

HBT: The HBT signal for the EOS softest point has been measured. Can we interpret it? What further analyses or calculations are needed?

H. Caines<sup>1</sup>, Y. Khyzhniak<sup>2</sup>, G. Nigmatkulov<sup>3</sup>

1. Yale University

2. The Ohio State University

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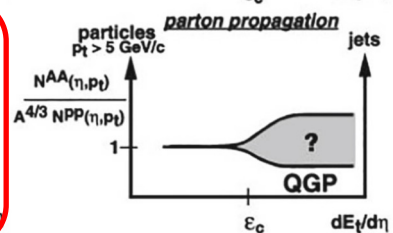
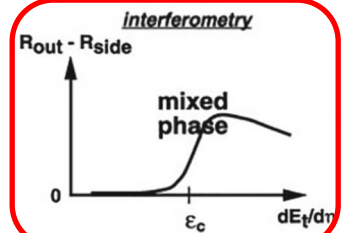
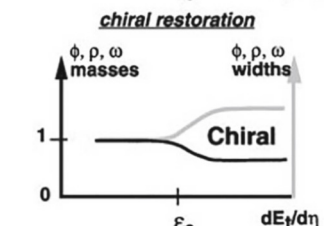
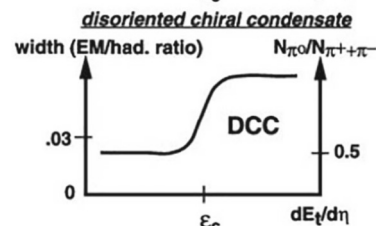
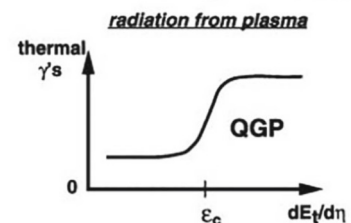
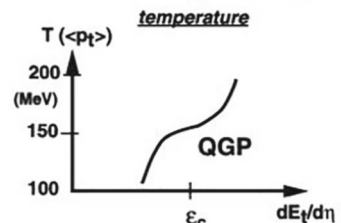
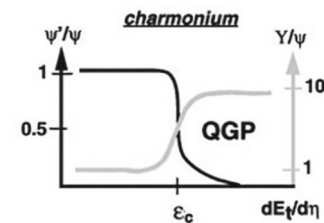
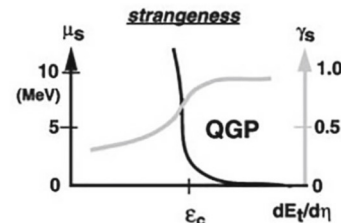
# Outline

- Introduction
- Experimental measurements (STAR BES-I)
  - Femtoscopic radii
  - Measurements with respect to the event plane
- Current theoretical predictions (transport, hybrids)
- Femtoscopy at low-collision energies
- Open questions & discussion

# MID-90S (PRE-RHIC ERA)

THE TRANSITION FROM A HADRON GAS TO A QUARK-GLUON PLASMA IS A DISCONTINUOUS, POSSIBLY FIRST-ORDER, PHASE TRANSITION. THE ASSUMPTION WAS MOTIVATED BY SIMPLIFIED MODELS AND BY LATTICE SIMULATIONS OF SU(3) GAUGE THEORY WITHOUT DYNAMICAL QUARKS

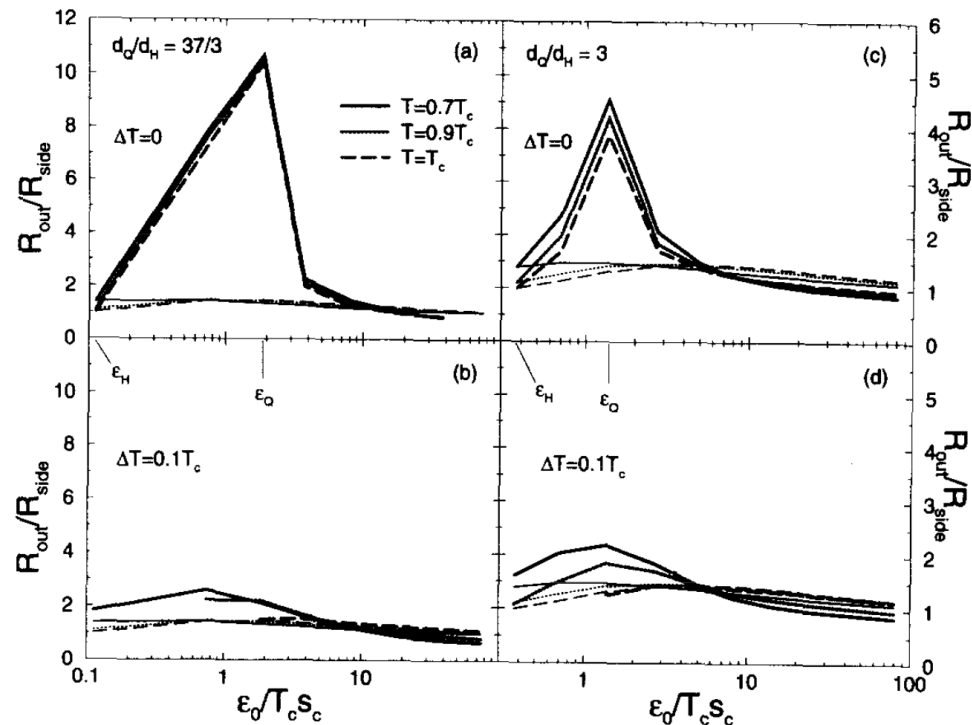
## SIGNATURES



# The 1<sup>st</sup>-order Phase Transition and Femtoscopy

- Ideal hydro (zero viscosity), spherically symmetric expansion of a fireball, as well as the cylindrically symmetric transverse expansion of a QGP with boost-invariant initial conditions along the beam axis
- Experimentally measurable **ratio  $R_{\text{out}}/R_{\text{side}}$**  as a function of  $\epsilon_0$  for the spherical fireball geometry **reflects** closely **the behavior of the lifetime of the system, independent of details in the equation of state such as the width of the transition region  $\Delta T$  or the latent heat of the transition** (which is proportional to  $(d_Q/d_H)$ )
- **For the case of a first-order transition,  $\Delta T = 0$**  (panels a and c), the enhancement in  $R_{\text{out}}/R_{\text{side}}$  over the ideal gas case is a factor of 3 to 7 (for  $d_Q/d_H = 3$  to  $37/3$ ) at  $\epsilon_0 \sim \epsilon_Q$
- **In the case of a smooth transition,  $\Delta T = 0.1T_c$**  (panels b and d), this is considerably reduced, but if the system freezes out at temperatures  $T_f \leq 0.7T_c$ , there is still a factor of 2 enhancement over the ideal gas case

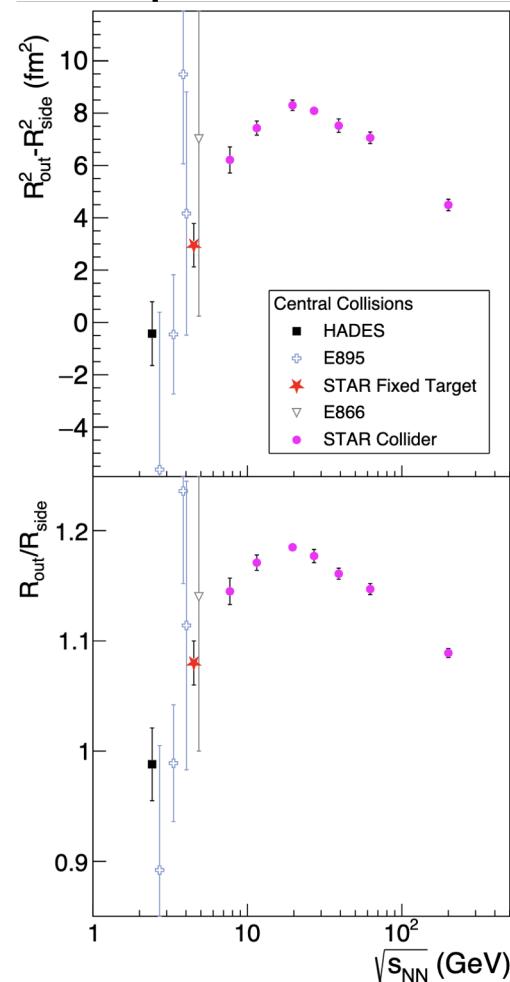
Nucl. Phys. A 608 (1996) 479



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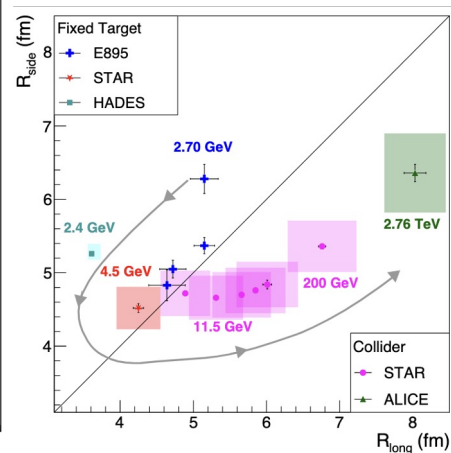
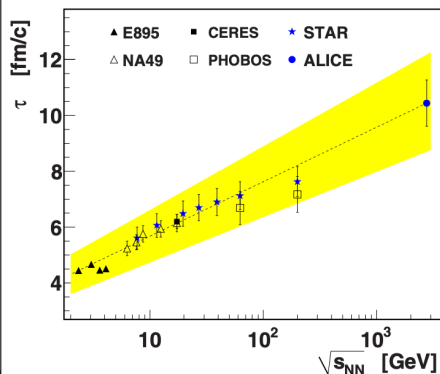
**Dogma**, in its broadest sense, is any belief held definitively and without the possibility of reform.

# Experiment Measurements: Radii



[Phys. Rev. C 92 \(2015\) 014904](#)

[Phys. Rev. C 103 \(2021\) 034908](#)



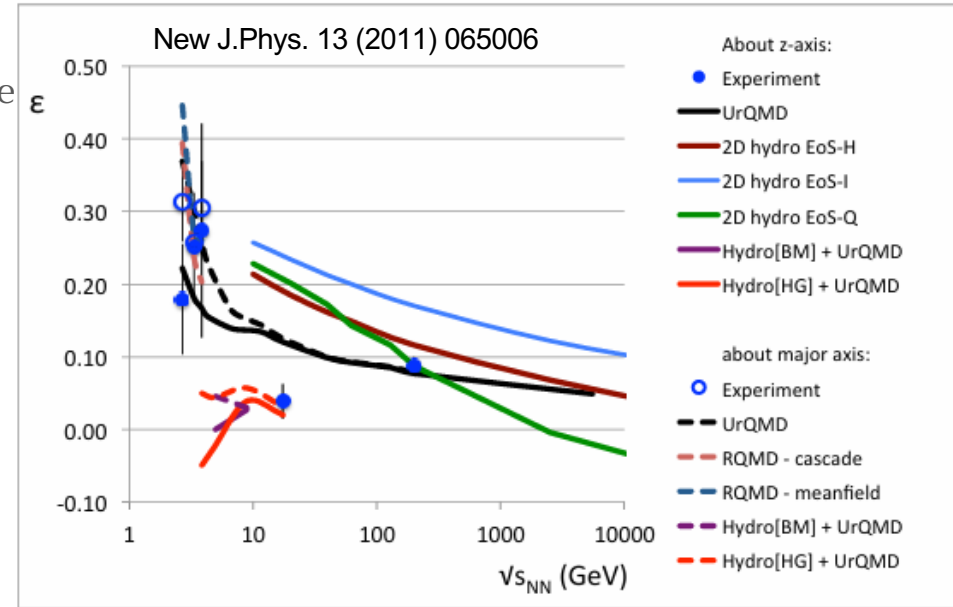
- $R_{side} \sim$  spatial extent of the particle-emitting region  
 $R_{out} \sim$  spatial extent + dynamics (related to the duration of particle emission)
- **The excitation function of  $R_{out}^2 - R_{side}^2$  shows a clear peak at  $\sqrt{s_{NN}} \approx 20$  GeV.** The values become smaller as  $\beta$  increases and even become negative at high  $\sqrt{s_{NN}}$  and  $m_T$ .  
*Assumptions (static, nonflowing source) used for the relation  $\beta\Delta\tau = R_{out}^2 - R_{side}^2$  are unreliable*
- **The  $R_{out}/R_{side}$  ratio has the advantage of removing the overall scale of the system + slightly more robust against flow effects. The ratio shows peaking structure with the maximum around at  $\sqrt{s_{NN}} \approx 20$  GeV**
- **No significant changes in the measured time (femtoscopic volume) of the maximal pion emission from AGS/SPS to LHC energies**
- Switch from in-plane to out-of-plane expansion at  $\sqrt{s_{NN}} \approx 4.5$  GeV as collision energy decreases from LHC to AGS and SIS-18

# Experimental Measurements with Respect to Event Plane

- In the blast-wave model, the relative to the reaction plane oscillations provide a good estimate of the source ellipticity at freeze-out

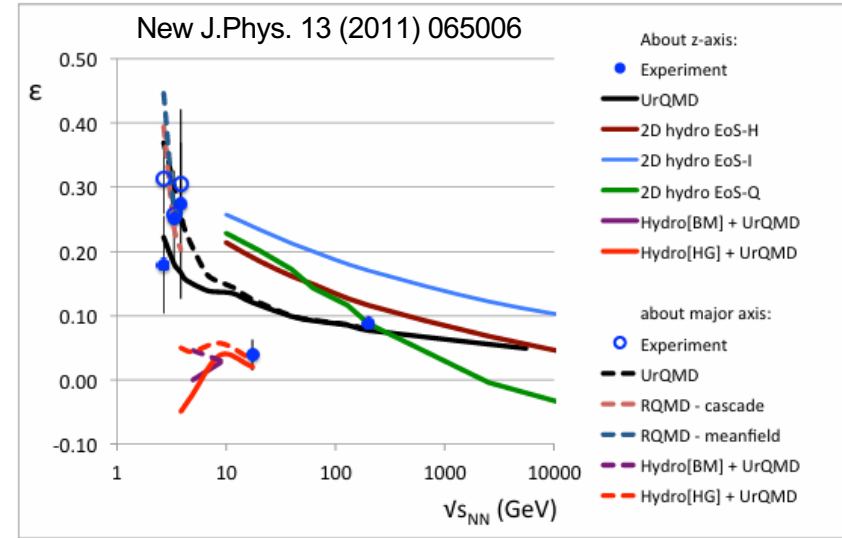
$$\epsilon \equiv \frac{(R_y^2 - R_x^2)}{(R_y^2 + R_x^2)} \approx \frac{1}{2} \cdot \frac{R_{s,2}^2}{R_{s,0}^2} \approx \frac{1}{2} \cdot \frac{R_{o,2}^2}{R_{s,0}^2} \approx \frac{1}{2} \cdot \frac{R_{os,2}^2}{R_{s,0}^2}.$$

- Where  $R_x$  and  $R_y$  are the source length scales in and out of the reaction plane, respectively. The approximation becomes exact in the case of vanishing transverse flow
- Eccentricity is positive  $\rightarrow$  Extended out of plane



# Experimental Measurements with Respect to Event Plane

- If the nature of the phase transition changes from a smooth **crossover** at high energy to a **first-order transition** at lower energy, the matter will evolve through a **mixed-phase regime** during which the **pressure gradients vanish** ( $c_s^2 = 0$ )
- Outside of the mixed-phase regime, the equation of state features even stronger pressure gradients — approximately  $c_s^2 = 1/3$  in the **QGP phase** compared to  $c_s^2 = 1/6$  in the **hadronic phase** ( $c_s^2$  quantifies the **stiffness of the equation of state (EoS)** — how strongly pressure gradients build up in response to compression or density variations)
- As the collision energy is varied, the system evolves along different trajectories through the  $T - \mu_B$  phase diagram. At low energy, the system may evolve through a first-order phase transition, and the **amount of time spent in each phase** can alter how much expansion occurs before freeze-out



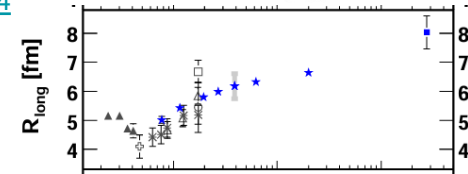
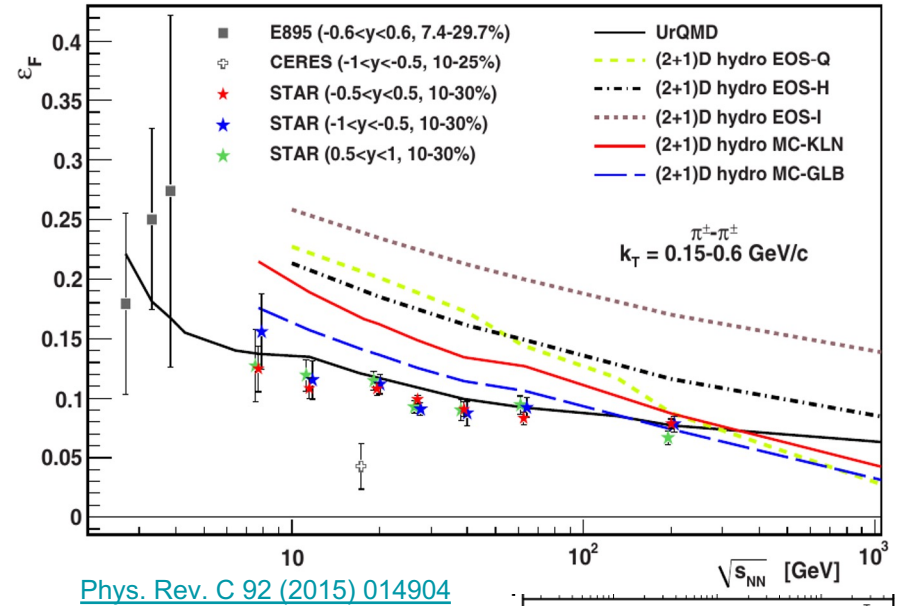
- **Minimum** observed in earlier measurements of the **freeze-out eccentricity** might correspond to **entering a mixed-phase regime** (where pressure gradients soften), followed by a **maximum** at higher energy once the system achieves **complete deconfinement** and the **strong pressure gradients reappear**.

=> **Energy dependence of the freeze-out shape** allows one to probe important physics related both to the **equation of state** and to the **dynamical processes** that drive the evolution of the collision.



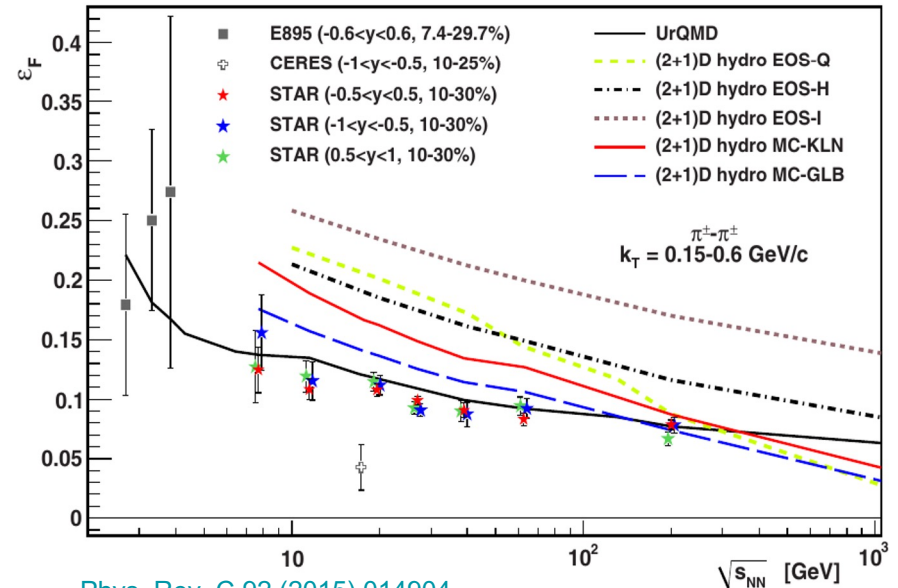
# Experimental Measurements with Respect to Event Plane

- STAR **monotonic decrease** excludes the scenario described earlier and is **consistent with an increased lifetime and/or stronger pressure gradients at higher collision energies**
- The **energy dependence of  $R_{long}$**  from the non-azimuthal analysis, together with the **lifetime**, also suggests that the system is **longer-lived at higher energies**
- All **models predict a monotonic decrease** in the **freeze-out shape** with increasing collision energy, **in agreement with the experimental data**
- The older (2+1)D, ideal hydrodynamical models [44], labeled EOS-H, EOS-I, and EOS-Q, all overpredict the data
- For EOS-Q, the slope changes, following EOS-H at low energies, but dropping more rapidly at higher energies.
- This is attributed to passage through a mixed-phase regime, which extends the lifetime, allowing the system to evolve to a rounder state at higher energies



# Experimental Measurements with Respect to Event Plane

- The older (2+1)D, ideal hydrodynamical models [44], labeled EOS-H, EOS-I, and EOS-Q, all overpredict the data.
- For EOS-Q, the slope changes, following EOS-H at low energies, but dropping more rapidly at higher energies. Attributed to passage through a mixed-phase regime
- MC-KLN and MC-GLB VISH2+1 model, get closer to the data (correspond to different initial conditions and are more realistic than the earlier results as they allow incorporation of viscous effects.
- The excitation function for freeze-out eccentricities has the potential to resolve ambiguities between models with different initial conditions and values of  $\eta/s$ . The two sets of initial conditions and  $\eta/s$  used here yield identical  $v_2$  but very different  $\epsilon_F$ .
- UrQMD follows the trajectories and interactions of all hadronic particles throughout the collision. Does not require assumptions about how freeze-out occurs. The model is 3D and does not require boost invariance; therefore, it is equally applicable at all the studied energies. This may be, at least partially, why the predictions from UrQMD more closely match the energy dependence of the data compared to the hydrodynamic predictions. While it does not explicitly contain a deconfined state, it does incorporate color degrees of freedom through the inclusion of the creation of color strings and their subsequent decay back into hadrons.

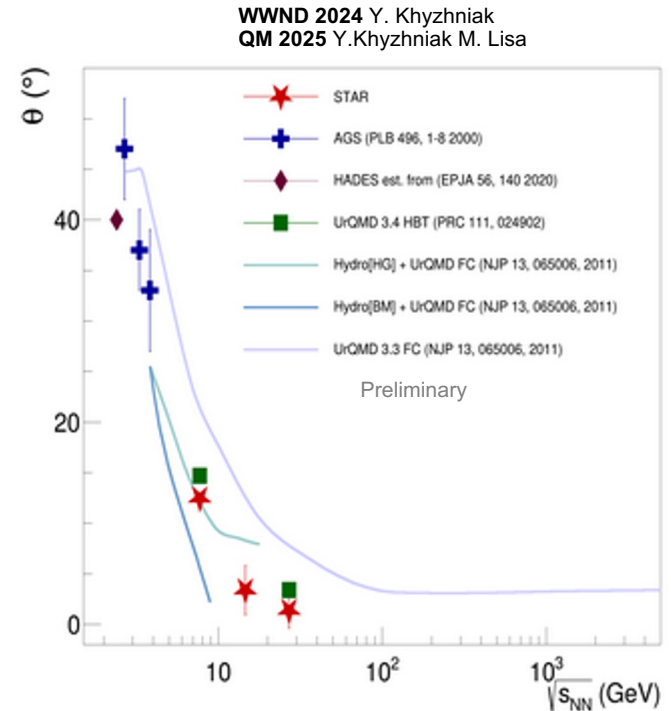


[Phys. Rev. C 92 \(2015\) 014904](#)

- UrQMD is the best to match the data
- UrQMD follows the trajectories and interactions of all hadronic particles throughout the collision, so it does not require assumptions about how freeze-out occurs. The model is 3D and does not require boost invariance; therefore, it is equally applicable at all the studied energies

# Experimental Measurements with Respect to Event Plane

- STAR results are consistent with the general trend observed in the AGS data: the tilt decreases with increasing collision energy, which is consistent with the expectation that the system becomes increasingly boost-invariant
- The STAR data lie slightly below the UrQMD3.4Cascade predictions
- The comparison between data and model calculations suggests that the tilt parameter is sensitive to the underlying equation of state (EoS).



# Results from BES-I (Interpretation Attempt)

[Phys. Rev. Lett. 114,\(2015\) 142301](#)

- The correlation length,  $\xi$ , diverges near the transition temperature ( $T_c$ ) as  $\xi \propto |\tau|^{-\nu}$  for an infinite volume
- The second-order phase transition is expected to show a pseudocritical point for correlation length  $\xi \approx L$  ( $L$  - system size).
- Leads to a characteristic power law volume ( $V$ ) dependence of the magnitude ( $\chi_T^{max}$ ), width ( $\delta_T$ ) and peak position ( $\tau_T$ ) of the susceptibility:

$$\chi_T^{max}(V) \sim L^{\gamma/\nu}$$

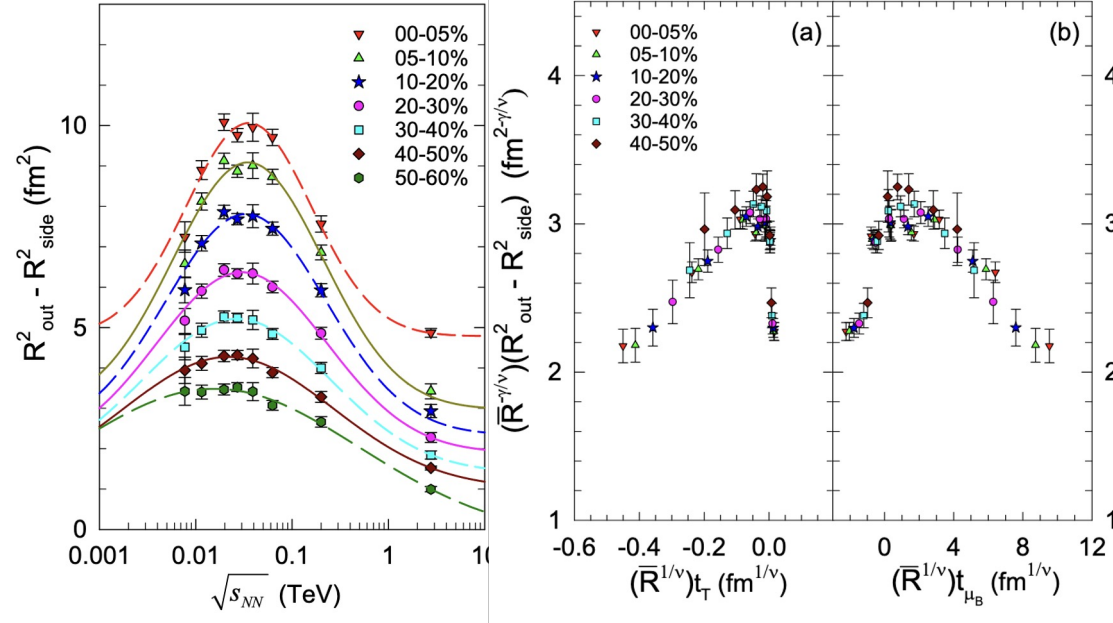
$$\delta T(V) \sim L^{-\frac{1}{\nu}}$$

$$\tau_T(V) \sim T^{cep}(V) - T^{cep}(\infty) \sim L^{-\frac{1}{\nu}}$$

where  $\nu$  and  $\gamma$  are critical exponents which characterize the divergence of  $\xi$  and  $\chi_T$  respectively

$$(R_{out}^2 - R_{side}^2)^{max} \propto \bar{R}^{\gamma/\nu}$$

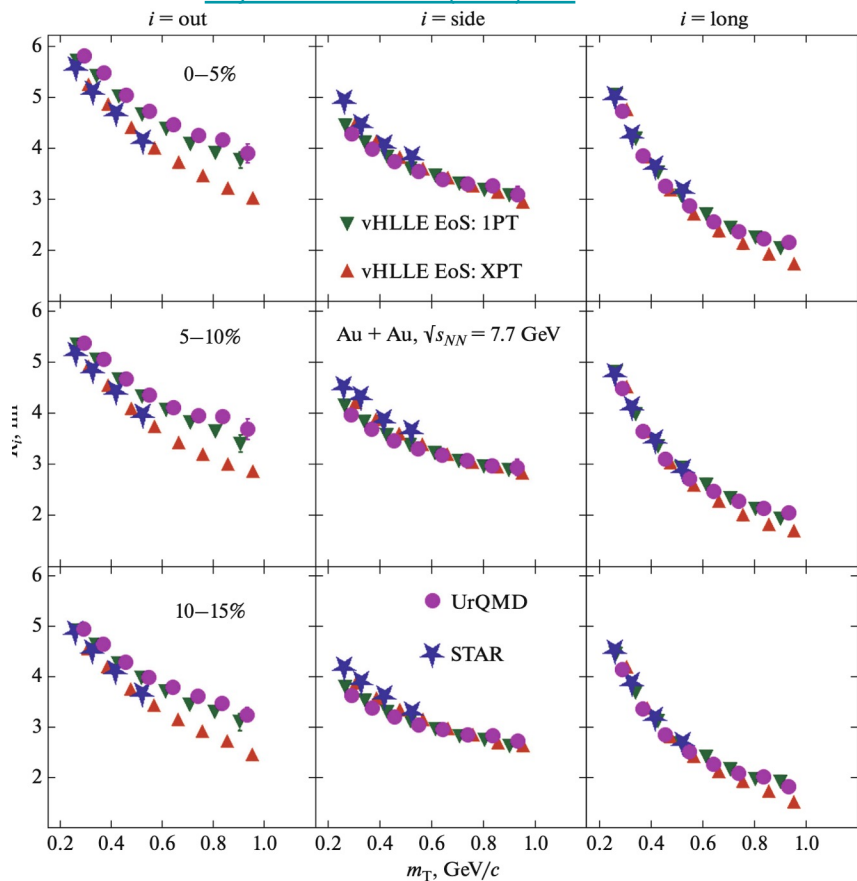
$$\sqrt{s_{NN}}(V) = \sqrt{s_{NN}}(\infty) - k \times \bar{R}^{-\frac{1}{\nu}}$$



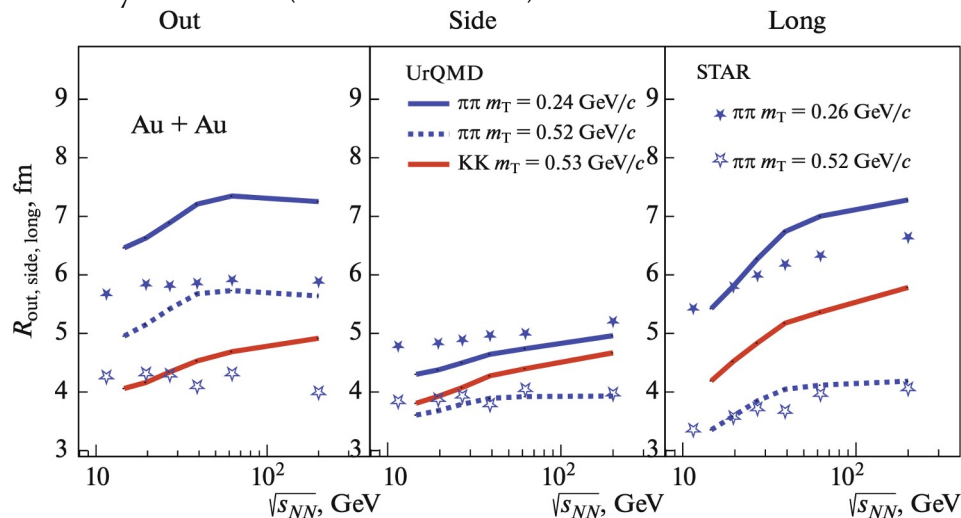
The location of the critical point to the extracted value  
 $\sqrt{s_{NN}}(\infty) \sim 47.5 \text{ GeV}$

# Results from BES-I (Comparing Exp. Data to Theory)

[Phys. Part. Nucl. 52 \(2021\) 624](#)



- Caveat: Not all MC generators contain parameters at kinetic freeze-out
- Modern MC generators qualitatively reproduce experimental data
- Introducing the femtoscopic parameter difference alone (without other observables) between simulations and data to the MC generators may not be very efficient (see next slides)



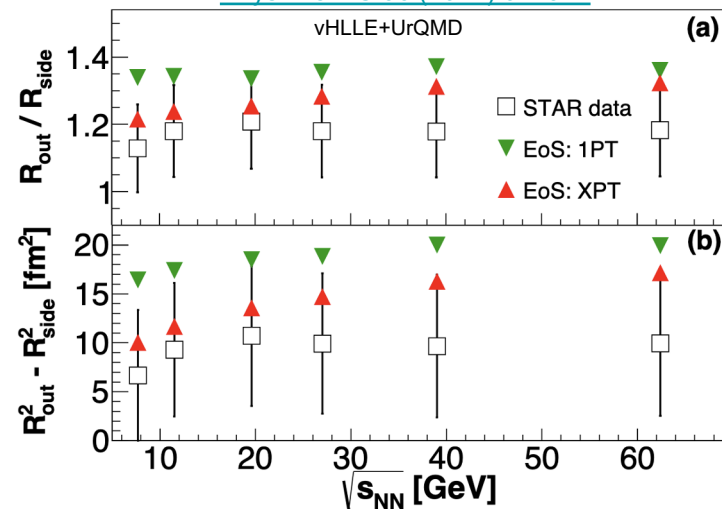
[Phys. Part. Nucl. 55 \(2024\) 879](#)

# Results from BES-I (Comparing Exp. Data to Theory)

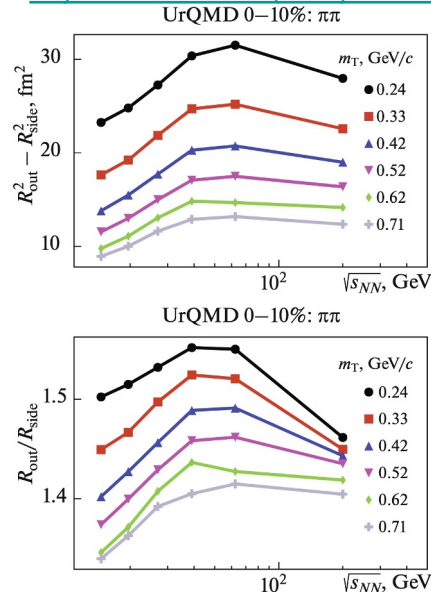
Reproducibility of the out-side behavior in models:

- **vHLE+UrQMD** – crossover is closer to the data than the 1<sup>st</sup>-order phase transition  
(demonstrates sensitivity to the phase transition type)
- **UrQMD** – qualitatively reproduces the shape (initial conditions or some dynamics?)
- **vHLE+SMASH** – shape is inverted?
  - Not clear difference

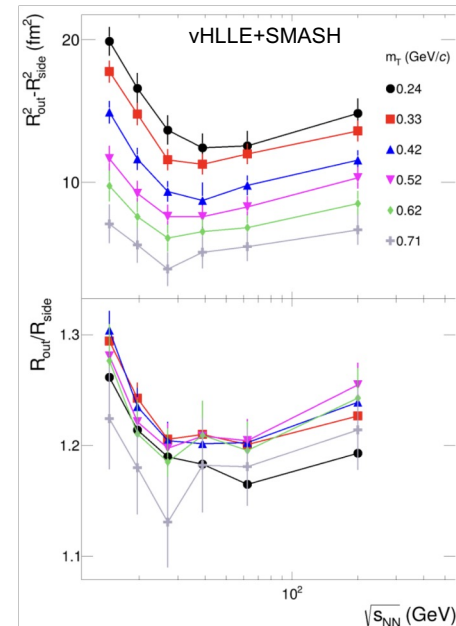
[Phys. Rev. C 96 \(2017\) 024911](#)



[Phys. Part. Nucl. 55 \(2024\) 879–883](#)



Talk at ISHEPP 2025



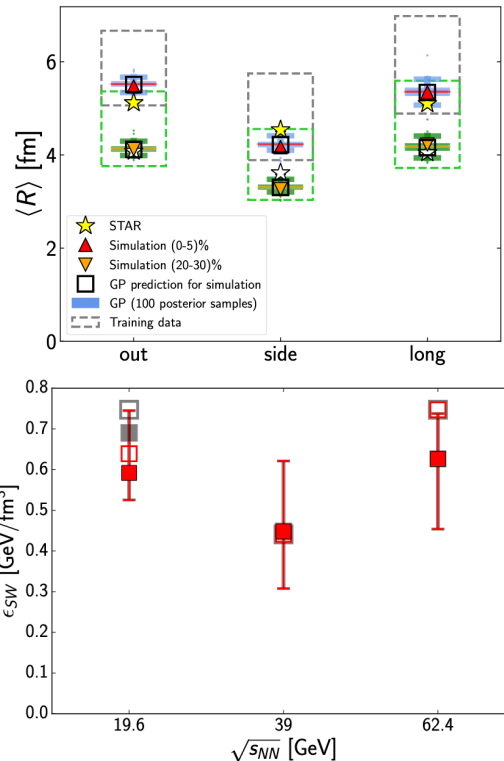
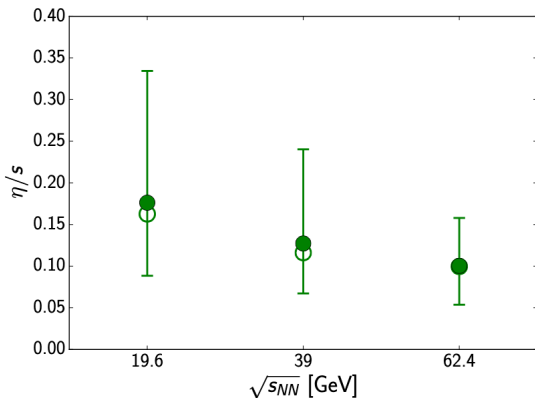
# Bayesian Analysis

- Bayesian analysis has been used to determine the probability distributions of the shear viscosity over the entropy density ratio  $\eta/s$  at  $\sqrt{s_{NN}} = 19.6, 39, \text{ and } 62.4 \text{ GeV}$  (vHLL+UrQMD):
  - $R_{\text{out}}$  is described well;  $R_{\text{long}}$  values from model output are systematically too large compared to data;  $R_{\text{side}}$  is notably smaller than the value determined from measurements
  - $R_{\text{long}}$  is more sensitive to  $\eta/s$  than  $\epsilon_{SW}$
- Recently, also for vHLL+SMASH ([Phys. Rev. C 112 \(2025\) 014910](#)); femtoscopic parameters are not utilized
- How sensitive are the properties of the quark-gluon matter to the femtoscopic radii from the Bayesian analysis (nondogmatic view)?
  - Can we learn about initial conditions?
  - Sensitivity to phase transition?
- Tests with JETSCAPE-like modular generators?

[Phys. Rev. C 97 \(2018\) 044905](#)

TABLE I. Calibration data at  $\sqrt{s_{NN}} = 19.6 \text{ GeV}$ .

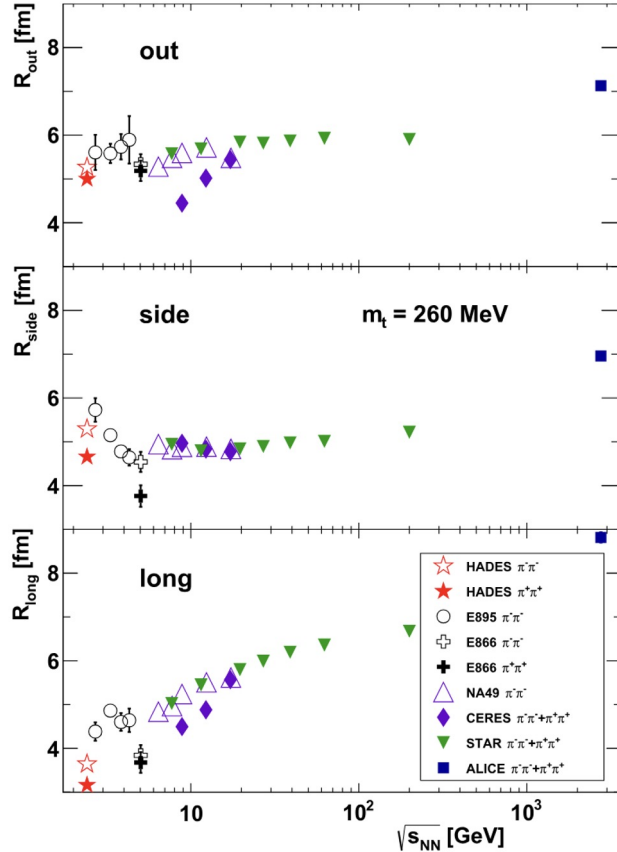
| Observable                                                                                                              | Centrality (%)          | Ref. |
|-------------------------------------------------------------------------------------------------------------------------|-------------------------|------|
| $dN_{\text{ch}}/d\eta$ at $\eta = 0.1$                                                                                  | 0–6, 6–15, 15–25        | [28] |
| $\langle \eta \rangle$ for $\eta > 0.0$                                                                                 | 0–6, 6–15, 15–25        | [28] |
| $N(\pi^+), N(\pi^-)$                                                                                                    | 0–5, 5–10, 10–20, 20–30 | [12] |
| $N(K^+), N(K^-)$                                                                                                        | 0–5, 5–10, 10–20, 20–30 | [12] |
| $\langle p_T \rangle$ for $\pi^+, \pi^-, K^+, K^-$                                                                      | 0–5, 5–10, 10–20, 20–30 | [12] |
| Charged particle $v_2\{2\}$ in $ \eta  < 0.5$                                                                           | 5–10, 10–20, 20–30      | [32] |
| $dN(\Omega)/dp_T$ at $p_T = 1.01 \text{ GeV}/c$                                                                         | 0–10                    | [33] |
| $\langle p_T \rangle$ for $\Omega$ at $p_T = 1.0–5.0 \text{ GeV}/c$                                                     | 0–10                    | [33] |
| $R_{\text{out}}, R_{\text{side}}, R_{\text{long}}$ of charged $\pi$ at $\langle k_T \rangle \approx 0.22 \text{ GeV}/c$ | 0–5, 20–30              | [11] |
| $\langle R_{\text{out}} \rangle, \langle R_{\text{side}} \rangle, \langle R_{\text{long}} \rangle$ of charged $\pi$     | 0–5, 20–30              | [11] |



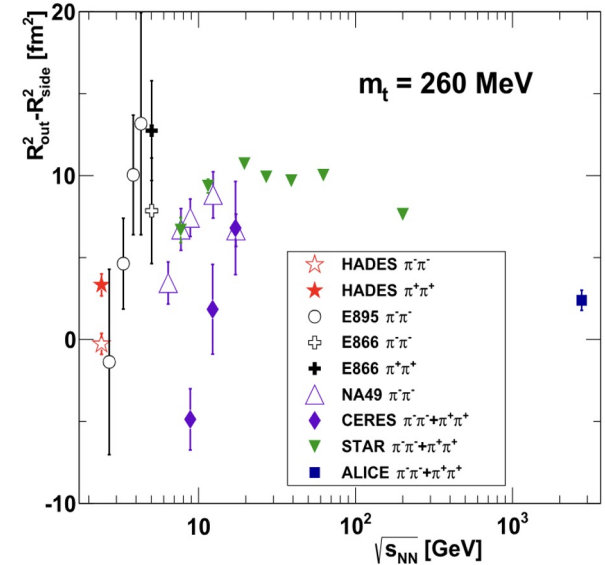


# Femtoscscopy at Low Collisions Energies

[Phys. Lett. B 795 \(2019\) 446](#)



Results from the HADES experiment demonstrate **substantial differences in the source radii for pairs of negatively and positively charged pions, especially at low transverse momenta**. The effect is hardly visible at higher collision energies

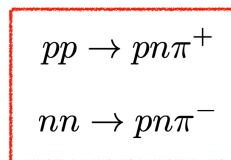




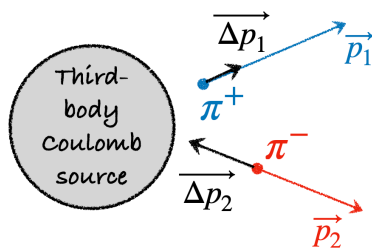
# The third-body Coulomb effect

Vinh Luong ([QM'25 talk](#))

- Clear difference in  $R_{\text{side}}$  and  $R_{\text{long}}$  radii between  $\pi^+\pi^+$  and  $\pi^-\pi^-$  at small pair transverse momentum ( $k_T$ )
- Possible effects causing the difference:
  - Third-body Coulomb effect
  - Isospin effect



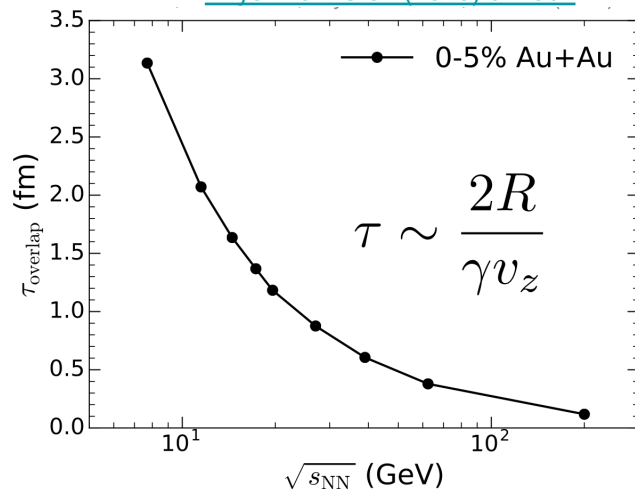
Isospin effect



Third-body Coulomb effect

| $\sqrt{s_{\text{NN}}}$ (GeV)      | 3.0   | 3.2  | 3.5  | 3.9  | 4.5  | 5.2  | 7.7  |
|-----------------------------------|-------|------|------|------|------|------|------|
| $\tau_{\text{overlap}}$ (fm/c)    | 10.11 | 9.14 | 8.02 | 6.94 | 5.80 | 4.90 | 3.18 |
| $Z_{\text{res.ch.}}^{\text{eff}}$ | 50    | 45   | 40   | 34   | 29   | 24   | 16   |

[Phys. Rev. C 97 \(2018\) 024907](#)



- What source is measured?
- Typical times of the maximum emission  $\sim 5$  fm/c

Accounting for the residual Coulomb field must be taken into account (for femtoscopy, see e.g.):

- [Phys. Rev. C 49 \(1994\) 349](#)
- [Nucl. Phys. A 604 \(1996\) 69](#)

# Femtoscscopy at Low Collisions Energies

- High-statistics data from the STAR FXT and BES-II programs allow to demonstrate the differences between positively and negatively charged femtoscopic correlations
- The 3<sup>rd</sup>-body Coulomb effect must be more pronounced in spectra
- “Simple” ad-hoc corrections allow to remove the difference between  $\pi^+\pi^+$  and  $\pi^-\pi^-$  femtoscopic radii

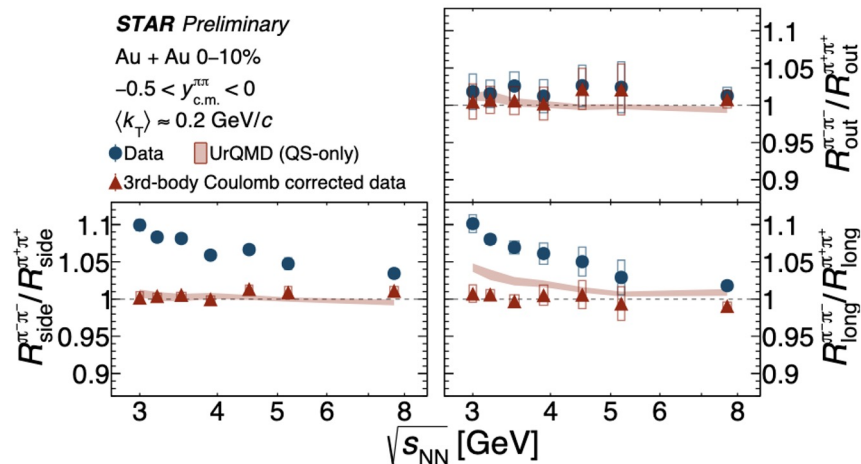
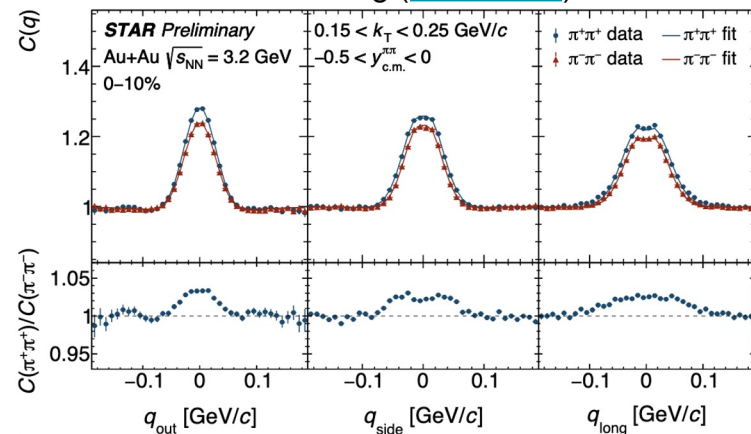
## Residual Coulomb field:

- Many observables may be affected. Simple parametrizations? Lessons from low-energy nuclear physics?
- Is it possible to introduce the effect in the MC generators?

## Influence of residual Coulomb field and initial isospin on observables:

- Is it possible to decouple the effects?
- The latter seems to be smaller than the former

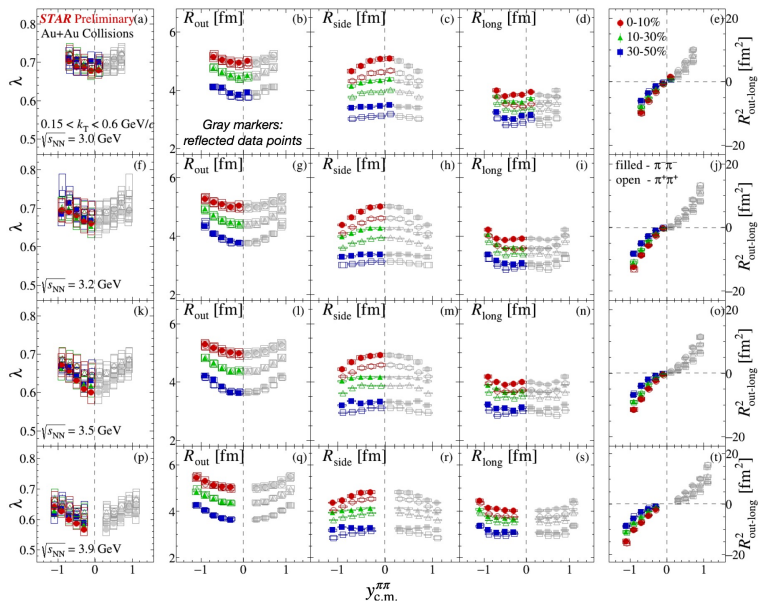
Vinh Luong (QM'25 talk)



# Femtoscscopy at Low Collisions Energies

Vinh Luong (QM'25 talk)

## Rapidity dependence of femtoscopic parameters



- The difference of  $R_{\text{side}}$  and  $R_{\text{long}}$  between positive and negative pion is observed
- $R_{\text{side}}$  decreases as  $|y_{\text{c.m.}}^{\pi\pi}|$  increases
  - ▶ Hint on boost-invariance breaking
- $|R_{\text{out-long}}^2|$  is larger for larger  $|y_{\text{c.m.}}^{\pi\pi}|$

- Boost-invariance braking at low collision energies?
- Implications?

# Other Femtoscopic Correlations

- During recent years, non-identical particle femtoscopic correlations have become a powerful tool to study the final state interaction parameters
  - Including recent p-d and d-d correlations measured in heavy-ion collisions ([Phys. Lett. B 864 \(2025\) 139412](#)) and hyperon interactions from pp collisions (e.g.,  $\Lambda\Xi$  [Phys. Lett. B 844 \(2023\) 137223](#))
  - Importance for astrophysics (neutron stars). What particle combinations are needed for the theory?
- Do non-identical particle correlations provide better sensitivity to the dynamics and type of the phase transition as compared to identical particle correlations?
- Other particle species? KK, pp,  $\bar{p}\bar{p}$ , ...?
  - KK – doable with the data from STAR
  - pp,  $\bar{p}\bar{p}$  - statistics demanding (1D analysis is possible for BES-II, 3D may be very limited at low collision energies). Need to know final state interaction in 3D (feasible? Analytical or numerical calculations using modern computers?)
- Many-particle femtoscopic correlations (only few studies done)
  - Sensitivity to coherence?
  - Sensitivity to phase transitions?

# Summary and Discussions

- Correlation femtoscopy provided quite a number of extremely important insights into the properties of quark-gluon matter evolution
- Experimental search for the 1<sup>st</sup>-order phase transition
  - Peak structure observed in the identical pion correlations
    - Are other particle species better?
    - Proton and antiproton identical pairs may shed light due to different production mechanisms (BES-II data from STAR are available) and be complementary to  $dv_1/dy$  and net-proton fluctuation measurements
  - Can be reproduced by some models even without phase transitions
  - Non-identical femtoscopic correlations for low- and middle-collision energies from RHIC are missing
- Theory
  - Theoretical calculations and guidance are needed
    - Initial conditions, modern parameters for the phase transitions, accurate description of the final scatterings. How to check the reliability of dynamics? Can we pin the components one by one?
    - How would the 1st-order phase transition devastate itself?
  - Bayesian analysis with femtoscopic radii is important. Do we need close collaboration between experimentalists and theorists in order to boost the work?

# Backup slides

# Maximum of Pion Emission Time and Interferometric Volume

[Phys. Rev. C 96 \(2017\) 024911](#)

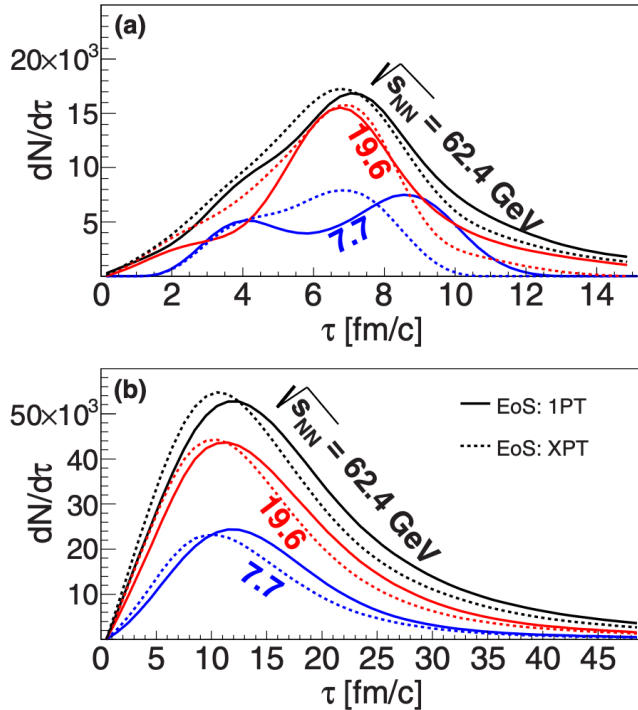
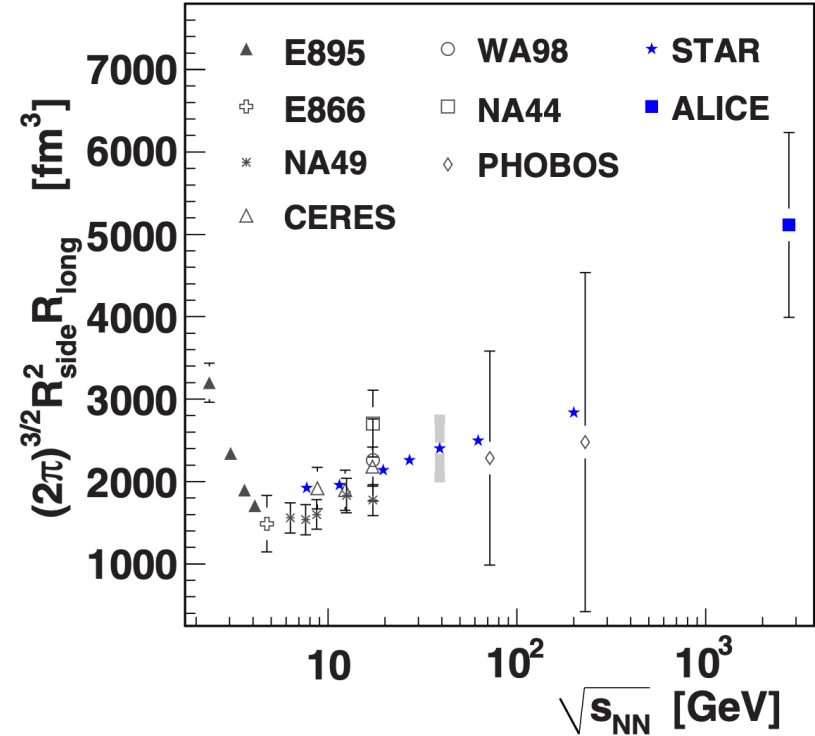
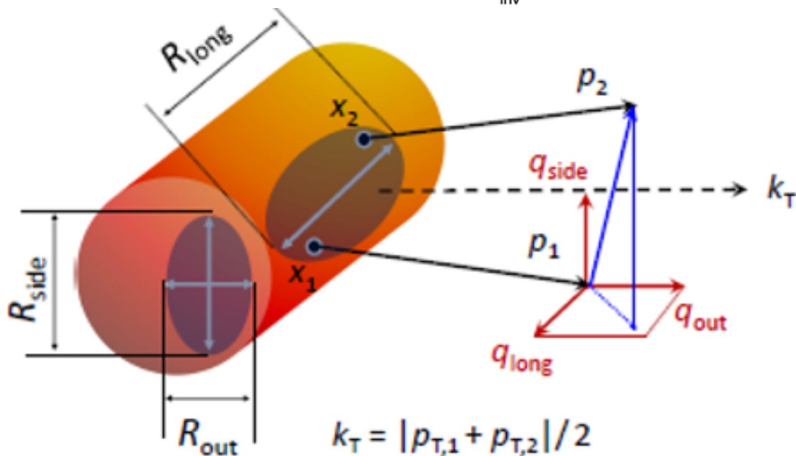
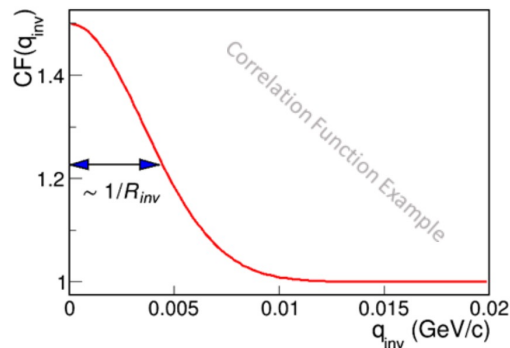


FIG. 2. Pion emission times at the particleization surface (a) and the last interactions (b) in the center-of-mass system of colliding gold nuclei at different values of  $\sqrt{s_{NN}}$ .

[Phys. Rev. C 92 \(2015\) 014904](#)



# Collision Evolution: Spatial and Temporal Information



- Experimentally, correlation function:  $C(\vec{q}) = A(\vec{q})/B(\vec{q})$ 
  - Relative momentum:  $\vec{q} = \vec{p}_1 - \vec{p}_2$
  - $A(\vec{q})$ : measured distribution of  $\vec{q}$  within the same event, containing quantum statistic (QS) correlation and final state interactions
  - $B(\vec{q})$ : background distribution of  $\vec{q}$  of two tracks from different events, where physical correlations are absent
- Projection of  $\vec{q}$  onto Bertsch-Pratt longitudinal co-moving system (LCMS):
  - $q_{out}$ : along pair transverse momentum ( $k_T$ )
  - $q_{long}$ : along beam direction
  - $q_{side}$ : perpendicular to the other two axes

S. Pratt, PRD **33**, 1314 (1986)  
G. Bertsch, M. Gong, M. Tohyama, PRC **37**, 1896 (1988)



# Results from vHLE+UrQMD

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