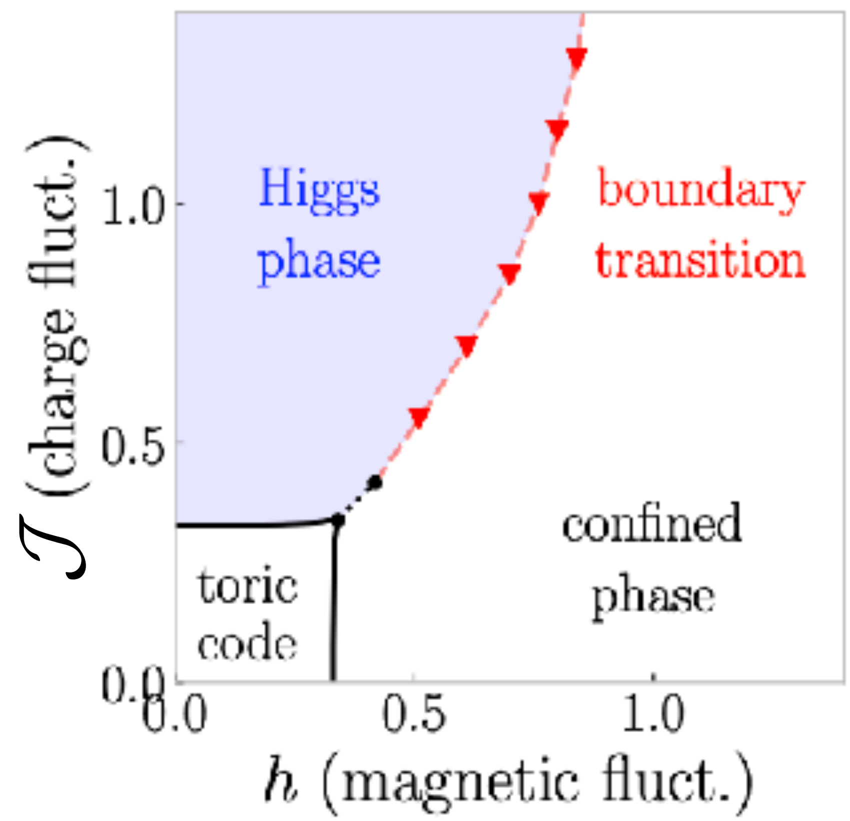
Higgs phase and SPT order

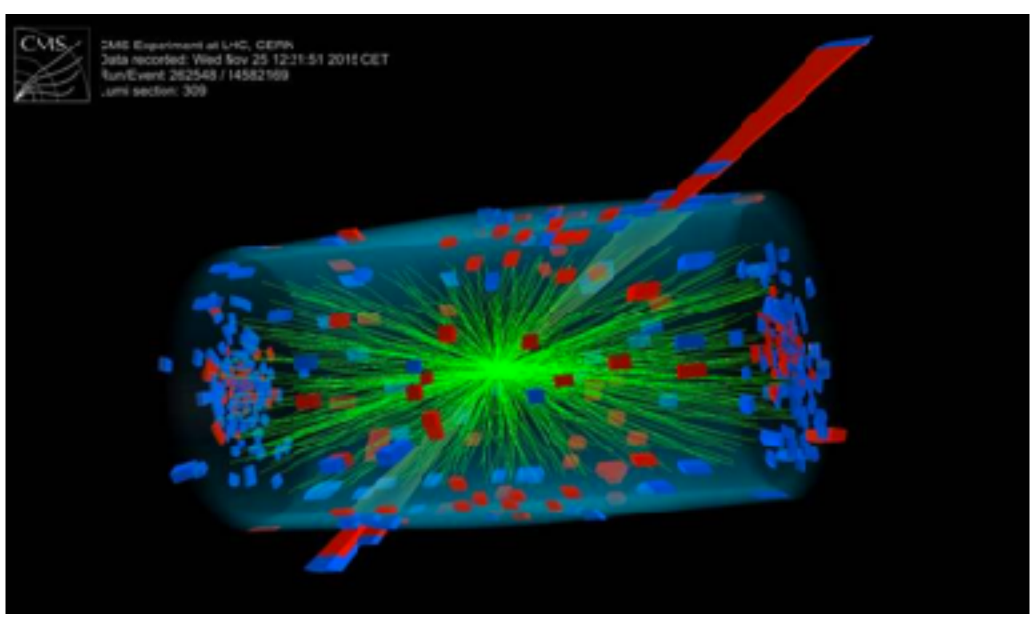


Sergej Moroz

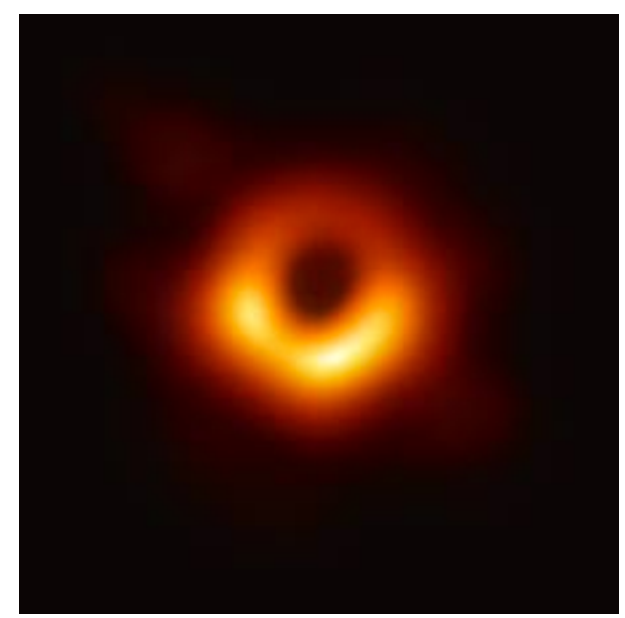
together with
Umberto Borla, Jeet Shah,
Ryan Thorngren, Ruben Verresen, Ashvin Vishwanath

Gauge theories

- General theory of relativity
- Standard model of particle physics

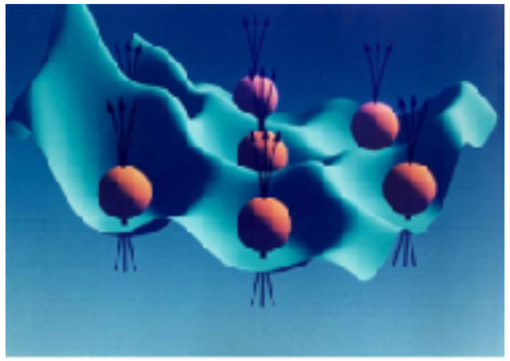


CMS, LHC

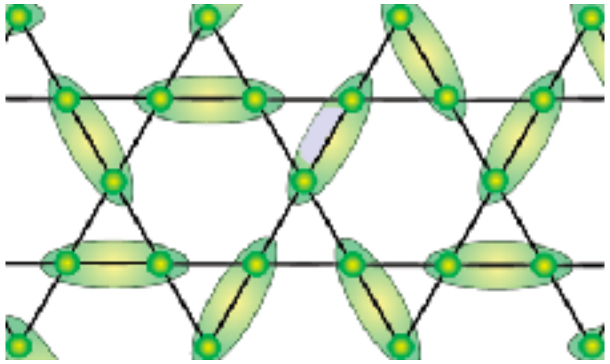


Event Horizon Telescope Team

Emergent gauge theories

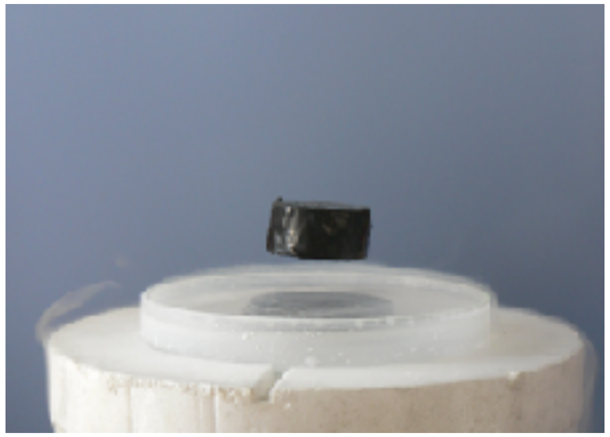


fractional quantum Hall fluids



L. Clark

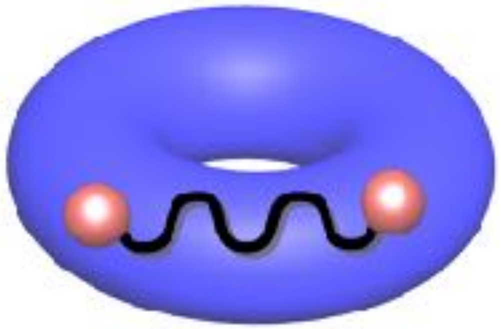
quantum spin liquids



Wikipedia

superconductors

- Anyons
- Ground state degeneracy
- Long-range entanglement



Wen
Kitaev

Z_2=Ising gauge theory

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 12, NUMBER 10 OCTOBER 1971

Duality in Generalized Ising Models and Phase Transitions without Local Order Parameters*

Franz J. Wegner†
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 29 March 1971)

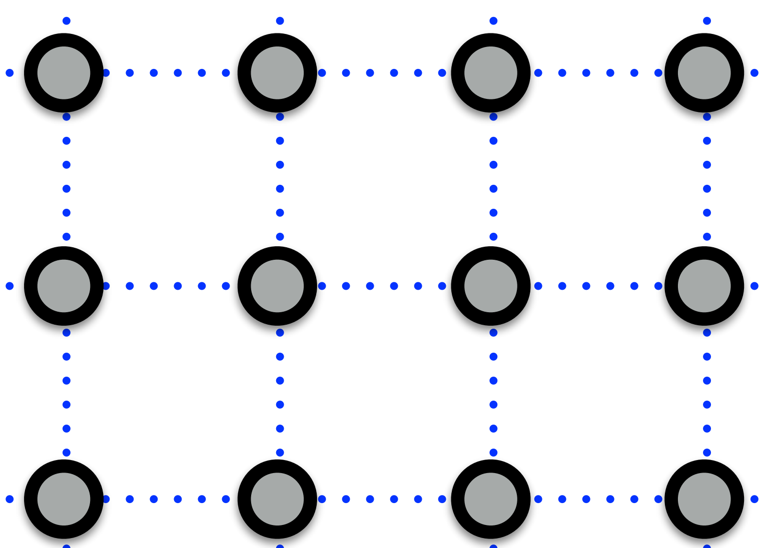
It is shown that any Ising model with positive coupling constants is related to another Ising model by a duality transformation. We define a class of Ising models M_{dn} on d-dimensional lattices characterized by a number $n = 1, 2, \ldots, d$ ($n = 1$ corresponds to the Ising model with two-spin interaction). These models are related by two duality transformations. The models with $1 < n < d$ exhibit a phase transition without local order parameter. A nonanalyticity in the specific heat and a different qualitative behavior of certain spin correlation functions in the low and the high temperature phases indicate the existence of a phase transition. The Hamiltonian of the simple cubic dual model contains products of four Ising spin operators. Applying a star square transformation, one obtains an Ising model with competing interactions exhibiting a singularity in the specific heat but no long-range order of the spins in the low temperature phase.

Simplest gauge theory we can define
on a lattice

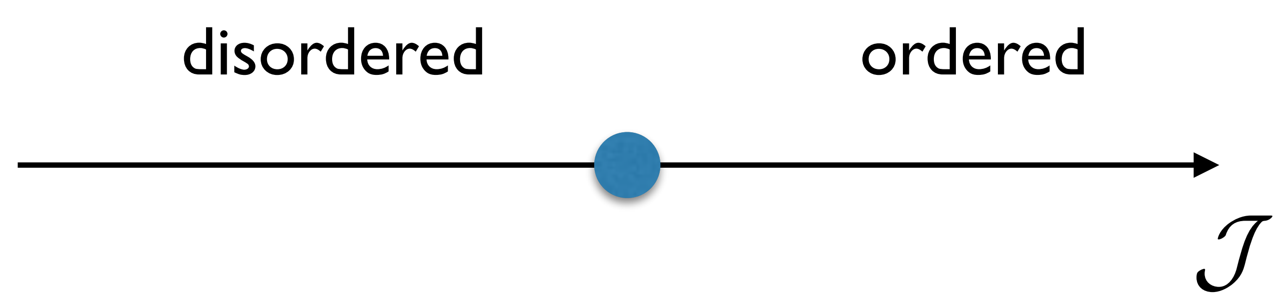
Quantum Ising model

Spin 1/2 operators acting on sites

$$H_I = -\mathcal{J}\sum_{\langle v,v'\rangle} Z_v Z_{v'} - \sum_v X_v$$



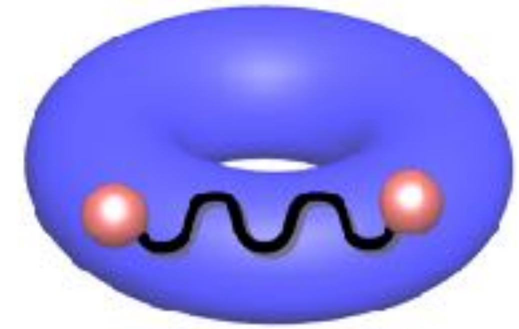
Ising symmetry: $P = \prod_v X_v$



$\lim_{|i-j|\to\infty}\langle Z_i Z_j\rangle = 0$ $\lim_{|i-j|\to\infty}\langle Z_i Z_j\rangle \neq 0$

Landau paradigm

Z_2 gauge theory



Discrete cousin of electrodynamics

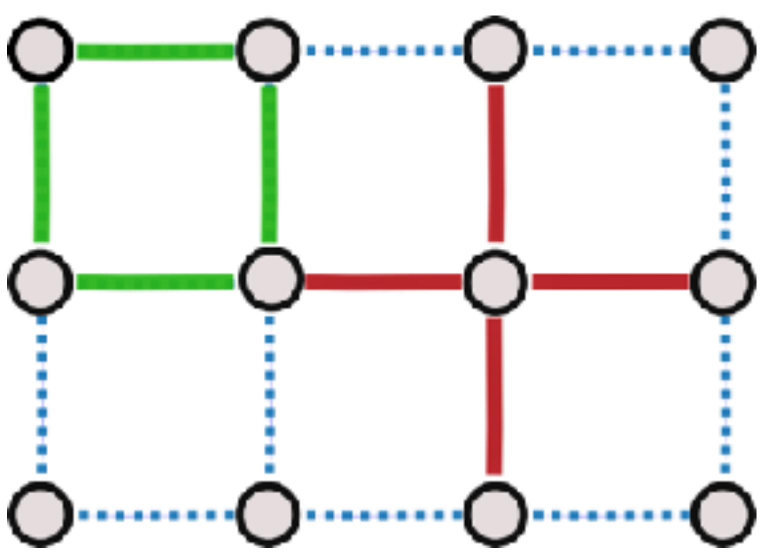
Wegner 1971
Kogut 1979

$$H = -J \sum_{\mathbf{r}*} \prod_{b \in \square_{\mathbf{r}*}} \sigma_b^z - h \sum_{\mathbf{r},\eta} \sigma_{\mathbf{r},\eta}^x$$

Correspondence

$$\sigma^z \sim e^{iA}$$

$$\sigma^x \sim e^{iE}$$



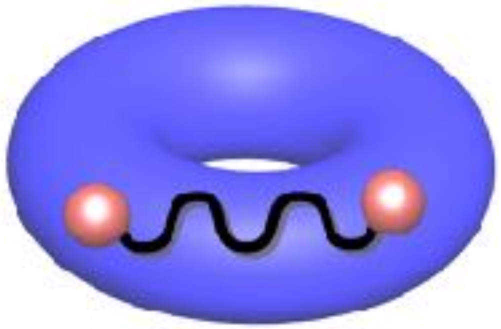
Z_2 gauge transformations

$$G_{\mathbf{r}} = \prod_{b \in +_{\mathbf{r}}} \sigma_b^x$$

Gauss' law: $G_{\mathbf{r}} = 1$

no static charges

Z_2 gauge theory



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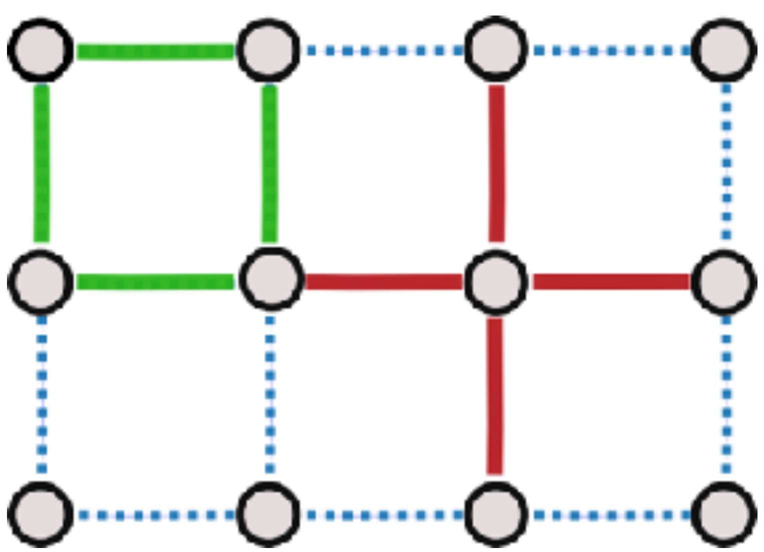


Phase transition without local order parameter

Z_2 topological order — no topological order

deconfined — confined — h/J

Lattice gauge theories

Confinement of quarks*

Kenneth G. Wilson

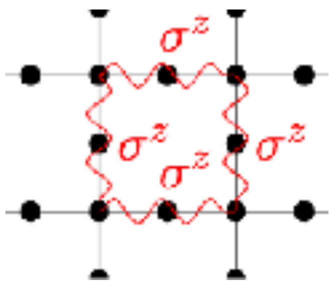
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

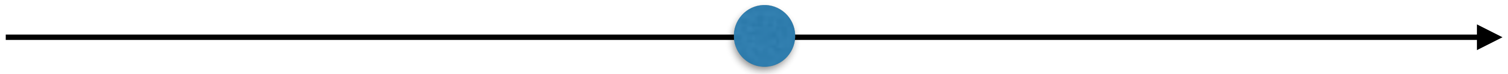
A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

Lattice gauge theories

- Systematic strong-coupling expansion
- Numerical simulations of gauge theories
- Confinement: Wegner-Wilson loop

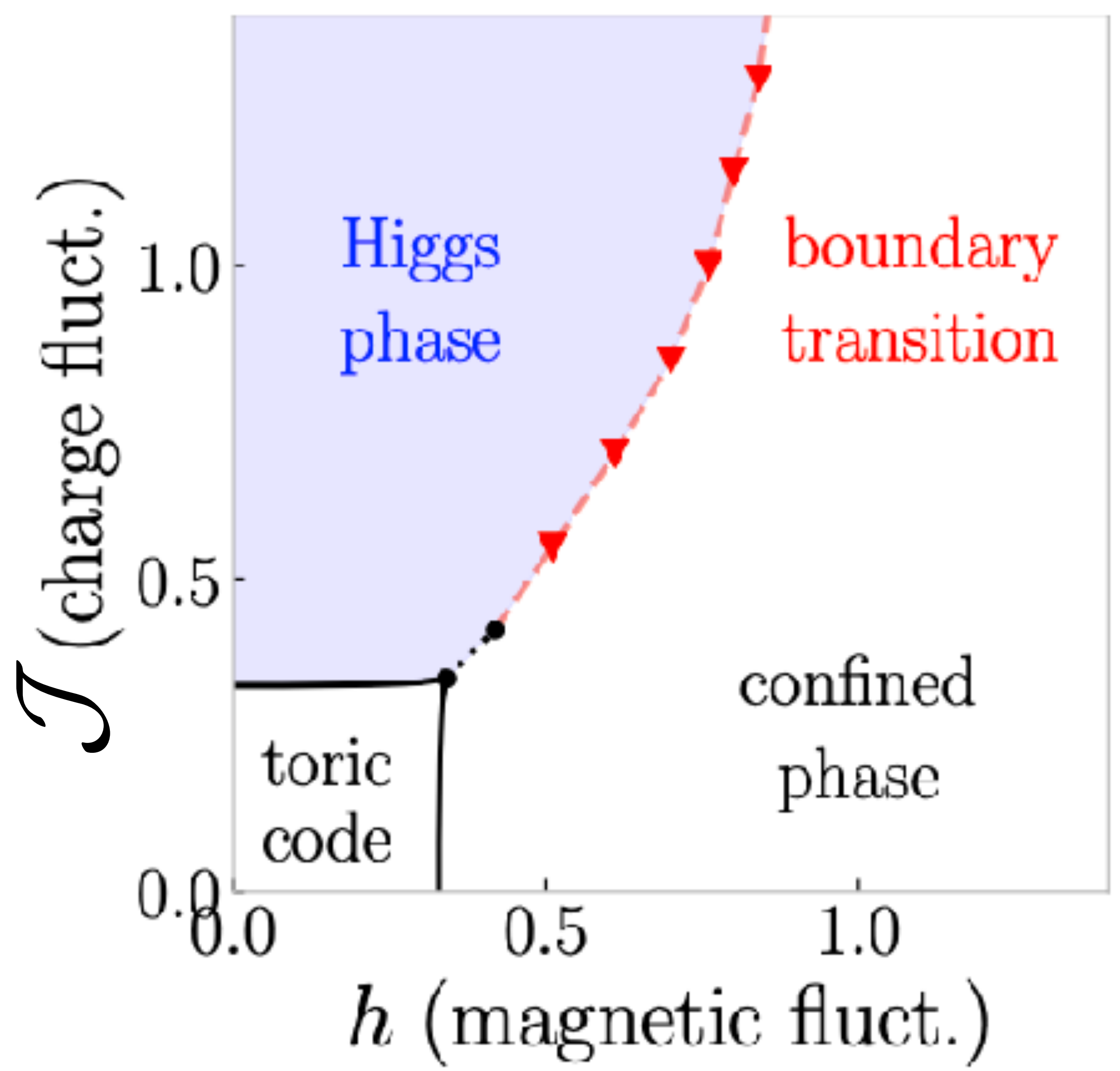


deconfined confined



$\langle W \rangle = \exp(-\#P)$ $\langle W \rangle = \exp(-\#S)$

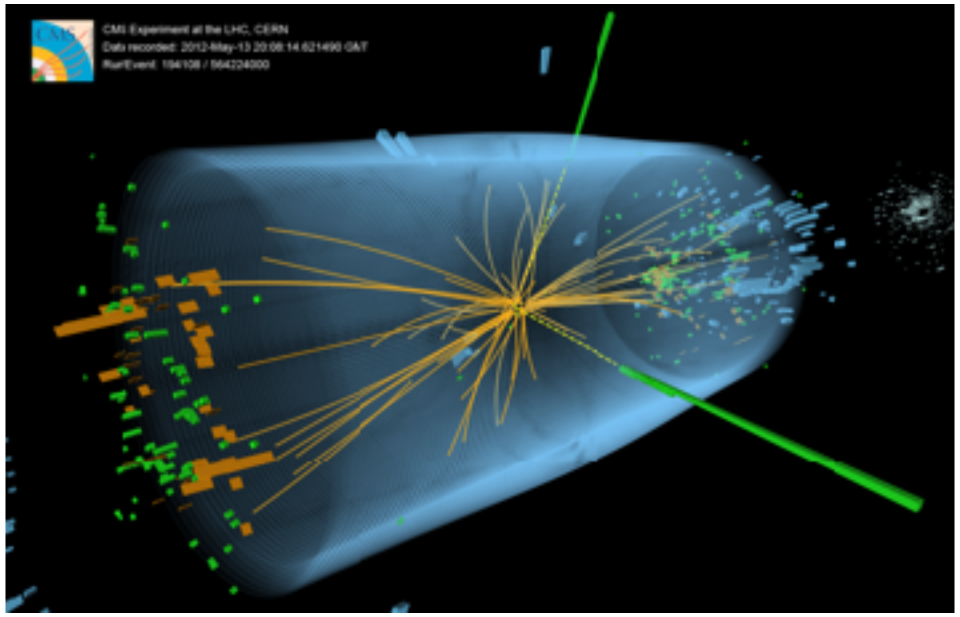
Higgs=SPT

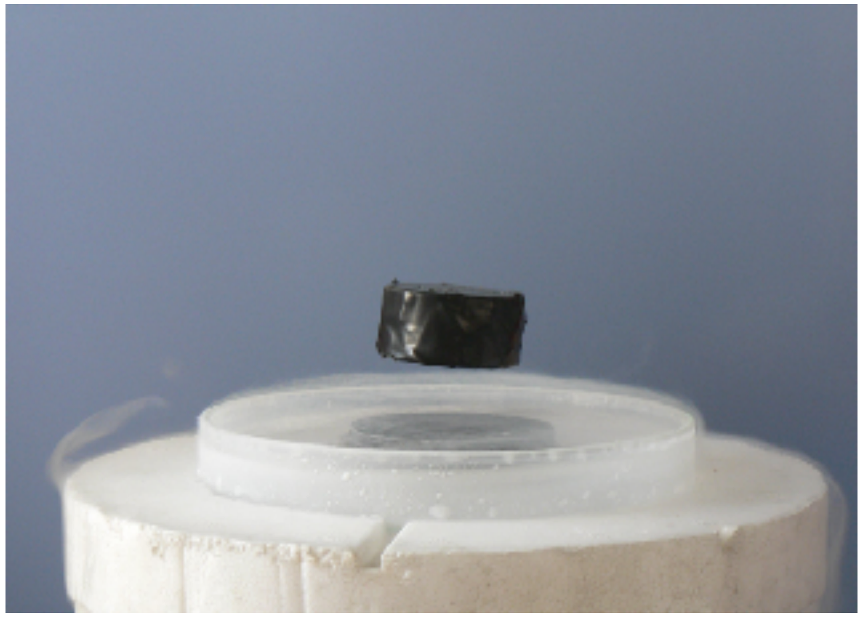


Umberto Borla, Ruben Verresen, Jeet Shah, SM
SciPost 2021

Ruben Verresen, Umberto Borla, Ashvin Vishwanth, SM, Ryan Thorngren
arXiv: 2211:01376

Higgs phase





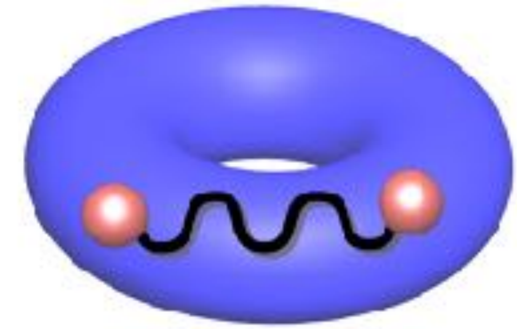
electroweak interactions

superconductors

massive W and Z bosons

massive photon

2d gauged Ising model

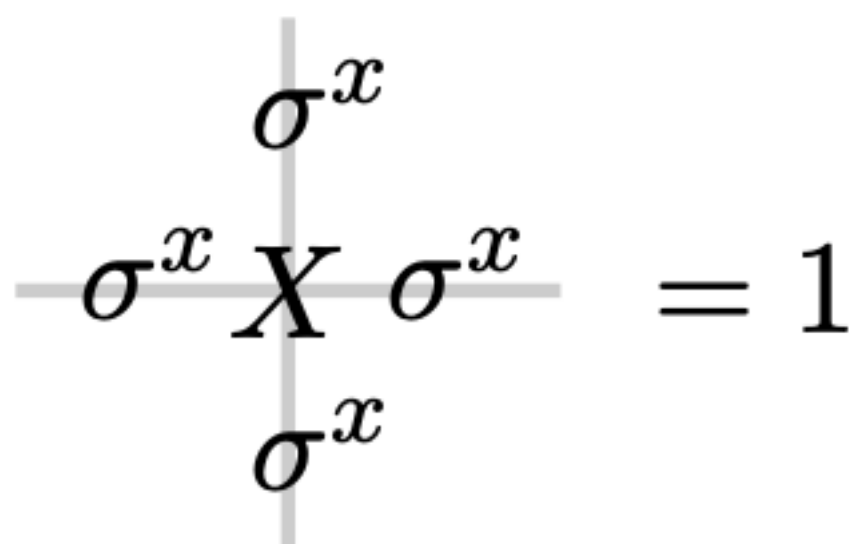


Adding dynamical Ising matter

Fradkin&Shenker 1979

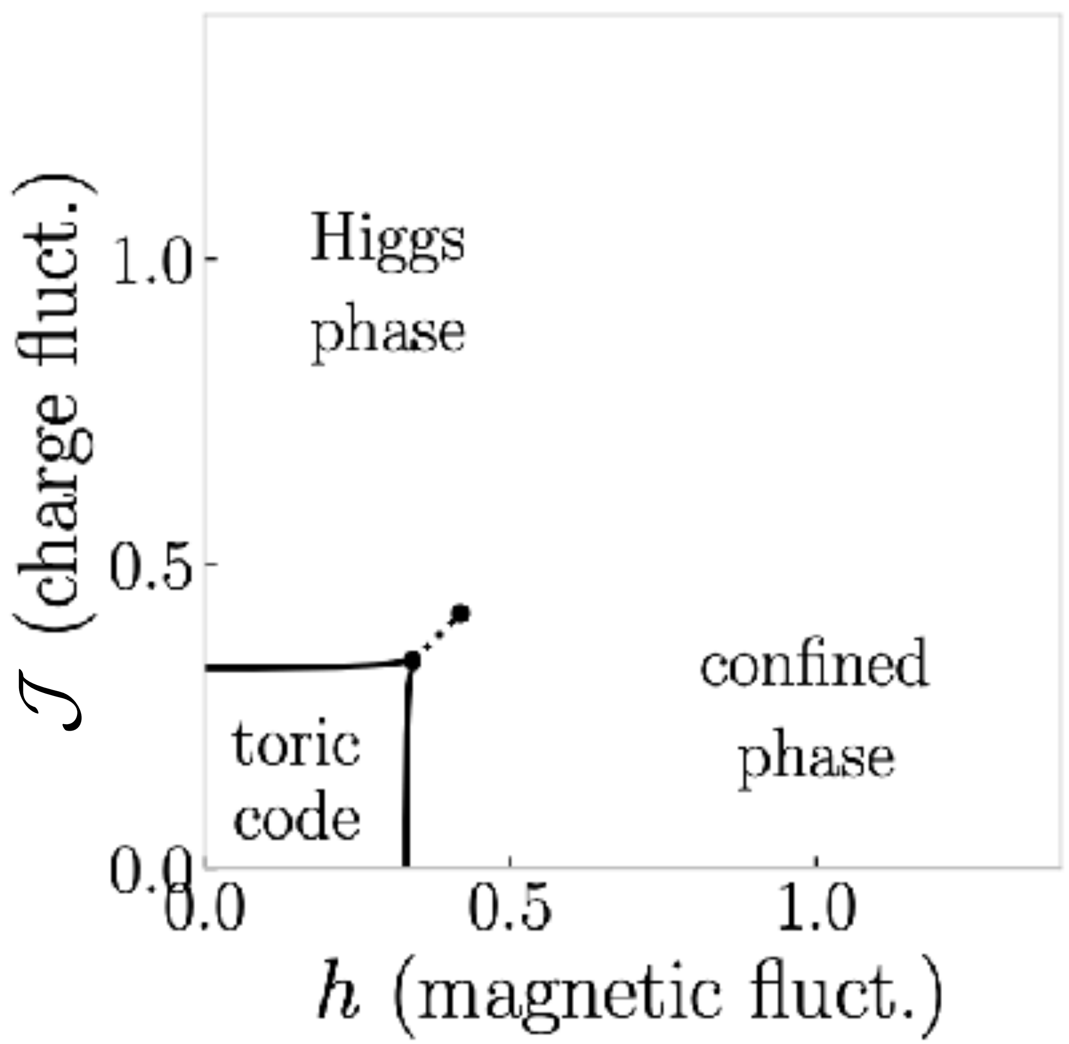
$$H_I = -\mathcal{J} \sum_{\langle v,v' \rangle} Z_v \sigma^z_{v,v'} Z_{v'} - \sum_v X_v$$

Gauss law



Z_2 gauge theory

Toric code in external magnetic field



Vidal et al 2009
Tupitsyn et al 2010
...

no phase transition
in the bulk

Higgs-confinement
continuity

Global symmetries

Two relevant global symmetries:

- Z_2 symmetry carried by matter charges
- Z_2 magnetic 1-form symmetry

Matter symmetry

Gauss law

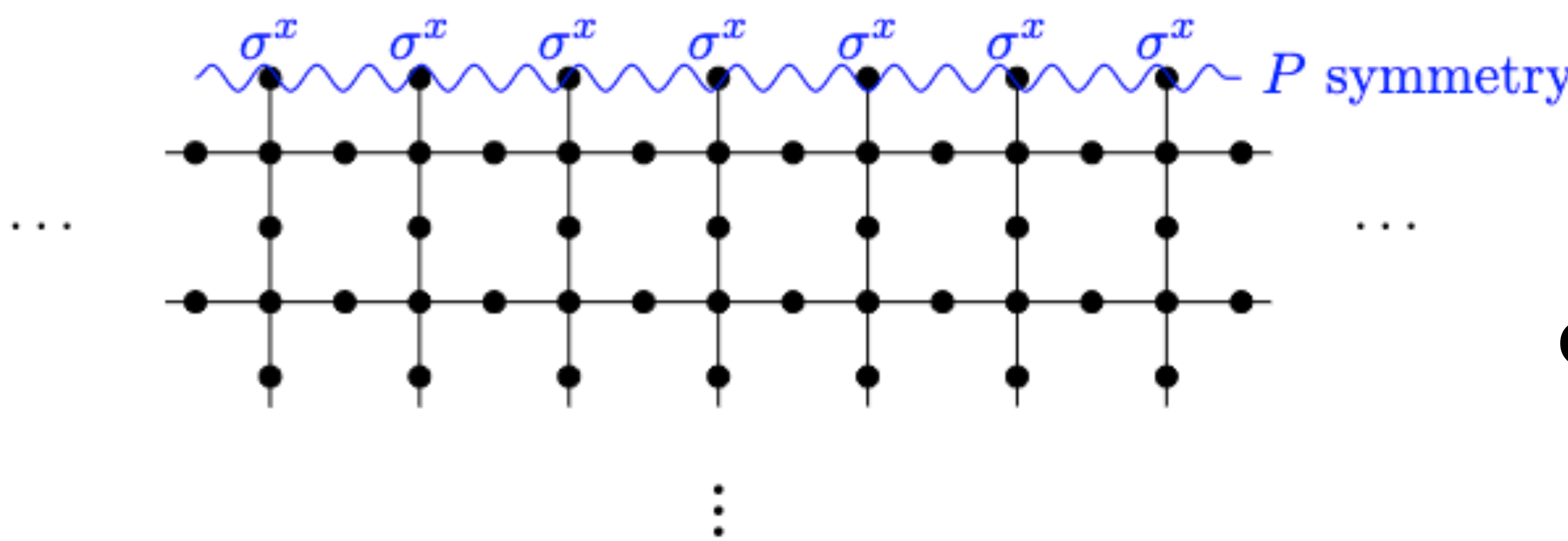
$$\begin{array}{ccc} & \sigma^x & \\ \sigma^x & X & \sigma^x \\ & \sigma^x & \end{array} = 1$$

On a closed surface: $P = \prod_v X_v = 1$

Ising symmetry
fully trivialized
after gauging

Matter symmetry

In presence of a <u>boundary</u>: $P = \prod_{v \in \Lambda} X_v = \prod_{l \in \partial\Lambda} \sigma_l^x$



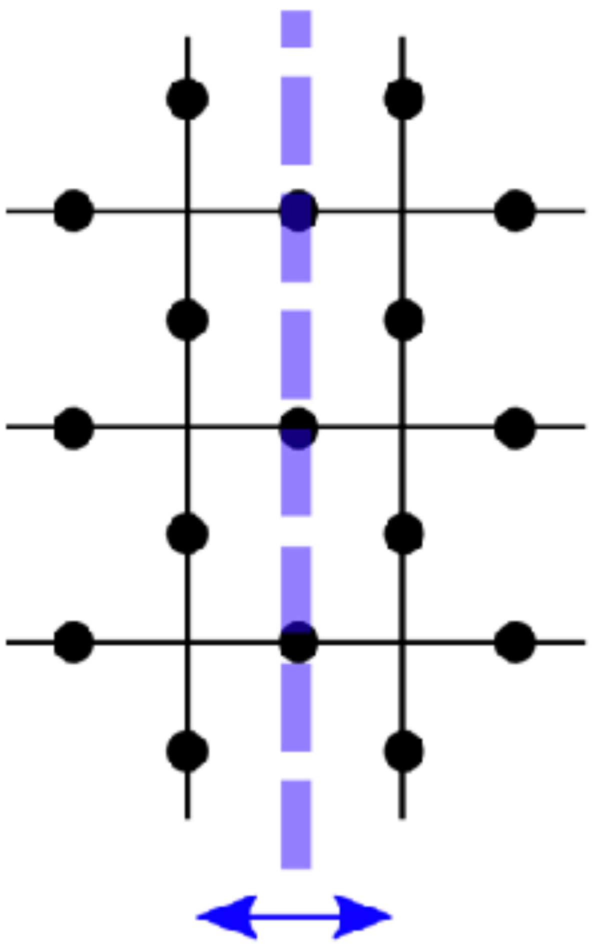
t'Hooft electric loop

Z_2 matter symmetry: no charged matter transported through boundary

After gauging, global symmetry survives, but acts only on <u>boundary</u> links

Matter symmetry

Alternatively, introduce insulating defect in the bulk:



no hopping across purple line

global Z_2 symmetry

$$P_L = \prod_{v \in \Lambda_L} X_v = \prod_{l \in \text{defect}} \sigma_l^x$$

Gentle gauging

Emergent gauge theory $G_v = X_v \Pi_{l \in +_v} \sigma_l^x$

$$H = -\sum_v X_v - J \sum_p B_p - \mathcal{J} \sum_{\langle v,v' \rangle} Z_v \sigma^z_{v,v'} Z_{v'} - K \sum_v G_v - \frac{1}{K} \sum_l \sigma_l^z$$

Ordinary Ising matter symmetry $P = \prod_v X_v$

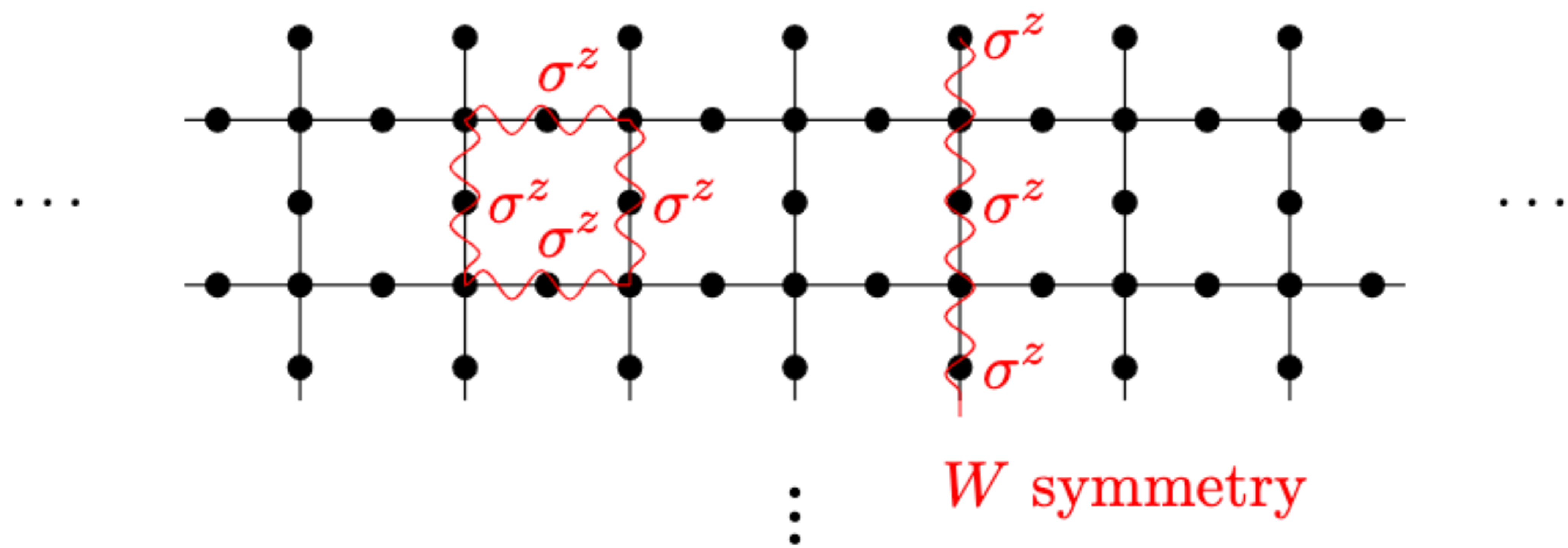
$K \to \infty$ gauged Ising model

$K \to 0$ ordinary Ising model

Magnetic symmetry

If no tension in electric strings, drop $-h\sum_l \sigma_l^x$

no vison creation or hopping



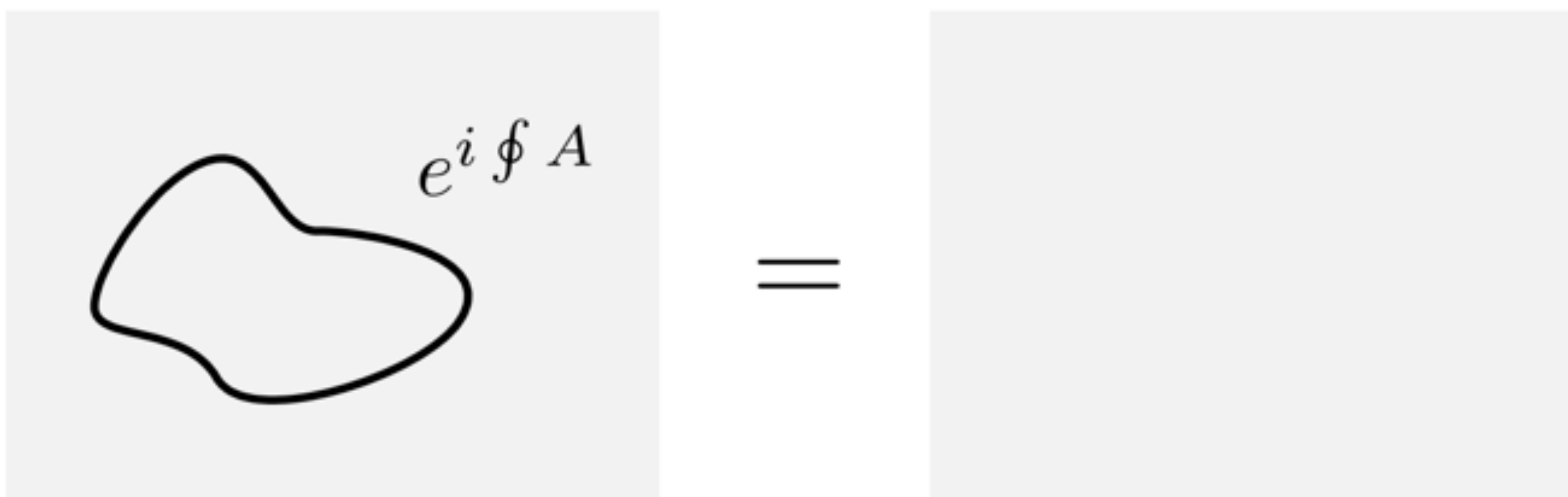
<u>1-form</u> magnetic symmetry generated by: $W_\gamma = \prod_{l\in\gamma} \sigma_l^z$

Gaiotto et al 2014

…

Deconfined phase

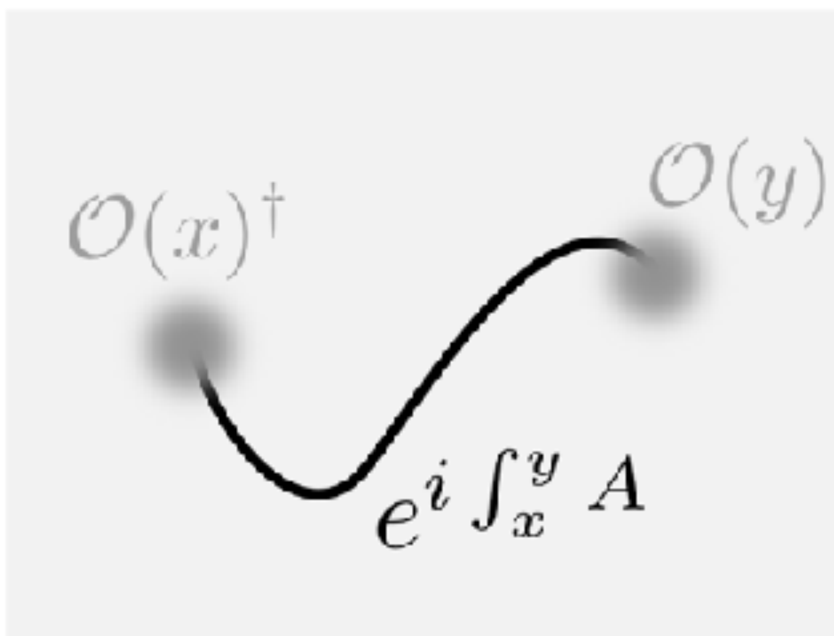
Z_2 topological order= SSB of magnetic 1-form symmetry



GSs are eigenstates of closed contractible loops,
but not of non-contractible loops

Deconfined phase

Topological order= SSB of magnetic 1-form symmetry



$\sim e^{-|x-y|/\xi}$

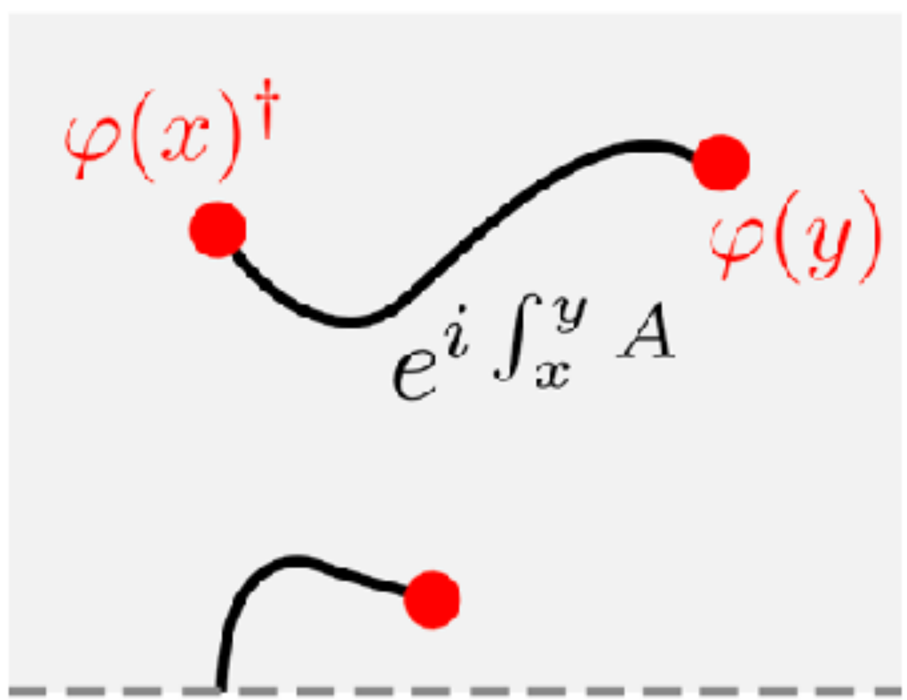
defects do not condense

robust under <u>explicit</u> breaking of magnetic symmetry!

review: McGreevy 2022

Higgs phase

In contrast to deconfined phase: charges condense

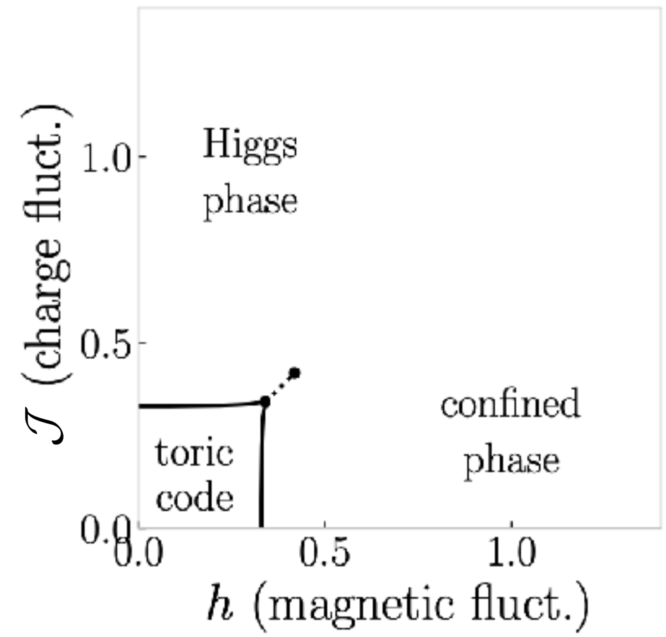


$\sim O(1)$

magnetic symmetry is not spontaneously broken

Higgs phase

In our Ising gauge theory:



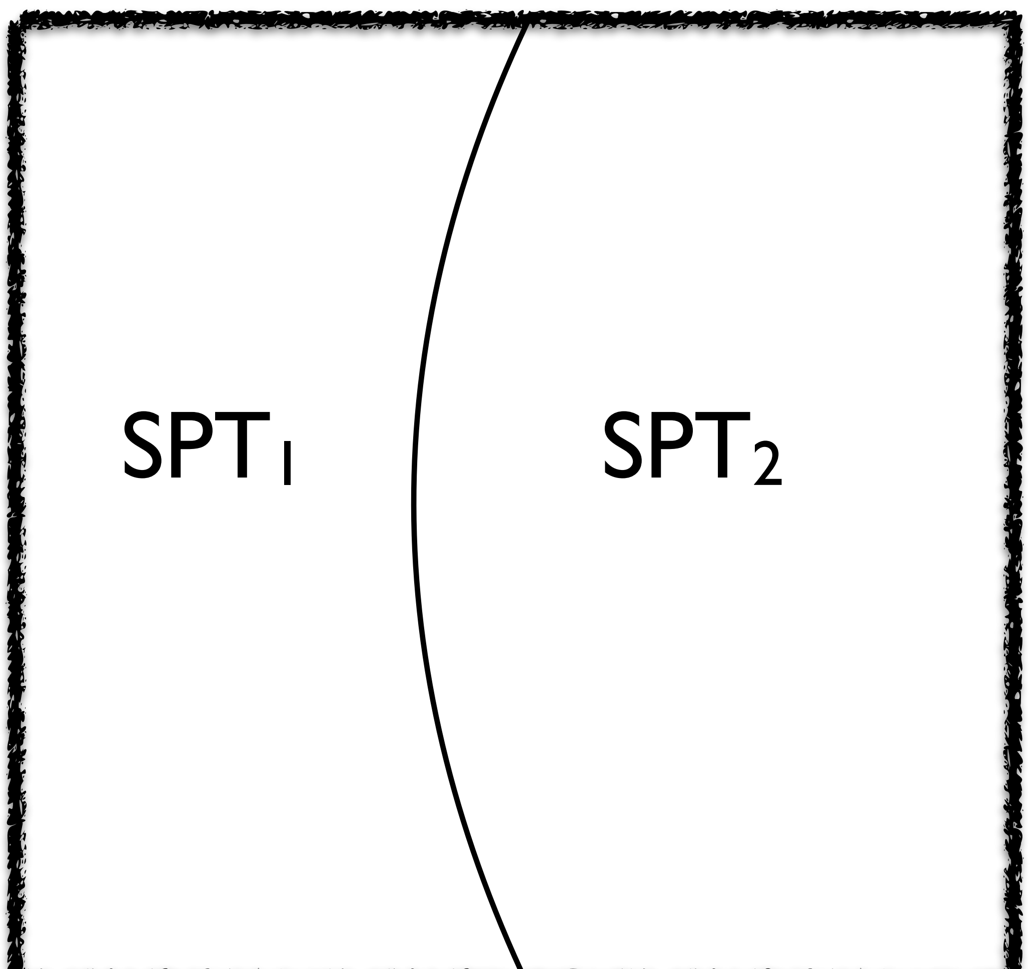
$$\left\langle Z \, \sigma^z \, \sigma^z \, \sigma^z \, \sigma^z \, \sigma^z \, Z \right\rangle \neq 0.$$

String order parameter detects SPT order

Higgs=SPT

Symmetry-protected topological order

Distinct short-range entangled states are separated by a phase transition provided symmetries are respected



Examples:

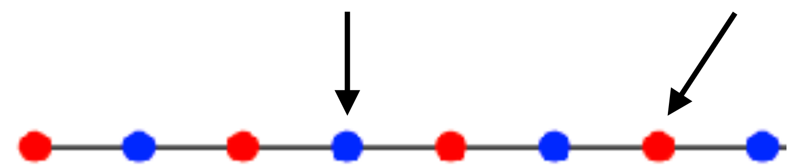
integer spin antiferromagnets

Kitaev chain

topological insulators

SPT: 1d example

1d chain of vertices and links: $\mathcal{H} = \mathcal{H}_{\text{vertices}} \otimes \mathcal{H}_{\text{link}}$



$$H = \lambda H_{\text{cluster}} + H_{\text{triv}}$$

$$H_{\text{cluster}} = -\sum_n \sigma^x_{n-1/2} X_n \sigma^x_{n+1/2} - \sum_n Z_n \sigma^z_{n+1/2} Z_{n+1}$$

$$H_{\text{triv}} = -\sum_n \left(X_n + \sigma^z_{n+1/2} \right)$$

Symmetry: $\mathbb{Z}_2 \times \mathbb{Z}_2$

$$P = \prod_n X_n \qquad W = \prod_n \sigma^z_{n+1/2}$$

SPT: 1d example

1d chain of vertices and links: $\mathcal{H} = \mathcal{H}_{\text{vertices}} \otimes \mathcal{H}_{\text{link}}$

$$H = \lambda H_{\text{cluster}} + H_{\text{triv}}$$

$$H_{\text{cluster}} = -\sum_n \sigma^x_{n-1/2} X_n \sigma^x_{n+1/2} - \sum_n Z_n \sigma^z_{n+1/2} Z_{n+1}$$

$$H_{\text{triv}} = -\sum_n \left(X_n + \sigma^z_{n+1/2} \right)$$

ground state respects the symmetry

trivial | SPT

λ

$\left\langle \prod_{i \le n < j} \sigma^z_{n+1/2} \right\rangle$ | $\lambda = 1$ | $\left\langle Z_i \prod_{i \le n < j} \sigma^z_{n+1/2} Z_j \right\rangle$

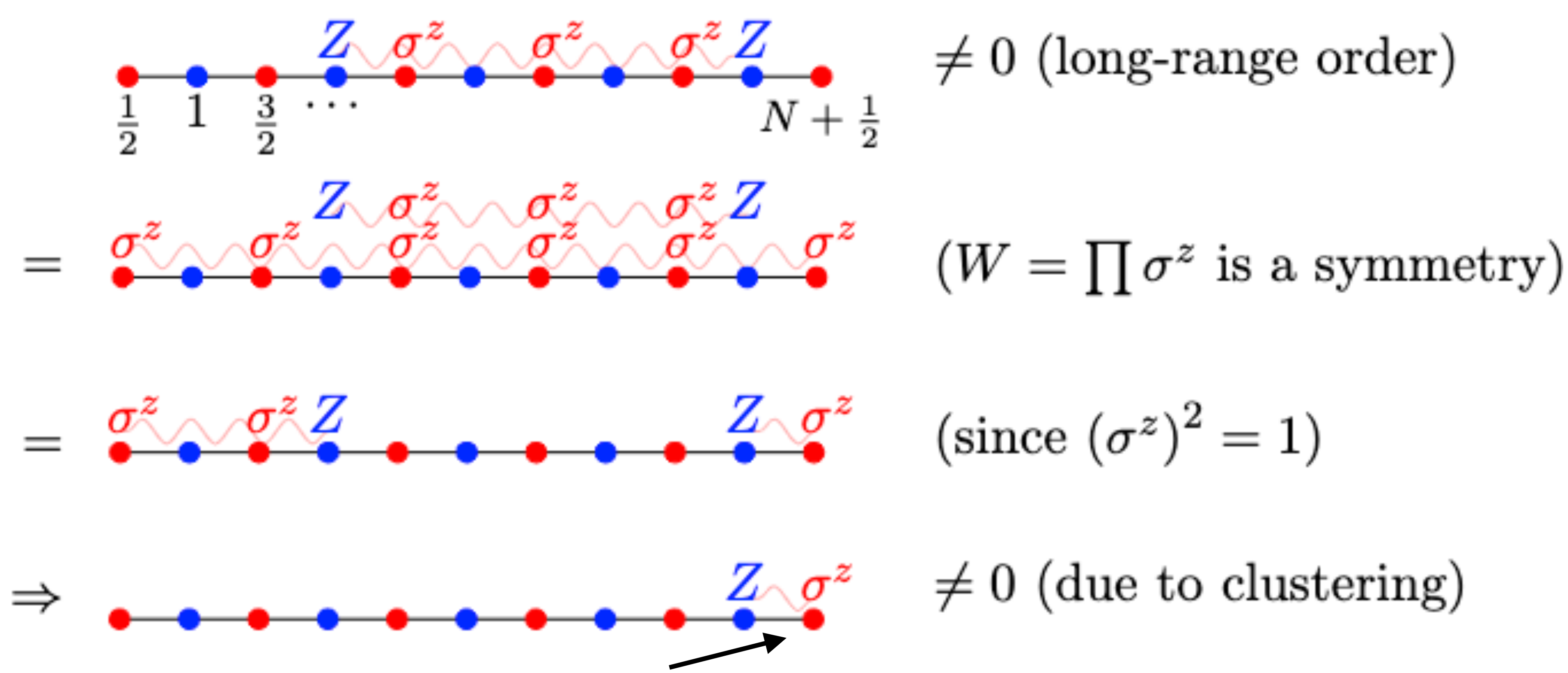
odd under P

SPT: 1d example

1d chain of vertices and links: $\mathcal{H} = \mathcal{H}_{\text{vertices}} \otimes \mathcal{H}_{\text{link}}$

$$H_{\text{cluster}} = -\sum_n \sigma^x_{n-1/2} X_n \sigma^x_{n+1/2} - \sum_n Z_n \sigma^z_{n+1/2} Z_{n+1}$$

Edge modes:



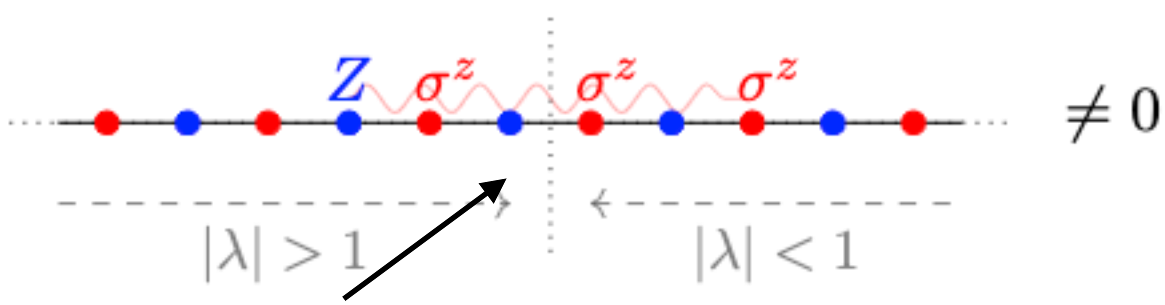
anticommutes with $P = \prod_n X_n$ $\longrightarrow$ GS edge degeneracy

SPT: 1d example

1d chain of vertices and links: $\mathcal{H} = \mathcal{H}_{\text{vertices}} \otimes \mathcal{H}_{\text{link}}$

$$H = \lambda H_{\text{cluster}} + H_{\text{triv}}$$

Cluster-trivial interface:



anticommutes with P $\longrightarrow$ GS edge degeneracy

Edge modes are robust manifestations of SPT

1d gauged Ising

$$H = -\mathcal{J}\sum_n Z_n \sigma^z_{n+1/2} Z_{n+1} - \sum_n X_n$$

with Gauss law: $G_n = \sigma^x_{n-1/2} X_n \sigma^x_{n+1/2} = 1$

Higgs phase $\mathcal{J} \gg 1$ is SPT phase

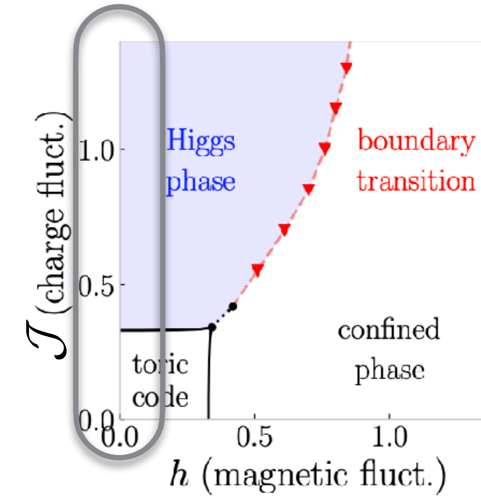
with Borla, Verresen, Shah SciPost 2021

while the phase with $\mathcal{J} \ll 1$ breaks spontaneously W

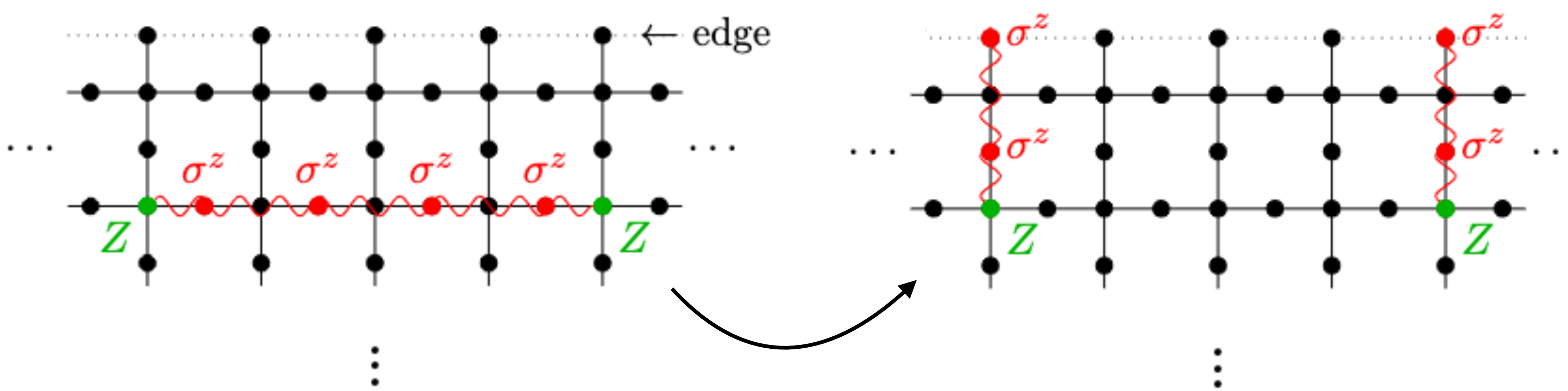
Finite electric string tension $\longrightarrow$ no SPT order

$$-h\sum_n \sigma^x_{n+1/2}$$

2d gauged Ising



Set first string tension to zero, h=0



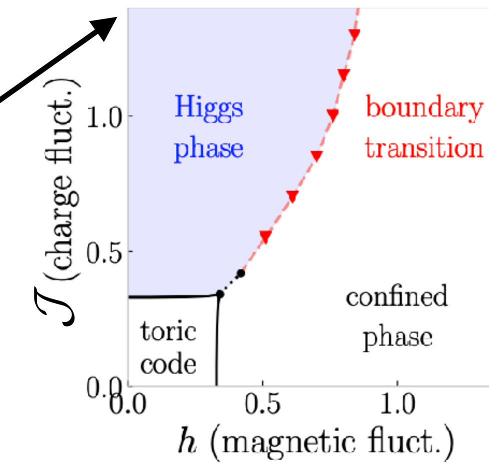
Local order parameter develops long-range order!

Matter symmetry $P = \prod_{v\in\Lambda} X_v$ is broken spontaneously in the Higgs phase

? How robust is it?

No magnetic symmetry

Go deep into the Higgs phase



$\mathcal{J} \to \infty$

$Z_v \sigma^z_{v,v'} Z_{v'} \to 1$

in the bulk

$$\lim_{\mathcal{J}\to\infty} U \left(H_{\text{open b.c.}}\right) U^\dagger = \underbrace{-\mathcal{J}\sum_{l\notin\partial\Lambda}\sigma^z_l}_{\text{bulk}} \underbrace{-J\sum_{\langle l,l'\rangle\in\partial\Lambda}\sigma^z_l\sigma^z_{l'} - h\sum_{l\in\partial\Lambda}\sigma^x_l}_{\text{boundary Ising model}}$$

Bulk is fully frozen, but at boundary we have Ising model

SSB of P-symmetry — P-symmetric GS

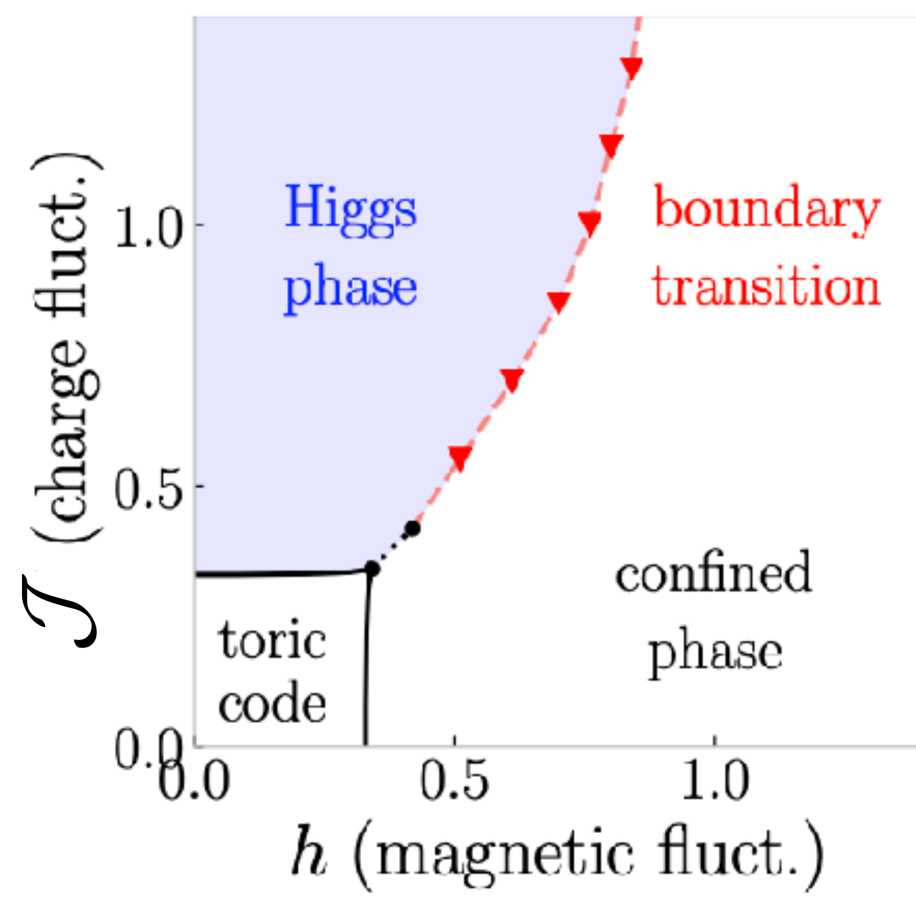
Higgs — confined

$\frac{h}{J}$

Robustness of edge modes

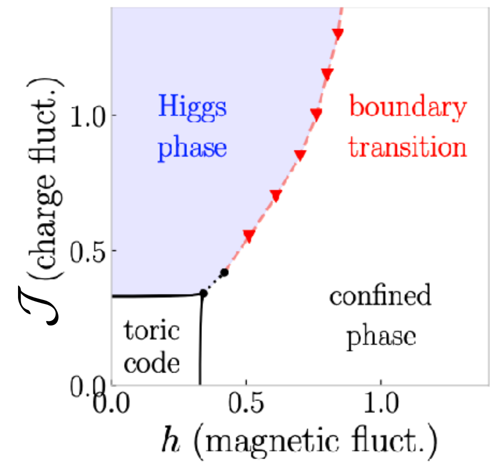
SSB of P-symmetry is robust under W-breaking terms

DMRG on infinite strip L=3



Higgs and confined phases separated by the Ising <u>boundary phase transition</u>

It matters which symmetry is broken



Start with exact magnetic and matter symmetries:

exact boundary degeneracy: $|\ldots\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\ldots\rangle, |\ldots\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\ldots\rangle$

- <u>magnetic symmetry breaking</u>: local vison pair creation

exp small energy splitting
parametrically suppressed by vison gap

- <u>matter symmetry breaking</u>: push charge through boundary

energy splitting is
linear in the perturbation strength

Anomaly

Bulk SPT $\longrightarrow$ boundary anomaly

as a result: no symmetric, short-range entangled state on the boundary

SSB or gapless or topological order

In the Higgs phase: $\mathbb{Z}_2 \times \mathbb{Z}_2[d-1]$ anomaly

due to anticommutation of the two symmetries on the boundary

Conclusions

- Discrete gauge theories can provide valuable insights
- Higgs=SPT in Ising gauge theory
- Generalizes to other gauge theories:
  U(1), non-abelian, …
- New insights into superconductors, quark-hadron continuity?