Two aspects of the three-body systems

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Accessing and Understanding the QCD Spectra,
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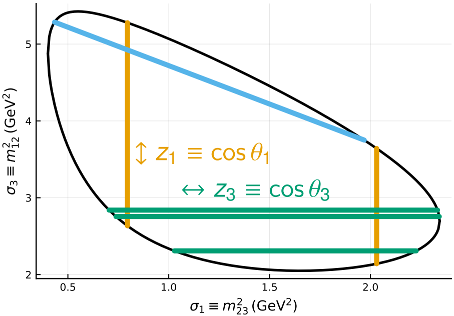
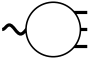
1. Three-body decay
2. Ladders and Resonances
3. T_{cc}^{+} model in the LHCb analysis
4. Analytic continuation
5. Effective range expansion

Three-body decay

Three-body decay



Decay amplitude –
$\langle p_1 p_2 p_3 | \hat{T} | p_0 \rangle$

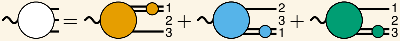
 $= D^{J*}_{M\lambda}(\alpha,\beta,\gamma) F_\lambda(s,\sigma_1,\sigma_2)$

Dalitz plot variables

- Subchannel resonances are bands.
- Angular distribution along the bands determined by angular momenta.

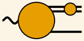
Partial-waves vs Isobar representation

Isobar representation



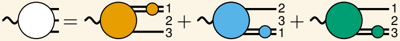
$$F(\sigma_1,\sigma_2) = F^{(1)}(\sigma_1,\sigma_2) + F^{(2)}(\sigma_1,\sigma_2) + F^{(3)}(\sigma_1,\sigma_2)$$

$$= \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_1) a_l^{(1)}(\sigma_1) + \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_2) a_l^{(2)}(\sigma_2) + \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_3) a_l^{(3)}(\sigma_3).$$

Simple **model**:  $= a_l^{(i)}(\sigma_1) \to c^{(i)}\mathrm{BW}(\sigma_1) =$ .

Partial-waves vs Isobar representation

Isobar representation



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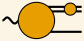
Simple **model**:  $= a_l^{(i)}(\sigma_1) \to c^{(i)}\mathrm{BW}(\sigma_1) =$ .

Partial-wave representation

 $= F(\sigma_1,\sigma_2) = \sum_l^{\infty} \sqrt{2l+1} P_l(z_1) f_l^{(1)}(\sigma_1)$

Why would someone do this? – theoretical constant to $f^{(1)}(\sigma_1)$ is straightforward.

Two-body unitarity and Khuri-Treiman model

Example of $f_0^{(1)}(\sigma_1)$ constraints:

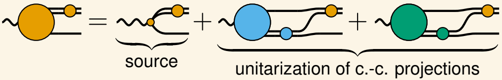
$$f_0^{(1)}(\sigma_1) = \underbrace{a_0^{(1)}(\sigma_1)}_{\text{same channel}} + \underbrace{\int_{-1}^{1} \frac{\mathrm{d}z_1}{2} \left(\sum_l \sqrt{2l+1} P_l(z_2) a_l^{(2)}(\sigma_2) + \sum_l \sqrt{2l+1} P_l(z_3) a_l^{(3)}(\sigma_3) \right)}_{\text{cross-channel(c.-c.) projections}}$$

Unitarity of $f_0^{(1)}(\sigma_1)$ – same RHC as $2 \to 2$ scattering amplitude, $\mathrm{BW}_0^{(1)}(\sigma_1)$

$\Rightarrow$ consistency relation the **direct term** and the **cross-channel projections**

$\Rightarrow$ $a_l^{(1)}(\sigma_1)$ obtains corrections from one seen in $2 \to 2$.

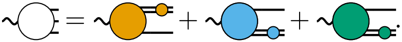
KT model: analytic continuation of **two-body** unitarity



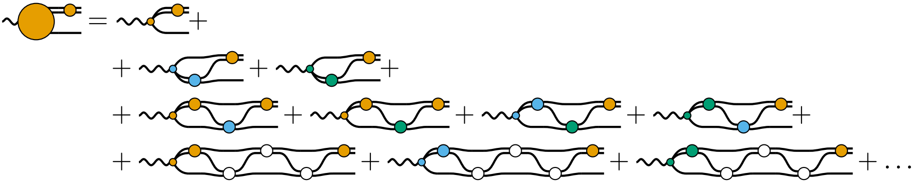
(the loop is a dispersive integral)

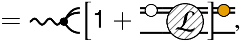
Diagramatic representation

Isobar representation with $a_l^{(i)}(\sigma_i) = \hat{a}_l^{(i)}(\sigma_i)\,\mathrm{BW}_l^{(i)}(\sigma_i)$

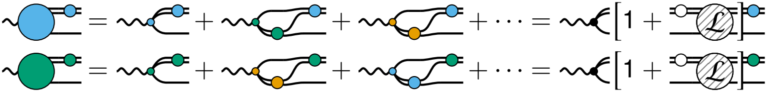


The amplitude prefactor is not constant: $a_l^{(i)}(\sigma_i) = c_l^i \mathrm{BW}_l^{(i)}(\sigma_i) + \dots$





the ladder – a sum of all possible exchanges



Systematic investigation of the FSI

[D. Stamen, T. Isken, B. Kubis, M. Mikhasenko, M. Niehus, arXiv:2212.11767]

- $X \to 3\pi$
- Identify J^{PC} sectors with ρ-dominated interaction:
  - $I^G J^{PC} = 0^- 1^{--}$ (ω, ϕ): $\rho^- \pi^+ + \rho^0 \pi^0 + \rho^+ \pi^-$
  - $I^G J^{PC} = 1^- 1^{-+}$ (π_1^+): $\rho^0 \pi^+ - \pi^+ \rho^0$
  - $I^G J^{PC} = 1^- 2^{++}$ (a_2^+): $\rho^0 \pi^+ + \pi^+ \rho^0$
  - $I^G J^{PC} = 0^- 0^{--}$ (ω_0): $\rho^- \pi^+ + \rho^0 \pi^0 + \rho^+ \pi^-$
- One subtraction (normalization) $\Rightarrow$ parameter-free distribution

Many efforts in the past:

$\eta \to 3\pi$: [Sebastian et al. (2011)], [P.Guo et al., JPAC (2015)], [Albaladejo, Moussallam (2017)]

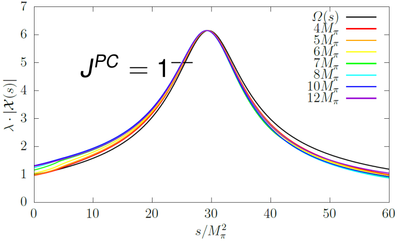
$\eta' \to \eta\pi\pi$: [Isken, Kubis (2017)]

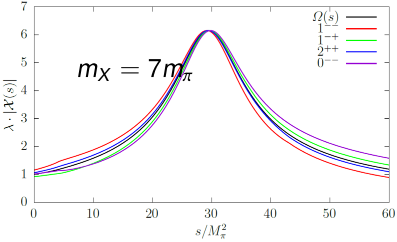
$\omega/\phi \to 3\pi$: [Niecknig, Kubis (2012)], [Danilkin et al., JPAC (2012)]

$a_1 \to 3\pi$: [Albaladejo et. al, JPAC (2020)]

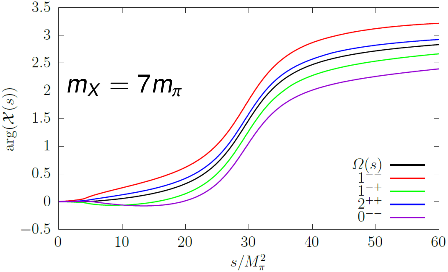
$D \to K\pi\pi$: [Niecknig, Kubis (2015)], [Moussallam (2023)]

Lineshape modifications





Phase shift of rescattered ρ



- Rescattering leads to skewness
- Phase shift get the largest impact

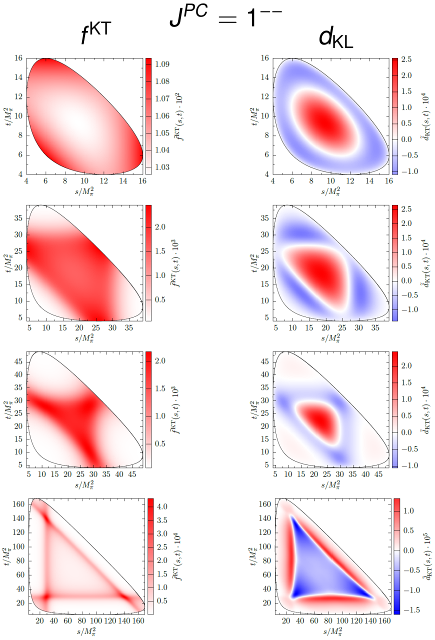
Strength of the effect

- Continuous ambiguities (zero modes)
  $\Rightarrow$ compare the Dalitz plots
- Comparison of two functions using **KL divergence**
  (likelihood difference on a finite sample)

$$d_{\mathrm{KL}} = \int_{\mathrm{Dalitz}} f^{\mathrm{KT}} \log \frac{f^{\mathrm{KT}}}{f^{\mathrm{Omnes}}}$$

- ▸ Rescattering model is truth
- ▸ Test of Naive (Omnes) model

$\Rightarrow$ of $\Delta\mathcal{L} > 0$ depending of sample size



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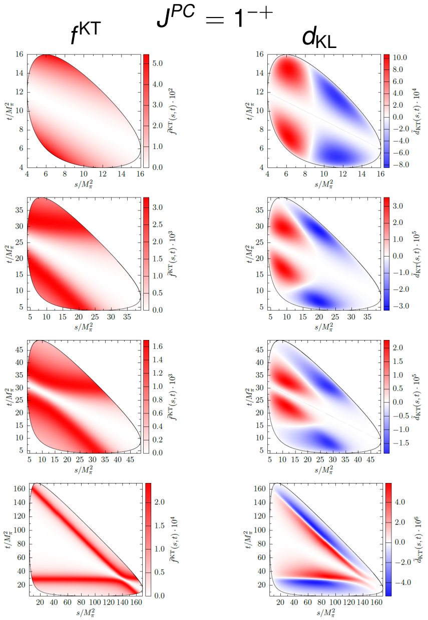

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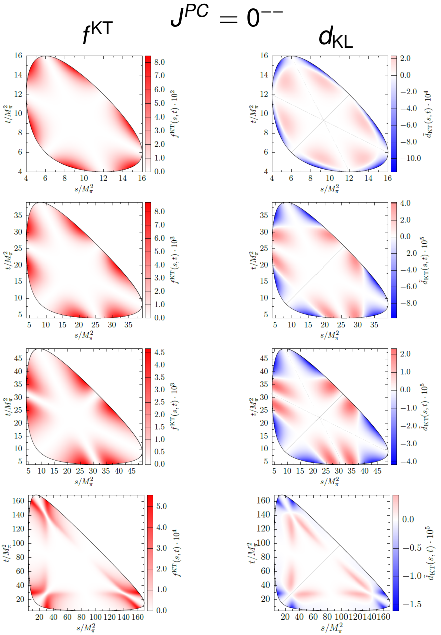

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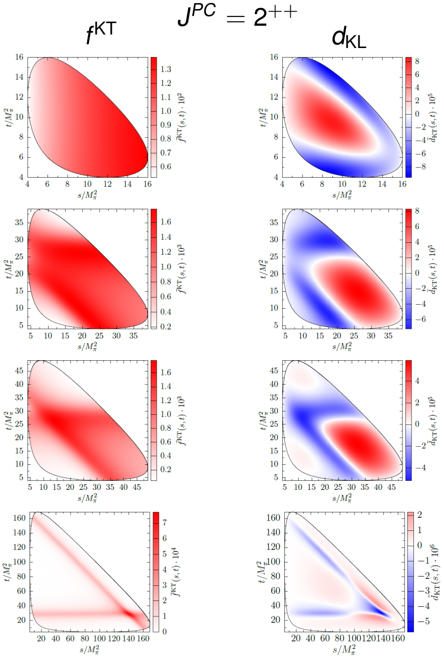
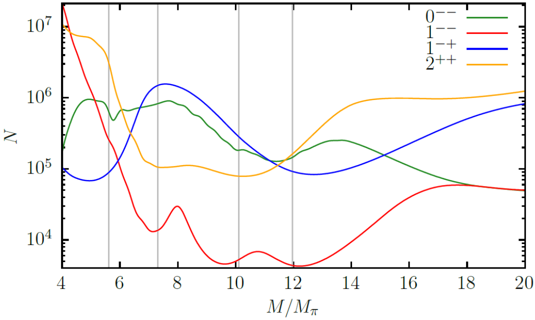
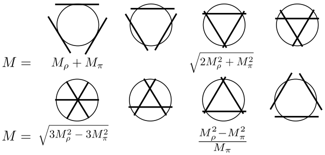


Searching for 3σ effect

Likelihood comparison of the KT and Omnes



- Effect strongly depends on J^{PC}:
  - 1^{++} is the easiest to spot $O(10^4)$ events for ϕ
  - $0^{--}, 1^{-+}, 2^{++}$ need $O(10^6)$



- The structures (bumps and dips) are related to the geometry of resonances on the Dalitz plot
- high m_X: elasticity breaks

“Ladders and Resonances”
Three-body formalism

Three-body resonances

- Many resonances dominantly decay to three-particle final state (e.g. $a_1(1260) \to 3\pi$, $X(3872) \to D^0\bar{D}^0\pi^0$, $T_{cc}^+(3872) \to D^0D^0\pi^+$, $\Lambda_{c/b}^* \to \Lambda_{c/b}\pi\pi$)
- Reaction amplitude need to include three-body effects (e.g. three-body threshold, dynamics of OPE, triangle singularity)
- Three-body unitarity [Fleming(1964)/Holman(1965)/Amado(1973)]:

$$\mathcal{T} - \mathcal{T}^\dagger = \mathcal{T}^\dagger\tau\mathcal{T} + \mathcal{T}^\dagger\tau^\dagger\mathcal{D}\tau\mathcal{T} + \mathcal{D}\tau\mathcal{T} + \mathcal{T}^\dagger\tau^\dagger\mathcal{D} + \mathcal{D}$$

  - the $\mathcal{D}$ is an imaginary part of the OPE
    it is non-zero in the physical region: $(\sigma_1, \sigma_2) \in$ Dalitz
  - the τ indicates the phase space the particle-pair mass
  - the $\tau^\dagger\mathcal{D}\tau$ is an integral over the Dalitz

[Mai et al., EPJA 53, 177 (2017)]

[Jackura et al. (JPAC), EPJC 79, 56 (2019)]

[MM et al. (JPAC), JHEP 08 (2019) 080]

Ladders and resonances [MM et al. (JPAC), JHEP 08 (2019) 080]

$$\mathcal{T} = \mathcal{T}_{\text{short range}} + \mathcal{T}_{\text{includes OPE}}$$

- $\mathcal{T}_{\text{includes OPE}}$ has a short log cut due to real-pion exchange
- Splitting is not unique: $\mathcal{T}_{\text{includes OPE}}$ must include contact term

Is the $\mathcal{T}$ what we are interested in?

- OPE process is an integral part of the three-to-three scattering, however,
- OPE is a "background" for the resonance pole

$$\mathcal{T} \xrightarrow{s \to s_{\text{pole}}} \frac{g^2}{s_{\text{pole}} - s} + \mathcal{B} + \dots$$

- Any production amplitude $2 \to 3$ neither include $\mathcal{B}$. It is convoluted with the source.

Denominator and self-energy

Two-body resonance

$$\hat{\mathcal{T}}_2(s) = \frac{g^2}{m^2 - s - ig^2\Phi_2(s)} \to \frac{1}{(m^2 - s)/g^2 - \Sigma_2(s)}$$

- Self-energy: $ig^2\Phi_2(s) \to \Sigma_2(s)$, Chew-Mandelstam to ensure analyticity.
- $\mathcal{K}$-matrix: $g^2/(m^2 - s)$ is uncontrolled real part

Three-body resonance

$$\hat{\mathcal{T}}_R(s) = \frac{1}{(m^2 - s)/g^2 - \Sigma(s)} = \frac{1}{\mathcal{K}_1^{-1}(s) - \Sigma(s)}$$

- Dispersion relation for the self-energy:

$$\Sigma(s) = \frac{s}{2\pi} \int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'(s' - s)} \int_{\text{Dalitz}(s')} \left|\hat{\mathcal{A}}_{R\to 1,2,3}(s', \sigma_1', \sigma_2')\right|^2 \mathrm{d}\Phi_3'$$

- The $\left|\hat{\mathcal{A}}_{R\to 1,2,3}(s, \sigma_1, \sigma_2)\right|^2$ is observable (+FSI) Dalitz-plot distribution

Iterative solution

Bethe-Salpeter / Blankenbeckler-Sugar / Lippmann-Schwinger / B-matrix (Mai/JPAC) approaches

$$\mathcal{T} = \mathcal{V} + \mathcal{V}\tau\mathcal{T}, \quad \mathcal{V} = \mathcal{V}_0 + \mathcal{V}_1$$

- Integral equation (system of eqs) that yields the unitary solution **if $\mathcal{V}$ includes OPE**, $\mathcal{V}_0$.
- The real part of $\mathcal{V}$ is unconstrained by unitarity ($\mathcal{K}$-matrix).

- Incorporates all three-body effects:
  - Three-body unitarity cut
  - Dynamics of the OPE
  - Triangle singularities
- Analyticity of $\mathcal{T}$ is not ensured
  - ? Spurious left-hand singularities
  - ? First-sheet wooly cut(©Aitchison)

Relation to BS equations

Two-potential decomposition [Nakano, PRC 26 (1982) 1123], [MM et al. JHEP 08 (2019) 080]

1. Potential has two terms ($\mathcal{V}_0$ includes OPE):

$$\mathcal{V} = \mathcal{V}_1 + \mathcal{V}_0\,.$$

2. Obtain $\mathcal{T}_0$:

$$\mathcal{T}_0 = \mathcal{V}_0 + \mathcal{V}_0\,\tau\,\mathcal{T}_0\,, \qquad \text{(the ladder: long-range)}$$

3. Find a solution in the form ($\mathcal{T}_0$ + something)

$$\mathcal{T} = \mathcal{T}_0 + (1 + \mathcal{T}_0\tau)\,\hat{\mathcal{T}}\,(1 + \tau\mathcal{T}_0)\,,$$

4. e.g. by solving,

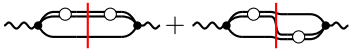
$$\hat{\mathcal{T}} = \mathcal{V}_0 + \mathcal{V}_0(\tau + \tau\mathcal{T}_0\tau)\hat{\mathcal{T}}\,, \qquad \text{(resonance: short-range)}$$

5*. can use algebraic inversion for $\hat{\mathcal{T}}$, ala K-matrix, $\hat{\mathcal{T}} - \hat{\mathcal{T}}^\dagger = \hat{\mathcal{T}}^\dagger X \hat{\mathcal{T}}$

$$\begin{aligned} X &= i\tau^\dagger \frac{1}{\rho_3}\tau + \tau\mathcal{T}_0\tau - \tau^\dagger\mathcal{T}_0^\dagger\tau^\dagger \\ &= (1 + \tau^\dagger\mathcal{T}_0^\dagger)\left[i\,\tau^\dagger\frac{1}{\rho_3}\tau + \tau^\dagger\mathcal{D}\tau\right](1 + \mathcal{T}_0\tau)\,. \end{aligned}$$

Physical meaning of the terms [MM et al. (JPAC), JHEP 08 (2019) 080]

$$\left|\hat{\mathcal{A}}_{R\to 1,2,3}\right|^2 = \underbrace{\left(1+\tau^\dagger \mathcal{T}_0^\dagger\right)}_{\text{initial-state interaction}} \overbrace{\left[\tau^\dagger \frac{1}{\rho_3}\tau + \tau^\dagger \mathcal{D}\tau\right]}^{\left|\hat{\mathcal{A}}^{(\text{naive})}_{R\to 1,2,3}\right|^2} \underbrace{\left(1+\mathcal{T}_0\tau\right)}_{\text{final-state interaction}}.$$



Relation to KT

- Khuri-Treiman framework: two-body unitarity continued to the decay domain
- Gives the rescattering effect, systematically accounts for triangle diagrams

$$\mathcal{F}_{\text{decay}} = \underbrace{\left(1+\mathcal{T}_{\text{KT}}\tau\right)}_{\text{final-state interaction}} \hat{C}$$

- Originally, s is a fixed parameter, however,
- $\mathcal{T}_{\text{KT}}$ is a valid construct for $\mathcal{T}_0$ [Aitchison(1986), Pasquier(1968)]

The $\left|\hat{\mathcal{A}}_{R\to 1,2,3}(s,\sigma_1,\sigma_2)\right|^2$ is observable (+FSI) Dalitz-plot distribution

“Ladders and resonances” proposal for the 3b unitarity:

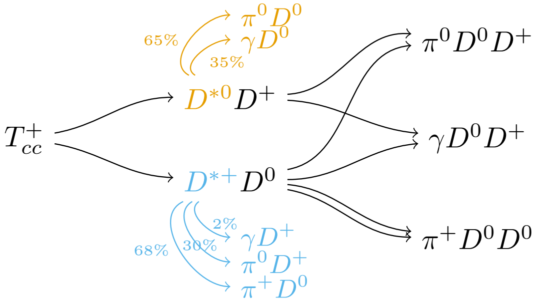
1. Dispersion relation for the self-energy:

$$\Sigma(s) = \frac{s}{2\pi} \int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'(s'-s)} \int_{\text{Dalitz}(s')} \left|\hat{\mathcal{A}}_{R\to 1,2,3}(s', \sigma'_1, \sigma'_2)\right|^2 \mathrm{d}\Phi'_3$$

2. Dalitz plot $\mathcal{A}_{R\to 1,2,3}(s', \sigma'_1, \sigma'_2)$ is corrected for rescattering (e.g. KT): resonances of “modified” lineshape

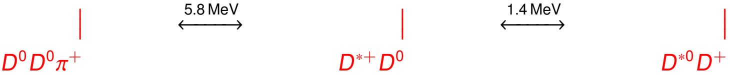
Application to T_{cc}^{+}

T_{cc}^{+} decay amplitude [LHCb, Nature Physics (2022) & Nature Communicatitions, 13, 3351]



Model assumptions:

- $J^P = 1^+$: S-wave decay to D^*D
- T_{cc}^{+} is an isoscalar: $\left|T_{cc}^{+}\right\rangle_{I=0} = \left\{\left|D^{*0}D^{+}\right\rangle - \left|D^{*+}D^{0}\right\rangle\right\} / \sqrt{2}$
- No isospin violation in couplings to $D^{*+}D^{0}$ and $D^{*0}D^{+}$



T_{cc}^{+} self-energy and hadronic reaction amplitude

[LHCb, Nature Physics (2022) & Nature Communicatitions, 13, 3351]

Dynamic amplitude of $D^*D \to D^*D$ scattering:

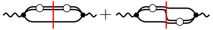
$$T_{2\times2}(s) = \frac{K}{1-\Sigma K} = \frac{K(m^2-s)}{m^2 - s - i\,g^2\left(\rho_{\text{tot}}(s) + i\xi(s)\right)}$$

where K is the isoscalar potential:

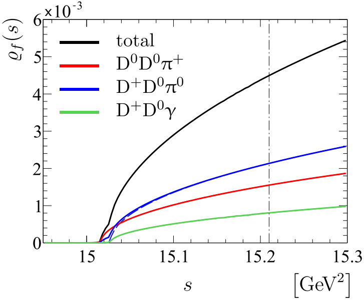
$$K = \frac{1}{m^2-s}\begin{pmatrix} g\cdot g & -g\cdot g \\ -g\cdot g & g\cdot g \end{pmatrix},$$

and Σ is the loop function:

$$\Sigma(s) = [D^*D \to DD\pi(\gamma) \to D^*D]$$

$= \Big[$  $\Big].$

$$\operatorname{Im}\left[\begin{pmatrix} g \\ -g\end{pmatrix}^{\dagger}\Sigma(s)\begin{pmatrix} g \\ -g\end{pmatrix}\right] = \rho(s)$$



D^* decays are accounted for.

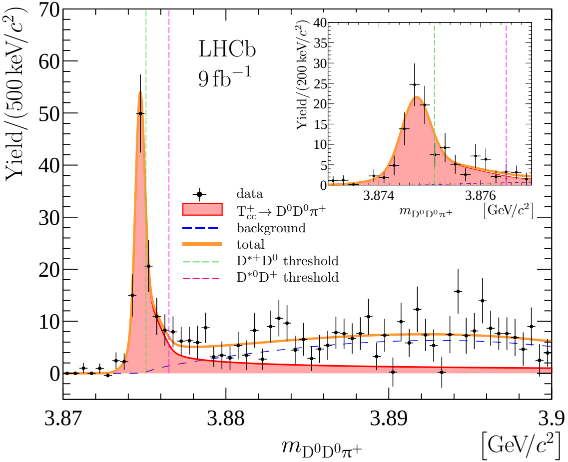
Model parameters: $|g|^2$ and m^2 – bare mass and coupling

Fit to the spectrum

Unitarized model

- The signal shape does not depend on $|g|$ for $|g| \to \infty$.
- The lower limit: $|g| > 7.7(6.2)\,\mathrm{GeV}$ at 90(95)% CL
- δm_U is the only parameter

Parameter	Value
N	186 ± 24
δm_{U}	$-359 \pm 40\,\mathrm{keV}/c^2$
$\|g\|$	3×10^4 GeV (fixed)



No direct sensitivity to the width, the value is driven by the model

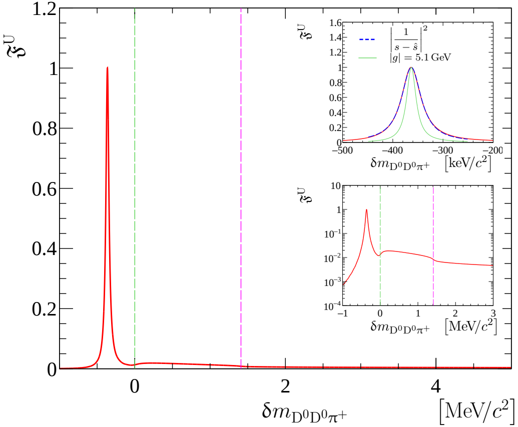
True mass spectrum

The resolution removed

[Nature Communicatitions, 13, 3351 (2022)]

- The narrow resonance peak below the lowest threshold
- Long tail with cusps at the the thresholds

$$T(s) = \frac{\text{production}}{\text{rescattering}(s\,|\,\delta m, g)}$$



The obtained analytic “formula” for matrix element enables us:

- Obtain mass and width – pole position *à la* [MM et al. (JPAC), PRD 98 (2018) 096021]
- Compute scattering parameters [MM, 2203.04622]

Analytic continuation

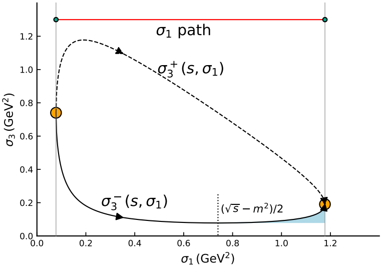
Details on analytic continuation [MM et al. (JPAC), PRD 98 (2018) 096021]

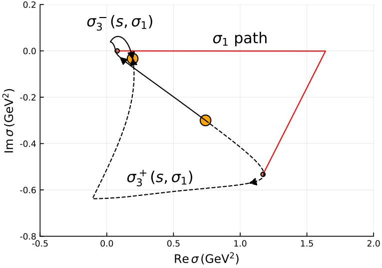
$$\hat{\mathcal{T}}^{-1} = \frac{m^2 - s}{g^2} - \Sigma(s)$$

- $\Sigma(s)$ is a dispersion integral of three-body integral

Function $\Sigma(s)$ on the unphysical sheet

$$\Sigma_{II}(s_{\text{complex}}) = \underbrace{\Sigma_{I}(s_{\text{complex}})}_{\text{dispersion integral (easy)}} + \underbrace{2i\rho(s_{\text{complex}})}_{\text{Dalitz integral (tough)}}$$

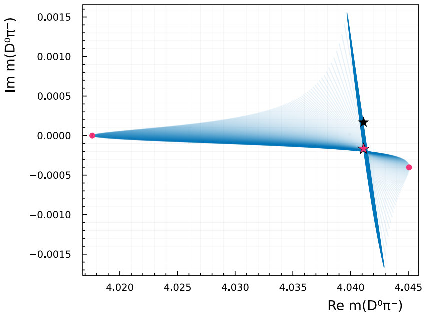


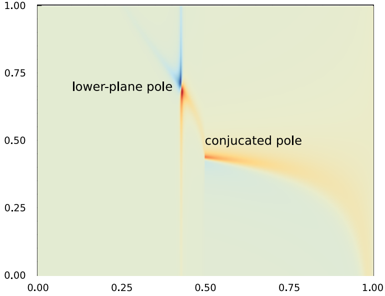


The T_{cc}^{+} Dalitz complex

$$\int_{\text{Dalitz}} [\mathcal{F}_{D^*}(\sigma_2) + \mathcal{F}_{D^*}(\sigma_3)] \cdot [\mathcal{F}^*_{D^*}(\sigma_2) + \mathcal{F}^*_{D^*}(\sigma_3)]\, \mathrm{d}\sigma_2 \mathrm{d}\sigma_3$$

- Conjugation implies changing the sheet, $\mathcal{F} \to \mathcal{F}^{I}$, $\mathcal{F}^* \to \mathcal{F}^{II}$,
- $\mathcal{F}_{D^*}(\sigma_2)\mathcal{F}^*_{D^*}(\sigma_2)$ is easy – no singularity in σ_3
- $\mathcal{F}^*_{D^*}(\sigma_2)\mathcal{F}_{D^*}(\sigma_3)$ is little tricky
  - pole below in σ_2,
  - pole above in σ_3



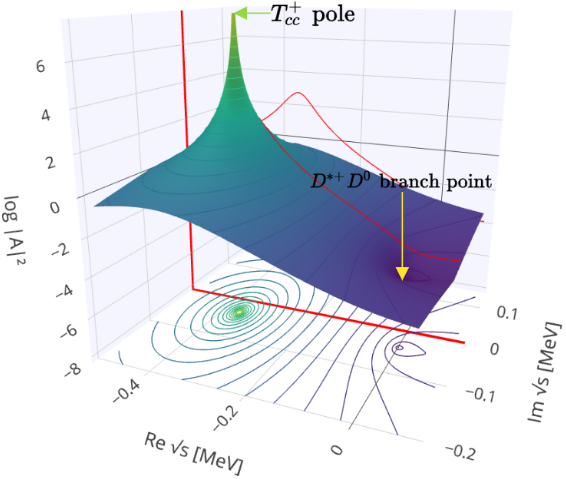


Fundamental resonance parameters

[interactive]

Mass and width – position of the complex pole of the reaction amplitude

- Analytic continuation is non-trivial due to three-body decays [MM et al. (JPAC), PRD 98 (2018) 096021]



The pole parameters:

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0}\,\text{keV}\,,$$
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\,\text{keV}\,.$$

Scattering length and effective range

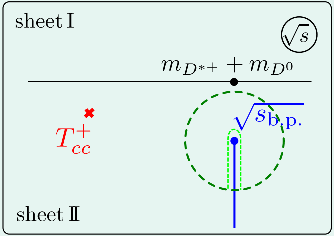
Expansion of the phase shift

$$k \cot \delta(k) = \frac{1}{a} + r\frac{k^2}{2} + O(a^3k^4)\,.$$

Equivalently,

$$T^{-1} = N\left(\frac{1}{a} + r\frac{k^2}{2} + O(a^3k^4) - ik\right)\,.$$

For 3b scattering (D^0D^{*+})



Is it possible to make a proper definition of the a^{-1} and r?

1. Match the analytic structure of T,
2. Make convergent series

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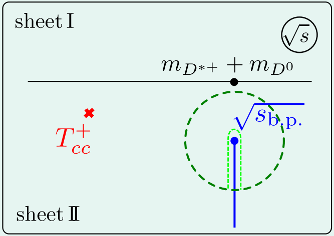


Is it possible to make a proper definition of the a^{-1} and r?

1. Match the analytic structure of T,
2. Make convergent series

1 – yes, $k(s, m_{D^0}, m_{D^{*+}} - i\Gamma_{D^{*+}}/2)$,
2 – yes(?)

The regular function is not linear

$$T^{-1}(s) = N\Big(R(s) - ik\Big), \quad R(s) = \frac{1}{a} + r\frac{k^2}{2} + O(a^3k^4)\,.$$

- *ik* is the singular part that defines the discontinuity
  - Normalization *N* is found by matching discontinuity on left and right
- $R(s)$ is the regular part at the complex $s_{\text{b.p}}$
  - Expansion is done using the Cauchy integrals

Nominal (LHCb) model:

$$r_{DD^*} < -4.4\,\text{fm}$$

From comparison of the models:

$$\Delta r_{D^{*0}D^+} = -3.73 + 0.30i\,\text{fm}\,,$$
$$\Delta r_{\text{ope}} = -0.94 - 0.28i\,\text{fm}\,.$$

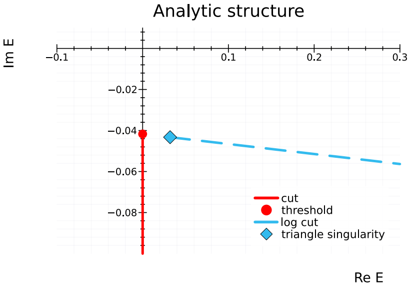
- Use the limit case, $g \to \infty$, to get upper limit, $r_{DD^*} < r_{\text{limit}}$

Model	$1/\Re\, a^{-1}$ [fm]	r [fm]
full	-6.0	$-4.37 + 0.47i$
full excl. D^{*0}	-7.3	$-0.64 + 0.17i$
$\pi^+ D^0 D^0$ with two D^{*+}	-7.3	$-0.79 + 0.27i$
$\pi^+ D^0 D^0$ with one D^{*+}	-7.6	$0.15 - 0.01i$
$D^{*+} D^0$ complex mass	-7.6	$0.16 - 0.01i$

Nearby Triangle Singularity

The Triangle Singularity is a logarithmic branch related to the pion-exchange.

- Located at the unphysical sheet, hidden by $D^0 D^{*+}$ branch point



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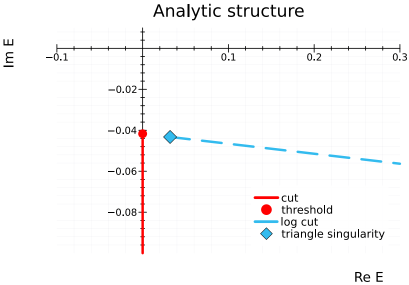
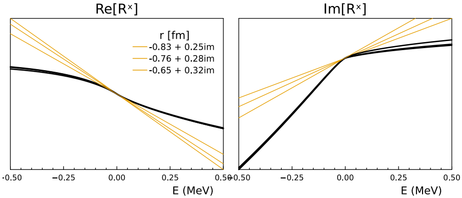


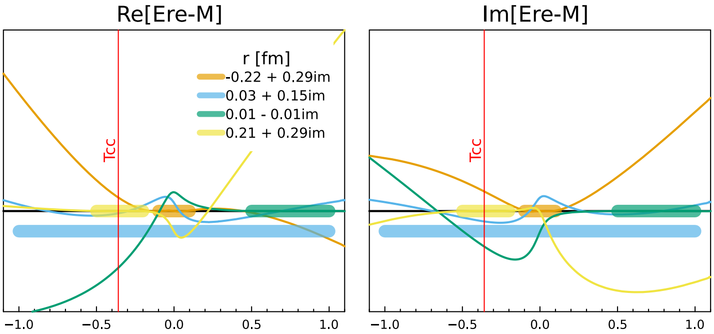


The regular function is not linear

Can I fit scattering parameters?

Often used approach is to fit T^{-1} by expansion series $N(a^{-1} + rk^2/2 - ik)$.

- Result strongly depends on the range of the fit



- Difference of T^{-1} and $N(a^{-1} + rk^2/2 - ik)$
- Can much two function only in small range < 0.1MeV

Summary

Three-body **rescattering** on Dalitz, and three-body **resonance dynamics** are closely connected

- [arXiv:2212.11767]: systematic studies of FSI in 3π
- [arXiv:2203.04622]: analytic continuation of T_{cc}^{+} amplitude, effective range discussion

Summary

Three-body **rescattering** on Dalitz, and three-body **resonance dynamics** are closely connected

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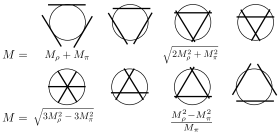
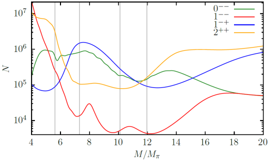
Backup

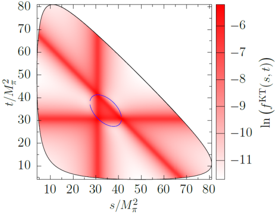
Interefence ring

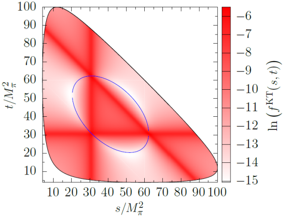
The structures (bumps and dips) are related to the geometry of resonances on the Dalitz plot

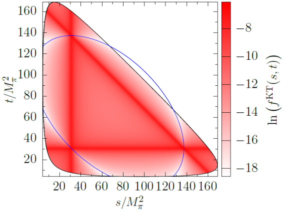
- One non-obvious is the **interference** ring





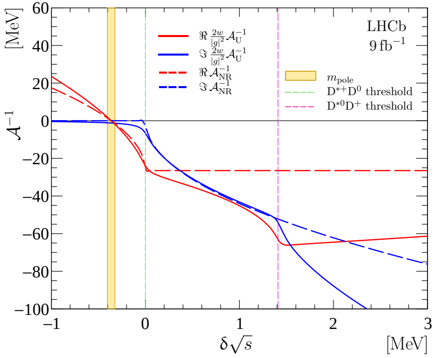






Effective range and Weinberg compositeness

[LHCb, arXiv:2109.01056]



$$\mathcal{A}_{NR}^{-1} = \frac{1}{a} + r\frac{k^2}{2} - ik + \mathcal{O}(k^4)\,,$$

$$\frac{2}{|g|^2}\mathcal{A}_U^{-1} = -\left[\xi(s) - \xi(m_U^2)\right] + 2\frac{m_U^2 - s}{|g|^2} - i\varrho_{tot}\,.$$

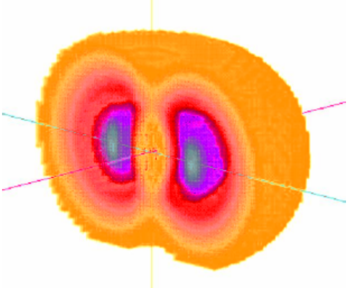
Matching:

- $r = 16w/|g|^2$, where w is a normalization factor between ρ and k
- w excludes the contribution of the second threshold
- does not have the $1/\sqrt{\delta}$ term

- T_{cc}^+: $a = (-7.16 \pm 0.51) + i(1.85 \pm 0.28)$ fm
- T_{cc}^+: r is negative in the model: $0 < -r < 11.9(16.9)$ fm at 90(95) % CL
- T_{cc}^+: $1 - Z > 0.48(0.42)$. T_{cc}^+ is consistent with the molecule

Comparison to the deuteron

Deuteron [Garcon, Van Orden(2001)]



- Presumably molecule
- $1 - Z \approx 1$
- $R_{\text{charge}} = 2.1\,\text{fm}$
- $R_{\text{matter}} = 1.9\,\text{fm}$
- $a = -5.42\,\text{fm}$
- $r = 1.75\,\text{fm}$

Tetraquark T_{cc}^{+} [LHCb, arXiv:2109.01056]

[compact cc core]

[$\bar{u}\bar{d}$ cloud]

- Expected to be atomic
- $1 - Z \geq 0.48$ at 90% CL
- $R_{\text{charge}} = ??$
- $R_{\text{matter}} = ??$
- $a = -7.16\,\text{fm}$
- $r > -11.9\,\text{fm}$ at 90% CL