A finite temperature expansion for the dense matter equation of state

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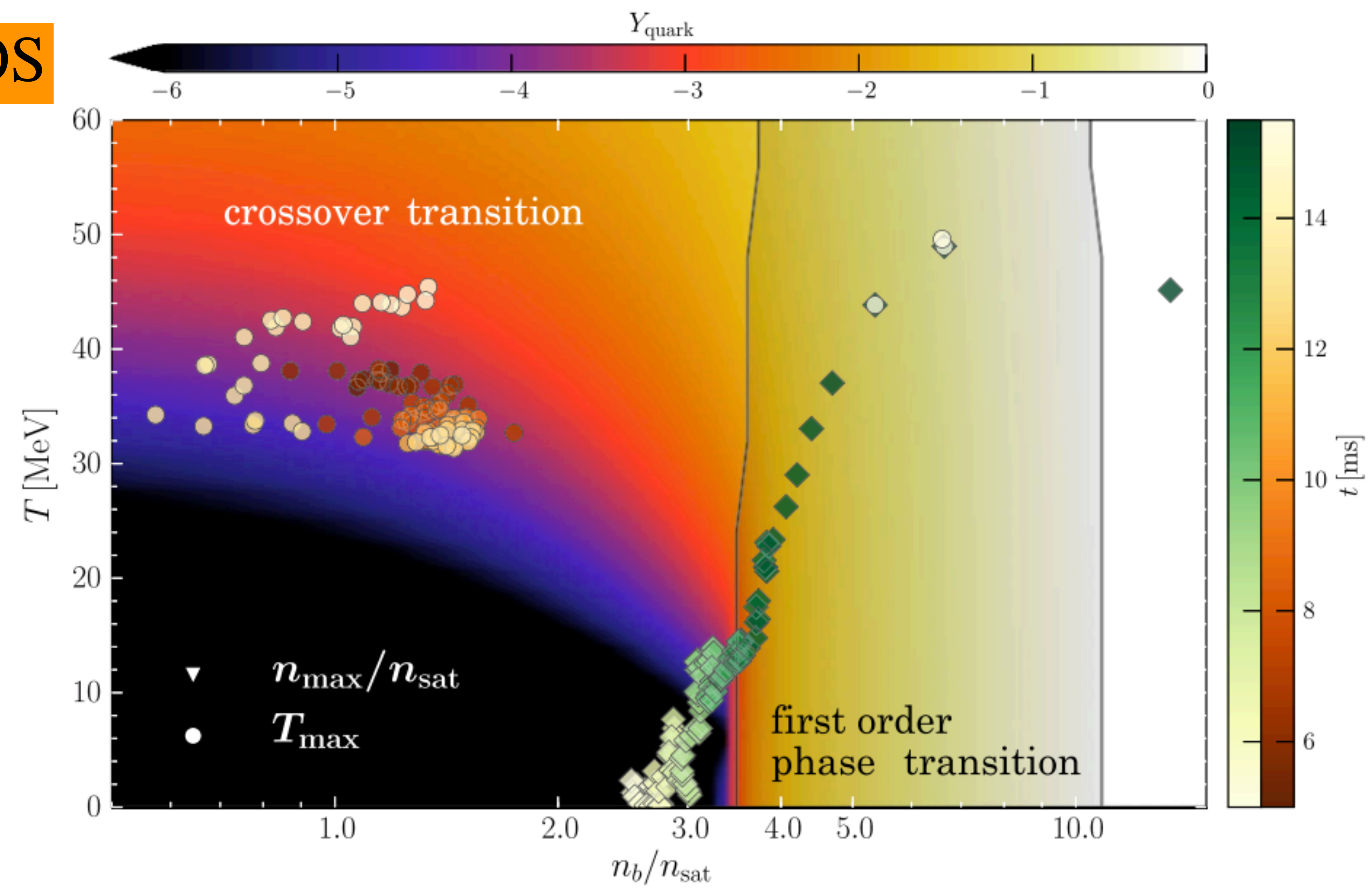
Relevant works:
PRC 113 (2026), D. Mroczek, N. Yao, K. Zine, and J. Noronha-Hostler (UIUC/ICASU), V. Dexheimer (Kent State), A. Haber (U. Southampton), L. Brodie (Wash. U.), E. Most (Caltech).
PRD 110 (2024), D. Mroczek, J. Noronha-Hostler, N. Yunes (UIUC/ICASU), M. Coleman Miller (Maryland).
arXiv: 2512.16720, H. Gholami, M. Hofmann (TUD), D. Mroczek, J. Noronha-Hostler (UIUC/ICASU).

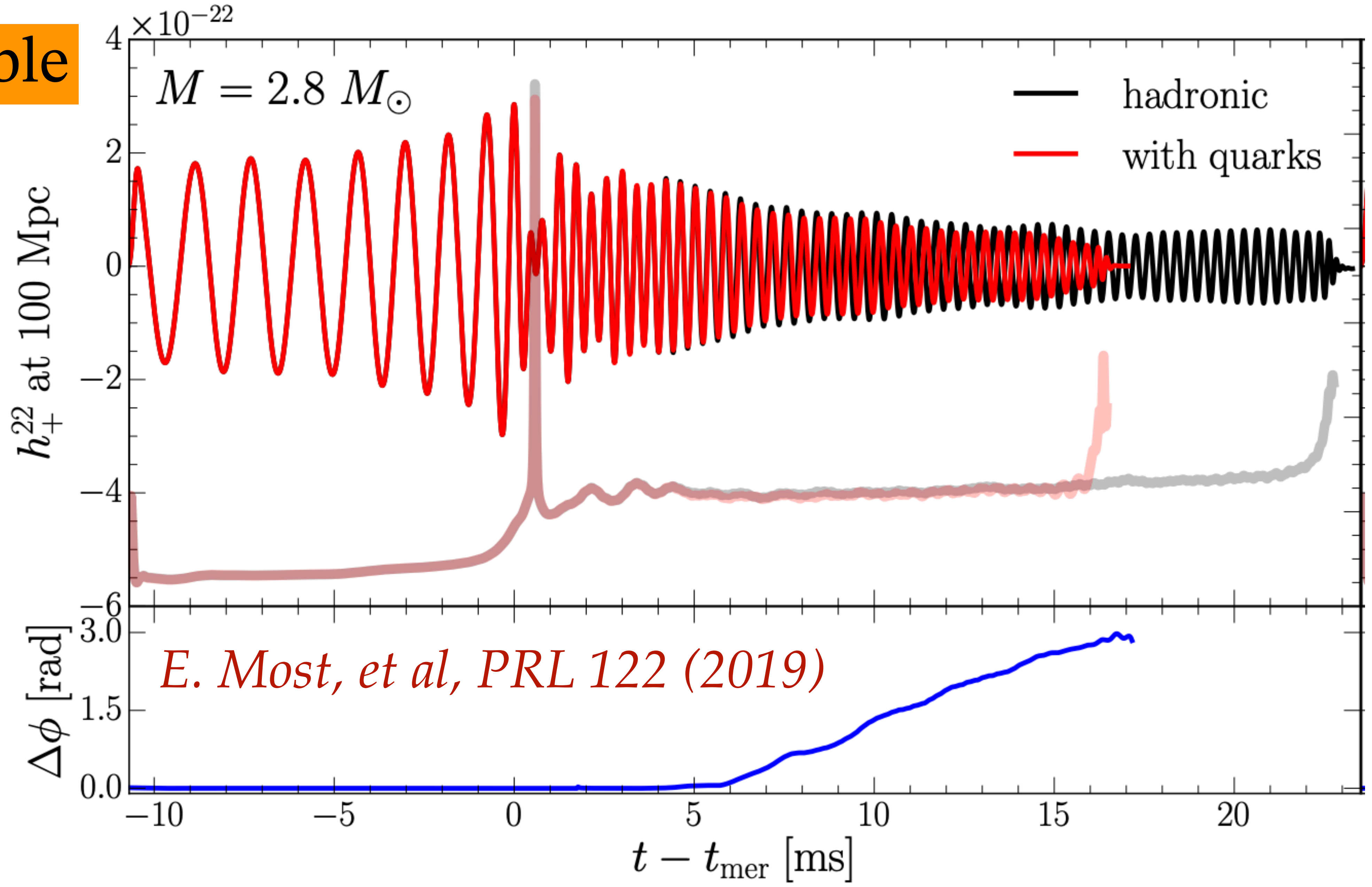
Why do we need a finite temperature expansion of the dense matter EOS?

EOS



????

Observable



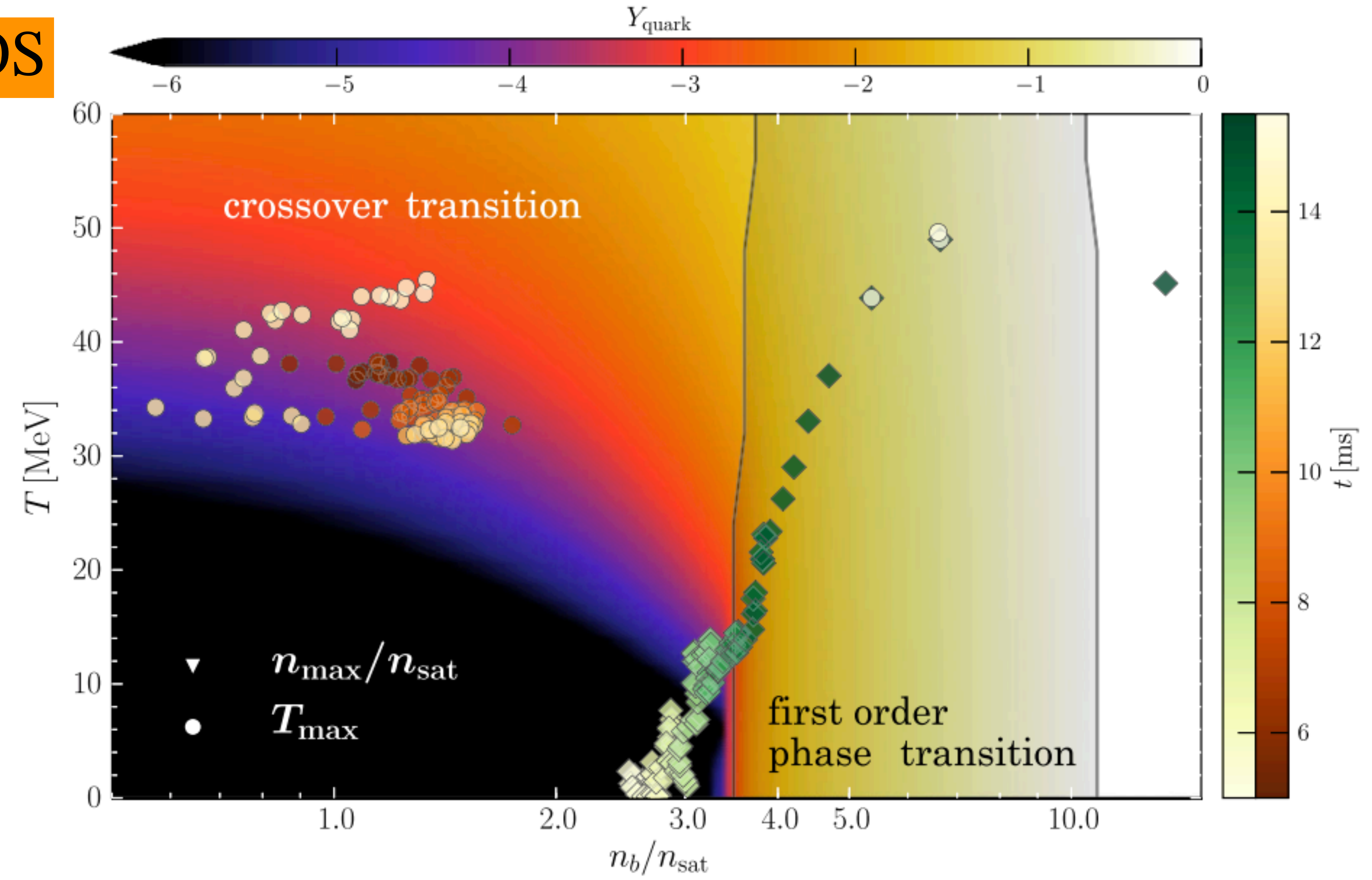
Example: **Dynamical** description of neutron star formation and their mergers requires a **3D** EOS

Why do we need a finite temperature expansion of the dense matter EOS?

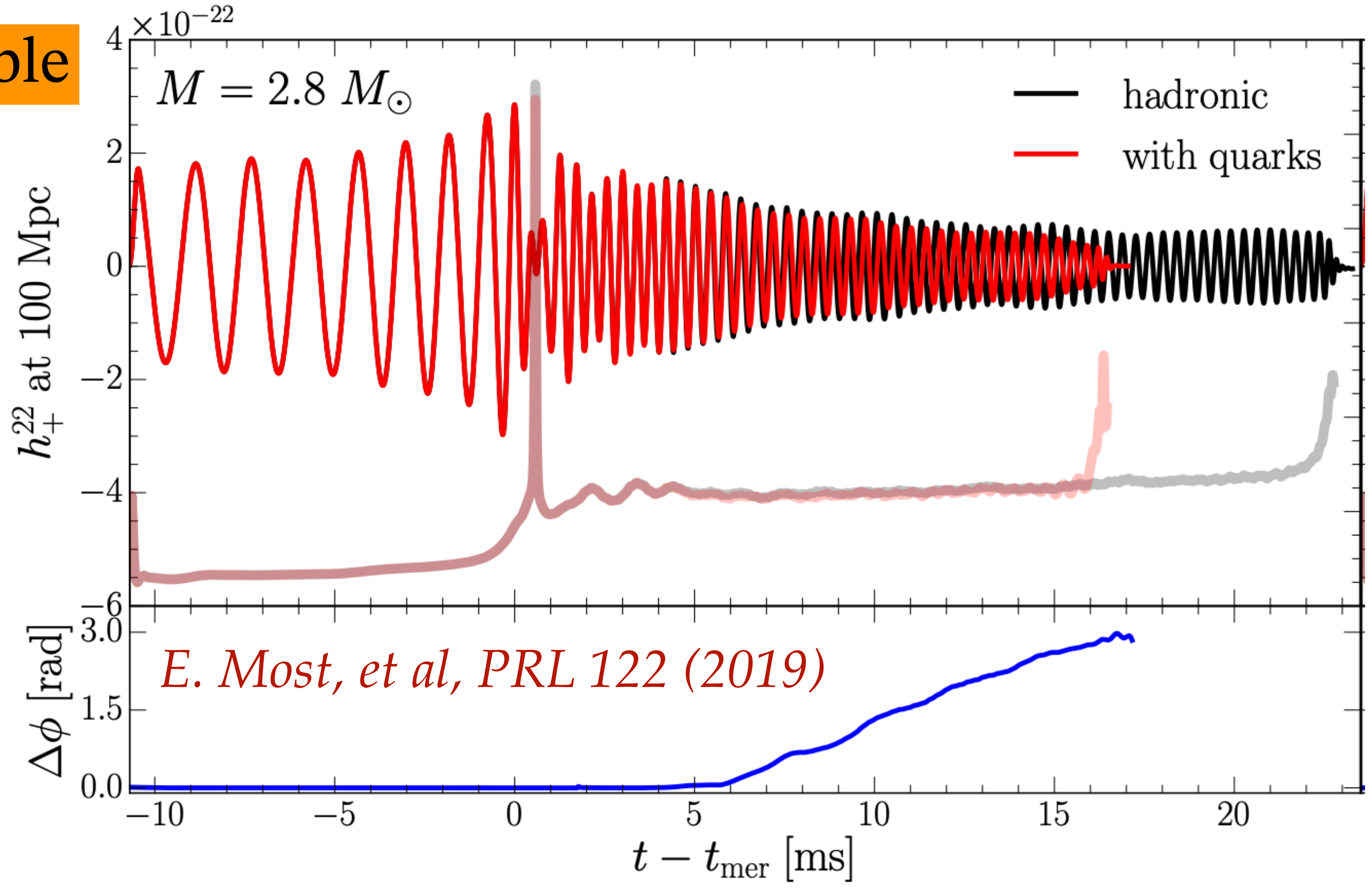
Inference

EOS model ➡ Observables vs. Data ➕ Theory

EOS



Observable



Dynamical systems are complex to model — what do we gain?

How do we discover new phases of matter in neutron stars?

EoS model

microscopic

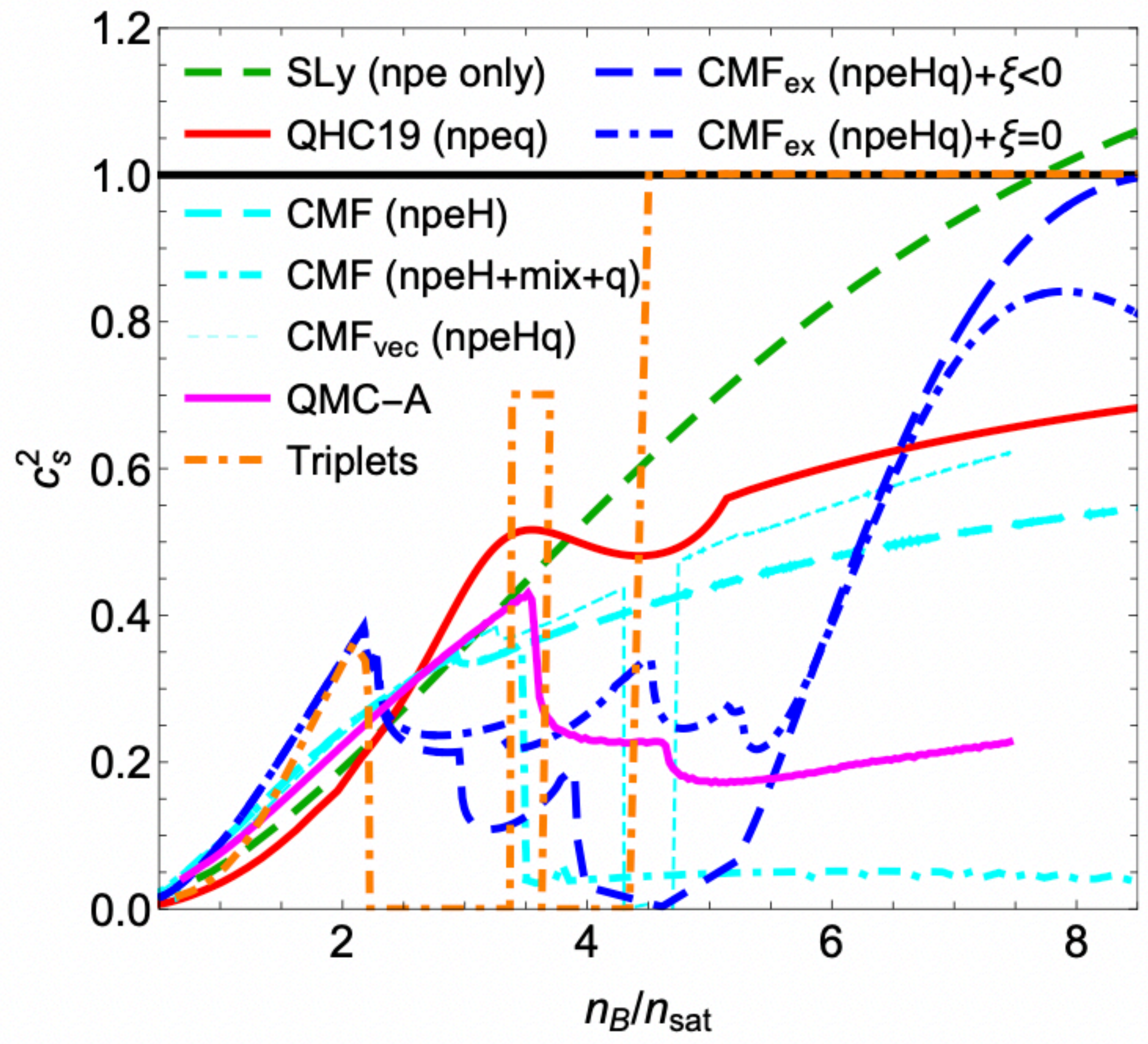
parametric/statistical (model agnostic)

Pros: interpretable
Cons: Fixed d.o.f. + interactions, costly evaluation

Pros: flexible, cheap
Cons: indirectly interpretable

- **Multi-scale correlations** characterize the onset of **new degrees of freedom**
- Not adequately captured by standard GPs or methods that fix the **correlation length**

from: Tan et al. PRD (2022), see for refs.

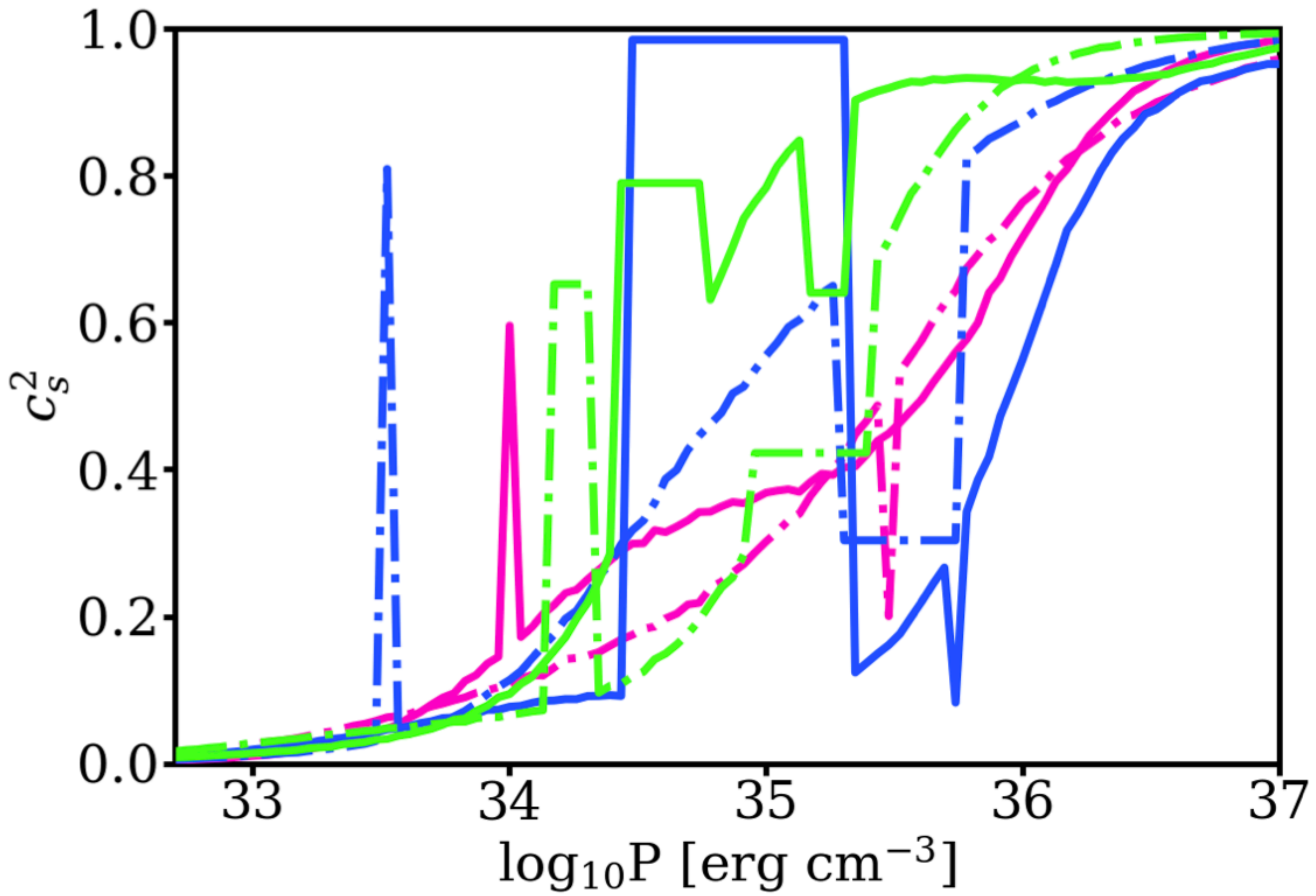


n: neutrons, p: protons, e: electrons, q: quarks, H: hyperons

How do we discover new phases of matter in neutron stars?

Modified Gaussian processes (mGP):

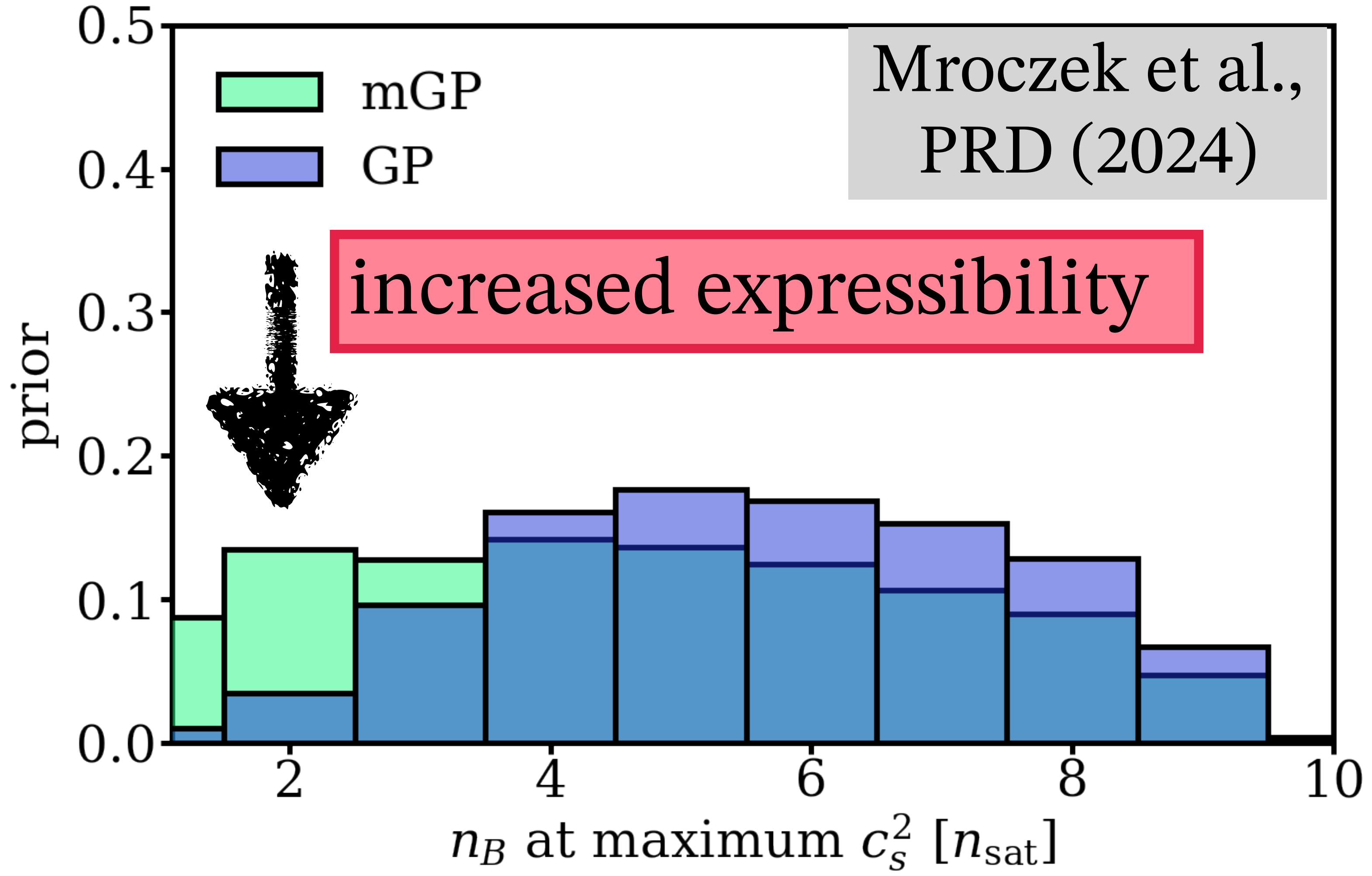
Accounts for **multi-scale correlations**

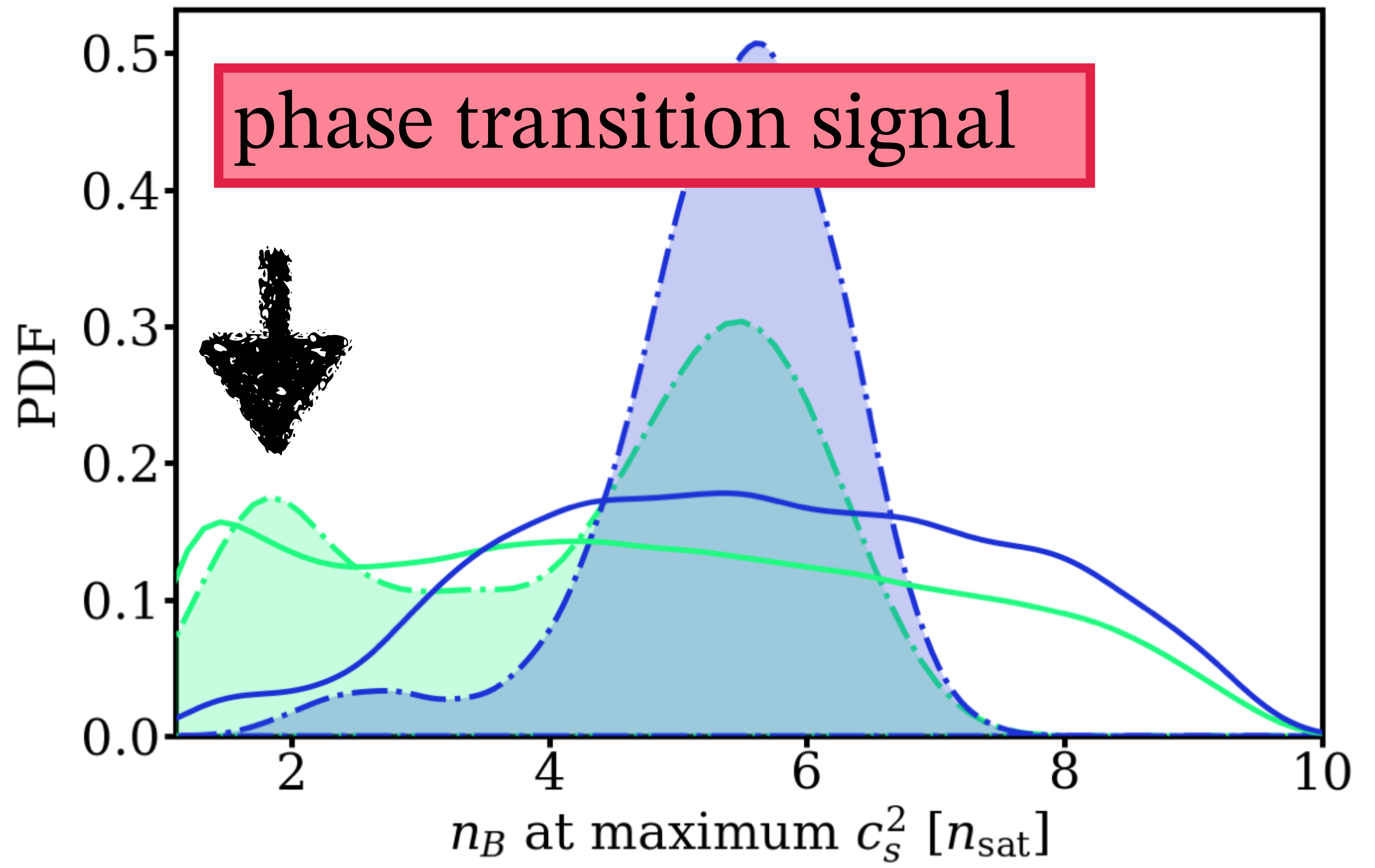


Long-range correlation (c_s^2 increases with density)

+ piecewise modifications

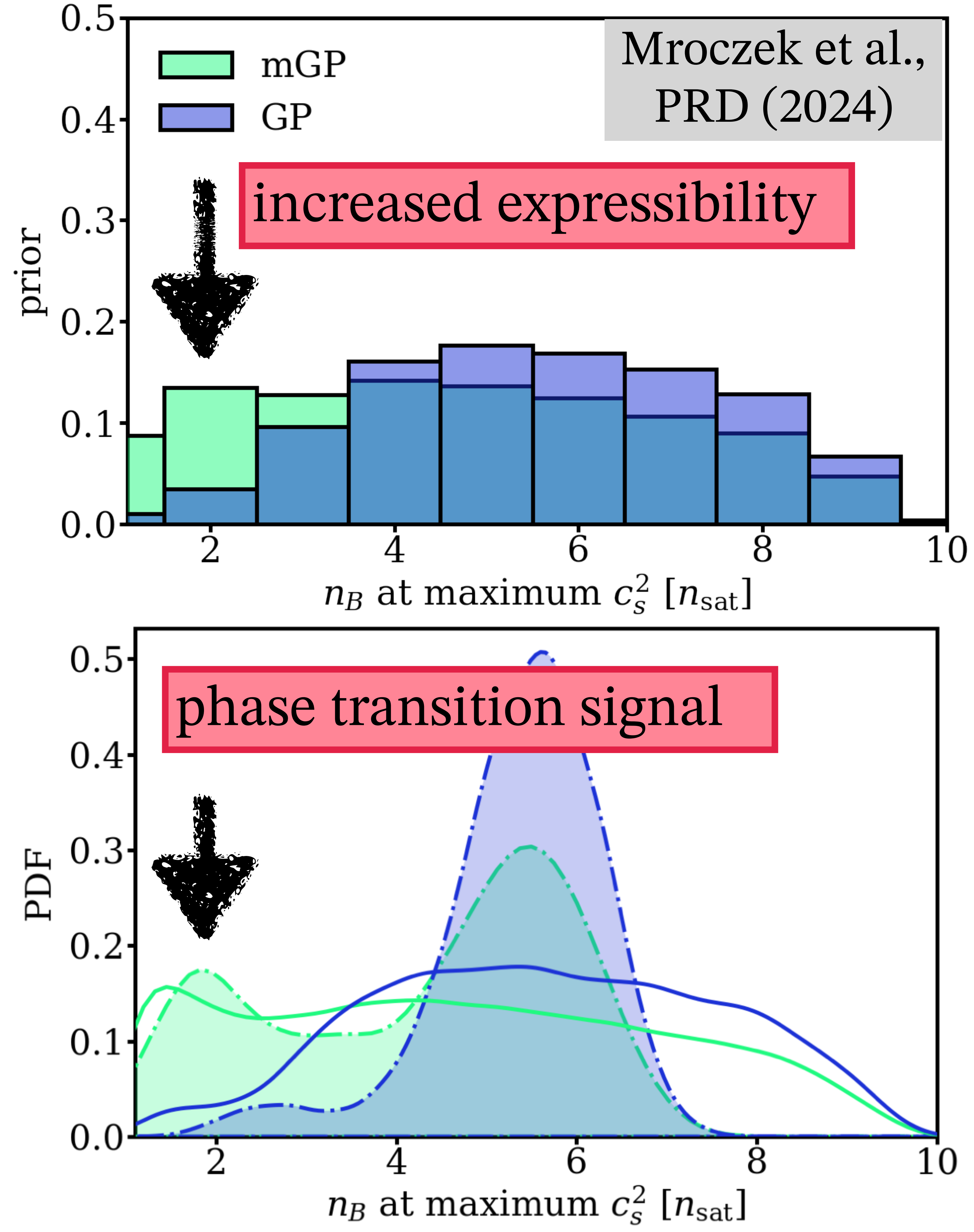
Steep rise in c_s^2 followed by a softening → maximum within NS densities





How do we discover new phases of matter in neutron stars?

Steep rise in c_s^2 followed by a softening → maximum within NS densities

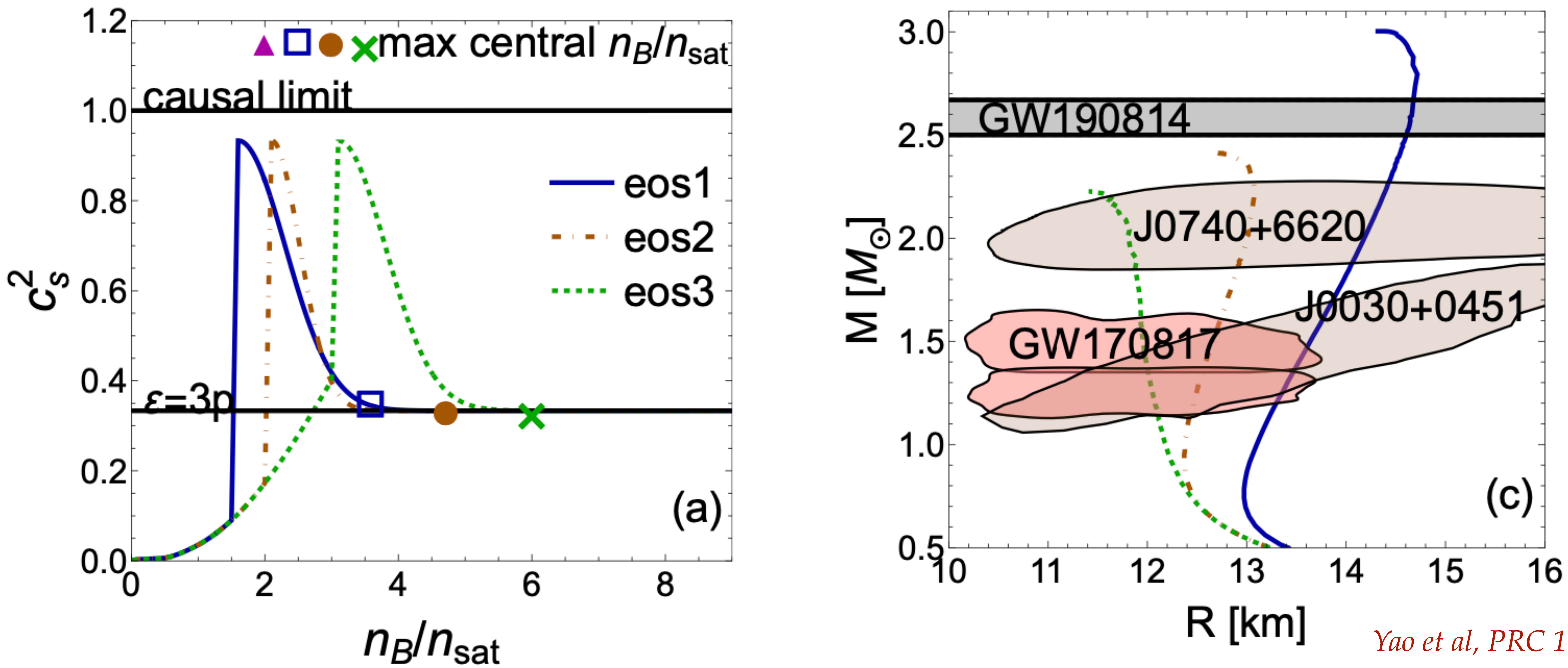


How to break the ambiguity?

1) More and more precise observations (time, NS population…)
2) Compare to other systems that probe the high density EoS

Case study: from neutron stars to heavy-ion collisions

Are EoS capable of producing super heavy ($M \geq 2.3\ M_\odot$) excluded by heavy-ion collisions?

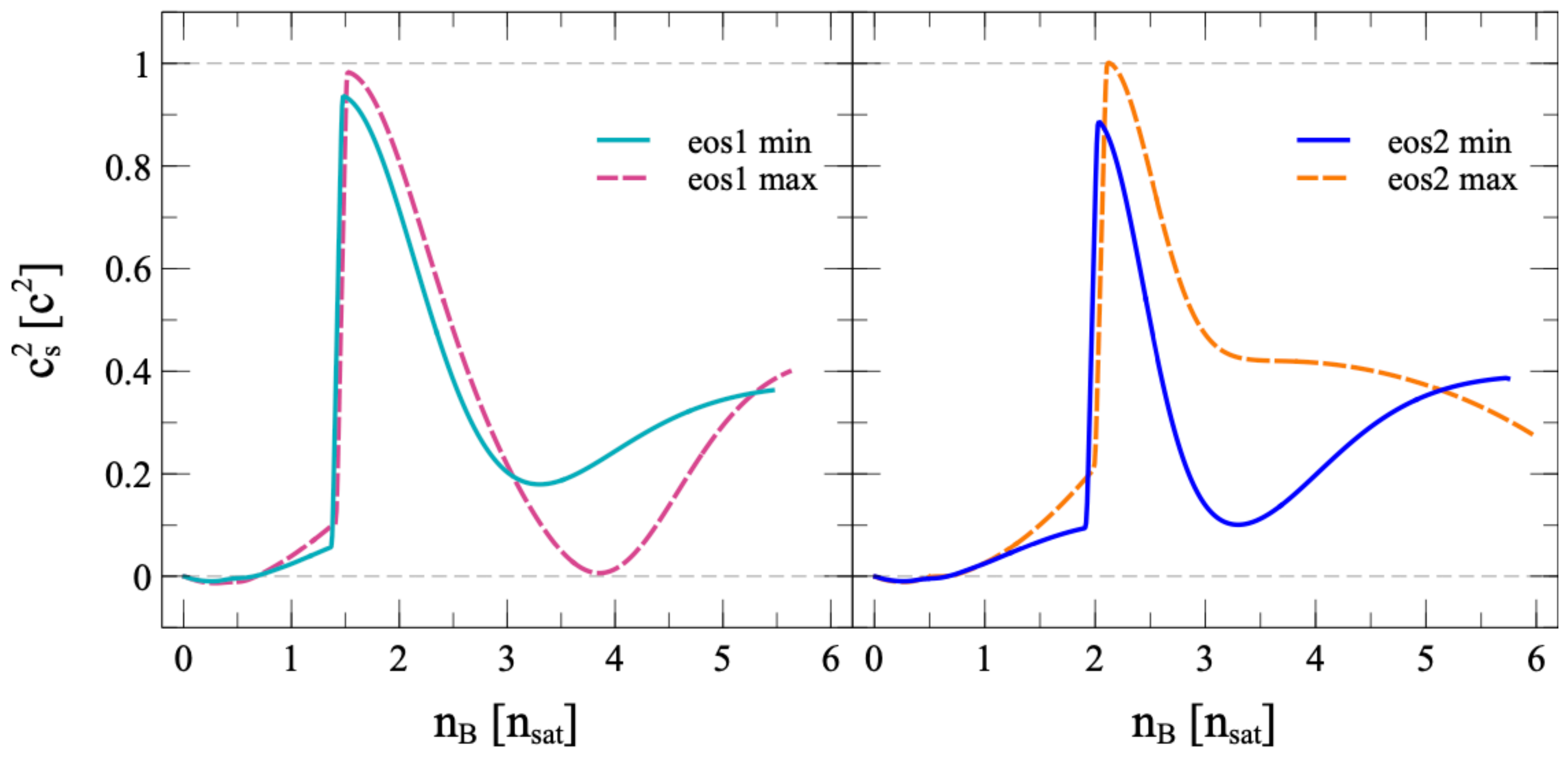


Yao et al, PRC 109 (2024)

Peak in c_s^2 can produce high $M_{\max}$ and satisfy observational constraints

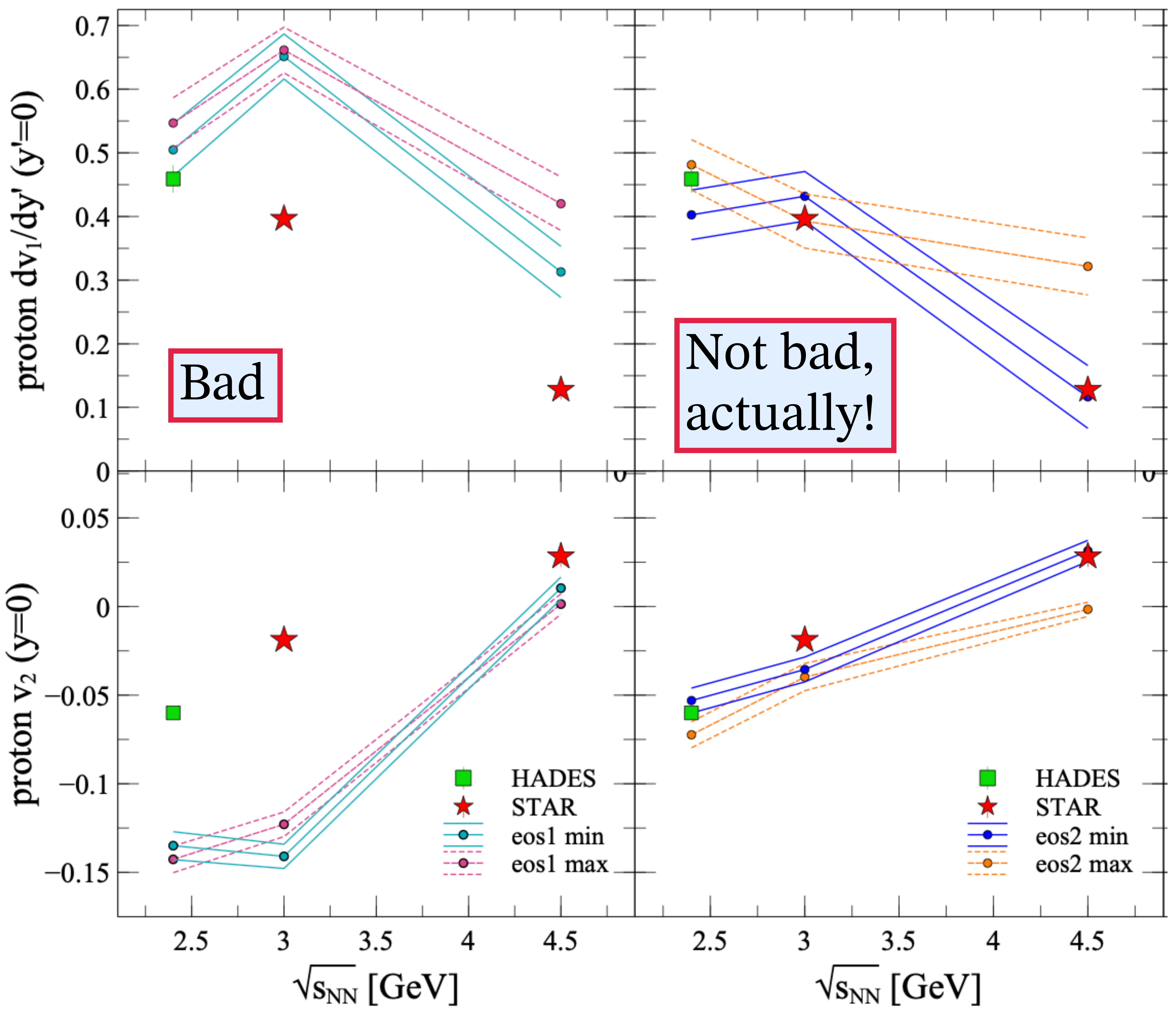
Super-heavy neutron stars vs. heavy-ion flow

- Set of representative NS EoS mapped to symmetric nuclear matter via symmetry energy expansion
- Compared to heavy-ion proton flow using hadronic transport



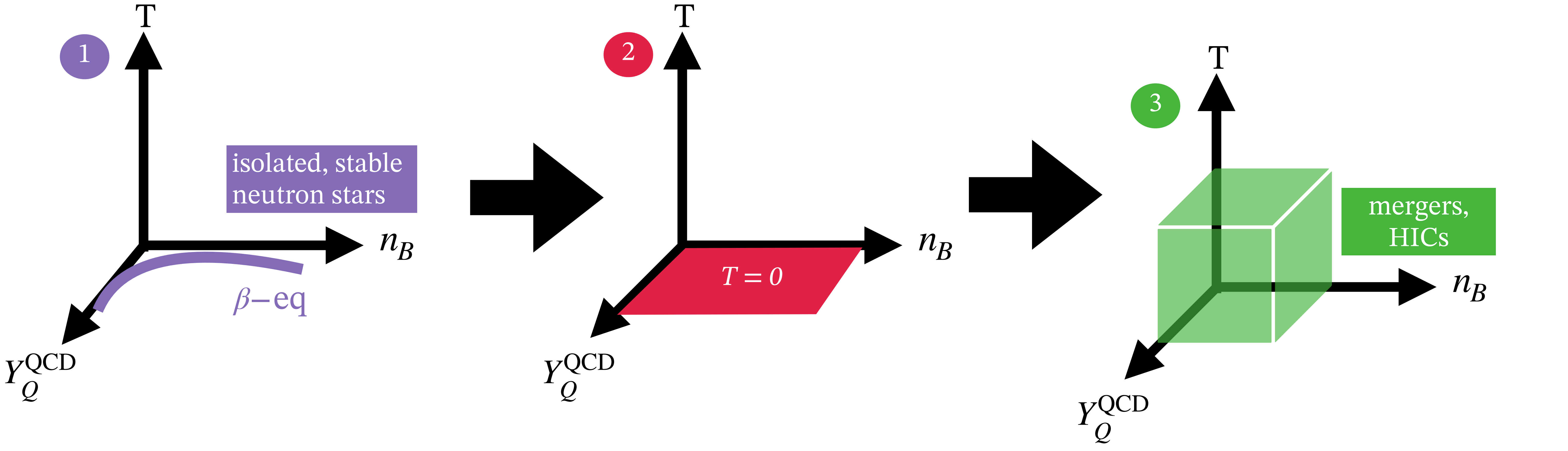
Starting from an **arbitrary NS EOS, how do we build a 3D EOS** for numerical relativity/HIC simulations?

Yao et al, PRC 109 (2024)



What is needed from a temperature expansion of the dense matter EOS?

<u>Dense matter</u> (in this work) → **hadron/quark** state of matter with **no strange degrees of freedom** in the regime relevant for **neutron stars.**



1) **Baryon number density** (isolated, stable NS)

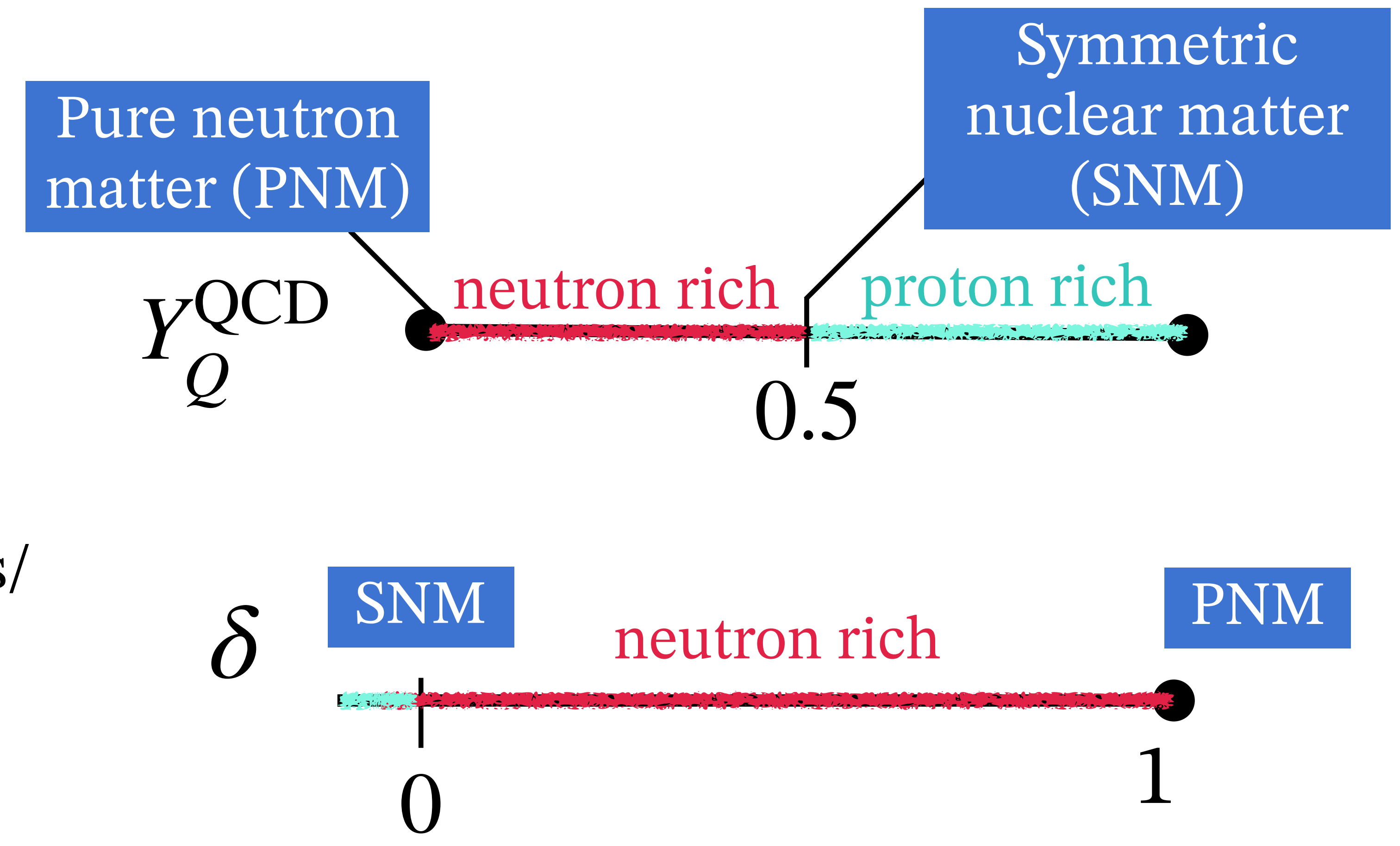
$$n_B = \left.\frac{\partial p}{\partial \mu_B}\right|_{\mu_Q}$$

Relevant scale: nuclear saturation density, $n_{\rm sat} \equiv 0.16\ {\rm fm}^{-3}$

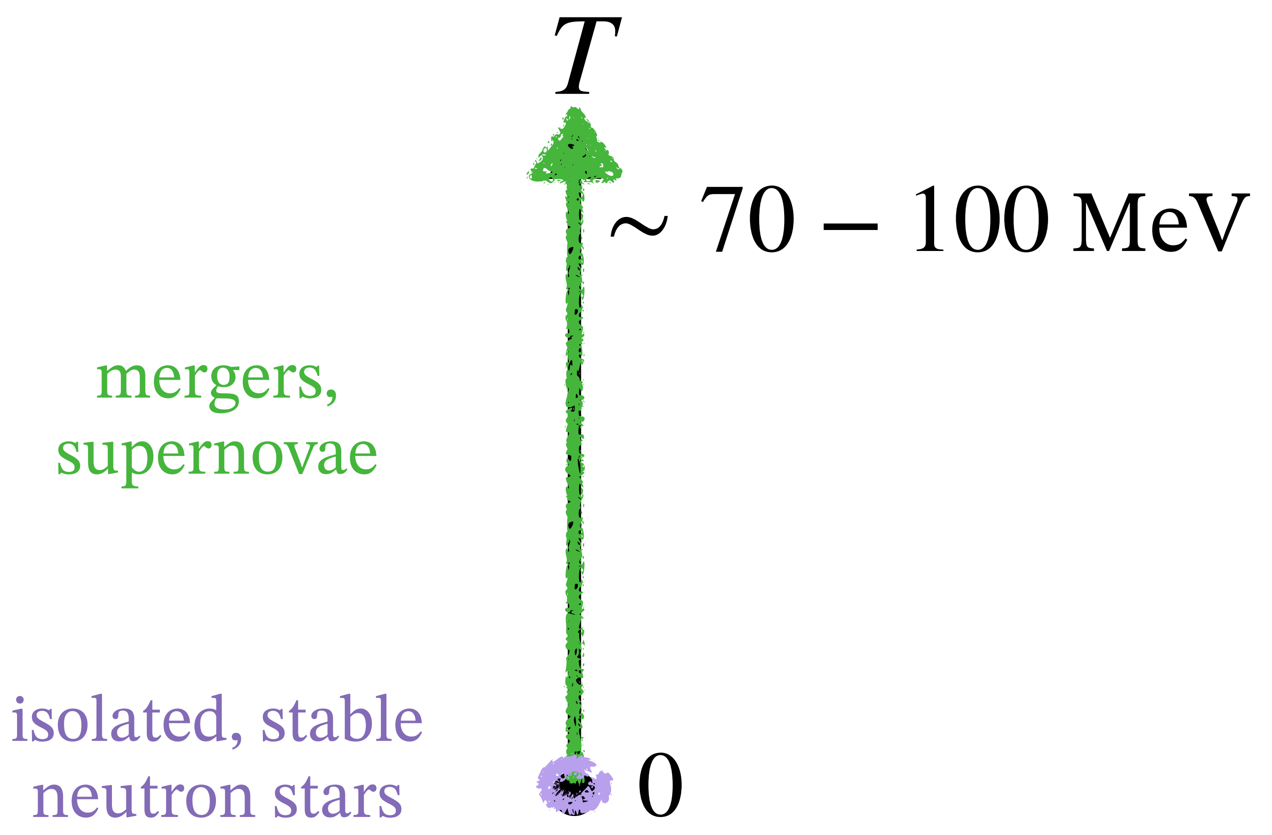
2) **Charge fraction / isospin asymmetry**

$$Y_Q^{\rm QCD} = \frac{n_Q^{QCD}}{n_B} \rightarrow \delta = 1 - 2Y_Q^{\rm QCD}$$

No leptons, hadrons/ quarks only.

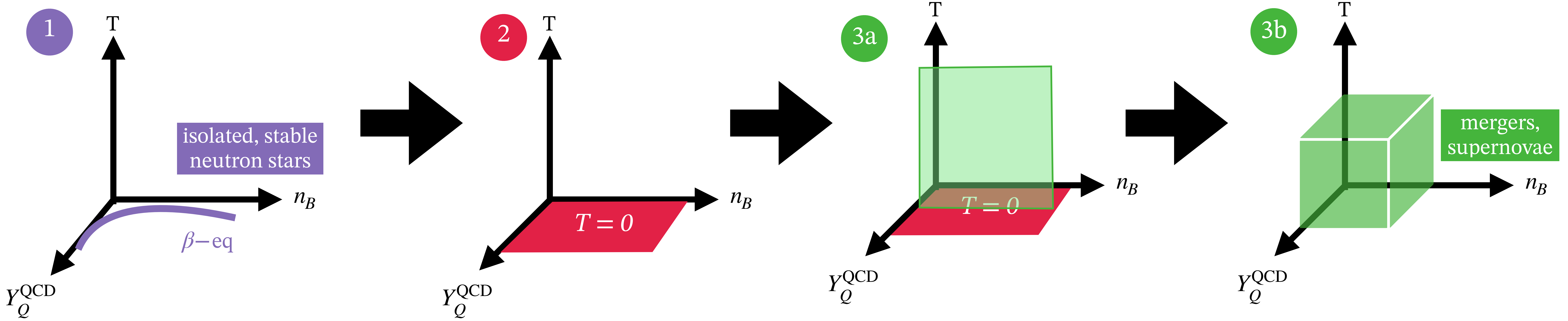


3) **Temperature**

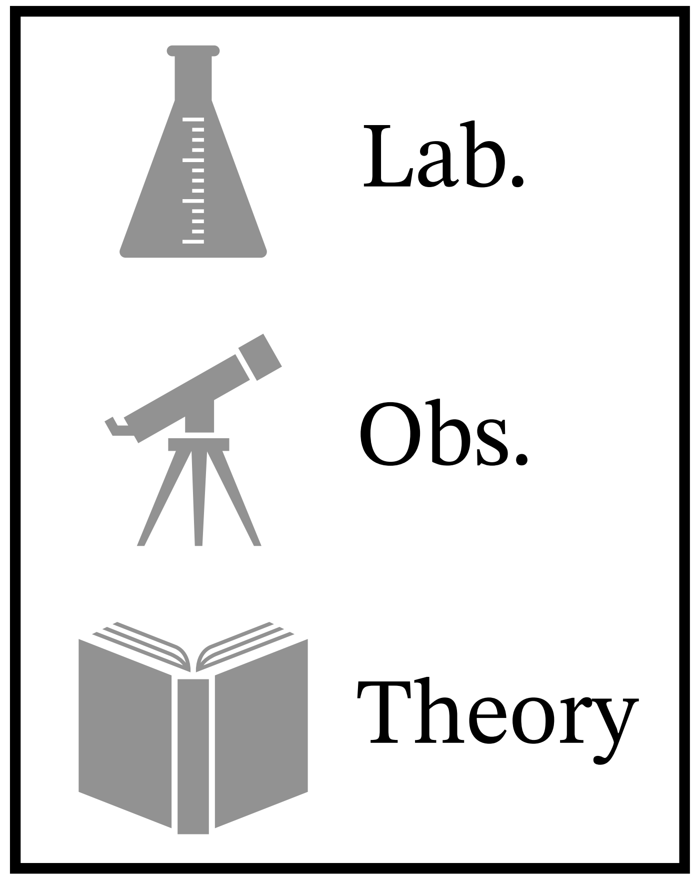


Mroczek et al., 2404.01658

What is needed (pt. 2) and our approach



- ☑ Thermodynamically consistent
- ☑ Beyond n+p degrees of freedom
- ☑ Connection to available experiments, observations, and theory predictions →



1 → 2: Expansion of the symmetry energy about NS EOS

Yao et al, PRC 109 (2024)

2 → 3a: Finite temperature expansion at fixed μ_Q

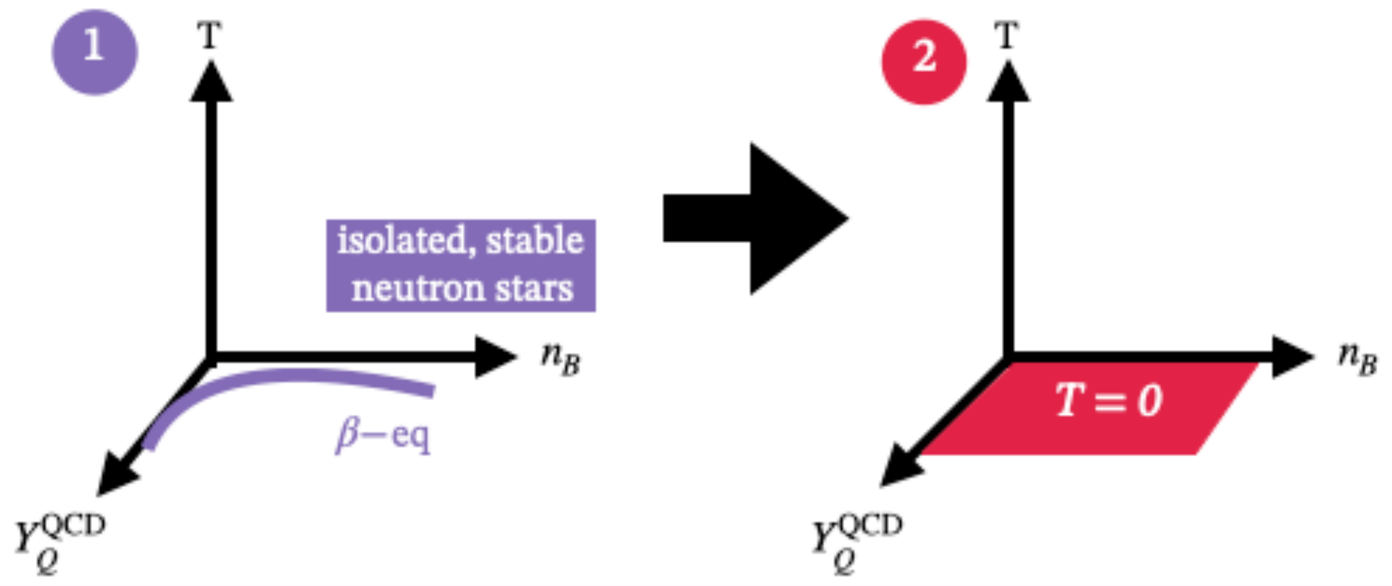
New!

3a → 3b: Expansion of charge fraction dependence of finite temperature effects

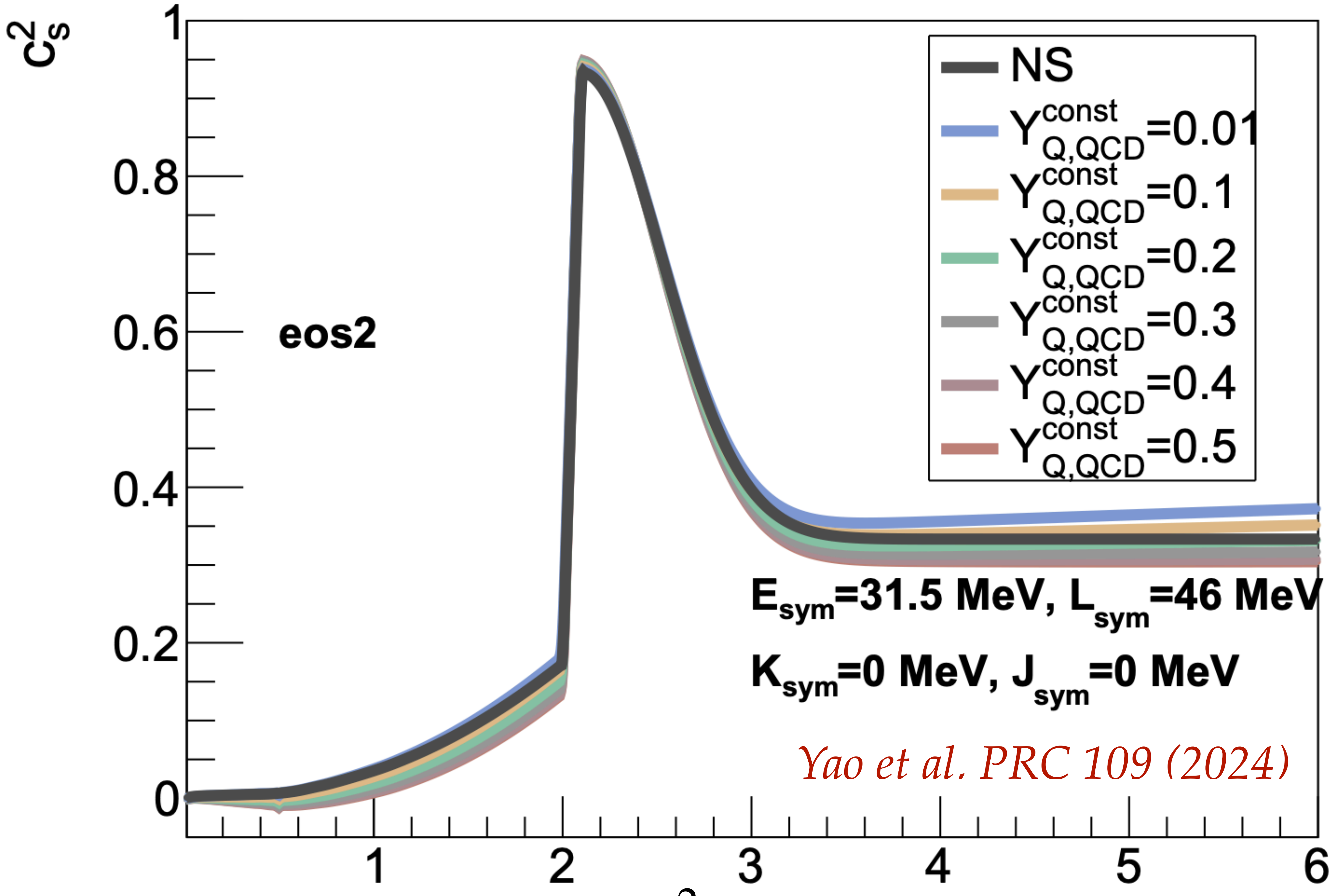
New!

From β-equilibrium to arbitrary charge fraction

- Symmetry energy expansion derived in Bombaci and Lombardo (1991), modified in Yao et al. (2024):



$$\frac{E_{\mathrm{HIC,sym}}}{N_B} = \underline{\frac{E_{\mathrm{NS,QCD}}}{N_B}} - \left[\underline{E_{\mathrm{sym,sat}}} + \underline{\frac{L_{\mathrm{sym,sat}}}{3}} \left(\frac{n_B}{n_0} - 1\right) + \underline{\frac{K_{\mathrm{sym,sat}}}{18}} \left(\frac{n_B}{n_0} - 1\right)^2 + \underline{\frac{J_{\mathrm{sym,sat}}}{162}} \left(\frac{n_B}{n_0} - 1\right)^3\right] (1-2Y_{Q,QCD})^2$$



Example: large bump in $c_s^2(n_B)$ of NS to SNM

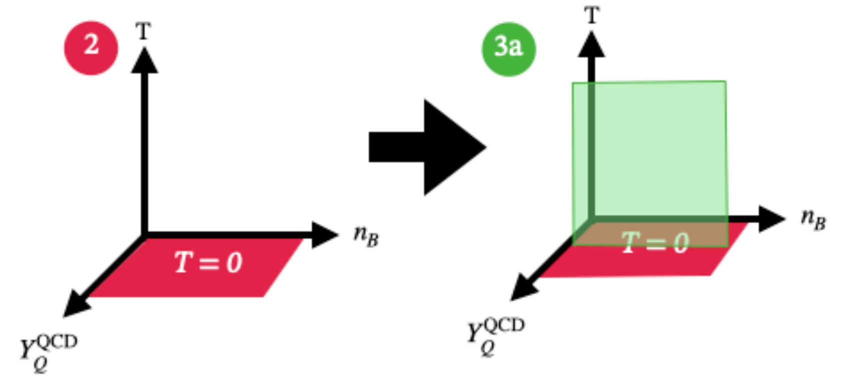
Input:

- NS EOS
- Symmetry energy coefficients

Coefficient	Definition	Range	References
$E_{\mathrm{sym,sat}}$	$\left(\frac{E_{\mathrm{PNM}}-E_{\mathrm{SNM}}}{N_B}\right)_{n_{\mathrm{sat}}}$	31.7 ± 3.2 [MeV]	Multiple data analyses from nuclear physics and astrophysics [121]
$L_{\mathrm{sym,sat}}$	$3n_{\mathrm{sat}}\left(\frac{dE_{\mathrm{sym},2}}{dn_B}\right)_{n_{\mathrm{sat}}}$	58.7 ± 28.1 [MeV]	Multiple data analyses from nuclear physics and astrophysics [121]
$K_{\mathrm{sym,sat}}$	$9n^2_{\mathrm{sat}}\left(\frac{d^2E_{\mathrm{sym},2}}{dn^2_B}\right)_{n_{\mathrm{sat}}}$	106 ± 37 [MeV]	PREXII [122, 123]
		-120^{+80}_{-100} [MeV]	Bayesian analyses inferred from GW170817 and PSR J0030+0451[124]
$J_{\mathrm{sym,sat}}$	$27n^3_{\mathrm{sat}}\left(\frac{d^3E_{\mathrm{sym},2}}{dn^3_B}\right)_{n_{\mathrm{sat}}}$	300 ± 500 [MeV]	Many-body nuclear theory [125]

Yao et al. PRC 109 (2024)

From T = 0 to finite T



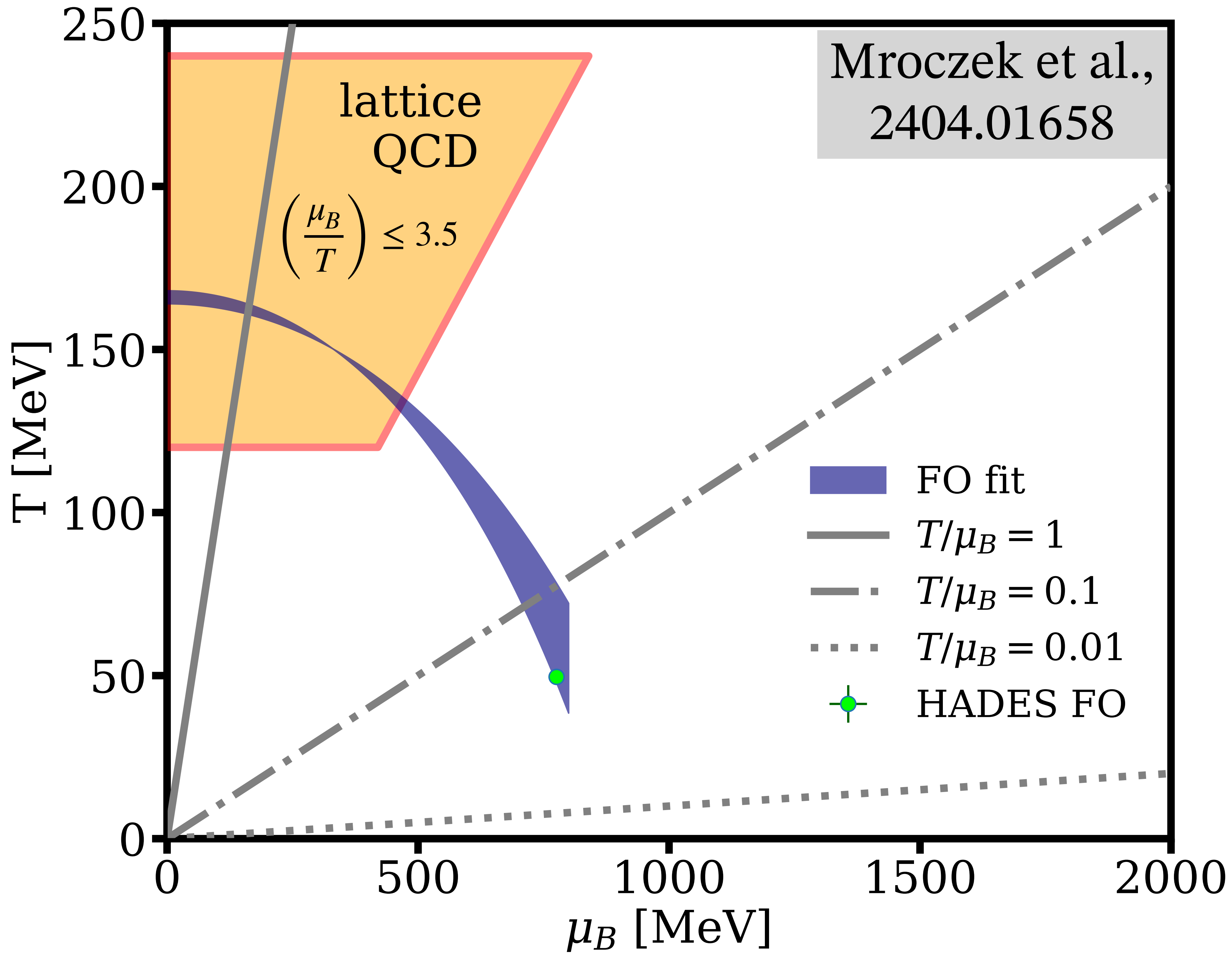
- Taylor expansion about $p(T=0,\, \mu_B,\, \mu_Q)$ New!

$$p(T, \vec{\mu}) = p_{T=0} + \cancelto{0}{\left.\frac{\partial p}{\partial T}\right|_{T=0,\vec{\mu}}} T + \frac{1}{2}\boxed{\left.\frac{\partial^2 p}{\partial T^2}\right|_{T=0,\vec{\mu}}} T^2 + \ldots$$

Entropy!
$s(T=0) = 0$

“Heat capacity” $\left.\frac{\partial s}{\partial T}\right|_{T=0} > 0$

$$\boxed{p(T, \vec{\mu}) \approx p_{T=0} + \frac{1}{2}\left.\frac{\partial s}{\partial T}\right|_{T=0,\vec{\mu}} T^2}$$



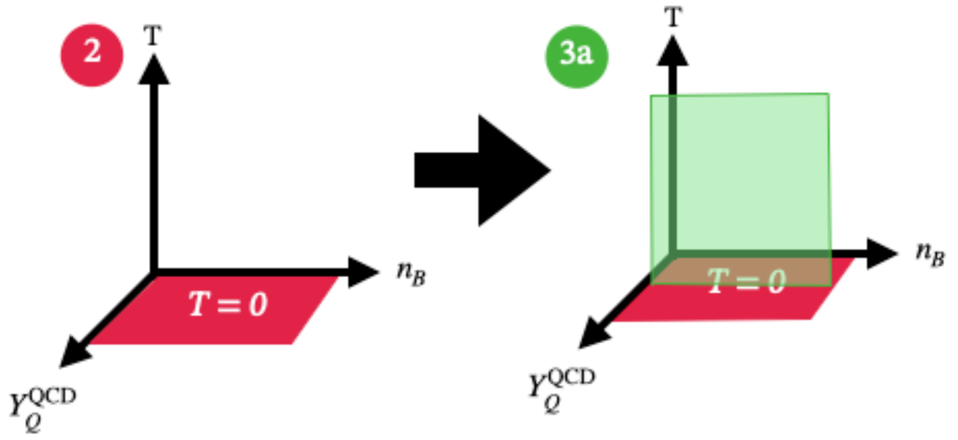
- Special case: Sommerfeld (1928) expansion
- Ideal Fermi systems at $T \ll T_F$,

  $p \approx p_{T=0} + aT^2 + bT^4 + \ldots$
- Fermionic **quasi-particles**

- Physical motivation
- Expansion parameter $(T/\mu_B) < 0.1$ in relevant regime
- Overlap with few-GeV $\sqrt{s_{NN}}$ freeze-out (FO)

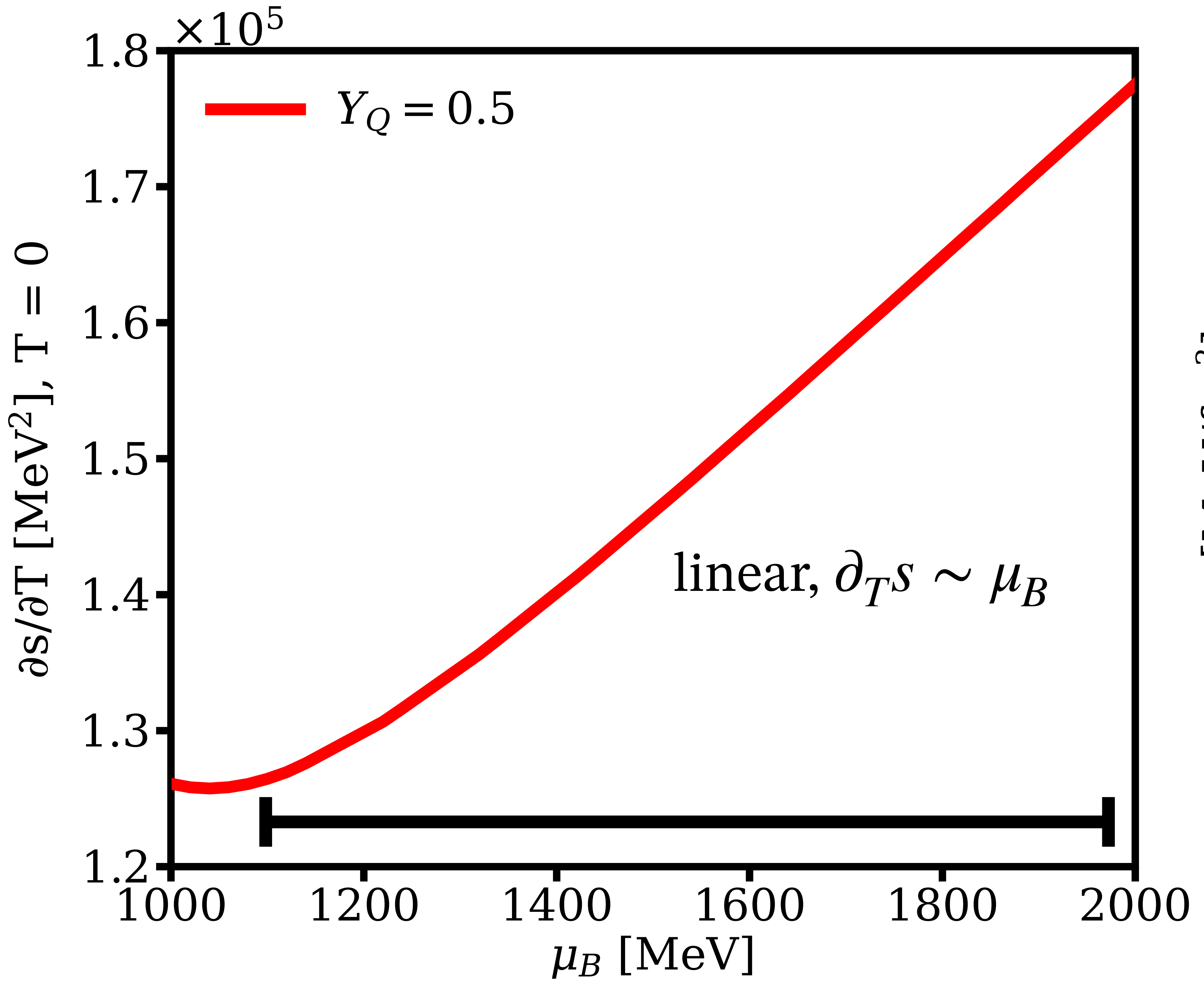
FO fit from Cleymans et al, PRC 73 (2006), HADES FO from Harabasz et al, PRC 102 (2020)

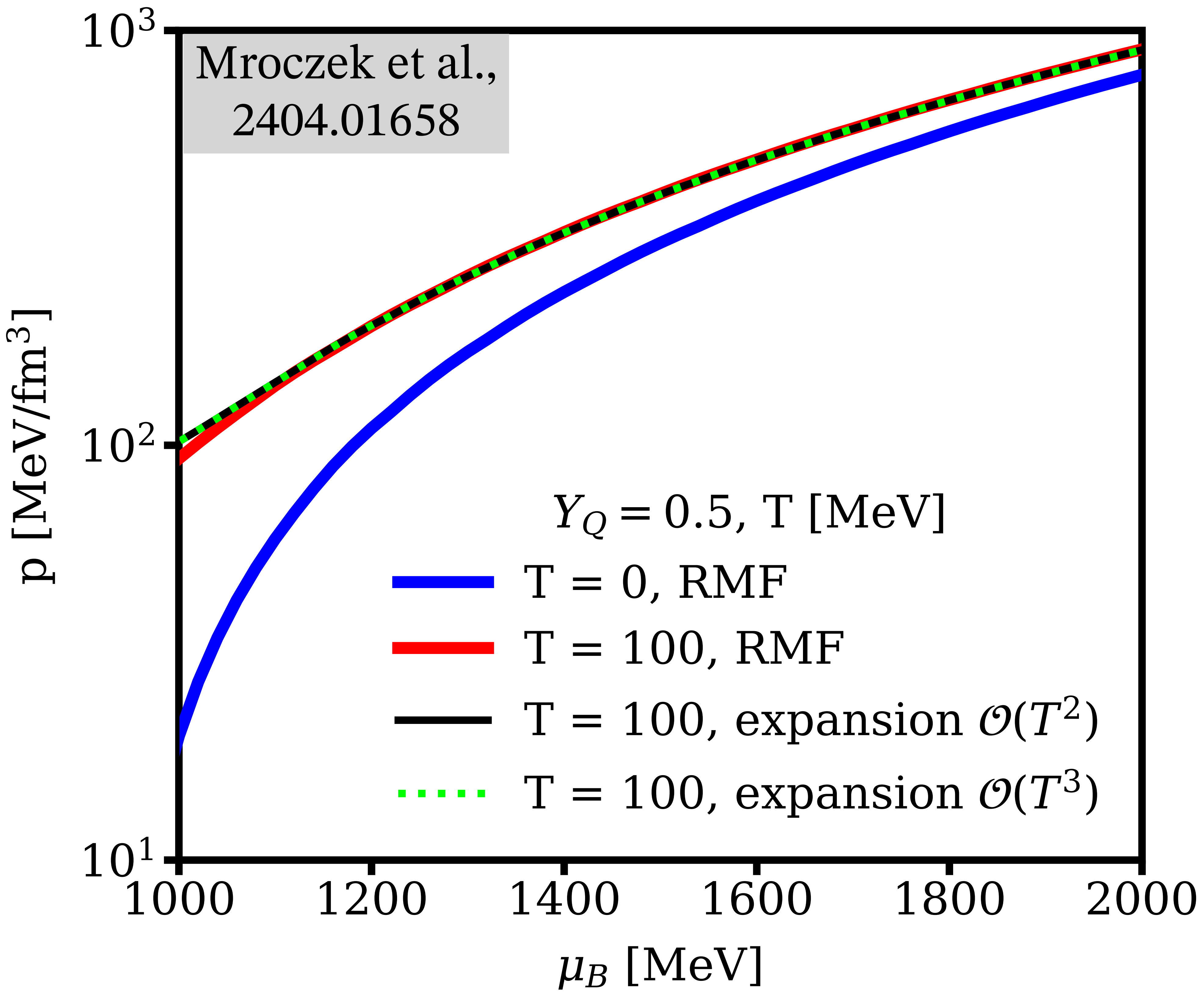
From T = 0 to finite T, test with microscopic model



- Numerical tests with relativistic mean-field (RMF) theory (n+p) well suited for the expansion

T^2 term captures the finite temperature behavior of the pressure to high accuracy when $\partial s/\partial T$ is known





- Breakdown near liquid-gas PT
- Linear coefficient

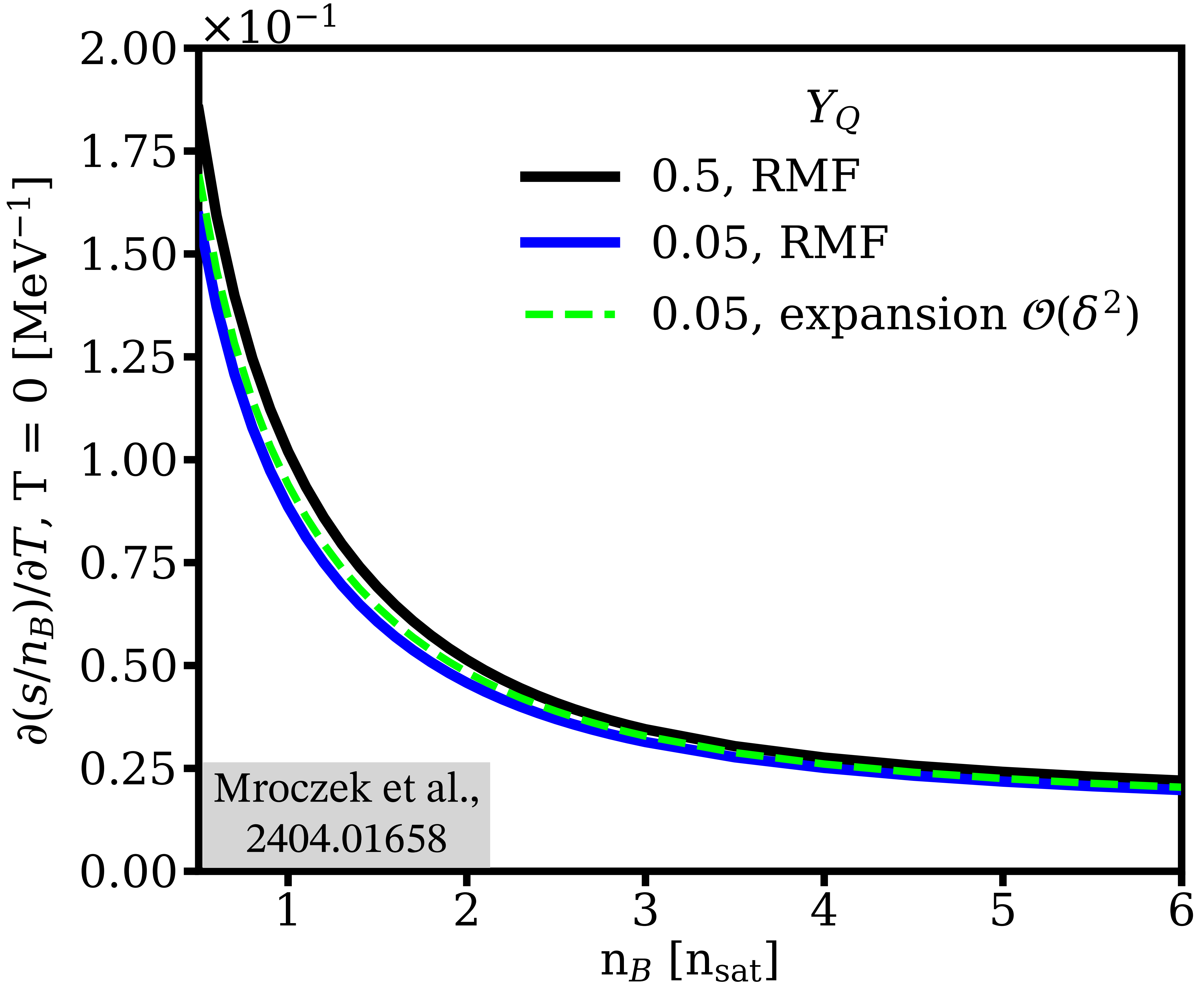
→ easy to parametrize

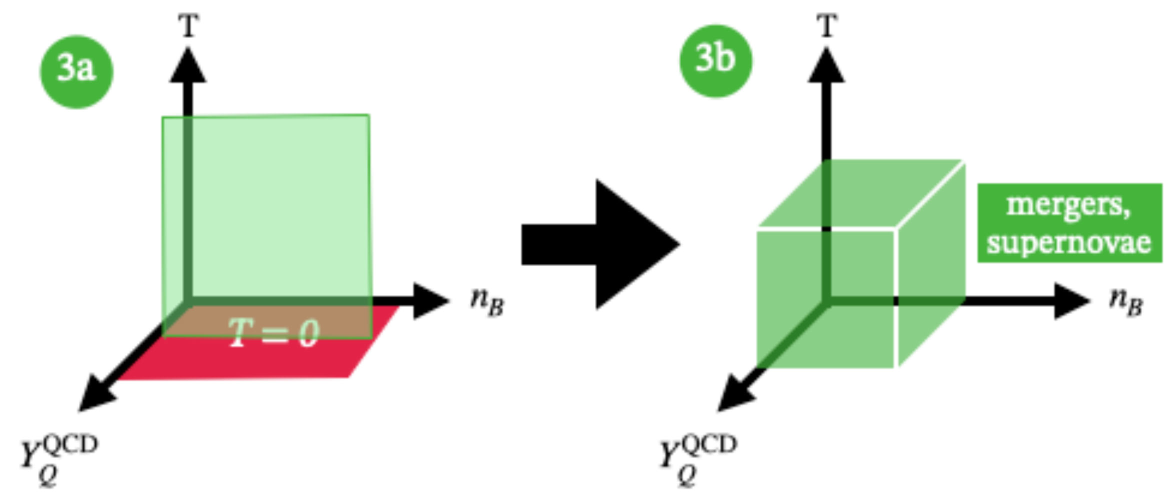
- T^2 term dominates

<u>But:</u> must know $\partial_T s$ for all μ_B, μ_Q

Microscopic model: RMF theory from Alford et. al PRC 106, (2022)

Charge fraction dependence of finite temperature effects





- Motivation: s/n_B for a given (Z/A, $\sqrt{s_{\mathrm{NN}}}$) can be extracted from thermal fits of particle yields

→ Expand $\partial_T(s/n_B)$ about SNM assuming isospin symmetry

- **New expansion**:

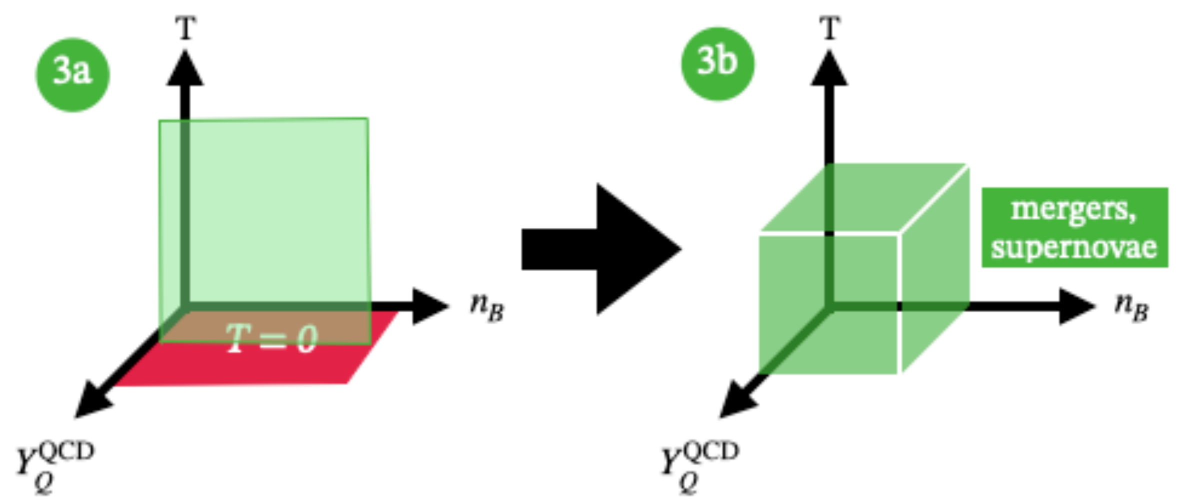
"Heat capacity" at $Y_Q^{\mathrm{QCD}} = 0.5$

$$\left.\frac{\partial \tilde{S}(T, n_B, Y_Q)}{\partial T}\right|_{T=0} = \frac{1}{n_B}\boxed{\left.\frac{\partial s_{\mathrm{SNM}}(T, n_B, Y_Q)}{\partial T}\right|_{T=\delta=0}} + \frac{1}{2}\left(1 - 2Y_Q\right)^2 \boxed{\left.\frac{\partial^3 \tilde{S}_{\mathrm{SNM},2}(T, n_B, \delta = 0)}{\partial T \partial \delta^2}\right|_{T=\delta=0}}$$

Heat capacity dependence on Y_Q

Heat capacity across all $\vec{\mu}$ can be extracted from microscopic models

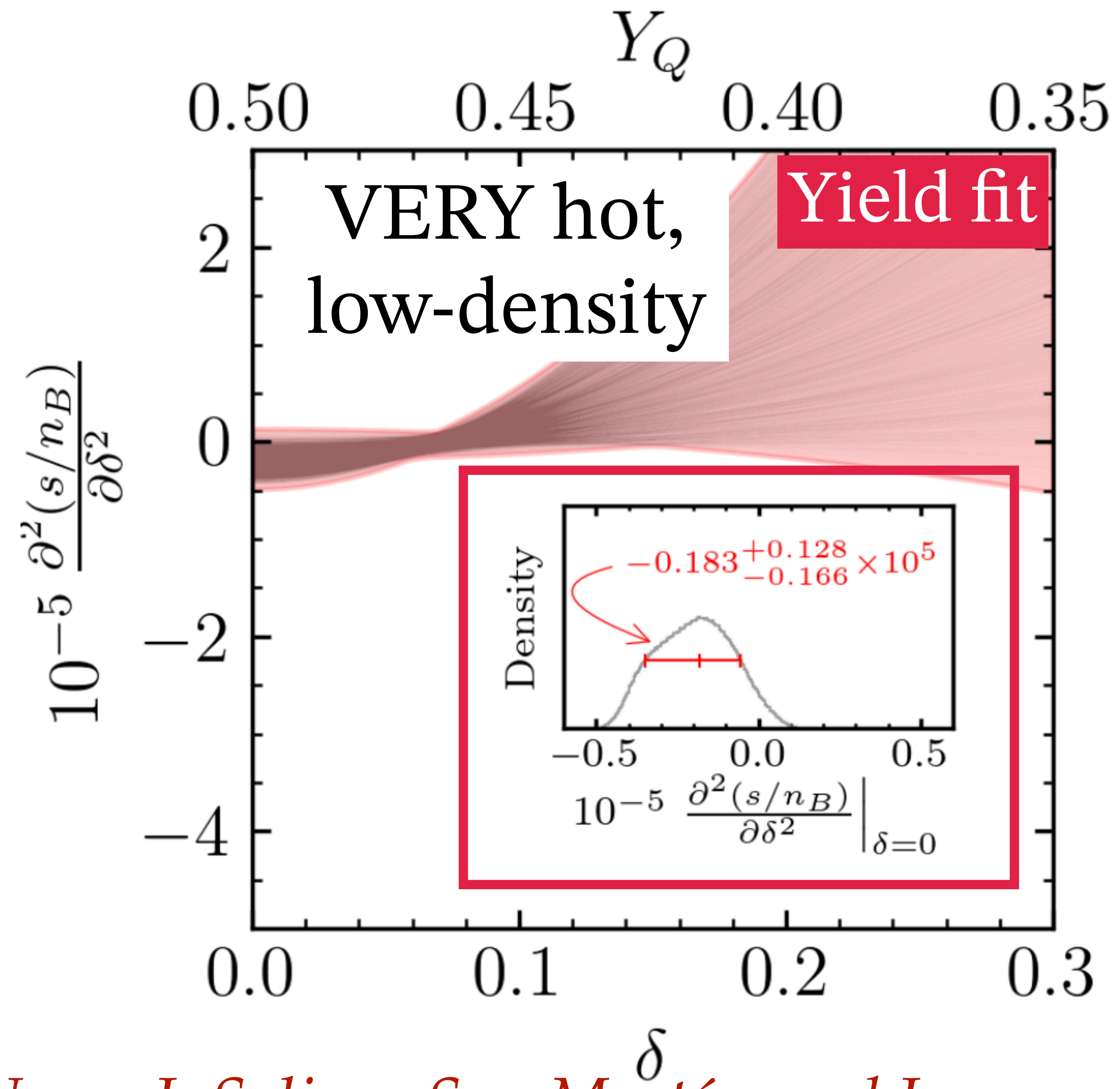
Connection to heavy-ion collisions: **system scan**



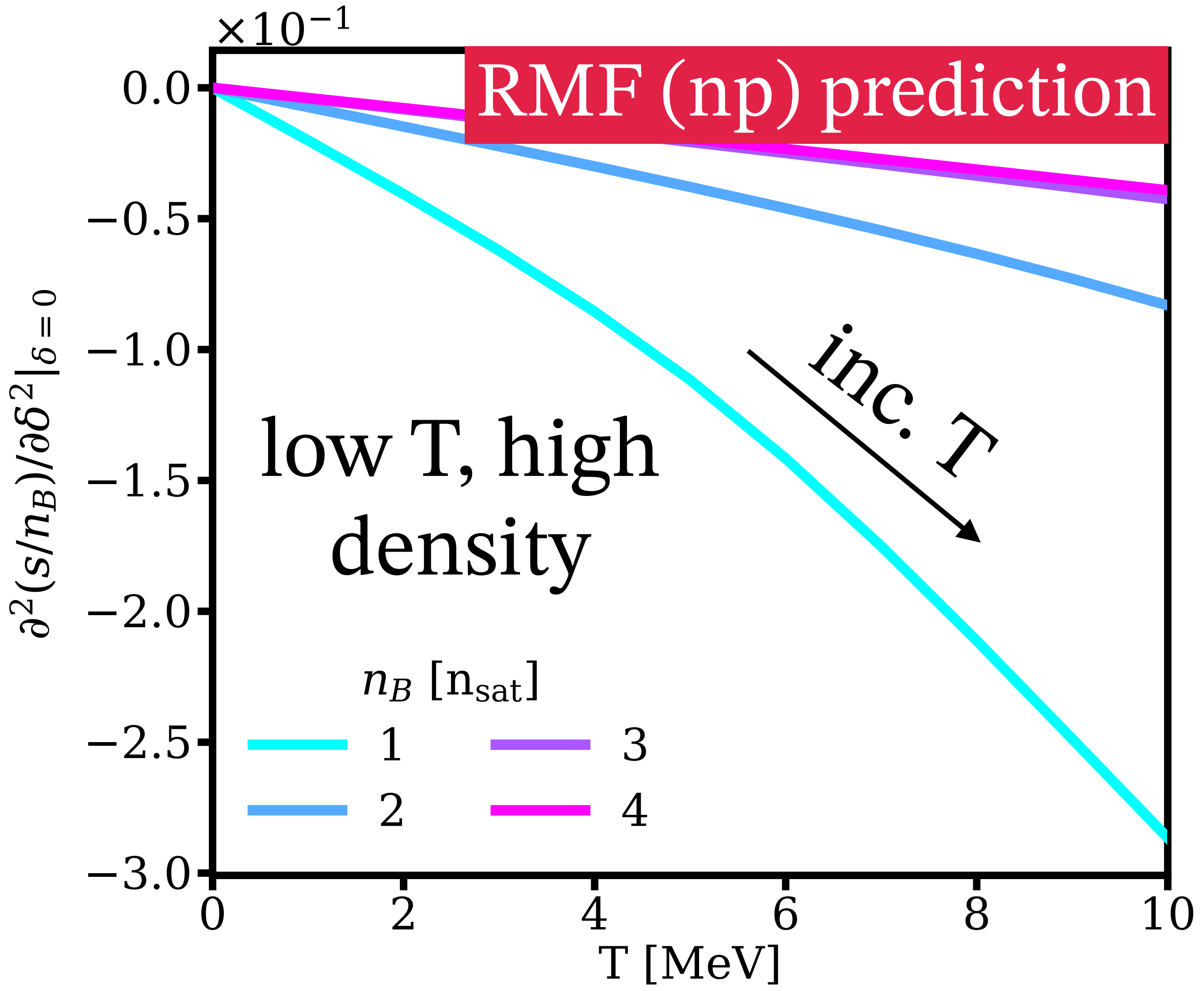
- Nana et al extracted $\partial^2(s/n_B)/\partial\delta^2$ from particle yields across different colliding species, central collisions at $\sqrt{s_{NN}} = 200$ GeV

System	Z	A	Y_Q	Published yield data?
O+O	8	16	0.500	no
Cu+Cu	29	63	0.460	yes
Ru+Ru	44	96	0.458	no*
Zr+Zr	40	96	0.417	no*
Au+Au	79	198	0.399	yes
U+U	92	238	0.387	yes

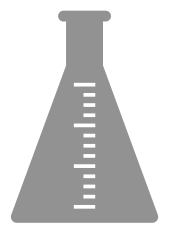
Fits predict a **large and negative** value for $\partial^2(s/n_B)/\partial\delta^2$ at $T_{FO} \sim 145$ MeV, $n_B \sim 0.025\ n_{sat}$, in **qualitative agreement** with RMF (n+p) results

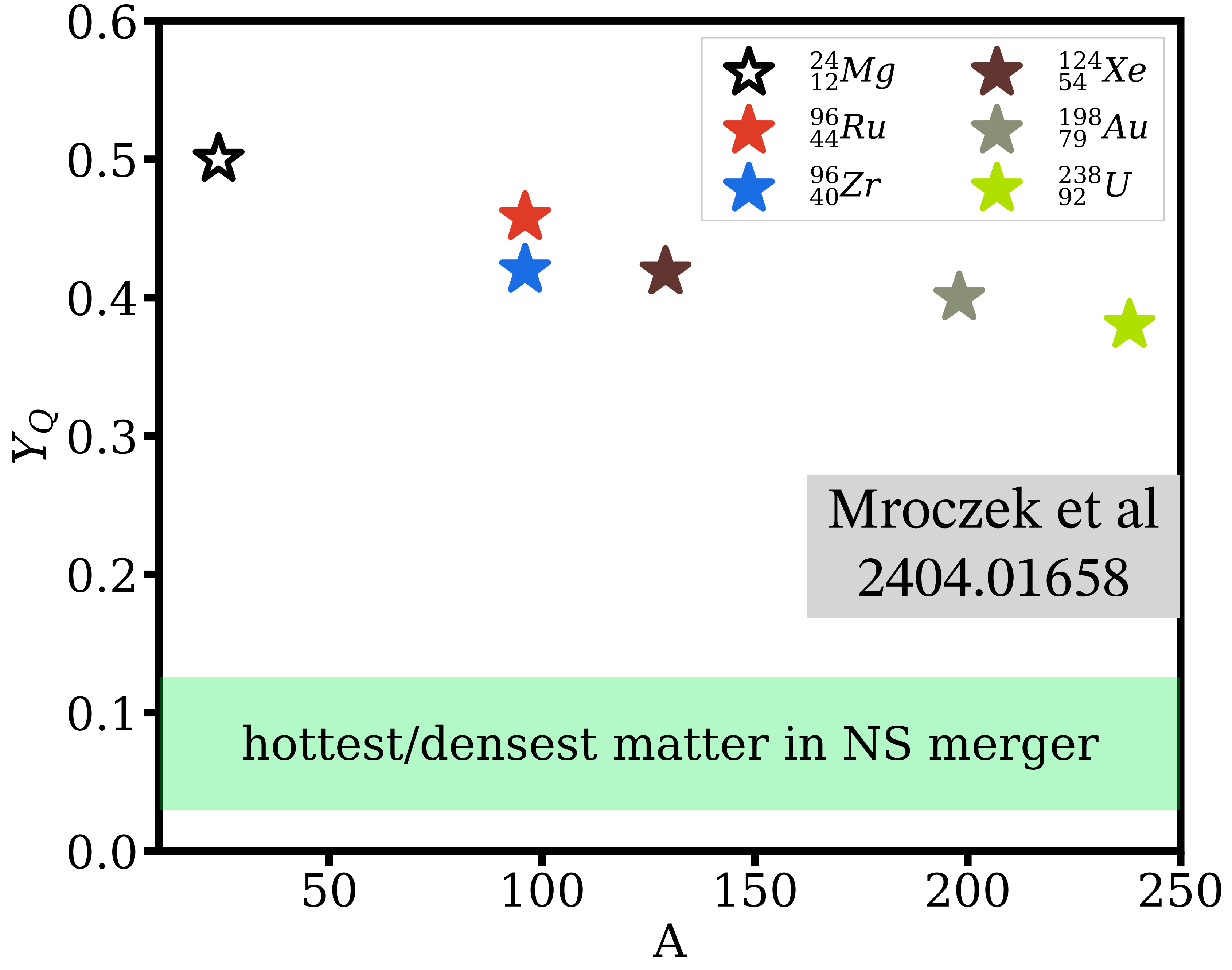


F. Nana, J. Salinas San Martín, and J. Noronha-Hostler, 2411.03705



Mroczek et al 2404.01658





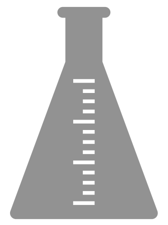
- Needed: **system + energy scan**
- **Symmetric nuclei**, e.g., O+O, crucial for extracting the expansion coefficient at $\delta = 0$

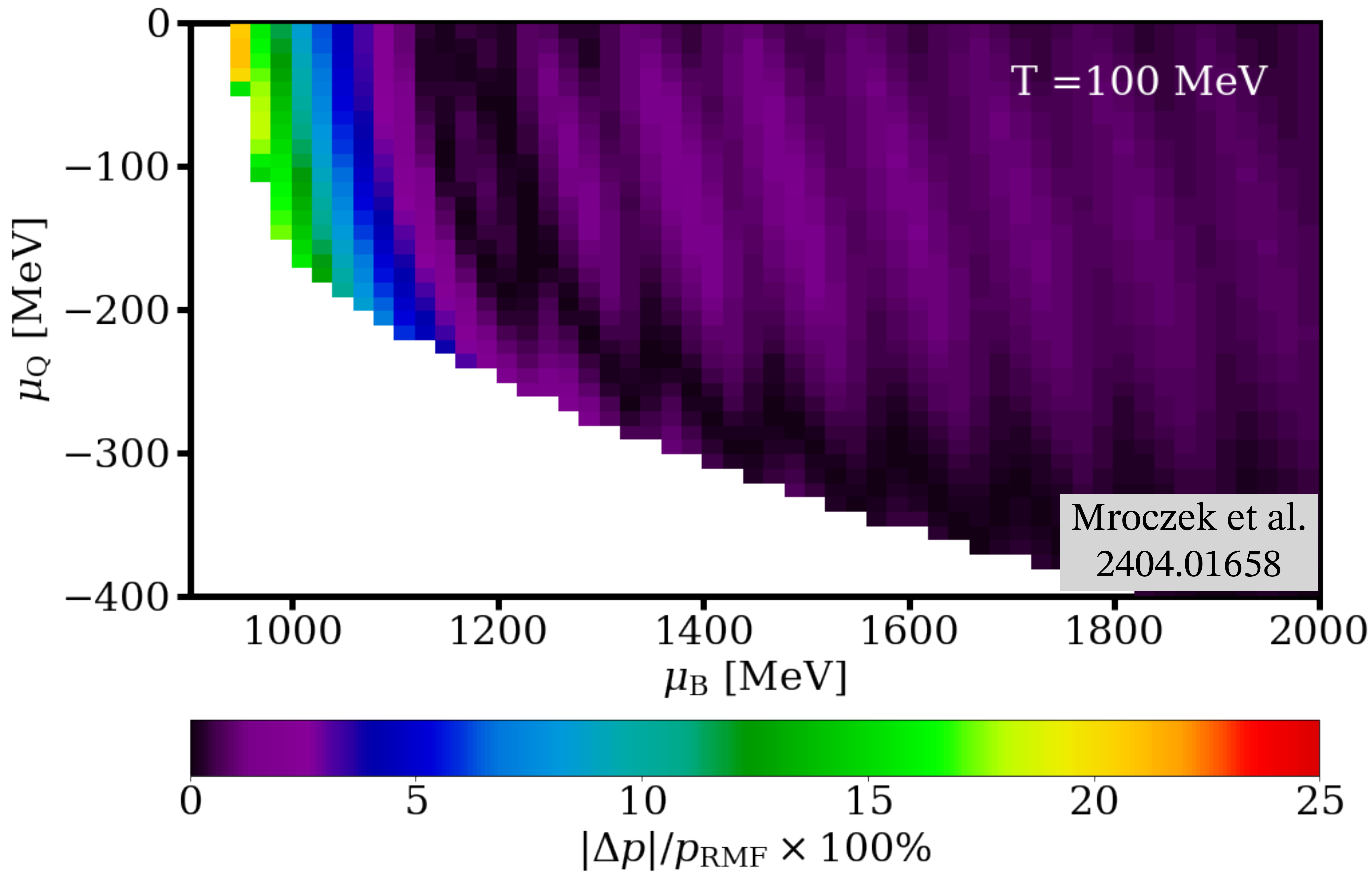
LHC, CBM @ FAIR?

Proof-of-principle with a microscopic EOS

- Expansions 2) and 3) tested against an RMF EOS
- Error introduced by finite T + s/n_B expansions **below 5%** across almost all μ_B, μ_Q at T=100 MeV
- Larger error: liquid-gas phase transition near $n_{\rm sat}$
- Did not account for uncertainty in expansion coefficients:

$$\left.\frac{\partial s_{\rm SNM}(T, n_B, Y_Q)}{\partial T}\right|_{T=\delta=0}, \qquad \left.\frac{\partial^3 \tilde{S}_{\rm SNM,2}(T, n_B, \delta)}{\partial T \partial \delta^2}\right|_{T=\delta=0}$$

 ✓ definitely  • hopefully



Note: n+p RMF is used as a proof-of-principle, 100 MeV is arbitrary. More realistic T for mergers is 50 MeV where we achieve <1% error.

Going beyond astrophysical observations with a unified EoS

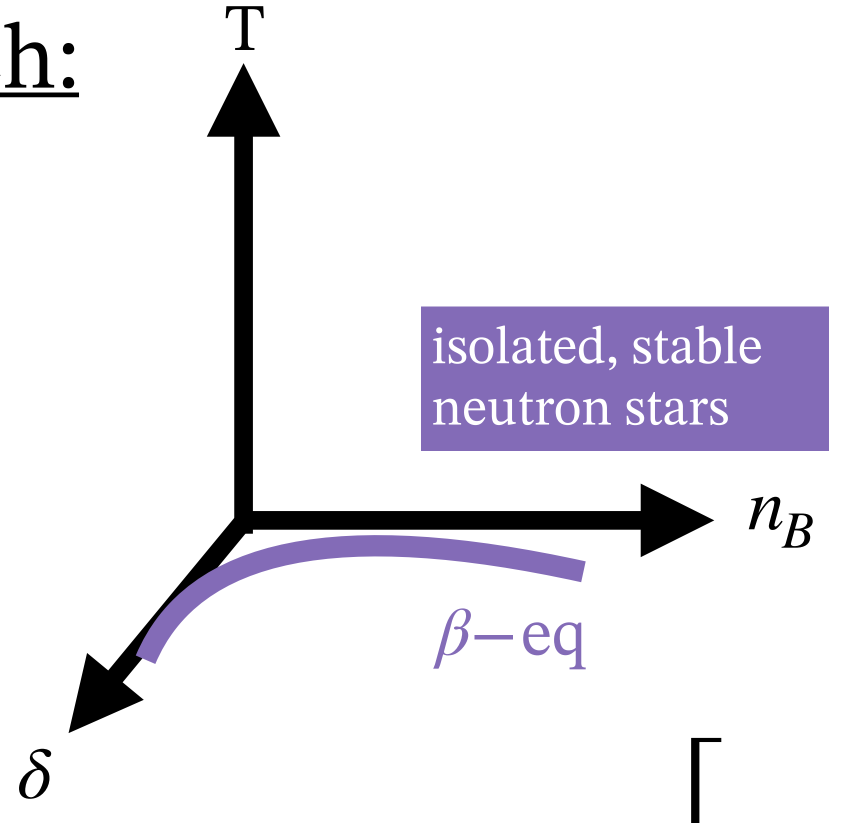
Combining knowledge from:
1) neutron star observations
2) HICs
3) Theory (perturbative, effective, lattice)

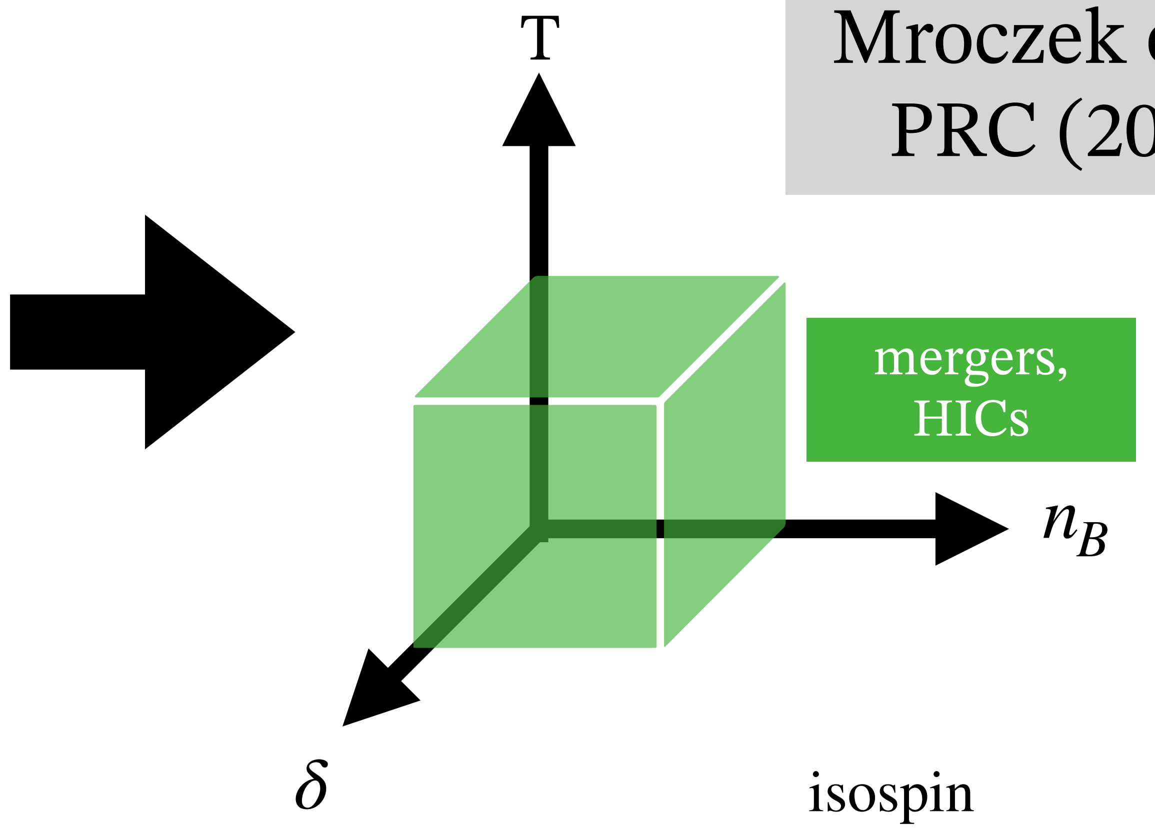
requires 3D EoS model in (T, n_B, δ)

- Collective flow in heavy-ion collisions (HICs) in the few-GeV region: **sensitive to the EoS above** n_{sat}
- Lattice, CEFT, perturbative calculations available

Our approach:

Mroczek et al., PRC (2025)





$$p(T, n_B, \delta) = \underbrace{\left(p_{\mathrm{NS}}(n_B) + \overbrace{c_\delta \delta^2}^{\text{isospin}}\right)}_{\text{cold}} + \underbrace{\frac{1}{2}\left[\frac{\partial s_{\mathrm{SNM}}(T, n_B)}{\partial T}\bigg|_{T=0} + \frac{n_{B,0}}{2}\overbrace{\frac{\partial^3 \tilde{S}(T, n_B, \delta)}{\partial T \partial \delta^2}\bigg|_{T=\delta=0}\delta^2}^{\text{isospin}}\right] T^2}_{\text{finite T}}$$

The expansion

- Is thermodynamically consistent + allows for **uncertainty propagation/quantification**
- Reproduces microscopic EoSs up to T=100 MeV within 5% error
- Was recently expanded to include 2SC and CFL phases (Gholami, Hofmann, **Mroczek**, arXiv:2512.1672)
- Open-source implementation code (out soon)