Heavy Hybrid decays to Quarkonia

INT Workshop: Accessing and Understanding the QCD spectra

Institute of Nuclear Theory

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arXiv:2212.09187 (Accepted in PRD)

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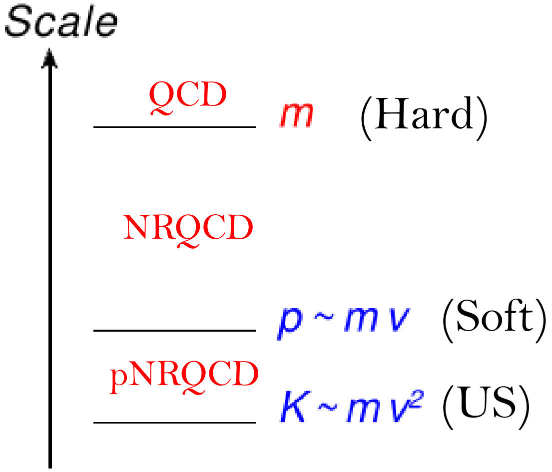
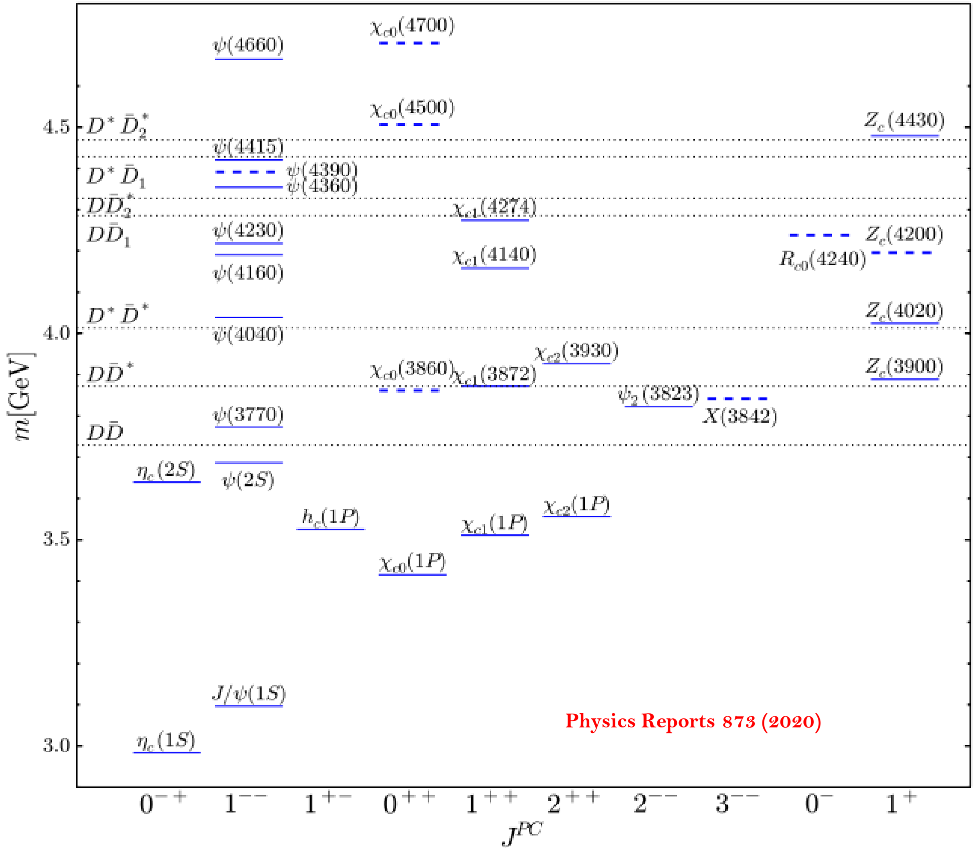
Quarkonium

- Color singlet bound state of $Q\bar{Q}$ (Q = c, b).
- Hierarchy of Energy Scales:
  - ✓ Mass : m
  - ✓ Relative separation: $r \sim 1/mv$
  - ✓ Non-perturbative physics: Λ_{QCD}
  - ✓ **Nonrelativistic bound-state:** $v \ll 1$ $(m \gg mv)$
  - ✓ Heavy Quark K.E scale: mv^2





$$m \gg mv \gg mv^2 \sim \Lambda_{QCD}$$

(perturbative dynamics: Weakly Coupled)

⬇

Heirarchy for low-lying states (far-away from threshold)

Ex. J/ψ, $\Upsilon(1S)$.

$$m \gg mv \sim \Lambda_{QCD} \gg mv^2$$

(nonperturbative dynamics: Strongly Coupled)

⬇

Hierarchy for all other cases.

- Potential-NRQCD (pNRQCD): EFT for quarkonium. **Describes physics at the scale mv^2.**
- pNRQCD: Schrödinger description for the quarkonium states. **QCD analogy** of Hydrogen atom!!

Mehen and Fleming, Phys. Rev. D73, 034502 (2005) Bodwin, Braaten & Lepage (1994) Luke, Manohar & Rothstein (2000) Brambilla, Pineda, Soto and Vairo, Rev. Mod. Phys 77, (2005)

Exotic Hadron

- Quark Model:

  Mesons: quark-antiquark states   Baryons: 3-quark states

- QCD spectrum: allows more complex structures called as Exotics.

- Exotic states: XYZ mesons (heavy-quark sector)

  ✓ Quarkonium-like states that don't fit traditional $Q\bar{Q}$ spectrum.

  ✓ In some cases exotic quantum numbers:

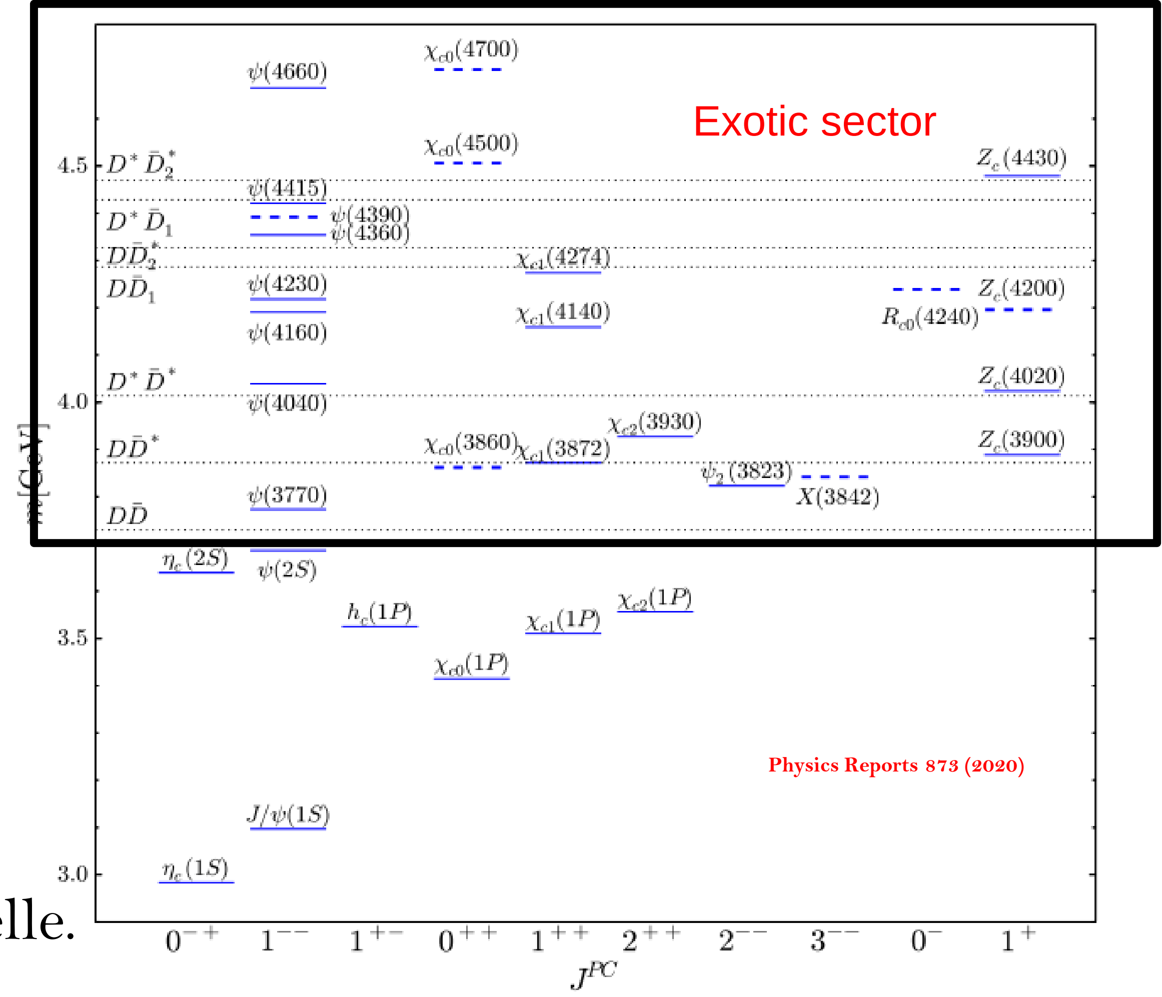
  - $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic

  - Charged Z_c and Z_b states: minimal 4-quark state: $Z_c(4430)^{\pm}$ $Z_b(10650)^{\pm}$

    Tetraquarks

For review see Brambilla et al. Phys. Reports. 873 (2020)

https://www.nikhef.nl/~pkoppenb/particles.html



- X(3872): First exotic state discovered in 2003 by Belle.

  Phys. Rev. Lett. 91, 262001 (2003)

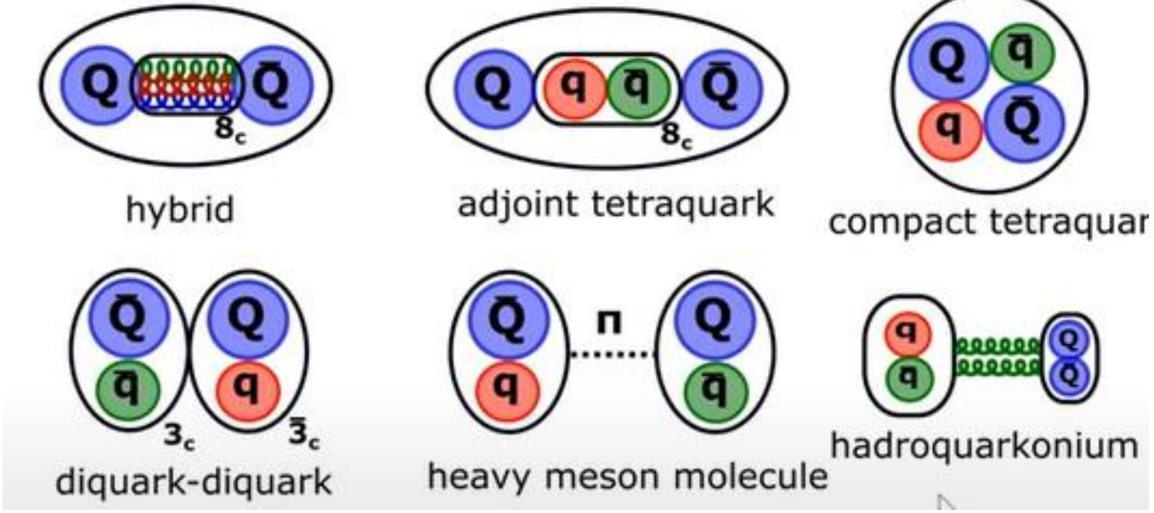
- Several **new** heavy quark exotic states (around 30 XYZ mesons) have been discovered since 2003 (masses & decay rates measured in various channels).

PDG 2022

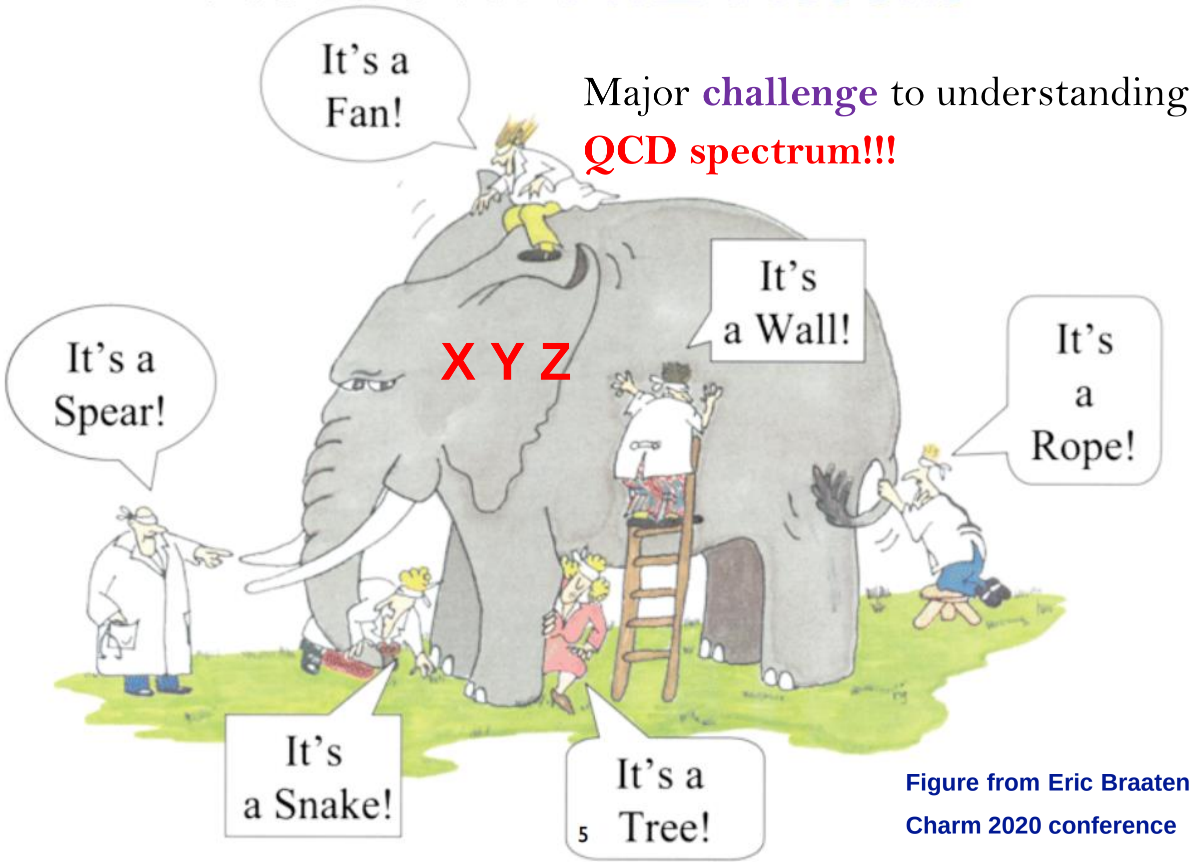
Exotic Hadron

- Exotics broadly classified as
  - ❖ Structures with active gluons ❖ Multiquark stat
- Multiple Models for Exotics:

Figure from J. Castellà talk



and many more ……….



Major **challenge** to understanding of **QCD spectrum!!!**

Figure from Eric Braaten talk: Charm 2020 conference

- Individual success in describing some XYZ hadrons. No success in revealing general pattern.

AIM: Coherent comprehensive framework based on QCD for all X Y Z hadrons !!!

Exotic Hadron

- Multiple Models for Exotics:

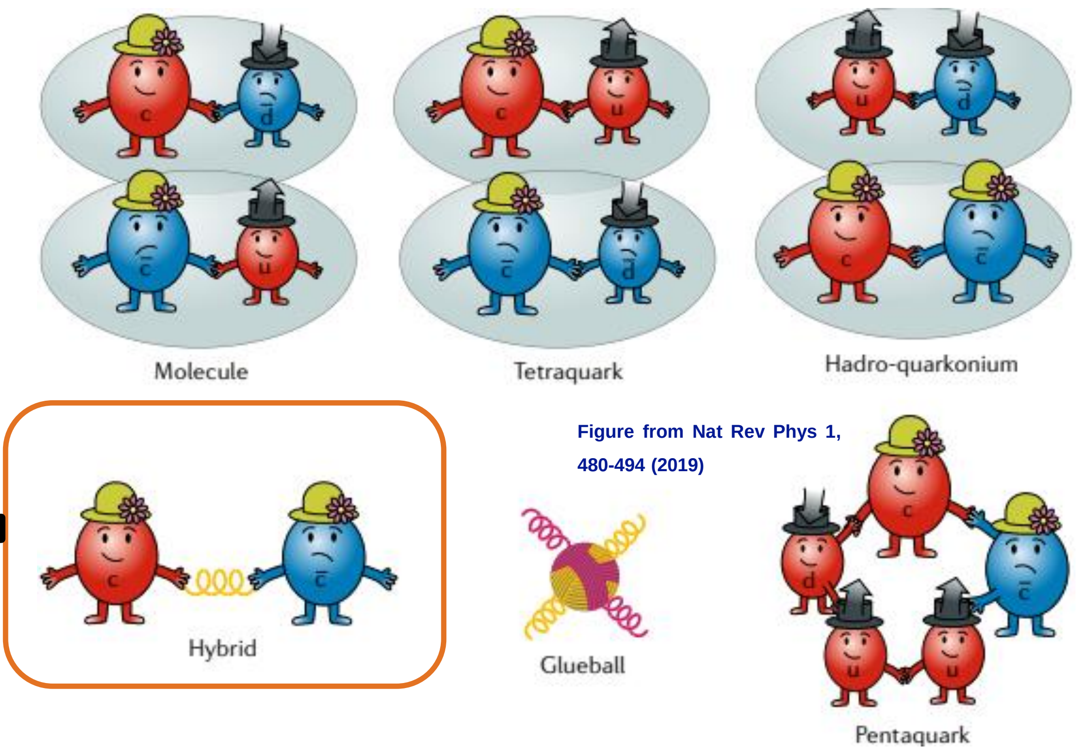
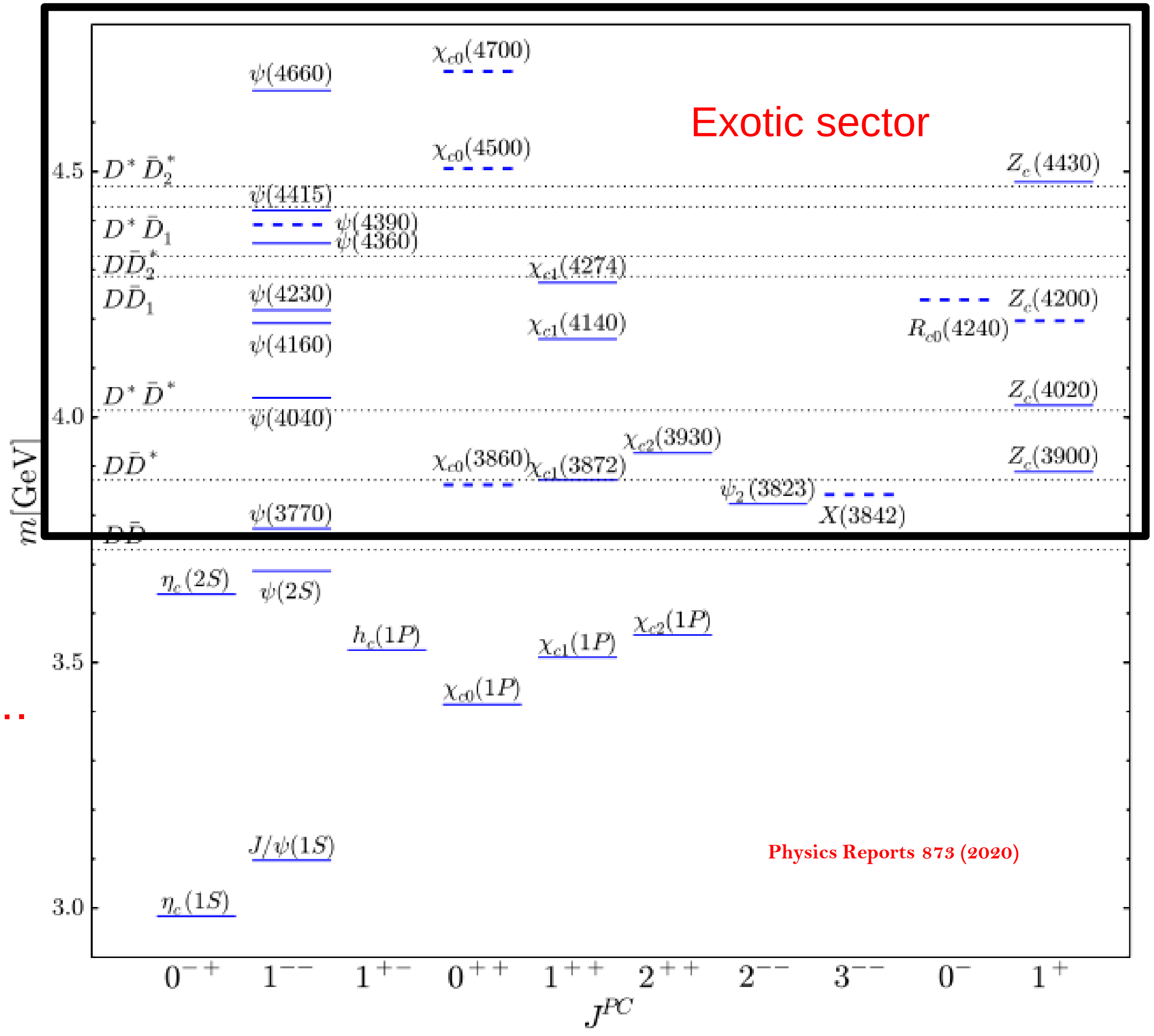


Figure from Nat Rev Phys 1, 480-494 (2019)



- No single model completely describes all XYZ states.

Hybrids ($Q\bar{Q}g$): Simplest extension of quarkonium. Isospin scalar exotic state. Focus of this work.

Use EFT + lattice to have model independent description

Exotic: Hybrid candidates

State (PDG)	State (Former)	M (MeV)	Γ (MeV)	J^{PC}	Decay modes
$\chi_{c1}(4140)$	$X(4140)$	4146.5 ± 3.0	19^{+7}_{-5}	1^{++}	$\phi J/\psi$
$X(4160)$		4153^{+23}_{-21}	136^{+60}_{-35}	$?^{??}$	$\phi J/\psi$, $D^* \bar{D}^*$
$\psi(4230)$	$Y(4230)$	4222.7 ± 2.6	49 ± 8	1^{--}	$\pi^+\pi^- J/\psi$, $\omega \chi_{c0}(1P)$,
	$Y(4260)$				$\pi^+\pi^- h_c(1P)$
$\chi_{c1}(4274)$	$Y(4274)$	4286^{+8}_{-9}	51 ± 7	1^{++}	$\phi J/\psi$
$X(4350)$		$4350.6^{+4.7}_{-5.1}$	13^{+18}_{-10}	$(0/2)^{++}$	$\phi J/\psi$
$\psi(4360)$	$Y(4360)$	4372 ± 9	115 ± 13	1^{--}	$\pi^+\pi^- J/\psi$,
	$Y(4320)$				$\pi^+\pi^- \psi(2S)$
$\psi(4390)$[a]	$Y(4390)$	4390 ± 6	139^{+16}_{-20}	1^{--}	$\eta J/\psi$, $\pi^+\pi^- h_c(1P)$
$\chi_{c0}(4500)$	$X(4500)$	4474 ± 4	77^{+12}_{-10}	0^{++}	$\phi J/\psi$
$Y(4500)$[b]		4484.7 ± 27.5	111 ± 34	1^{--}	
$X(4630)$[c]		4626^{+24}_{-111}	174^{+137}_{-78}	$?^{?+}$	$\phi J/\psi$
$\psi(4660)$	$Y(4660)$	4630 ± 6	72^{+14}_{-12}	1^{--}	$\pi^+\pi^- \psi(2S)$, $\Lambda_c^+ \bar{\Lambda}_c^-$,
	$X(4660)$				$D_s^+ D_{s1}(2536)$
$\chi_{c1}(4685)$[d]		4684^{+15}_{-17}	126^{+40}_{-44}	1^{++}	$\phi J/\psi$
$\chi_{c0}(4700)$	$X(4700)$	4694^{+17}_{-5}	87^{+18}_{-10}	0^{++}	$\phi J/\psi$
$Y(4710)$[e]		4704 ± 87	183 ± 146	1^{--}	
$\Upsilon(10753)$		$10752.7^{+5.9}_{-6.0}$	36^{+18}_{-12}	1^{--}	$\pi\pi\Upsilon(1S, 2S, 3S)$
$\Upsilon(10860)$	$\Upsilon(5S)$	$10885.2^{+2.6}_{-1.6}$	37 ± 4	1^{--}	$\pi\pi\Upsilon(1S, 2S, 3S)$,
					$\pi^+\pi^- h_b(1P, 2P)$,
					$\eta\Upsilon(1S, 2S)$, $\pi^+\pi^-\Upsilon(1D)$
					(see PDG listings)
$\Upsilon(11020)$	$\Upsilon(6S)$	11000 ± 4	24^{+8}_{-6}	1^{--}	$\pi\pi\Upsilon(1S, 2S, 3S)$,
					$\pi^+\pi^- h_b(1P, 2P)$,
					(see PDG listings)

- ✓ Isoscalar neutral meson states above the open-flavor thresholds which are potential candidates for hybrids
- ✓ Table adapted from PDG 2022
- ✓ Y(4500): New state recently seen by BESIII experiment.

M. Ablikim et al, Chin.Phys.C,46,111002(2022).

- ✓ X(4630): New state recently seen by LHCb experiment.
- ✓ χ_{c1}(4685): New state recently seen by LHCb experiment.

R. Aaji et al, Phys. Rev. Lett. 127, 082001 (2021)

- ✓ Y(4710): New state recently seen by LHCb experiment.

M. Ablikim et al, arXiv: 2211.08561.

Brambilla, Lai, AM, Vairo arXiv:2212.09187

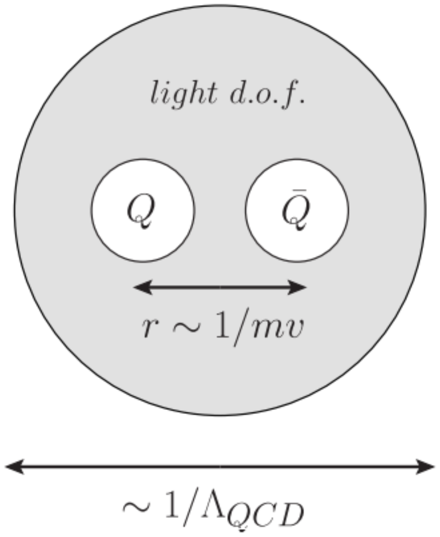
Hybrids: BOEFT

- **Hybrids ($Q\bar{Q}g$)**: Color singlet combination of color octet $Q\bar{Q}$ + gluonic excitations.
- Hierarchy of scales in hybrids:

$$m \gg mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$$

- ❖ Mass of heavy quark: $\boldsymbol{m}$
- ❖ Energy scale for light d.o.f: $\boldsymbol{\Lambda_{\rm QCD}}$
- ❖ Relative separation between heavy quarks: $r \sim 1/mv$
- ❖ Hybrids are extended objects: $< r > \gtrsim 0.7\ fm$
- ❖ Heavy Quark K.E scale: mv^2

❖ **Nonrelativistic bound-state system: $\boldsymbol{v \ll 1}$**



- Time-scale for dynamics of $Q\bar{Q}$: $\sim \dfrac{1}{mv^2} \gg \dfrac{1}{\Lambda_{\rm QCD}}$

Born-Oppenheimer Approximation

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

- **Born-Oppenheimer EFT (BOEFT):** EFT for hybrids. **Describes physics at the scale $\boldsymbol{mv^2}$.**

$${\rm QCD} \to {\rm NRQCD} \to {\rm pNRQCD/BOEFT}$$

- BOEFT: Extension of pNRQCD for hybrid states.

R. Oncala, J. Soto, Phys. Rev. D96 (2017) Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

BOEFT

- Born-Oppenheimer EFT: Describes $Q\bar{Q}$ pair systems at energy scale mv^2.
  - **d.o.f:** quarkonium and hybrid fields.
  - Focus only on low-lying hybrid states: lowest static energies Σ_u^- and Π_u corresponding to gluon quantum # $\kappa = 1^{+-}$. Ignore mixing with hybrid states built out of higher static energies.
  - Calculation of spectrum reduces to solving **Schrödinger equation** with potential $V(r)$.
- BOEFT Lagrangian: $$L_{\rm BOEFT} = L_\Psi + L_{\Psi_{\kappa\lambda}} + L_{\rm mixing},$$

Trace over spin indices.

Quarkonium:

$$L_\Psi = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \mathrm{Tr}\left[\Psi^\dagger(\mathbf{r},\mathbf{R},t)\left(i\partial_t + \frac{\nabla_r^2}{m_Q} - V_\Psi(r)\right)\Psi(\mathbf{r},\mathbf{R},t)\right]$$

Hybrid:

$$L_{\Psi_{\kappa\lambda}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)\left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m_Q}P^i_{\kappa\lambda'}\right]\Psi_{\kappa\lambda'}(\mathbf{r},\mathbf{R},t)\right\}$$

Hybrid-Quarkonium mixing:

$$L_{\rm mixing} = -\int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda} \mathrm{Tr}\left[\Psi^\dagger V^{\rm mix}_{\kappa\lambda}\Psi_{\kappa\lambda} + \mathrm{h.c.}\right]$$

r: relative coordinate

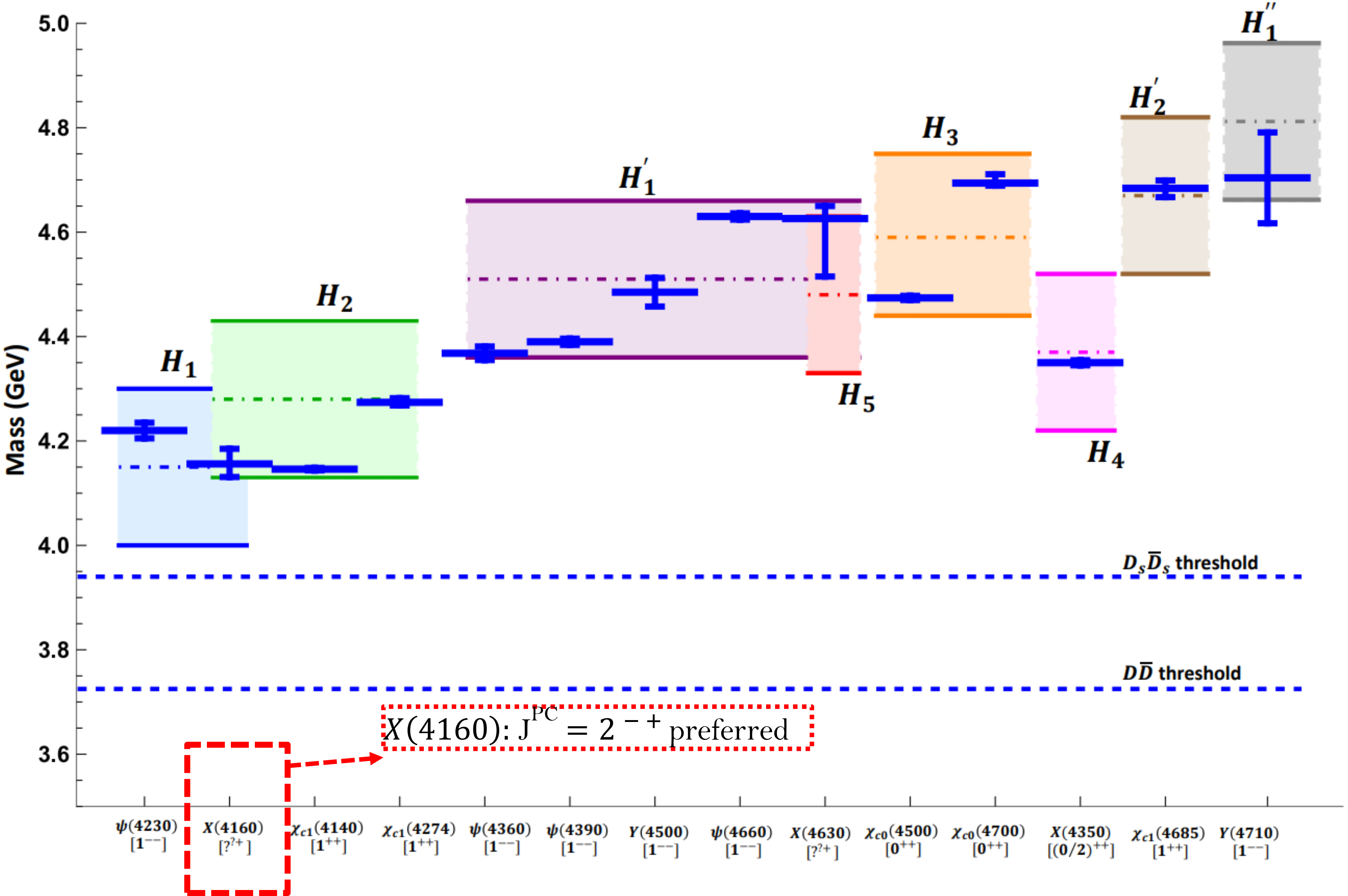
R: COM coordinate

- No lattice calculations on mixing potential. Current work, ignore mixing, $V^{\rm mix}_{\kappa\lambda} = 0$

Schrödinger equation with potentials from lattice as inputs

BOEFT: Hybrids

- **Charmonium hybrids**: comparison with experimental results:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

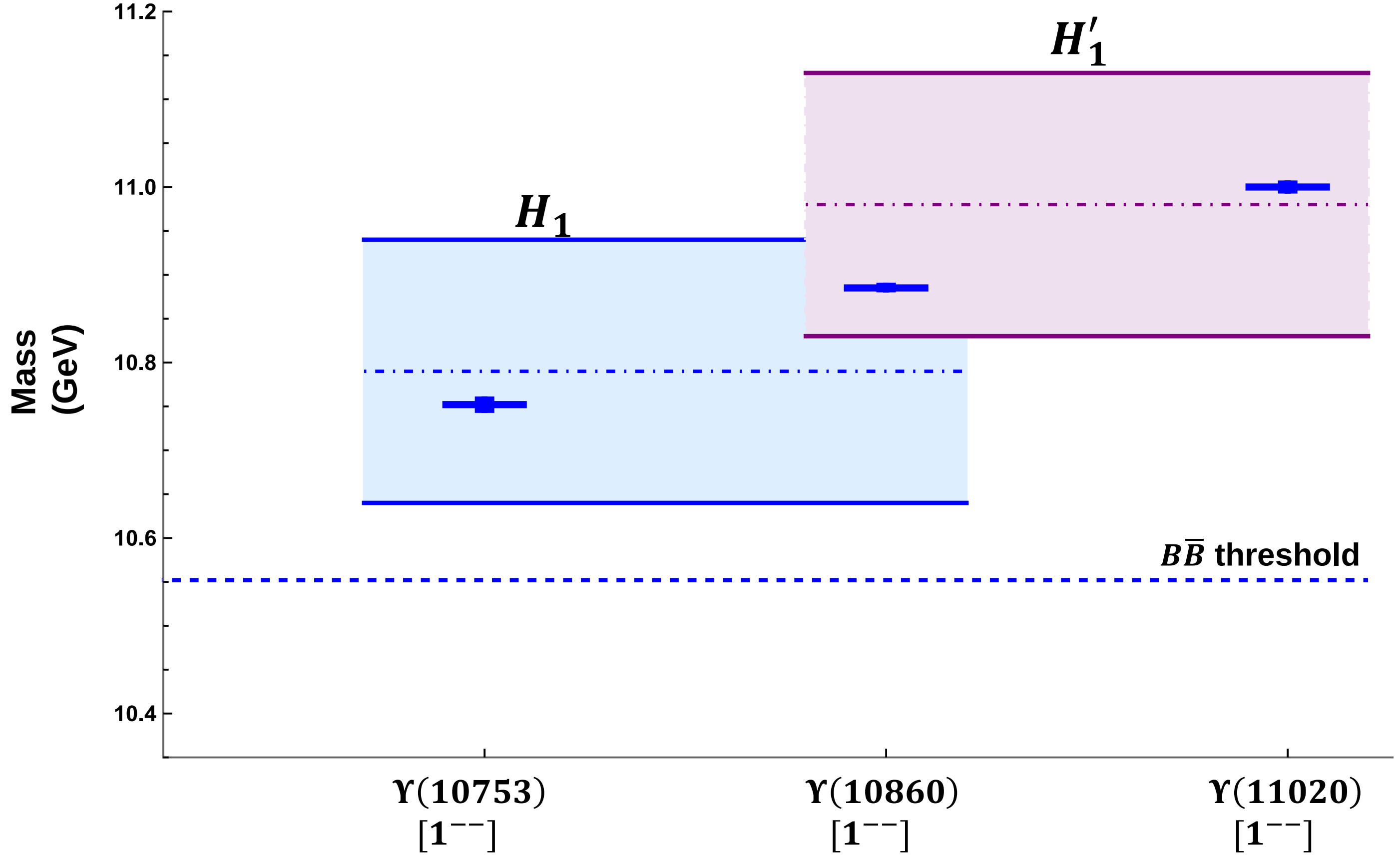
- Λ- doubling: opposite parity states are non-degenerate.

PDG 2022

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- **Bottomonium hybrids**: comparison with experimental results:



PDG 2022

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

- Dozens of XYZ states: clear theoretical understanding is still missing!!

- Most of the exotic states discovered from decays to low-lying quarkonium. So, decays might provide information on the structure of XYZ.

- Consider the semi-inclusive process: $\boldsymbol{H_m \to Q_n + X}$; H_m: low-lying hybrid, Q_n: low-lying quarkonium (states below threshold) and X: light hadrons.

  ✓ ΔE: Large energy difference ⇒ $\Delta E \equiv \boldsymbol{E_{H_m} - E_{Q_n} \gtrsim 1}$ **GeV**.

  ✓ Assume hierarchy of scales: $\Lambda_r \gg \Delta E \gg \Lambda_{QCD} \gg mv^2$

Energy scale related to decay

$$\Lambda_r^{-1} \equiv |\langle Q_n|\mathbf{r}|H_m\rangle|$$

.Hierarchy required for performing perturbative calculation !!!

- In BOEFT, all energy scales above $\mathbf{mv^2}$ are integrated out. So, scale ΔE must be integrated out. This gives imaginary contribution to hybrid potential:

Optical theorem: $$\sum_n \Gamma(H_m \to Q_n) = -2\,\mathrm{Im}\,\langle H_m|V|H_m\rangle$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

- Imaginary piece of hybrid potential: determined from matching weakly-coupled **pNRQCD** and **BOEFT** effective theories.

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

- Weakly-coupled pNRQCD Lagrangian: d.o.f are perturbative octet and singlet states:

Weakly-coupled pNRQCD Lagrangian

$$L_{\rm pNRQCD} = \int d^3R \Big\{ \int d^3r \Big({\rm Tr}\left[{\rm S}^\dagger \left(i\partial_0 - h_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - h_o \right) {\rm O} \right]$$
$$+ g{\rm Tr}\left[{\rm S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,{\rm O} + {\rm O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,{\rm S} + \frac{1}{2}{\rm O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},{\rm O}\} \right] + \frac{g}{4m}{\rm Tr}\left[{\rm O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},{\rm O}] \right]$$
$$+ \frac{gc_F}{m}{\rm Tr}\left[{\rm S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,{\rm O} + {\rm O}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,{\rm S} + {\rm O}^\dagger \boldsymbol{S}_1\cdot\boldsymbol{B}\,{\rm O} - {\rm O}^\dagger \boldsymbol{S}_2 {\rm O}\cdot\boldsymbol{B} \right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\, a} \Big\}$$

Two decay channels:

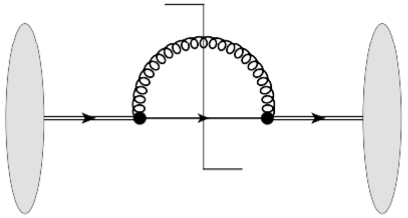
- **Spin preserving decays $[O(r^2)]$**
- **Spin flipping decays $[O(1/m^2)]$**

- BOEFT (only hybrid term):

$$L_{\Psi_{\kappa\lambda}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda\lambda'} {\rm Tr}\left\{ \Psi^\dagger_{\kappa\lambda}(\mathbf{r},\,\mathbf{R},\,t) \left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\nabla^2_r}{m_Q}P^i_{\kappa\lambda'} \right] \Psi_{\kappa\lambda'}(\mathbf{r},\,\mathbf{R},\,t) \right\}$$

Potential term in BOEFT

- BOEFT and pNRQCD matching: additional contribution to $V_{\kappa\lambda\lambda'}$.



Decays are computed from local imaginary terms in the BOEFT Lagrangian.

decay rate ∝ $Im\ (V_{\kappa\lambda\lambda'})$

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

- **pNRQCD Lagrangian**: d.o.f are the perturbative singlet and octet fields and gluons of energy scale **mv^2**.

$$\mathrm{S} = S\mathbb{I}_c/\sqrt{N_c}$$
$$\mathrm{O} = O^a T^a/\sqrt{T_F}$$

Weakly-coupled pNRQCD Lagrangian

$$L_{\text{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger \left(i\partial_0 - h_s\right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o\right) \mathrm{O} \right]$$
$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\}\right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}]\right]$$
$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O} + \mathrm{O}^\dagger(\boldsymbol{S}_1-\boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{S} + \mathrm{O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,\mathrm{O} - \mathrm{O}^\dagger\boldsymbol{S}_2\mathrm{O}\cdot\boldsymbol{B}\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}\Big\}$$

- Connection with non-perturbative fields: quarkonium and hybrid in short-distance limit **$\mathbf{r} \to 0$**

Fields:

$$S(\mathbf{r},\mathbf{R},t) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$
$$P^{i\dagger}_{\kappa\lambda}O^a(\mathbf{r},\mathbf{R},t)\,G^{ia}_\kappa(\mathbf{R},t) \to Z_\kappa^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \dots,$$
$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi} r^2 + \dots$$

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- **pNRQCD Lagrangian**: d.o.f are the perturbative singlet (S) and octet (O) fields and gluons of energy scale $\mathbf{mv^2}$.

$$\mathrm{S} = S\mathbb{I}_c/\sqrt{N_c}$$

$$\mathrm{O} = O^a T^a/\sqrt{T_F}$$

Weakly-coupled pNRQCD Lagrangian

$$L_{\text{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger\,(i\partial_0 - h_s)\,\mathrm{S} + \mathrm{O}^\dagger\,(iD_0 - h_o)\,\mathrm{O}\right]$$

$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\}\right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}]\right]$$

$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O} + \mathrm{O}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{S} + \mathrm{O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,\mathrm{O} - \mathrm{O}^\dagger\boldsymbol{S}_2\mathrm{O}\cdot\boldsymbol{B}\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}\Big\}$$

- Connection with non-perturbative fields: quarkonium and hybrid in short-distance limit $\mathbf{r} \to \mathbf{0}$

Fields:

$$S(\mathbf{r},\mathbf{R},t) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$

$$P^{i\dagger}_{\kappa\lambda}O^a(\mathbf{r},\mathbf{R},t)\,G^{ia}_\kappa(\mathbf{R},t) \to Z_\kappa^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

G^{ia}_κ: **Gluon fields**

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+}r^2 + \ldots,$$

$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi}r^2 + \ldots$$

V_s & V_o: singlet and octet potential

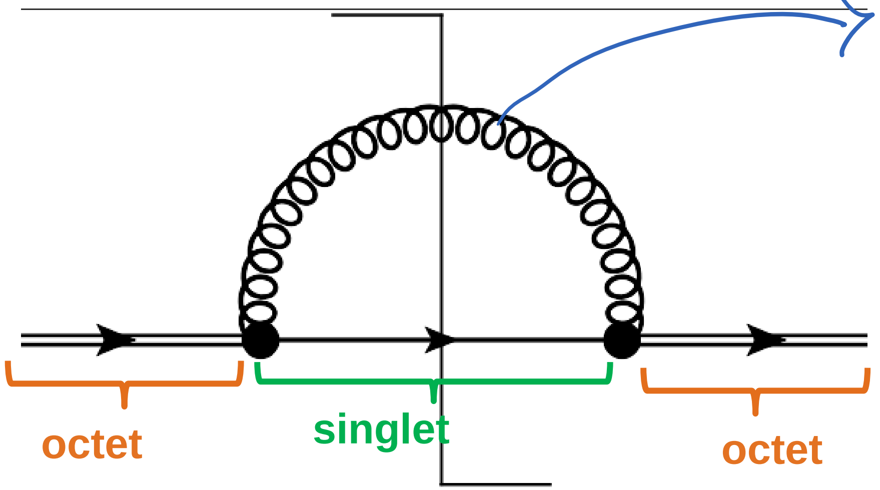
Λ: gluelump mass

Non-perturbative parameters

Hybrid Decays

- Conceptually, for process: $\boldsymbol{H_m \to Q_n + X}$, ΔE (energy gap) is large enough that gluon can **resolve color configuration of** $Q\bar{Q}$ pair in hybrid and quarkonium:

Color configuration of $Q\bar{Q}$ pair:
Quarkonium ------> Singlet
Hybrid ------> Octet



gluon $\Delta E = E_{H_m} - E_{Q_n}$

- For this work, assume: **quarkonium = singlet, hybrid = octet.**

Generally not true. Holds only in short-distance limit.

Future work: Relax this assumption: compute decay to threshold states **(?)** and hence inclusive rate.

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

Brambilla, Lai, AM, Vairo arXiv:2212.09187

○ Spin-conserving decay due to $\boldsymbol{r} \cdot \boldsymbol{E}$ term :

$|S_H = 1 > ---\rightarrow |S_Q = 1 >$
$|S_H = 0 > ---\rightarrow |S_Q = 0 >$

$$\Gamma(H_m \rightarrow Q_n) = \frac{4\alpha_s\,(\Delta E)\,T_F}{3N_c}\, T^{ij}\,(T^{ij})^\dagger\,\Delta E^3$$

$$T^{ij} \equiv \langle H_m | r^j | Q_n \rangle = \int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, r^j\, \Phi^{Q\bar{Q}}_{(n)}(\mathbf{r})$$

J. Castellà, E. Passemar, Phys. Rev. D104, 034019 (2021)

$$\langle H_m | \boldsymbol{r} | Q_n \rangle = \sqrt{T^{ij}(T^{ij})^\dagger}$$

R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017).

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

$\Psi^{i}_{(m)}$: Hybrid wf

Φ^{Q}_{n} : Quarkonium wf

$|\chi_H\rangle$: Hybrid spin wf

$|\chi_Q\rangle$: Quarkonium spin wf

○ Spin-flipping decay due to $\boldsymbol{S}.\boldsymbol{B}$ term:

$|S_H = 1 > ---\rightarrow |S_Q = 0 >$
$|S_H = 0 > ---\rightarrow |S_Q = 1 >$

$$T^{ij} \equiv \langle H_m | \left(S_1^j - S_2^j\right) | Q_n \rangle = \left[\int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, \Phi^{Q}_{(n)}(\mathbf{r})\right] \langle \chi_H | \left(S_1^j - S_2^j\right) | \chi_Q \rangle$$

- Depends on overlap of quarkonium and hybrid wavefunctions.

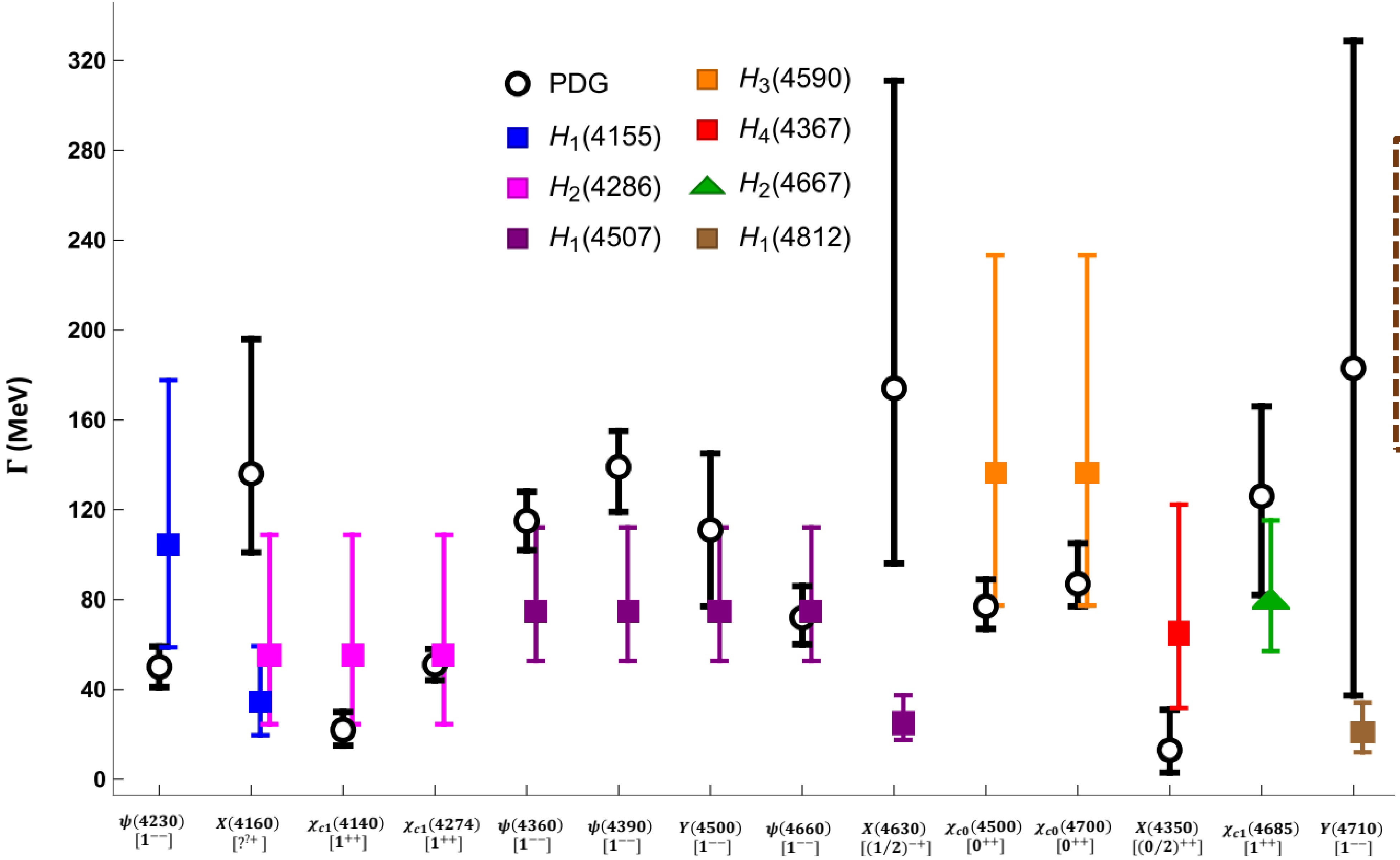
Hybrid-to-Quarkonium transition decay rate
= **spin-conserving** + **spin-flipping** decay rates.

Our estimate of decay rate are **lower-bounds** for the **total width of hybrids**
and thus lower-bound on inclusive width of XYZ exotic states

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: charm exotic states with corresponding charmonium hybrid state:

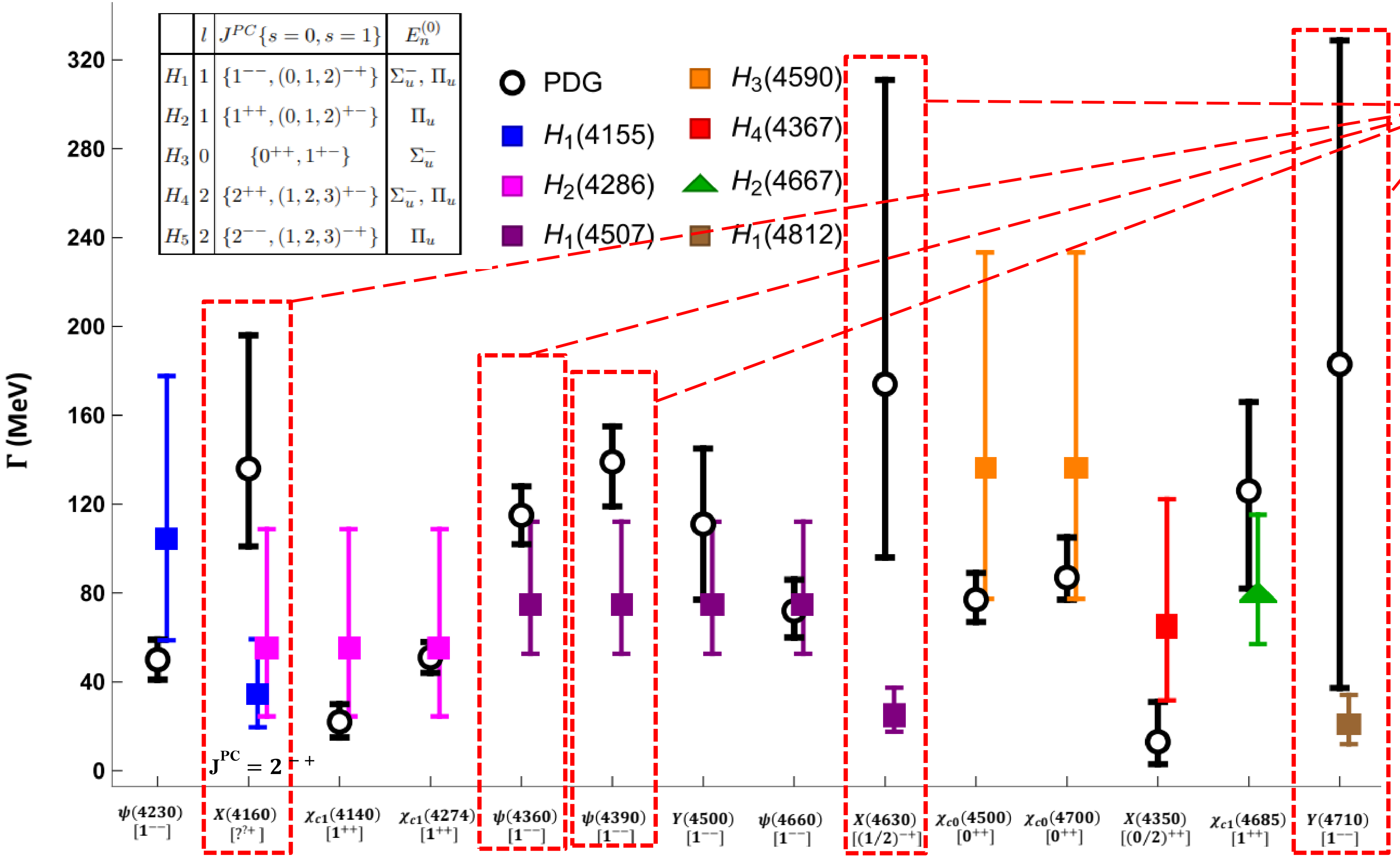


spin-conserving + spin-flipping decays = **lower bound on the total decay widths of hybrids** which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrid-to-quarkoniunium transition widths:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Could have a significant hybrid component.

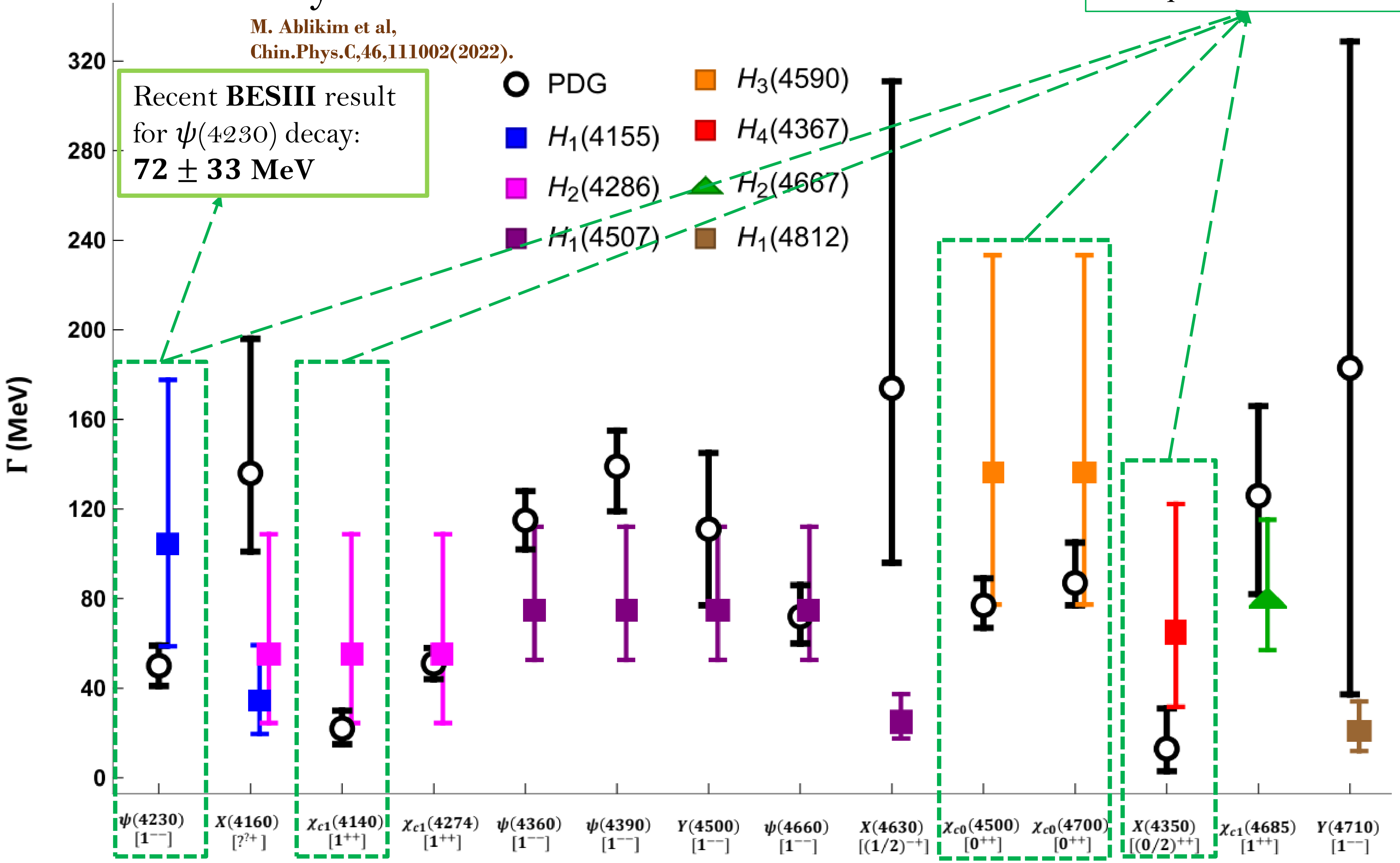
- ✓ Decay to open-threshold states seen but no information on branching ratio (BR). Require experimental inputs.
- ✓ Decay to open-threshold states could be significant or expected to be large (?).

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: charm exotic states with corresponding charmonium hybrid state:

Disfavors hybrid interpretation !



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
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H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

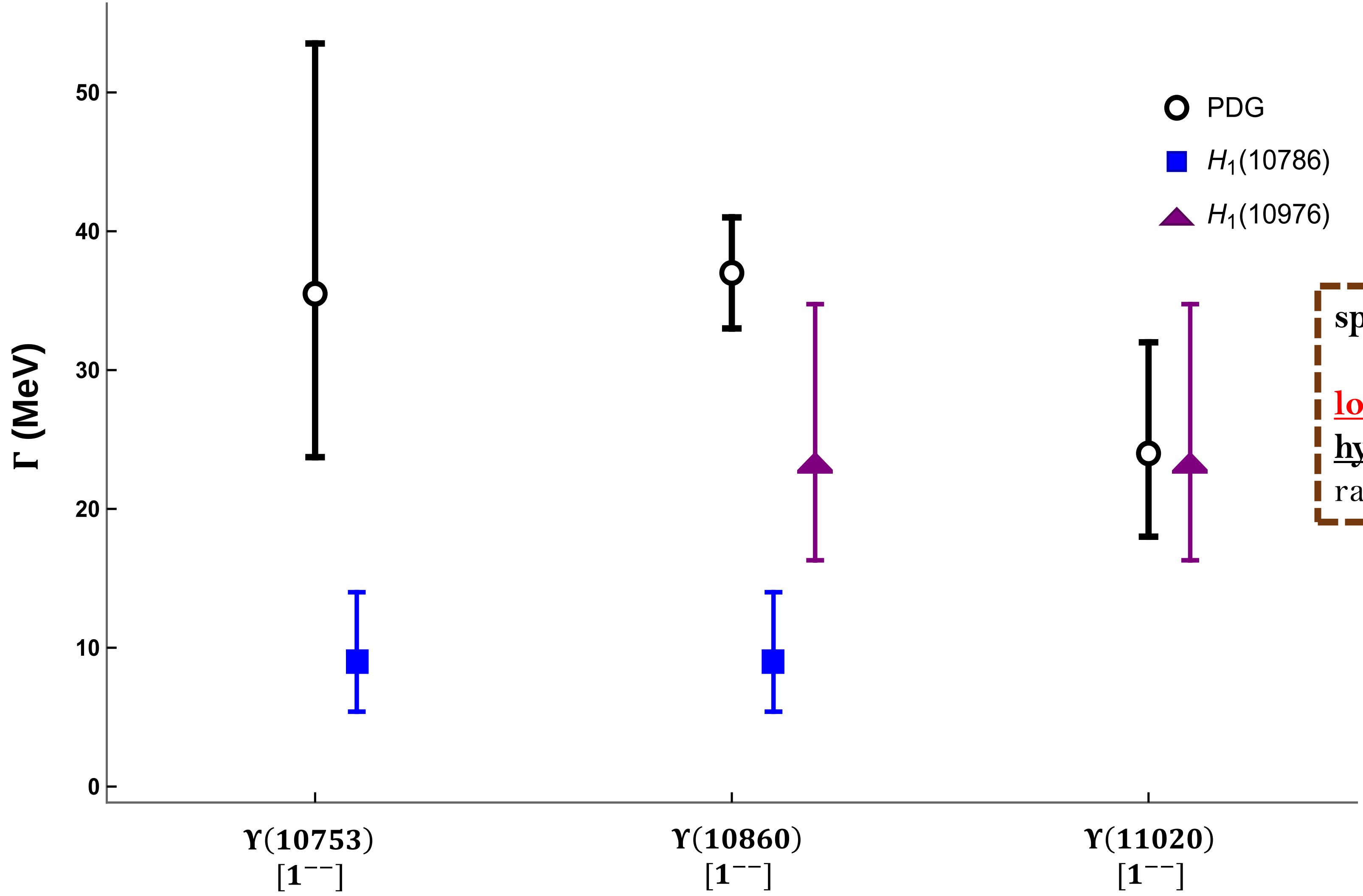
$\chi_{c0}(4500)$ & $\chi_{c0}(4700)$

Decay to $\chi_c(1P)$ not seen !!!. Major contribution to theoretical estimate from this decay channel.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: bottom exotic states with corresponding bottomonium hybrid state:

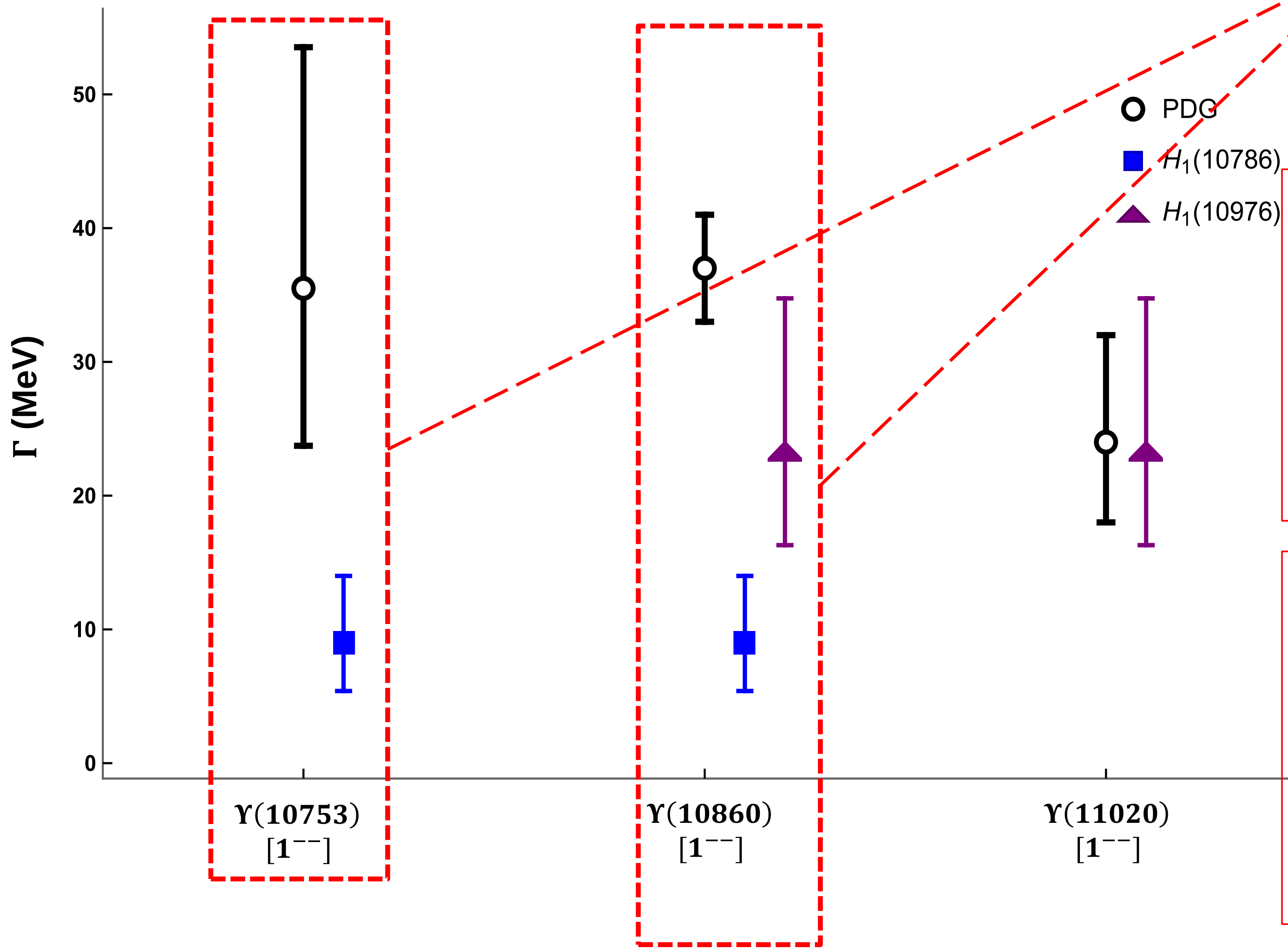


spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrid-to-quarkoniunium transition widths:



Could have a significant hybrid component.

$\Upsilon(10860)$

- ✓ Inclusive rate: **37 ± 4** MeV. **Branching fraction for decay to open-bottom mesons**: **76.6%**. Decay rate to quarkonium: **$8.8^{+2.6}_{-1.8}$** MeV. Good agreement with our lower-bound estimate for $H_1[1^{--}](10786)$.
- ✓ Candidate for $\Upsilon(5S)$ state

$\Upsilon(11020)$ & $\Upsilon(10753)$

Large branching fraction for decays to open-bottom mesons expected. Different quark model calculations predict branching fraction in the range from 30-70 %. Need experimental input on branching fraction.

$\Upsilon(10753)$: Candidate for $\Upsilon(3D)$ state.

Results

- Spin-flipping transitions: suppressed by powers of the heavy-quark mass due to the heavy-quark spin symmetry;

- Relative comparison between **spin-conserving** and **spin-flipping decays**: $H_m \to Q_n + X$:
  - ✓ Size of energy gap **ΔE**: final quarkonium states are different in both the decay process.
  - ✓ Depends on relative magnitude of matrix element (radial): $|\langle Q_n|r|H_m\rangle|$ & $|\langle Q_n|H_m\rangle|/m$

    Ratio : $m\,|\langle Q_n|\mathbf{r}|H_m\rangle|/|\langle Q_n|H_m\rangle|$

    No obvious hierarchy relation between the two-decay process.
  - ✓ For bottom hybrids: spin-flipping transitions are smaller compared to spin-conserving.
  - ✓ For charm hybrids: spin-flipping transitions are not necessarily small: $m\,|\langle Q_n|\mathbf{r}|H_m\rangle|/|\langle Q_n|H_m\rangle| \sim 1$

    Spin-flipping ~ spin-conversing: indicating heavy-quark spin-symmetry violations!

- Quarkonium transition: $Q_m \to Q_n + X$:spin-flipping decay suppressed by $O(v)^2$ compared to spin- conserving.

$$|\langle Q_n|Q_m\rangle|/m\,|\langle Q_n|\mathbf{r}|Q_m\rangle| \sim v^2 \ll 1$$

Summary/Outlook

- BOEFT provides a model-independent & systematic way to study heavy quark hybrids (exotic) and decays.
- For the decay process, the hierarchy of energy scales :

$$\boxed{1/|\langle Q_n|\mathbf{r}|H_m\rangle| \gg \Delta E} \gg \boxed{\Lambda_{QCD} \gg m_Q v^2}$$

pNRQCD and BOEFT matching

Neglect hybrids of higher gluonic excitations and mixing.

- Our results for hybrid-to-quarkonium transition widths **sets lower-bounds** on the inclusive rate of physical exotic states, if interpreted as pure hybrid states .

Hybrid-to-Quarkonium transition decay rate = **spin-conserving** + **spin-flipping** decay rates.

- Our analysis disfavors: $\psi(4230)$, $\chi_{c1}(4140)$, $\chi_{c0}(4500)$, $\chi_{c0}(4700)$, and $X(4350)$ as pure hybrid states.
- Our analysis suggests:
  - $\boldsymbol{X(4160)}$: could be the **charm hybrid** $\boldsymbol{H_1[2^{-+}](4155)}$.
  - $\boldsymbol{\psi(4390)}$: could be the **charm hybrid** $\boldsymbol{H_1[1^{--}](4507)}$.
  - $\boldsymbol{X(4630)}$: could be the **charm hybrid** $\boldsymbol{H_1[(1/2^{-+})](4507)}$.
  - $\boldsymbol{\psi(4710)}$: could be the **charm hybrid** $\boldsymbol{H_1[(1^{--})](4812)}$.
  - $\boldsymbol{\Upsilon(10753)}$: could be the **bottom hybrid** $\boldsymbol{H_1[(1^{--})](10786)}$.
  - $\boldsymbol{\Upsilon(10860)}$: could be the **bottom hybrid** $\boldsymbol{H_1[(1^{--})](10786)}$.
  - Nothing conclusive can be said about other exotic states.

Brambilla, AM, Vairo,.... (in progress)

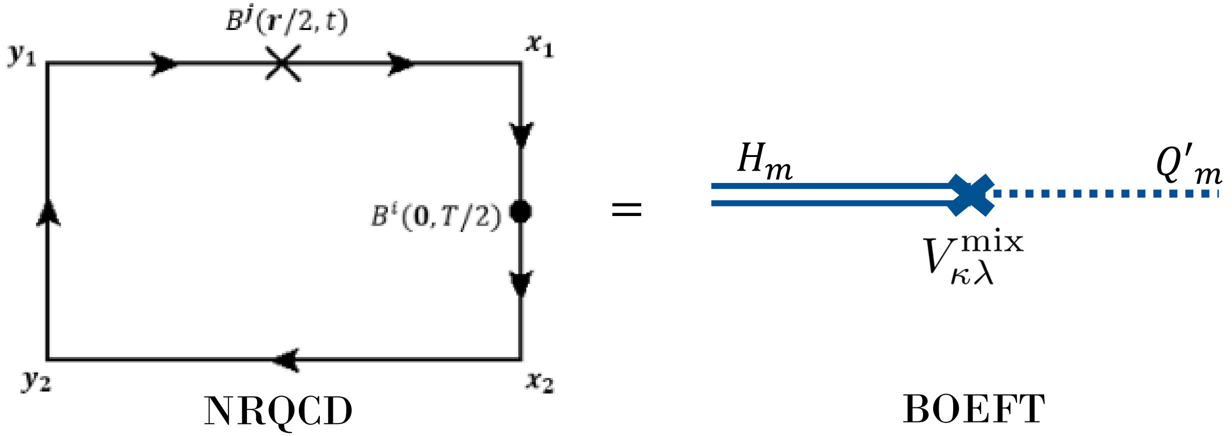
Hybrid-quarkonium mixing (in progress)

- Hybrid states in the same energy range and same quantum #'s as quarkonium can mix.
- Mixing can impact spectrum and decay properties of hybrid. Implications on hybrid interpretation for exotics.

Ex. $H_1\left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$

Effect on decay: $H_m \leftrightarrow Q_m^{'} \rightarrow (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Hybrids with gluon quantum # $\boldsymbol{\kappa} = \mathbf{1}^{+-}$, mix with quarkonium through heavy-quark spin dependent operator. Mixing potential at $O(1/m)$ in BOEFT.
- Mixing potential $V_{\kappa\lambda}^{\text{mix}}$: determined from matching NRQCD and BOEFT at $O(1/m)$



Expression after matching:

$$V_{|\lambda|}^{\text{mix}} = -\frac{gc_F}{2m_Q}\, {}^{(0)}_{\lambda}\langle 1|B^j\left(\mathbf{r}/2,0\right)|0\rangle^{(0)} P_\lambda^j,$$

Above expression can be computed on lattice if we identify:

$$|0\rangle^{(0)} = |\Sigma_g^+\rangle$$

$$|1\rangle_{\lambda=0}^{(0)} = |\Sigma_u^-\rangle, |1\rangle_{|\lambda|=1}^{(0)} = |\Pi_u\rangle$$

Ongoing/Future prospects

- Extending the BOEFT framework to include hybrid-quarkonium mixing.

  Hybrid states in the same energy range as quarkonium can mix (same quantum #'s). O (1/m) term in BOEFT.

  Impact on decay: $H_m \leftrightarrow Q'_m \rightarrow Q_n + X$; n & m denotes quantum #'s

- Computing decays to open-flavor states.
- **Extending BOEFT framework to study quarkonium tetraquarks and pentaquarks and their decay properties** (in progress).
- Studying evolution of exotics (say hybrids) in medium using open-quantum systems.

BOEFT: A unified framework for XYZ exotics (hybrids, tetraquarks, molecules and pentaquarks) based on QCD !!??.

Thank you!!

Backup Slides

Quarkonium hybrids: BOEFT

- Static limit (m→ ∞): Quantum #'s for hybrid

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \dots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \dots \; (\Sigma, \Pi, \Delta, \Phi, \dots)$
- Eigenvalue of CP: $\eta = +1\,(g),\; -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

- Static Energies (Σ, Π, Δ): Eigenvalue of NRQCD Hamiltonian in the static limit.

- For r→ 0: static energies are degenerate. Characterized by O(3)×C symmetry group.

Labelled by: $(K^{PC}, \Lambda_\eta^\sigma)$

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Gluonic static energies

M. Foster and C. Michael, Phys. Rev. D59 (1999)

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

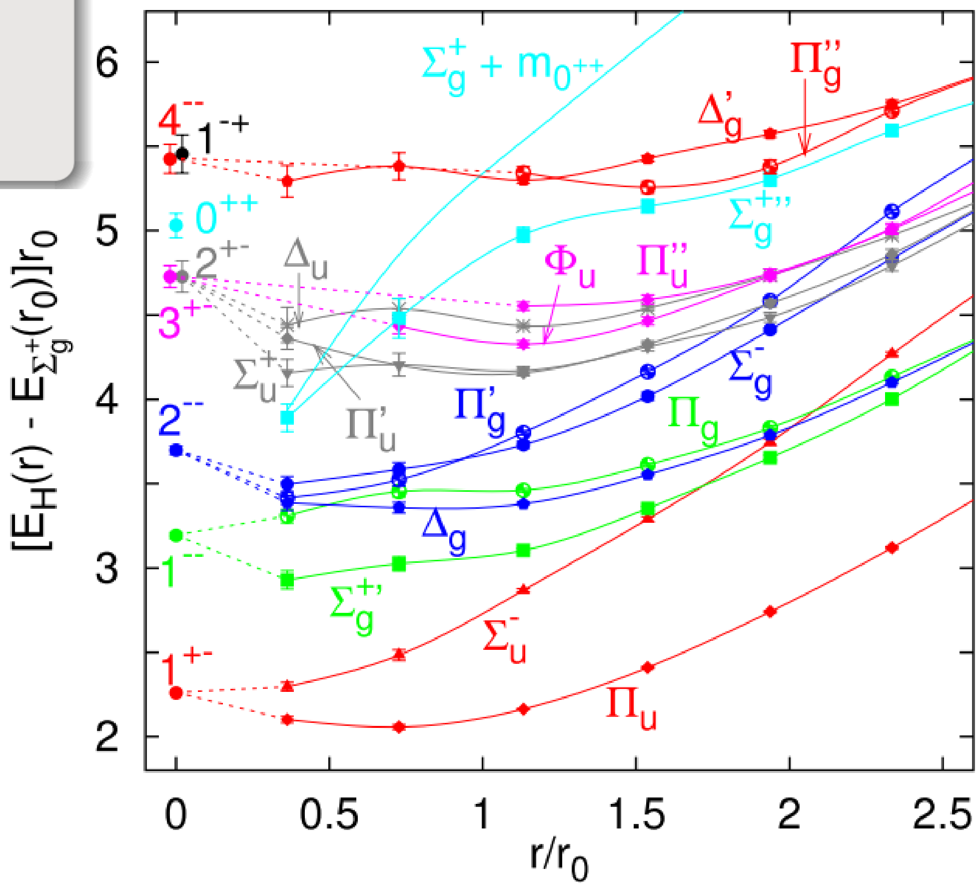


Fig from G. S. Bali and A. Pineda, Phys. Rev. D69 (2004)

Quarkonium hybrids: BOEFT

- Static limit (m→ ∞): Quantum #'s for hybrid

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g),\ -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

Gluonic operators characterizing Hybrids in Wilson loop

- Static Energies (Σ, Π, Δ): Eigenvalue of NRQCD Hamiltonian in the static limit.

- For r→ 0: static energies are degenerate. Characterized by O(3)×C symmetry group.

Labelled by: $(K^{PC}, \Lambda_\eta^\sigma)$

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

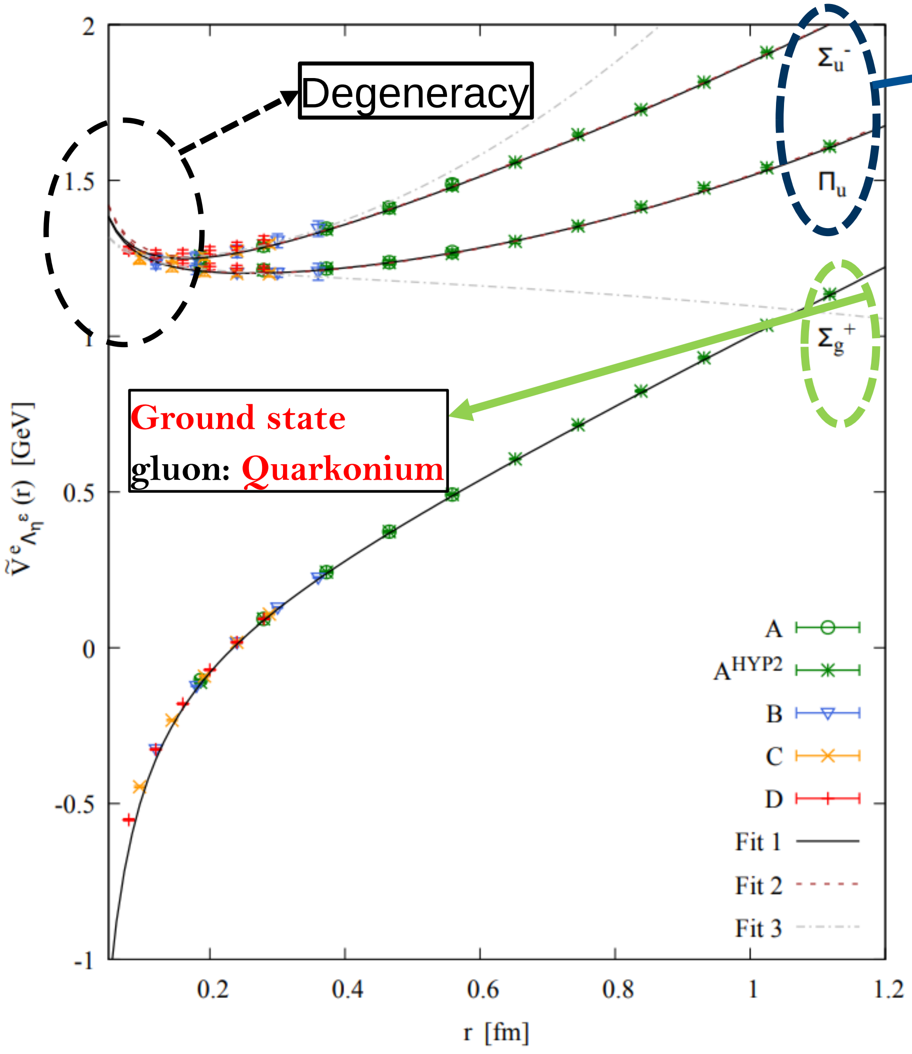
Λ_η^σ	K^{PC}	O_n
Σ_u^-	1^{+-}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{B},\ \hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{E})$
Π_u	1^{+-}	$\hat{\boldsymbol{r}} \times \boldsymbol{B},\ \hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{E},\ \hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{B})$
Π_g	1^{--}	$\hat{\boldsymbol{r}} \times \boldsymbol{E},\ \hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{B})$
Σ_g^-	2^{--}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{B})$
Π_g'	2^{--}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{B} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{B}))$
Δ_g	2^{--}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i (\hat{\boldsymbol{r}} \times \boldsymbol{B})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j (\hat{\boldsymbol{r}} \times \boldsymbol{B})^i$
Σ_u^+	2^{+-}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{E})$
Π_u'	2^{+-}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{E} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{E}))$
Δ_u	2^{+-}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i (\hat{\boldsymbol{r}} \times \boldsymbol{E})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j (\hat{\boldsymbol{r}} \times \boldsymbol{E})^i$

Focus on these two for low lying hybrids

BOEFT

- Lattice potentials for solving the Schrödinger Eq:

Schlosser and Wagner Phys. Rev. D. 105, (2022)



Excited gluon static energies: Hybrid

Quarkonium Potential:

$$V_{\Sigma_g^+}(r) = -\frac{\kappa_g}{r} + \sigma_g r + E_g^{Q\bar{Q}}$$

$$\kappa_g = 0.489\,, \quad \sigma_g = 0.187\,\mathrm{GeV}^2 \quad E_g^{c\bar{c}} = -0.254\,\mathrm{GeV},\ E_g^{b\bar{b}} = -0.195\,\mathrm{GeV},$$

Hybrid Potential:

$$E_n^{(0)}(r) = \begin{cases} V_o^{RS}(\nu_f) + \Lambda_H^{RS}(\nu_f) + b_n r^2, & r < 0.25\ fm \\ \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4, & r > 0.25\ fm \end{cases}$$

$$a_1^\Sigma = 0.000\,\mathrm{GeVfm}\,, \quad a_2^\Sigma = 1.543\,\mathrm{GeV}^2/\mathrm{fm}^2, \quad a_3^\Sigma = 0.599\,\mathrm{GeV}^2, \quad a_4^\Sigma = 0.154\,\mathrm{GeV}\,,$$

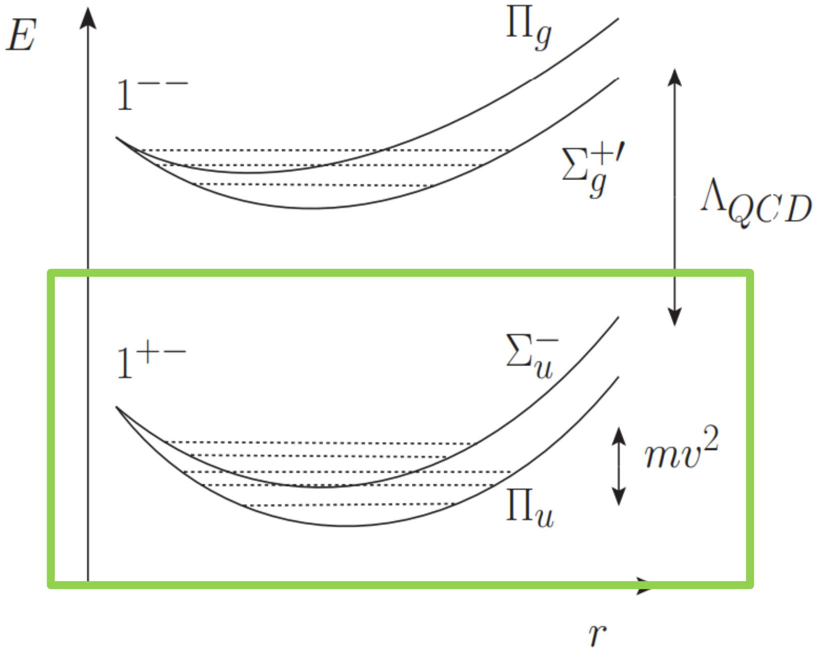
$$a_1^\Pi = 0.023\,\mathrm{GeVfm}\,, \quad a_2^\Pi = 2.716\,\mathrm{GeV}^2/\mathrm{fm}^2, \quad a_3^\Pi = 11.091\,\mathrm{GeV}^2, \quad a_4^\Pi = -2.536\,\mathrm{GeV}\,,$$

$$b_\Sigma = 1.246\,\mathrm{GeV}/\mathrm{fm}^2, \quad b_\Pi = 0.000\,\mathrm{GeV}/\mathrm{fm}^2$$

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

Brambilla, Lai, AM, Vairo arXiv:2212.09187



- Degeneracy at short distances $r \to 0$, mixes hybrid states corresponding to Σ_u^- and Π_u potential

- Coupled Schrödinger Eq: Dynamics of $Q\bar{Q}$ at scale $mv^2 \ll \Lambda_{QCD}$

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_u^-} & 0 \\ 0 & E_{\Pi_u} \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma^{(m)} \\ \psi_{-\Pi}^{(m)} \end{pmatrix} = E_m^{Q\bar{Q}g}\begin{pmatrix} \psi_\Sigma^{(m)} \\ \psi_{-\Pi}^{(m)} \end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2}\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{m_Q r^2} + E_{\Pi_u}\right]\psi_{+\Pi}^{(m)} = E_m^{Q\bar{Q}g}\,\psi_{+\Pi}^{(m)}$$

Hybrid quantum numbers

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Hybrid Spectrum:

Multiplet	J^{PC}	$M_{c\bar{c}g}$	$M_{b\bar{b}g}$
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	4155	10786
H_1'		4507	10976
H_1''		4812	11172
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	4286	10846
H_2'		4667	11060
H_2''		5035	11270
H_3	$\{0^{++}, 1^{+-}\}$	4590	11065
H_3'		5054	11352
H_3''		5473	11616
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	4367	10897
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	4476	10948

Λ- doubling: opposite parity states non-degenerate.

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018) Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Exotic Hadron

- **conventional quarkonium**, which consists of a color-singlet heavy quark-antiquark pair: $(Q\bar{Q})_1$,
- **quarkonium hybrid meson**, which consists of a color-octet $Q\bar{Q}$ pair to which a gluonic excitation is bound: $(Q\bar{Q})_8 + g$,
- **compact tetraquark** [7], which consists of a $Q\bar{Q}$ pair and a light quark q and anti-quark $\bar{q}$ bound by inter-quark potentials into a color singlet: $(Q\bar{Q}q\bar{q})_1$,
- **meson molecule** [8], which consists of color-singlet $Q\bar{q}$ and $\bar{Q}q$ mesons bound by hadronic interactions: $(Q\bar{q})_1 + (\bar{Q}q)_1$,
- **diquark-onium** [9], which consists of a color-antitriplet Qq diquark and a color-triplet $\bar{Q}\bar{q}$ diquark bound by the QCD color force: $(Qq)_{\bar{3}} + (\bar{Q}\bar{q})_3$,
- **hadro-quarkonium** [10], which consists of a color-singlet $Q\bar{Q}$ pair to which a color-singlet light-quark pair is bound by residual QCD forces: $(Q\bar{Q})_1 + (q\bar{q})_1$. An essentially equivalent model is a quarkonium and a light meson bound by hadronic interactions.
- **quarkonium adjoint meson** [11], which consists of a color-octet $Q\bar{Q}$ pair to which a light quark-antiquark pair is bound: $(Q\bar{Q})_8 + (q\bar{q})_8$.

Braaten, Langmack, and Smith Phys. Rev. D90, 014044 (2014)

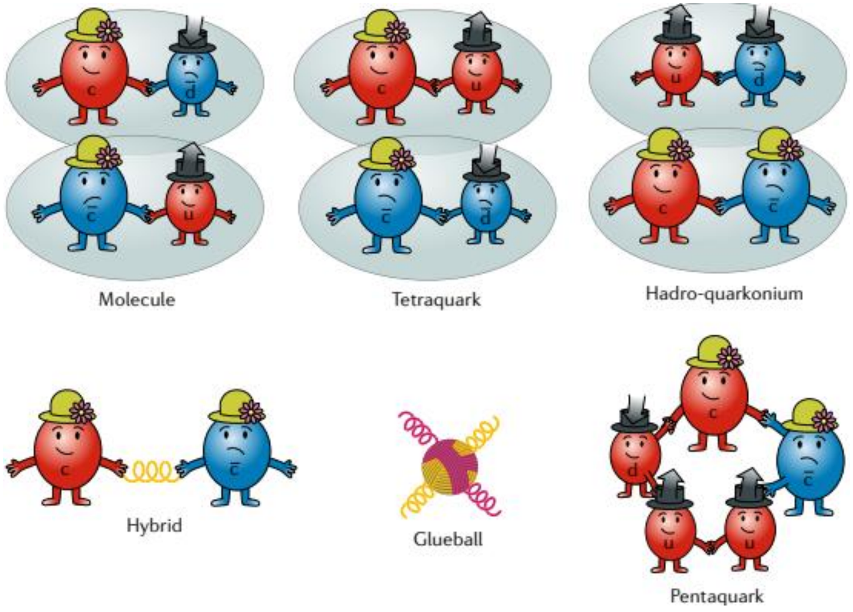
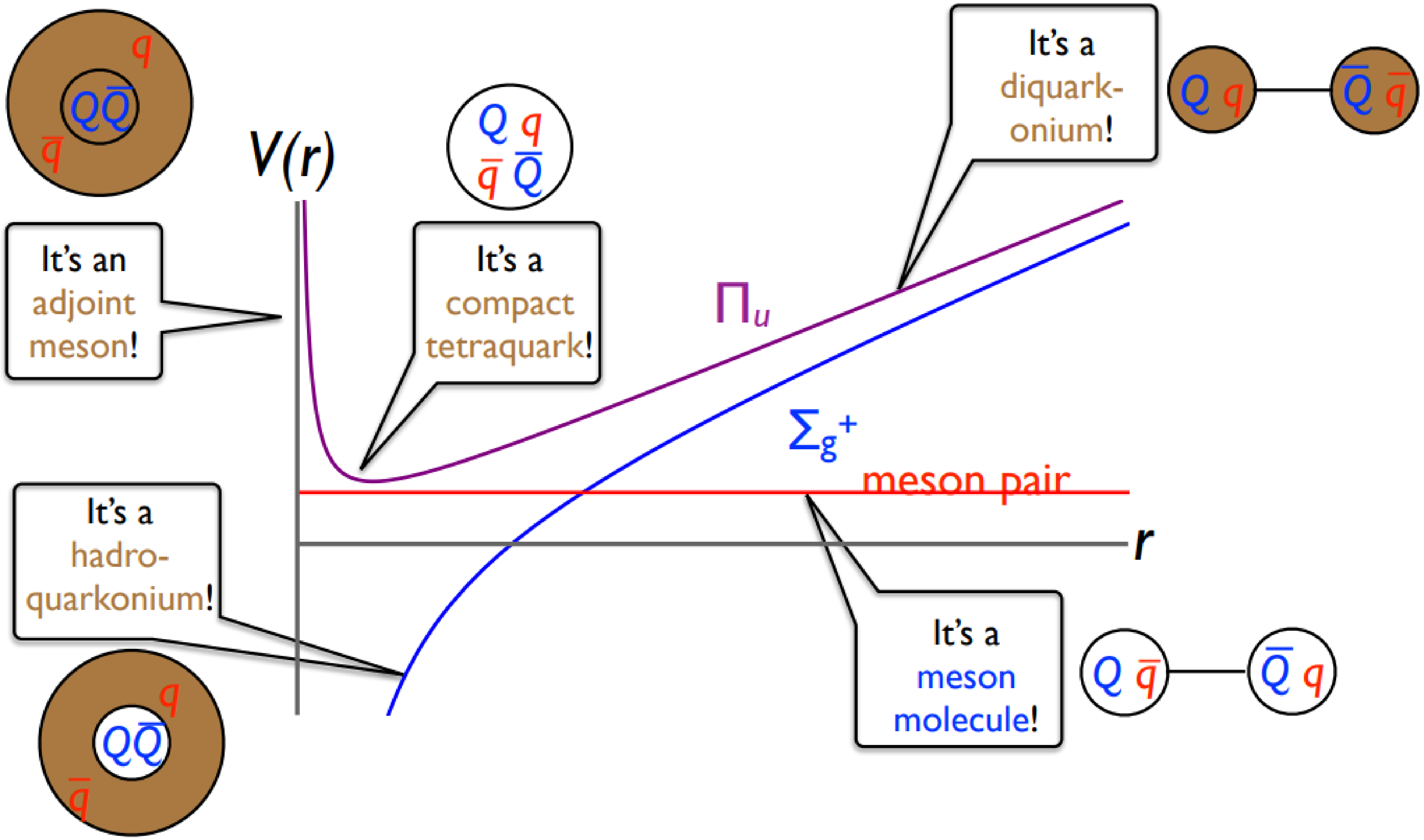


Figure from Nat Rev Phys 1, 480-494 (2019)

<u>"Common Names" *(with exceptions)*</u>

Y(mass): produced in $e^+e^- \rightarrow Y$
Z(mass): has non-zero isospin
X(mass): everything else

What is an XYZ Meson?



Each model describes some region of the Born-Oppenheimer wavefunction

Figure from Eric Braaten talk:
Charm 2020 conference