Scalar and vector type momentum-dependent potentials in the relativistic molecular dynamics and the anisotropic flows

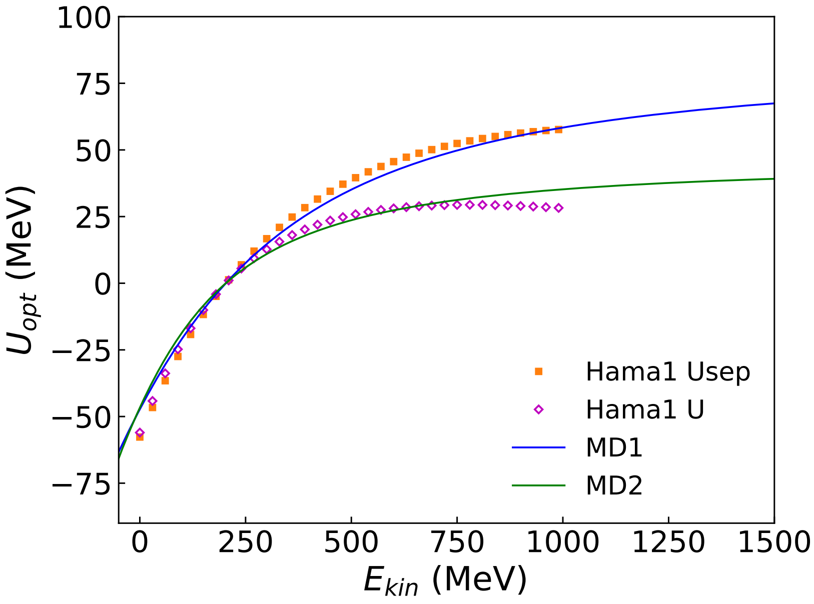
Yasushi Nara (Akita International Univ.)

- Introduction
- Relativistic quantum molecular dynamics (RQMDv)
- RQMDv with the Skyrme + momentum-dependent (MD) potential
- RQMDv with chiral mean-field (CMF) + MD potential (crossover)
- RQMDv with vector density functional (VDF) + MD potential (1st order phase transition)

JAM2.1: git@gitlab.com:transportmodel/jam2.git

INT workshop Dense nuclear matter equation of state from heavy-ion collisions 2022/12/5

Momentum-dependent (MD) potential



Experimental information on the MD potential is only up to Elab = 1GeV.
MD potential has large effects on the flow, and determination of the MD potential at higher energies is necessary to constrain EoS from heavy ion collisions.

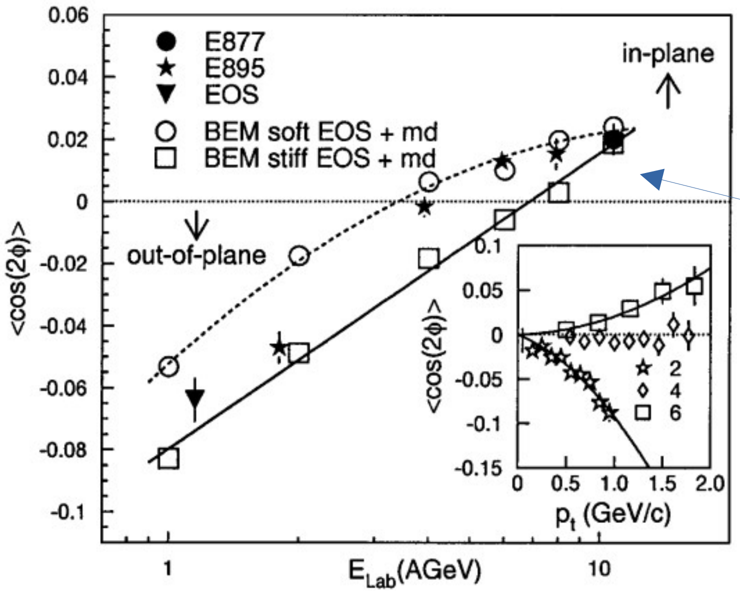
For the description of anisotropic flow:

$$\mathrm{Vector} \neq \mathrm{Vector} + \mathrm{MD} \approx \mathrm{Scalar} + \mathrm{Vector} \approx \mathrm{Scalar} + \mathrm{Vector} + \mathrm{MD}$$

Determination of EoS from flows

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\phi - \Phi_r)]\right)$$

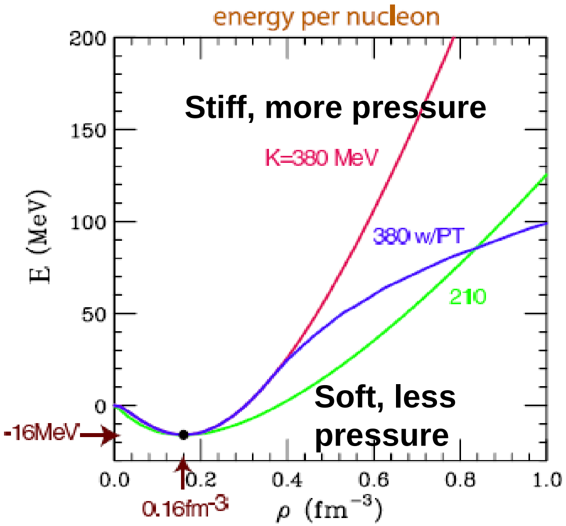
$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



E895,PRL83(1999)

Softening of EoS?

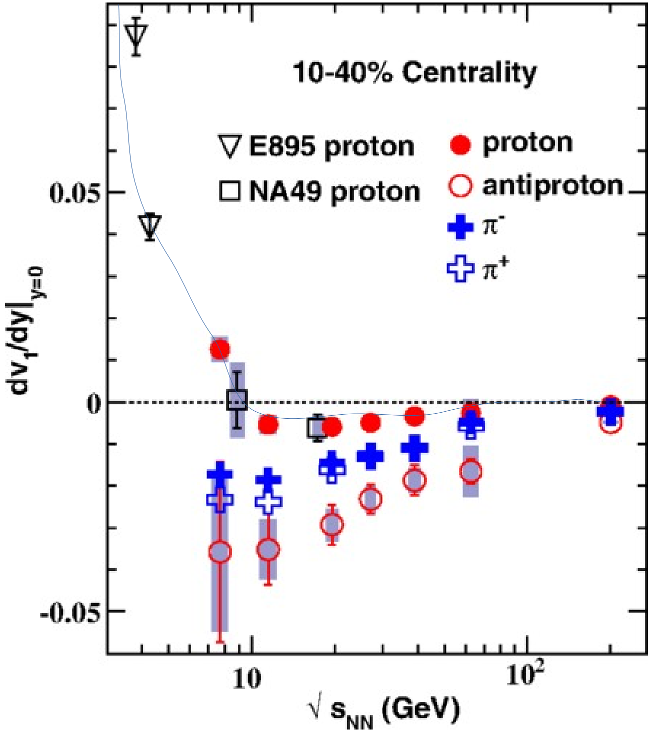
Transport model predicts strong sensitivities of the elliptic flows to the EoS.



P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002) 1592

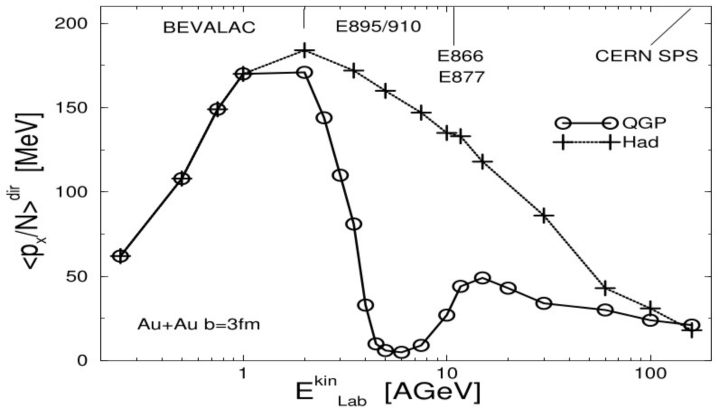
Beam energy dependence of v1: discovery of negative flow

L. Adamczyk et al. (STAR Collaboration) Phys. Rev. Lett. 112, 162301 – 23 April 2014



Proton slope changes sign from positive to negative at 11.5GeV. Signal of a 1st-order phase transition?

But so far, no model with 1st-order phase transition reproduces beam energy dependence of the proton v1 data.



$$v_1(y) = Fy + F_3y^3, \quad dv_1/dy|_{y=0} = F$$

1FD: D.H.Rischke, et.al
Heavy Ion Phys.1, 309 (1995)

Quantum molecular dynamics

J. Aichelin, Phys. Rep. 202 (1991)233

Quantum molecular dynamics (QMD) → N-body theory

$$\frac{d\boldsymbol{r}_i}{dt} = \frac{\partial\langle H\rangle}{\partial\boldsymbol{p}_i}, \quad \frac{d\boldsymbol{p}_i}{dt} = -\frac{\partial\langle H\rangle}{\partial\boldsymbol{r}_i} \quad + \text{Boltzmann type collision term}$$

$$\langle H\rangle = \langle\Phi|H|\Phi\rangle, \quad \Phi = \text{Gaussian wave packets}$$

One-particle potential V(n) use in QMD is related with the single-particle potential U(n) as

$$V(n) = \frac{1}{n}\int_0^n dn' U(n')$$

Relativistic quantum molecular dynamics

JAM2: RQMD equations of motion: scalar and vector potential

$$\dot{x}_i = \frac{p_i^*}{p_i^{*0}} + \sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial p_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial p_i} \right), \quad \dot{p}_i = -\sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial r_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial r_i} \right)$$

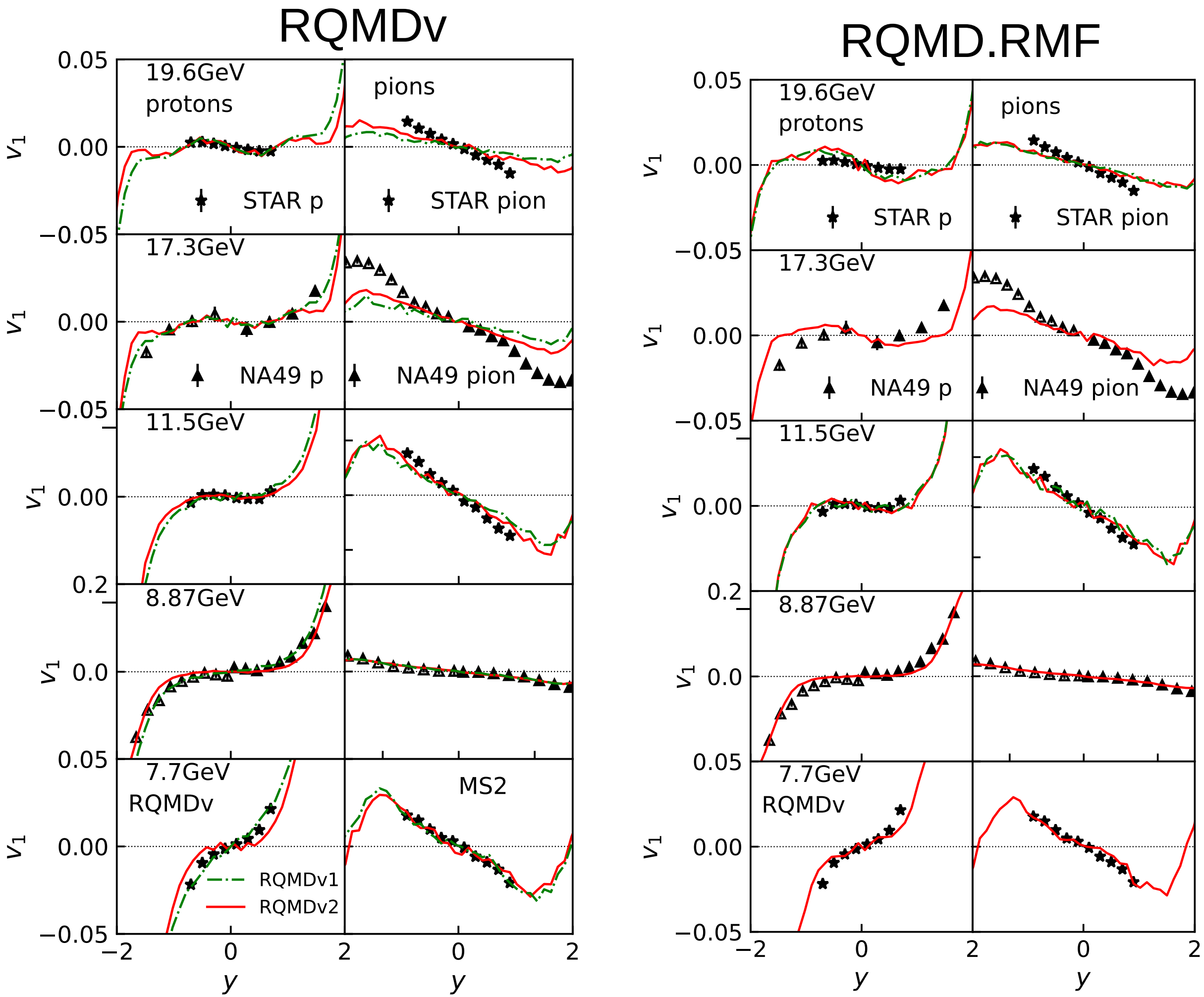
$$m^* = m - S(x,p), \quad p_\mu^* = p_\mu - U_\mu(x,p)$$

- RQMD.RMF: σ-ω model, PRC (2019),(2020)
- RQMDv: Lorentz vector Skyrme potential PRC(2022)

$$U_{\rm sk}(\rho) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^\gamma, \qquad U_m^\mu(p) = \frac{C}{\rho_0}\int d^3p' \frac{p^{*\prime\mu}}{e^*} \frac{f(x,p')}{1+[(\boldsymbol{p}-\boldsymbol{p}')/\mu_k]^2},$$

Collision term: hadronic resonances and strings.

Beam energy dependence of v1 from RQMD

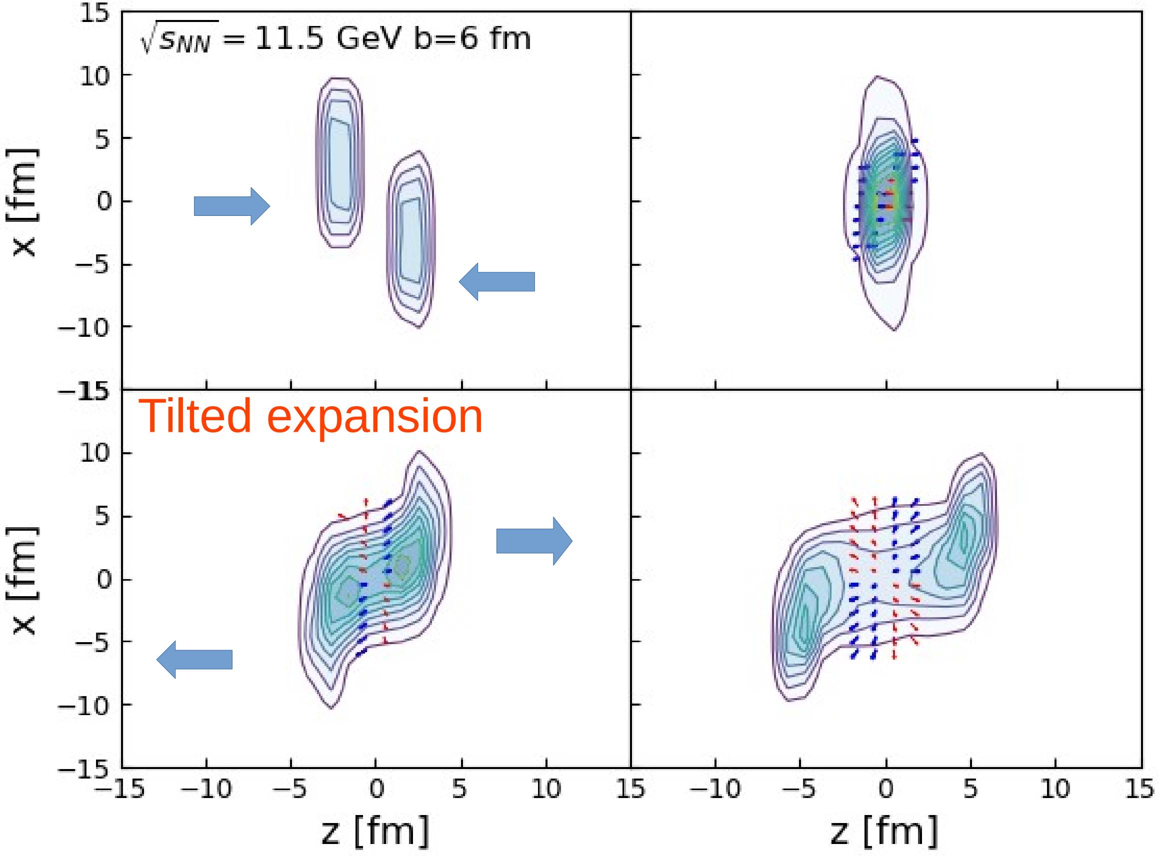


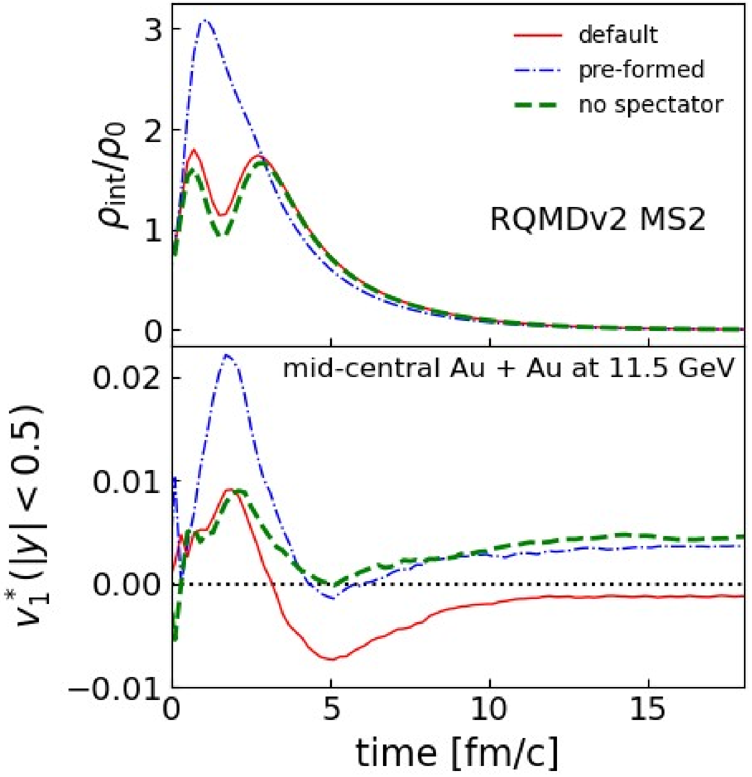
Beam energy dependence of v1 is explained by a mean-field both Skyrme type and sigma-omega.

Y.N, A. Ohnishi, PRC (2022)

Time evolution of v1 at 11.5GeV

Time evolution of the baryon density in Au+Au mid-central collision (b=6fm)



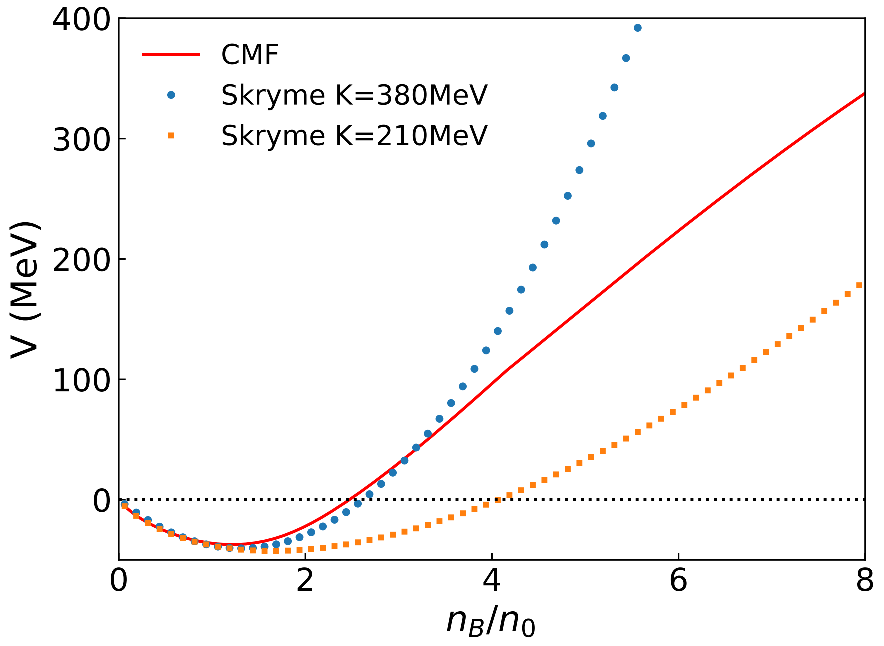


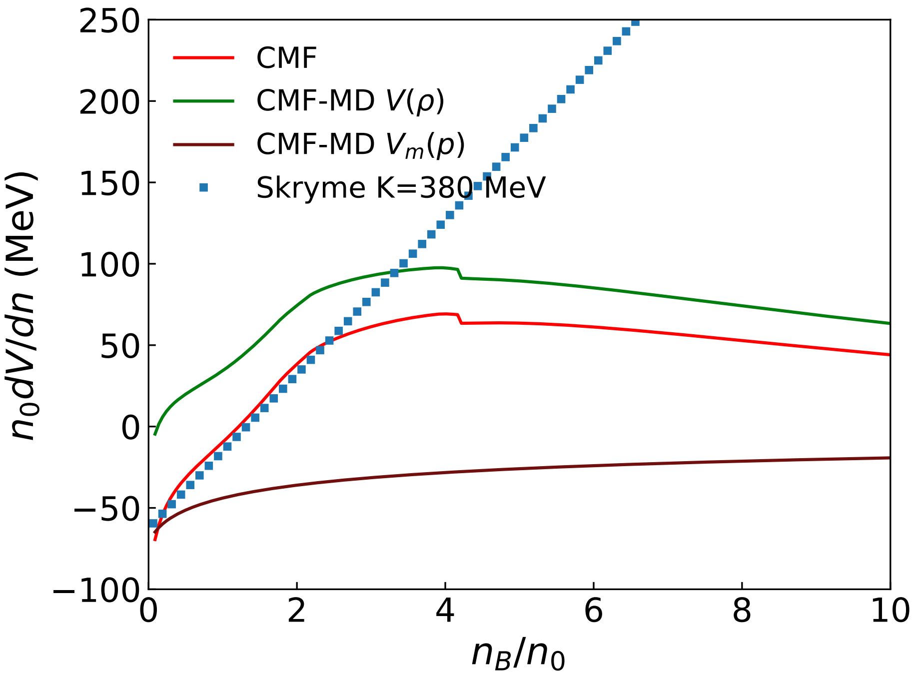
Positive v1 at compression, while negative v1 at expansion.

$$v_1^* = \int_{-0.5}^{0.5} dy v_1(y)\,\mathrm{sgn}(y)$$

QMD potential from the Chiral Mean Field model (CMF)

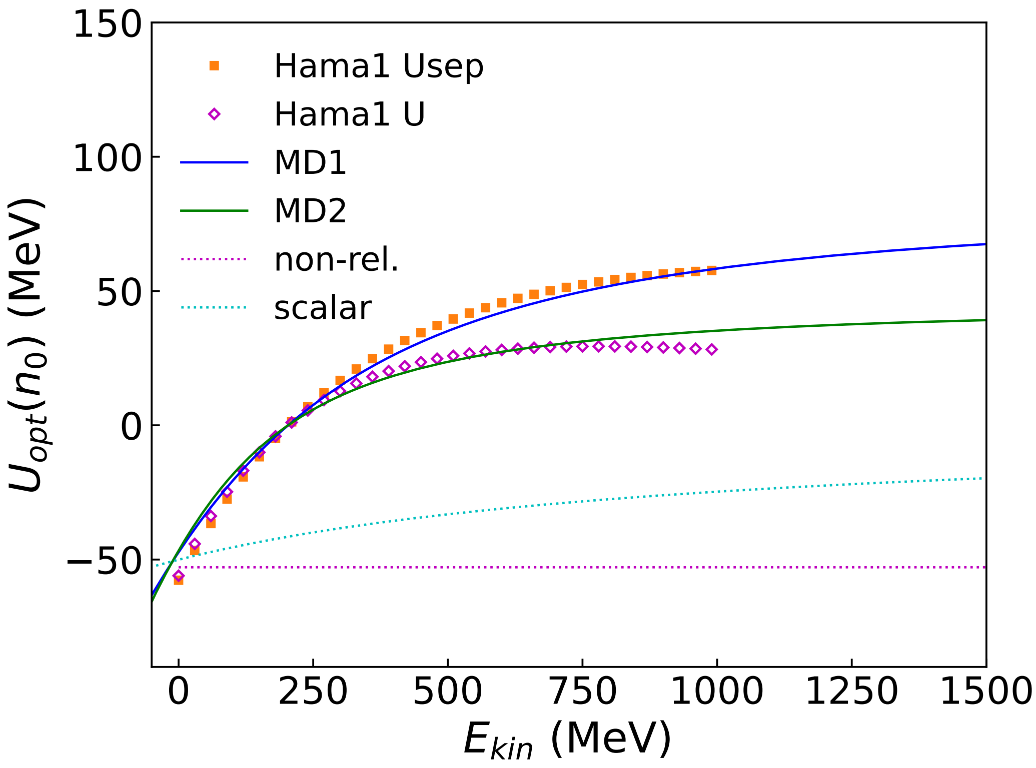
A. Motornenko,et.al PRC103,054908(2021), J.Steinheimer,et.al, EPJC82,911(2022)





$$V = \frac{1}{n}\left[\epsilon - \epsilon_{\text{free}}\right] \qquad n\frac{dV}{dn} = U - V \qquad U = \text{single particle energy} - \sqrt{m_N^2 + k_F^2}$$

Momentum-dependent potential



Schrödinger-equivalent potential : $U_{\rm sep}$

$$V^0(n) = V'^0(n) + V^0_m(n,p)$$

$$\frac{dV^0(n)}{dn} = \frac{dV'^0(n)}{dn} + \frac{dV^0_m(n,p)}{dn}$$

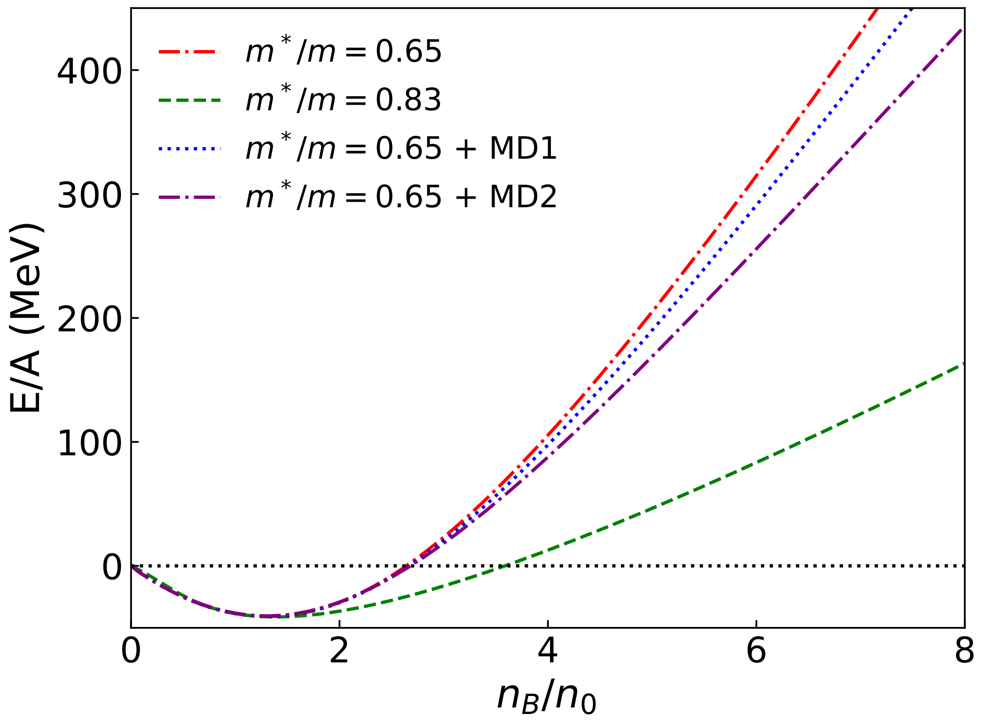
$$V^0_m = \frac{1}{2}\frac{g}{(2\pi)^3}\int d^3p\, U^0_m(p)$$

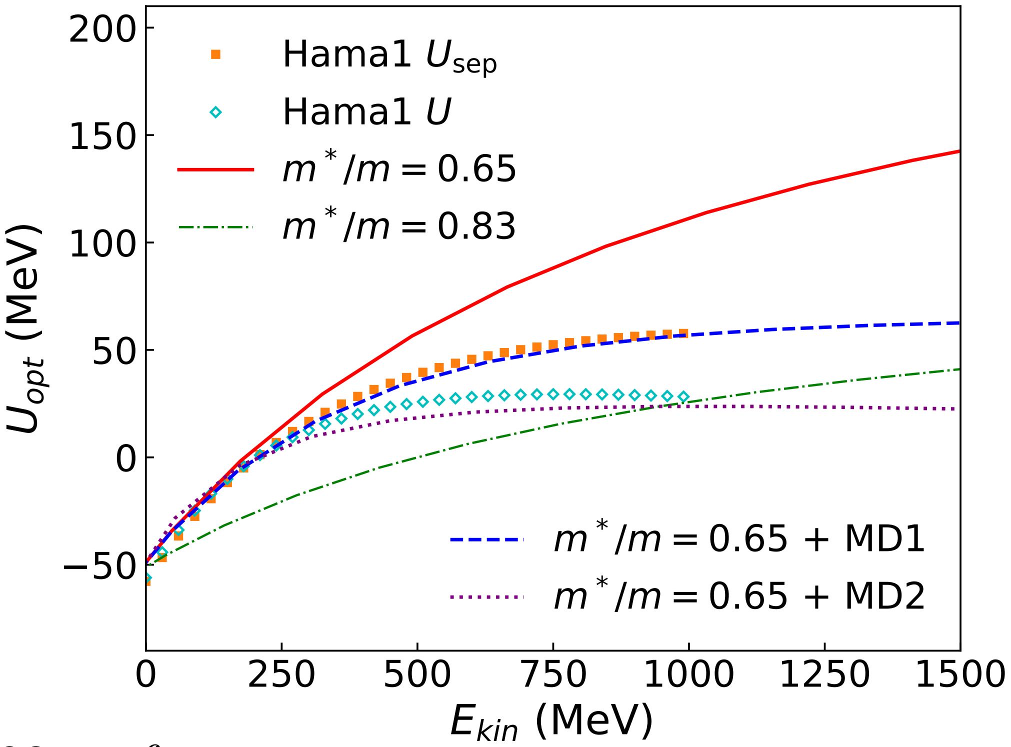
$$U^\mu_m(p) = \frac{C}{n_0}\int d^3p'\frac{p^{*\prime\mu}}{p'^{*0}}\frac{f(x,p')}{1+[(\boldsymbol{p}-\boldsymbol{p}')/\mu_k]^2},$$

$$U = e - \sqrt{m_N^2 + p^2}$$

Relativistic mean field (RMF)

Sigma + omega type Relativistic mean field



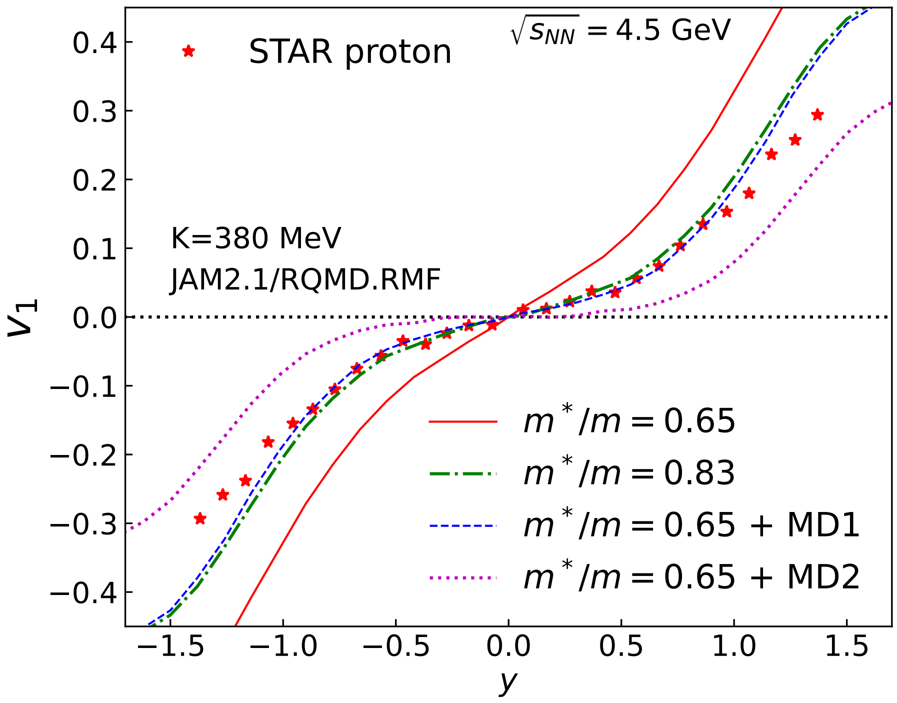


$m^*(n_0)/m = 0.65$: hard $\quad$ $m^*(n_0)/m = 0.83$: soft

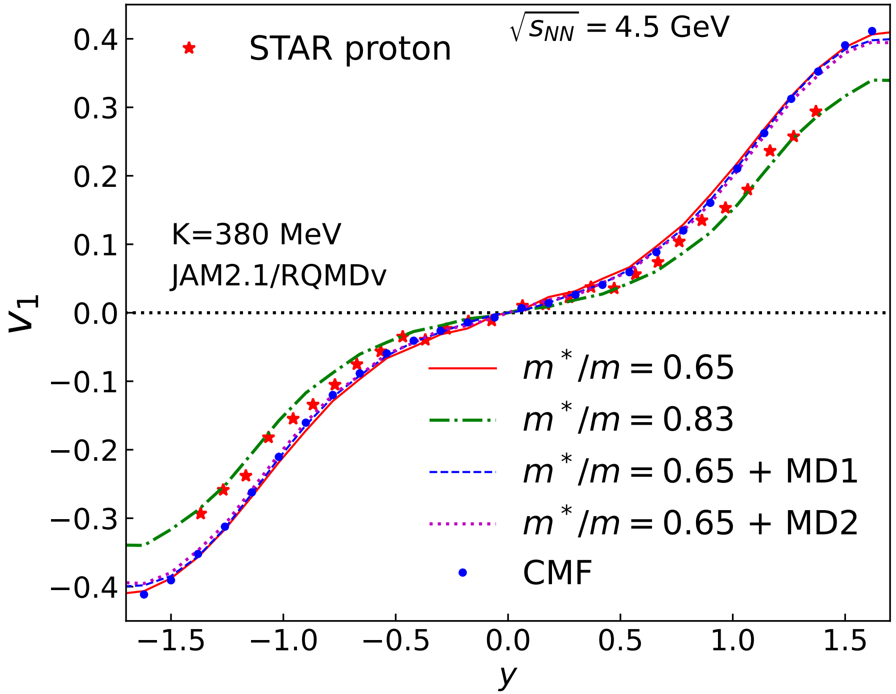
In RMF, stiffness of the EoS is controlled by the effective mass at ground state.

Single potential reduction of RMF

Relativistic mean field



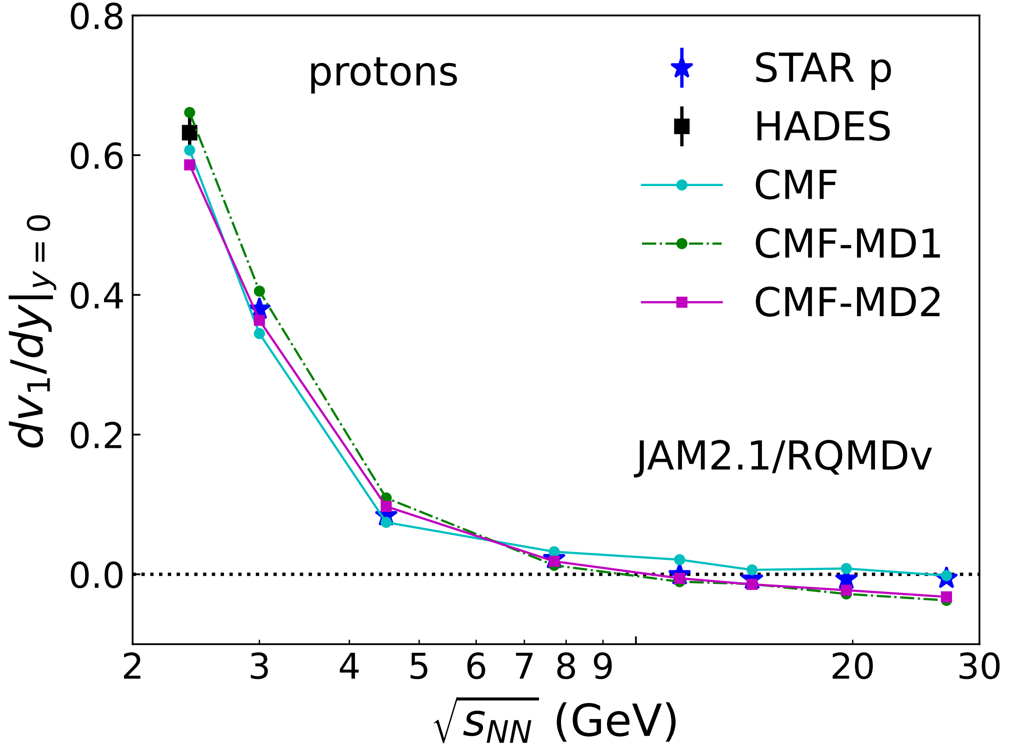
Vector potential

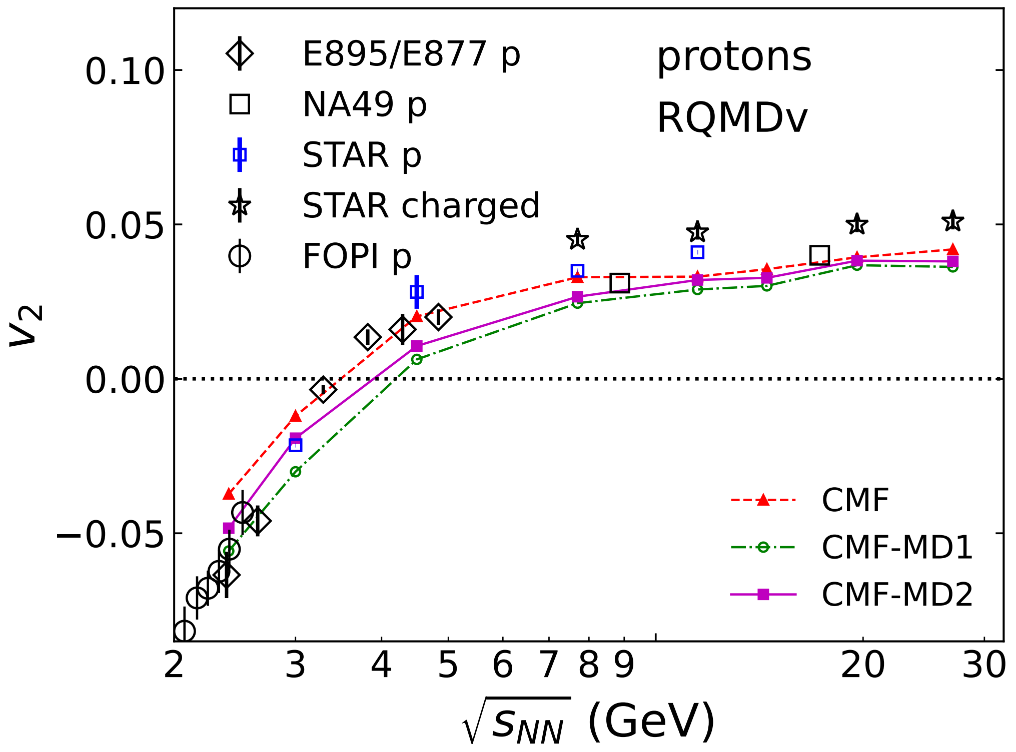


All RMF EoS with m*/m=0.65 predict the same v1 in the RQMDv model.

RQMD.RMF=RQMD.RMF-MD = RQMDv+MD

v1 and v2 from JAM2 RQMD mode



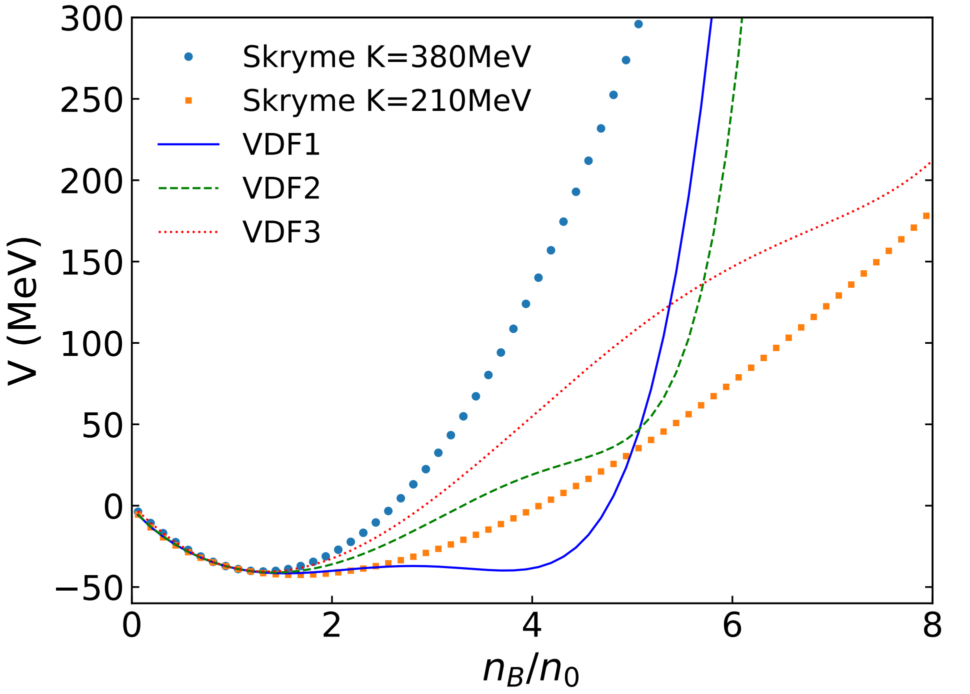


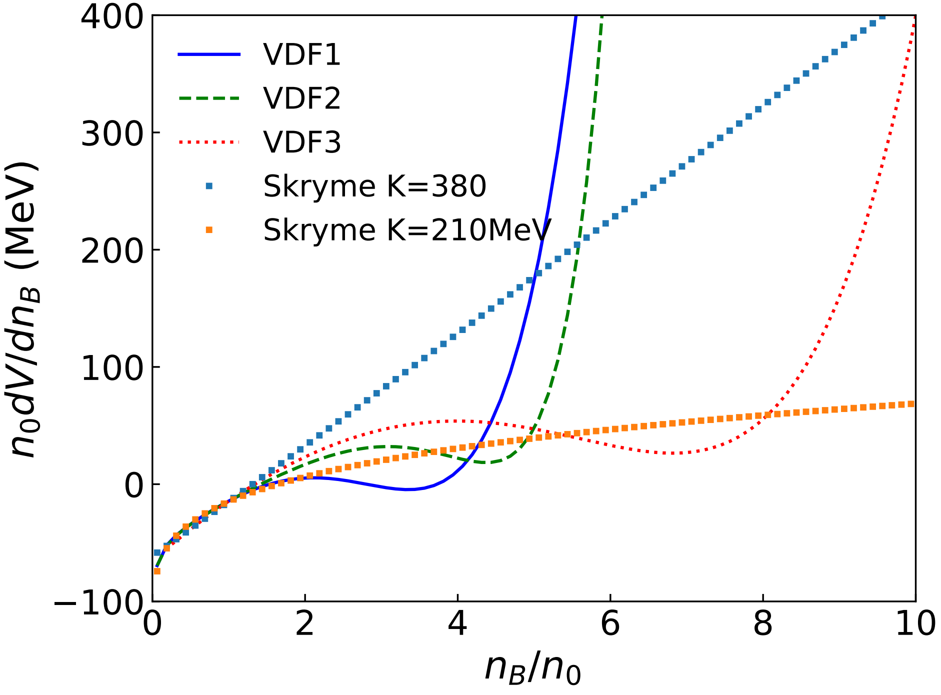
$$V^{\mu} = V(n_B)u^{\mu}, \quad u^{\mu} = \frac{J_B^{\mu}}{n_B}, \quad n_B = \sqrt{J_B^2}$$

MD potential improves the description of v1 and v2.

The vector density functional model (VDF)

A. Sorensen, V. Koch, Phys. Rev. C104,034904(2021)

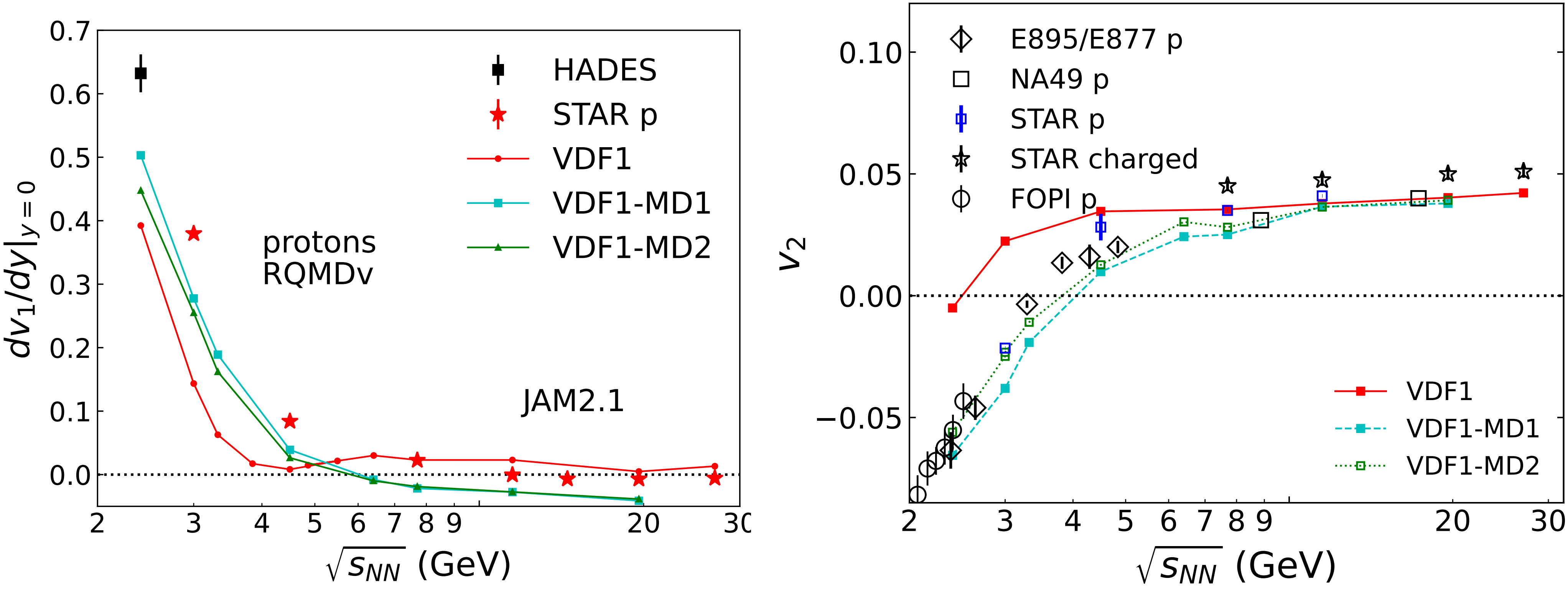




$$V^{\mu}_{\rm VDF} = \sum_{i=1}^{4} \frac{C_i}{b_i} \left(\frac{n_B}{n_0}\right)^{b_i - 1} \cdot \frac{J^{\mu}}{n_B}$$

$$P = P_{\rm kin} + n^2 \frac{dV}{dn}$$

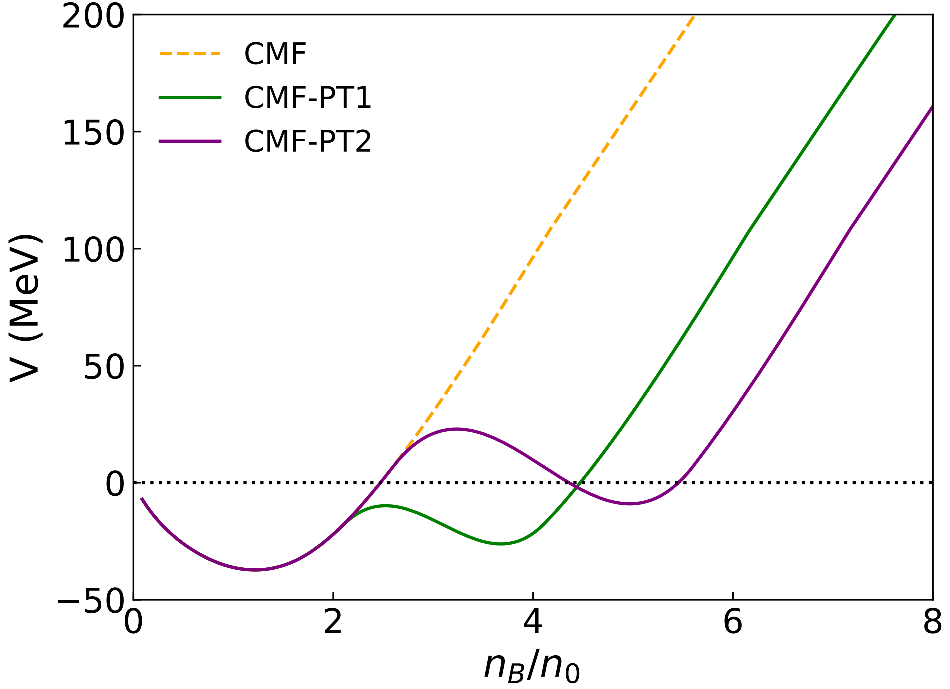
v1 and v2 from RQMD + VDF

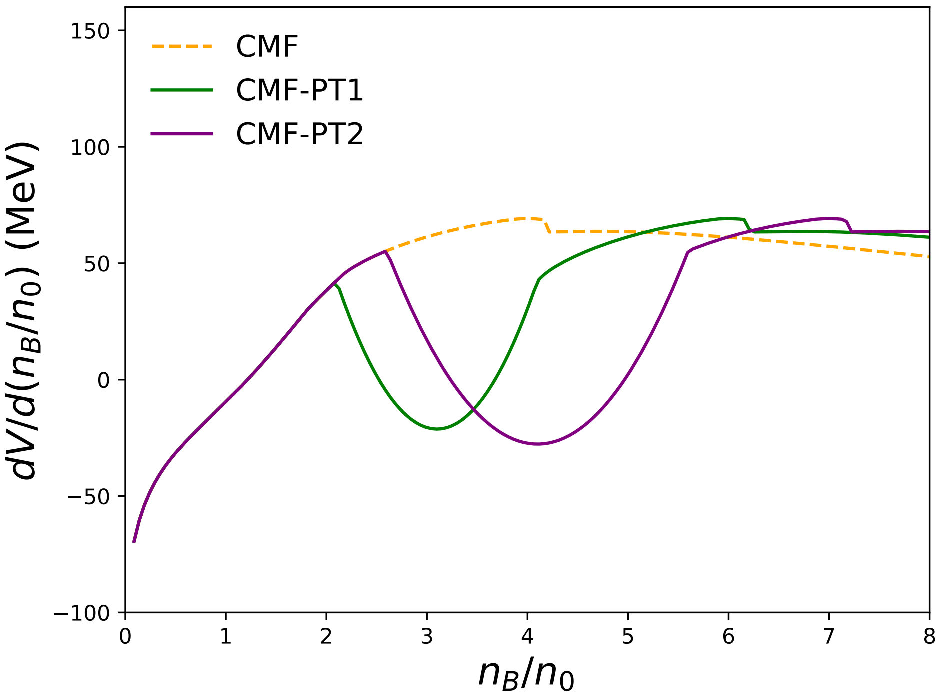


Minium in the slope of the directed flow due to 1st order PT disappears by the MD potential.

Enhancement of v1 by a first-order phase transition disappears by MD potential.

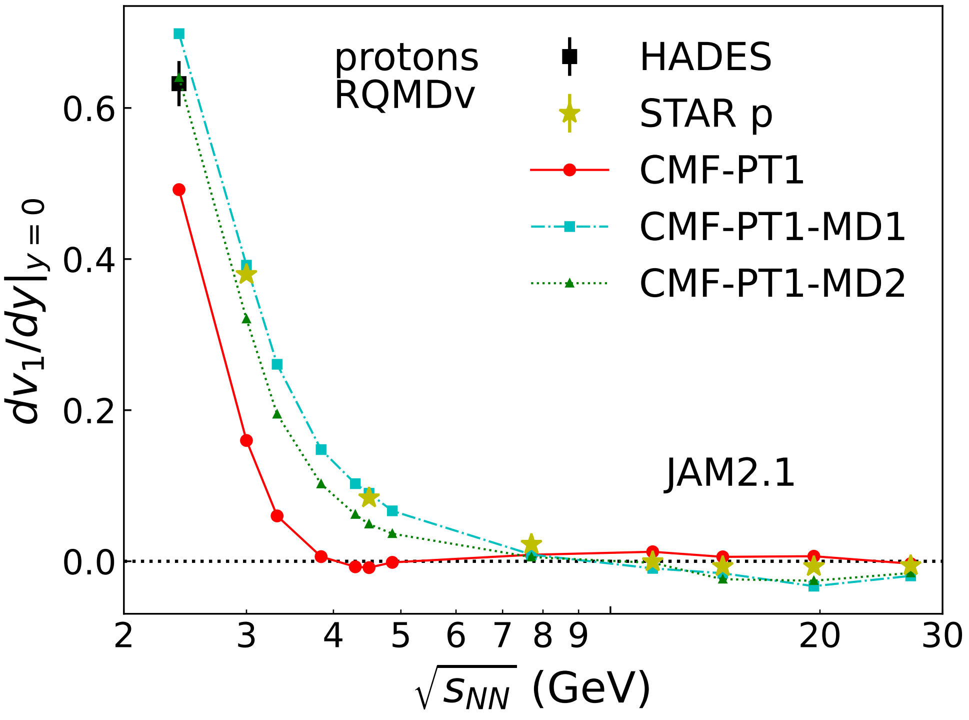
CMF + 1stOPT

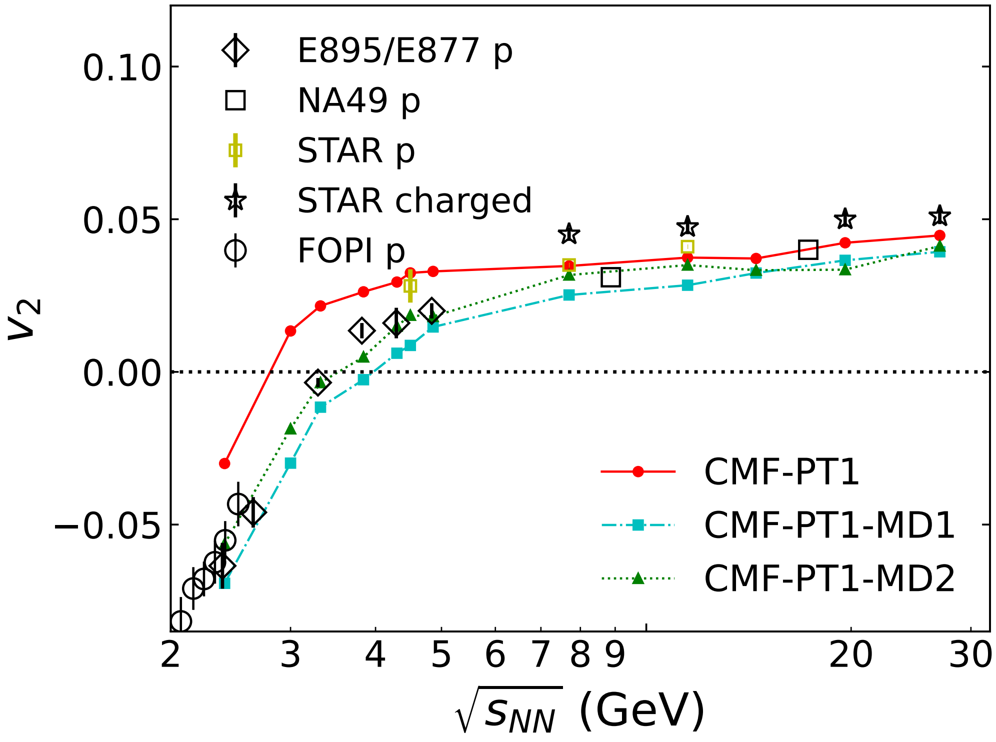




Introduce 1stOPT into the CMF model J.Steinheimer, et.al. Eur. Phy.C82(2022)911

V1 and v2 from CMF + 1st OPT





Collapse of v1 slope disappears by the momentum-dependent interaction.

Summary

- Directed flow (v1) is determined by the interplay between positive v1 generated in the compression stages and the negative v1 generated during the expansion stage.
- RQMDv with CMF + MD potential agrees with the data on v1 and v2.
- Collapse of directed flow for a 1st OPT EoS in the beam energy dependence of v1 disappears, when momentum-dependent interaction is introduced.

Buck up

Relativistic quantum molecular dynamics (RQMD) approach

RQMD was developed based on the constrained Hamiltonian dynamics:
H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192, 266 (1989).

T. Maruyama, et. al. Prog. Theor. Phys. 96, 263 (1996).

Manifestly covariant way: four-vectors $q_i^\mu, \quad p_i^\mu \quad (i = 1, N)$

For the description of N-particle system, we have 8N dimension.
In order to reduced the dimension from 8N to 6N, we need 2N constraints.

Hamiltonian is a linear combinations of the constraints, and equations of motion are given by

$$H = \sum_i \lambda_i \phi_i \qquad \frac{dq_i}{d\tau} = \{H, q_i\}, \quad \frac{dp_i}{d\tau} = \{H, p_i\}$$

2N constraints: On-mass shell condition and time fixation.

$$\phi_i = (p_i - V_i)^2 + (m_i - S_i)^2 = p_i^{*2} + m_i^{*2} = 0, \quad (i = 1, \cdots, N)$$

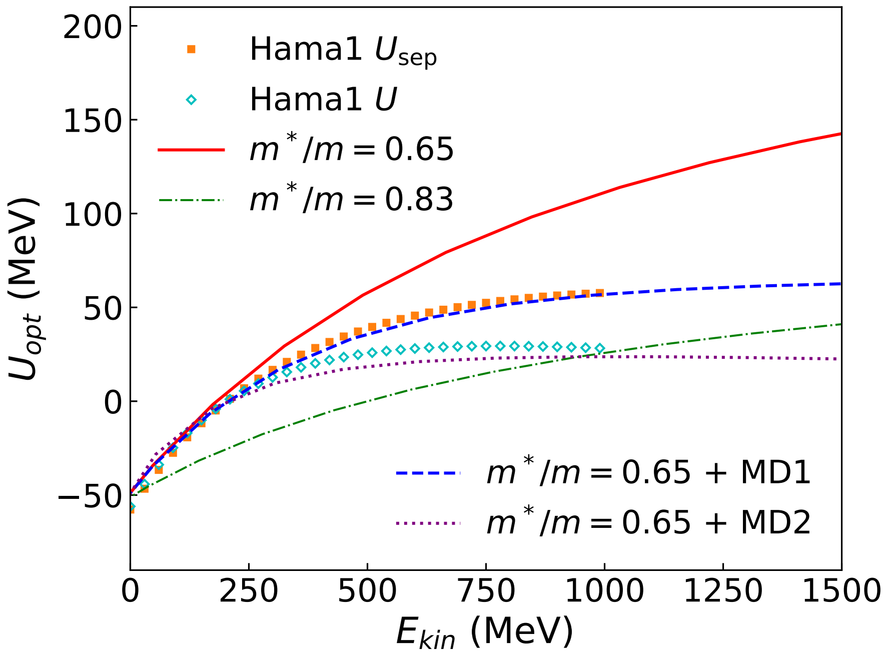
JAM2:micro-macro transport model

- Fortran77 → C++
- Pythia6 → Pythia8
- Update of collision term: include new pp data.
  - ✓ New total hadronic cross section at high energies (PDG2016)
  - ✓ New resonance cross section (Ecm< 4GeV)
  - ✓ New string excitation low (4 < Ecm < 20 GeV)
  - ✓ New multiple-parton scattering (Pythia8) (Ecm > 20GeV)

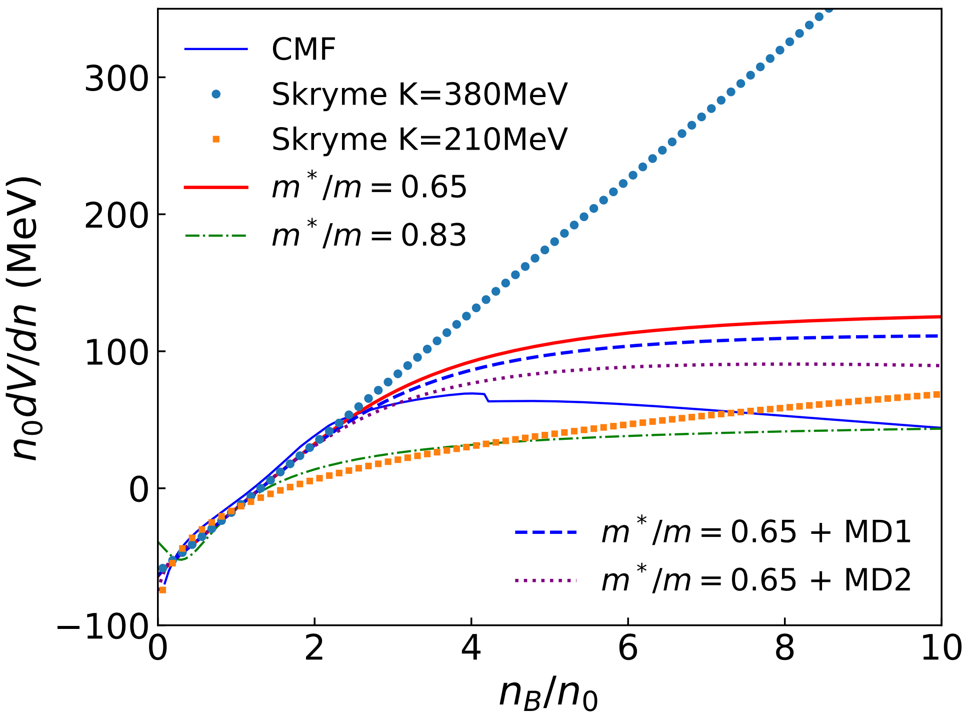
- Quantum Molecular Fluid Dynamics (QMFD): 3D perfect hydro + RQMD model
- RQMD with Skyrme force (Lorentz scalar and vector)
- RQMD.RMF with momentum-dependent potential
- Speeding up computational time by introducing expanding box for both collision term and potential evaluation

Single potential reduction of RMF

Sigma + omega type Relativistic mean field



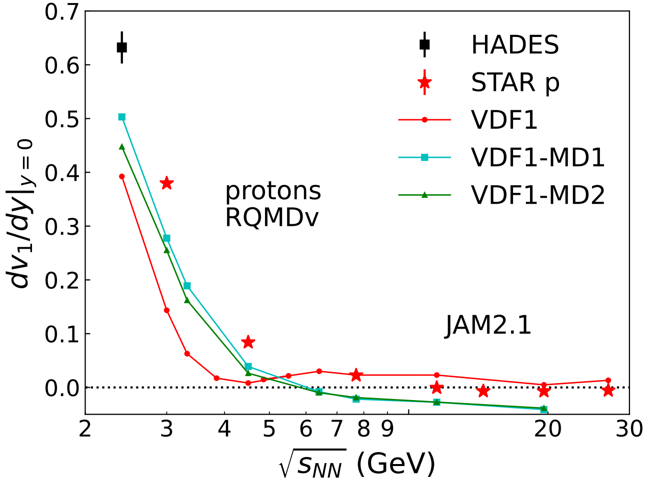
derivatives of the QMD potential

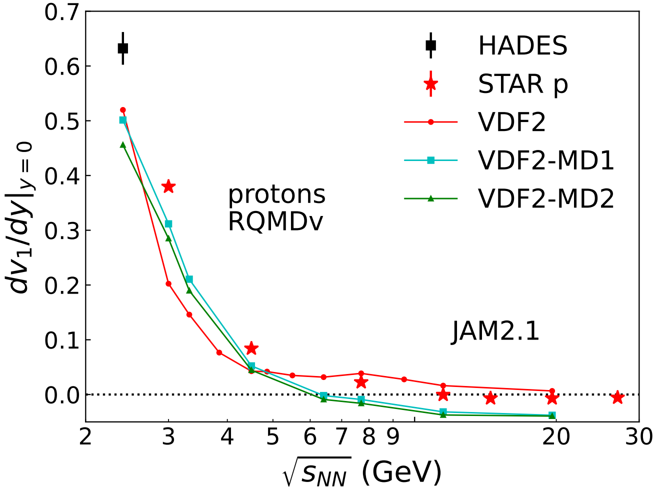


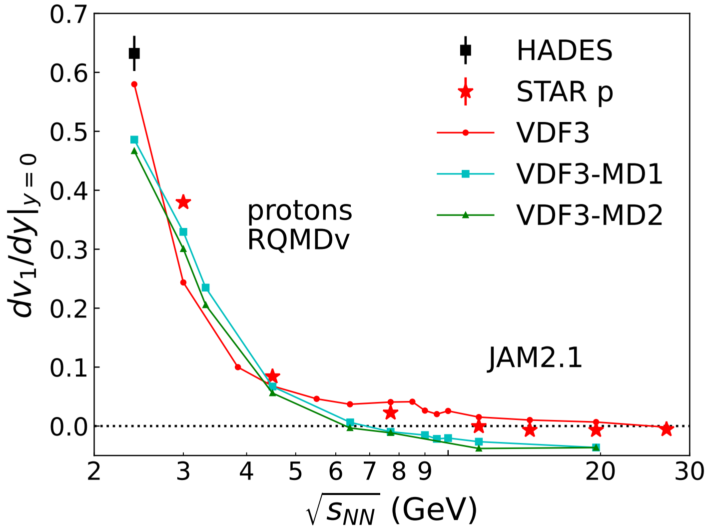
$$m^*(n_0)/m = 0.65 : \text{hard}$$

$$m^*(n_0)/m = 0.83 : \text{soft}$$

v1 from RQMD + VDF

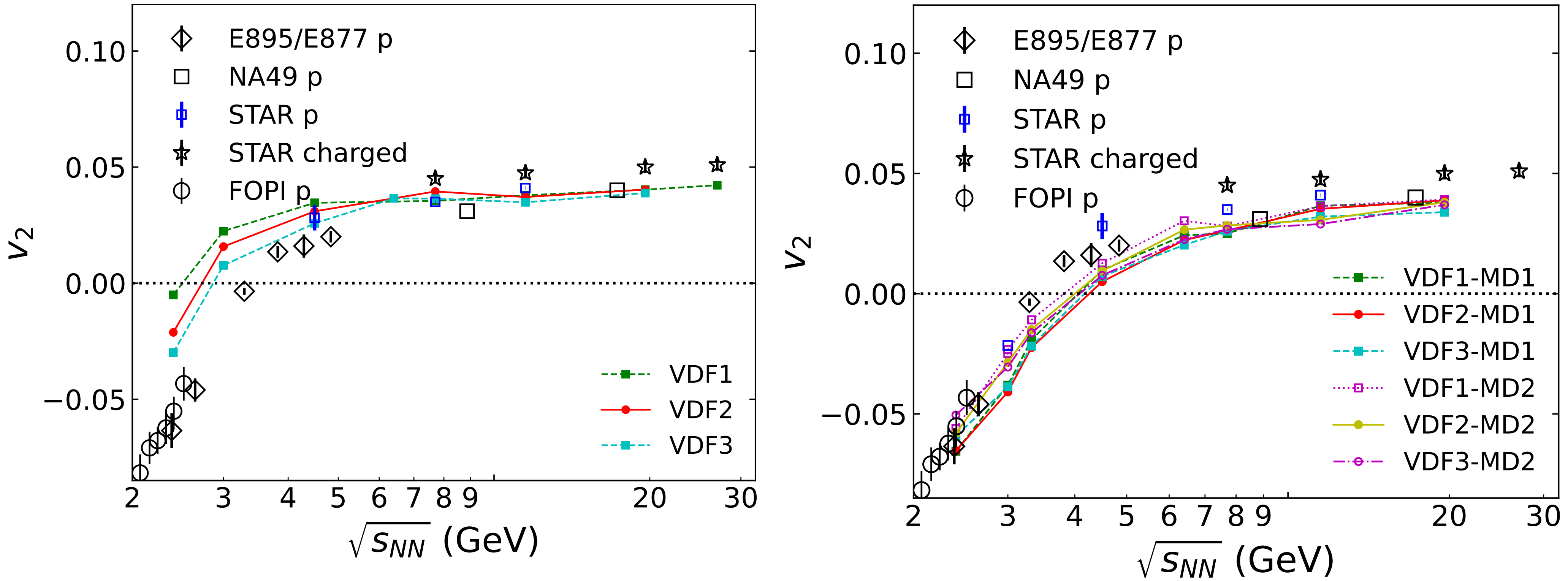






Minium in the slope of the directed flow due to 1st order PT disappears by the MD potential.

v2 from RQMD + VDF

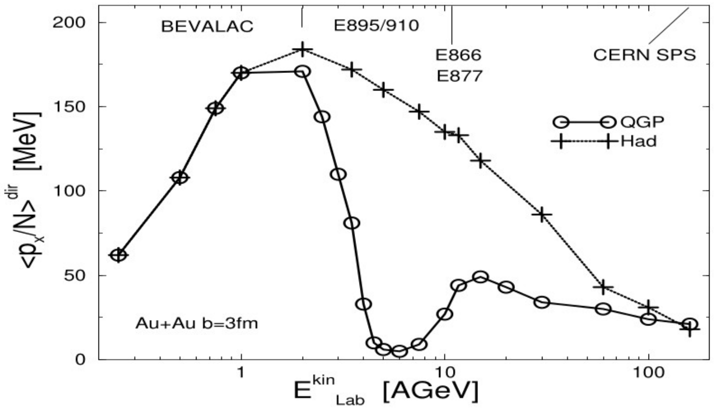


Enhancement of v1 by a first-order phase transition disappears by MD potential.

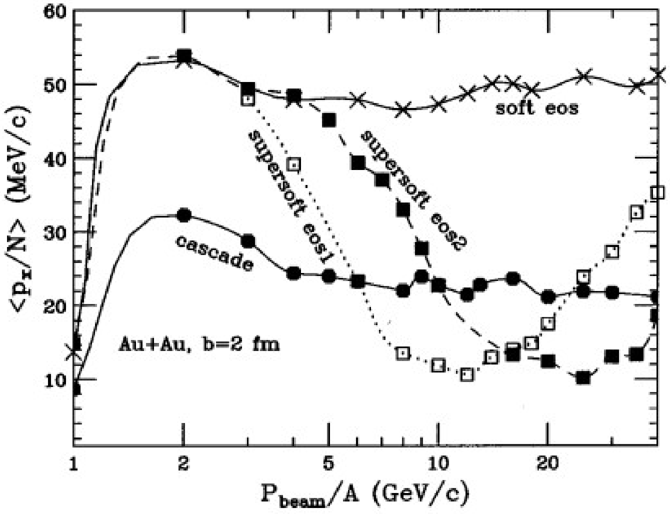
The softest point in the EoS

$$\langle p_x/N\rangle^{dir} = \frac{1}{N}\int_{-y_{CM}}^{y_{CM}} \mathrm{d}y\ \langle p_x/N\rangle(y)\ \frac{\mathrm{d}N}{\mathrm{d}y}\ \mathrm{sgn}(y)$$

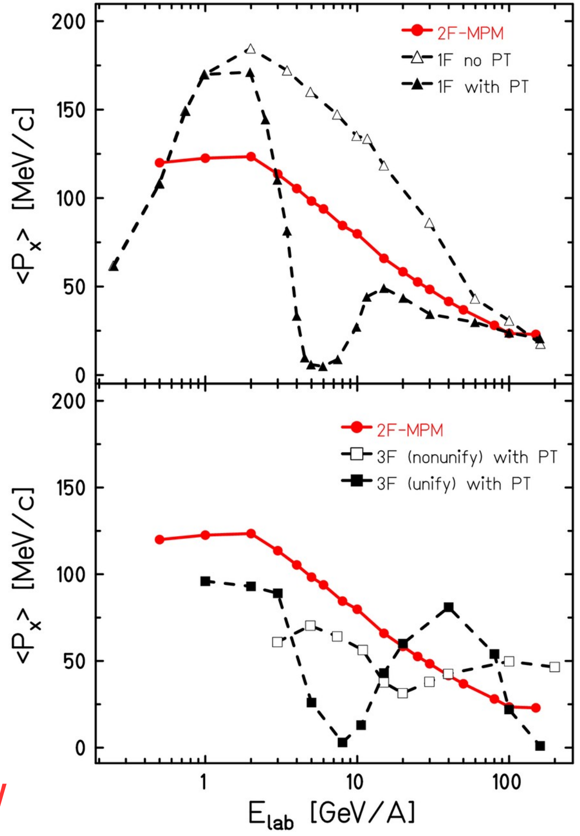
3FD:Ivanov Heavy Ion Phys.15:117-130,2002



1FD: D.H.Rischke, et.al
Heavy Ion Phys.1, 309 (1995)



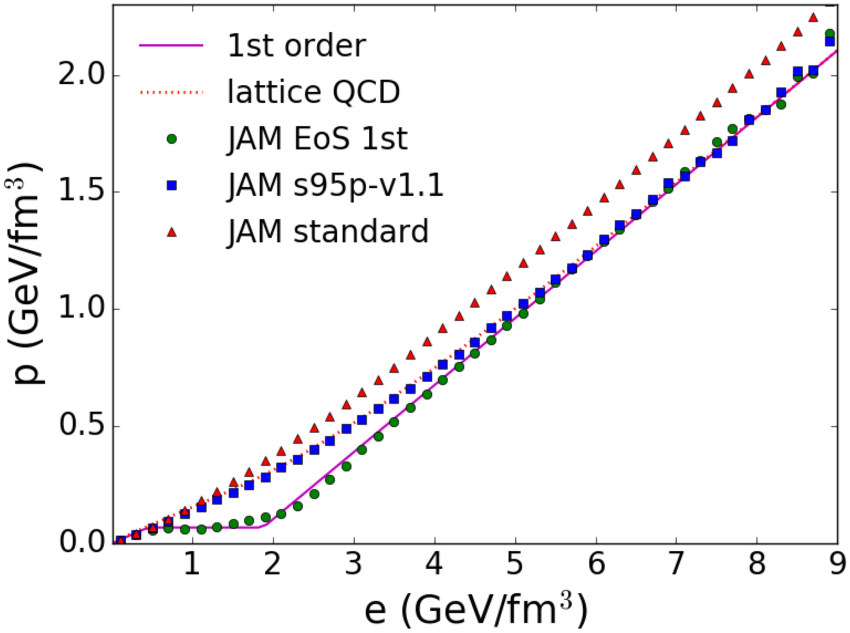
ART:B-A. Li and C.M. Ko,
PRC58(1998)R1382



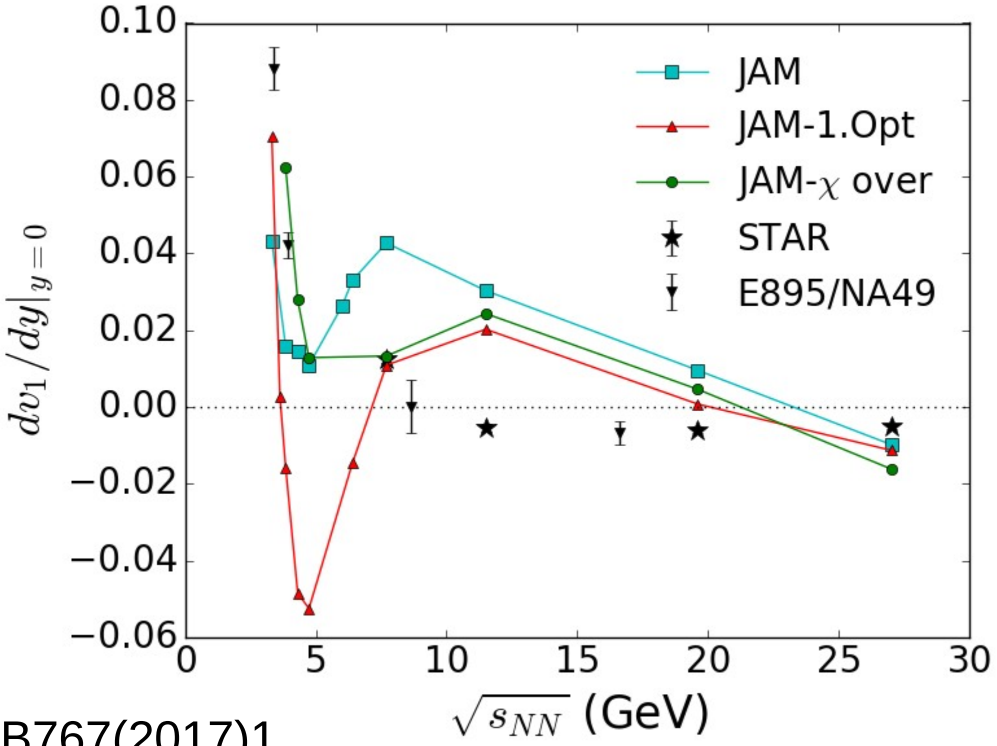
A minimum is predicted in the excitation function of the directed flow

v1 from EoS modified collision term

EoS modified collision term
provides efficient method
to control EoS in a
microscopic transport model.



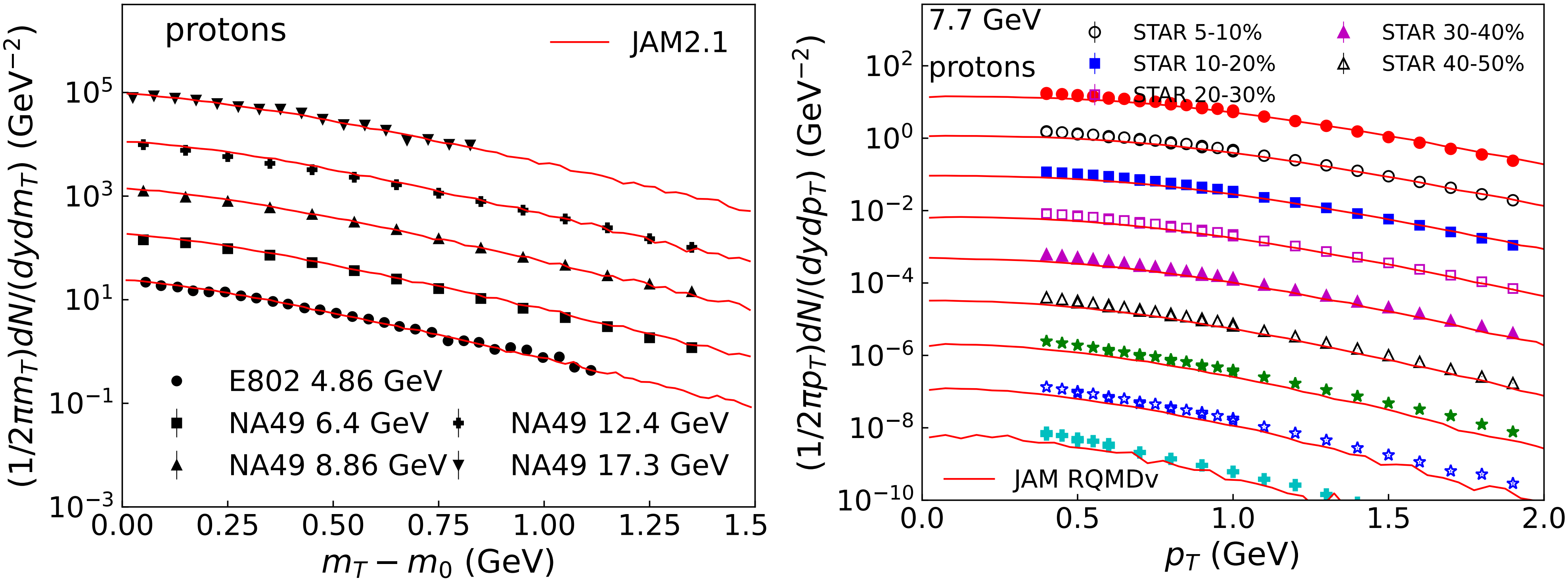
JAM with fully baryon density dependent EoS meets hydro



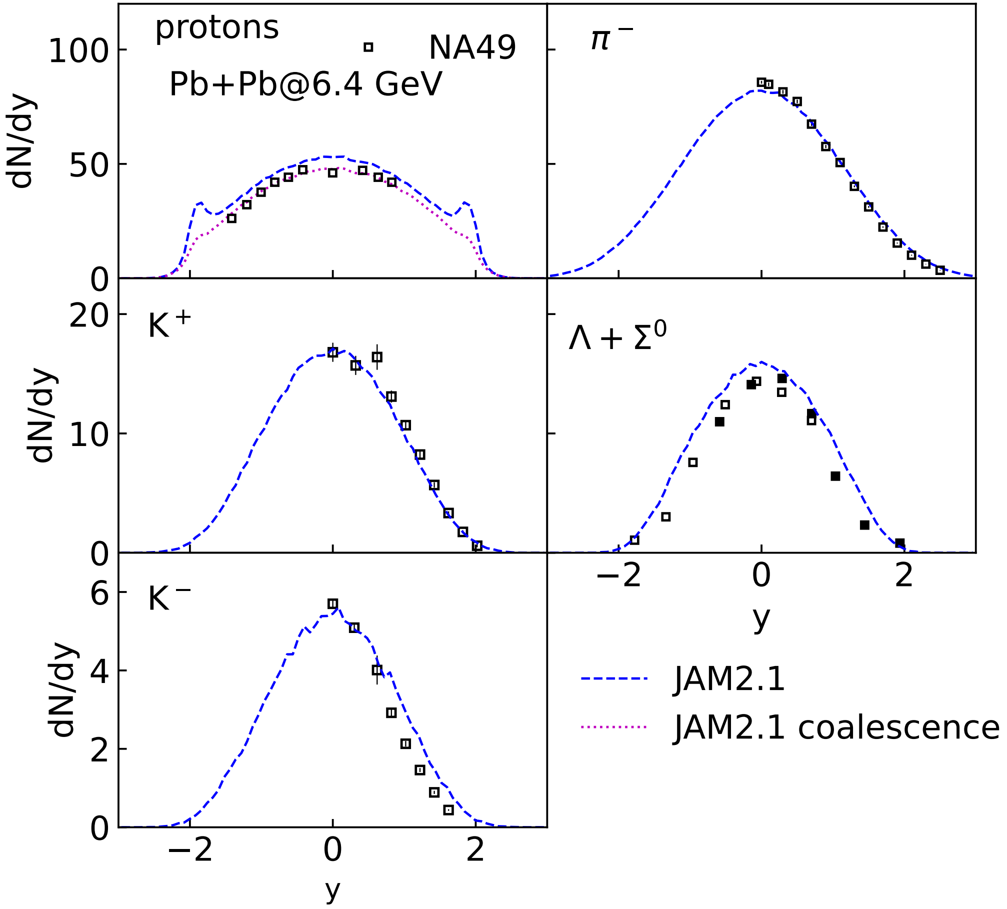
Y.Nara, PLB767(2017)1

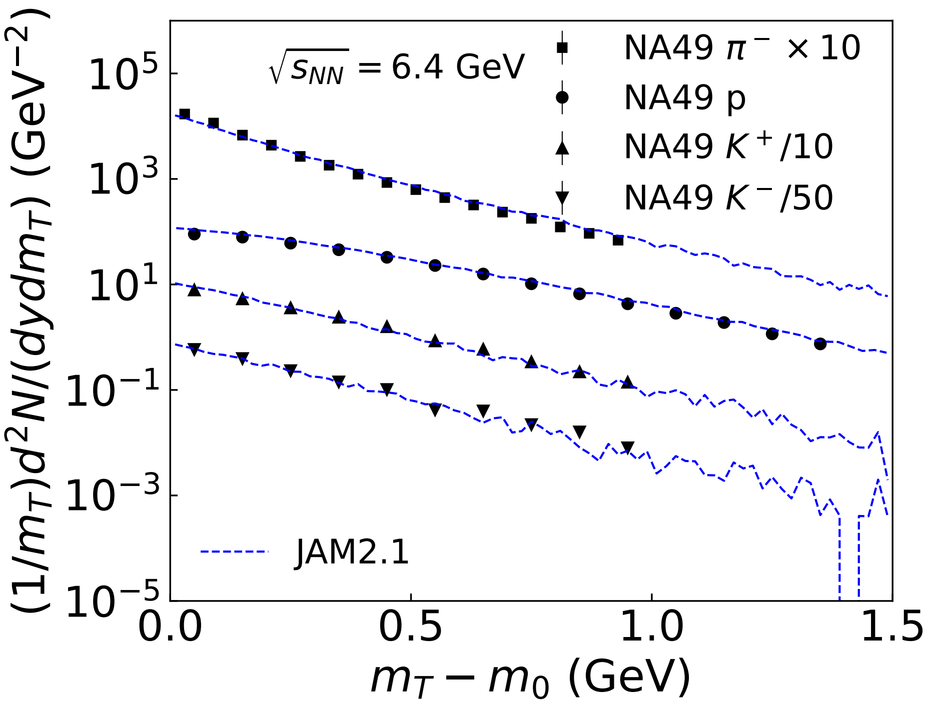
Proton spectra

JAM2.108



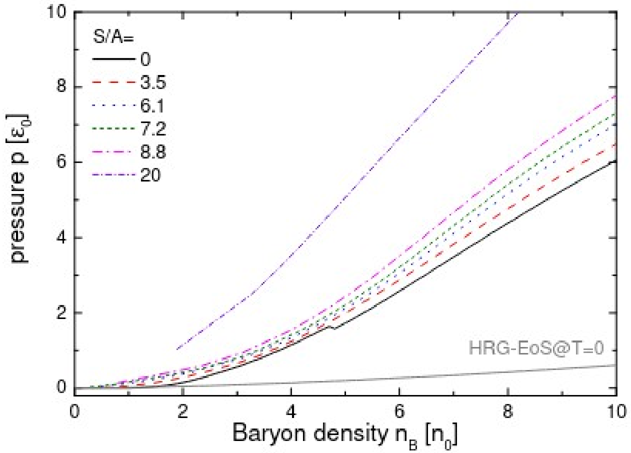
Pb + Pb at Elab=20AGeV



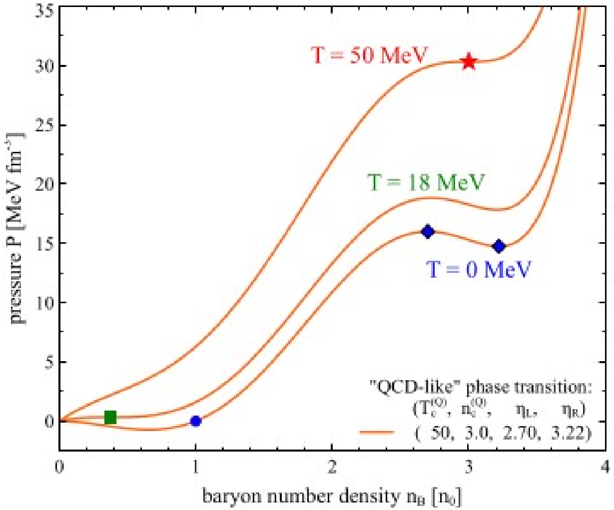


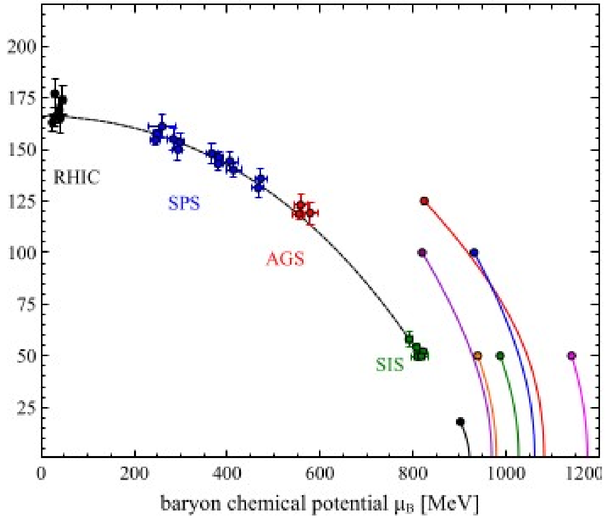
Recent progress in microscopic transport approaches

- SMASH with critical point EoS, A. Sorensen and V. Koch, PRC104(2021)034904
- Chiral mean-field (CMF) EoS in UrQMD, M. Omana Kuttan, et. al. nucl-th2201.01622
- Microscopic transport model with the Parity doublet model
  - ✓ DJBUU: M. Kim, et.al PRC101(2020)064614
  - ✓ GiBUU: A.B.Larionov and L. von Smekal, nucl-th2109.03556
- PHQMD (Parton hadron quantum molecular dynamics) J.Aichelin, PRC101(2020)044905
- JAM RQMD.RMF (2019),(2020), RQMDv (2022)



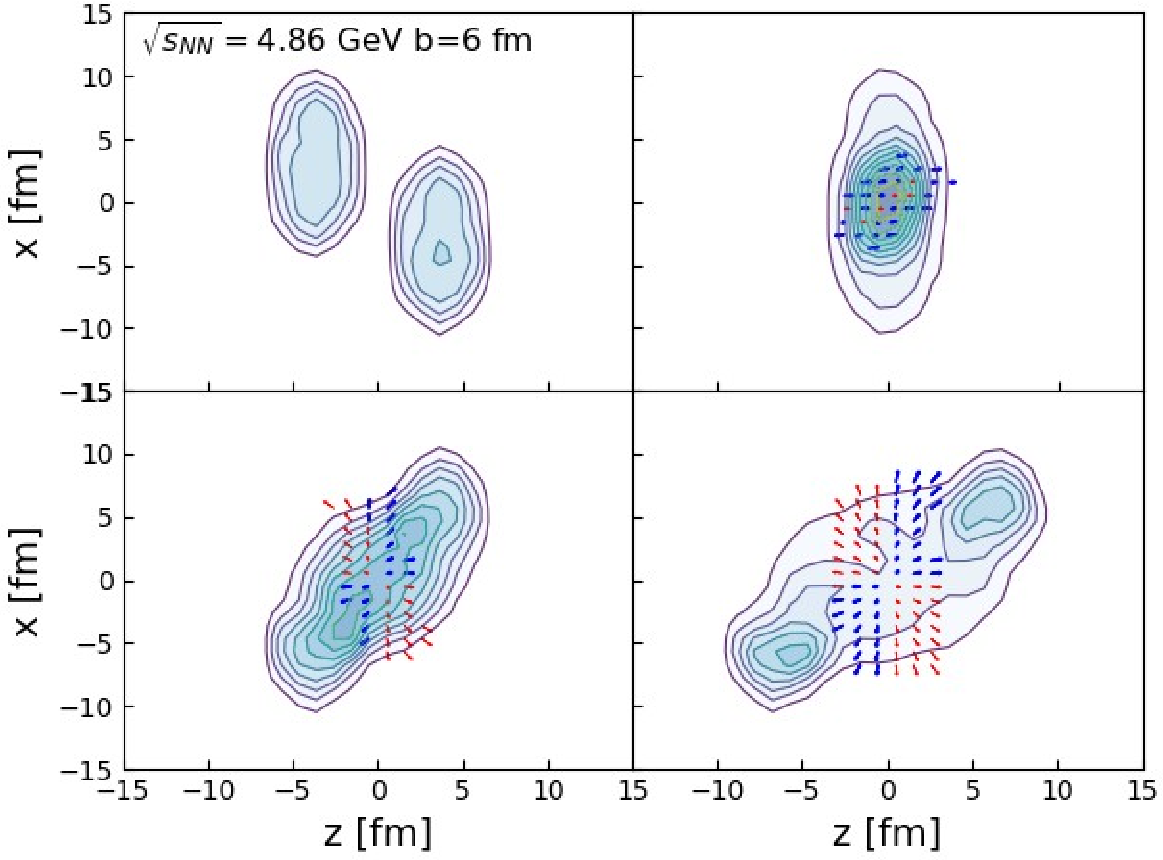
CMF

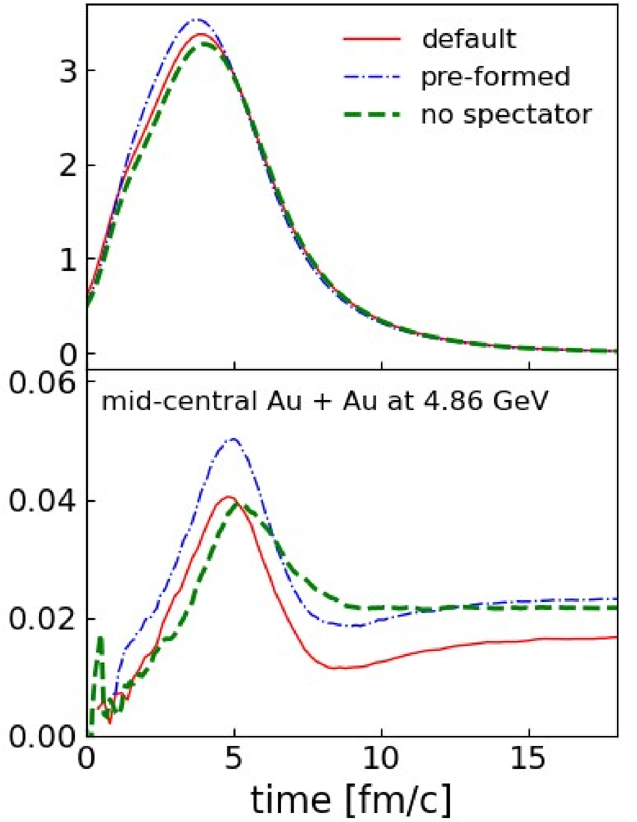




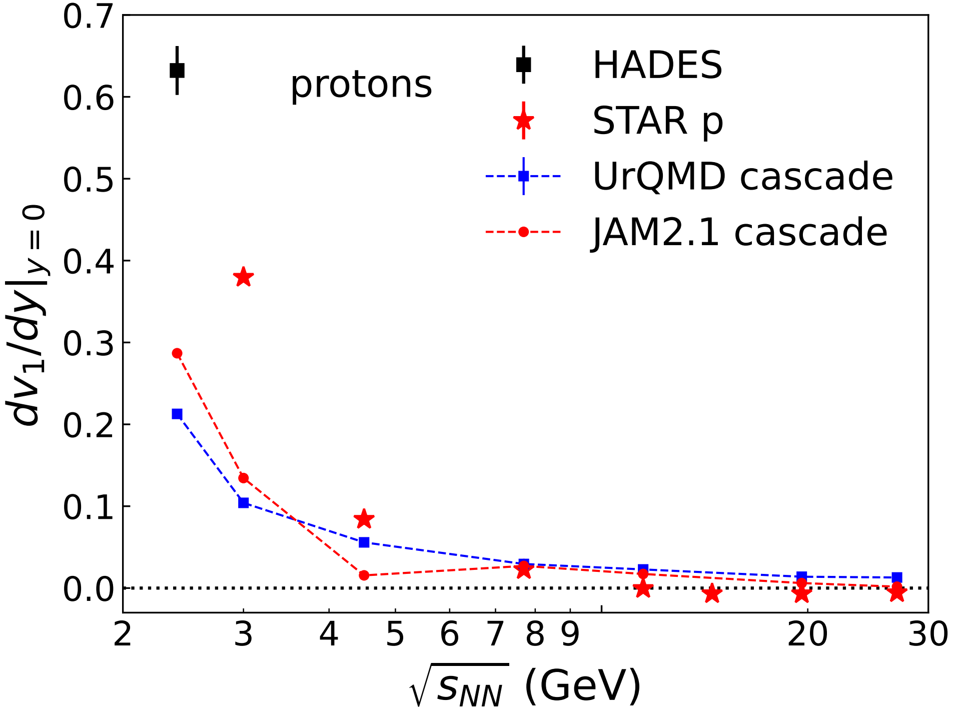
Time evolution of v1 at 4.86 GeV

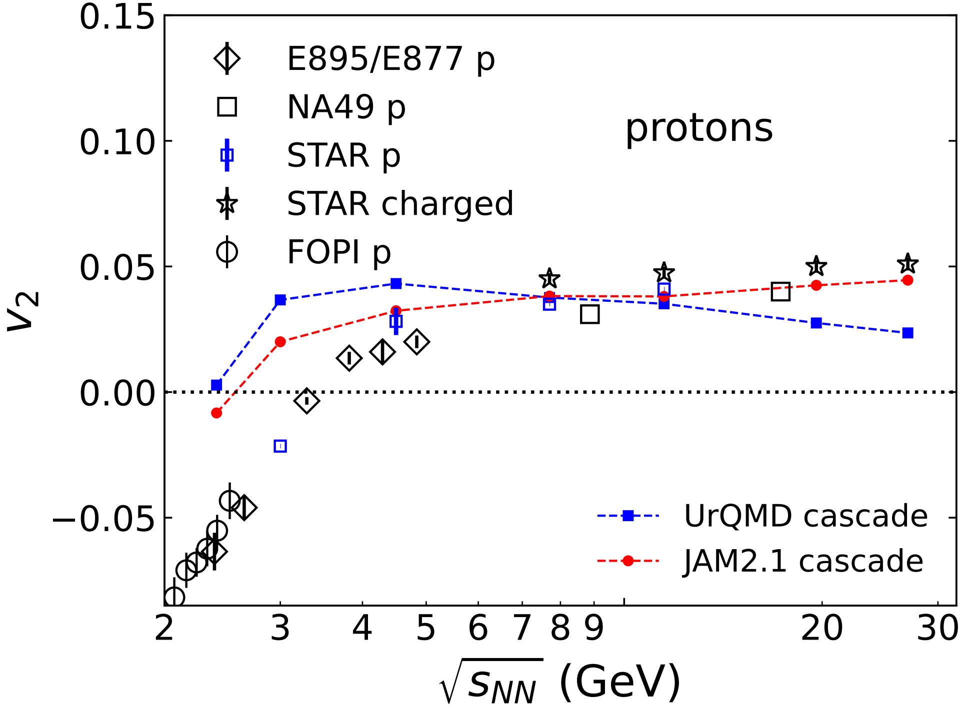
Time evolution of the baryon density in Au+Au mid-central collision (b=6fm)



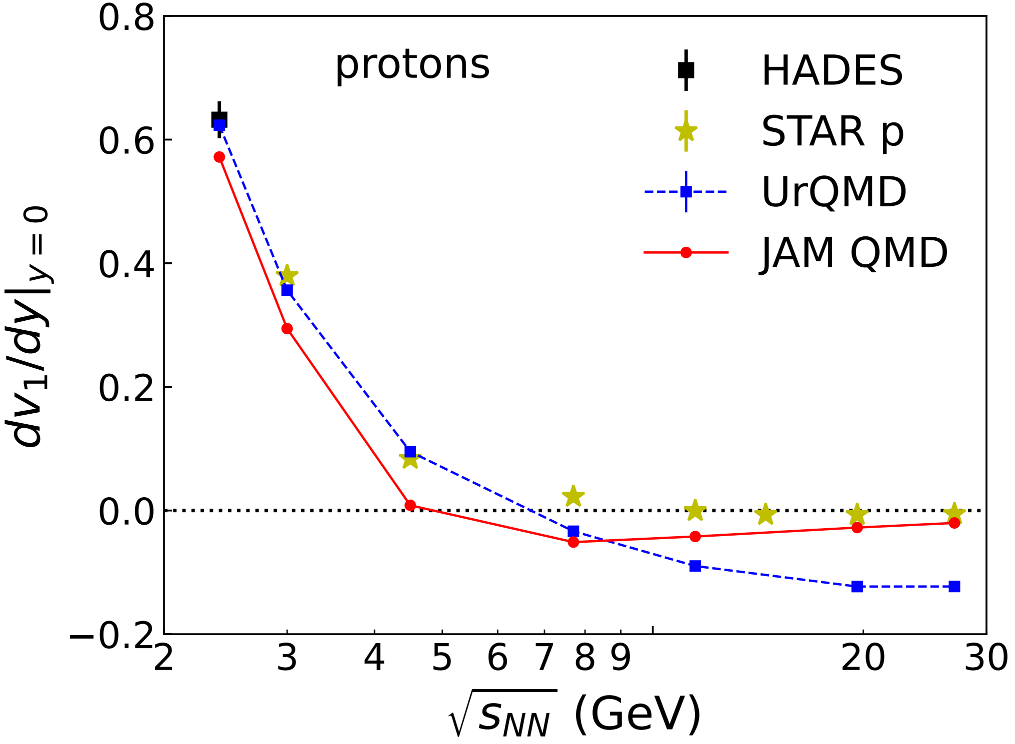


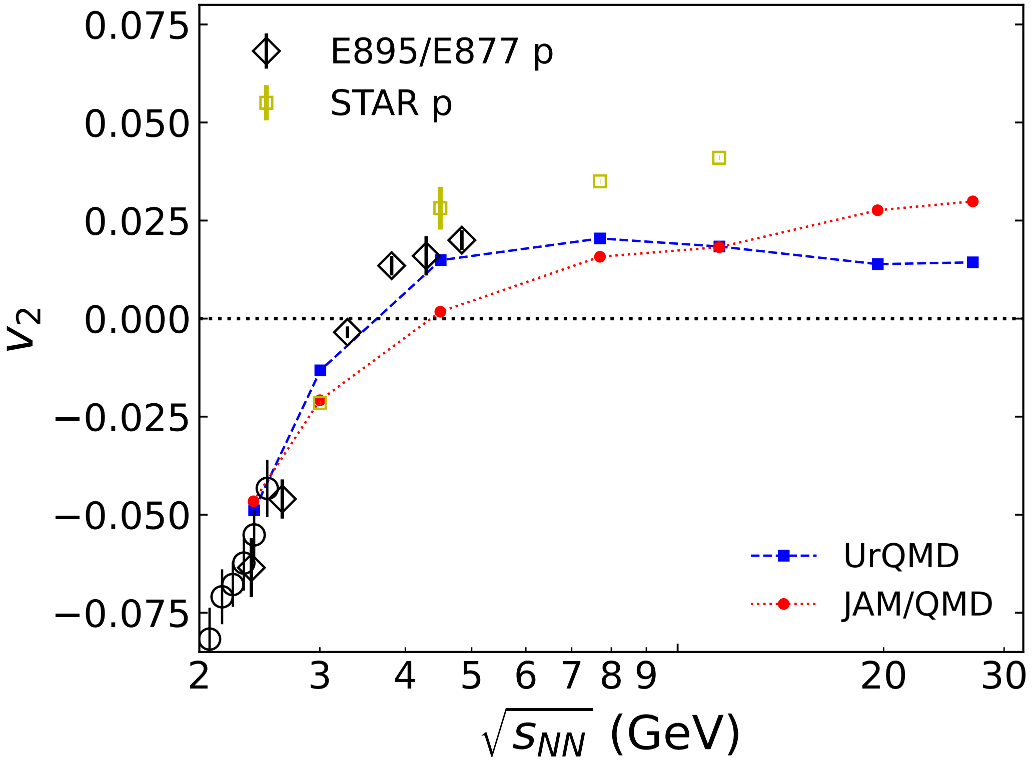
Beam energy dependence of v1 and v2 from cascade mode



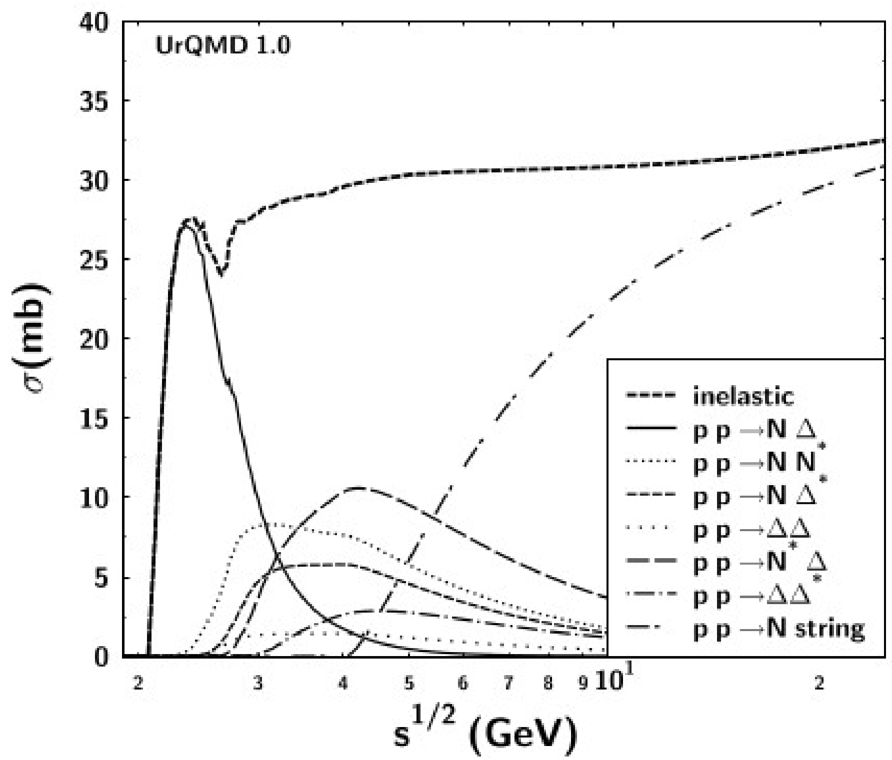


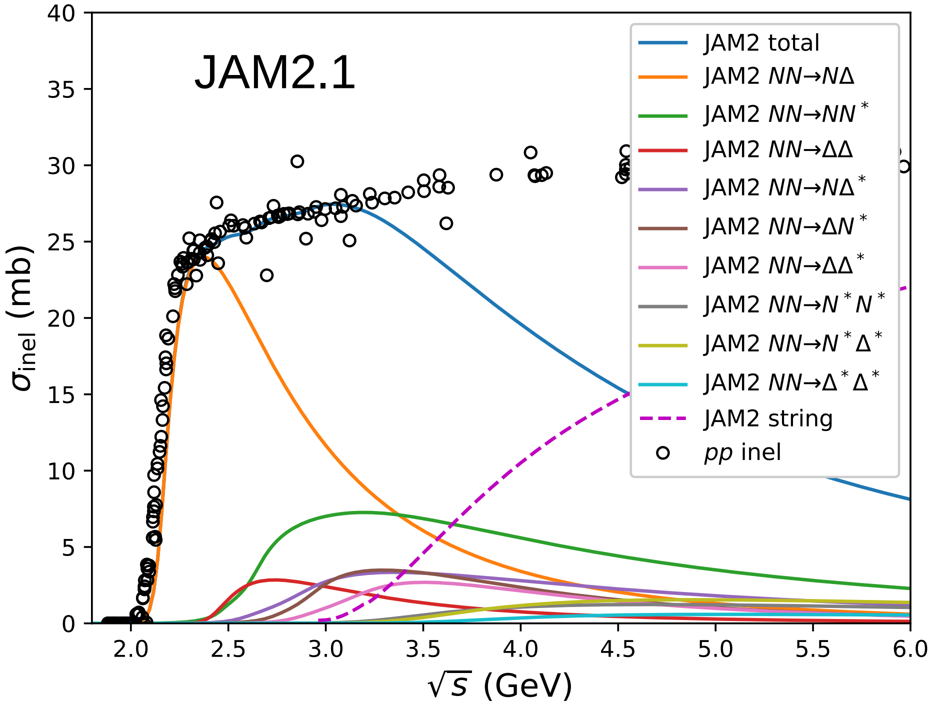
Beam energy dependence of v1 and v2 from QMD mode





UrQMD and JAM cross sections

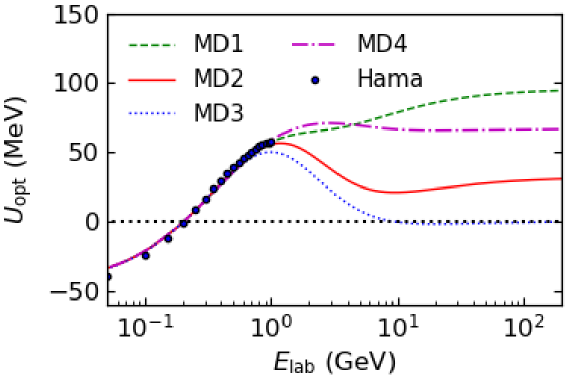


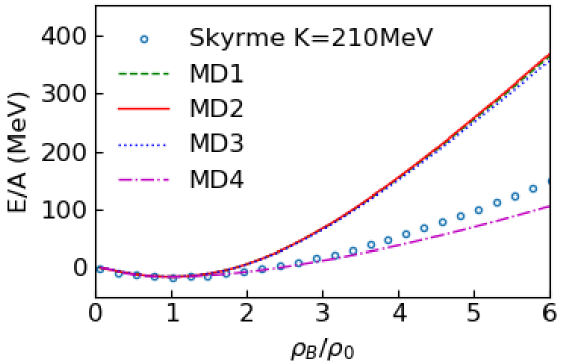


different parametrization

Momentum-dependent potential

K. Weber, B. Blaettel, W. Cassing, H. C. Doenges, V. Koch,
A. Lang and U. Mosel, Nucl. Phys. A 539, 713 (1992).





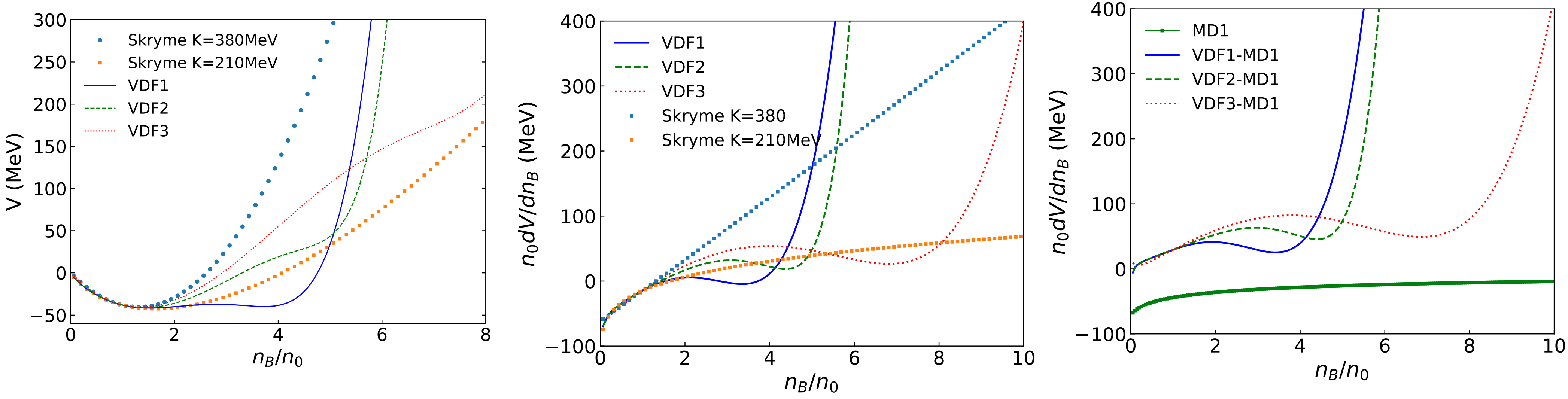
$$V_s^{\rm MD} = \frac{\bar{g}_s^2}{m_s^2} \int d^3p \frac{m^*}{p_0^*} \frac{f(x,p)}{1+(p-p')^2/\Lambda_s^2} \quad V_\mu^{\rm MD} = \frac{\bar{g}_v^2}{m_v^2} \int d^3p \frac{p_\mu^*}{p_0^*} \frac{f(x,p)}{1+(p-p')^2/\Lambda_v^2}$$

		MD1	MD2	MD3	MD4
K	(MeV)	380	380	380	210
m^*/m		0.65	0.65	0.65	0.83
$U_{\rm opt}(\infty)$	(MeV)	95	30	-0.4	67
g_s		9.030	9.233	5.439	4.059
g_v		6.740	3.888	0.0	5.632
g_2	(1/fm)	4.218	4.012	-15.59	-160.3
g_3		6.667	5.520	391.9	2684
$\bar{g}_s$		3.186	2.502	7.711	5.544
$\bar{g}_v$		8.896	10.43	11.22	3.926
Λ_s	(GeV)	0.641	0.4897	1.702	0.704
Λ_v	(GeV	1.841	2.489	1.898	4.252

Y.N. T.Maruyama,H.Stoecker,PRC(2020)

The vector density functional model (VDF)

A. Sorensen, V. Koch, Phys. Rev. C104,034904(2021)

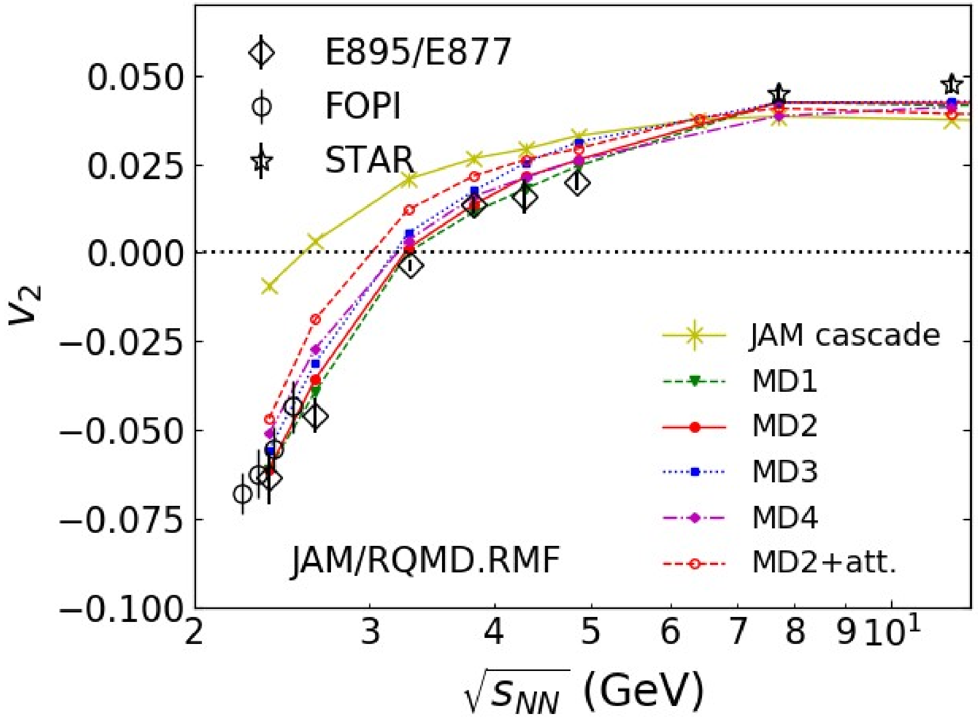


$$V^{\mu}_{\mathrm{VDF}} = \sum_{i=1}^{4} \frac{C_i}{b_i} \left(\frac{n_B}{n_0}\right)^{b_i - 1} \cdot \frac{J^{\mu}}{n_B}$$

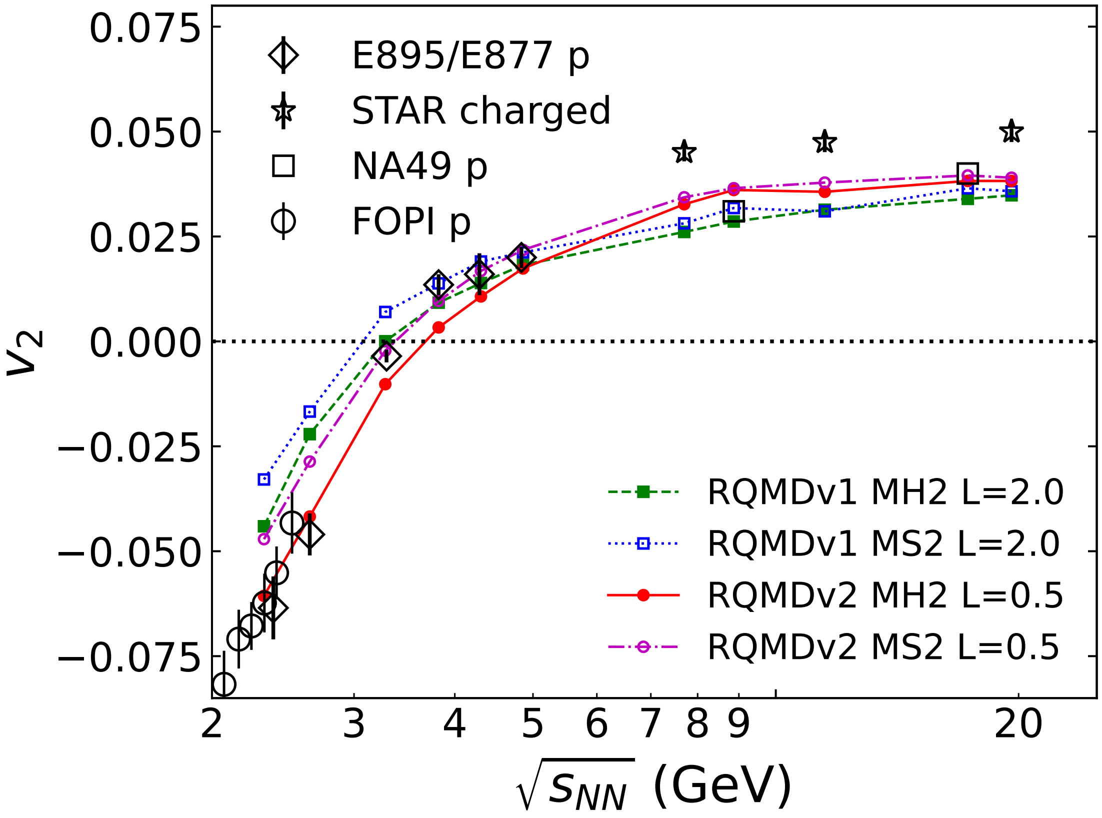
$$P = P_{\mathrm{kin}} + n^2 \frac{dV}{dn}$$

v2 from RQMD.RMF

Beam dependence of proton v1 at mid-rapidity



V2 from JAM2/RQMDv



Importance of momentum-dependent potential

PHYSICAL REVIEW C VOLUME 35, NUMBER 5 MAY 1987

Heavy-ion collision theory with momentum-dependent interactions

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We examine the influence of momentum-dependent interactions on the momentum flow in 400 MeV/nucleon heavy ion collisions. Choosing the strength of the momentum dependence to produce an effective mass $m^* = 0.7m$ at the Fermi surface, we find that the characteristics of a stiff equation of state can be obtained with a much softer compressibility.

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Importance of Momentum-Dependent Interactions for the Extraction of the Nuclear Equation of State from High-Energy Heavy-Ion Collisions

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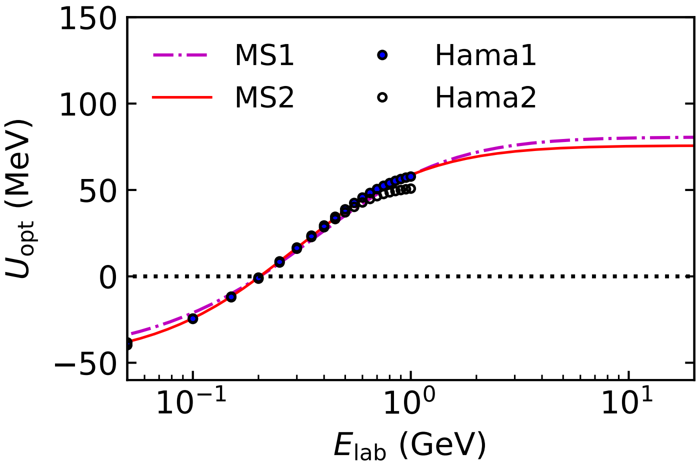
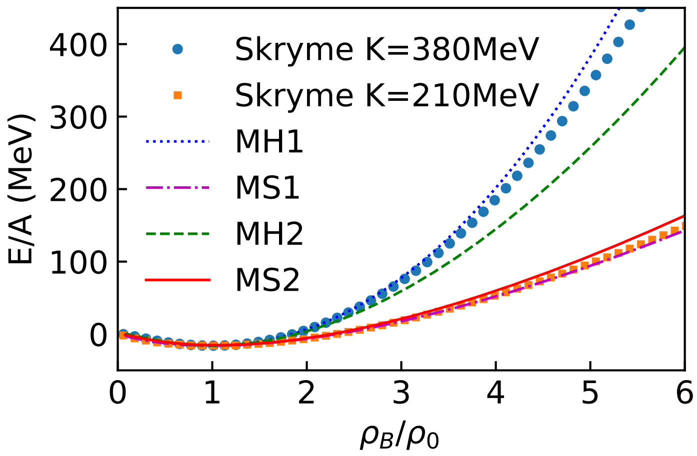
(Received 4 November 1986; revised manuscript received 19 February 1987)

We demonstrate that momentum-dependent nuclear interactions (MDI) have a large effect on the dynamics and on the observables of high-energy heavy-ion collisions: A soft potential with MDI suppresses pion and kaon yields much more strongly than a local hard potential and results in transverse momenta intermediate between soft and hard local potentials. The collective-flow angles and the deuteron-to-proton ratios are rather insensitive to the MDI. Only simultaneous measurements of these observables can give clues on the nuclear equation of state at densities of interest for supernova collapse and neutron-star stability.

EoS in the JAM2 RQMDv mode





Y.N. and A. Ohnishi, PRC(2022)

$$\mathrm{p}^{*\mu} = p^{\mu} - U^{\mu}_{\rm sk}(\rho) - U^{\mu}_{m}(p).$$

$$U_{\rm sk}(\rho) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma},$$

$$U^{\mu}_{m}(p) = \frac{C}{\rho_0}\int d^3p'\,\frac{p^{*\prime\mu}}{e^*}\,\frac{f(x,p')}{1+[(\boldsymbol{p}-\boldsymbol{p}')/\mu_k]^2},$$