Transverse momentum-dependent partons from lattice QCD

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Seattle, USA

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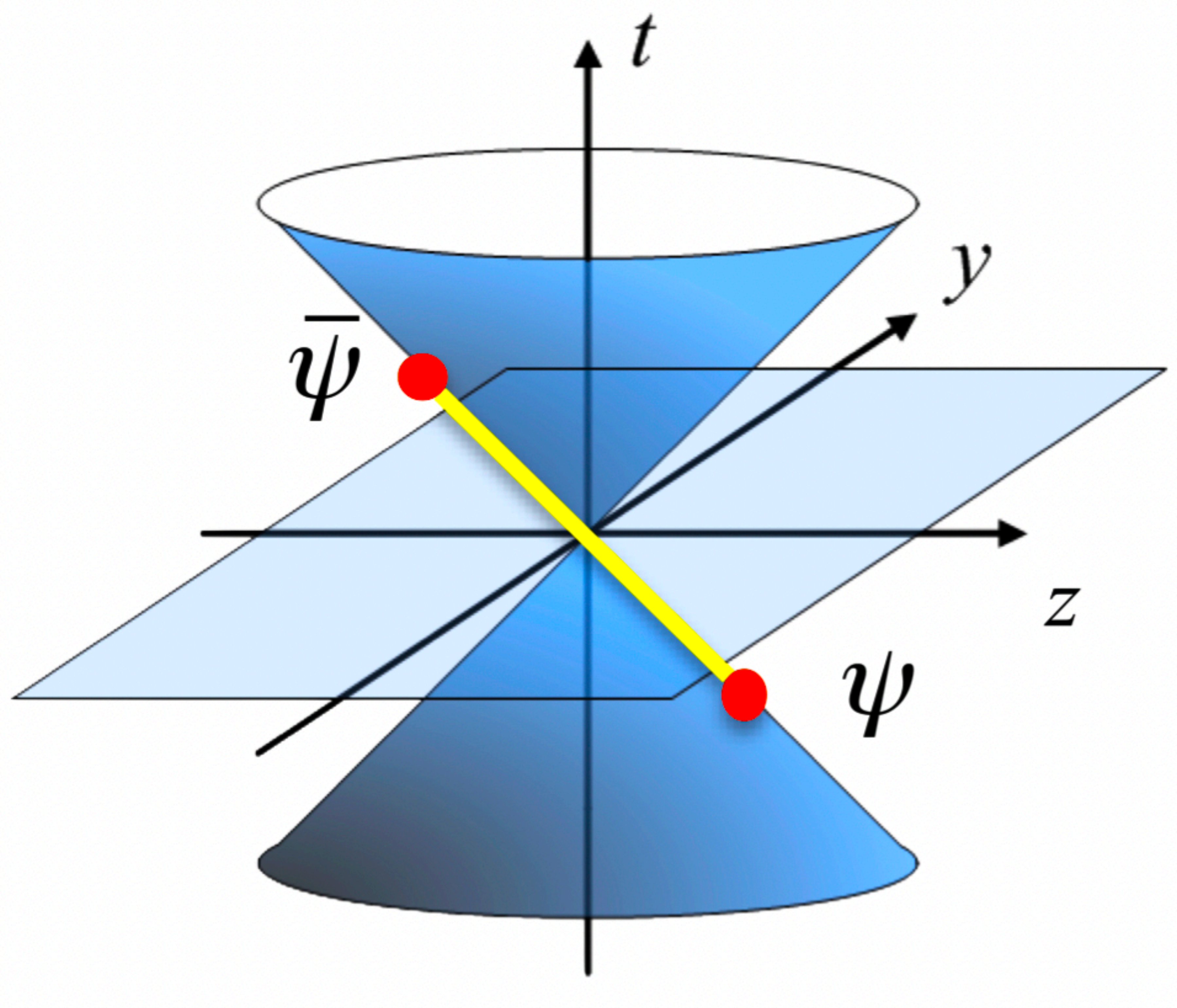
partonic image of hadron

regularize QCD after taking the lightcone, $P_z \to \infty$ / $z^2 \to 0$, limit

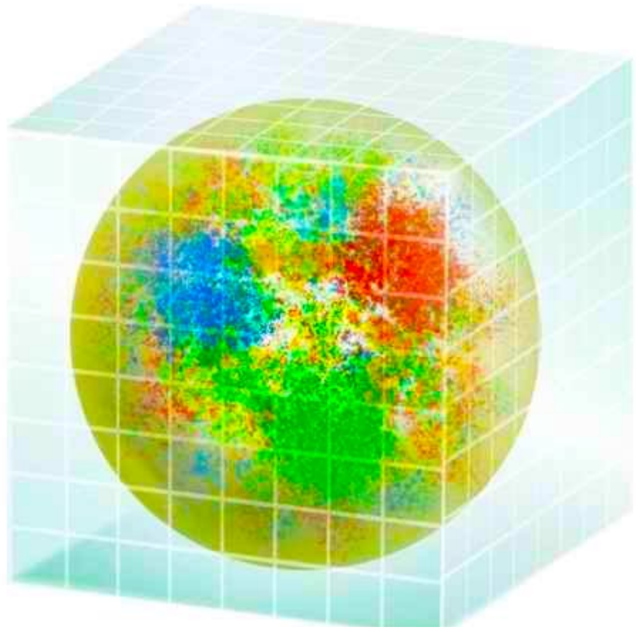


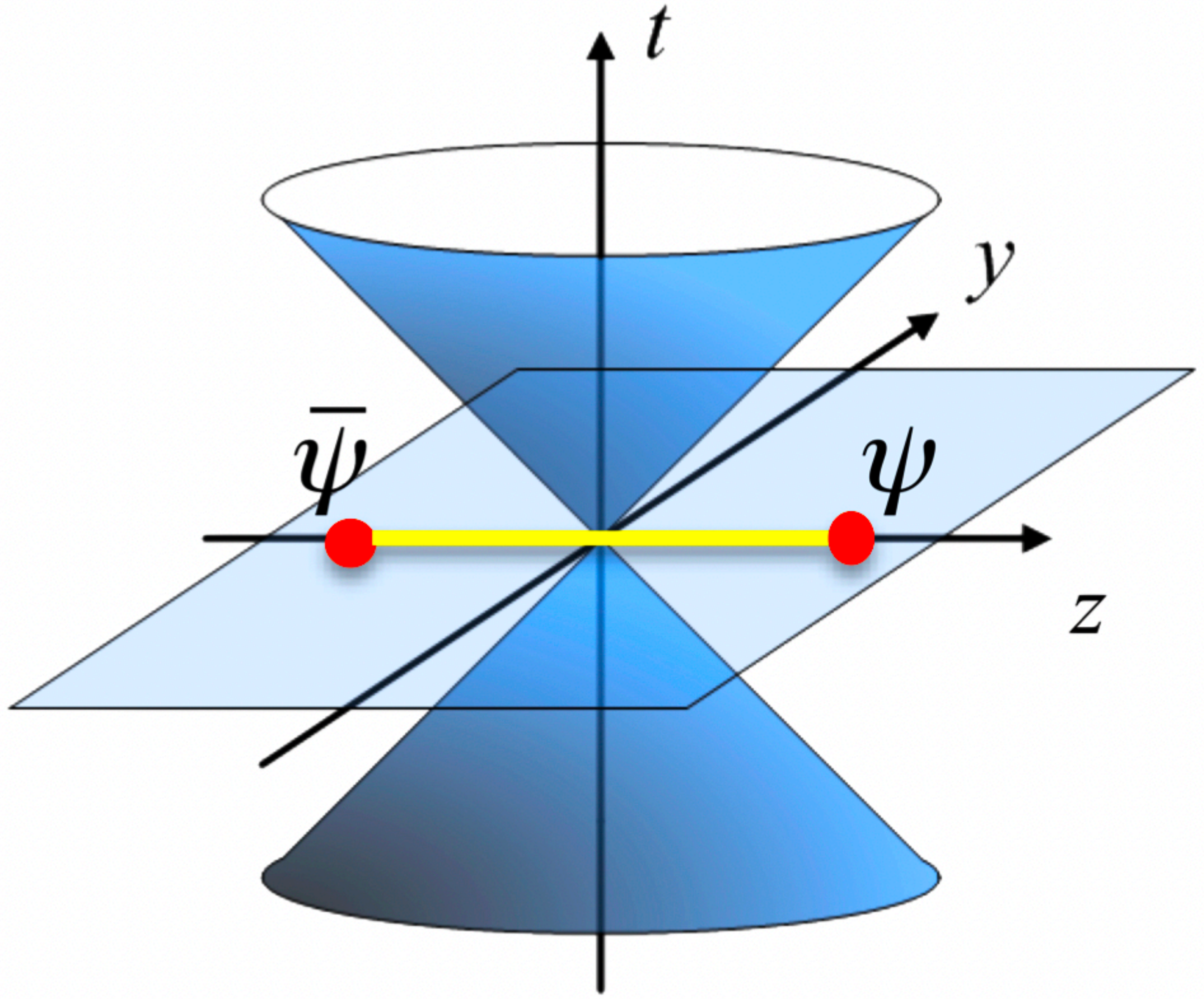
$$f(x,\mu) \sim \left\langle H(P_z) \,|\, \hat{O}(z^-,\mu) \,|\, H(P_z) \right\rangle$$

timelike separated bilocal operator

partonic structures from lattice QCD

hadron at rest



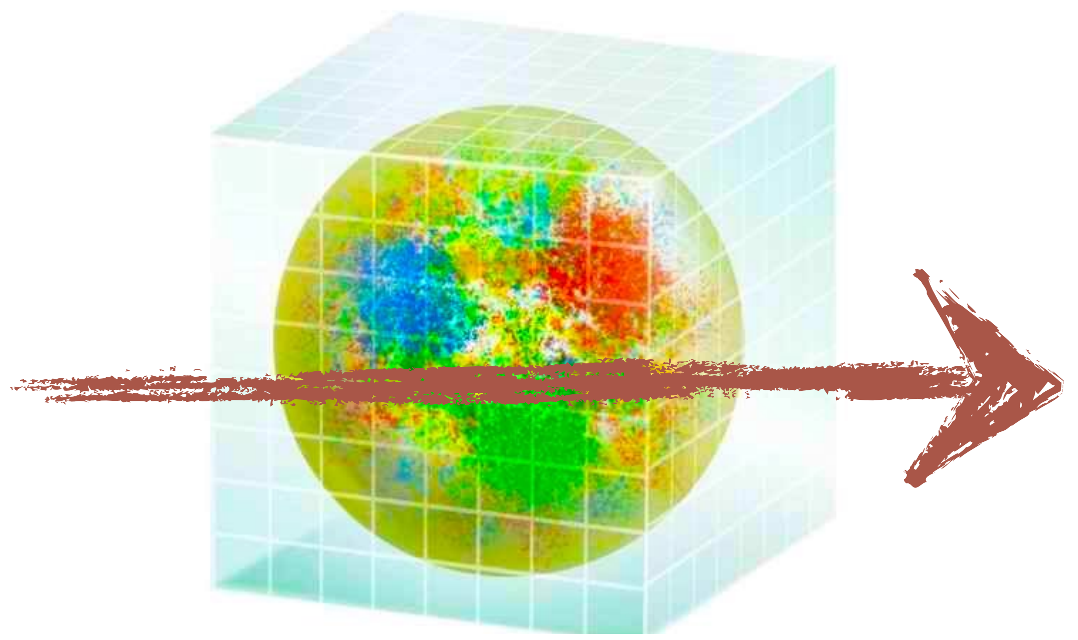


renormalize: scale μ

$$M(z^2, \mu) \sim \left\langle H(0) \,|\, \hat{O}(z, \mu) \,|\, H(0) \right\rangle$$

spacelike separated bilocal operator

fast-moving hadron



P_z

$P_z \approx E$

$$\left\langle H(0)\,|\,\hat{O}(z,\mu)\,|\,H(0)\right\rangle \qquad \left\langle H(P_z)\,|\,\hat{O}(z^-,\mu)\,|\,H(P_z)\right\rangle$$

factorization of $M(y, \mu, P_z) \sim$ perturbative $\otimes$ non-perturbative

$\tilde{c}(y, x, \mu, P_z) \otimes \tilde{f}(x, \mu)$

momentum space

$\tilde{c}(y, x, \mu, z^2) \otimes \tilde{f}(x, \mu)$

position space

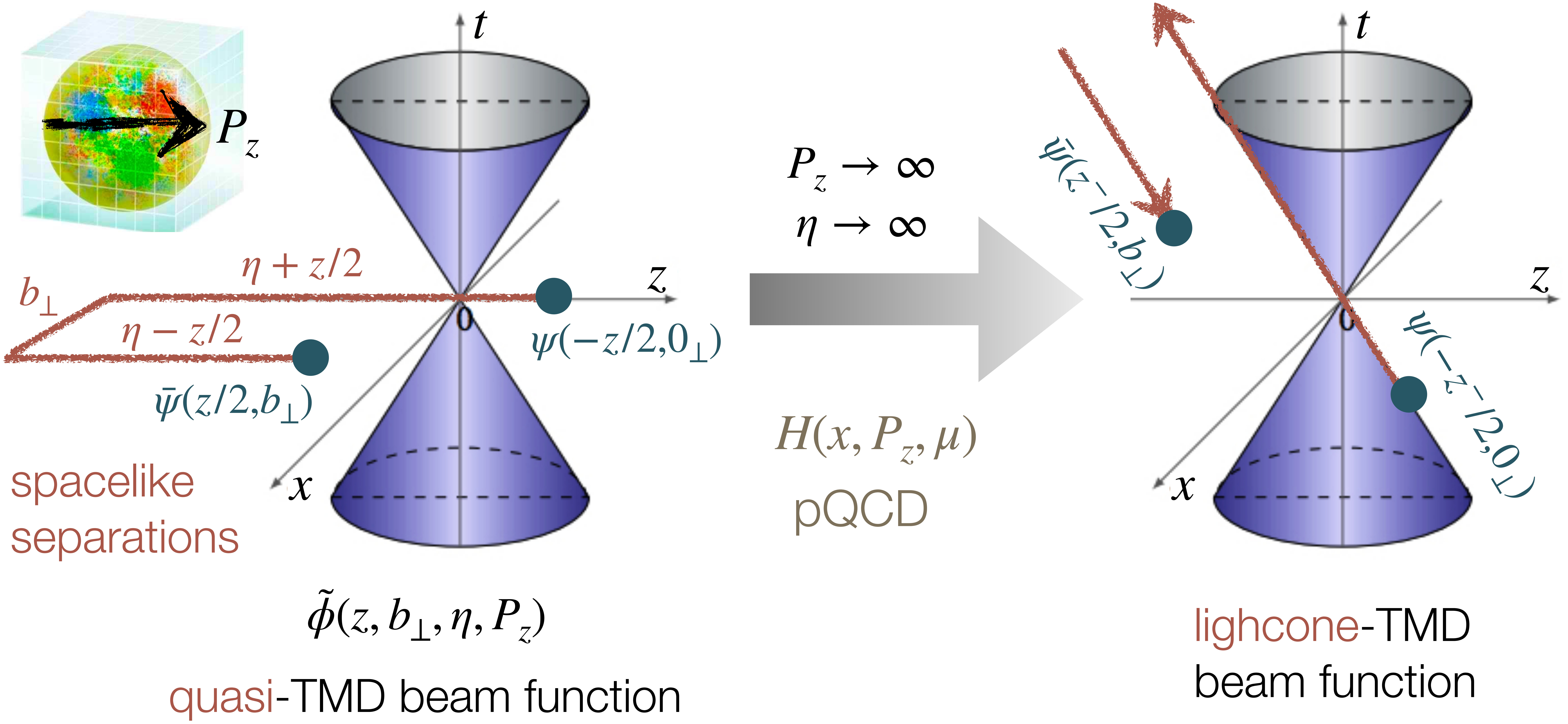
nonperturbative objects on the lightcone, $f(x, \mu)$, and from lattice QCD, $\tilde{f}(x, \mu)$, shares same infrared singularities, i.e. governed by same evolution equations factorization: perturbative $\otimes$ non-perturbative

$$M(y,\mu,P_z) \sim \tilde{\sigma}(y,x,\mu,P_z) \otimes \tilde{f}(x,\mu) \qquad M(y,\mu,z^2) \sim \tilde{\sigma}(y,x,\mu,z^2) \otimes \tilde{f}(x,\mu)$$

regularize QCD on a lattice, then $P_z \to \infty$ / $z^2 \to 0$; opposite order of limits from light-cone quantization

difference is UV physics, can be taken care of through perturbative matching

TMD distributions from lattice QCD



TMD factorization of LQCD beam function

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T,\mu)} \cdot \tilde{f}(x, b_T, P_z, \mu) = H(x, P_z, \mu) \cdot \exp\left[\frac{1}{2}\ln\frac{(2xP_z)^2}{\zeta}\gamma^{\overline{\rm MS}}(b_T,\mu)\right] \cdot f(x, b_T, \zeta, \mu)$$

intrinsic soft factor

pQCD kernel

nonperturbative Collins-Soper kernel

b_T

$b_T \sim \Lambda_{\mathrm{QCD}}^{-1}$ ↑ nonperturbative

$\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu)$

$b_T \ll \Lambda_{\mathrm{QCD}}^{-1}$ ↓ perturbative

JLAB | EIC | RHIC | LHC

$\sqrt{\zeta}$

nonperturbative Collins-Soper kernel from LQCD

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T,\mu)} \cdot \tilde{f}(x,b_T,P_z,\mu) = H(x,P_z,\mu) \cdot \exp\left[\frac{1}{2}\ln\frac{(2xP_z)^2}{\zeta}\gamma^{\overline{\text{MS}}}(b_T,\mu)\right] \cdot f(x,b_T,\zeta,\mu)$$

intrinsic soft factor

pQCD kernel

universal CS kernel

$$\gamma^{\overline{\text{MS}}}(b_T,\mu) = \frac{1}{\ln(P_2/P_1)}\ln\left[\frac{\tilde{f}(x,b_T,P_2,\mu)}{\tilde{f}(x,b_T,P_1,\mu)}\right] + \delta\gamma^{\overline{\text{MS}}}(b_T,\mu,P_1,P_2)$$

LQCD

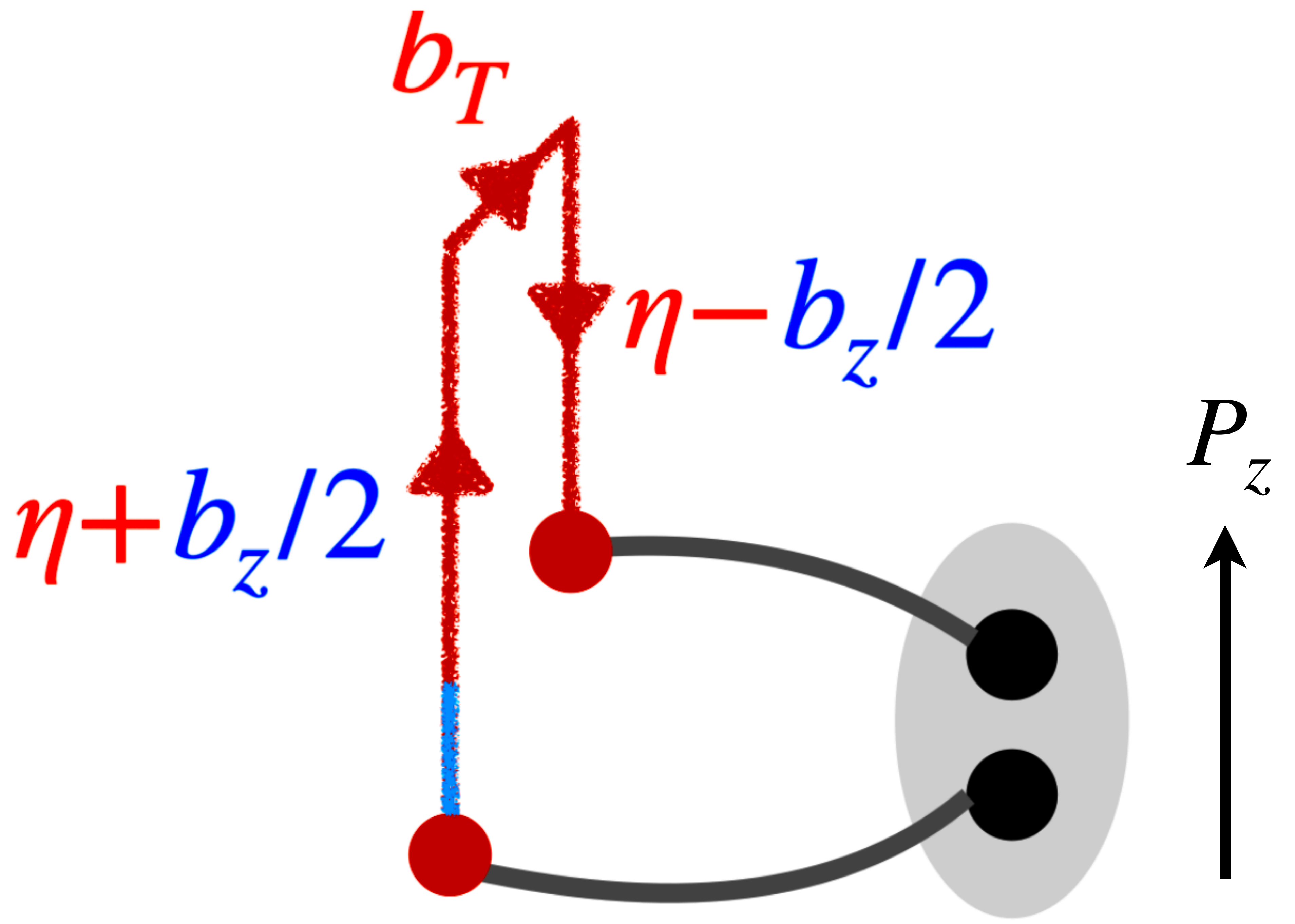
pQCD kernel

lattice QCD calculations of CS kernel

simplest choice for the quasi-TMD beam function $\tilde{\phi}(b_z, b_\perp, \eta, P_z)$

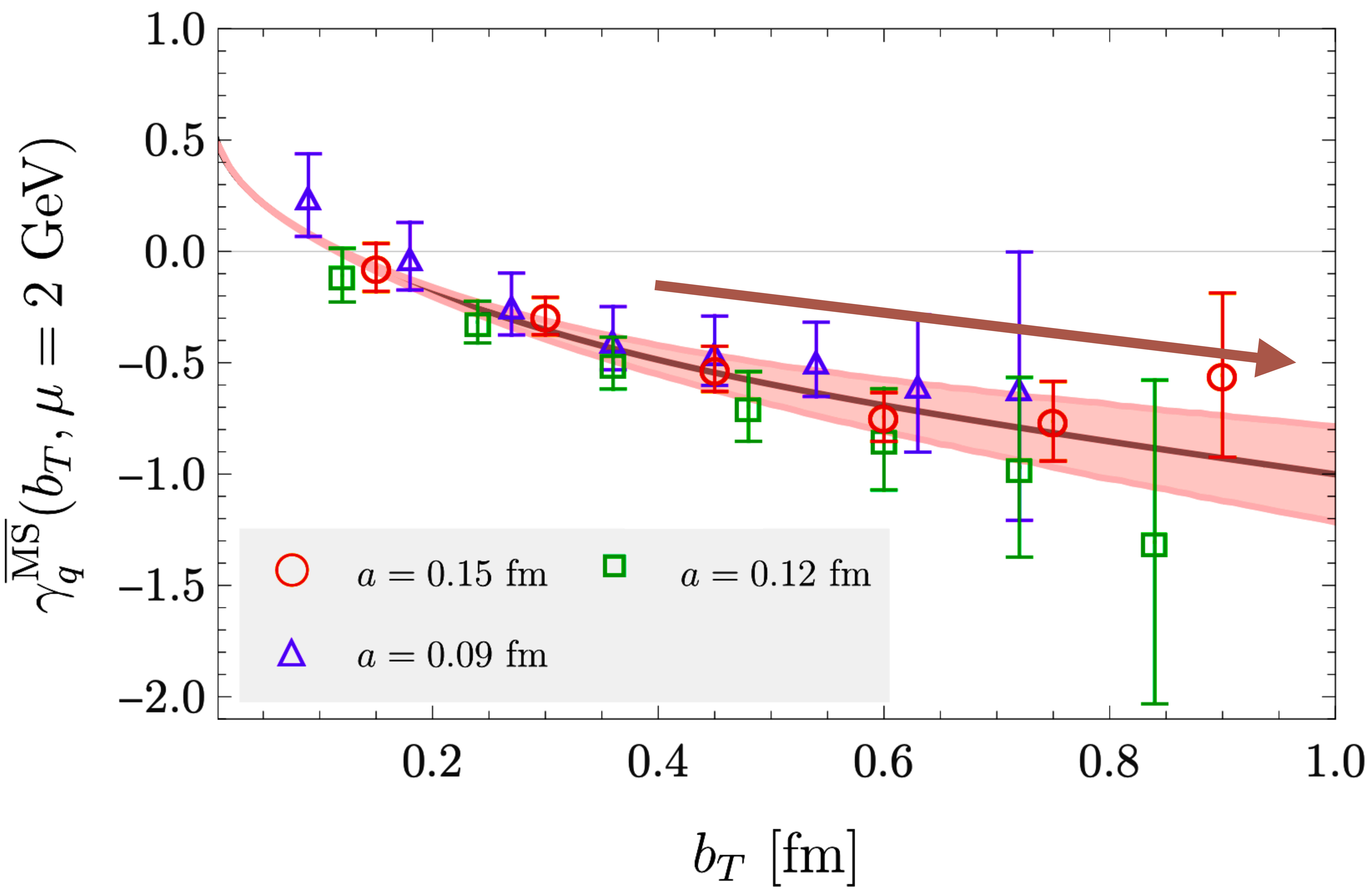
pion TMD wave function (TMDWF)

$$\langle \Omega | \bar{\psi}(\frac{b_z}{2}, b_\perp) \Gamma W_{\sqsupset}(\frac{\mathbf{b}}{2}, -\frac{\mathbf{b}}{2}, \eta) \psi(-\frac{b_z}{2}, 0) | \pi^+, P_z \rangle$$



the challenge

rapidly growing errors with increasing $b_\perp$

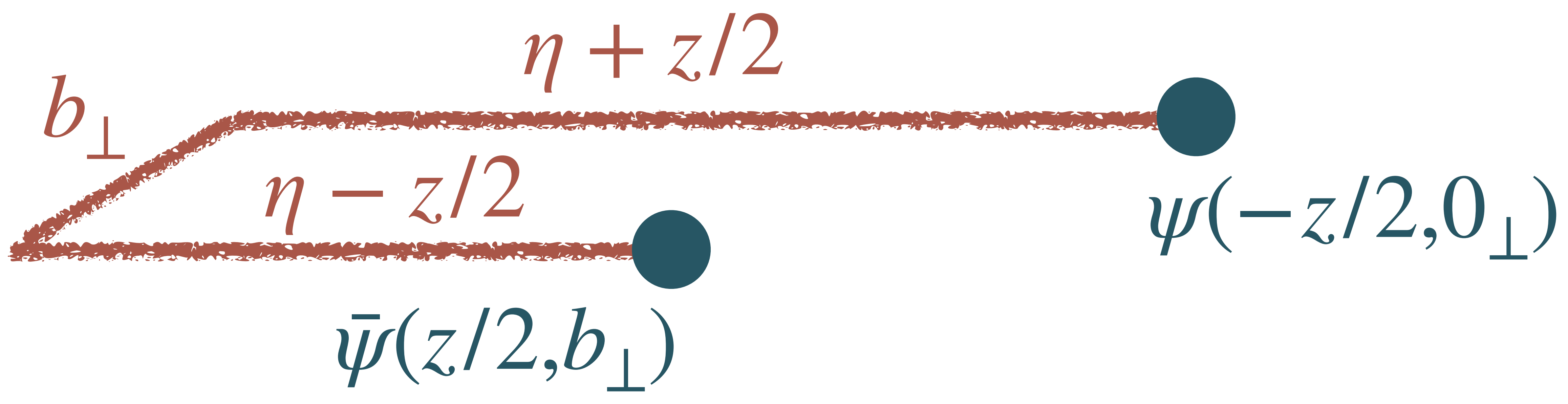


Avkhadiev et al., Phys. Rev. Lett. 23, 231901 (2024)

understanding the challenge

multiplicative renormalization factor of the Wilson line:

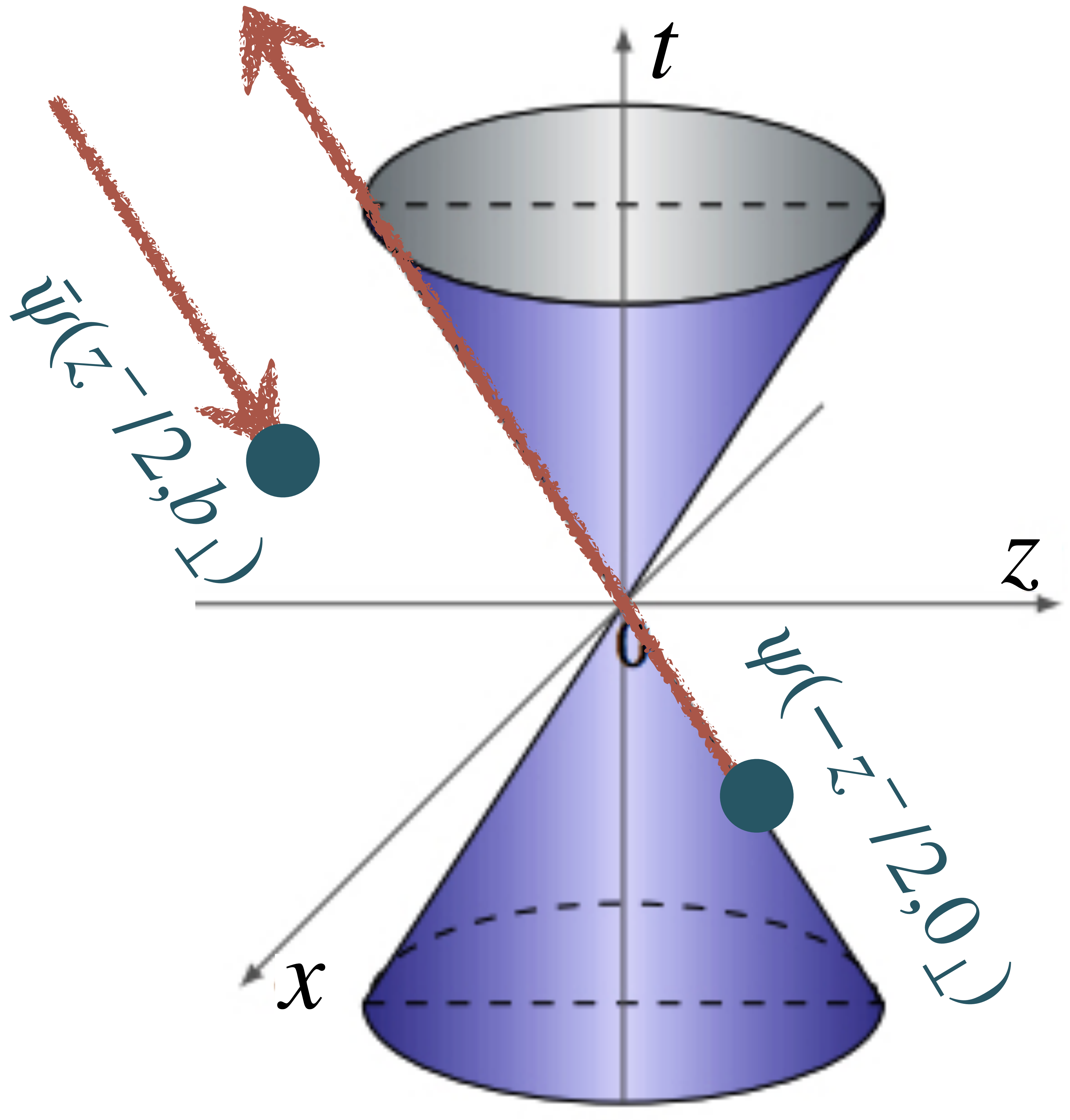
$$\sim e^{-\delta m(\eta + b_\perp)}$$



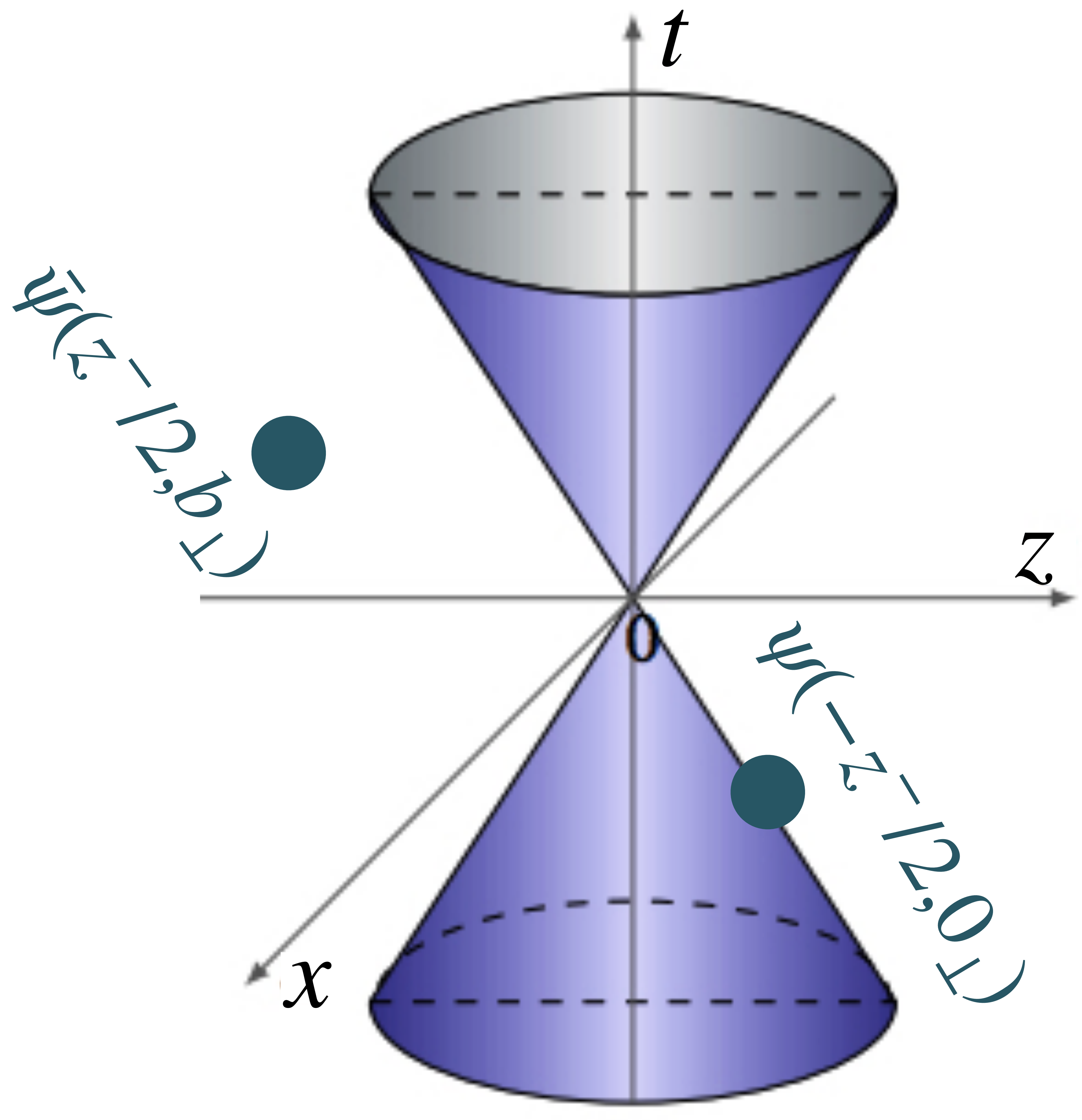
exponential decrease of signal for large η and increasing $b_\perp$

overcoming the challenge

physical lightcone gauge $A^+ = 0$



$\equiv$



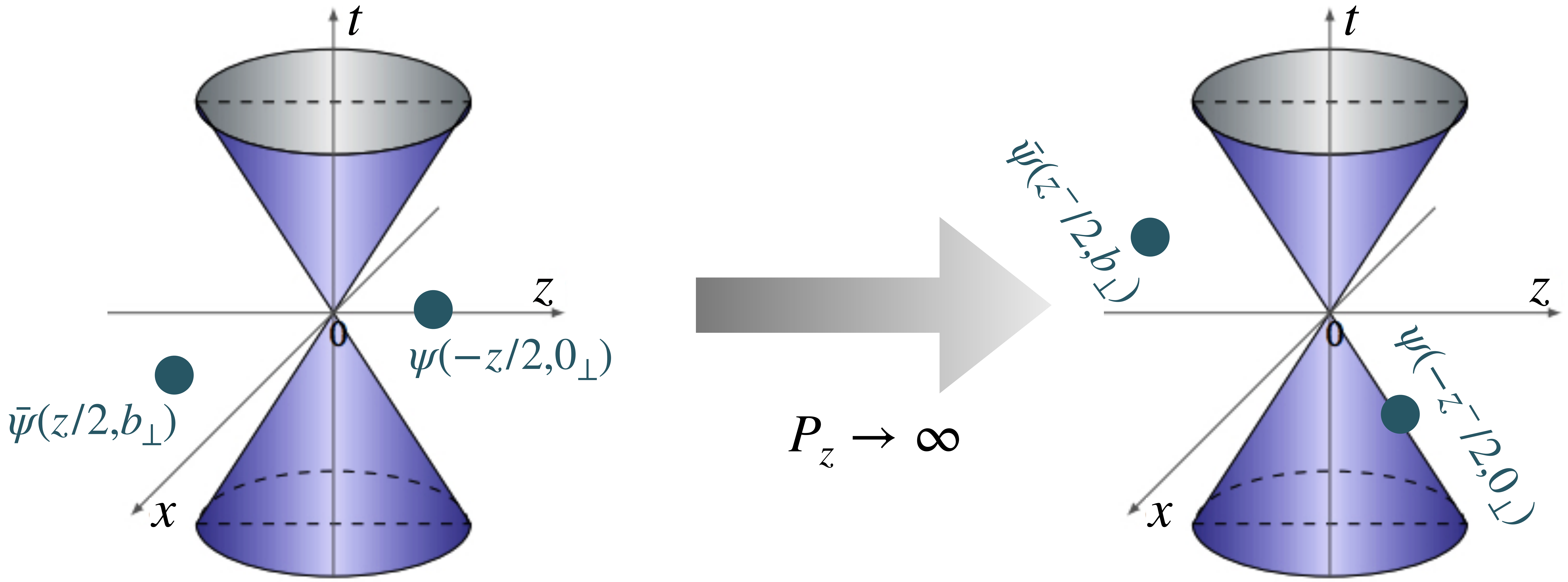
how can we access $A^+ = 0$ in lattice QCD calculations ?

find a gauge that becomes equivalent to $A^+ = 0$ in the limit $P_z \to \infty$

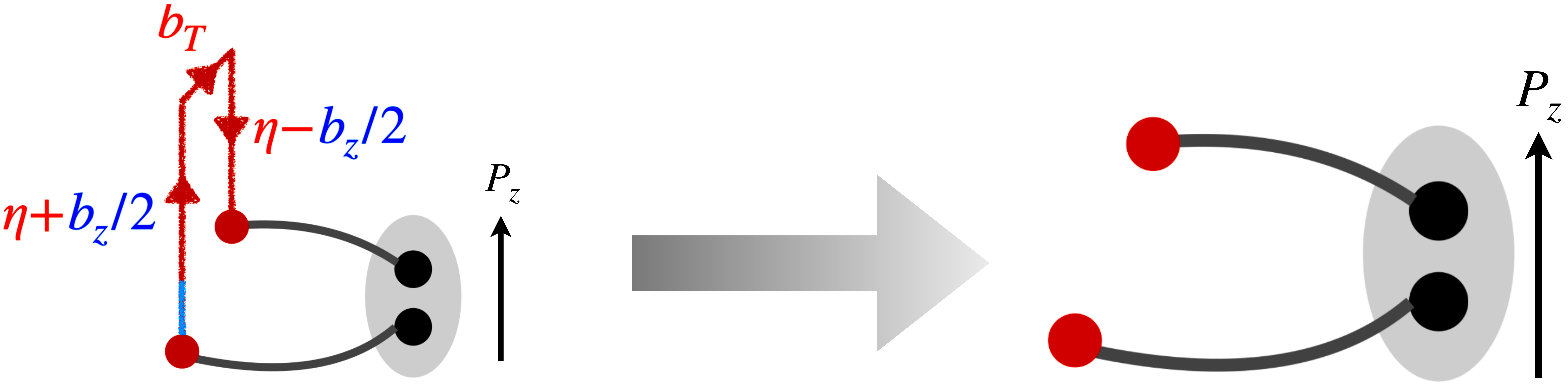
Coulomb gauge	$P_z \to \infty$	lightcone gauge
$\vec{\nabla} \cdot \vec{A} = 0$	⟹	$A^+ = 0$

quasi-TMD beam function in Coulomb gauge (CG)

$$\bar{\psi}(z/2, b_\perp)\Gamma\psi(-z/2, 0_\perp)\Big|_{\nabla\cdot A=0}$$



CG quasi-TMD beam function

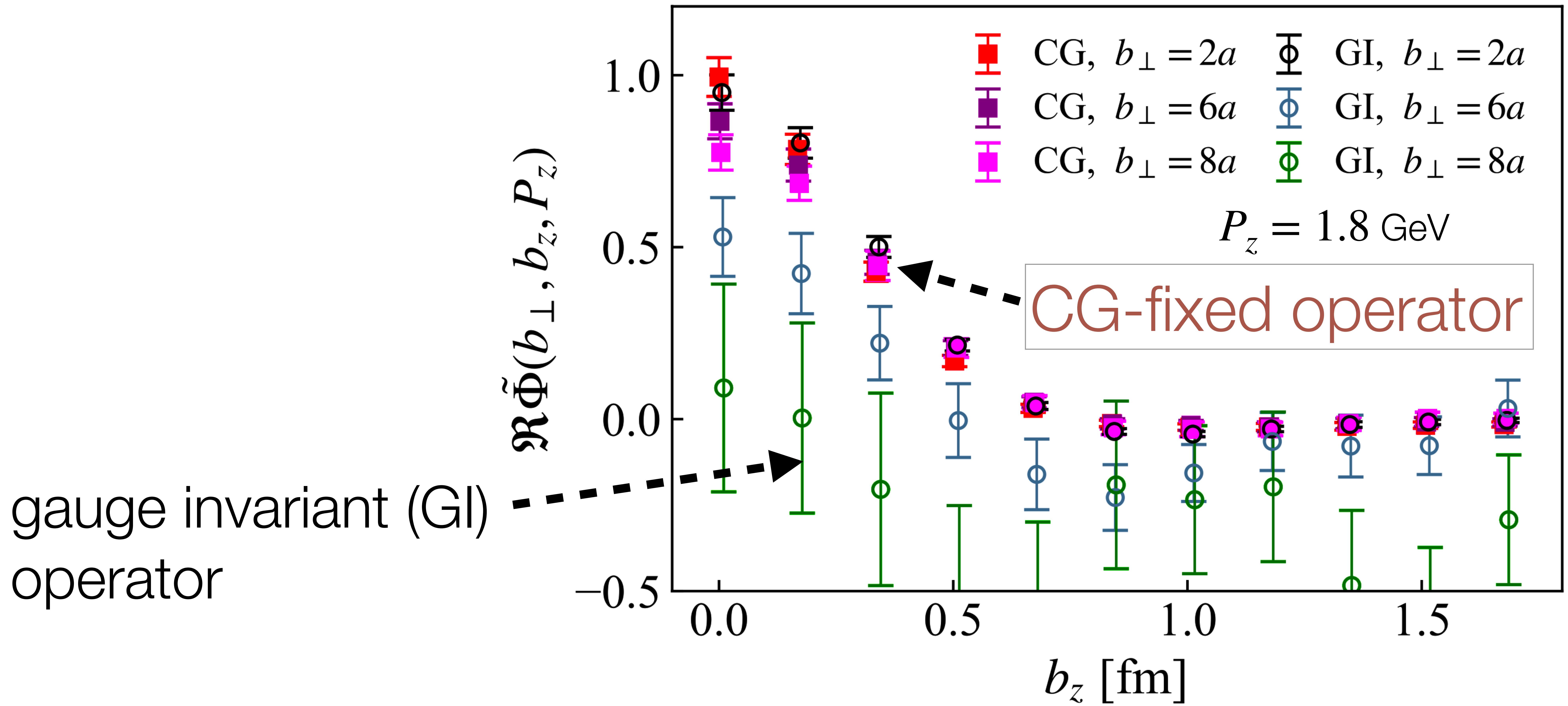


+ re-computation of pQCD matching function $\delta\gamma^{\overline{\rm MS}}(x,\mu,P_1,P_2)$

next-to-leading-log (NLL) accuracy

Y. Zhao, Phys. Rev. Lett. 133, 241904 (2024)

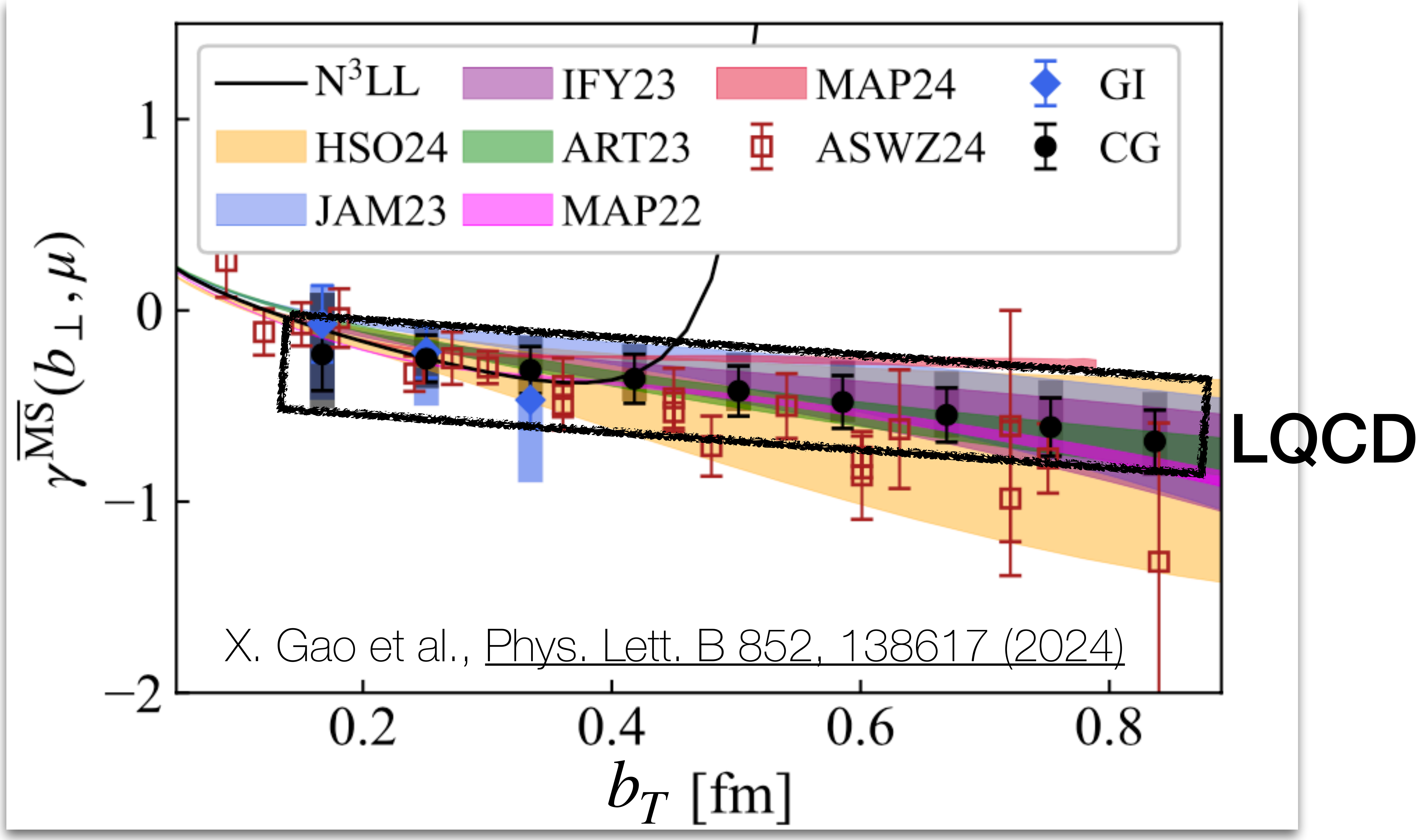
renormalized quasi-TMD beam functions



unitary chiral (Domain Wall) fermions, physical pion mass, lattice spacing a=0.085 fm

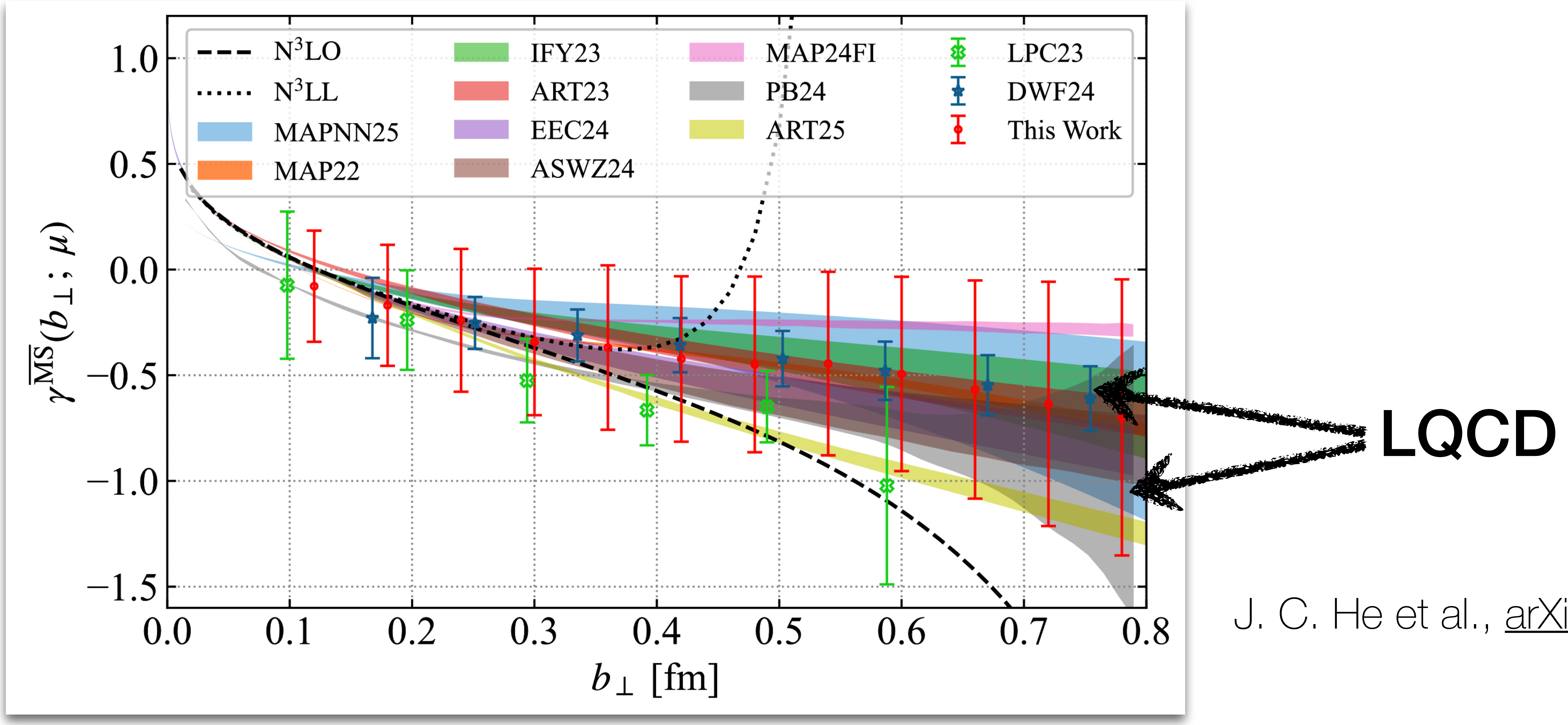
Bollweg et al.: Phys. Lett. B 852, 138617 (2024)

nonperturbative Collins-Soper kernel from LQCD



unitary chiral quarks, physical mass

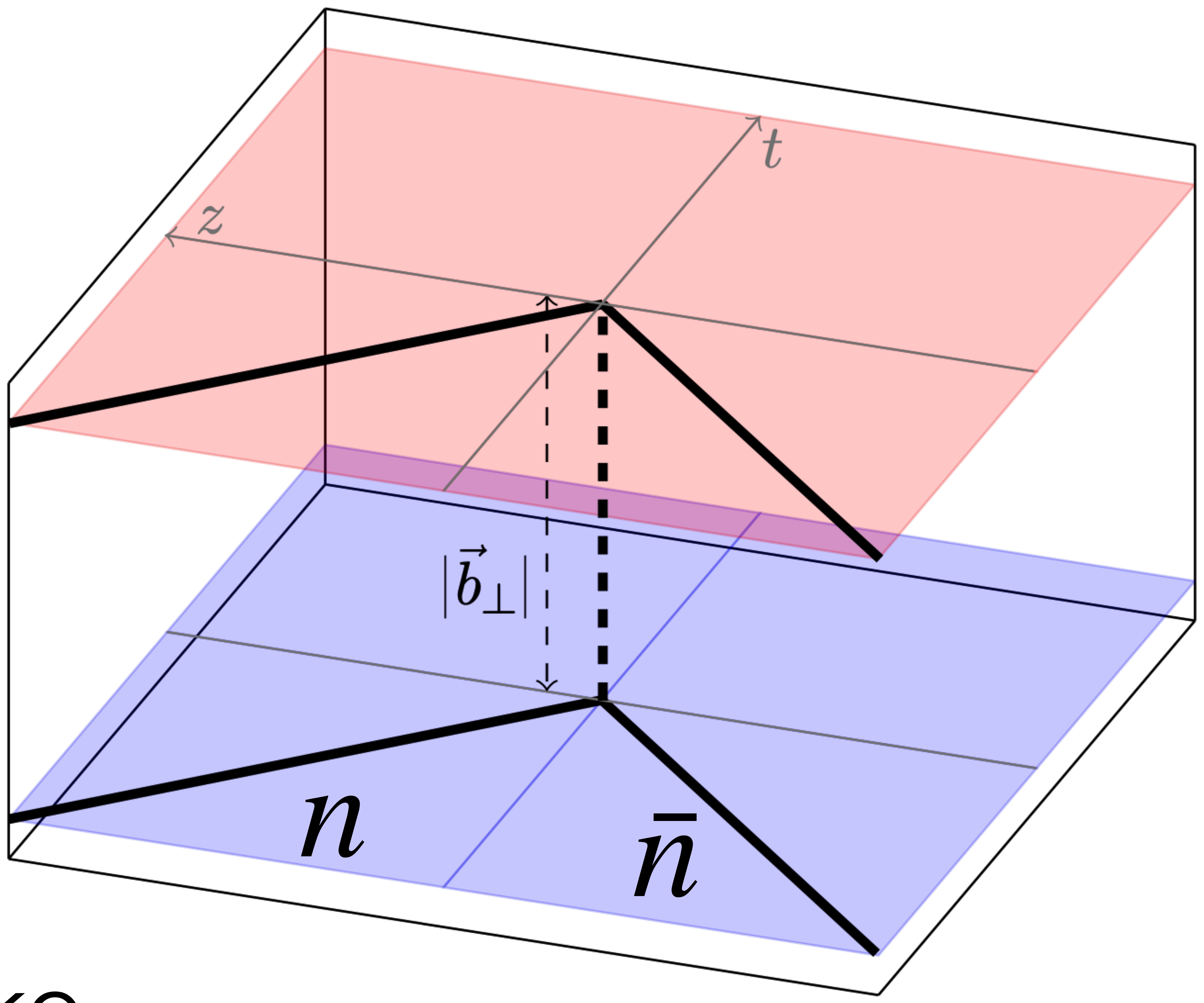
nonperturbative Collins-Soper kernel from LQCD



J. C. He et al., arXiv:2504.04625

check university through independent calculations to larger momenta, larger pion mass, and different lattice discretization

intrinsic soft factor



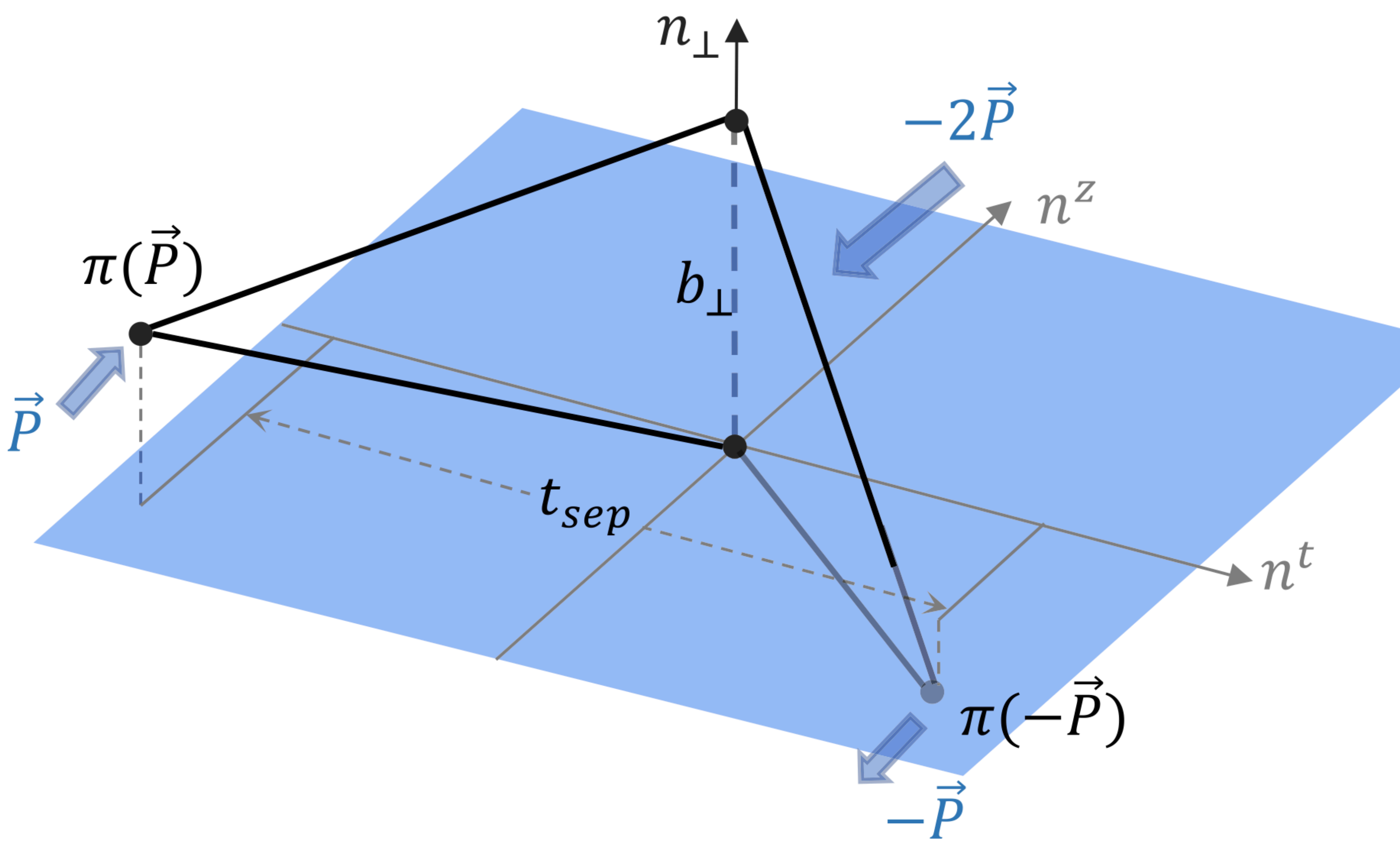
operator involves two lightcone directions

impossible to obtain by computing a space-like operator within a hadron boosted in a direction

need an alternative indirect approach

form factor of two transversely-separated currents within boosted pions from LQCD

$$F(b_T, P_z) \sim \langle P_z | [\bar{q}(b_T)\gamma_T q(b_T)][\bar{q}(0)\gamma_T q(0)] | -P_z \rangle$$



factorizes into pion TMD wave function

$$F(b_T, P_z) \sim H_F(x_1, x_2, P_z, \mu) \otimes \phi^\dagger(x_1, b_T, \mu, \zeta_1, \bar{\zeta}_1) \otimes \phi(x_2, b_T, \mu, \zeta_2, \bar{\zeta}_2)$$

pQCD

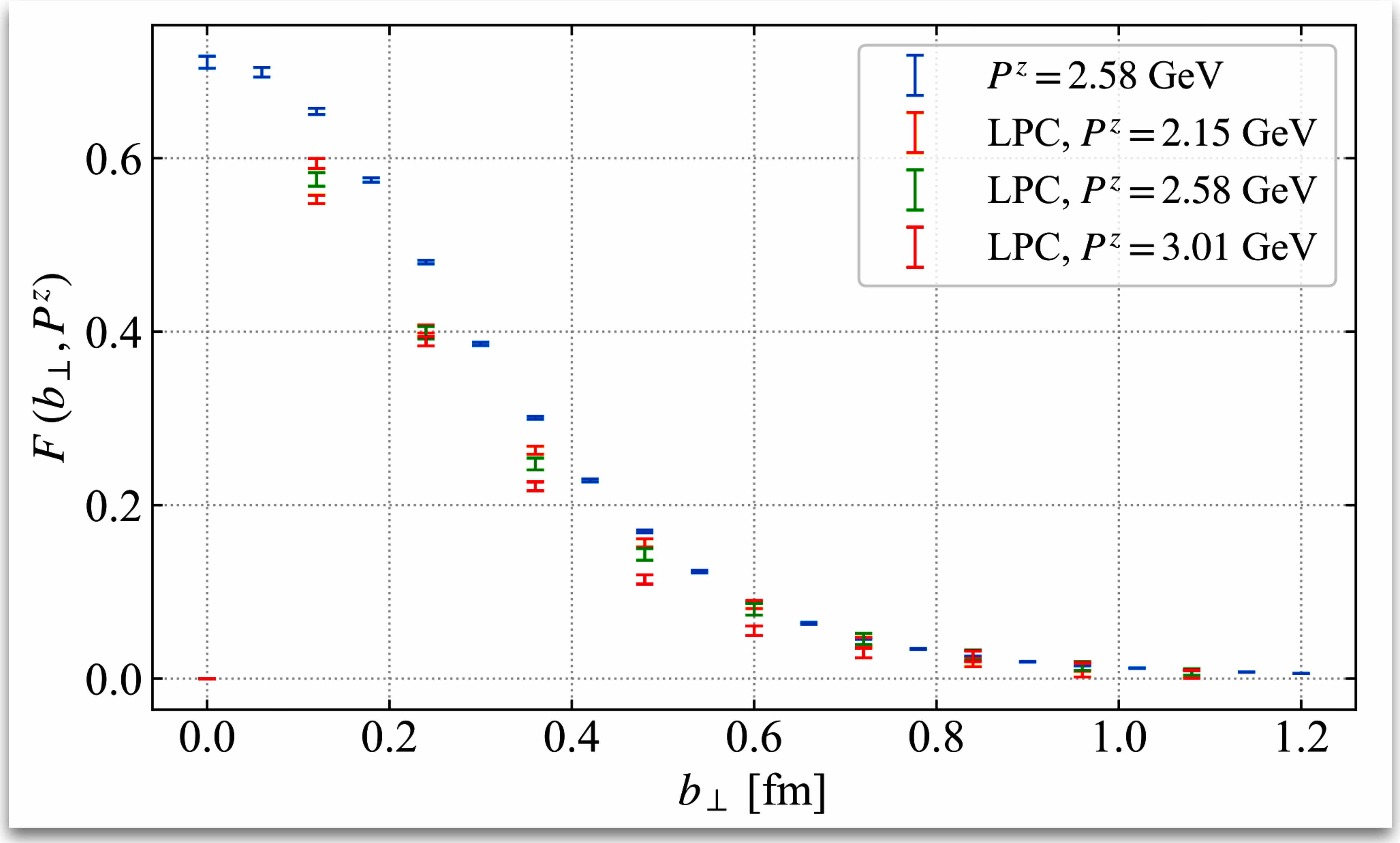
pion TMD wave function from LQCD

$$\sqrt{S_I(b_T, \mu)} \cdot \tilde{\phi}(x, b_T, P_z, \mu) = H_\phi(x, \bar{x}, P_z, \mu) \cdot \phi(x, b_T, \mu, \zeta, \bar{\zeta})$$

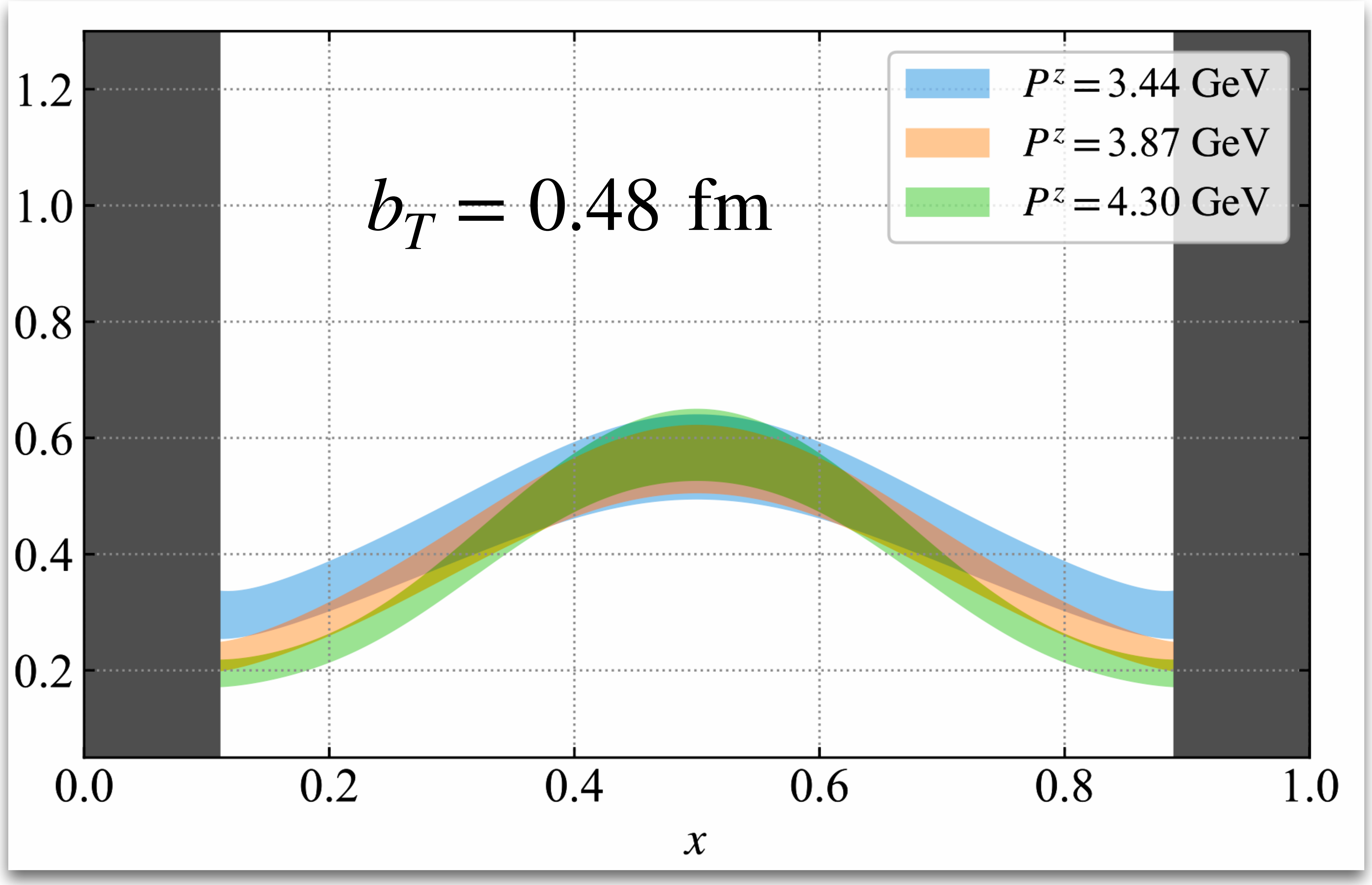
intrinsic soft factor

pQCD

form factor

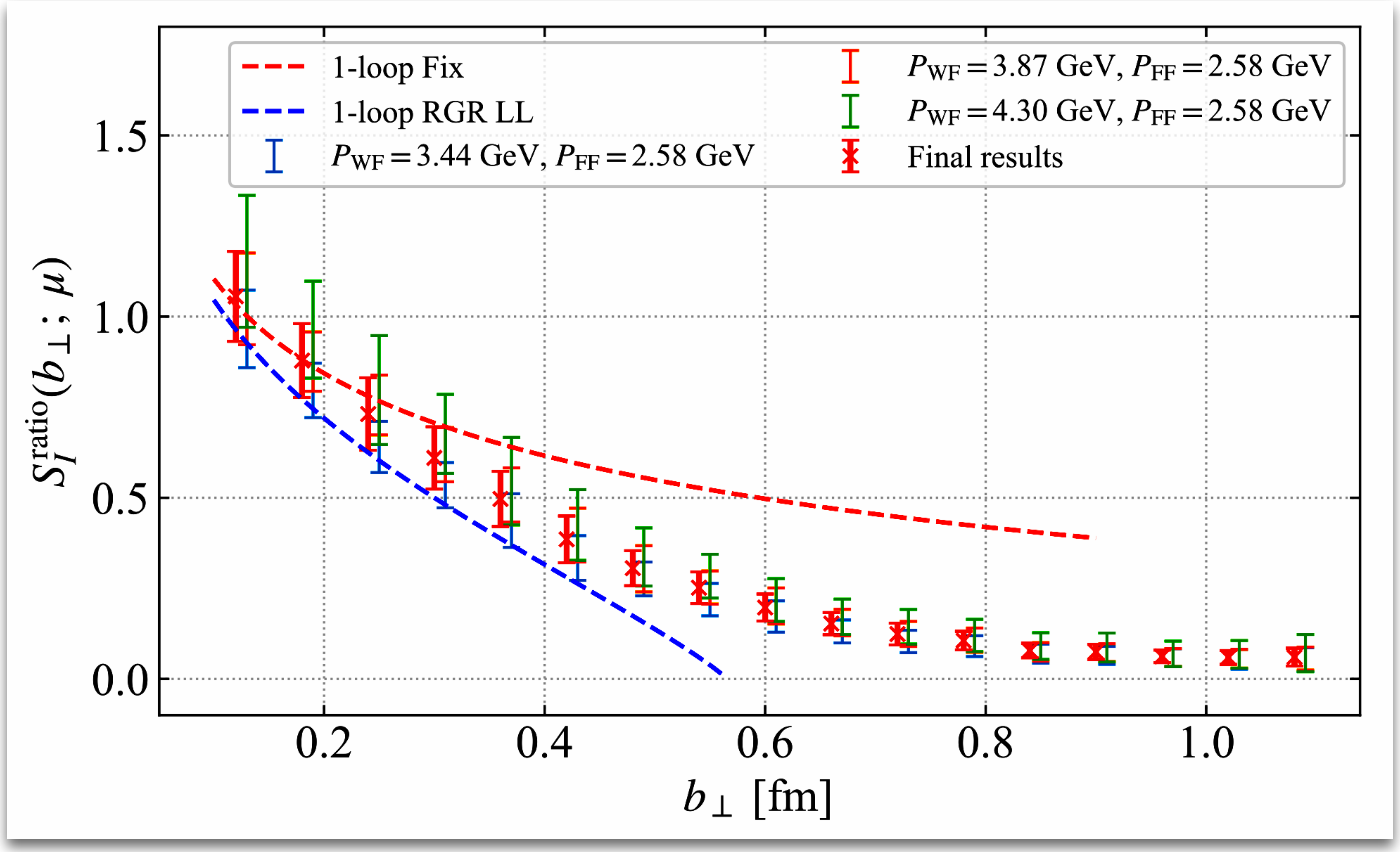


pion TMD wave function



J. C. He et al., arXiv:2504.04625

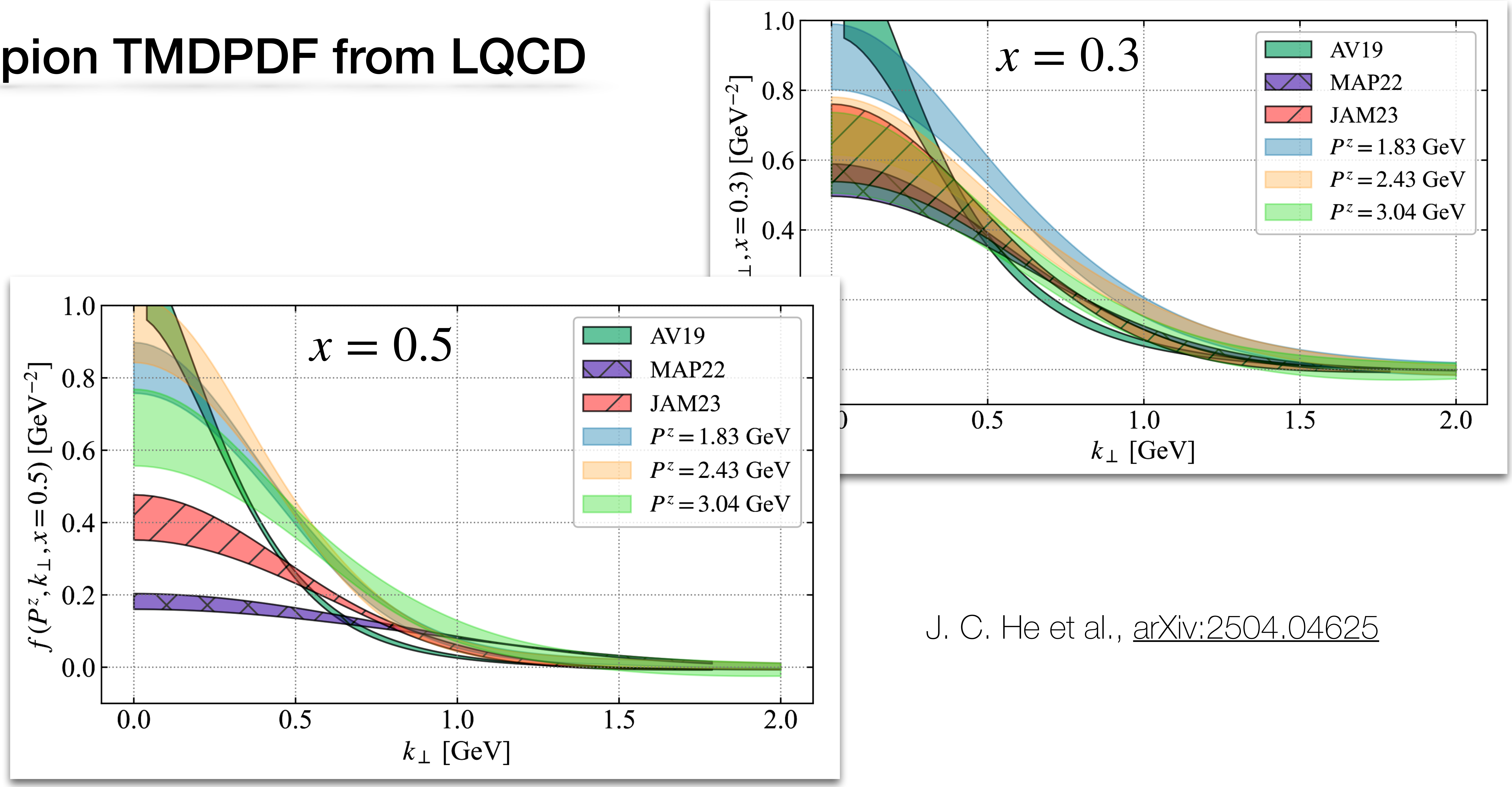
intrinsic soft factor



scheme dependent

J. C. He et al., arXiv:2504.04625

pion TMDPDF from LQCD



J. C. He et al., arXiv:2504.04625

TMD factorization of LQCD beam function

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T,\mu)} \cdot \tilde{f}(x,b_T,P_z,\mu) = H(x,P_z,\mu) \cdot \exp\left[\frac{1}{2}\ln\frac{(2xP_z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}(b_T,\mu)\right] \cdot f(x,b_T,\zeta,\mu)$$

intrinsic soft factor

pQCD kernel

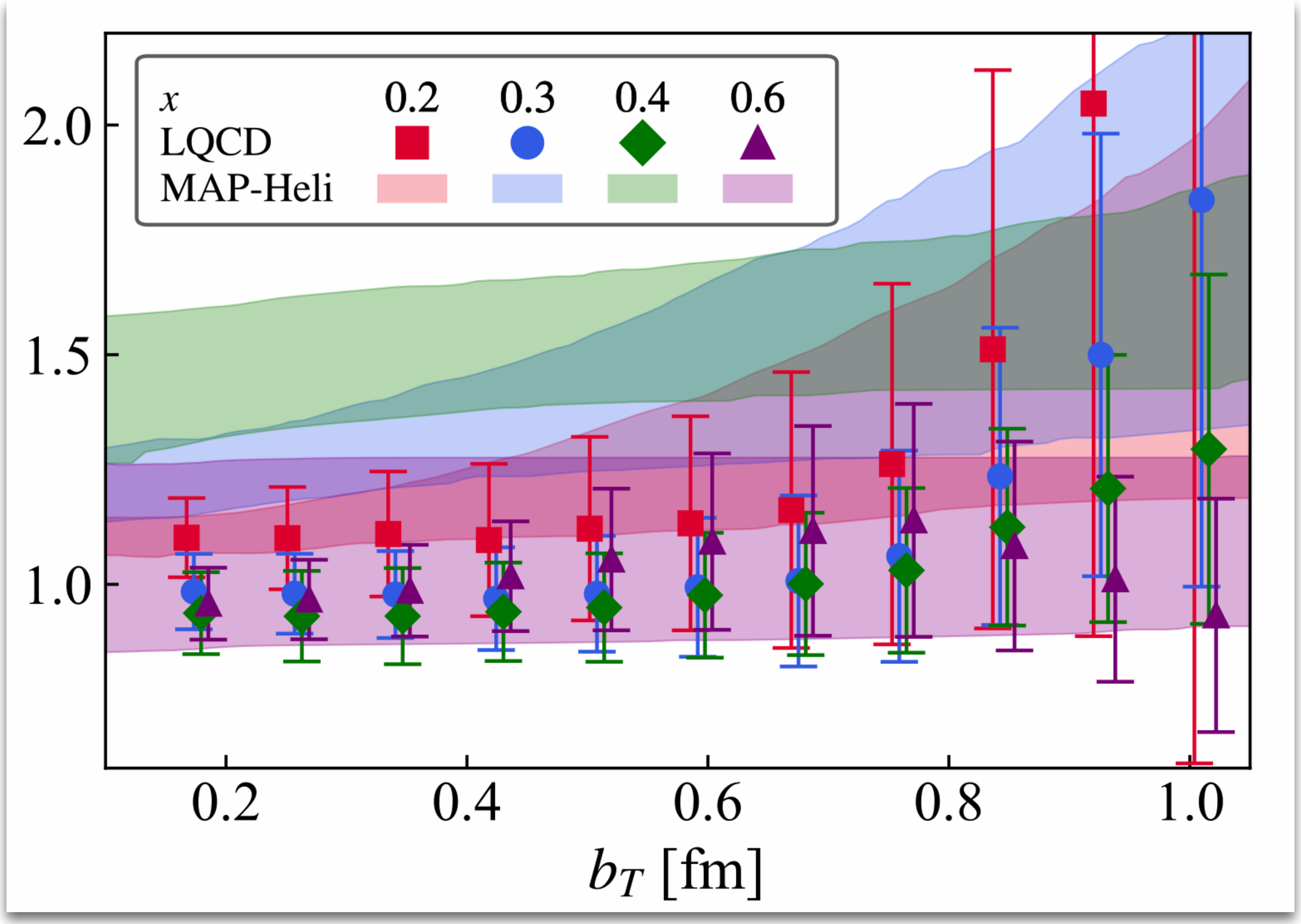
scale-independent ratios of TMDPDF

$$\frac{f_a(x,b_T,\zeta,\mu)}{f_b(x,b_T,\zeta,\mu)} = \frac{\tilde{f}_a(x,b_T,P_z,\mu)}{\tilde{f}_b(x,b_T,P_z,\mu)}$$

TMDPDF of proton: helicity to unpolarized TMDPDF

$$\frac{g_{1L}^{\Delta u_+ - \Delta d_+}(x, b_T, \zeta, \mu)}{g_A \cdot f_1^{u_v - d_v}(x, b_T, \zeta, \mu)}$$

unitary chiral quarks, physical mass

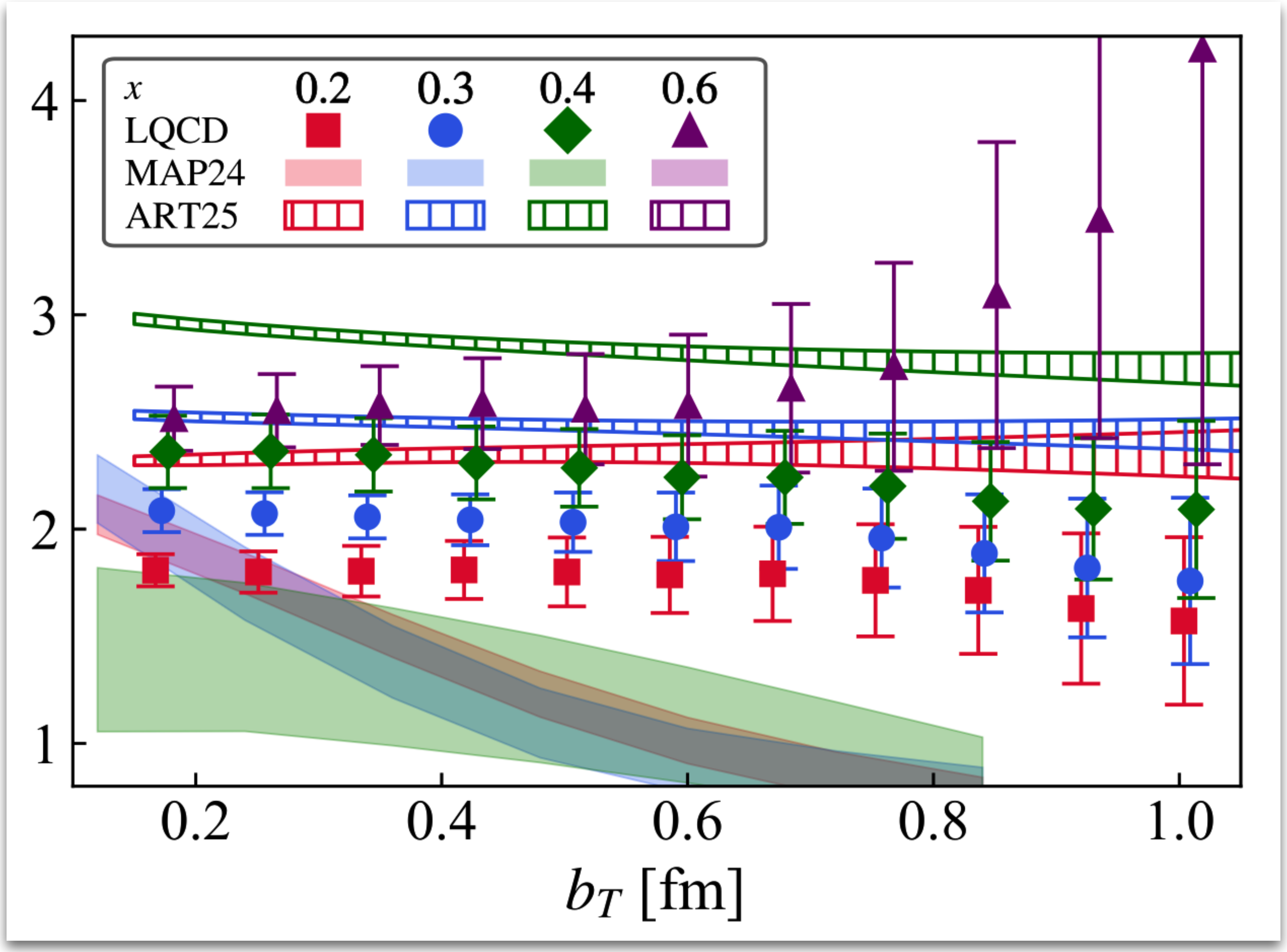


X. Gao et al., arXiv:2505.18430

TMDPDF of proton: up to down unpolarized TMDPDF

$$\frac{f_1^{u_v}(x, b_T, \zeta, \mu)}{f_1^{d_v}(x, b_T, \zeta, \mu)}$$

unitary chiral quarks, physical mass

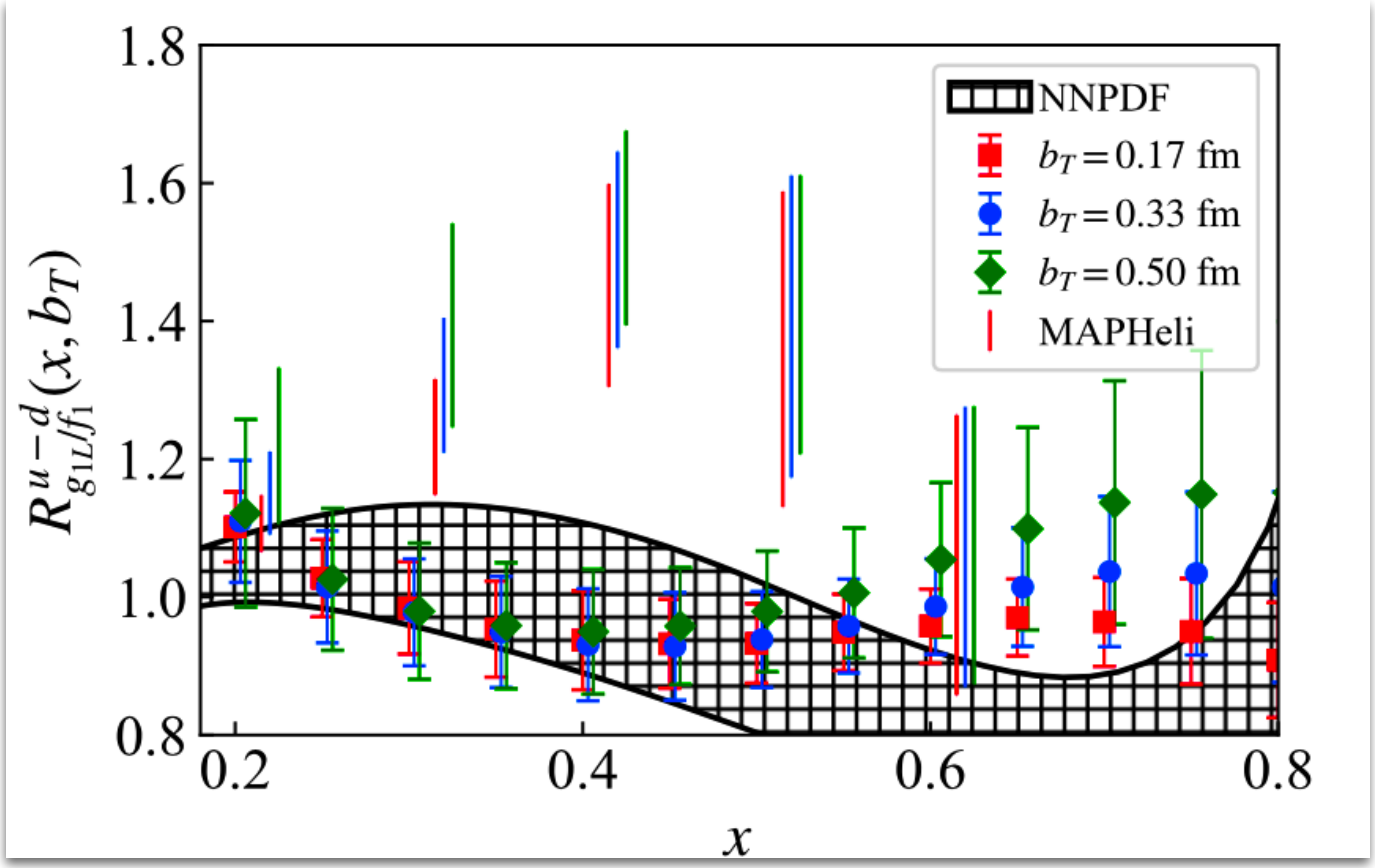


X. Gao et al., arXiv:2505.18430

dirty laundries / foods for thought

helicity to unpolarized TMDPDF

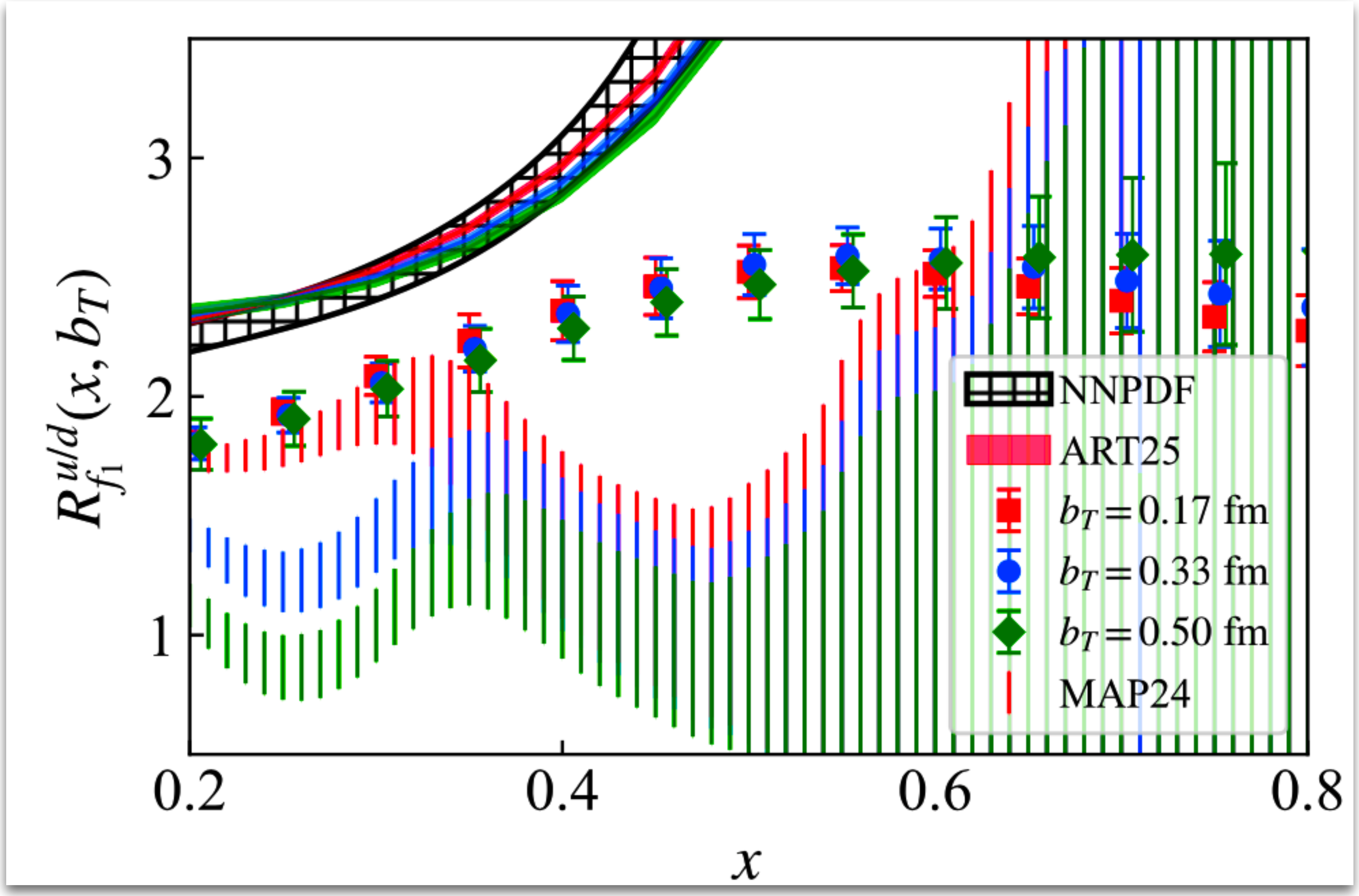
$$\frac{f_1^{u_v}(x, b_T, \zeta, \mu)}{f_1^{d_v}(x, b_T, \zeta, \mu)}$$



NNPDF: $\mu = 2$ GeV

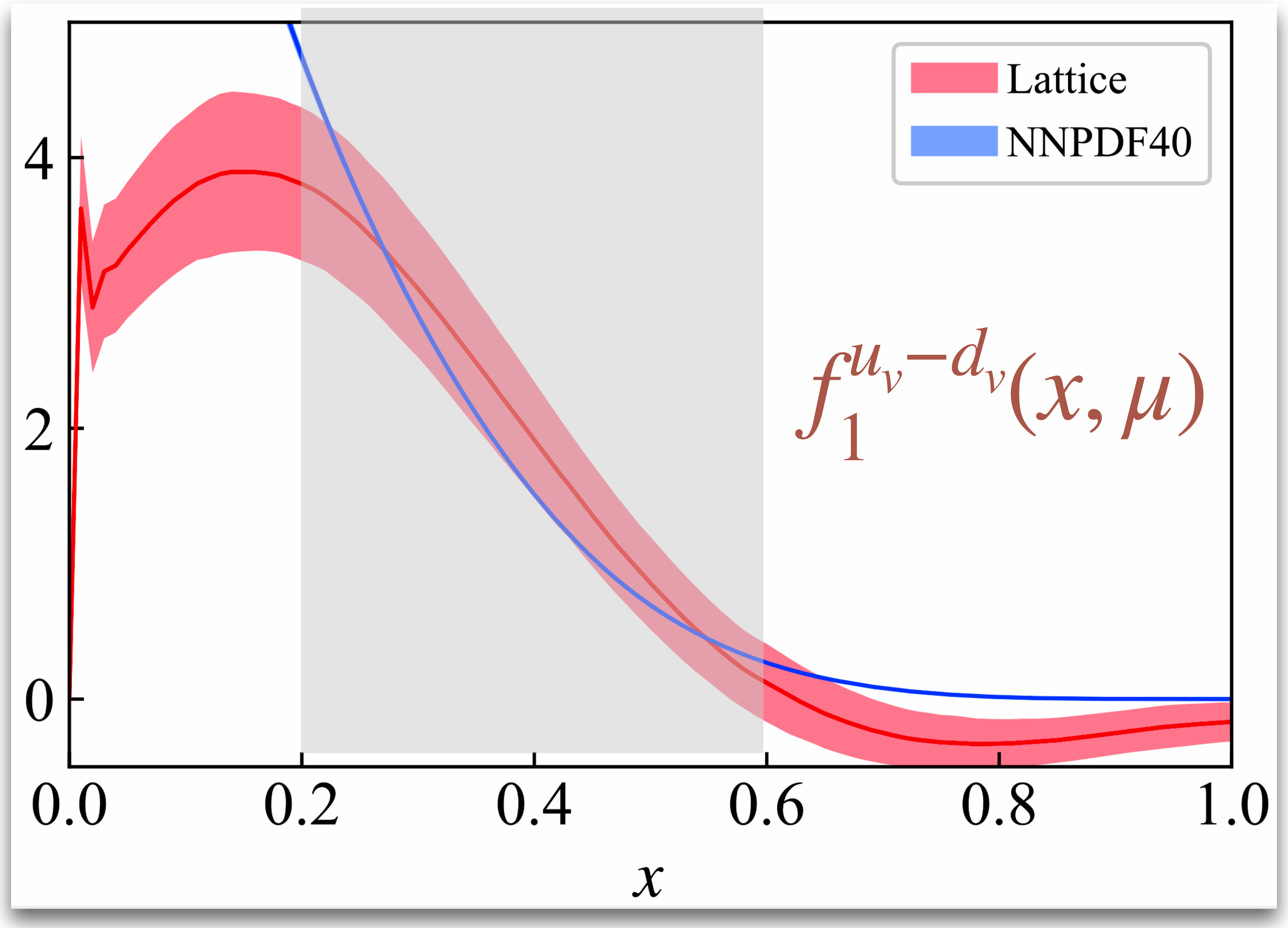
up to down unpolarized TMDPDF

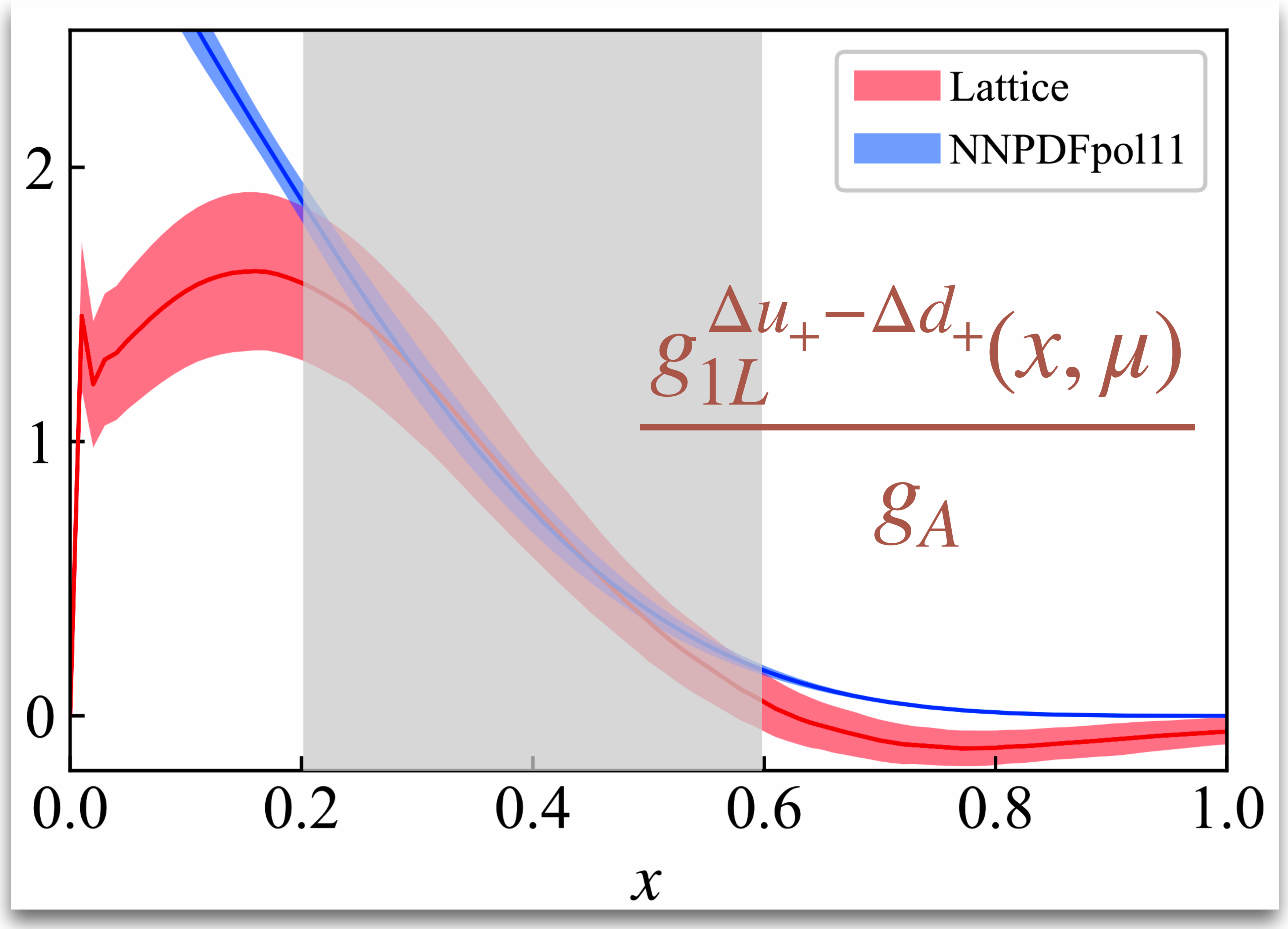
$$\frac{f_1^{u_v}(x, b_T, \zeta, \mu)}{f_1^{d_v}(x, b_T, \zeta, \mu)}$$

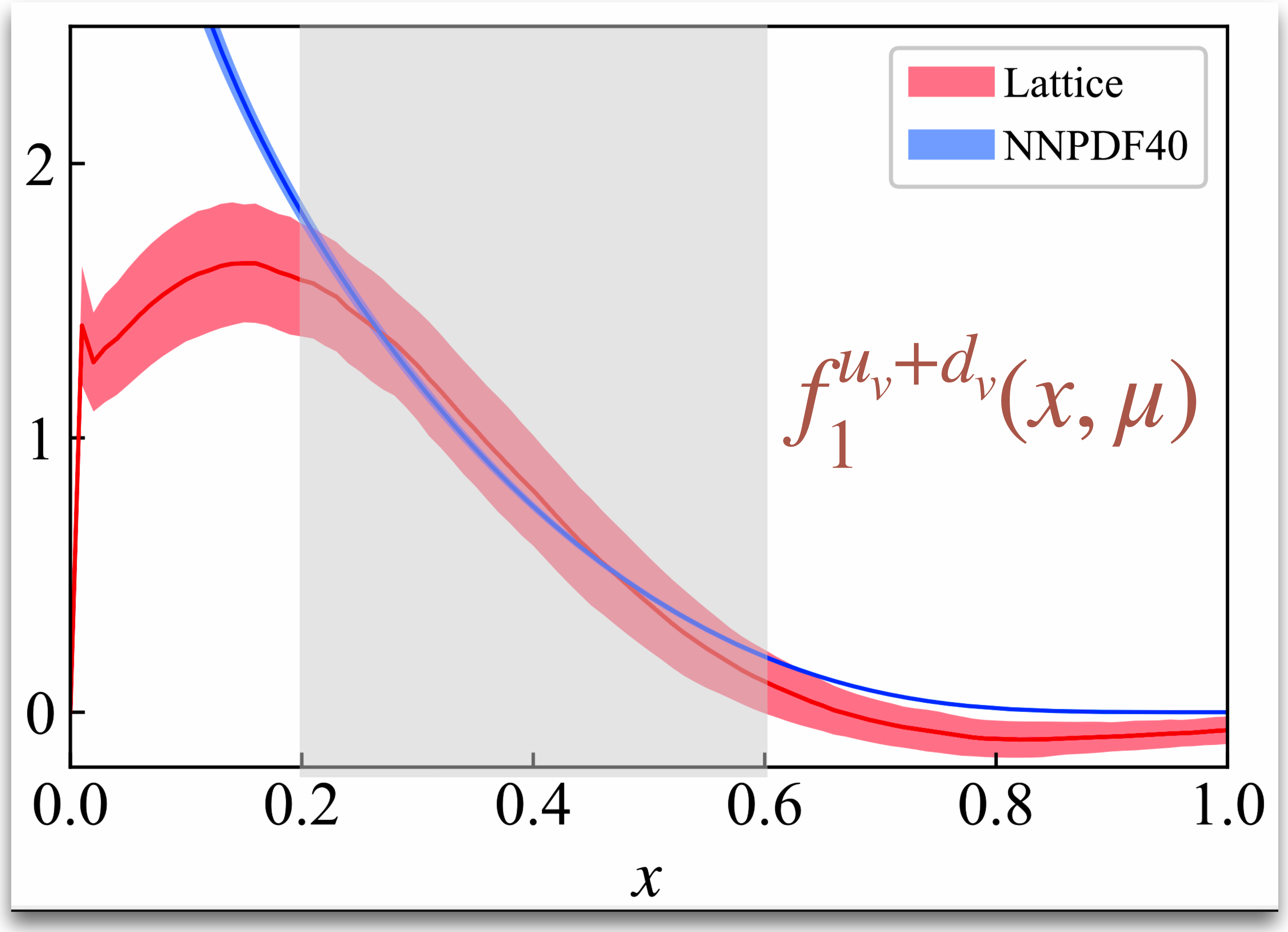


NNPDF: $\mu = 2$ GeV

PDFs


Lattice
NNPDF40
$f_1^{u_v-d_v}(x,\mu)$
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0
x


Lattice
NNPDFpol11
$\frac{g_{1L}^{\Delta u_+ - \Delta d_+}(x,\mu)}{g_A}$
2
1
0
0.0
0.2
0.4
0.6
0.8
1.0
x


Lattice
NNPDF40
$f_1^{u_v+d_v}(x,\mu)$
2
1
0
0.0
0.2
0.4
0.6
0.8
1.0
x

$\mu = 2$ GeV

CG vs gauge-invariant TMDPDFs