Nucleon mass in models (that I’ve worked on)

Gerald A Miller UW

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My models were used not used to calculate the mass of the nucleon!
We computed various form factors in many papers
New Talk

Nucleon mass in models (that I've worked on)

Outline

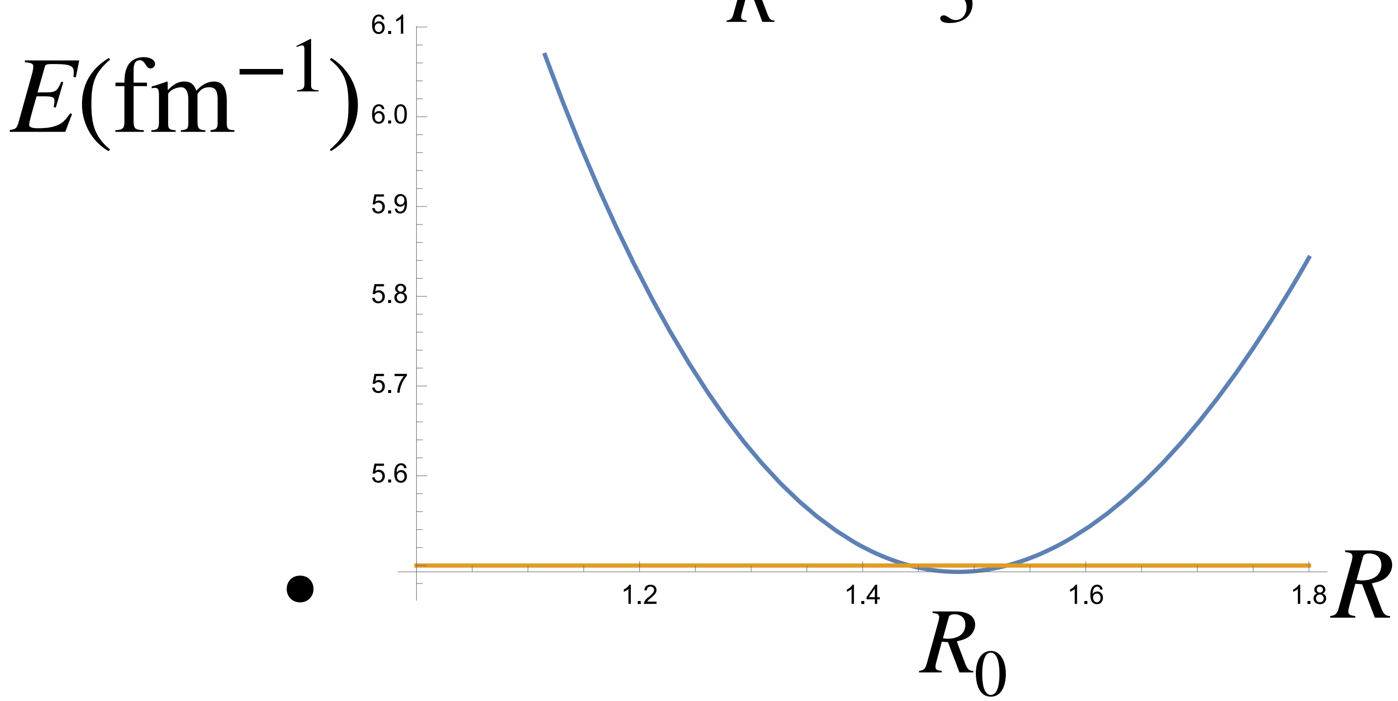
MIT and Cloudy Bag Models -$M = \langle H \rangle$

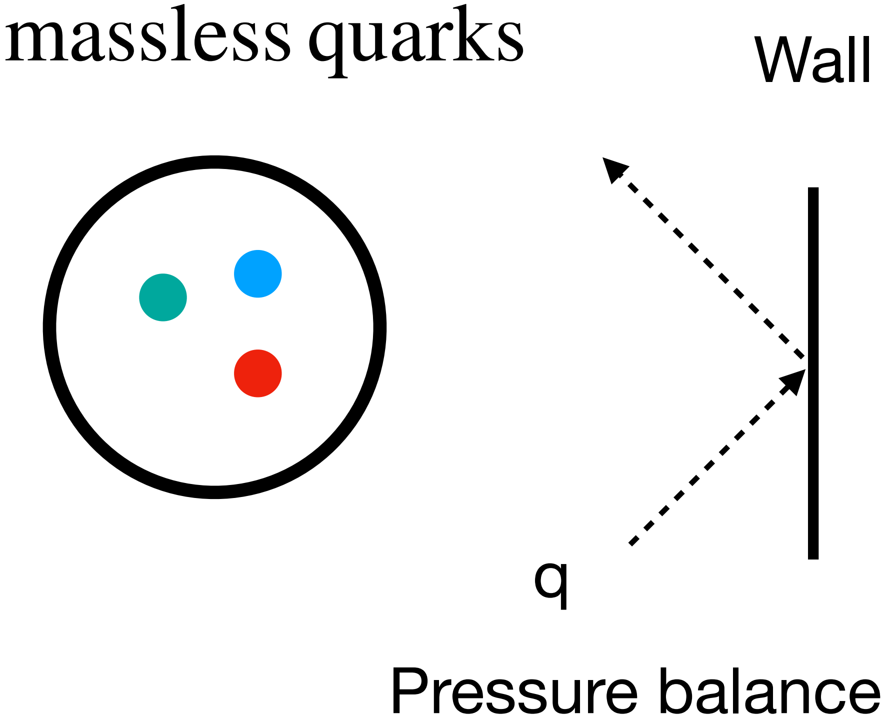
't Hooft model $M = \langle P^{-} \rangle$

Use of models- examples to display the possibilities & ideas to interpret lattice calculations

MIT bag model

$$E(R) = \frac{3\omega}{R} + \frac{4\pi}{3}BR^3, \quad \omega = 2.04 \quad \text{massless quarks}$$





$$M = E(R_0), \quad \text{quark energy} = 3\text{bag energy}$$

Problems

$$R_0 = 1.5\ \mathrm{fm}$$

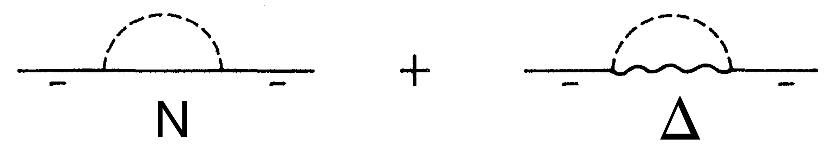
Infinite potential breaks chiral symmetry breaking

Axial vector current discontinuity at bag surface

Solution -pions carry axial vector current outside the bag

Cloudy bag model

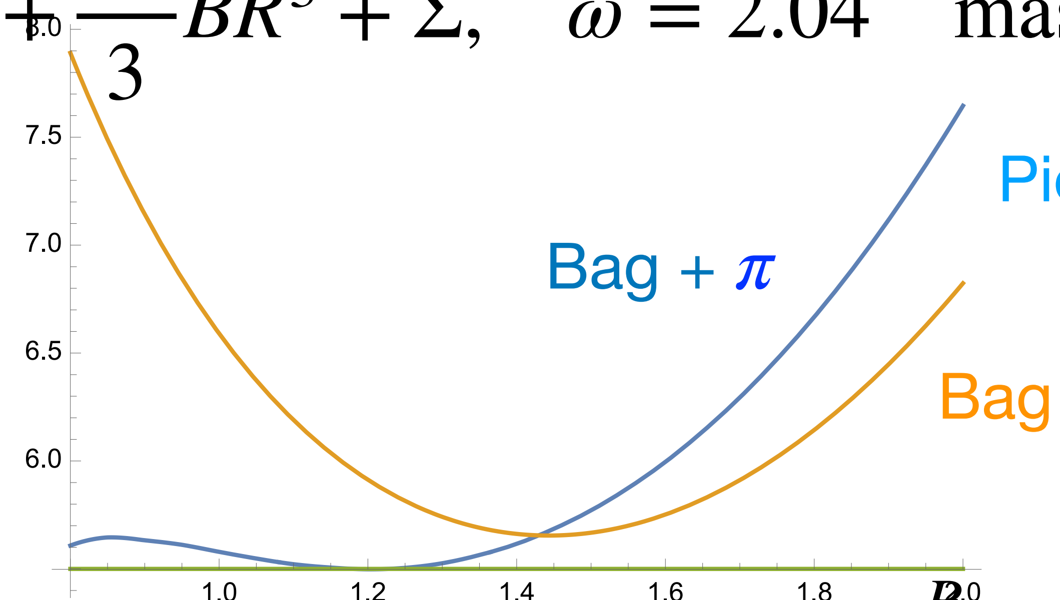
Thomas Theberge Miller PRD 24,216

$\Sigma =$ 

$\Sigma \sim \frac{-\lambda}{R^3}$ comes from pion-field and quark-pion interactions <0

$$E(R) = \frac{3\omega}{R} + \frac{4\pi}{3} B R^3 + \Sigma, \quad \omega = 2.04 \quad \text{massless quarks}$$

$E(\mathrm{fm}^{-1})$



R_0

Pions outside squeeze bag
Reduce the radius

$$R_0 = 1.2\,\mathrm{fm}$$

$$\frac{3\omega}{R_0} = 5.1, \quad \frac{4\pi}{3} B R_0^3 = 1.01, \quad \Sigma = -0.601 \quad (\mathrm{fm}^{-1})$$

Now quark kinetic energy is 90 %, not 75 % of nucleon mass

Summary of bag models

- Mass =H comes from quark kinetic energy, Bag energy, pion energy

- Difficult to compute gravitational form factors because the bag is a sphere fixed in space, it is not a momentum eigenstate

't Hooft model

Gerard 't Hooft, "A Two-Dimensional Model for Mesons," Nucl. Phys. B **75**, 461–470 (1974).

- QCD in (1+1) space-time, Large N_c,
- gauge : $\mathrm{A}^+ = 0$, no gluon-gluon interactions
- Pro- Model is solvable, Con- (1+1), Large N_c

$$x_\mu p^\mu = x^+ p^- + x^- p^+ = x_- p_+ + x_+ p_- \quad \text{light} - \text{front}$$

$$\mathcal{L} = -\tfrac{1}{2}\mathrm{Tr}\,(\partial_- A_+)^2 - \bar{q}^a(\gamma\partial + m_{(a)} + g\gamma_- A_+)\,q^a \;. \tag{7}$$

There is no ghost in this gauge. If we take x^+ as our time direction, then we notice that the field A_+ is not an independent dynamical variable because it has no time derivative in the Lagrangian. But it does provide for a (non-local) Coulomb force between the Fermions.

$$\partial_- \partial_- A_+ = g \bar{q} \gamma_- q$$

Poisson's eq. in 1 dim-> linear potential

't Hooft model

$H = P^+P^-, \quad \mathrm{P}^- \text{ dynamical} \quad \mathrm{P}^+ \text{ kinematic}$

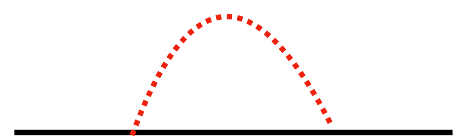
Schroedinger eq is mass decomposition

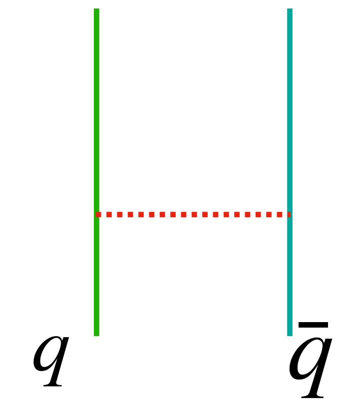
$$\mu^2\phi(x) = \frac{\gamma - 1}{x(1-x)}\phi(x) - P\int_0^1 dy \frac{\phi(y)}{(x-y)^2},$$

$$\gamma = \pi\frac{m^2}{g^2}$$

$x = \dfrac{k^+}{P^+}$, momentum fraction

Quark kinetic energy quark self-energy Potential energy





$$\frac{P}{(y-x)^2} \equiv \frac{1}{2}\left[\frac{1}{(y-x+i\epsilon)^2} + \frac{1}{(y-x-i\epsilon)^2}\right] = \frac{-1}{2}\int_{-\infty}^{\infty} d\tilde{z}\,|\,\tilde{z}\,|\,e^{i(y-x)\tilde{z}}e^{-|\epsilon|\tilde{z}}$$

't Hooft Prin. Value is linear potential in coordinate space $\tilde{z}$ is coordinate canonically conjugate to x: Miller & Brodsky PRC 102,(2020) 022201 , 1912.08911

$$\tilde{z} = x^- P^+$$

Schroedinger equation can be re-expressed:

$$\mu^2\phi(x) = \frac{\gamma}{x(1-x)}\phi(x) + P\int_0^1 dy \frac{\phi(x)-\phi(y)}{(x-y)^2}$$

Chiral limit: $\gamma = 0, \phi(x) = 1, \mu^2 = 0$ only analytic solution

’t Hooft end-point analysis

$$\mu^2\phi(x) = \frac{\gamma}{x(1-x)}\phi(x) + P\int_0^1 dy\frac{\phi(x)-\phi(y)}{(x-y)^2}$$

PRD 19, 3024
Numerically difficult to solve
For small masses

t Hooft postulated

$$\phi(x) = x^\beta(1-x)^\beta\frac{\sqrt{\Gamma(4\beta+2)}}{\Gamma(2\beta+1)}$$

handle end-point singularity,
constant: $\int \phi^2(x)dx = 1$

$$\text{Integral } (\mathrm{x} \to 0) \approx \mathrm{x}^{\beta-1}[\pi\beta\cot\pi\beta - 1]$$

Match $x^{\beta-1}$ terms $\rightarrow \beta = \sqrt{\frac{3}{\pi}\frac{m}{g}}$

Integrate $\mu^2 = m_\pi^2 = \gamma\frac{\int_0^1 \frac{\phi(x)}{x(1-x)}dx}{\int_0^1 \phi(x)dx}$

$$\boxed{m_\pi^2 = 2\sqrt{\frac{\pi}{3}}mg + 4m^2}$$

Uses full range of x
? How accurate

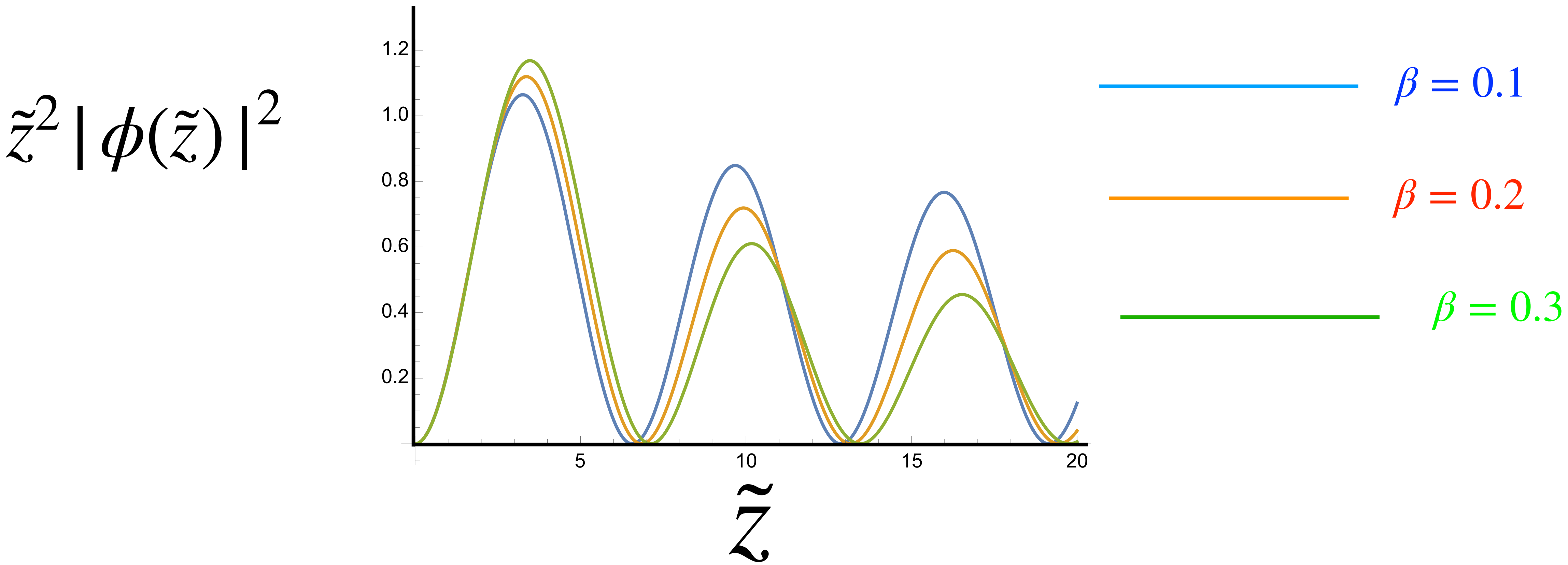
GMOR relation in 1+1 spacetime

Potential energy in coordinate space

$$V(\beta) \equiv \int_0^1 dx \int_0^1 dy \phi(x) \frac{P}{(x-y)^2} \phi(y)$$
$$= \frac{1}{2} \int_{-\infty}^{\infty} \phi^*(\tilde{z}) |\tilde{z}| \tilde{\phi}(\tilde{z}) d\tilde{z}, \qquad (9)$$

with

$$\tilde{\phi}(\tilde{z}) = \int_0^1 \phi(y) e^{iy\tilde{z}}\, dy. \qquad (10)$$



Increasing β suppresses momentum increases distance

Get wave function is it

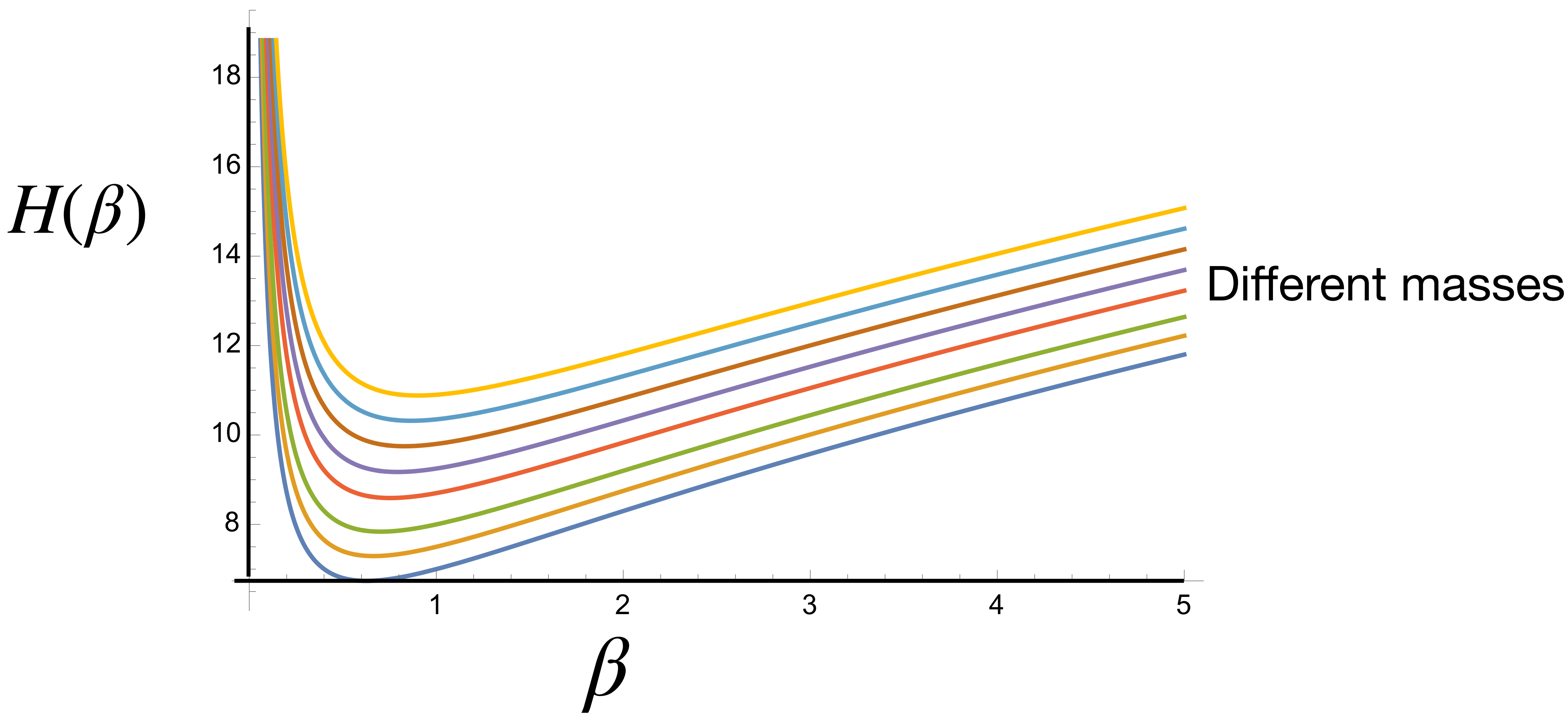
$$\phi(x) = x^{\beta}(1-x)^{\beta}\frac{\sqrt{\Gamma(4\beta+2)}}{\Gamma(2\beta+1)}$$

Logic -no nodes, good guess

Procedure - use above as variational wave function compute $H(\beta)$ and minimize

$$\mu^2\phi(x) = \frac{\gamma-1}{x(1-x)}\phi(x) - P\int_0^1 dy\frac{\phi(y)}{(x-y)^2},\qquad H(\beta) = \int_0^1 dx\phi(x)H\phi(x) = (\gamma-1)(4+\frac{1}{\beta}) + V(\beta)$$

$$V(\beta) = \frac{1}{2}\int_{-\infty}^{\infty}\phi^*(\tilde{z})\,|\tilde{z}|\,\tilde{\phi}(\tilde{z})d\tilde{z} = \pi^2\,2^{-8\beta}\frac{\Gamma(2+4\beta)}{\beta\Gamma^4(1/2+\beta)} \rightarrow_{\beta\rightarrow\infty} 2\sqrt{2\pi}\sqrt{\beta}$$



Mass P^- decomposition

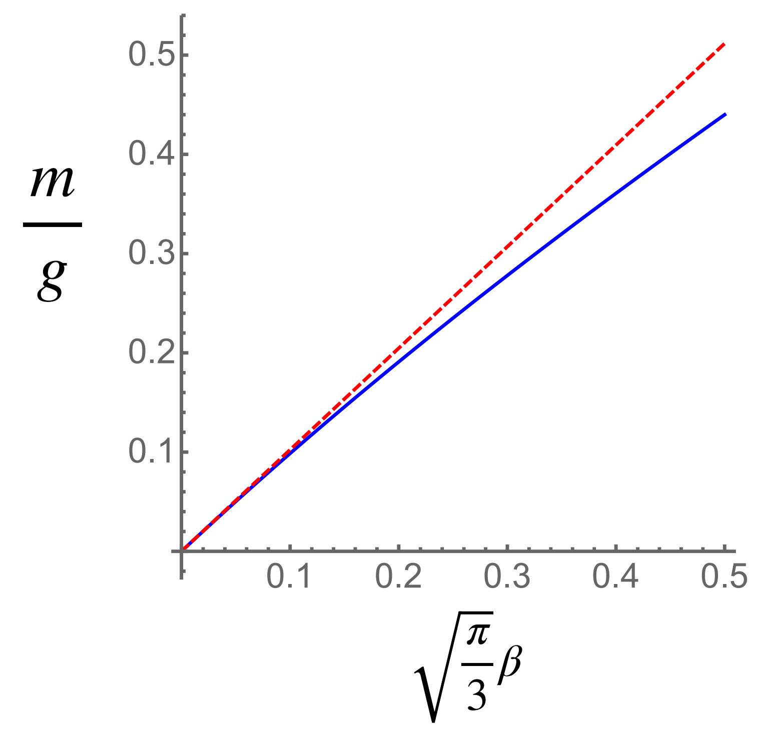


FIG. 1: The ratio m/g versus $\sqrt{\frac{\pi}{3}}\beta$. The solid (blue) curve shows the value of β determined by minimizing $H(\beta)$ with respect to β. The dashed (red) curve shows the line $m/g = \sqrt{\pi/3}$.

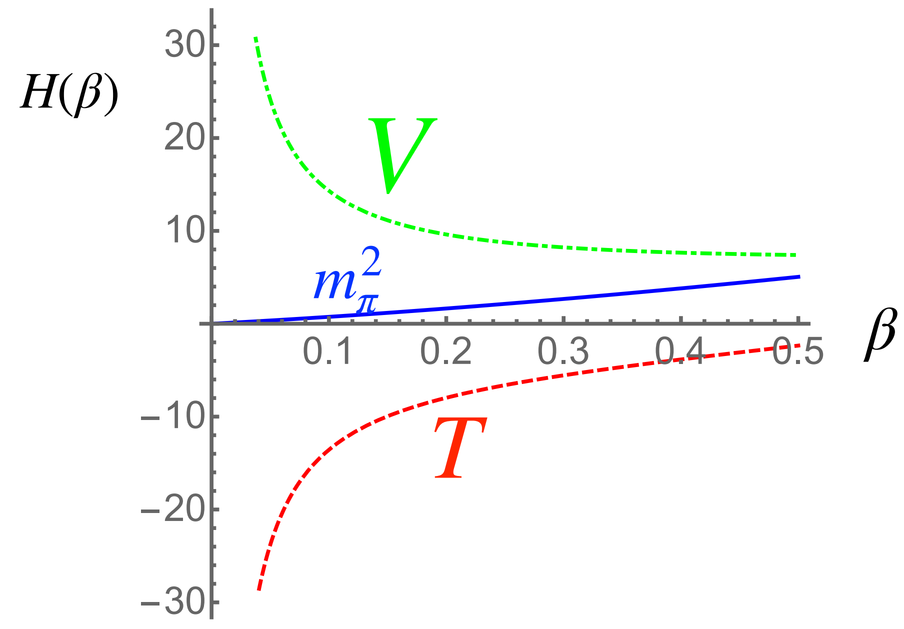


FIG. 2: Different components of m_π^2 versus β. The solid (blue) curve shows m_π^2 The dashed curve (red) shows the kinetic energy which is negative due to the quark self-enegy. The dot-dashed curve shows the potential energy.

Wave function ~ok for $m/g < 0.1$

Summary- complete cancellation of potential and self energy terms in the chiral limit
GMOR-type expression for small quark masses
Potential energy increases with increasing suppression of $\phi(x)$ at end-points
P^- decomposition is possible, relation to $T^{\mu\nu}$ to be done with Adam Freese