Quark Counting, Drell-Yan West, and the Pion Wave Function

Mary Alberg (SU,UW) and Gerald A. Miller (UW)

2403.03356 (hep-ph)   PRC 110, L042201

Understanding the pion is important

Plans to measure elastic form factor at JLAB and EIC

Eur. Phys. J. A (2019) **55**: 190
DOI 10.1140/epja/i2019-12885-0

THE EUROPEAN PHYSICAL JOURNAL A

Review

Pion and kaon structure at the electron-ion collider

Arlene C. Aguilar[1], Zafir Ahmed[2], Christine Aidala[3], Salina Ali[4], Vincent Andrieux[5,6], John Arrington[7], Adnan Bashir[8], Vladimir Berdnikov[4], Daniele Binosi[9], Lei Chang[10], Chen Chen[11], Muyang Chen[10], João Pacheco B.C. de Melo[12], Markus Diefenthaler[13], Minghui Ding[9,10], Rolf Ent[13,a], Tobias Frederico[14], Fei Gao[15], Ralf W. Gothe[16], Mohammad Hattawy[17], Timothy J. Hobbs[18], Tanja Horn[4,b], Garth M. Huber[2], Shaoyang Jia[19], Cynthia Keppel[13], Gastão Krein[20], Huey-Wen Lin[21], Cédric Mezrag[22], Victor Mokeev[13], Rachel Montgomery[23], Hervé Moutarde[24], Pavel Nadolsky[18], Joannis Papavassiliou[25], Kijun Park[13], Ian L. Pegg[4], Jen-Chieh Peng[5], Stephane Platchkov[24], Si-Xue Qin[26], Khépani Raya[10], Paul Reimer[7], David G. Richards[13], Craig D. Roberts[27,28,c], Jose Rodríguez-Quintero[29], Nobuo Sato[13], Sebastian M. Schmidt[30], Jorge Segovia[31], Arun Tadepalli[13], Richard Trotta[4], Zhihong Ye[7], Rikutaro Yoshida[13,d], and Shu-Sheng Xu[32]

EIC yellow report

The EIC can allow a pion form factor measurement up to $Q^2 = 35\ \mathrm{GeV}^2$

Quark distribution to be measured at COMPASS LOI

Lattice QCD calculations

Relate quark distribution to elastic form factor

$$q(x) = x^\alpha (1-x)^\beta \text{ value of } \beta(\mathrm{x}, \mathrm{Q}^2)??$$

Drell-Yan (PRL 24,181) & West(PRL 24,1206) 1970

- Relate relate $q(x)$ to $F_1(t)$ large $x, t = -\Delta^2$ of proton
- Proof applies to pion as well
- $\lim_{x \to 1} q(x) = (1-x)^{n_H} \qquad \lim_{\Delta^2 \to \infty} F_1(\Delta^2) \propto \frac{1}{(\Delta^2)^{(n_H+1)/2}}$

n_H = number of partons in hadron

From model wave functions

Quark counting rules (N & π)
Farrar &Jackson PRL 35,1416 ('75), 43,246('79)

$$\lim_{x\to 1} q(x,Q^2) \propto (1-x)^{2n_H-3+2|\Delta_s|+\Delta\gamma}$$

n_H minimum # of constituents of hadron,

Δ_s =diff between z components of spin quarks and hadrons,

$\Delta\gamma$ accounts for evolution from starting scale ζ_H^2

$$\lim_{\Delta^2\to\infty} F_H(\Delta^2) \propto \frac{1}{(\Delta^2)^{n_H-1}}$$

From counting propagators in Feynman diagrams

Summary: Drell-Yan West & quark counting

Two sets of predictions for pion

Drell-Yan West $q(x) \sim (1-x)^2, F(\Delta^2) \sim 1/\Delta^3$

Quark counting $q(x) \sim (1-x)^2, F(\Delta^2) \sim 1/\Delta^2$

Our goal: study the connection between

$q(x)$ and $F(\Delta^2)$ given current knowledge in 2023

Light-front analysis

- Hadronic wave functions depend on factorization scale ζ
- Several refs argue that there is a scale ζ_H at which hadron is made of dressed valence quarks that carry all of the hadron momentum. For pion u and dbar each carry 1/2 of pion momentum
- From Phys. Rev. D 101, 054014 (2020), arXiv:1905.05208 [nucl-th]. "The result of every calculation of pionic properties that respects Poincare covariance, and the Ward- Green-Takahashi identities along with the consequences of dynamical symmetry breaking inherent in the quark gap-equation has these features."

.

$$q(x) = \frac{1}{\pi}\int \frac{d^2k_\perp}{x(1-x)}\,|\,\Phi(x,k_\perp)\,|^2 + \cdots$$

At ζ_π, $\Phi(x,k_\perp) \to \Phi(z \equiv \frac{k_\perp^2 + M^2}{x(1-x)}),\ M = \text{ constituent quark mass}$

From Terent'ev and Strikman-Frankfurt

Light-front analysis

- Hadronic wave functions depend on factorization scale ζ

- Several refs argue that there is a scale ζ_H at which hadron is made of dressed valence quarks that carry all of the hadron momentum. For pion u and dbar each carry 1/2 of pion momentum

- From Phys. Rev. D 101, 054014 (2020), arXiv:1905.05208 [nucl-th]. "The result of every calculation of pionic properties that respects Poincare covariance, and the Ward- Green-Takahashi identities along with the consequences of dynamical symmetry breaking inherent in the quark gap-equation has these features."

$$q(x) = \frac{1}{\pi}\int \frac{d^2k_\perp}{x(1-x)}\,|\,\Phi(x,k_\perp)\,|^2 + \cdots$$

At ζ_π, $\Phi(x,k_\perp) \to \Phi(z \equiv \frac{k_\perp^2 + M^2}{x(1-x)})$, $M =$ constituent quark mass

From Terent'ev and Strikman-Frankfurt

Light-front analysis-II

$$q(x) = \frac{1}{\pi}\int \frac{d^2k_\perp}{x(1-x)}\,|\,\Phi(x,k_\perp)\,|^2 + \cdots$$

Poincare invariance

At $\zeta_\pi\,\Phi(x,k_\perp) \to \Phi(z \equiv \frac{k_\perp^2 + M^2}{x(1-x)}),\ M =$ constituent quark mass

From Terent'ev and Strikman & Frankfurt

Change variables to z , neglect $+\cdots$

$$q(x) = \int_{\frac{M^2}{x(1-x)}}^{\infty} dz\,|\,\Phi(z)\,|^2,$$

q(Large x) is related to high-momentum component of wave function

q(x) must be evolved to compare with DIS data

Light-front analysis-III

$$F(\Delta^2) = \frac{1}{\pi}\int_0^1 dx \int \frac{d^2k_\perp}{x(1-x)}\Phi(\frac{k_\perp^2+M^2}{x(1-x)})\Phi(\frac{(\mathbf{k}_\perp+(1-x)\mathbf{\Delta})^2+M^2}{x(1-x)})$$

Start with power law (PL) $|\Phi^{\rm PL}(z)|^2 \to \frac{K}{(z)^{n+1}}, q^{\rm PL} \sim (1-x)^n$

n	$\lim_{\Delta^2\to\infty} F\ (\Delta^2)$	DY & W	Quark counting
1	$6\left(\frac{\ln^2(\Delta^2)-4\ln(\Delta^2)+8}{2\Delta^2} - \frac{2(\ln(\Delta^2)+2)}{\Delta^4}\right)$	$\frac{1}{\Delta^2}$	$\frac{1}{\Delta^3}$
2	$180\sqrt{\pi}\left(\frac{(\Delta^2-6)\ln\left(\sqrt{\Delta^2}+\sqrt{\frac{\Delta^2}{4}+1}\right)}{(\Delta^2)^{5/2}} + \frac{16-5\sqrt{\Delta^2+4}}{2\Delta^4}\right)$	$\frac{1}{\Delta^3}$	$\frac{1}{\Delta^4}$
3	$840\left(\frac{3(\ln^2(\Delta^2)-3\ln(\Delta^2)+7)}{\Delta^6} + \frac{3\ln(\Delta^2)-14}{6\Delta^4}\right)$	$\frac{1}{\Delta^4}$	$\frac{1}{\Delta^5}$

No logs

Why?

$$F_n(\Delta^2) \propto \int_0^1 dx \int_0^1 du \frac{(x(1-x))^n(u(1-u))^{(n-1)/2}}{(1+\Delta^2(1-x)^2u(1-u))^n}$$

End points!!!

Realistic models-

Model1 Cui et al EPJA58,10 ('22), EPJC 80,,1064 ('20)

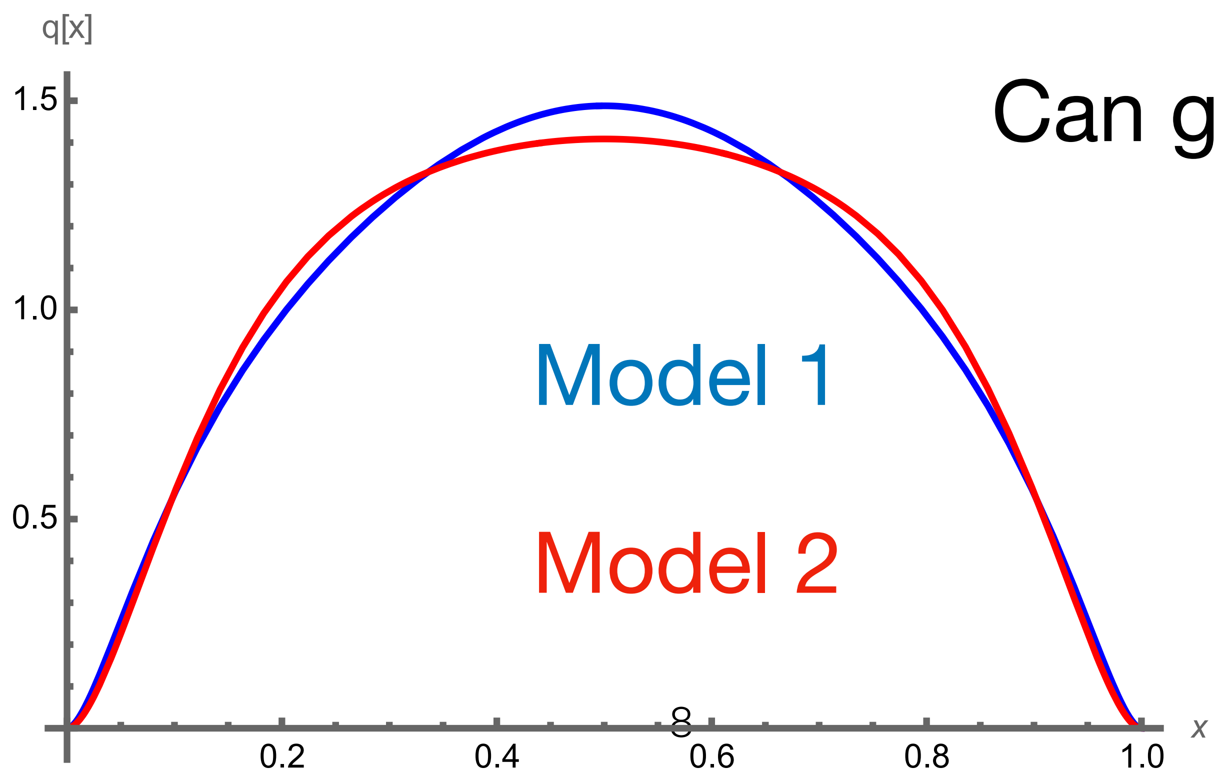
"parameter-free prediction valence-quark distribution"

$$q(x) = \sum_{N=4}^{8} C_N(x(1-x))^{N/2}, \quad \Phi(z) = \frac{1}{\sqrt{\pi}} \sum_{n=3}^{5} \frac{A_n}{z^{n/2}}. \qquad z = \frac{k_\perp^2 + M^2}{x(1-x)}$$

A_n determined from C_N

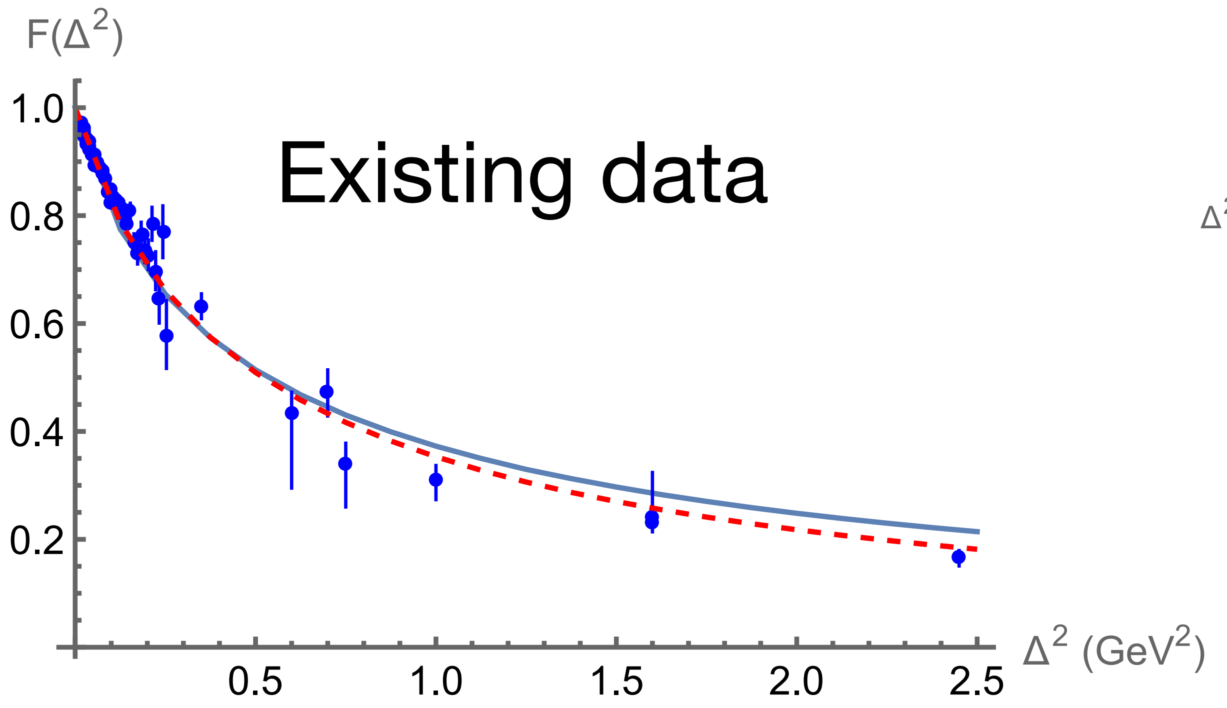
Model 2 Ding et al PRD101,054014 ('20) Both Craig Roberts group

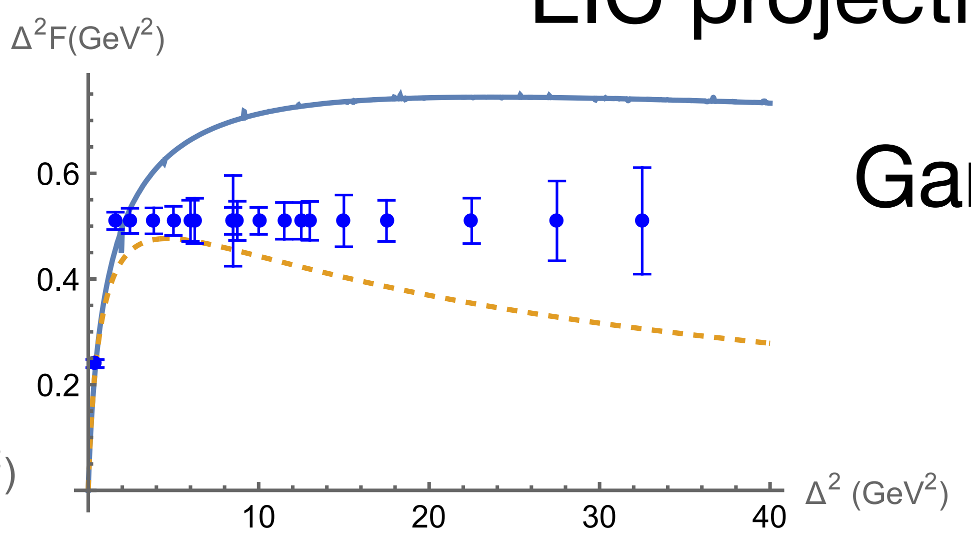
Similar form, different coefficients $q(x) \sim (1-x)^{3.27}$



Can get $\Phi(z)$ from $q(x)$

Results





Garth Huber

Two models agree with existing data for low Δ^2, disagree strongly at higher Δ^2 to be measured in future experiments

$$\lim_{x\to 1} q(x) = (1-x)^n \to F(\Delta^2) \sim \frac{\log(\Delta^2)}{(\Delta^2)^{(n+1)/2}}$$

Summary

- $q(x)$ and $F(\Delta^2)$ are connected- increases interest in both
- New version of DY W - logs in numerator
- Pion electromagnetic form factor is not yet determined