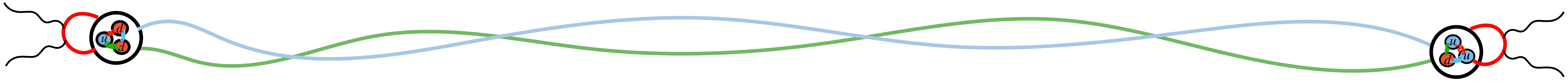
Form Factors & Matrix Elements from LQCD

Two Roads from the lattice to $0\nu\beta\beta$ decay



Joseph Moscoso

University of Maryland, College Park

NSF Hub for $0\nu\beta\beta$

INT Program 26-2a: Recommended Values for $0\nu\beta\beta$ Nuclear Matrix Elements
July 9, 2026

INT Program 26-2a: Recommended Values for $0\nu\beta\beta$ Nuclear Matrix Elements
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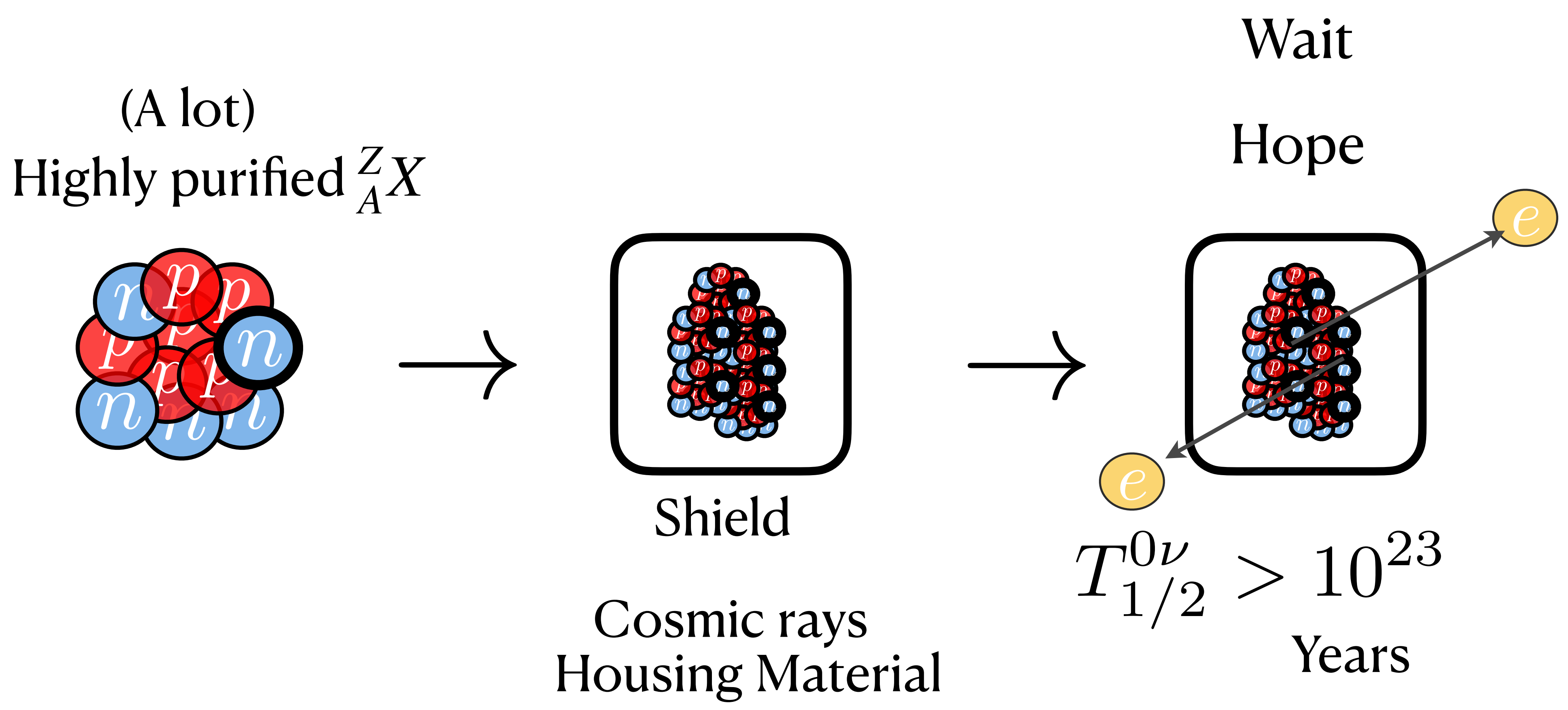
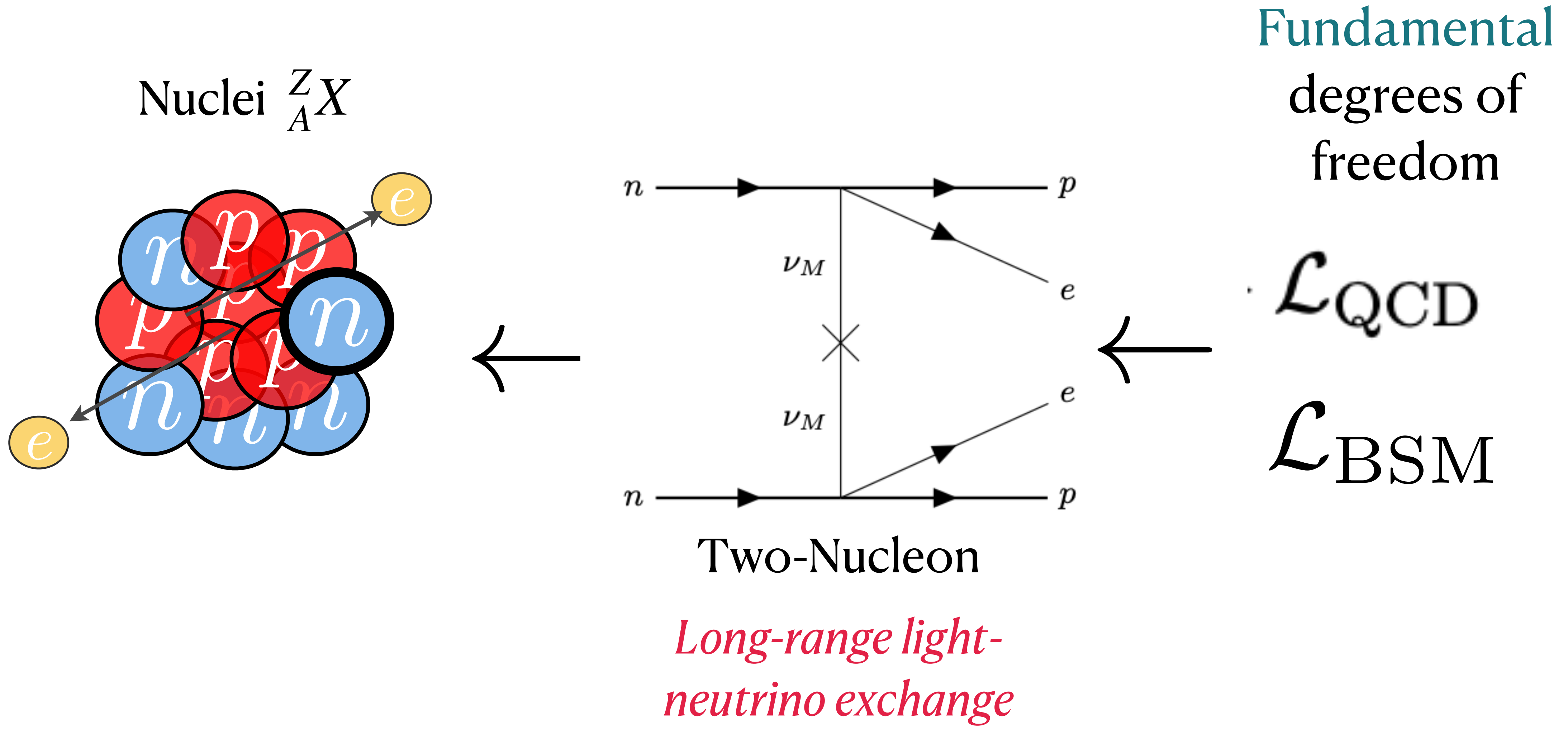
A sensitive, forbidden signal to detect



BSM process to calculate at precision at multiple scales



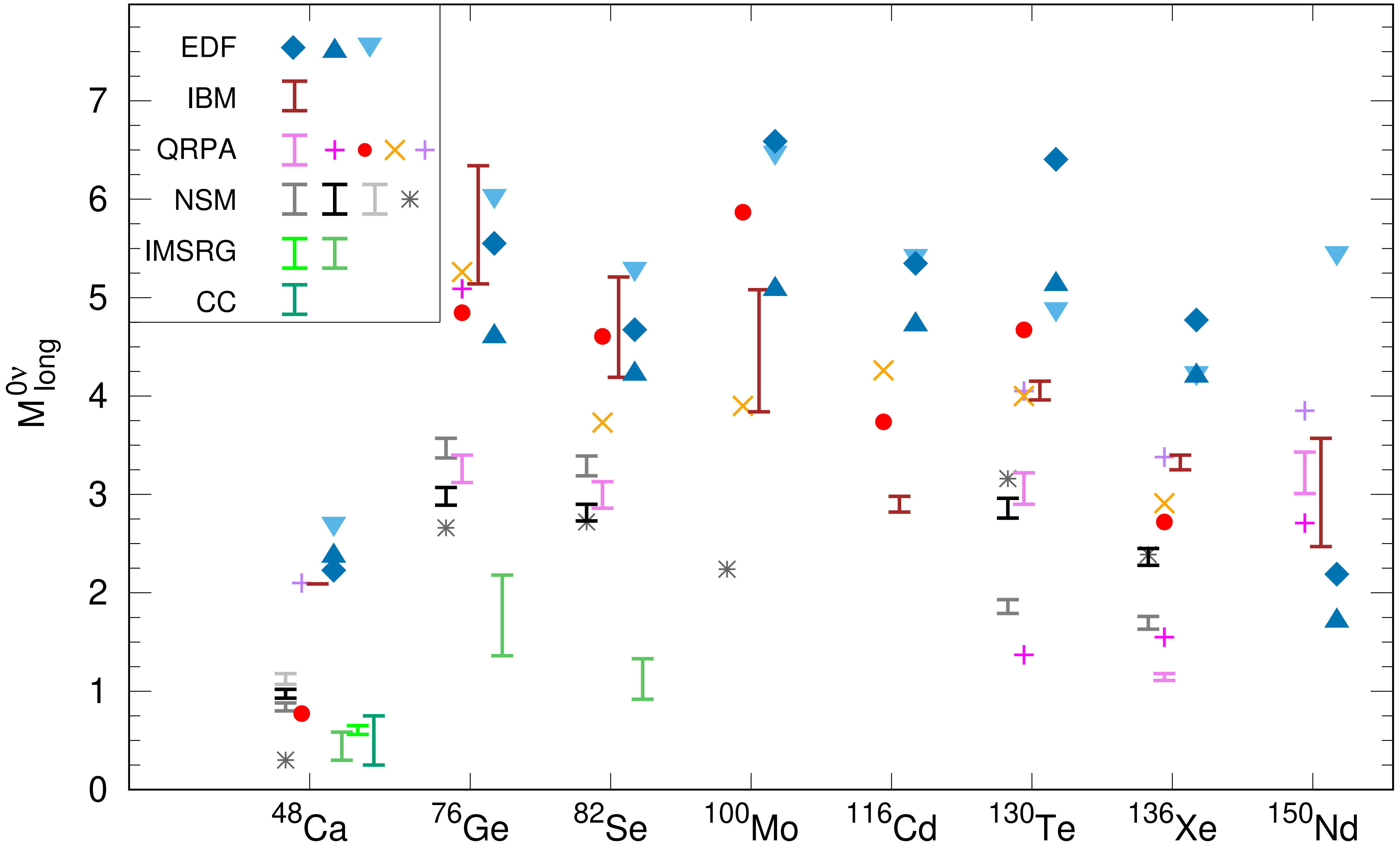
CUPID LEGEND

^{100}Mo ^{76}Ge

@NDB
Advancing Theory for Nuclear Double-Beta Decay

INT Program 26-2a: Recommended Values for $0\nu\beta\beta$ Nuclear Matrix Elements
July 9, 2026

The problem this program exists to solve



Agostini, Benato, Detwiler, Menéndez, Vissani, RMP 95, 025002 (2023) [2202.01787]

- Factor ~3 spread in $M^{0\nu}$ $\Rightarrow$ factor ~10 in $T_{1/2}$
- Part of the spread: many-body methods
- Part of it: hadronic input buried in every calculation: g_A, form-factor shapes, short-range couplings

This talk: anchoring the hadronic layer in QCD

From half-life to BSM physics: EFT tower

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{0\nu}\,\left|M^{0\nu}\right|^2 \left|\frac{m_{\beta\beta}}{m_e}\right|^2$$ (light Majorana exchange; master formula generalizes)

Λ_{BSM}: LNV mechanism $\to$ SMEFT dim-5, 7, 9 operators

$\downarrow$

Λ_χ: chiral EFT operators, LECs $(g_A, g_\nu^{NN}, g_{1..5}^{\pi\pi}, \dots)$ $\leftarrow$ **QCD lives here**

$\downarrow$

k_F: two-body transition operators, neutrino potential V_ν

$\downarrow$

Many-body: shell model / QRPA / IMSRG / CC $\to M^{0\nu}$

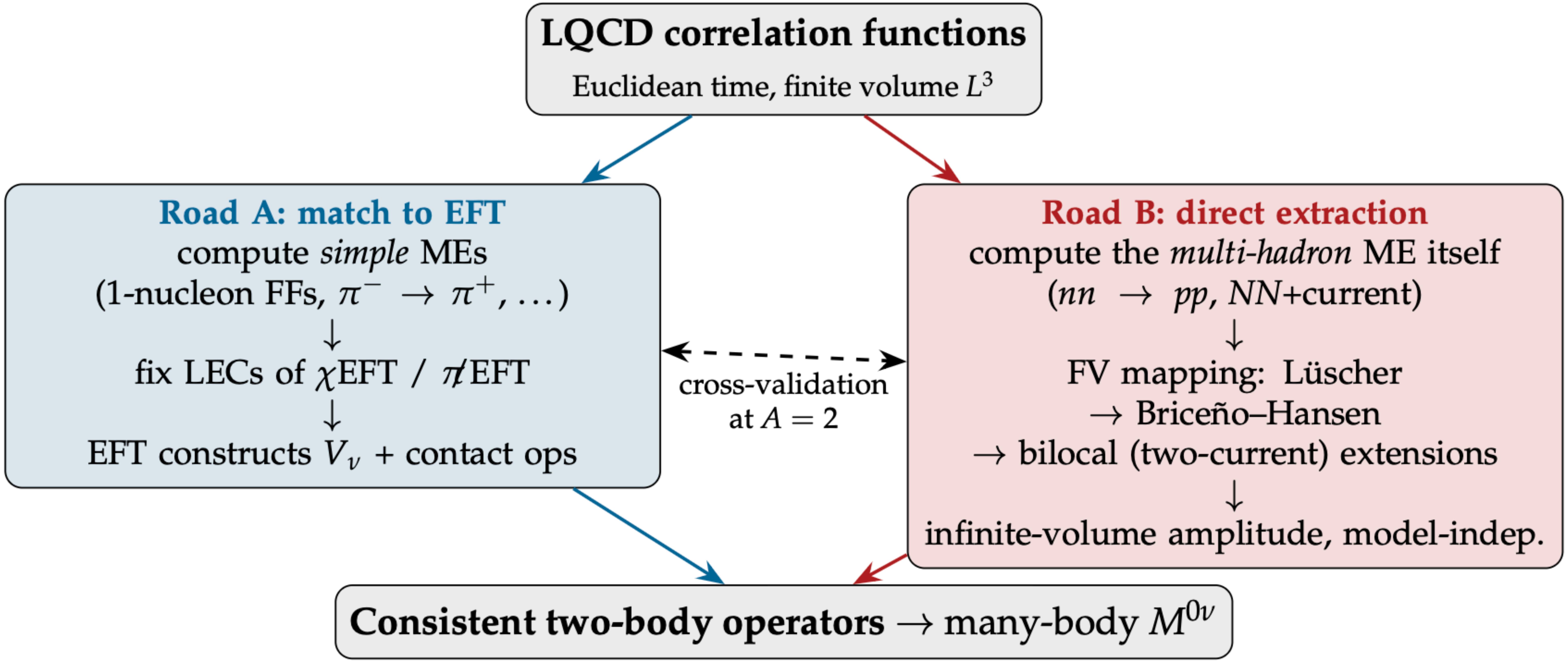
Cirigliano, Dekens, de Vries, Graesser, Mereghetti, JHEP 12 (2018) 097 [1806.02780]

Roads from QCD to Nuclei

Mechanism	Hadronic input	Enters via	Lattice status
Long-range ($m_{\beta\beta}$)	$g_V(q^2)$, $g_A(q^2)$, $g_M(q^2)$, $g_P(q^2)$	V_ν	**maturing**
LO contact	g_ν^{NN}	V_ν counterterm	**first results** (Anthony)
Short-range dim-9	$\pi^- \to \pi^+$ MEs ($\mathcal{O}_{1..5}$)	$g_i^{\pi\pi}$	**done, phys. point**
Short-range dim-9	πN MEs	$g_i^{\pi N}$	**missing (naturalness est.)**
Short-range dim-9	NN MEs	g_i^{NN}	**missing**
Dim-7 / scalar–tensor	g_S, g_T (+ g'_P, PCAC)	non-std currents	**done, phys. point***
Long-range, direct	$nn \to pp$ amplitude	full A=2 input	**proof of principle**

Operator basis & LECs: [1806.02780]; LO contact: [1802.10097]; per-entry refs on the slides that follow. *charges: g_T to $\sim$3–4%, g_S to $\sim$10% (FLAG); q^2-dependence of S/T form factors still partial; g'_P chirally enhanced, fixed by PCAC/GMOR

Two Roads from QCD to Nuclei



QCD to Nuclei

- Compute matrix elements that are *clean* on the lattice; equate to the EFT expression at the same kinematics:

$$\mathcal{A}^{\mathrm{LQCD}}(q^2;\, a\to 0,\, L\to\infty,\, m_\pi^{\mathrm{phys}}) \;=\; \mathcal{A}^{\chi\mathrm{EFT}}(q^2;\,\mathrm{LECs})$$

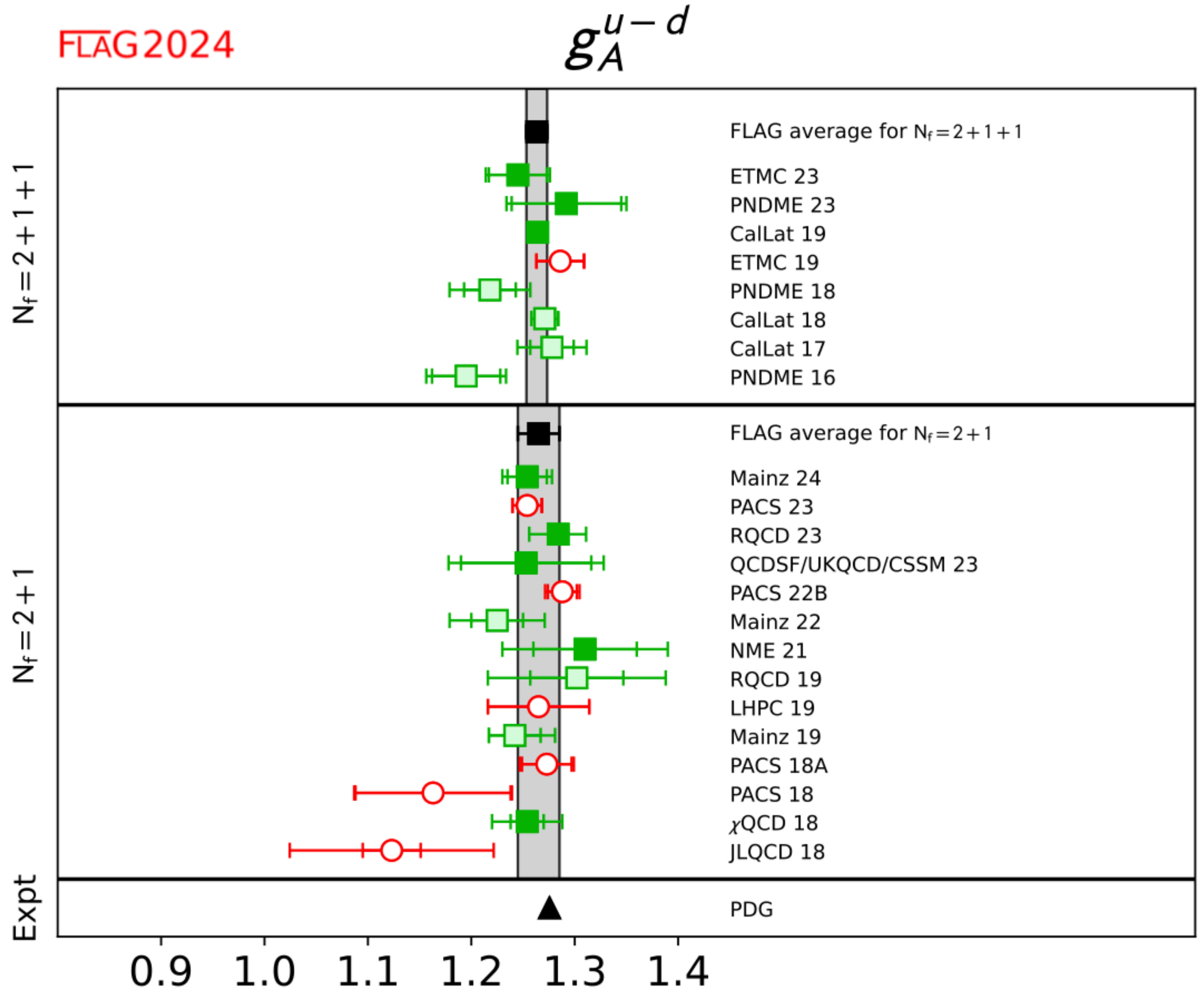
- LECs are then *QCD predictions*, inherited by every many-body calculation using that EFT
- Requirement: same scheme/regulator as the nuclear interaction — a convention this program could fix in the white paper

Deliverables on this road

g_A and form-factor shapes; $g^{\pi\pi}_{1..5}$; g_S, g_T; g_ν^{NN} (Anthony)

→ Open question for the community

Week 2 is the operator week: what LEC delivery format does this community need? Standard cutoffs + conversion formulas? One reference scheme + recipe? Or sidestep it — match on the $A{=}2$ amplitude itself (scheme-agnostic ⇒ Road B)? (Cirigliano et al., PRC 97, 065501 [1710.01729]; PRC 100, 055504 [1907.11254])



g_A: nature's golden standard

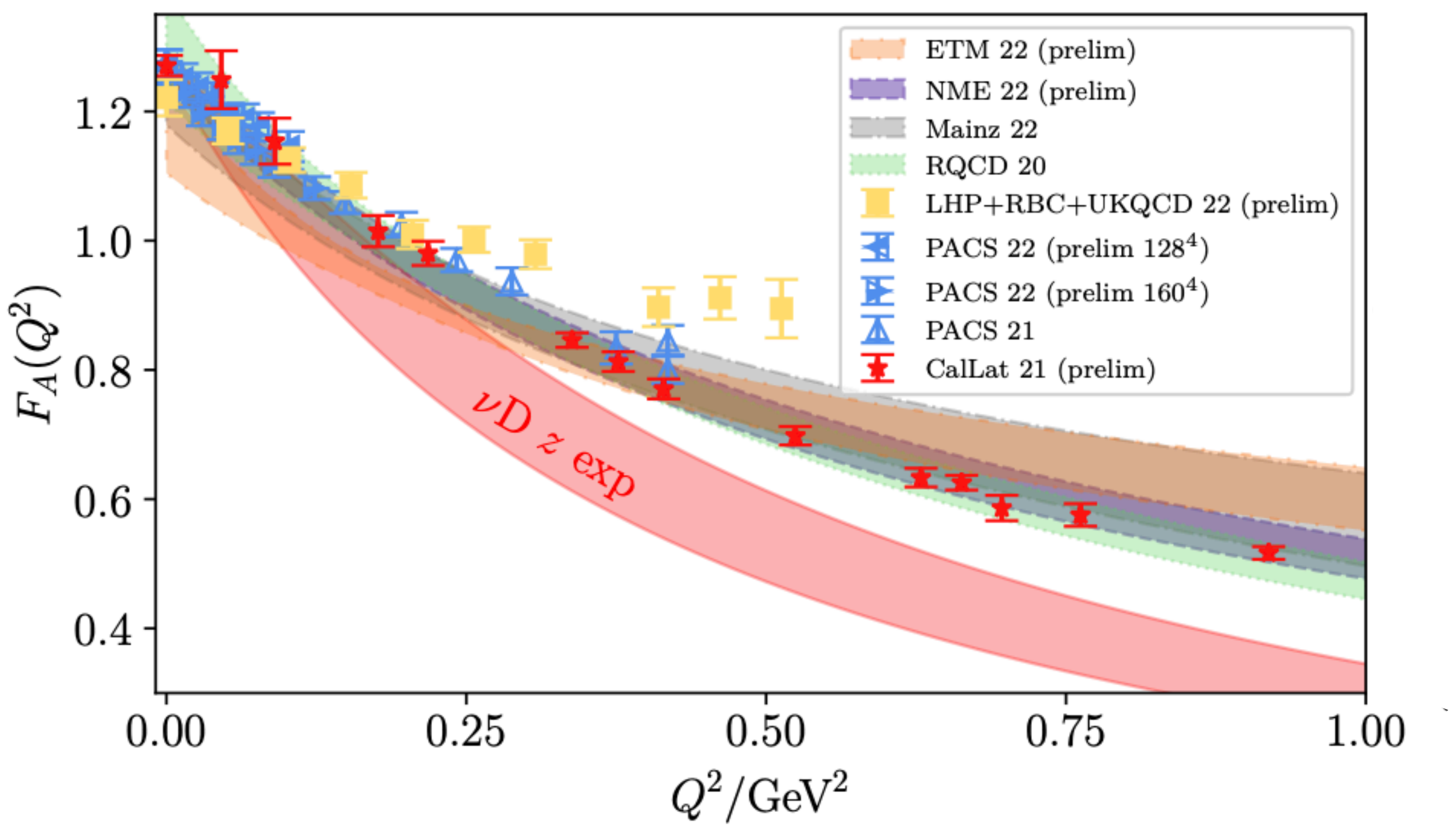
- Every other LEC on the shopping list has no experimental counterpart — g_A is where the matching procedure is graded against data
- Verdict: $g_A = 1.27$ at the percent level, agrees with neutron decay
- g_A is the field's standard candle: every rate calculation runs through it — at the free value or as a quenched g_A^{eff} — so the nucleon-level benchmark anchors both uses

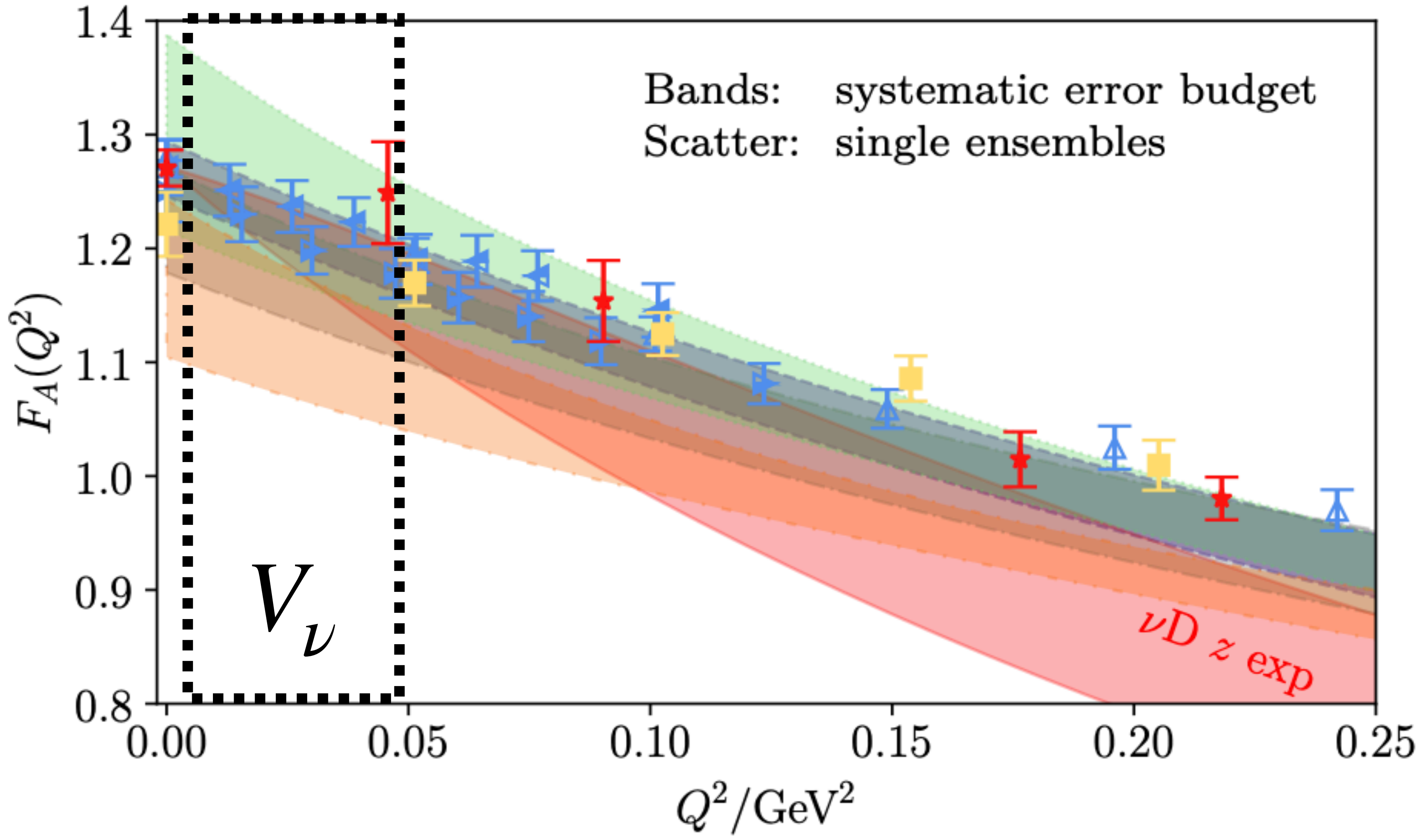
→ Open question for the community

Nucleon-level g_A is settled at 1%. How should the recommended values treat "effective g_A" — a phenomenological calibration within truncated model spaces, or physics now computed explicitly by ab initio + two-body currents?

CalLat, Nature 558, 91 (2018) [1805.12130]; FLAG 24, Tab. 67 (g_A^{u-d}) [2411.04268]; newest: Hall et al., PRC 113, 055206 [2503.09891] (FV corrections); g_A^{eff} vs 2-body currents: Gysbers et al., Nat. Phys. 15, 428 [1903.00047]

Axial & Induced Form Factors @ $0\nu\beta\beta$ Kinematics

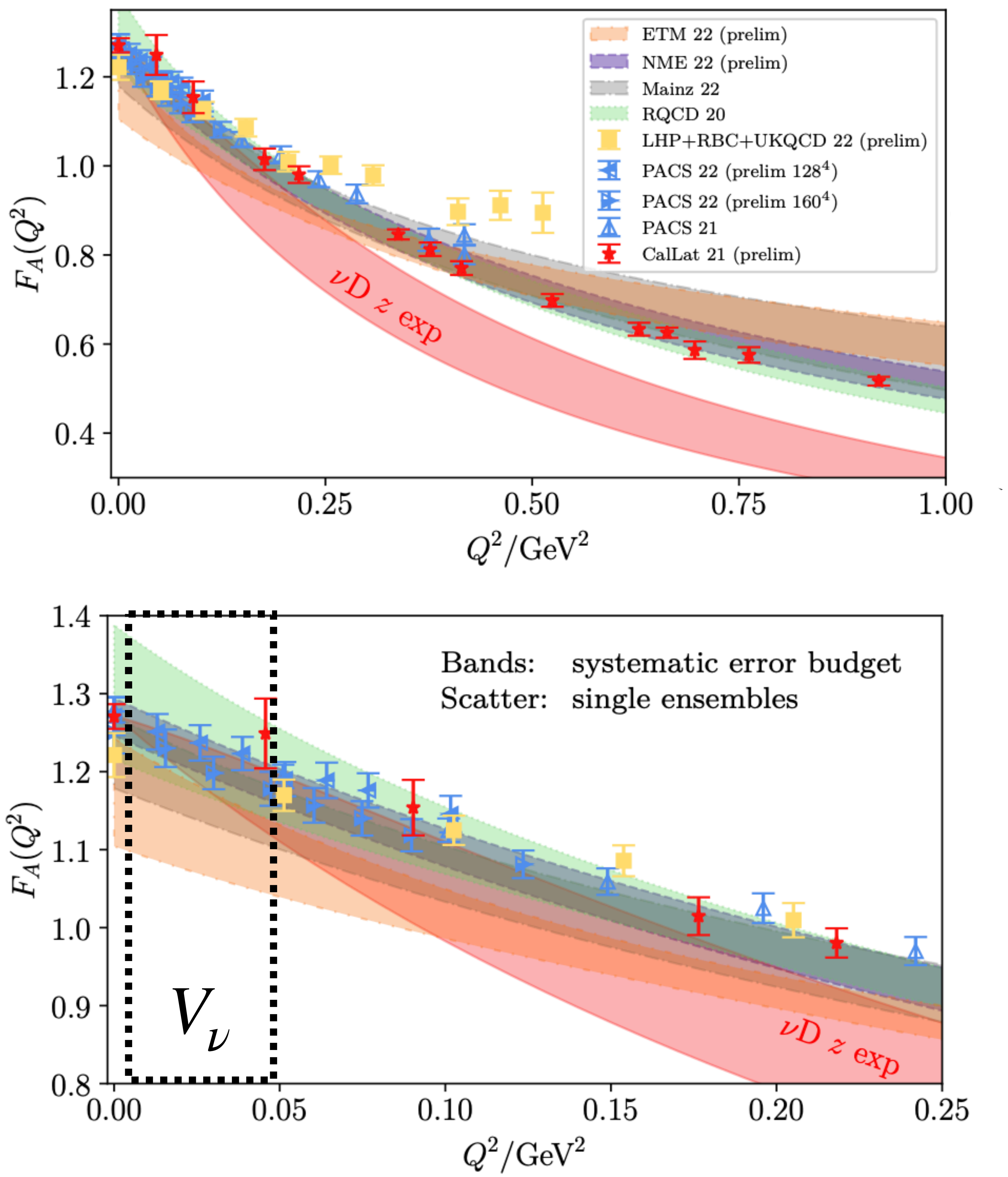




- V_ν weight peaks at $q \sim 100$–200 MeV ($\sim m_\pi$); lattice brackets the window — $Q^2{=}0$ directly, first box momenta $2\pi/L$ just above — so crossing it is interpolation, not extrapolation
- Dipole with $\Lambda_A = 1.086$ GeV is an *assumption*; lattice F_A comes out stiffer [2201.01839] — and MINERvA's $\bar{\nu}$–H measurement sides with the lattice (Nature 614, 48 (2023))
- Induced pseudoscalar $g_P(q^2)$: pion pole / PCAC tested directly
- Weak magnetism $g_M(q^2)$ from vector FFs

Recent lattice $F_A(q^2)$ at physical point: Mainz [2207.03440], ETMC [2309.05774], + PNDME, RQCD, NME

Which F_A parametrization



- Dipole = one-parameter ansatz: fits νd data but underestimates the shape uncertainty by construction
- z-expansion refit of the same deuterium data (Meyer, Betancourt, Gran, Hill, PRD 93, 113015 [1603.03048]): honest errors on $F_A(q^2)$ grow to $\mathcal{O}(10\%)$
- $\Rightarrow$ lattice–dipole "tension" reframed: the data never pinned the shape; lattice now does
- Proposal: adopt z-exp coefficients + *covariance* (lattice, or lattice+νd combined); g_P tied to F_A via PCAC, not fit independently

Recent lattice $F_A(q^2)$ at physical point: Mainz [2207.03440], ETMC [2309.05774], + PNDME, RQCD, NME

Do form-factor shapes matter for $M^{0\nu}$

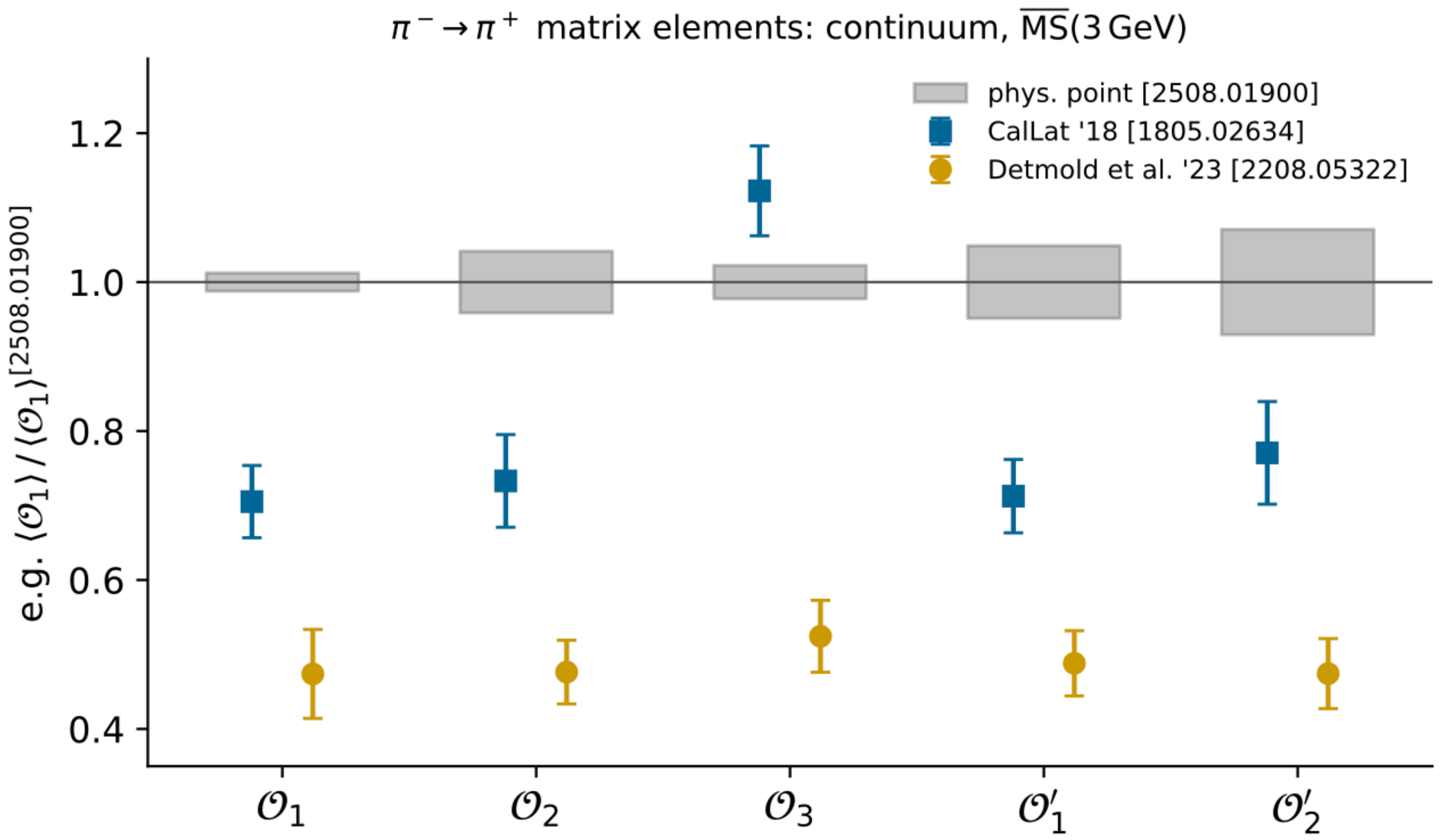
$$V_\nu(q) \;\propto\; \frac{1}{q\,(q+\bar{E})}\left[g_V^2(q^2)\,h_F + g_A^2(q^2)\,h_{GT}(q^2) + \ldots\right]$$

- Open question of impact: how much does $M^{0\nu}$ move when dipole $\to$ z-exp/lattice FFs?
- The *same* FFs enter V_ν for every nucleus $\Rightarrow$ their errors shift all $M^{0\nu}$ together (correlated across isotopes: don't average down in combinations, cancel in ratios) — the off-diagonal entries the white paper's correlation matrix needs (Week 3)

→ Open question for the community

Standard phenomenological calculations (QRPA, SM, IBM) carry a dipole with $\Lambda_A = 1.086$ GeV in V_ν (Šimkovic et al. convention); ab initio truncates the chiral operator instead. Is it worthwhile to update to a standard lattice parametrization (z-expansion + covariance) and rerun a benchmark NME to quantify the shift — or is the effect provably below other uncertainties?

$\pi^+ \to \pi^-$ Results

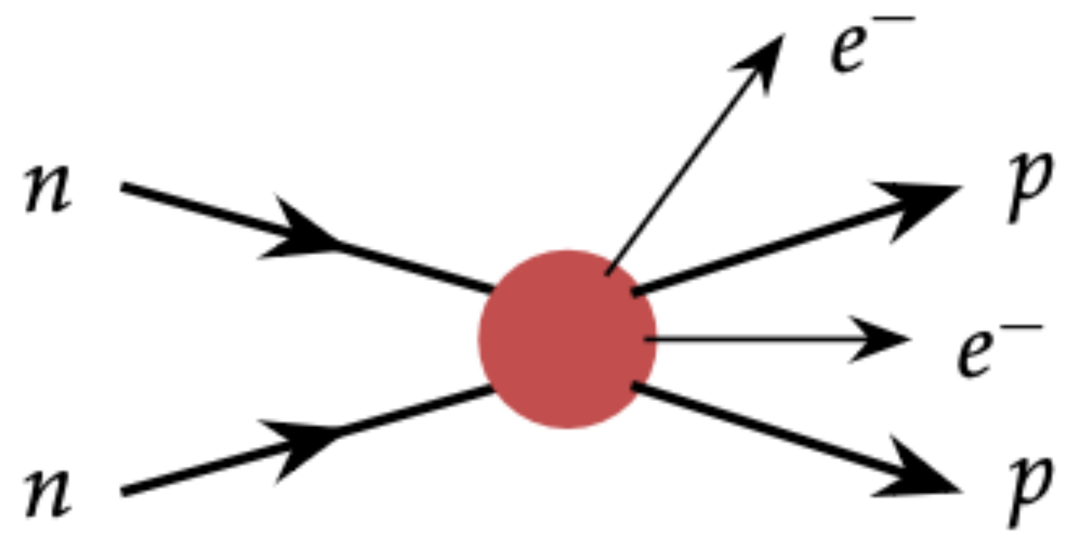


- Dim-9 short-range mechanisms: LO contribution is $\pi\pi$ exchange
- $\langle \pi^+ | \mathcal{O}_{1..5} | \pi^- \rangle$ at/near physical point, few-% errors, continuum-extrapolated
- $\Rightarrow g^{\pi\pi}_{1..5}$: LECs handed to χEFT — **done**
- Long-distance $\pi^- \to \pi^+ e^- e^-$ (Tuo–Feng–Jin, PRD 100, 094511 [1909.13525]; Detmold–Murphy [2004.07404]): validates χPT convergence for the bilocal amplitude
- New: short-range $\pi^- \to \pi^+ ee$ directly at physical m_π (arXiv:2508.01900)

CalLat, PRL 121, 172501 [1805.02634]; Detmold, Jay, Murphy, Oare, Shanahan, PRD 107, 094501 [2208.05322]

g_ν^{NN} LO Contact Term

- g_ν^{NN} required at LO for renormalization (Cirigliano et al., PRL 120, 202001 [1802.10097])
- Dispersive/Cottingham estimate: PRL 126, 172002 [2012.11602] + JHEP 05 (2021) 289 [2102.03371]
- Lattice determination → Anthony's talk yesterday
- Redundant determinations (lattice vs dispersive) = exactly the robustness "recommended values" need

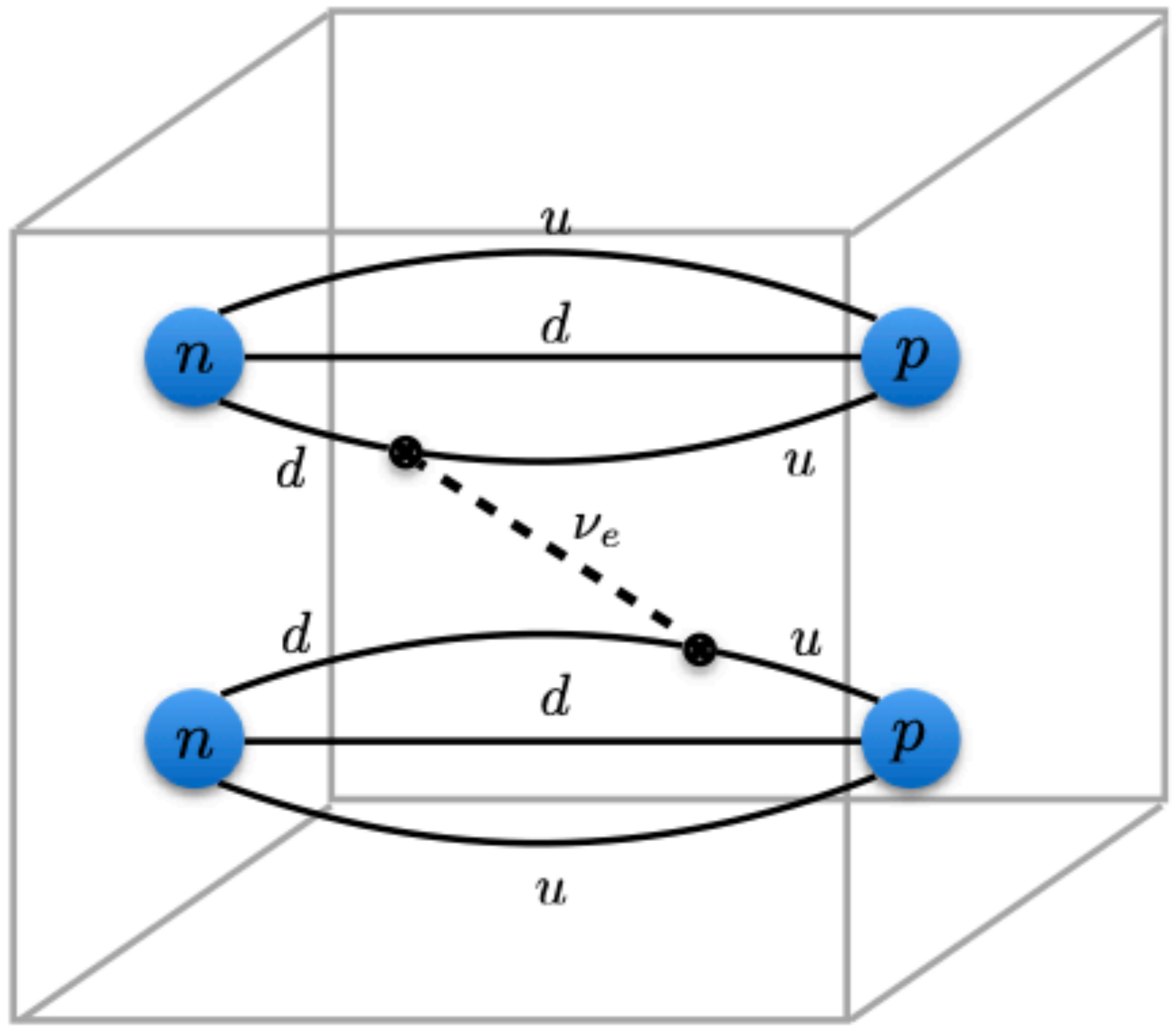


LO contact: g_ν^{NN}

→ Open question for the community

What precision must a lattice g_ν^{NN} reach to be worth using? The many-body impact is large and sign-sensitive, so a lattice value earns its place only once its total uncertainty is at or below the dispersive/Cottingham estimate — otherwise the dispersive value stays the reference. (impact: Wirth–Yao–Hergert, PRL 127, 242502; Jokiniemi et al., arXiv:2107.13354; dispersive: [2012.11602, 2102.03371])

Multi-Hadron Matrix Elements



- Finite volume: no asymptotic states, only discrete levels
- Euclidean time: no direct Minkowski amplitudes (Maiani–Testa)
- FV energies and MEs are not nuisances — they *encode* infinite-volume amplitudes through calculable relations
- Power-law L-dependence: the naive ratio-of-correlators estimate is *wrong*, not just imprecise

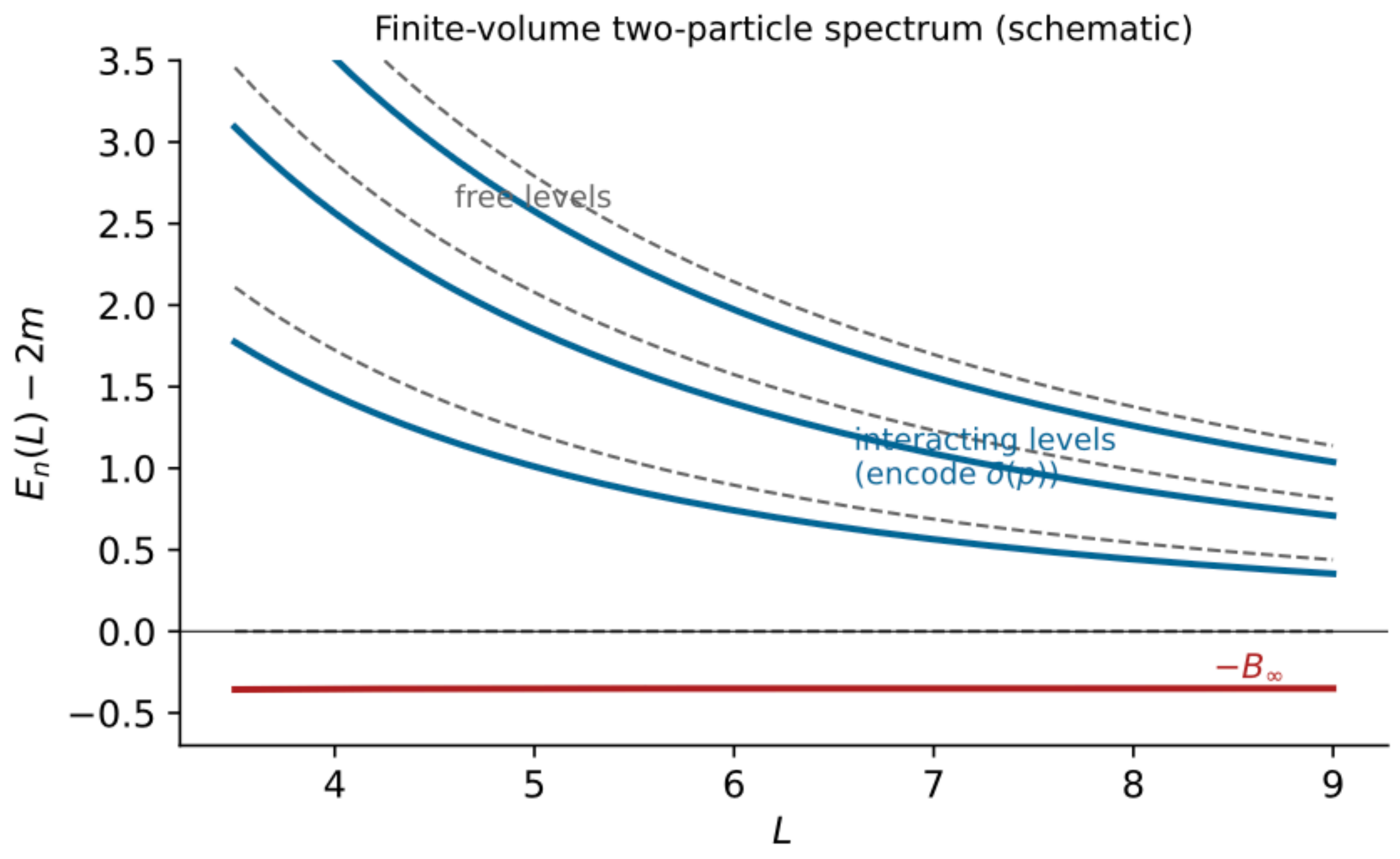
Maiani–Testa, PLB 245, 585 (1990)

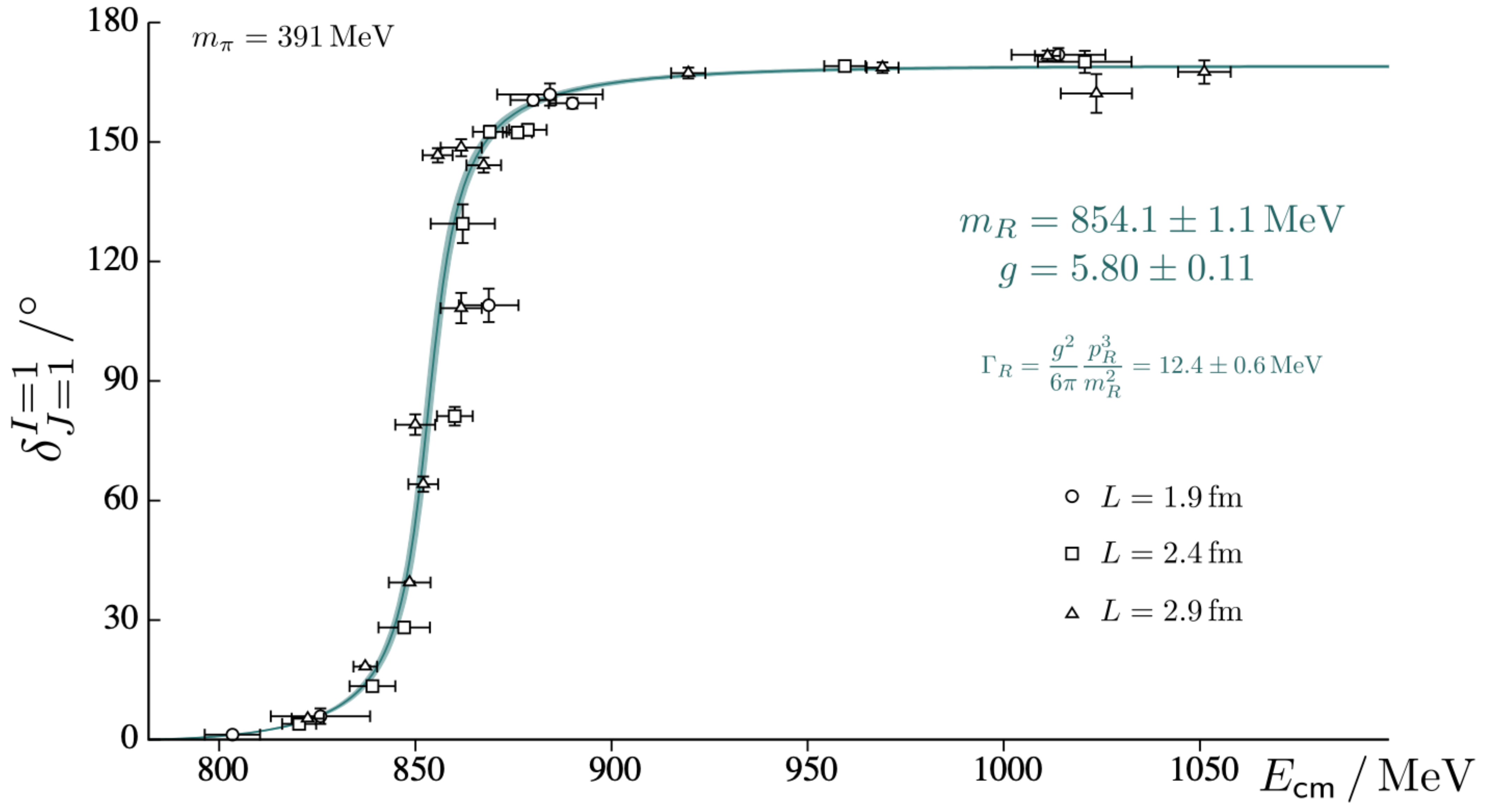
Multi-Hadron Matrix Elements *Mapping I. Spectrum —> Scattering*

$$\det\left[F^{-1}(E_n, L) + \mathcal{M}(E_n)\right] = 0$$

- Model-independent: FV spectrum $\{E_n(L)\} \leftrightarrow$ scattering amplitude $\mathcal{M}(E)$
- Exact up to $e^{-m_\pi L}$ corrections (lightest exchanged particle wrapping the volume): sub-% for $m_\pi L \gtrsim 4$ — the power-law L-dependence is the signal, the exponentials a controlled residual
- Lüscher (1986, 1991) $\to$ moving frames, coupled channels, three particles

Lüscher, CMP 105, 153 (1986); NPB 354, 531 (1991); review: Briceño–Dudek–Young, RMP 90, 025001 (2018) [1706.06223]





Multi-Hadron Matrix Elements *Mapping II. Currents*

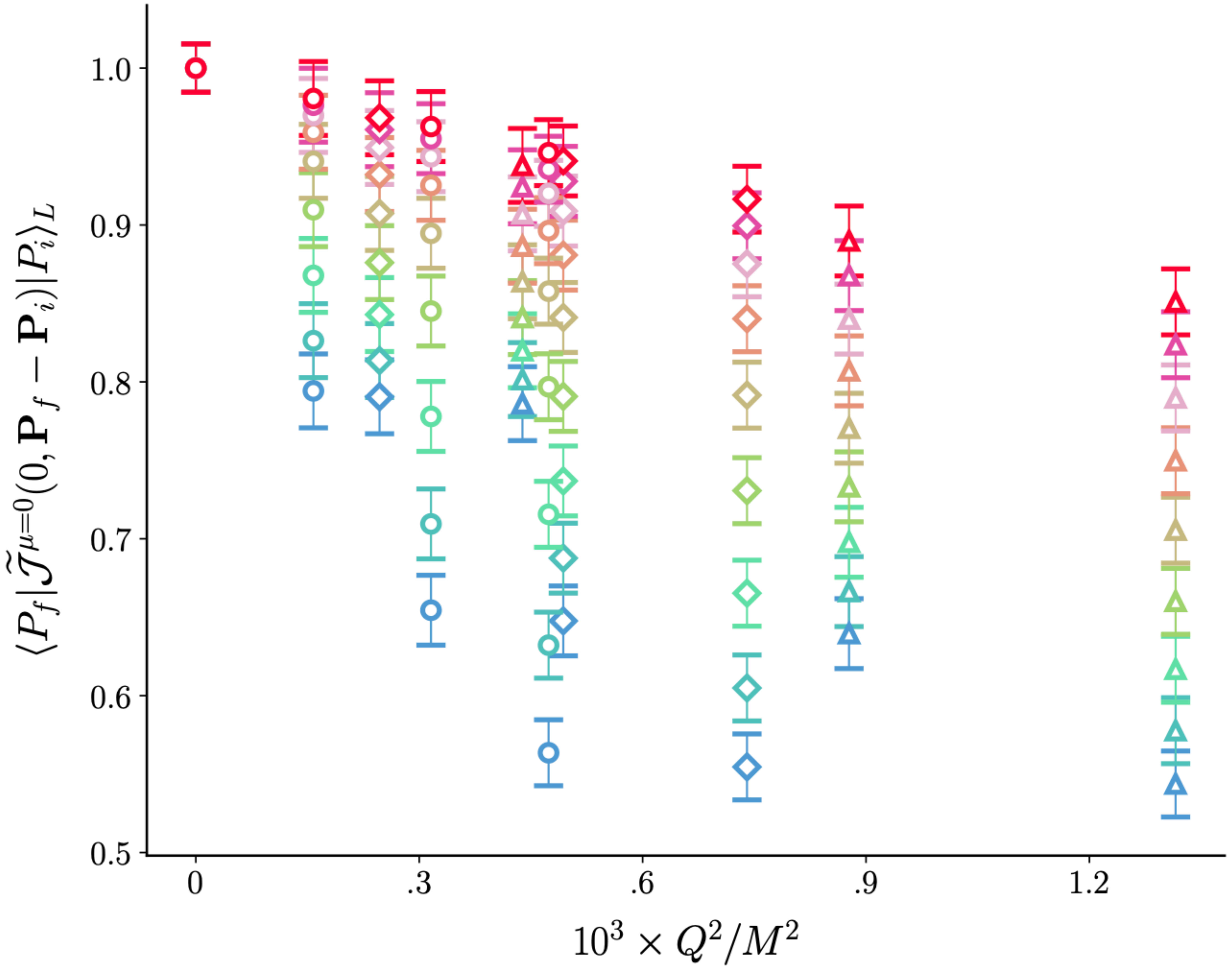
$$\left|\langle E_n, L \,|\, \mathcal{J}(0) \,|\, E_m, L\rangle\right|^2 = \underbrace{\mathcal{R}(E_n)\,\mathcal{R}(E_m)}_{\text{LL-type factors}} \left|\,\mathcal{W}_{\rm df}(E_n, E_m, Q^2) + \dots\right|^2$$

- $1 \to 2$ ($K \to \pi\pi$): Lellouch–Lüscher (2001)
- $2 \to 2$ with external current: Briceño–Hansen (PRD 94, 013008); covariant, implementation-ready framework: BBHOG [1812.10504] — FV ME $\leftrightarrow \mathcal{W}_{\rm df}$, the divergence-free $2{+}\mathcal{J} \to 2$ amplitude
- Bound-state poles of $\mathcal{W}_{\rm df} \Rightarrow$ elastic form factors of composite states
- EFT counterpart: solving the EFT *in the box* (Davoudi–Kadam-type formulations) generates these LL factors order by order — model-independent relations and FV-EFT matching must agree within EFT truncation

Lellouch–Lüscher [hep-lat/0003023]; Briceño–Hansen [1509.08507]; Baroni–Briceño–Hansen–Ortega-Gama, PRD 100, 034511 [1812.10504]; checks: [1909.10357], [2002.00023]

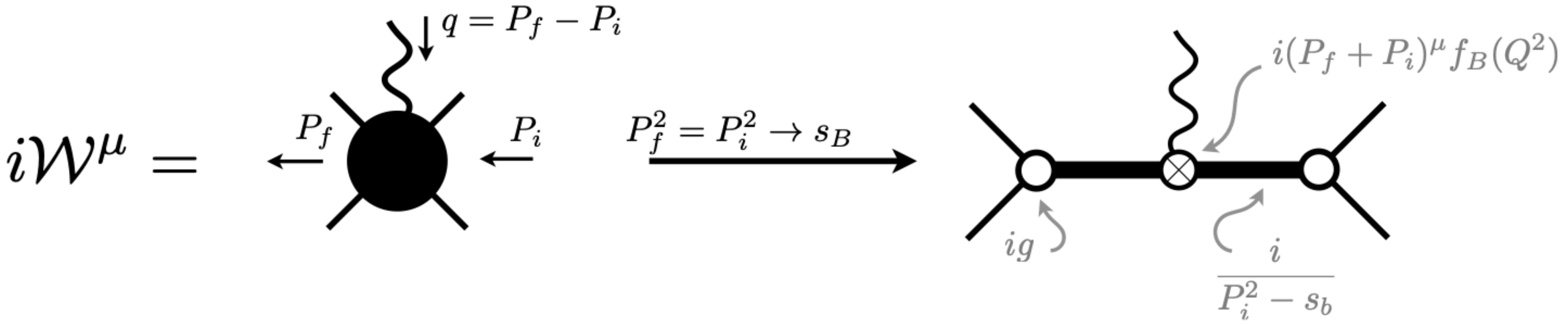
Multi-Hadron Matrix Elements *Towards LQCD Calculations*

- First lattice-QFT calculation of a $2 \to 2$ matrix element of a local current
- LO pionless EFT of two nucleons in a 3D box; Hamiltonian diagonalized *exactly* at moderate L
- Coupling tuned $\to$ deuteron-like bound state
- Every ingredient computable both directly *and* through the FV formalism $\Rightarrow$ end-to-end



JM, F. Ortega-Gama, R. Briceño, A. Jackura, C. Kacir, A. Nicholson — arXiv:2602.20373

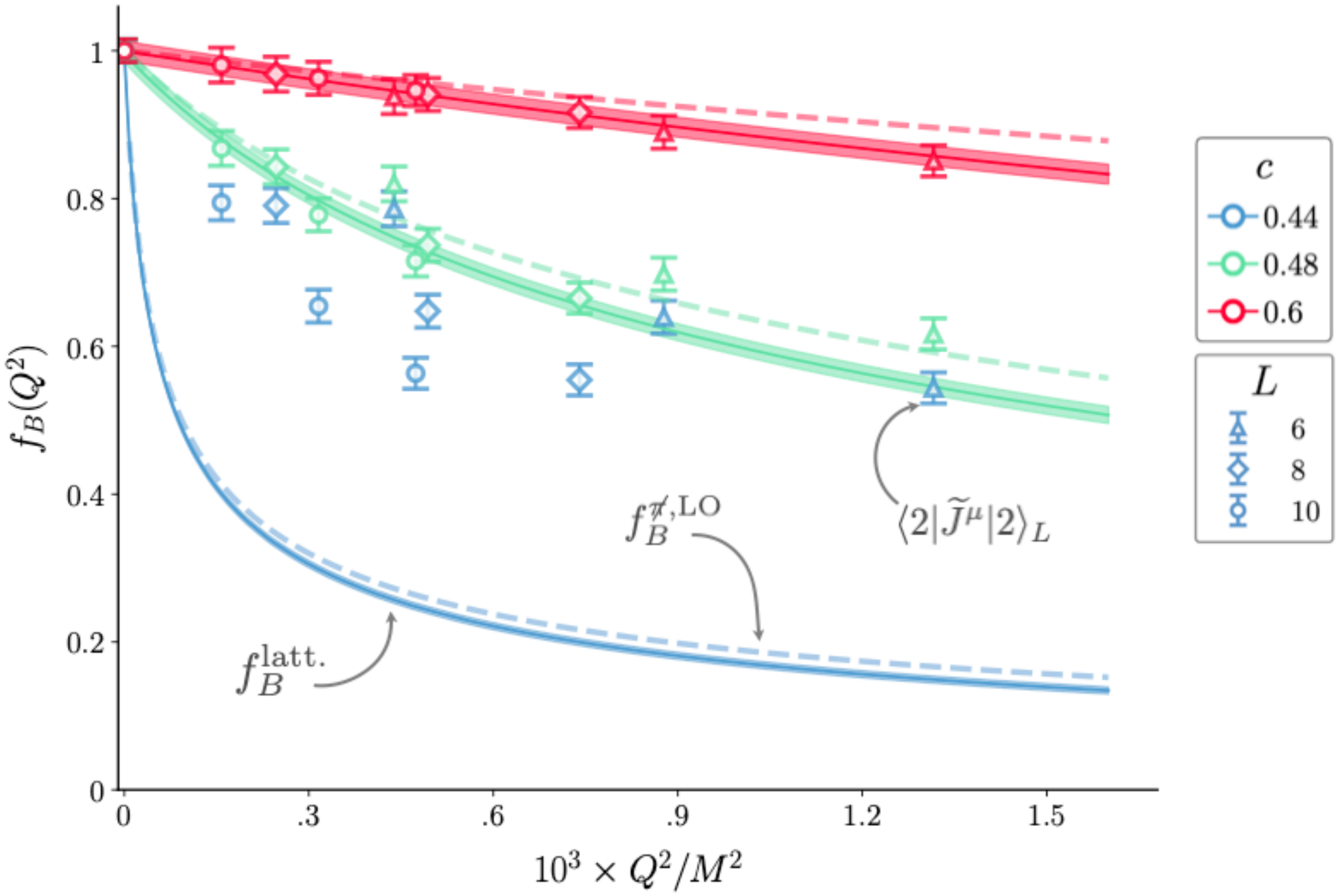
Multi-Hadron Matrix Elements *Bound-state structure*



- Elastic form factor of the deuteron-like state from FV data — the exact maneuver needed for real nuclei on the lattice
- Lessons: size of FV corrections at realistic L; which kinematic points are clean
- Naive extractions carry $\mathcal{O}(e^{-\kappa_B L})$ errors for bound states (κ_B = binding momentum: parametrically enhanced for shallow states) — corrections the formalism guarantees to remove

JM, F. Ortega-Gama, R. Briceño, A. Jackura, C. Kacir, A. Nicholson — arXiv:2602.20373

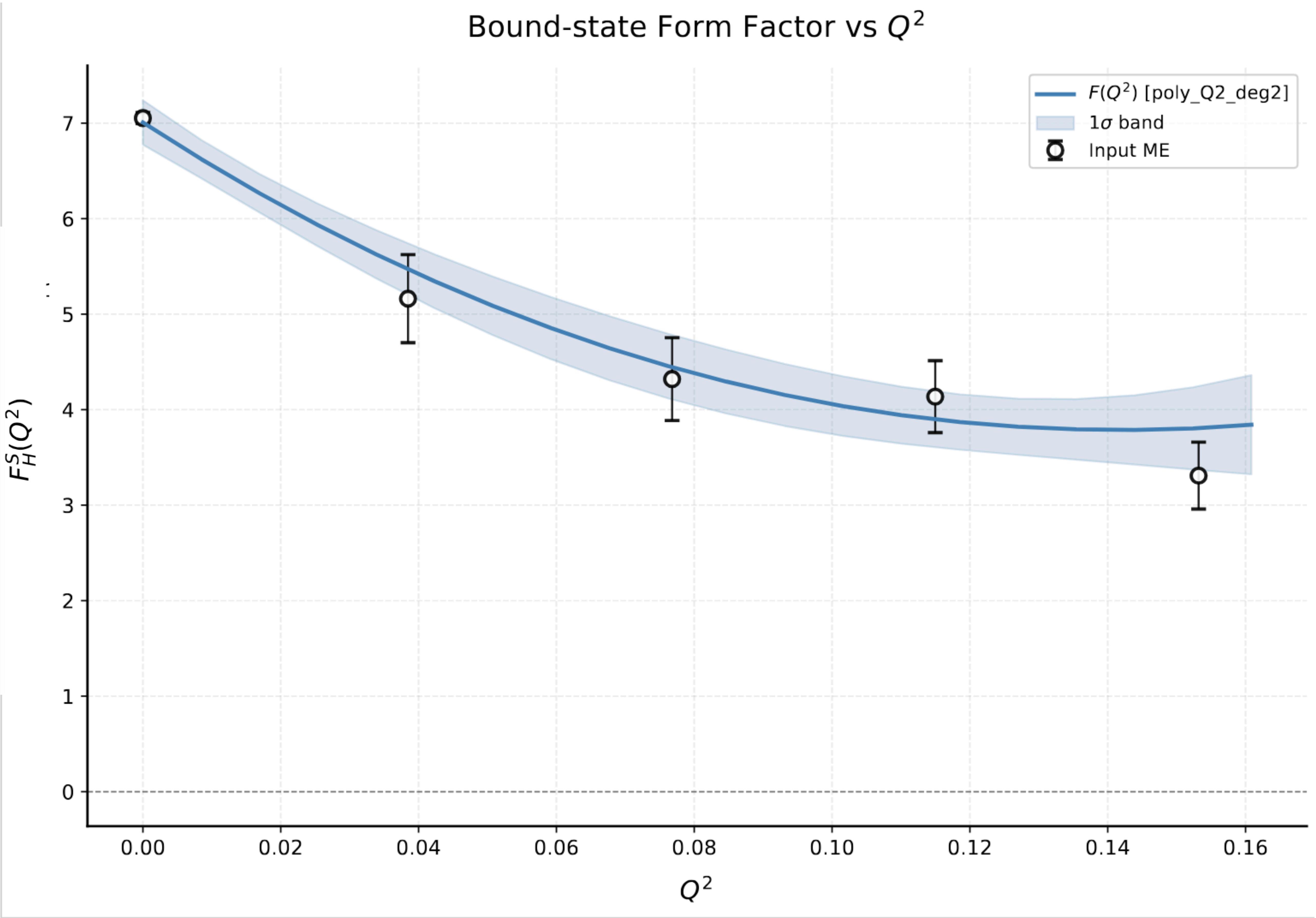
Multi-Hadron Matrix Elements *Towards LQCD Calculations*



- FV spectra $\xrightarrow{\text{Lüscher}}$ hadronic amplitude $\mathcal{M}$
- FV vector-current MEs $\xrightarrow{\text{Briceño–Hansen}}$ the physical current–two-nucleon transition amplitude: one function of (E, E', Q^2)
- The box samples it at discrete kinematics; the bound state lives below threshold $\Rightarrow$ parametrize, fit, continue — the ρ workflow, one level up (form varied as a systematic)
- Formalism works, including in the presence of a bound state

JM, F. Ortega-Gama, R. Briceño, A. Jackura, C. Kacir, A. Nicholson — arXiv:2602.20373

Multi-Hadron Matrix Elements *Scalar Hyperon Form Factors*



- Same Briceño–Hansen machinery as the testbed, now applied to QCD two-nucleon systems (large m_π)
- What changes from the testbed: statistics and excited states now in play — the testbed's map of clean kinematic points tells us where to fit
- Still heavy m_π: deliverable is *methods* + *FV/EFT systematics*, physical point to follow

JM, Z. Davoudi, M. Wagman, A. Grebe (@NDB Hub) — in progress; *NN* energy bounds: [2601.22272]

Towards Two Currents $\langle \mathcal{O}_{pp}(t_+) J_\mu(x) J_\mu(y) \mathcal{O}^\dagger_{nn}(t_-) \rangle$

- Two insertions ⇒ new FV physics: intermediate states between the currents (below threshold: exponentially growing in Euclidean time) — the single-current formalism does not suffice
- Davoudi–Kadam: FV-correlator ↔ amplitude matching in $\not\pi$EFT, at NLO for $2\nu\beta\beta$ [2007.15542]; extended to the $0\nu\beta\beta$ short-distance piece ⇒ prescription to extract g_ν^{NN} from FV data [2012.02083]

Layer	Formalism	Character	Status
spectrum → amplitude	Lüscher	model-indep.	mature
one current, 2 → 2	BH / BBHOG	model-indep.	**validated** [2602.20373]
two currents	Davoudi–Kadam	$\not\pi$EFT matching	ready; applied @ 806 MeV [2402.09362]
two currents	BH-type bilocal	model-indep.	**open**
any current	χEFT in a box	EFT matching	**open** as LQCD interface

Open Frontier: Where does matching happen

Route 1 (direct): infinite volume first

FV data $\to$ Lüscher / Briceño–Hansen $\to$ physical amplitude $\to$ match EFT

$+$ model-independent, EFT-agnostic

$-$ dedicated formalism per observable; bilocal (0ν) case involved

"Lüscher-based matching formalism for any EFT?"

Route 2 (match): EFT in the box

Solve the EFT in the *same* finite volume; match FV energies & MEs directly

$+$ natural for bound states; demonstrated in $\pi\!\!\!/$EFT (FVEFT: Detmold–Shanahan [2102.04329, 2202.03530])

$-$ consistent FV chiral EFT not yet formulated

"Chiral EFT in a box?"

- $0\nu\beta\beta$ needs bilocal two-current operators in FV — done via Route 2 in $\pi\!\!\!/$EFT (Davoudi–Kadam); model-independent Route 1 version still open
- Heavy-m_π precedent for Route 2: $\pi\!\!\!/$EFT fits to lattice nuclei (Barnea et al., PRL 114, 052501 [1311.4966])

Run both routes on the same heavy-m_π data — where they must agree — before the physical point.

Cross Validation at $A = 2$

Road A (match)	**Road B (direct)**
available **now**: LECs at physical point (FFs, $g_i^{\pi\pi}, g_\nu^{NN}$)	model-independent; no EFT truncation
inherits EFT truncation & scheme dependence	expensive; physical point ~3–5 yrs
scales to any A via many-body methods	limited to $A \lesssim 4$ for the foreseeable future

The payoff of running both

Direct $nn \to pp$ amplitude (B) vs EFT amplitude with matched LECs (A) — the comparison the community roadmaps call for: validate the EFT treatment of the $0\nu\beta\beta$ operators against QCD — does the chiral expansion converge, and is the LO contact term as large as claimed?

test advocated in the community roadmaps: [2203.12169], [1904.09704]; matching path: [2012.02083]

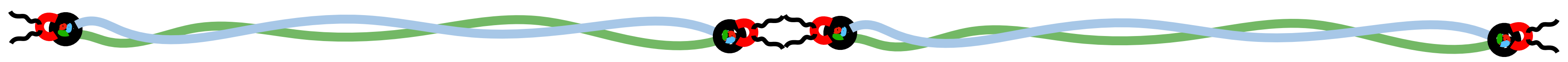
Direct Amplitudes —> Many-Bodies

- Many-body methods are calibrated to data — and for $\Delta L = 2$, nature provides none: Road B's $nn \to pp$ amplitude *is* the missing datum
- **How it reaches heavy nuclei**: each framework fits its *own* contact coupling (own scheme, own regulator) to reproduce the $A{=}2$ amplitude — observable-level matching, no scheme conversion — already standard practice via the "synthetic datum" (Wirth–Yao–Hergert [2105.05415]); Road B upgrades that datum from dispersive to QCD
- **At $A \leq 3$**: direct head-to-head benchmarks — ab initio computes the *same* amplitudes with its own operators
- **Error propagation**: the amplitude's uncertainty enters downstream like an experimental error bar

> $A \lesssim 3$ is not the limitation — it is the location of the experiment: the datum that anchors calculations of ^{76}Ge and ^{136}Xe

$\pi\!\!\!/$EFT chain lattice $\to$ nuclei: [1311.4966]; roadmap language: [2203.12169]

UQ: Weighing Hadronic Input on $M^{0\nu}$



Input	Impact on $M^{0\nu}$ (light Majorana)	Lattice status	Priority
g_ν^{NN} contact	15–50% (shell model), 30–80% (pnQRPA), sign-sensitive	first results	1
$F_A(q^2)$, g_P shapes	few–10% (needs systematic study)	maturing	2
$g_i^{\pi N}$, g_i^{NN} (dim-9)	$\mathcal{O}(1)$ for BSM interpretation	missing	3
g_A value	$\sim$1% $\Rightarrow$ negligible at nucleon level	settled	—
$g_i^{\pi\pi}$	heavy-mechanism limits & discrimination	converging	—

g_ν^{NN} impact: [2107.13354], [2105.05415]; FF impact: open study (this talk); dim-9: [1806.02780]

The form recommended values need

For the recommended values, the useful deliverable is an input vector (g_A, F_A/g_P shape coefficients, g_ν^{NN}, $g^{\pi\pi}$, $g_{S,T}$) *with a covariance*, in a fixed scheme — it can propagate through your emulators and is common to every isotope.

UQ Prospects

- LEC errors: mature FLAG-style budgets (statistics, excited states, $a \to 0$, m_π) — already delivered with covariance for several inputs
- EFT truncation: Bayesian order-by-order models give correlated truncation priors (BUQEYE: [1904.10581])
- Propagation to $M^{0\nu}$: eigenvector-continuation emulators make global sensitivity analysis feasible [1711.07090, 1909.08446]; demonstrated end-to-end for ^{76}Ge — Belley et al., PRL 132, 182502 [2308.15634]; updated analysis [2605.21657]
- Open question: how do we put a *principled* prior on the operator-expansion truncation and scheme choice? — cutoff variation bounds it today, but a calibrated estimate is still missing

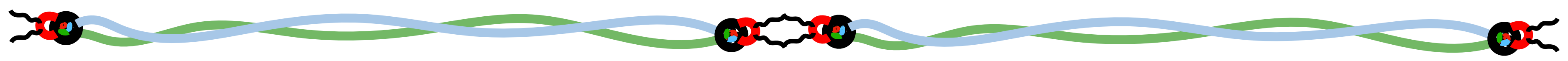
UQ Prospects

- Budget: statistics (S/N problem), excited states → rigorous two-sided energy bounds [2601.22272], FV-mapping truncations quantified in the exact testbed [2602.20373], plus standard $a \to 0, m_\pi$
- No EFT truncation error at $A = 2$ — replaced by mapping systematics that are *measurable*, not estimated
- Downstream, the amplitude's total error behaves like an experimental uncertainty in the many-body fit (previous slide) — the cleanest error propagation available to this program
- Still open: end-to-end propagation is under active investigation — multiple error sources (mapping, $a \to 0$, m_π, spectral/excited-state) and parametrization choices are still present and not yet folded in with correlations

The comparison as a UQ instrument

The A–B difference at $A = 2$ *measures* the χEFT operator-truncation error directly — a data-driven prior for exactly the Bayesian truncation models many-body UQ already uses. Road B doesn't just check Road A; it calibrates Road A's error bar.

What Lattice must supply to the program



The deliverables to be pinned down

- g_A and the axial/induced form-factor *shapes* with a full covariance, at the physical point
- $\pi^- \to \pi^+$ and the $\pi N/NN$ dim-9 LECs; scalar and tensor charges g_S, g_T
- Physical-point $nn \to pp$: long-distance piece + LO contact, matched to interaction LECs with a complete error budget
- g_ν^{NN} obtained *redundantly* — dispersive input and a cross-check between Road A and Road B at $A = 2$

- Lattice FF parametrizations & LEC values adopted *with correlations*
- A single fixed scheme-matching convention (lattice ↔ interaction)
- A $2\nu\beta\beta$ end-to-end validation gate (2007.15542; 0ν–2ν correlations 2605.19479)
- Lattice ME *ratios* for mechanism discrimination — light-Majorana vs dim-9 (Niţescu, next talk)

Mechanism	Hadronic input	Enters via	Lattice status
Long-range ($m_{\beta\beta}$)	$g_V(q^2)$, $g_A(q^2)$, $g_M(q^2)$, $g_P(q^2)$	V_ν	**maturing**
LO contact	g_ν^{NN}	V_ν counterterm	**first results** (Anthony)
Short-range dim-9	$\pi^- \to \pi^+$ MEs ($\mathcal{O}_{1..5}$)	$g_i^{\pi\pi}$	**done, phys. point**
Short-range dim-9	πN MEs	$g_i^{\pi N}$	**missing (naturalness est.)**
Short-range dim-9	NN MEs	g_i^{NN}	**missing**
Dim-7 / scalar–tensor	g_S, g_T (+ g'_P, PCAC)	non-std currents	**done, phys. point***
Long-range, direct	$nn \to pp$ amplitude	full A=2 input	**proof of principle**

Two Roads from QCD to Nuclei

1. **Road A is productive now**: several hadronic inputs (g_A, FF shapes, $g_i^{\pi\pi}$, g_ν^{NN}) are QCD-anchored at or near the physical point — increasingly usable as inputs rather than model parameters
2. **Road B is opening up**: the FV mapping formalism is validated end-to-end (2602.20373), a first QCD application — two-nucleon bound-state form factors — is underway, and the FV matching question ("chiral EFT in a box?") is now sharply posed
3. **The roads meet at $A = 2$**: their comparison could offer a first-principles handle on the EFT power counting — a natural candidate for the white paper's error budget


THIS IS THE END
THANK YOU FOR
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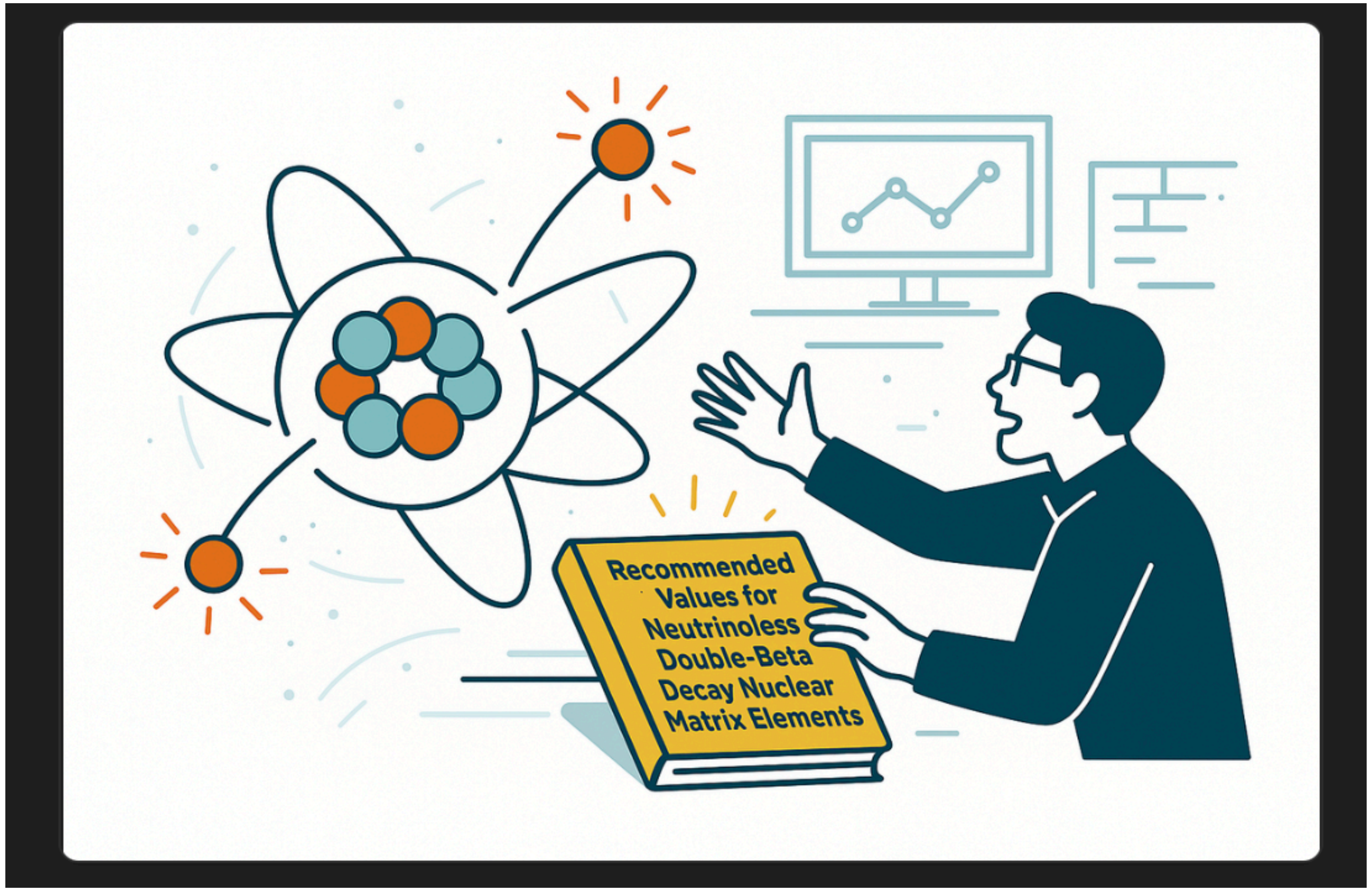
Recommended
Values for
Neutrinoless
Double-Beta
Decay Nuclear
Matrix Elements