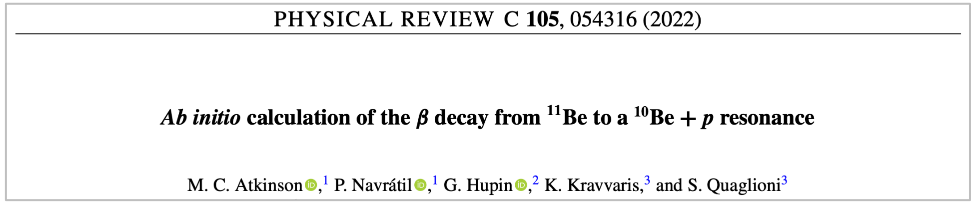


NCSMC extended to describe exotic ^{11}Be βp emission, supports large branching ratio due to narrow ½⁺ resonance

$^{11}\text{Be} \rightarrow (^{10}\text{Be+p}) + \beta^{-} + \bar{\nu}_e$ GT transition

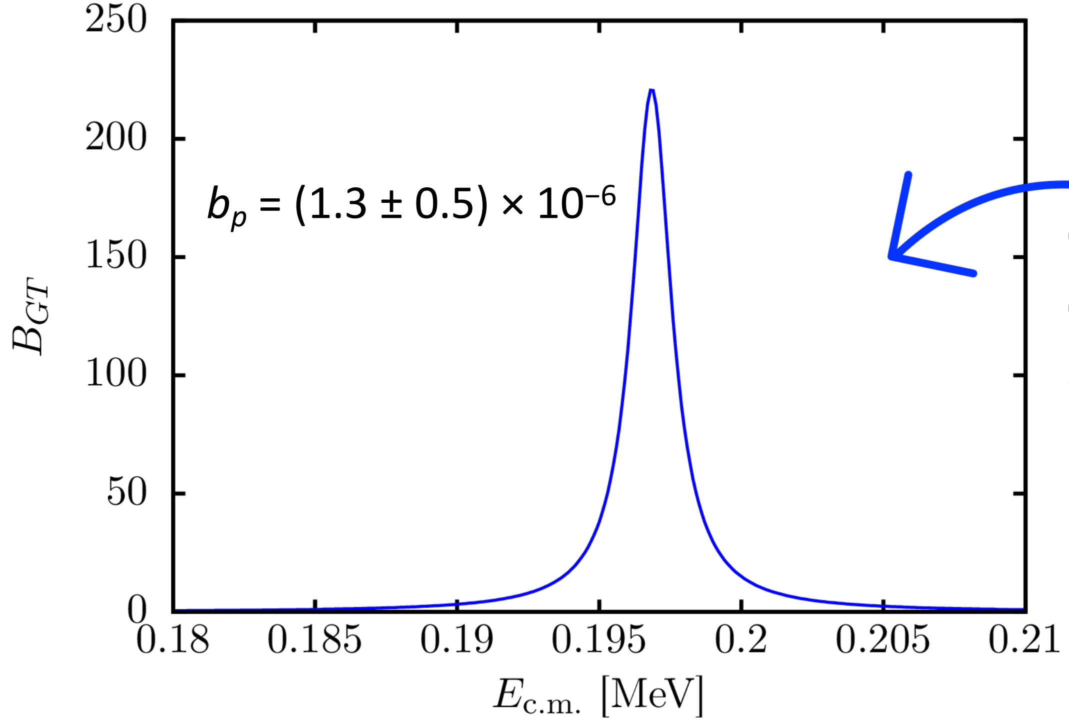
p+^{10}Be Scattering Phase Shifts



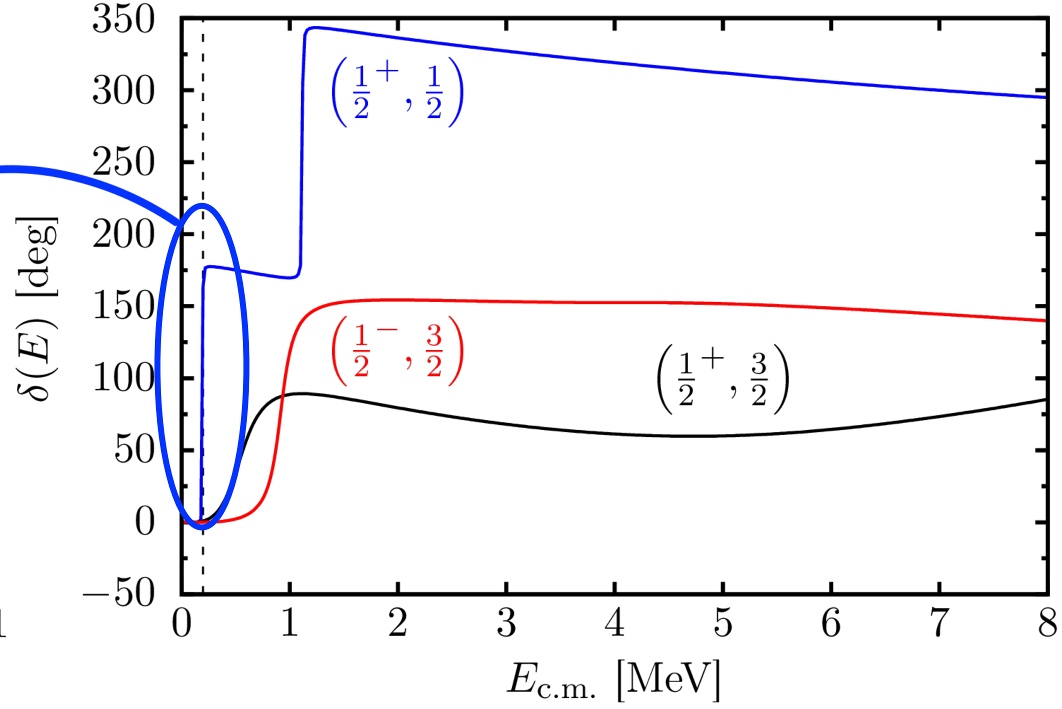


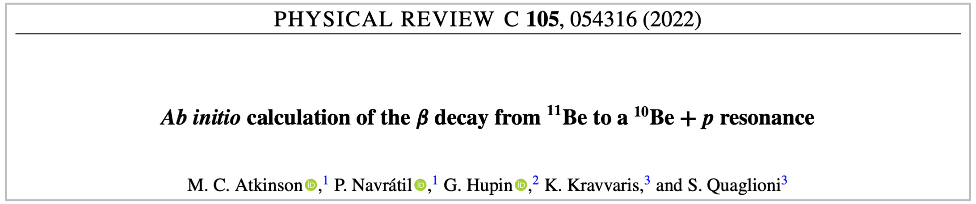
NCSMC extended to describe exotic ^{11}Be βp emission, supports large branching ratio due to narrow ½⁺ resonance

$^{11}Be \rightarrow (^{10}Be+p) + \beta^{-} + \bar{\nu}_e$ GT transition



p+^{10}Be Scattering Phase Shifts

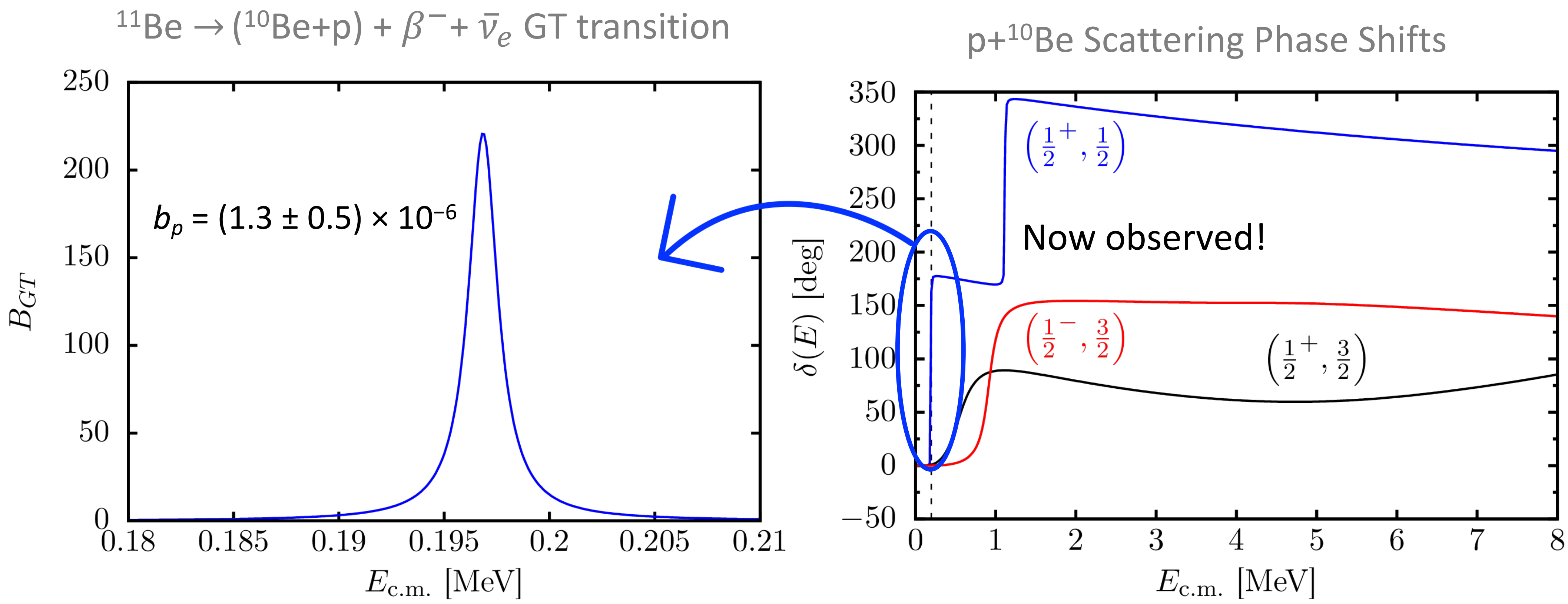



PHYSICAL REVIEW C **105**, 054316 (2022)

***Ab initio* calculation of the β decay from ^{11}Be to a ^{10}Be + p resonance**

M. C. Atkinson,[1] P. Navrátil,[1] G. Hupin,[2] K. Kravvaris,[3] and S. Quaglioni[3]

NCSMC extended to describe exotic ^{11}Be βp emission, supports large branching ratio due to narrow ½$^+$ resonance



PHYSICAL REVIEW LETTERS 129, 012502 (2022)

Observation of a Near-Threshold Proton Resonance in ^{11}B

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PHYSICAL REVIEW LETTERS 129, 012501 (2022)

Evidence of a Near-Threshold Resonance in ^{11}B Relevant to the β-Delayed Proton Emission of ^{11}Be

Y. Ayyad,[1,2,*] W. Mittig,[2,3] T. Tang,[2] B. Olaizola,[4] G. Potel,[5] N. Rijal,[2] N. Watwood,[2] H. Alvarez-Pol,[1] D. Bazin,[2,3] M. Caamaño,[1] J. Chen,[6] M. Cortesi,[2] B. Fernández-Domínguez,[1] S. Giraud,[2] P. Gueye,[2,3] S. Heinitz,[7] R. Jain,[2,3] B. P. Kay,[6] E. A. Maugeri,[7] B. Monteagudo,[2] F. Ndayisabye,[2,3] S. N. Paneru,[2] J. Pereira,[2] E. Rubino,[2] C. Santamaria,[2] D. Schumann,[7] J. Surbrook,[2,3] L. Wagner,[2] J. C. Zamora,[2] and V. Zelevinsky[2,3]

PHYSICAL REVIEW C 105, 054316 (2022)

***Ab initio* calculation of the β decay from ^{11}Be to a ^{10}Be + p resonance**

M. C. Atkinson,[1] P. Navrátil,[1] G. Hupin,[2] K. Kravvaris,[3] and S. Quaglioni[3]

β-delayed proton emission in ^{11}Be

- New FRIB experiment measuring proton emission led by Jason Surbrook reports branching ratio b_p ~ 8(4) x 10^{-6}
  - Lower but still consistent with Ayyad TRIUMF experiment

- More experiments planned!

- NCSMC calculations will be extended by including the $^{7}Li+\alpha$ mass partition

Conclusions

- We used *ab initio* nuclear theory and the χEFT framework to analyze the nuclear-structure corrections to ^{6}He β-decay observables
  - The angular correlation coefficient
  - Nuclear structure term with an inverse energy spectral dependence, imitating a Fierz interference term
    - We find a non-zero Fierz term comparable to an effect of interference between SM and a TeV-scale BSM currents
    - The achieved uncertainty of ~15% is dominated by the neglect of the weak magnetism two-body currents $\rightarrow$ the next thing to focus on

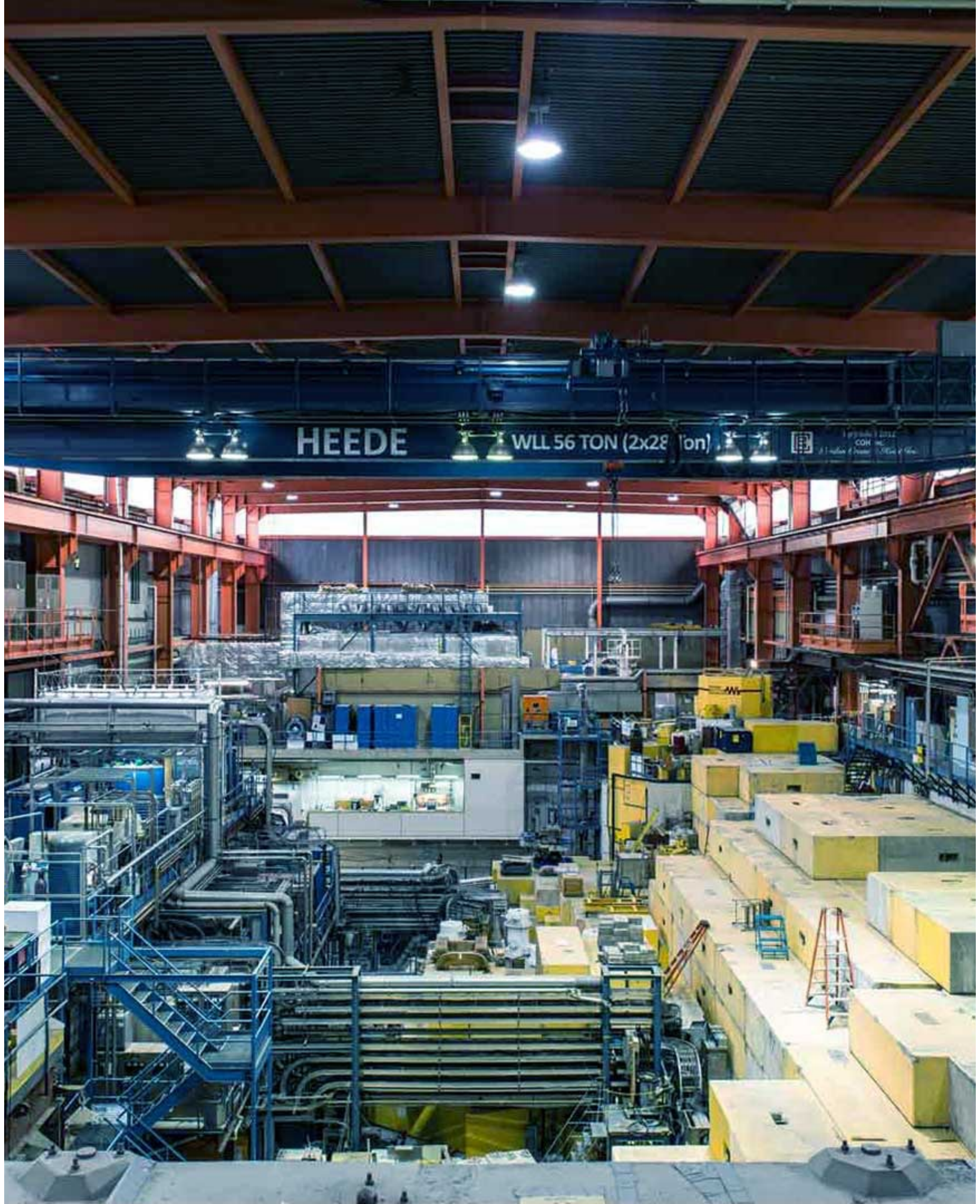
- *Ab initio* investigation of the fist forbidden unique ^{16}N → ^{16}O beta decay ongoing
  - Electron spectrum sensitive to BSM physics

- *Ab initio* calculations of the structure corrections for the extraction of the V_{ud} matrix element from the ^{10}C → ^{10}B Fermi transition under way
  - Preliminary result for δ_{NS}
  - The same approach will be applied to ^{14}O → ^{14}N Fermi transition and possibly also to ^{18}Ne → ^{18}F and ^{22}Mg → ^{22}Na

- Applications of NCSMC to ^{11}Be β decay with the proton emission
  - Supports large branching ratio due to a narrow ½$^{+}$ resonance

TRIUMF

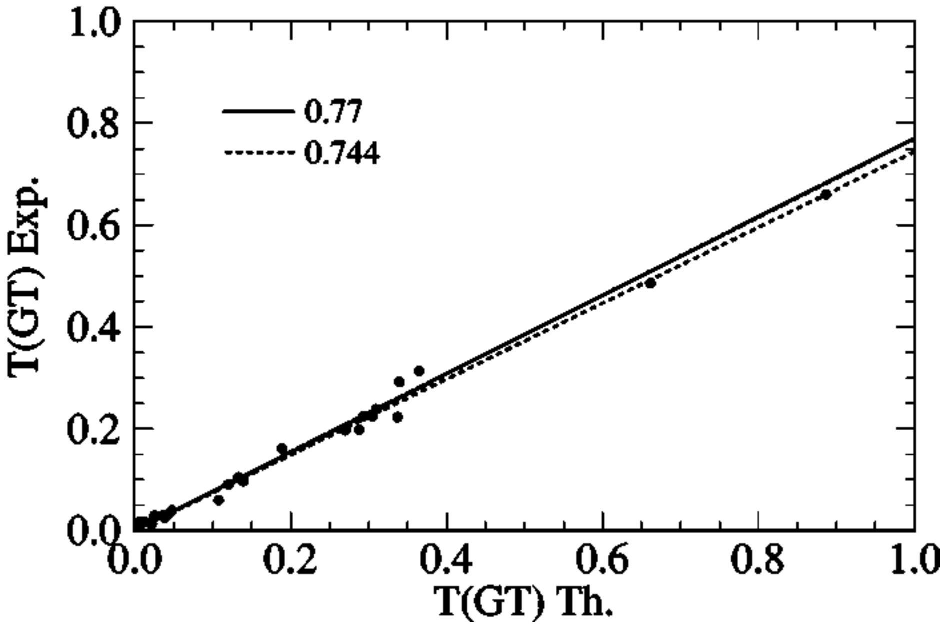
Backup slides



Discovery, accelerated

Applications to β decays in p-shell nuclei and beyond

- Problem of quenching of the axial-vector coupling constant g_A in shell model calculations of GT transitions



REVIEWS OF MODERN PHYSICS, VOLUME 77, APRIL 2005

The shell model as a unified view of nuclear structure

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nature physics LETTERS
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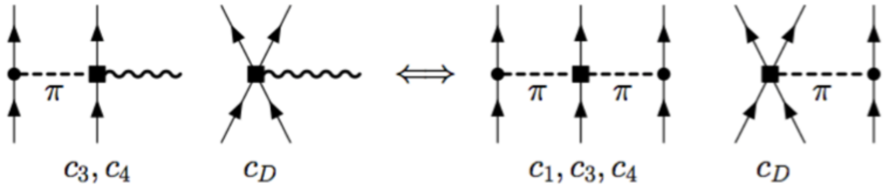
Discrepancy between experimental and theoretical β-decay rates resolved from first principles

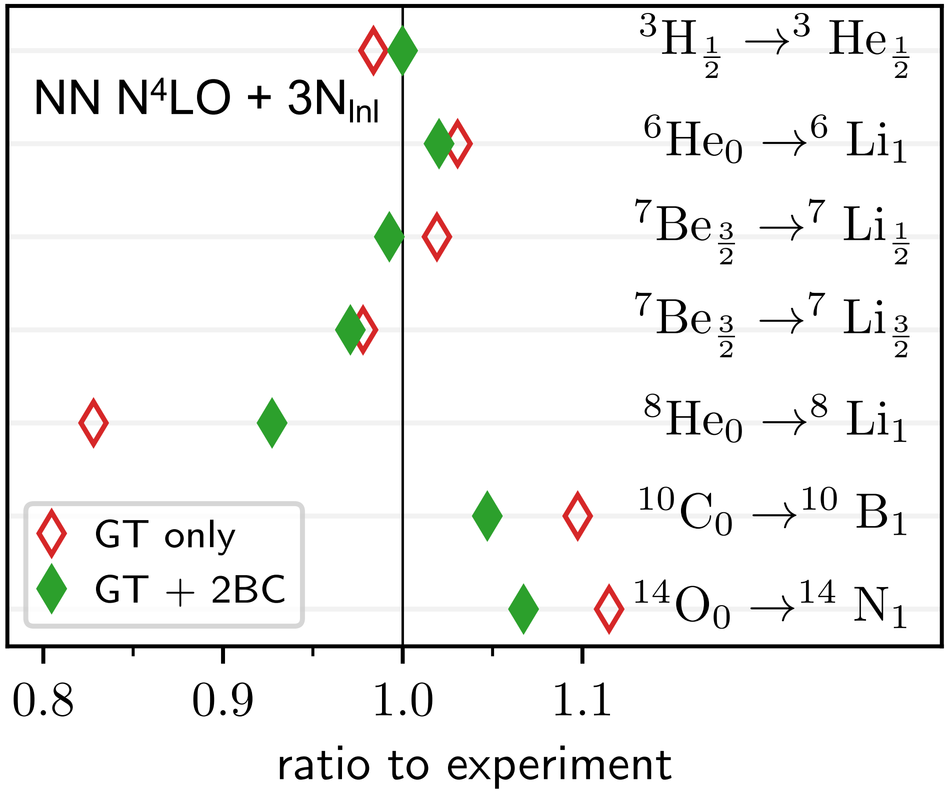
P. Gysbers[1,2], G. Hagen[3,4*], J. D. Holt[1], G. R. Jansen[3,5], T. D. Morris[3,4,6], P. Navrátil[1], T. Papenbrock[3,4], S. Quaglioni[7], A. Schwenk[8,9,10], S. R. Stroberg[1,11,12] and K. A. Wendt[7]

Applications to β decays in p-shell nuclei and beyond

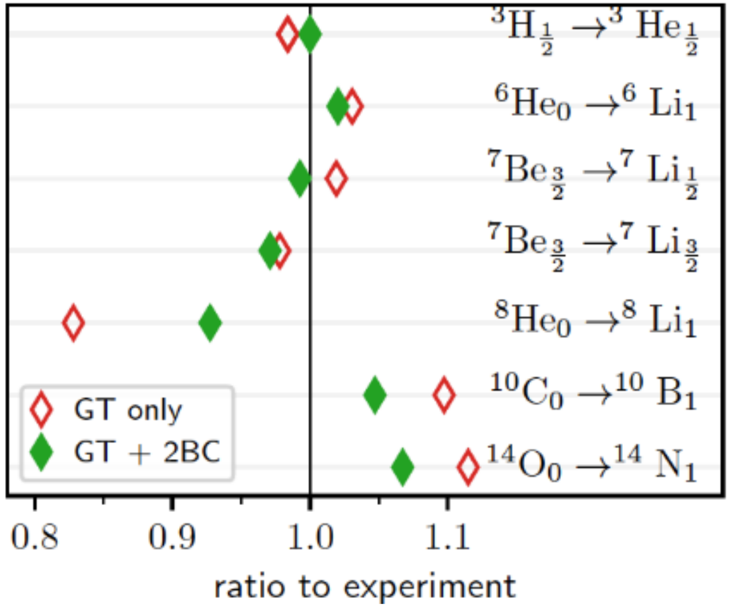
- Does inclusion of the MEC explain g_A quenching?
- In light nuclei correlations present in *ab initio* (NCSM) wave functions explain almost all of the quenching compared to the standard shell model
  - MEC inclusion overall improves agreement with experiment
- The effect of the MEC inclusion is greater in heavier nuclei
- SRG evolved matrix elements used in coupled-cluster and IM-SRG calculations (up to ^{100}Sn)

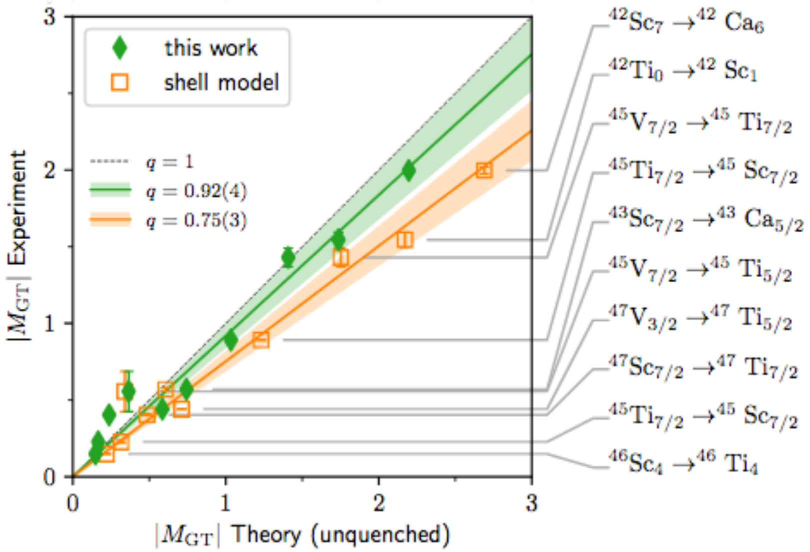
Hollow symbols – GT
Filled symbols – GT+MEC
Both Hamiltonian and operators SRG evolved
Hamiltonian and current consistent parameters

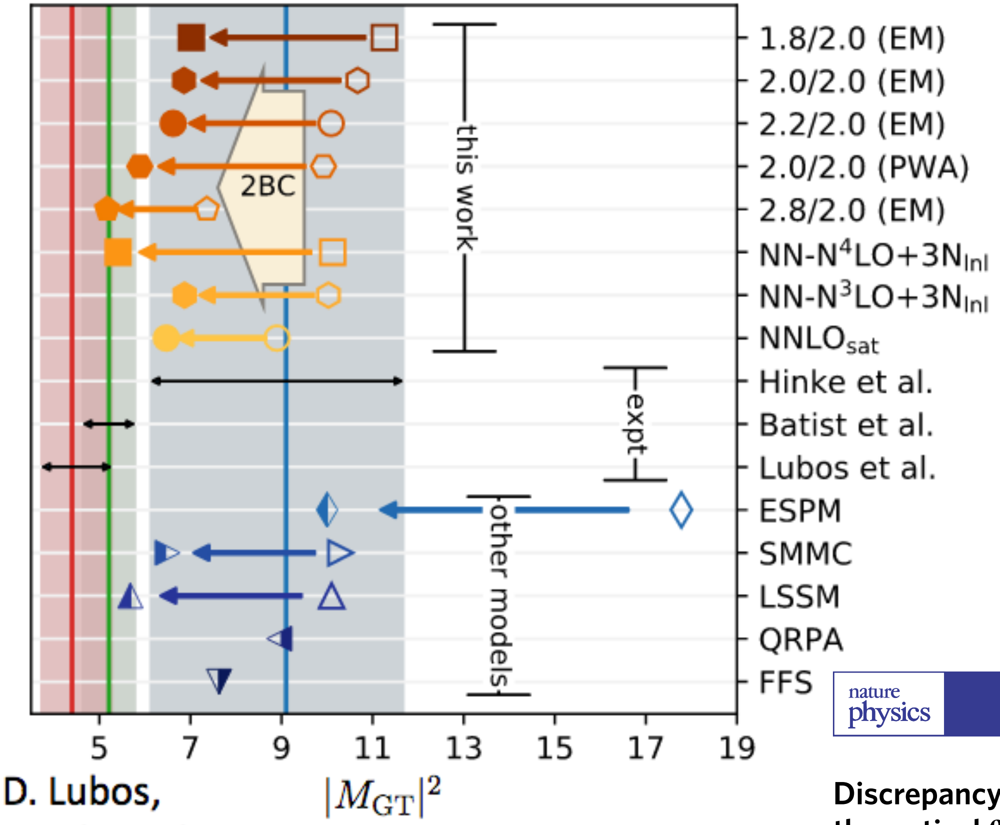




50 year old puzzle of quenched beta decays resolved from first principles







D. Lubos, PRL (2019)

nature physics LETTERS
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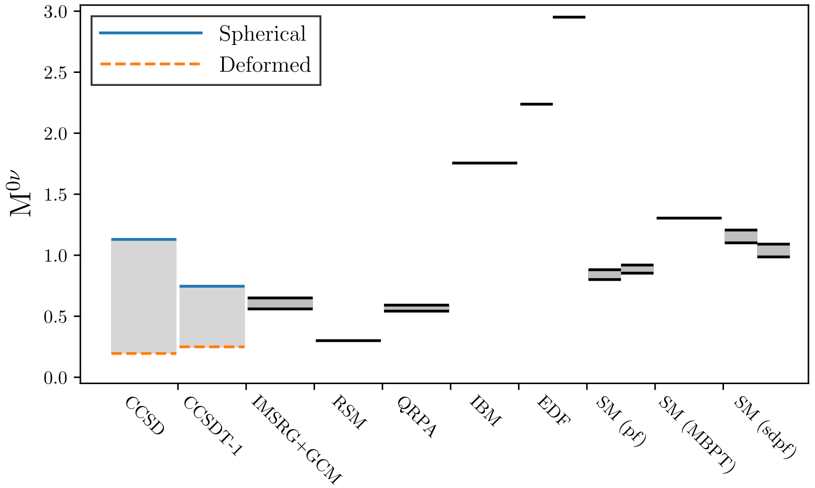
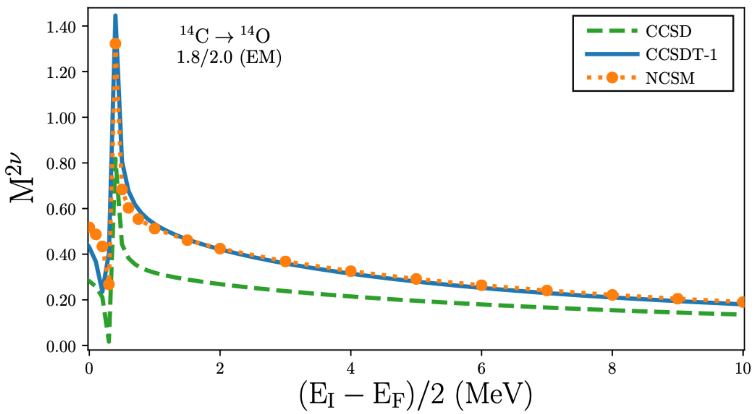
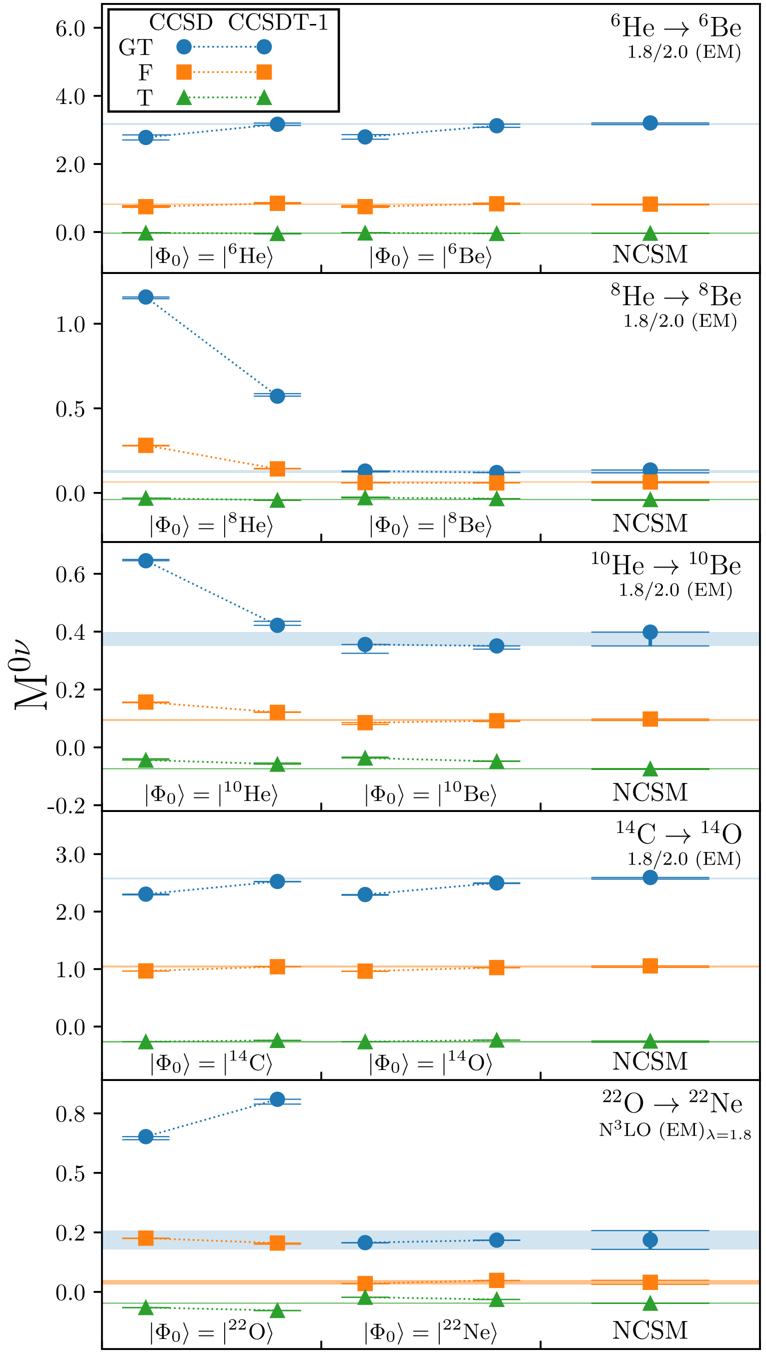
Strong nuclear correlations and two-body currents solve the beta decay quenching problem

Ab initio calculations of the $^{48}Ca \rightarrow {}^{48}Ti$ neutrinoless double beta decay matrix elements

- Benchmarks for light nuclei: NCSM & Coupled-Cluster
  - Both two-neutrino and neutrinoless double beta decay
- Coupled-Cluster $^{48}Ca \rightarrow {}^{48}Ti$ results







PHYSICAL REVIEW LETTERS **126,** 182502 (2021)

Coupled-Cluster Calculations of Neutrinoless Double-β Decay in ^{48}Ca

S. Novario,[1,2] P. Gysbers,[3,4] J. Engel,[5] G. Hagen,[2,1,3] G. R. Jansen,[6,2] T. D. Morris,[2] P. Navrátil,[3] T. Papenbrock,[1,2] and S. Quaglioni[7]