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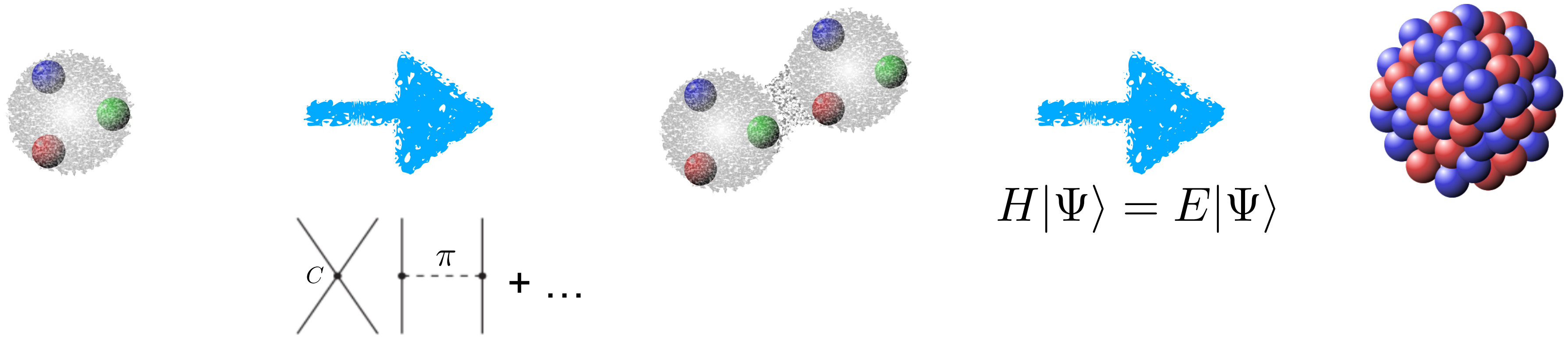
Electroweak properties of medium-mass nuclei and a new emulator



Takayuki Miyagi (University of Tsukuba, Japan)

Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond @ INT, Seattle, April 27 - May 29, 2026

NuHamil code

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- NuHamil code (https://github.com/Takayuki-Miyagi/NuHamil-public)
  - NN and 3N matrix elements expressed with the harmonic oscillator basis.
- NuHamil
  - NN interactions up to N4LO (EMN)
  - 3N interaction up to N2LO
  - NN current at momentum transfer 0 (magnetic moment, Gamow-Teller transition, etc.)
  - NN current at finite momentum transfer (ongoing…)
  - 0vbb transition operator (3N neutriono potential is ongoing…)
  - Parity-violating NN interactions (anapole moment, Schiff moment, …)
- Please push your implementation!

The matrix element files will be available @ Japan Lattice Data Grid (JLDG)

Theoretical framework

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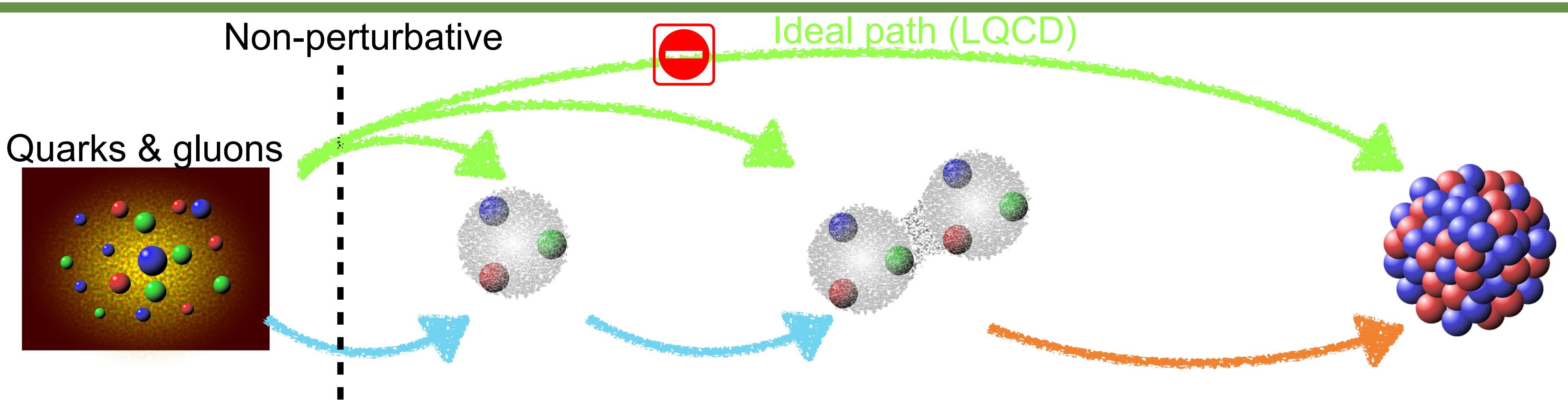
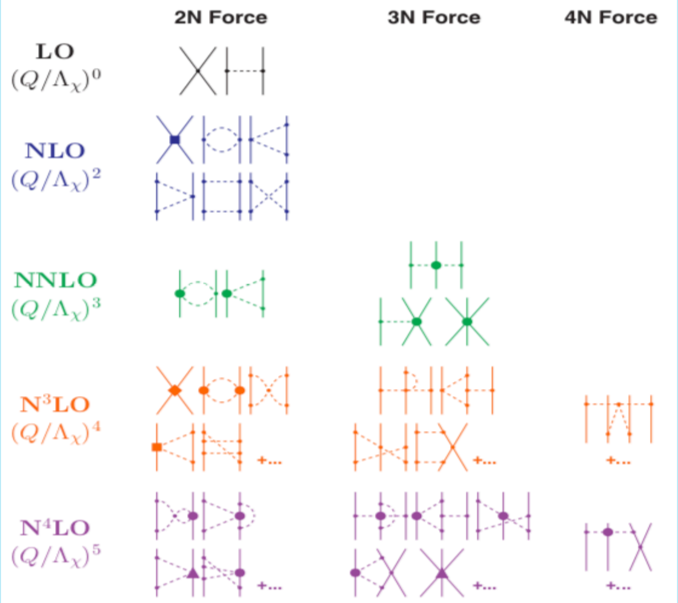




Nuclear many-body problem

- Green’s function Monte Carlo
- No-core shell model
- Nuclear lattice effective field theory
- Self-consistent Green’s function
- Coupled-cluster
- In-medium similarity renormalization group
- Many-body perturbation theory
- …

Nuclear interaction from chiral EFT

Weinberg, van Kolck, Kaiser, Epelbaum, Glöckle, Meißner, Entem, Machleidt, …

- Lagrangian construction
  - Chiral symmetry
  - Power counting
- Systematic expansion
  - Unknown LECs
  - Many-body interactions
  - Estimation of truncation error

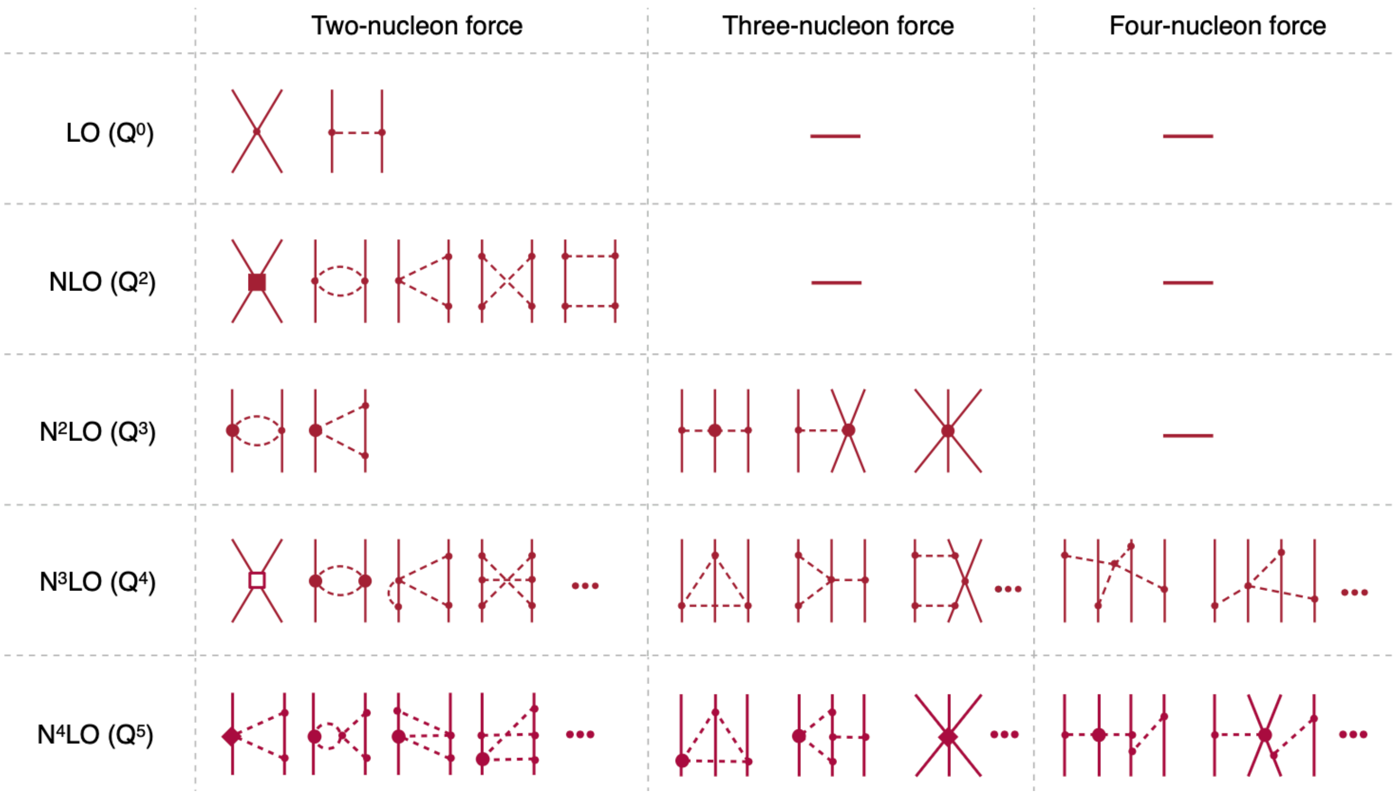
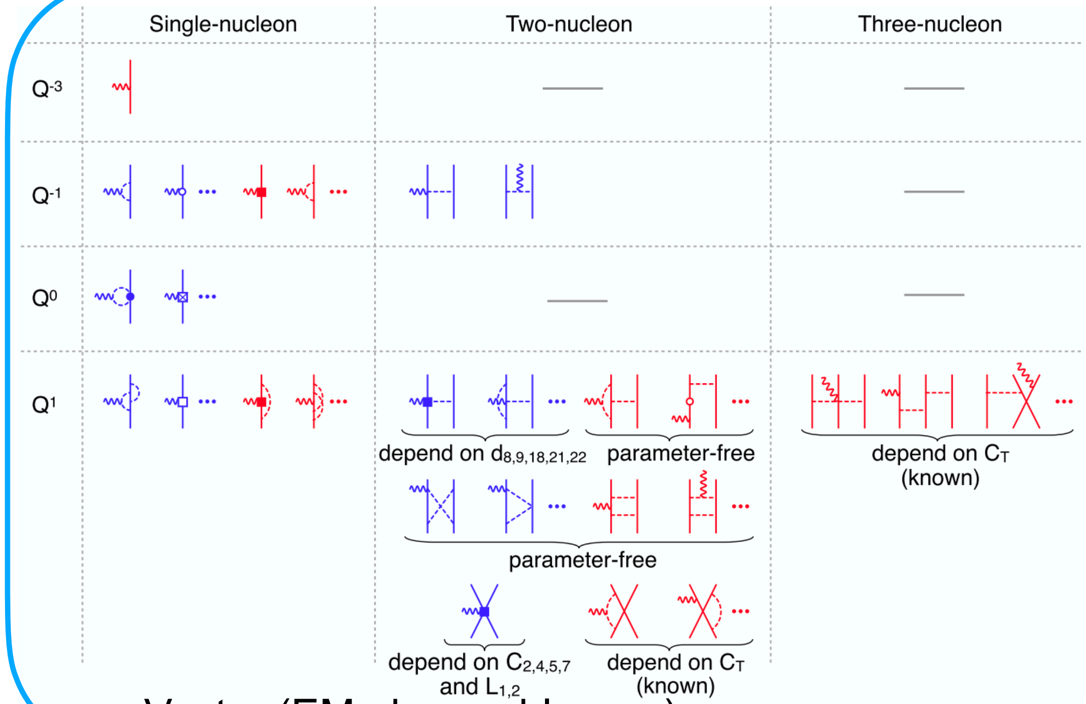


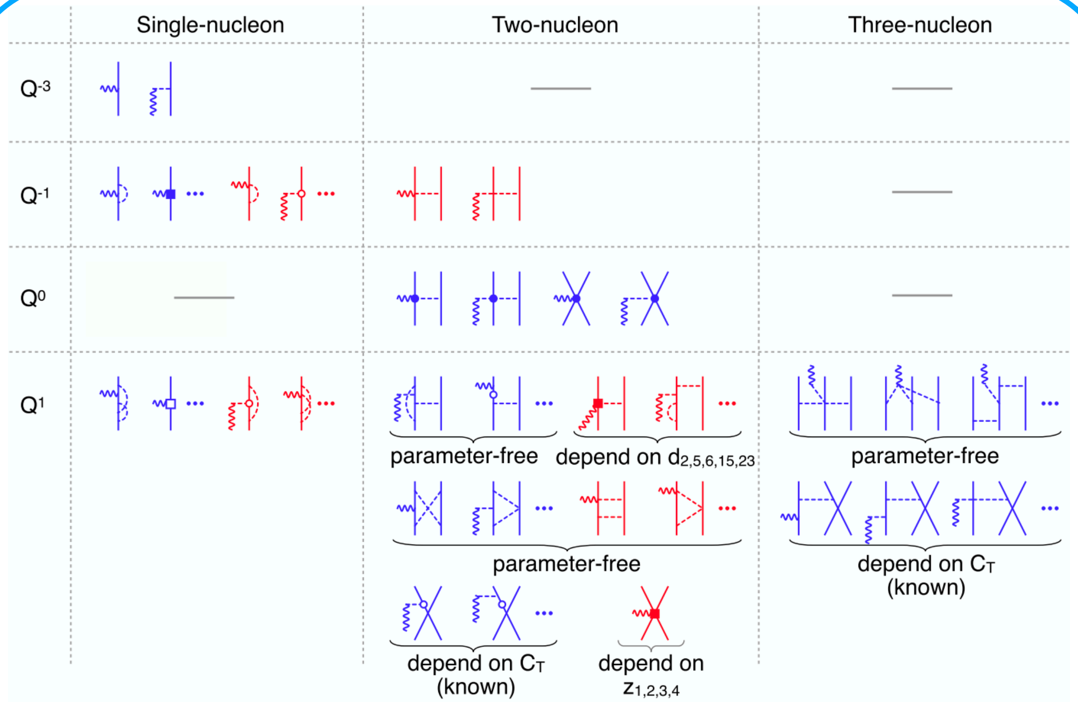
Figure is from E. Epelbaum, H. Krebs, and P. Reinert, Front. Phys. 8, 1 (2020).

Nuclear currents from ChEFT

- Electroweak properties are related to current operators. $\mathcal{H} \propto l_\mu J^\mu_{\rm nucl}$
- Chiral EFT allows us a systematic expansion for charge and current operators.



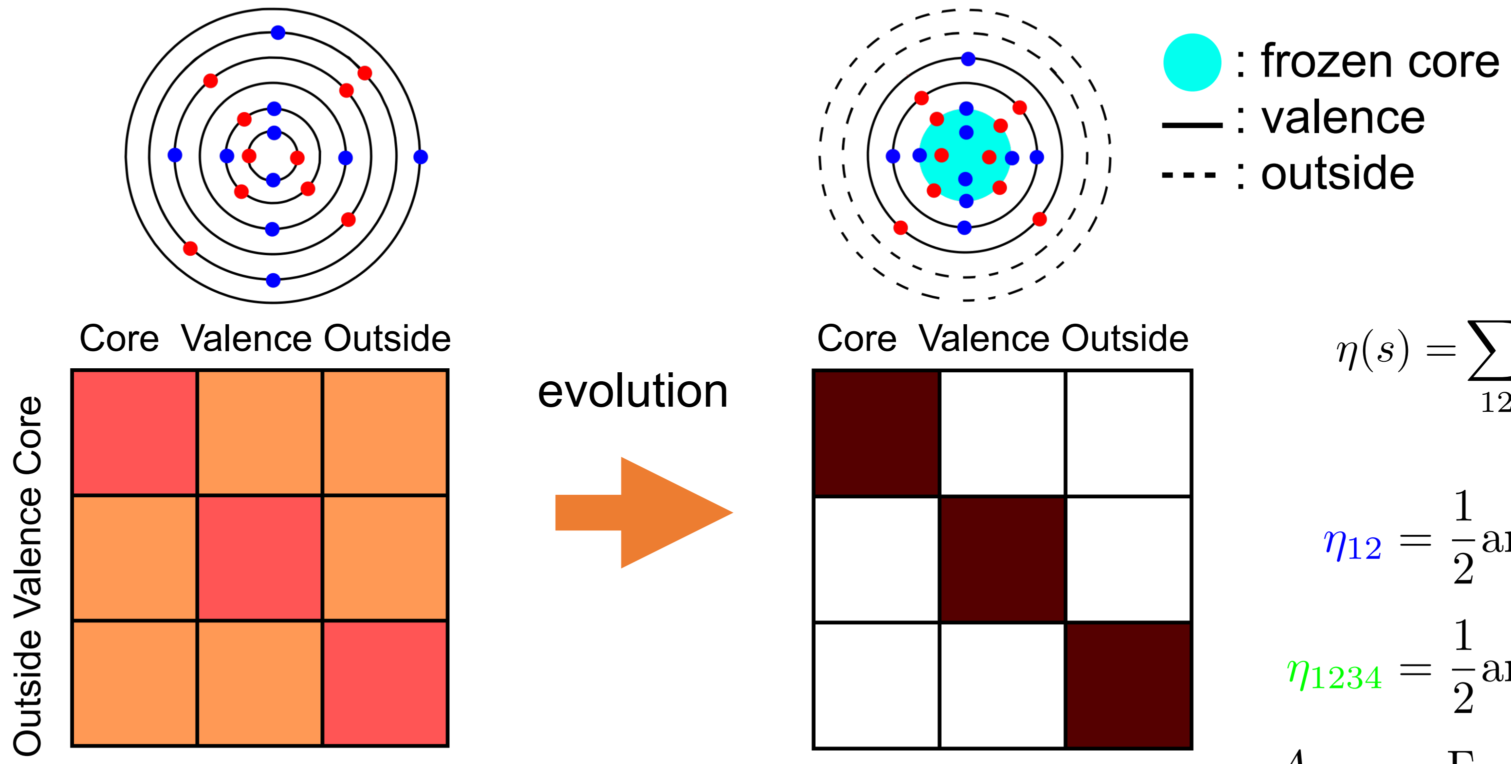
Vector (EM observables, …)



Axial vector (GT transition, …)

E. Epelbaum, arXiv:1908.09349.

Valence-Space In-Medium Similarity Renormalization Group



Similarity transformation

H $\qquad$ $H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$

$$H(s) \approx E(s) + \sum_{12} f_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4}\sum_{1234} \Gamma_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

s: flow parameter

$$\frac{d\Omega}{ds} = \eta(s) - \frac{1}{2}[\Omega(s), \eta(s)] + \cdots$$

$$\eta(s) = \sum_{12} \eta_{12}(s)\{a_1^\dagger a_2\} + \sum_{1234} \eta_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

$$\eta_{12} = \frac{1}{2}\arctan\left(\frac{2f_{12}}{f_{11} - f_{22} + \Gamma_{1212} + \Delta}\right)$$

$$\eta_{1234} = \frac{1}{2}\arctan\left(\frac{2\Gamma_{1234}}{f_{11} + f_{22} - f_{33} - f_{44} + A_{1234} + \Delta}\right)$$

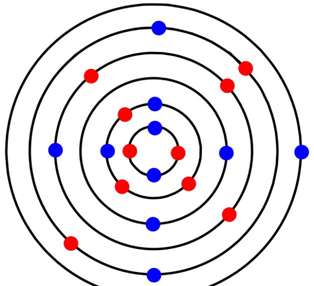
$$A_{1234} = \Gamma_{1212} + \Gamma_{3434} - \Gamma_{1313} - \Gamma_{2424} - \Gamma_{1414} - \Gamma_{2323}$$

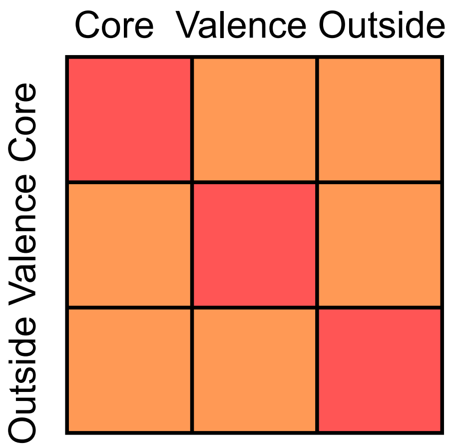
f_{12}, Γ_{1234} : matrix elements to be suppressed

$$\mathcal{O}(s) = e^{\Omega(s)}\mathcal{O}e^{-\Omega(s)} \approx \mathcal{O}^{[0]}(s) + \sum_{12}\mathcal{O}^{[1]}_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4}\sum_{1234}\mathcal{O}^{[2]}_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

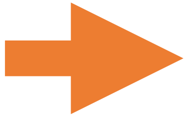
H. Hergert, S. K. Bogner, T. D. Morris, A. Schwenk, and K. Tsukiyama, Phys. Rep. **621**, 165 (2016).
S. R. Stroberg, H. Hergert, S. K. Bogner, and J. D. Holt, Annu. Rev. Nucl. Part. Sci. **69**, 307 (2019).

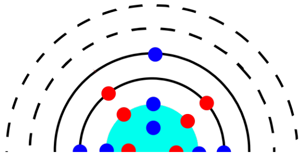
Valence-Space In-Medium Similarity Renormalization Group





evolution

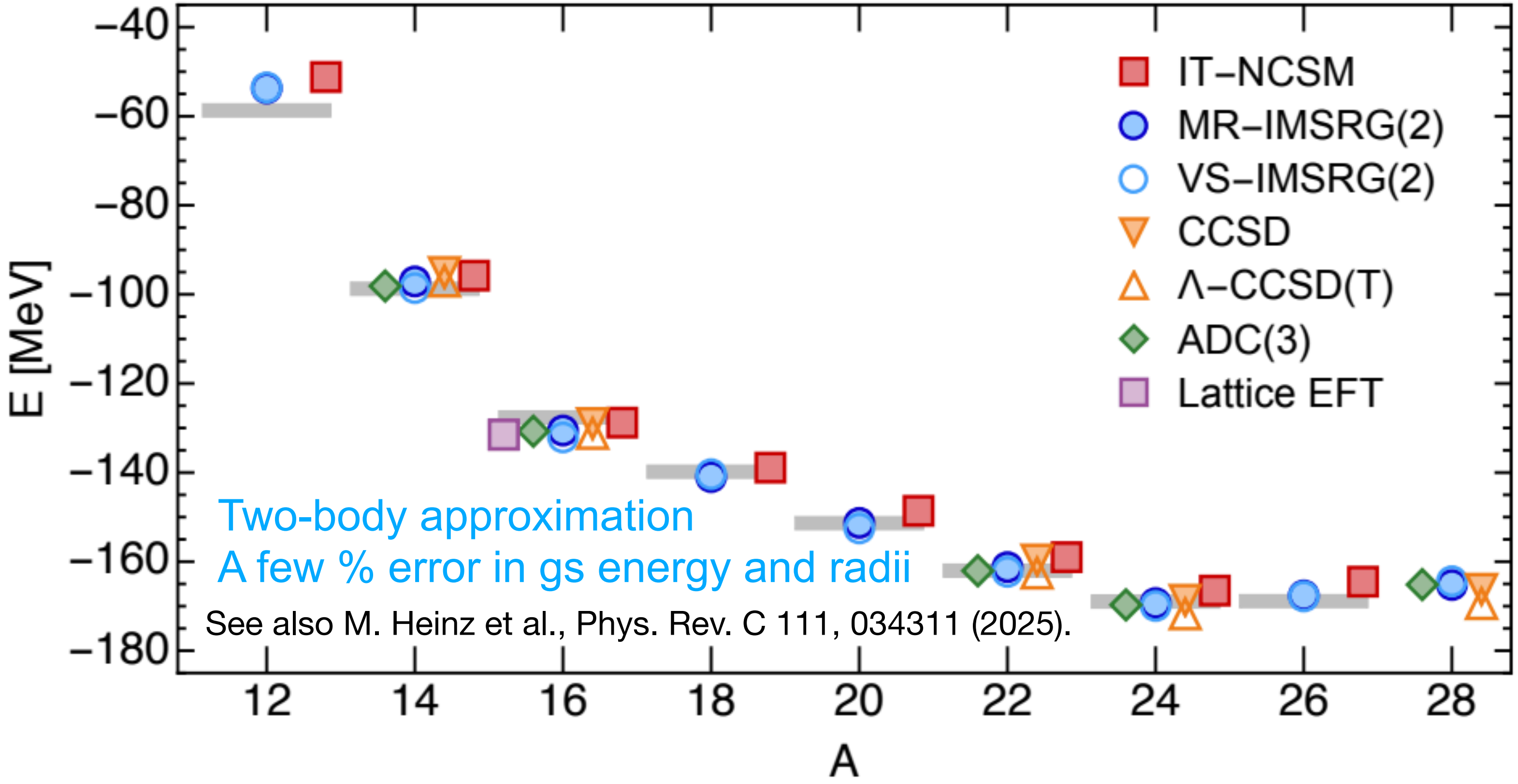




: frozen core

: valence

H. Hergert, Front. Phys. 8, 1 (2020).



Similarity transformat

H

$$H(s) \approx E(s) + \sum_{12} f_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4} \quad {}_{1234}$$

s: flow parameter

H. Hergert, S. K. Bogner, T. D. Morris, A. Schwenk, and K. Tsukiyama, Phys. Rep. **621**, 165 (2016).

S. R. Stroberg, H. Hergert, S. K. Bogner, and J. D. Holt, Annu. Rev. Nucl. Part. Sci. **69**, 307 (2019).

Ca radii puzzle

- Charge radii of Ca isotopes
  - 48 to 52
  - 40 to 48

ARTICLES
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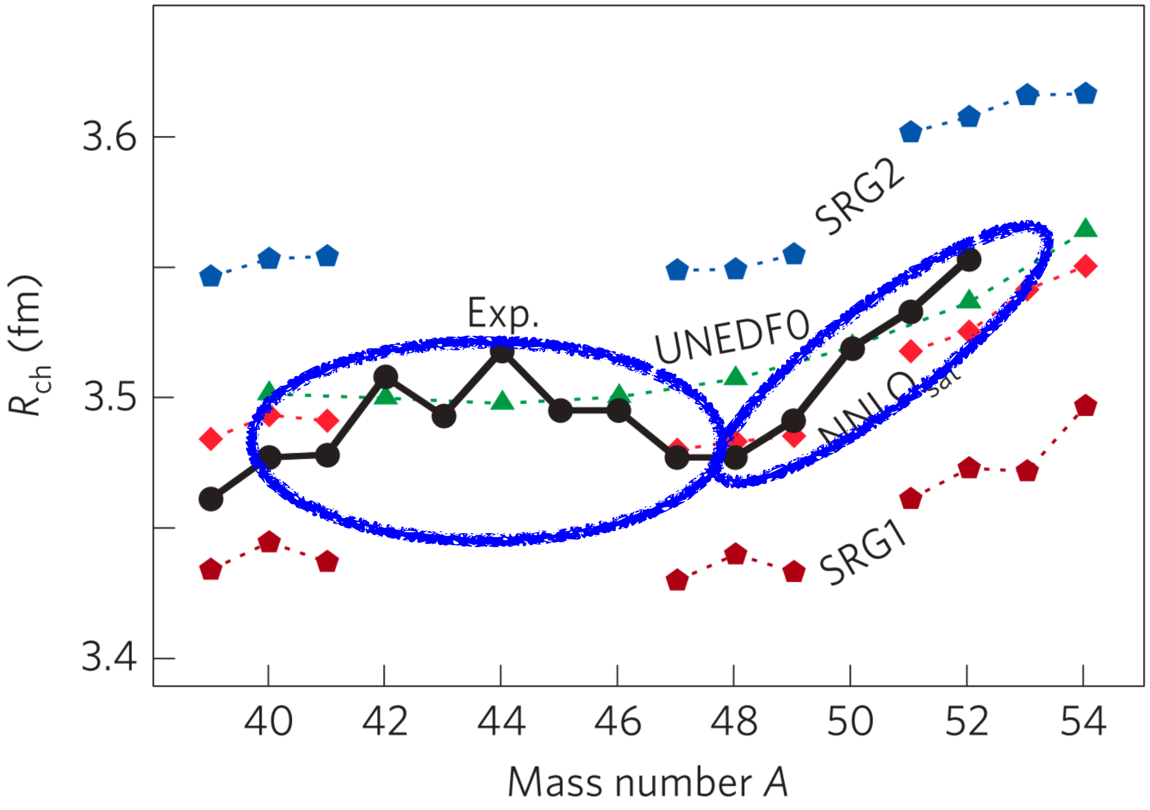
nature physics

Unexpectedly large charge radii of neutron-rich calcium isotopes

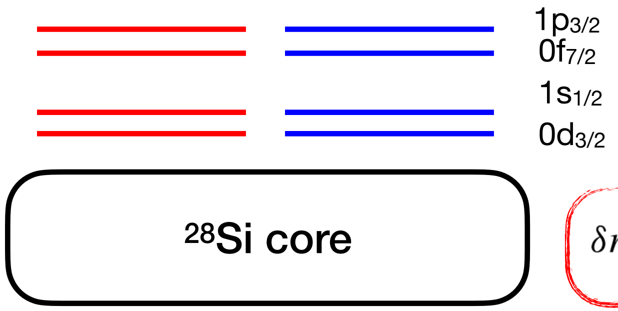
R. F. Garcia Ruiz[1]*, M. L. Bissell[1,2], K. Blaum[3], A. Ekström[4,5], N. Frömmgen[6], G. Hagen[4], M. Hammen[6], K. Hebeler[7,8], J. D. Holt[9], G. R. Jansen[4,5], M. Kowalska[10], K. Kreim[3], W. Nazarewicz[4,11,12], R. Neugart[3,6], G. Neyens[1], W. Nörtershäuser[6,7], T. Papenbrock[4,5], J. Papuga[1], A. Schwenk[3,7,8], J. Simonis[7,8], K. A. Wendt[4,5] and D. T. Yordanov[3,13]



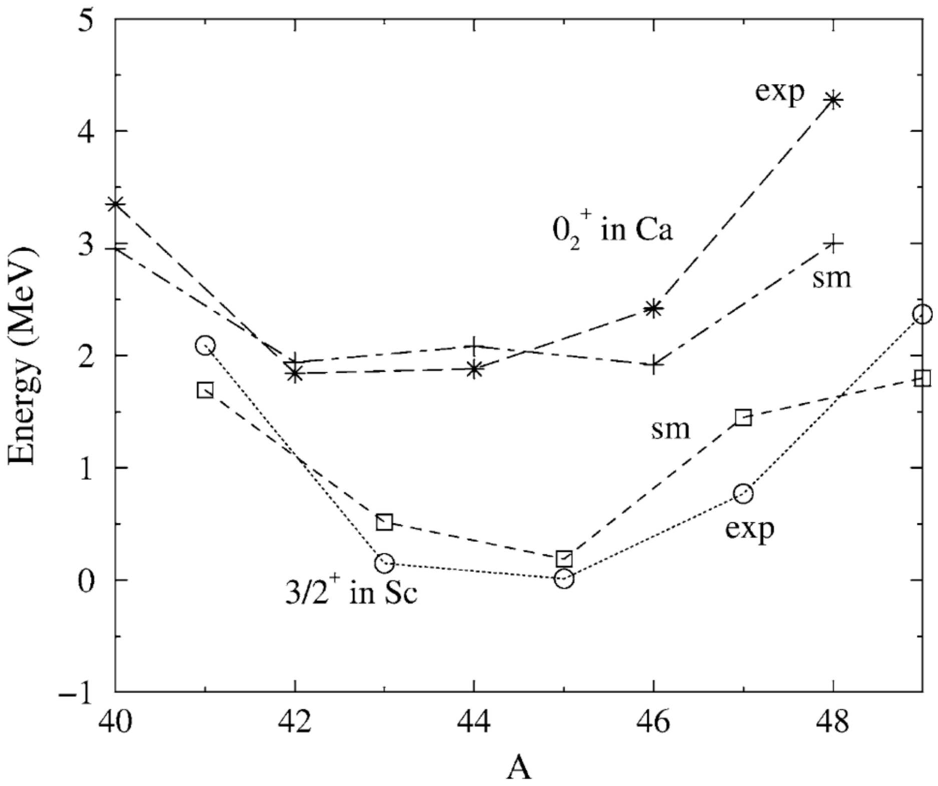
Ca radii puzzle

- Shell-model calculation

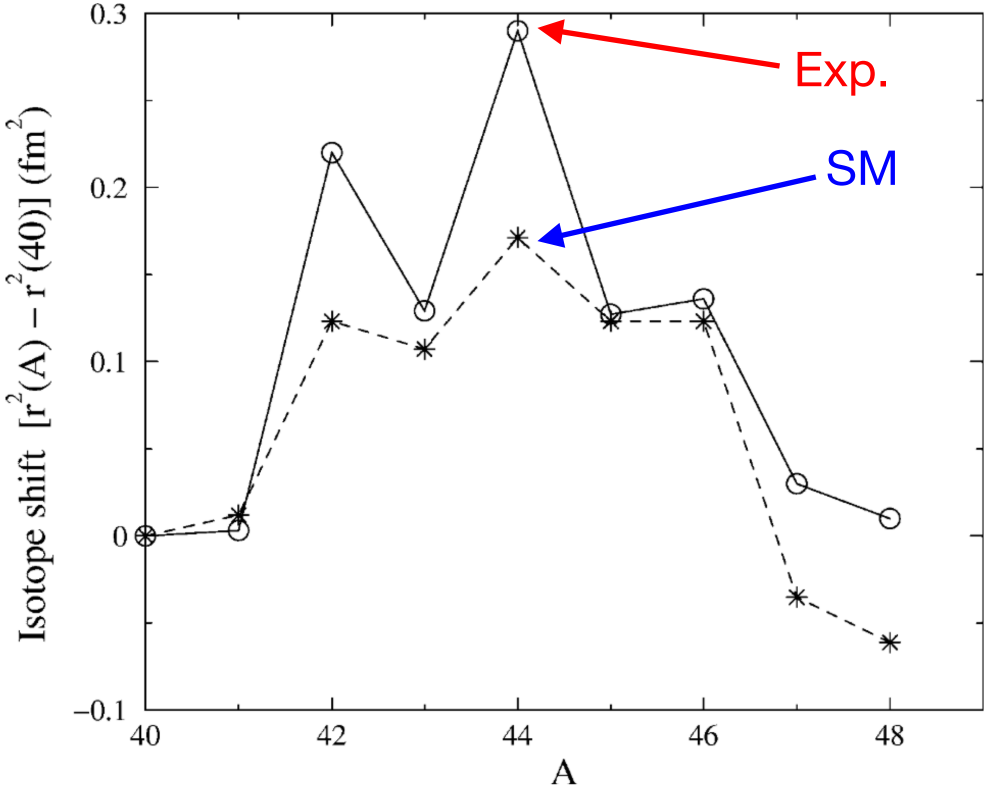
  Explicit inclusion of excitations across the N=Z=20 gap seems essential.



$$\delta r_c^2(A) = \frac{1}{Z} n_{fp}^{\pi}(A) b^2,$$



E. Caurier et al., Phys. Lett. B 522, 240 (2001).



Ca radii puzzle

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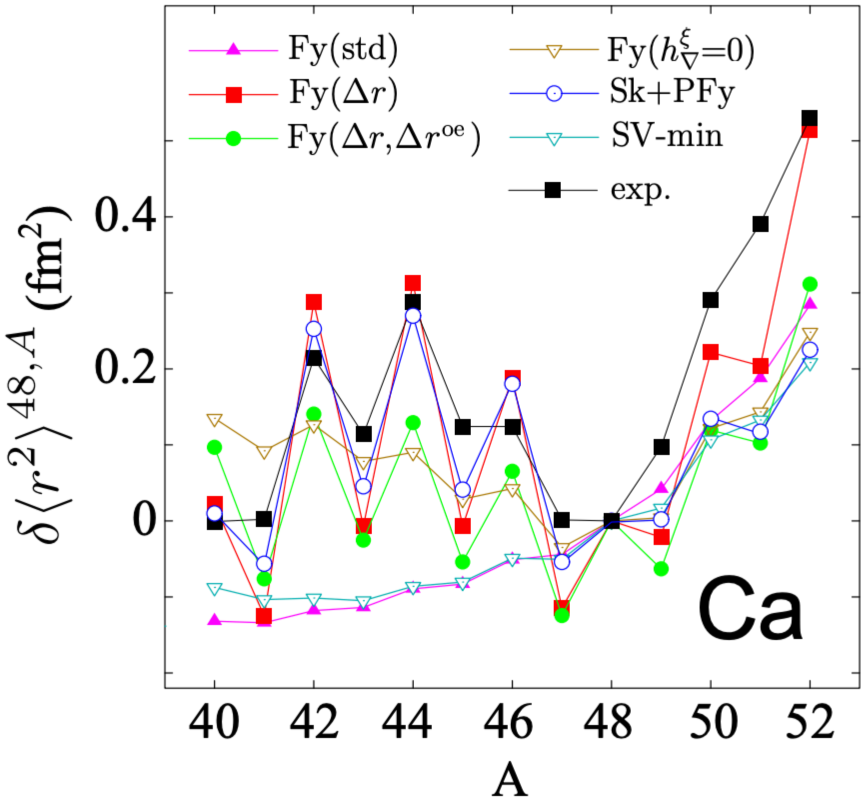
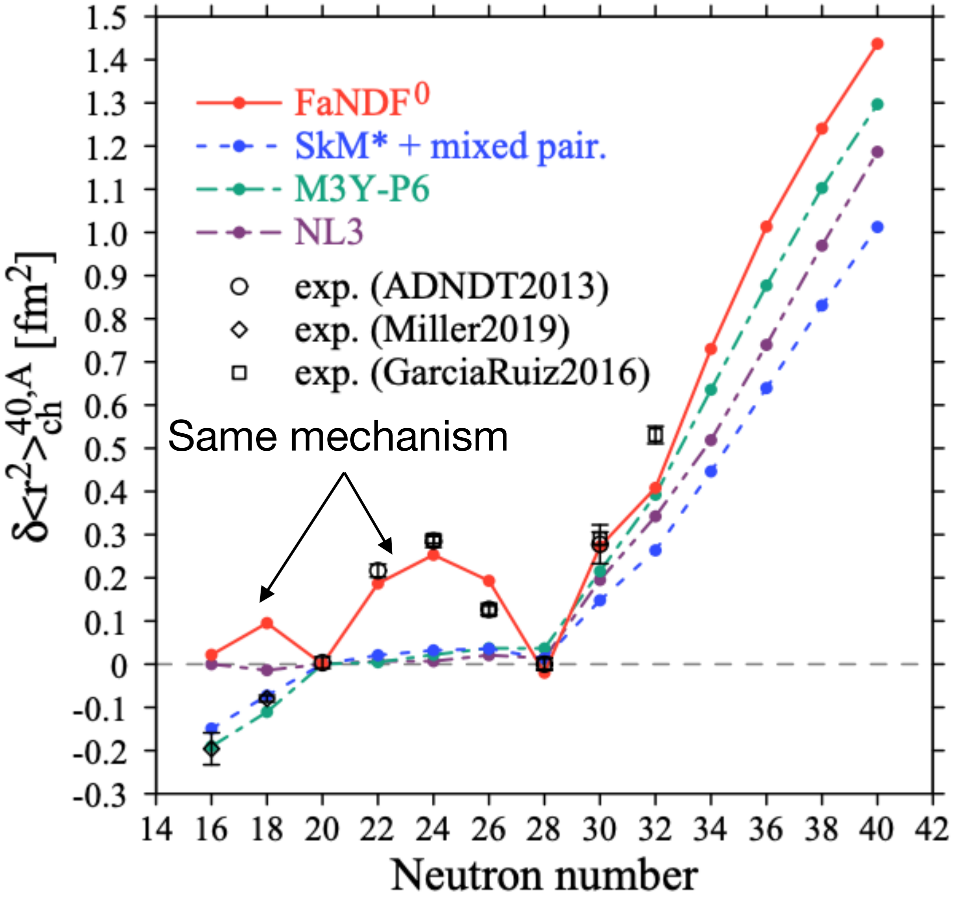
- DFT results

20<=N<=48 and N<20 cannot be explained simultaneoulsly.

Fayans functional





P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C 95, 064328 (2017).

T. Inakura et al., Phys. Rev. C 110, 054315 (2024).

Ca radii with one major-shell valence space

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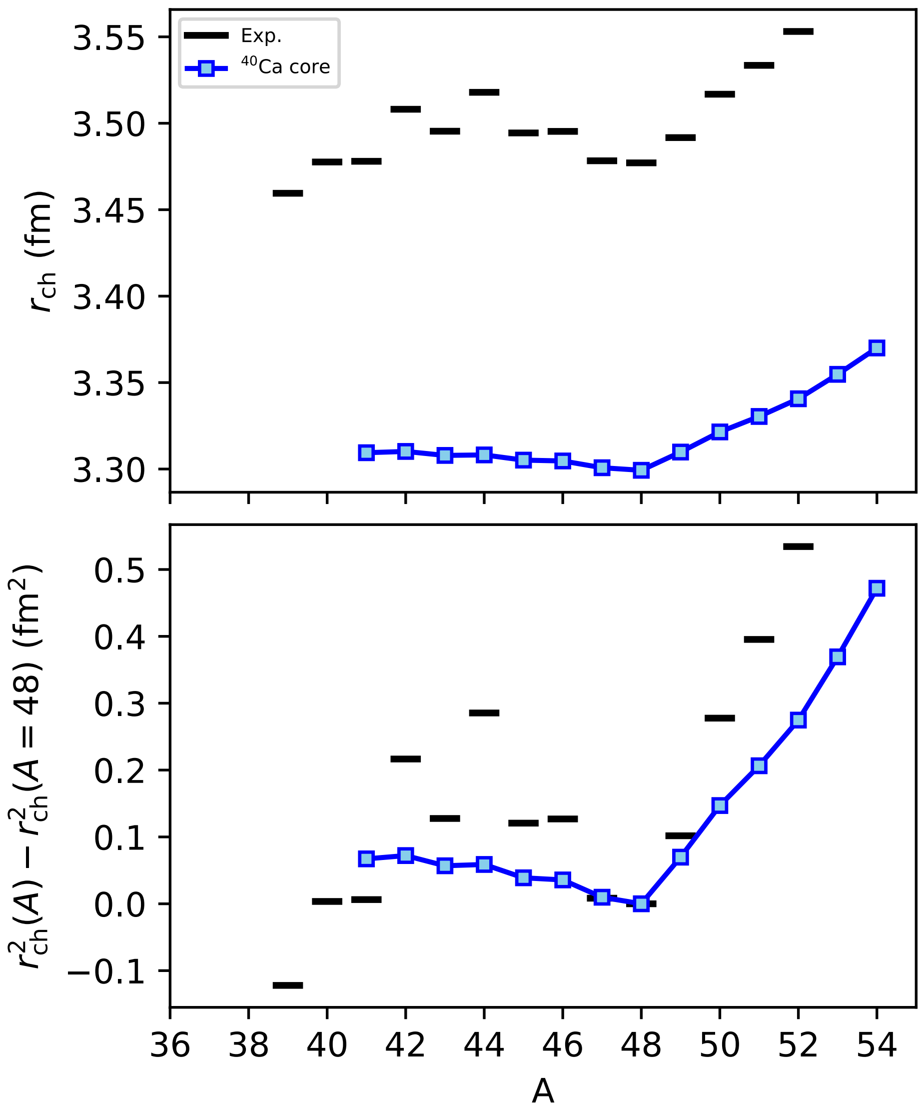
- The computed radii are significantly smaller than the experiments due to the 1.8/2.0 (EM) interaction.
- 40 < A <= 48 behavior is totally off from the data.
- The enhancements in A>48 can be observed, but not enough.
- Explicit excitations across Z=N=20 gap?

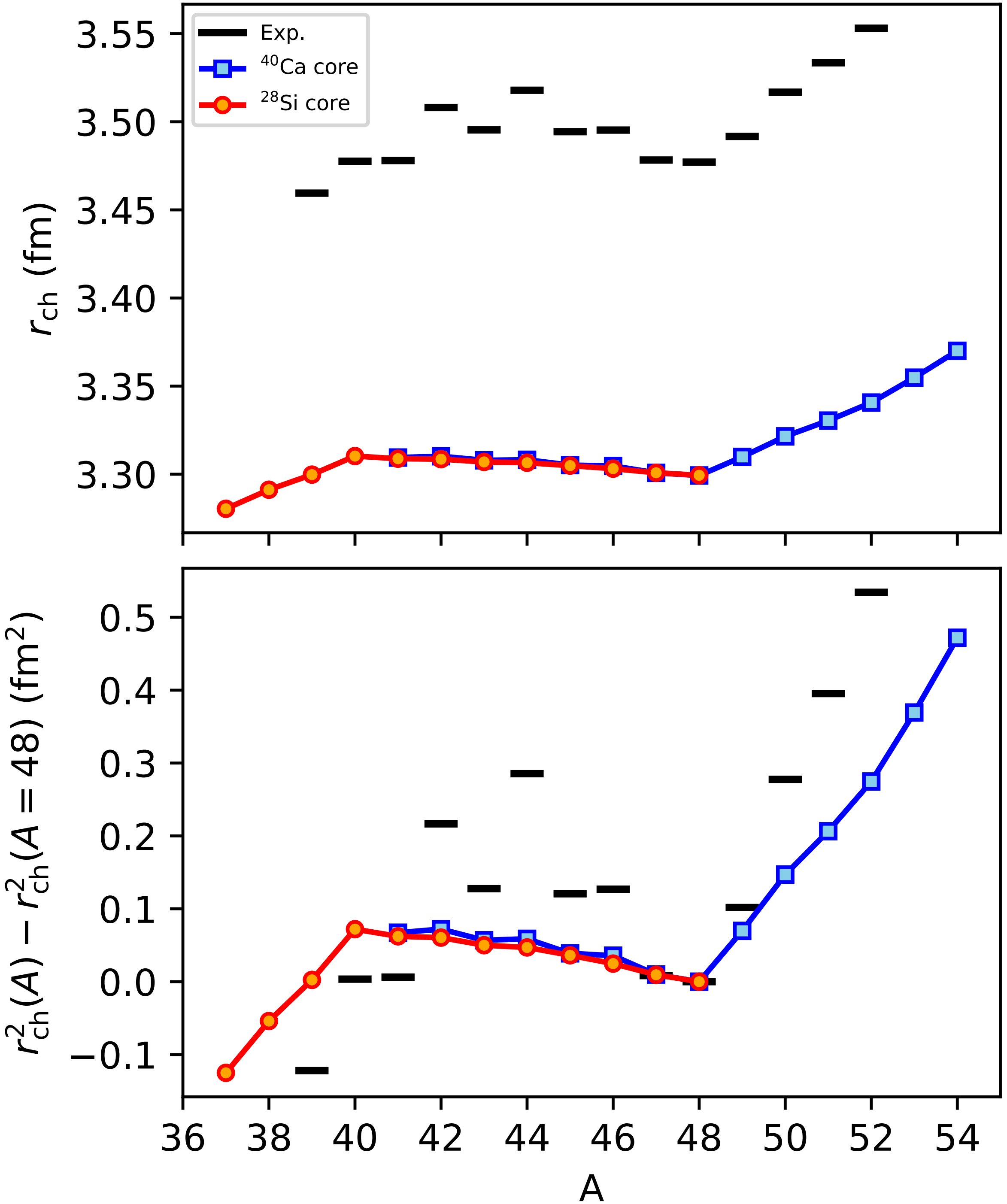
E. Caurier et al., Phys. Lett. B 522, 240 (2001).



Ca radii

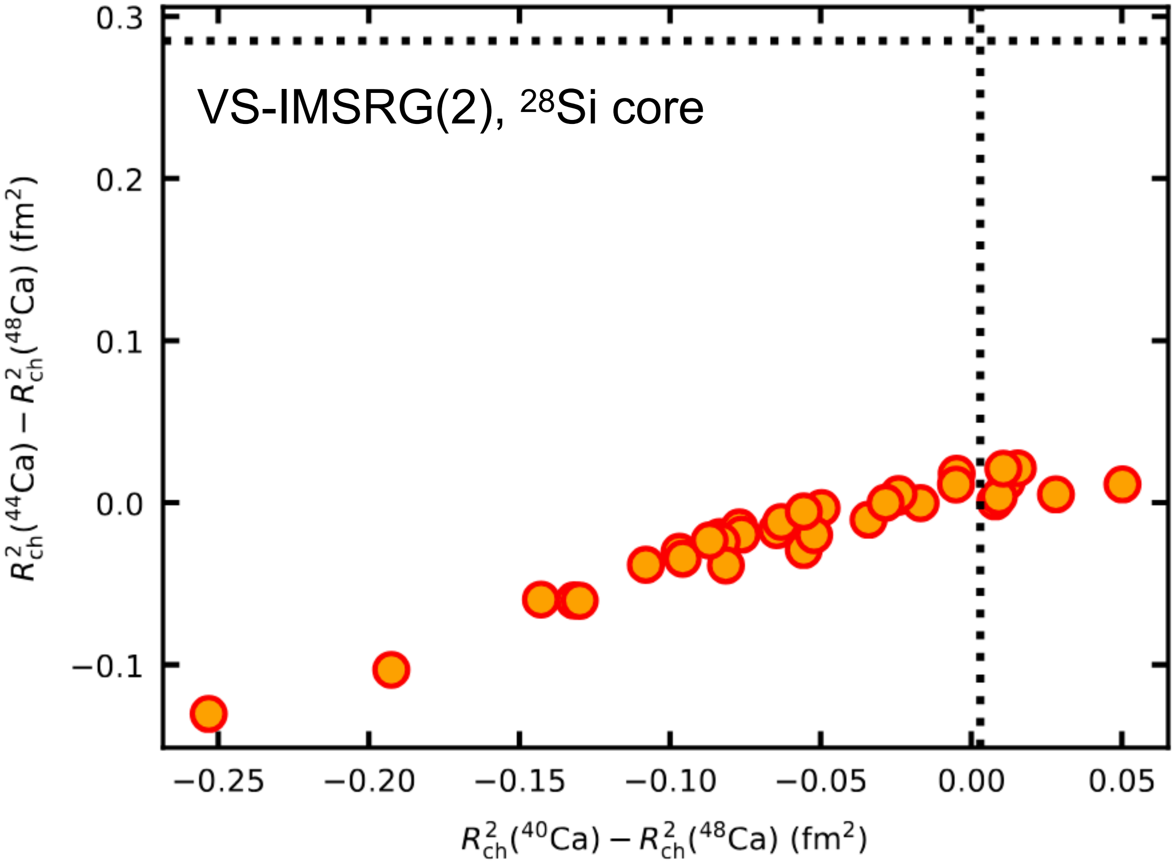
- The ^{40}Ca and ^{28}Si core results are almost the same.

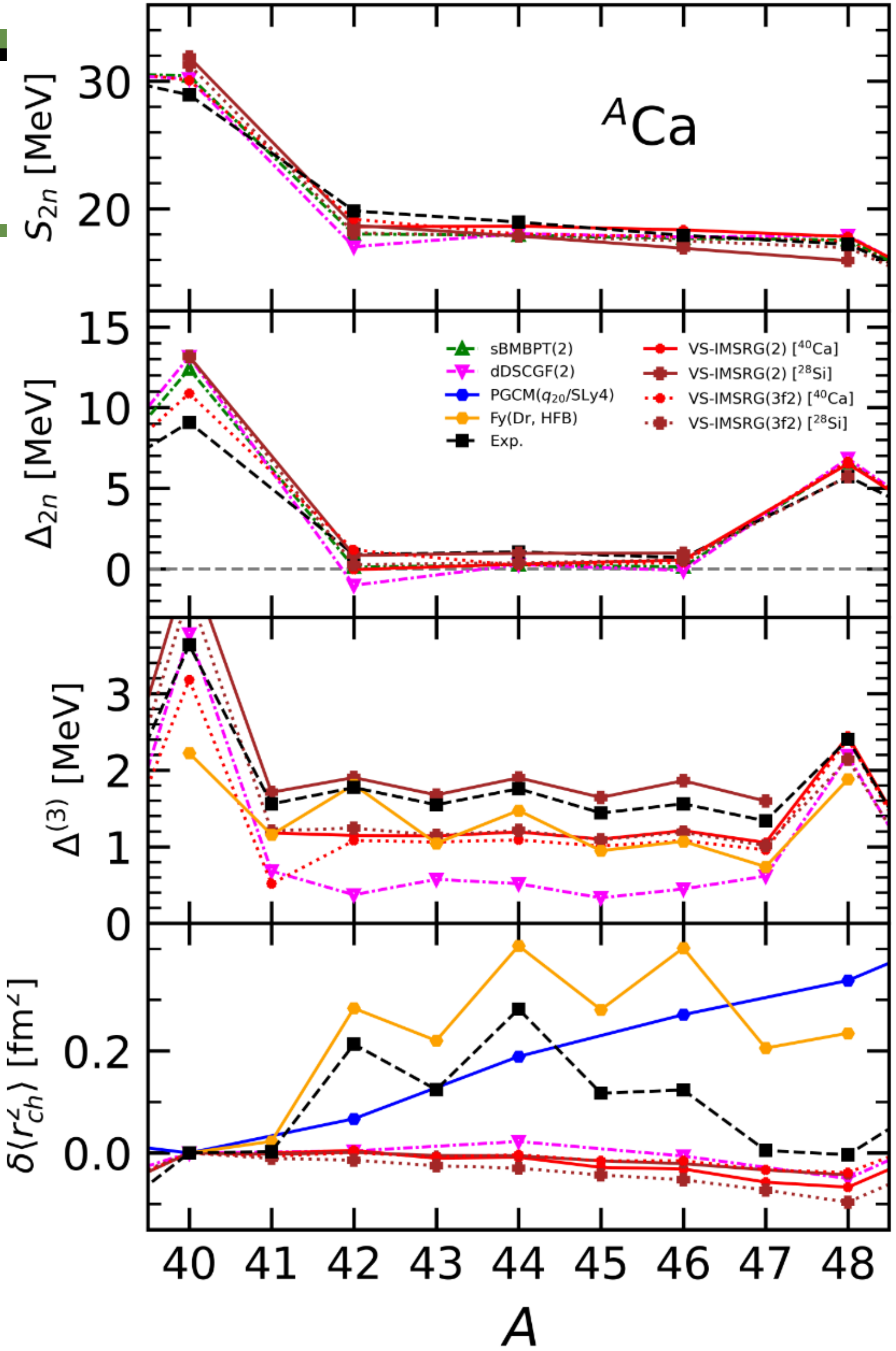
- 0hw components:
  - ^{40}Ca: 0.49
  - ^{42}Ca: 0.50
  - ^{44}Ca: 0.54
  - ^{46}Ca: 0.65
  - ^{48}Ca: 0.81
- Note Sc isomer states are significantly overestimated



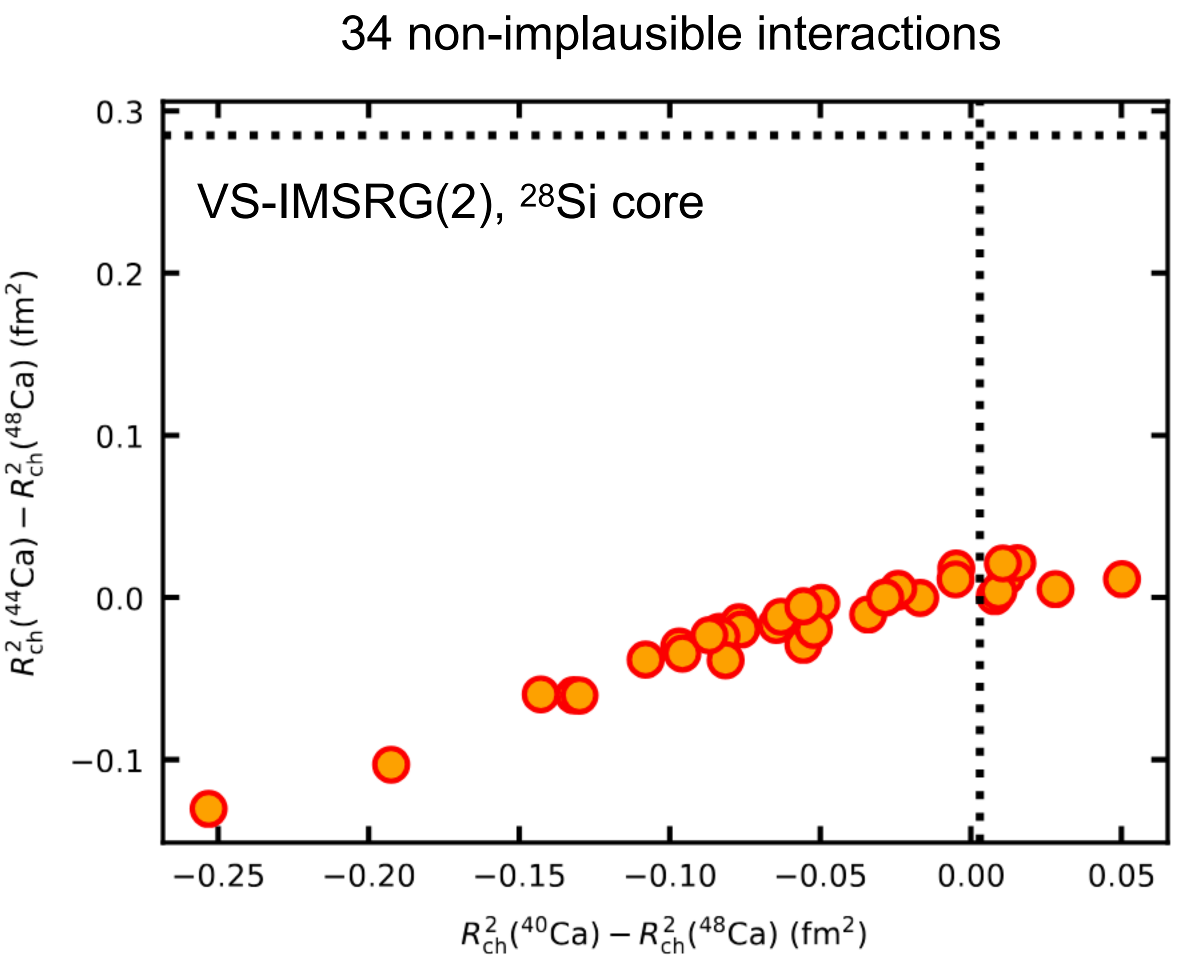
Other observables

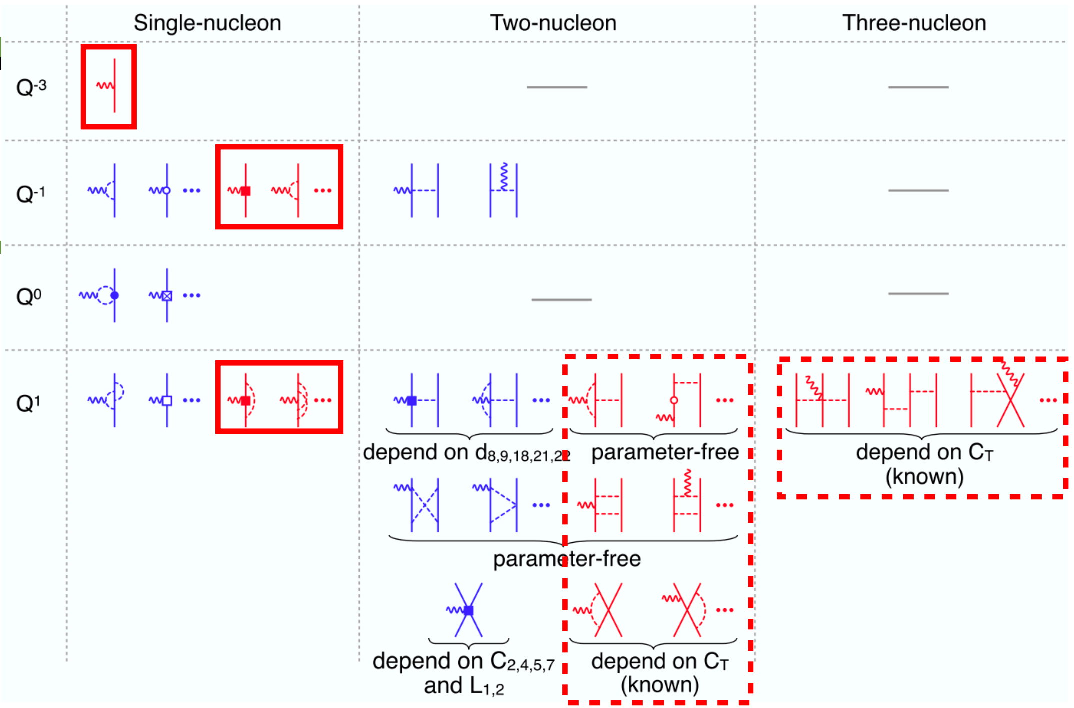
34 non-implausible interactions





Other observables





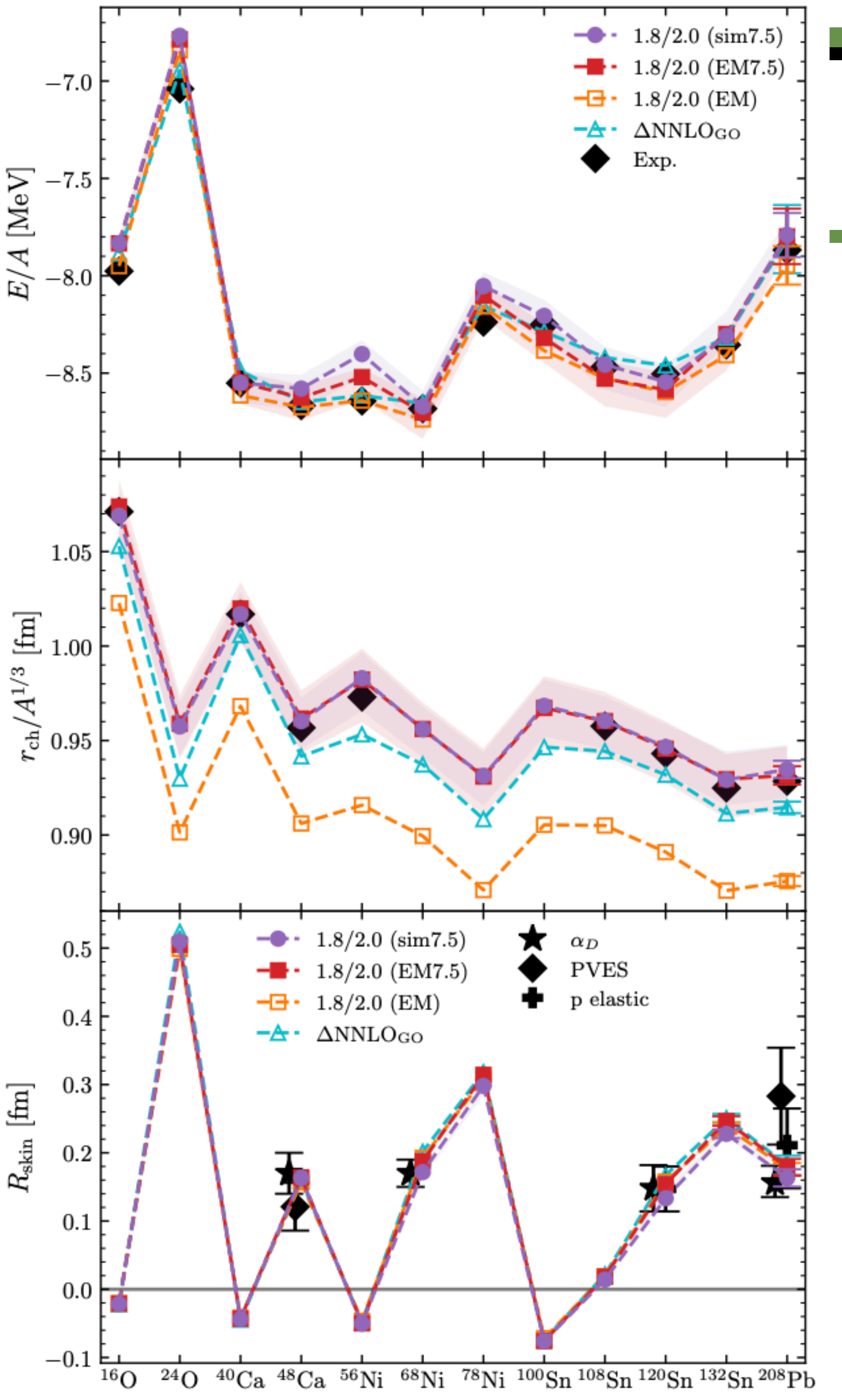
- Many-body effect
  - ✦ Cluster, octupole, …
- many-body charge density
- N3LO Hamiltonian structure

New interactions

Neutron-rich nuclei and neutron skins from chiral low-resolution interactions

P. Arthuis,[1,2,*] K. Hebeler,[1,2,3,†] and A. Schwenk[1,2,3,‡]
[1] *Technische Universität Darmstadt, Department of Physics, 64289 Darmstadt, Germany*
[2] *ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*
[3] *Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*

cD = 7.5



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The neutron dripline in calcium isotopes from a chiral interaction

B. S. Hu,[1,2,*] A. Ekström,[3] C. Forssén,[3] G. Hagen,[2,4] W. G. Jiang,[5] T. Miyagi,[6] and T. Papenbrock[4]

[1] *Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA*
[2] *Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
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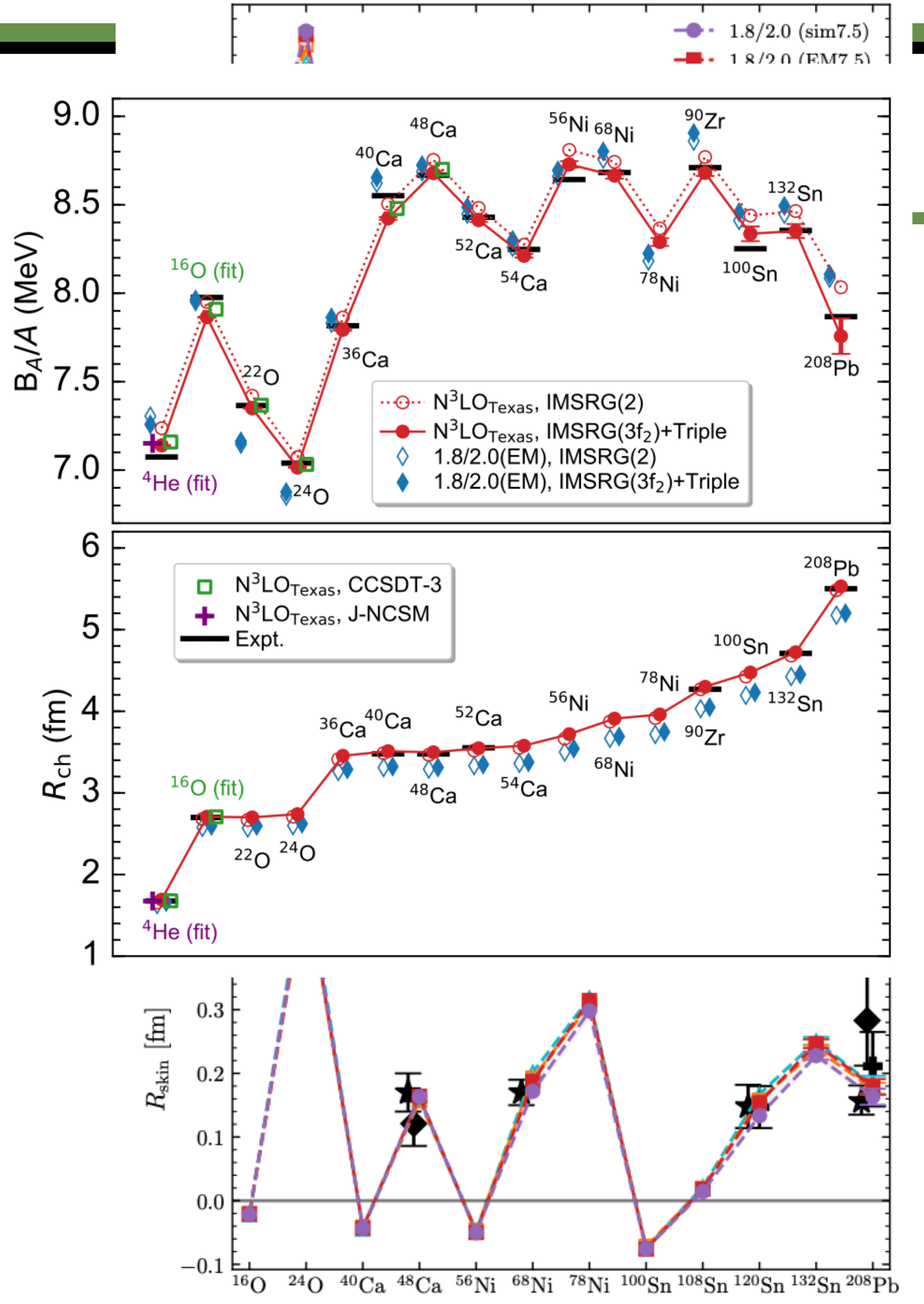
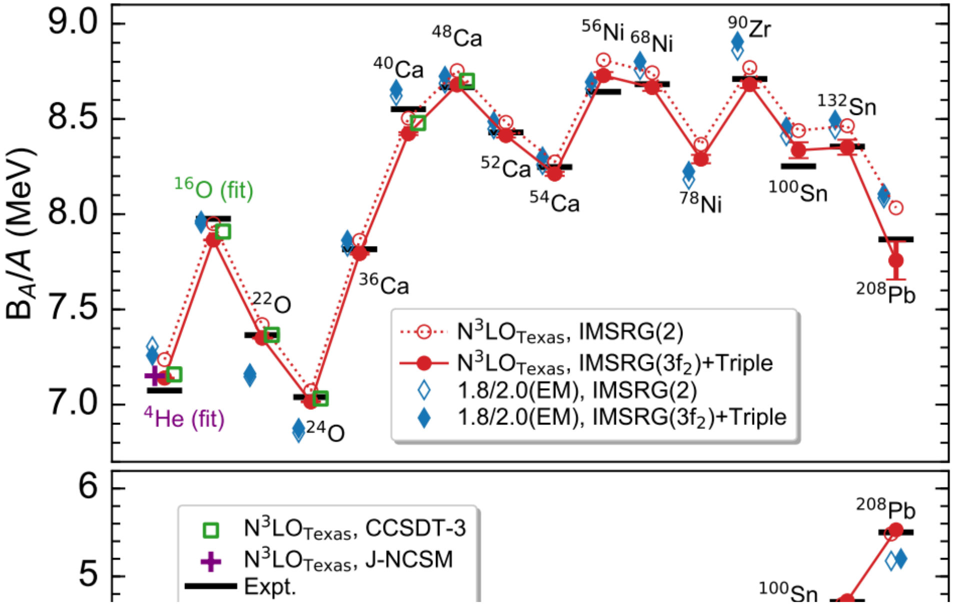
[4] *Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[5] *Mainz Institut für Theoretical Physics and PRISMA+ Cluster of Excellence, Johannes Gutenberg-Universität, Mainz 55128, Germany*
[6] *Center for Computational Sciences, University of Tsukuba, 1-1-1 Tennodai, Tsukuba 305-8577, Japan*

cD = -5.06



New interactions



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Texas A&M University, C...
[2] *Physics Division, Oak Ridge National*
[3] *Department of Physics, Chalmers Unive*
[4] *Department of Physics and Astronomy, Univ*
[5] *Mainz Institut für Theoretical Ph*
Johannes Gutenberg-Uni
[6] *Center for Computational Sciences, University*

cD

* number in parentheses: 2B SRG evolved GT op

Interaction	GT(1B) / GT(Exp)	GT(1+2B) / GT(Exp)
1.8/2.0 (EM)	1.002 (0.989)	0.952 (0.967)
1.8/2.0 (EM7.5)	0.998 (0.984)	1.065 (1.077)
$N3LO_{Texas}$	0.999	0.932
$N3LO_{lnl}$	0.979	1.001

- A large cD is essential to reproduce binding energy and radius simultaneously?
- GT matrix element?

$$\boldsymbol{A}_{2\mathrm{B}} = \boldsymbol{A}_{\pi} - \frac{c_D}{4 f_\pi^2 \Lambda_\chi} \left\{ \left[\boldsymbol{\sigma}_1 - \frac{\boldsymbol{Q}(\boldsymbol{Q}\cdot\boldsymbol{\sigma}_1)}{Q^2 + m_\pi^2} \right] \boldsymbol{\tau}_1 + (1 \leftrightarrow 2) \right\}$$

Beta decay half-lives around N = 50

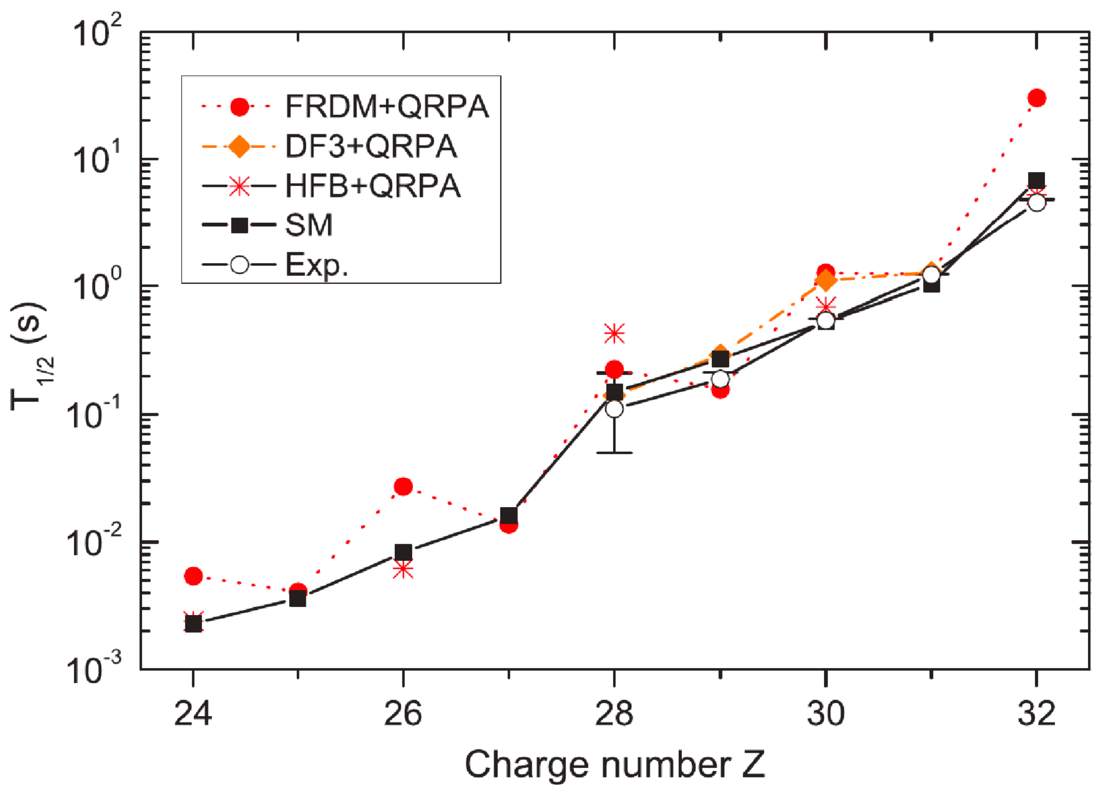
- Beta-decay half-lives are inputs of r-process simulations
  - ✦ Neutron-rich nuclei are experimentally less known
  - ✦ Theoretical inputs are needed

$$T_{1/2}^{-1} = \sum_f t_{fi}^{-1}$$

$$t_{fi}^{-1} = \frac{1}{\kappa}\int_1^{W_0} dW C_{fi}(W) F(Z,W)\sqrt{W^2-1}W(W_0-W)^2$$

- Different theory models yield different results
  - ✦ Uncertainty of each result is unclear…
  - ✦ What about nuclear ab initio calculations?

Zhi et al., Phys. Rev. C 87, 025803, (2013).



Beta decay half-lives around N = 50

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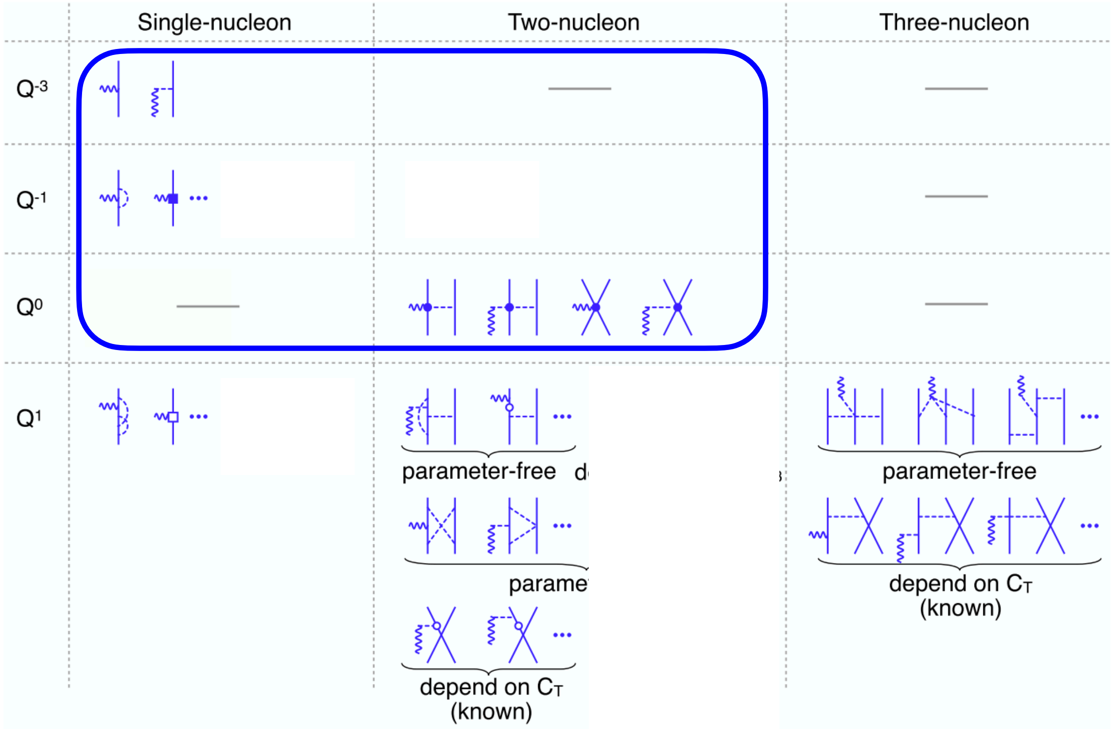
- Partial inverse half-life: $t_{fi}^{-1} = \frac{1}{\kappa} \int_1^{W_0} dW C_{fi}(W) F(Z,W) \sqrt{W^2-1} W (W_0 - W)^2$

  κ (constant): 6144 s
  W: energy in the unit of electron mass
  F(Z, W): Electron wave function, an analytically approximated expression is used
  $C_{fi}(W)$: Nuclear structure part

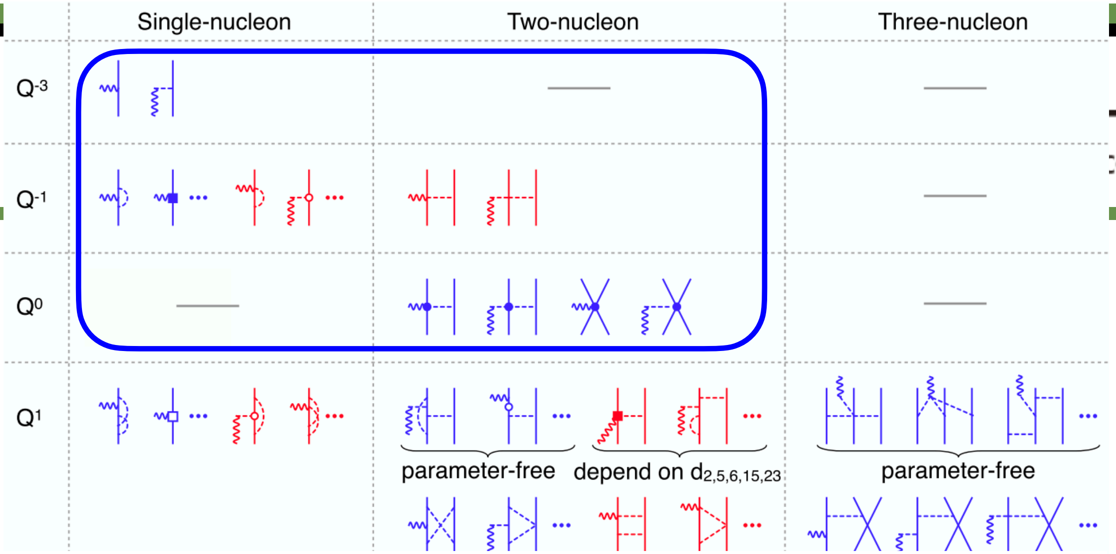
- (W-independent) Gamow-Teller transition:

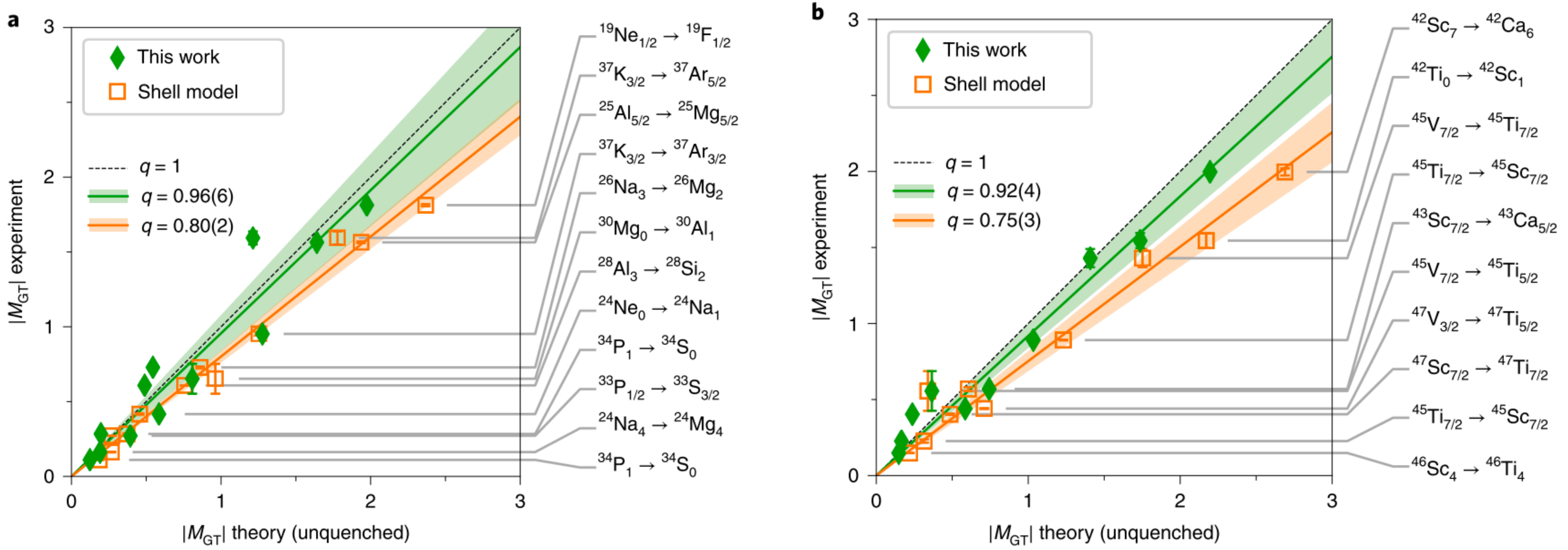
$$C_{fi} \propto |\langle J_f || A_1(\boldsymbol{Q} \to 0) || J_i \rangle|^2$$



Beta decay half-lives around N = 50

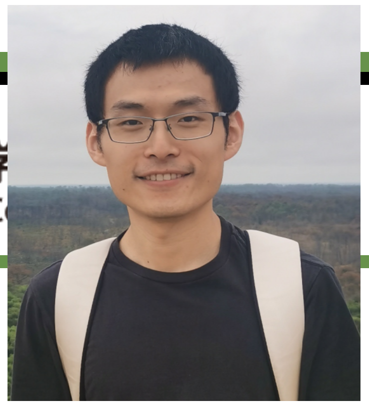
- Gamow-Teller quenching problem: $g_A^{\text{eff}} \sim 0.8 g_A^{\text{free}}$
- Artificial quenching factor is not needed! (q ~ 1)



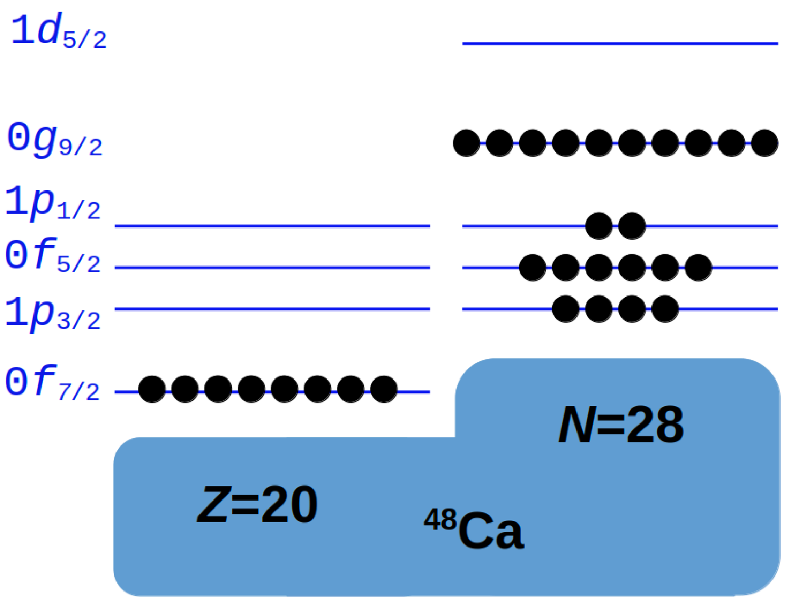


Beta decay half-lives around N = 50

Zhen Li
Postdoc @ TU Darmstadt

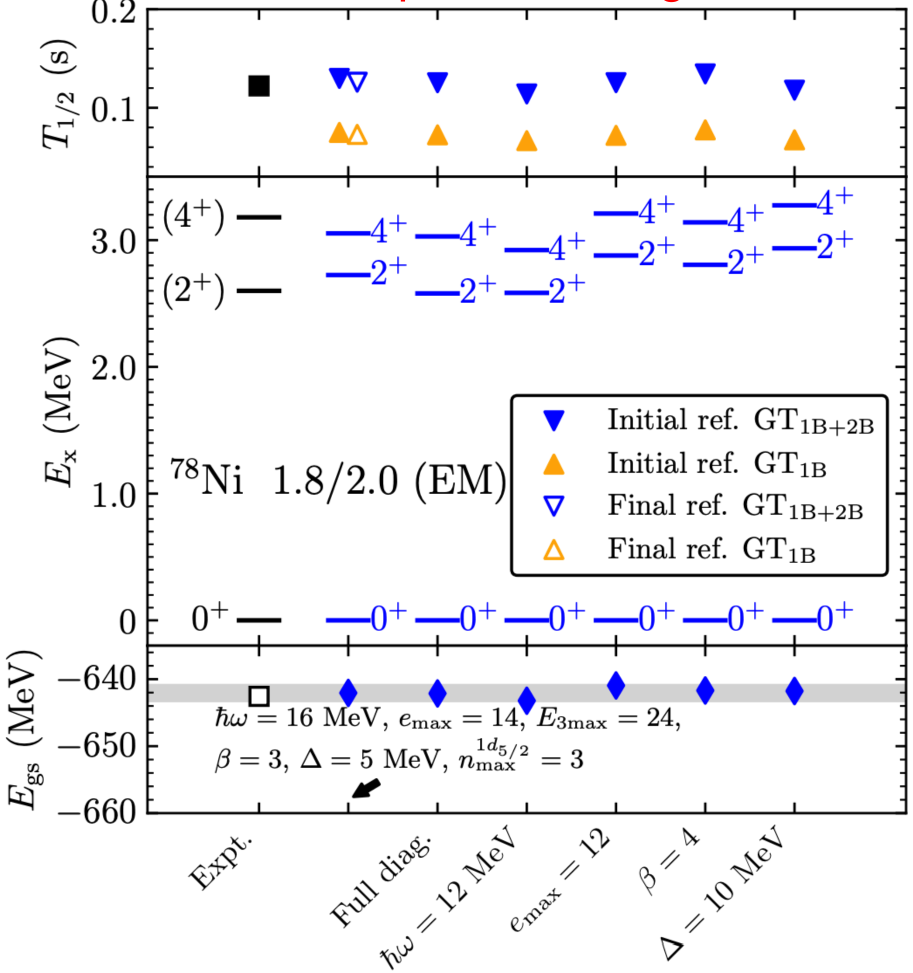


- VS-IMSRG with allowed GT transitions
- Valence-space choice:



- Hamiltonian choices:
  - Delta-less EFT: 1.8/2.0 (EM)

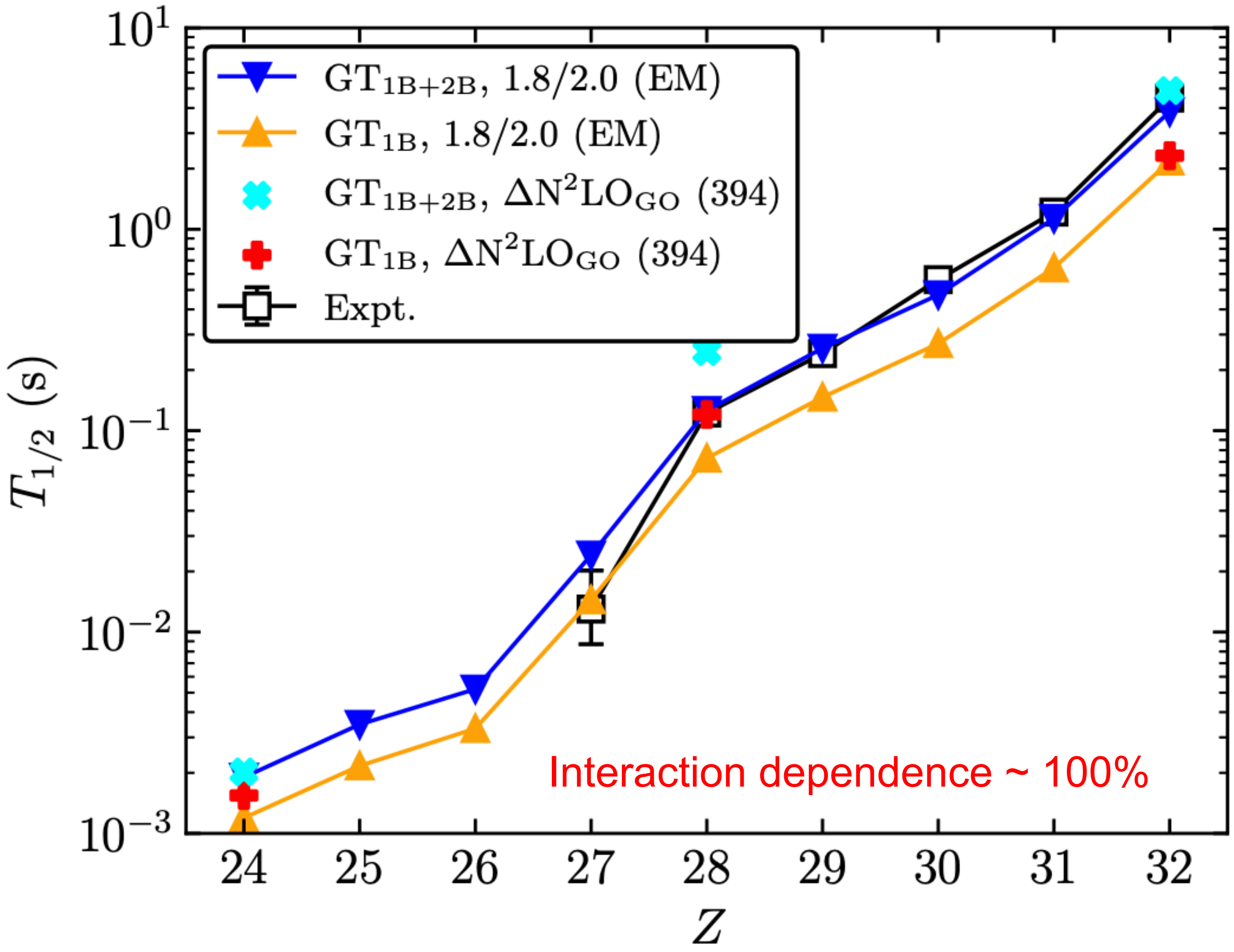
model-space convergence



Beta decay half-lives around N = 50

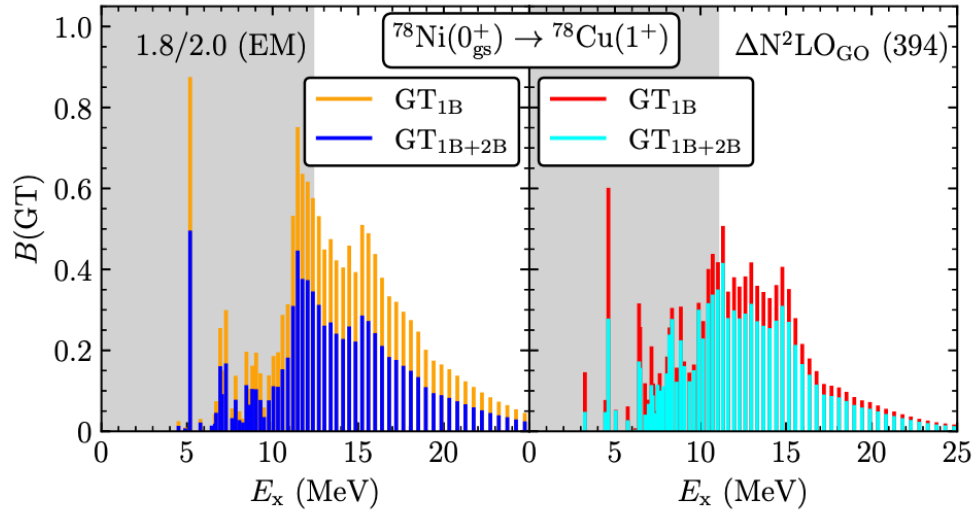
- VS-IMSRG with allowed GT transitions
- The 2BCs make half-lives longer (reduce transition probability)
- Hamiltonian dependence is significant
- Running sum suggests that the agreement of 1.8/2.0 (EM) and ΔN2LOGO in Ge is accidental
- Forbidden decays with 2BC

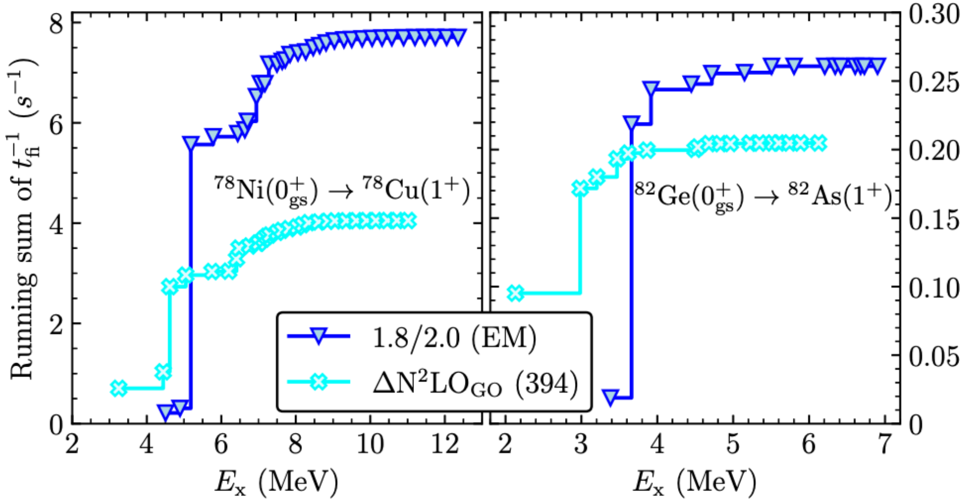
Z Li et al., arXiv:2509.06812.



Beta decay half-lives around N = 50

- In general, 2BC suppresses the GT strength.
- The suppression patterns are similar in 1.8/2.0 (EM) and ΔN2LO$_{GO}$.
- Running sum suggests that the agreement of 1.8/2.0 (EM) and ΔN2LO$_{GO}$ in Ge is accidental
  - ✦ contributions look different
- Interaction dependence needs to be investigated more carefully.
- Many-body uncertainty is unclear…





Z Li et al., arXiv:2509.06812

MEP emulator

- Multiparameter Eigenvalue Problem (MEP) emulator

$$P(\tilde{D}|D,I) = \int d\boldsymbol{\theta} P(\tilde{D}|\boldsymbol{\theta},I) P(\boldsymbol{\theta}|D,I)$$

Model parameters (LECs in ChEFT) — Model prediction — Posterior parameter dist.

- Posterior parameter distribution
  - One can use Bayes' theorem: $P(\boldsymbol{\theta}|D,I) \propto P(D|\boldsymbol{\theta},I) P(\boldsymbol{\theta})$

    Likelihood — Prior parameter dist.

- An emulator is a key ingredient to realizing meaningful UQ
- But still, sampling could be a non-trivial task as the distribution can be complicated.

Emulator: eigenvector continuation

- Affine structure of Hamiltonian: $H(\boldsymbol{\theta}) = H_0 + \sum_{i=1}^{n_{\mathrm{LEC}}} \theta_i H_i$
- Schrödinger equation for target vector: $H(\boldsymbol{\theta}_\odot)|\Psi(\boldsymbol{\theta}_\odot)\rangle = E(\boldsymbol{\theta}_\odot)|\Psi(\boldsymbol{\theta}_\odot)\rangle$
- If the eigenvector does not change drastically with respect to θ, the target wave function could be expanded with the snapshot vectors:

$$|\Psi(\boldsymbol{\theta}_\odot)\rangle = \sum_{i=1}^{n_{\mathrm{train}}} c_i |\Psi(\boldsymbol{\theta}_i)\rangle$$

- Subspace Schrödinger equation: $\left[H_0^{\mathrm{sub}} + \theta_{\odot,1} H_1^{\mathrm{sub}} + \cdots + \theta_{\odot,n_{\mathrm{LEC}}} H_{n_{\mathrm{LEC}}}^{\mathrm{sub}}\right] \boldsymbol{c} = E N^{\mathrm{sub}} \boldsymbol{c}$

$$H_i^{\mathrm{sub}} = \begin{pmatrix} \langle\Psi(\boldsymbol{\theta}_1)|H_i|\Psi(\boldsymbol{\theta}_1)\rangle & \cdots & \langle\Psi(\boldsymbol{\theta}_1)|H_i|\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})\rangle \\ \vdots & \ddots & \vdots \\ \langle\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})|H_i|\Psi(\boldsymbol{\theta}_1)\rangle & \cdots & \langle\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})|H_i|\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})\rangle \end{pmatrix} \qquad N^{\mathrm{sub}} = \begin{pmatrix} \langle\Psi(\boldsymbol{\theta}_1)|\Psi(\boldsymbol{\theta}_1)\rangle & \cdots & \langle\Psi(\boldsymbol{\theta}_1)|\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})\rangle \\ \vdots & \ddots & \vdots \\ \langle\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})|\Psi(\boldsymbol{\theta}_1)\rangle & \cdots & \langle\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})|\Psi(\boldsymbol{\theta}_{n_{\mathrm{train}}})\rangle \end{pmatrix}$$

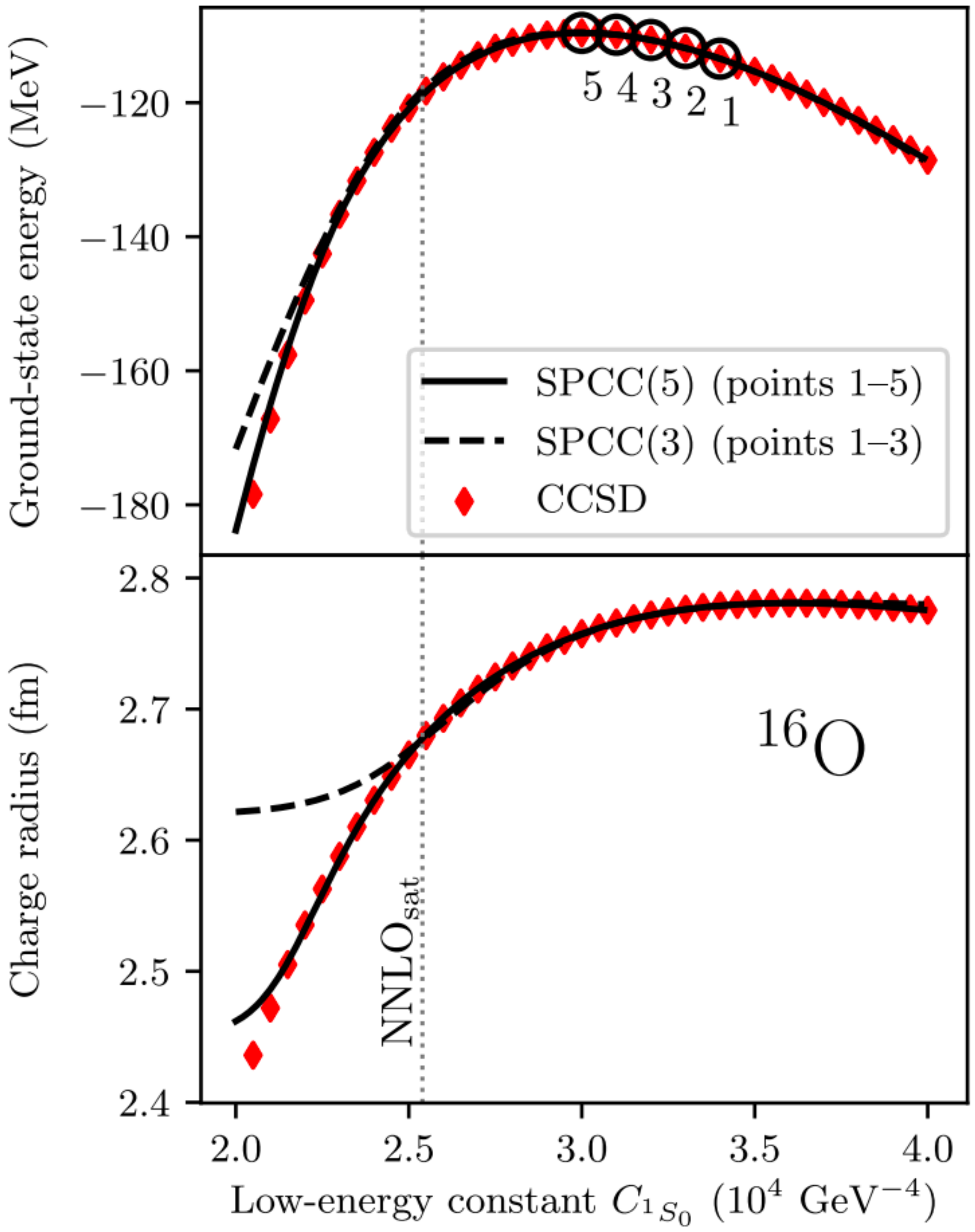
Emulator: eigenvector continuation

- Coupled-cluster method

$$\langle\tilde{\Psi}'|H(\vec{\alpha}_\odot)|\Psi\rangle = \langle\Phi_0|(1+\Lambda')e^{-T'}H(\vec{\alpha}_\odot)e^{T}|\Phi_0\rangle$$
$$= \langle\Phi_0|(1+\Lambda')e^{X}\bar{H}(\vec{\alpha}_\odot)|\Phi_0\rangle,$$

$$\langle\tilde{\Psi}'|\Psi\rangle = \langle\Phi_0|(1+\Lambda')e^{X}|\Phi_0\rangle,$$

- T' and T include only ph excitation operators
  - ✦ Expansion terminates at finite order.



A. Ekström and G. Hagen, Phys. Rev. Lett. 123, 252501 (2019).

Parametric matrix model

- EC works well.
  - ✦ But not so easy for IMSRG

- Parametric matrix model (PMM)

P. Cook et al., Nature Communications 16, 5929 (2025).

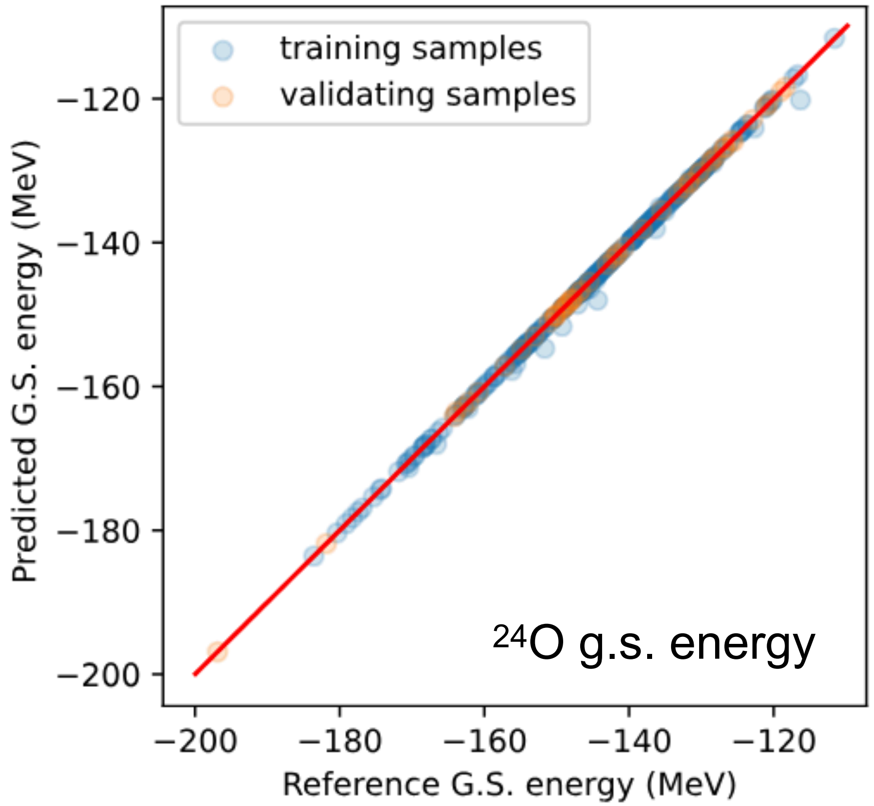
$$H|\Psi\rangle = E|\Psi\rangle$$

$$H = H_0 + \tilde{C}_{^1S_{0,nn}} H_{^1S_{0,nn}} + \tilde{C}_{^1S_{0,np}} H_{^1S_{0,np}} + \tilde{C}_{^1S_{0,pp}} H_{^1S_{0,pp}} + \cdots$$

Numerically optimized such that the training data are reproduced

*The idea is similar to EC, but the matrices are obtained with a data-driven way

Original IMSRG problem → O(10) matrix diagonalization



MEP emulator

- A set of eigenvalue problems:

$$\left(H_0^{[1]} + \sum_{i=1}^{n_{\mathrm{par}}} c_i H_i^{[1]} - E^{[1]} N^{[1]}\right) \boldsymbol{y}^{[1]} = 0$$
$$\left(H_0^{[2]} + \sum_{i=1}^{n_{\mathrm{par}}} c_i H_i^{[2]} - E^{[2]} N^{[2]}\right) \boldsymbol{y}^{[2]} = 0$$
$$\vdots$$
$$\left(H_0^{[n_{\mathrm{par}}]} + \sum_{i=1}^{n_{\mathrm{par}}} c_i H_i^{[n_{\mathrm{par}}]} - E^{[n_{\mathrm{par}}]} N^{[n_{\mathrm{par}}]}\right) \boldsymbol{y}^{[n_{\mathrm{par}}]} = 0$$
$$\vdots$$
$$\left(H_0^{[m]} + \sum_{i=1}^{n_{\mathrm{par}}} c_i H_i^{[m]} - E^{[m]} N^{[m]}\right) \boldsymbol{y}^{[m]} = 0$$

- Multiparameter eigenvalue problem:

$$\left(O_j + \sum_{i=1}^{m} \alpha_i A_{ij}\right) \boldsymbol{y}_j = 0$$

Inputs: matrices O and A

Outputs: alpha and vector y

- $\{c_{i=1,\ldots,n}\} \rightarrow \{E_{j=1,\ldots,m}\}$ (forward problem)

$$O_j = H_0^{[j]} + \sum_{i=1}^{n_{\mathrm{par}}} c_i H_i^{[j]}, \quad A_{ij} = \delta_{ij} N^{[j]}, \quad \alpha_i = E^{[i]}$$

MEP emulator

- A set of eigenvalue problems:

$$\left(H_0^{[1]} + \sum_{i=1}^{n_{\text{par}}} c_i H_i^{[1]} - E^{[1]} N^{[1]}\right) \boldsymbol{y}^{[1]} = 0$$

$$\left(H_0^{[2]} + \sum_{i=1}^{n_{\text{par}}} c_i H_i^{[2]} - E^{[2]} N^{[2]}\right) \boldsymbol{y}^{[2]} = 0$$

$$\vdots$$

$$\left(H_0^{[n_{\text{par}}]} + \sum_{i=1}^{n_{\text{par}}} c_i H_i^{[n_{\text{par}}]} - E^{[n_{\text{par}}]} N^{[n_{\text{par}}]}\right) \boldsymbol{y}^{[n_{\text{par}}]} = 0$$

$$\vdots$$

$$\left(H_0^{[m]} + \sum_{i=1}^{n_{\text{par}}} c_i H_i^{[m]} - E^{[m]} N^{[m]}\right) \boldsymbol{y}^{[m]} = 0$$

- Multiparameter eigenvalue problem:

$$\left(O_j + \sum_{i=1}^{m} \alpha_i A_{ij}\right) \boldsymbol{y}_j = 0$$

Inputs: matrices O and A

Outputs: alpha and vector y

- $\{E_{i=1,\ldots,n}\} \rightarrow \{c_{j=1,\ldots,n}, E_{j=n+1,\ldots,m}\}$ (inverse problem)

$$O_j = \begin{cases} H_0^{[j]} - E^{[j]} N^{[j]} & 1 \le j \le n_{\text{par}} \\ H_0^{[j]} & n_{\text{par}} < j \le m \end{cases},$$

$$A_{ij} = \begin{cases} H_i^{[j]} & 1 \le j \le n_{\text{par}} \\ -\delta_{ij} N^{[i]} & n_{\text{par}} < j \le m \end{cases},$$

$$\alpha_i = \begin{cases} c_i & 1 \le i \le n_{\text{par}} \\ E^{[i]} & n_{\text{par}} < i \le m \end{cases}.$$

Some details about the MEP emulator

- MEP: $\left(O_j + \sum_{i=1}^{m} \alpha_i A_{ij}\right) \boldsymbol{y}_j = 0$

$$K = \begin{vmatrix} G_{11} & \cdots & G_{m1} \\ \vdots & \ddots & \vdots \\ G_{1m} & \cdots & G_{mm} \end{vmatrix}_{\otimes}$$

$$= \sum_{\sigma \in S_m} \operatorname{sgn}(\sigma) G_{\sigma(1)1} \otimes G_{\sigma(2)2} \ldots \otimes G_{\sigma(m)m}$$

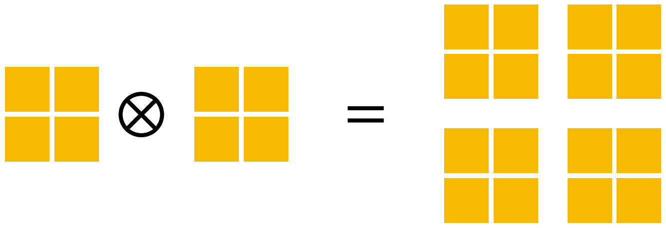
$$\equiv \sum_{\sigma \in S_m} \operatorname{sgn}(\sigma) K_\sigma \,.$$

- Formal solution of MEP: $(K_i - \alpha_i K_0)\boldsymbol{y}_\otimes = 0, \quad \boldsymbol{y}_\otimes = \bigotimes_{j=1}^{m} \boldsymbol{y}_j$

- The matrices are defined as

$$K_i = \begin{vmatrix} A_{11} & \cdots & A_{(i-1)1} & O_1 & A_{(i+1)1} & \cdots & A_{m1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{1m} & \cdots & A_{(i-1)m} & O_m & A_{(i+1)m} & \cdots & A_{mm} \end{vmatrix}_{\otimes} \qquad K_0 = \begin{vmatrix} A_{11} & \cdots & A_{m1} \\ \vdots & \ddots & \vdots \\ A_{1m} & \cdots & A_{mm} \end{vmatrix}_{\otimes}$$

- The problem is intractably large.
- Unclear how we can select the eigenvalue we are interested in.



$M \times M$ $\qquad\qquad$ $M^m \times M^m$

MEP emulator

- The solution MEP is known, and one needs to solve a huge generalized eigenvalue problem.
- One can construct a reduced-order model for MEP: $(\mathcal{K}_i - \alpha_i \mathcal{K}_0)\,\boldsymbol{y} = 0$
- The matrix element of K matrices can be computed through training snapshots.
- Also, it is possible to prove the following affine form:

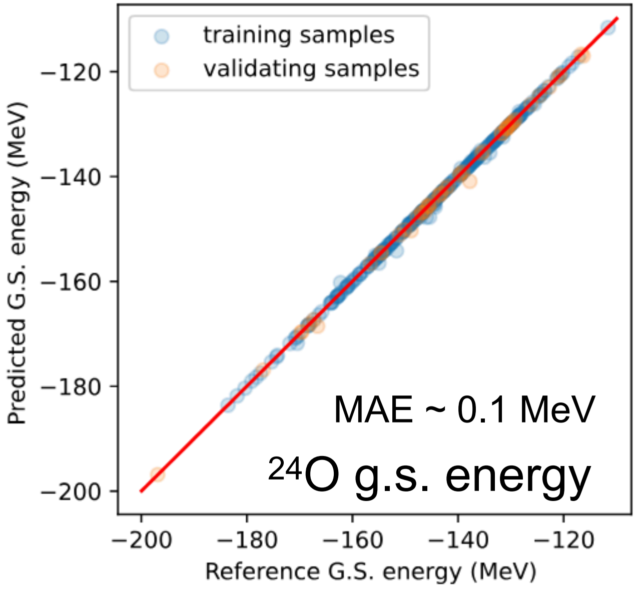
$$(\mathcal{K}_{i0} + E^{[1]}\mathcal{K}_{i1} + \cdots + E^{[n_{\mathrm{par}}]}\mathcal{K}_{in_{\mathrm{par}}} - \alpha_i \mathcal{K}_0)\boldsymbol{y} = 0$$

The matrices can be trained through numerical optimization

Hang Yu and TM, arXiv: 2503.01684 (2025)

MEP emulator: UQ

- A new emulator that takes observables as inputs.

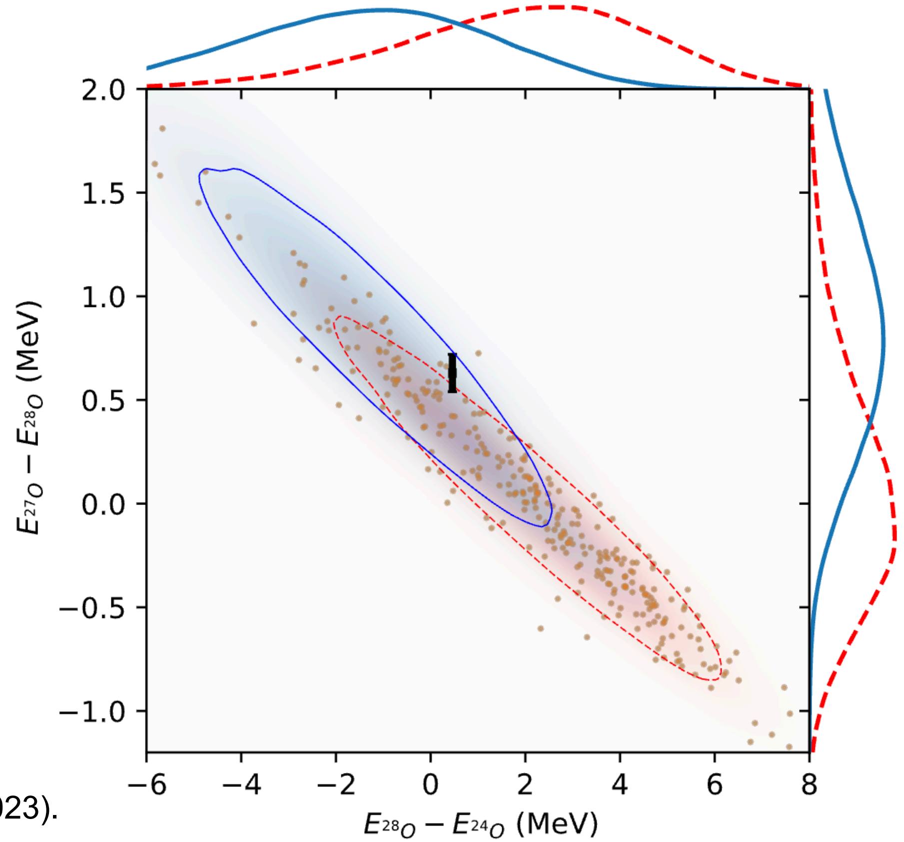


- Mathematically, one can prove

$$(\mathcal{K}_0 + E_1\mathcal{K}_1 + \cdots + E_{n_{\rm par}}\mathcal{K}_{n_{\rm par}} - E_{\rm out}\mathcal{L})\boldsymbol{y} = \mathbf{0}$$

Numerically optimized such that the training data are reproduced
Input (Energy observables)
Output (This can be LECs as well.)

- A function $f : \{D\} \to D_{\odot}$ allows us to avoid the prior assumption, and we can sample $P(D_{\odot} \,|\, \{D\})$ directly.

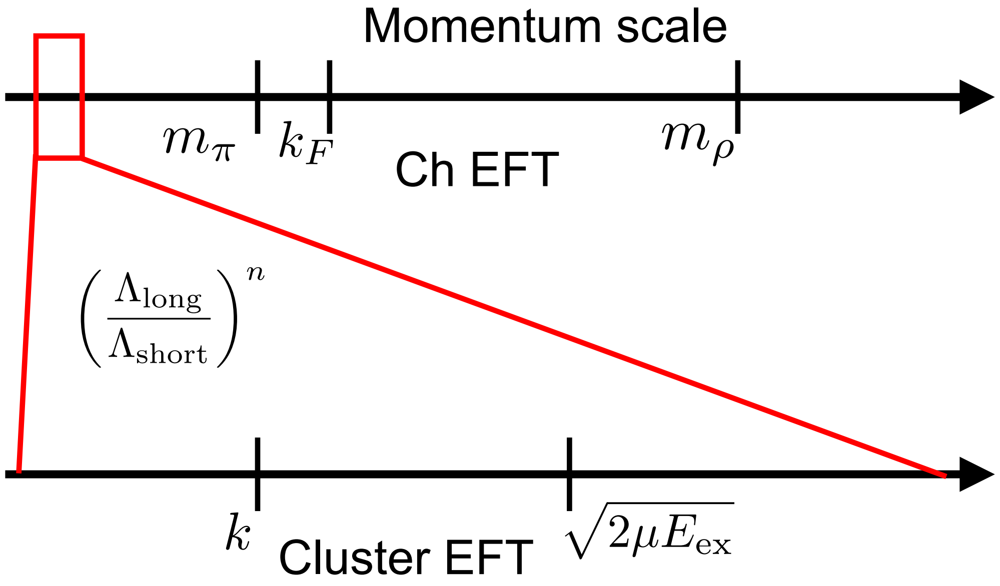


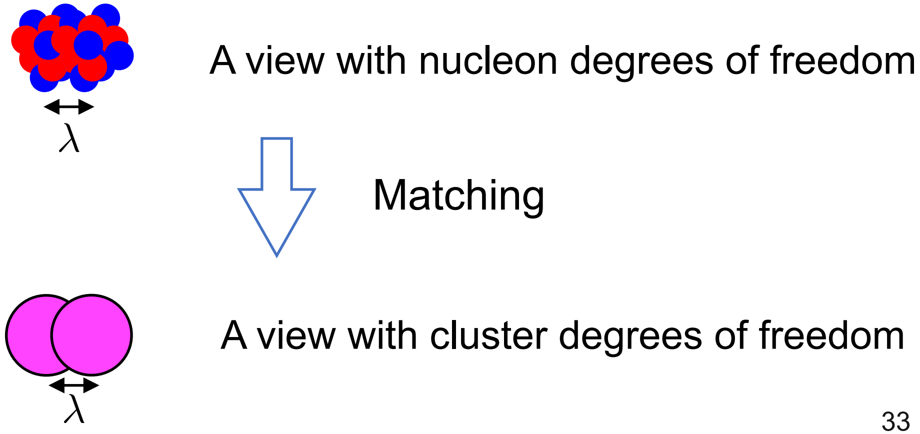
- We can also extract the LEC distributions as well.

Consistent with the CC result in
Y. Kondo et al., Nature 620, 965 (2023).

MEP emulator application: Matching two EFTs

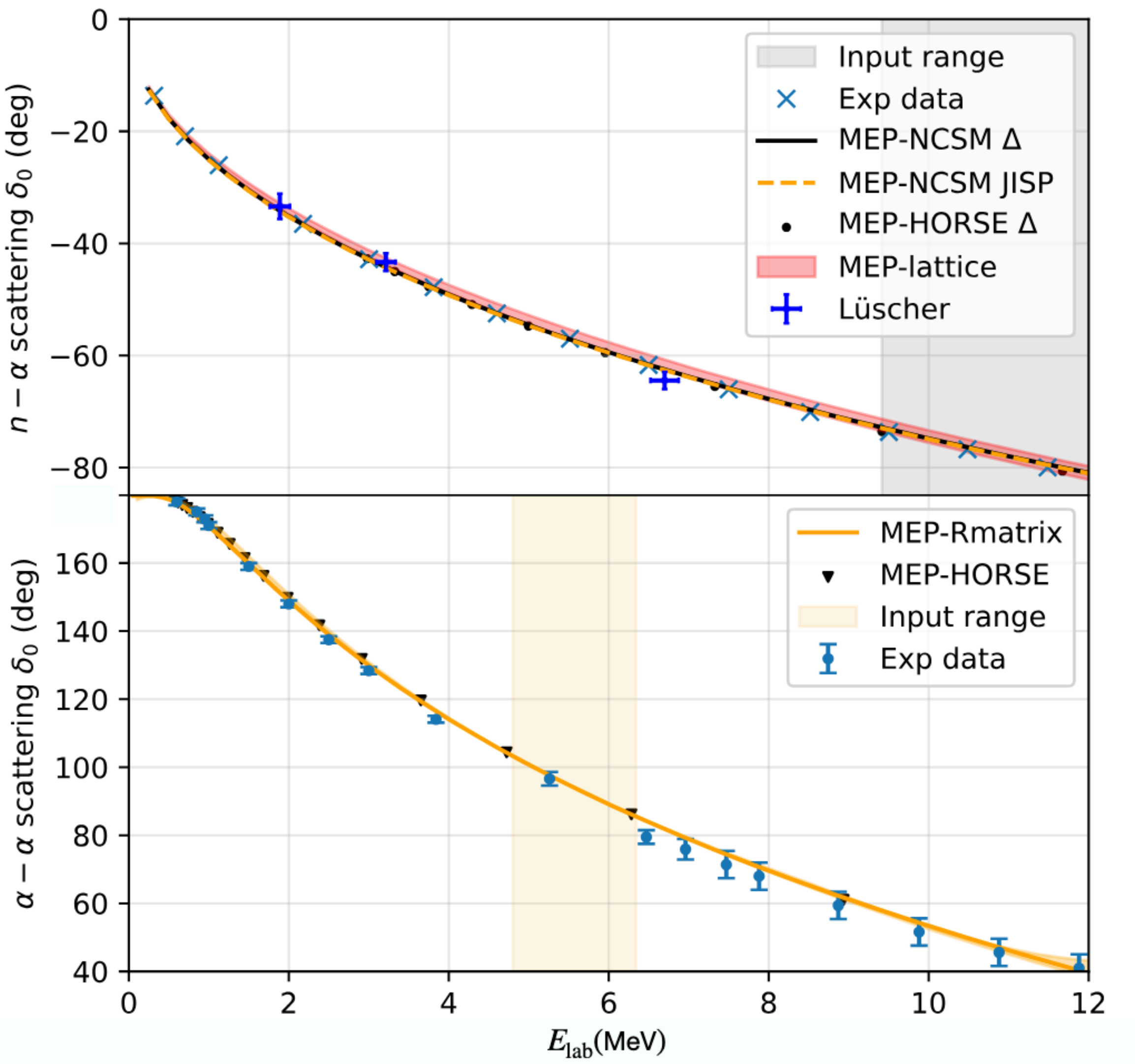
- As long as we are interested in physics significantly lower than the nuclear excitation scale, one can integrate out the nuclear excitation degree of freedom.
- Cluster/Halo EFT: $V = C_0 + C_2(\boldsymbol{p}^2 + \boldsymbol{p}'^2) + \cdots$
- Usually, the low-energy constants are optimized with experimental data.
- If we have an accurate theory, the LECs can be fitted to the theory calculation results.
- MEP emulator allows us to extract the LECs.



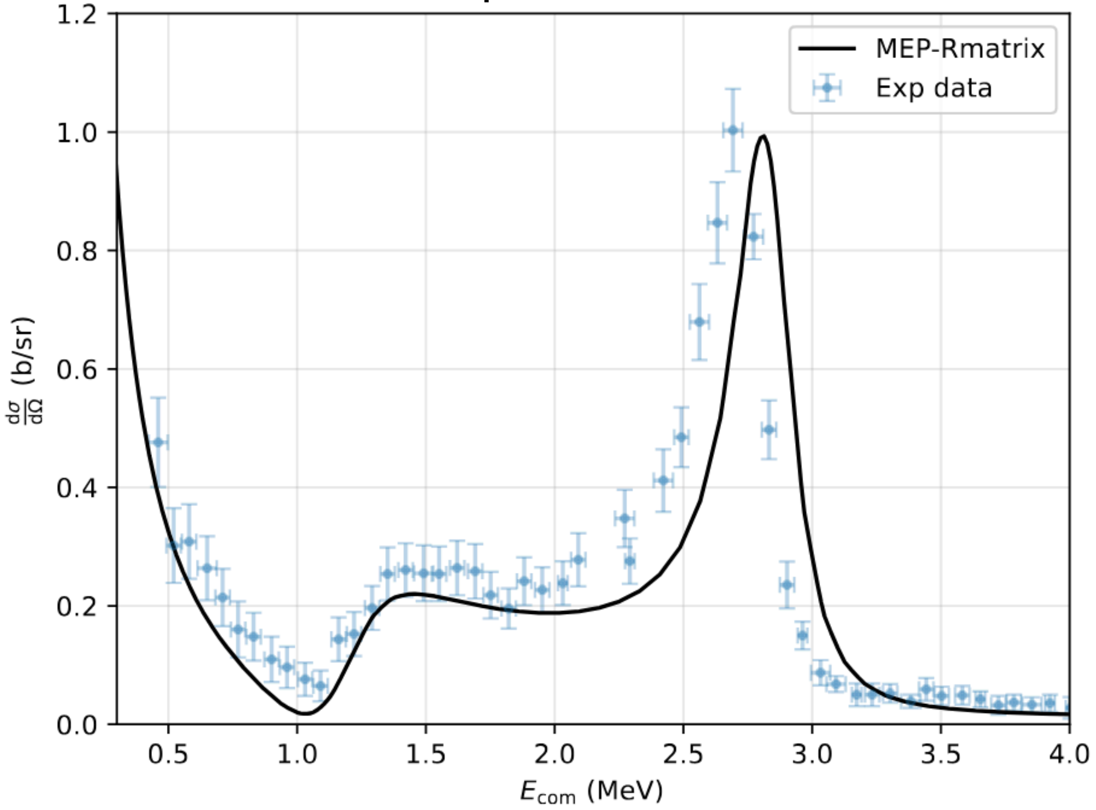


MEP emulator application: Matching two EFTs

n + α ($1/2^+$)



p + ^{14}O



J^π	E_R^{exp} (MeV)	Γ^{exp} (keV)	E_R^{MEP} (MeV)	Γ^{MEP} (keV)
$1/2^+$	1.27(2)(2)	374(70)(+200)	1.25(3)(10)	379(14)(50)
$5/2^+$	2.81(12)	251(26)	2.83(5)(20)	265(20)(50)

Hang Yu et al. in prep.

Summary

筑波大学 計算科学研究センター Center for Computational Sciences

- There are some issues reproducing data.
  - Charge radii in Ca isotopes
    - → many-body (cluster, octupole, …) or higher-order operator?
  - Beta-decay halflives depend on employed Hamiltonians(?)
    - → useful to constrain Hamiltonian? (But it is a complicated observable)
    - Any observable (relatively easy to compute) correlates with the beta-decay halflives?
- An emulator would play an important role in improving the nuclear forces.
  - MEP emulator allows us to skip potential statistical issues
  - (and MEP emulator is also applicable to the scattering problem)