Cos $2 \oplus asymmetry$ in J/ψ and jet production at the EIC and effect of TMD evolution

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GLUONS	unpolarized	circular	linear
U	$\left(f_{1}^{g}\right)$		$h_1^{\perp g}$
L		$\left(egin{array}{c} g_{1L}^{g} \end{array} ight)$	$h_{_{1L}}^{\perp g}$
Т	$f_{1T}^{\perp g}$	$g^g_{\scriptscriptstyle 1T}$	$h^g_{1T}, h^{\perp g}_{1T}$

 $h_1^{\perp g}$: Linearly polarized gluon distribution in unpolarized hadron; T even



Gluon Sivers function in Transversely polarized proton

Vanish under p_T integration

In contrast to quark TMDs, very little is known about gluon TMDs

 $h_1^g \equiv h_{1T}^g + \frac{p_T^2}{2M_p^2} h_{1T}^{\perp g}$

Angeles-Martinez et al., Acta Phys, Pol. B46 (2015) Mulders, Rodrigues, PRD 63 (2001) Meissner, Metz, Goeke, PRD 76 (2007)

 $\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P, S | \operatorname{Tr}_{c} \left[F^{+\nu}(0) \mathcal{U}^{\mathcal{C}}_{[0,\varepsilon]} F^{+\mu}(\xi) \mathcal{U}^{\mathcal{C}'}_{[\varepsilon,0]} \right] | P, S \rangle$

Gluon TMDs need two gauge links for gauge invariance

Linearly polarized Gluon distributions

Linearly polarized gluon distributions were first introduced in

Mulders and Rodrigues, PRD 63, 094021 (2001)

Operator structure of unintegrated gluon distributions can be different in different processes. In the literature, at small x, Weizsacker-Williams (WW) gluon distribution contains both past or both future pointing gauge links and dipole distributions contain one past and one future pointing gauge link. These are also called f and d type distributions, contribute in different processes

Extensive literature on unintegrated gluon distributions.

Linearly polarized gluon TMD : Measures an interference between an amplitude when the active gluon is polarized along x (or y) direction and a complex conjugate amplitude with the gluon polarized in y (or x) direction in an unpolarized hadron

Affects unpolarized cross section as well as generates a cos 2¢ asymmetry

Gluon TMDs in J/\psi production processes

Semi-inclusive J/ψ production in eP collision is a good channel to probe gluon TMDs Godbole, Misra, AM, Rawoot, PRD (2012); AM and Rajesh EPJC (2017)

For low transverse momentum region, TMD factorization is expected to hold and for large transverse momentum collinear factorization is applicable. In the intermediate region, results from these two formalisms should match

TMD factorized description of the process needs smearing effects to be taken into account in the form of TMD shape functions. The perturbative tail of the shape function can be obtained through a matching procedure.

M. G. Echevarria, JHEP (2019), Boer et al, JHEP (2023)

Also gluon TMDs can be probed in back-to-back production of J/ψ and photon/jet/pion, TMD factorization is expected to be valid. The small scale is provided by the transverse momentum of the pair. By varying the invariant mass of the pair scale evolution of the TMDs can be studied

So far the smearing effects and the shape functions are not calculated by matching procedure

Production of J/\psi in NRQCD

In NRQCD the heavy quark pair is produced in the hard process either in color octet or in color singlet configuration

Then they hadronize to form a color singlet quarkonium state of given quantum numbers through soft gluon emission

Hard process is calculated perturbatively and soft process is given in terms of long distance matrix elements (LDMEs) that are determined from data

The LDMEs are categorized by performing an expansion in terms of the relative velocity of the heavy quark v in the limit v << 1

The theoretical predictions are arranged as double expansions in terms of v as well as α_s .

C. E. Carlson and R. Suaya, Phys. Rev. D 14, 3115 (1976).
E. L. Berger and D. L. Jones, Phys. Rev. D 23, 1521 (1981).
R. Baier and R. Ruckl, Phys. Lett. B 102B, 364 (1981).
R. Baier and R. Ruckl, Nucl. Phys. B201, 1 (1982).
E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327

(1995). P. L. Cho and A. K. Leibovich, Phys. Rev. D 53, 150 (1996).

G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 51, 1125 (1995); 55, 5853(E) (1997).

Production of J/\psi in NRQCD

 J/ψ is a bound state of charm quark and anti-quark ($Q\bar{Q}$)



G. T. Bodwin et al, PRD51 (1995), Lepage 95 Long distance matrix elements (LDMEs) : Describes hadronization of of $Q\bar{Q}[n]$ states into final quarkonium state

NRQCD factorization

$$d\sigma^{ab\to J/\psi} = \sum_{n} d\hat{\sigma}[ab \to c\bar{c}(n)] \langle 0 \mid \mathcal{O}_{n}^{J/\psi} \mid 0 \rangle$$
Perturbative short distance coefficient

Subprocess cross section for formation of heavy quark pair in particular color, angular momentum and spin state "n": ${}^{2S+1}L_J$, calculated by perturbative QCD

Cos 2 \oplus **asymmetry in almost back-to-back** production of J/ψ and jet in ep collision

 $e(l) + p(P) \rightarrow e(l') + J/\psi(P_{\psi}) + jet(P_i) + X$

z<1: energy fraction of the virtual photon carried by the hadron in proton rest frame

Use TMD factorization in the kinematics where the outgoing J/ψ and (gluon) jet are almost back-to back

$$\mathbf{q}_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}, \qquad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}.$$
$$|\mathbf{q}_t| \ll |\mathbf{K}_t|.$$

Use NRQCD to calculate the J/ψ production

Also compare with the color singlet (CS) model result

-p c.o.m frame

 $Q^2 = -q^2$, $s = (P+l)^2$, $W^2 = (P+q)^2$,

 $x_B = \frac{Q^2}{2P \cdot q}, \qquad y = \frac{P \cdot q}{P \cdot l}, \qquad z = \frac{P \cdot P_{\psi}}{P \cdot q}.$



J/ψ and jet in ep collision : Diagrams



FIG. 1. Feynman diagrams for the partonic process $\gamma^*(q) + g(p_g) \rightarrow J/\psi(\mathbf{P}_{\psi}) + g(\mathbf{P}_j)$.

Calculation of amplitude using NRQCD

The amplitude can be written as

$$M(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}](P_{\psi}) + g)$$

$$= \sum_{L_z S_z} \int \frac{d^3k}{(2\pi)^3} \Psi_{LL_z}(k) \langle LL_z; SS_z | JJ_z \rangle \operatorname{Tr}[\mathcal{O}(q, p, P_{\psi}, k) \mathcal{P}_{SS_z}(P_{\psi}, k)]$$

$$\text{Amplitude for production of } Q\bar{Q} \text{ pair}: \qquad \mathcal{O}(q, p, P_{\psi}, k) = \sum_{k=1}^{8} C_m \mathcal{O}_m(q, p, P_{\psi}, k)$$

m=1

The spin projection operator, $\mathcal{P}_{SS_z}(P_{\psi}, k)$, projects the spin triplet and spin singlet states of $Q\bar{Q}$ pair

$$\mathcal{P}_{SS_{Z}}(P_{\psi},k) = \sum_{s_{1}s_{2}} \left\langle \frac{1}{2}s_{1}; \frac{1}{2}s_{2} \middle| SS_{Z} \right\rangle \nu \left(\frac{P_{\psi}}{2} - k, s_{1} \right) \bar{u} \left(\frac{P_{\psi}}{2} + k, s_{2} \right) \qquad \Pi_{SS_{Z}} = \gamma^{5} \text{ for spin singlet } (S = 0)$$

$$= \frac{1}{4M_{\psi}^{3/2}} \left(-P_{\psi} + 2k + M_{\psi} \right) \Pi_{SS_{Z}}(P_{\psi} + 2k + M_{\psi}) + O(k^{2}) \qquad \Pi_{SS_{Z}} = \epsilon_{S_{Z}}^{\mu}(P_{\psi}) \gamma_{\mu} \text{ for spin triplet } (S = 1)$$

Almost back-to-back production of J/ψ and jet

Contribution comes from the color singlet state $({}^{3}S_{1}^{(1)})$ and color octet states $({}^{3}S_{1}^{(8)}, {}^{1}S_{0}^{(8)}, {}^{3}P_{J(0,1,2)}^{(8)})$

In NRQCD, k, the relative momentum of the charm quark is small. We have Taylor expanded the amplitude about k=0. The first term gives the S wave contribution and second term the p wave contribution $P_{\rm W} \sim K$

Formation of the bound state J/ψ from the heavy quark pair is encoded in the non-perturbative long distance matrix elements (LDMEs). These are obtained by fitting data



Upper bound of the asymmetries : U. D'Alesio, F. Murgia, C. Pisano, and P. Taels *Phys. Rev. D* 100 (2019) 9, 094016

Asymmetry

$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = \frac{\int \mathbf{q}_t \mathrm{d}\mathbf{q}_t \frac{\mathbf{q}_t^2}{M_p^2} \mathbb{B}_0 h_1^{\perp g}(x, \mathbf{q}_t^2)}{\int \mathbf{q}_t \mathrm{d}\mathbf{q}_t \mathbb{A}_0 f_1^g(x, \mathbf{q}_t^2)}.$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}y\mathrm{d}x_{B}\mathrm{d}^{2}\mathbf{q}_{t}\mathrm{d}^{2}\mathbf{K}_{t}} = \frac{1}{(2\pi)^{4}} \frac{1}{16sz(1-z)Q^{4}} \left\{ (\mathbb{A}_{0} + \mathbb{A}_{1}\cos\phi_{\perp} + \mathbb{A}_{2}\cos2\phi_{\perp})f_{1}^{g}(x,\mathbf{q}_{t}^{2}) + \frac{\mathbf{q}_{t}^{2}}{M_{P}^{2}}h_{1}^{\perp g}(x,\mathbf{q}_{t}^{2})(\mathbb{B}_{0}\cos2\phi_{t} + \mathbb{B}_{1}\cos(2\phi_{t} - \phi_{\perp}) + \mathbb{B}_{2}\cos2(\phi_{t} - \phi_{\perp})) + \mathbb{B}_{3}\cos(2\phi_{t} - 3\phi_{\perp}) + \mathbb{B}_{4}\cos(2\phi_{t} - 4\phi_{\perp})) \right\}.$$

Gaussian parametrization of TMDs :

 $f_1^g(x, \mathbf{q}_t^2) = f_1^g(x, \mu) \frac{1}{\pi \langle \mathbf{q}_t^2 \rangle} e^{-\mathbf{q}_t^2 / \langle \mathbf{q}_t^2 \rangle},$

Spectator model :

Spectral function

$$F^{g}(x,\mathbf{q}_{t}^{2}) = \int_{M}^{\infty} dM_{X}\rho_{X}(M_{X})\hat{F}^{g}(x,\mathbf{q}_{t}^{2};M_{X}). \quad \rho_{X}(M_{X}) = \mu^{2a} \left[\frac{A}{B+\mu^{2b}} + \frac{C}{\pi\sigma}e^{-\frac{(M_{X}-D)^{2}}{\sigma^{2}}}\right],$$

$$h_1^{\perp g}(x, \mathbf{q}_t^2) = \frac{M_p^2 f_1^g(x, \mu)}{\pi \langle \mathbf{q}_t^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \frac{\mathbf{q}_t^2}{r \langle \mathbf{q}_t^2 \rangle}},$$

Boer and Pisano, PRD, 2012

 $\langle \mathbf{q}_t^2 \rangle = 0.25 \text{ GeV}^2.$ r=1/3

$$\begin{aligned} \hat{f}_1^g(x, \mathbf{q}_t^2; M_X) &= -\frac{1}{2} g^{ij} [\Phi^{ij}(x, \mathbf{q}_t, S) + \Phi^{ij}(x, \mathbf{q}_t, -S)] \\ &= [(2Mxg_1 - x(M + M_X)g_2)^2 [(M_X - M(1 - x))^2 + \mathbf{q}_t^2] \\ &+ 2\mathbf{q}_t^2 (\mathbf{q}_t^2 + xM_X^2)g_2^2 + 2\mathbf{q}_t^2 M^2 (1 - x)(4g_1^2 - xg_2^2)] [(2\pi)^3 4x M^2 (L_X^2(0) + \mathbf{q}_t^2)^2]^{-1}, \end{aligned}$$

$$\hat{h}_{1}^{\perp g}(x, \mathbf{q}_{t}^{2}; M_{X}) = \frac{M^{2}}{\varepsilon_{t}^{ij} \delta^{jm}(p_{t}^{j} p_{t}^{m} + g^{jm} \mathbf{q}_{t}^{2})} \varepsilon_{t}^{ln} \delta^{nr}[\Phi^{nr}(x, \mathbf{q}_{t}, S) + \Phi^{nr}(x, \mathbf{q}_{t}, -S)]$$
$$= [4M^{2}(1-x)g_{1}^{2} + (L_{X}^{2}(0) + \mathbf{q}_{t}^{2})g_{2}^{2}] \times [(2\pi)^{3}x(L_{X}^{2}(0) + \mathbf{q}_{t}^{2})^{2}]^{-1}.$$

A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taels, Eur. Phys. J. C 80, 733 (2020).

TMD Evolution

Also incorportated TMD evolution in the asymmetry

TMD evolution is done in impact parameter space

Aybat and Rogers, PRD 83, 114042 (2011)

$$\begin{split} \hat{f}(x,\mathbf{b}_{i}^{2},Q_{f}^{2}) &= \frac{1}{2\pi} \sum_{p=q,\bar{q},\bar{q},g} (C_{g/p} \otimes f_{1}^{p})(x,Q_{i}^{2}) \\ &\times e^{-\frac{1}{2}s_{\lambda}(\mathbf{b}_{i}^{2},Q_{i}^{2},Q_{i}^{2})} e^{-S_{np}(\mathbf{b}_{i}^{2},Q_{i}^{2})}, \quad S_{A}(\mathbf{b}_{i}^{2},Q_{j}^{2},Q_{i}^{2}) &= \frac{C_{A}}{\pi} \int_{Q_{i}^{2}}^{Q_{f}^{2}} \frac{d\eta^{2}}{\eta^{2}} \alpha_{s}(\eta) \left(\log \frac{Q_{f}^{2}}{\eta^{2}} - \frac{11-2n_{f}/C_{A}}{6}\right) \\ & S_{np} &= \frac{A}{2} \log \left(\frac{Q_{f}}{Q_{np}}\right) \mathbf{b}_{i}^{2}, \quad Q_{np} = 1 \text{ GeV}. \\ \\ & \text{Scarpa, Boer, Echevarria, Lansberg, Pisano, and Taels, JHEP} \\ (2020) 40. \\ & Q_{f} &= \sqrt{M_{\psi}^{2} + K_{t}^{2}}. \quad A = 2.3 \text{ GeV}^{2} \\ & Q_{i} &= 2e^{-\gamma_{E}}/b_{t} \quad Q_{i} \text{ larger than } Q_{t} \text{ for low } \mathbf{b}_{t} \\ & \text{Final expressions are :} \\ \\ \frac{s_{g1}^{q}(x,\mathbf{q}_{t}^{2}) &= \frac{1}{2\pi} \int_{0}^{\infty} \mathbf{b}_{t} d\mathbf{b}_{t} J_{0}(\mathbf{b}_{t}q_{t}) \left\{ f_{1}^{g}(x,Q_{f}^{2}) - \frac{\alpha_{s}}{2\pi} \left[\left(\frac{C_{A}}{2} \log^{2} \frac{Q_{f}^{2}}{Q_{i}^{2}} - \frac{11C_{A} - 2n_{f}}{6} \log \frac{Q_{f}^{2}}{Q_{i}^{2}} \right) f_{1}^{g}(x,Q_{f}^{2}) \\ & + \left(P_{gg} \otimes f_{1}^{g} + P_{gi} \otimes f_{1}^{i}\right)(x,Q_{f}^{2}\right) \log \frac{Q_{f}^{2}}{Q_{i}^{2}} - 2f_{1}^{g}(x,Q_{f}^{2}) \right] \right\} \times e^{-S_{np}(\mathbf{b}_{i}^{2})}. \\ \end{array}$$

Numerical estimate of the asymmetry



y = 0.3 In upper panels $\sqrt{s} = 140 \ GeV$ $K_t = 0.2 \ GeV$ In lower panels

Result in spectator model in the kinematics considered overlaps with the upper bound saturating the positivity bound

Result is Gaussian parametrization lower than in spectator model

Asymmetries in CS smaller than in NRQCD

Raj Kishore, AM, Amol Pawar, M. Siddiqah, *Phys.Rev.D* 106 (2022) 3, 034009

TMD evolution

In impact parameter space, the perturbative part of the TMDs evolved from initial to final scale can be written as

Aybat and Rogers (2011)

$$\widehat{F}(x, b_T, \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T; \zeta, \zeta_0, \mu, \mu_0)} \widehat{F}(x, b_T, \zeta_0, \mu_0)$$

 S_A the perturbative Sudakov factor that resums large UV and rapidity logs : same for unpolarized and polarized TMD valid in the perturbative domain: $|b_T| \ll 1/\Lambda_{OCD}$

$$S_{A}(\mathbf{b}_{t}^{2}, Q_{f}^{2}, Q_{i}^{2}) = \frac{C_{A}}{\pi} \int_{Q_{i}^{2}}^{Q_{f}^{2}} \frac{d\eta^{2}}{\eta^{2}} \alpha_{s}(\eta) \left(\log \frac{Q_{f}^{2}}{\eta^{2}} - \frac{11 - 2n_{f}/C_{A}}{6} \right)$$
For $b_{T} \ll \Lambda_{QCD}^{-1}$, perturbative tails of TMDs at the initial scale can be expressed by OPE:

$$= \frac{C_{A}}{\pi} \alpha_{s} \left(\frac{1}{2} \log^{2} \frac{Q_{f}^{2}}{Q_{i}^{2}} - \frac{11 - 2n_{f}/C_{A}}{6} \log \frac{Q_{f}^{2}}{Q_{i}^{2}} \right).$$
For $b_{T} \ll \Lambda_{QCD}^{-1}$, perturbative tails of TMDs at the initial scale can be expressed by OPE:

$$\widehat{f}_{g/A}(x, b_{T}, \mu_{0}, \zeta_{0}) = \sum_{j=q, \bar{q}, g} C_{g/j}(x, b_{T}; \mu_{0}, \zeta_{0}) \otimes f_{j/A}(\hat{x}, \mu_{0})$$
Perturbative Wilson coeffs are dependent on specific TMD
Holds in small b_T region

TMD evolution

Wilson coefficients can be expanded in powers of α_S as

$$C_{g/a}(x;\mu_b^2) = \delta_{ga}\delta(1-x) + \sum_{k=1}^{\infty} C_{g/a}^k(x) \left(\frac{\alpha_s(\mu_b)}{\pi}\right)^k$$

We need to introduce a non-perturbative Sudakov factor that freezes the perturbative contribution slowly as b_T gets larger.

$$\widehat{F}(x, b_T, \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T^*; \zeta, \zeta_0, \mu, \mu_0)} \widehat{F}(x, b_T^*, \zeta_0, \mu_0) e^{-S_{NP}(x, b_T)}$$

Fourier transform needs entire b_T region. But applicability of the perturbative expression confined in the range $b_0/Q < b_T < b_{T_{max}}$.

For small b_T region μ_b exceeds hard scale, so evolution should stop.

TMD evolution

$$b_T^*(b_c(b_T)) = \frac{b_c(b_T)}{\sqrt{1 + \left(\frac{b_c(b_T)}{b_{T_{\text{max}}}}\right)}} \qquad b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{Q}\right)^2}. \qquad b_{T_{\text{max}}} = 1.5 \text{ GeV}^{-1}.$$

This ensures that $\mu_{b^*} = \frac{b_0}{b_T^*(b_c)}$ Always lies between Q and $b_0/b_{T_{\text{max}}}$ When b_T tends to 0 and infinity respectively

Non-perturbative factor suppresses the perturbative contribution for large b_T , should be equal to 1 for small b_T for large b_T should decrease monotonically to zero typically within the confinement distance.

Result of TMD evolution cannot be uniquely predicted until the non-perturbative part is extracted from the data : plays an important role in the evolution. Different ansatz exist constrained by the above conditions.

Non-perturbative factor (gluon TMD)



(a)
$$S_{NP}(b_c(b_T)) = \frac{A}{2} \ln\left(\frac{Q}{Q_{NP}}\right) b_c^2(b_T), \qquad Q_{NP} = 1 \text{ GeV}.$$

Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, Eur. Phys. J. C 80, 87 (2020).

(b)
$$S_{NP}(b_T; Q) = \left[A \ln \frac{Q}{Q_{NP}} + B(x)\right] b_T^2,$$

J. Bor and D. Boer, Phys. Rev. D 106, 014030 (2022).

 $b_{t;lim}$ Defined from the non-pert factor 'such that exp(-S_{NP}) becomes negligible at a distance

A controls the width of the nonperturbative Sudakov factor for a particular Q. Obtained at $b_T = b_{T, lim}$

TABLE I. Value of parameter A used in $e^{-S_{NP}}$ evaluated at Q = 4.47 GeV.

$A (\text{GeV}^2)$	$b_{t;\text{lim}} \; (\text{GeV}^{-1})$	<i>r</i> (fm)
2.2697727	2	0.2
0.57415574	4	0.4
0.14395514	8	0.8

TABLE II. Values of the parameters A and B used in $e^{-S_{NP}}$ evaluated at Q = 12 GeV.

$A (\text{GeV}^2)$	$b_{t;\lim}$ (GeV ⁻¹)	<i>r</i> (fm)	x	B(x)
0.80	2	0.2	0.003	0.6080
0.20	4	0.4	0.01	0.5211
0.05	8	0.8	0.05	0.4655

Approach-A

Expand $e^{-\frac{1}{2}S_A} \rightarrow 1 - S_A/2$ and coefficient function $C_{g/j}$ to $\sigma(\alpha_S)$

$$S_A(\boldsymbol{b}_T^2, \mu^2) = \frac{C_A}{\pi} \int_{\mu_b^2}^{\mu^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2} \alpha_s(\bar{\mu}) \left(\ln \frac{\mu^2}{\bar{\mu}^2} - \frac{11 - 2n_f/C_A}{6} \right)$$
$$= \frac{C_A}{\pi} \alpha_s \left(\frac{1}{2} \ln^2 \frac{\mu^2}{\mu_b^2} - \frac{11 - 2n_f/C_A}{6} \ln \frac{\mu^2}{\mu_b^2} \right). \quad (1)$$

D. Boer, U. D'Alesio, F. Murgia, C. Pisano, and P. Taels, J. High Energy Phys. 09 (2020) 040.

Running of coupling is neglected)

$$\begin{split} \hat{f}_{1}^{g}(x,b_{T};\mu) &= f_{g/A}(x;\mu_{b}) \\ &- \frac{\alpha_{S}}{2\pi} \left[\left(\frac{C_{A}}{2} \ln^{2} \frac{\mu^{2}}{\mu_{b}^{2}} - \frac{11C_{A} - 2n_{f}}{6} \ln \frac{\mu^{2}}{\mu_{b}^{2}} \right) f_{g/A}(x;\mu_{b}) + \sum_{i=q,\bar{q},g} \int_{x}^{1} \frac{d\hat{x}}{\hat{x}} C_{g/i}^{1}(x;\mu_{b}) f_{i/A}\left(\frac{\hat{x}}{x};\mu_{b}\right) \right] e^{-S_{NP}(x,b_{T})} \\ &+ \sigma(\alpha_{S}^{2}) \end{split}$$

At inpute scale,
$$\mu_b$$
: $C_{g/g}^1 = -\frac{\pi^2}{12}\delta(1-x)$ $C_{g/q}^1 = C_{g/\bar{q}}^1 = C_F x$

Approach-A

$$\begin{split} \hat{h}_{1}^{\perp g}(x, b_{T}; \mu) \\ &= \left[\frac{C_{A} \alpha_{S}(\mu_{b})}{\pi} \int_{x}^{1} \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/A}(\hat{x}, \mu_{b}^{2}) + \frac{C_{F} \alpha_{S}(\mu_{b})}{\pi} \sum_{j=q,\bar{q}} \int_{x}^{1} \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{j/A}(\hat{x}, \mu_{b}^{2}) \right] e^{-S_{NP}(x, b_{T})} + \sigma(\alpha_{S}^{2}) \\ C_{g/g}^{[1]} &= \frac{\alpha_{s}}{\pi} C_{A} \left(\frac{\hat{x}}{x} - 1 \right), \qquad S_{NP}(b_{T}; Q) = A \ln\left(\frac{Q}{Q_{NP}} \right) b_{c}^{2}(b_{T}), \qquad Q_{NP} = 1 \text{ GeV} \\ C_{g/i=q,\bar{q}}^{[1]} &= \frac{\alpha_{s}}{\pi} C_{F} \left(\frac{\hat{x}}{x} - 1 \right). \qquad b_{c}(b_{T}) = \sqrt{b_{T}^{2} + \left(\frac{b_{0}}{Q} \right)^{2}} \quad and \quad b_{T}^{*}(b_{T}) = \frac{b_{c}}{\sqrt{1 + (b_{c}/b_{Tmax})^{2}}} \end{split}$$

A is fixed by defining a b_{Tlim} such that $e^{-S_{NP}}$ becomes negligible (~10⁻³) for a given Q

To estimate uncertainty, we consider $b_{Tlim} = 2, 4$ and 8 GeV^{-1}

TABLE I. Value of parameter A used in $e^{-S_{NP}}$ evaluated at Q = 4.47 GeV.

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Approach-B

Perturbative tails of f_1^g and $h_1^{\perp g}$ given by integrated PDF; consider only leading order terms:

$$\hat{f}_1^g(x, b_T; \mu_0, \zeta_0) = f_{g/A}(x; \mu_0) + \sigma(\alpha_S) + \sigma(b_T \Lambda_{QCD})$$

 $h_1^{\perp g}$ requires an additional gluon exchange (helicity flip); perturbative tails starts at $\sigma(\alpha_s)$

$$\hat{h}_{1}^{\perp g}(x, b_{T}; \mu_{0}, \zeta_{0}) = \frac{C_{A}\alpha_{S}(\mu_{0})}{\pi} \int_{x}^{1} \frac{\mathrm{d}\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{g/A}(\hat{x}, \mu_{0}^{2}) + \frac{C_{F}\alpha_{S}(\mu_{0})}{\pi} \sum_{j=q,\bar{q}} \int_{x}^{1} \frac{\mathrm{d}\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{j/A}(\hat{x}, \mu_{0}^{2}) + \sigma(\alpha_{S}) + \sigma(b_{T}\Lambda_{QCD})$$

Perturbative Sudakov factor S_A , as well as one-loop running of α_S ; we set $\mu \sim \sqrt{\zeta} \sim Q$ and $\mu_0 \sim \sqrt{\zeta_0} \sim \mu_b$ $S_A(b_T; Q, \mu_b) = \frac{36}{33 - 2n_f} \left[\ln \frac{Q^2}{\mu_b^2} + \ln \frac{Q^2}{\Lambda_{OCD}^2} \ln \left(1 - \frac{\ln(Q^2/\mu_b^2)}{\ln(Q^2/\Lambda_{OCD}^2)} \right) + \left(\frac{11 - 2n_f/C_A}{6} \right) \ln \left(\frac{\ln(Q^2/\Lambda_{QCD}^2)}{\ln(\mu_b^2/\Lambda_{OCD}^2)} \right) \right] + \sigma(\alpha_S^2)$

Same for both unpol and linearly polarized TMDs

Approach-B

For semi-inclusive production of J/ψ , one also needs to include the shape function in TMD framework

This incorporates the smearing effect due to the transverse momentum of the soft gluon emitted during the formation of the bound state-also pays a role in the resummation of soft gluons

Shape functions can in general be process dependent

J. Bor and D. Boer, Phys. Rev. D 106, 014030 (2022), D. Boer, J. Bor, L. Maxia, C. Pisano, and F. Yuan, J. High Energy Phys. 08 (2023) 105, M. G. Echevarria, J. High Energy Phys. 10 (2019) 144, S. Fleming, Y. Makris, and T. Mehen, J. High Energy Phys. 04 (2020) 122; L. Maxia, D. Boer, J. Bor 2504.19617 [hep-ph]

 μ_h

We have chosen an initial scale μ'_{b}

And a slightly different b^*_T prescription

$$S_{NP}(b_T; Q) = \left[A \ln \frac{Q}{Q_{NP}} + B(x)\right] b_T^2,$$

D. Boer and W.J. den Dunnen, Nucl. Phys. B886, 421 (2014).

Approach-A and B

$$\begin{split} f_1^g(x, \boldsymbol{q}_T^2, \mu) &= \int_0^\infty \frac{db_T}{2\pi} b_T J_0(b_T q_T) f_1^g(x; \mu'_{b^*}) e^{-S_A(b_T^*; \mathcal{Q}, \mu'_{b^*})} e^{-S_{NP}(b_T; \mathcal{Q})}, \\ \frac{q_T^2}{2M_P^2} h_1^{\perp g}(x, \boldsymbol{q}_T^2, \mu) &= \int_0^\infty \frac{db_T}{2\pi} b_T J_2(b_T q_T) \left[\frac{\alpha_s(\mu'_{b^*})}{\pi} \int_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1 \right) \left\{ C_A f_1^g(x'; \mu'_{b^*}) \right. \\ &+ C_F \sum_{i=q,\bar{q}} f_1^i(x'; \mu'_{b^*}) \right\} \right] e^{-S_A(b_T^*; \mathcal{Q}, \mu'_{b^*})} e^{-S_{NP}(b_T; \mathcal{Q})}. \end{split}$$

Approach A : expanded the evolution kernel $e^{-1/2S_A}$, at leading log in resummation, to fixed order in α_s and considered the perturbative part of TMDs up to O(α_s).

Approach B : considered only LO terms in the perturbative tails of TMDs multiplied with the evolution kernel, in the exponent only the leading order terms. Included the effect of the running of α_s .



Significant different in the two approaches for larger b_{t,lim} : due to difference in non-perturbative factor



Numerical Results



Same non-perturbative factor

$$S_{NP}(b_T;Q) = \left[A\ln\frac{Q}{Q_{NP}} + B(x)\right]b_T^2,$$

Not so much difference between scheme A and B at low values of Q²

.

Non-perturbative Sudakov factor mainly affects the asymmetry , asymmetry smaller at smaller value of x

Numerical results



Asymmetry plotted at a higher value of Q² : there is significant difference between the two approaches in two different S

Asymmetries are different for larger values of $b_{t,lim}$: upper part of the band

Shows the effect of higher powers of the large logarithmic terms in the perturbative Sudakov kernel, in this kinematical region, which were not included in approach A as we expanded the exponent to a fixed order in α_s .

Sudakov factors



Sudakov factors in the two approaches for two different values of b_{t,lim}

Same non-perturbative factor

$$S_{NP}(b_T;Q) = \left[A\lnrac{Q}{Q_{NP}} + B(x)
ight]b_T^2,$$

R. Kishore, AM, A. Pawar, S. Rajesh, M. Siddiqah, PRD 111, 014003 (2025)

Behaviour similar for low $b_{t,lim}$ but large logs are more pronounced when $b_{t,lim}$ is large : this causes the difference in the asymmetry

Effect of LDME set



CMSWZ : K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, and Y.-J.Zhang, Phys. Rev. Lett. 108, 242004 (2012).BK : M. Butenschön and B.A. Kniehl, Phys. Rev. Lett. 106,

022003 (2011).

SV : R. Sharma and I. Vitev, Phys. Rev. C 87, 044905 (2013). ZSSL : Zhang, Z. Sun, W.-L. Sang, and R. Li, Phys. Rev. Lett. 114, 092006 (2015). Uncertainty in the LDME sets introduces uncertainty in the prediction of the asymmetry

BK : gives largest asymmetry

Effect of LDME set



Asymmetry for two different LDME sets

Uncertainty band due to the parameters also depends on the LDME set chosen. Uncertainty more for CMSWZ set

Summary and conclusion

Presented a calculation of cos 2ϕ azimuthal asymmetry in ep -> J/ ψ + jet at the EIC : useful for the extraction of linearly polarized gluon distribution

Investigated the effect of TMD evolution in detail on the asymmetry in the kinematics of EIC

Approach A : expanded the evolution kernel at leading log in resummation, to fixed order in α_s and considered the perturbative part of TMDs up to $O(\alpha_s)$. Approach B : considered only LO terms in the perturbative tails of TMDs multiplied with the evolution kernel, in the exponent only the leading order terms. Included the effect of the running of α_s .

The non-perturbative factor for gluon TMDs is largely unknow, we investigated the effect of two different forms. Significant uncertainty band was seen in the case of S_{NP} with a larger width, allowing more contribution of the perturbative part in the higher b_T region.

We found the the perturbative part in both approaches have a similar influence on the asymmetry at a relatively low scale, but the effect is different at a larger scale,

LDMEs introduce uncertainty in the prediction of the asymmetry.

EIC-India Groups





EIC-India Groups

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EIC-India Theory Alliance (EITA)

Efforts from the Indian theory groups towards EIC-Science

SI. No.	Institutions	Primary Research Area	
1	Aligarh Muslim University, Aligarh	 Generalized Parton Distributions (CPDs) and 	
2	Indian Institute of Technology (IIT), Bombay	Nucleon Spin	
3	Indian Institute of Technology (IIT), Kanpur	 Transverse Momentum 	
4	Indian Institute of Science Education and Research (IISER), Berhampur	Dependents (TMDs)	
5	National Institute of Science Education and Research (NISER), Jatni	 Color Glass Condensate (CGC) and Gluon Saturations 	
6	National Institute of Technology (NIT), Kurukshetra	 Small-x physics 	
7	National Institute of Technology (NIT), Jalandhar	 Jets in EIC 	
8	Tata Institute of Fundamental Research (TIFR), Mumbai	Precision eP physics	
9	The Institute of Mathematical Sciences (IMSc), Chennai	riccision er physics	
10	Vellore Institute of Technology (VIT), Vellore	 Hadron spectroscopy in lattice 	