Moments of multiplicity distribution in relativistic heavy-ion collisions: Insights on thermalization and nuclei production

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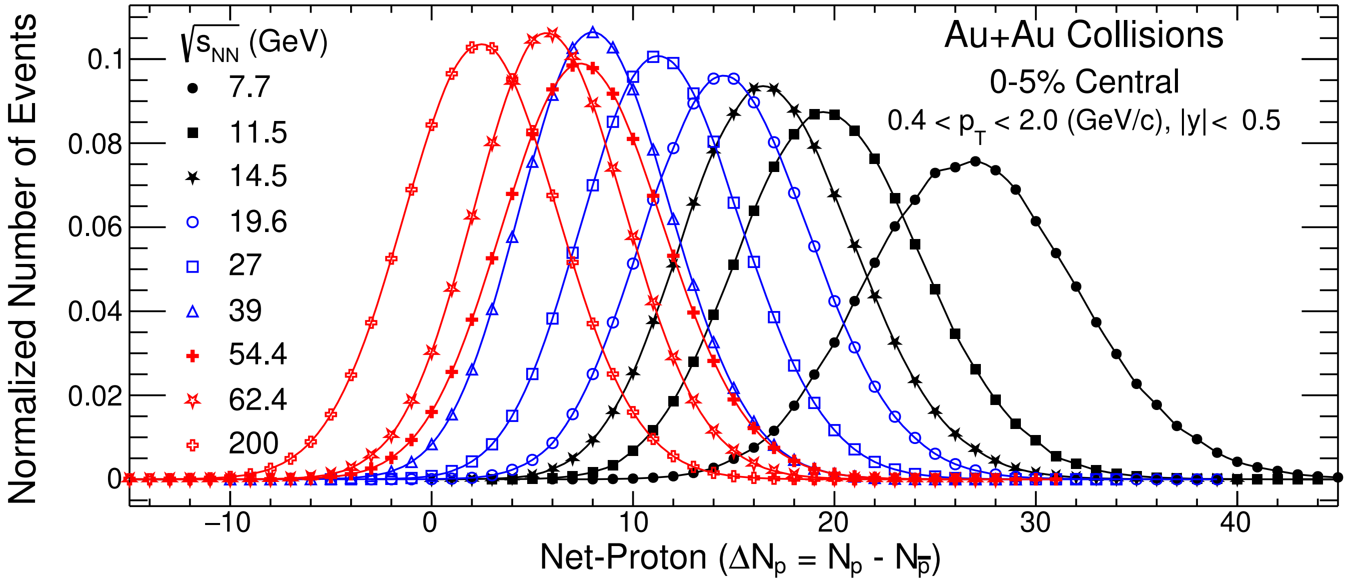
(NISER and CERN)

Outline

- Thermalization
- Nuclei production

Institute for Nuclear Theory workshop "Chirality and Criticality: Novel Phenomena in Heavy-Ion Collisions"

Moments of multiplicity distributions



PHYSICAL REVIEW LETTERS 126, 092301 (2021)

$C_1 = < N >$
$C_2 = < (\delta N)^2 >$
$C_3 = < (\delta N)^3 >$
$C_4 = < (\delta N)^4 > -3 < (\delta N)^2 >^2$

Connect to theory.
correlation length & susceptibility

Sensitive to:
(1)Nature of transition
(2)Critical point
(3)Freeze-out
(4)Thermalization
(5)Initial EM fields

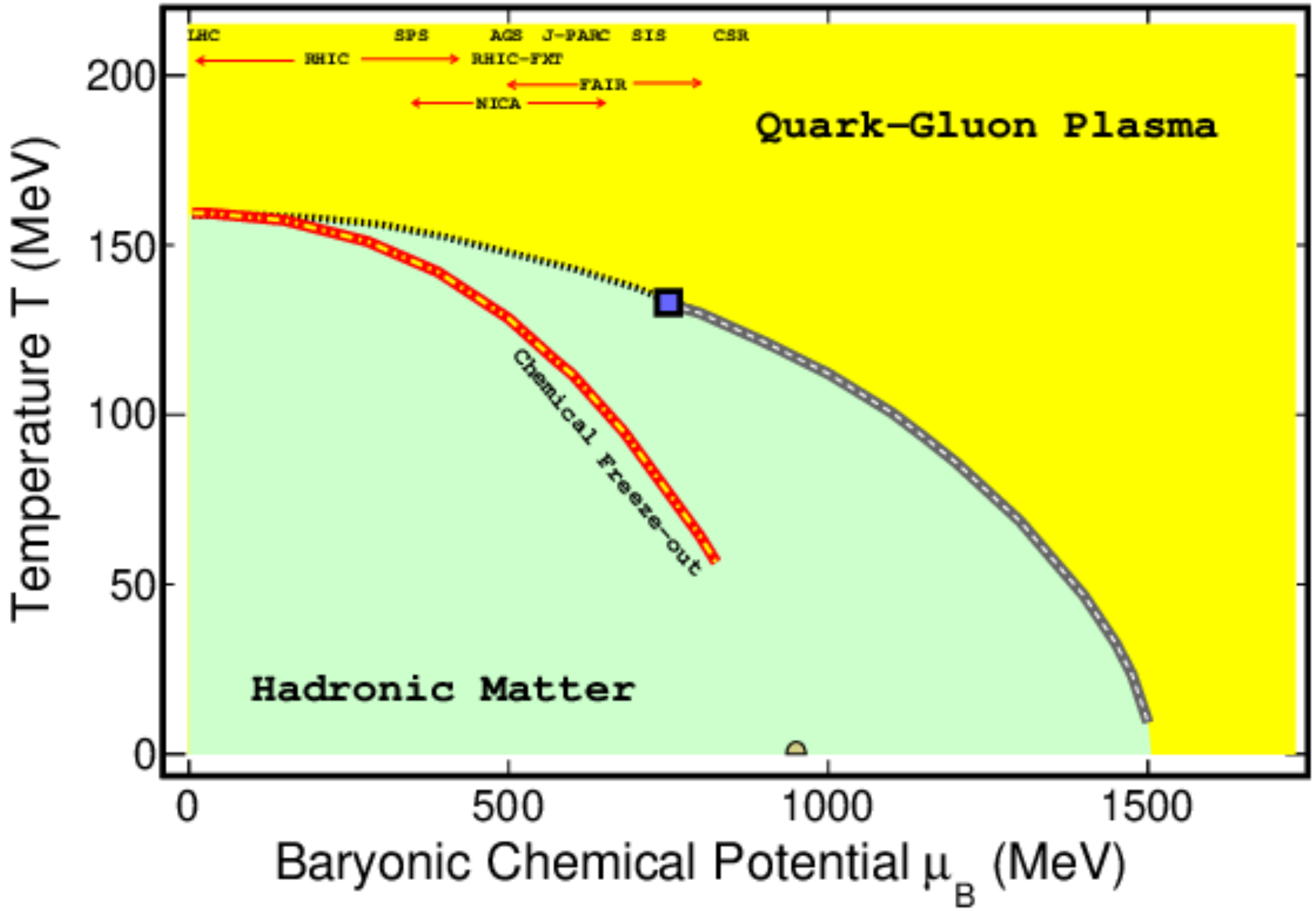
Key topics in the field

- $C_2 \sim \xi^2 \qquad C_4 \sim \xi^7$
- $\frac{\chi_q^{(4)}}{\chi_q^{(2)}} = \kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} \qquad \frac{\chi_q^{(3)}}{\chi_q^{(2)}} = S\sigma = \frac{C_{3,q}}{C_{2,q}}$

*PRL*105, 22303(10); *ibid*, 112, 032302(14) *PLB633*, 275(06); *PRL*102, 032301(09); *PLB695*,136(11);*PLB696*, 459(11)

Thermalization

- Why address this topic ?
  - To establish quark-gluon plasma
  - To establish the QCD phase diagram
  - To understand several physics conclusions at Relativistic Heavy Ion Collisions



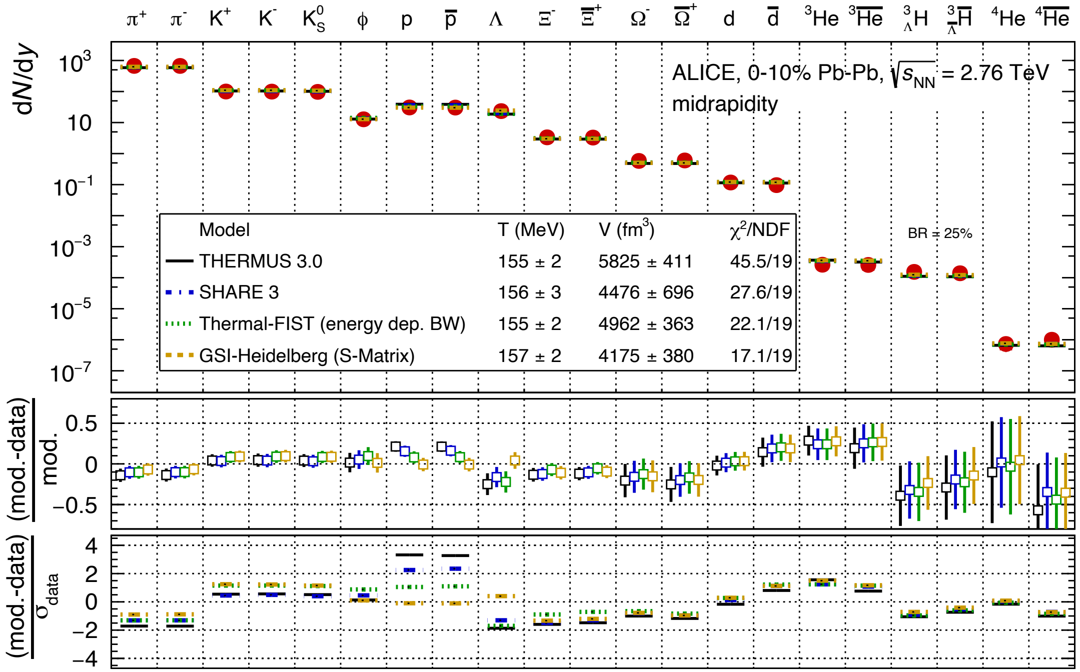
Prog.Part.Nucl.Phys. 125 (2022) 103960

Thermalization

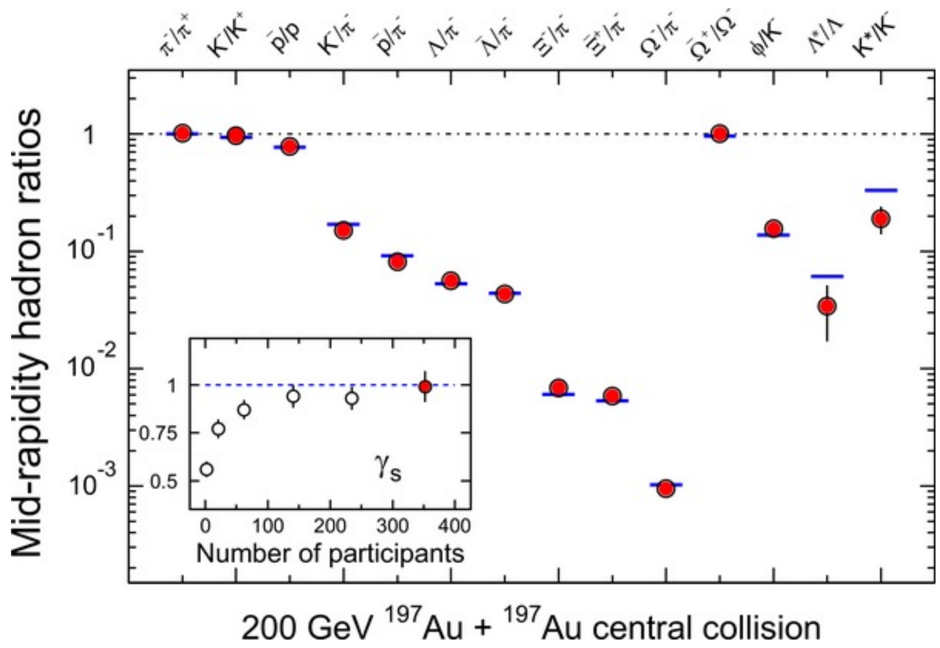
- Some of the ways to address the topic
  - Maximum entropy - $dS/dt = 0$ (Our system short lived) -- *To show experimentally is challenging (impossible?)*
  - Interactions among constituents saturate. (State in thermal equilibrium has no knowledge of past) -- *Can we demonstrate this experimentally ?*
  - Space-momentum distributions reach equilibrium value -- *Can we access this experimentally* ?

Thermalization

ALICE: arXiv:2211.04384



STAR white papers - 2005, Nucl. Phys. A757, STAR: p102



Mean yields have been successfully explained by thermal models. But that is not the full distribution. Distributions described by moments - are all orders of moments thermal ?

Moments of multiplicity distribution and statistical hadron resonance gas model

$$\ln Z^{GC}(T, V, \{\mu_i\}) = \sum_{\text{species } i} \frac{g_i V}{(2\pi)^3} \int d^3p \ln(1 \pm e^{-\beta(E_i - \mu_i)})^{\pm 1}$$

$$N_i^{GC} = T \frac{\partial \ln Z^{GC}}{\partial \mu_i} = \frac{g_i V}{2\pi^2} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \frac{m_i^2 T}{k} K_2 \left(\frac{k m_i}{T} \right) \times e^{\beta k \mu_i}$$

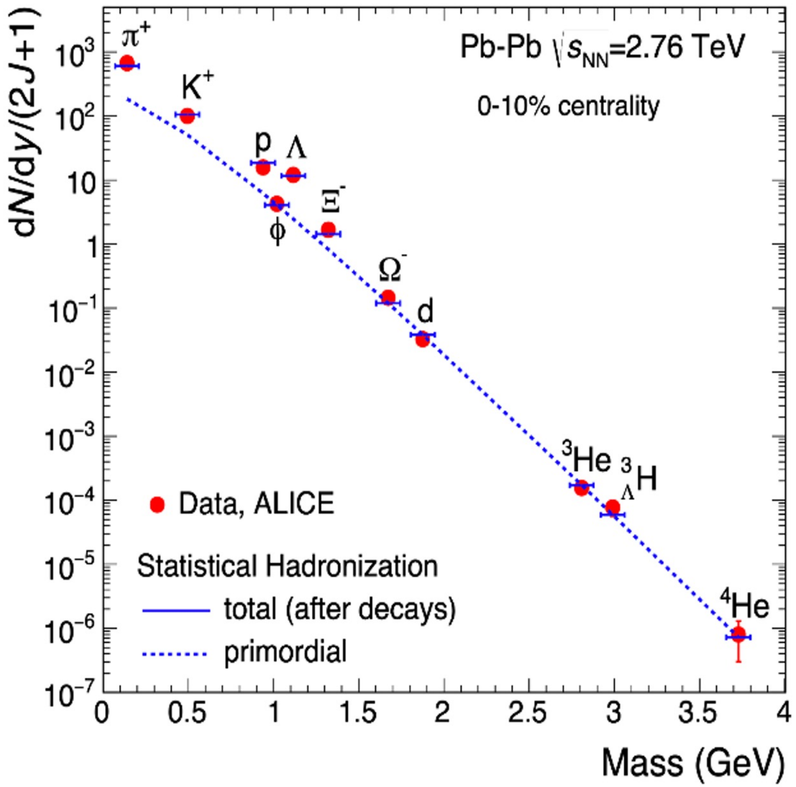
$$C_n^X = (V/T) T^n \chi_X^{(n)}$$

$$C_{1,1}^{X,Y} = \frac{V}{T} T^2 \chi_{X,Y}^{(1,1)} = \frac{V}{T} T^2 \frac{d^2 P}{d\mu_X d\mu_Y}$$

$$\chi_X^{(n)} = d^n P / d\mu_X^n$$

$$P(T, \mu_B, \mu_Q, \mu_S, V) = \frac{T}{V} \sum_i \ln Z_i = \sum_i \pm \frac{T g_i}{2\pi^2} \int k^2 dk \ln\{1 \pm \exp[(\mu_i - E)/T]\}$$

- **Assumptions:**
  - Thermal equilibrium
  - Point like hadrons
  - Conservation laws applied on average

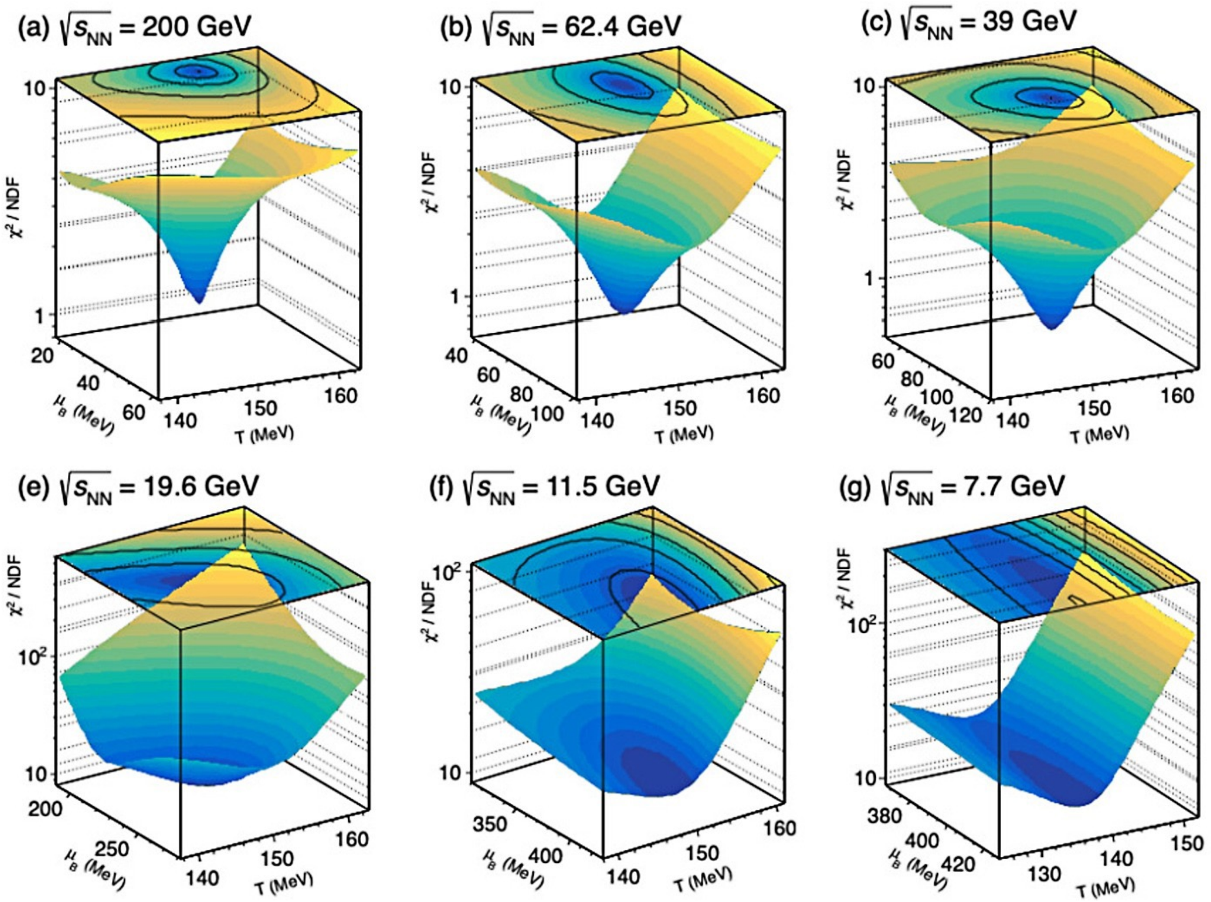


A. Andronic et al., Nature vol. 561, (2018) 321
HotQCD Collaboration, Phys, Lett. B 795 (2019) 15

Observables

all observables	$C_1^{\pi^\pm}, C_1^{K^\pm}, C_1^{p}, C_1^{\bar{p}}$, $C_2^{NQ}/C_1^{NQ}, C_2^{NK}/C_1^{NK}, C_2^{NP}/C_1^{NP}, C_{1,1}^{NP,NK}/C_2^{NK}$, $C_3^{NQ}/C_2^{NQ}, C_3^{NK}/C_2^{NK}, C_3^{NP}/C_2^{NP}$, $C_4^{NQ}/C_2^{NQ}, C_4^{NK}/C_2^{NK}, C_4^{NP}/C_2^{NP}$.
13 observable set	$C_1^{\pi^\pm}, C_1^{K^\pm}, C_1^{p}, C_1^{\bar{p}}$, $C_2^{NQ}/C_1^{NQ}, C_2^{NK}/C_1^{NK}, C_2^{NP}/C_1^{NP}, C_{1,1}^{NP,NK}/C_2^{NK}$, $C_3^{NQ}/C_2^{NQ}, C_3^{NK}/C_2^{NK}, C_3^{NP}/C_2^{NP}$.
11 observable set	$C_1^{\pi^\pm}, C_1^{K^\pm}, C_1^{p}, C_1^{\bar{p}}$, $C_2^{NQ}/C_1^{NQ}, C_2^{NK}/C_1^{NK}, C_{1,1}^{NP,NK}/C_2^{NK}$, $C_3^{NQ}/C_2^{NQ}, C_3^{NK}/C_2^{NK}$.

Moments of multiplicity distribution and hadron gas model



Phys.Lett.B 829 (2022) 137021

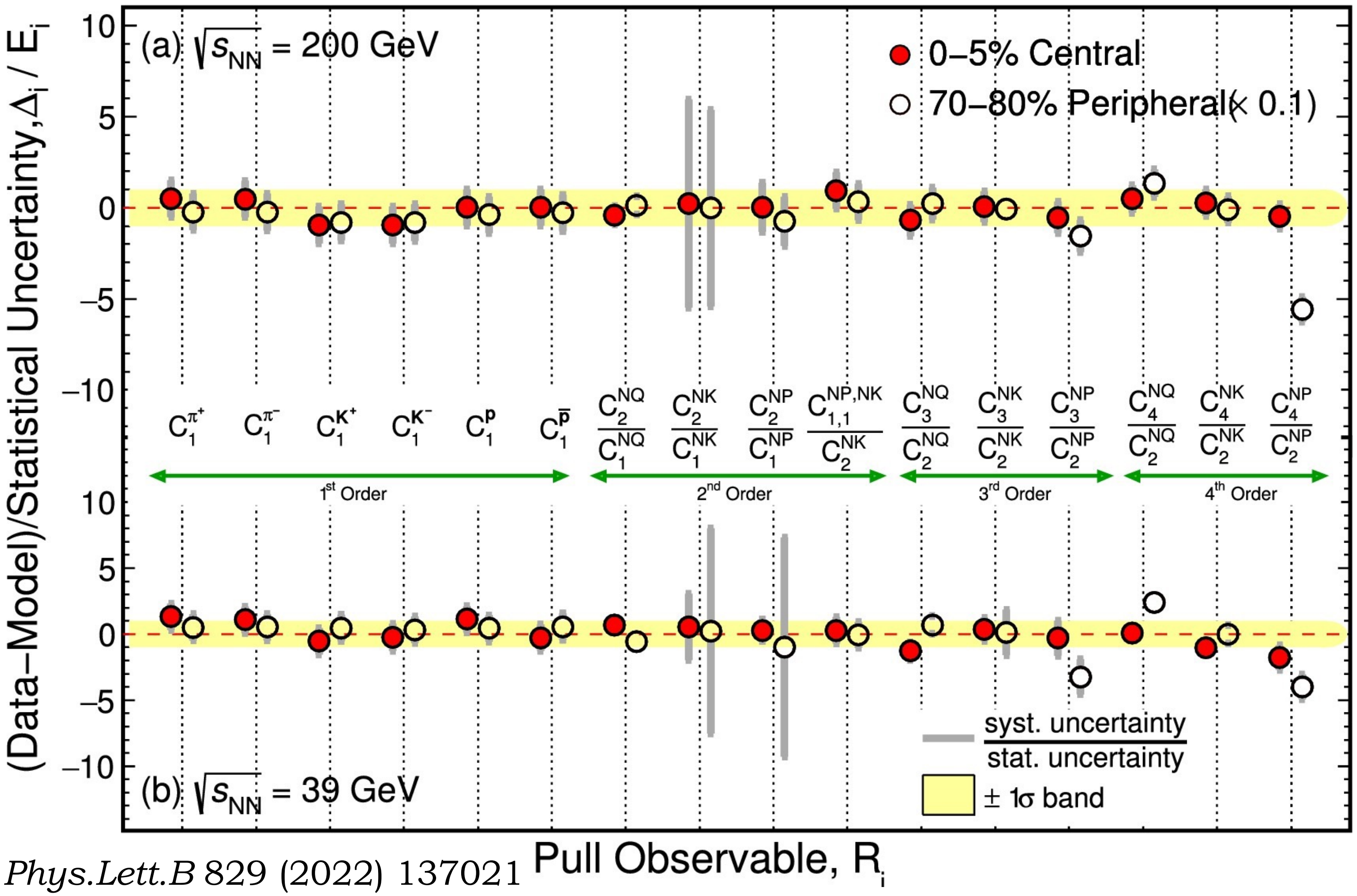
$$\chi^2 = \sum_{i=1}^{N} \left(\frac{\Delta_i}{E_i}\right)^2 \quad \text{where} \quad \Delta_i = R_i^{\text{exp}} - R_i^{\text{HRG}}$$

χ^2 /NDF is close to unity

Except may be 7.7 GeV

Fits to moments of multiplicity distributions

Moments of multiplicity distribution and hadron gas model

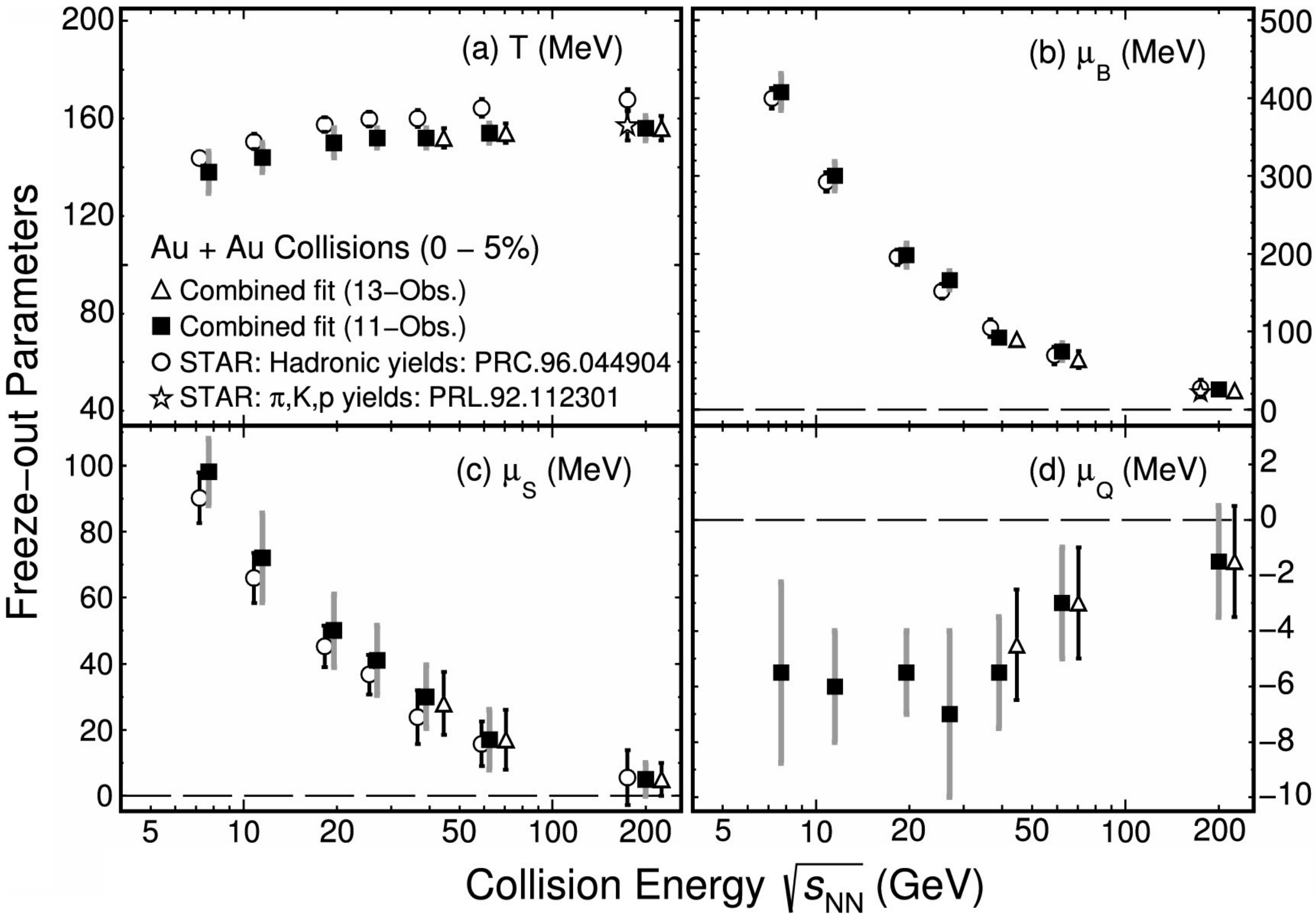


Phys.Lett.B 829 (2022) 137021

Measurements in central collisions agrees with thermal model

Peripheral collisions do not

Moments of multiplicity distribution and hadron gas model

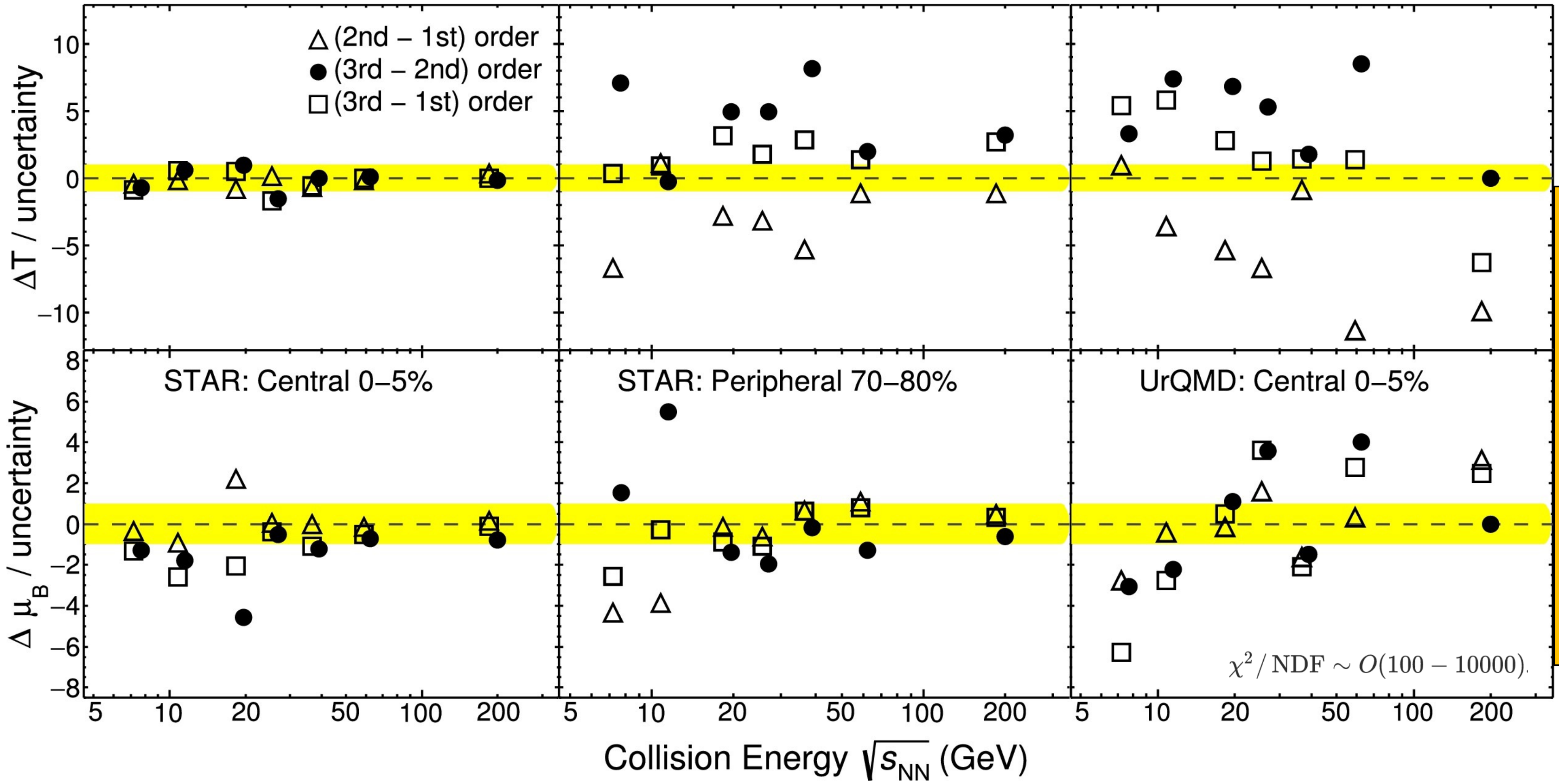


Phys.Lett.B 829 (2022) 137021

Thermal model parameters

13 observables only for higher energies

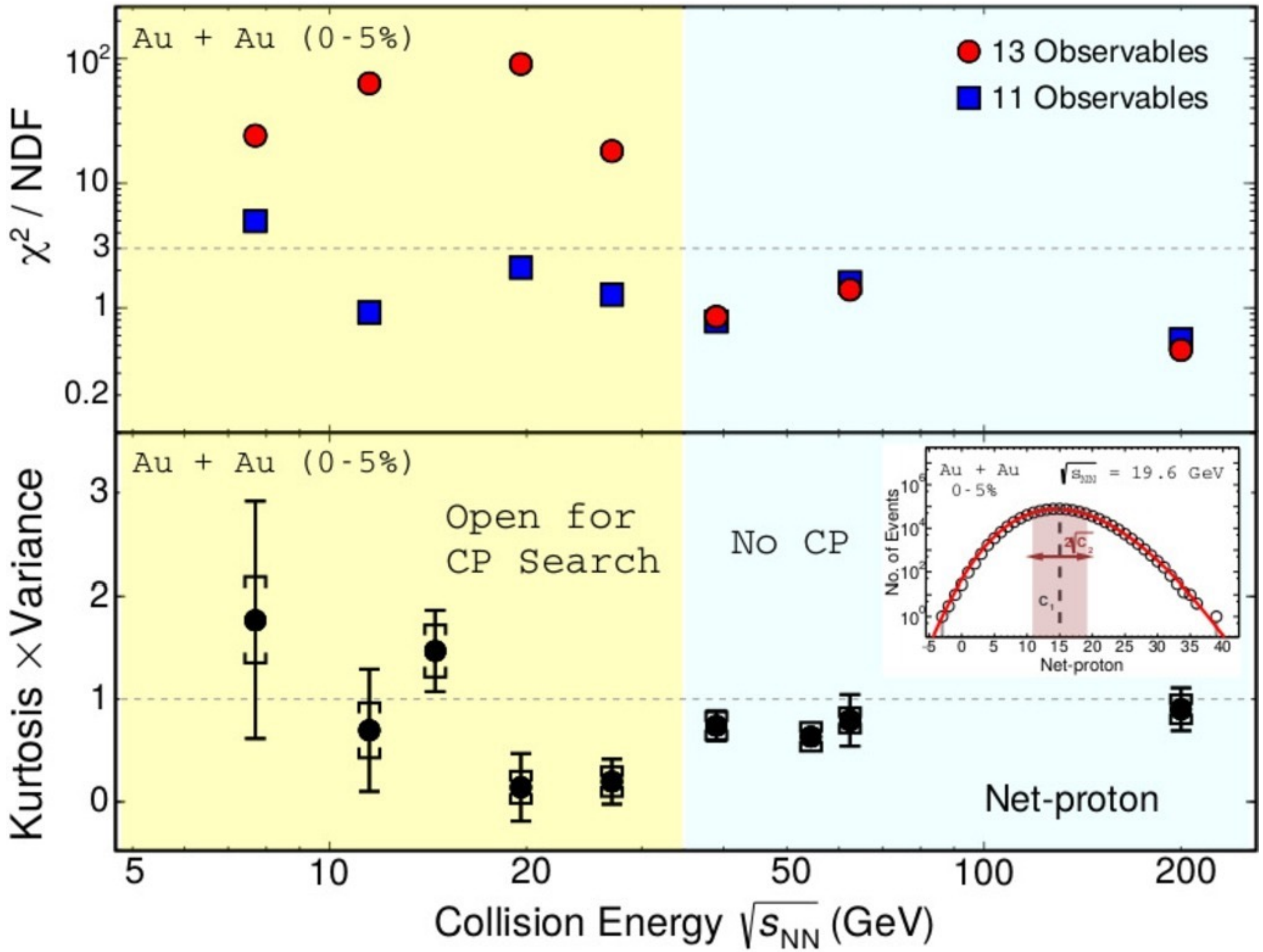
Moments of multiplicity distribution and hadron gas model



Order-by-order thermal model parameters

Phys.Lett.B 829 (2022) 137021

Discussion



Favors a thermal system for collision energies > 30 GeV

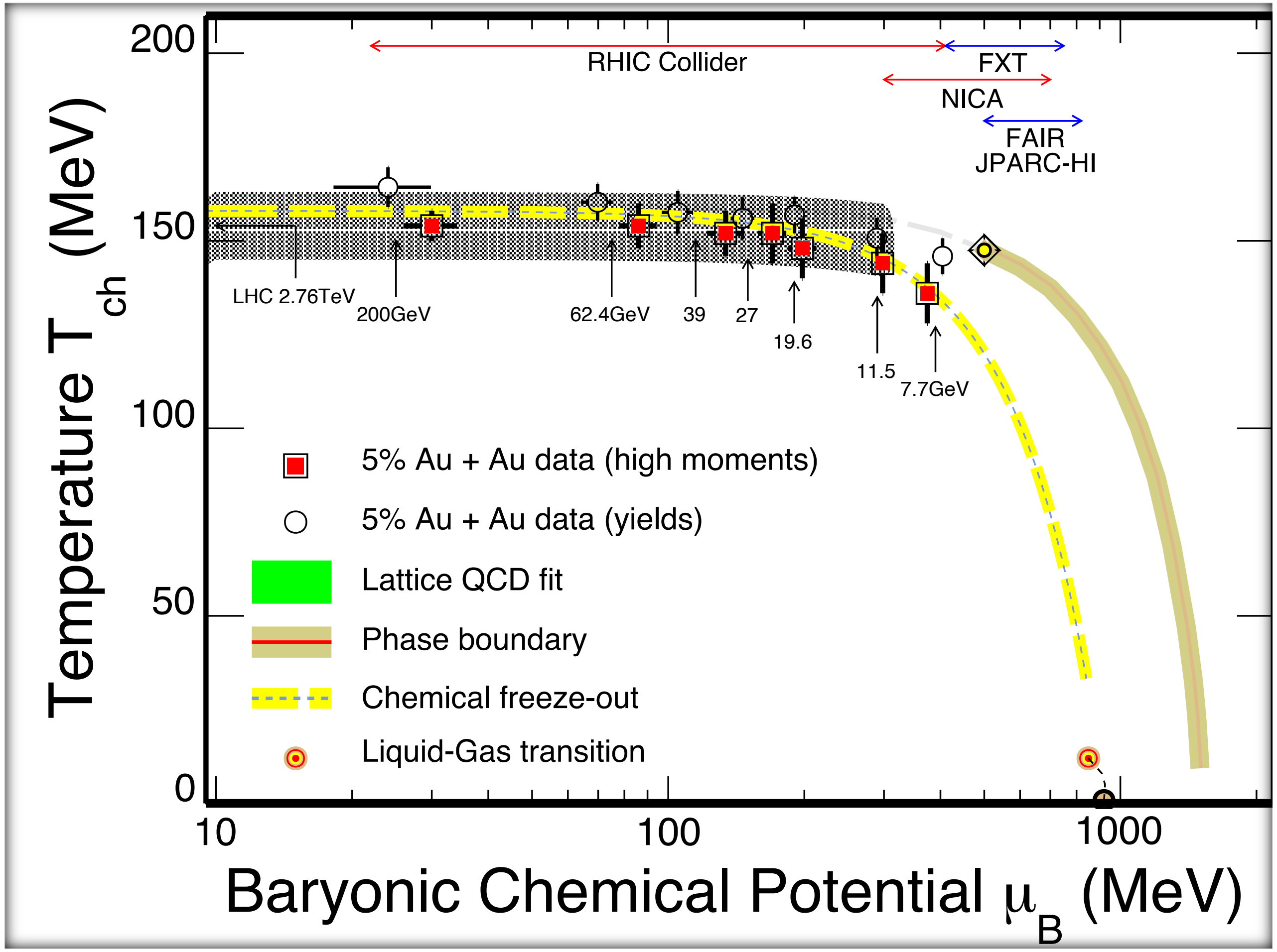
Future: (1) Try canonical approach specially at lower collision energies. (2) Fixed target energies and (3) Try the approach in multiplicity dependent proton-proton collisions

Summary # Tests of thermalization

AAPPS Bull. 31 (2021) 1

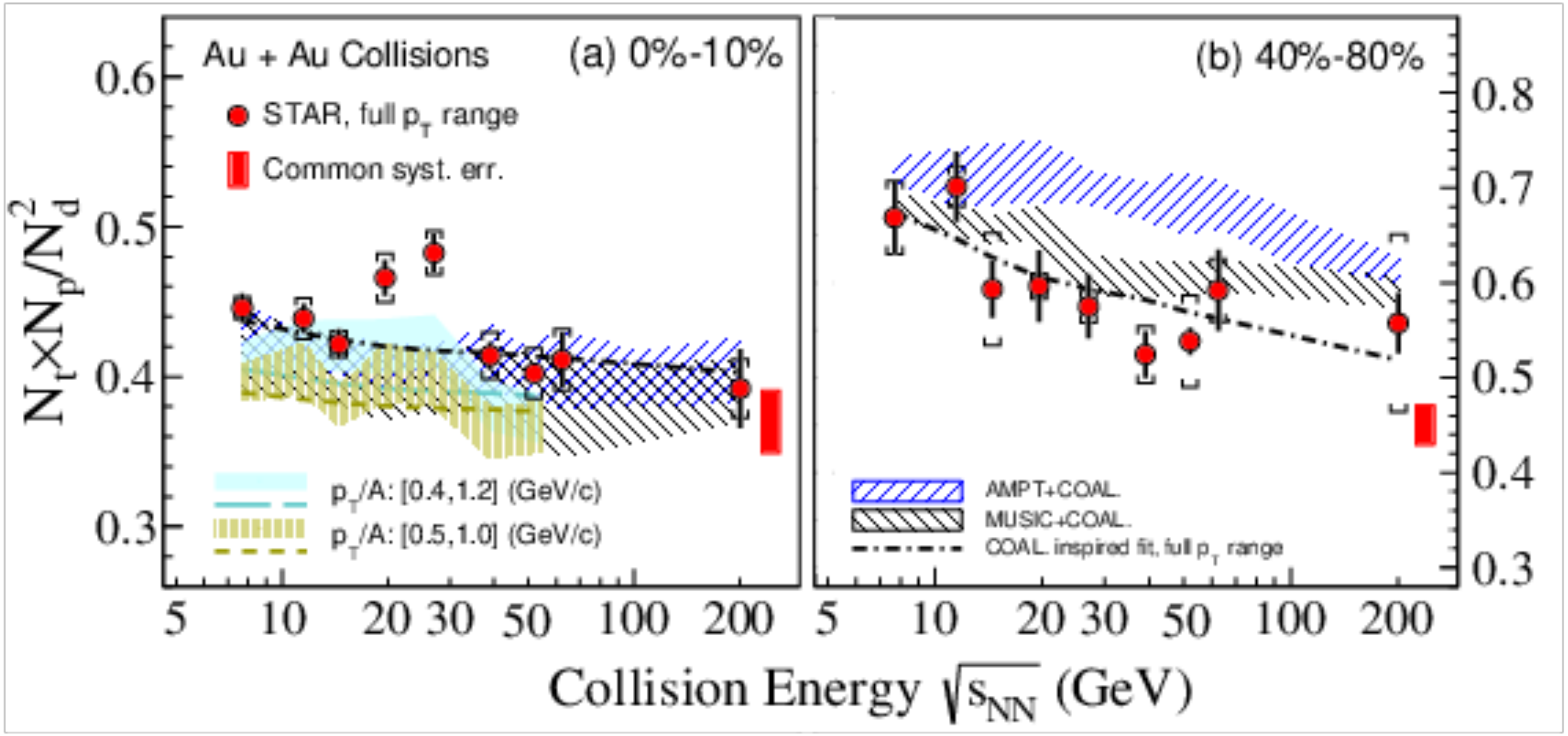
Statistical thermal model of hadrons and resonances with grand canonical ensemble tested using moments of multiplicity distributions up to 4th order.

The distributions look thermal for central collisions for collision energies above 30 GeV



Temperature versus Baryonic chemical potential

Nuclei production and criticality

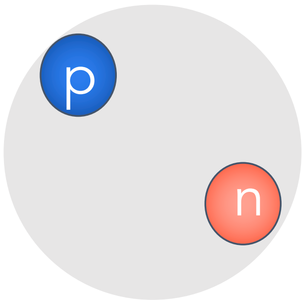


STAR: Phys.Rev.Lett. 130 (2023) 202301

Sensitive to the neutron density fluctuations

Nuclei production

Hadronization for light nuclei is not well understood

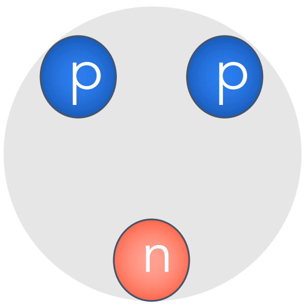


Deuteron

E_b = 2.22 MeV

$\sqrt{\langle R_c^2 \rangle}$ = 2.13 fm

P. J. Mohr et al., Rev. Mod. Phys. 88 (2016) 035009



^{3}He

E_b = 7.72 MeV

$\sqrt{\langle R_c^2 \rangle}$ = 1.96 fm

Nucl. Data Sheets 130, 1 (2015)

Typical energy scales

Hadronic yields and spectra are fixed around temperature ~ 90 - 160 MeV.

Binding energy of deuteron ~ 2 MeV.

Light nuclei in hadron gas

Hadron gas is a very hostile environment for light nuclei (reminder: binding energy ≈ few MeV)

- typical hadronic momentum transfer > 100 MeV/*c*

$$\sigma_{\pi d} > 100 \text{ mb} = 10 \text{ fm}^2$$

From SAID database

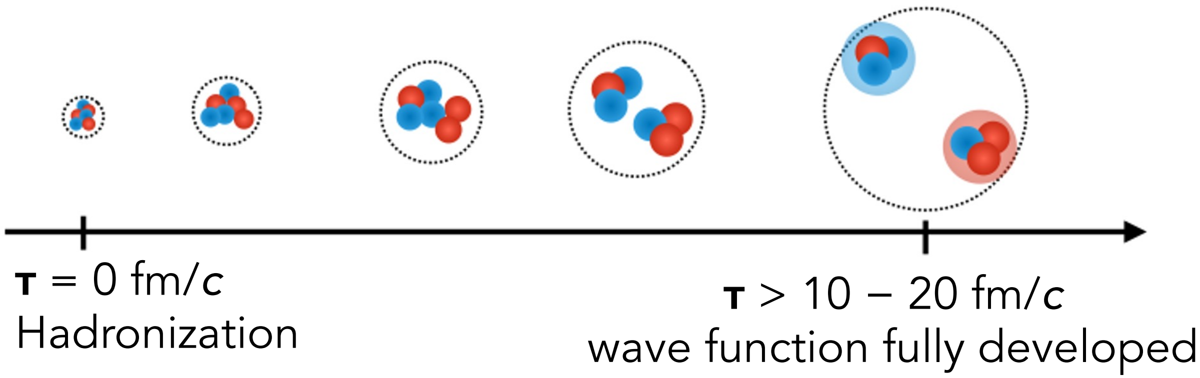
$$\lambda_d = \frac{1}{n\sigma} < \frac{1}{\frac{0.05}{\text{fm}^3} \times 10 \text{ fm}^2} = 2 \text{ fm}$$

should exceed ≈ 10-15 fm for deuteron survival!

Density at kinetic freeze-out
(when elastic interactions cease)

Assumptions:

- Light nuclei produced as compact (colorless) quark systems
  > Negligible interaction with hadrons
- Formation time > τ hadronic phase



Nuclei production two mechanisms

GCE Thermal model

Yield of deuteron:

$N_d = \frac{g_d V}{\pi^2} m_d^2 T K_2(m_d/T) \exp(\mu_d/T)$

where, g_d: degeneracy, μ_d: chemical potential.

Deuteron is treated as a free and point particle.

Degeneracy, mass and baryon number are inputs.

Coalescence model

Invariant yield:

$$E_d \frac{d^3N_d}{dp_d^3} = B_2 (E_p \frac{d^3N_p}{dp_p^3})(E_n \frac{d^3N_n}{dp_n^3})$$

Elliptic flow:

$$v_2^d(p_T) \approx 2v_2^p(\frac{p_T}{2})$$

Light nuclei created using protons and neutrons.

Simple coalescence model expectations

Simplified Coalescence Model

Probability of deuteron formation, $\lambda_d = B_2 n_p n_n$

Assume, proton (n_p) and neutron (n_n) follow Poisson distributions,

- At low $\sqrt{s_{NN}}$, B_2 increases.
- Larger value of n_p and n_n at low $\sqrt{s_{NN}}$.
- Results in rise of scaled moments of deuteron number.

Scaled Moments: $\sigma^2/M = C_2/C_1$, $S\sigma = C_3/C_2$, $\kappa\sigma^2 = C_4/C_2$

Two assumptions in the model:

Model A: Correlated p and n (n_p=n_n).

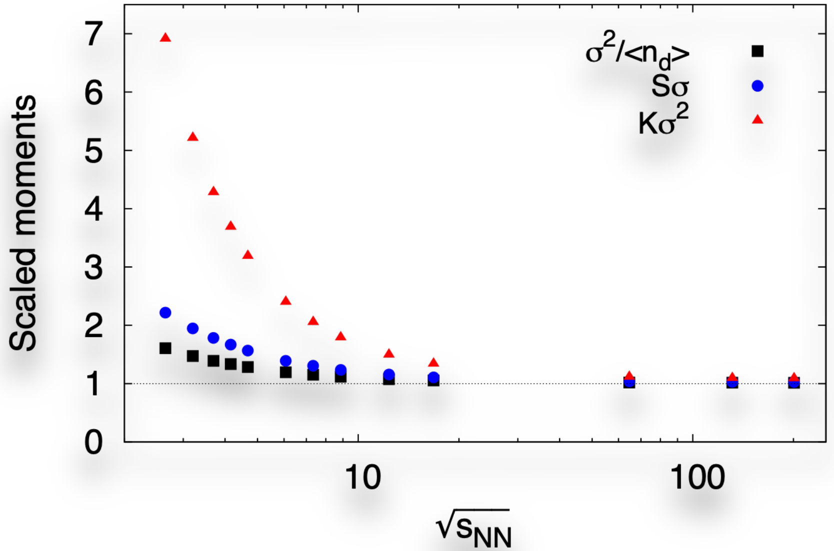
$$\lambda_d = B_2 n_p^2$$

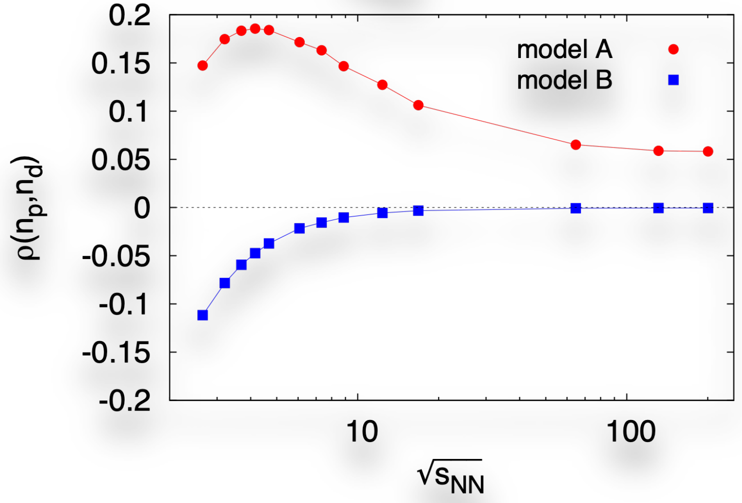
Model B: Independent p and n.

$$\lambda_d = B_2 n_p n_n$$

$$\rho(n_p, n_d) = \frac{\langle (n_p - \langle n_p \rangle)(n_d - \langle n_d \rangle) \rangle}{\sigma_p \sigma_d}$$

☑ Model A: $\rho > 0$ Model B: $\rho < 0$





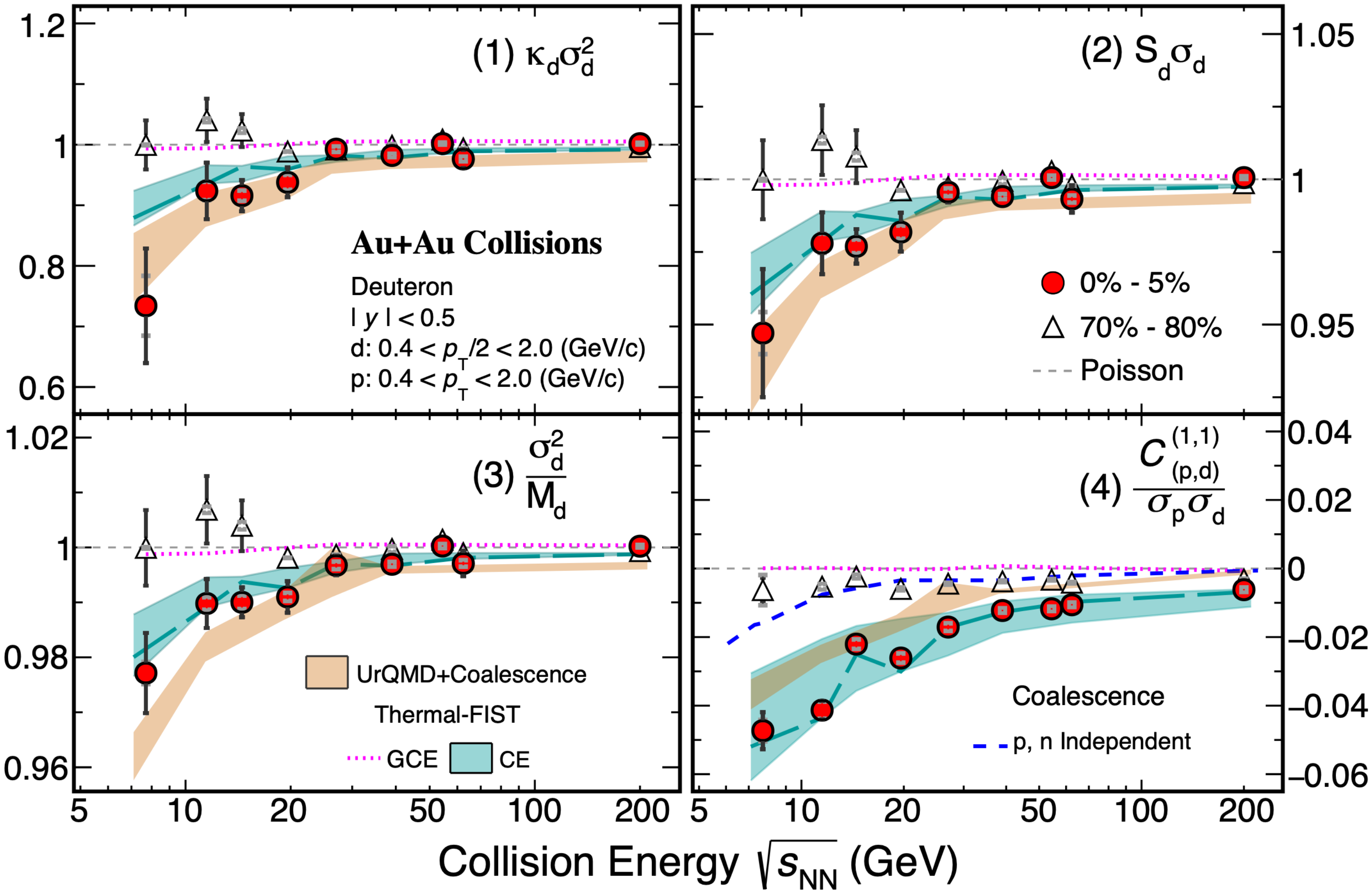
Z. Fecková, J. Steinheimer, B. Tomášik and M. Bleicher: Phys. Rev. C 93, 054906 (2016)

Results from RHIC

STAR: e-Print: 2304.10993 [nucl-ex]

Thermal-FIST: V. Vovchenko et al., Comput. Phys. Commun. 244, 295 (2019)
Simple Coalescence: Z. Fecková et. al,: PRC 93, 054906 (2016)

Black bars: statistical uncertainties
Grey caps: systematic uncertainties



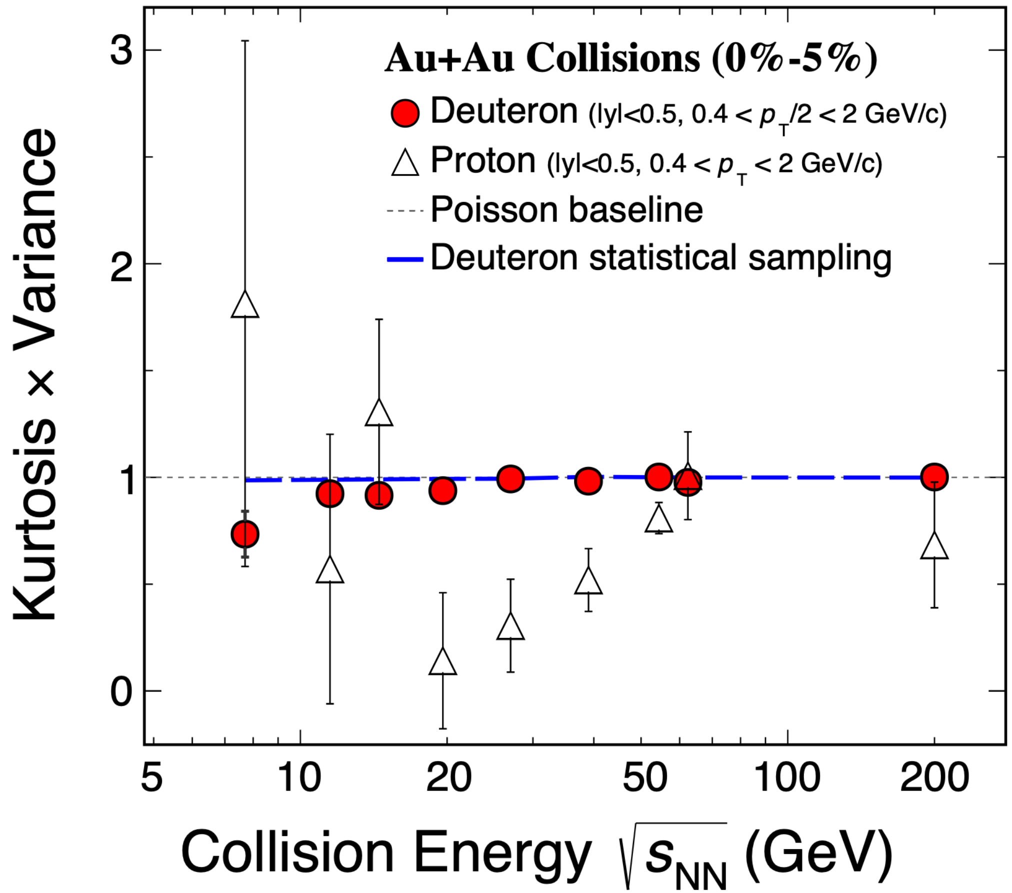
GCE thermal model seems to fail to describe the cumulant ratios for lower $\sqrt{s_{NN}}$.

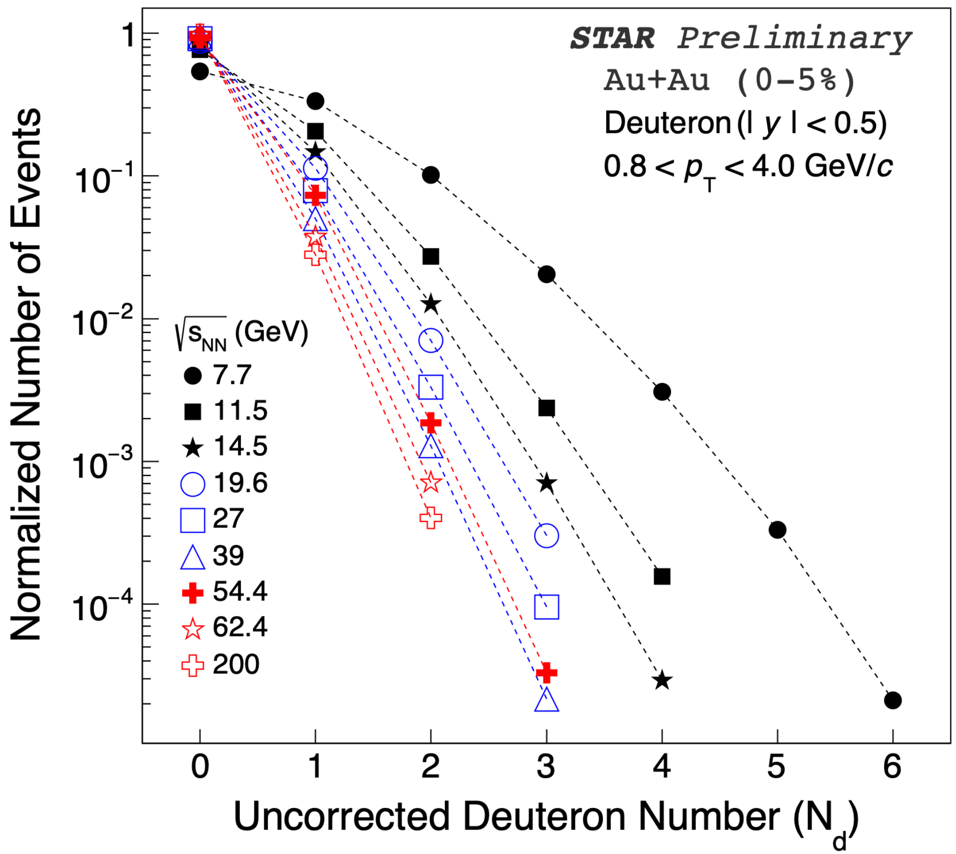
CE thermal model qualitatively reproduce collision energy dependence.

UrQMD+Coalescence also reproduces the trend and shows better agreement with the cumulant ratios.

Neither correlated nor independent assumption for proton and neutron in the toy model from reproduce the data.

$\kappa\sigma^2$: Deuterons vs. Protons

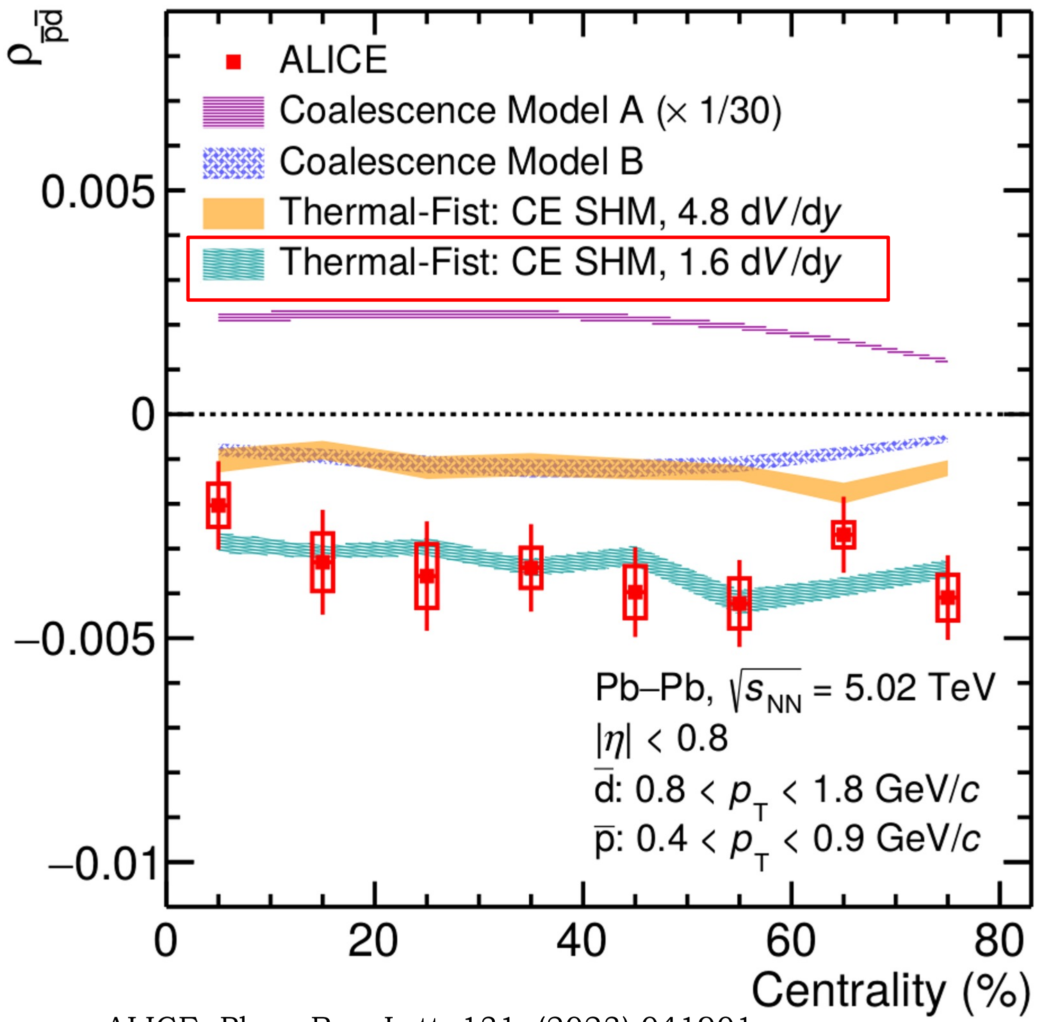




Deuteron cumulants : Monotonic energy dependence in contrast to protons

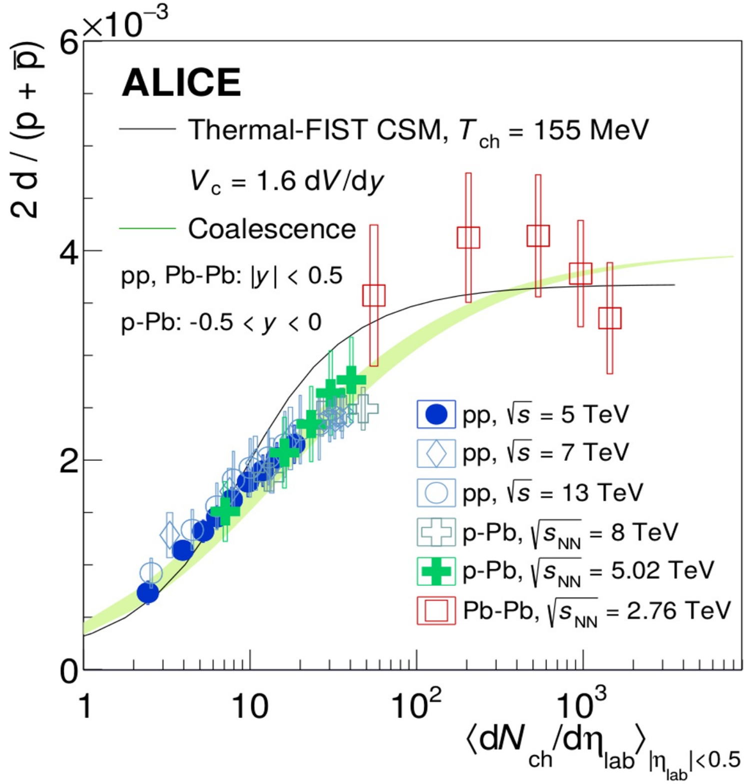
Statistical test: Due to low event-by-event yield of deuterons

Results from LHC



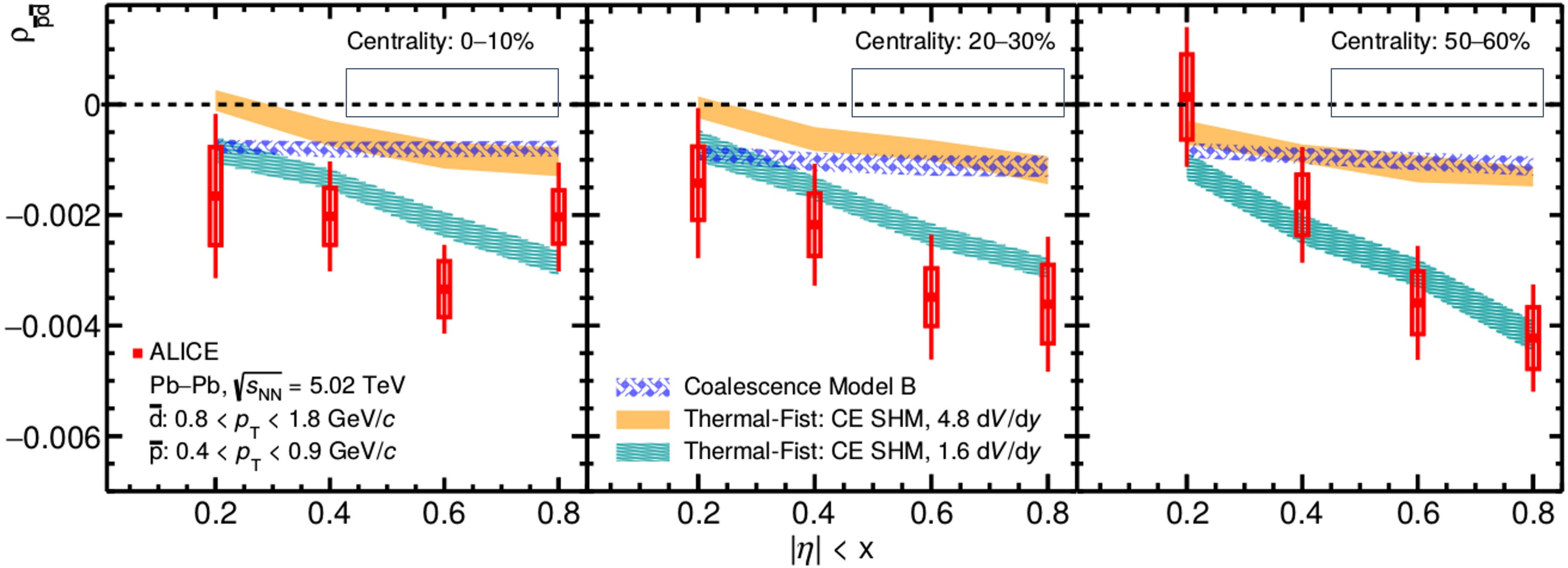
ALICE, Phys. Rev. Lett. 131, (2023) 041901

Correlation volume of 1.6 ± 0.3 dV/dy best describes the data



ALICE, Phys. Rev. C 107 (2023) 064904

Results from LHC

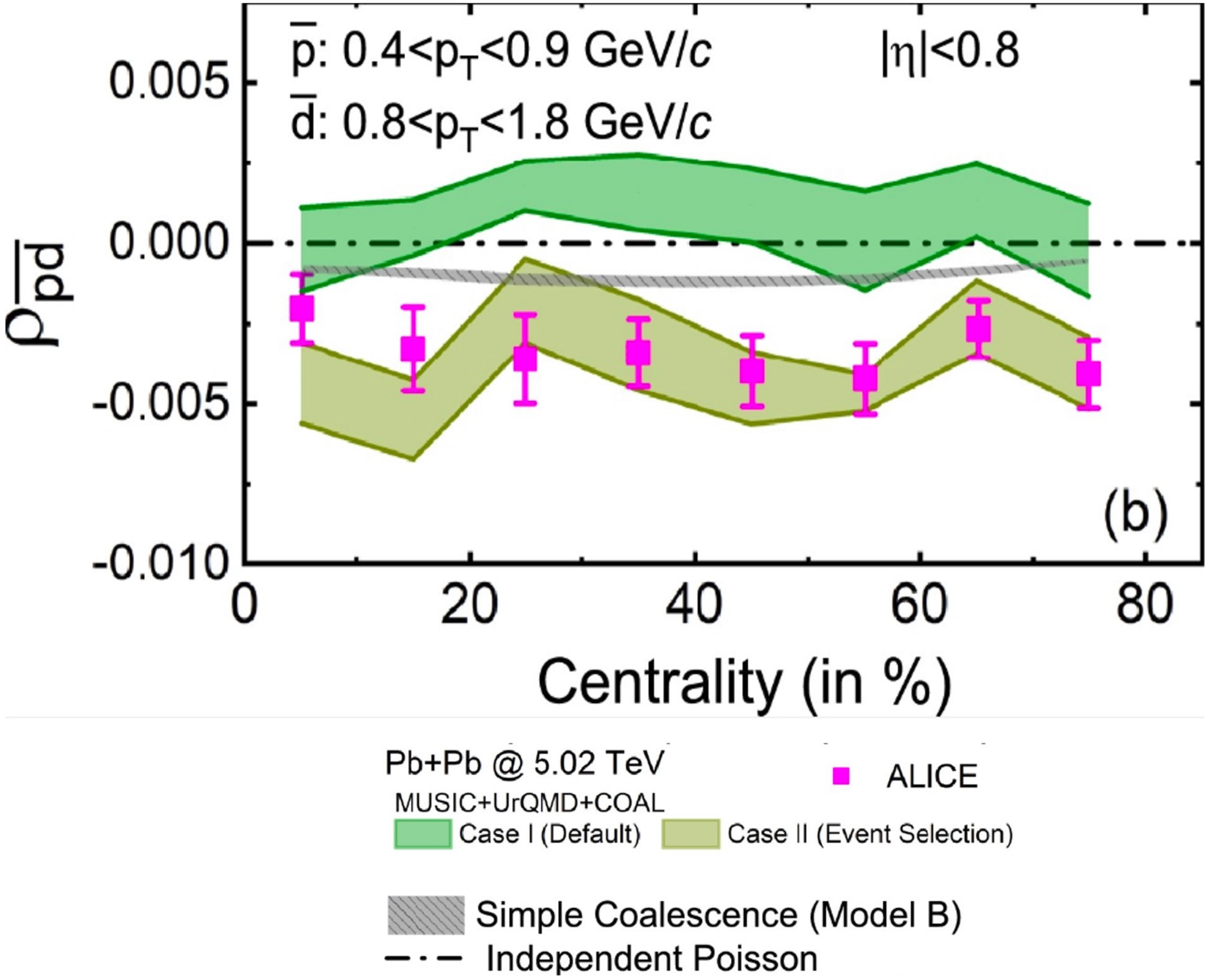


ALICE, Phys. Rev. Lett. 131, (2023) 041901

- Data: strong acceptance dependence of correlation strength
- SHM: describes data
- Coalescence: ~flat with acceptance, strength depends on the nucleon phase space density or d/p ratio

Results from LHC

State of art coalescence model: MUSIC + UrQMD + Coalescence + implement correlation between nucleons using ALICE net-proton fluctuation measurement



Kai-Jia Sun et al. Phys. Lett. B 840 (2023) 137864

Summary # Nuclei production

Moments of nuclei multiplicity distributions are sensitive observables for their production mechanism.

Still no clear differentiation between two approaches – thermal and coalescence

Additional correlation measurements required.

[S. Mrówczyński and P. Słoń, Acta Phys. Polon. B 51, 1739-1755 (2020)
S. Mrówczyński and P. Słoń, Phys. Rev. C 104, 024909 (2021)
[S. Bazak and S. Mrowczynski, Eur. Phys. J. A 56, 193 (2020)

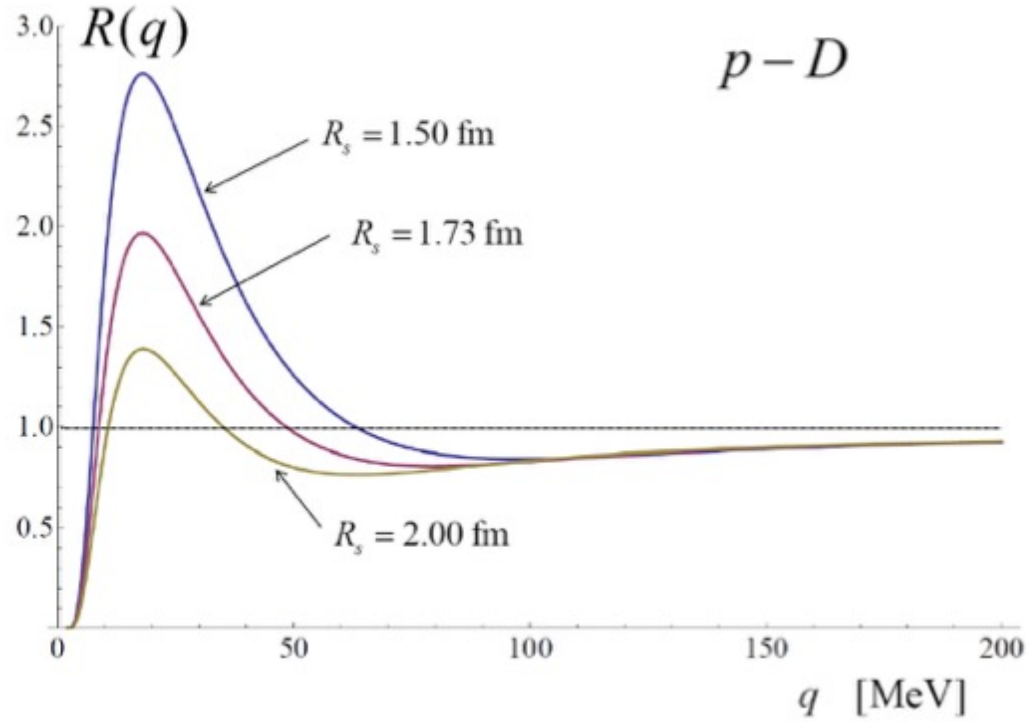


Fig. 2. $p-D$ correlation function

Femtoscopic correlations depends on source size

Thank you

Acknowledgements:

All members of STAR Collaboration

All members of ALICE Collaboration

Sourendu Gupta and Dipak Mishra