The double-pole nature of the $\Lambda(1405)$ from Lattice QCD

Daniel Mohler

Technische Universität Darmstadt

Seattle,
24. March, 2023

People

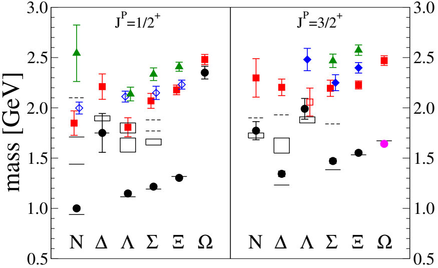
- Diverse subgroup of people doing spectroscopy using CLS ensembles:
  - DESY Zeuthen $\rightarrow$ Bochum: John Bulava
  - BNL: Andrew Hanlon
  - Intel: Ben Hörz
  - North Carolina: Amy Nicholson, Joseph Moscoso
  - TU Darmstadt/GSI: Daniel Mohler, Barbara Cid Mora
  - CMU: Colin Morningstar, Sarah Skinner
  - MIT: Fernando Romero-López
  - LBNL: André Walker-Loud
- All results are still preliminary, but many analysis details are settled

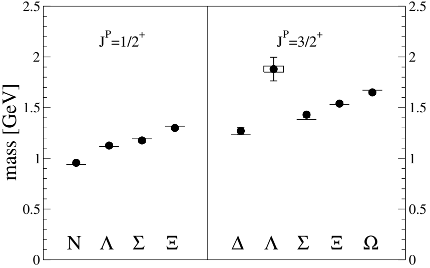
Lattice QCD and quark-model puzzles

- Various kind of exotic/unconventional states (examples)
  - Mesons: light scalar resonances, $D^*_{s0}(2317)$, $D_{s1}(2460)$ and b-quark cousins, XYZ states, hybrid mesons
  - Baryons: Roper resonance; $\Lambda(1405)$, Pentaquark states
  - Glueballs, ...
- Hadron-hadron scattering: Different challenges
  - Need for all-to-all propagators (at least timeslice-to-timeslice) for meson-meson and meson-baryon scattering
  - Noise problem particularly severe for calculations involving baryons
  - Cost of contractions/correlation functions much larger for systems with baryons
- $\Lambda(1405)$: Difficult but likely feasible with current methods

Baryon bound-states and resonances: Ancient history

Engel, Lang, DM, Schäfer, PRD 87 074504 (2013)





- Spectra resulted from 3-quark interpolators only
  mostly no indications of multiparticle levels
- Sensible for
  - refining methods
  - getting an idea about the number of states
  - spectra at very heavy quark masses
  - some narrow states (i.e. high spin)
- We need to make clear what they were used for!

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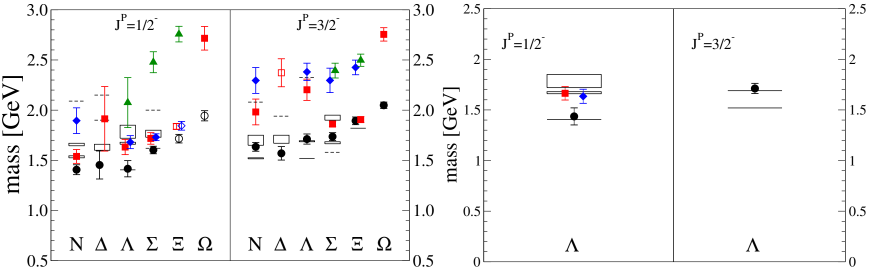

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Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)

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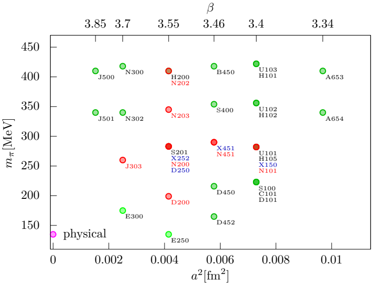

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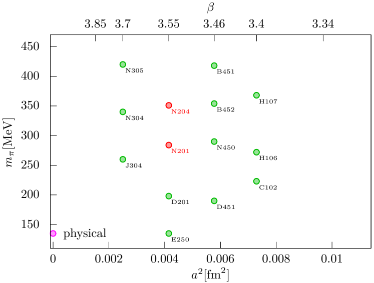

plot style by Jakob Simeth, RQCD

Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Taking the *infinite volume limit*: $L \to \infty$
- Calculation at (or extrapolation to) physical quark masses

CLS gauge field ensembles

Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)

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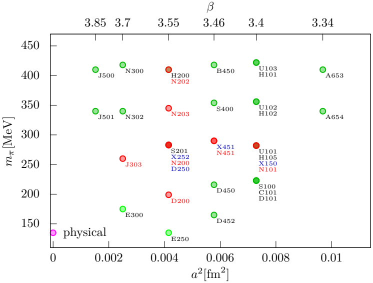

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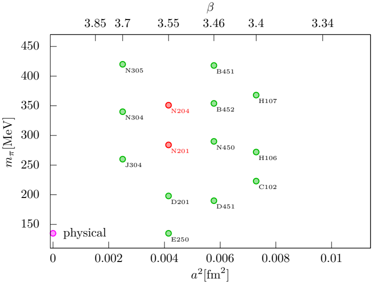

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Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Want to exploit (power law) finite volume effects (keeping exponential effects small)
- Calculation at (or extrapolation to) physical quark masses

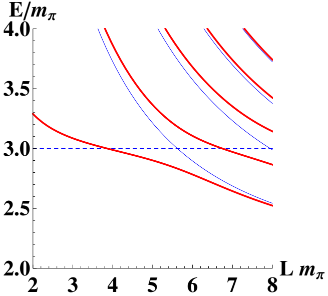
Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite-volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes L^3
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \to \pi\gamma^*$)
- Significant progress for 3-particle scattering



Please refer to the other talks!

An old puzzle: $\Lambda(1405)$, $J^P = \frac{1}{2}^-$

- PDG (4 star resonance)

$$M_\Lambda = 1405^{+1.3}_{-1.0} MeV \qquad \Gamma_\Lambda = 50.5 \pm 2.0$$

  (Some) quark models struggled to accommodate this state.
- However
  - Unitarized χPT + Model input yields 2 poles with $\Re \approx 1400$ MeV
    $\rightarrow$ Now new PDG state
  - CLAS observes different line shapes for $\Sigma^-\pi^+$, $\Sigma^+\pi^-$ and $\Sigma^0\pi^0$
    Interference between $I = 0$ and $I = 1$ amplitudes is the likely reason
  - Even the $\Sigma^0\pi^0$ is badly described by a single Breit-Wigner
  - CLAS data consistent with popular 2-pole picture
  - No satisfactory lattice results (although claims exist)
- Relevant channels: $\Sigma\pi$, $N\bar{K}$ (and maybe $\Lambda\eta$); simulation in isospin limit
- Goal: Explore coupled channel problem and extract scattering amplitudes from the low-lying energy spectrum

$\Lambda(1405)$ – Experimental developments

- Angular analysis of the process $\gamma + p \to K^+ + \Sigma + \pi$ by CLAS strongly favors the assignment of quantum numbers $J^P = \frac{1}{2}^-$

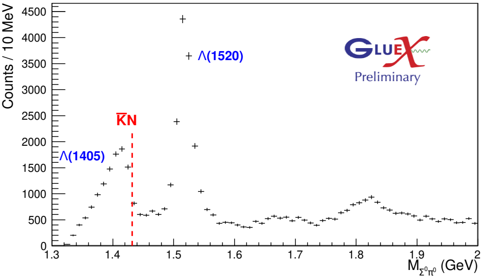
  Moriya *et al.*, PRC 87 035206 (2013)

- $K^- p$ scattering length determined by the SIDDHARTHA collaboration

  Bazzi *et al.*,PLB 704 (2011) 113

- A glimpse of the future: Preliminary analysis at GlueX

  Wickramaarachchi *et al.*, arXiv:2209.06230



Excited state energies and the variational method

Matrix of correlators projected to fixed momentum (will assume 0)

$$C(t)_{ij} = \sum_n \mathrm{e}^{-tE_n} \langle 0|O_i|n\rangle \left\langle n|O_j^\dagger|0\right\rangle$$

Solve the generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$
$$\lambda^{(k)}(t) \propto \mathrm{e}^{-tE_k}\left(1 + \mathcal{O}\left(\mathrm{e}^{-t\Delta E_k}\right)\right)$$

At large time separation: only a single state in each eigenvalue.
Eigenvectors can serve as a fingerprint.

Michael Nucl. Phys. B259, 58 (1985)

Lüscher and Wolff Nucl. Phys. B339, 222 (1990)

Blossier *et al.* JHEP 04, 094 (2009)

The "Distillation" method

Peardon et al. PRD 80, 054506 (2009)
Morningstar et al. PRD 83, 114505 (2011)

- Idea: Construct separable quark smearing operator using low modes of the 3D lattice Laplacian
  Spectral decomposition for an $N \times N$ matrix:

$$f(A) = \sum_{k=1}^{N} f(\lambda^{(k)})\, v^{(k)} v^{(k)\dagger}.$$

  With $f(\nabla^2) = \Theta(\sigma_s^2 + \nabla^2)$ (Laplacian-Heaviside (LapH) smearing):

$$q_s \equiv \sum_{k=1}^{N} \Theta(\sigma_s^2 + \lambda^{(k)}) v^{(k)} v^{(k)\dagger}\, q \;=\; \sum_{k=1}^{N_v} v^{(k)} v^{(k)\dagger}\, q\,.$$

- Advantages: momentum projection at source; large interpolator freedom, small storage
- Disadvantages: expensive; unfavorable volume scaling
- Stochastic approach (partly) eliminates bad volume scaling

Ensemble and group theory

Current data on CLS Ensemble D200

a [fm]	$T \times L^3$	m_π [MeV]	m_K [MeV]	$m_\pi L$	N_{cnfg}
0.0633(4)(6)	128×64^3	200	480	4.3	2000

Lattice irreducible representations for a given J^P

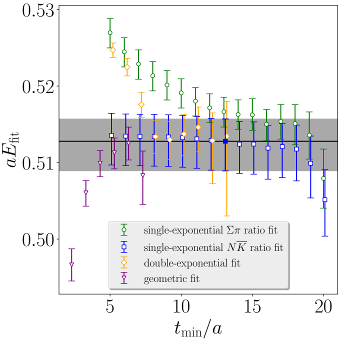
see Morningstar *et al.* arXiv:1303.6816

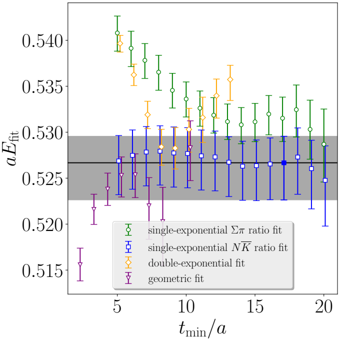
J^P	[000]	[00n]	[0nn]	[nnn]	
$\frac{1}{2}^+$	G_{1g}	G_1	G	G	Λ, $\Lambda(1600)$
$\frac{1}{2}^-$	G_{1u}	G_1	G	G	$\Lambda(1405)$, $\Lambda(1670)$
$\frac{3}{2}^+$	H_g	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1690)$
$\frac{3}{2}^-$	H_u	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1520)$, $\Lambda(1690)$

Specific setup on D200

- Combined basis of simple 3-quark structures and 2 hadron interpolators with the lowest few momentum combinations in each irrep
- Distillation setup:
  - $n_{ev} = 448$ eigenmodes of the Lattice Laplacian
  - Quark lines connecting source and sink:
    Noise dilution scheme with $(TF, SF, LI16)$ and 6 noises
  - Lines starting end ending on the same time slice:
    Noise dilution scheme with $(TI8, SF, LI16)$ and 2 noises
  - Four source time slices
  - Lattice Laplacian constructed on stout smeared links with $(\rho, n) = (0.1, 36)$

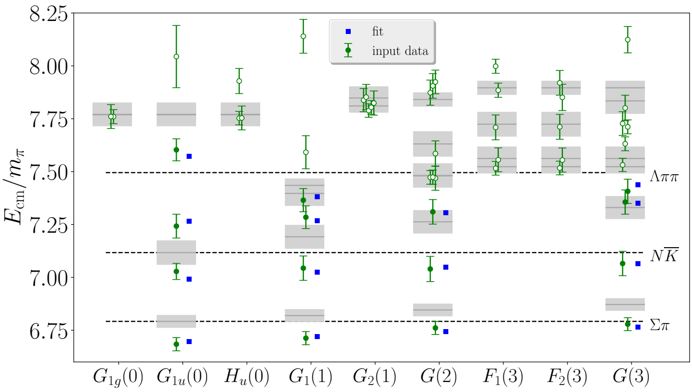
Extracting the spectrum (examples)





- We used various methods/cross checks
- Geometric series fit: $C(t) = \frac{Ae^{-E_0 t}}{1-Be^{-\Delta E t}}$
- Two students with two slightly different analysis methods

Finite volume spectra (preliminary)



- Full symbols will be used in our $\Lambda(1405)$ analysis
- Amplitude analysis uses ratios to extract energy differences with regard to non-interacting levels
- Blue squares indicate results from our preferred amplitude fit

A family of simple parameterizations

Blatt-Biederharn parameterization

$$\tilde{K}^{-1} = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix} \begin{pmatrix} \frac{k}{M_\pi}\cot\delta_1 & 0 \\ 0 & \frac{k}{M_\pi}\cot\delta_2 \end{pmatrix} \begin{pmatrix} \cos\epsilon & -\sin\epsilon \\ \sin\epsilon & \cos\epsilon \end{pmatrix}.$$

Each quantity can be parameterized by an effective range expansion (ERE):

$$\frac{k}{M_\pi}\cot\delta_1 = \sqrt{s}(A_1 + B_1\Delta_{NK} + ...), \quad \Delta_{NK} = \frac{s-(M_N+M_K)^2}{(M_N+M_K)^2}$$

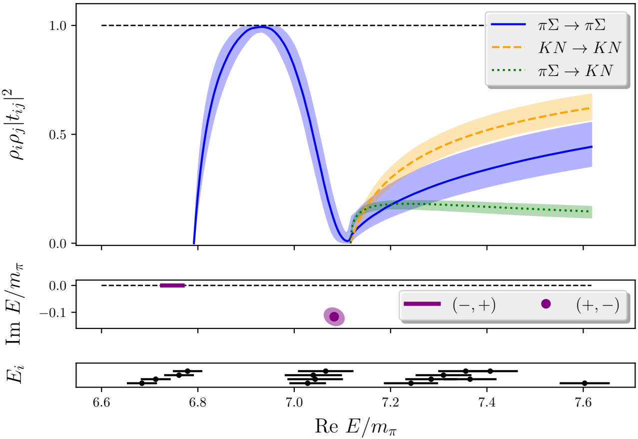
$$\frac{k}{M_\pi}\cot\delta_2 = \sqrt{s}(A_2 + B_2\Delta_{\pi\Sigma} + ...), \quad \Delta_{\pi\Sigma} = \frac{s-(M_\pi+M_\Sigma)^2}{(M_\pi+M_\Sigma)^2}$$

and

$$\epsilon = \epsilon_0 + \epsilon_1\Delta_{\pi\Sigma}$$

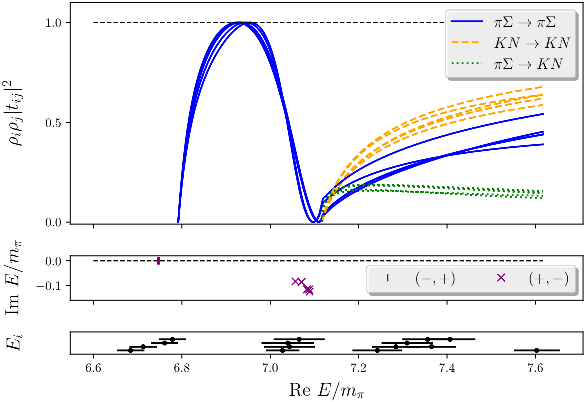
$\Delta_{\pi\Sigma}$ measures the distance from the $\pi\Sigma$ threshold

Our preferred amplitude and resulting poles



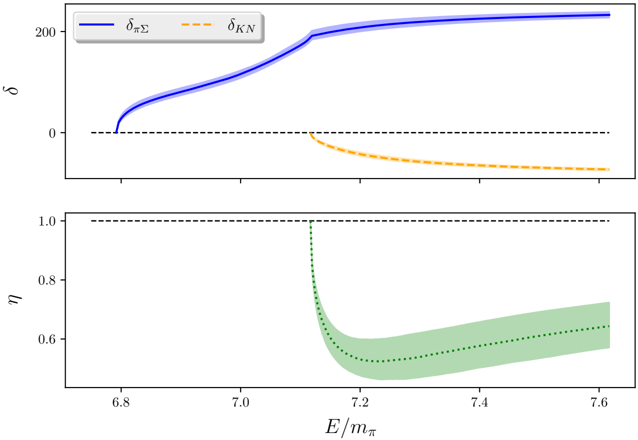
- Sub-threshold levels pose strong constraints on the amplitude
- Limited data and therefore limited possibility to vary parameterizations (suggestions welcome)

Some Variations of the used amplitude



- Results from varying the terms in the parameterization/ from omitting the highest data point
- Some simpler options, including single Flatté checked (not displayed)
- We also explored simple constraints for higher partial waves (negligible effect in range used)

Same thing different: Phases and inelasticity



- Alternative way of showing our results: 2 phases and inelasticity η

Pole positions and expectations from the literature

	Pole II (sheet)	Pole I (sheet)
three parameters fit with $\sqrt{s}$	$E = 6.747(21)\ (+,-)$	$E = 7.083(22) - 0.116(35)i\ (-,+)$
w/o highest energy level	$E = 6.747(19)\ (+,-)$	$E = 7.071(19) - 0.086(35)i\ (-,+)$
three params + B_1	$E = 6.745(22)\ (+,-)$	$E = 7.088(25) - 0.117(37)i\ (-,+)$
three params + B_2	$E = 6.747(17)\ (+,-)$	$E = 7.057(23) - 0.084(38)i\ (-,+)$
three parameters fit w/o $\sqrt{s}$	$E = 6.749(21)\ (+,-)$	$E = 7.089(24) - 0.125(38)i\ (-,+)$

- Poles labeled as $(\pm, \pm)$ depending on the signs of $(k_{KN}, k_{\pi\Sigma})$
- Rough conversion to physical units yields

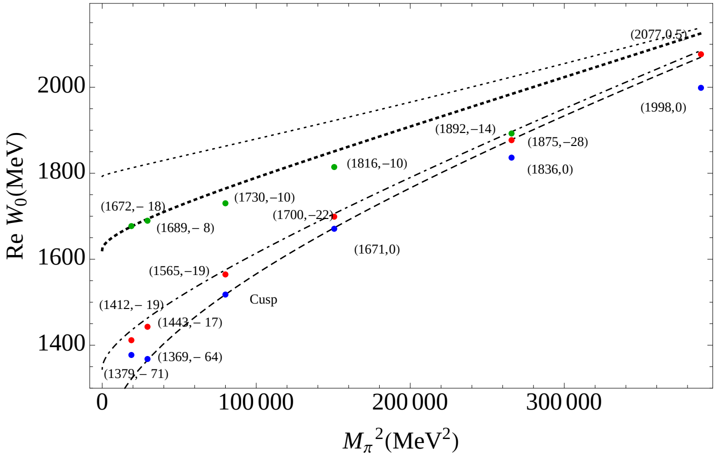
$$\begin{array}{ll} \text{Pole I} & 1460(20) - i24(7)\text{MeV} \\ \text{Pole II} & 1390(20)\text{MeV} \end{array}$$

- Examples from the PDG review

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. [14, 15], NLO	$1424^{+7}_{-23} - i\ 26^{+3}_{-14}$	$1381^{+18}_{-6} - i\ 81^{+19}_{-8}$
Ref. [17], Fit II	$1421^{+3}_{-2} - i\ 19^{+8}_{-5}$	$1388^{+9}_{-9} - i\ 114^{+24}_{-25}$
Ref. [18], solution #2	$1434^{+2}_{-2} - i\ 10^{+2}_{-1}$	$1330^{+4}_{-5} - i\ 56^{+17}_{-11}$
Ref. [18], solution #4	$1429^{+8}_{-7} - i\ 12^{+2}_{-3}$	$1325^{+15}_{-15} - i\ 90^{+12}_{-18}$

Expected quark-mass dependence

Molina, Döring, PRD 94 056010 (2016)



- Qualitative agreement with regard to expected behavior

Conclusions and Outlook

- First coupled-channel LQCD calculation in the baryon sector
- Suitable K-matrix parameterizations suggest two poles at our m_π
- Masses remarkably similar to physical situation in Unitarized χPT
  Consequence of $\mathrm{Tr}(M) = \mathrm{const.}$?
- We would like to explore the quark-mass dependence.
- We would like to calculate more comprehensive spectra.
- More lattice data from additional volumes?
- Other channels with strangeness?
- Inconvenient things: discretization effects, chiral extrapolation, better parameterizations
- To be provocative: Any Lattice QCD calculation at a single lattice spacing is just a lattice model!

Backup slides