Neutron star studies put constraints on the QCD EOS: What can they tell us about the CP of nearly-isospin symmetric matter?

C. Ecker, M. Marczenko, D. Mroczek, T. Zhao

Neutron star studies put constraints on the QCD EOS

Yes.

What can they tell us about the CP of nearly-isospin symmetric matter?

We all agree, right now, nothing.

What can we learn about QCD from neutron stars

For isolated, slowly-rotating neutron stars:

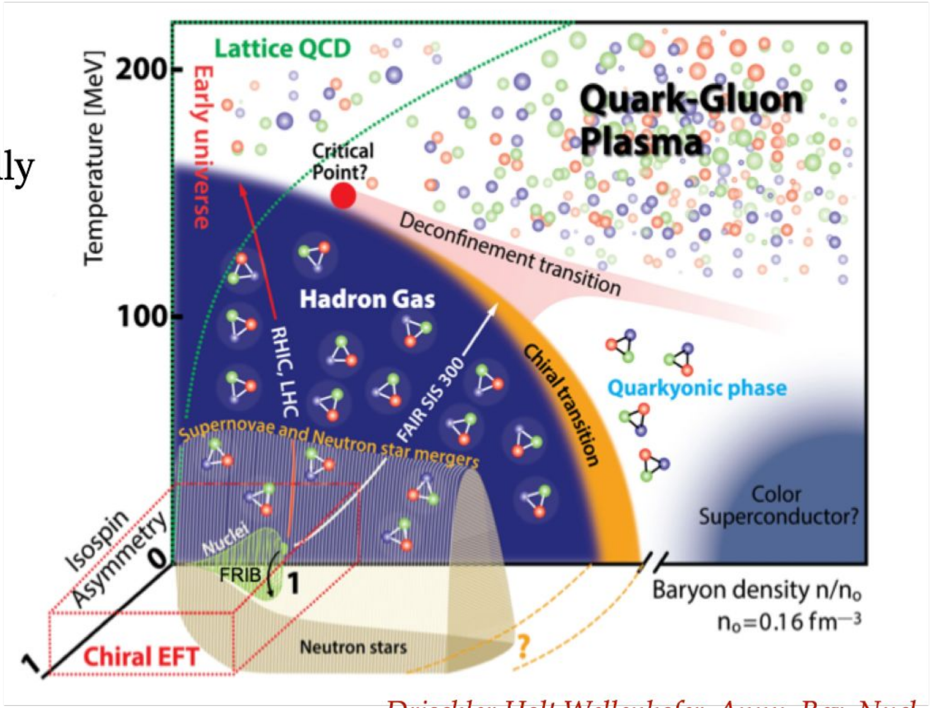
1) $T = 0$, since $T_F(\sim 10^{12}\ K) \gg T(\sim 10^{8-10}\ K)$

2) β-equilibrium, producing neutrons is energetically favorable at high densities.

$$\text{Neutron decay}: n \rightarrow p + e^- + \bar{\nu}_e$$
$$\text{Electron capture}: p + e^- \rightarrow n + \nu_e$$

$\rightarrow$ fraction of charged baryons, $Y_Q^{\rm QCD} = n_Q^{\rm QCD}/n_B$, is a function of density

3) The star is electrically neutral $\rightarrow n_{l^-} = n_Q^{\rm QCD}$

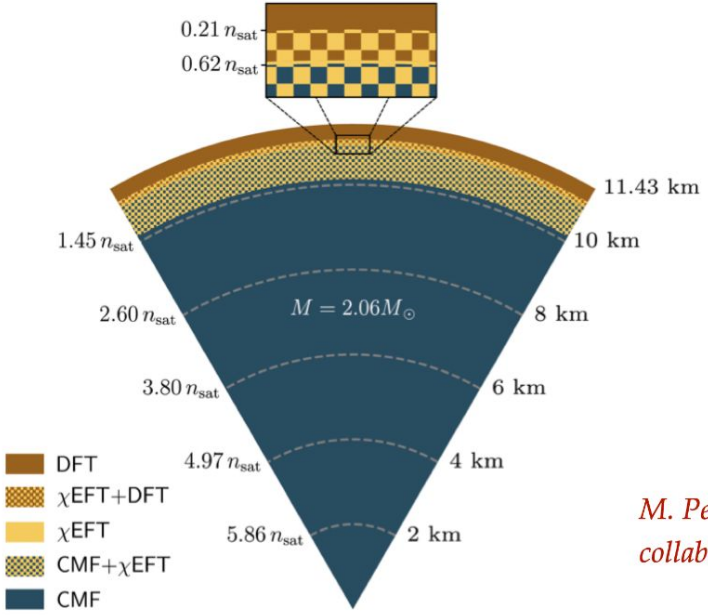


Drischler, Holt, Wellenhofer, Annu. Rev. Nucl. Part. Sci. (2021)

Baryon number density (isolated, stable NS)

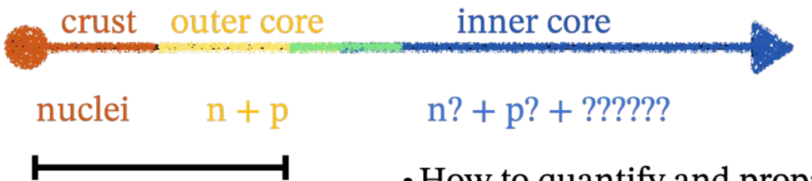
$$n_B = \frac{\partial p}{\partial \mu_B}\bigg|_{\mu_Q}$$

Relevant scale: nuclear saturation density, $n_{sat} \equiv 0.16\ \mathrm{fm}^{-3}$



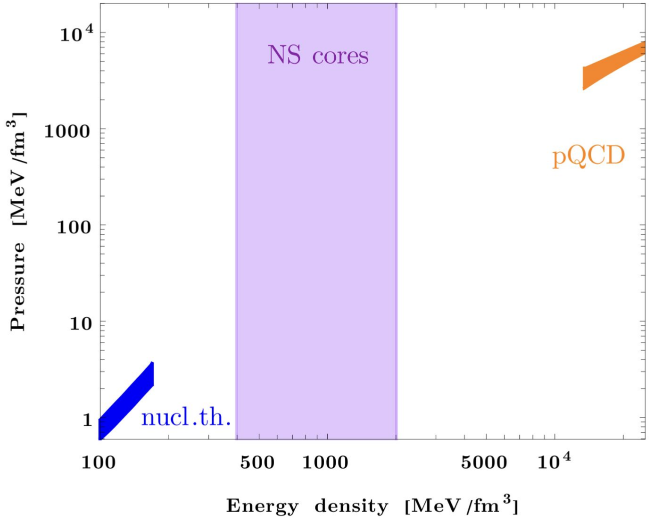
M. Pelicer et al (MUSES collaboration), 2502.07902

What are the relevant degrees of freedom and interactions?



Effective theories available →

- How to quantify and propagate theory uncertainties?
- What are the relevant parameters/interactions?



What is available from observations?

1. Shapiro-delay: lower bound on mass: $M_{\rm TOV} \gtrsim 2M_\odot$

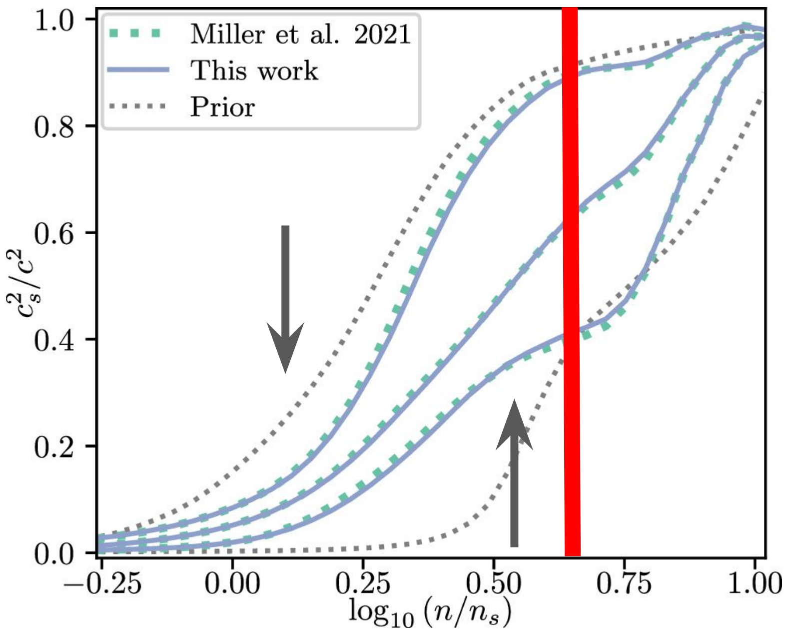
2. GW170817 + GW190425: $\tilde{\Lambda} < 720$

3. Joint mass-radius inference (NICER)

Table 10 Empirical constraints related to mass-radii from NICER.

Neutron Star	M ($M_\odot$)	Radius (km)	Reference
PSR J0030+0451	$1.34^{+0.15}_{-0.16}$	$12.71^{+1.14}_{-1.19}$	Riley et al (2019a)
PSR J0740+6620	$2.072^{+0.067}_{-0.066}$	$12.39^{+1.30}_{-1.98}$	Riley et al (2021a)
PSR J0030+0451	$1.44^{+0.15}_{-0.14}$	$13.02^{+1.24}_{-1.06}$	Miller et al (2019)
PSR J0740+6620	$2.08^{+0.07}_{-0.07}$	$13.7^{+2.6}_{-1.5}$	Miller et al (2021b)

Modified from Dittmann et al ApJ (2024)



Possible ways we could learn about the CP of isospin-symmetric matter

1) Finding strong observational evidence for a first order phase transition (still leaves open the question of isospin dependence).

2) Finding strong evidence against a first order phase transition.

The role of astrophysical measurements

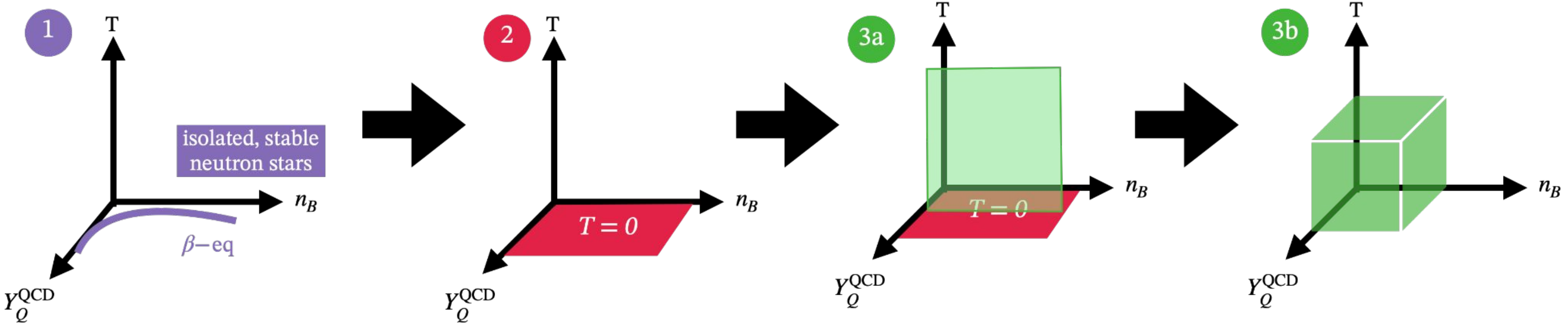
- Extensive literature on the role of current and future astrophysical measurements.
- First order PTs, crossover to quark or exotic (beyond n+p) hadronic states neither ruled out nor required by current observations.
- Not much hope from GW inspiral (tidal deformability) with current instruments.
- NICER requires many years of observation time to significantly improve uncertainties, new sources unlikely in near future.
- Possible paths: improved GW sensitivities (merger, post-merger), progress in theory.

Phase Transition Phenomenology with Nonparametric Representations of the Neutron Star Equation of State

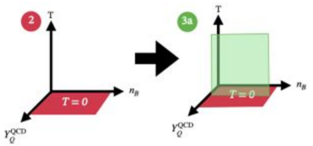
Reed Essick,[1,2,3,4,*] Isaac Legred,[5,6,†] Katerina Chatziioannou,[5,6,‡] Sophia Han (韩君),[7,8,9,§] and Philippe Landry[1,¶]

branches and/or one moderate phase transition. We also showed that we will not be able to confidently detect the presence of a phase transition with catalogs of $\leq$ 100 GW events. Although we do not directly estimate how many events will be needed for computational reasons, extrapolating Fig. 8 suggests that we may need several hundred events to reach Bayes factors $\gtrsim$ 100, often taken as a rule-of-thumb for confident detections [79]. We can, however, expect to confidently rule out the presence of multiple stable branches at low masses after 100 events. While the exact rates of NS coalescences and future GW-detector sensitivities are still uncertain, it is unlikely that we will obtain a catalog of this size within the lifetime of the advanced LIGO and Virgo detectors [65].

Starting from an **arbitrary NS EOS, reconstruct a 3D EOS**



From T = 0 to finite T



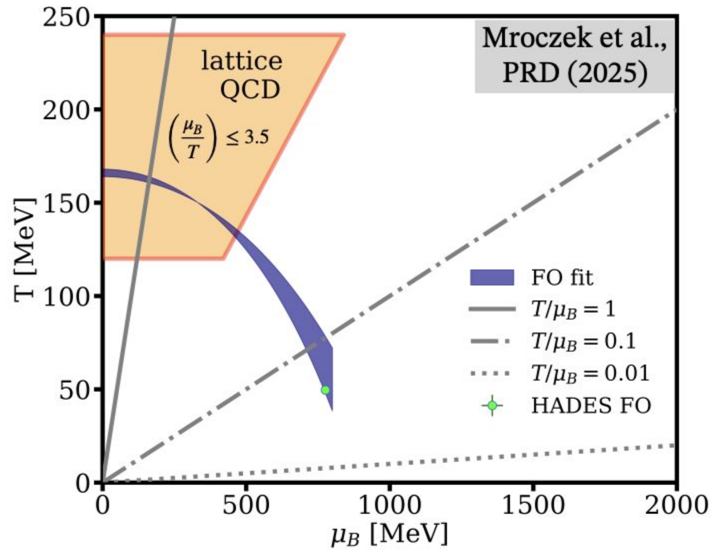
- Taylor expansion about $p(T=0,\, \mu_B,\, \mu_Q)$

$$p(T,\vec{\mu}) = p_{T=0} + \cancelto{0}{\left.\frac{\partial p}{\partial T}\right|_{T=0,\vec{\mu}}} T + \frac{1}{2}\boxed{\left.\frac{\partial^2 p}{\partial T^2}\right|_{T=0,\vec{\mu}}} T^2 + \ldots$$

Entropy!
$s(T=0)=0$

Heat capacity $\left.\frac{\partial s}{\partial T}\right|_{T=0} > 0$

$$\boxed{p(T,\vec{\mu}) \approx p_{T=0} + \frac{1}{2}\left.\frac{\partial s}{\partial T}\right|_{T=0,\vec{\mu}} T^2}$$



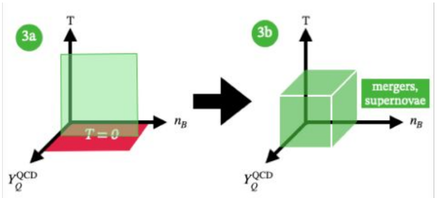
- Special case: Sommerfeld (1928) expansion
- Ideal Fermi systems at $T \ll T_F$,

  $p \approx p_{T=0} + aT^2 + bT^4 + \ldots$
- Fermionic **quasi-particles**

✳ Physical motivation
✳ Expansion parameter $(T/\mu_B) < 0.1$ in relevant regime
✳ Overlap with few-GeV $\sqrt{s_{\rm NN}}$ freeze-out (FO)

FO fit from Cleymans et al, PRC 73 (2006), HADES FO from Harabasz et al, PRC 102 (2020)

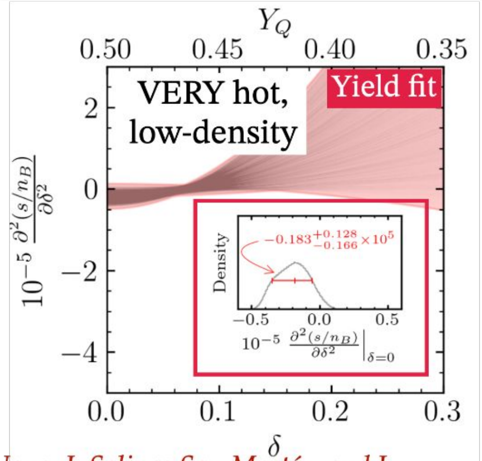
Connection to heavy-ion collisions: system scan



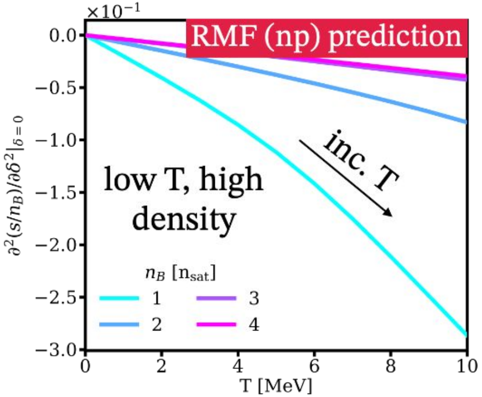
- Nana et al extracted $\partial^2(s/n_B)/\partial\delta^2$ from particle yields across different colliding species, central collisions at $\sqrt{s_{NN}} = 200$ GeV

System	Z	A	Y_Q	Published yield data?
O+O	8	16	0.500	no
Cu+Cu	29	63	0.460	yes
Ru+Ru	44	96	0.458	no*
Zr+Zr	40	96	0.417	no*
Au+Au	79	198	0.399	yes
U+U	92	238	0.387	yes

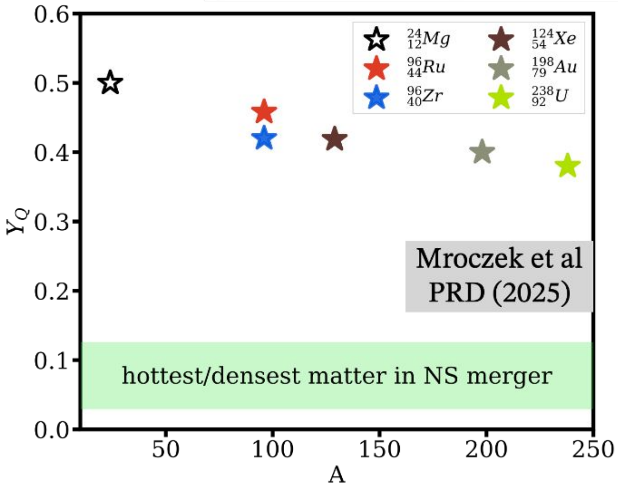
Fits predict a **large and negative** value for $\partial^2(s/n_B)/\partial\delta^2$ at $T_{FO} \sim 145$ MeV, $n_B \sim 0.025\ n_{sat}$, in **qualitative agreement** with RMF (n+p) results



F. Nana, J. Salinas San Martín, and J. Noronha-Hostler, 2411.03705



Mroczek et al PRD (2025)



- Needed: **system + energy scan**
- **Symmetric nuclei**, e.g., O+O, crucial for extracting the expansion coefficient at $\delta = 0$

LHC, CBM @ FAIR?

V-QCD hybrid EOS

- Holographic bottom-up model in Veneziano limit ($N_c, N_f \to \infty, \frac{N_f}{N_c} = 1$).

  Järvinen, Kiritsis 1112.1261 (JHEP)

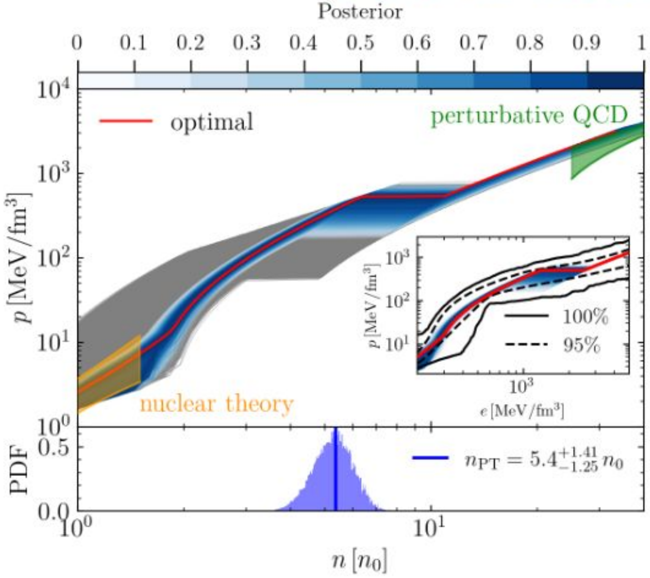
- Baryons, quark matter and transition between them from the same action

$$S_{\mathrm{V-QCD}} = S_g + S_f + S_{nm}$$

similar hybrid construction as in: Demircik, CE, Järvinen 2112.12157 (PRX)

- Bayesian analysis of 10^5 models variants with astro and χEFT constraints.

For details see: CE, Jokela, Järvinen, 2506.10065



Static neutron star properties

- Strong first-order transition $\Rightarrow$ no stable quark matter in static stars.
- Mass-radius estimates:

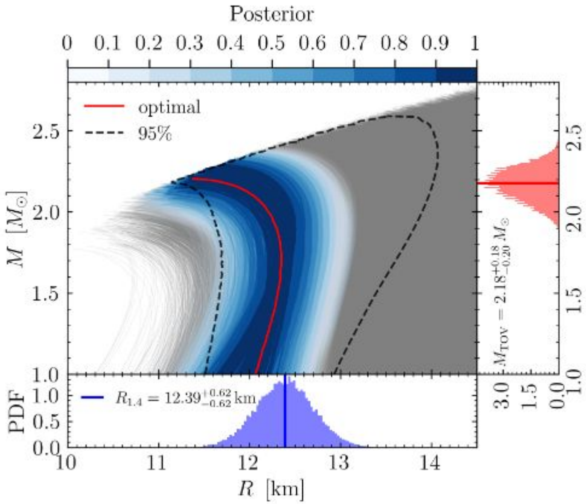
$$R_{1.4} = 12.39^{+0.62}_{-0.62}\,\mathrm{km}\,, \quad M_{\mathrm{TOV}} = 2.18^{+0.18}_{-0.20}\,M_\odot\,.$$

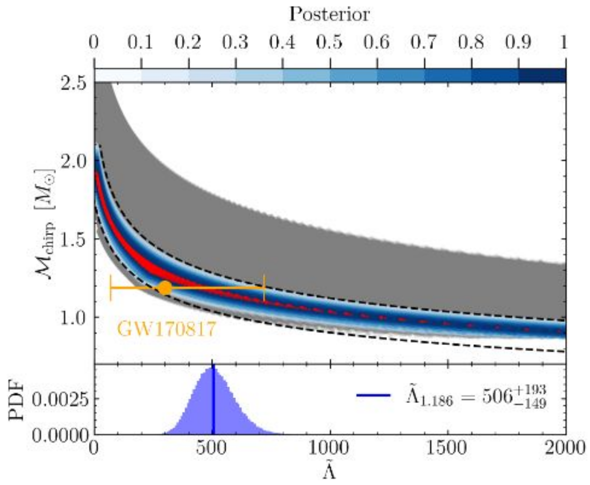
- Bounds on binary tidal deformability parameter:

V-QCD: $\tilde{\Lambda}_{1.186} = 506^{+193}_{-149}$ vs. model-agnostic (no PT): $\tilde{\Lambda}_{1.186} = 384^{+306}_{-158}$.

via physics informed priors: Magnal, CE, Rezzolla, Lasky, Goode 2504.21526 (ApJL)

- Smaller error and larger mean value, because specific model with stiff NM.



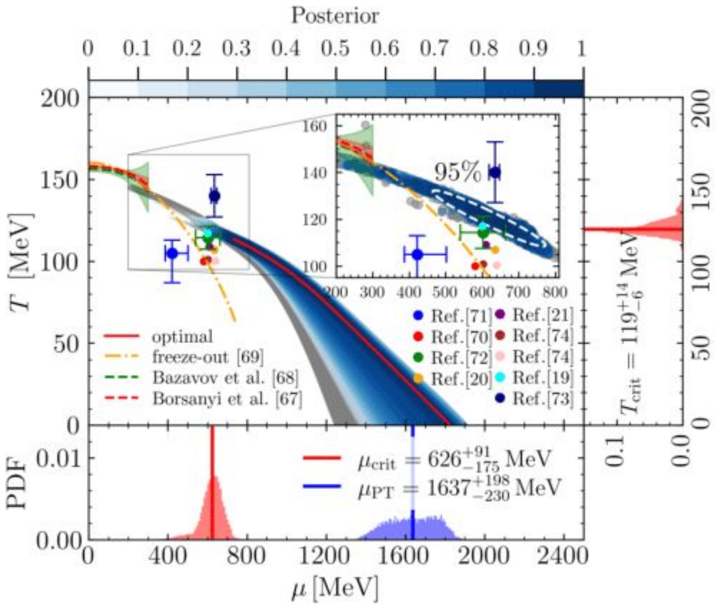


Phase diagram and critical endpoint estimate

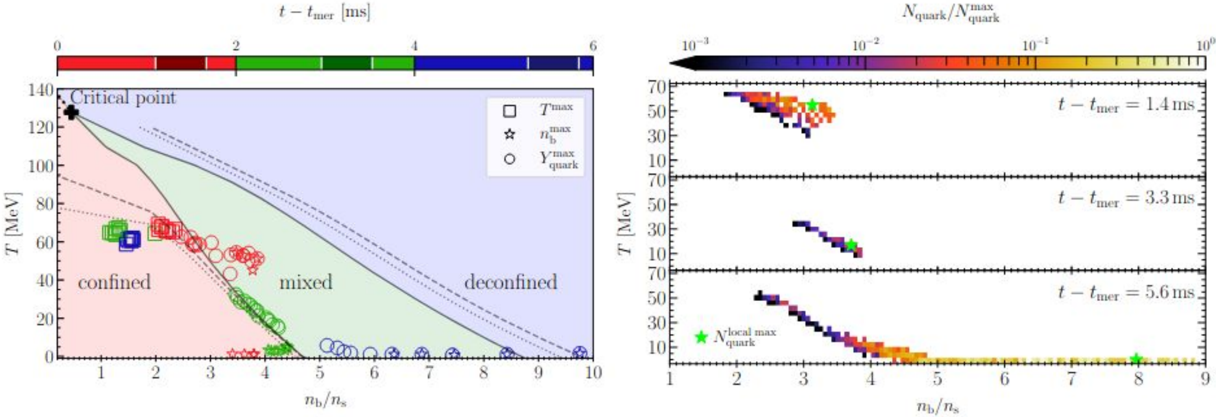
- Excluded-volume corrected van der Waals model matched at $T = 0$.
  Vovchenko, Motornenko, Alba, Gorenstein, Satarov, Stöcker 1707.09215 (PRC)
- Critical endpoint where (extrapolated) latent heat vanishes across PT

$$\Delta e(T_{\mathrm{crit}}, \mu_{\mathrm{crit}}) = 0\,, \quad \Delta e(T, \mu) = e_{\mathrm{QM}}(T, n^*_{\mathrm{QM}}(\mu)) - e_{\mathrm{NM}}(T, n^*_{\mathrm{NM}}(\mu))$$

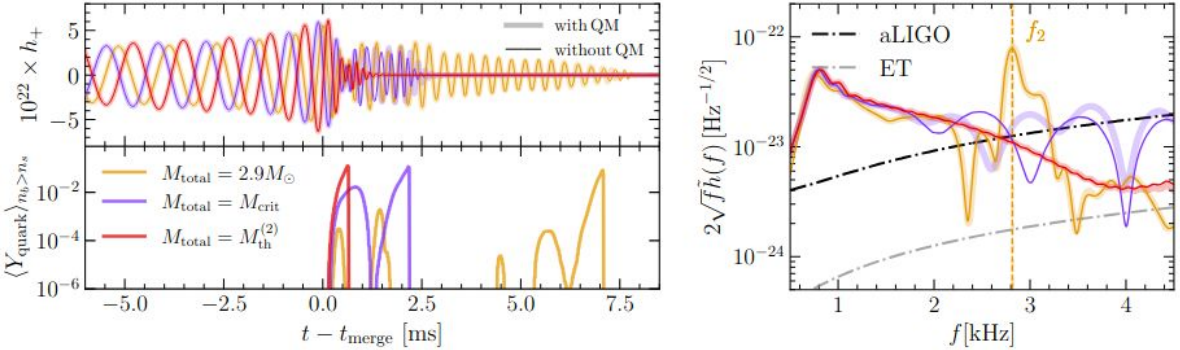
$$(\mu_{\mathrm{crit}}, T_{\mathrm{crit}}) = (626^{+91}_{-175}, 119^{+14}_{-6})\,\mathrm{MeV}\,.$$



CEP probably not reached in BNS mergers



Tootle, CE, Topolski, Demircik, Järvinen, Rezzolla 2205.05691 (SciPost)



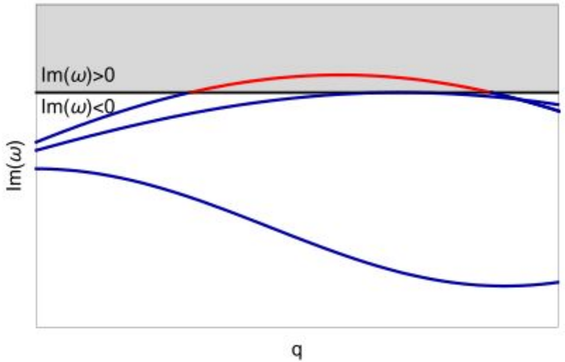
CE, Topolski, Järvinen, Stehr 2402.11013 (PRD)

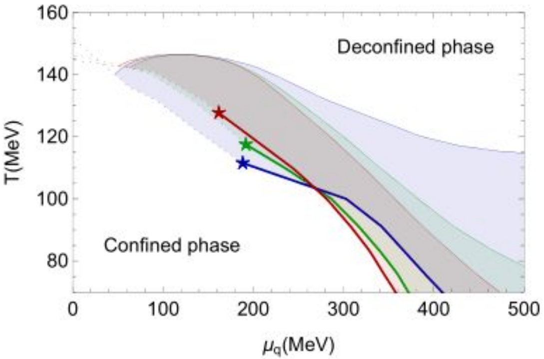
Modulated Instability

- Nakamura-Ooguri-Park instability to spatially modulated chiral currents.
  Demircik, Jokela, Järvinen, Piispa 2405.02392, Nakamura, Ooguri, Park 1011.4144 (PRL)
- Fluctuation analysis reveals $\mathrm{Im}(\omega) < 0$ at finite momentum

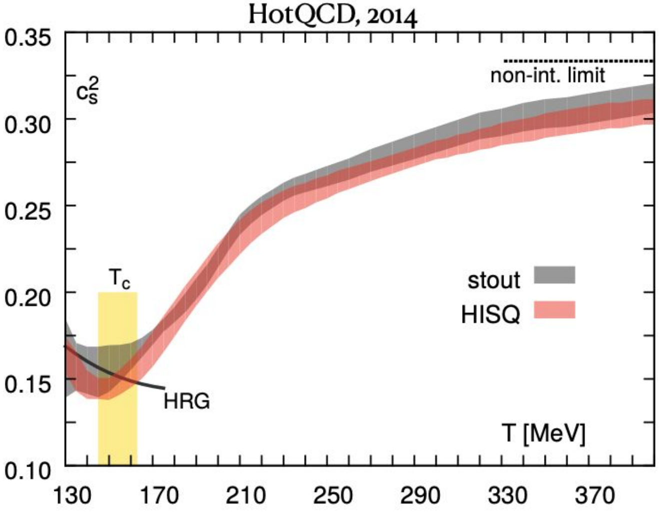
$$\delta A_L^k(r,t,z) = e^{-i(\omega t - qz)} \delta A_L^{ka}(r) t^a \ , \quad (k = x, y)$$

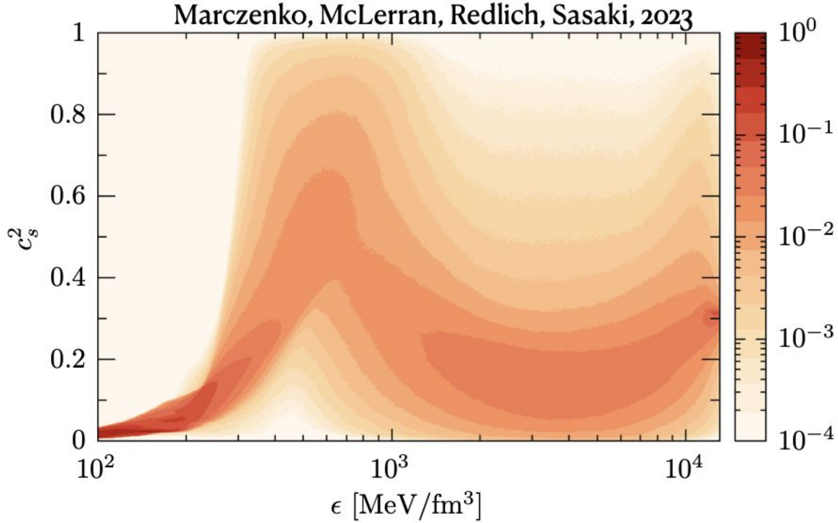
- Instability driven by Chern-Simons term fixed by flavor anomalies of QCD.
- Ground state can have spatially modulated fields $A_L^{xa}(r,z)$, dual QCD phase has modulated currents: $\langle \bar{\psi} \gamma^x (1-\gamma_5) t^a \psi \rangle$.





Demircik, Jokela, Järvinen, Piispa 2405.02392





$$c_s^2 = \frac{S}{T}\frac{dT}{dS} < \frac{1}{3}$$

$$c_s^2 = \frac{n}{\mu}\frac{d\mu}{dn} > \frac{1}{3}$$

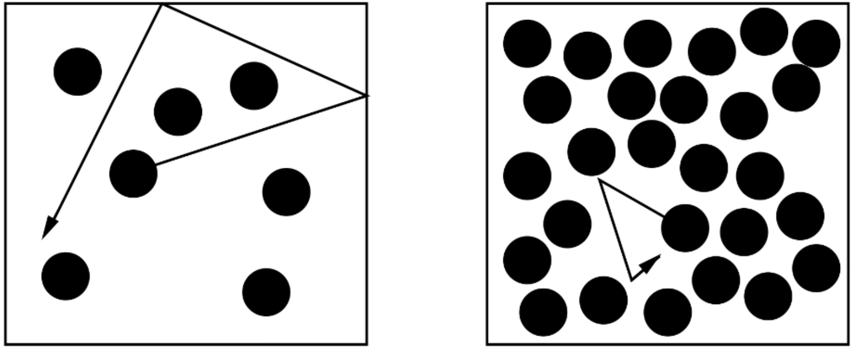
- Attractive interactions with resonance formation
- Chiral symmetry restoration and deconfinement

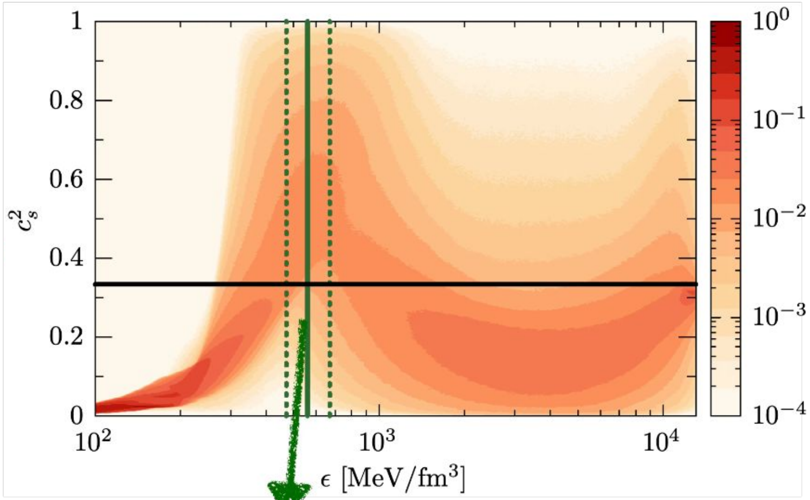
- Dominance of repulsive interactions
- Onset of quark or quarkyonic or another form of matter?

Non-monotonicity ⟷ Change of medium composition

Percolation theory vs speed of sound

see e.g. Satz, 1998; Castorina et al, 2009; Fukushima, 2020





Percolation theory: $n_c = 1.22/V_0$

$$n_{\text{peak}} = 0.54^{+0.09}_{-0.07}\ \text{fm}^{-3}$$

Avg. proton radius: $R_0 = 0.80 \pm 0.05\ \text{fm}$ → $n_c = 0.57^{+0.12}_{-0.09}\ \text{fm}^{-3}$

Wang et al 2022

Pb-Pb collisions at $\sqrt{s} = 2.76\ \text{TeV}$ → $n_c = 0.60 \pm 0.07\ \text{fm}^{-3}$

Andronic et al 2018

Measure of conformality

Trace Anomaly measure

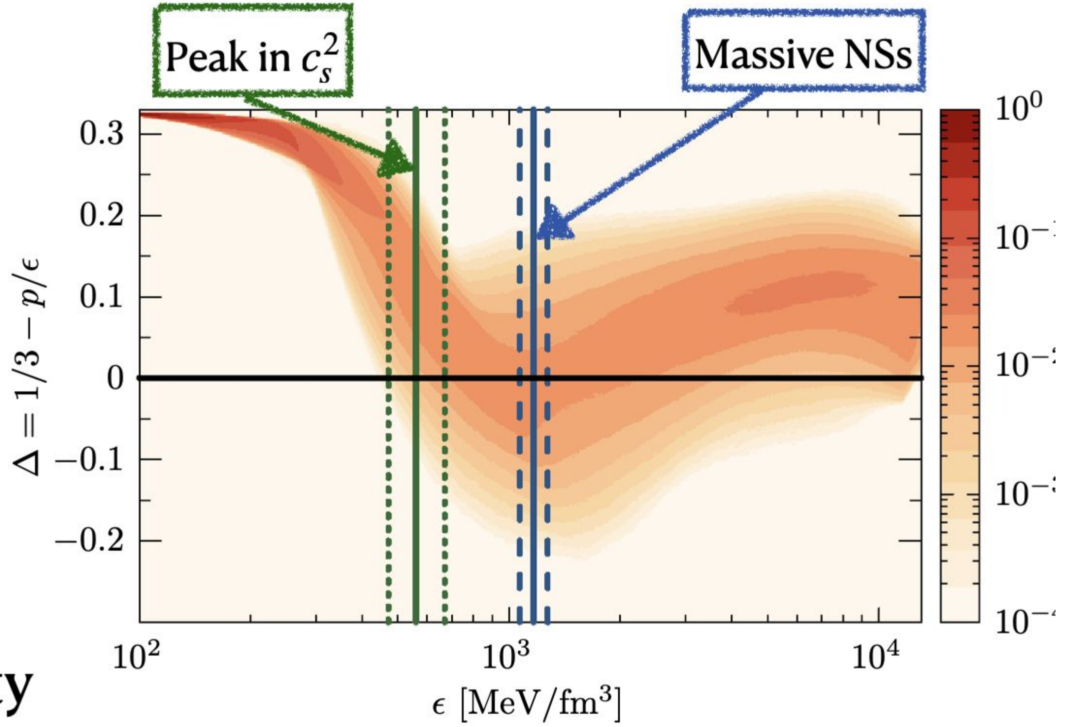
$$\Delta = \frac{1}{3} - \frac{p}{\epsilon}$$

Δ monotonic up to $\simeq \epsilon_{\rm TOV}$

$$c_s^2 = \frac{1}{3} - \Delta - \epsilon\frac{d\Delta}{d\epsilon}$$

Maximum in c_s^2

Fast approach to comformality



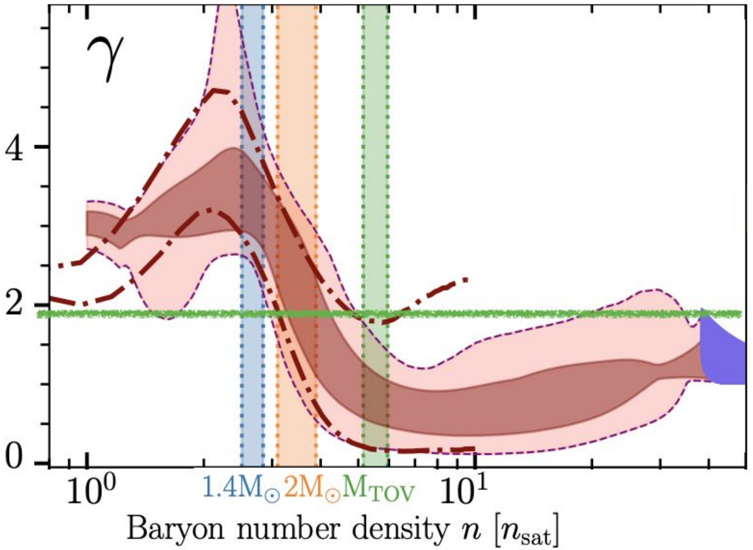
$\Delta \simeq 0$ at $\epsilon \simeq 1$ GeV/fm^3

Changeover to nearly-conformal regime

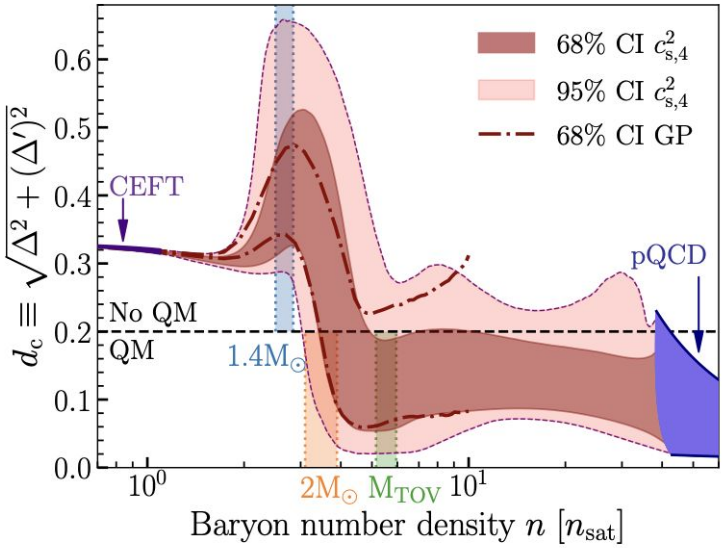
$$\gamma = \frac{\epsilon}{p} c_s^2 = \frac{c_s^2}{1/3 - \Delta} \lesssim 1.75$$

$$d_c = \sqrt{\Delta^2 + (\epsilon \Delta')^2} \lesssim 0.2$$

Annala et al 2020



Annala et al 2023



Curvature of the energy per particle

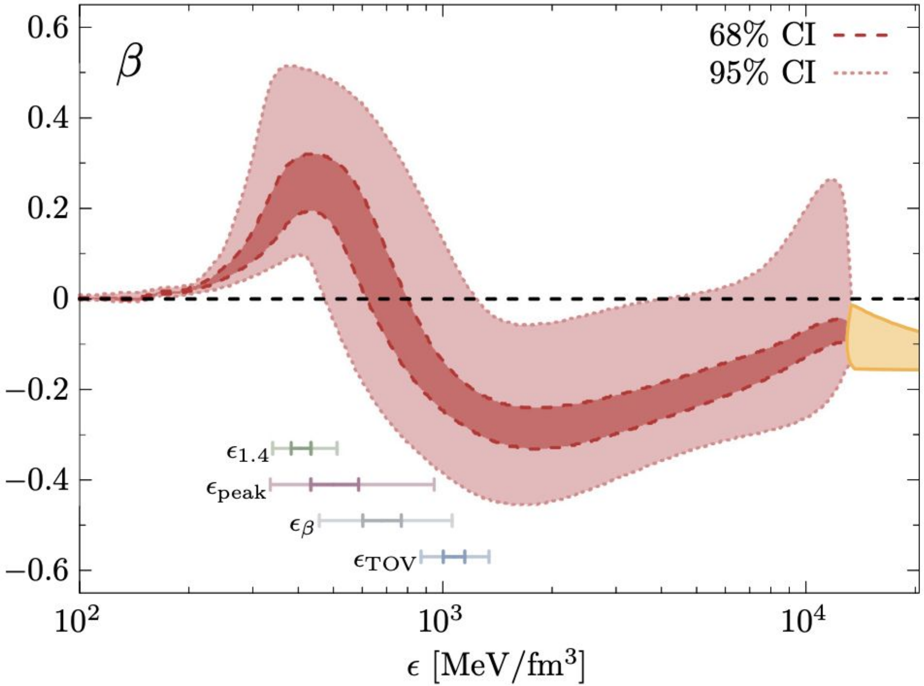
Pressure from $\frac{\epsilon}{n} \rightarrow p = n^2 \frac{d\epsilon/n}{dn}$

$$c_s^2 = \frac{1}{\mu}\frac{dp}{dn} = \alpha + \beta$$

$$\alpha = 2\frac{n}{\mu}\frac{d\epsilon/n}{dn} = 2\frac{1/3 - \Delta}{4/3 - \Delta}$$

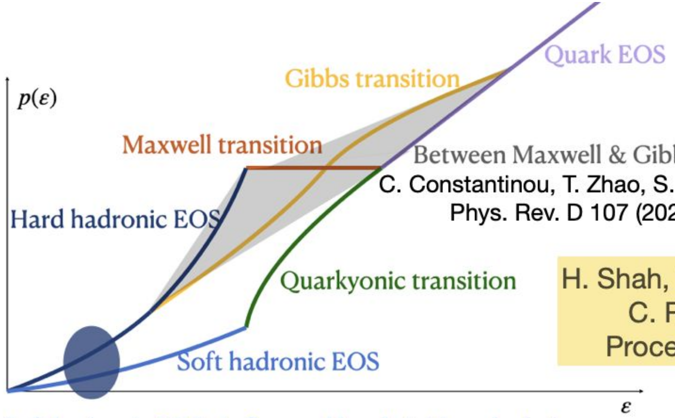
$$\boxed{\beta = \frac{n^2}{\mu}\frac{d^2\epsilon/n}{dn^2} = c_s^2 - \alpha}$$

Marczenko, Redlich, Sasaki, 2024



$\beta = 0 \rightarrow$ changeover to conformal regime

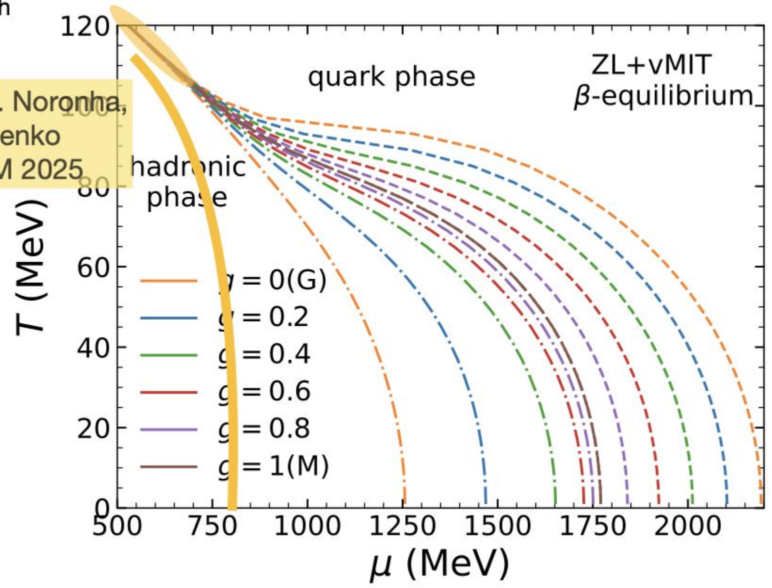
x



C. Constantinou, T. Zhao, S. Han, M. Prakash
Phys. Rev. D 107 (2023) 074013

Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

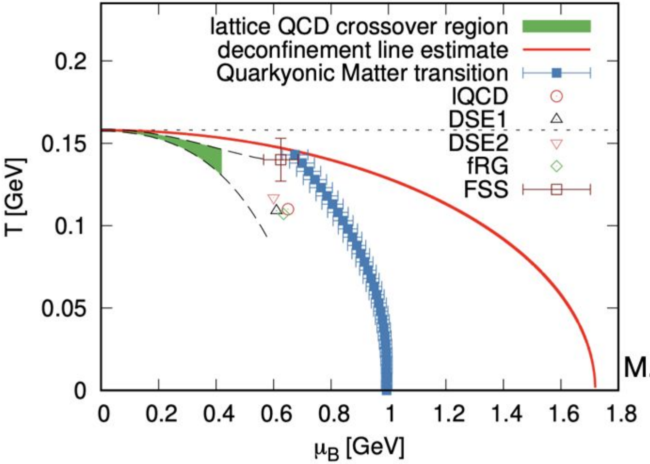
Between Maxwell & Gibbs



H. Shah, M. Hippert, J. Noronha, C. Ratti, V. Vovchenko
Proceedings for QM 2025

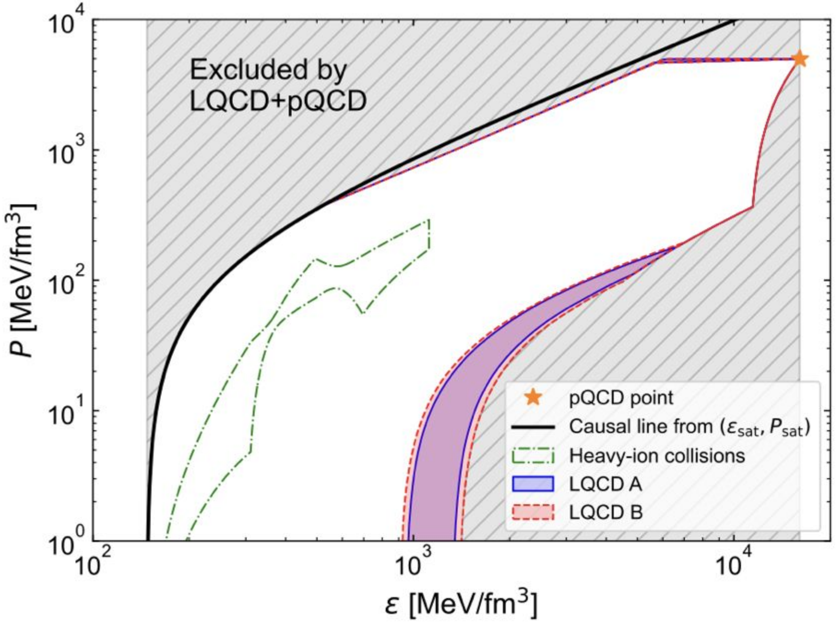
Based on C. Constantinou, M. Guerrini, T. Zhao, S. Han, M. Prakash
Accepted Phys. Rev. D (2025)

Quakyonic



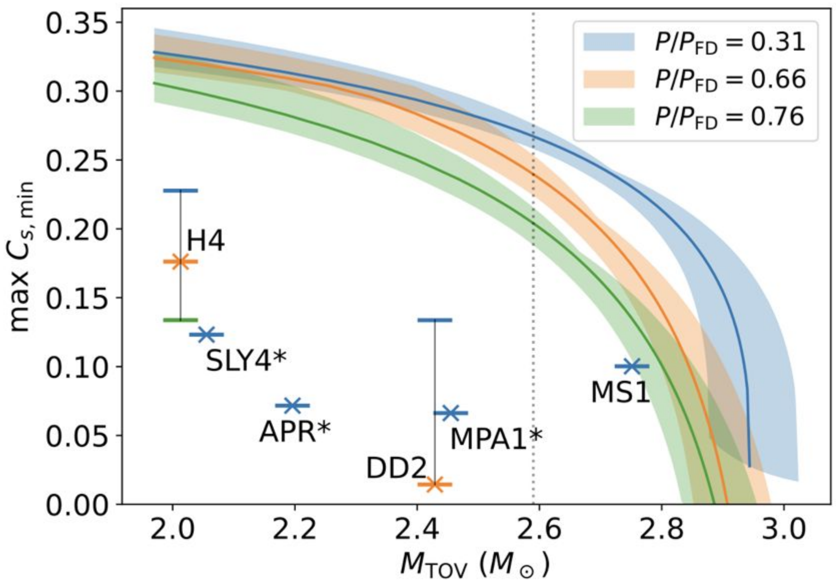
M. Bluhm, Y. Fujimoto, L. McLerran,M. Nahrgang
Phys. Rev. C 111 (2025) 044914

Bounds from LQCD with finite μ_I



Y. Fujimoto, S. Reddy
Phys. Rev. D 109 (2024) 014020

Bounds sound speed beyond TOV density



D. Zhou arXiv: 2408.16738

Bulk Viscosity

- A small step away from the ideal fluid.

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (p+\Pi)h^{\mu\nu} + q^{(\mu}u^{\nu)} + \pi^{\mu\nu}$$

$$\Pi = -\zeta[\Theta + \beta_0\dot{\Pi} + \frac{1}{2}T\nabla_\mu(\frac{\beta_0}{T}u^\mu)\Pi - \alpha_0\nabla_\mu q^\mu - \gamma_0 T q^\mu \nabla_\mu(\frac{\alpha_0}{T})$$

- Realized from hadron or Gluon splitting in the HRG (QGP) region:

$$\pi + \pi \longleftrightarrow \pi + \pi + \pi + \pi$$

$$g + g + g \longleftrightarrow g + g$$

- Realized from (modified) Urca process,

in npe neutron star matter:

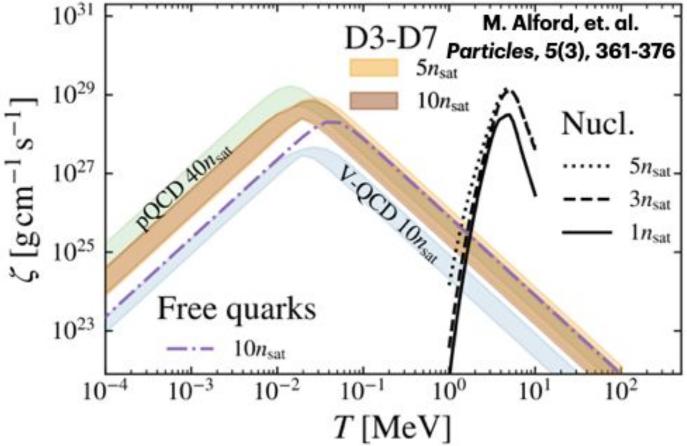
$$p + e(+X) \longleftrightarrow n(+X)$$

in ude quark star matter:

$$u + e(+X) \longleftrightarrow d(+X)$$

- Realized from the nonleptonic $W^\pm$ process,

$$u + d \longleftrightarrow u + s$$



J. Rojas, T. Gorda, C. Hoyos, N. Jokela, M. Järvinen, A. Kurkela, R. Paatelainen, S. Säppi, A. Vuorinen Phys. Rev. Lett. 133 (2024) 071901

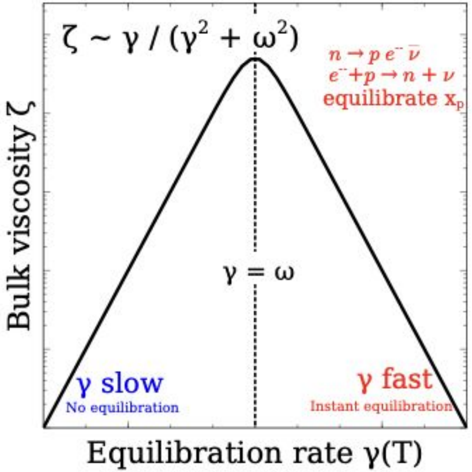
J. Hernandez, C. Manuel, L. Tolos Phys. Rev. D 109 (2024) 123022

Bulk Viscosity of npe matter

Bulk viscosity $\zeta = \frac{\varepsilon + p}{\omega}\mathrm{Im}\left[c_{dy}^2\right] = \frac{\gamma}{\omega^2 + \gamma^2}(c_{ad}^2 - c_{eq}^2)(\varepsilon + p)$, where $\gamma = -\frac{1}{n_B}\frac{\partial(\Gamma_{n\to p} - \Gamma_{p\to n})}{\partial \delta_\mu}\bigg|_{n_B,S}\frac{\partial \delta_\mu}{\partial \delta_x}\bigg|_{n_B,S}$

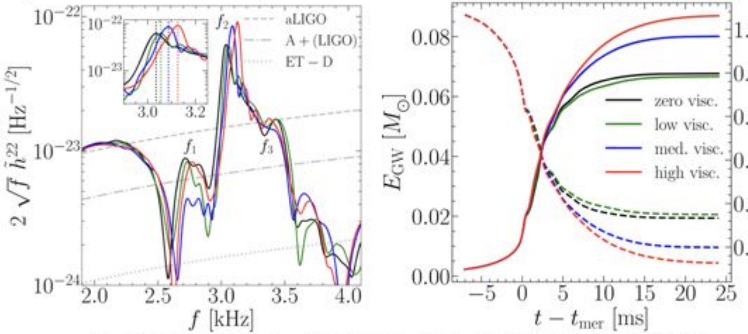
quantifies energy dissipation from compressional flow,

$$\frac{d\varepsilon}{dt} = -\zeta(\nabla\cdot\mathbf{v})^2.$$

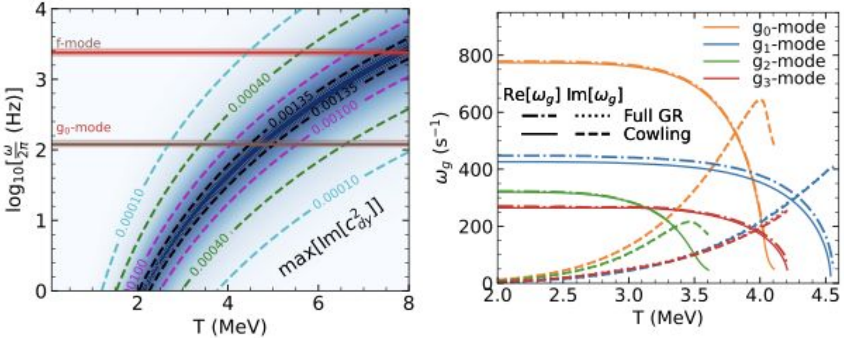


Depends on EOS, e.g.,
increases with symmetry energy slope L.

Y. Yang, M. Hippert, E. Speranza, J. Noronha
Phys. Rev. C 109 (2024) 015805



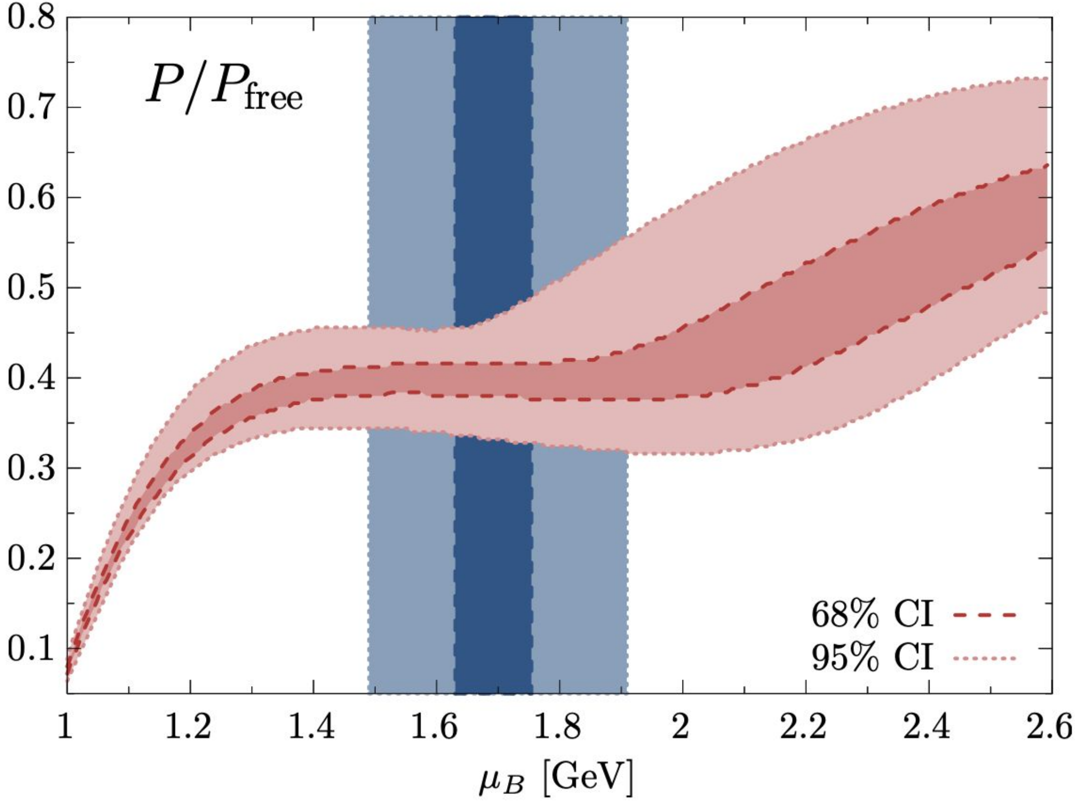
M. Chabanov, L. Rezzolla PRL 134 (2025) 071402



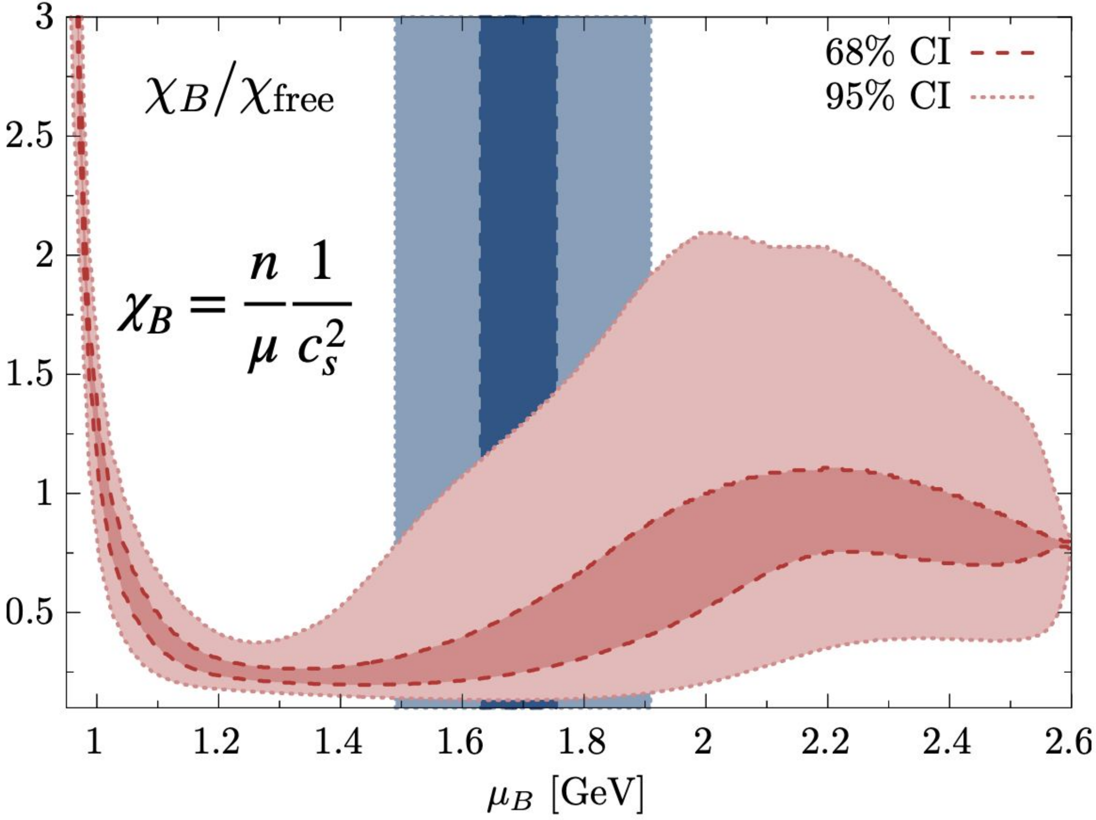
T. Zhao, P. Rau, A. Haber, S. Harris,
C. Constantinou, S. Han ApJ 993 (2025) 161

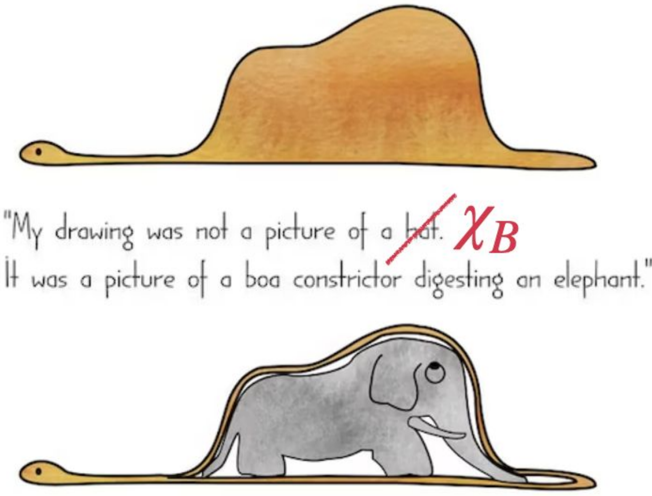
Backup

Equation of State



Net-baryon number susceptibility





Charge current processes

- Nonleptonic process dominates at low T,
  $u + d \longleftrightarrow u + s \qquad \Gamma \propto \mu^5 T^3$

  $\Gamma_{sd} - \Gamma_{ds} \propto (\mu_s - \mu_d)\mu_d^5 T^2$
  $\zeta = \dfrac{\gamma}{\gamma^2 + \omega^2} \times$ [lots of susceptibilities]

- Direct URCA process modifies low-density,
  $u + e(+\bar{\nu}_e) \longleftrightarrow d(+\nu_e) \qquad \Gamma \propto \mu^3 T^5$
  $\Gamma_{ud} - \Gamma_{du} \propto (\mu_d - \mu_u - \nu_e)\mu_d \mu_u \mu_e T^4$
  with both processes, $\zeta = \dfrac{\kappa_1 + \kappa_2 \omega^2}{\kappa_3 + \kappa_4 \omega^2 + \omega^4}$

EOS, Susceptibilities

- With free strange quark matter.

- With MIT bag model + $O(\alpha_s)$, the renomolization scale $\Lambda = 2\mu_s$

- With pQCD, the renormalization parameter $X \in [1/2,2]$

- "Pocket formula" with holographic D3-D7 model,
$$p = \frac{1}{4\pi^2}\sum_{i=u,d,s}(\mu_i^2 + M_i^2)^2, M_i \in [435,540]MeV$$