A Finite Temperature Meta-Model for Nuclear Matter and Neutron Stars

Gabriele Montefusco
LPC CAEN - CNRS

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Meta-modelling of the EoS

Originally presented in [PRC 97, 025805 (2018)]

Parametric representation of the energy density $\epsilon_X(n_n, n_p, ...)$ as a function of the different species

The variation of X makes possible to explore the largest EoS space compatible with the composition hypothesis

Both nuclear and Astro observables are accessible
with minimal microscopic bias

r_i, BE_i, GR_i ⇒ $e_X(n_n, n_p, ...)$ ⇐ M, R, Λ

Nuclear observables

Astro observables

Asymptotically causal meta-model: Analytical representation of the energy density

Montefusco et al [arXiv:2604.00196]

Causality asymptotically implemented

Starting ansatz:

$$\epsilon(n, x_e, x_\mu) = \epsilon_k(n, x_e, x_\mu) + n\left[e_0(n) + \delta^2 e_2(n) + \delta^4 e_4(n)\right]$$

Nuclear asymmetry

$$\delta = 1 - 2(x_e + x_\mu)$$

Quartic correction

$$e_4(n) = A\frac{n/n_0}{1 + \left(n/n_0\right)^B}$$

free Fermi gas energy density for $npe\mu$ matter

Nucleonic Potential (per baryon)

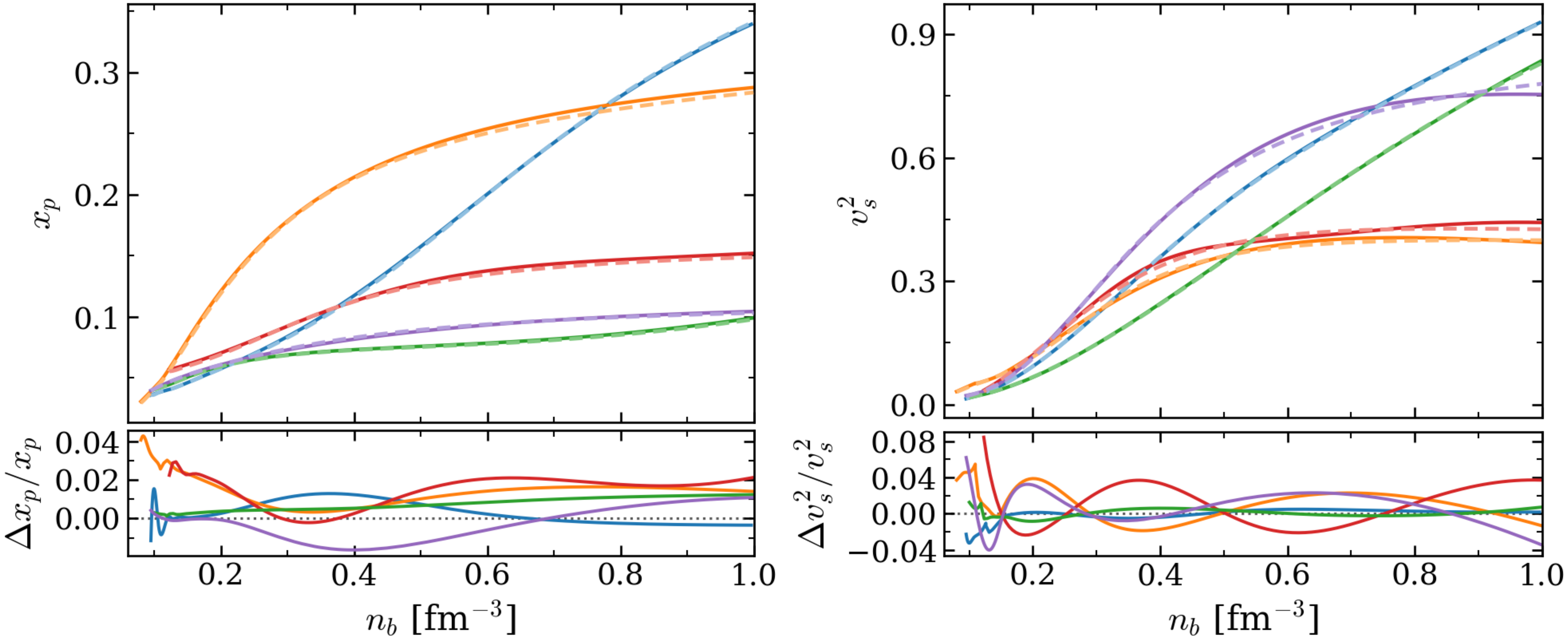
$$e_0(x) = V_0(x) + \frac{h_0 + h_1 x + h_2 x^2 + h_3 x^3}{(1 + a_0 x)(1 + b_0 x)(1 + c_0 x)}$$

Asymptotically causal meta-model: EoS reconstruction

Test the **flexibility** of the model to reproduce β-equilibrated EoS

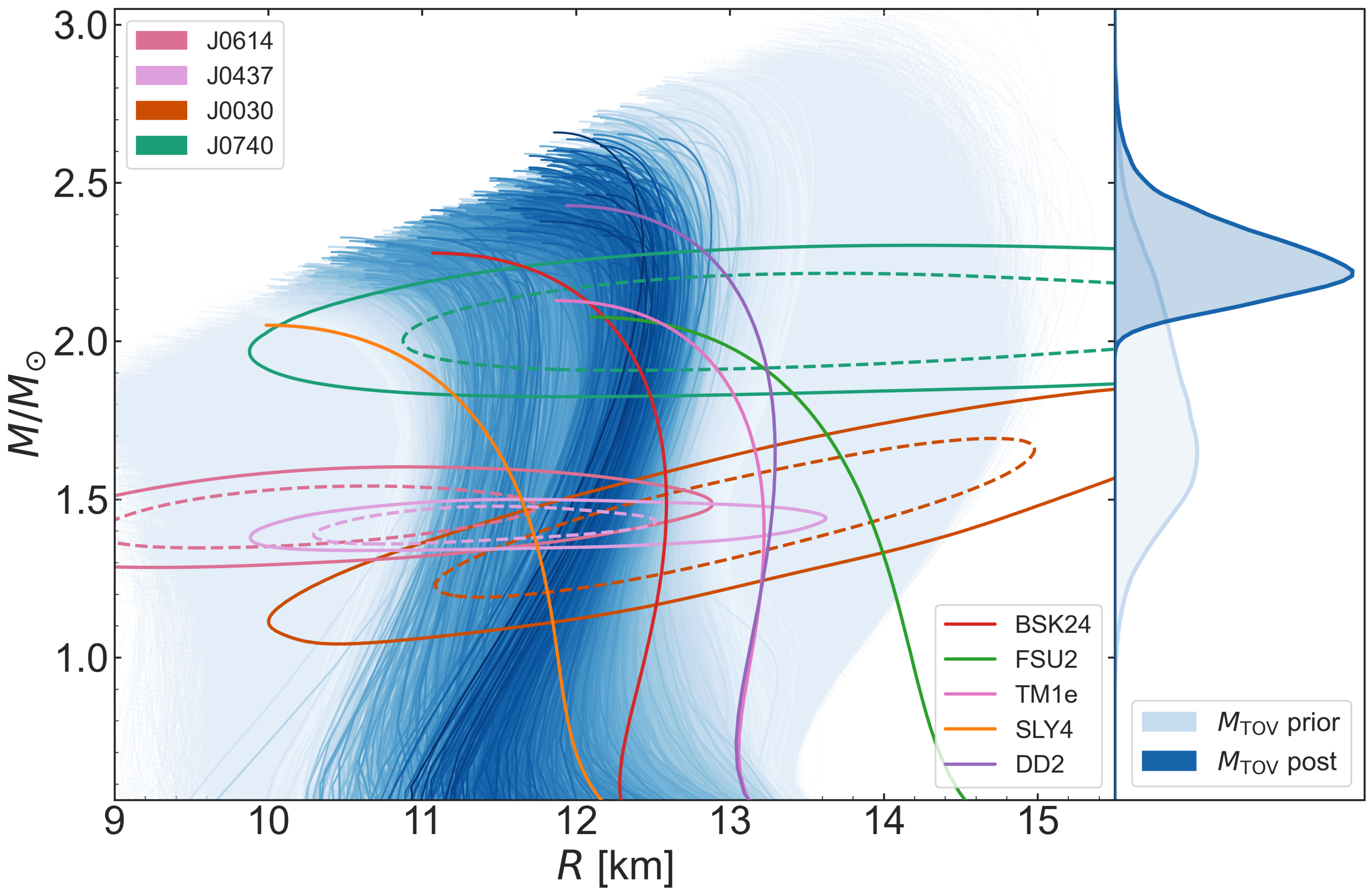
Constrain the space of the auxiliary parameters

We have chosen: SLy4, BSk24, DD2, FSU2 and TM1e



Asymptotically causal meta-model: Mass - Radius

We perform **Bayesian inference** with nuclear and astro information

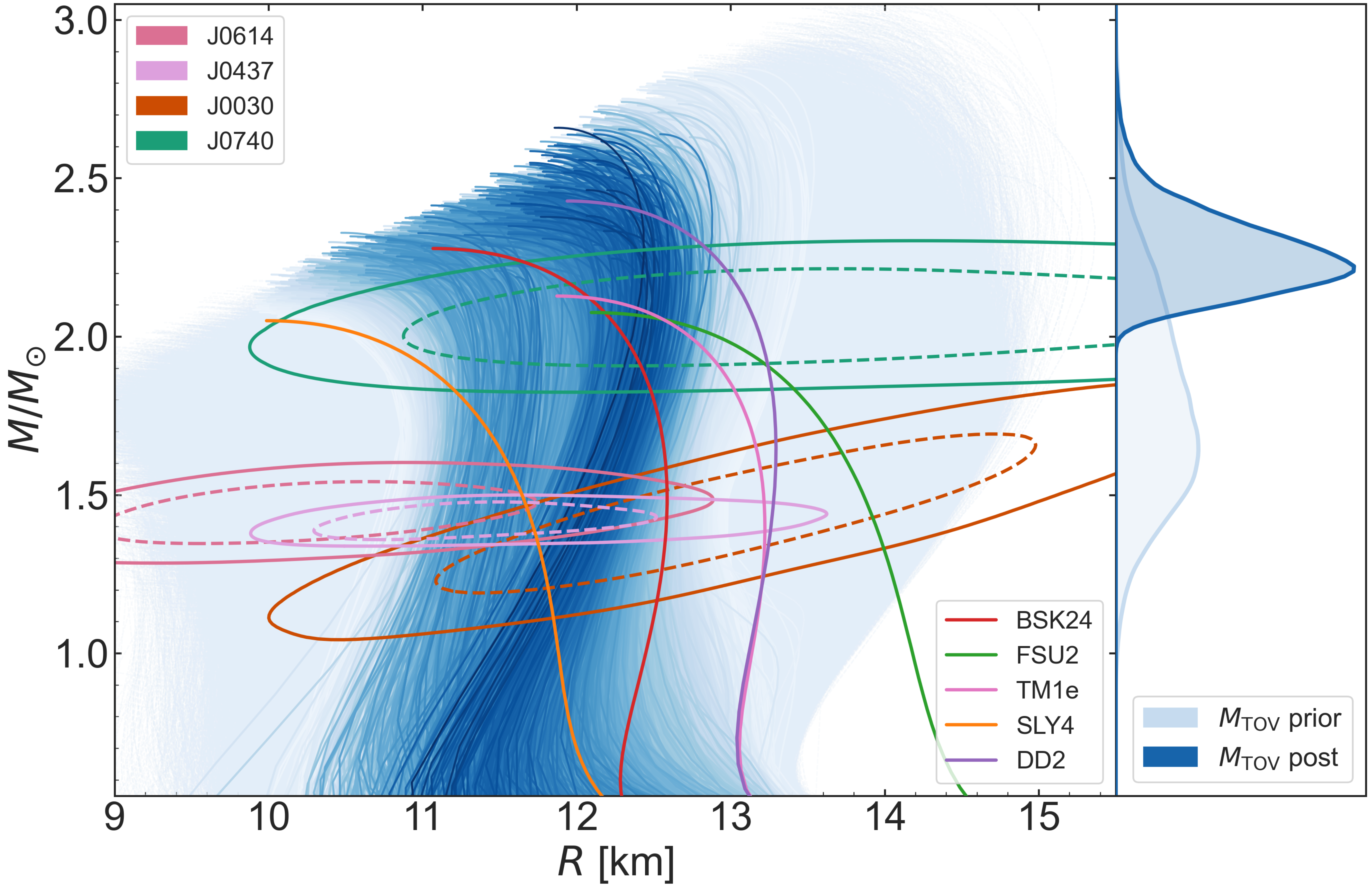


Asymptotically causal meta-model: Mass - Radius

We perform **Bayesian inference** with nuclear and astro data

The model covers a range of masses, radii and tidal similar to agnostic models

We obtain a **large set of composition-aware EoS** constrained by nuclear and astro observations



Symmetry-energy constraint suppresses direct Urca cooling

χ_{EFT}

[Phys. Rev. C 103, 025803 (2021)]
Pure neutron matter information

Heavy-ion data

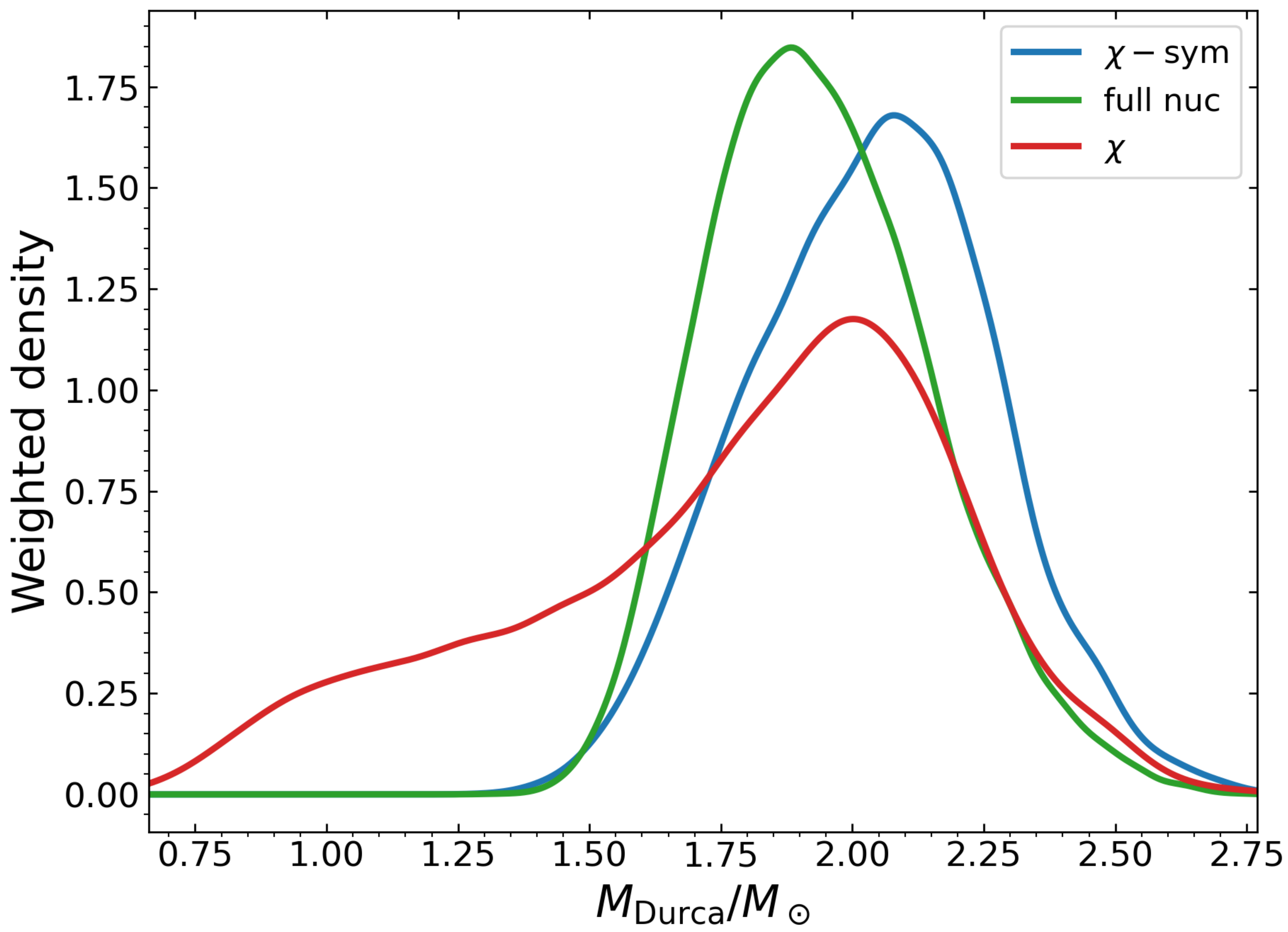
[PLB:139815]

Symmetry energy constraint

Nuclear structure

[arXiv:2604.11358]
Symmetric matter and low asymmetry around saturation

Montefusco et al [in prep]



How to add finite T without losing flexibility and analyticity?

Hadronic thermal contribution in mean-field approximation

Cold Static Potential

zero-temperature interaction sector fixed by the cold meta-model

Thermal Quasi-particle

finite-T quasi-particle gas with effective masses $m_q^*(n_B, \delta)$

Finite-T free energy

$$f^{MM}(n_B, \delta, T) \;=\; v^{MM}(n_B, \delta) \;+\; f_{\rm kin}(n_B, \delta, T)$$

The thermal contribution is fully controlled by the effective mass

Hadronic thermal contribution in mean-field approximation

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Thermal Quasi-particle

finite-T quasi-particle gas with effective masses $m_q^*(n_B, \delta)$

Finite-T free energy

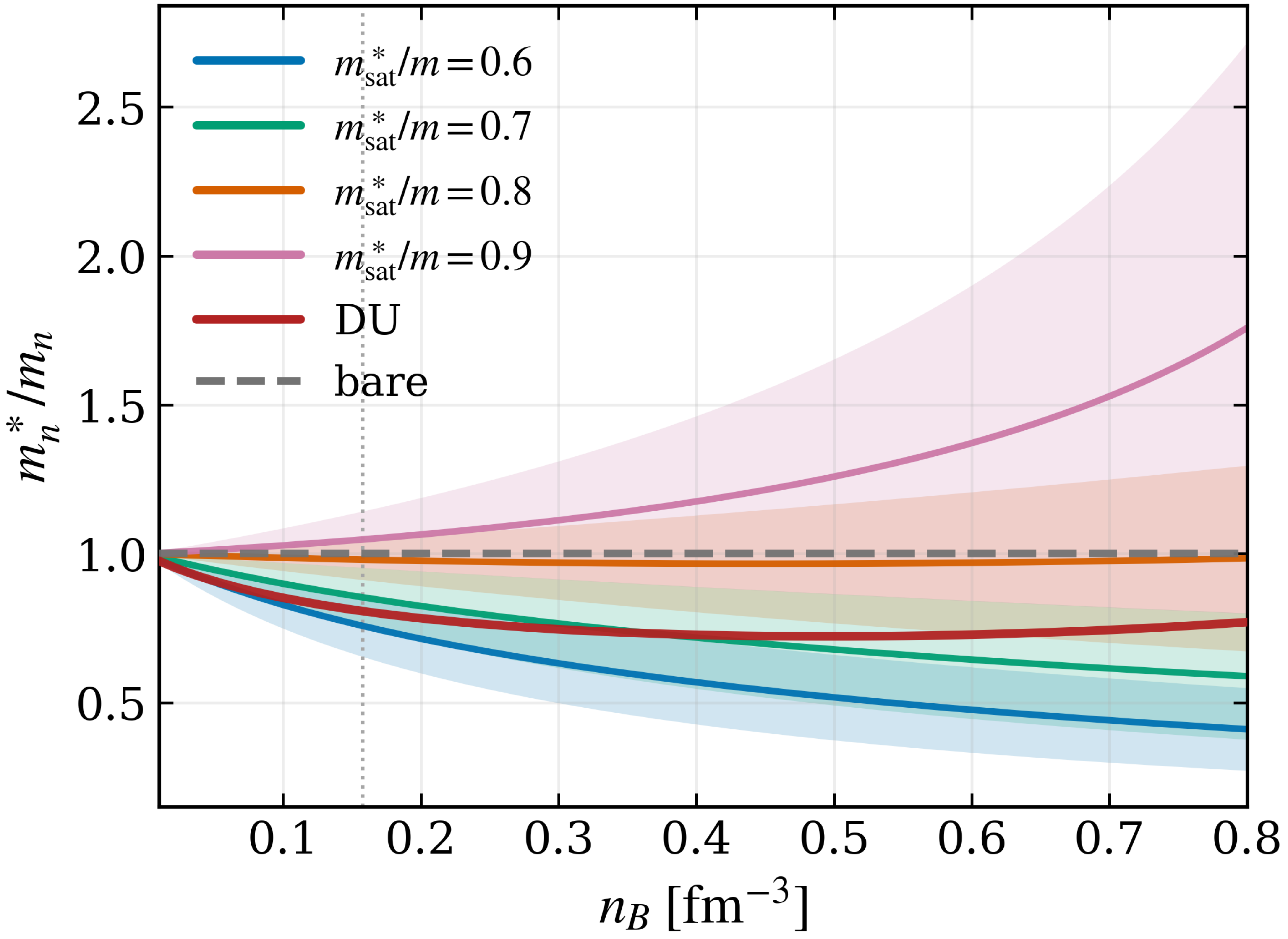
$$f^{MM}(n_B, \delta, T) \; = \; v^{MM}(n_B, \delta) \; + \; f_{\text{kin}}(n_B, \delta, T)$$

This term involves Fermi integrals, we would like to make it a **flexible analytical parametrization** dependent only on $m^*(n_B, \delta)$

Effective Mass Models and cold baseline

We want to isolate the impact of different m^* prescriptions at finite temperature:

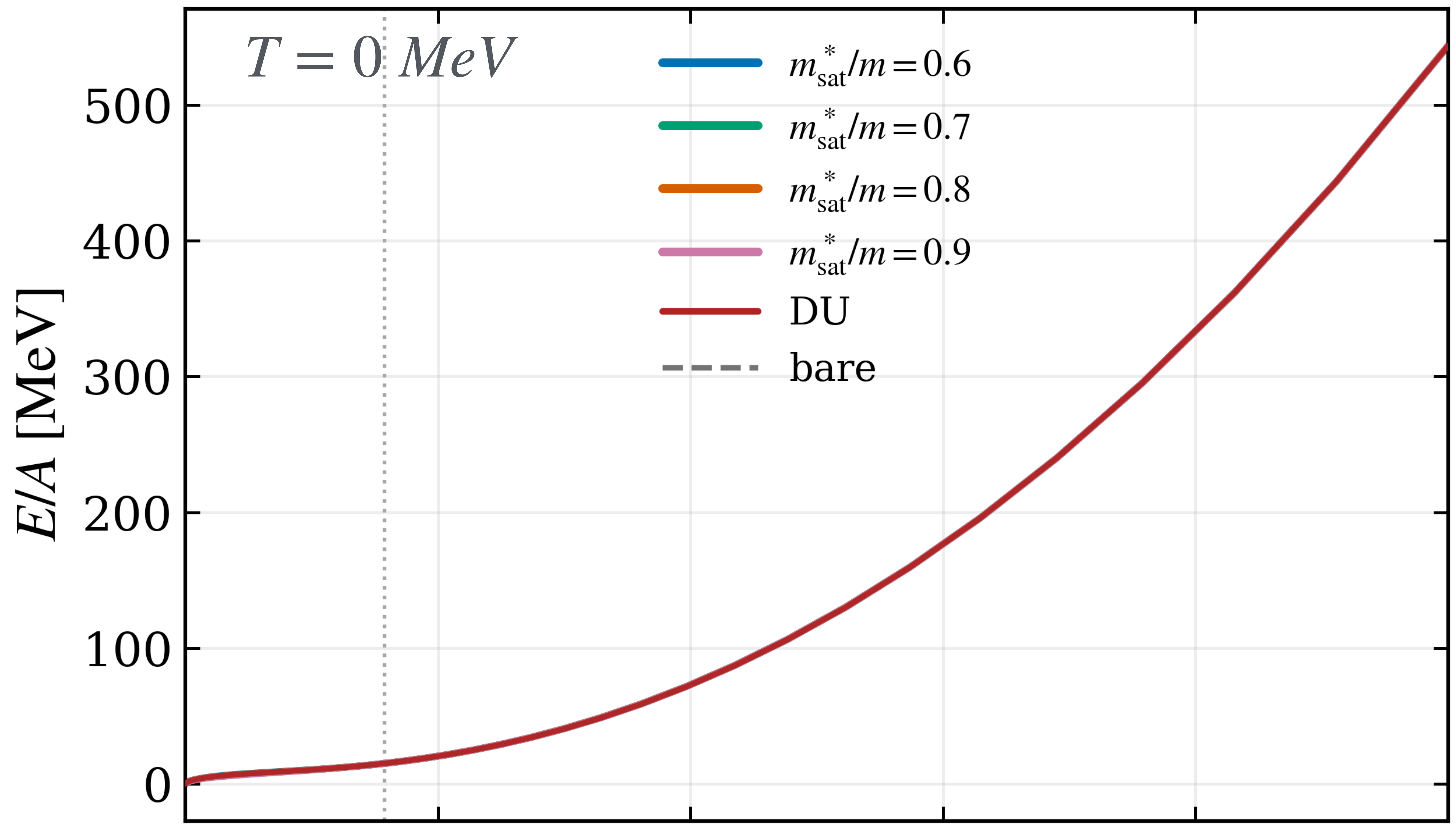
- The bare mass as a reference
- A phenomenological linear family labelled by m^*_{sat}/m
- A fit to BHF microscopic calculation by Duan et Urban [PhysRevC.110.065806] labelled DU



Effective Mass Models and cold baseline

We want to isolate the impact of different m^* prescriptions at finite temperature

We fit the same nuclear model at zero temperature varying m^*



Finite temperature effects

We want to isolate the impact of different m^* prescriptions at finite temperature

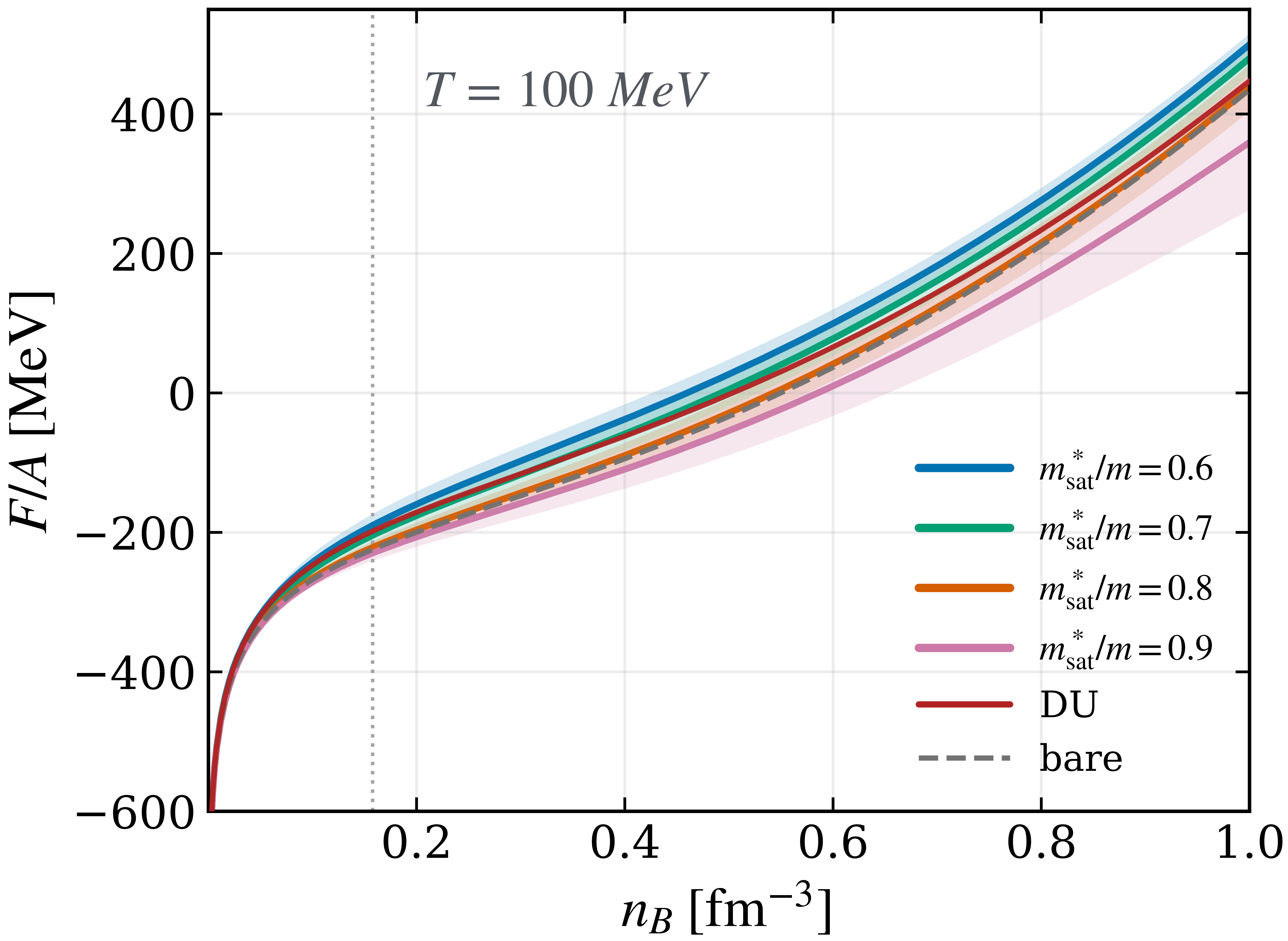
We fit the same nuclear model at zero temperature varying m^*

We explore:

$10^{-8} < n_B < 1 \ \ fm^{-3}$

$0.5 \leq T \leq 100$ MeV

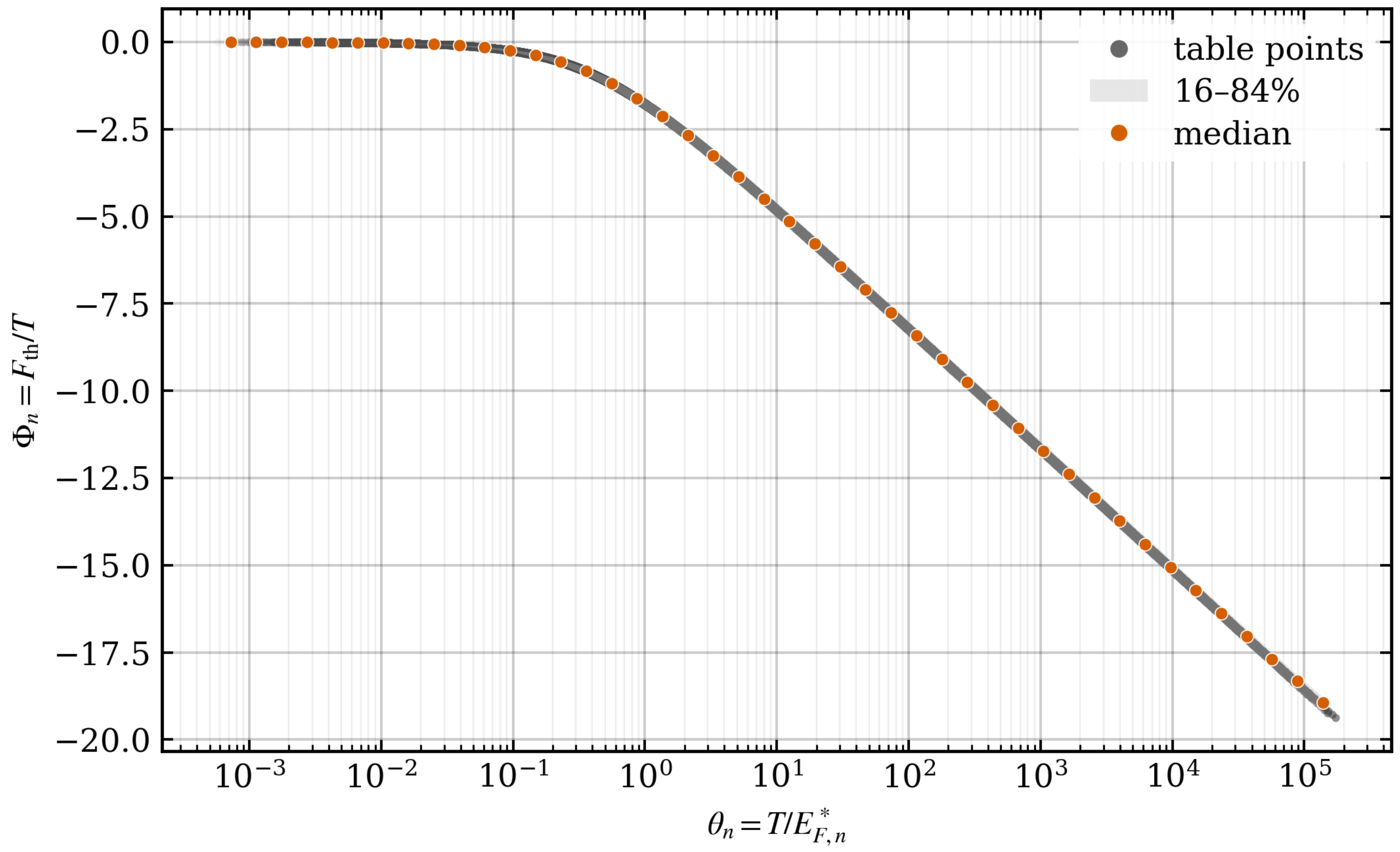
$x_p = 0$ $x_p = 0.25$ $x_p = 0.5$



F_{th}/T collapses onto one effective-mass-scaled curve

We found a quasi-universal collapse plotting:

$$\Phi(\theta) = \frac{F_{th}/A}{T} \qquad \theta \equiv \frac{T}{E_F^*},$$



F_{th}/T collapses onto one effective-mass-scaled curve

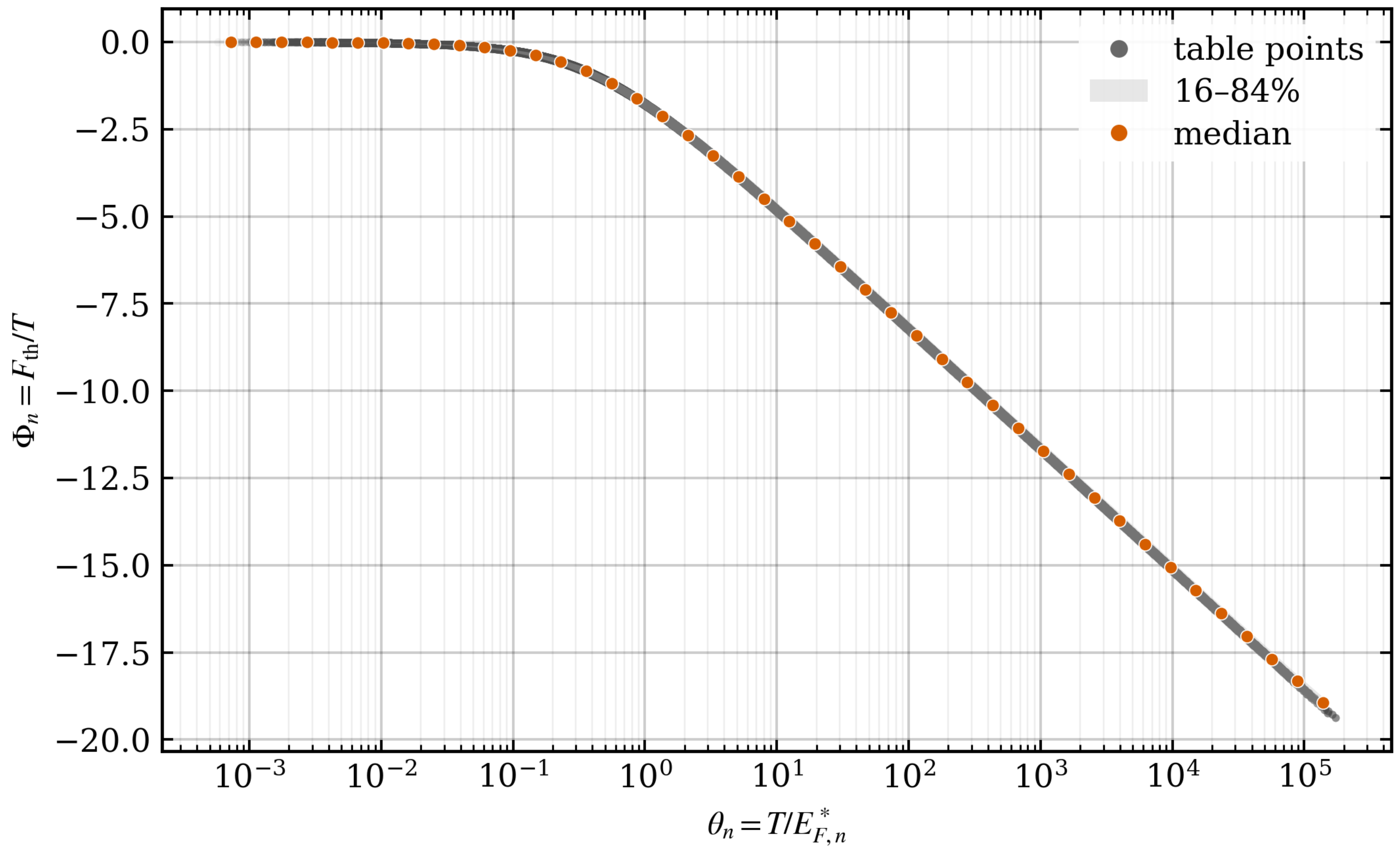
We found a quasi-universal collapse plotting:

$$\Phi(\theta) = \frac{F_{th}/A}{T} \qquad \theta \equiv \frac{T}{E_F^*},$$

We propose an analytical ansatz to reproduce the curve

$$\Phi(\theta) = -\frac{\pi^2}{4}\frac{\theta}{\sqrt{1+a\theta^2}} - \frac{3}{2p}\ln\left(1+b\theta^p\right) - \frac{3}{5}\frac{\theta^3}{\theta^4+\eta^4} + R_N(\theta)$$

This function is constructed to reproduce the degenerate and classical limit by construction



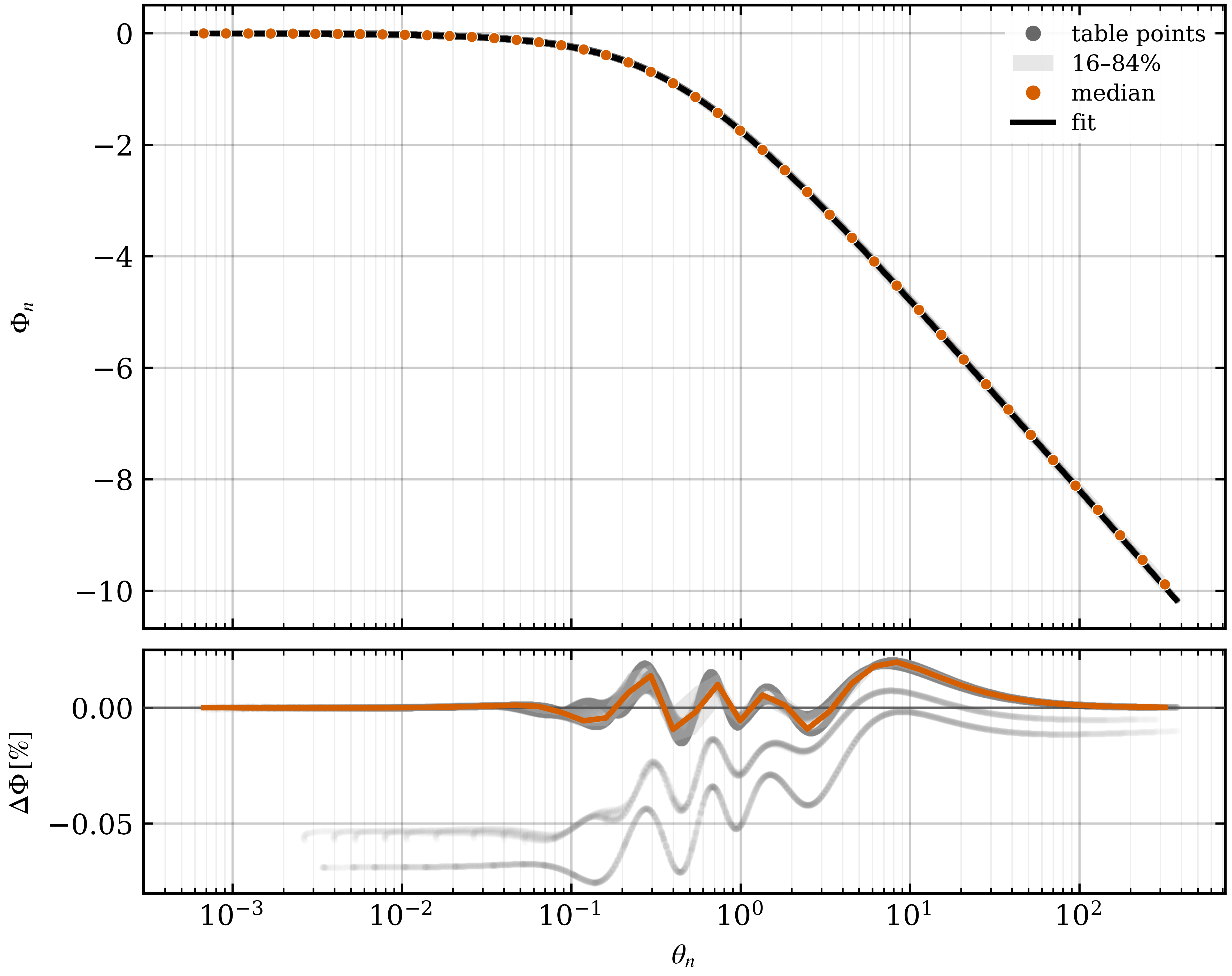
Fitting the quasi-universal curve

Since the thermal contribution is species-additive we fit:

$$\frac{F_{\rm th}^{\rm fit}}{A} = T \sum_{q=n,p} x_q \Phi_N(\theta_q) \qquad x_q = \frac{n_q}{n_B}$$

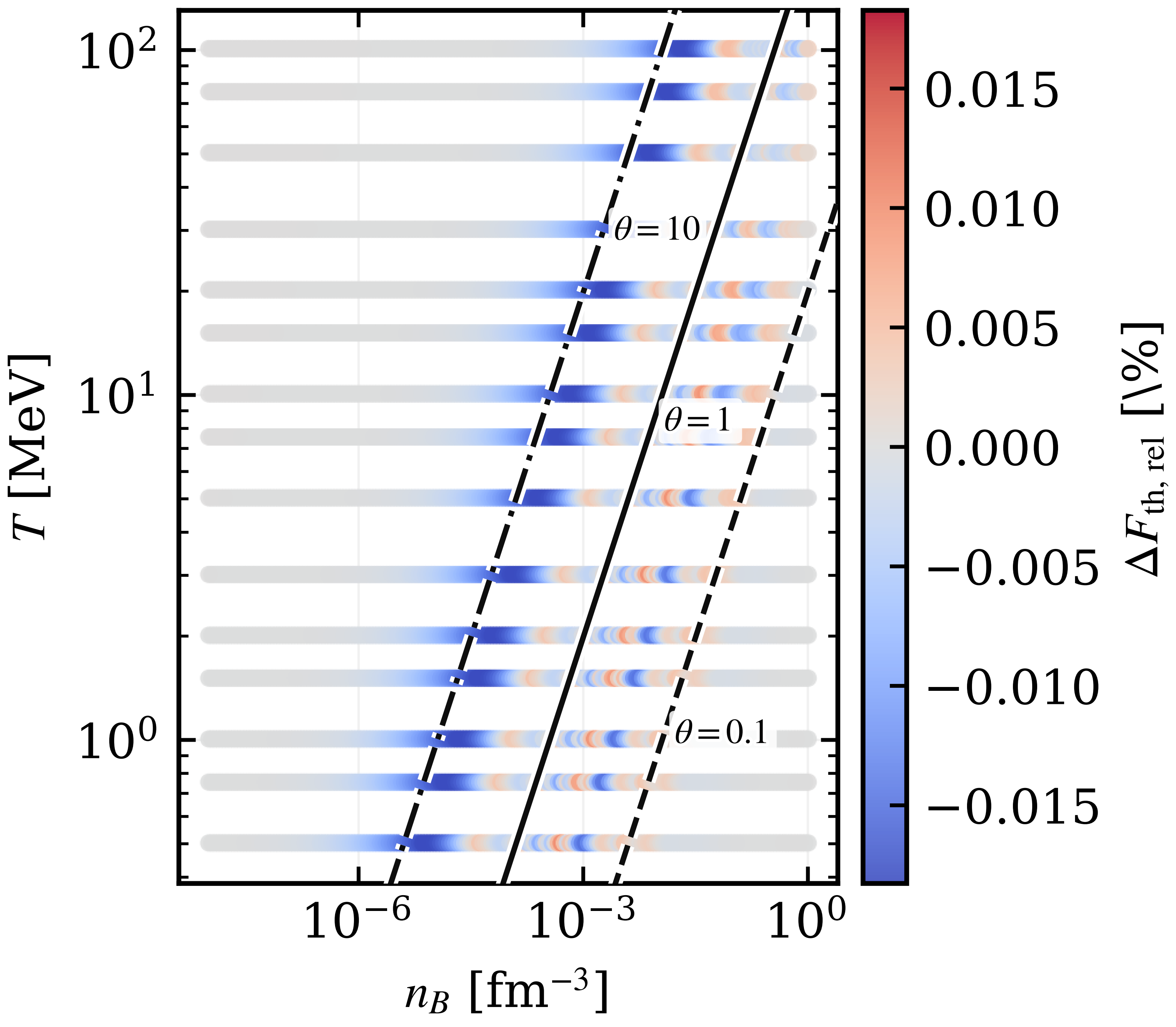
One fitted function Φ_N, applied species by species, reproduces F_{th} at the $\sim 10^{-4}$ level

The two offset curves which display $\Delta\Phi \sim 0.05\,\%$ correspond to the microscopic m^* with $x_p \neq 0$



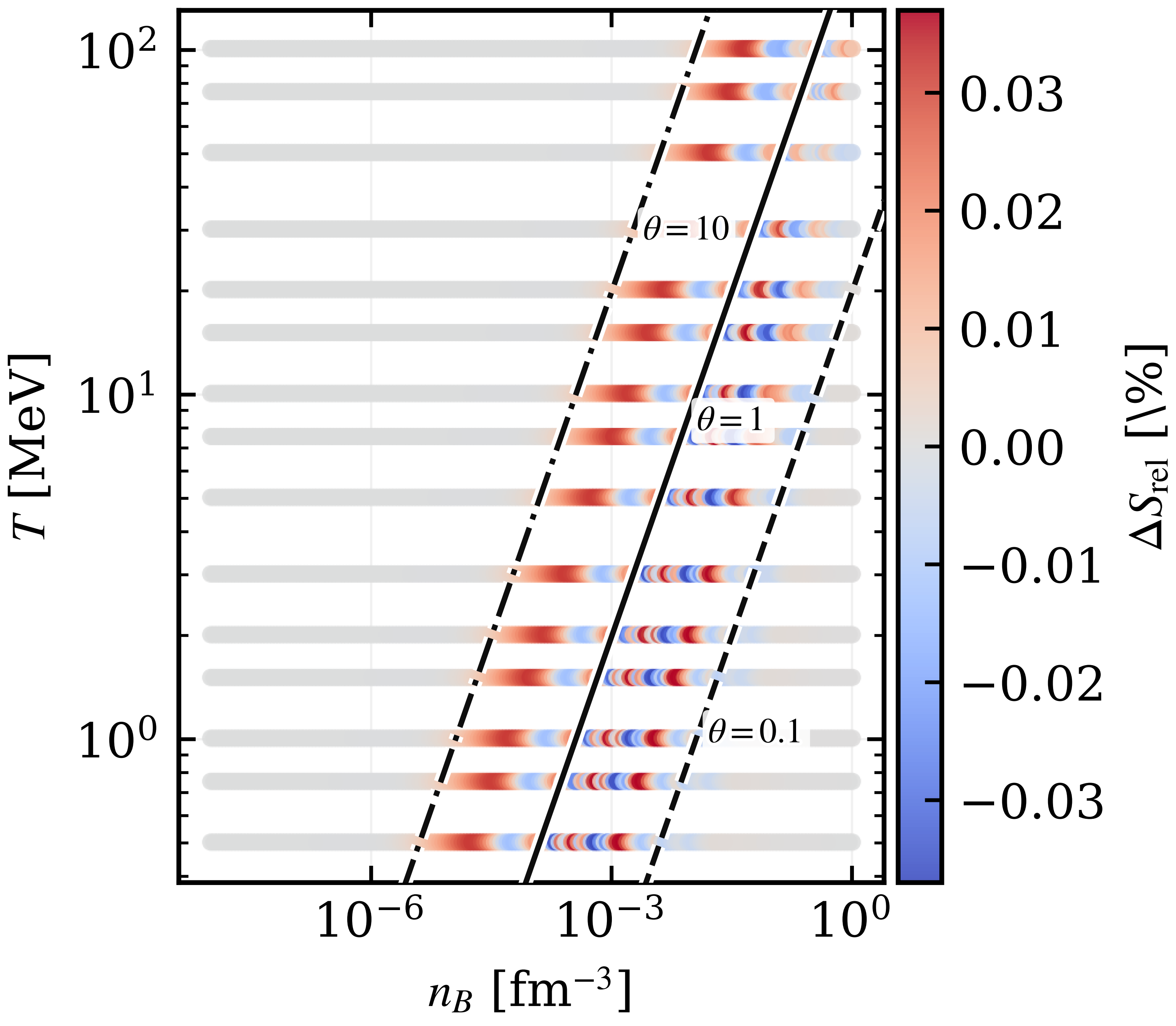
One fitted potential generates all thermal quantities

Thermal free energy



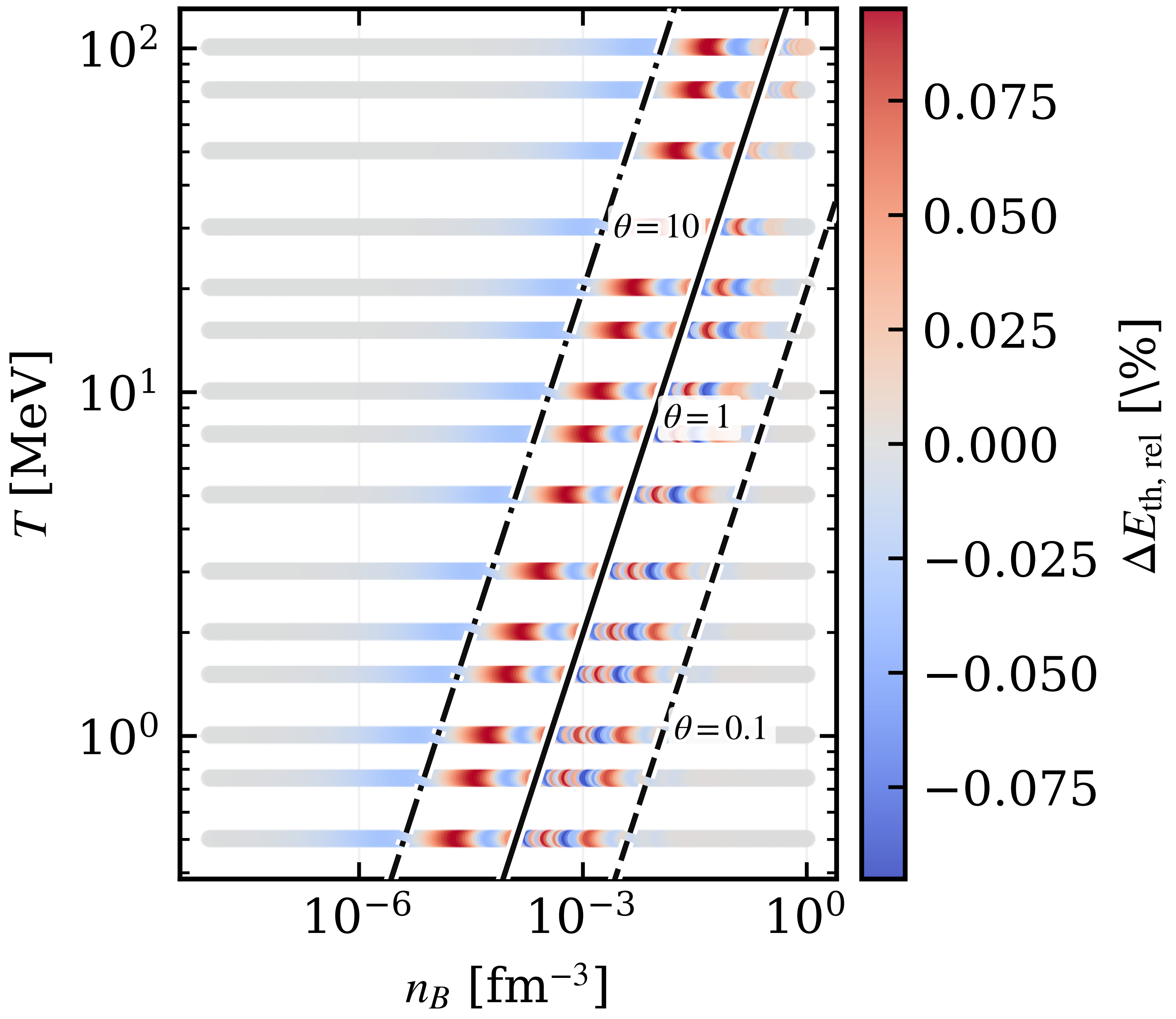
Entropy

$$s = -\left(\frac{\partial F}{\partial T}\right)_{n_B,\delta}$$



Thermal energy

$$E_{th} = F_{th} + Ts$$



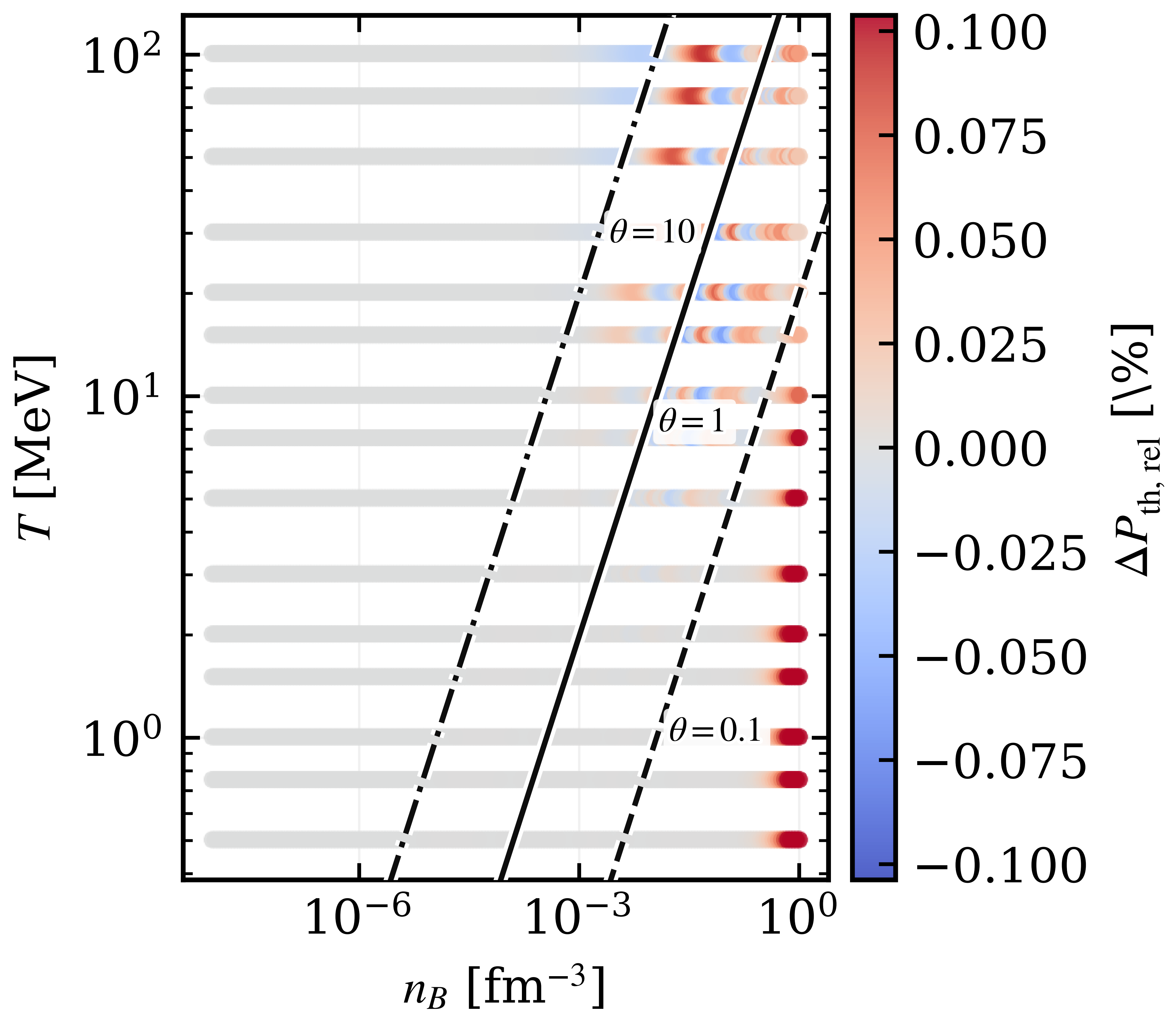
Pressure is the stringent test: it differentiates the effective mass

The pressure can be reconstructed from:

$$P_{\rm th}^{\rm fit} = -\, n_B^2 T \sum_{q=n,p} x_q \theta_q \Phi_N'(\theta_q) \left(\frac{\partial \ln E_{F,q}^*}{\partial n_B} \right)_{x_p}$$

It involves density derivative of the the effective Fermi energy, and thus of m_q^*

The relative error remains below $\sim 0.1\,\%$



Final result: an analytic finite-T generating potential

We can finally write an **analytical finite-T hadronic sector of the MM**

$$f(n_n, n_p, T) = f_0(n_n, n_p) + f_{\rm th}(n_n, n_p, T),$$

with

$$f^{TH}(n_n, n_p, T) = T \sum_{q=n,p} n_q \, \Phi_N(\theta_q)$$

and

$$\theta_q = \frac{T}{E^*_{F,q}} \qquad E^*_{F,q} = \frac{\hbar^2}{2m^*_q(n_n, n_p)} \left(3\pi^2 n_q\right)^{2/3}$$

This provides a **single flexible generating potential** from which **all finite-T thermodynamic quantities can be derived analytically and consistently**

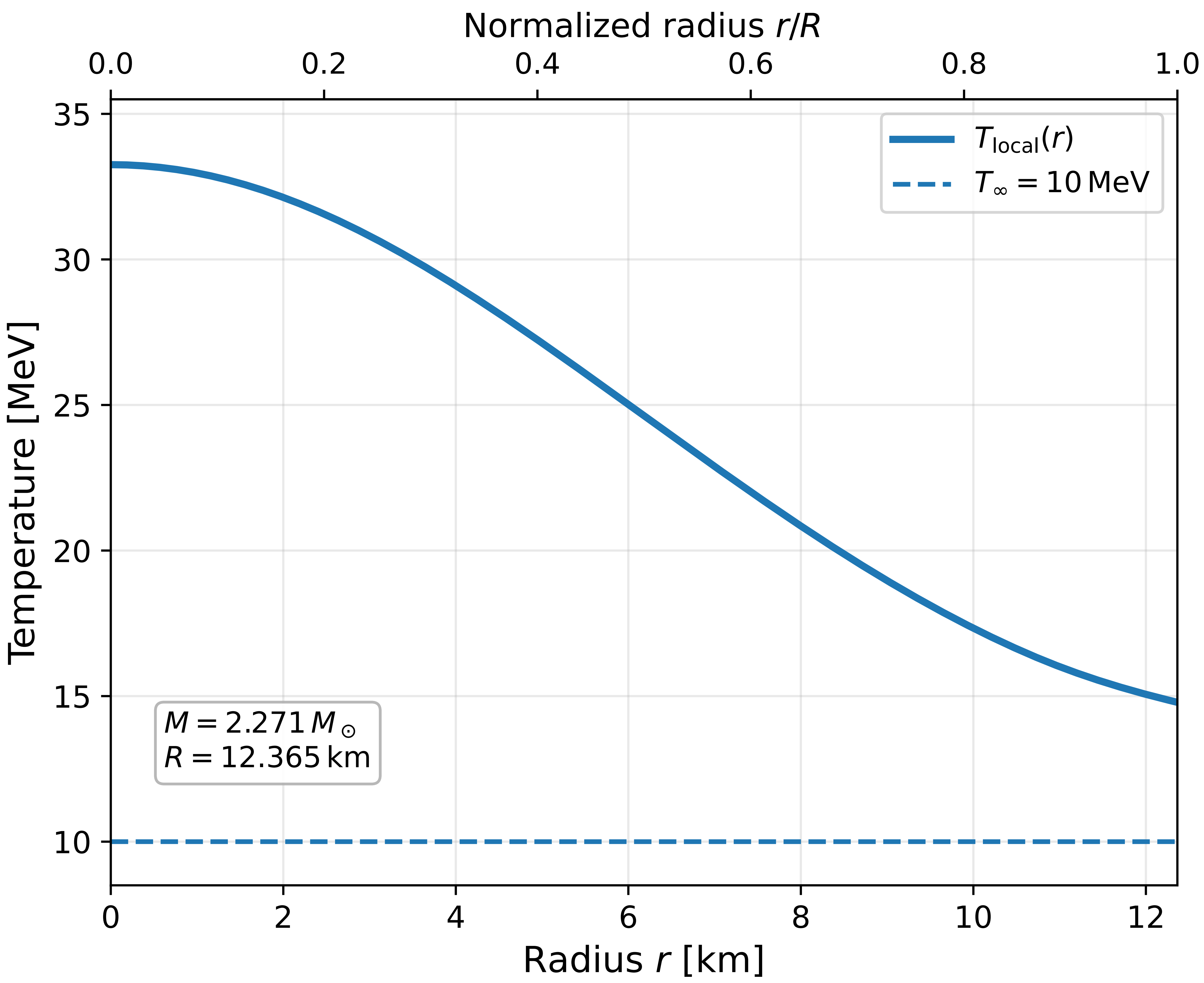
An illustrative application: Isothermal star

We consider a red-shifted isothermal star

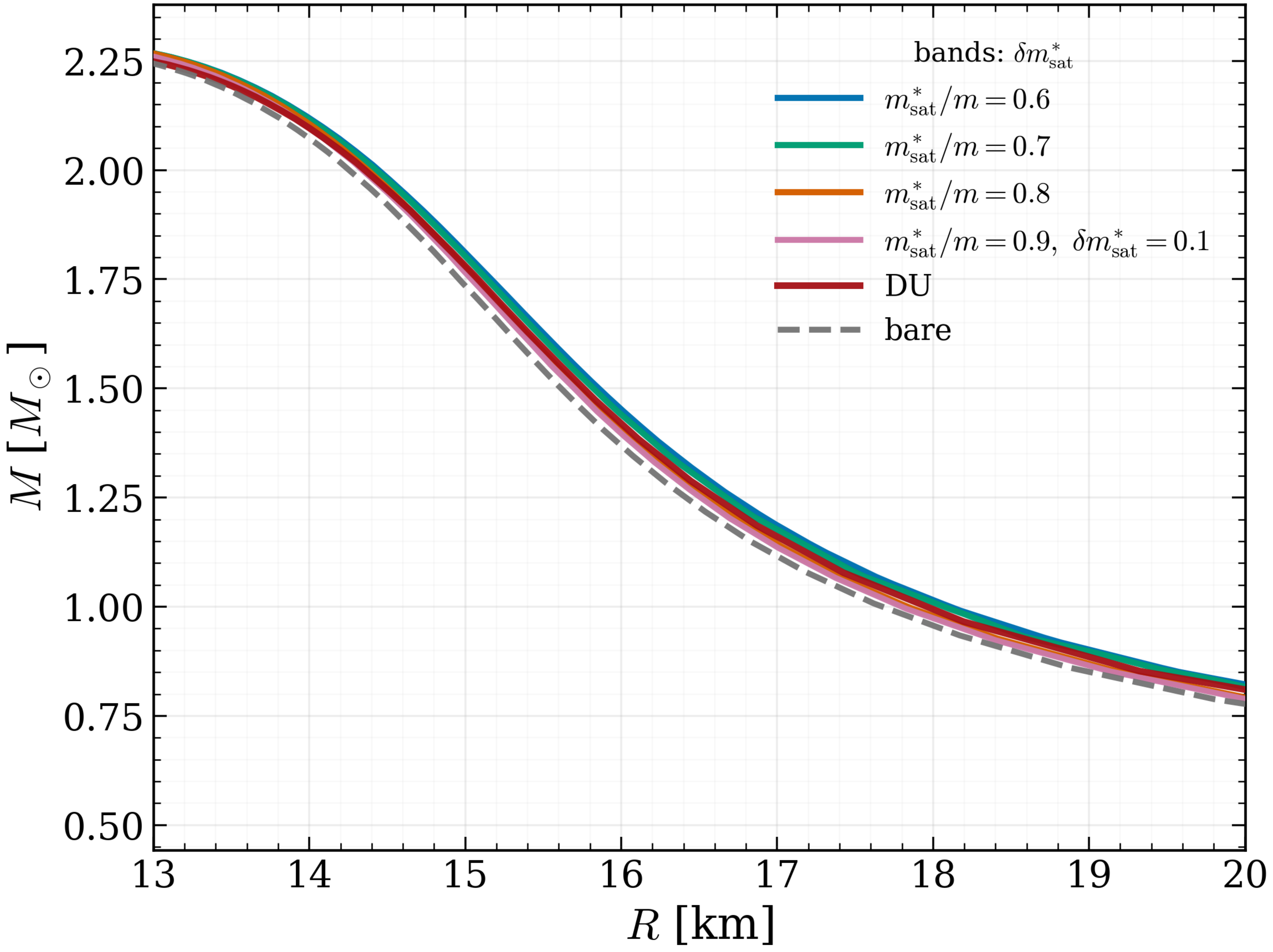
$$T_\infty = T(r)\, e^{\Phi(r)} = \text{const}$$

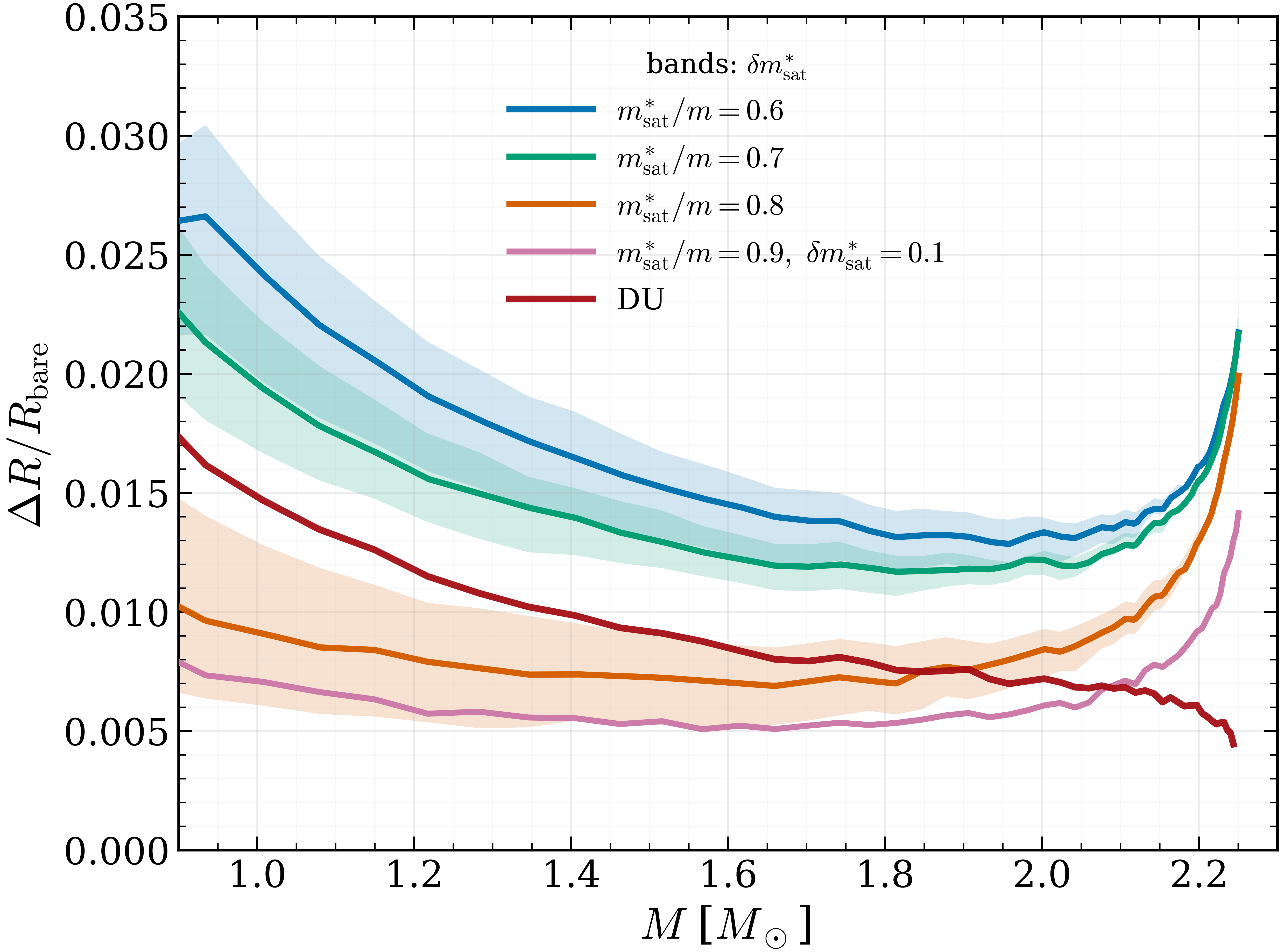
Consisting of homogeneous neutrino-free matter in β-equilibrium

Any inhomogeneous phase is neglected and the radius is defined with a density cutoff at $10^{-6} fm^{-3}$

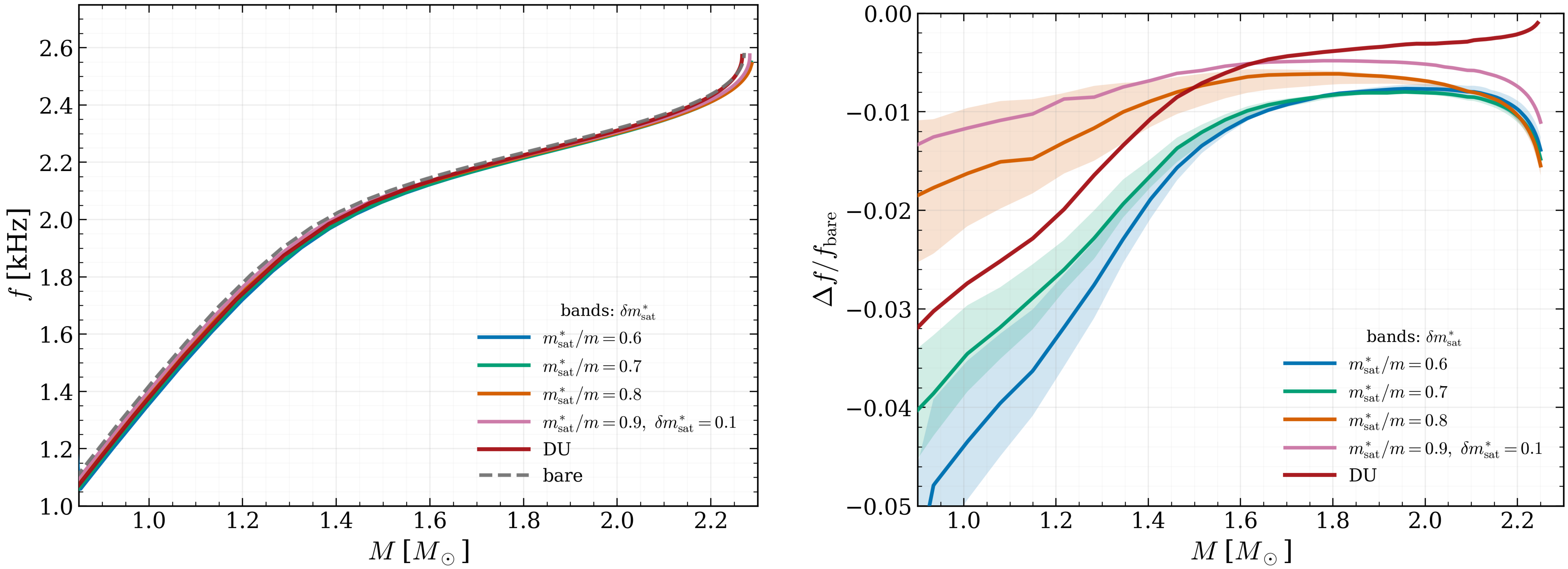


Isothermal star: Mass-Radius





Isothermal star: *f*-mode



Summary

- At $T = 0$, the asymptotically causal MM provides a flexible, composition-aware cold EoS generator constrained by nuclear and astrophysical information

- The thermal free energy is organized by the effective-mass-scaled degeneracy parameter $\theta_q = T/E^*_{F,q}$ as $f_{\rm th} = T \sum_{q=n,p} n_q \Phi_N(\theta_q)$

- All thermal observables follow consistently from this potential; pressure provides the most stringent derivative test.

- Global properties of isothermal star in β-equilibrium are only weakly sensitive to the thermal sector of the EoS

- This analytical form could be explored in simulations as an alternative to interpolating finite-T tables for the homogeneous hadronic sector.

BACKUP SLIDES

Limiting behavior

$$\Phi(\theta) = \frac{F_{th}/A}{T} \qquad \theta \equiv \frac{T}{E_F^*}$$

$$\Phi(\theta) = -\frac{\pi^2}{4}\frac{\theta}{\sqrt{1+a\theta^2}} - \frac{3}{2p}\ln\left(1+b\theta^p\right) - \frac{3}{5}\frac{\theta^3}{\theta^4+\eta^4} + \frac{\theta^3}{\left[1+(\theta/\theta_c)^2\right]^3}\sum_{k=0}^{N} c_k\left[\frac{(\theta/\theta_c)^2}{1+(\theta/\theta_c)^2}\right]^k$$

$\theta \ll 1$

Degenerate limit: low temperature, high density

$$\Phi_N(\theta) = -\frac{\pi^2}{4}\theta + O(\theta^3)$$

$$\frac{F_{\rm th}^{\rm fit}}{A} \simeq -\frac{\pi^2}{4}T^2\sum_{q=n,p}\frac{x_q}{E_{F,q}^*}$$

$\theta \gg 1$

Classical limit: high temperature, low density

$$\Phi_N(\theta) = -\frac{3}{2}\ln\theta + C_F + O(\theta^{-1})$$

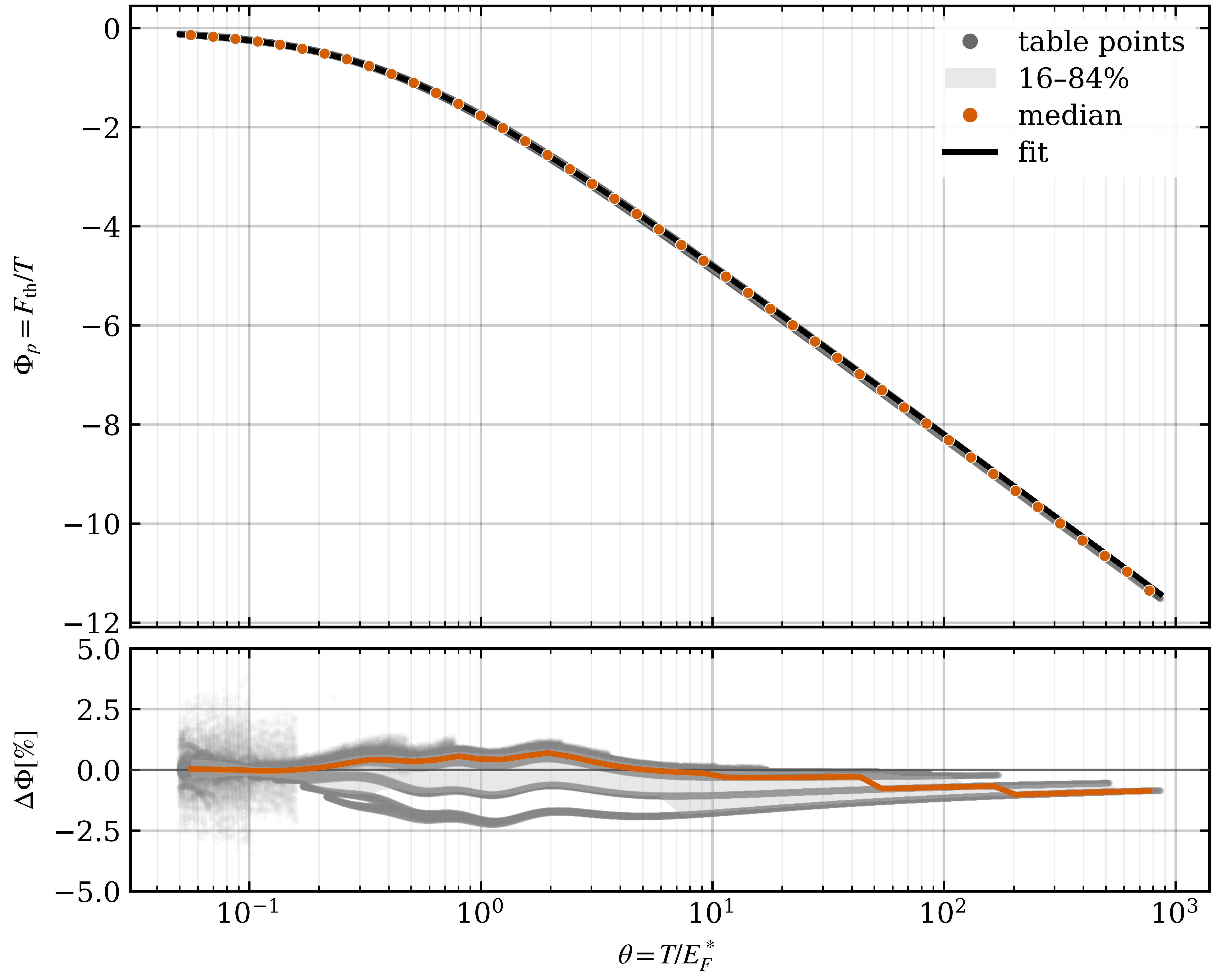
$$\frac{F_{\rm th}^{\rm fit}}{A} \simeq T\sum_{q=n,p}x_q\left[\ln\left(\frac{n_q\lambda_q^{*3}}{g}\right) - 1\right]$$

Exploratory RMF extension

$$\frac{F_{\rm th}^{\rm fit}}{A} = T \sum_{q=n,p} x_q \Phi_N(\theta_{rel,q}) \qquad x_q = \frac{n_q}{n_B}$$

$$\theta_{\rm rel} = \frac{T}{E_{F,\rm L}^{\rm rel}}, \qquad E_{F,\rm L}^{\rm rel} = \frac{k_F^2}{2E_F^{\rm rel}}, \qquad E_F^{\rm rel} = \sqrt{k_F^2 + m_D^{*2}}$$

We should modify $\mathbf{\Phi}$ to incorporate the correct degenerate and classical limits

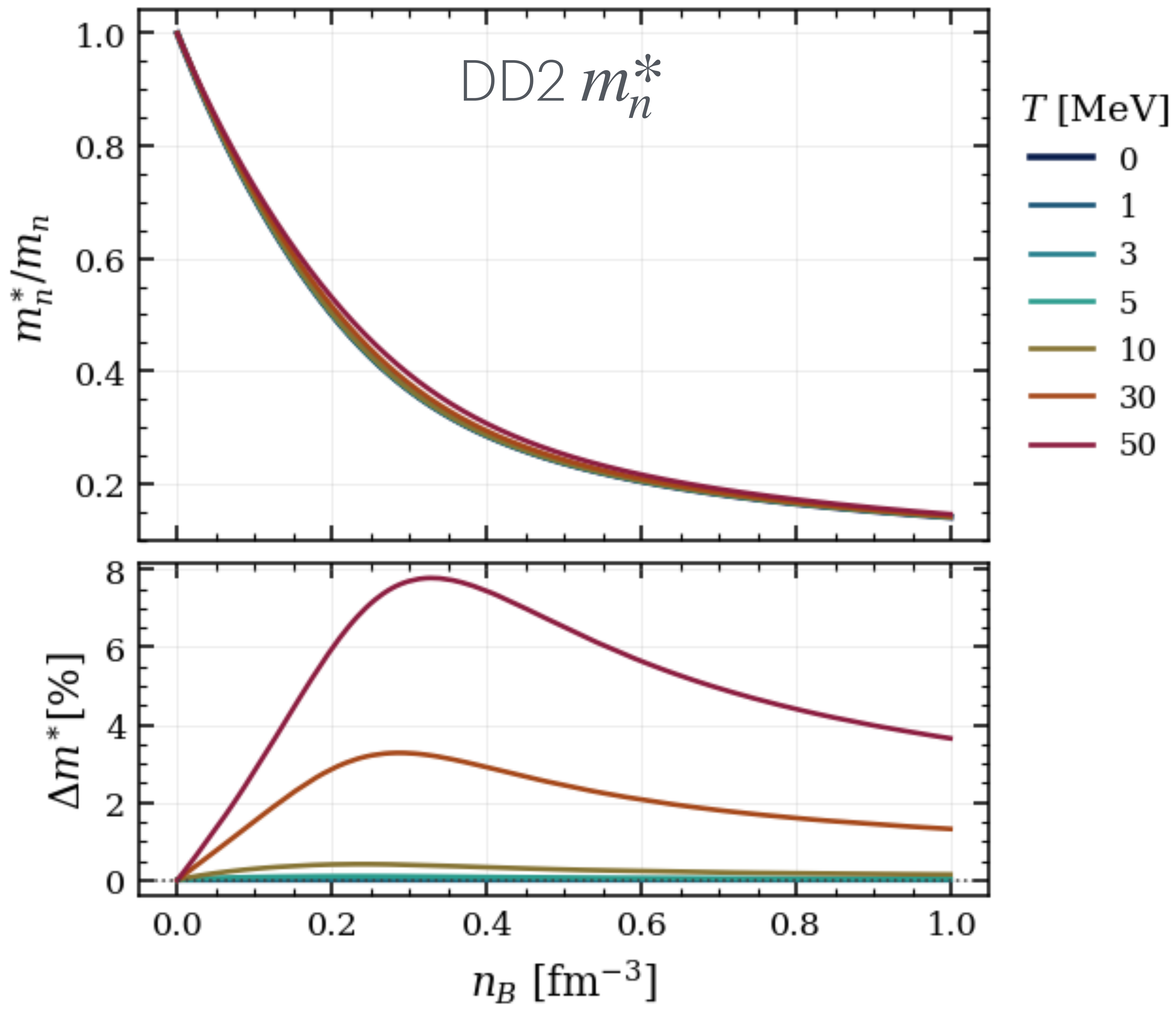


T-dependent *m**

$$\theta_q = \frac{T}{E^*_{F,q}(n_n, n_p, T)}, \qquad E^*_{F,q} = \frac{\hbar^2 k^2_{F,q}}{2m^*_q(n_n, n_p, T)}$$

We have extra chain rules terms for temperature derivatives

$$\frac{S_{\text{fit}}}{A} = -\sum_q x_q \left[\Phi_N(\theta_q) + \theta_q \Phi'_N(\theta_q)\left(1 + T\frac{\partial \ln m^*_q}{\partial T}\right)\right]$$



Symmetry energy constraint from Indra-Fazia collaboration

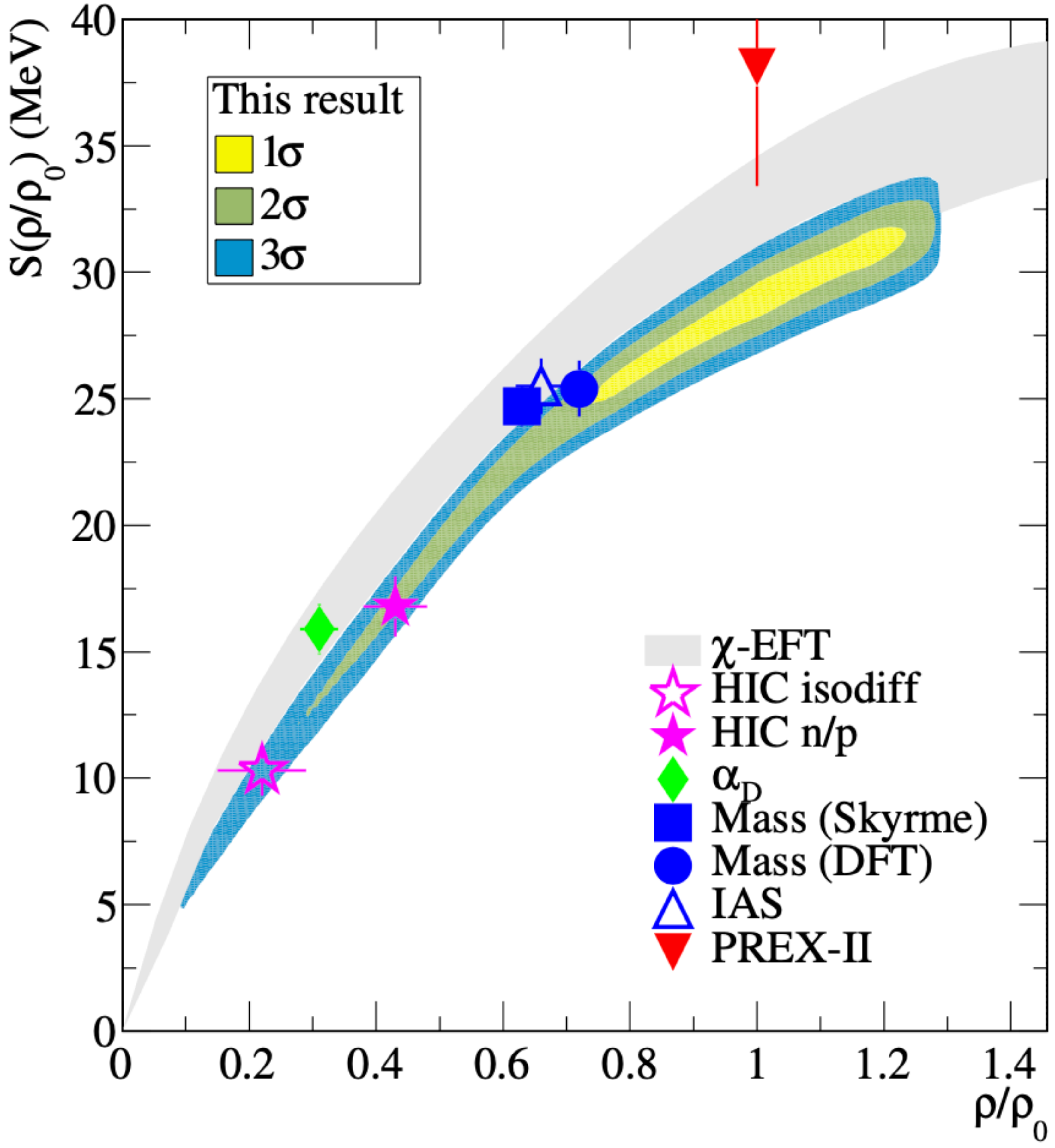
Data from $^{58,64}Ni + {}^{58,64}Ni$ collision at 32 MeV/nuc

They **probe the symmetry energy** defined as

$$e_{sym} = \frac{\partial^2 e}{\partial \delta^2}\Big|_{x=0,\delta=0}$$

Sensitive density region is **close to saturation density**

Ciampi et al [PLB:139815]

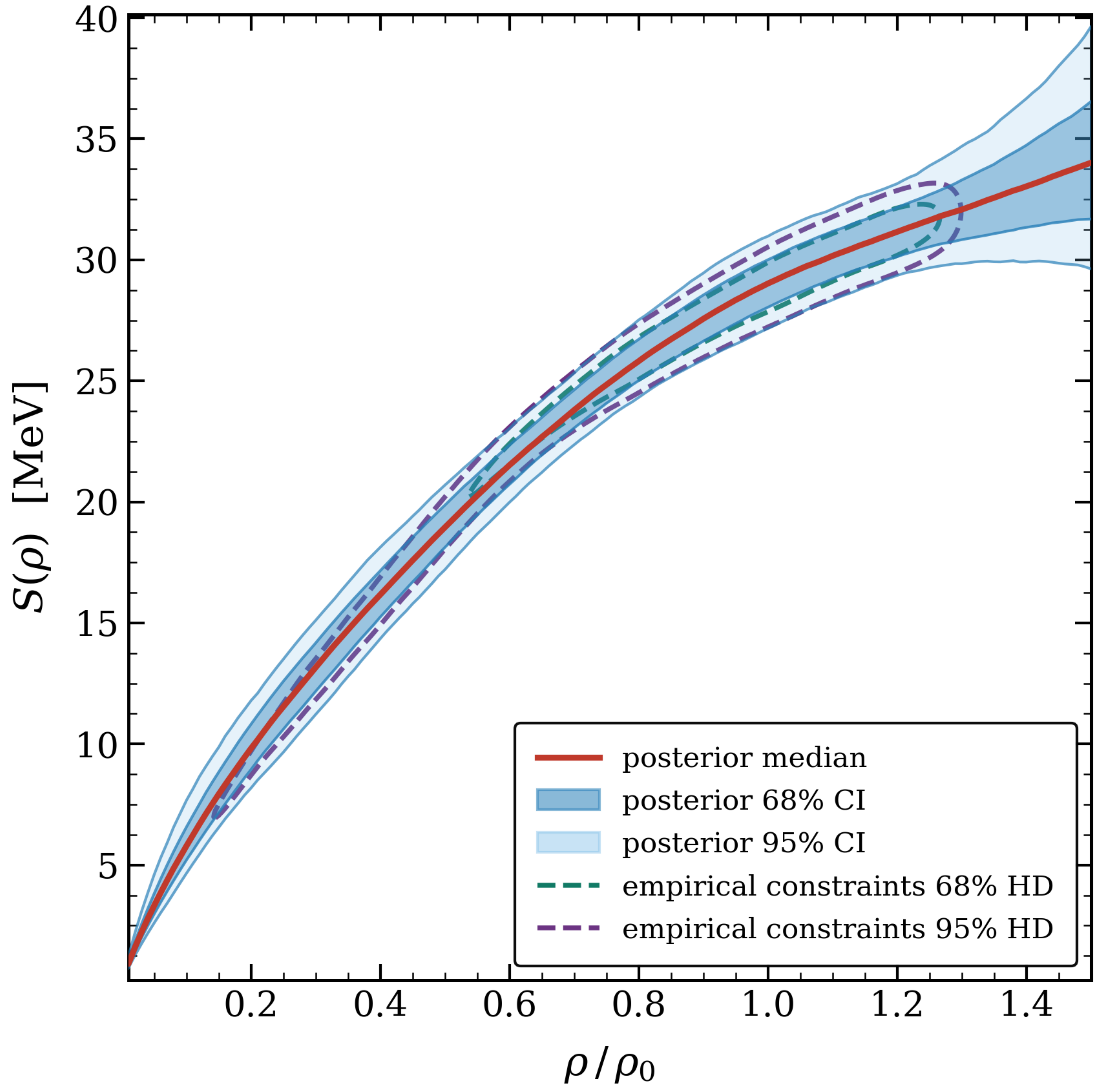


Sampling the symmetry energy

The data are used to build a **probability density in the** $(\rho/\rho_0, e_{sym})$ **plane**

The **asymptotic causal meta-model is sampled** using this probability

The **posterior is validated** against the original band



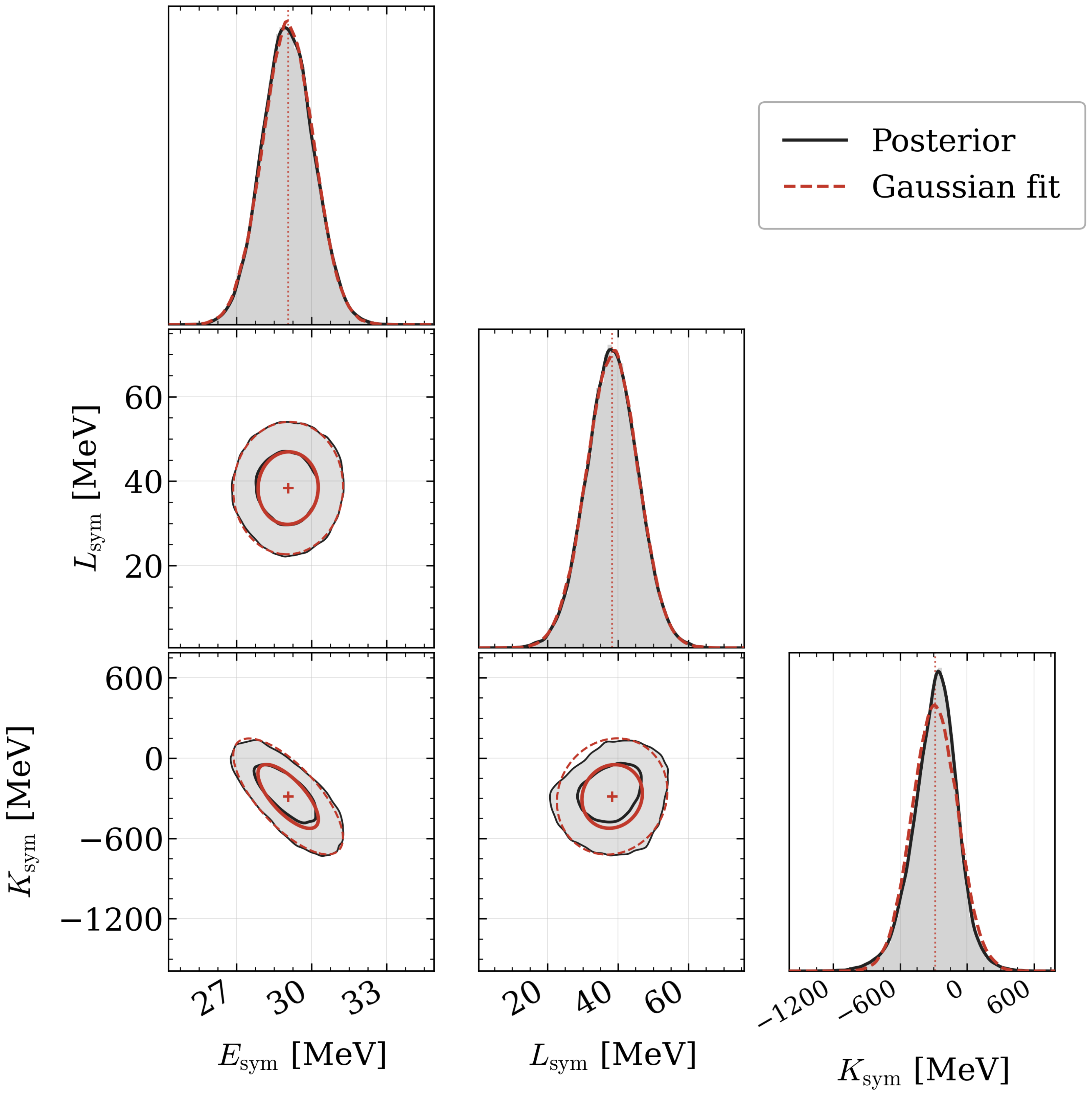
Sampling the symmetry energy

Recalling the **meta-model ansatz**:

$$\epsilon(n, x_e, x_\mu) = \epsilon_k(n, x_e, x_\mu) + n\left[e_0(n) + \delta^2 e_2(n) + \delta^4 e_4(n)\right]$$

we are sampling directly e_2 which coincides with e_{sym} by construction

We can extract a prediction on the isovector sector of the nuclear matter parameters



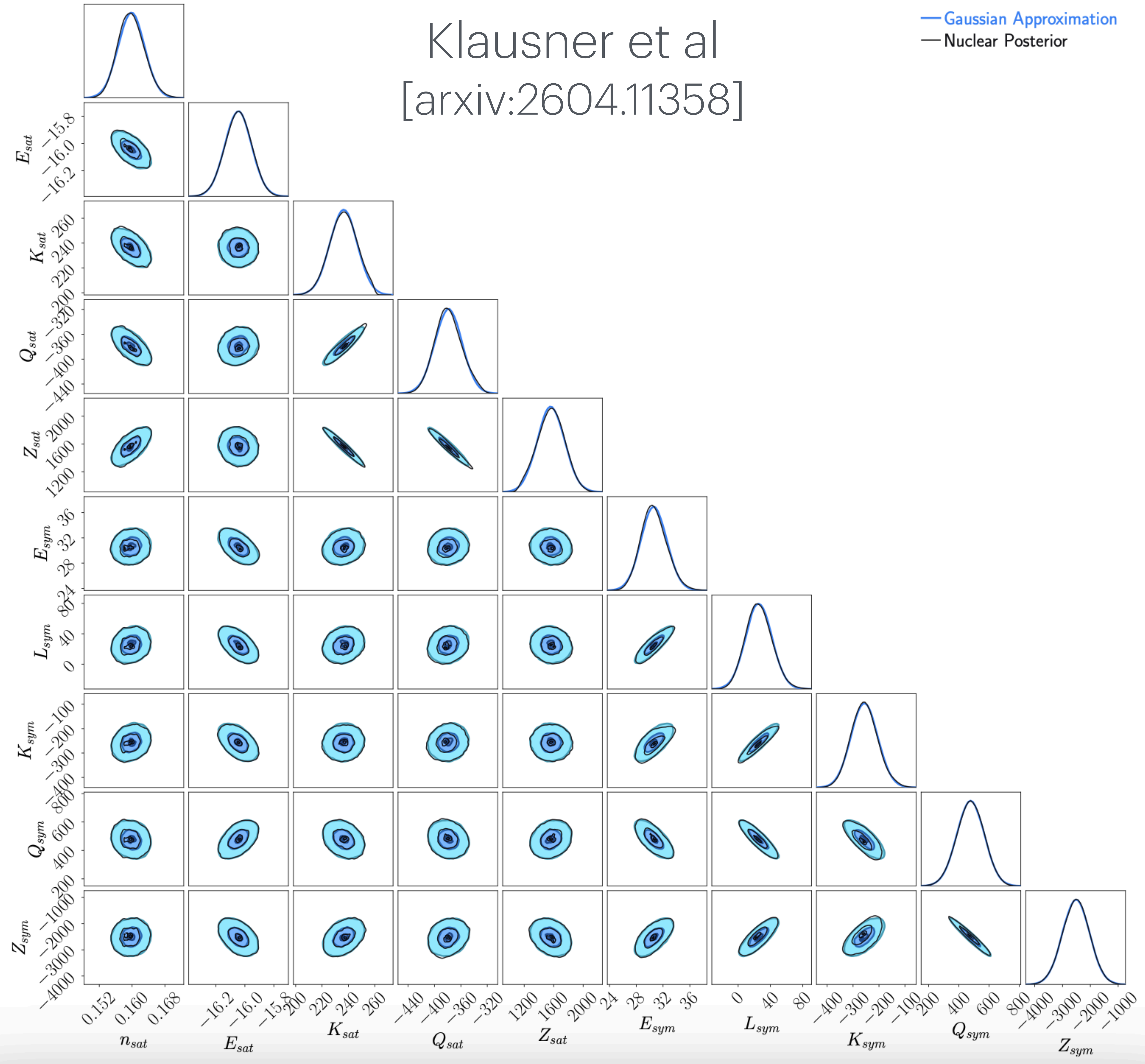
Nuclear structure inference with Skyrme functional

Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	1636.4 ± 2.0	5.50 ± 0.05	2.02 ± 0.50
^{48}Ca	416.0 ± 2.0	3.48 ± 0.05	1.72 ± 0.50
^{68}Ni	590.4 ± 2.0	-	-
^{132}Sn	1102.8 ± 2.0	4.71 ± 0.05	-
^{90}Zr	783.9 ± 2.0	4.27 ± 0.05	-

Isoscalar resonances		
	E^{IS}_{GMR} [MeV]	E^{IS}_{GQR} [MeV]
^{208}Pb	13.5 ± 0.5	10.9 ± 0.5
^{90}Zr	**18.7 ± 0.5**	-

Isovector properties			
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	550 ± 18
^{48}Ca	2.07 ± 0.22	-	2668 ± 113

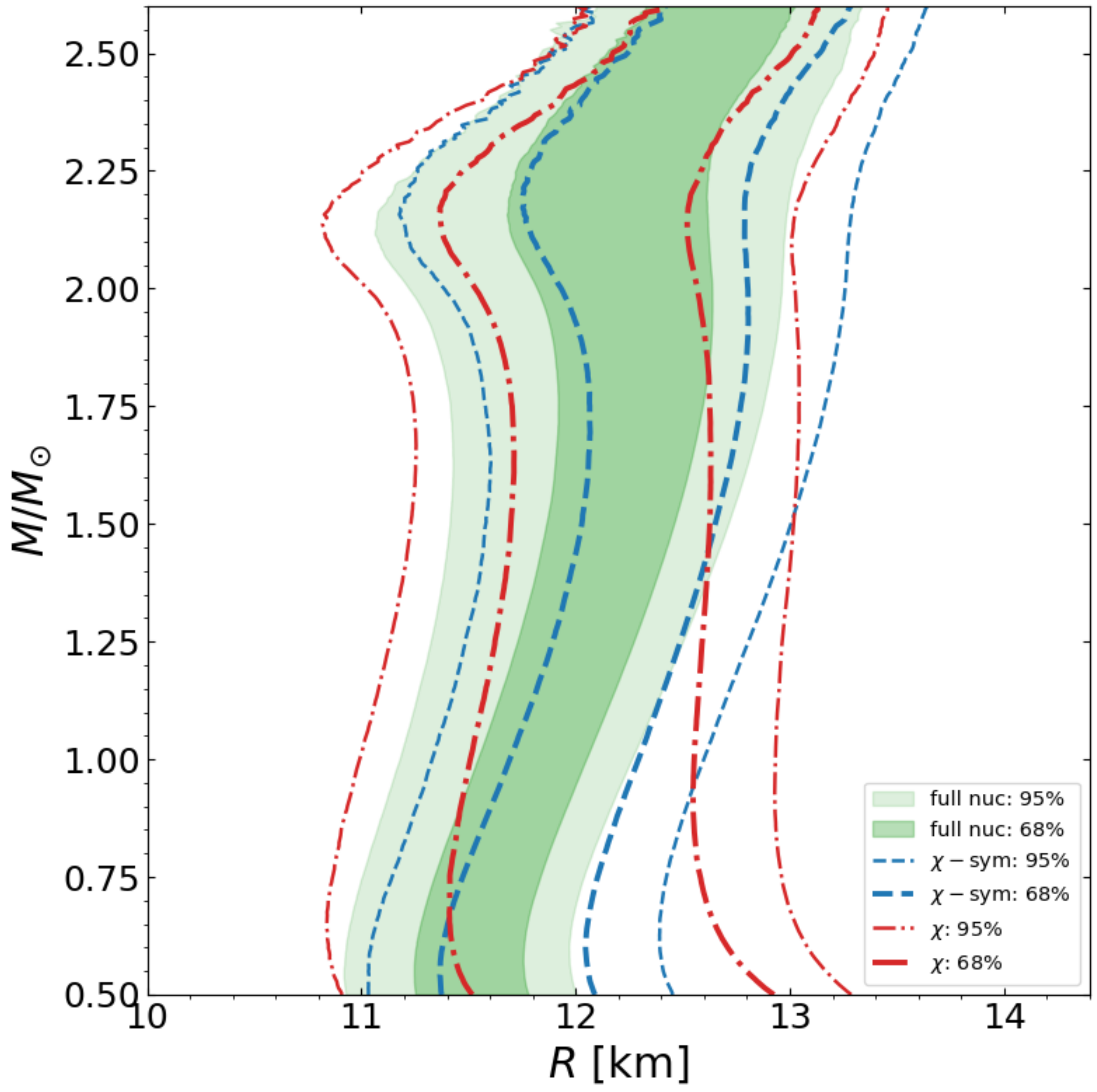
New Data from OS nuclei			
	$B.E.$ [MeV]	R_{ch} [fm]	Δ_n [MeV]
^{50}Ca	427.5 ± 2.0	3.52 ± 0.05	-
^{46}Ca	398.8 ± 2.0	-	-
^{44}Ca	381.0 ± 2.0	-	-
^{42}Ca	361.9 ± 2.0	-	-
^{120}Sn	1020.5 ± 2.0	4.65 ± 0.05	1.3 ±0.2
^{112}Sn	953.5 ± 2.0	-	-
^{124}Sn	1050.0 ± 2.0	-	-



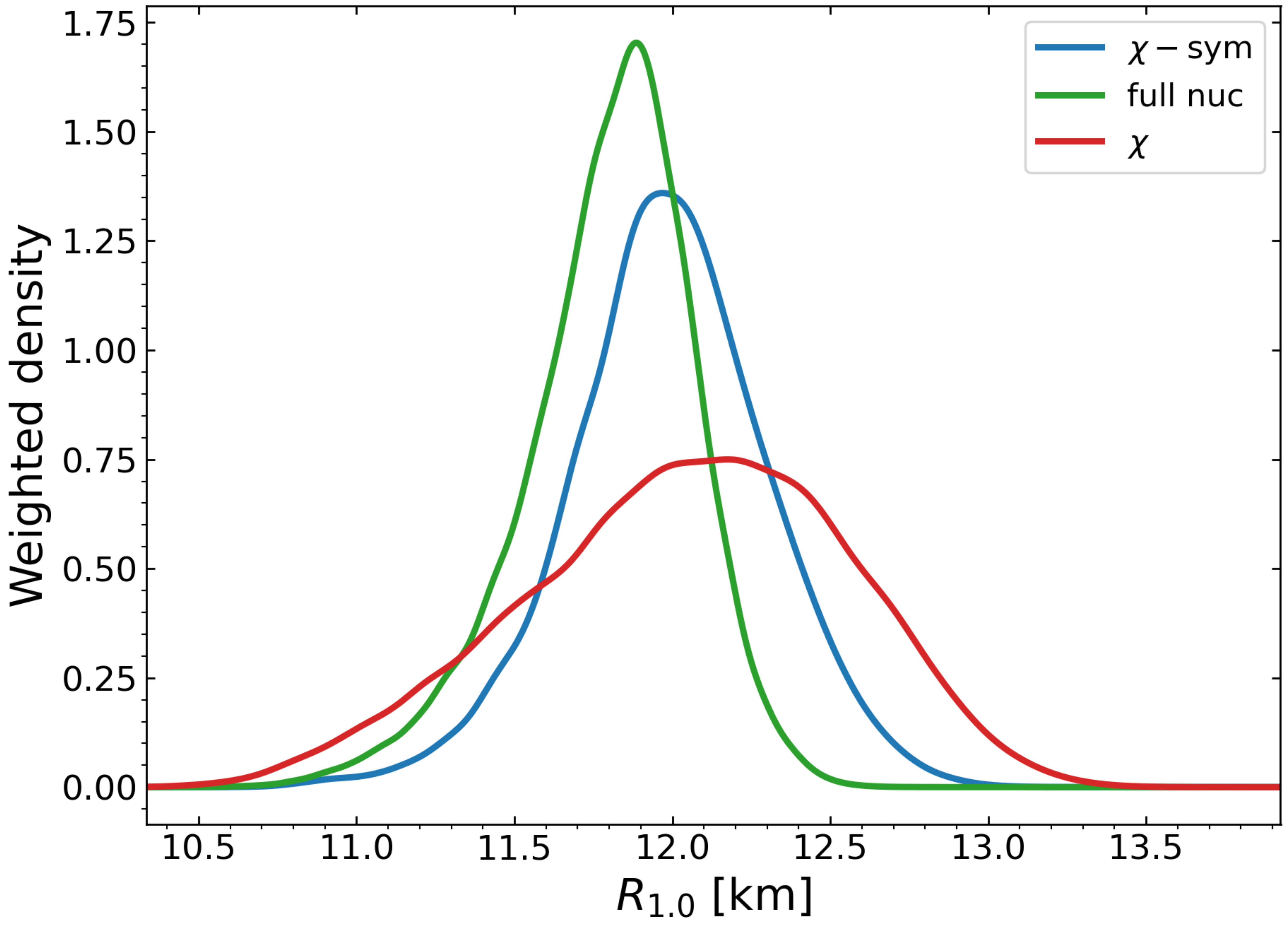
Mass-Radius impact of nuclear likelihoods

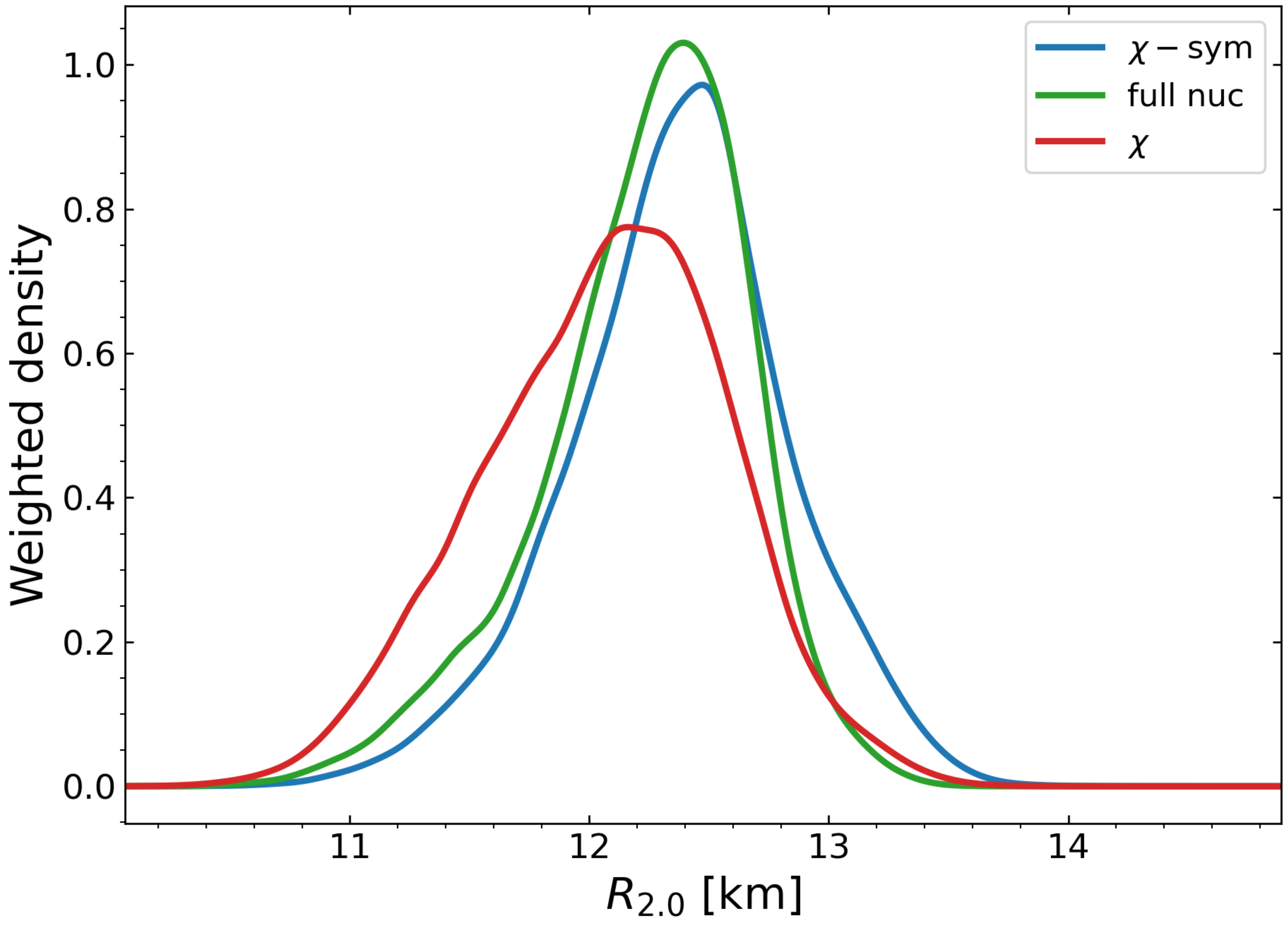
Low mass star radius distribution is more affected

The effects become smaller with increasing mass

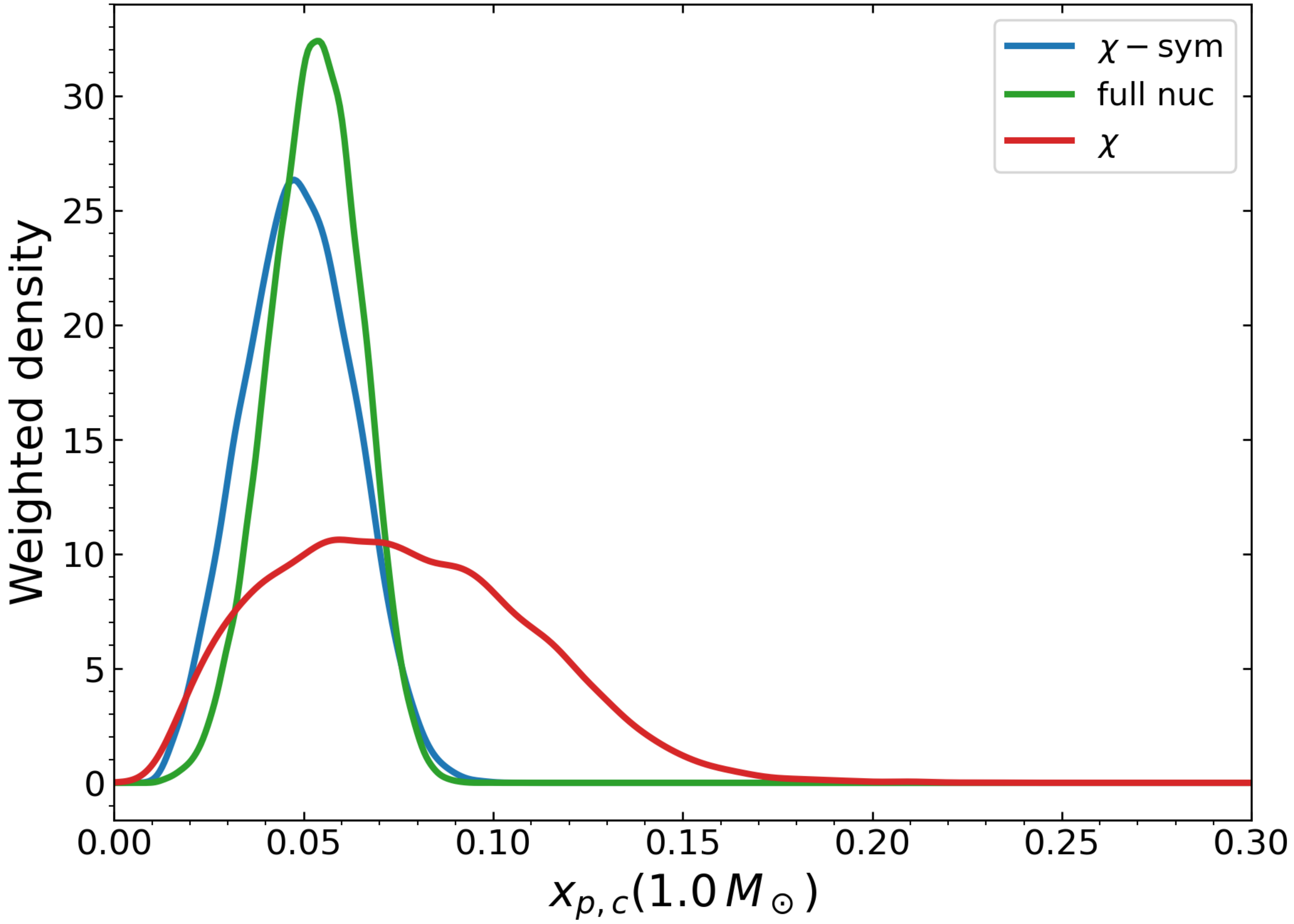


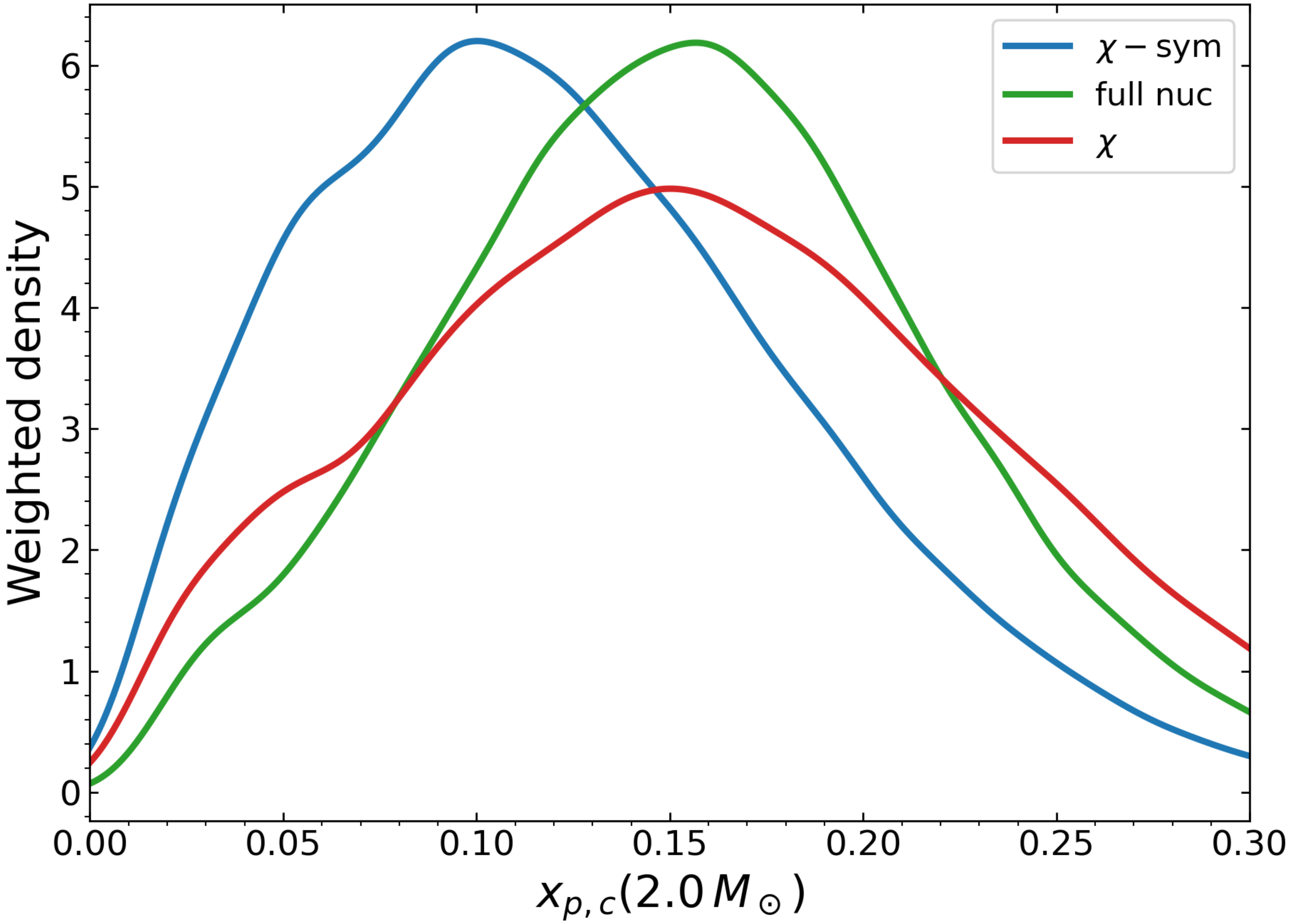
Radius changes respect to nuclear likelihoods





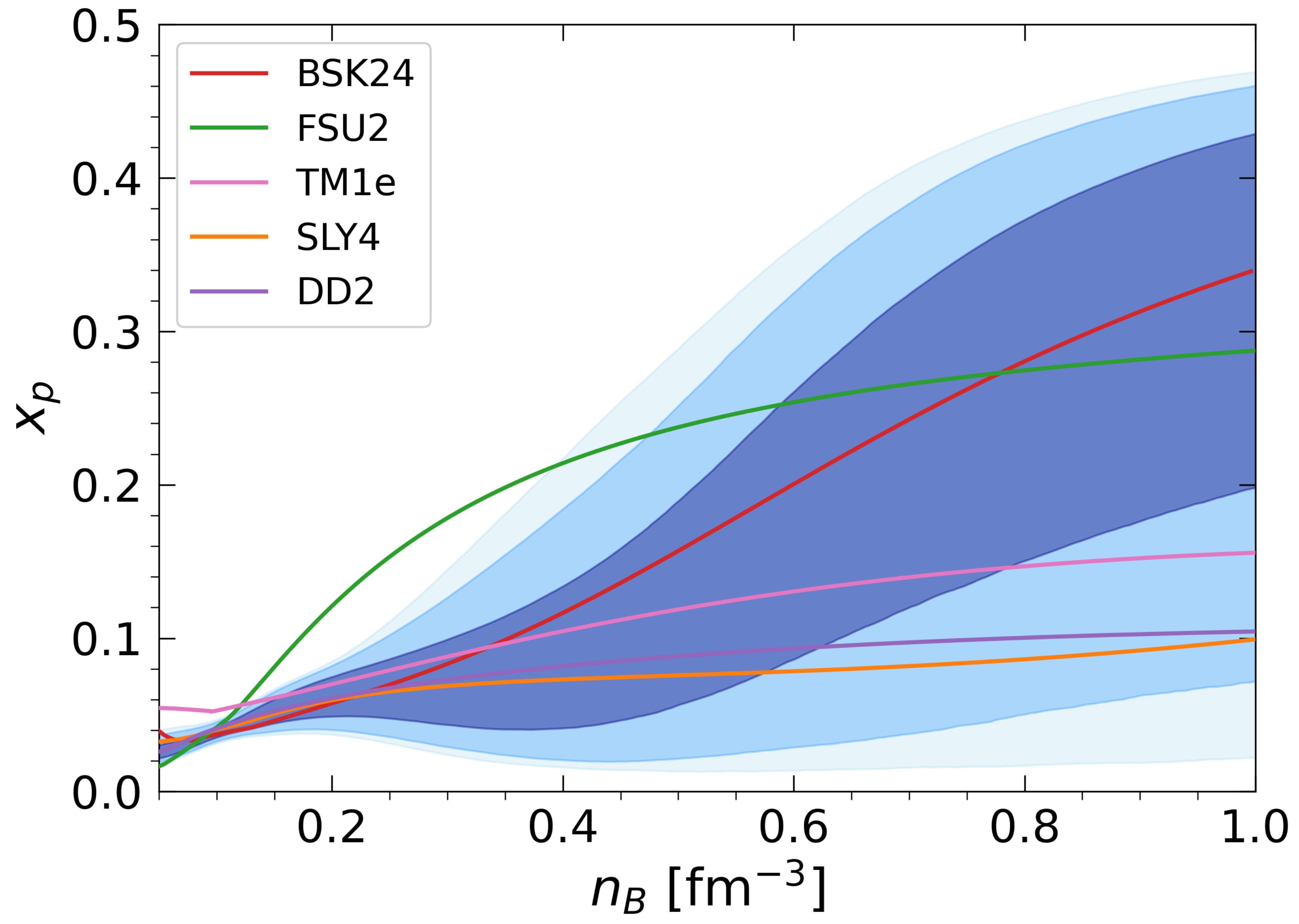
Central proton fractions changes



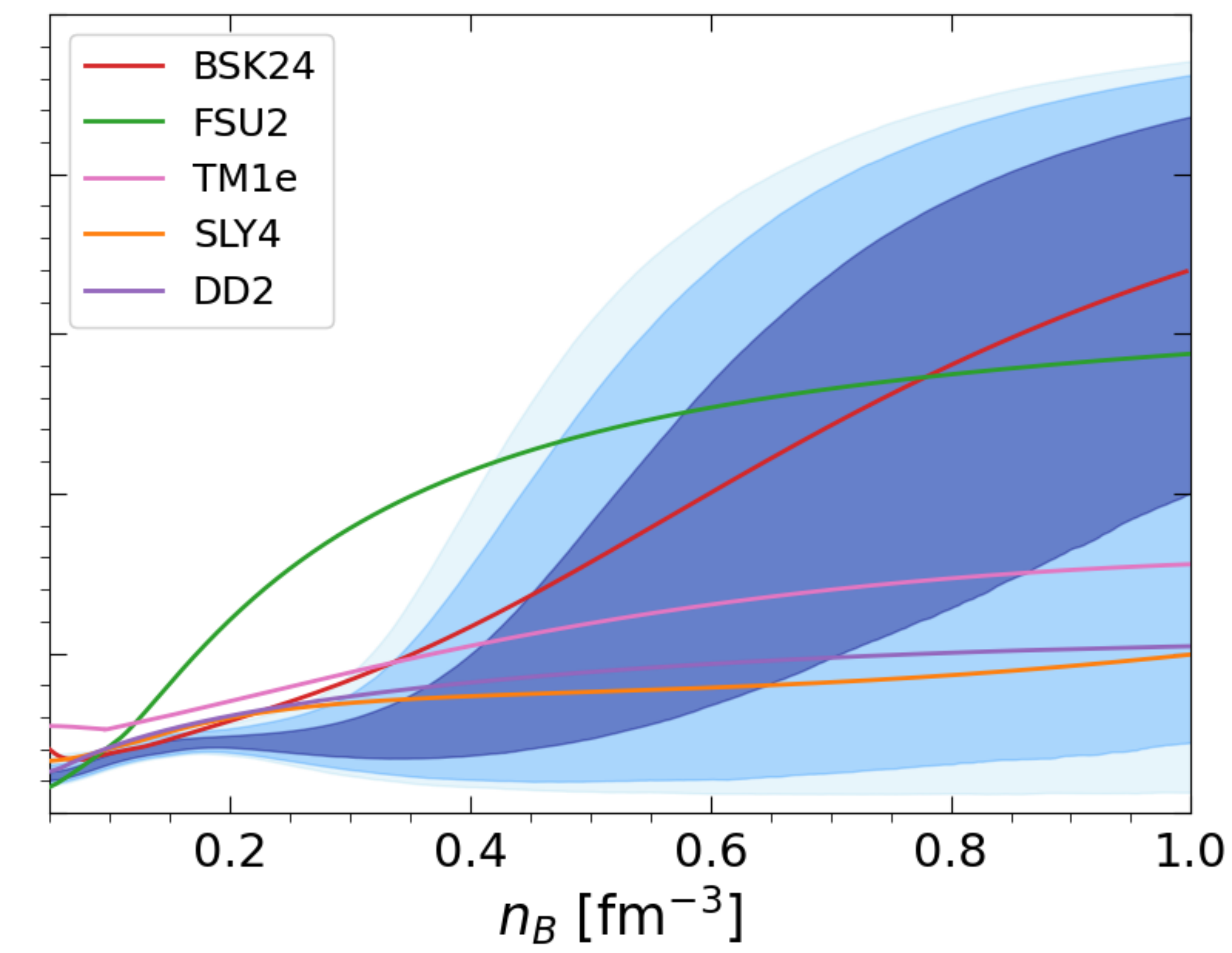


Proton fractions for each dataset

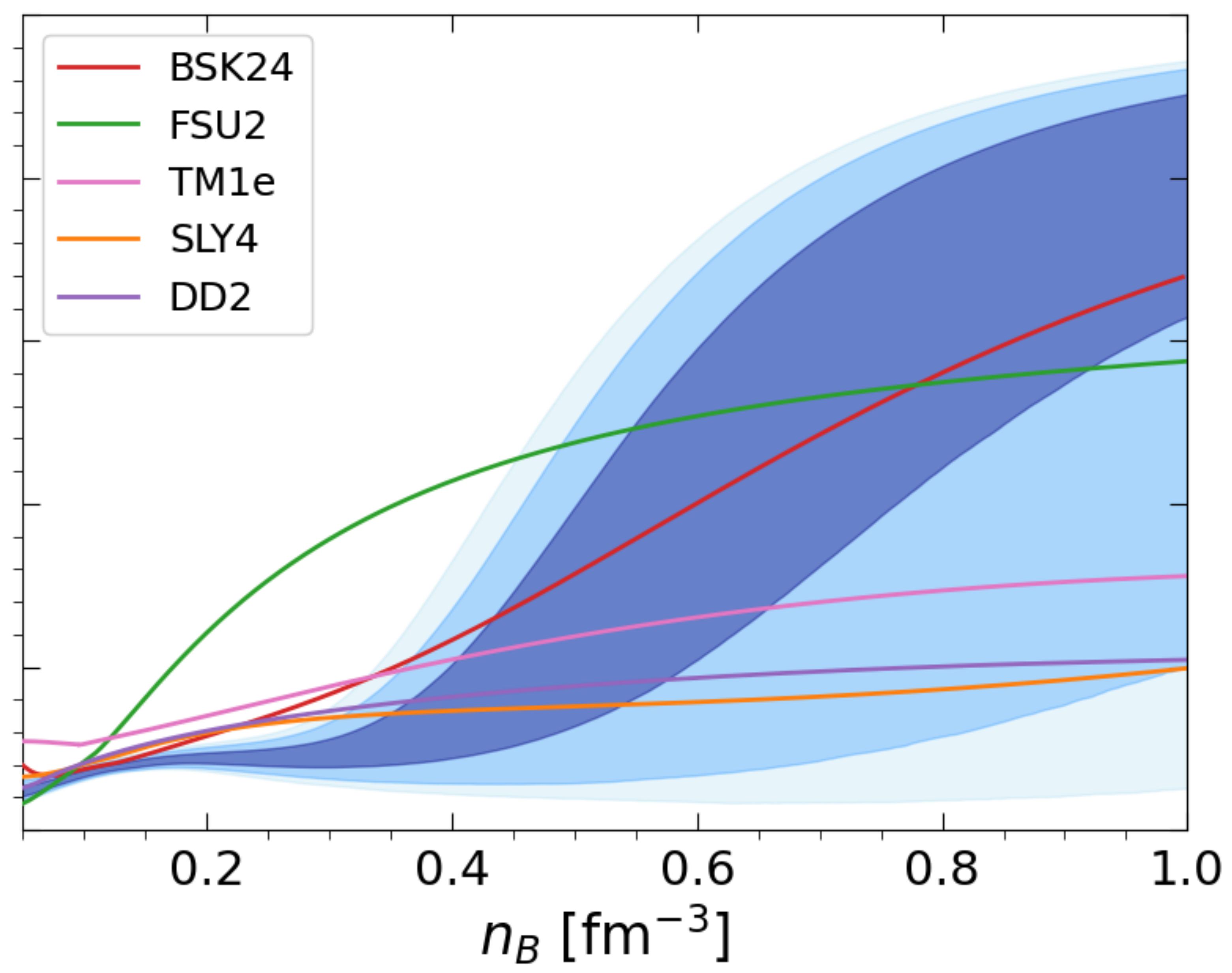
χ_{EFT}



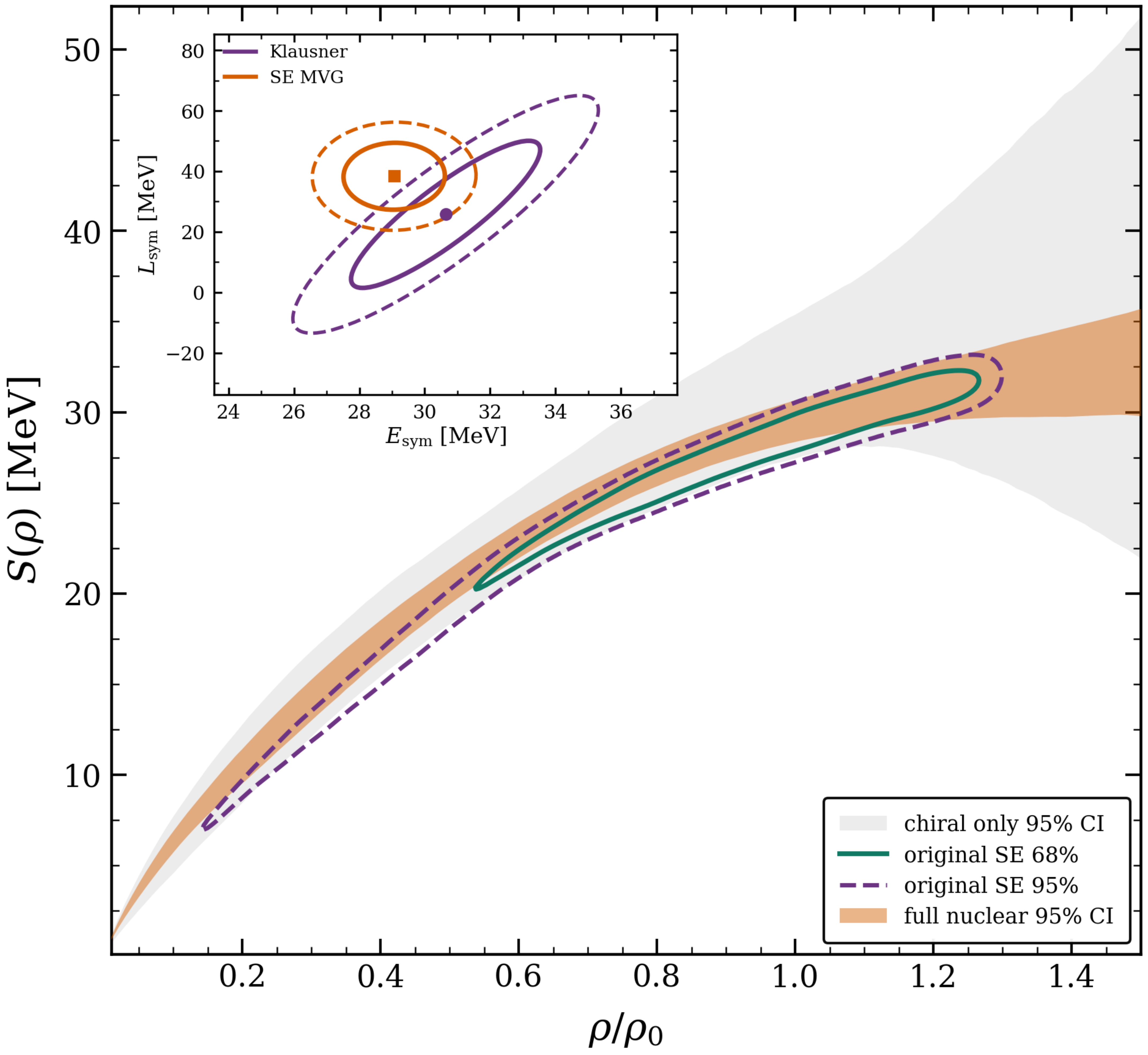
Indra-Fazia data



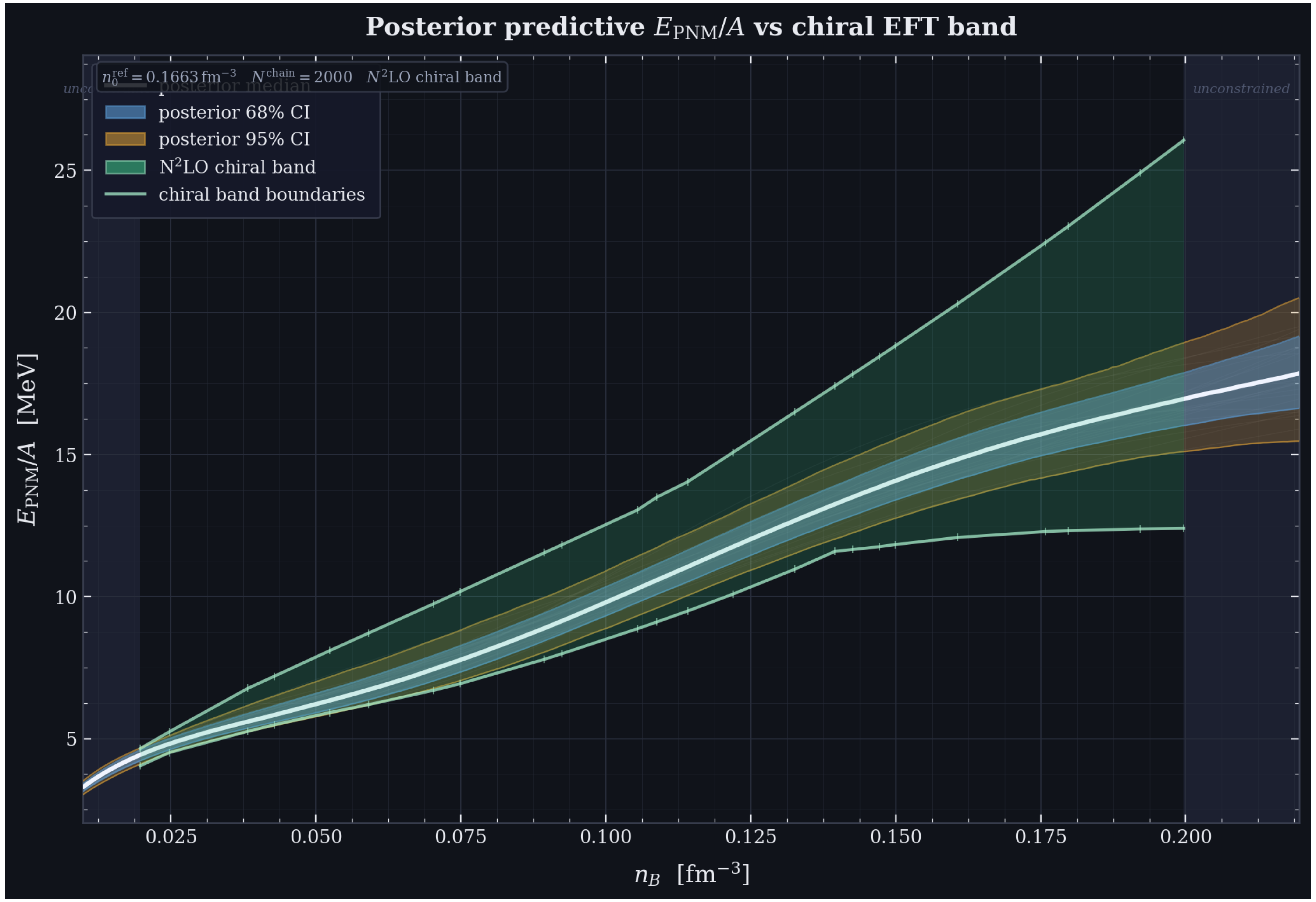
Nuclear structure



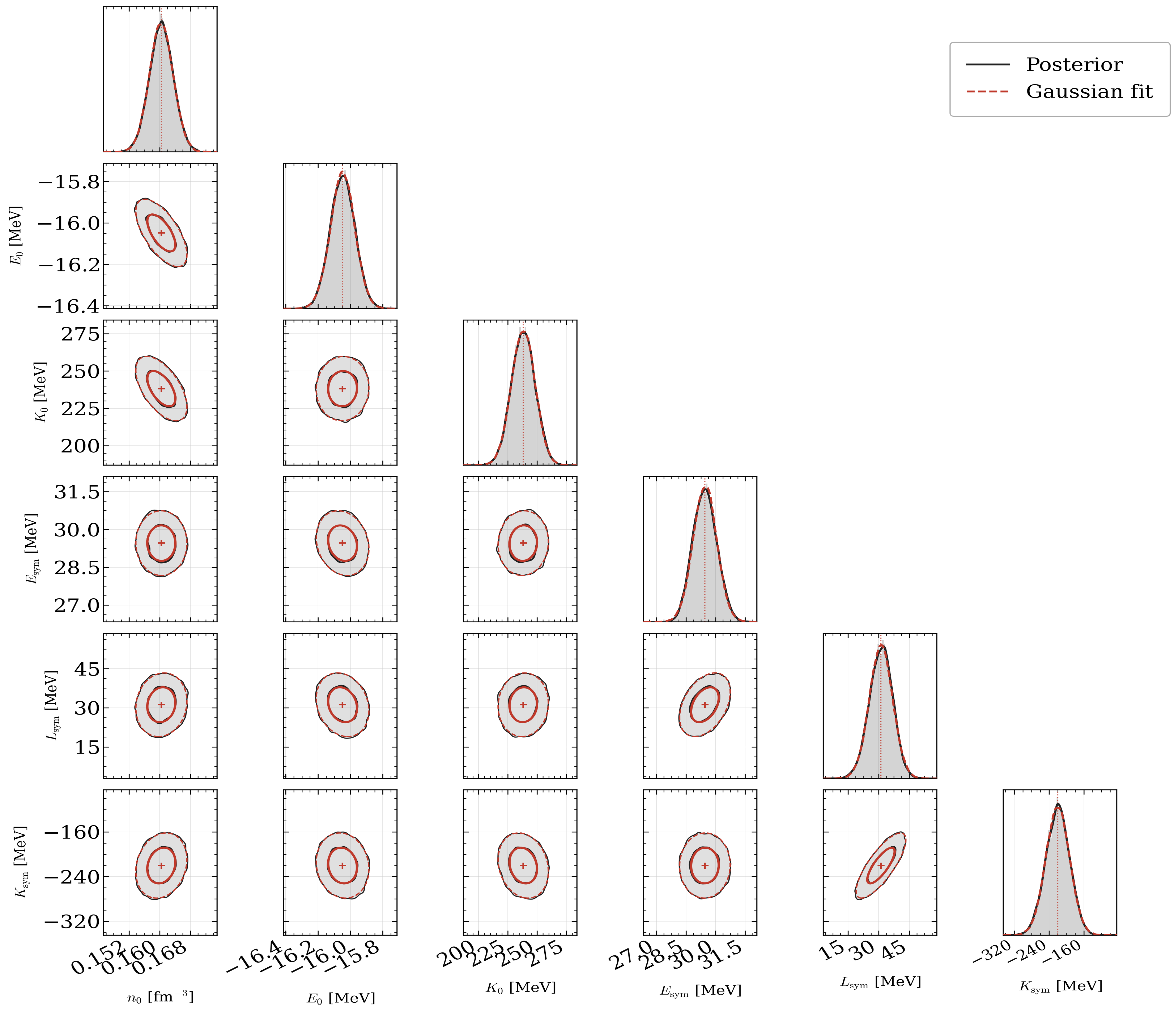
Full nuclear predictions: symmetry energy



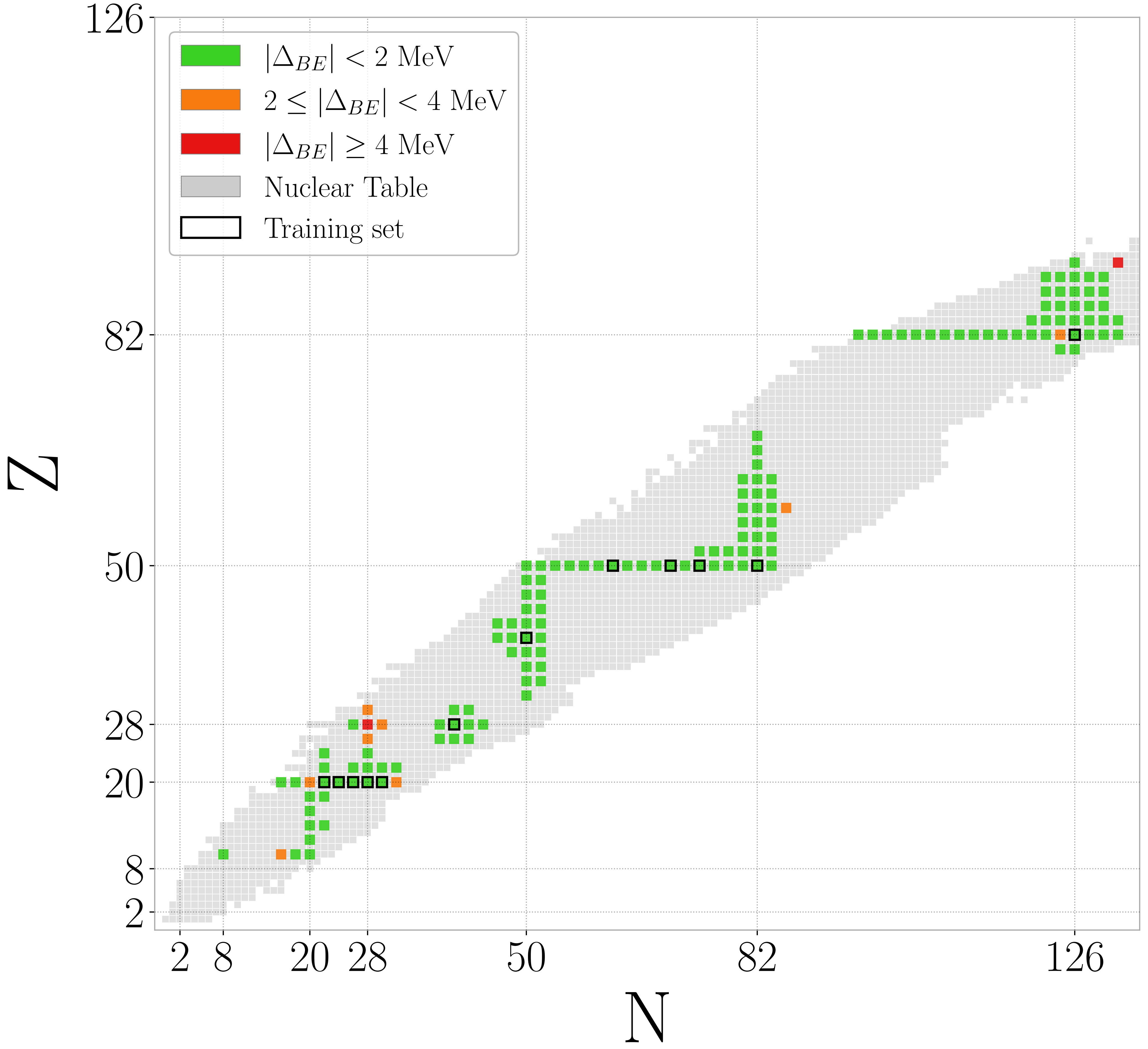
Full nuclear predictions: χ_{EFT}



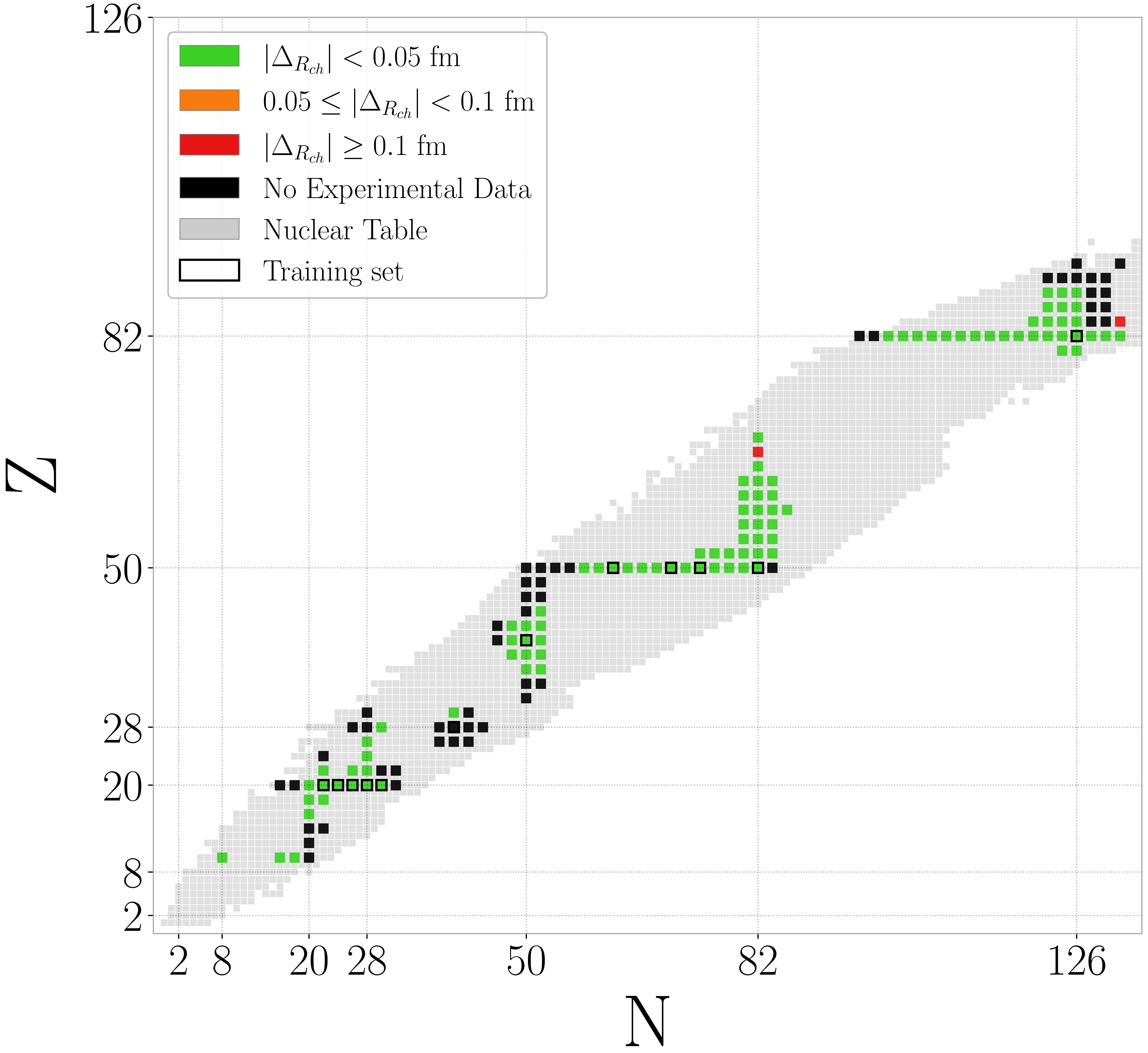
Full nuclear predictions: NMP cornerplot



Full nuclear predictions: Skyrme Functional nuclear structure



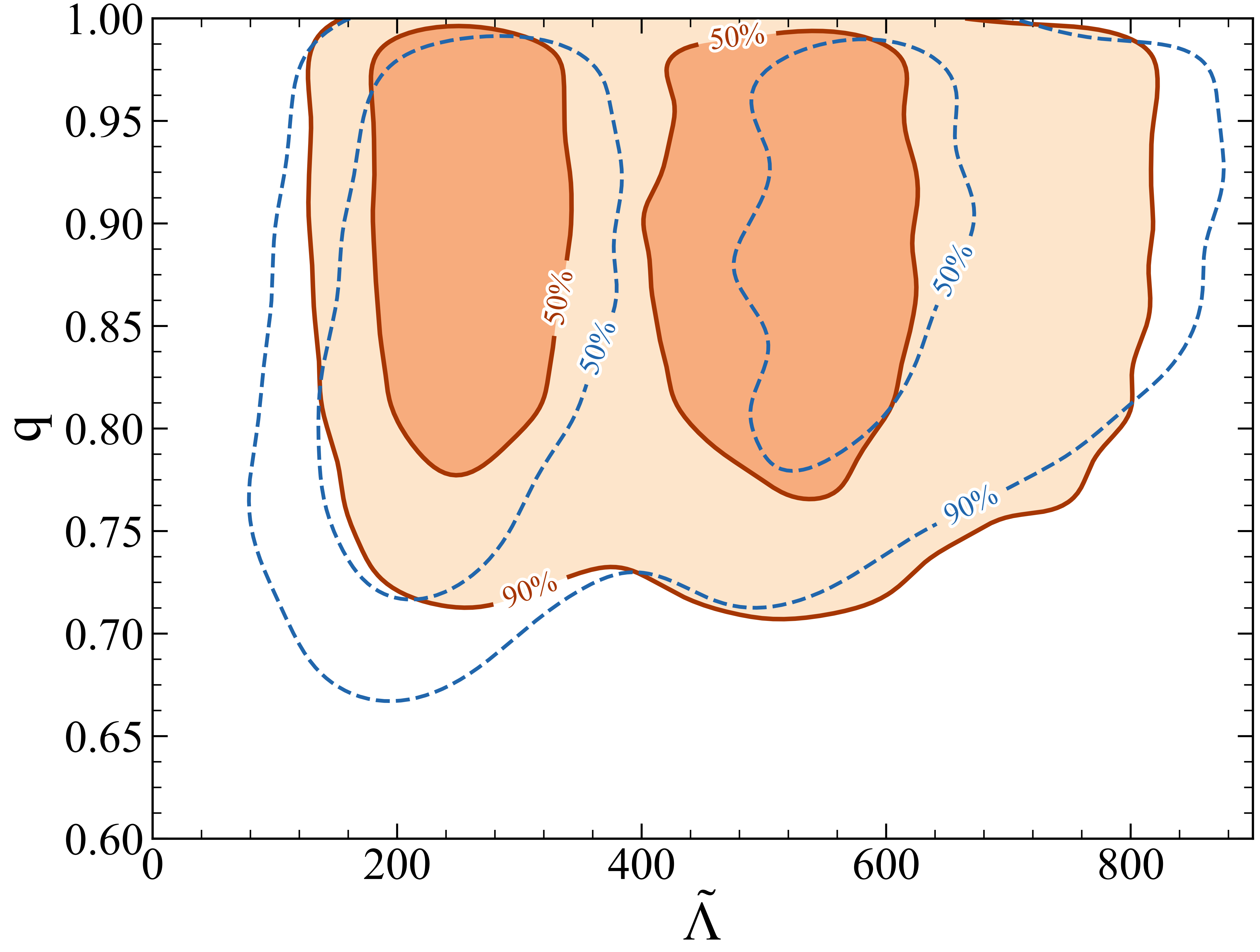
Spherical nuclei considered: 150; RMS = 1.19 MeV; $|\Delta_{BE}| = 0.89$ MeV (green (93%), orange (5%), red (1%))



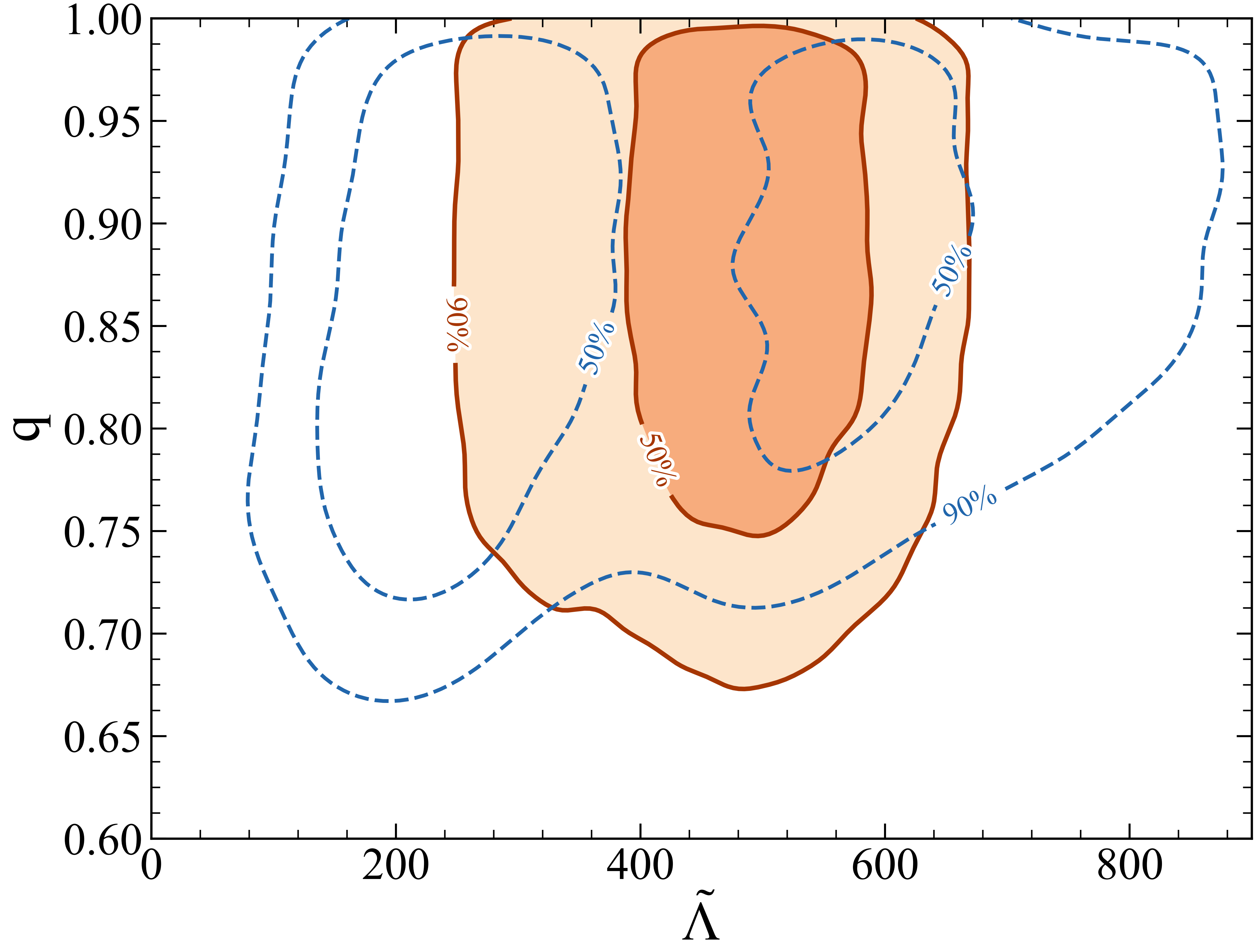
Spherical nuclei considered: 150; RMS = 0.02 fm; $|\Delta_{R_{ch}}| = 0.01$ fm (green (65%), orange (0%), red (1%), black (34%

Tidal posterior predictive

χ_{EFT} **+ GW170817 likelihood**

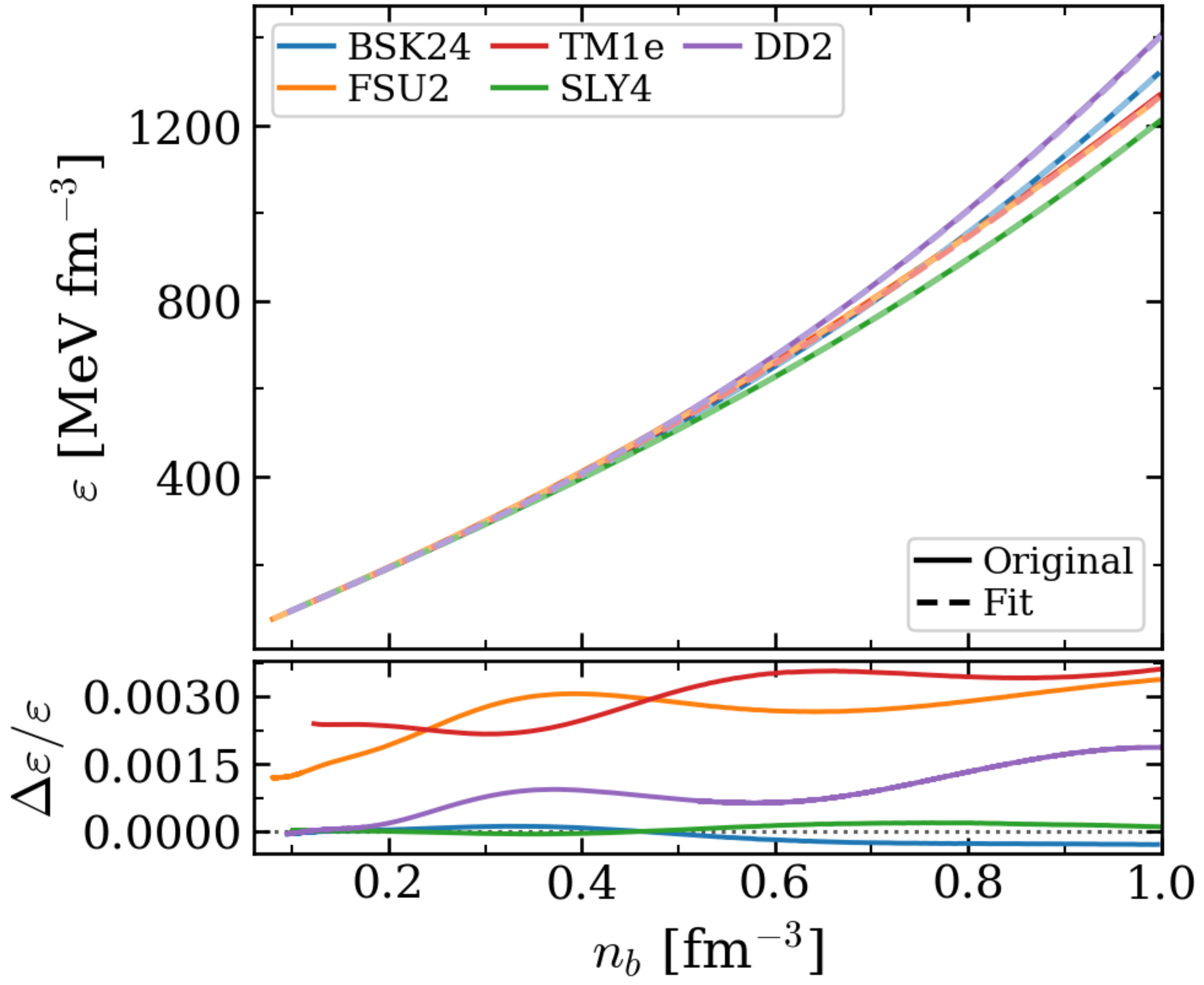


All likelihoods applied



Asymptotically causal meta-model: EoS reconstruction

Energy Density



Pressure