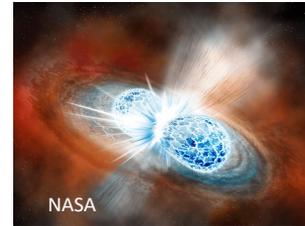
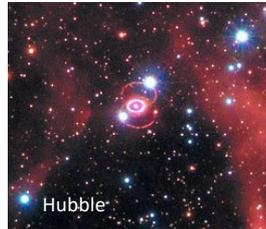


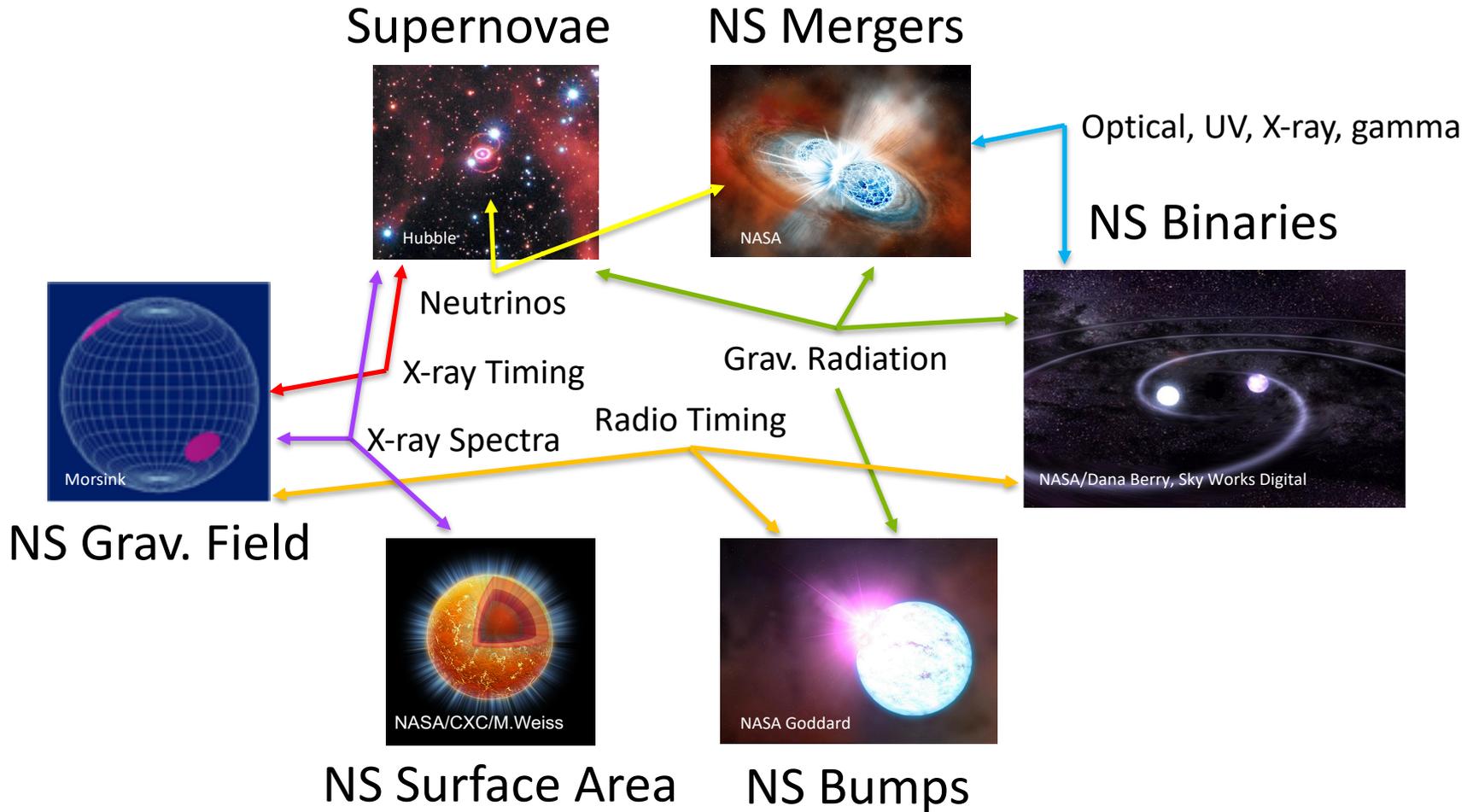
# Astrophysical Constraints on Neutron Star Masses and Radii



Sharon Morsink

Department of Physics, University of Alberta, Edmonton, Canada

# Types of Neutron Star Observations (Past, Present, Future)

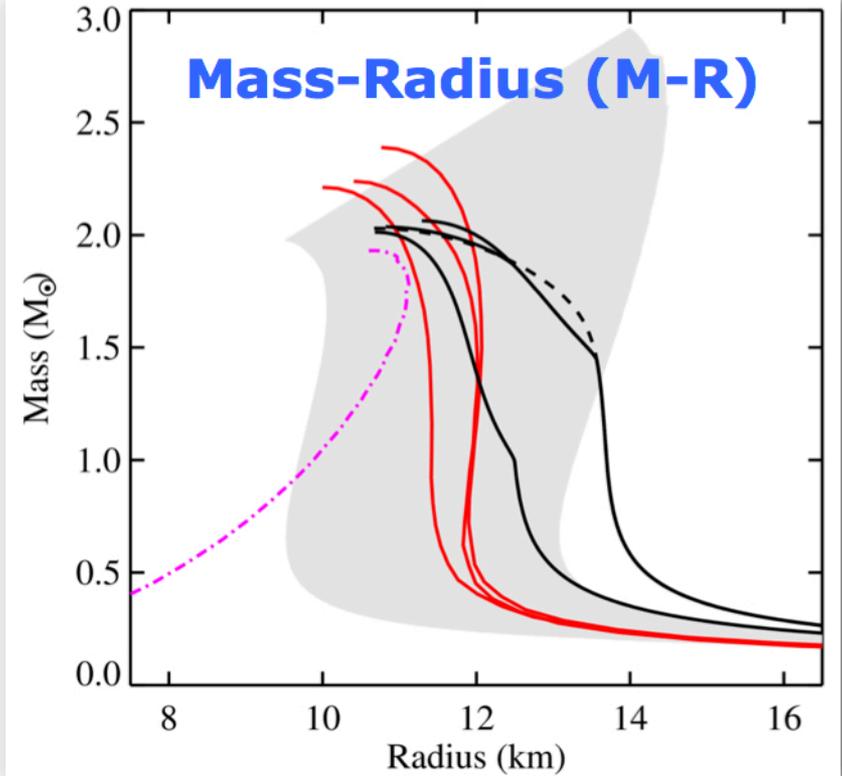
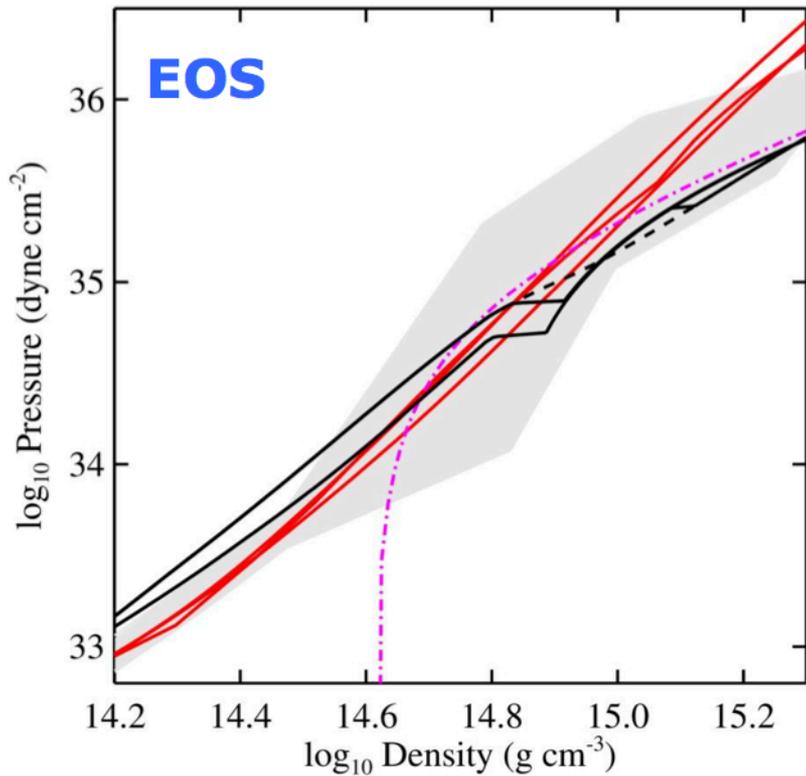




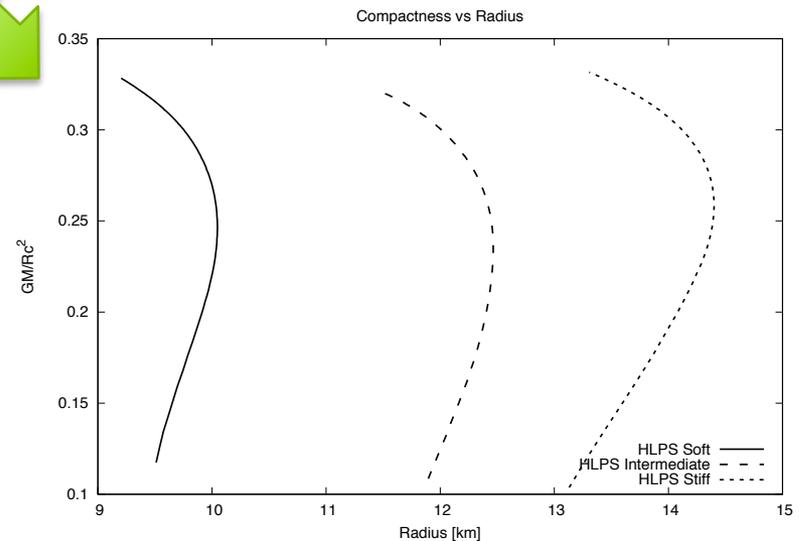
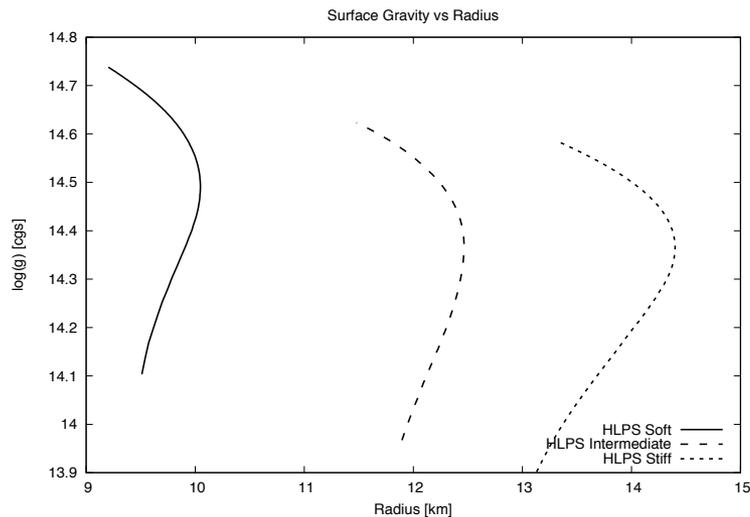
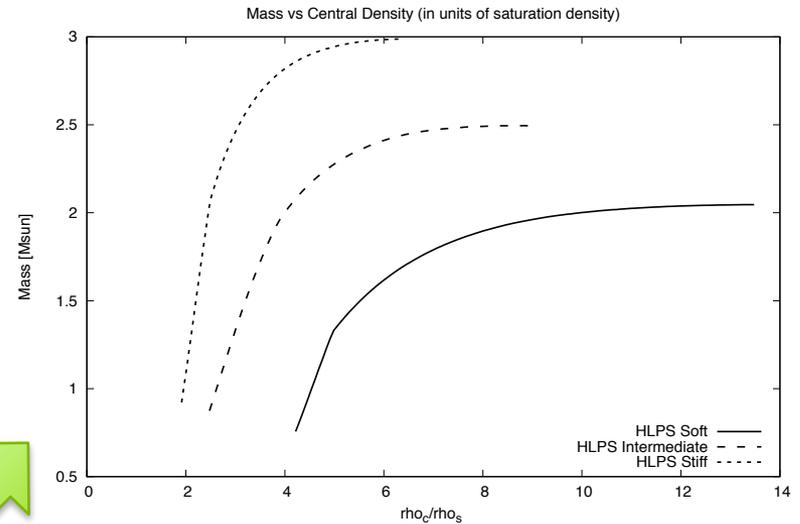
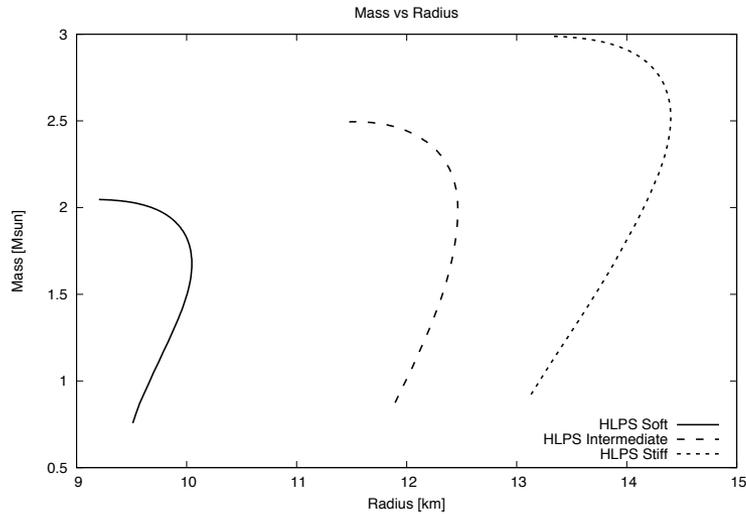
Mariner Conspiracy Board: Star Trek Lower Decks (CBS All Access)

# Equations of State vs Mass-Radius Curves

Stellar structure equations



# Equivalent Macroscopic NS Parameters: $M$ , $R$ , $\rho$ , $I$ , $\Lambda$ , $Q$ , $z$ , $g$ , ect...

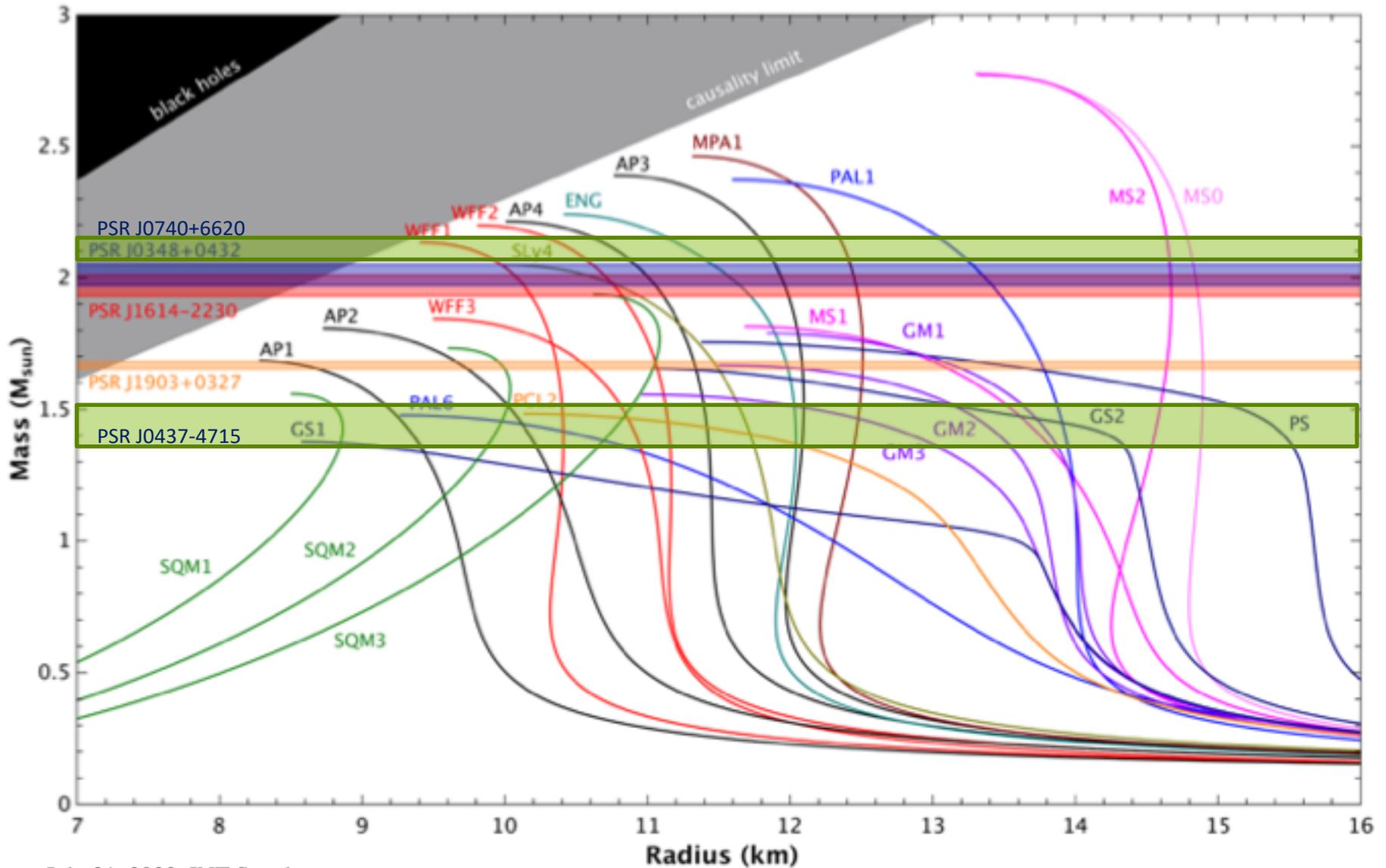


# What do we measure???

- We talk about “measuring” mass and radius to constrain the EOS
- But what do we mean by a measurement?
- We don’t really measure mass or radius – Instead we measure other quantities and use a physical model that depend on  $M$  and/or  $R$
- Models range from simple (just gravity) to complicated (atmosphere models; stellar astrophysics; transport properties; dynamical gravity; etc...)

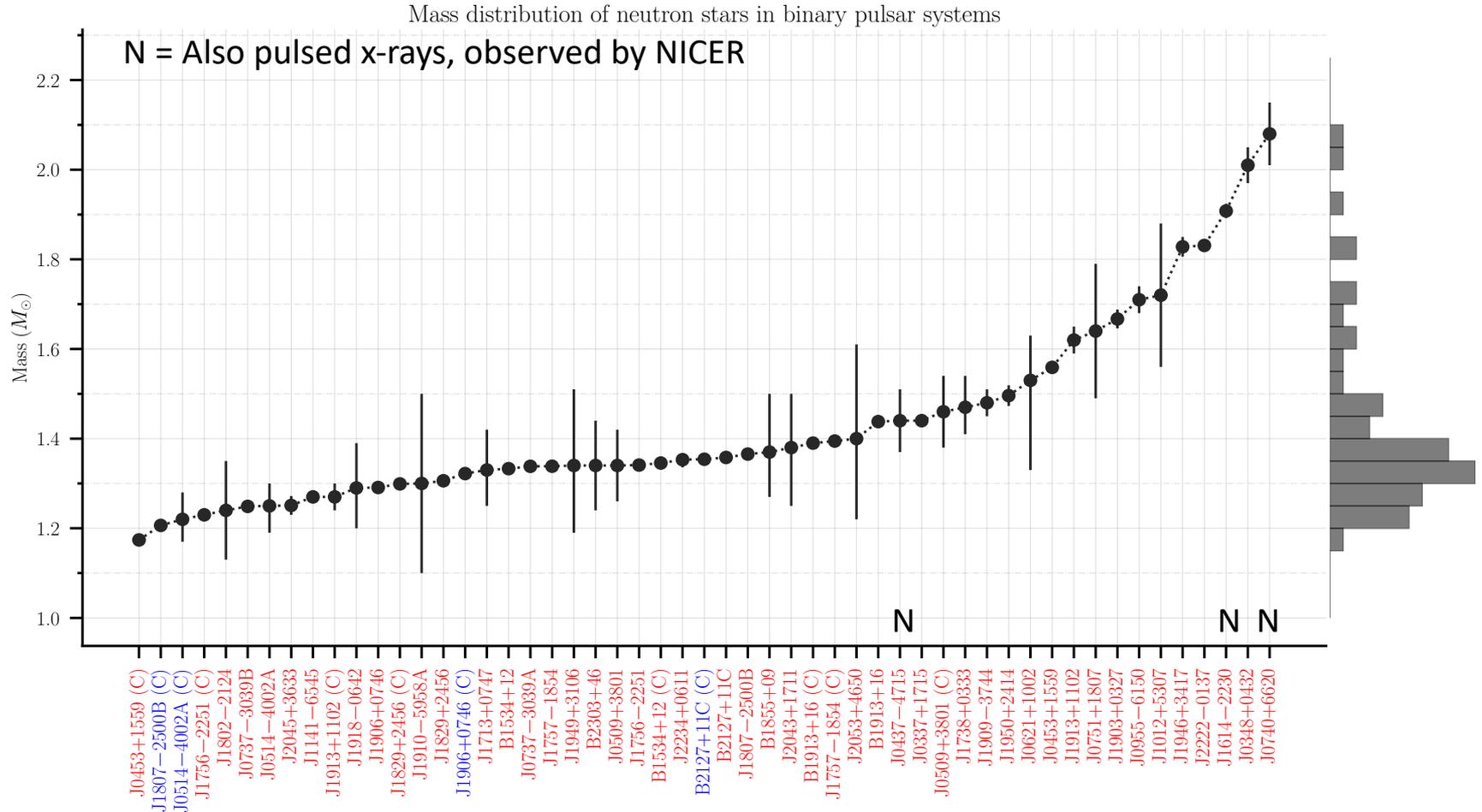
# **MASS MEASUREMENTS**

# The neutron star mass-radius relation



# Mass Measurements from Radio Pulse Timing:

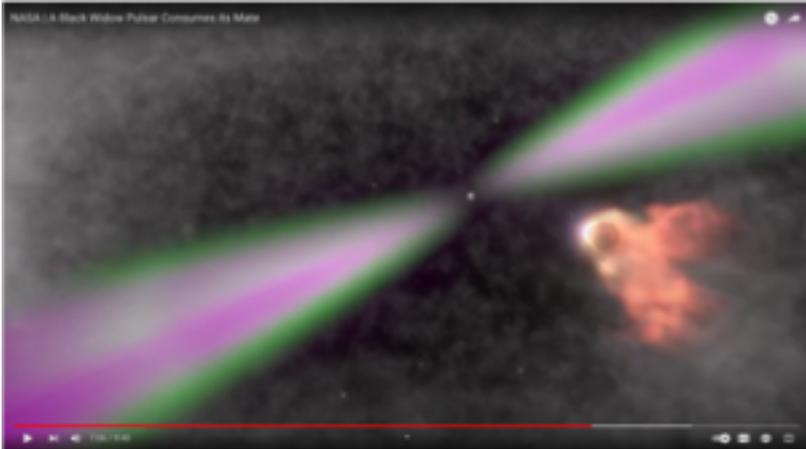
Gravity – Kepler; Periastron Precession; GW Orbital Decay; Shapiro Delay  
Astro Uncertainty - galactic dynamics (small); Non-accreting systems!



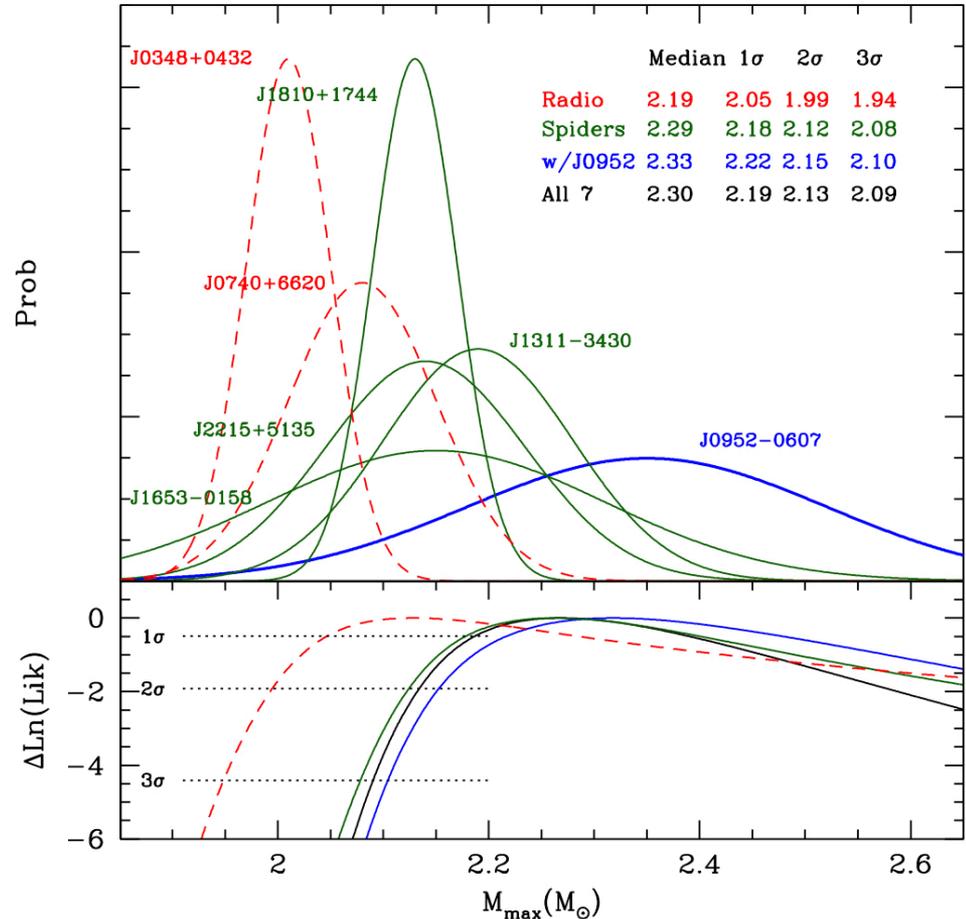
Vivek V. Krishnan and Paulo Freire

[https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS\\_masses.html](https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html)

# “Spider” Binaries (Black Widows and Redbacks)



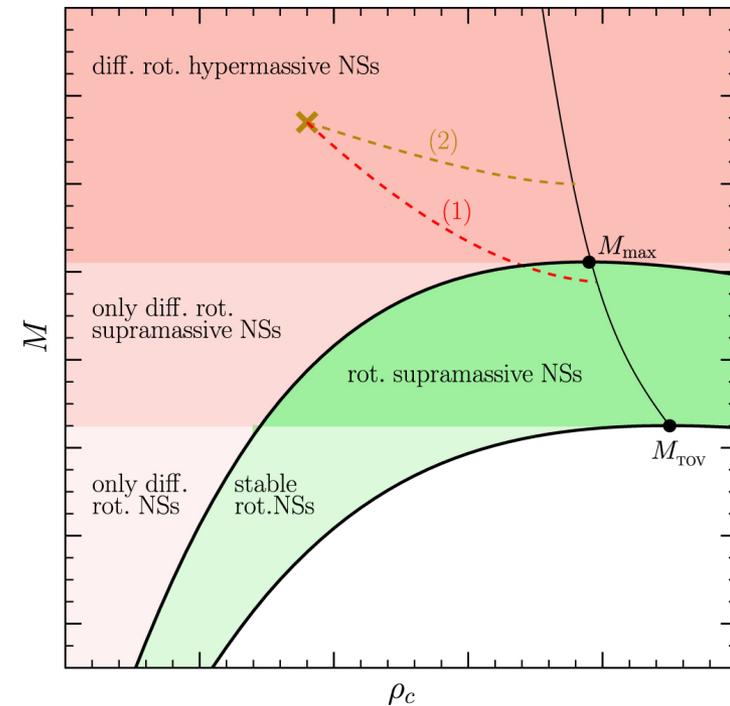
- Pulsar heats up and slowly eats its companion.
- Observe companion light curve (optical light)
- Model heating and distortion of companion to infer the masses
- PSR J0952-0607,  $M = 2.35 M_{\text{sun}}$
- Romani+ astro-ph/2207.05124



# Maximum NS Mass from GW170821/Kilonova

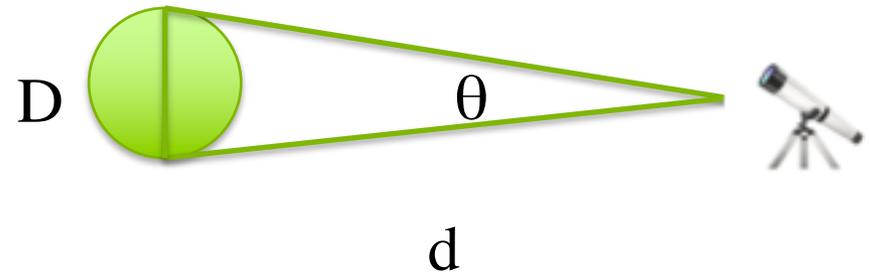
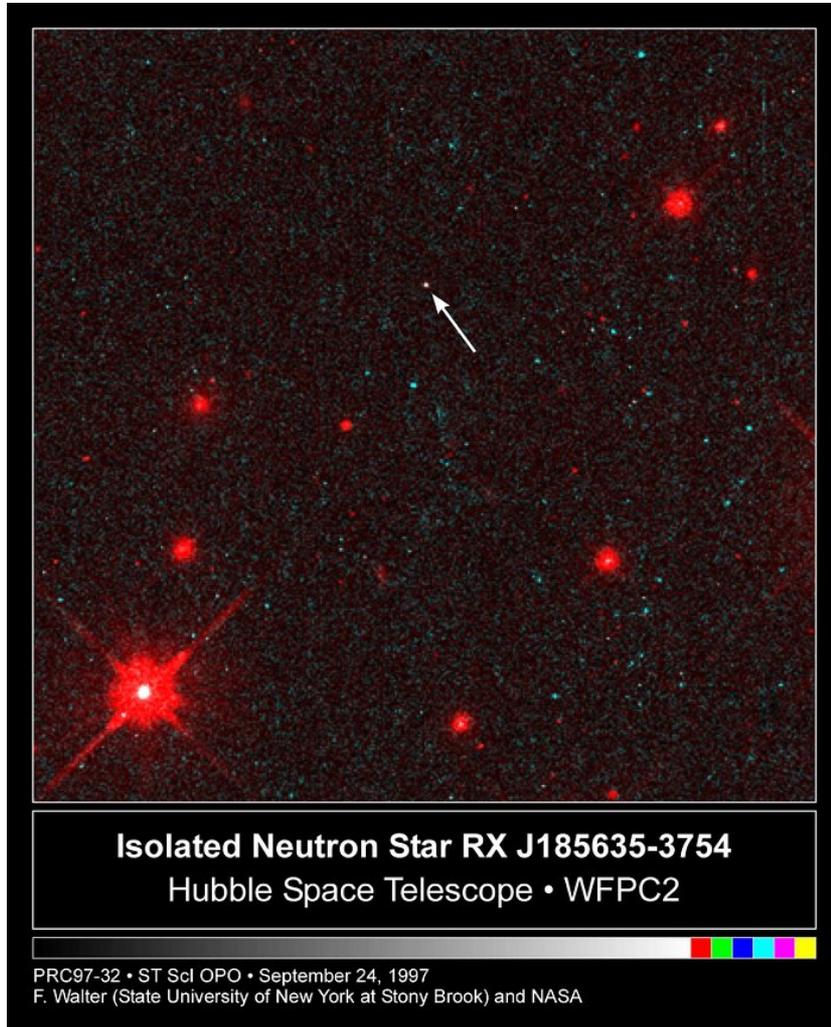
- The two merging NSs in GW170821 have a combined mass of  $2.74^{+0.04}_{-0.01}$  Msun
- Rezzolla+2018ApJL:
  - If remnant first creates hypermassive NS, then collapses to BH
  - Kilonova obs with assumptions about ejected mass leads to maximum NS mass  $M_{\text{TOV}} < 2.16 + 0.17 - 0.15$  Msun
  - An example of a very model-dependent limit to the maximum NS mass

(Similar types of arguments by others, eg Margalit & Metzger ApJL 2017)



# RADIUS MEASUREMENTS

# Hubble's Best NS Photo



Direct geometrical measurement:  
 $D = \theta d$  not possible!

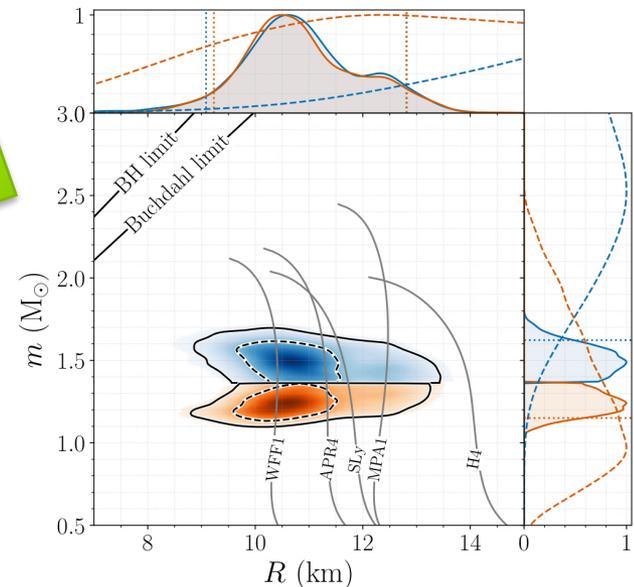
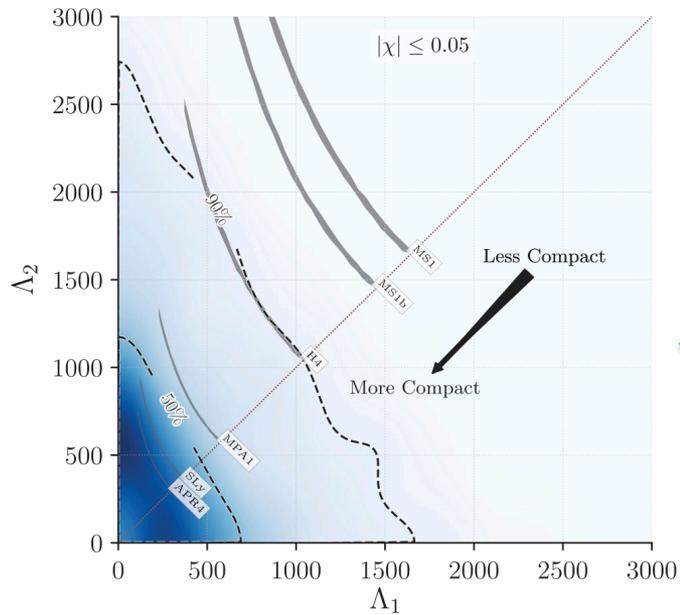
# FINITE SIZE EFFECTS IN BINARIES

# NS Finite Size Effects in NS-NS Inspiral (GW measurements)

- NS is not a point particle!
- Orbital energy during binary inspiral is “wasted” in tidal deformations or exciting oscillation modes
- Compare GW phase evolution with point particle predictions to find the NS finite size
- Leads to limits on Tidal Deformability,  $\Lambda$



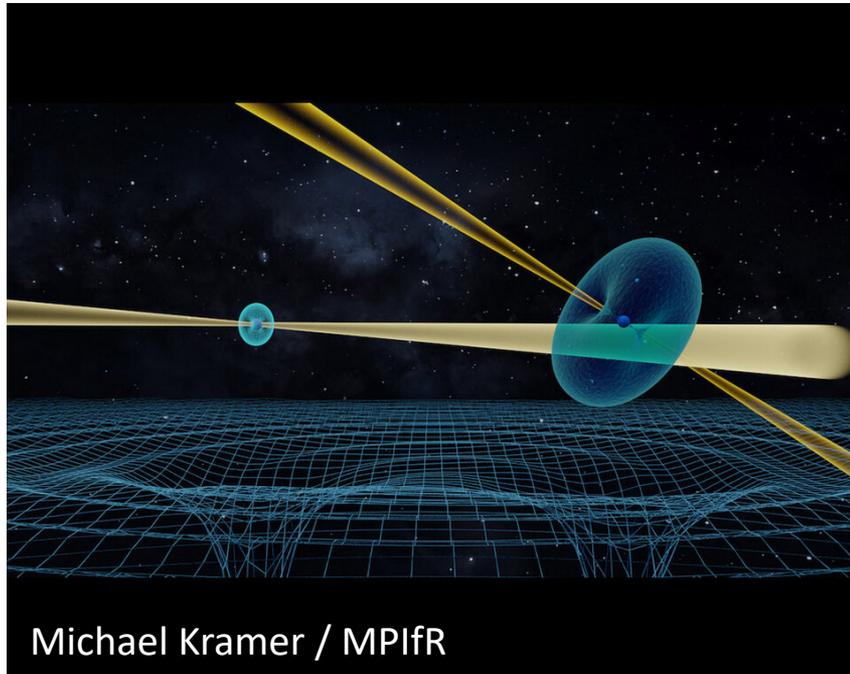
# Love Number related to NS Radius



LIGO observations “prefer” smaller NS radii!

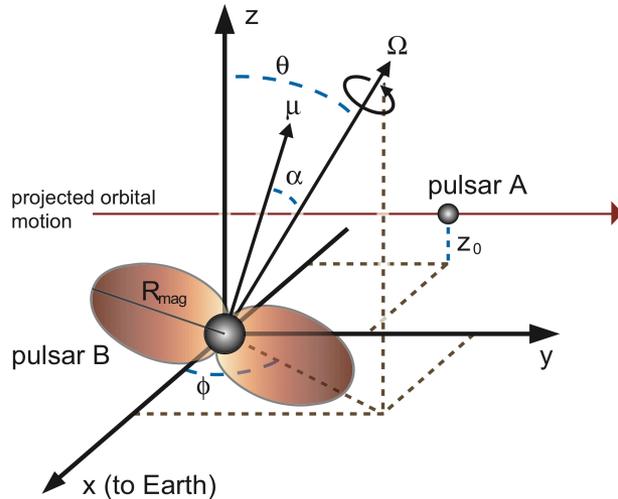
# MOMENT OF INERTIA

# Moment of Inertia – The Double Pulsar J0737-3039

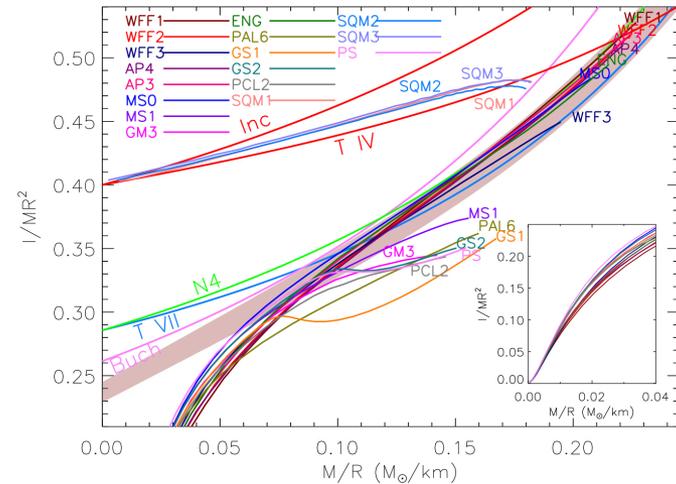


- Discovered in 2003
- Orbital Period = 2.45 h
- Overconstrained system perfect for testing general relativity
- GW orbital period decay; periastron precession, Shapiro delay, etc...
- Almost 20 years of data!

# Spin-Orbit Coupling & Precession



Breton+ Science2008 Fig. 1

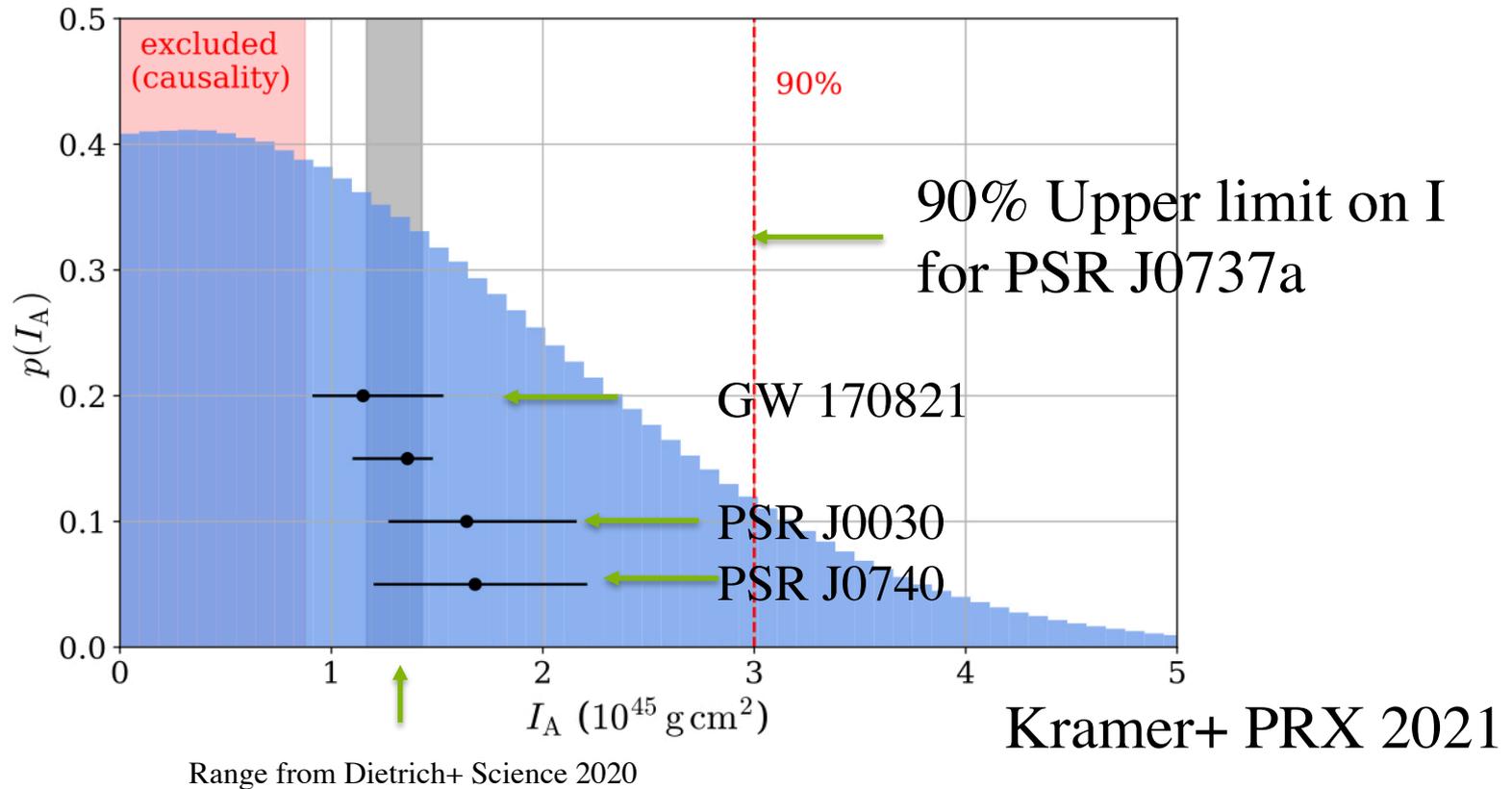


Lattimer & Schutz ApJ 2005

- Precession of the Pulsar Spin around the orbital angular momentum vector depends on the moment of inertia
- Also contributes a small correction to the periastron precession!

# Limits on Moment of Inertia

Not yet constraining, but will improve with time...



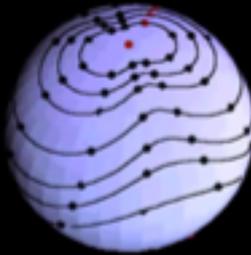
# OSCILLATION MODES

# Oscillation Modes?

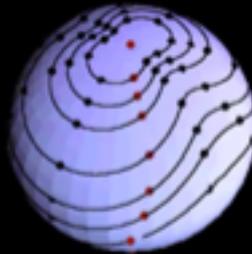
## Ian Jones' Talk

### Instability: the CFS mechanism

GW emission can drive the  $r$ -mode unstable, a two-stream instability.



View from the inertial frame.



View from the corotating frame

Movie credit: Hanna & Owen

24



Institute For Nuclear Theory

# Suppose that you could detect gravitational radiation from an “r-mode”

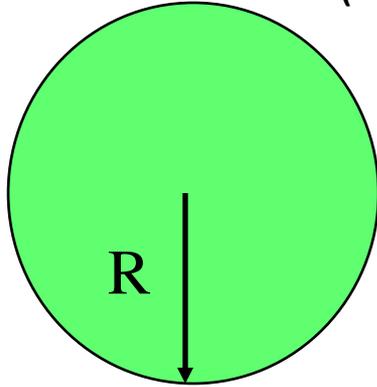
- Oscillation frequency  $f$
- $f = \frac{2}{3} f_{\text{spin}} + O(M/R)$
- If you know the spin frequency of the NS (since it’s a pulsar, for example) and you measure the GW frequency the relativistic correction will be measured! This could place limits on the NS’s  $M/R$
- Also interesting for understanding transport properties

# LUMINOSITY RADIUS

# Surface Area Measurements

(Recall Sebastian Guillot's Talk last week)

T



d



In Newtonian physics, if  $d$ , temperature ( $T$ ) and flux ( $F$ ) are measured then the “Luminosity Radius” is given by:

$$R_L = \frac{d}{T^2} \sqrt{\frac{F}{\sigma}}$$

In General Relativity, the gravitational redshift Effect must be corrected for, so the actual radius is:

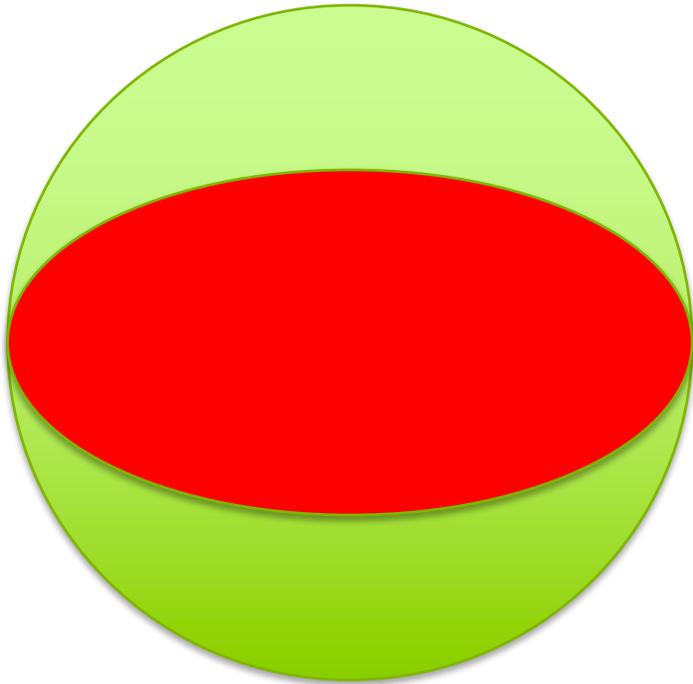
$$R = R_L \sqrt{1 - \frac{2M}{R}}$$

- Real NS aren't pure blackbodies, requires more complicated spectrum

# Luminosity Radius Measurements

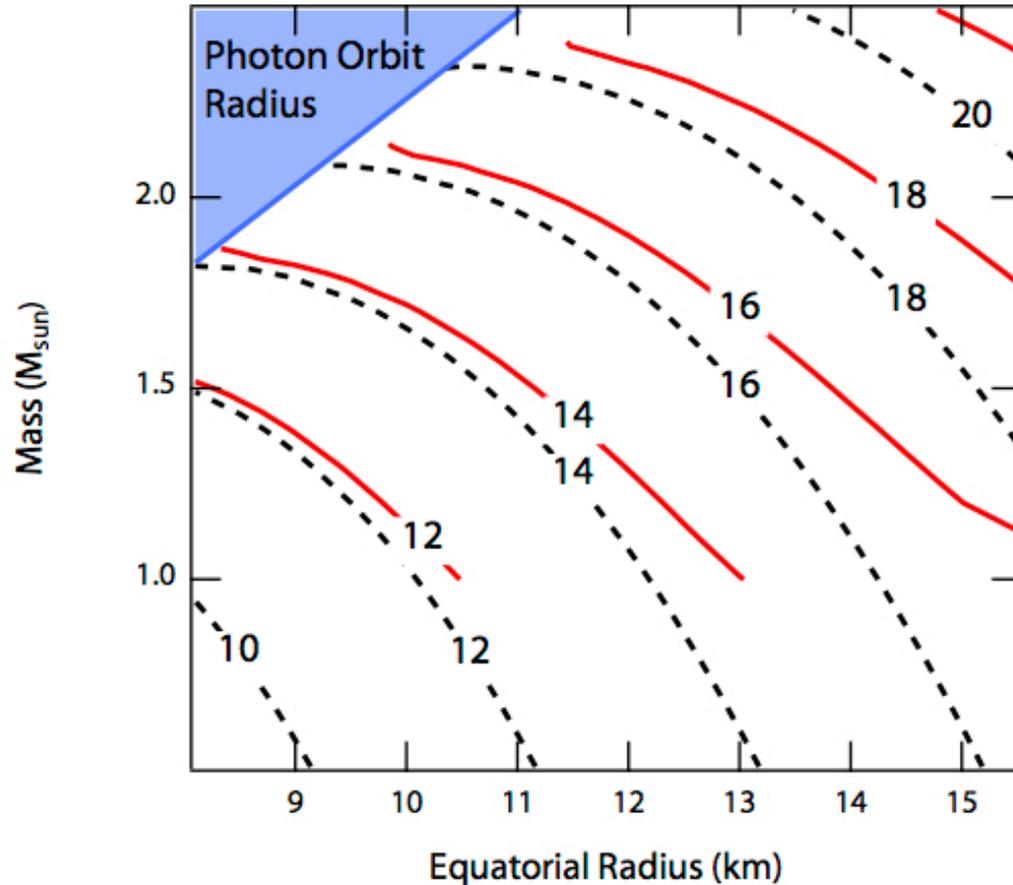
- Quiescent Low Mass X-ray Binaries (qLMXBs)  
NS in quiet periods between accretion events
  - Long observations (if you can get the time) since steady-state
- Thermonuclear X-ray bursts and “cooling tails”
  - Short lived nuclear explosion; evolves with time
  - During the cooling tail the atmosphere cools, while the surface area stays constant allowing for extraction of the surface area

# Rotational Effect on Luminosity Radius



- An oblate star with the same equatorial radius as a spherical star has a smaller area  $A$
- Flux  $\sim A$  so assuming a sphere underestimates the equatorial radius of the star

# Luminosity Radius vs “Real” Radius



Baubock, Ozel, Psaltis,  
Morsink, ApJ 2015

(Calculation is for pure  
blackbody, also includes  
Doppler boosting effects)

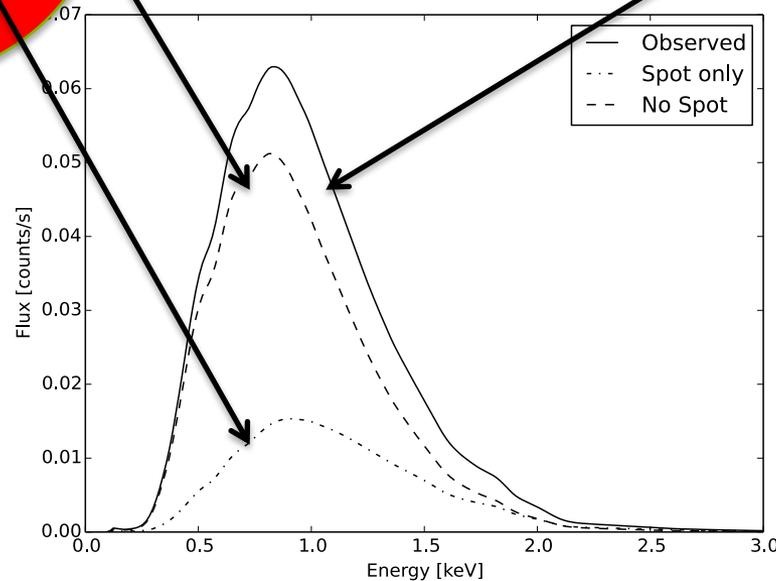
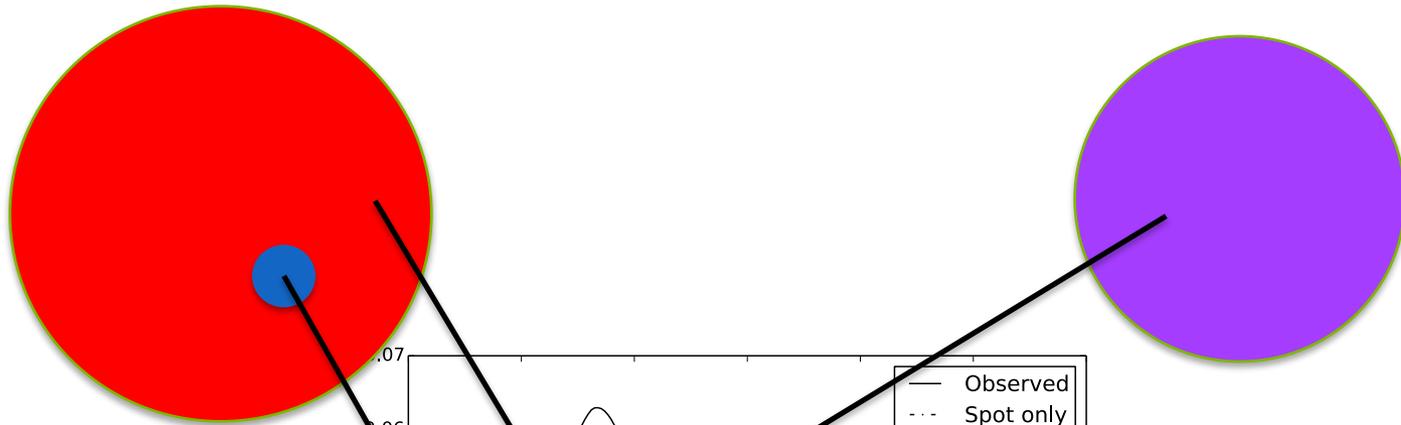
Assuming a spherical star  
could lead to  
underestimating the radius  
by 3-5%

- Luminosity Radius for zero spin  $R_L = R(1-2M/R)^{-1/2}$
- Luminosity Radius for spinning star (600 Hz)

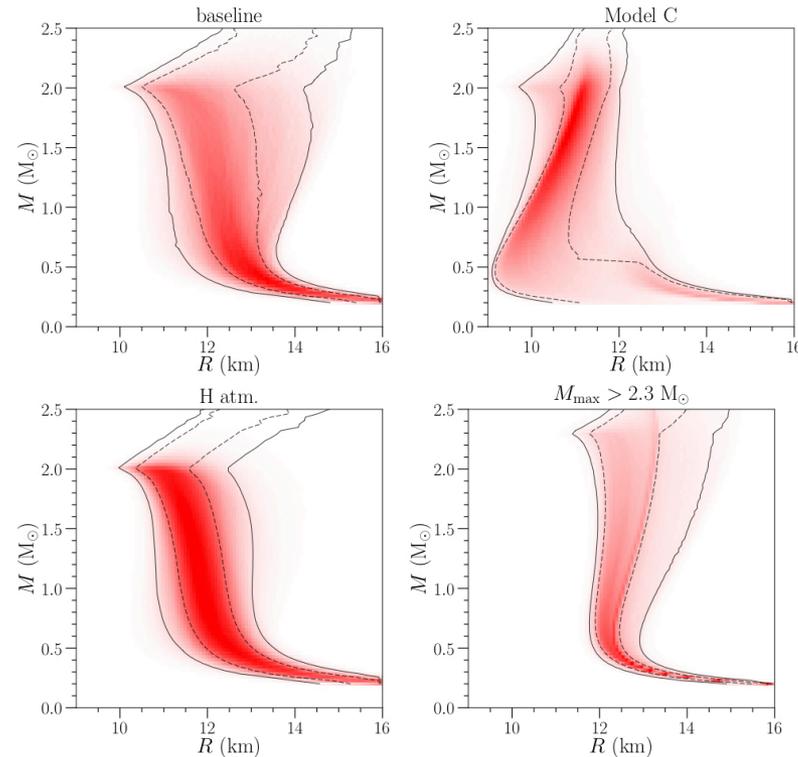
# What happens if we can't tell that the star has a hot spot?

Reality

What we infer: hotter T and smaller R



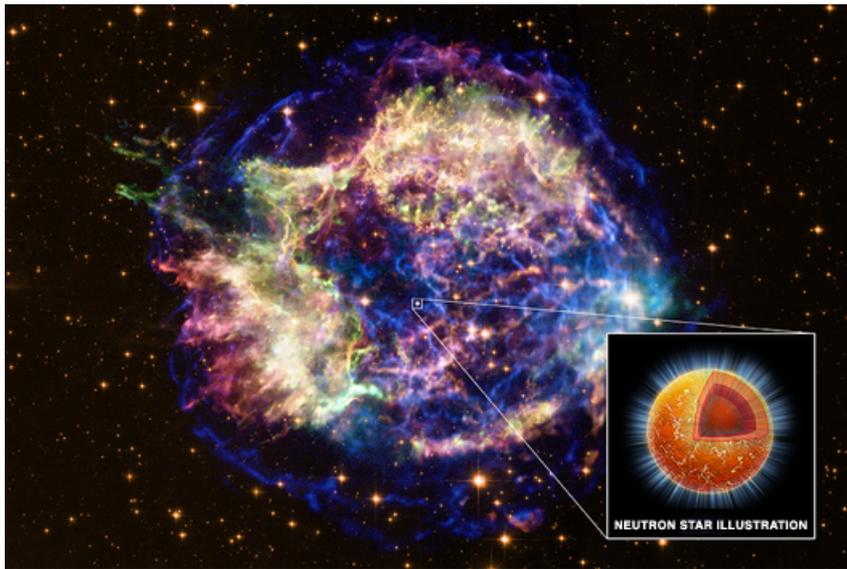
# qLMXB Mass-Radius Constraints from 7 Globular Cluster NSs



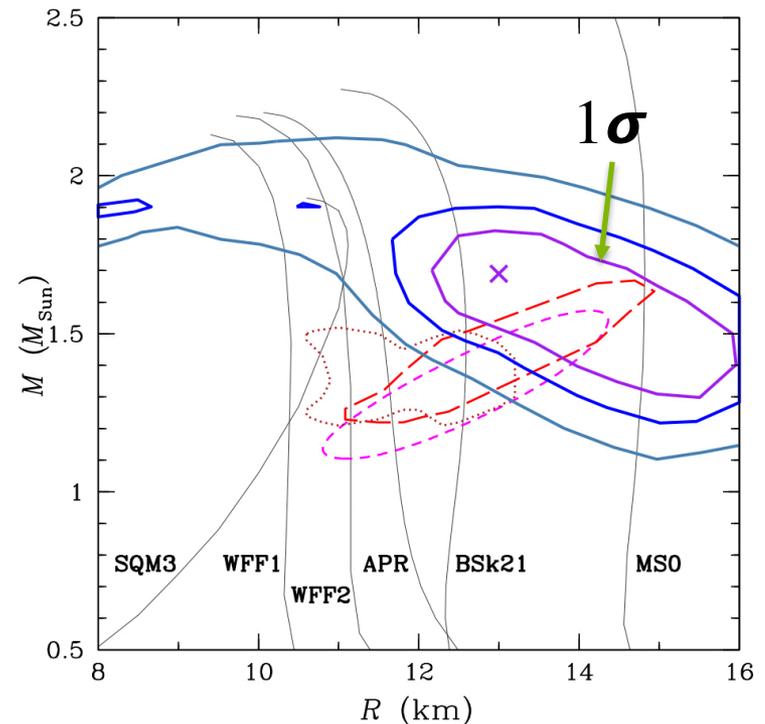
Radius of 1.4 Msun NS lies in range 10 - 14 km

Steiner, Heinke, Bogdanov, Li, Ho, Bahramian, Han MNRAS 2018

# Application to NS in Cas A Supernova Remnant



19 years of Chandra + XMM data;  
C atmosphere; cooling



Ho, Zhao, Heinke, Kaplan, Shternin, Wijngaarden MNRAS 2021

# Core Collapse Supernova?



It's 35 years since SN 1987a!

A believable EM NS detection would be nice!

Surely it's time for another galactic core-collapse SN!

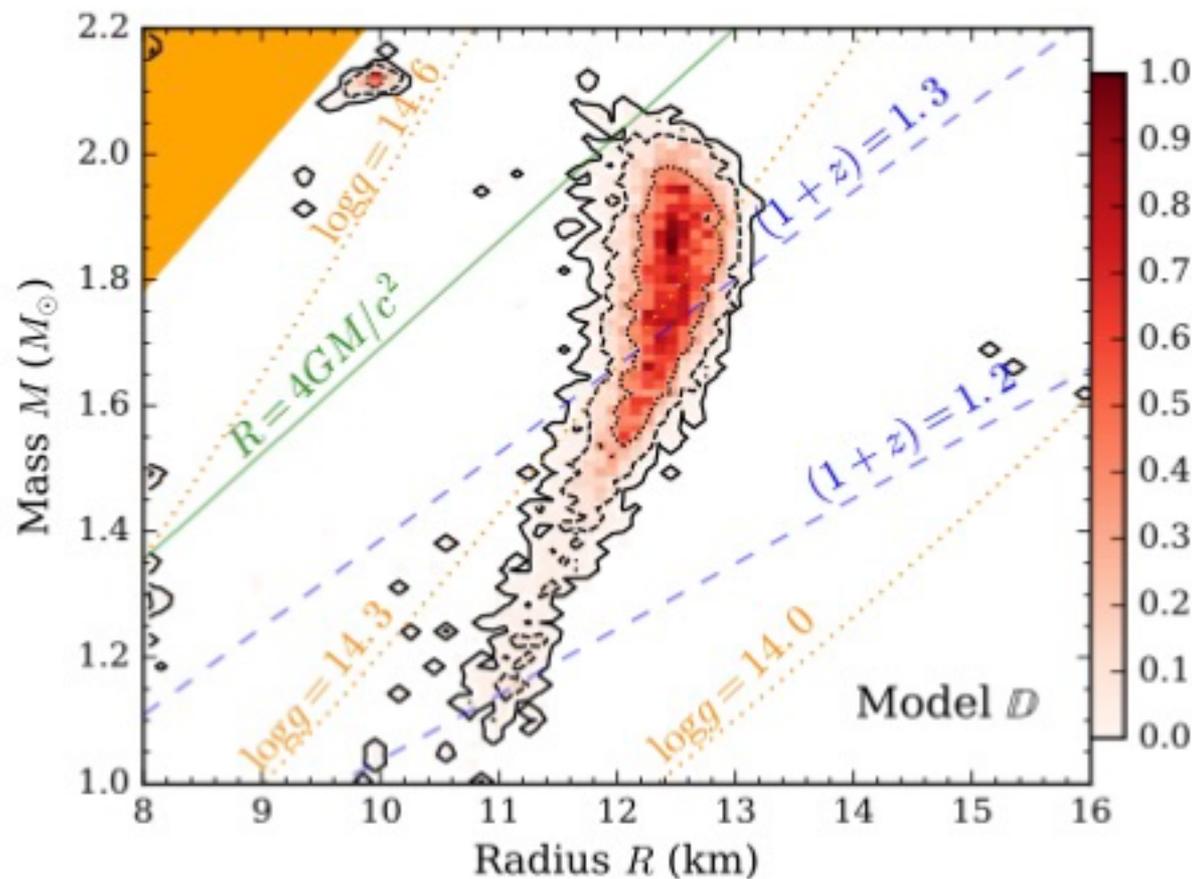
With GW detectors and sensitive neutrino detectors we're ready!

# **LUMINOSITY RADIUS PART II**

## **– X-RAY BURSTS**

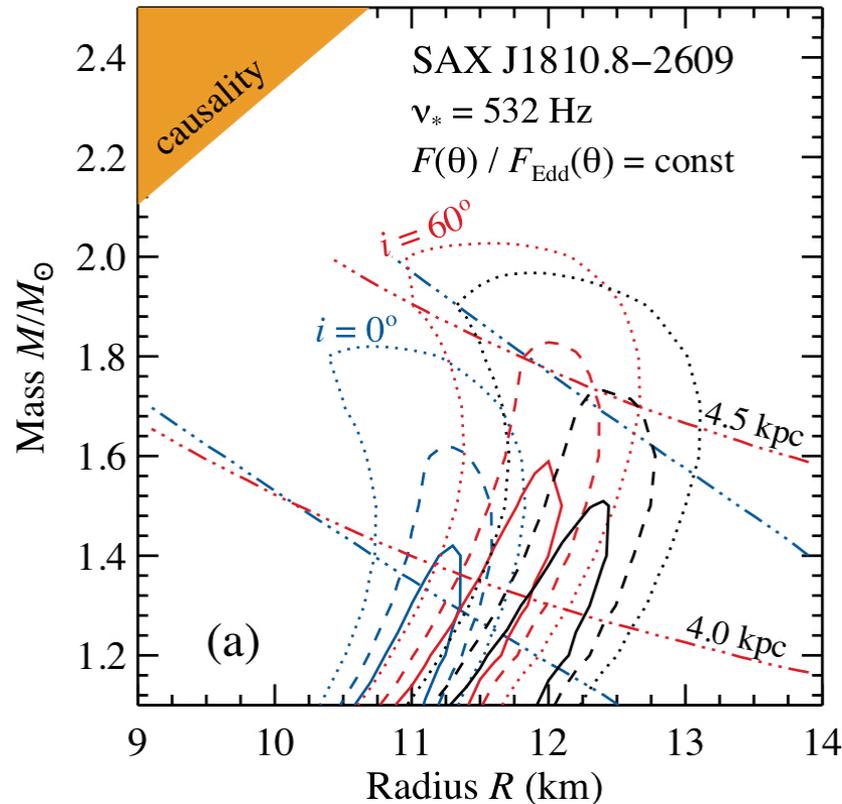
# Eg. Cooling Tail Method

## X-ray bursts from 4U 1702-429



Natilla, Miller, Steiner, Kajava, Suleimanov, Poutanen, 2017 A&A  
Star spins at 330 Hz

# Rotational Effects on Cooling Tail Method



Unknown inclination angle means that rotation increases the range of probable radii by a couple km.

Suleimanov, Poutanen, Werner, A&A 2020

# Upcoming X-ray Spectroscopy and Imaging Telescopes



JAXA/NASA/ESA collaboration; Launch date in 2023



Athena ESA/NASA;  
Launch date 2035

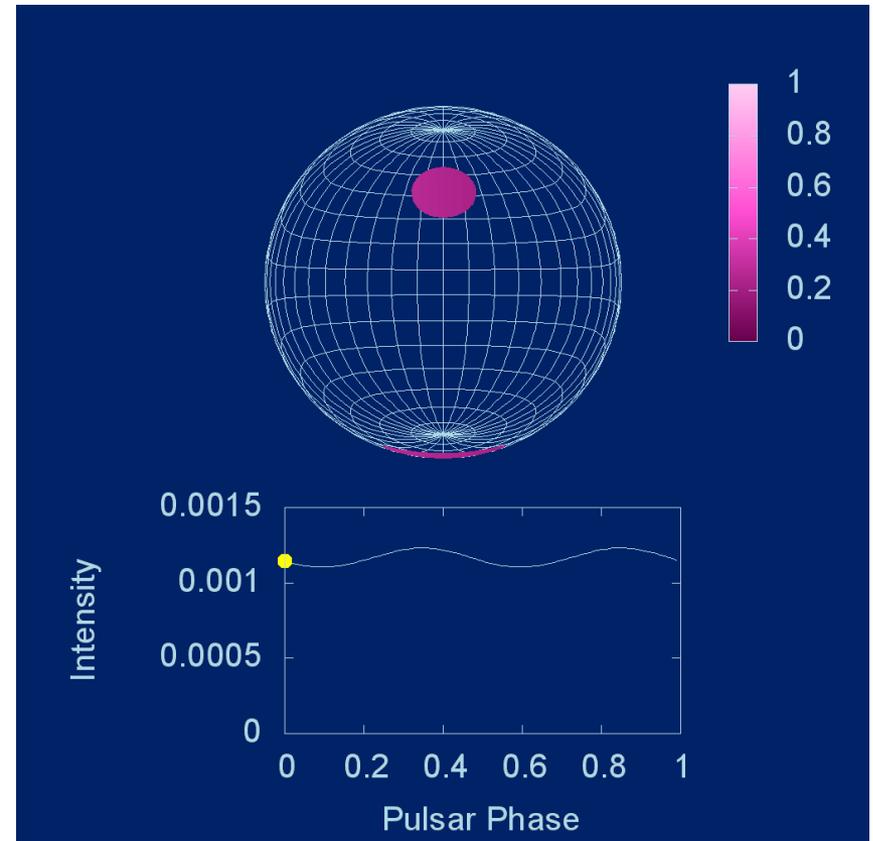
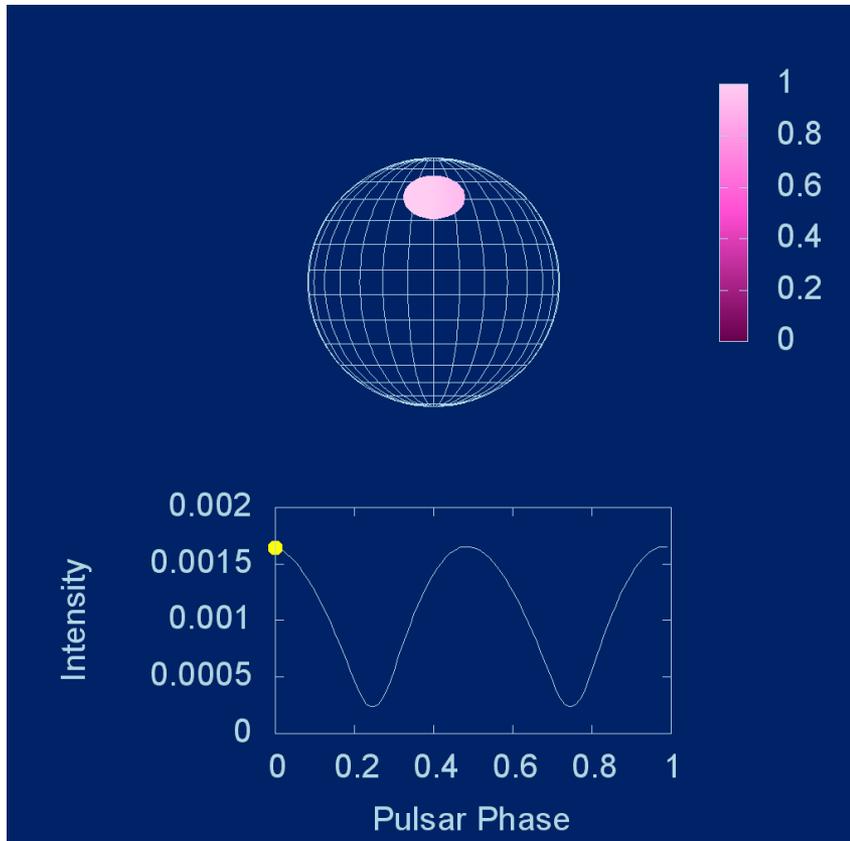
# **PULSE PROFILE MODELLING**

# Pulse Profile Modelling

- Hot spots on rotating neutron star
- X-rays feel the NSs gravitational field while travelling from star to telescope
- Gravitational Potential  $\sim M/R$  causes light to travel on curved path
- Doppler Boosting  $\sim \Omega R/c$  adds timing asymmetry and harmonics

# Dependence of Pulse Profiles on M/R

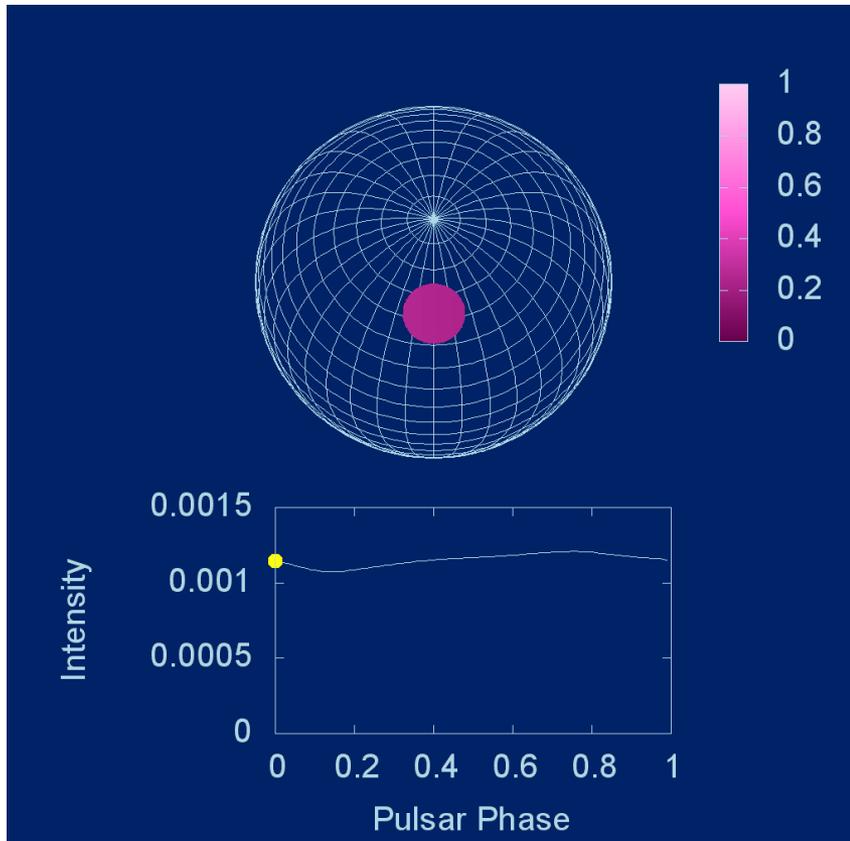
M/R = “compactness” affects light-bending  
Larger M/R  $\rightarrow$  less modulation



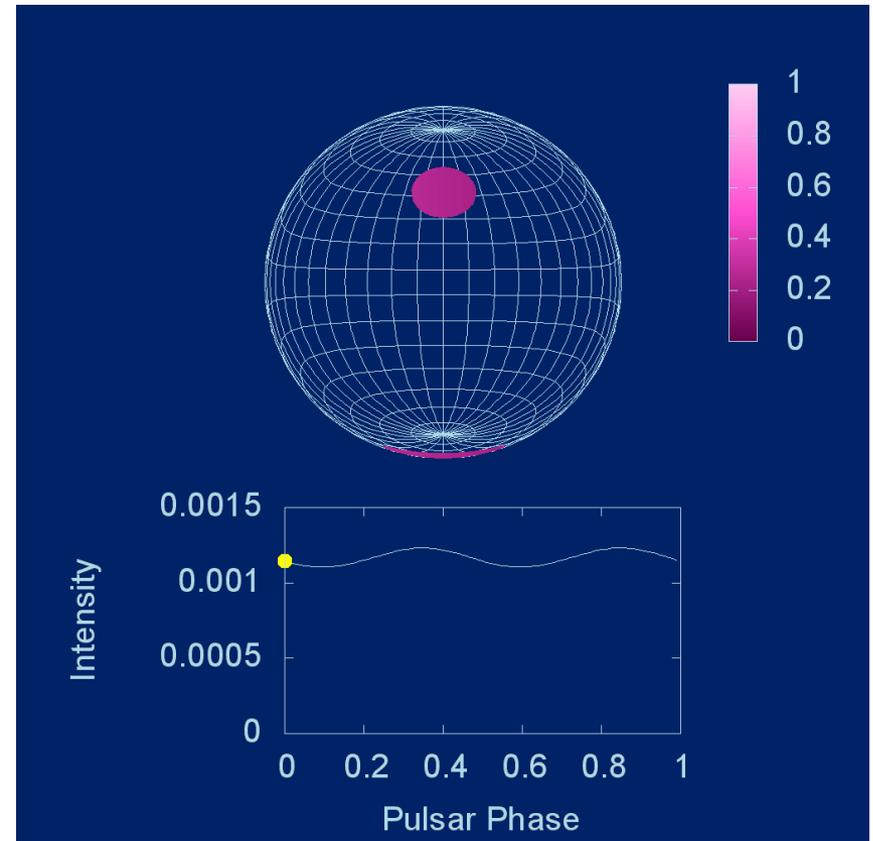
Newtonian Gravity

General Relativity M/R = 0.25

# Effect of Observer's Viewing Angle

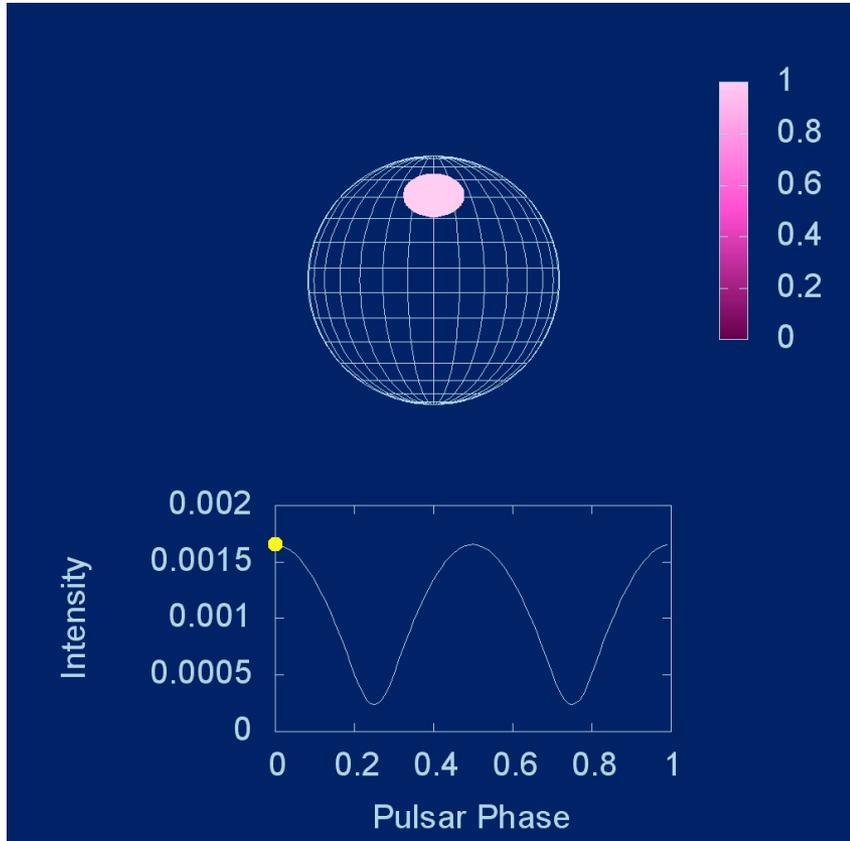


30 deg from Spin Axis

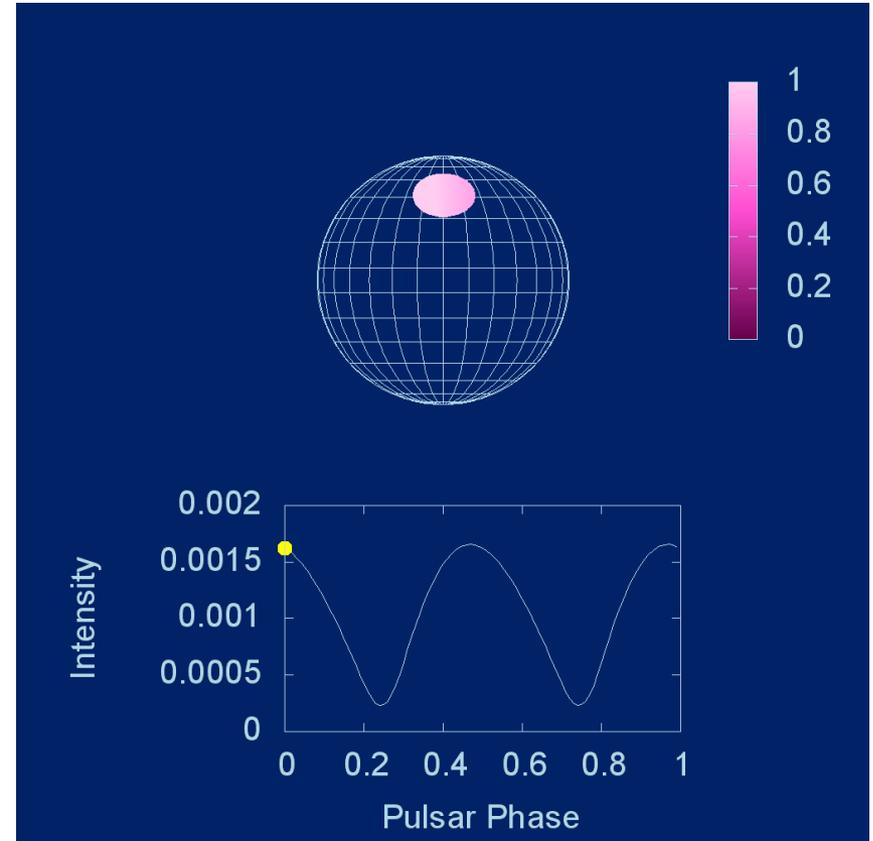


90 deg from Spin Axis

# Effect of Rotational Speed $\propto R \sin i \sin \theta$



$v/c = 0.01$



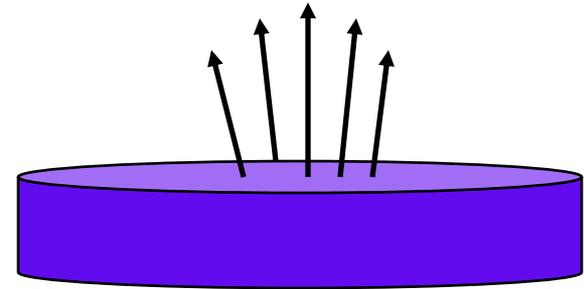
$v/c = 0.2$  (harmonics)

# Anisotropic Emission

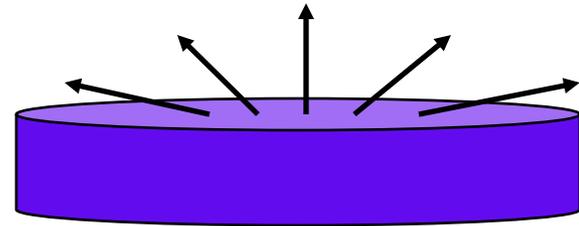
- Modulation: Normal beaming (A) gives higher pulsed fraction than anti-beaming (C)
- timing asymmetries: peak emission occurs earlier for C than for A
- Pulse shape: double-peaks or flattened peaks possible with C

Anisotropy depends on the photon wavelength.

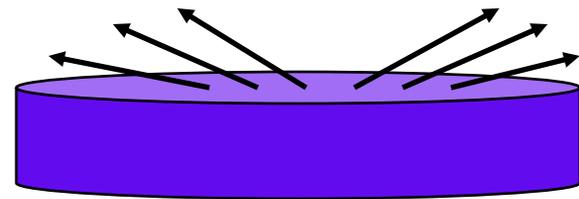
We require phase resolved spectroscopy!



A = Beamed towards the normal



B = Isotropic emission

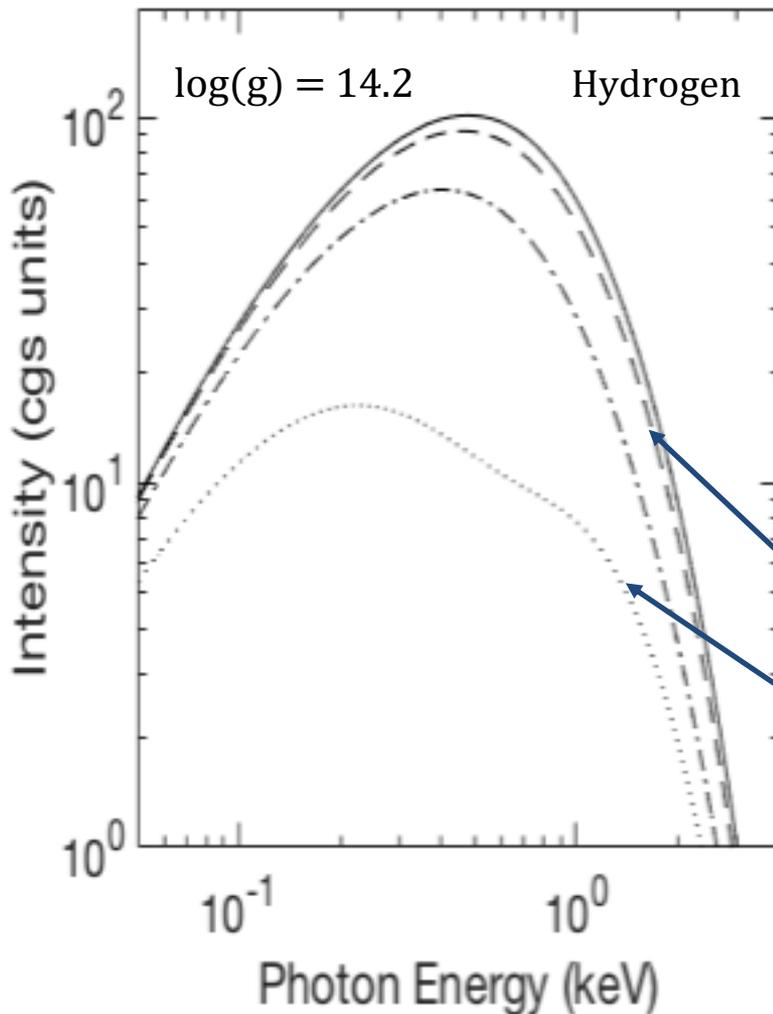


C = Beamed towards the surface

# Hot Spot Model Atmospheres

Magnetic polar caps of MSPs are heated by energetic return current from magnetosphere

$$T = 10^6 \text{ K}$$

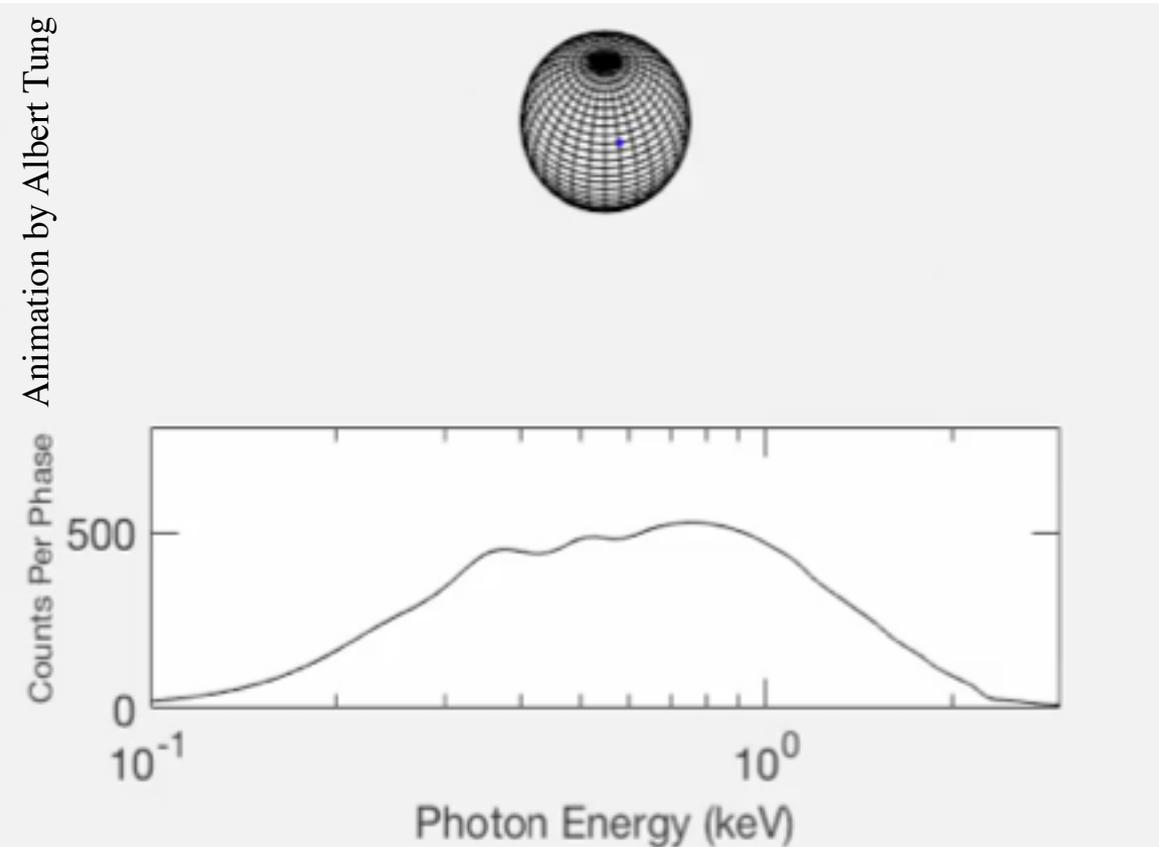


- Surface likely covered by light element atmosphere (see Zavlin, Pavlov, & Shibano 1996)
- The NICER team employs NSX non-magnetic atmosphere models, H and He, for pulse profile fitting (Ho & Lai 2001; Ho & Heinke 2009)
- Variation of surface gravity due to rotation (AlGendy & Morsink 2014)

Emission normal to surface

Emission tangent to surface

# Synthetic Data for J0437



Synthetic data inspired  
by XMM Observations

- Two hot spots
- One small and hot
- One large and cool
  
- Includes background  
from AGN and  
diffuse sky
- TBNew ISM  
absorption
- 2014 NICER  
Response Matrix

# X-ray Timing Telescopes

- RXTE – Rossi X-ray Timing Explorer (1995-2012)  
X-ray timing
- XMM – great energy resolution, +timing mode
- AstroSAT – Indian RXTE-like mission
- **NICER – great timing AND spectroscopy –  
designed for pulse profile observations!!!!**
- Future:
  - eXTP, StrobeX = RXTE x 10 + spectra  
+ **polarization (eXTP)!!!!**

# The Neutron Star Interior Composition Explorer



PI: Keith Gendreau

Science Lead: Zaven Arzoumanian

July 21, 2022, INT Seattle

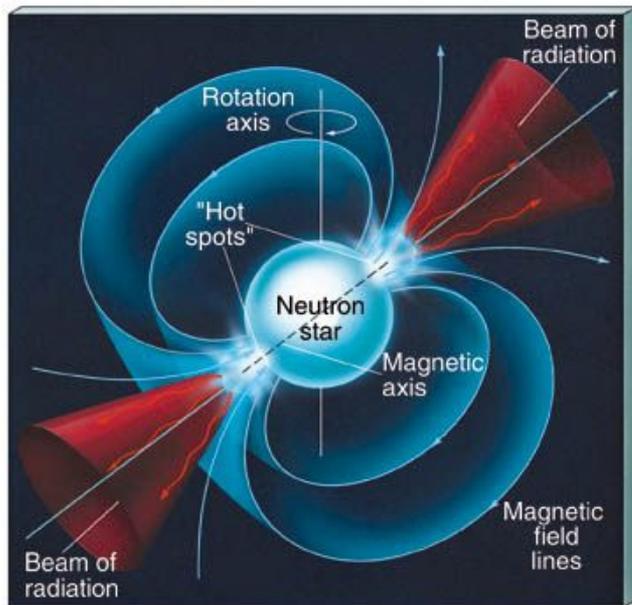
Neutron Rich Matter in Heaven and Earth



Installed on the ISS in June 2017

# NICER 's Key Science Objective

Constrain the equation of state of bulk nuclear matter through precise mass and radius measurements of several neutron stars.

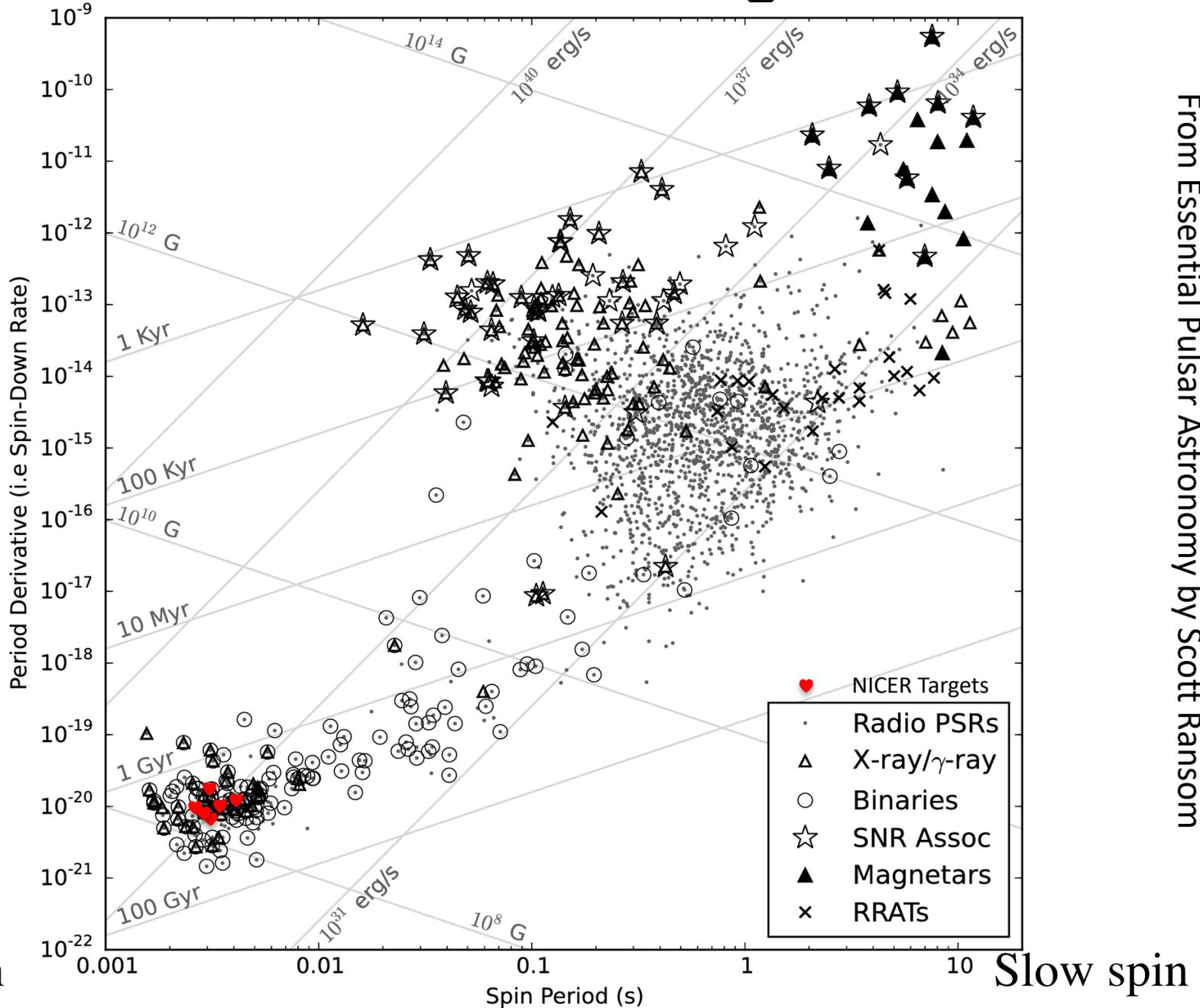


Targeting (mainly) thermal emission from **rotation-powered MSPs** and using the pulse profile modeling technique

- $\dot{E} \approx 10^{33-34}$  erg/s,  $L_X \approx 10^{30-31}$  erg/s
- Soft, thermal X-ray emission from hot spots
- Non-magnetic (0 G, effectively  $B < 10^{10}$  G) hydrogen or helium atmosphere
- **Non-transient (always "on") and non-variable**

# Pulsar P-Pdot Diagram

Rapid  
Change  
in Spin



Stable  
spin

Fast spin

From Essential Pulsar Astronomy by Scott Ransom

Slow spin

## NICER Target List for M-R Constraints

|                | Spin Period<br>(ms) | Distance (pc)        | Mass *<br>( $M_{\odot}$ ) | NICER Rate<br>(photons/ks) |
|----------------|---------------------|----------------------|---------------------------|----------------------------|
| PSR J0437–4715 | 5.76                | $156.79 \pm 0.25$    | $1.44 \pm 0.07$           | 1319                       |
| PSR J0030+0451 | 4.87                | $325 \pm 9$          | isolated                  | 314                        |
| PSR J1231–1411 | 3.68                | 440                  | ?                         | 210                        |
| PSR J2124–3358 | 4.93                | $410_{-70}^{+90}$    | isolated                  | 100                        |
| PSR J0614–3329 | 3.10                | ~ 550                | ?                         | 27                         |
| PSR J1614–2230 | 3.15                | $670_{-40}^{+50}$    | $1.928 \pm 0.016$         | 18                         |
| PSR J0740+6620 | 2.89                | $1140_{-150}^{+170}$ | $2.08 \pm 0.07$           | 15                         |

\* Masses from radio timing for pulsars in a binary.

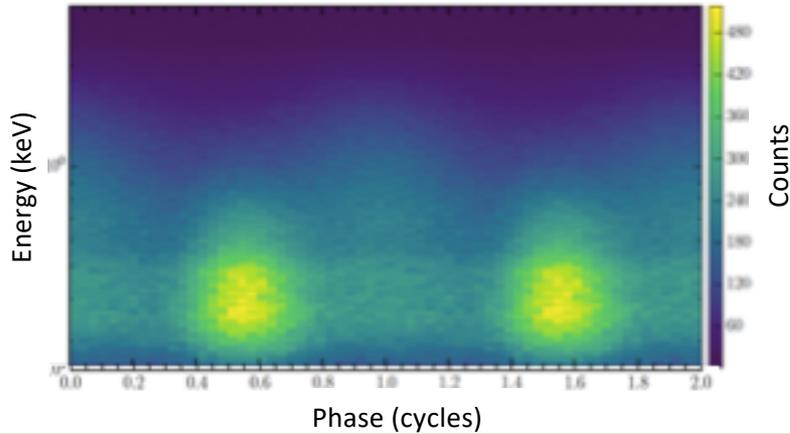
# NICER Target List for M-R Constraints

|   | Spin Period<br>(ms) | Distance (pc)                        | Mass *<br>( $M_{\odot}$ ) | NICER Rate<br>(photons/ks) |
|---|---------------------|--------------------------------------|---------------------------|----------------------------|
| PSR J0437–4715<br><i>Expect results later in 2022! Best statistics on this one!</i>                               | 5.76                | 156.79±0.25                          | 1.44±0.07                 | 1319                       |
| PSR J0030+0451<br><i>ApJL 2019, inferred mass near 1.4 <math>M_{\text{sun}}</math>, 1.9 Msec exposure in 2019</i> | 4.87                | 325±9                                | isolated                  | 314                        |
| PSR J1231–1411<br><i>Fermi LAT gamma ray pulsar, X-ray pulsations 1<sup>st</sup> detected by NICER!</i>           | 3.68                | 440                                  | ?                         | 210                        |
| PSR J2124–3358  | 4.93                | 410 <sub>-70</sub> <sup>+90</sup>    | isolated                  | 100                        |
| PSR J0614–3329  | 3.10                | ~ 550                                | ?                         | 27                         |
| PSR J1614–2230  | 3.15                | 670 <sub>-40</sub> <sup>+50</sup>    | 1.928±0.016               | 18                         |
| PSR J0740+6620<br><i>ApJL 2021, high mass, denser interior important for supra-nuclear physics!</i>               | 2.89                | 1140 <sub>-150</sub> <sup>+170</sup> | 2.08±0.07                 | 15                         |

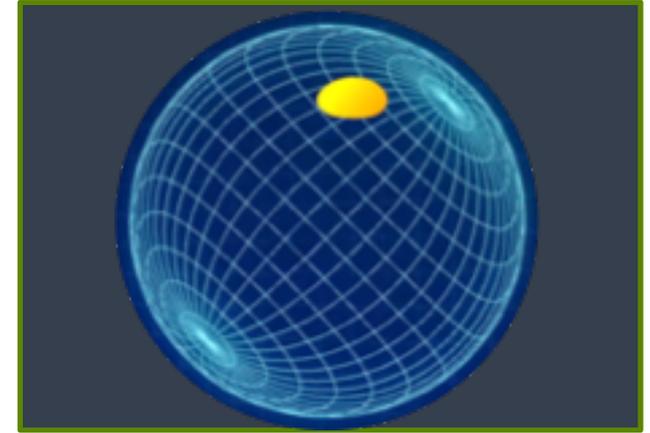
\* Masses from radio timing for pulsars in a binary.

# The Pulse Profile Modelling process

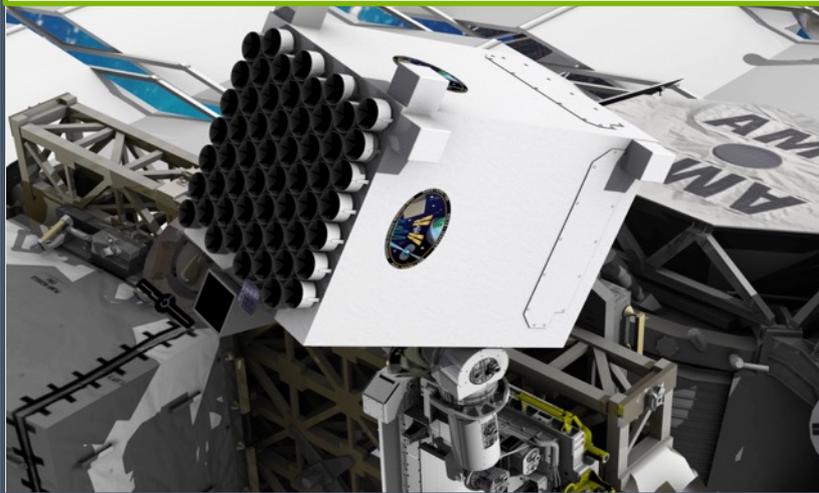
Pulse profile data: Phase, Energy



Lightcurve model:  
Emission, Relativistic ray-tracing



Instrument properties

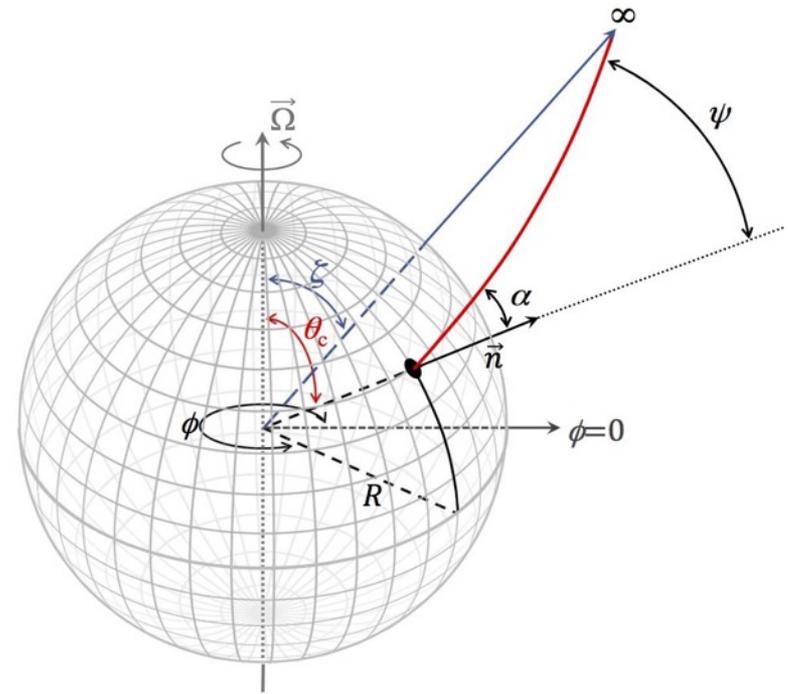


Inference code:  
Likelihood calculation,  
statistical sampling

Mass-radius  
EOS (nuclear physics)

# Model Geometry and Relativistic Effects

- Rotating star with oblate shape
- Two or more X-ray emitting "hot-spots"
- Relativistic effects:
  - Light bending in a Schwarzschild geometry
  - Gravitational redshift
  - Doppler shifts
  - Relativistic aberration
  - Propagation time differences
- Spot **co-latitudes**  $\theta_{c1}$  ,  $\theta_{c2}$  , ...
- Relative phase of the spots
- Spot angular radii  $\rho_{1,2}$
- Observer inclination  $\zeta$
- Relation between  $\psi$  and  $\alpha$  depends on  $M/R$

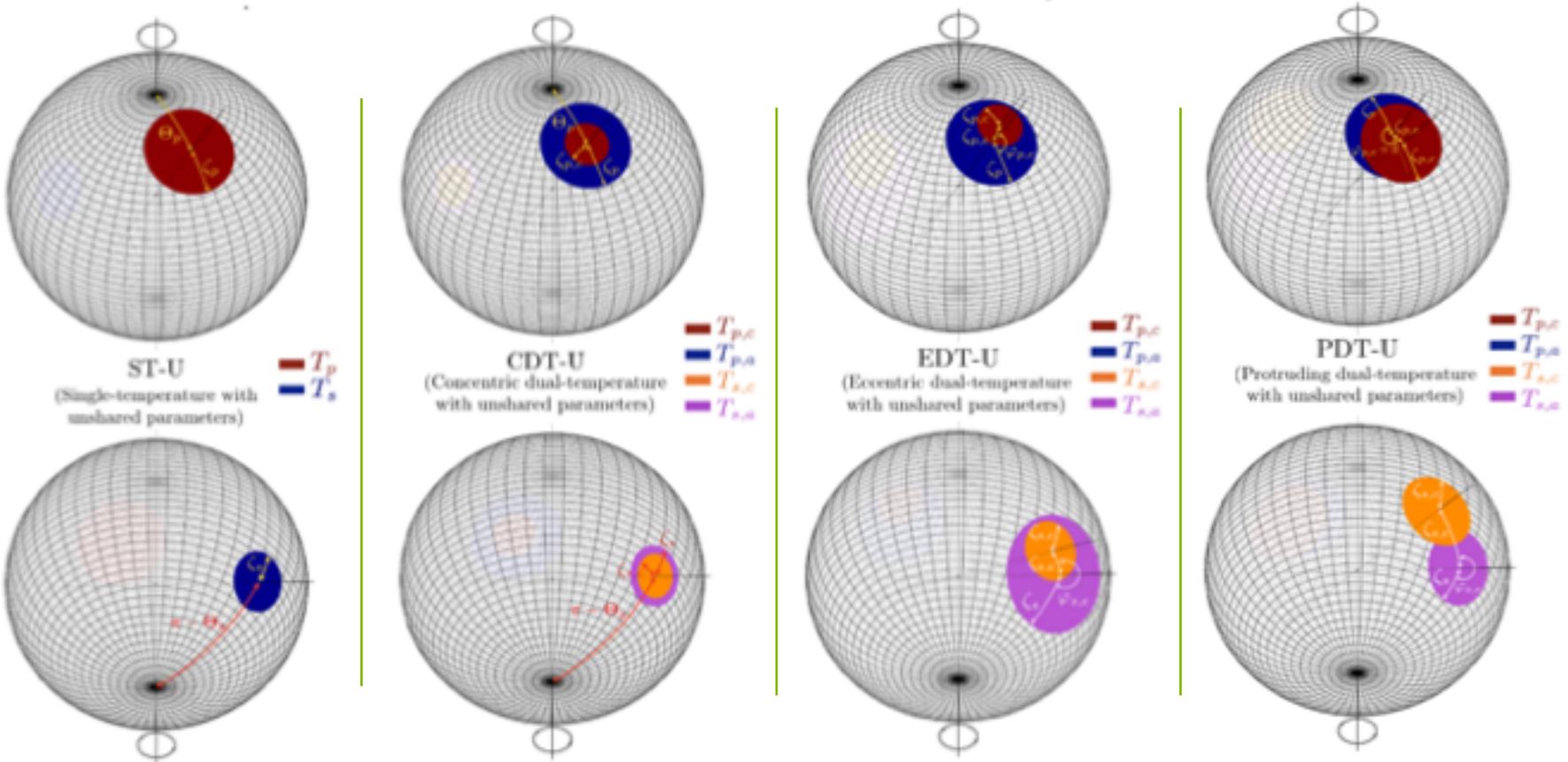


(Miller & Lamb 1998; Beloborodov 2002; Poutanen & Gierlinski 2003; Poutanen & Beloborodov 2006; Morsink et al. 2007; Lo et al. 2013; Miller & Lamb 2015; Bogdanov; Ozel & Psaltis et al. ; Strohmayer & Mahmoodifar; Watts et al. , ... )

# Amsterdam Spot Shapes

- Two-cap models of increasing surface pattern complexity.

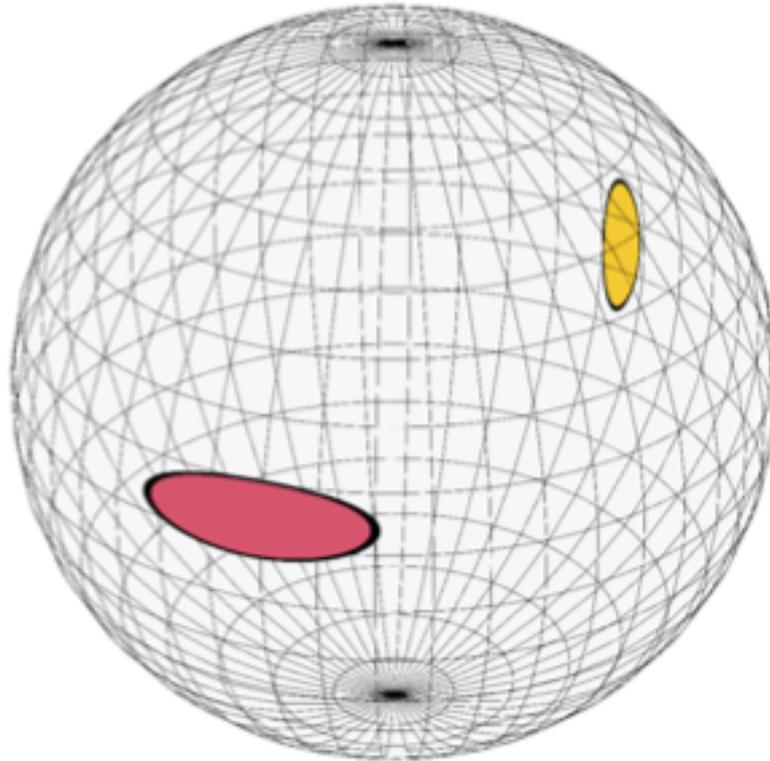
*Northern rotational hemisphere*



*Southern rotational hemisphere*

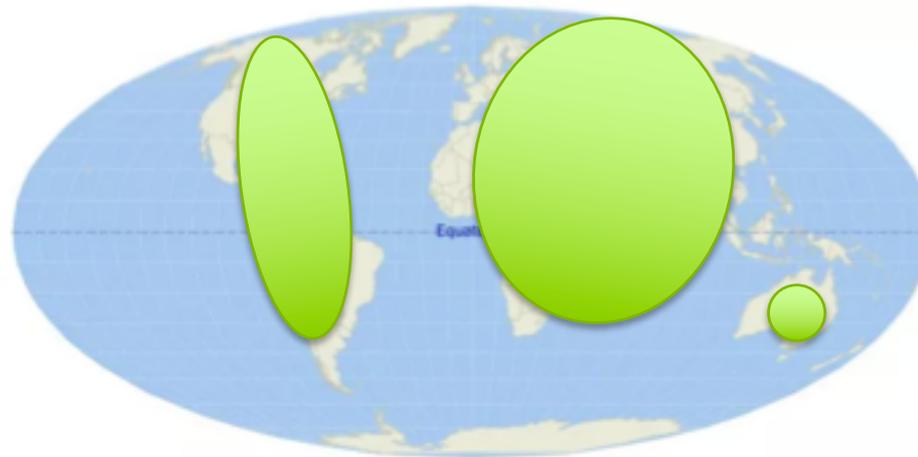
# Illinois-Maryland Spot Shapes

Two or more hot spots, allowing for elongated spots with arbitrary overlap



Courtesy of Alex Dittmann, Fred Lamb, & Cole Miller

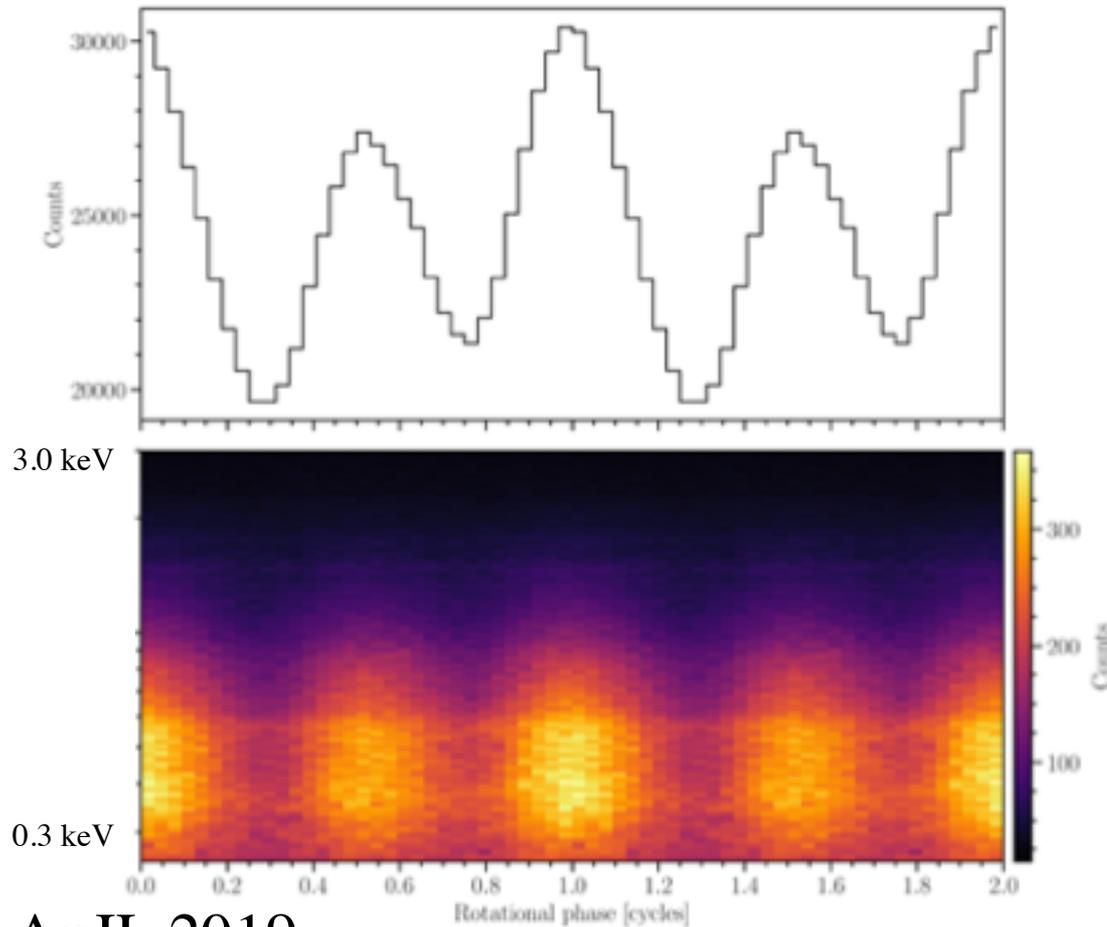
# Spot Shapes



Model vs Reality



# J0030 Lightcurve (2019 dataset)



Bogdanov+, ApJL 2019

# First Results on J0030

- No independent radio mass measurement
- Two independent analyses (crescents or ovals) Riley+ (ApJL 2019) and Miller+ (ApJL 2019)

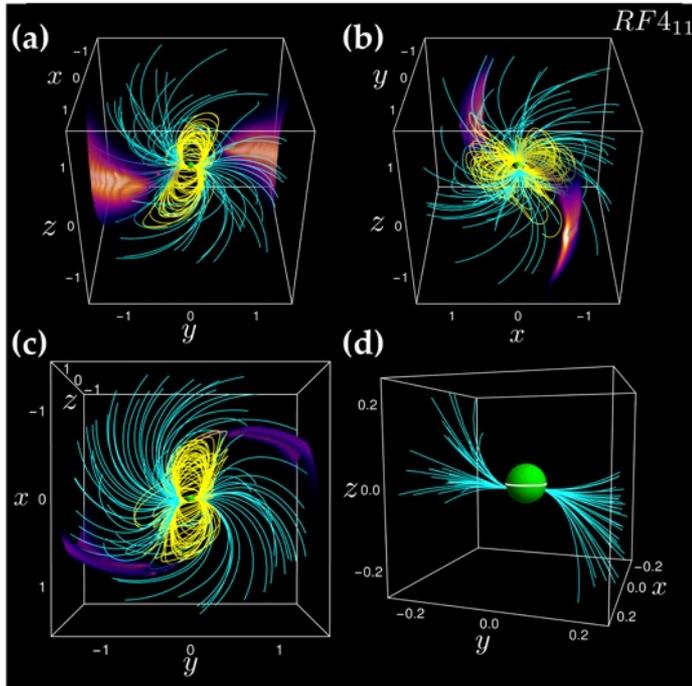
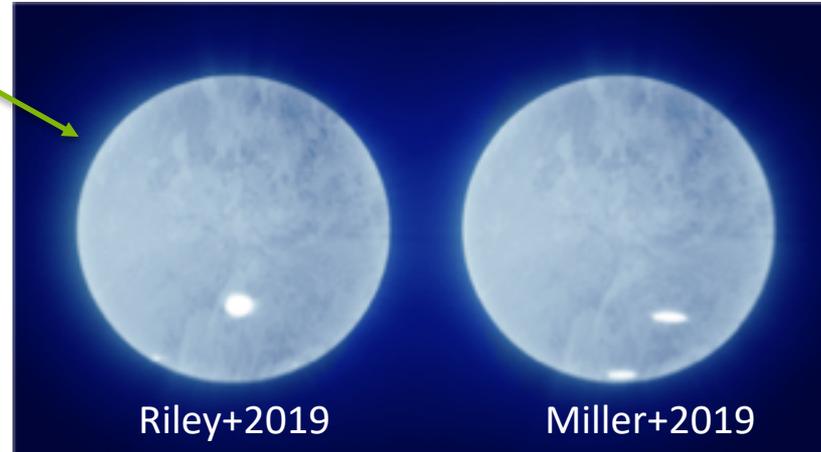
$$M = 1.34_{-0.16}^{+0.15} M_{\text{sun}} \quad R = 12.71_{-1.19}^{+1.14} \text{ km} \quad (\text{Riley+})$$

$$M = 1.44_{-0.14}^{+0.15} M_{\text{sun}} \quad R = 13.02_{-1.06}^{+1.24} \text{ km} \quad (\text{Miller+})$$

- Similar observer inclinations, and spot locations
- Differences in M, R values show systematics in modelling choices
- Updated values (more observing time, improved background) expected later in 2022
- Error regions will shrink with better statistics!

# Inferred Spot Geometries

observer

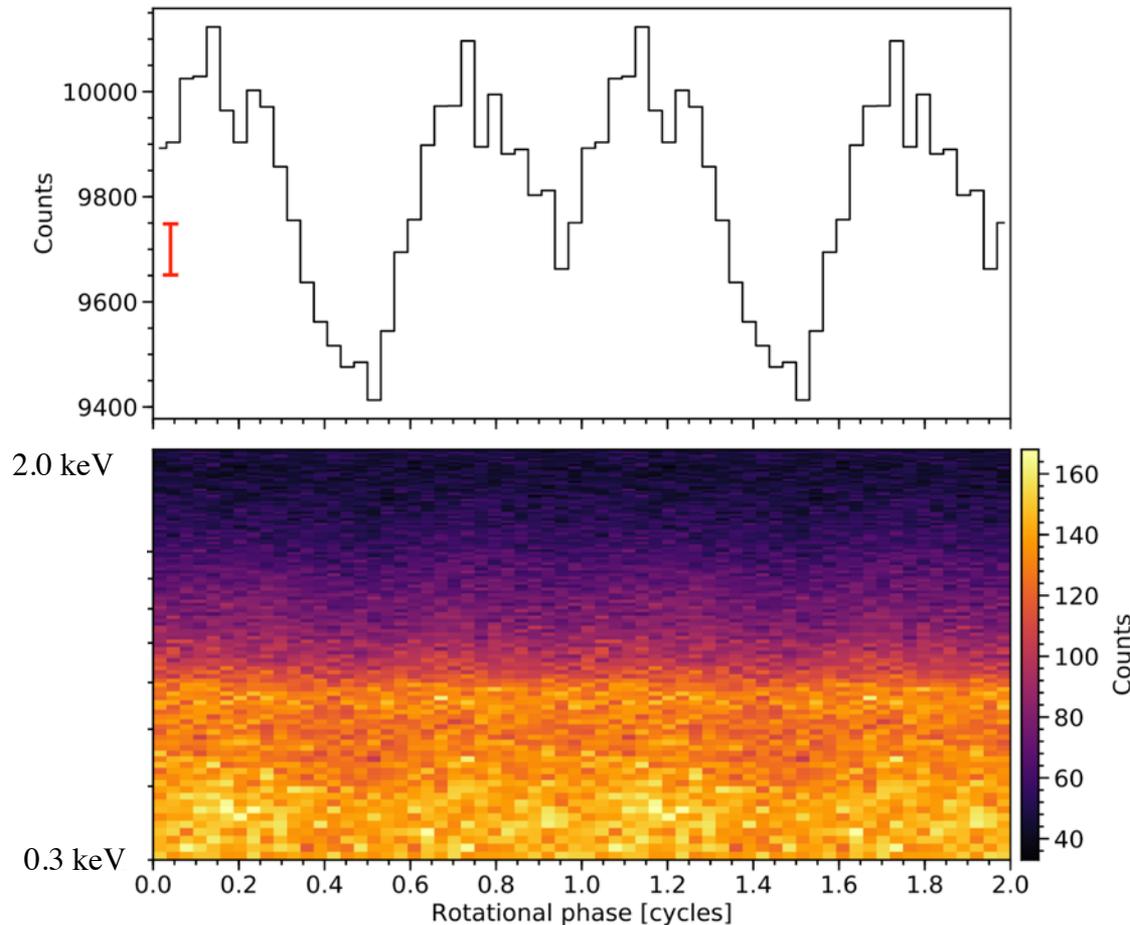


Kalapothisarakos + 2021

- Non-dipole magnetic field
- Example: Offset dipole + quadrupole
- Kalapothisarakos, Wadiasingh, Harding, & Kazanas ApJ (2021)

# PSR J0740 Lightcurve

Known High Mass =  $2.08 \pm 0.07$  Msun



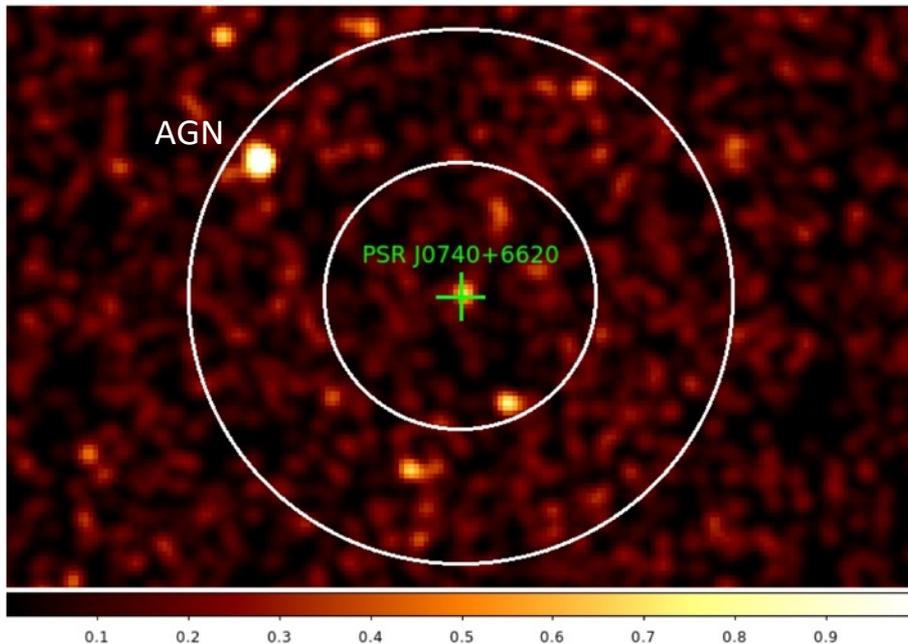
X-ray pulsations  
detected at  $15 \sigma$   
significance!

Wolff+ ApJL 2021

July 21, 2022, INT Seattle

Neutron Rich Matter in Heaven and Earth

# XMM Image of Region Near PSR J0740



- NICER is a “one-pixel” telescope
- No information about location of unpulsed emission
- J0740 is very faint
- Bright (harder) Active Galactic Nucleus and diffuse background

Wolff+ ApJL 2021

# First Results on J0740

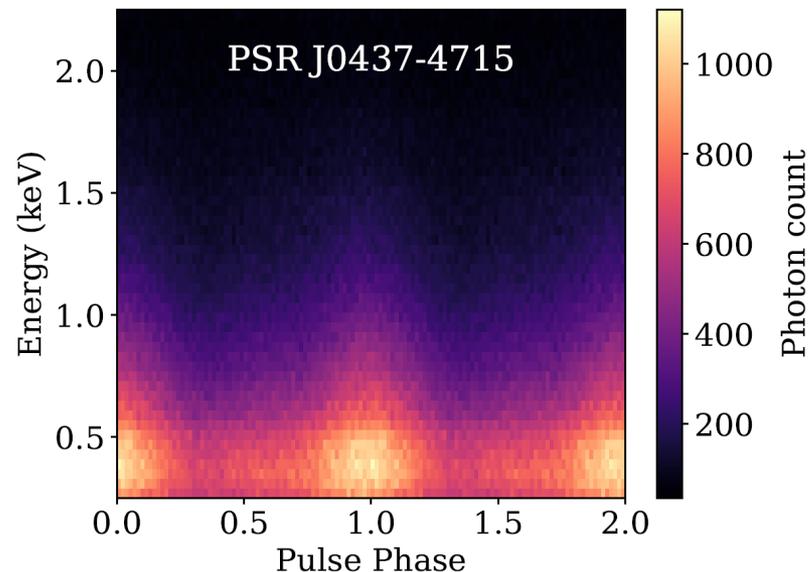
- Mass measurement from NANOGrav + CHIME:  
 $M = 2.08 \pm 0.07 M_{\odot}$  (Shapiro Delay, Fonseca+2021)
- Inclination close to 90 degrees (from radio obs)
- 2 Circular spots, closer to dipole than J0030
- Different treatments of XMM  
background/normalizations/priors:
  - $R = 12.4_{-1.0}^{+1.3}$  km (Riley+ ApJL 2021)
  - $R = 13.7_{-1.5}^{+2.6}$  km (Miller+ ApJL 2021)
- **1-sigma lower limits on radius:**  
 **$R_{1\sigma} = 11.4$  km (Riley+);  $R_{1\sigma} = 12.2$  km (Miller+)**

# Updates on J0740

- Tuomo Salmi, lead author (Monday's talk)
- Salmi + 2022 (ApJ submitted)
  - Improved background treatment with original data
  - Better agreement on 1 sigma lower limits from the two groups:  
 **$R_{1\sigma} = 11.93$  km (Ams);  $R_{1\sigma} = 11.98$  km (Ill-MD)**
- Salmi + (in prep) longer observation timeline, will have smaller statistical error

# Next: J0437

- We expect best radius measurement
- Precisely known value of mass and observer's viewing angle from radio observations
- Complication of contamination from background AGN in same field of view
- Results from J0437 and other pulsars in 2022/23



# NICER Results on Core Science

- First precise measurement of mass through pulse-profile modelling (J0030)
- Radius inferred for 2 pulsars (J0030, J0740) with masses that differ by  $0.5 M_{\text{sun}}$
- Later this year
  - New results for J0437 (with precise radio prior for mass, inclination, distance)
  - Updated mass & radius results for J0030 & J0740 with improved precision

## *THE NICER LIGHTCURVE MODELING WORKING GROUP*

**Slavko Bogdanov (chair)**, Zaven Arzoumanian, Keith Gendreau, Anna Bilous, Deepto Chakrabarty, Devarshi Choudhury, Alexander Dittmann, Sebastien Guillot, Alice Harding, Wynn Ho, Fred Lamb, Jim Lattimer, Renee Ludlam, Simin Mahmoodifar, Cole Miller, Sharon Morsink, Chanda Prescod-Weinstein, Paul Ray, Ron Remillard, Thomas Riley, Tuomo Salmi, Tod Strohmayer, Serena Vinciguerra, Anna Watts, Michael Wolff, Kent Wood.

With thanks to Teru Enoto, Andrea Lommen, Matthew Kerr, Michi Bauböck, Feryal Özel, Craig Markwardt, Dimitrios Psaltis, Jack Steiner, & others

# Summary

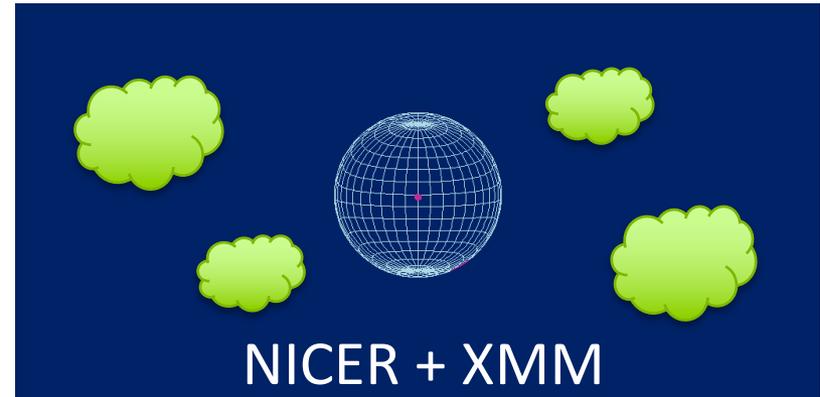
- We are in the golden era of NS observations!
- Amazing instruments today:  
LIGO/VIRGO/KAGRA, NICER, XMM, Chandra
- Bigger and better instruments planned for future both for Gravitational Radiation, and EM observations!
- Multiple methods for determining radius coming together with consistent values!

# Extra Slides

# Effect of Adding Information about XMM Background



- Some of unpulsed signal comes from the spot pattern
- Inferred unpulsed background is small
- Smaller radius, larger spots



- Less unpulsed signal comes from the spot pattern
- Inferred unpulsed background is larger
- Larger radius, smaller spots

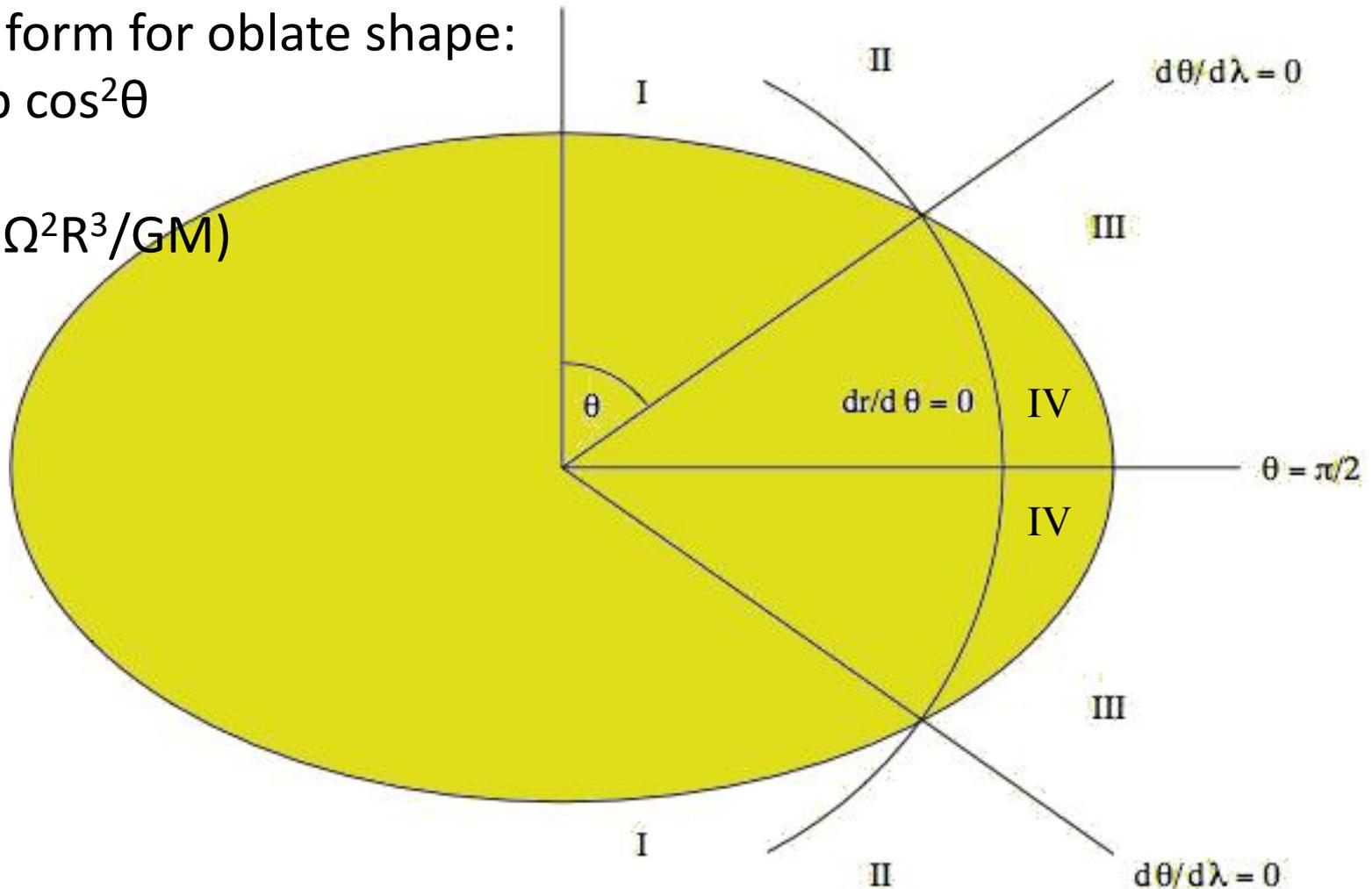
Adding XMM background information **INCREASES** inferred radius.

# Stellar Oblateness

“Universal” form for oblate shape:

$$R(\theta) = R_e - b \cos^2\theta$$

$$b = b(M/R, \Omega^2 R^3/GM)$$

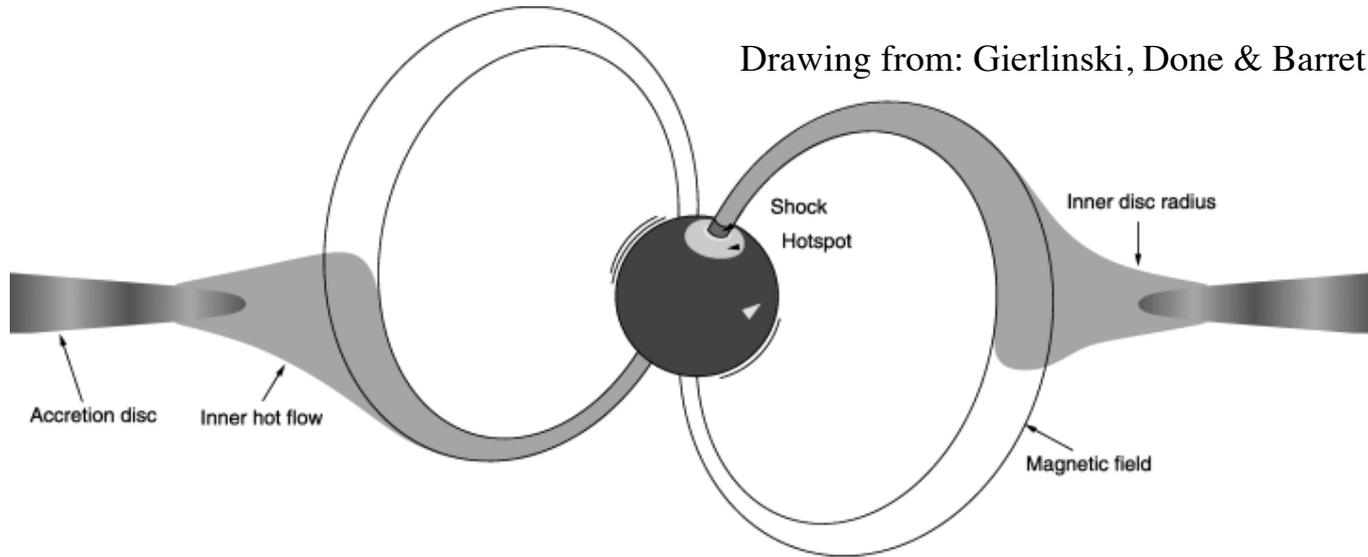


Morsink, Leahy, Cadeau & Braga 2007 ApJ

\*\*\* Adding correct oblate shape does not add extra parameters!!!!

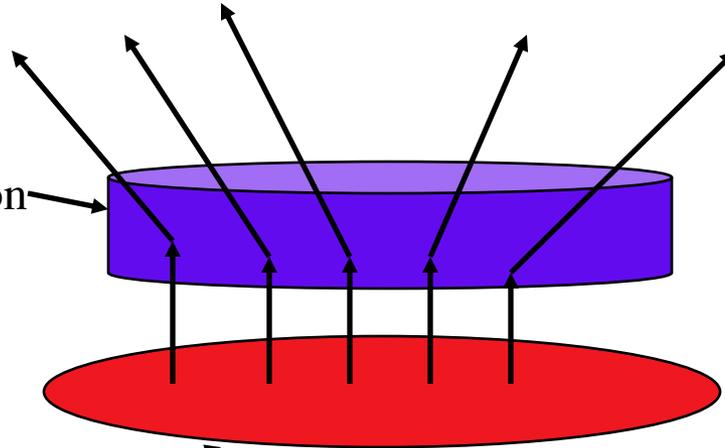
# Hot Spot Model for Accreting X-ray Pulsars

Drawing from: Gierlinski, Done & Barret, 2002, MNRAS



## Side View of Hot Spot:

Electron plasma above spot Compton scatters "seed" blackbody photons



Anisotropic emission (anti-beaming) Sunyaev & Titarchuk

Isotropic emission

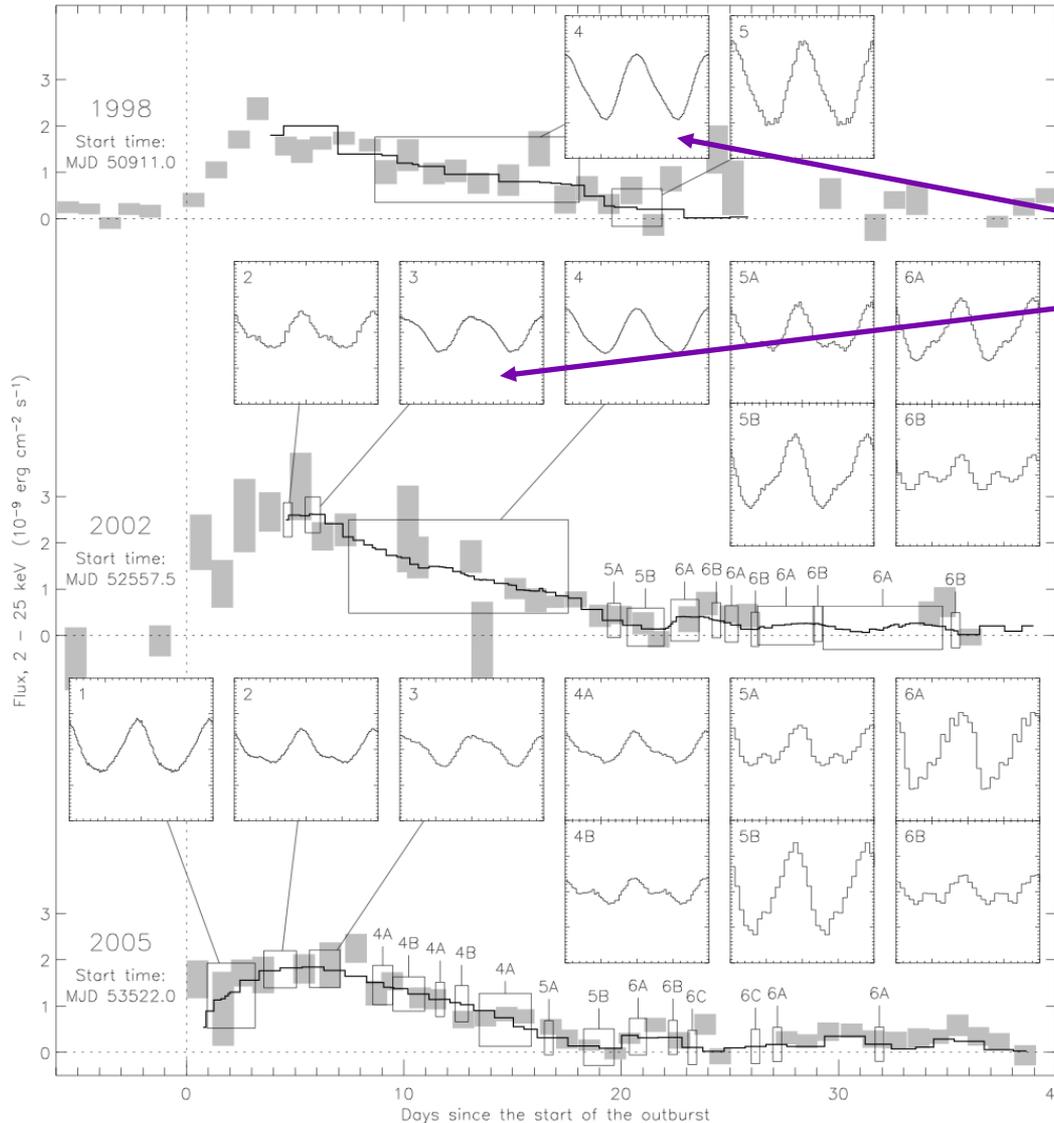
Blackbody emission from surface of star

# Accretion-powered ms-Period Pulsars are VARIABLE!!!!

## Pulse Shape Variability of SAX J1808 (Jacob Hartman et al 2008)

Timing of SAX J1808.4-3658

9



Joint Fits with data using  
1998 Box4 and 2002 Box 3

Require Mass, Radius,  
Inclination to stay  
constant.

Figure from Hartman et al 2008

FIG. 3.— A comprehensive view of the 2–15 keV pulse profiles observed from SAX J1808.4–3658. Each pulse profile was calculated by folding the observations within the indicated time intervals using the best-fit constant-frequency model of each outburst, so any movement