Model Calculations of the Mechanical Properties of the Nucleon

ASMITA MUKHERJEE

Indian Institute of Technology Bombay

Origin of the Visible Universe: Unraveling the Proton Mass
June 13-17, 2022, INT, Seattle

Based on :

Chakrabarti, Mondal, AM, Nair, Zhao; Phys.Rev.D 102 ,113011 (2020)

More, AM, Nair, Saha ; Physical Review D 105, 056017 (2022)

Supported by DST-SERB POWER FELLOWSHIP, India
(**Promoting Opportunities for Women in Exploratory Research)**

Gravitational Form Factors (GFFs)

$$\langle P', S'|T_i^{\mu\nu}(0)|P, S\rangle = \bar{U}(P', S')\Big[-B_i(q^2)\frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2) + B_i(q^2))\frac{1}{2}(\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu)$$
$$+ C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu}\Big] U(P, S),$$

$$\bar{P}^\mu = \frac{1}{2}(P' + P)^\mu,\ q^\mu = (P' - P)^\mu$$

We choose Drell-Yan frame $Q^2 = -q^2 = \vec{q}_\perp^{\,2}$

$$P = (P^+, P_\perp, P^-) = \left(P^+, 0, \frac{M^2}{P+}\right),$$

$$P' = (P'^+, P'_\perp, P'^-) = \left(P^+, q_\perp, \frac{q_\perp^2 + M^2}{P+}\right)$$

$$q = P' - P = \left(0, q_\perp, \frac{q_\perp^2}{P+}\right),$$

GFFs give how matter couples to gravity

GFFS

$A(Q^2)$ and $B(Q^2)$ are related to the mass and angular momentum of the proton

$$\int dx x[H_q(x,0,0) + E_q(x,0,0)] = A_q(0) + B_q(0) = 2J_q$$

X. Ji, PRD, 1997

H_q and E_q are generalized parton distributions (GPDs) that can be accessed in exclusive processes like DVCS or deeply virtual meson production

Poincare invariance imposes the constraint :

$$\sum_{a=q,g} A_a(0) = 1, \quad \sum_{a=q,g} B_a(0) = 0, \quad \sum_{a=q,g} \bar{C}_a(t) = 0.$$

Total gravitomagnetic moment is zero follows from the equivalence principle of GTR

$\bar{C}(Q^2)$ arises due to non-conservation of EM tensor separately for quarks and gluons, and must vanish when summed over both

Lorce, Moutarde, Trawinski, EPJC (2019)

GFFs and Pressure Distribution

However, $C(Q^2)$, also called the D-term, is not related to any Poincare generator and is unconstrained

D term is related to the pressure and shear force distributions inside the nucleon

Polyakov and Schweitzer, IJMPA (2018)

Recent result from Jlab showed that the pressure distribution at the center of the nucleon is repulsive and confining towards the outer region

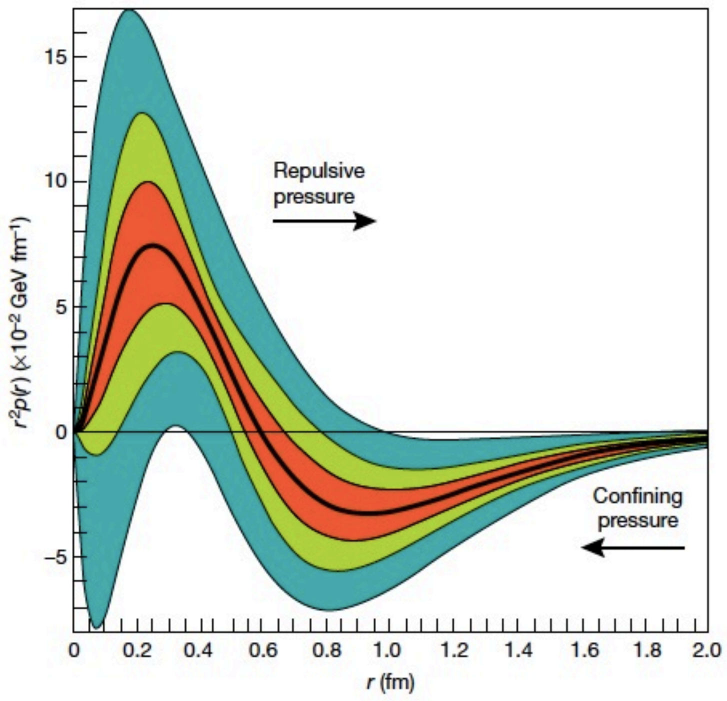
Burkert, Elouadrhiri, Girod, Nature(2018)

This also connects a set of collider observables (GPDs) to the investigation of the equation of state (EoS) of neutron stars

Rajan, Gorda, Liuti, Yagi (2018)

Quite a lot of theoretical calculations in recent days

Pressure distribution inside the nucleon



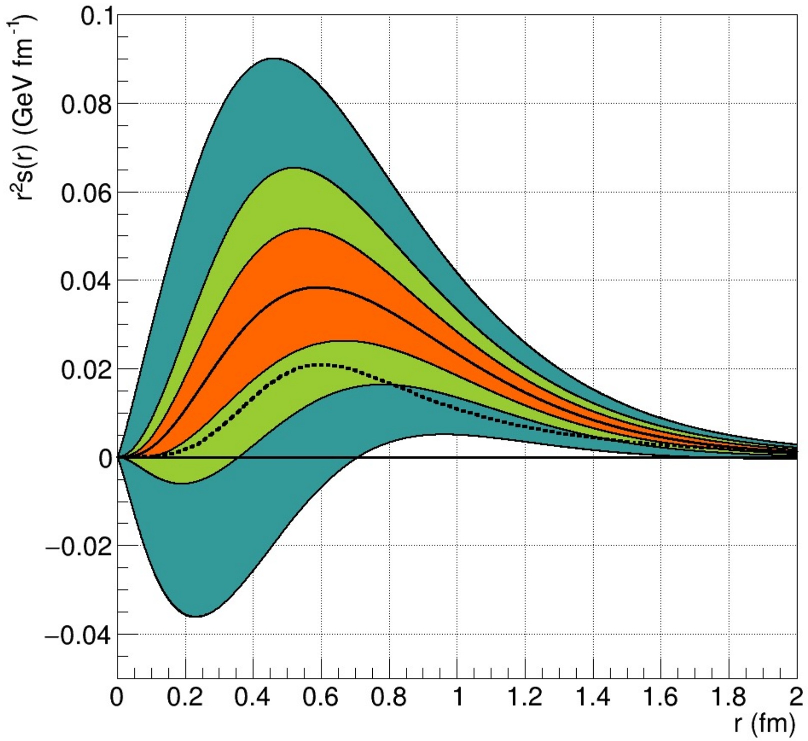
Pressure distribution obtained from fits to Jlab data to extract the GPDs, in particular the D-term

Pressure distribution is repulsive at the center of the nucleon and confining in the outer region

At the core it exceeds the pressure density of the most dense object that is neutron star , average peak pressure 10^{35} Pascals

Burkert, Elouadrhiri, Girod, Nature(2018)

Shear Distribution Inside the Nucleon



Shear (tangential) force inside the nucleon from DVCS data at JLab

Maximum shear force at 0.6 fm from the center of the nucleon : confinement may be dominant

Shear forces change direction at r=0.45 fm from the center

Burkert, Elouadrhiri, Girod, 2104.02031[nucl-ex]

GFFs and Pressure Distribution

Jlab result triggered a lot of interest : theoretical model calculations of the pressure distributions

Polyakov and Schweitzer, IJMPA (2018)

Most calculations are done in the Breit frame and are subject to relativistic corrections

2-D distributions in the infinite momentum frame or light-front formalism introduced in

Lorce, Moutarde, Trawinski, EPJC (2019), Freese and Miller, PRD(2021)

Because of transverse Galilean symmetry on the light-front these are free from relativistic corrections

Connection between 2D and 3D pressure distributions can be established through Abel transformation

Panteleeva and Polyakov, 2021

Model calculation of GFFs

Light-front quark diquark model, where the two-particle LFWFs are modelled from the solution of soft-wall Ads/QCD

Chakrabarti, Mondal (2013)

Included only scalar diquarks in this work

$$|P;\uparrow(\downarrow)\rangle = \sum_q \int \frac{dx\, d^2\mathbf{k}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \left[\psi_{+q}^{\uparrow(\downarrow)}(x,\mathbf{k}_\perp)|+\frac{1}{2},0;xP^+,\mathbf{k}_\perp\rangle + \psi_{-q}^{\uparrow(\downarrow)}(x,\mathbf{k}_\perp)|-\frac{1}{2},0;xP^+,\mathbf{k}_\perp\rangle\right].$$

$$\psi_{+q}^{\uparrow}(x,\mathbf{k}_\perp) = \varphi_q^{(1)}(x,\mathbf{k}_\perp),$$
$$\psi_{-q}^{\uparrow}(x,\mathbf{k}_\perp) = -\frac{k^1+ik^2}{xM}\varphi_q^{(2)}(x,\mathbf{k}_\perp),$$
$$\psi_{+q}^{\downarrow}(x,\mathbf{k}_\perp) = \frac{k^1-ik^2}{xM}\varphi_q^{(2)}(x,\mathbf{k}_\perp).$$
$$\psi_{-q}^{\downarrow}(x,\mathbf{k}_\perp) = \varphi_q^{(1)}(x,\mathbf{k}_\perp),$$

$$\varphi_q^{(i)}(x,\mathbf{k}_\perp) = N_q^{(i)}\frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\,x^{a_q^{(i)}}(1-x)^{b_q^{(i)}}\exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

Ads/QCD scale parameter $\kappa = 0.4\ \mathrm{GeV}$

Parameters are obtained from fits of electromagnetic form factors

Model Calculations of GFFs

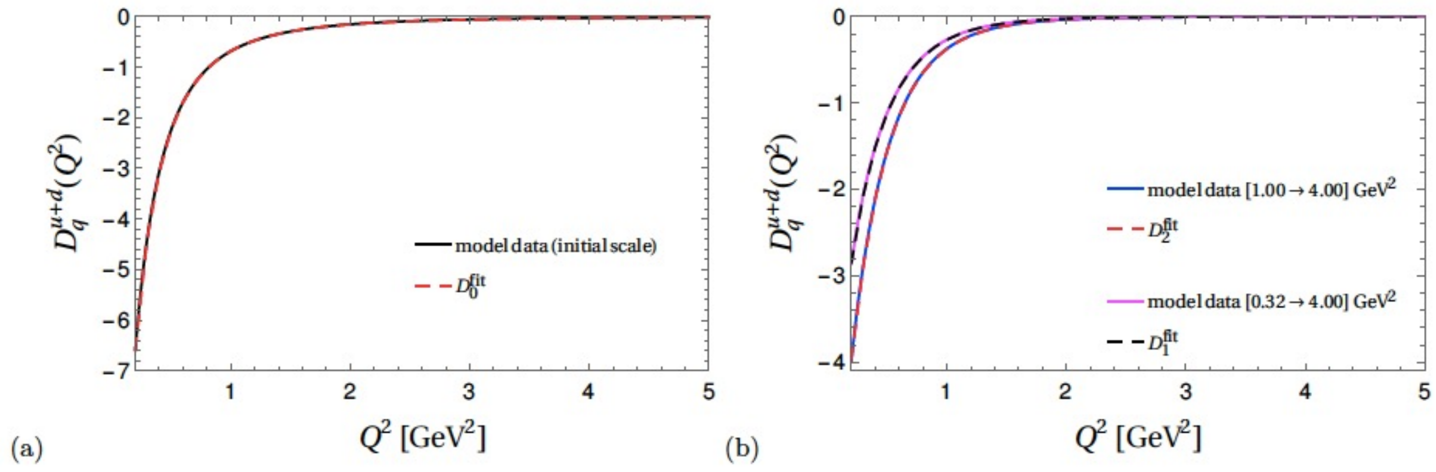
$$A^q(Q^2) = \mathcal{I}_1^q(Q^2),$$
$$B^q(Q^2) = \mathcal{I}_2^q(Q^2),$$
$$C^q(Q^2) = -\frac{1}{4Q^2}\left[2M^2\mathcal{I}_1^q(Q^2) - Q^2\mathcal{I}_2^q(Q^2) - \mathcal{I}_3^q(Q^2)\right],$$
$$\bar{C}^q(Q^2) = -\frac{1}{4M^2}\left[\mathcal{I}_3^q(Q^2) - \mathcal{I}_4^q(Q^2)\right],$$

$C(Q^2)$ diverges as Q^2=0 ; artifact of the LFWFs

To avoid this we fit $C(Q^2)$ in the region Q^2>0.1 GeV^2 and analytically continue to low Q^2

$$\mathcal{I}_1^q(Q^2) = \int dx x\Big[{N_1}^2 x^{2a_1}(1-x)^{2b_1+1} + {N_2}^2 x^{2a_2-2}(1-x)^{2b_2+3}\frac{1}{M^2}\Big(\frac{\kappa^2}{\log(1/x)} - \frac{Q^2}{4}\Big)\Big]\exp\Big[-\frac{\log(1/x)}{\kappa^2}\frac{Q^2}{4}\Big],$$
$$\mathcal{I}_2^q(Q^2) = 2\int dx\ N_1 N_2 x^{a_1+a_2}(1-x)^{b_1+b_2+2}\exp\Big[-\frac{\log(1/x)}{\kappa^2}\frac{Q^2}{4}\Big],$$
$$\mathcal{I}_3^q(Q^2) = 2\int dx N_2 N_1 x^{a_1+a_2-2}(1-x)^{b_1+b_2+2} \times\Big[\frac{4(1-x)^2\kappa^2}{\log(1/x)} + Q^2(1-x)^2 - 4m^2\Big]\exp\Big[-\frac{\log(1/x)}{\kappa^2}\frac{Q^2}{4}\Big],$$
$$\mathcal{I}_4^q(Q^2) = -2\int dx\ N_2 N_1 x^{a_1+a_2-2}(1-x)^{b_1+b_2+2}\Big[\frac{\kappa^2(1-x)^2}{\log(1/x)} + \frac{Q^2(1-x)^2}{4} + m^2\Big]\exp\Big[-\frac{\log(1/x)}{\kappa^2}\frac{Q^2}{4}\Big].$$

GFF $C(Q^2)$



$C(Q^2)$ diverges as $Q^2=0$; artifact of the LFWFs

To avoid this we fit $C(Q^2)$ in the region $Q^2>0.1$ GeV^2 and analytically continue to low Q^2

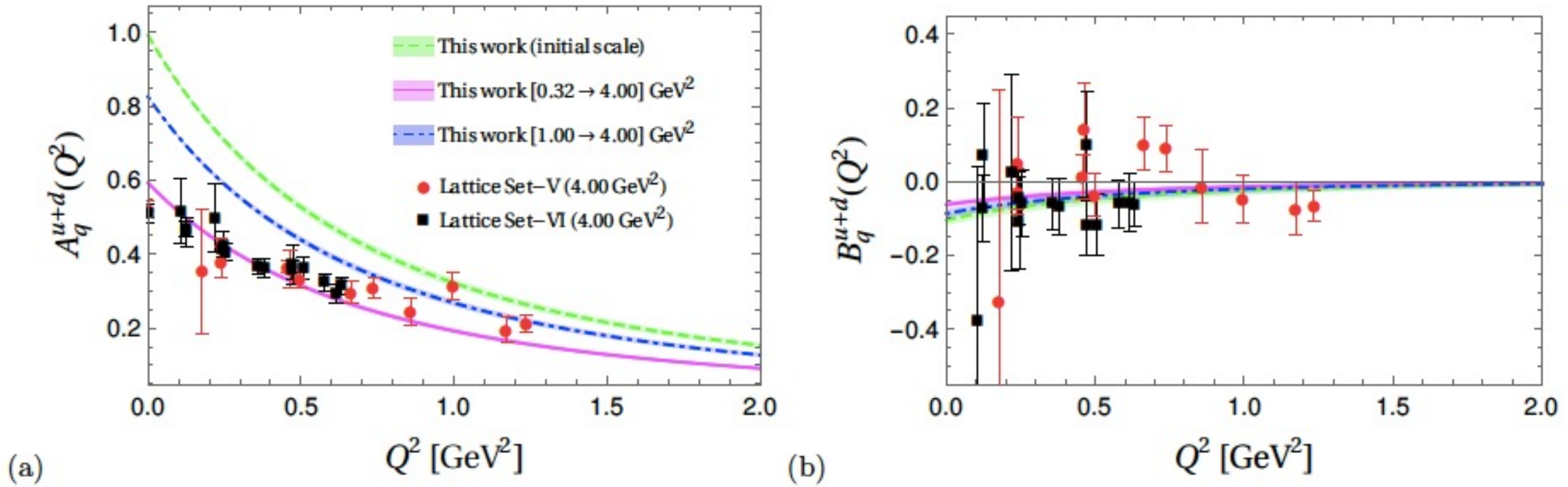
Multipole function used for the fit

$$D^q_{\text{fit}}(Q^2) = 4C^q_{\text{fit}}(Q^2) = \frac{a_q}{(1+b_q Q^2)^{c_q}},$$

Scale evolution is encoded in the model parameters which are considered to be scale dependent, and evolved in a way that the pdfs obey the DGLAP equation; following the approach of

Maji, Mondal, Chakrabarti; PRD (2017)

Numerical results for the GFFs

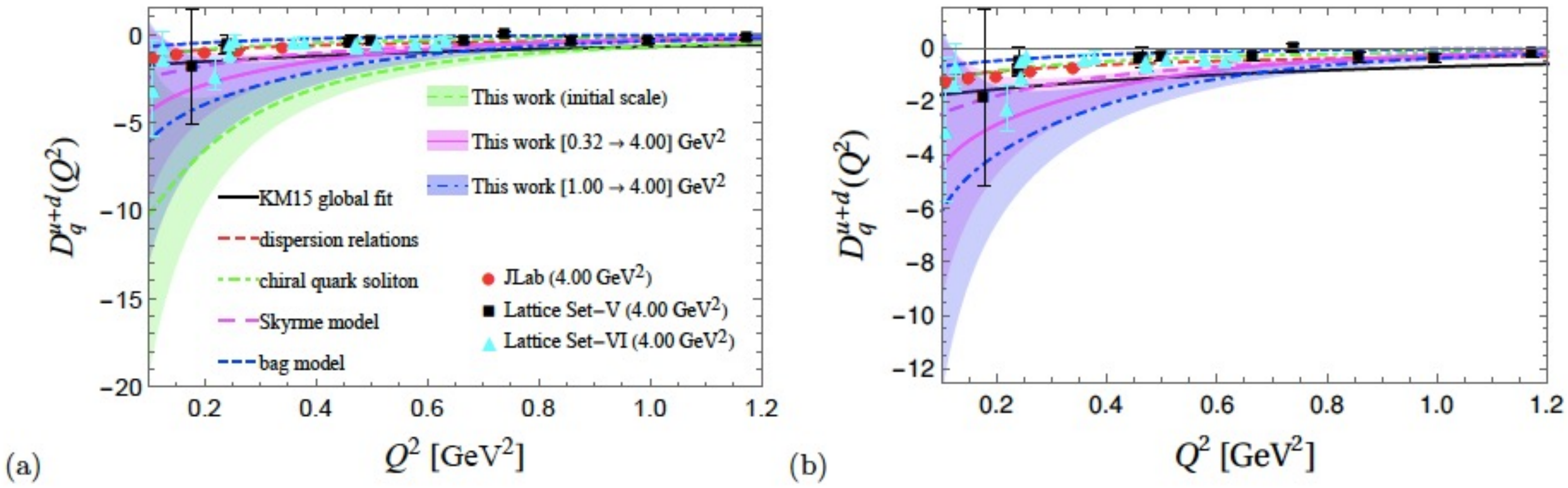


Lattice: LHPC Collaboration, PRD (2008)

To compare with lattice result we evolve our results to $\mu^2 = 4$ GeV^2 using DGLAP evolution (integrands in our model calculation are $xH_q(x, Q^2)$ and $x\,E_q(x, Q^2)$ respectively)

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

D form factor in a diquark model



$$D_q^{u+d}(Q^2) = 4C_q^{u+d}(Q^2)$$

KM15 : Kumericki and Muller (2016)

Dispersion relation : Pasquini, Polayakov and Vanderhaeghen (2014)

Chiral Quark Soliton : K. Goeke et al (2007)

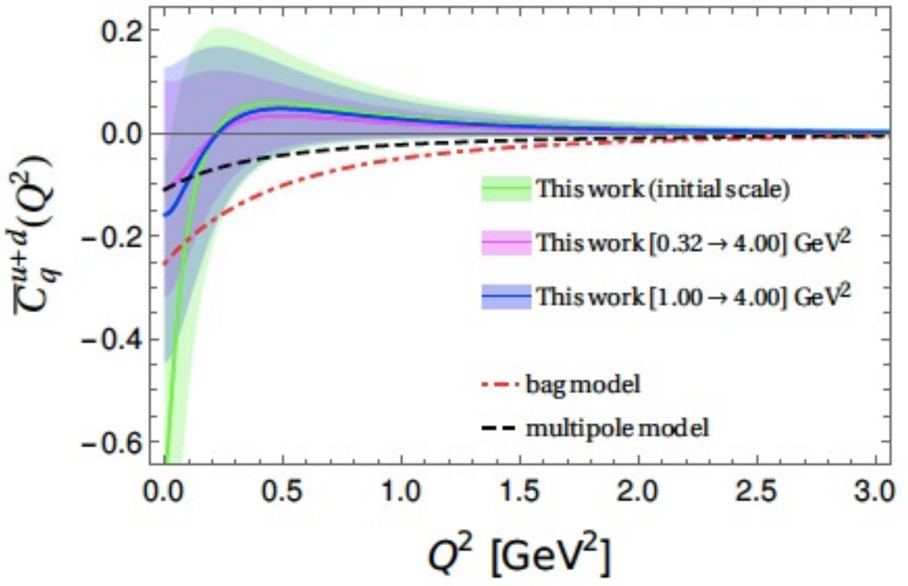
Skyrme mdel : Cebulla et al (2007)

Bag model : X. Ji et al (1997)

Uncertainty shows the GFFs depend strongly on model parameters

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

GFFs in a diquark model



Multipole model : Lorce, Moutarde, Trawinski (2019)

Bag model : X. Ji et al (1997)

$\bar{C}(Q^2)$ Is negative for small Q^2 but positive for higher values of Q^2

In contrast with other models, where it is negative

Error bands correspond to 2% uncertainty in model parameters

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

Pressure and Energy Density at the Center

Pressure and energy density at the center of the nucleon

$$p_0 = -\frac{1}{24\pi^2 M_n}\int_0^\infty \mathrm{d}Q^2\ Q^3\ D(Q^2),$$

$$\mathcal{E} = \frac{M_n}{4\pi^2}\int_0^\infty \mathrm{d}Q^2 \left(A(Q^2) + \frac{Q^2}{4M_n^2}D(Q^2)\right),$$

Mechanical radius

$$\langle r^2_{\mathrm{mech}}\rangle = 6D_{\mathrm{fit}}(0)\Big[\int_0^\infty \mathrm{d}Q^2\ D(Q^2)\Big]^{-1}.$$

TABLE III. The mechanical properties: pressure, energy density, and mechanical radius of nucleon.

Approaches/Models	p_0 [GeV/fm^3]	$\mathcal{E}$ [GeV/fm^3]	$\langle r^2_{\mathrm{mech}}\rangle$ [fm^2]
This work ($\sqrt{0.32}$ GeV $\to$ 2 GeV)	0.29	3.21	0.74
This work (1.00 GeV $\to$ 2 GeV)	0.40	4.58	0.74
QCDSR set-I (1 GeV) [8]	0.67	1.76	0.54
QCDSR set-II (1 GeV) [8]	0.62	1.74	0.52
Skyrme model [10]	0.47	2.28	-
modified Skyrme model [11]	0.26	1.445	-
χQSM [7]	0.23	1.70	-
Soliton model [9]	0.58	3.56	-
LCSM-LO [4]	0.84	0.92	0.54

Pressure and Shear Distribution

Pressure and shear force distributions are defined as

$$p(b) = \frac{1}{6M_n}\frac{1}{b^2}\frac{d}{db}b^2\frac{d}{db}\tilde{D}(b),$$
$$s(b) = -\frac{1}{4M_n}b\frac{d}{db}\frac{1}{b}\frac{d}{db}\tilde{D}(b),$$

Where

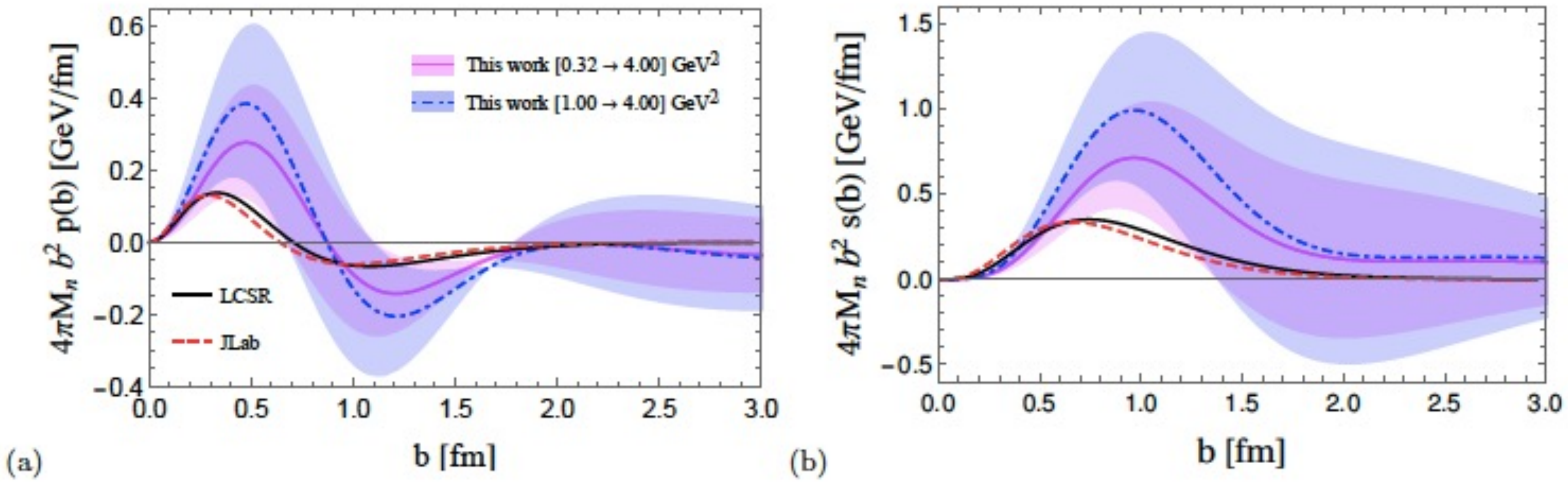
$$\tilde{D}(b) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^3} e^{i\vec{q}_\perp\cdot\vec{b}_\perp} D(Q^2)$$

The pressure distribution has to obey the von Laue condition for stability $\int_0^\infty db\, b^2\, p(b) = 0.$

This is a consequence of the conservation of energy-momentum tensor

b represents the impact parameter

Pressure and Shear Distribution



Pressure and shear distribution compared with Burkert, Elouadrhiri, Girod, Nature(2018) and Light cone sum rule approach Anikin, PRD (2019)

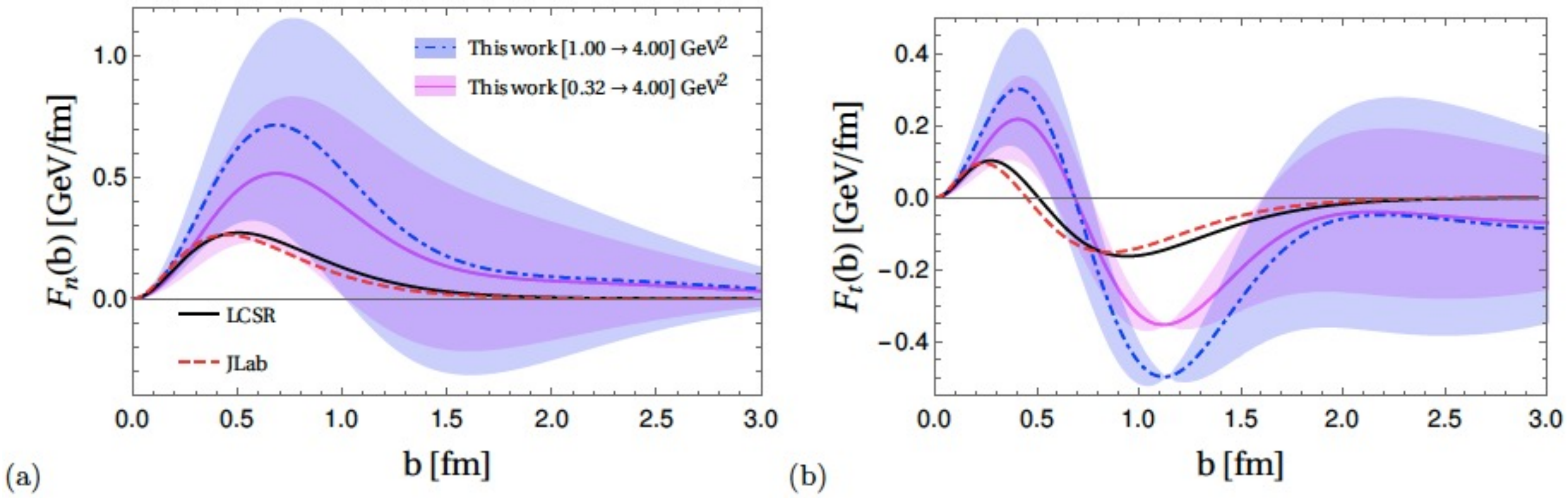
Qualitative behavior similar to other calculations

Pattern ensures mechanical stability: repulsive core prevents the system from collapsing and the attractive Force away from center binds the system.

Shear distribution : related to surface tension and surface energy; which are positive in stable hydrostatic systems.

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

Normal and tangential force



A spherical shell of radius b inside the nucleon experiences radial and tangential force

$$F_n(b) = 4\pi M_n b^2 \left(p(b) + \frac{2}{3} s(b) \right),$$

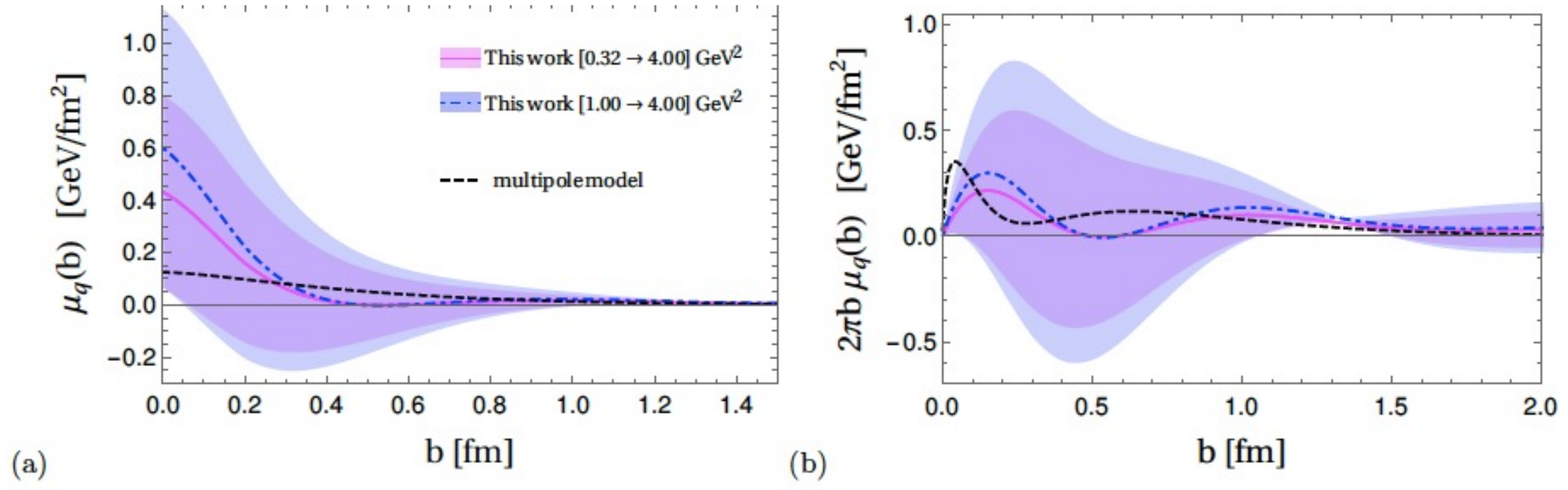
$$F_t(b) = 4\pi M_n b^2 \left(p(b) - \frac{1}{3} s(b) \right).$$

Qualitative behavior of radial and tangential force similar to light cone sum rule and fits to Jlab data

Tangential force positive near the center, negative in the outer region

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

2D energy density



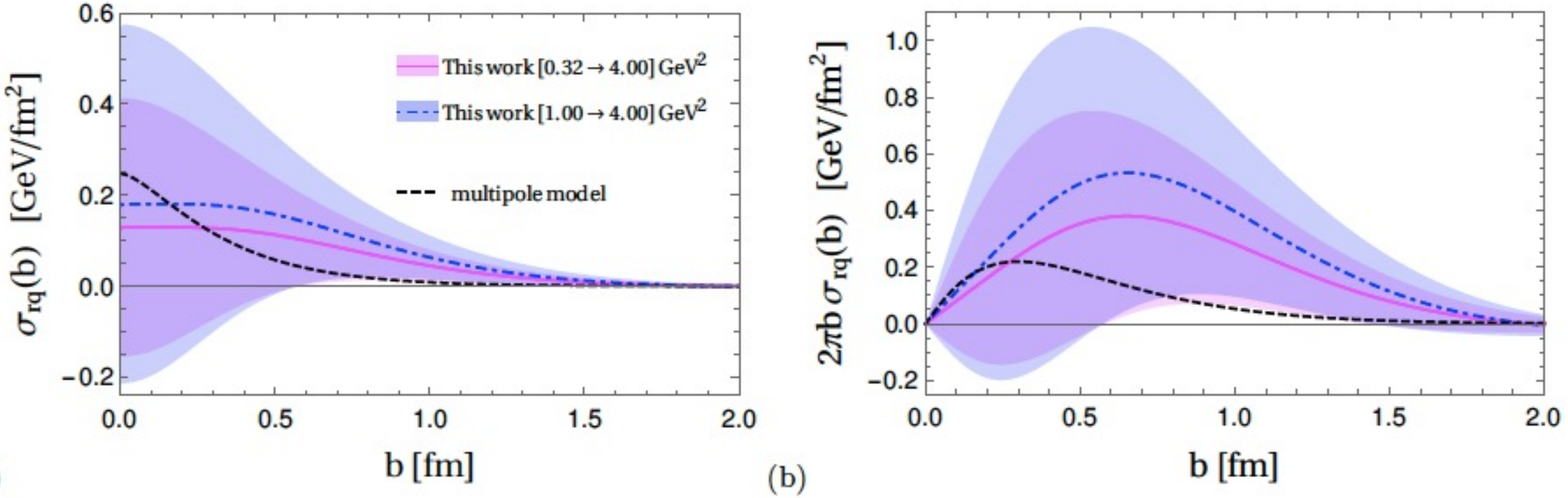
Energy density has a peak at the center, consistent with the simple multipole model of C. Lorce et al EPJC (2019)

$$\mu_a(b) = M_n\left\{\frac{A_a(b)}{2} + \bar{C}_a(b) + \frac{1}{4M_n^2}\frac{1}{b}\frac{d}{db}\left(b\frac{d}{db}\left[\frac{B_a(b)}{2} - 4C_a(b)\right)\right]\right\}$$

$$\chi(b) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{i\vec{q}_\perp . \vec{b}_\perp}\chi(q^2).$$

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

2D radial pressure



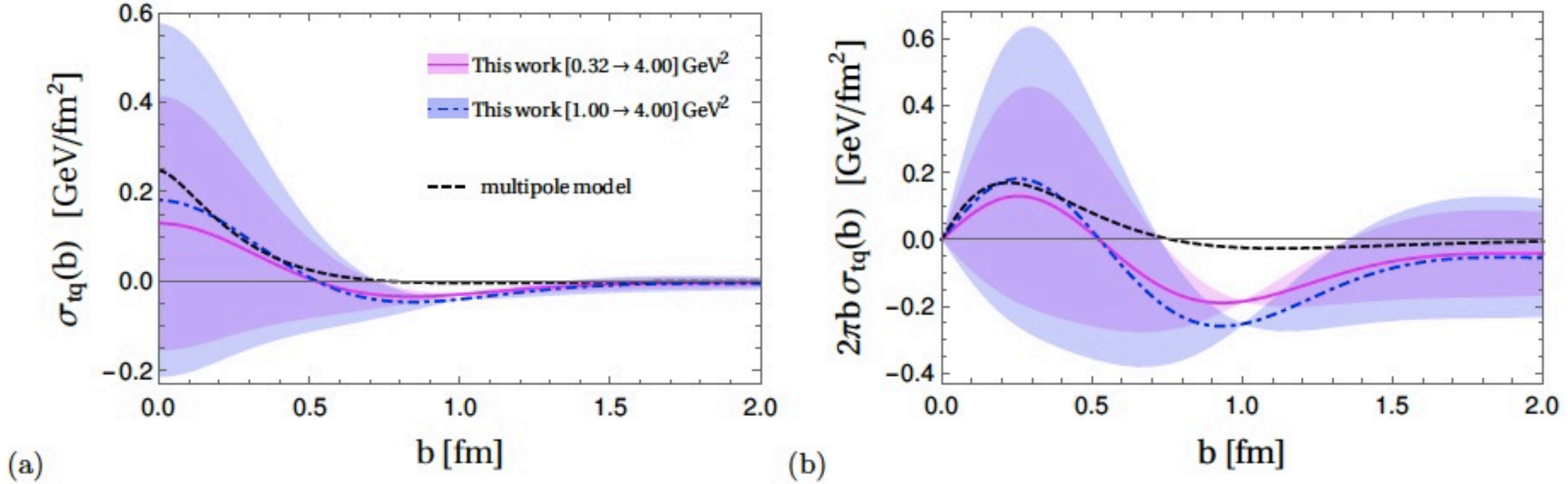
(a) (b)

$$\sigma_{r,a}(b) = M_n\left\{ -\bar{C}_a(b) + \frac{1}{M_n^2}\frac{1}{b}\frac{dC_a(b)}{db}\right\}$$

$$\chi(b) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{i\vec{q}_\perp.\vec{b}_\perp}\chi(q^2).$$

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

2D tangential pressure

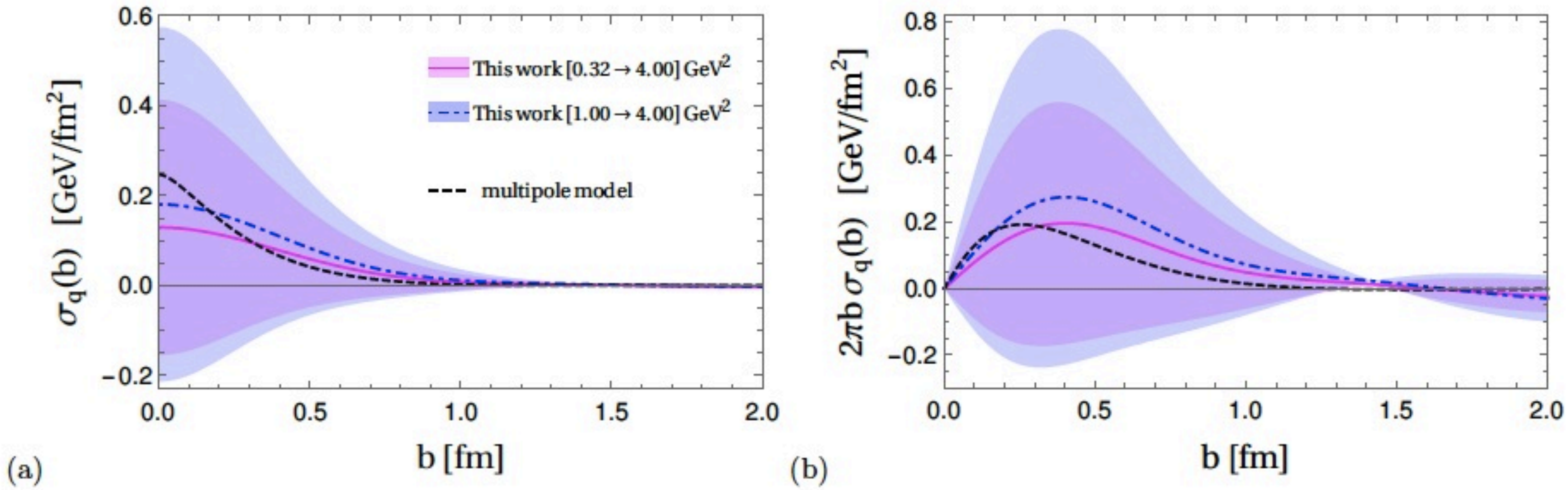


$$\sigma_{t,a}(b) = M_n \Big\{ -\bar{C}_a(b) + \frac{1}{M_n^2} \frac{d^2 C_a(b)}{db^2} \Big\}$$

$$\chi(b) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp . \vec{b}_\perp} \chi(q^2).$$

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

Isotropic pressure

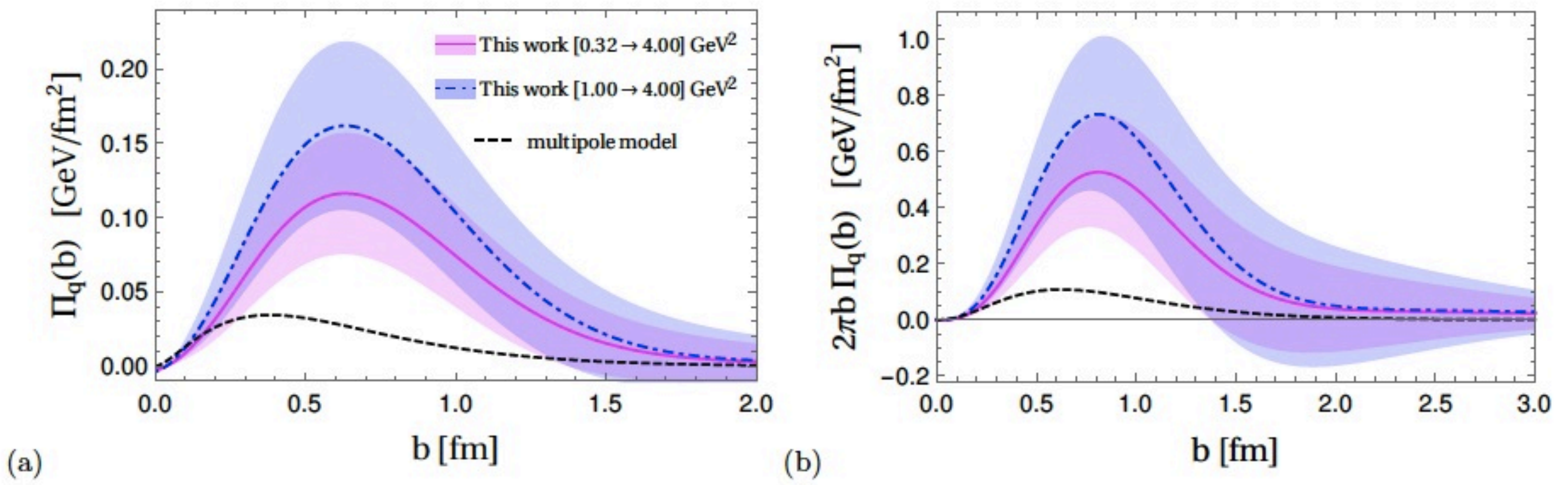


$$\sigma_a(b) = M_n \left\{ -\bar{C}_a(b) + \frac{1}{2}\frac{1}{M_n^2}\frac{1}{b}\frac{d}{db}\left(b\frac{dC_a(b)}{db}\right) \right\}$$

$$\chi(b) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{i\vec{q}_\perp.\vec{b}_\perp}\chi(q^2).$$

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

2D Pressure Anisotropy



Vanishes at center due to spherical symmetry

Positive away from center : radial pressure larger than tangential pressure

$$\Pi_a(r) = \sigma_{ra}(r) - \sigma_{ta}(r)$$

$$\Pi_a(b) = M_n\Big\{ - \frac{1}{M_n^2} b \frac{d}{db}\left(b \frac{dC_a(b)}{db}\right)\Big\}$$

$$\chi(b) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{i\vec{q}_\perp . \vec{b}_\perp} \chi(q^2).$$

Chakrabarti, Mondal, Mukherjee, Nair, Zhao; PRD(2020)

Quark state dressed with a gluon

$A(Q^2)$ and $B(Q^2)$ involves the 'good' components of the energy momentum tensor. But $C(Q^2)$ and $\bar{C}(Q^2)$ involves 'bad' components with the interaction terms

Instead of a proton state we take a simpler state with a gluon degree of freedom : a dressed quark

State is expanded in Fock space in terms of multiparton light-front wave functions (LFWFs) : two particle LFWF consists of a quark and a gluon

Fully relativistic spin -1/2 composite state that incorporates a gluonic degree of freedom

Two-particle LFWF can be calculated analytically using light front Hamiltonian perturbation theory

In order to calculate the interaction terms we use the two component formalism in light cone gauge

Two-component formalism

Fermionic field is decomposed as $\psi = \psi_+ + \psi_-$, $\psi_\pm = \Lambda_\pm \psi$ $\Lambda_\pm$: projection operators

In light cone gauge one uses a particular representation of gamma matrices so that

$$\psi_+ = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \qquad \psi_- = \begin{bmatrix} 0 \\ \eta \end{bmatrix},$$

ξ and **η** are two component fields

$$\eta(y) = \left(\frac{1}{i\partial^+}\right)[\sigma^\perp \cdot (i\partial^\perp + gA^\perp(y)) + im]\xi(y),$$

Constrained field, can be eliminated using the equation of constraint

$$\xi(y) = \sum_\lambda \chi_\lambda \int \frac{[k]}{\sqrt{2(2\pi)^3}}[b_\lambda(k)e^{-ik\cdot y} + d^\dagger_{-\lambda}(k)e^{ik\cdot y}],$$

Dynamical field

W. M . Zhang and A. Harindranath, PRD 48,4881 (1993)

Two-component formalism

$$A^{\perp}(y)=\sum_{\lambda}\int\frac{[k]}{\sqrt{2(2\pi)^3k^+}}[\epsilon_{\lambda}^{\perp}a_{\lambda}(k)e^{-ik\cdot y}+\epsilon_{\lambda}^{\perp *}a_{\lambda}^{\dagger}(k)e^{ik\cdot y}].$$

Dynamical components of gauge field

The state of momentum P and helicity λ can be expanded in Fock space

$$|P,\lambda\rangle=\psi_1(P,\lambda)b_{\lambda}^{\dagger}(P)|0\rangle+\sum_{\lambda_1,\lambda_2}\int[k_1][k_2]\sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2)\psi_2(P,\lambda|k_1,\lambda_1;k_2,\lambda_2)b_{\lambda_1}^{\dagger}(k_1)a_{\lambda_2}^{\dagger}(k_2)|0\rangle,$$

LFWFs can be written in terms of the relative momenta x_i and κ_i

$$k_i^+=x_iP^+,\qquad \boldsymbol{k}_i^{\perp}=\boldsymbol{\kappa}_i^{\perp}+x_i\boldsymbol{P}^{\perp},$$

$$\begin{aligned}\phi_{\lambda_1,\lambda_2}^{\lambda a}(x_i,\boldsymbol{\kappa}_i^{\perp})=&\left[\frac{x(1-x)}{\boldsymbol{\kappa}^{\perp 2}+m^2(1-x)^2}\right]\frac{g}{\sqrt{2(2\pi)^3}}\frac{T^a}{\sqrt{1-x}}\chi_{\lambda_1}^{\dagger}\\&\times\left[\frac{-2(\boldsymbol{\kappa}^{\perp}\cdot\epsilon_{\lambda_2}^{\perp *})}{1-x}-\frac{1}{x}(\tilde{\sigma}^{\perp}\cdot\boldsymbol{\kappa}^{\perp})(\tilde{\sigma}^{\perp}\cdot\epsilon_{\lambda_2}^{\perp *})\right.\\&\left.+im(\tilde{\sigma}^{\perp}\cdot\epsilon_{\lambda_2}^{\perp *})\frac{1-x}{x}\right]\chi_{\lambda}\psi_1^{\lambda}\end{aligned}\qquad(8)$$

Gravitational Form Factors

$$\theta^{\mu\nu} = \frac{1}{2}\bar{\psi} i[\gamma^\mu D^\nu + \gamma^\nu D^\mu]\psi - F^{\mu\lambda a}F^{\nu}_{\lambda a} + \frac{1}{4}g^{\mu\nu}(F_{\lambda\sigma a})^2 - g^{\mu\nu}\bar{\psi}(i\gamma^\lambda D_\lambda - m)\psi.$$

$$\mathcal{M}^{++}_{\uparrow\uparrow} + \mathcal{M}^{++}_{\downarrow\downarrow} = 2(P^+)^2 A_Q(q^2),$$

$$\mathcal{M}^{++}_{\uparrow\downarrow} + \mathcal{M}^{++}_{\downarrow\uparrow} = \frac{iq^{(2)}}{M}(P^+)^2 B_Q(q^2).$$

$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} + \mathcal{M}^{22}_{\uparrow\downarrow} + \mathcal{M}^{22}_{\downarrow\uparrow} = i\left[B_Q(q^2)\frac{q^2}{4M} - C_Q(q^2)\frac{3q^2}{M} + \bar{C}_Q(q^2)2M\right]q^{(2)}.$$

GFFs are extracted by calculating the matrix elements of the energy momentum tensor

$$\mathcal{M}^{\mu\nu}_{SS'} = \frac{1}{2}[\langle P', S'|\theta^{\mu\nu}_Q(0)|P, S\rangle]$$

$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} - \mathcal{M}^{22}_{\uparrow\downarrow} - \mathcal{M}^{22}_{\downarrow\uparrow} = i\left[\frac{B_Q(q^2)}{4M} - \frac{C_Q(q^2)}{M}\right]((q^{(1)})^2 q^{(2)} - (q^{(2)})^3),$$

Gravitational form factors

The form factors obtained from the equations are :

$$A_Q(q^2) = 1 + \frac{g^2 C_F}{2\pi^2}\left[\frac{11}{10} - \frac{4}{5}\left(1 + \frac{2m^2}{q^2}\right)\frac{f_2}{f_1} - \frac{1}{3}\log\left(\frac{\Lambda^2}{m^2}\right)\right]$$

$$B_Q(q^2) = \frac{g^2 C_F}{12\pi^2}\frac{m^2}{q^2}\frac{f_2}{f_1},$$

$$D_Q(q^2) = \frac{5g^2 C_F}{6\pi^2}\frac{m^2}{q^2}(1 - f_1 f_2) = 4C_Q(q^2),$$

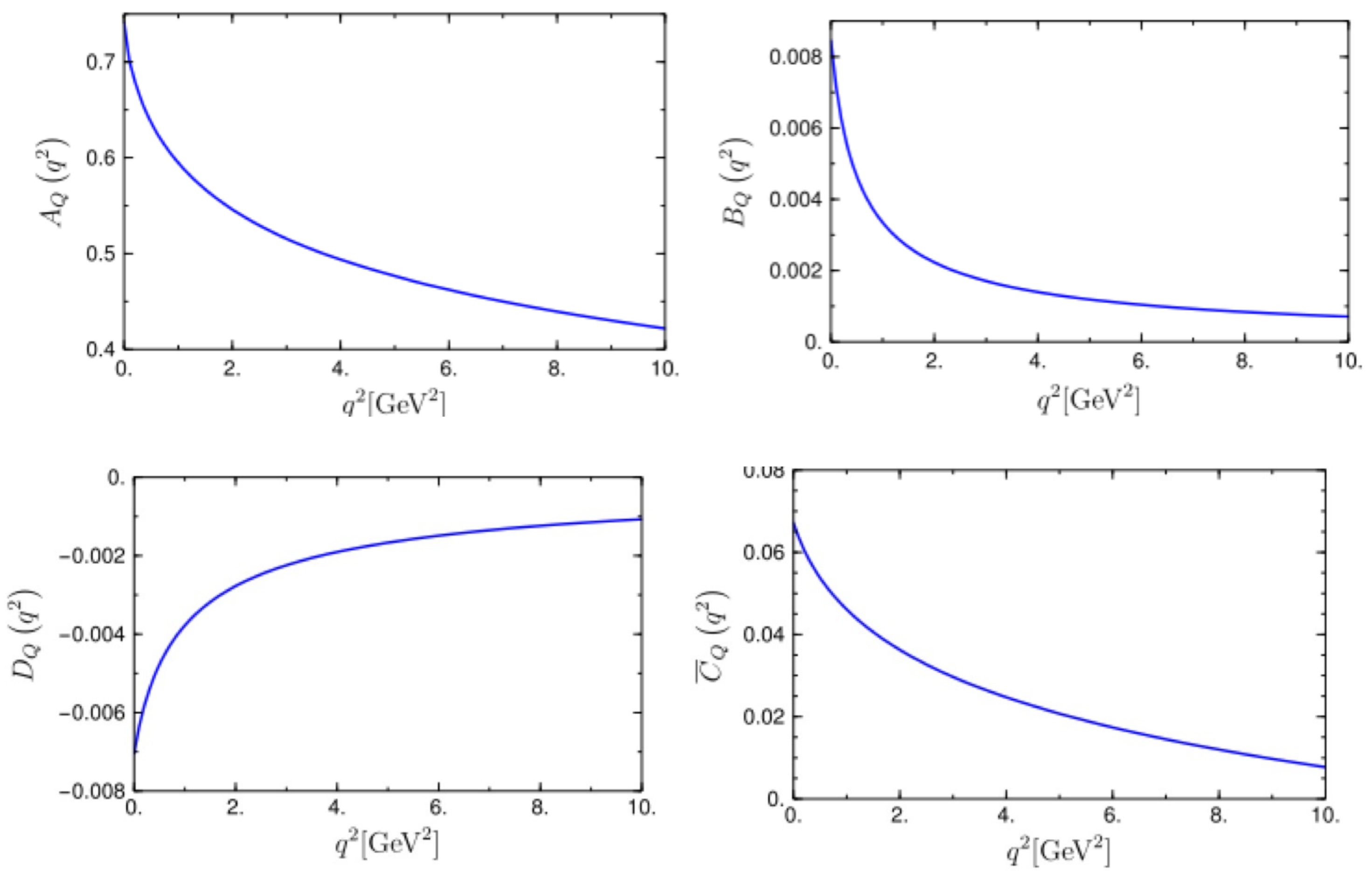
$$\bar{C}_Q(q^2) = \frac{g^2 C_F}{72\pi^2}\left(29 - 30 f_1 f_2 + 3\log\left(\frac{\Lambda^2}{m^2}\right)\right)$$

GFFs calculated from the quark part of the EM tensor : depend on the renormalization scale. In this approach it is Λ (upper cutoff of transverse momentum integration)

$$f_1 := \frac{1}{2}\sqrt{1 + \frac{4m^2}{q^2}},$$

$$f_2 := \log\left(1 + \frac{q^2(1 + 2f_1)}{2m^2}\right).$$

GFFs for a dressed quark



M=m=0.3, g =1, Λ =10^3 GeV

Our result for the D form factor for an electron agrees with Metz, Pasquini, Rodini, PLB 820, 136501 (2021)

GFFs

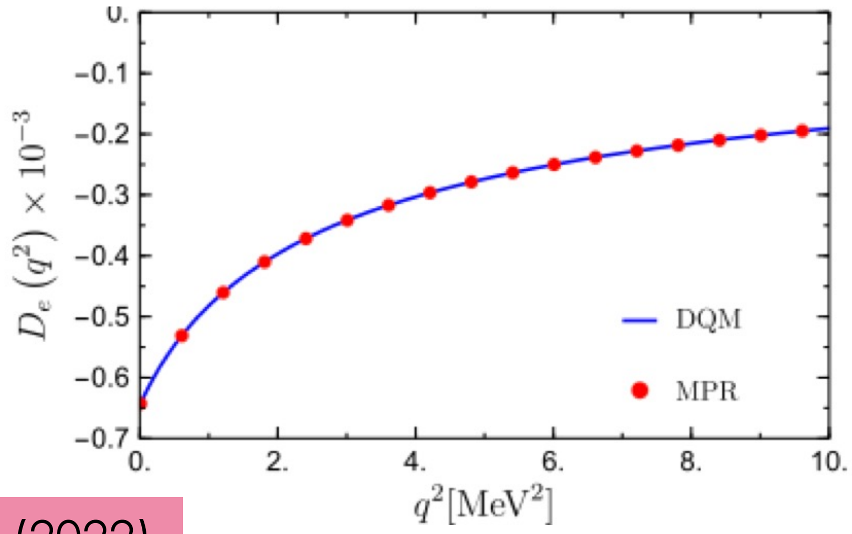
$B_Q(0) = 0.0084$. Contribution from gluons is expected to be negative so that total anomalous gravitomagnetic moment vanishes

For the electron, $B_e(0) = \alpha/3\pi$ satisfied

$D(Q^2)$ is negative, so the dressed quark behaves like a bound system.

$\bar{C}(Q^2)$ For quarks is positive, which means the contribution from the gluonic part of the EM tensor in this model is expected to be negative so that the sum is zero : ongoing work

Comparison of D-term in QED with Metz, Pasquini, Rodini, PLB 820, 136501 (2021)



More, AM, Nair, Saha ; PRD 105, 056017 (2022)

Pressure and Shear Distribution

Two- dimensional pressure and shear force distributions are defined as

$$p(b^\perp) = \frac{1}{2Mb^\perp}\frac{d}{db^\perp}\left[b^\perp \frac{d}{db^\perp}D_Q(b^\perp)\right] - M\bar{C}_Q(b^\perp)$$

$$s(b^\perp) = -\frac{b^\perp}{M}\frac{d}{db^\perp}\left[\frac{1}{b^\perp}\frac{d}{db^\perp}D_Q(b^\perp)\right],$$

Where

$$F(b^\perp) = \frac{1}{(2\pi)^2}\int d^2q^\perp e^{-iq^\perp b^\perp}\mathcal{F}(q^2)$$
$$= \frac{1}{2\pi}\int_0^\infty dq^{\perp 2} J_0(q^\perp b^\perp)\mathcal{F}(q^2),$$

The pressure distribution has to obey the von Laue condition for stability

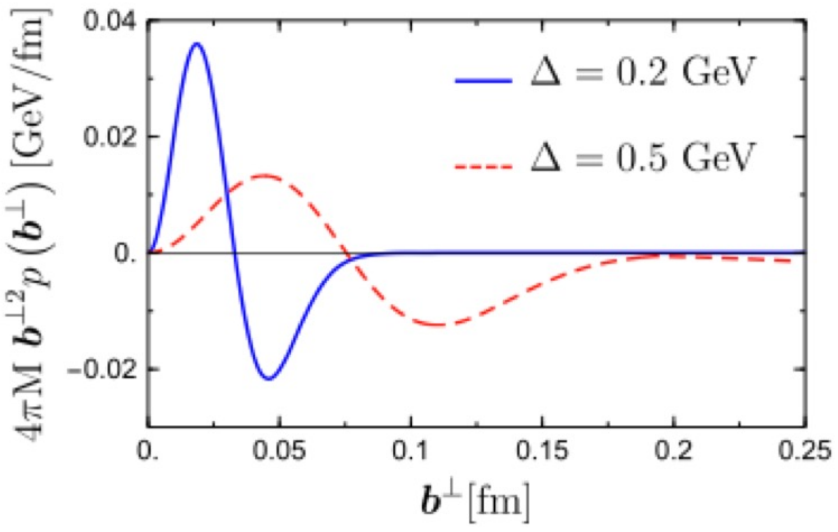
$$\int_0^\infty d^2b^\perp p(b^\perp) = 0.$$

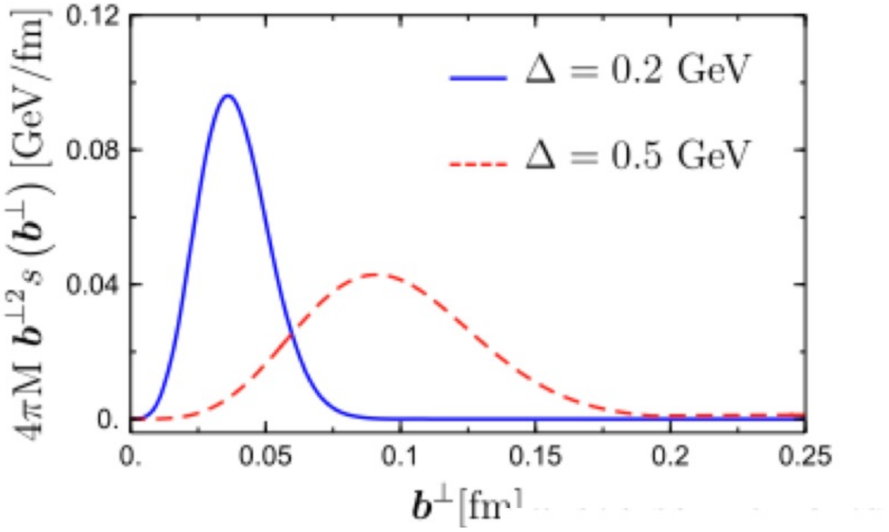
This is a consequence of the conservation of energy-momentum tensor

$b_\perp$ represents the impact parameter

In order to study the spatial distributions, we take a wave packet state Gaussian in form

Pressure and Shear Distribution





Wave packet state used

$$\phi(p) = p^+ \delta(p^+ - p_0^+)\phi(p^\perp)$$

$$\phi(\boldsymbol{p}^\perp) = e^{-\frac{\boldsymbol{p}^{\perp 2}}{2\Delta^2}}$$

$$\frac{1}{16\pi^3}\int \frac{d^2\boldsymbol{p}^\perp dp^+}{p^+}\phi(p)|p^+, \boldsymbol{p}^\perp, \lambda\rangle$$

Pressure distribution satisfies Von Laue condition

More, AM, Nair, Saha ; PRD 105, 056017 (2022)

Pressure and Shear Distribution

Von Laue condition requires at least one node of the pressure distribution

Pressure is repulsive near the center and confining in the outer region, maintaining stability

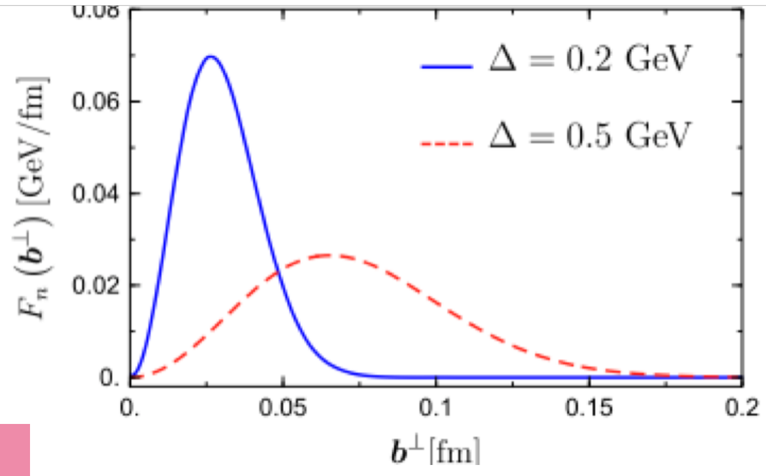
Shear distribution is positive in b space : this is also seen is stable hydrostatic system

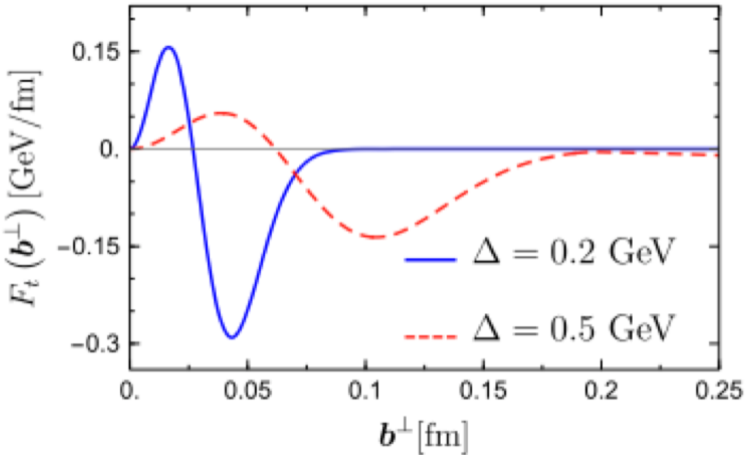
As the width of the Gaussian increases the peak value of the distribution decreases and shifts away from the center in b space

Normal and tangential force

$$F_n(b^\perp) = 2\pi b^\perp\left(p(b^\perp) + \frac{1}{2}s(b^\perp)\right),$$

$$F_t(b^\perp) = 2\pi b^\perp\left(p(b^\perp) - \frac{1}{2}s(b^\perp)\right).$$





More, AM, Nair, Saha ; PRD 105, 056017 (2022)

Energy Density and Pressure Distribution

$$\mu_i(\boldsymbol{b}^\perp) = M\left[\frac{1}{2}A_i(\boldsymbol{b}^\perp) + \bar{C}_i(\boldsymbol{b}^\perp) + \frac{1}{4M^2}\frac{1}{\boldsymbol{b}^\perp}\frac{d}{d\boldsymbol{b}^\perp} \times \left(\boldsymbol{b}^\perp \frac{d}{d\boldsymbol{b}^\perp}\left[\frac{1}{2}B_i(\boldsymbol{b}^\perp) - 4C_i(\boldsymbol{b}^\perp)\right]\right)\right],$$

← . Galelian energy density

$$\sigma_i^r(\boldsymbol{b}^\perp) = M\left[-\bar{C}_i(\boldsymbol{b}^\perp) + \frac{1}{M^2}\frac{1}{\boldsymbol{b}^\perp}\frac{dC_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^\perp}\right],$$

← Radial pressure

Lorce, Moutarde, Trawinski, EPJC (2019)

$$\sigma_i^t(\boldsymbol{b}^\perp) = M\left[-\bar{C}_i(\boldsymbol{b}^\perp) + \frac{1}{M^2}\frac{d^2C_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^{\perp 2}}\right],$$

← Tangential pressure

$$\sigma_i(\boldsymbol{b}^\perp) = M\left[-\bar{C}_i(\boldsymbol{b}^\perp) + \frac{1}{2}\frac{1}{M^2}\frac{1}{\boldsymbol{b}^\perp}\frac{d}{d\boldsymbol{b}^\perp}\left(\boldsymbol{b}^\perp\frac{dC_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^\perp}\right)\right]$$

← Isotropic pressure

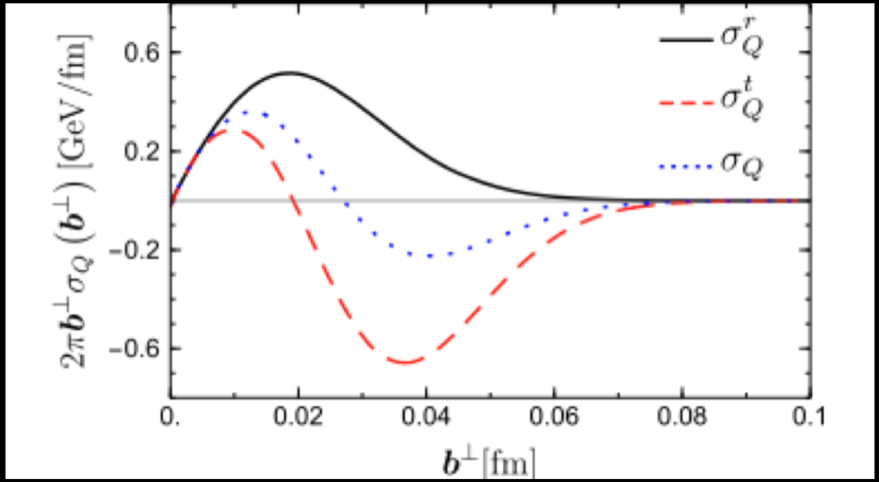
$$\sigma_Q = \frac{(\sigma_Q^r + \sigma_Q^t)}{2},$$

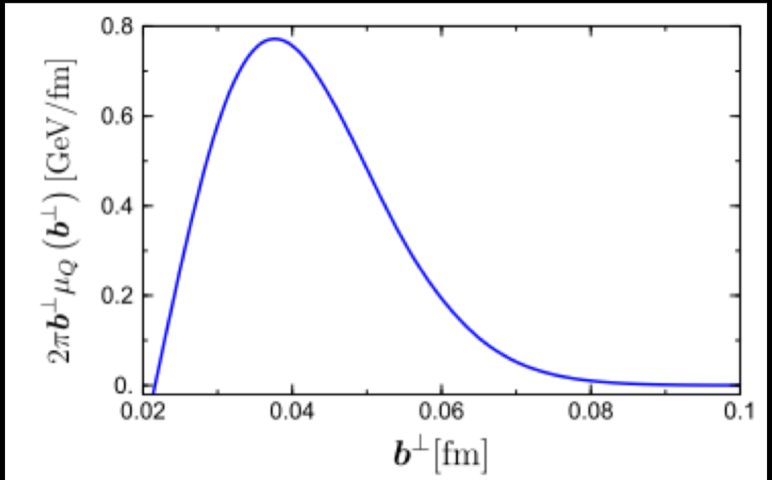
$$\Pi_i(\boldsymbol{b}^\perp) = M\left[-\frac{1}{M^2}\boldsymbol{b}^\perp\frac{d}{d\boldsymbol{b}^\perp}\left(\frac{1}{\boldsymbol{b}^\perp}\frac{dC_i(\boldsymbol{b}^\perp)}{d\boldsymbol{b}^\perp}\right)\right].$$

← Pressure anisotropy

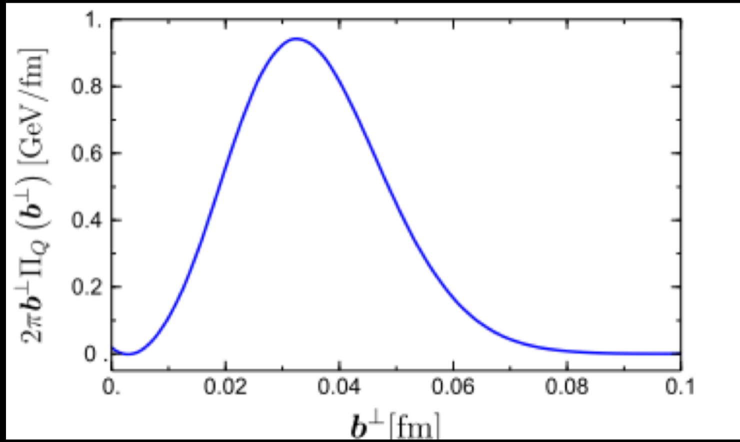
$$\Pi_Q = \sigma_Q^r - \sigma_Q^t.$$

Numerical results





Plots show contributions from the quark part of the energy momentum tensor, including the interaction term



More, AM, Nair, Saha ; PRD 105, 056017 (2022)

Conclusion-1

We presented a calculation of the GFF and pressure distributions in the proton using a light-front quark-diquark model with Ads/QCD predicted wave functions

$A(Q^2)$ and $B(Q^2)$ in this model are comparable with lattice result

Qualitative behaviour of $D(Q^2)$ agrees with other model calculations. $D(Q^2)$ is negative and the magnitude at $Q^2=0$ is larger than obtained in other models. $D(Q^2)$ can be described by a multipole form.

$\bar{C}(Q^2)$ Is negative for lower values of Q^2 but positive for higher values; in contrast with other model calculations

Presented the pressure and energy distributions and mechanical radius : compared with other calculations

Qualitative behavior of the pressure and shear distributions agree with other model calculations as well as that observed from Jlab data

Conclusion-II

We presented a calculation of the GFFs and mechanical properties of a simple relativistic spin ½ state, namely a quark dressed with a gluon

Used Light-front Hamiltonian approach. In light cone gauge eliminated the constrained dof

Perturbative model incorporating quark-gluon interaction in the energy-momentum tensor : involves 'bad' light cone components of the energy-momentum tensor

Ongoing work : to calculate the effect of the gluonic part of the E-M tensor and the check the constraints coming from its conservation

Would help to give insight in the role of gluons in the pressure, shear and energy distributions of a bound state in QCD like a nucleon