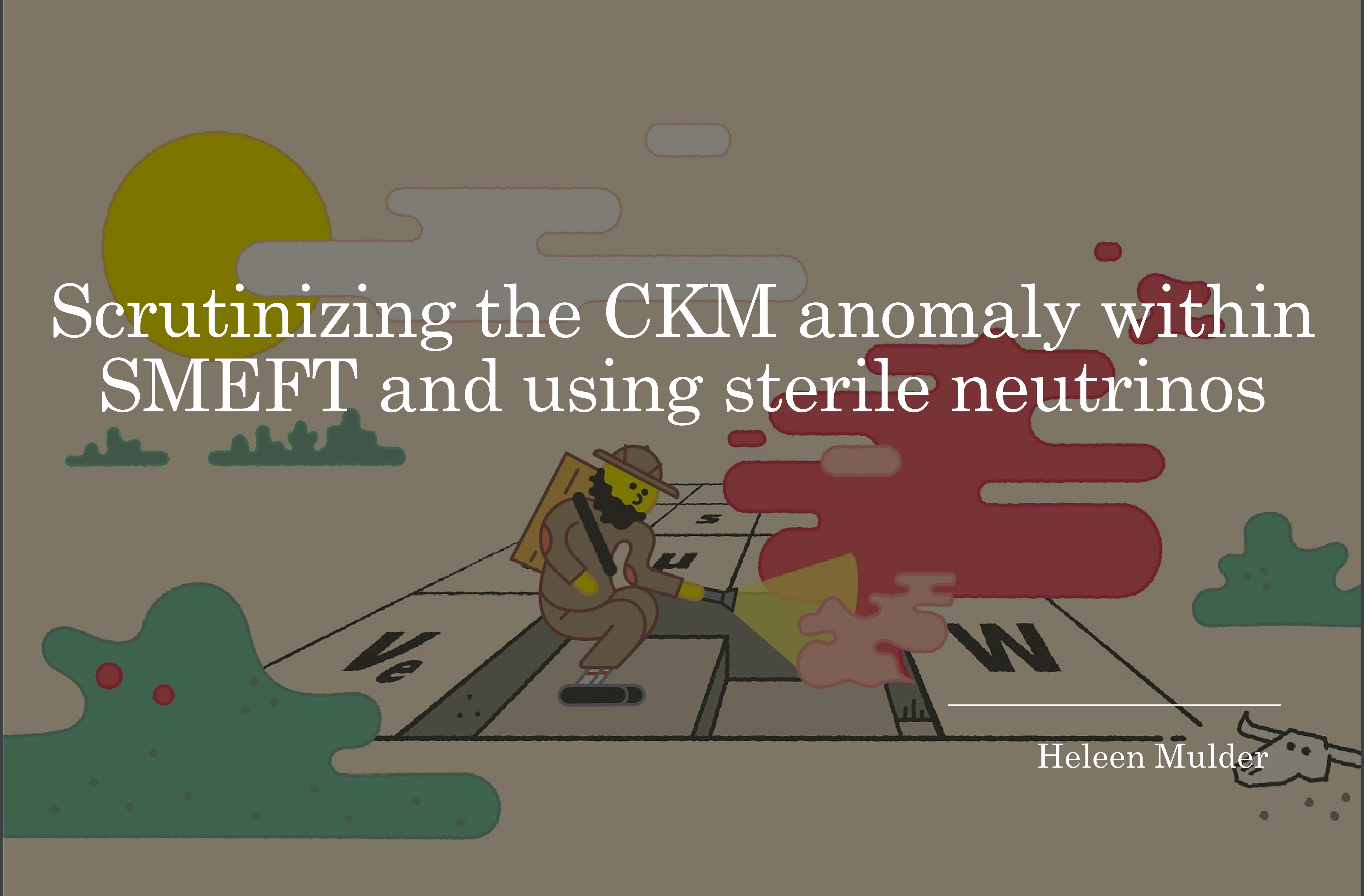


Scrutinizing the CKM anomaly within SMEFT and using sterile neutrinos

Heleen Mulder

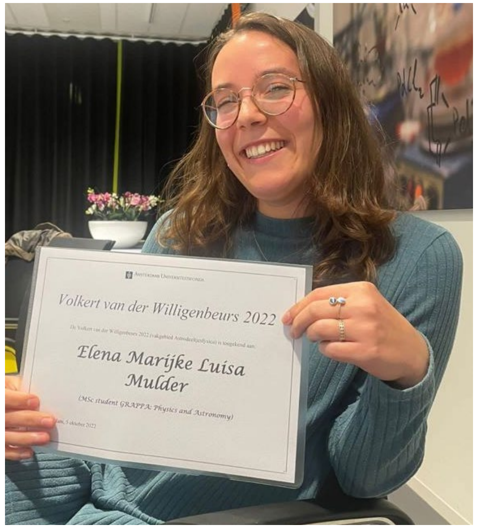
New Physics Searches at the Precision Frontier, INT, 5-5-2023

About me

- MSc student at University of Amsterdam
- Travel grant for visit to INT

Master's thesis in the Nikhef theory group, with Jordy de Vries

Nikhef



Important paper: Cirigliano, Crivellin, Hoferichter & Moulson, *Scrutinizing CKM unitarity with a new measurement of the* $K_{\mu 3}/K_{\mu 2}$ *branching fraction* (2022).

Contents

Part 1: the anomaly

- Relevant observables
- Development of the CKM anomaly

Part 2: the solution (hopefully…)

- SMEFT & LEFT
- Results from various NP models
  - Right-handed currents
  - Pseudoscalar currents
  - **4th sterile neutrino** (WIP)

As included in: Cirigliano, Dekens, De Vries, Fuyuto, Mereghetti & Ruiz, *Leptonic anomalous magnetic moments in νSMEFT* (2021).

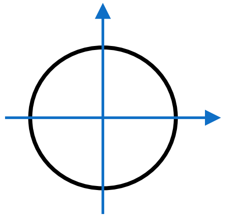
Part 1: the anomaly

CKM anomaly

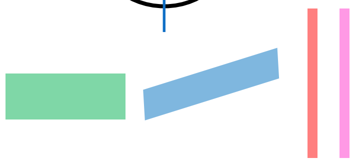
SM prediction:

$$|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2 + \cancel{|V_{\mathrm{ub}}|^2} = 1$$

What do we see?



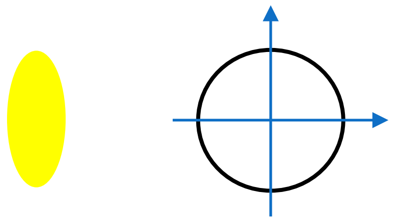
- SM prediction
- Experimental constraints



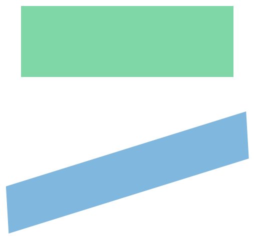
- Best-fit from experimental constraints (1σ)

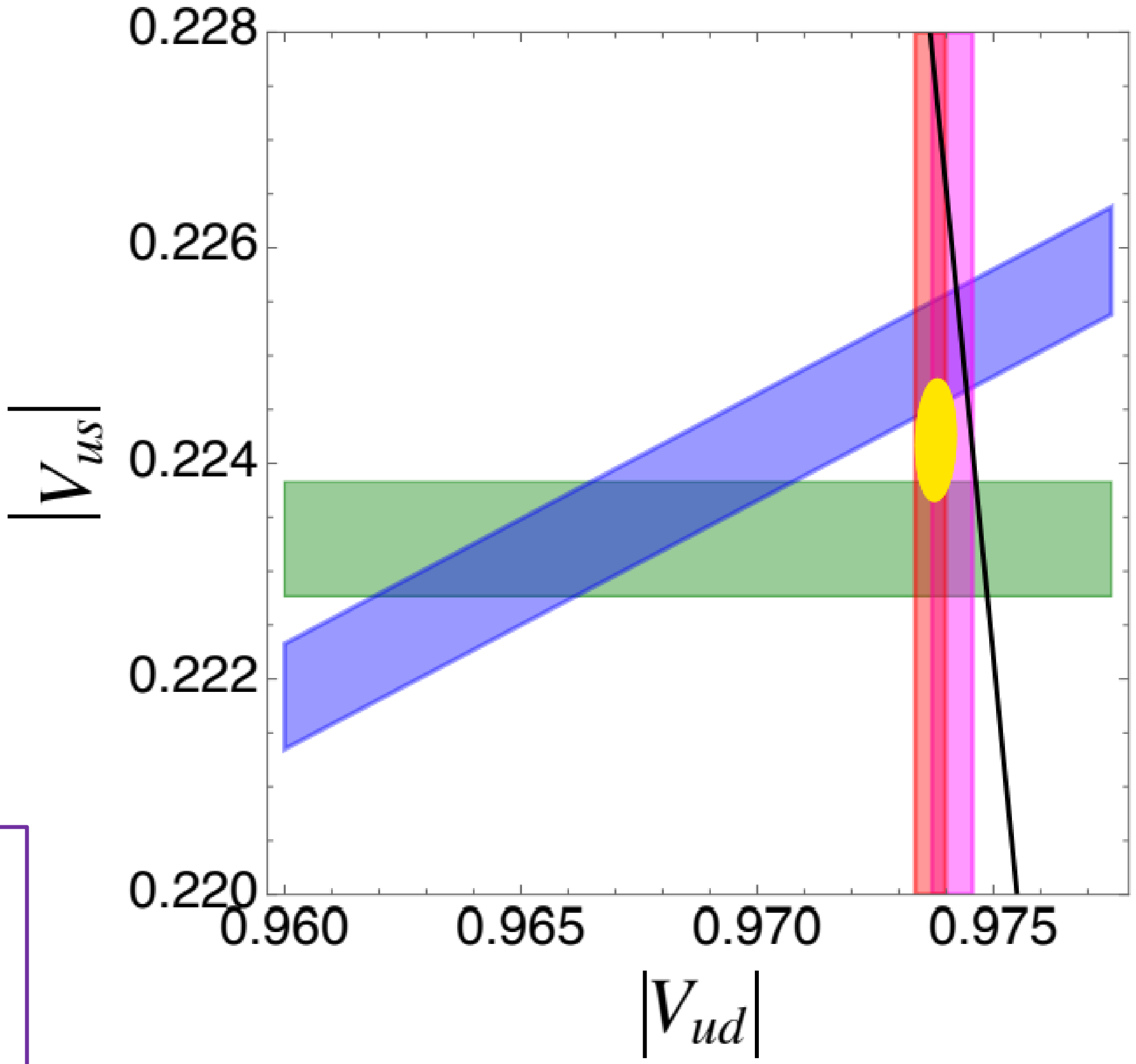
An anomaly with **2 aspects** (both 3σ):

1. Best-fit vs unitarity



2. Meson sector



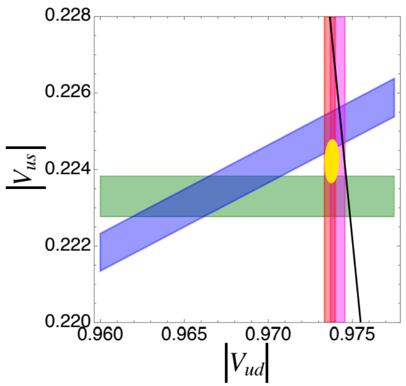


How do we obtain these constraints?

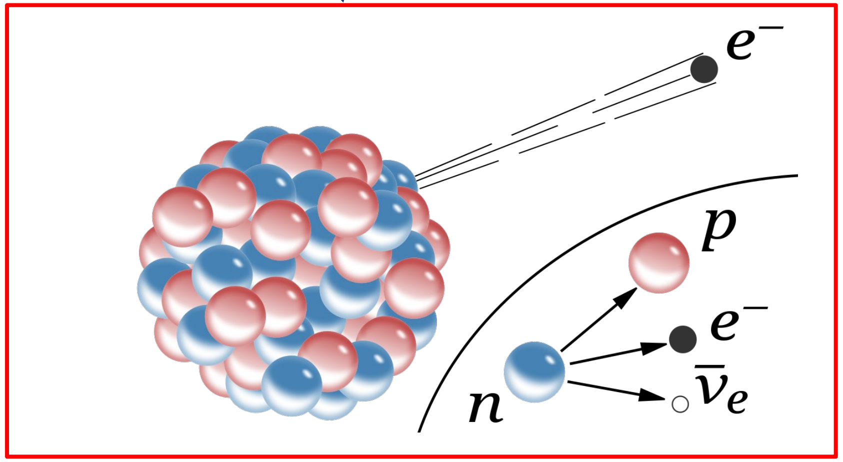
β decays → $|V_{\mathrm{ud}}|$

Hardy & Towner (2020), Seng et al. (2018), Seng et al. (2019), Czarnecki et al. (2019), Seng et al. (2020), Hayen (2021), Shiells et al. (2021)

Cirigliano et al. (2022)



$$|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2 + |V_{\mathrm{ub}}|^2 = 1$$

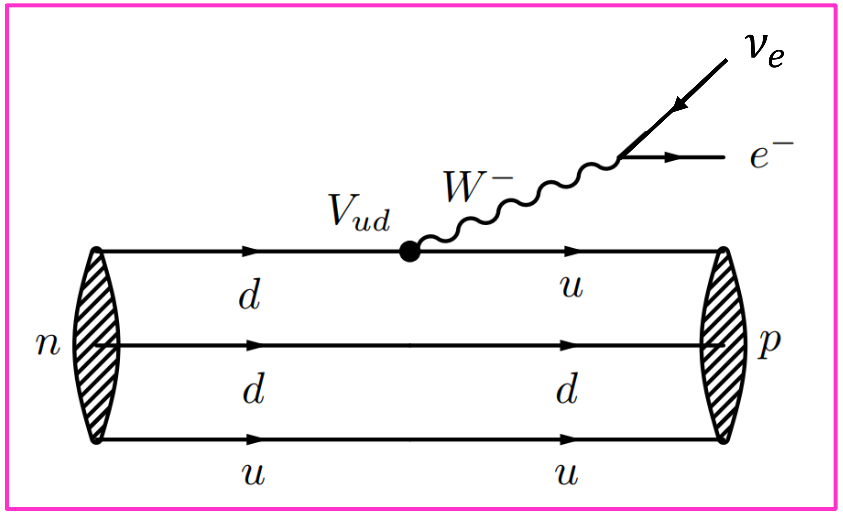


Superallowed nuclear β decays ($0^+ \to 0^+$)

- $\langle 0^+|V - A|0^+\rangle$: only V survives → no g_A uncertainty
- But: nuclear uncertainties

$$\left|V_{\mathrm{ud}}^{0^+\to 0^+}\right| = 0.97367[32]_{\mathrm{total}}$$

Uncorrelated (to good approximation)



Neutron β decay

- No nuclear uncertainties
- But: g_A/g_V and neutron lifetime uncertainties

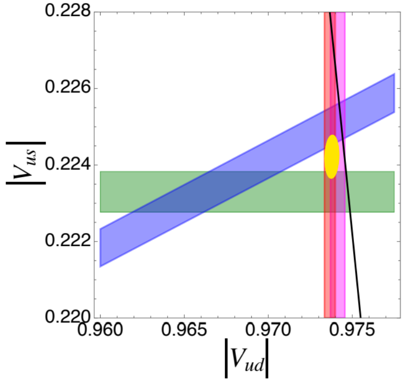
$$|V_{\mathrm{ud}}^{\mathrm{n}}| = 0.97413[43]_{\mathrm{total}}$$

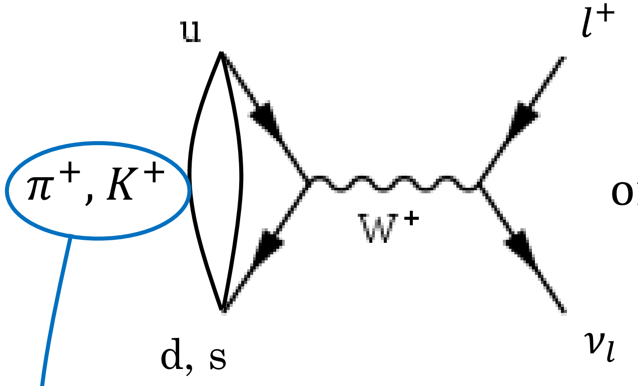
Note: I did not calculate these myself.

$K \rightarrow l\nu$ $\pi \rightarrow l\nu$

$K_{l2}/\pi_{l2} \rightarrow |V_{\rm us}|/|V_{\rm ud}|$

Cirigliano et al. (2022)





or $\pi^-, K^- \rightarrow l^- \, \overline{\nu_l}$

$\Gamma_{K_{l2}} \propto |V_{\rm us}|$ and $\Gamma_{\pi_{l2}} \propto |V_{\rm ud}|$, so we obtain $|V_{\rm us}|/|V_{\rm ud}|$:

My result (only for $l = \mu$):

$$\left.\frac{|V_{\rm us}|}{|V_{\rm ud}|}\right|_{K_{\mu 2}/\pi_{\mu 2}} = 0.23110[56]_{\rm total}$$

From where?

$\Gamma_{K_{l2}} \propto f_K$ and $\Gamma_{\pi_{l2}} \propto f_\pi$:

- biggest source of uncertainty: f_K/f_π

FLAG Review 2021 & ref. therein

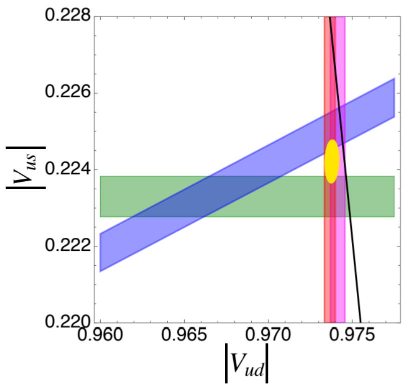
Also: isospin-breaking corrections, experiment.

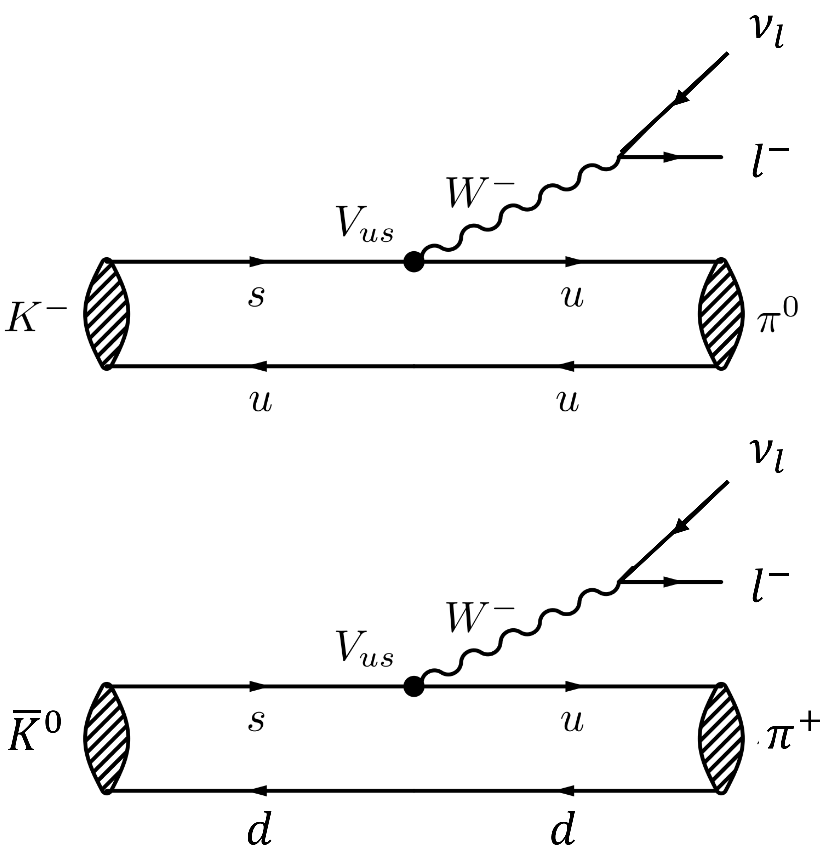
1904.08731v2, Di Carlo et al. (2019)

K_{l3} decays → $|V_{\mathrm{us}}|$

$$|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2 + |V_{\mathrm{ub}}|^2 = 1$$

Cirigliano et al. (2022)





or $K^+ \to \pi^0 l^+ \nu_l$

or $K^0 \to \pi^- l^+ \nu_l$

5 relevant decays:

K^+_{e3}, $K^+_{\mu3}$, $(K_{\mathrm{S}})_{e3}$, $(K_{\mathrm{L}})_{e3}$, $(K_{\mathrm{L}})_{\mu3}$

Combine in χ^2 analysis → my result:

$$|V_{\mathrm{us}}^{K_{l3}}| = 0.22275[54]_{\mathrm{total}}$$

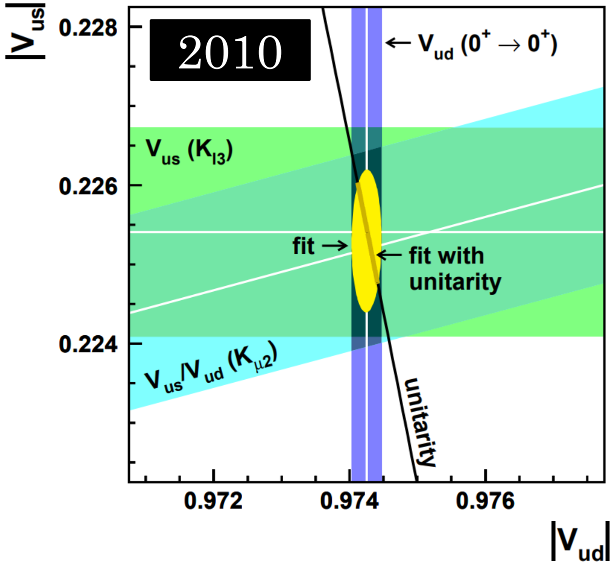
Inputs introducing uncertainties:

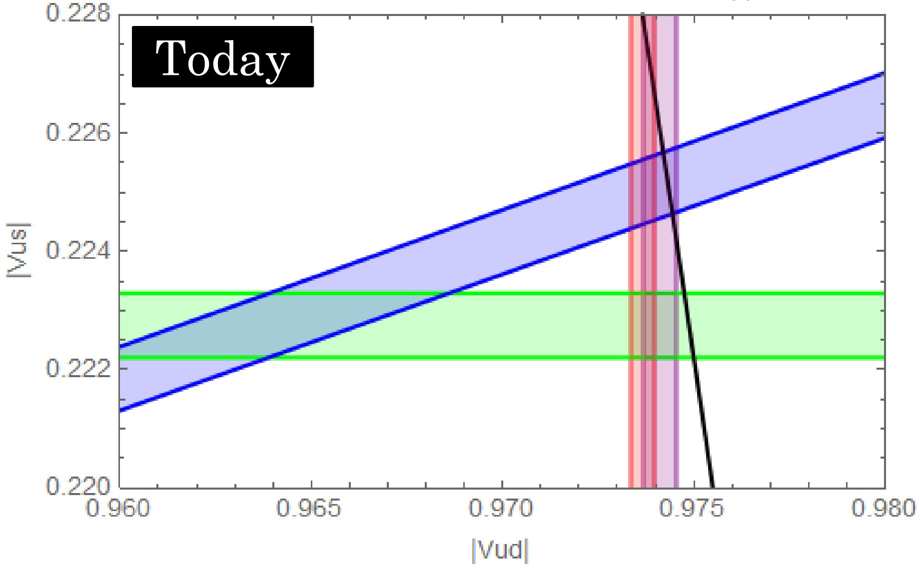
- Form factor parameters (mostly $f_+(0)$)

  FLAG Review 2021 & ref. therein
- Corrections: short-distance EW, radiative, isospin-breaking

  Seng et al., 2203.05217v2 (2022) & 2107.14708 (2021)
- Experimental parameters

Once upon a time in 2010… no anomaly!





Upper plot taken from Antonelli, Cirigliano et al. (2010).

What do we see?

1. $|V_{us}|$ shifted downwards
2. $|V_{us}|/|V_{ud}|$ became more narrow

} CKM anomaly

What changed?

1. $\Gamma_{K_{l3}} = |V_{us}|^2 \, (1 + \delta) \ldots$

Form factor parameters increased

Corrections (mostly) increased

2. $\frac{\Gamma_{K_{l2}}}{\Gamma_{\pi_{l2}}} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K}{f_\pi}(1 + \delta) \ldots$

Corrections increased

$\frac{f_K}{f_\pi}$ increased, uncertainty decreased

Part 2: the solution

SMEFT

- Standard **M**odel **E**ffective **F**ield **T**heory

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d\geq 5} \sum_{i=1}^{n_d} C_i^{(d)} Q_i^{(d)}$$

  - Degrees of freedom: same as SM
  - Symmetries: same as SM
  - EFT expansion in small $\delta \sim M_{EW}/\Lambda$

Wilson Coefficients

$$C_i^{(d)} \sim \Lambda^{4-d}$$

where Λ is the NP scale.

Local, gauge-invariant operators of dimension d, built out of SM fields.

A tower of EFTs

Typical energy
Λ
? TeV

?

EFT operators

$\mathcal{L}_{\text{SMEFT}}$ — RG flow

LHC physics

$M_{EW} \sim v \sim M_{W,Z,H,t}$
100 GeV

Integrate out heavy SM fields

$\mathcal{L}_{\text{LEFT}}$ — RG flow

B + D physics

$\Lambda_\chi \sim M_N$
1 GeV

Non-perturbative QCD regimes

Lattice QCD, chiral EFT

100 MeV - eV

Our observables: K, π, β decays

Low-energy EFT: "LEFT"

- EFT for E < $M_{EW} \sim v \sim M_{W,Z,H,t}$ → integrate out W & Z bosons, Higgs and top
- Effect on SM term relevant for K, π & β decays ($d_j \in \{d, s\}$):

$$-\frac{G_F}{\sqrt{2}} V_{\mathrm{ud_j}}\, \bar{l}\gamma^\mu(1-\gamma_5)\nu_l\, \underline{\bar{u}\gamma_\mu(1-\gamma_5)d_j} + \mathrm{h.c.}$$

LH (V-A)

- General $\mathcal{L}_{\mathrm{LEFT}}$ for K, π & β decays:

$$\begin{aligned}
\mathcal{L}_{\mathrm{LEFT}} = & -\frac{G_F}{\sqrt{2}} V_{ud_j} \times \Bigg[\left(1+\epsilon_L^{\ell j}\right) \bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \cdot \underline{\bar{u}\gamma^\mu(1-\gamma_5)d_j} \\
& + \epsilon_R^{\ell j}\ \bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \cdot \underline{\bar{u}\gamma^\mu(1+\gamma_5)d_j} \\
& + \epsilon_S^{\ell j}\ \bar{\ell}(1-\gamma_5)\nu_\ell \cdot \underline{\bar{u}d_j} \\
& - \epsilon_P^{\ell j}\ \bar{\ell}(1-\gamma_5)\nu_\ell \cdot \underline{\bar{u}\gamma_5 d_j} \\
& + \epsilon_T^{\ell j}\ \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \cdot \underline{\bar{u}\sigma^{\mu\nu}(1-\gamma_5)d_j} \Bigg] + \mathrm{h.c.}
\end{aligned}$$

LH (V-A)

RH (V+A)

S

PS

T

Can translate to SMEFT basis, e.g.:

$$V_{ud_j}\, \epsilon_R^{\ell j} = \frac{v^2}{2}\left[C_{Hud}\right]_{1j}$$

($d = 6$ operator)

Model 0: the SM

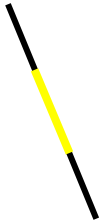
1. Don’t impose unitarity

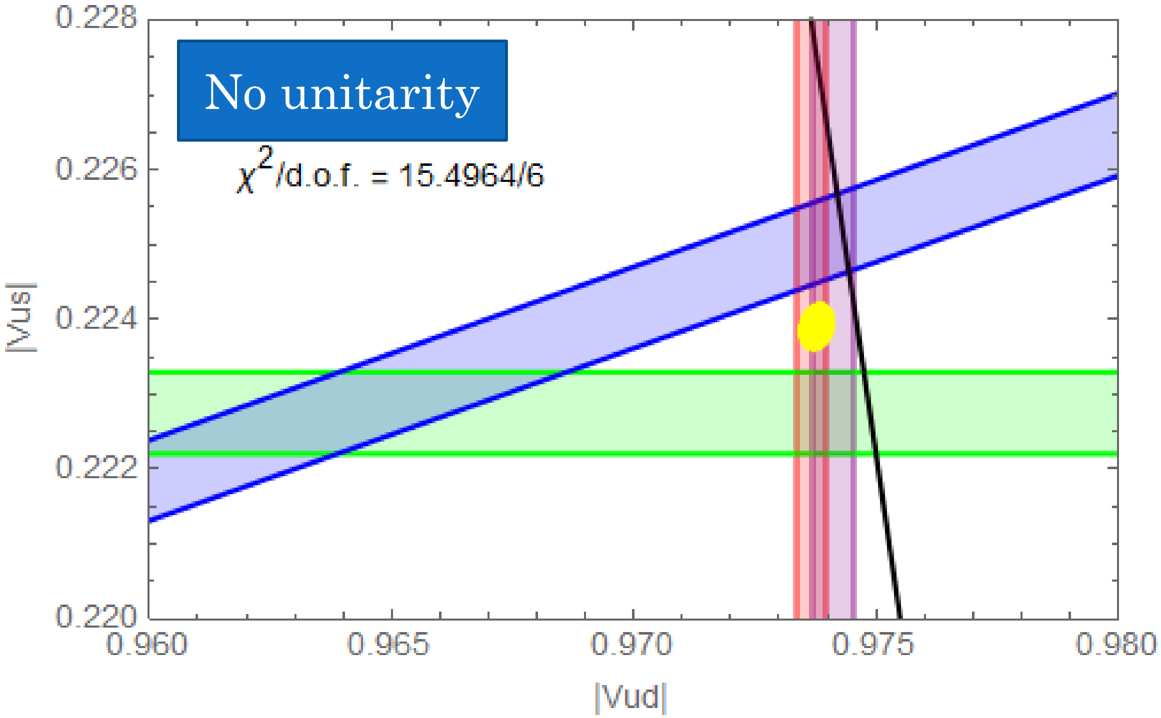
Fit parameters are $|V_{ud}|$ and $|V_{us}|$.

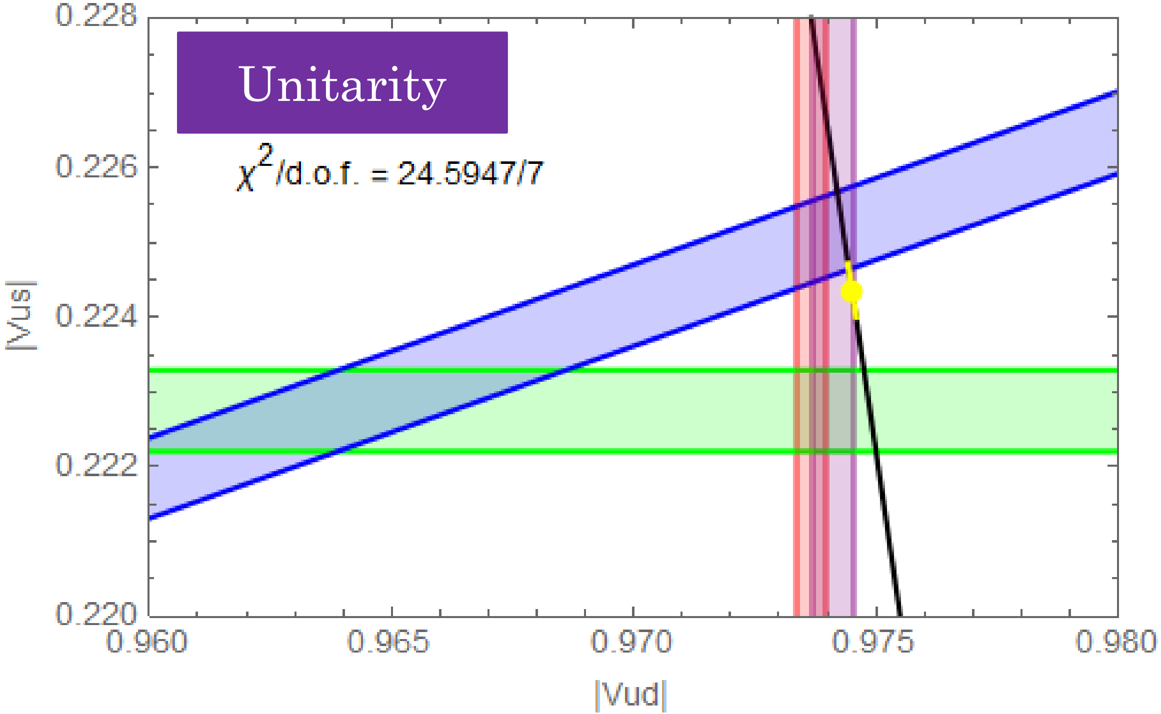
Plot the 2D 1σ ellipse

2. Impose unitarity

Fit parameter is $|V_{us}|$.

Plot the 1D 1σ range 





$$L^\mu \equiv \bar{l}\gamma^\mu(1-\gamma_5)\nu_l$$

Model 1: RH currents

ϵ_R & $\epsilon_R^{(s)}$ quantify RH currents in the non-strange & strange sector.

Simple effect: overall factor in decay rates!

$$V_{\mathrm{ud_j}}\left(L^\mu \bar{u}\gamma_\mu(1-\gamma_5)d_j + \epsilon_R^{(j)} L^\mu \bar{u}\gamma_\mu(1+\gamma_5)d_j\right)$$

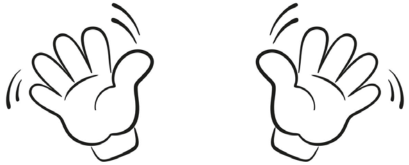
LH (V-A) RH (V+A)

→ $\left(1+\epsilon_R^{(j)}\right)$ for V currents: K_{l3} and β decays

→ $\left(1-\epsilon_R^{(j)}\right)$ for A currents: K_{l2} and π_{l2} decays

Run χ^2 analysis:

- Impose unitarity → 3 fit parameters: $|V_{\mathrm{us}}|$, ϵ_R & $\epsilon_R^{(s)}$
- Linearize all decay rates in ϵ to ensure EFT consistency



Results

$\chi^2/\mathrm{d.o.f.} = 6.5458/5$	$\epsilon_R = -0.00076[27]$
$\|V_{\mathrm{us}}\| = 0.22408[37]$	$\epsilon_R^{(s)} = -0.0059[17]$

What does this mean?

$$M_{W_R} \sim \sqrt{\frac{v^2}{\epsilon}}$$

so these ϵ give $M_{W_R} \sim 5-10$ TeV

$$L^\mu \equiv \bar{l}\gamma^\mu(1-\gamma_5)\nu_l$$

$$L \equiv \bar{l}(1-\gamma_5)\nu_l$$

Model 2: PS currents

ϵ_P & $\epsilon_P^{(s)}$ quantify PS currents in the non-strange & strange sector.

Again simply an overall factor in the decay rates: not all of them though…

$$V_{\mathrm{ud_j}}\left(L^\mu \bar{u}\gamma_\mu(1-\gamma_5)d_j + \epsilon_P^{(j)} L\bar{u}\gamma_5 d_j\right)$$

LH (V-A) — PS

→ 1 for V currents: K_{l3} and β decays

→ $\left(1-\epsilon_P^{(j)}\right)$ for A currents: K_{l2} and π_{l2} decays

The χ^2 is only sensitive to the difference
$$\Delta\epsilon_P = \epsilon_P^{(s)} - \epsilon_P,$$
because only the $K_{\mu 2}/\pi_{\mu 2}$ term is impacted & we linearize in all ϵ's.

Run χ^2 analysis:

- Impose unitarity → 2 fit parameters: $|V_{us}|$ & $\Delta\epsilon_P$
- Linearize all decay rates in ϵ to ensure EFT consistency

Results

$\chi^2/\mathrm{d.o.f.} = 19.0267/6$

$\Delta\epsilon_P = -0.0074[34]$

$|V_{us}| = 0.22308[30]$

Model 3: a 4th sterile neutrino

WIP

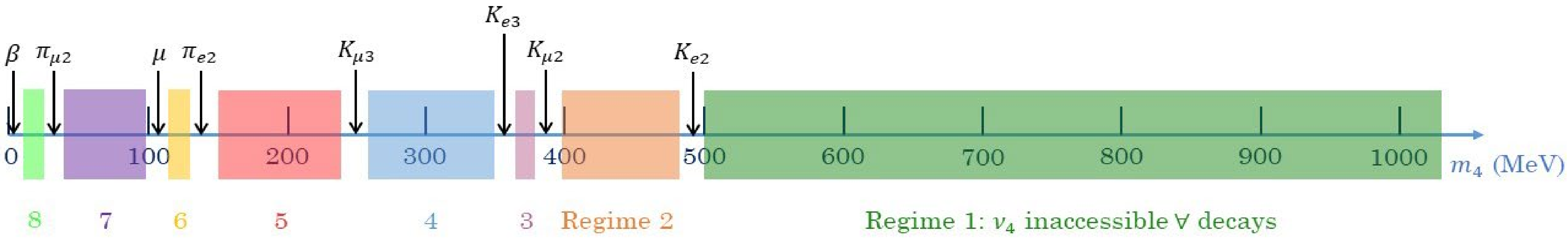
- Add d.o.f. to the SM: right-handed neutrino ν_R
- My model: $\mathcal{L}_{SM} + \underline{\bar{L}\tilde{H}y_\nu \nu_R} + \underline{\overline{\nu_R^C} M_R \nu_R}$ where $L = (\nu_L, e_L)^T$

Dirac mass term | Majorana mass term

- Rotate to mass basis: $\nu_l = U_{li}\nu_i$ where now $i \in \{1,2,3,4\}$

→ $m_{1,2,3} \approx 0$ and m_4 is free (within the scope of this project…)

- Size of m_4 w.r.t Q-values of decays determine kinematic accessibility of ν_4:



- Effect of tuning m_4? Different effect on K_{l3} and K_{l2}/π_{l2}, or on $l = e$ or μ final states?

Cirigliano, Dekens, De Vries, Fuyuto, Mereghetti & Ruiz, *Leptonic anomalous magnetic moments in νSMEFT* (2021).

WIP

Model 3: a 4th sterile neutrino

Regime 1: the simplest

- E.g. $m_4 \sim 1$ GeV
- All decay rates modified by overall factor: e.g. $K^+ \to \pi^0 e^+ \nu_e$,

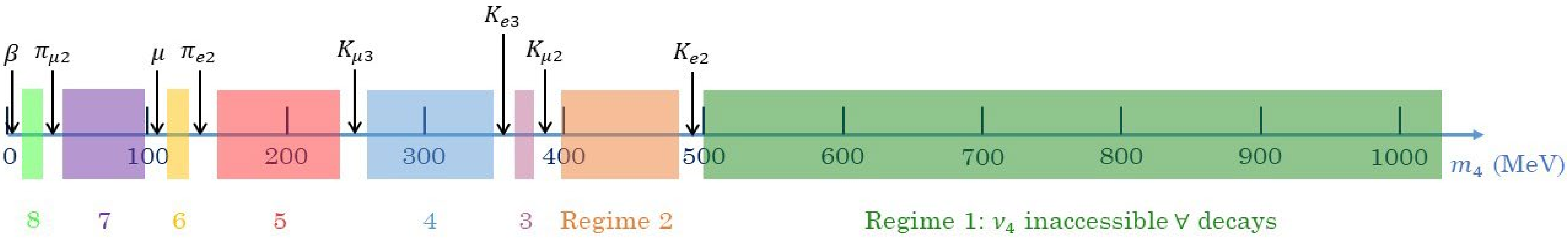
SM decay rate

$$\nu_e = U_{ei}\nu_i \text{ so } \Gamma(K_{e3}^+) \sim \sum_{i=1}^{3} |M(m_i)|^2 |U_{ei}|^2 \sim |M(0)|^2 (1 - |U_{e4}|^2)$$

- Also, $\Gamma \sim G_F^2$ and G_F from $\mu \to e\nu_e\nu_\mu$: $G_F^{(\mu)} = G_F^{(0)}(1 - \frac{1}{2}|U_{e4}|^2 - \frac{1}{2}|U_{\mu 4}|^2)$

$\Gamma(K_{e3}^+) \sim |V_{us}|^2 (1 + |U_{\mu 4}|^2)$

- But: this changes $|V_{ud}|^2 + |V_{us}|^2$ from 1 (SM) to > 1 → makes CKM anomaly worse
- Result: fit sets $|U_{e4}|$ and $|U_{\mu 4}|$ to 0



Model 3: a 4th sterile neutrino

WIP

What happens in the lower regimes? Suppose m_4 accessible in K_{e3}^+:

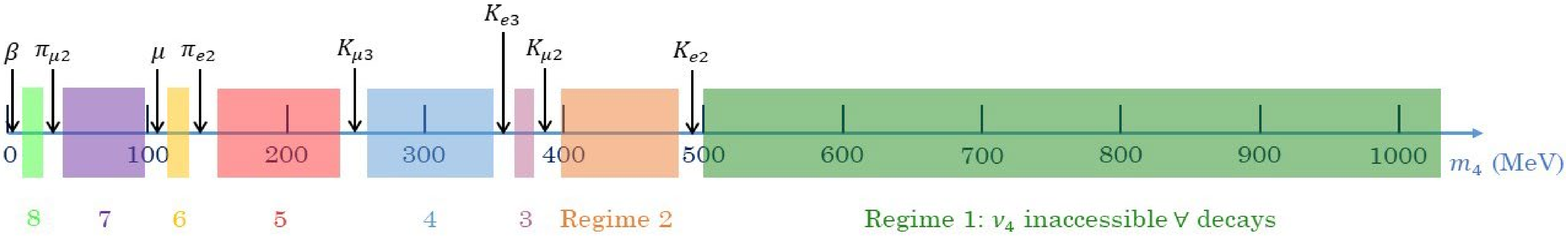
$$\Gamma(K_{e3}^+) \sim \sum_{i=1}^{4} G_F^2 |M(m_i)|^2 |U_{ei}|^2 \sim |M(0)|^2 \,\cancel{(1-|U_{e4}|^2)}\, + |M(m_4)|^2 |U_{e4}|^2$$

$$(1+|U_{\mu 4}|^2)$$

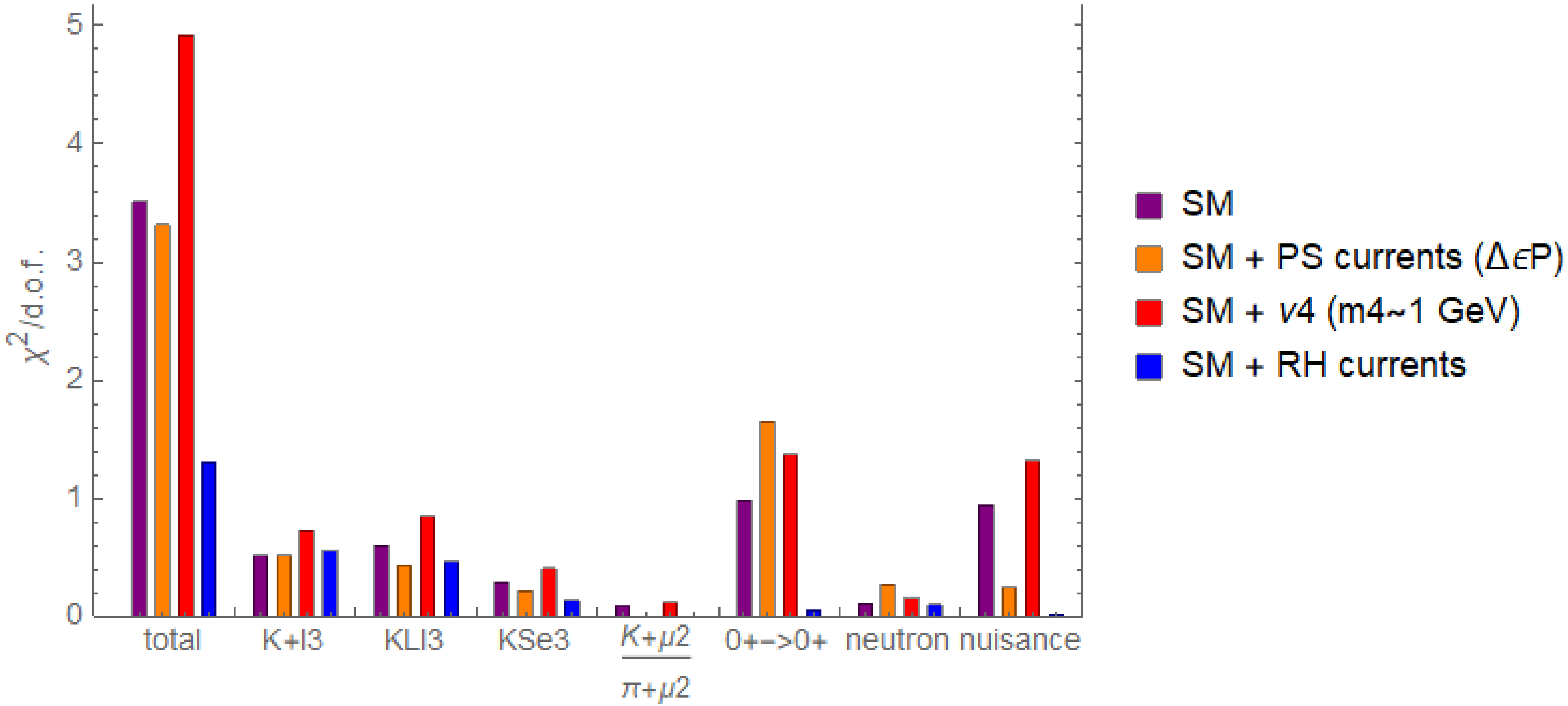
So: other terms besides those that increase $|V_{\mathrm{ud}}|$ and $|V_{\mathrm{us}}|$.

Speculation: the lower m_4, the more of these other terms…

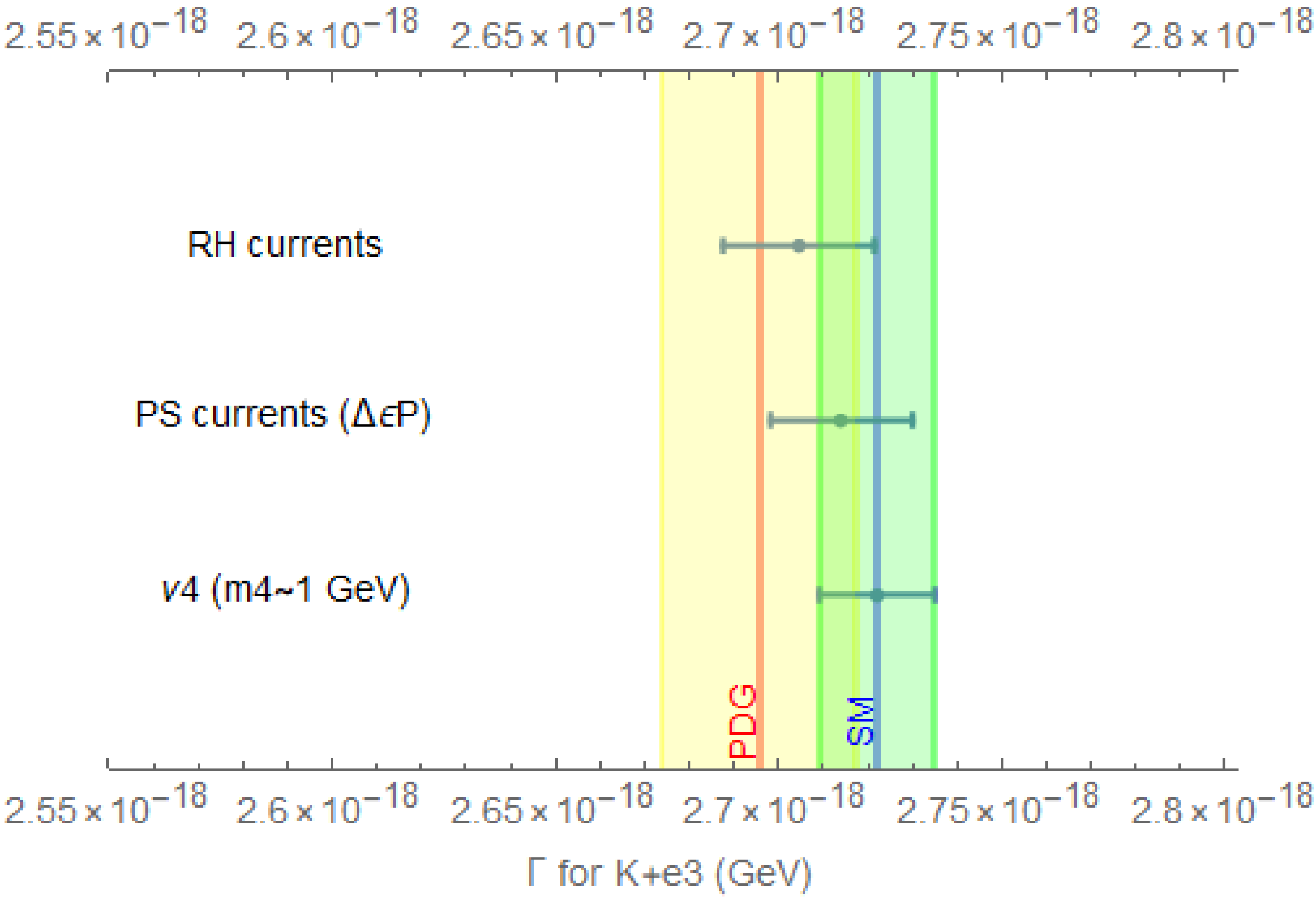
→ bigger chance of alleviating the CKM anomaly at lower m_4?



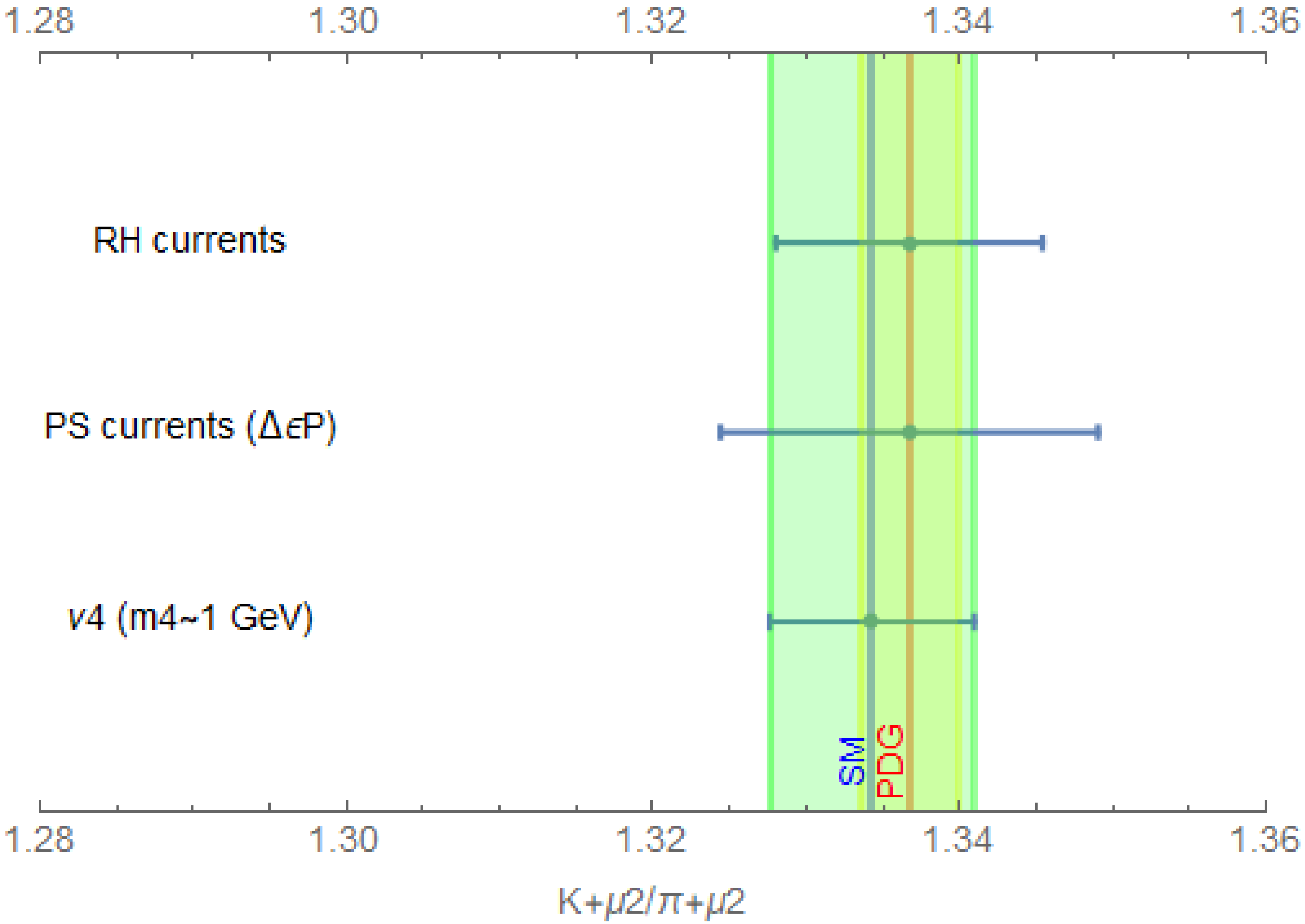
Compare χ^2/d.o.f. in the 3 NP models



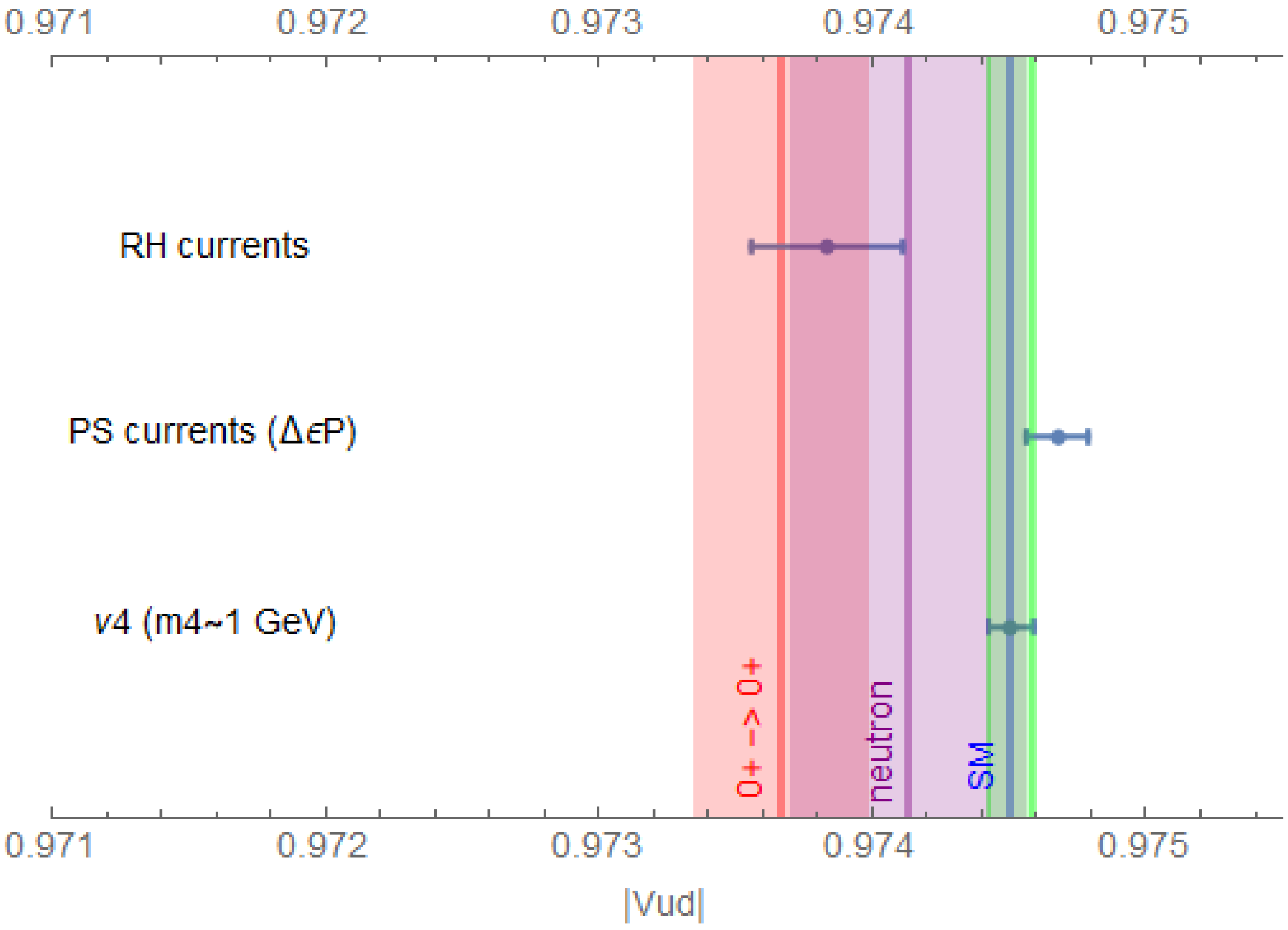
Compare $(K^+)_{e3}$ in the 3 NP models



Compare $K_{\mu2}/\pi_{\mu2}$ in the 3 NP models



Compare $|V_{\mathrm{ud}}|$ in the 3 NP models



In closing

Interesting to play around with NP & CKM anomaly.

RH model helps the anomaly, PS & Regime 1 ν_4 don't.

To-do:
- Work out lighter sterile neutrino regimes

Thoughts:
- What full NP models could generate these low-E effects?
- How do these models relate to the rest of the SM & other anomalies?
- What other NP models could be interesting?

Thank you for listening!
Any questions?

Backup slides

Determination of $|V_{\mathrm{ud}}|$

- Full result from superallowed nuclear β decays ($0^+ \to 0^+$):

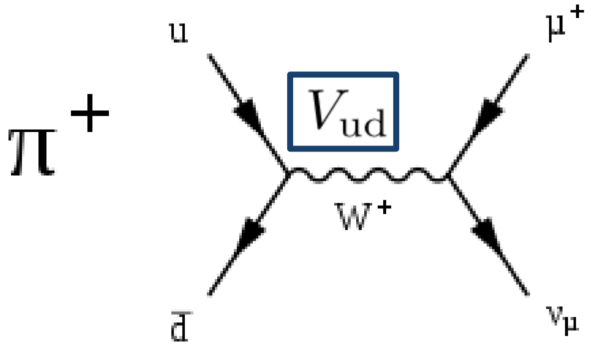
$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\mathrm{exp}}(13)_{\Delta_V^R}(27)_{\mathrm{NS}}[32]_{\mathrm{total}}$$

- Full result from neutron β decay:

$$V_{ud}^{\mathrm{n,\,PDG}} = 0.97441(3)_f(13)_{\Delta_R}(82)_\lambda(28)_{\tau_n}[88]_{\mathrm{total}}$$

$$V_{ud}^{\mathrm{n,\,best}} = 0.97413(3)_f(13)_{\Delta_R}(35)_\lambda(20)_{\tau_n}[43]_{\mathrm{total}}$$

- Other possibilities:
  - Hadronic τ decays
  - Pion decay: $\pi^+ \to l^+\nu_l$ or $\pi^0 \to \pi^- l^+ \nu_l$



Both of these currently have larger uncertainty than β decays.

Determination of $|V_{\text{us}}|$

- Full result from K_{l3} decays (Cirigliano et al. (2022)):

$$V_{us}^{K_{\ell 3}} = 0.22330(35)_{\text{exp}}(39)_{f_+}(8)_{\text{IB}}[53]_{\text{total}}$$

- Other possibilities:
  - Hadronic τ decays
  - Hyperon decays $\Lambda \to p\, e\, \bar{\nu}$ (as for neutron β decay: both V and A currents contribute)

  These are currently not competitive with kaon decays.

- Also: full result from K_{l2}/π_{l2} decay ratio (Cirigliano et al. (2022)):

$$\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}} = 0.23108(23)_{\text{exp}}(42)_{F_K/F_\pi}(16)_{\text{IB}}[51]_{\text{total}}$$

Parameter values for P_{l2} decays

Parameter	Decay / Particle	2010 value [5]	2023 value
f_K/f_π	K/π	1.193 ± 0.006	1.1978 ± 0.0022 [6]
$\delta_{\mathrm{EM}} + \delta_{\mathrm{IB}}$	K/π	-0.0070 ± 0.0018	-0.0127 ± 0.0021 [8] (eq. 2 & 3)
B.R.	$K_{\mu 2}$	0.6347 ± 0.0018	0.6356 ± 0.0011 [2]
τ	K	$(1.2384 \pm 0.015) \times 10^{-8}$ s	$(1.2380 \pm 0.0020) \times 10^{-8}$ s [2]
Partial Γ	$\pi_{\mu 2}$	$(38.408 \pm 0.007) \times 10^{6}$ s^{-1}	$(38.408 \pm 0.007) \times 10^{6}$ s^{-1} [2]

Table 7: Values of the decay constant ratio, corrections and branching ratios + lifetimes (for $K_{\mu 2}$) and partial decay rates (for $\pi_{\mu 2}$) as used in the 2010 and 2023 analyses. Note: [5] only lists an overall correction to the K/π ratio, while [8] lists an overall correction for the K and π decay rates separately. I have calculated the correction to the ratio from the values listed in eq. (2) and (3) of [8] using standard error propagation. The reason I put the partial decay rate for $\pi_{\mu 2}$ is that [5] only lists that. I calculated the partial decay rate for 2023 from the B.R. and τ in [2], again using standard error propagation.

Plots of $|V_{us}|f_+(0)$: 2010 vs today

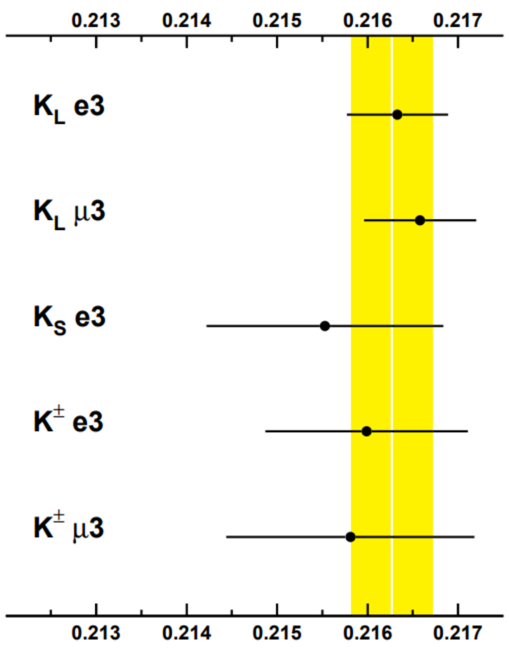
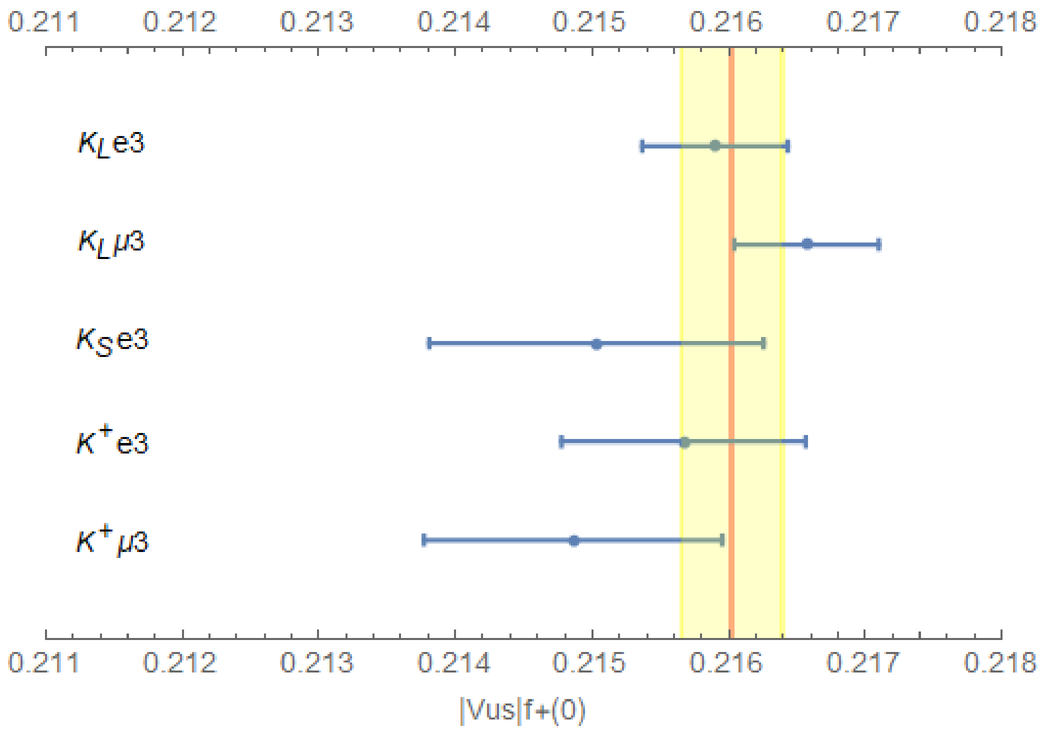


Fig. 9. Comparison of values for $|V_{us}|f_+(0)$ for all channels. Our average is indicated by the yellow band.

2010 (Antonelli et al.)



Today

Parameter values for K_{l3} decays (1)

Form factors: $f_i(q^2) = f_+(0)\left(1 + \lambda_i \frac{q^2}{m_\pi^2}\right)$ (where i = +, 0)

Parameter	2010 value [5]	2023 value
$f_+(0)$	0.959(5) (eq. (17))	0.9698(17) ([6])
λ_+ [3]	For $K^\pm$: 2.566×10^{-2} For K_L and K_S: 2.566×10^{-2}	For $K^\pm$: $(2.959\pm0.025) \times 10^{-2}$ ([2]) For K_L (and K_S): $(2.82\pm0.04) \times 10^{-2}$ ([2])
λ_0	For $K^\pm$: 1.29754×10^{-2} For K_L and K_S: 1.37131×10^{-2}	For $K^\pm$: $(1.76\pm0.25) \times 10^{-2}$ ([2]) For K_L (and K_S): $(1.38\pm0.18) \times 10^{-2}$ ([2])

Parameter	Decay / Particle	2010 value [5]	2023 value [2]
B. R.	$K^\pm_{e3}$	(5.078 ± 0.031) %	(5.07 ± 0.04) %
	$K^\pm_{\mu3}$	(3.359 ± 0.032) %	(3.352 ± 0.033) %
	$(K_S)_{e3}$	$(7.05 \pm 0.08) \times 10^{-4}$	$(7.04 \pm 0.08) \times 10^{-4}$
	$(K_L)_{e3}$	(40.56 ± 0.09) %	(40.55 ± 0.11) %
	$(K_L)_{\mu3}$	(27.04 ± 0.10) %	(27.04 ± 0.07) %
τ	$K^\pm$	$(1.2384 \pm 0.0015) \times 10^{-8}$ s	$(1.2380 \pm 0.0020) \times 10^{-8}$ s
	K_S	$(0.8959 \pm 0.0006) \times 10^{-10}$ s	$(0.8954 \pm 0.0004) \times 10^{-10}$ s
	K_L	$(5.116 \pm 0.021) \times 10^{-8}$ s	$(5.116 \pm 0.021) \times 10^{-8}$ s

Parameter values for K_{l3} decays (2)

Parameter	Decay	2010 value [5]	2023 value
S_{EW}	all	1.0232(3)	1.0232(3) [3]
δ_{EM}	$K^{\pm}_{e3}$	$(1 \pm 2.5) \times 10^{-3}$	$(2.1 \pm 0.5) \times 10^{-3}$ [4] [3]
	$K^{\pm}_{\mu3}$	$(0.16 \pm 2.5) \times 10^{-3}$	$(0.5 \pm 0.5) \times 10^{-3}$ [3]
	$(K_S)_{e3}$	$(9.9 \pm 2.2) \times 10^{-3}$	$(11.6 \pm 0.3) \times 10^{-3}$ [3]
	$(K_L)_{e3}$	$(9.9 \pm 2.2) \times 10^{-3}$	$(11.6 \pm 0.3) \times 10^{-3}$ [3]
	$(K_L)_{\mu3}$	$(14 \pm 2.2) \times 10^{-3}$	$(15.4 \pm 0.4) \times 10^{-3}$ [3]
δ_{IB}	$K^{\pm}_{e3}$	0.058 ± 0.008	0.0457 ± 0.0020 [7] [5]
	$K^{\pm}_{\mu3}$	0.058 ± 0.008	0.0457 ± 0.0020 [7]
	$(K_S)_{e3}$	0	0
	$(K_L)_{e3}$	0	0
	$(K_L)_{\mu3}$	0	0

Correlation matrices for K_{l3} decays

From [3]: $\delta_{\text{EM}} = \begin{pmatrix} \delta_{\text{EM}}^{K^0 e} & \delta_{\text{EM}}^{K^+ e} & \delta_{\text{EM}}^{K^0 \mu} & \delta_{\text{EM}}^{K^+ \mu} \end{pmatrix}^T$

$$\text{Corr}(\delta_{\text{EM}}) = \begin{pmatrix} 1 & -0.050 & 0.069 & -0.043 \\ & 1 & -0.049 & 0.079 \\ & & 1 & 0.088 \\ & & & 1 \end{pmatrix}$$

From [7]: $K_+^{\text{exp}} = \begin{pmatrix} \text{BR}(K^+ e) & \text{BR}(K^+ \mu) & \Gamma_{K^+} \end{pmatrix}^T$ $\quad K_L^{\text{exp}} = \begin{pmatrix} \text{BR}(K_L e) & \text{BR}(K_L \mu) & \Gamma_{K_L} \end{pmatrix}^T$

$$\text{Corr}(K_+^{\text{exp}}) = \begin{pmatrix} 1 & 0.8959847 & 0.01425396 \\ & 1 & 0.01376368 \\ & & 1 \end{pmatrix}$$

$$\text{Corr}(K_L^{\text{exp}}) = \begin{pmatrix} 1 & -0.2149932 & -0.2676291 \\ & 1 & -0.08759115 \\ & & 1 \end{pmatrix}$$

References for parameter values

[2] R. L. Workman and Others, “Review of Particle Physics,” PTEP, vol. 2022, p. 083C01, 2022. doi: 10.1093/ptep/ptac097.

[3] C.-Y. Seng, D. Galviz, M. Gorchtein, and U.-G. Meißner, “Complete theory of radiative corrections to Kℓ3 decays and the Vus update,” Journal of High Energy Physics, vol. 2022, no. 7, 2022. doi: 10.1007/jhep07(2022)071. [Online]. Available: https://arxiv.org/pdf/ 2203.05217.pdf.

[5] M. Antonelli et al., “An evaluation of |Vus| and precise tests of the Standard Model from world data on leptonic and semileptonic kaon decays,” The European Physical Journal C, vol. 69, no. 3-4, pp. 399–424, 2010. doi: 10.1140/epjc/s10052- 010- 1406- 3. [Online]. Available: https://arxiv.org/pdf/1005.2323v2.pdf.

[6] V. Cirigliano, A. Crivellin, M. Hoferichter, and M. Moulson, Scrutinizing CKM unitarity with a new measurement of the Kμ3/Kμ2 branching fraction, 2022. doi: 10.48550/ARXIV.2208. 11707. [Online]. Available: https://arxiv.org/abs/2208.11707.

[7] C.-Y. Seng, D. Galviz, W. J. Marciano, and U.-G. Meißner, “Update on |Vus| and |Vus/Vud| from semileptonic kaon and pion decays,” Phys. Rev. D, vol. 105, no. 1, p. 013 005, 2022. doi: 10.1103/PhysRevD.105.013005. arXiv: 2107.14708 [hep-ph].

[8] M. D. Carlo et al., “Light-meson leptonic decay rates in lattice QCD+QED,” Physical Review D, vol. 100, no. 3, 2019. doi: 10.1103/physrevd.100.034514. [Online]. Available: https: //arxiv.org/abs/1904.08731.

SMEFT operators

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex	
$[O_{H\ell}]_{IJ}$	$i\bar{\ell}_I \bar{\sigma}_\mu \ell_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \sigma^i \bar{\sigma}_\mu \ell_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
$[O_{He}]_{IJ}$	$ie^c_I \sigma_\mu \bar{e}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hq}]_{IJ}$	$i\bar{q}_I \bar{\sigma}_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar{q}_I \sigma^i \bar{\sigma}_\mu q_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
$[O_{Hu}]_{IJ}$	$iu^c_I \sigma_\mu \bar{u}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hd}]_{IJ}$	$id^c_I \sigma_\mu \bar{d}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hud}]_{IJ}$	$iu^c_I \sigma_\mu \bar{d}^c_J \tilde{H}^\dagger D_\mu H$

Dipole	
$[O^\dagger_{eW}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \sigma^i \ell_J W^i_{\mu\nu}$
$[O^\dagger_{eB}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \ell_J B_{\mu\nu}$
$[O^\dagger_{uG}]_{IJ}$	$u^c_I \sigma_{\mu\nu} T^a \widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{uW}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{uB}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O^\dagger_{dG}]_{IJ}$	$d^c_I \sigma_{\mu\nu} T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{dW}]_{IJ}$	$d^c_I \sigma_{\mu\nu} \bar{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{dB}]_{IJ}$	$d^c_I \sigma_{\mu\nu} H^\dagger q_J\, B_{\mu\nu}$

Table 4.2: Two-fermion D=6 operators in the Warsaw basis. The flavor indices are denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operators) the corresponding complex conjugate operator is implicitly included.

$\Delta_{CKM}^{(i)}$ as in Cirigliano et al. (2022)

Definition:

$$\Delta_{\rm CKM}^{(1)} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1,$$
$$\Delta_{\rm CKM}^{(2)} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 2}/\pi_{\ell 2},\beta}\right|^2 - 1,$$
$$\Delta_{\rm CKM}^{(3)} = \left|V_{ud}^{K_{\ell 2}/\pi_{\ell 2},\, K_{\ell 3}}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1,$$

RH currents:

$$\Delta_{\rm CKM}^{(1)} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2,$$
$$\Delta_{\rm CKM}^{(2)} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2,$$
$$\Delta_{\rm CKM}^{(3)} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2).$$

PS currents: $\Delta\epsilon_P = \epsilon_P^{(s)} - \epsilon_P$

$$\Delta_{\rm CKM}^{(1)} = 0,$$
$$\Delta_{\rm CKM}^{(2)} = -2\Delta\epsilon_P V_{us}^2,$$
$$\Delta_{\rm CKM}^{(3)} = 2\Delta\epsilon_P (1 - V_{us}^2),$$

Sketch of the χ^2 analysis

Structure:

$$\chi^2 = \sum_{i=1}^{8+10} \chi_i^2$$

- 5 terms from 5 K_{l3} decay rates
- 1 term from $K_{\mu 2}/\pi_{\mu 2}$ ratio
- 2 terms for $|V_{\mathrm{ud}}|$ from $0^+ \rightarrow 0^+$ and neutron β decay
- 10 terms for the nuisance parameters

- Assume Gaussian errors
- For K_{l3}: include correlations between δ_{EM}, B.R. & τ

Outlook:
include more
correlations?

$L^\mu \equiv \bar{l}\gamma^\mu(1-\gamma_5)\nu_l$

$L \equiv \bar{l}(1-\gamma_5)\nu_l$

Model 2: PS currents

ϵ_P & $\epsilon_P^{(s)}$ quantify PS currents in the non-strange & strange sector.

Again simply an overall factor in the decay rates: not all of them though…

$$V_{\mathrm{ud_j}}\left(L^\mu \bar{u}\gamma_\mu(1-\gamma_5)d_j + \epsilon_P^{(j)} L\,\bar{u}\gamma_5 d_j\right)$$

LH (V-A) | PS

→ 1 for V currents: K_{l3} and β decays

→ $\left(1-\epsilon_P^{(j)}\right)$ for A currents: K_{l2} and π_{l2} decays

Run χ^2 analysis:

- Impose unitarity → 2 fit parameters: $|V_{\mathrm{us}}|$ & $\Delta\epsilon_P$
- Linearize all decay rates in ϵ to ensure EFT consistency

The χ^2 is only sensitive to the difference

$$\Delta\epsilon_P = \epsilon_P^{(s)} - \epsilon_P,$$

because only the $K_{\mu 2}/\pi_{\mu 2}$ term is impacted & we linearize in all ϵ's.

Results

χ^2/d.o.f. = 19.0267/6

$\Delta\epsilon_P = -0.0074[34]$

$|V_{\mathrm{us}}| = 0.22308[30]$

Outlook: back to ϵ_P & $\epsilon_P^{(s)}$, add observables to χ^2 to constrain them?