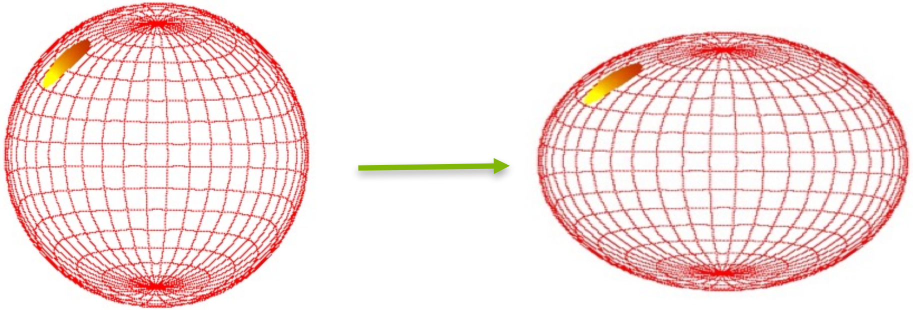
How does rapid rotation affect the inference of the neutron star equation of state?



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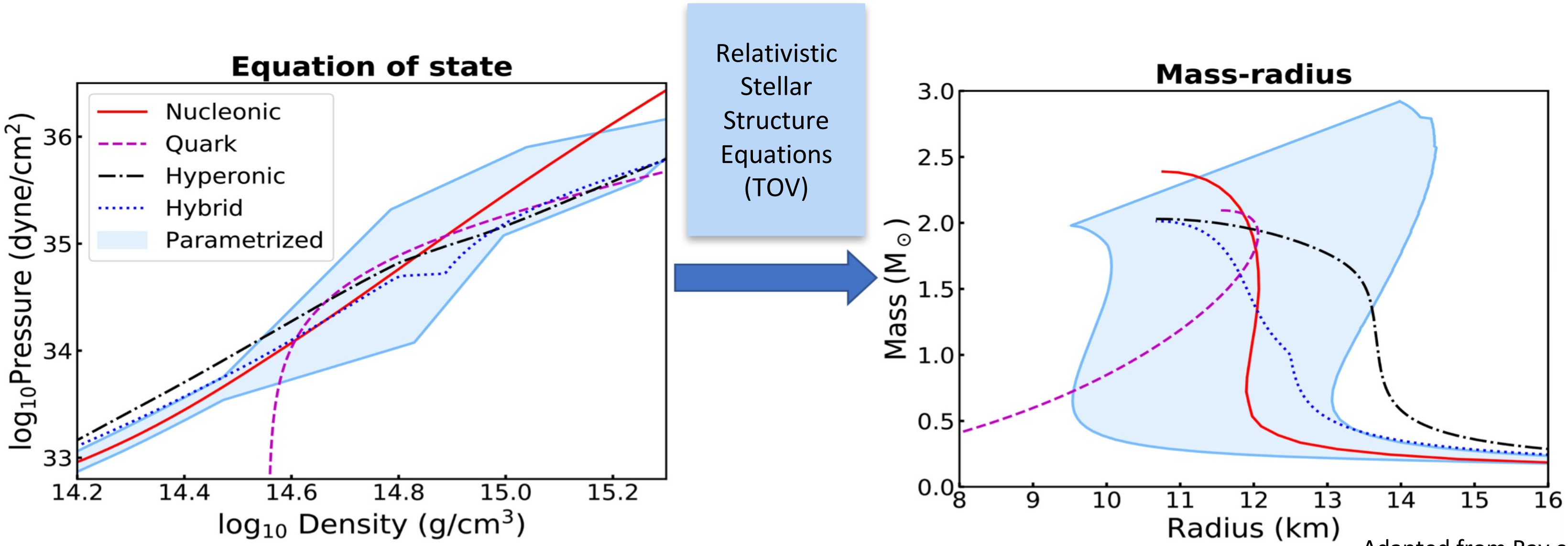
Neutron Rich Matter on Heaven and Earth, Institute for Nuclear Theory, Seattle WA, June 29, 2023

Equations of State (EOS) vs Mass-Radius Curves

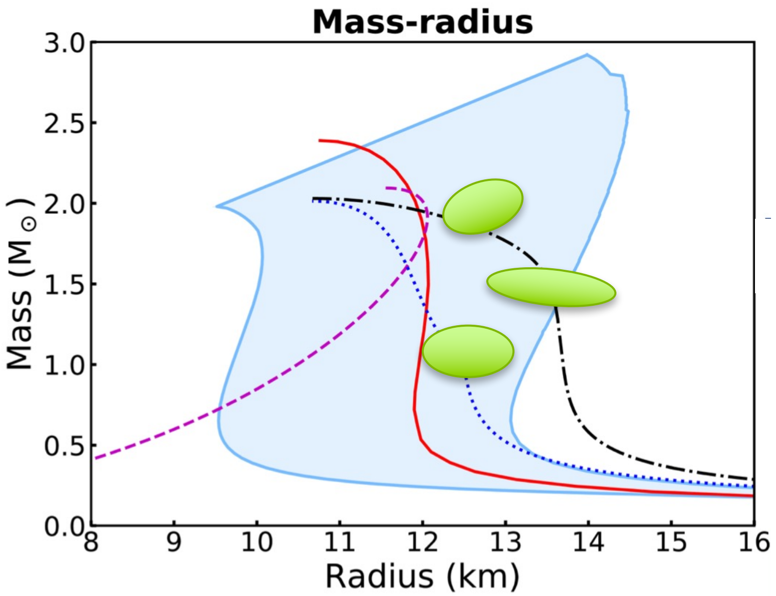


Adapted from Ray et al (2019)

1. Choose an EOS
2. Choose a value for the density at the centre of the star, ε_c
3. Solve TOV equations using EOS and $\varepsilon \leq \varepsilon_c$
4. Results in TOV mass and radius of this star M*, R*
5. Repeat 2-4 for different central densities to find full mass radius curve for the chosen EOS.

EOS Inference – 1. Observations

- We have observed N neutron stars, the i^{th} NS has a posterior probability $P_i(M,R)$ that its mass and radius is M and R
  - This could be coming from pulse profile modelling from NICER, or through X-ray flux measurements of qLMXBs or photospheric radius expansion bursts (eg Chandra, XMM)
  - Or measurements of other macroscopic properties like moment of inertia or tidal deformability instead of radius

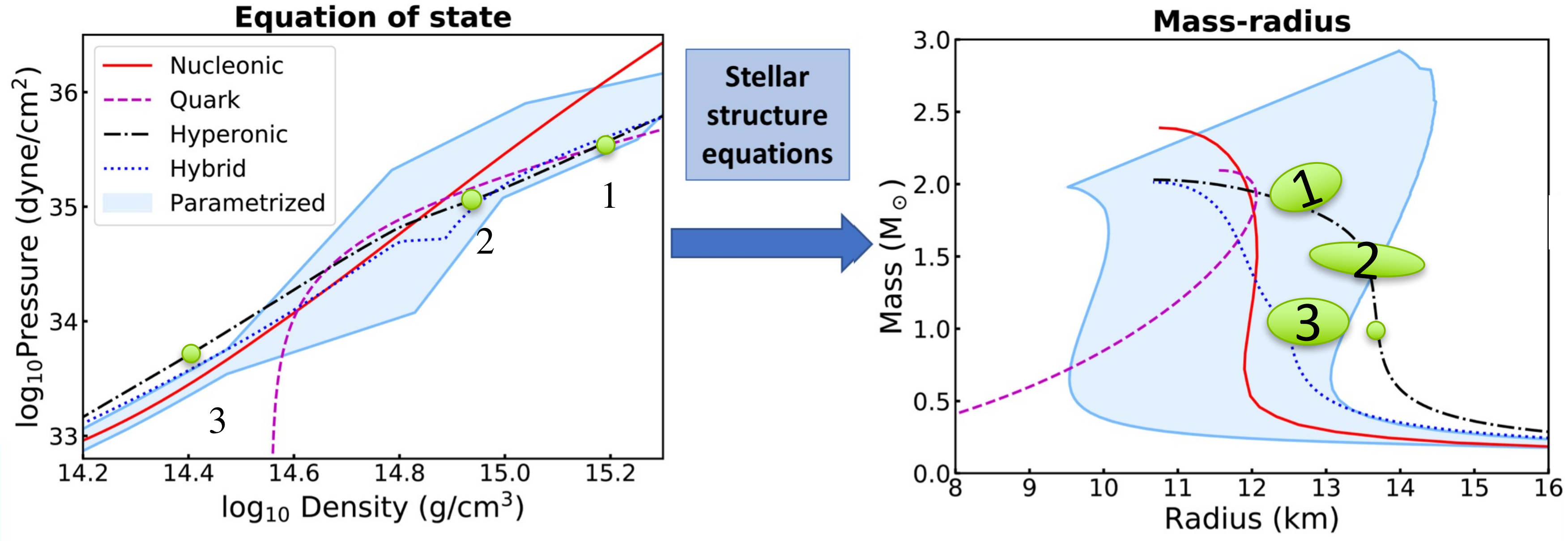


Example: 3 fake measurements with 1 sigma contours

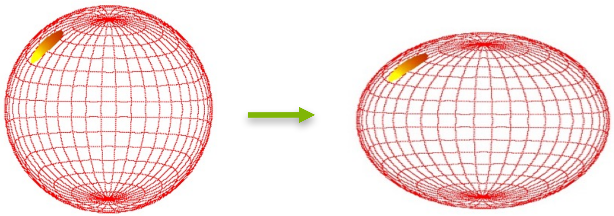
EOS Inference – 2. EOS Parameterization

- Choose some type of EOS parametrization: eg. piecewise polytropes, speed of sound, Gaussian process
- Fix the EOS by choosing a set of parameter values
- Each of the observed neutron stars should be described by the same EOS, but will have a different central density $\varepsilon_{c,i}$
- Solve TOV equations for M^*_i and R^*_i and find $P(M^*_{i,} R^*_i)$
- Find the overall likelihood that this EOS describes the set of N observations

“Dash-dot” EOS has poor likelihood



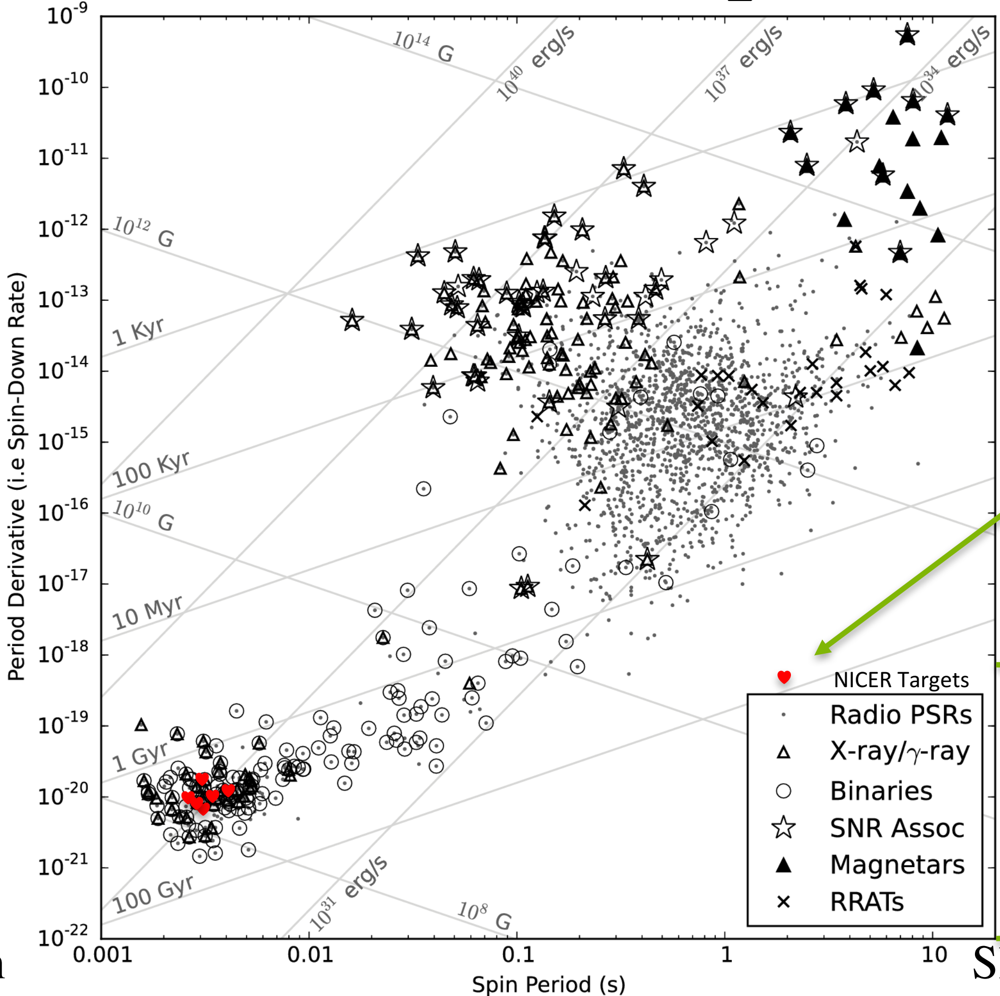
BUT… Neutron Stars Spin!!!



- The neutron stars we observe often spin very fast (ie rotational kinetic energy ~ 0.1 gravitational binding energy)
- Rotation increases the mass and the equatorial radius
- Techniques such as pulse profile modelling estimate the equatorial radius and mass of the rotating star!

Pulsar P-Pdot Diagram

Rapid Change in Spin



Stable spin

Fast spin

Slow spin

Examples: PSR J0030, PSR J0740,
Talks by Bas Dorsman, Serena Vinciguerra, Devarshi Choudhury, Tuomo Salmi

Non-accreting NS Spin-down due to B field

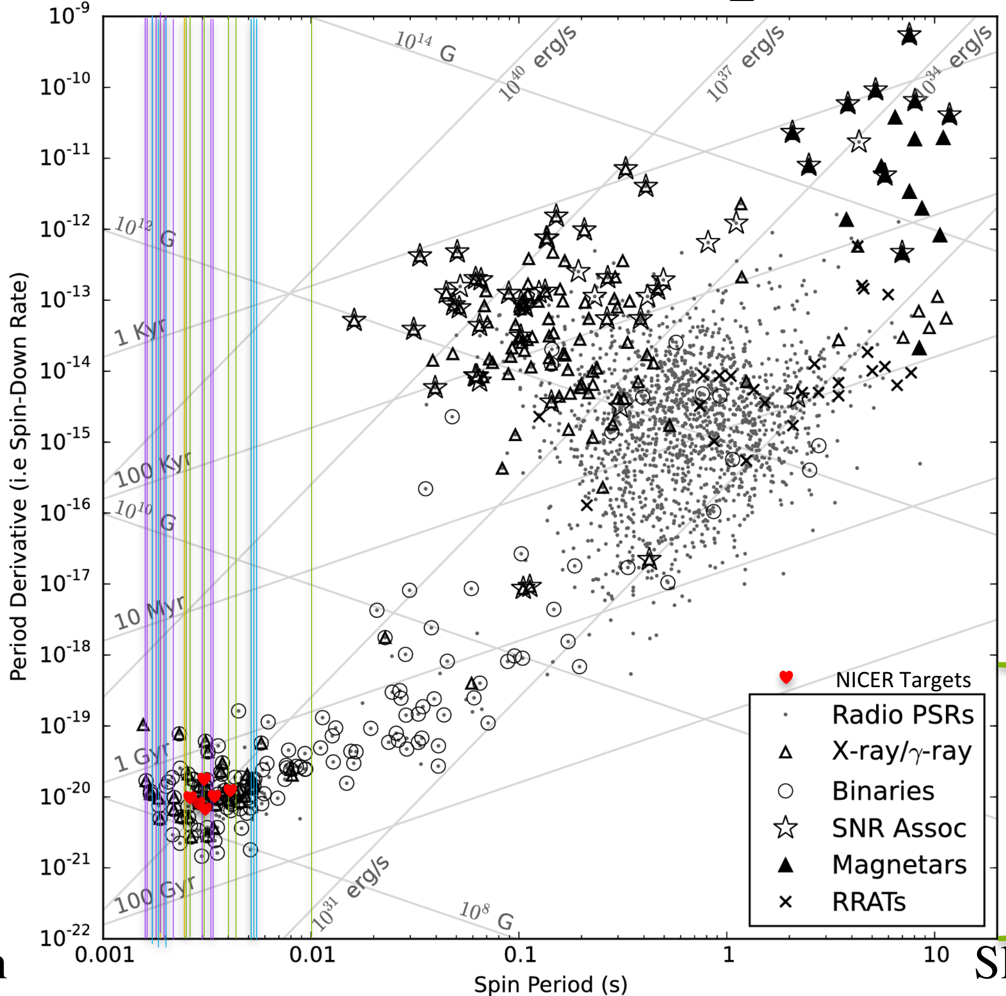
From Essential Pulsar Astronomy by Scott Ransom

Pulsar P-Pdot Diagram

Accreting NS

- Persistent accretion-powered pulsars with burst oscillations
- Intermittent accretion-powered pulsars with burst oscillations
- Burst oscillation sources without persistent pulsations
- Accretion-powered pulsars without burst oscillations

Examples: MXB 1659 cooling (Farrukh Fattoyev); Pulse profile modelling by Yves Kini



Fast spin

Slow spin

Non-accreting NS Spin-down due to B field

From Essential Pulsar Astronomy by Scott Ransom

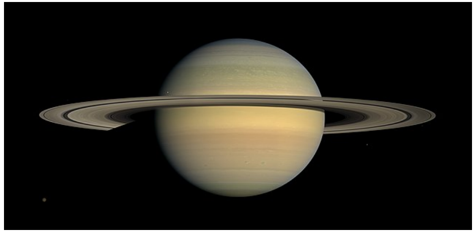
Relative Spins

Fraction of Breakup Frequency: $\frac{2\pi}{P}\sqrt{\frac{R^3}{GM}}$

Planet images: NASA

P = 9.9 hrs

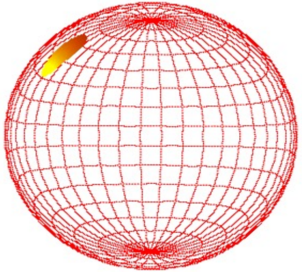
$$\frac{2\pi}{P}\sqrt{\frac{R^3}{GM}} = 0.3$$



P = 10.5 hrs

$$\frac{2\pi}{P}\sqrt{\frac{R^3}{GM}} = 0.4$$

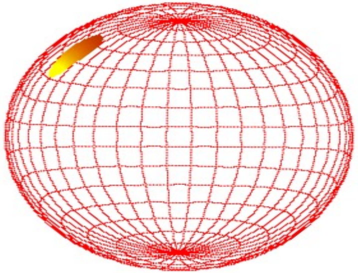
Neutron Stars with M=1.4 M_{sun}, R=12 km



P = 3 ms

$$\frac{2\pi}{P}\sqrt{\frac{R^3}{GM}} = 0.2$$

v/c = 0.085



P = 1.5 ms

$$\frac{2\pi}{P}\sqrt{\frac{R^3}{GM}} = 0.4$$

v/c = 0.17

How to combine observations of many NSs with different spins in EOS inference?

- Make use of "universal" relations that map non-rotating star's M* and R* (from TOV) to rotating star's M and R.
- Find mapping for **fixed central density**
- Compute dimensionless fractional changes
- Makes use of rns: 2D relativistic equilibrium code for uniform rotation (Stergioulas & Friedman 1995) but other codes such as Lorene (Gourgoulhon et al) also exist

Andreas Konstantinou and Sharon M. Morsink 2022 *ApJ*

Neutron Stars Have "Hair"

- Given a mass and spin (M and Ω) different EOS predict different Radii (R)
- However, given M, R, Ω dimensionless quantities: $x = GM/Rc^2$ $y = \Omega^2 R^3/GM$
- Many secondary NS properties depend only on x, y.
- I Love Q (Moment of Inertia, Love number, Quadrupole moment) relationships
- "Neutron Star Universality" (Yagi & Yunes 2013)

Example: Moment of Inertia

- Expect Moment of Inertia of the form:

  $I = \beta M R^2$

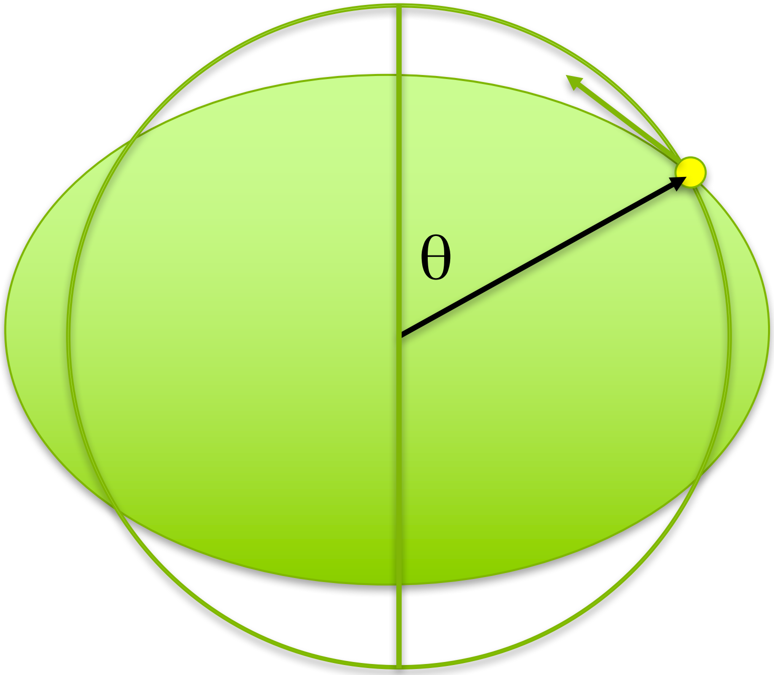
  Where β depends on how density varies inside the star. (Ravenhall & Pethick, 1994; Lattimer & Prakash, 2001)

- For Neutron Stars $\beta = \beta(M/R, \Omega^2 R^3/GM)$ with β a known function
- Similar functions for quadrupole moment, ellipticity, acceleration due to gravity, oscillation mode frequencies
- Applications to gravitational radiation from NS mergers (Raithel, et al; Raithel & Most)

Stellar Oblateness Effects on Raytracing

“Universal” form for
oblate shape:
$R(\theta) = R_e - b\cos^2\theta$

$b = b(M/R_e, \Omega^2 R_e^3/GM)$

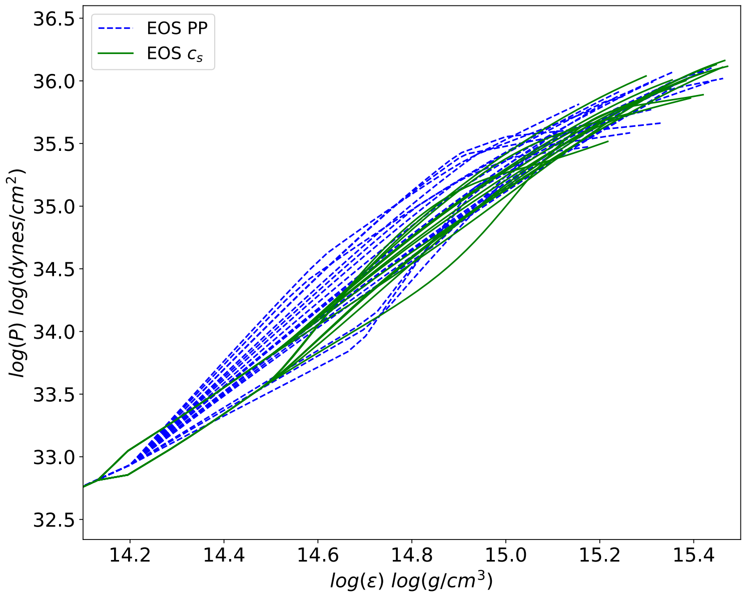


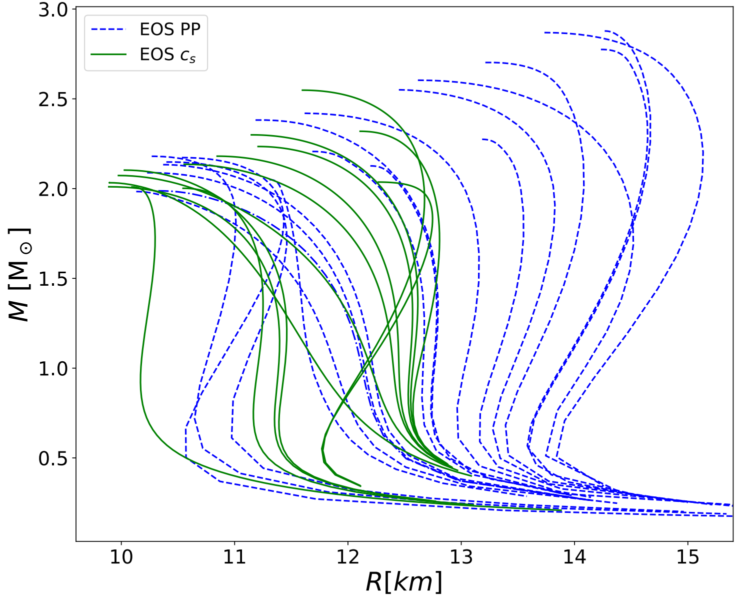
Morsink, Leahy, Cadeau & Braga 2007 ApJ

Randomly Generated EOS

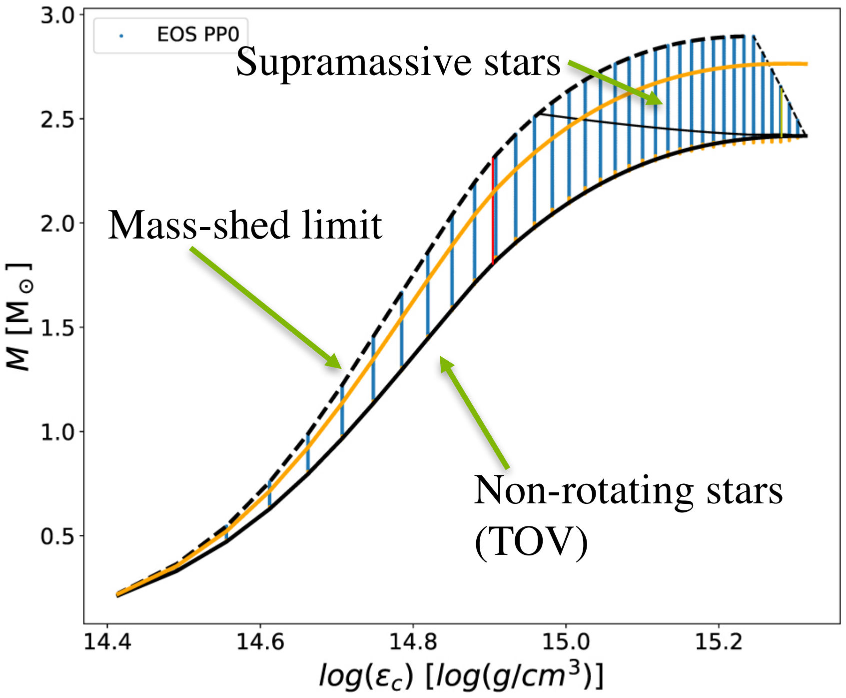
- We created a library of randomly generated EOS and computed rotational changes in M and R
- Dense EOSs generated using either:
  - Piecewise Polytrope (Read+ 2009) using Hebeler+ 2013 parametrization
  - Speed of Sound parametrization (Greif+ 2019)
- Matching to Chiral EFT bands for $n < 1.1\ n_0$
- Matching to BPS or NV crust for $n < 0.5\ n_0$
- 32 EOS generated (19 PP and 13 CS) providing converged universal relations

Randomly Generated EOS Library



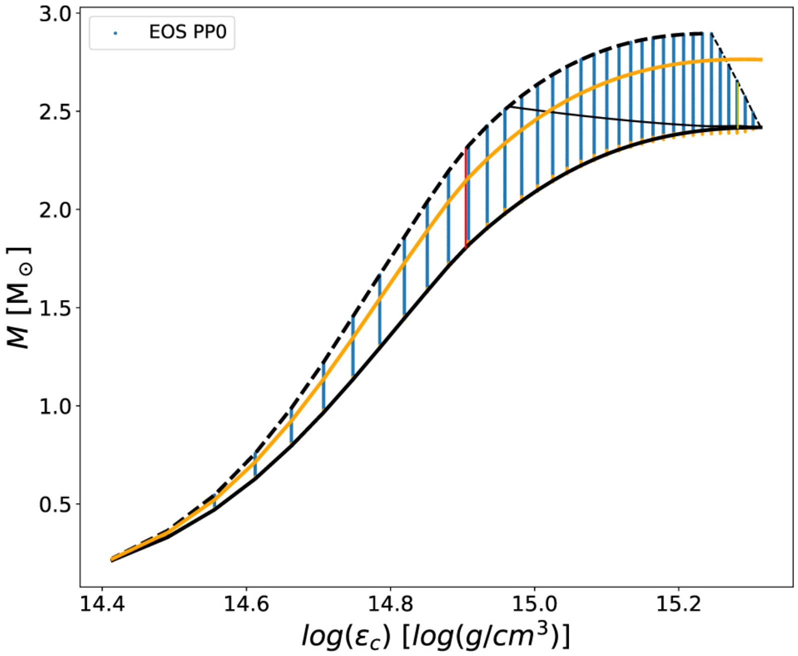


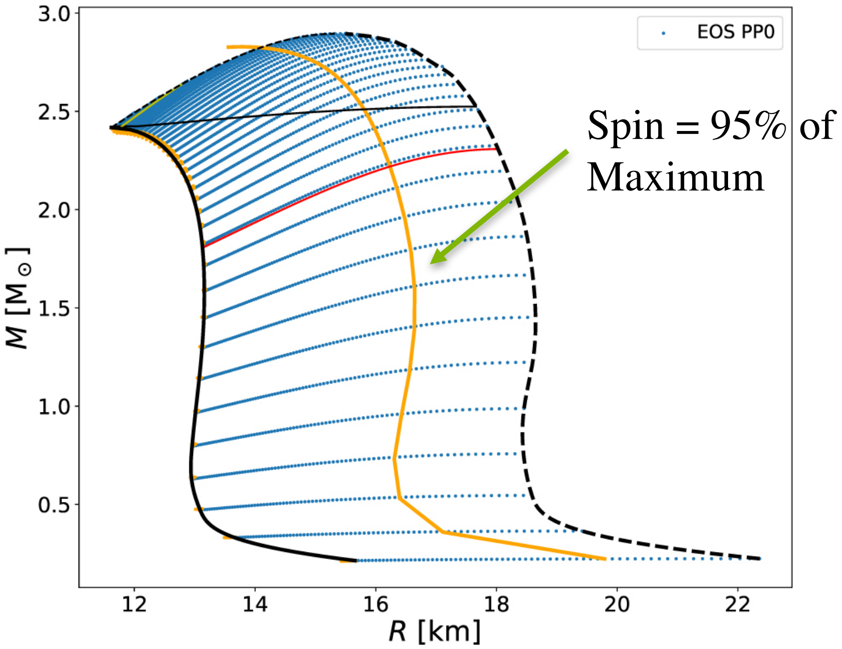
Effect of Spin on one EOS (Example)



- Blue vertical dots are sequences of rotating stars with the same central density
- Spin increases upwards
- Density dependent phenomena (eg: phase transition; direct URCA cooling; Cooper pairing) occur in all stars in a sequence if they occur in the zero spin star

Effect of Spin on one EOS (Mass-Radius Region)



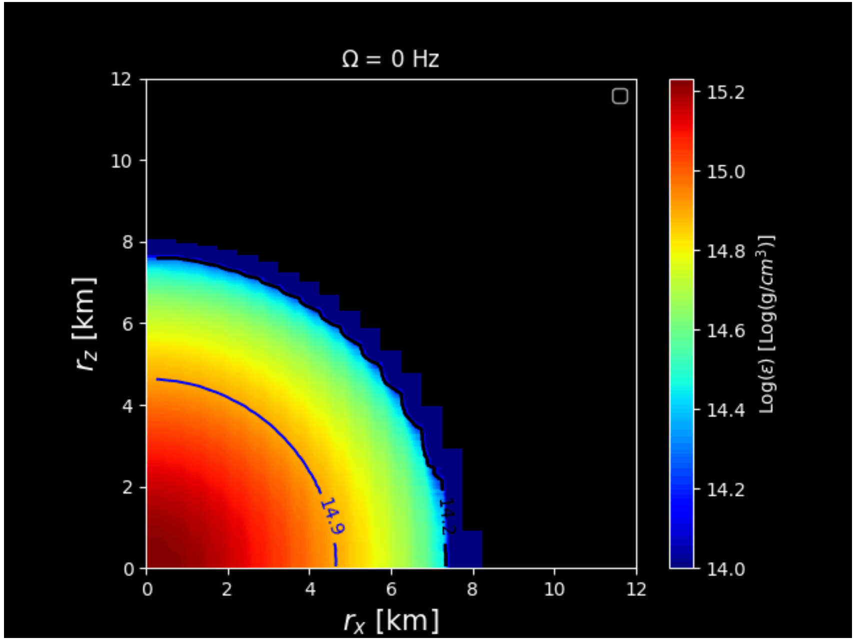


Equatorial Radius

Effect of Rotation on a Neutron Star

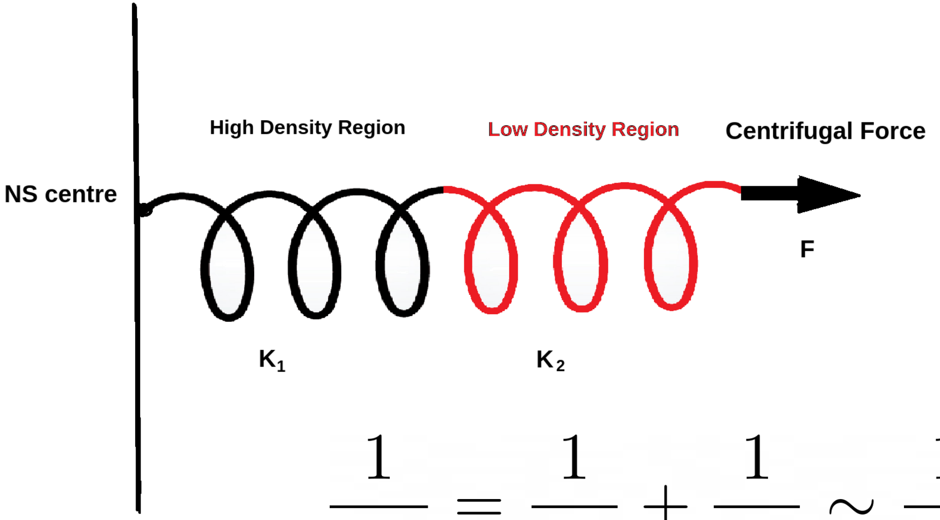
- Central Density is kept constant
- Mass and equatorial radius increase as spin increases
- Densest region $n > 3\ n_0$ is barely deformed
- Most deformation occurs in low density region $n < n_0$

Constant Central Density Sequence



Relativistic Computations/Animations by Andreas Konstantinou (U of A)

Overly Simplified Spring Model



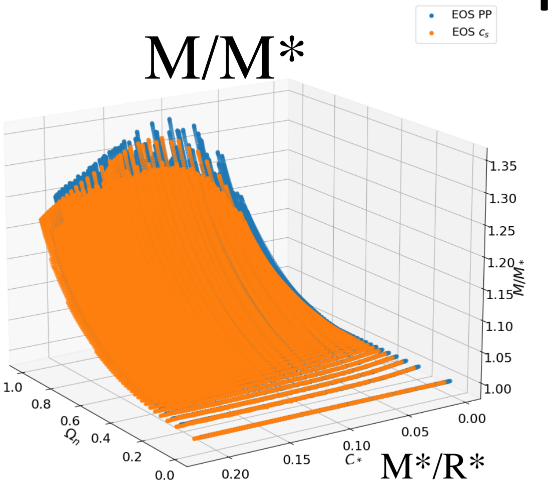
If $K_2 << K_1$ then:

$$\frac{1}{K_T} = \frac{1}{K_1} + \frac{1}{K_2} \sim \frac{1}{K_2}$$

Most of the strain is dominated by lower density region with a smaller spring constant.

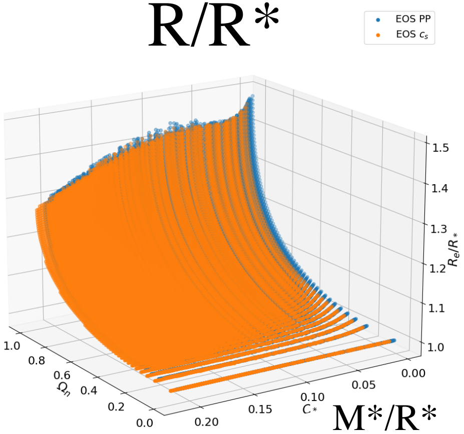
Increases in M and R due to Spin versus dimensionless spin and compactness

M/M*



M*/R*

R/R*



M*/R*

$$\frac{R_e}{R_*} = 1 + (e^{A_r \Omega_n^2} - 1 + B_r \left[\ln(1 - (\frac{\Omega_n}{1.1})^4) \right]^2) \times \left(1 + \sum_{i=1}^{5} a_{r,i} C_*^i \right)$$

$$\frac{M}{M_*} = 1 + (e^{A_m \Omega_n^2} - 1) \times \left(\sum_{i=0}^{4} a_{m,i} C_*^i \right)$$

Best fit equations for the surfaces with coefficients depending only on M*, R*, Ω

Revised EOS Inference for Spinning NS

- Choose EOS
- Compute M*, R* with TOV equations for each central density
- For each NS with known spin, compute the new spin-corrected M and R using the formulae on the previous page.
- Use the spin corrected M and R to compare with observations
- This method is computationally cheap to implement!

Is this really necessary????

- Right now NICER observes relatively slowly rotating NS with spins < 300 Hz
- Corrections due to spin are smaller than the uncertainty in the NICER mass-radius measurements so NOT REALLY REQUIRED
- But…
  - Accreting NS spin faster, up to around 600 Hz: larger spin corrections
  - One day Strobe-X and eXTP will hopefully fly, and their improved precision may make these corrections necessary

M/R Dependence

- Although M and R both increase with spin, their ratio is **almost** constant for these sequences
- This occurs because:
  - Decrease in polar radius is **almost** balanced by increase in equatorial radius
  - Newtonian volume of ellipsoid: $$V = \frac{4\pi R_e^2 R_p}{3}$$
  - Change in volume:

$$\frac{\Delta V}{V_*} = \frac{R_e}{R_*}\left(2\frac{\Delta R_e}{R*} - \frac{\Delta R_p}{R*}\right) \simeq \frac{\Delta R_e}{R_*}$$

M/R Dependence

– Change in volume: $$\frac{\Delta V}{V_*} = \frac{R_e}{R_*}\left(2\frac{\Delta R_e}{R*} - \frac{\Delta R_p}{R*}\right) \simeq \frac{\Delta R_e}{R_*}$$

– Change in average density:

$$\frac{\Delta\rho}{\rho_*} = \frac{\Delta M}{M_*} - \frac{\Delta V}{V_*} \simeq \frac{\Delta(M/R)}{M_*/R_*}$$

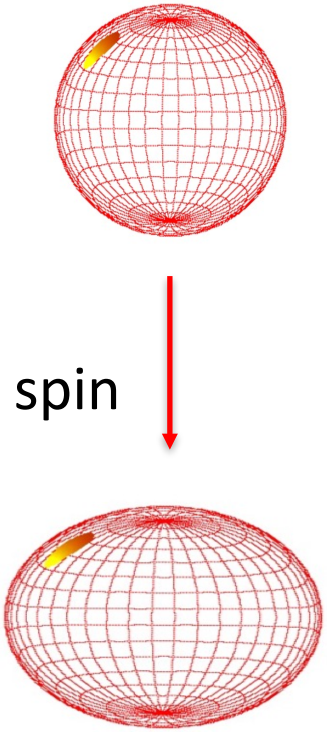
– Average density stays approximately constant along a constant central density spin sequence (typical of pure Newtonian polytropes)

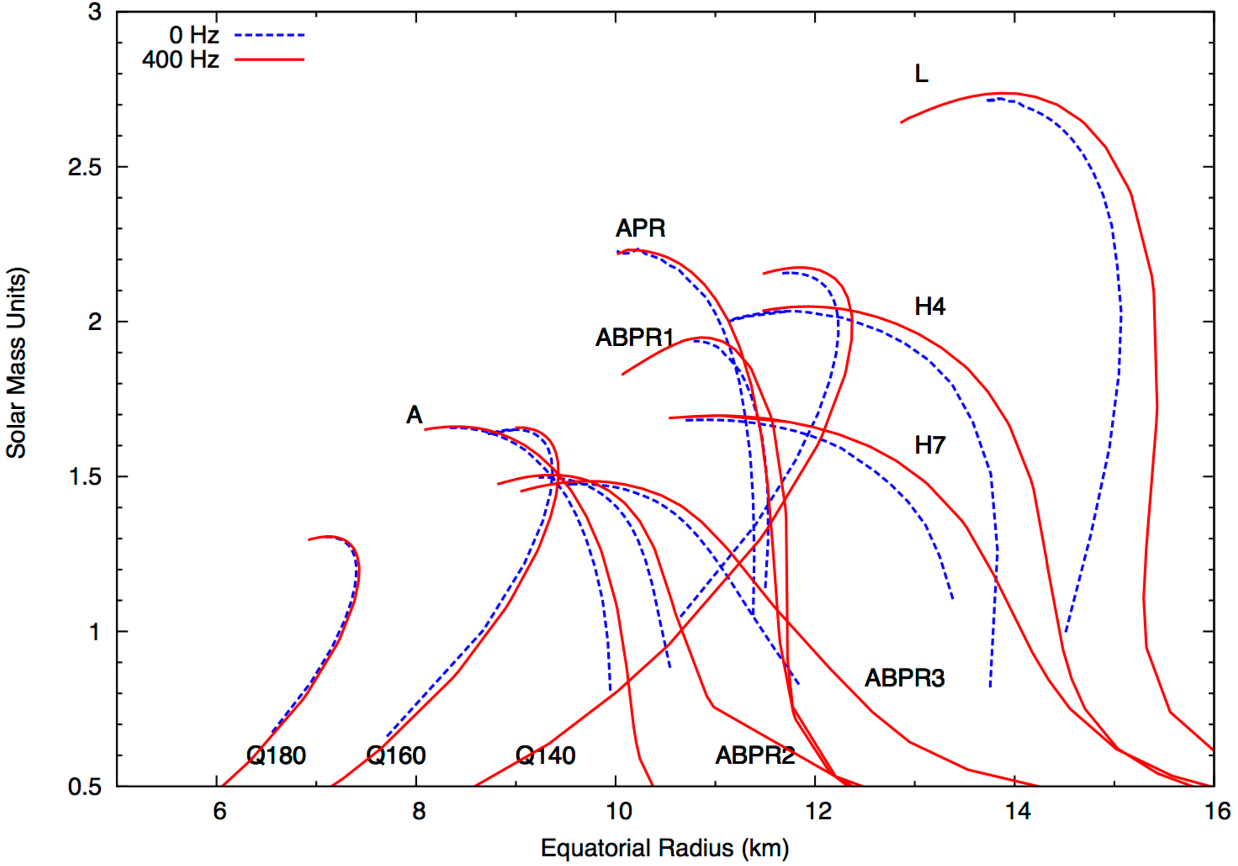
Summary

- Neutron stars can spin very fast! We have introduced a method to correct for spin for EOS inference.
- Caveats: crust could be treated better! Modern crust EOS, inclusion of solid matter in crust stress tensor should be investigated
- The corrections to mass and radius due to rotation are much smaller than NICER's measurement precision.
- New instruments with better precession, Athena, eXTP, Strobe-X will require better treatment of rotation even for quiescent systems.

Extra Slides

Mass – Radius Curves



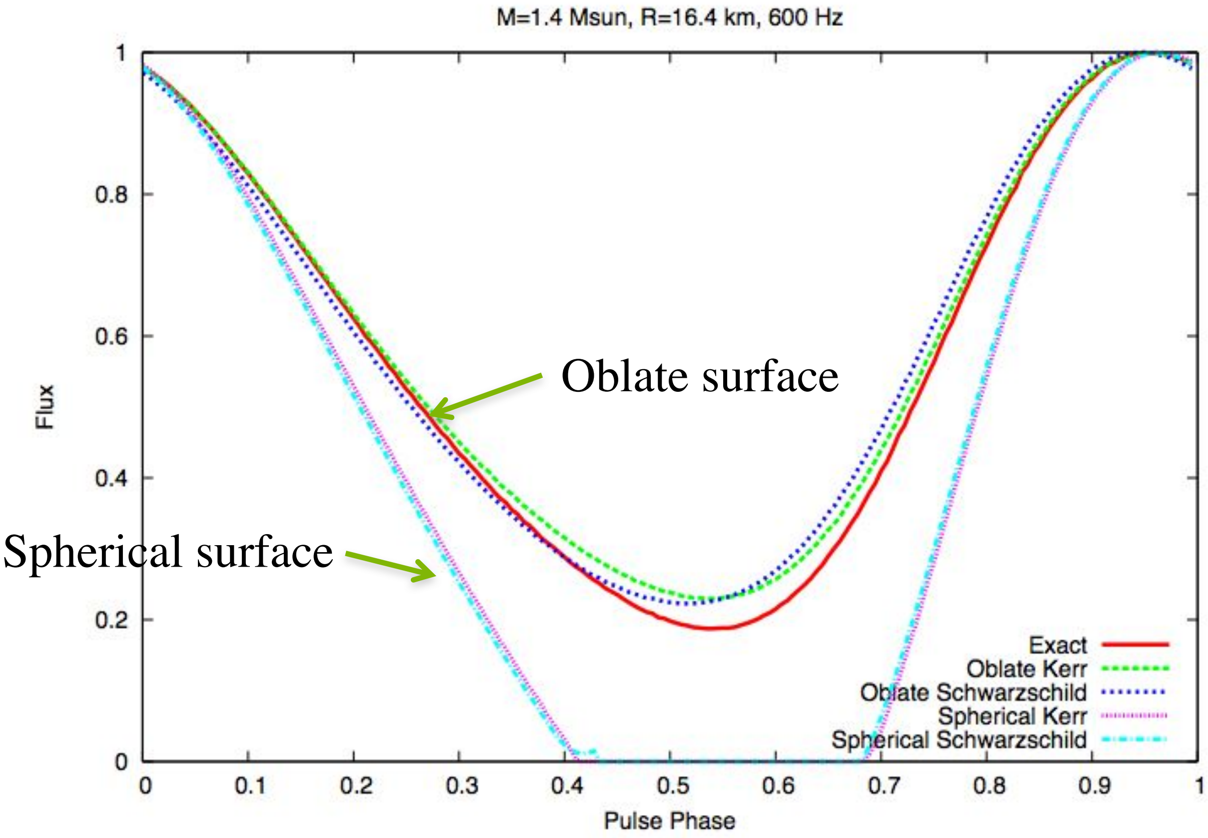


Metric for axisymmetric and stationary star

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}\left(dr^2 + r^2 d\theta^2\right) + e^{\gamma-\rho}r^2\sin^2\theta(d\phi - \omega dt)^2$$

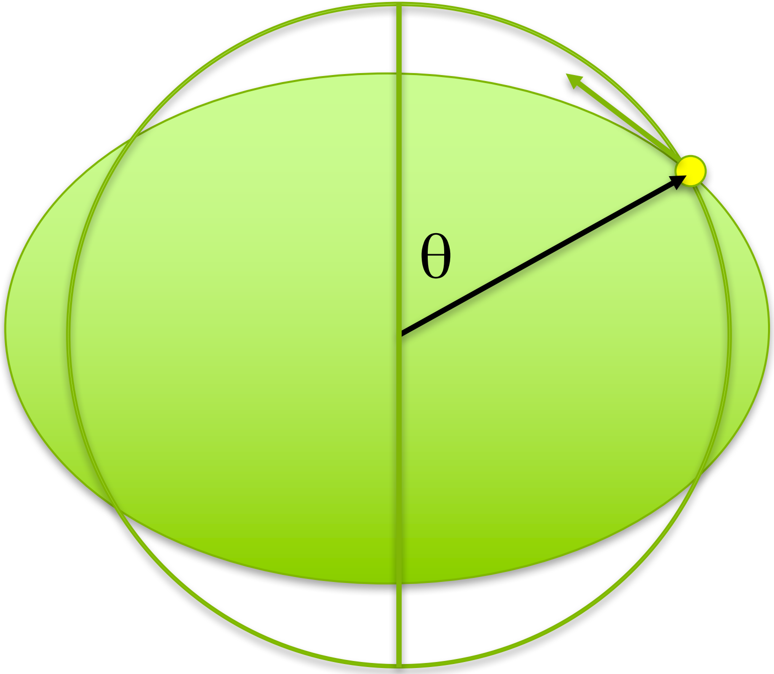
- Metric functions γ, ρ, α, ω are functions of r and θ
- Metric computed numerically on 2D grid using Green function method (rns code by Nikolaos Stergioulas, based on code by Greg Cook and method by Komatsu, Eriguchi and Hachisu, 1989)
- Raytracing on numerical background to construct pulse shapes to compare with approximations based on Schwarzschild and Kerr metrics. (Cadeau, Morsink, Leahy, & Campbell ApJ 2007)
- Test of Schwarzschild + Doppler Approximation

Spot at 15^{o} from North Pole Observer at 100^{o} from North Pole



(Cadeau, Morsink, Leahy, & Campbell ApJ 2007)

Stellar Oblateness



Morsink, Leahy, Cadeau & Braga 2007 ApJ

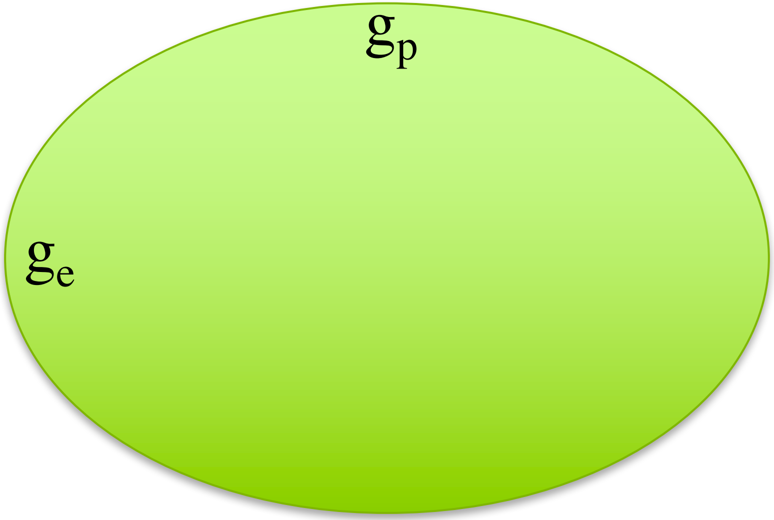
Effective Acceleration due to Gravity

- Simple EOS-independent formula for the variation of g on the surface of a rotating neutron star (AlGendy & Morsink, ApJ 2014)
- Depends only on the rotating star's M, R_e, Ω
- For realistic M, R, and a spin of 600 Hz,

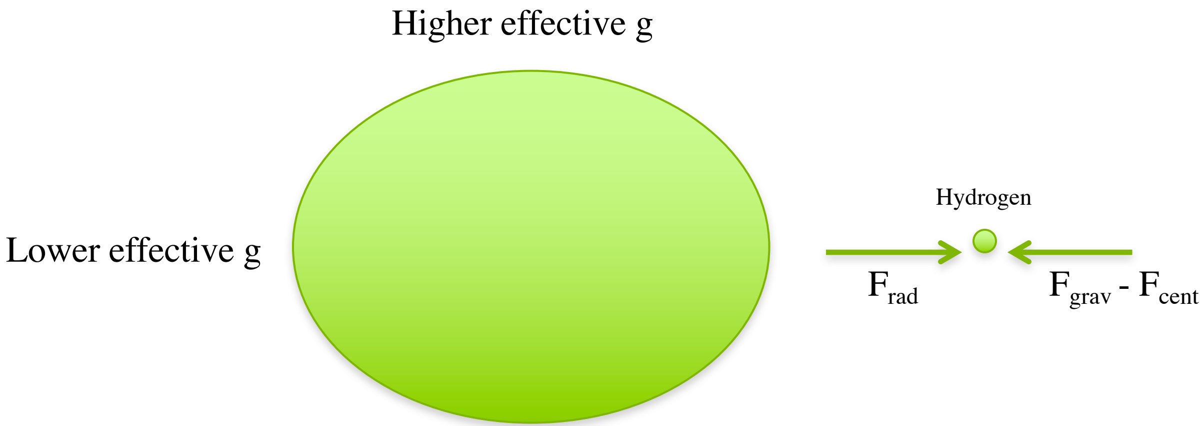
20% variation in the effective g over the surface

$g_p - g_e \sim 0.2\, g_e$

- Formula allows the use of atmosphere models that depend on g without adding any new parameters
- Used in NICER analysis, and in X-ray Burst cooling tail method (Suleimanov et al 2020)



Eddington Limit – Rotating Star

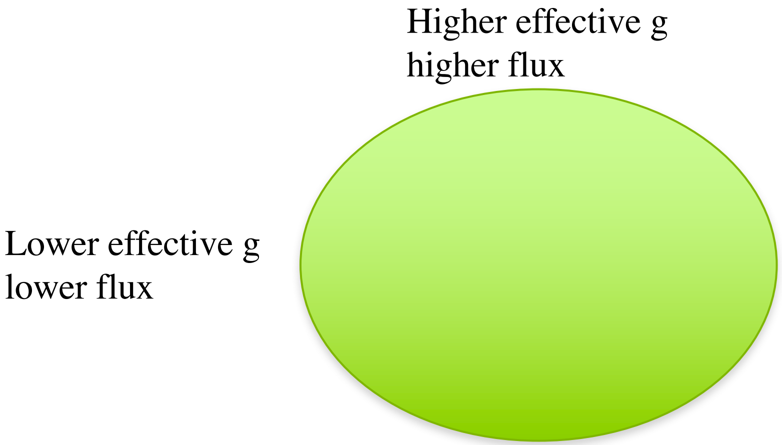


At Equator: $F_{Edd} = F_0\,(1\text{-}2M/R_e)^{1/2}\,(1 - a\,\Omega^2\,R^3/GM)\,f_L$

(1) $a > 0$ centrifugal reduction in effective surface gravity
(2) $f_L < 1$ Luminosity radius reduction factor (due to rotation)

For 600 Hz, both corrections reduce Eddington by up to 15%
(AlGendy & Morsink, ApJ 2014)

``Gravitational Darkening" Exponents

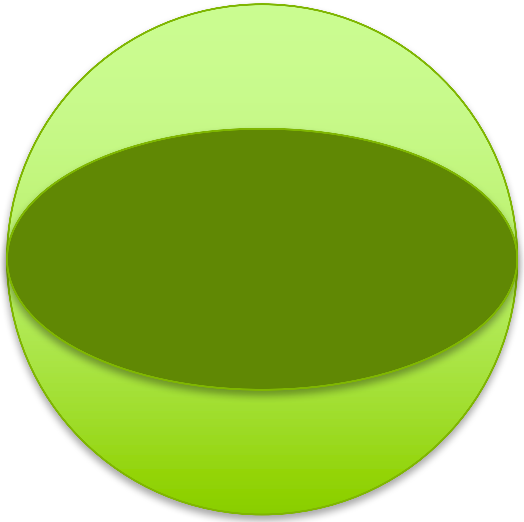


Von Zeipel's Law (1924): assuming radiative transport in the atmosphere and blackbody radiation: Flux(θ) ~ g(θ) [known to be different power of g if convective]

Generalization with realistic Hydrogen atmosphere & opacities could be useful for NS without climate change/weather (eg. NS in Quiescent Low Mass X-ray Binaries) in luminosity radius method.

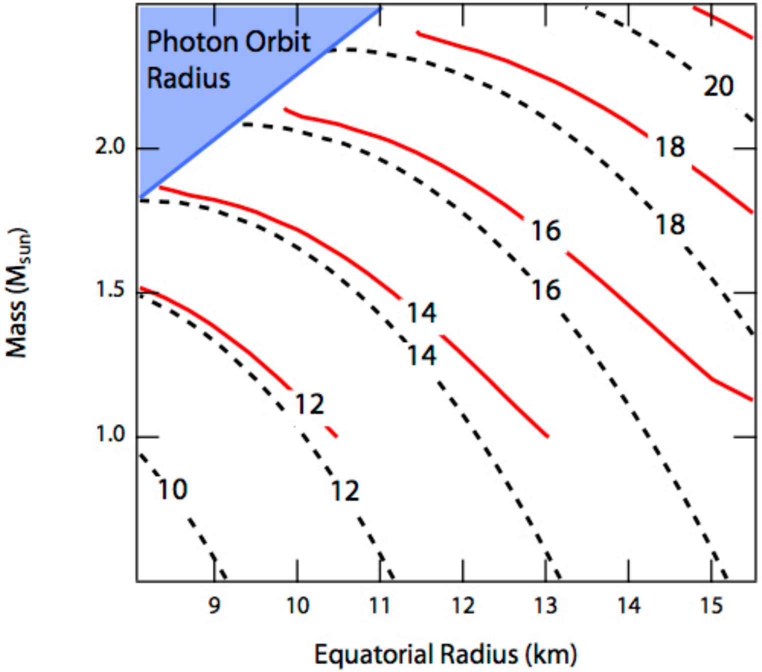
Application to X-ray bursts (Majczyna et al 2022)

Rotational Effect on Luminosity Radius



- An oblate star with the same equatorial radius as a spherical star has a smaller cross-sectional area A
- Flux ~ A so assuming a sphere underestimates the equatorial radius of the star

Luminosity Radius vs "Real" Radius



Baubock, Ozel, Psaltis, Morsink, ApJ 2015

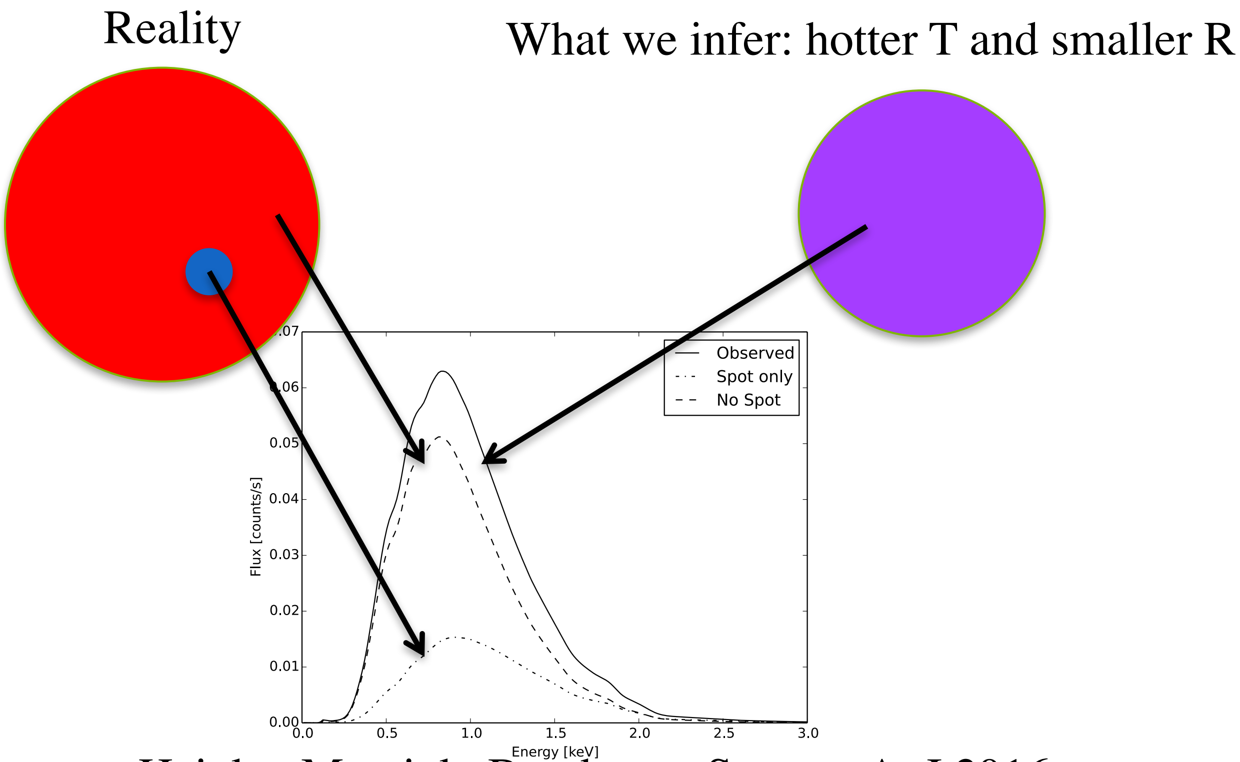
(Calculation is for pure blackbody, also includes Doppler boosting effects)

Assuming a spherical star could lead to underestimating the radius by 3-5%

-------- Luminosity Radius for zero spin $R_L = R(1-2M/R)^{-1/2}$

—— Luminosity Radius for spinning star (600 Hz)

What happens if we can’t tell that the star has a hot spot?



Elshamouty, Heinke, Morsink, Bogdanov, Stevens ApJ 2016