Calculations of structure and reaction observables with the Gamow Shell Model

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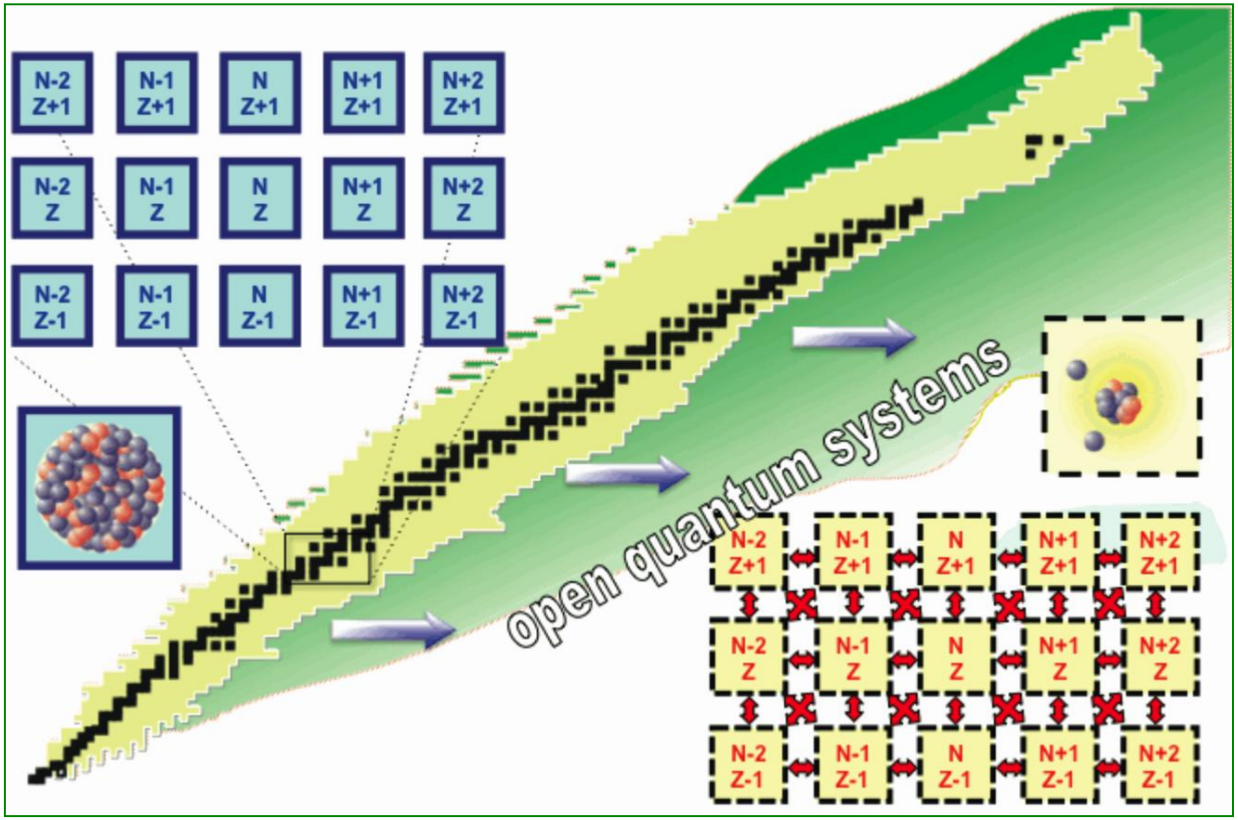
INT, The Quantum Physics of Stars, August 17th - 21st 2026

Scientific context

Nuclear chart

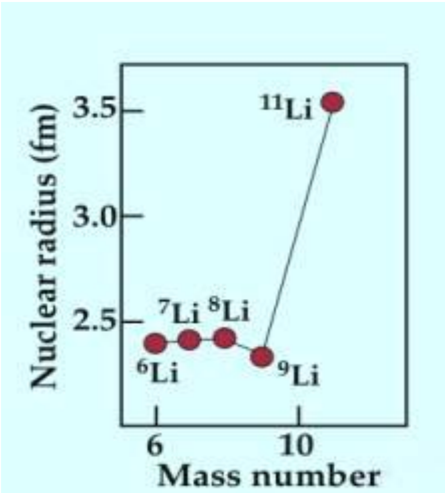


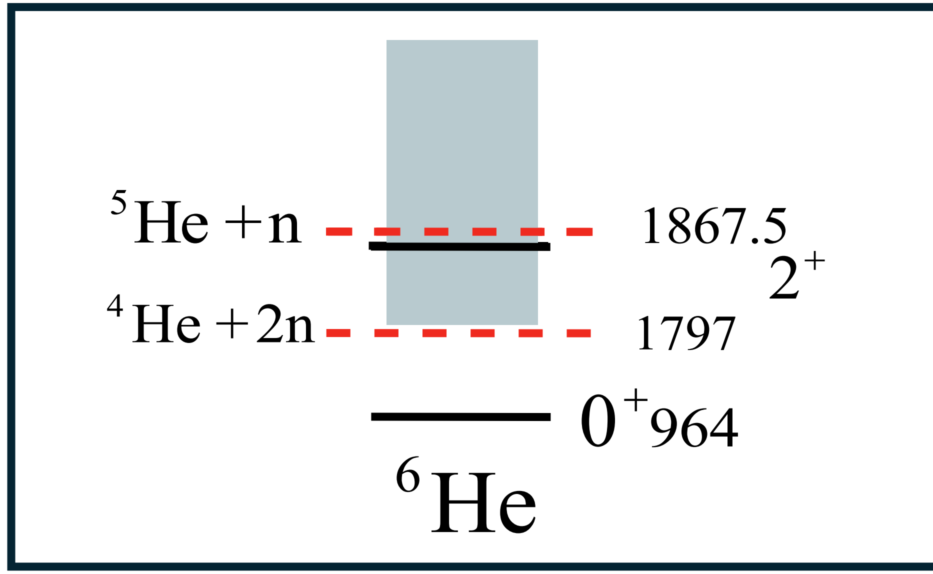
Experimental interest
Study of nuclei far from the valley of stability
Many efforts made to study drip-line nuclei

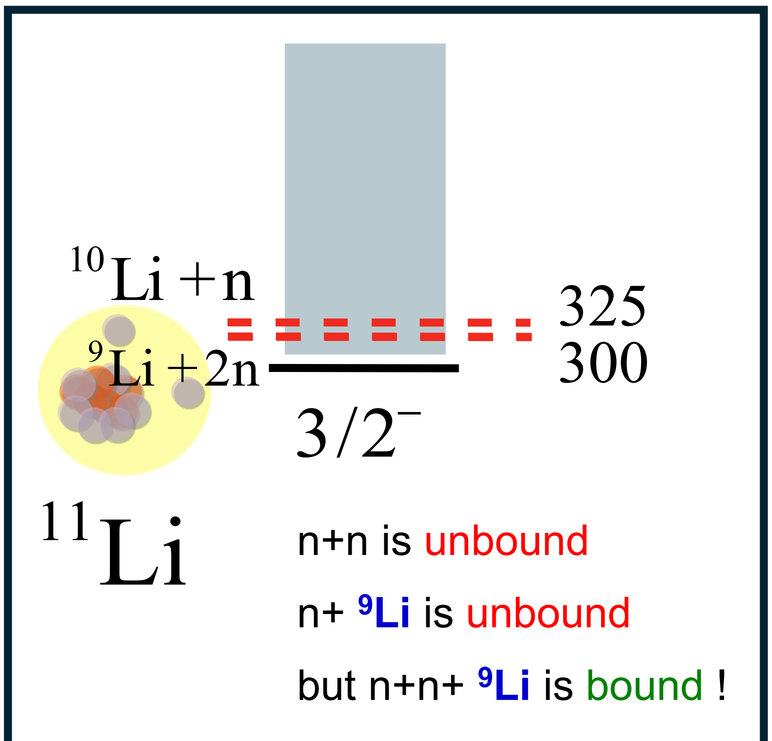
N. Michel, W. Nazarewicz, J. Okołowicz, M. Płoszajczak,
J. Phys. G: Nucl. Part. Phys., **37** 064042 (2010)

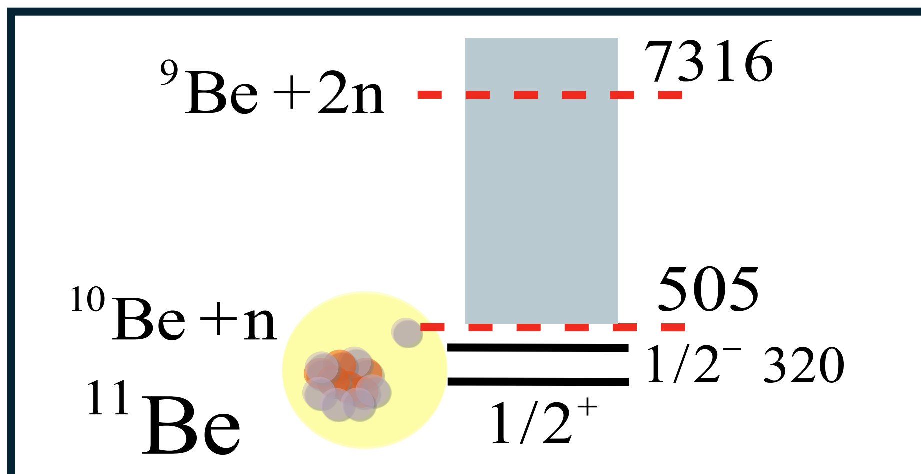
Scientific context

Halos, resonances





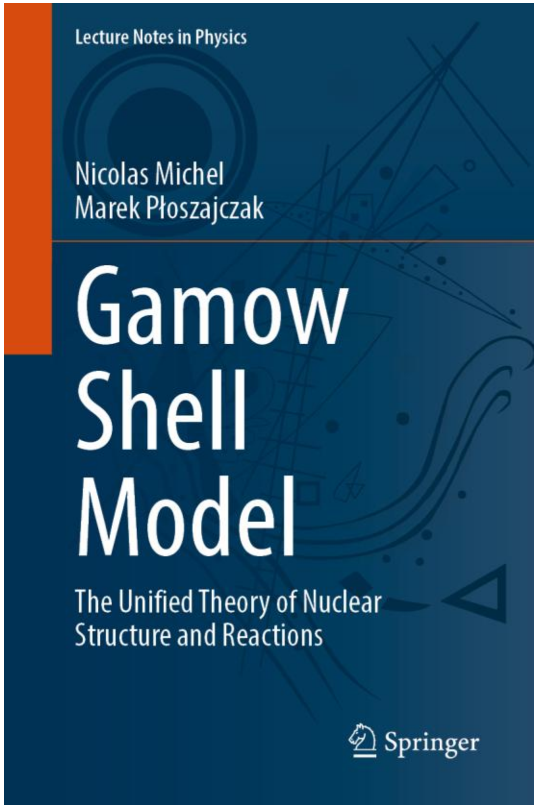




The Gamow Shell Model: the unified theory of nuclear structure and reactions

Authors : N. Michel and M. Płoszajczak

Publisher : Lectures Notes in Physics (Springer)



Background

Functional analysis, linear algebra, differential equations, standard quantum mechanics

Main topics

Introduction with one-body and two-body systems

Many-body theory of complex-energy physics

Halos and resonances in molecules and nuclei

Nuclear structure and reactions

Exercises and codes

Theoretical details about used methods

Codes available from internet

Problematic

Nuclear structure and reactions

Drip-line nuclei

Stable, beta unstable nuclei : well understood from effective/realistic interactions

Nuclei at drip lines : continuum coupling, halos, resonances, shell gaps modification, new magic numbers

Theoretical estimates of nuclear reaction rates often significantly differ from experimental data

Cross sections : influence of low-lying resonances in the nuclear spectrum

Nuclei at drip-lines cannot be reproduced with the standard tools of nuclear theory : new models needed

No-core shell model with continuum

Continuum coupling, realistic interactions in a no-core picture

Advantage : reliable description of the structure of light nuclei

Inconvenient : very expensive numerically, only one mass partition used in the expansion

It cannot be utilized for A > 16 even with two-body forces only.

Gamow shell model

Configuration interaction picture with the Berggren basis; Hermitian Hamiltonian but not in Hilbert space

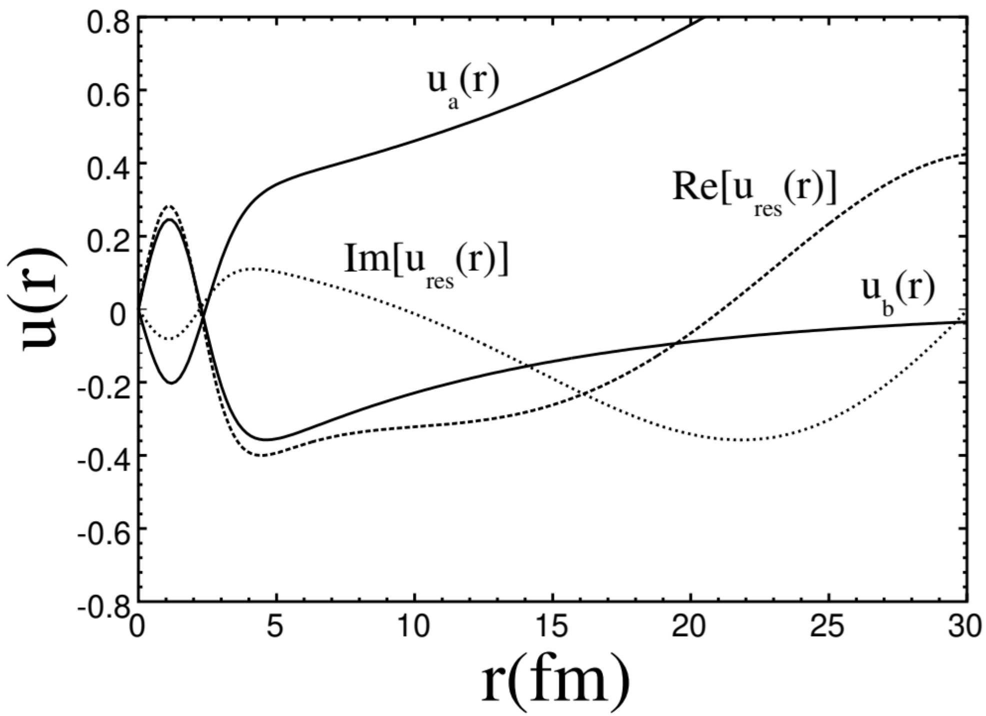
Inter-nucleon correlations + continuum coupling

Core and no core approaches, effective and realistic interactions available

Versatile models for light and medium nuclei

Gamow states

Quasi-stationary solutions of the time-dependent Schrödinger equation



$$\Psi(x,t) = \Phi(x) e^{i\tilde{E}t/\hbar}$$
$$H|\Phi\rangle = \tilde{E}|\Phi\rangle$$

$$\tilde{E} = E - i\Gamma/2$$

$$u''(k,r) = \left(\frac{\ell(\ell+1)}{r^2} + v_l(r) - k^2\right) u(k,r)$$

$$u(k,r) = C^+ H^+_{\ell,\eta}(kr)$$
$$r \geq R$$

Resonances : unbound in time-independent formalism

Bound and resonance states: same object

Same qualitative behavior inside the nucleus

Asymptote is different for bound, antibound, resonance states

Unbound states diverge on the real axis

Complex scaling necessary

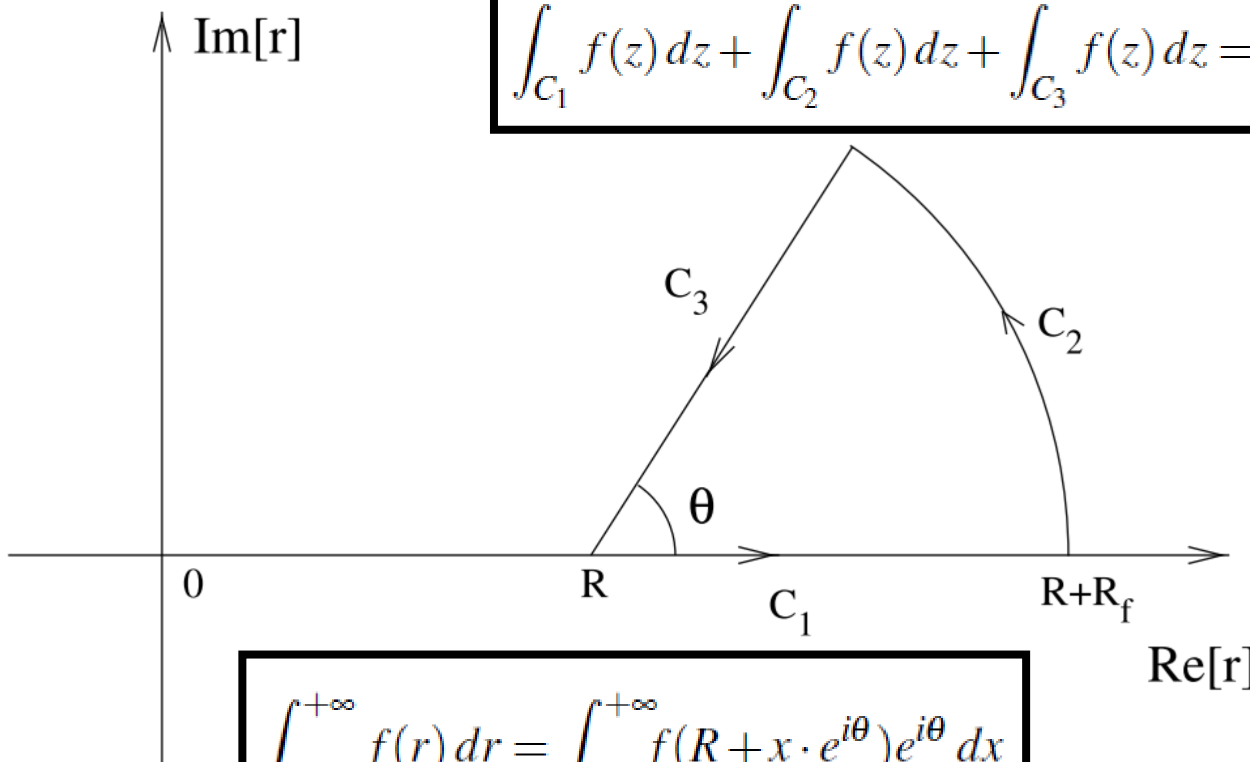
N. Michel, M. Płoszajczak,

The Gamow Shell Model: the unified theory of nuclear structure and reactions, Lecture Notes in Physics, Springer International Publishing (2021)

Gamow states

Complex scaling

$$\int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz + \int_{C_3} f(z)\,dz = 0$$



$$\int_R^{+\infty} f(r)\,dr = \int_0^{+\infty} f(R + x \cdot e^{i\theta}) e^{i\theta}\,dx$$

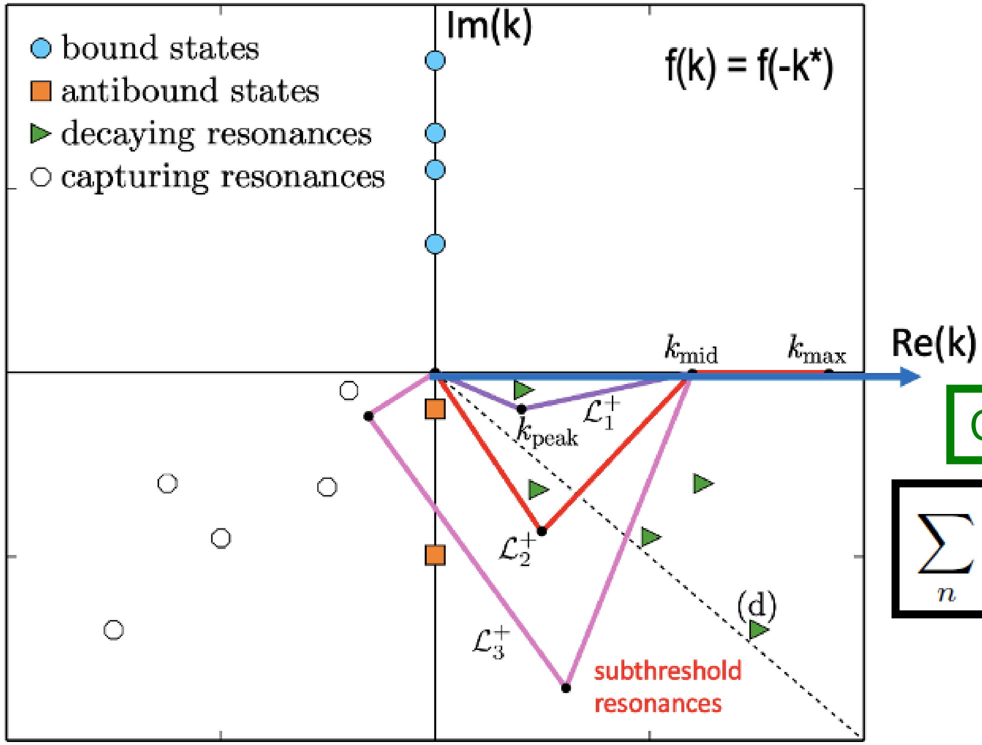
Unbound states do not belong to the Hilbert space
Resonance states: localized in the complex plane

Complex scaling method to calculate matrix elements
Bound and resonance states: normalized
Scattering states: normalized with a Dirac delta

N. Michel, M. Płoszajczak,
The Gamow Shell Model: the unified theory of nuclear structure and reactions,
Lecture Notes in Physics, Springer International Publishing (2021)

Gamow states

Berggren basis



Completeness relation of a partial wave

$$\sum_n u_n(r)u_n(r') + \int_{L+} u_k(r)u_k(r')\,dk = \delta(r-r')$$

Efficient discretization of the L^+ contour with Gauss-Legendre quadrature

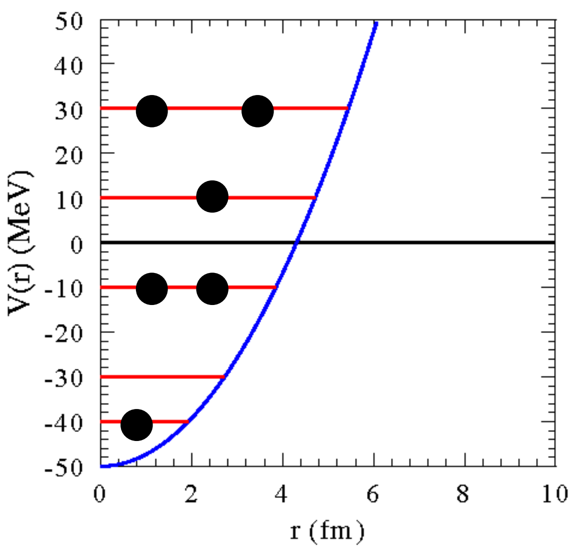
$$\sum_n u_n(r)u_n(r') + \sum_{i=1}^{N_d} u_i(r)u_i(r') \simeq \delta(r-r')$$

N. Michel, M. Płoszajczak,
The Gamow Shell Model: the unified theory of nuclear structure and reactions,
Lecture Notes in Physics, Springer International Publishing (2021)

Gamow Shell Model

Standard shell model

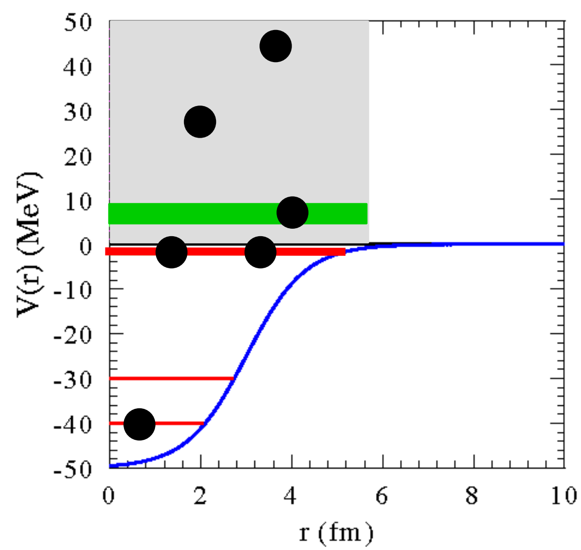
Closed quantum system description



Localized states
No continuum

Gamow Shell Model

Open quantum system description



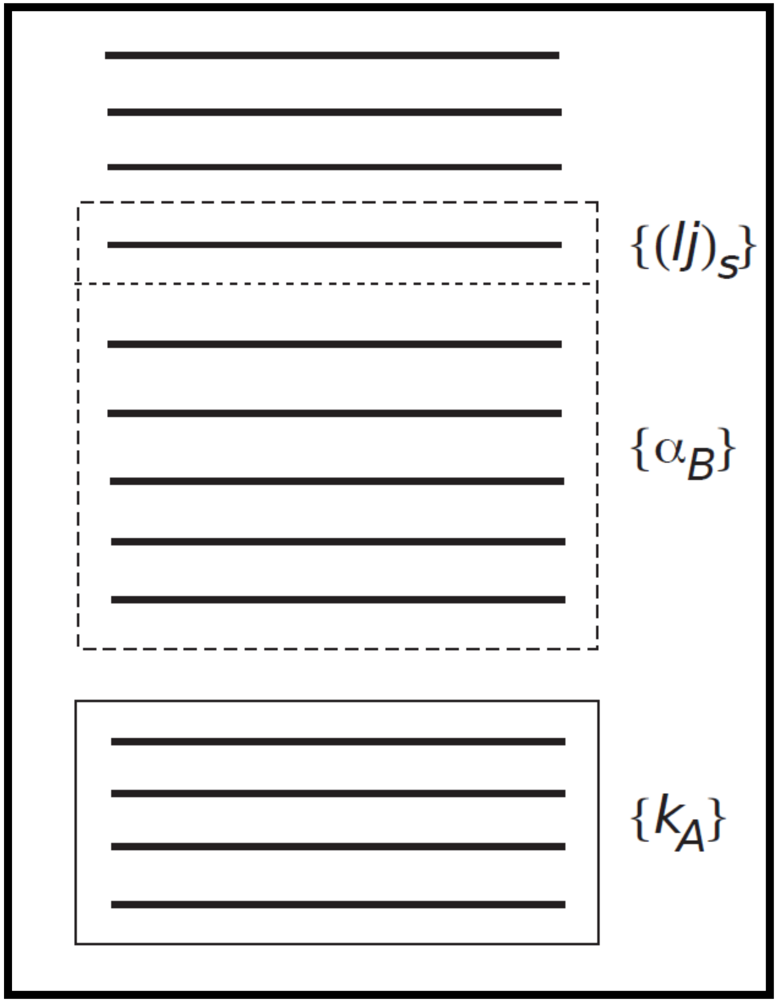
Localized states
Weakly bound/resonant states
Halo + resonance many-body states

Discretized many-body completeness relation

$$\sum_n |\Psi_n\rangle \langle\widetilde{\Psi_n}| \simeq \hat{1}$$

Gamow Shell Model

Density matrix renormalization group



Successive addition of degrees of freedom
Scattering one-body states added one by one in GSM

Basis recalculation and Hamiltonian recalculated
when the dimension reaches a given limit

k_A : fixed reference space
i_B : density matrix renormalization group space
generated by adding new (lj) shells

Truncation criterion from the density matrix $\langle\psi \,||\, [a^+_\alpha\, a_\beta]^j \,||\, \psi\rangle$
Density matrix : eigenvectors with the largest eigenvalues kept

$$|\Psi_J\rangle = \sum_{k_A, i_B} c_{i_B}^{k_A} \{|k_A\rangle \otimes |i_B\rangle\}^J$$

J. Rotureau et al., Phys. Rev. Lett. **97** 110603 (2006)
J. Rotureau et al., Phys. Rev. C **79**, 014304 (2009)

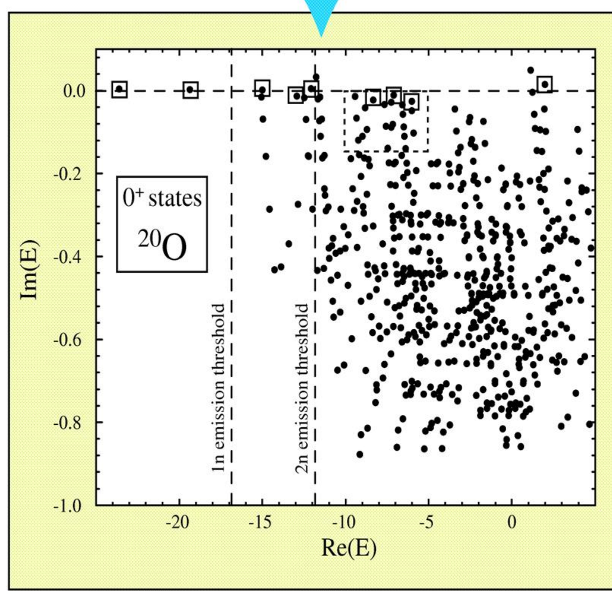
Gamow Shell Model

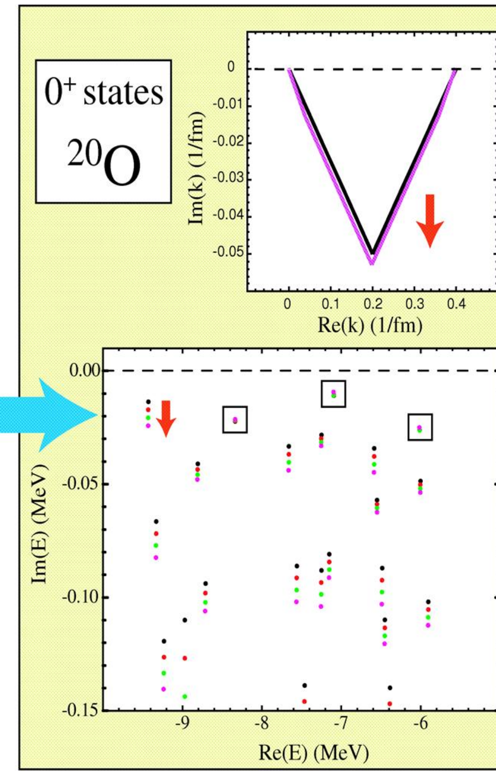
Diagonalization of GSM matrices

Complex symmetric Hamiltonian matrix
from basis states asymptotes

Resonant eigenstates
hidden among scattering eigenstates

Stability of the 'physical' states





1) Diagonalization in pole space

2) Diagonalization in full space

3) Identification with the overlap method

N. Michel, W. Nazarewicz, M. Płoszajczak, J. Okołowicz,
Phys. Rev. C, **67** 054311 (2003)

Gamow Shell Model

Cluster Orbital Shell Model

Problematic

3A degrees of freedom (particle coordinates)

3(A-1) physically (translational invariance) → spurious states

$$\hat{H} = \hat{T} + \hat{U} + \hat{V} = \sum_{i\in \mathrm{val}}\left(\frac{\mathbf{p}_i^2}{2\mu_i} + \hat{U}_i(\mathbf{r}_i)\right) + \frac{1}{M_{\mathrm{core}}}\sum_{(i<j)\in \mathrm{val}}\mathbf{p}_i\cdot\mathbf{p}_j + \sum_{(i<j)\in \mathrm{val}}\hat{V}_{i,j}$$

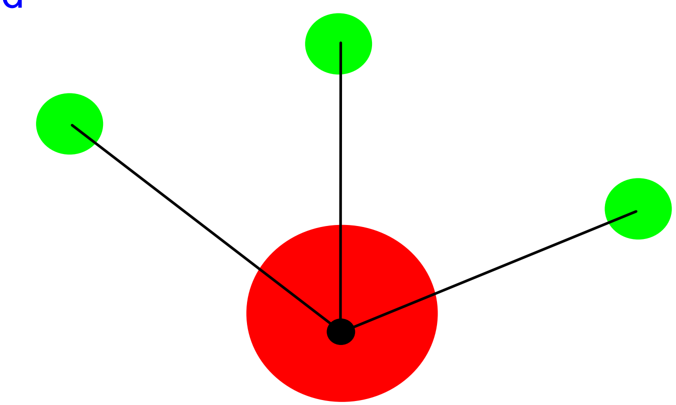
Y. Suzuki, K. Ikeda, Phys. Rev. C **38**, 410 (1988)

Standard shell model

Lawson method (no-core shell model) : spuriosity removed

Calculation in a major shell : non spurious

Harmonic oscillator basis only



$$\vec{r} = \vec{r}_{lab} - \vec{R}_{core}$$

Cluster orbital shell model (COSM)

Relative core coordinates → no center of mass excitation

Center of mass handled by a recoil term in the Hamiltonian

Formal use identical to laboratory coordinates

Definition of Hamiltonian directly in COSM frame

Calculations with COSM and Jacobi coordinate models very close

Gamow Shell Model - coupled-channels

Reaction formalism

GSM many-body scattering states : complex asymptotic behavior from Hamiltonian diagonalization

Decomposition in incoming and outgoing channels impossible to devise in practice

GSM unsuitable for reaction observables : another model must be devised.

Channel representation of the Schrödinger equation

$$\hat{H}\,|\Psi^{J_A}_{M_A}\rangle = E\,|\Psi^{J_A}_{M_A}\rangle$$

$$|\Psi^{J_A}_{M_A}\rangle = \hat{\mathscr{A}}\{|\Psi^{J_T}_{T}\rangle \otimes |\Psi^{J_P}_{P}\rangle\}^{J_A}_{M_A}$$

$$|\Psi^{J_A}_{M_A}\rangle = \sum_c \int_0^{+\infty} \left(\frac{u_c(r)}{r}\right) |(c,r)\rangle\, r^2 dr$$

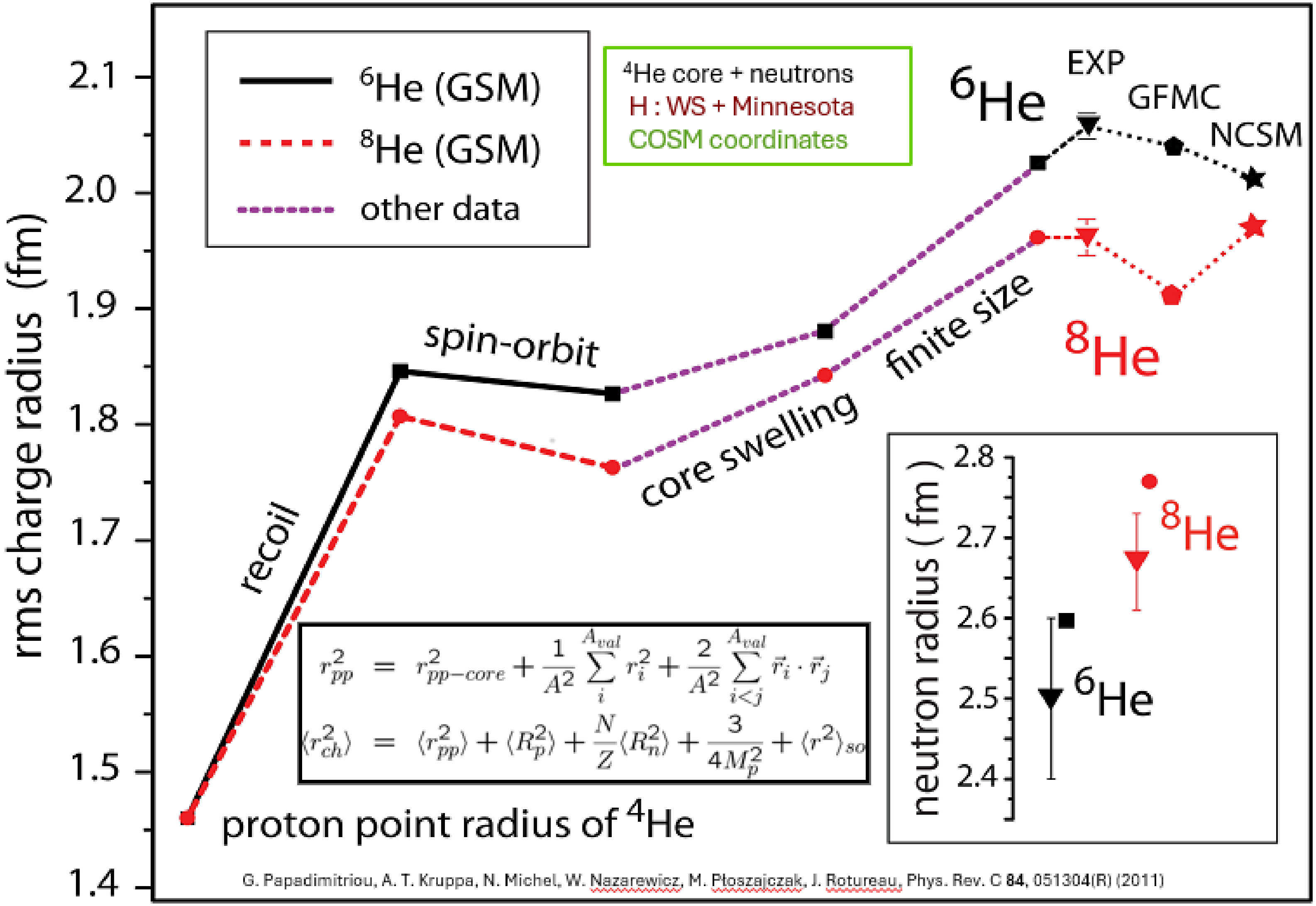
$$\sum_c \int_0^{+\infty} \left(H_{cc'}(r,r') - E\; N_{cc'}(r,r')\right) u_c(r')dr' = 0$$

$$H_{cc'}(r,r') = rr'\,\langle (c,r)|\,\hat{H}\,|(c',r')\rangle$$
$$N_{cc'}(r,r') = rr'\,\langle (c,r)|(c',r')\rangle$$

$$H_{cc'}(r,r') = \left(\frac{\hbar^2}{2m}\left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2}\right) + E_T\right)\delta(r-r')\;\delta_{cc'} + U_{\text{basis}}(r,r')\;\delta_{cc'} + V_{cc'}(r,r')$$

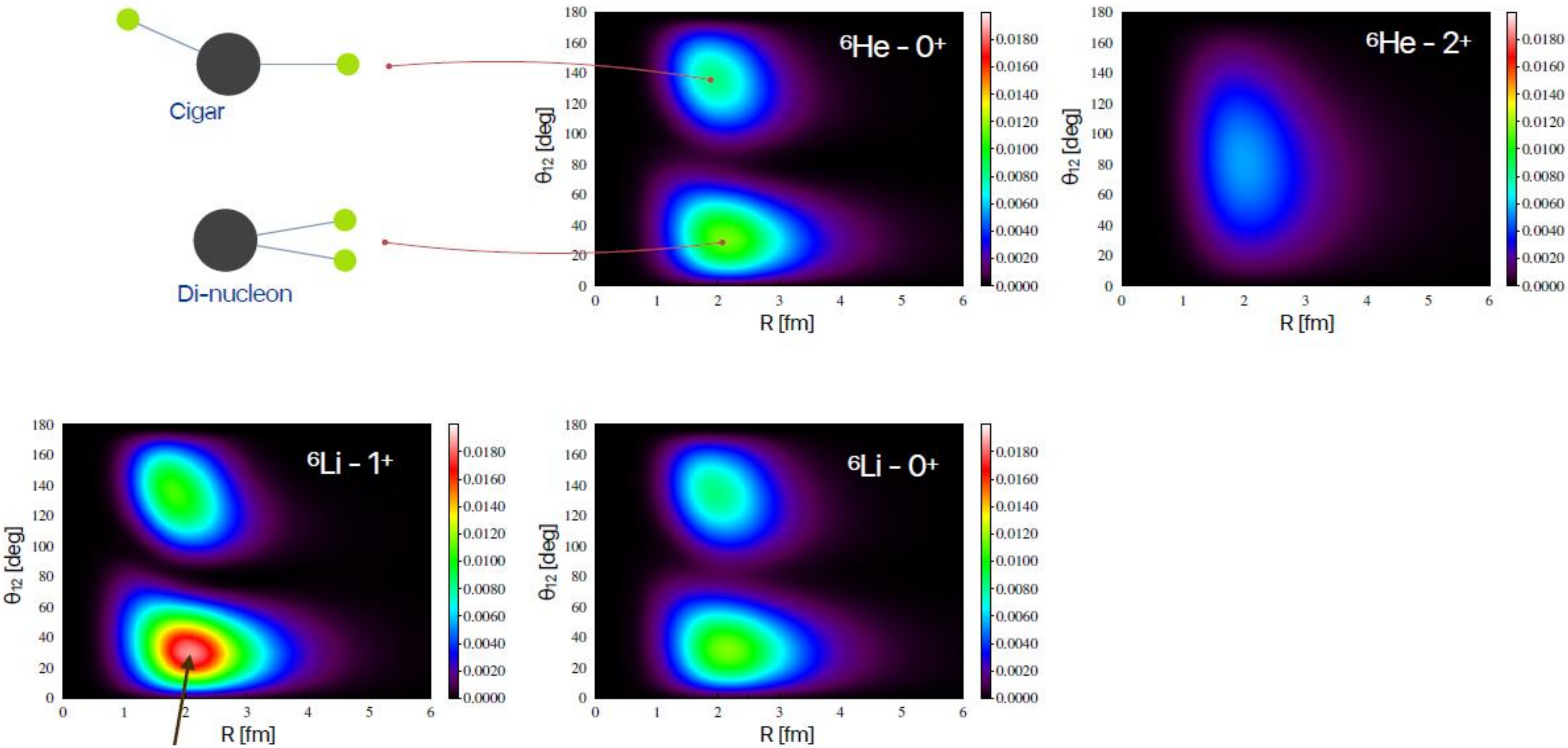
Gamow Shell Model applications

^{6,8}He charge radii



Gamow Shell Model applications

Correlation densities

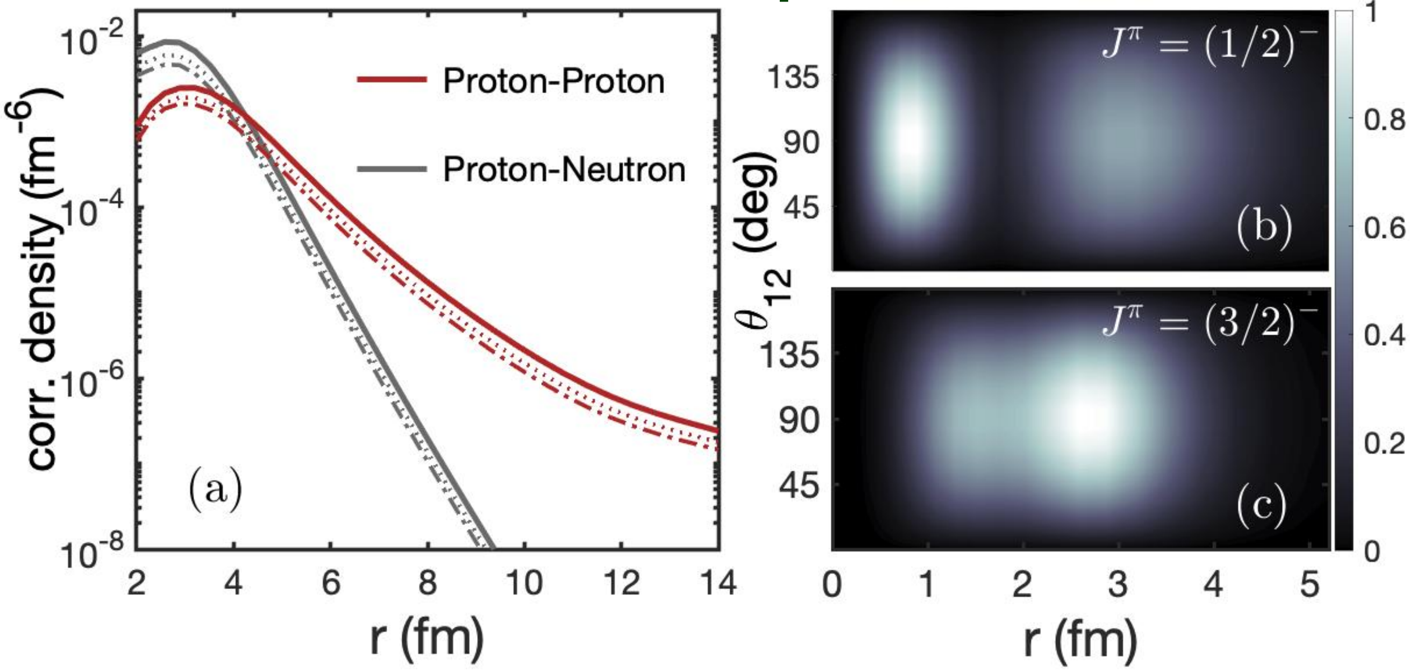


Strong T=0 deuteron clusterization/correlation

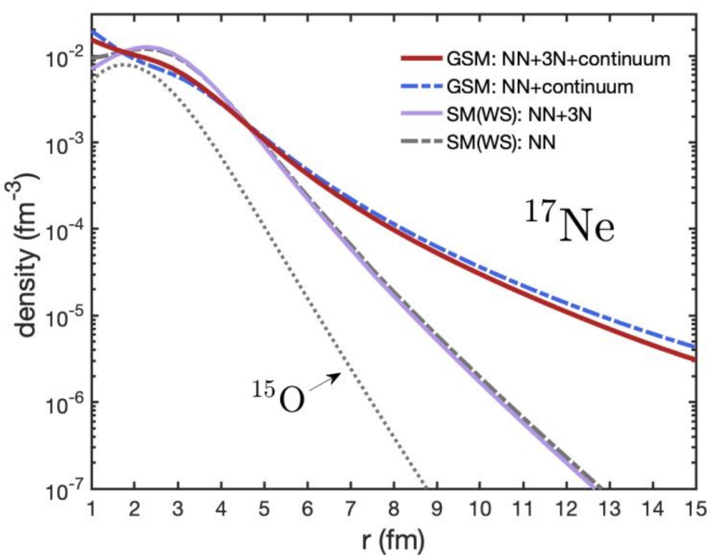
Y. Jaganathen, R. M. Id Betan, N. Michel, W. Nazarewicz, M. Płoszajczak, Phys. Rev. C **96**, 054316 (2017)

Gamow Shell Model applications

Two-proton halo of ^{17}Ne



Y.Z. Ma, F.R. Xu, N. Michel,
S. Zhang, J.G. Li, B.S. Hu,
L. Coraggio, N. Itaco, A. Gargano,
Phys. Lett. B **808**, 135673 (2020)

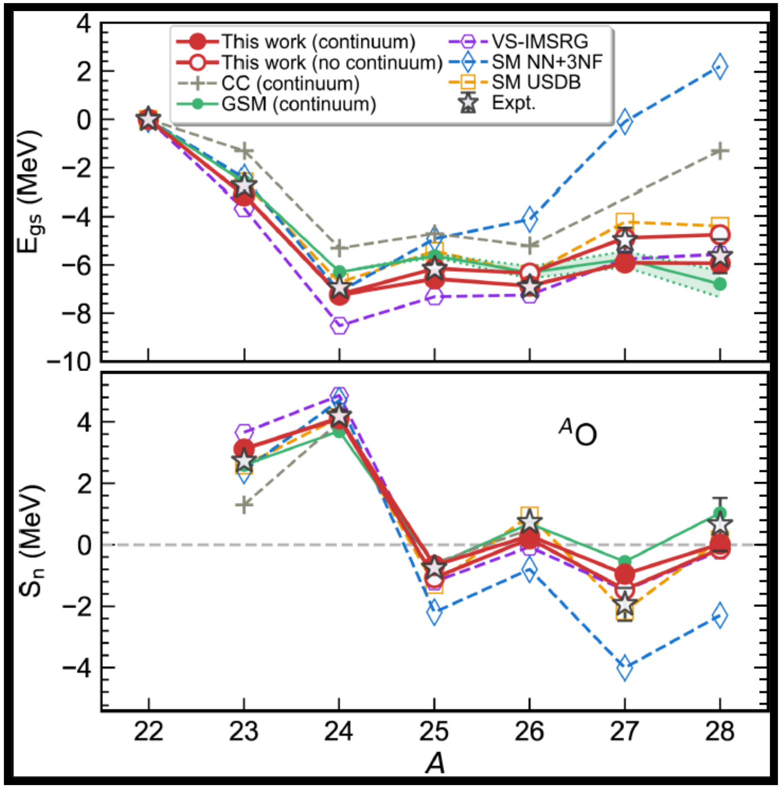


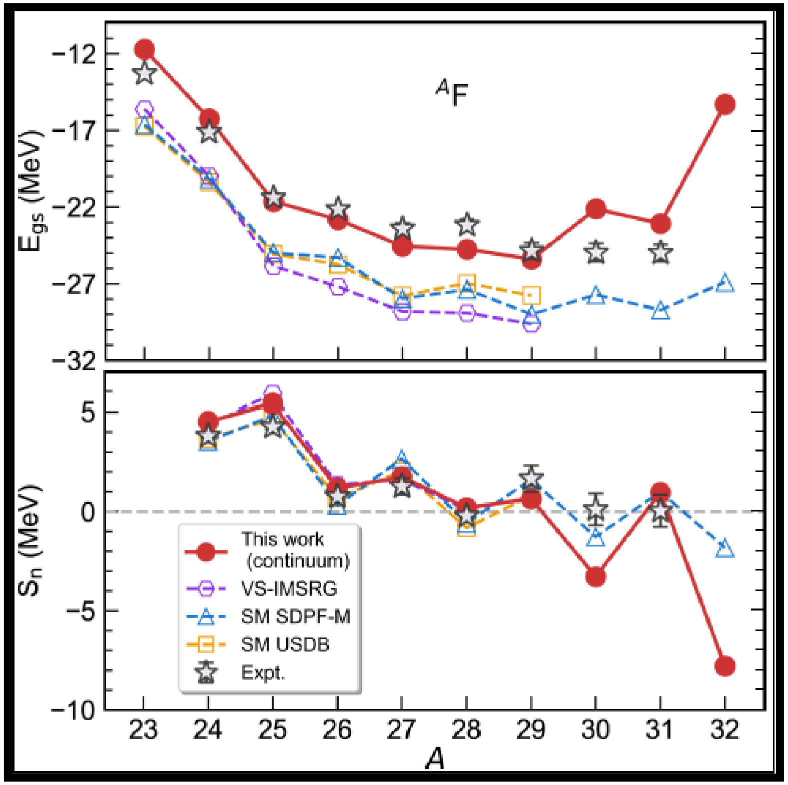
Realistic chiral N^{3}LO potential: Q-box with ^{14}O core
Two-nucleon and three nucleon forces included
Neutrons: bound $0p_{1/2}$ + sd shell
Protons: $s_{1/2}$ and $d_{5/2}$ Berggren basis states

Inter-nucleon correlations + continuum coupling
^{17}Ne two-proton halo supported by GSM calculations
Shell model with WS basis, 2N+3N : no halo
Three-nucleon forces: not prominent in halo formation

Gamow Shell Model applications

Fluorine and oxygen chain of isotopes



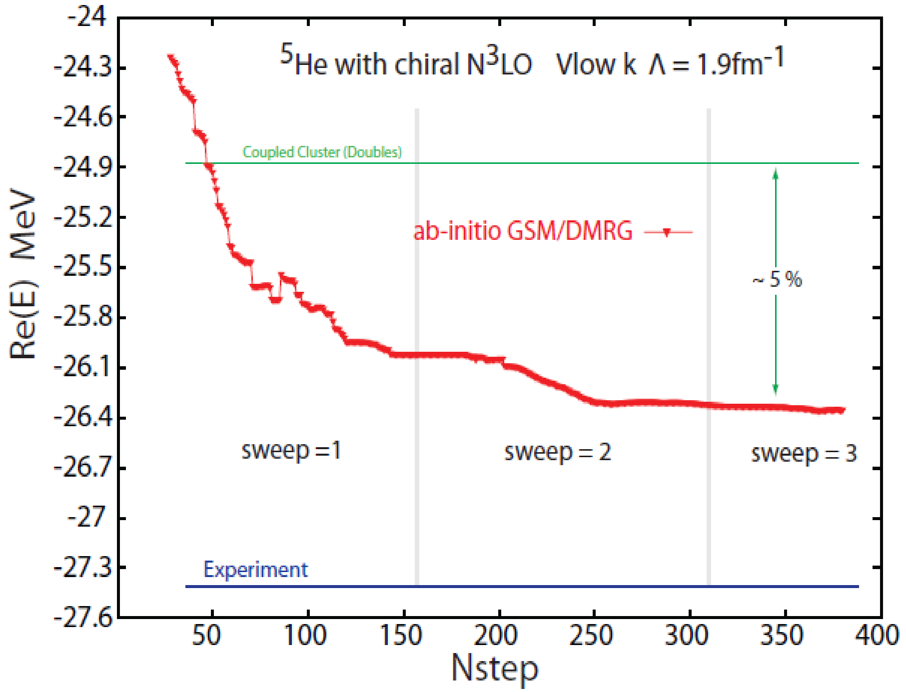


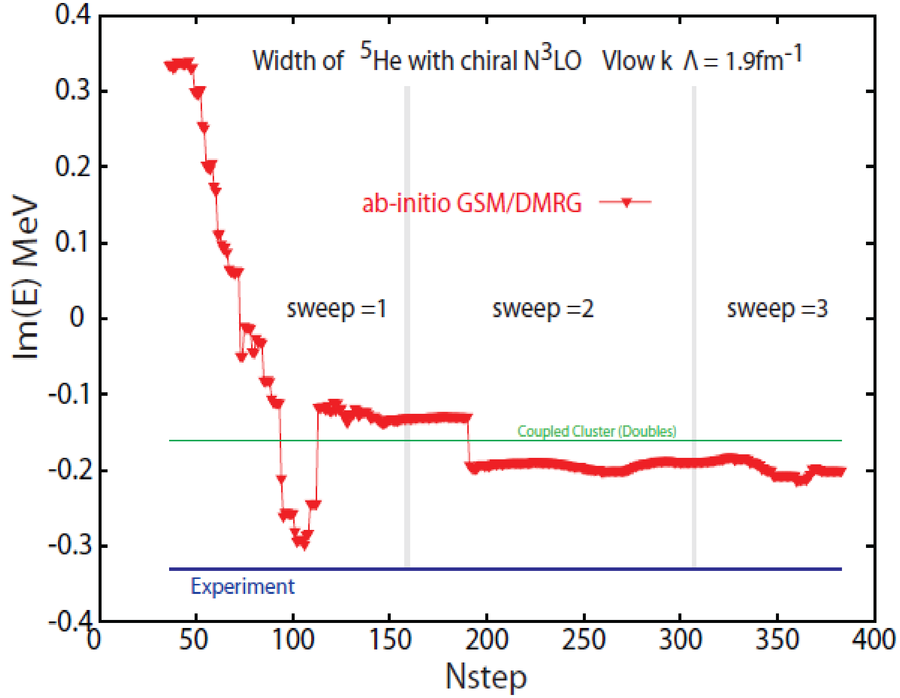
Effective interactions from the Q-box method with ^{22}O core (GSM)
In-medium similarity renormalization group in valence space (VS-IMSRG)
Shell model with 2N, 3N forces, USDB and SDPF-M interactions (SM NN+3NF, SM USDB, SM SDPF-M)
GSM provides the best overall reproduction of nuclear ground states at drip lines

B.S. Hu, Q. Wu, J.G. Li, Y.Z. Ma, Z.H. Sun, N. Michel, F.R. Xu, Phys. Lett. B **802**, 135206 (2020)

No core GSM applications

^{5}He with density matrix renormalization group





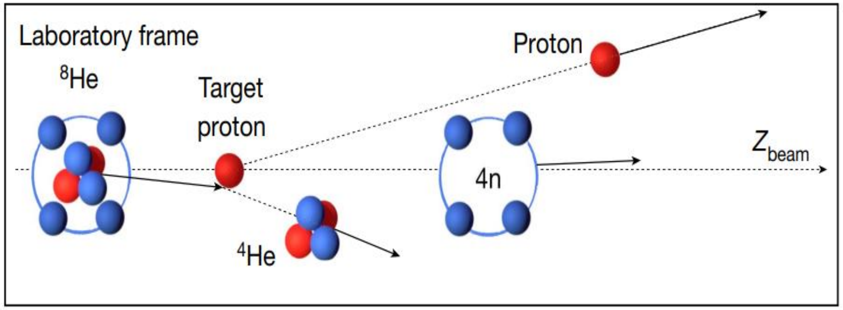
First example of resonant nucleus calculated with ab-initio GSM: ^{5}He

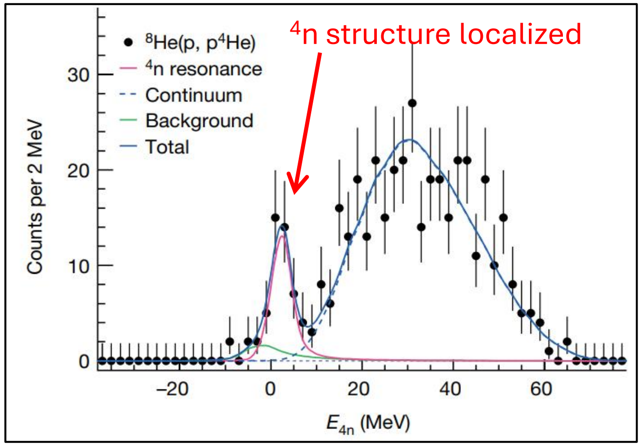
Local fluctuations in energy convergence: consequence of the generalized variational principle

G. Papadimitriou, J. Rotureau, N. Michel, M. Płoszajczak, B. R. Barrett, Phys. Rev. C **88**, 044318 (2013)

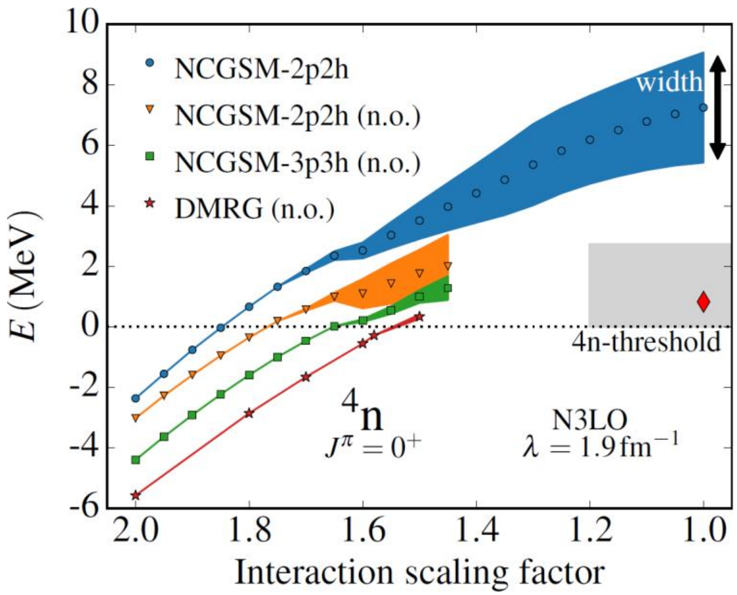
No core GSM applications

Tetraneutron





Exp. data : M.Duer et al., Nature **606**, 678–682 (2022)



K. Fossez, J. Rotureau, N. Michel, M. Płoszajczak, Phys. Rev. Lett. **119**, 032501 (2017)

Tetraneutron
Long standing question about its existence
Many-body effect of nuclear interaction

NCGSM
N^3LO interaction
GSM, DMRG, natural orbitals

GSM with 2p-2h incomplete
Natural orbitals with 3p-3h quickly imprecise
DMRG unstable in the unbound region
DMRG extrapolates to the experimental region

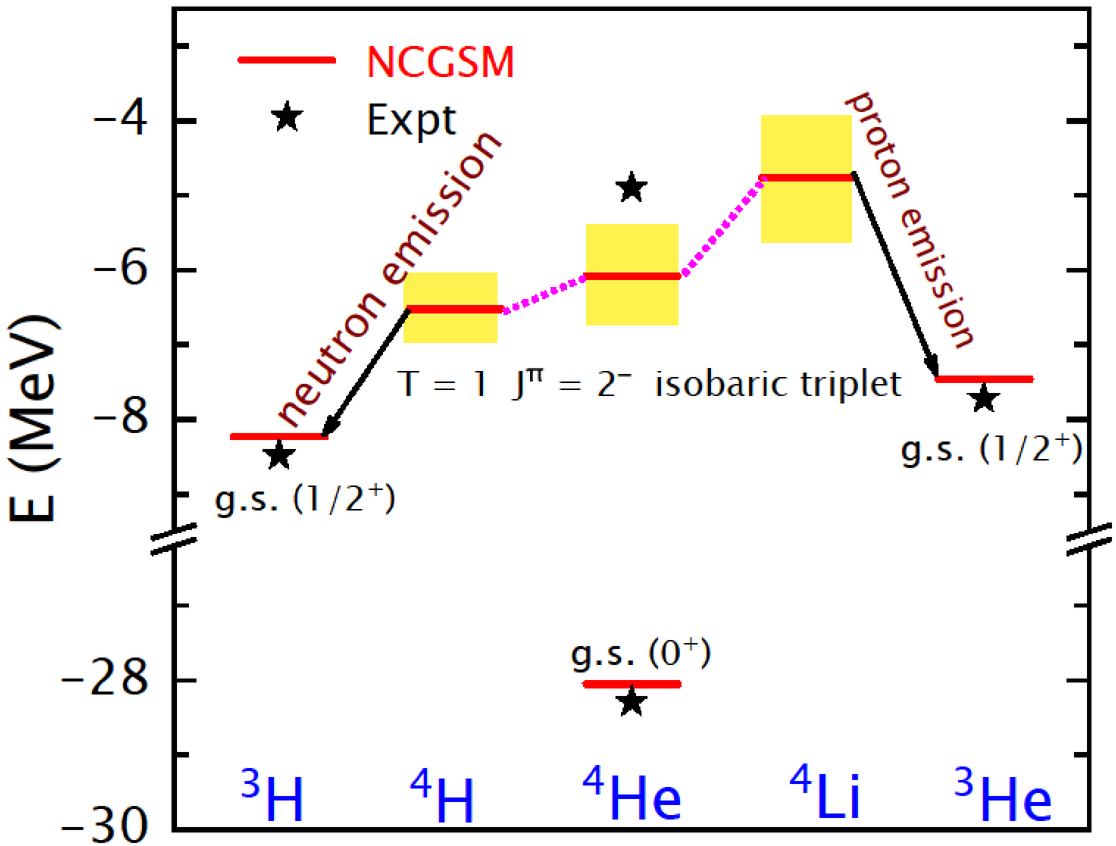
Few-body models
Experimental strength function reproduced
Enhanced presence of 4n in the asymptotic region
Tetraneutron not found as a resonance

R. Lazauskas, E. Hiyama, J. Carbonell,
Phys. Rev. Lett., 130, 102501 (2023)

Possible existence of a 4n resonance still debated

No core GSM applications

Isospin symmetry breaking in A=4 T=1 resonances



Coulomb interaction

No Coulomb force in ^{4}H
Coulomb force moderate in ^{4}He
Coulomb force largest in ^{4}Li

Width increases from ^{4}H to ^{4}Li
due to Coulomb force

Isospin symmetry breaking

T=1 $J^\pi = 2^-$ many body states
Isospin multiplet in A=4 states

Broad resonances in T=1 multiplet

T = 1 in ^{4}H and ^{4}Li
T ~ 0.71 in ^{4}He

Isospin symmetry strongly broken in ^{4}He
Origin still unknown

N. Michel, J. G. Li, F. R. Xu, W. Zuo, Phys. Rev. C **104**, 024319 (2021)

GSM-CC applications

Overview

Channel representation : products of target and projectile states

Target and projectile states from GSM

Projectiles: proton/neutron (scattering, radiative capture),

d, ^{3}He, t, ^{4}He (scattering, transfer, radiative capture)

Potentials calculated with GSM framework

Coupled-channel equations to solve : GSM-CC

Same Hamiltonian in GSM and GSM-CC (plus a few corrective factors)

Binary mass partitions only

Unification of structure and reaction models

Applications

Direct reactions of low energy

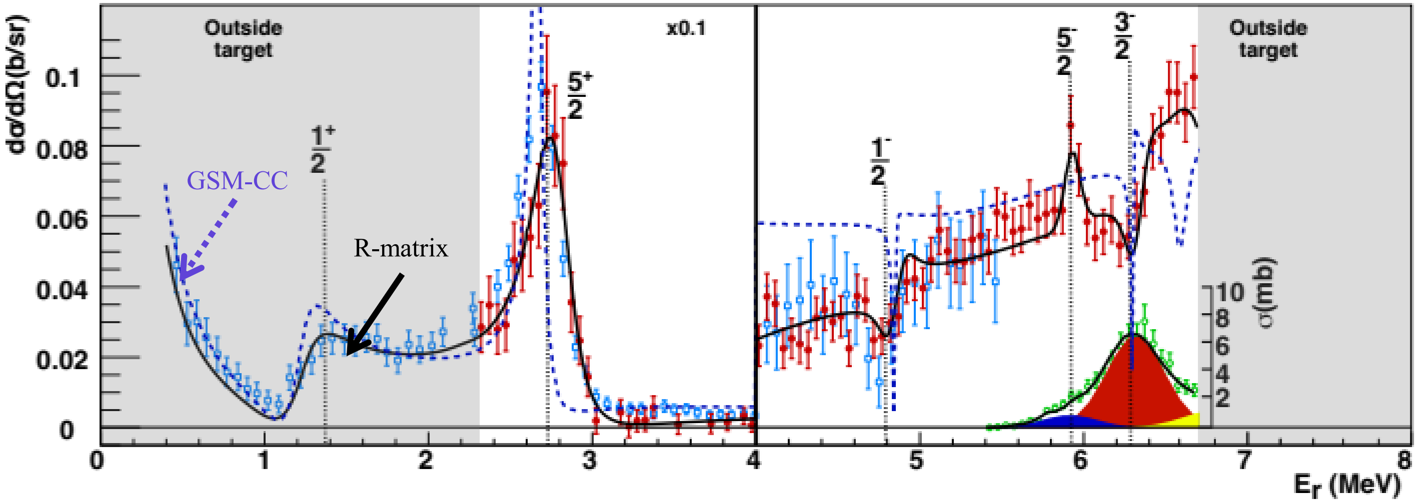
Elastic scattering reactions

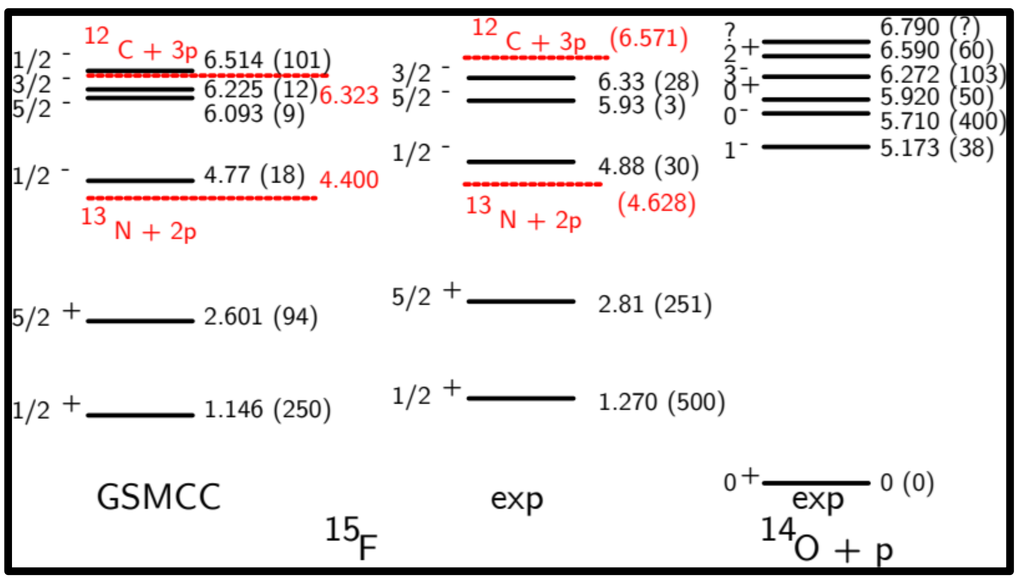
One-nucleon and many-nucleon projectiles

No-core GSM-CC

GSM-CC applications

$^{14}O(p,p)$ scattering reaction





<u>Narrow resonances and cross sections</u>

$1/2^-, 3/2^-, 5/2^-$ states found at high energy

Small width : ~ 15-30 keV

Spectrum + cross sections with same Hamiltonian

Fitted spectrum only: predictive model

^{14}O core + valence protons

Hamiltonian : WS + FHT interaction

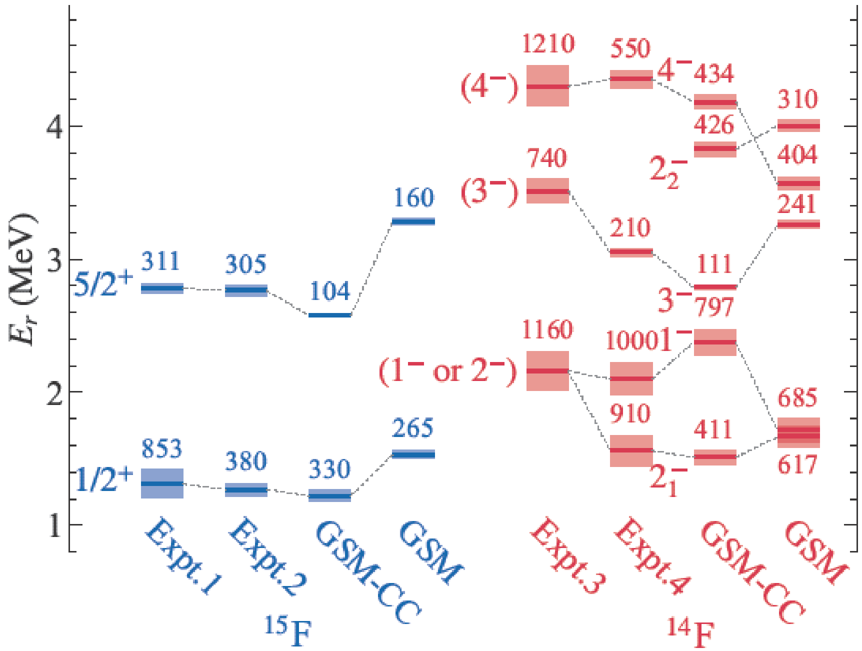
Collective structure due to continuum coupling

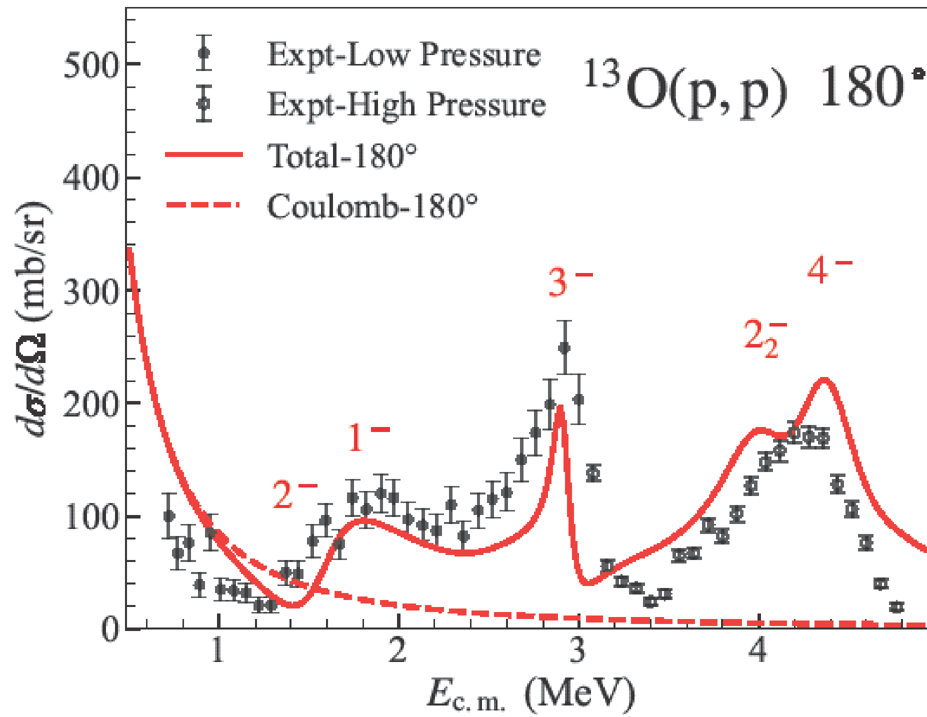
1p emission suppressed

V. Girard-Alcindor et al., Phys. Rev. C **105**, L051301 (2022)

GSM-CC applications

^{13}O(p,p) scattering reaction





^{8}C core + EFT interaction

^{14}F spectrum: proton-emitting resonances, not well known

Excited 2^-(2) resonance not identified in previous measurements

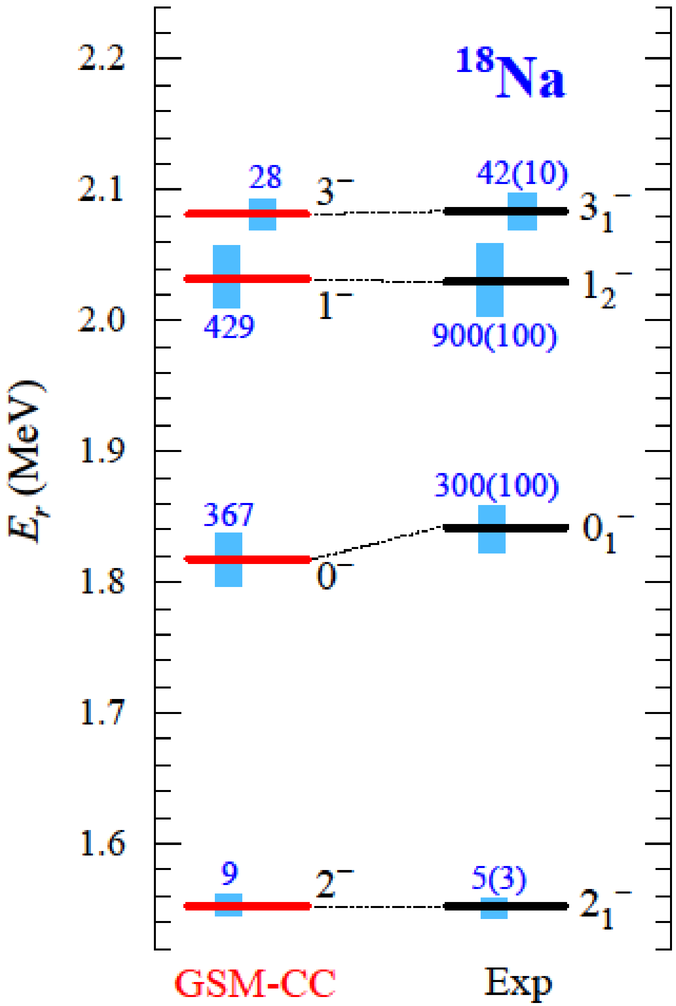
Consistent with previous phenomenological studies (potential model)

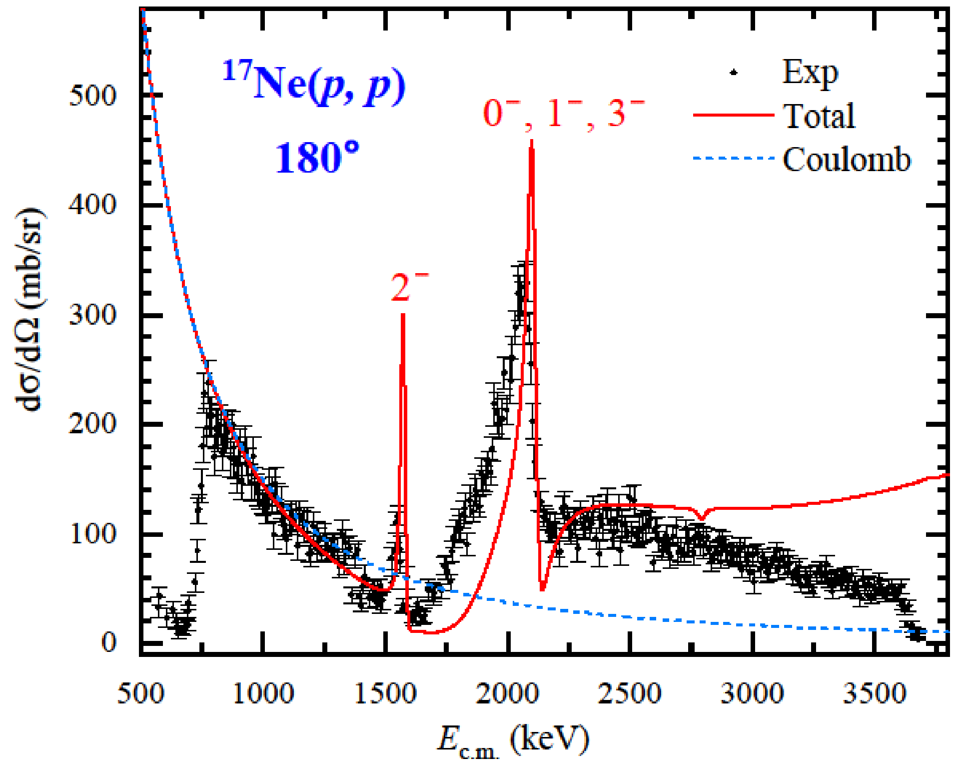
Broad enhancement near 4 MeV from combined 2^- and 4^- contributions

N. Chen, J.G. Li, N. Michel, M.R. Xie, Q. Yuan, W. Zuo , Phys. Rev. C **114**, 024306 (2026)

GSM-CC applications

^{17}Ne(p,p) scattering reaction



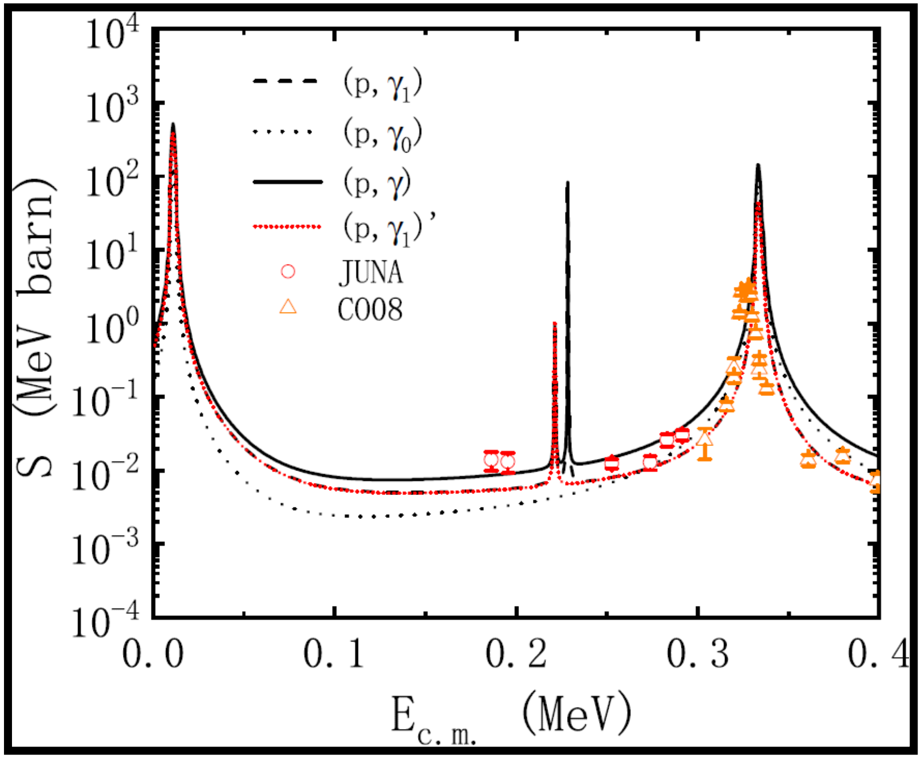


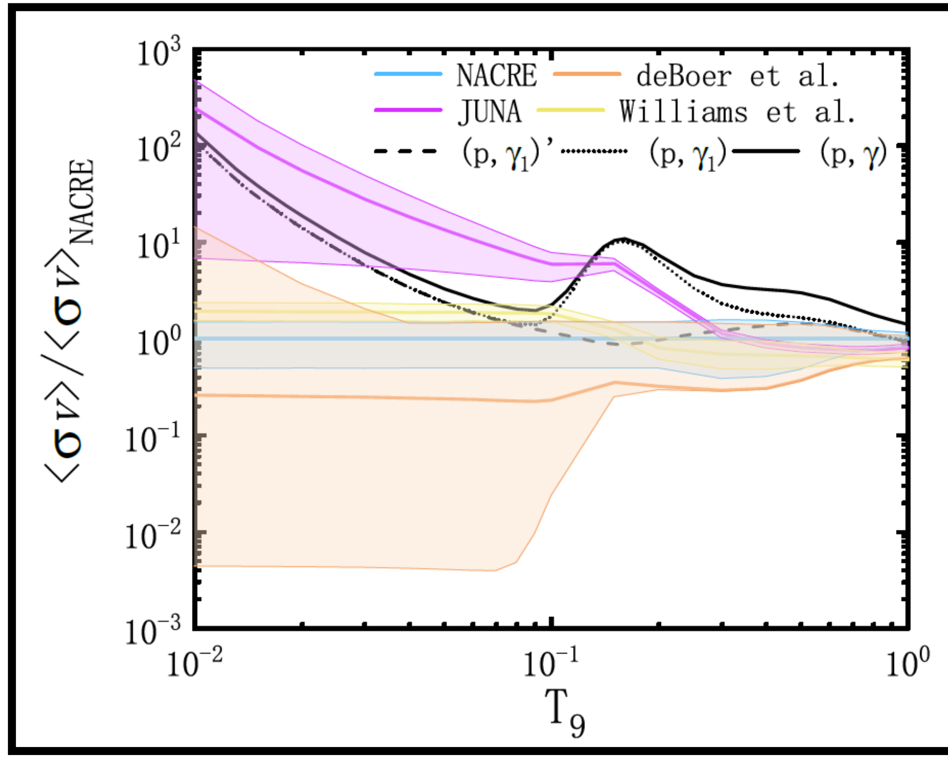
- ^{18}Na: intermediate nucleus in the sequential decay of ^{19}Mg
- Competing sequential and cluster emission in ^{19}Mg
- ^{14}O core + EFT interaction
- Good reproduction of spectrum and cross section (except above 3 MeV)
- Interplay between continuum and Coulomb interaction important

N. Chen, J.G. Li, K.H. Li, N. Michel, P.Y. Wang, W. Zuo , Phys. Rev. C **112**, 034319 (2026)

GSM-CC applications

$^{19}F(p, \gamma)$ radiative capture





Calcium abundance puzzle in stars

Recommended NACRE rates : 100 times lower than in metal-poor stars

GSM-CC exceeds the NACRE recommendation by over 100 times

Aligns with JUNA rate for T in [0.06 : 0.08] GK

Astrophysical factor

$$\mathscr{S}(E_{CM}) = \sigma(E_{CM})\ E_{CM}\ e^{2\pi\eta_{CM}}$$

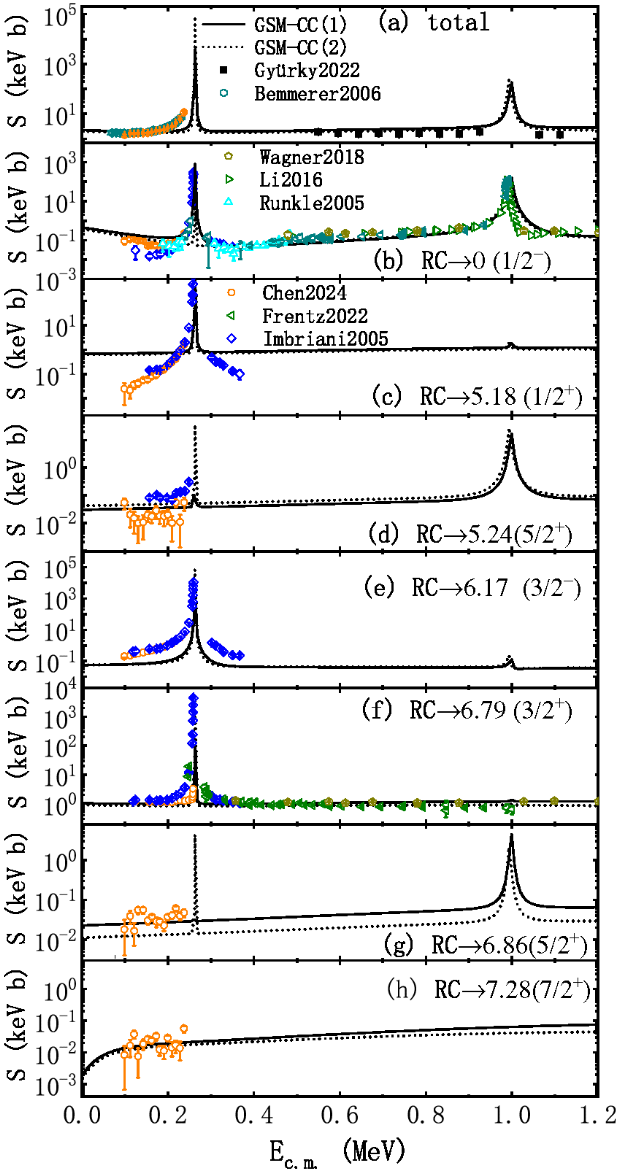
Valence nucleons above a ^{12}C core

psd Berggren space

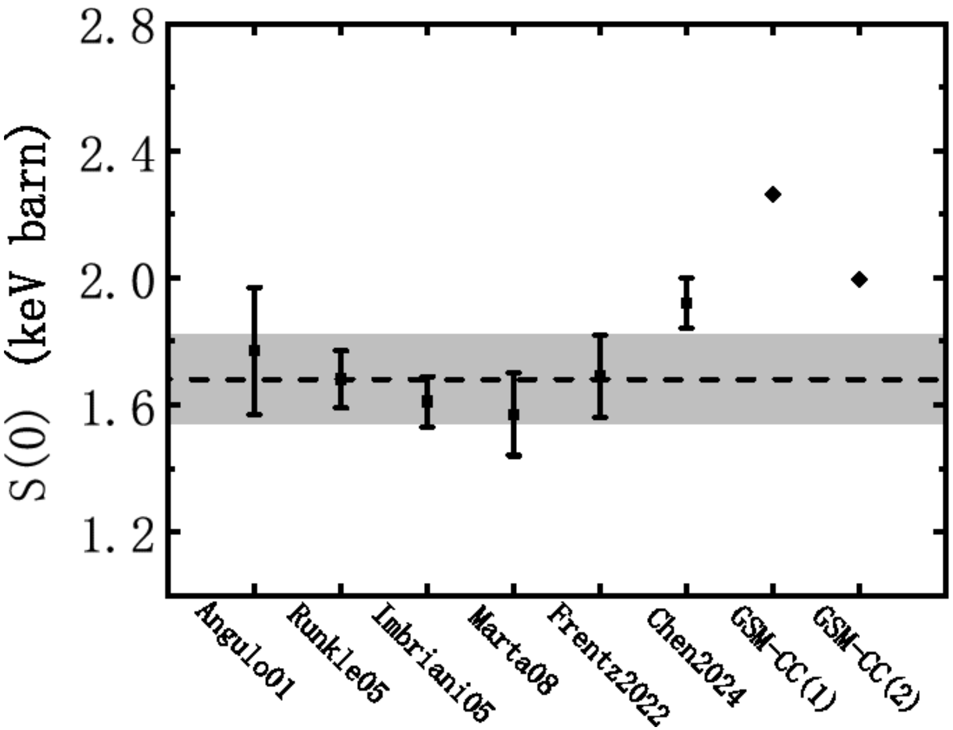
FHT interaction

G.X. Dong, X.B. Wang, N. Michel, M. Ploszajczak, Phys. Rev. C **110**, L061601 (2024)

GSM-CC applications



$^{14}N(p,\gamma)^{15}O$ radiative capture



Solar metallicity: ratio of elements heavier than He to H in the Sun

30-40% reduction compared to the traditional models

$^{14}N(p,\gamma)^{15}O$ direct measurement impossible: GSM-CC is the tool of choice

^{4}He core + FHT interaction

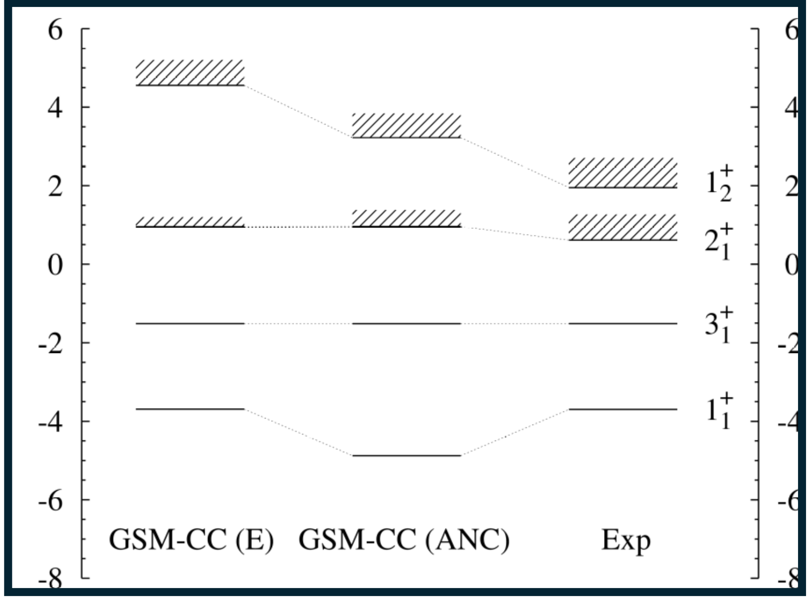
Good reproduction of cross section and S(0), albeit larger than experimental data

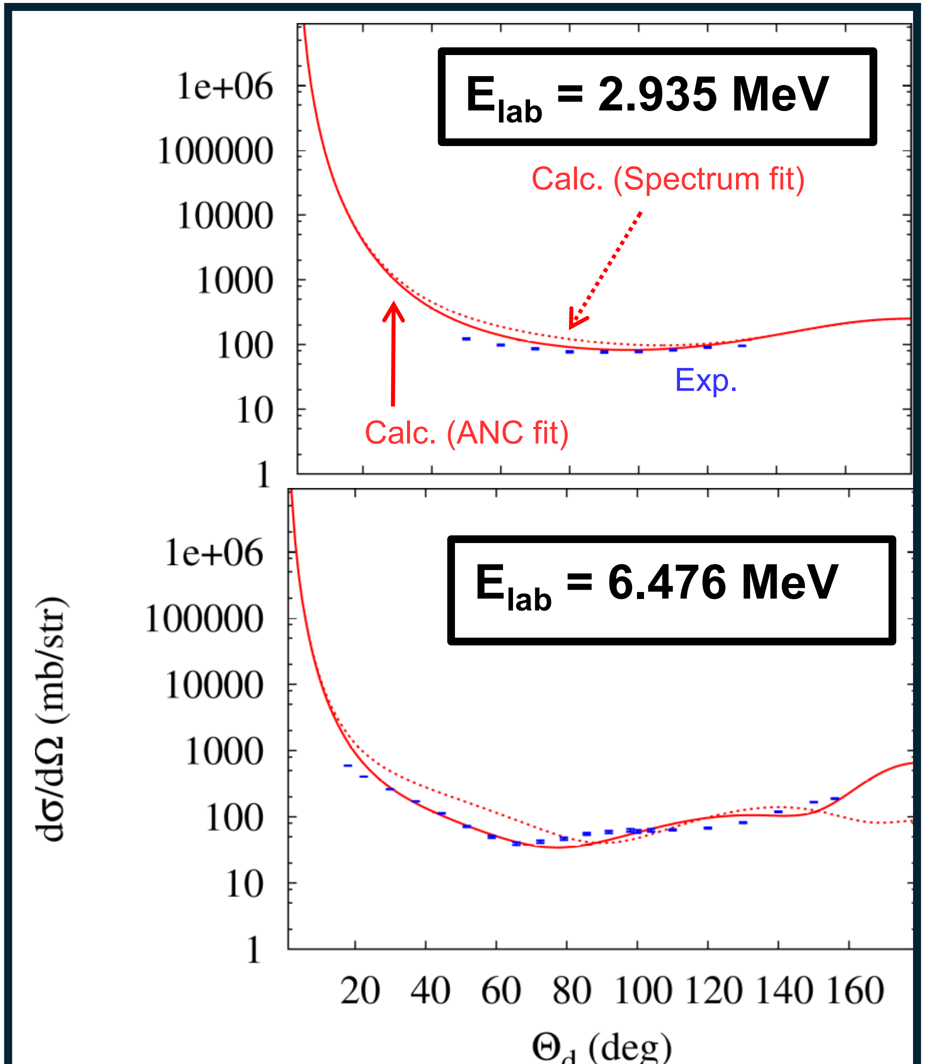
CN abundances in GSM-CC: $3\text{-}4.10^{-4}$ to compare to $4\text{-}6.10^{-4}$ experimentally

G.X. Dong, X.B. Wang, N. Michel, M. Płoszajczak , Phys. Rev. C **113**, L032801 (2026)

GSM-CC applications

$^{4}He(d,d)$ scattering reaction





Valence nucleons above an alpha core

Partial waves up to l=4

FHT interaction between valence nucleons

Structure and reactions features

^{6}Li: spectrum fit vs ANC fit

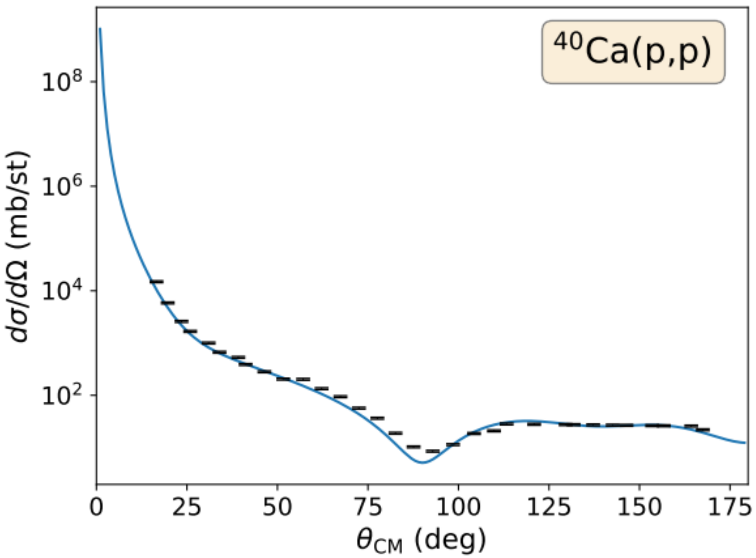
Deuteron structure from N^3LO interaction

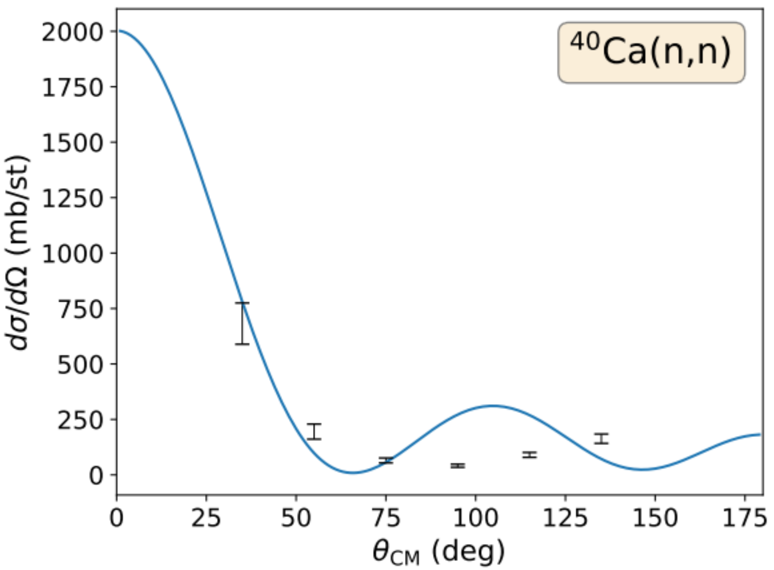
Break-up included with scattering deuteron states

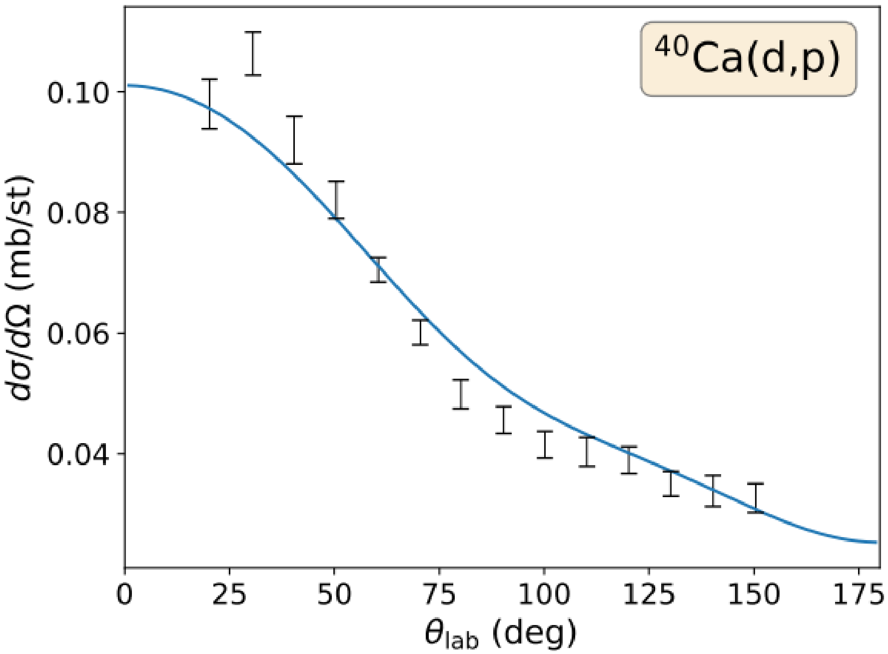
A. Mercenne, N. Michel, M. Płoszajczak, Phys. Rev. C **99**, 044606 (2019)

GSM-CC applications

^{40}Ca(d,p) transfer reaction







Valence nucleons above a ^{40}Ca core

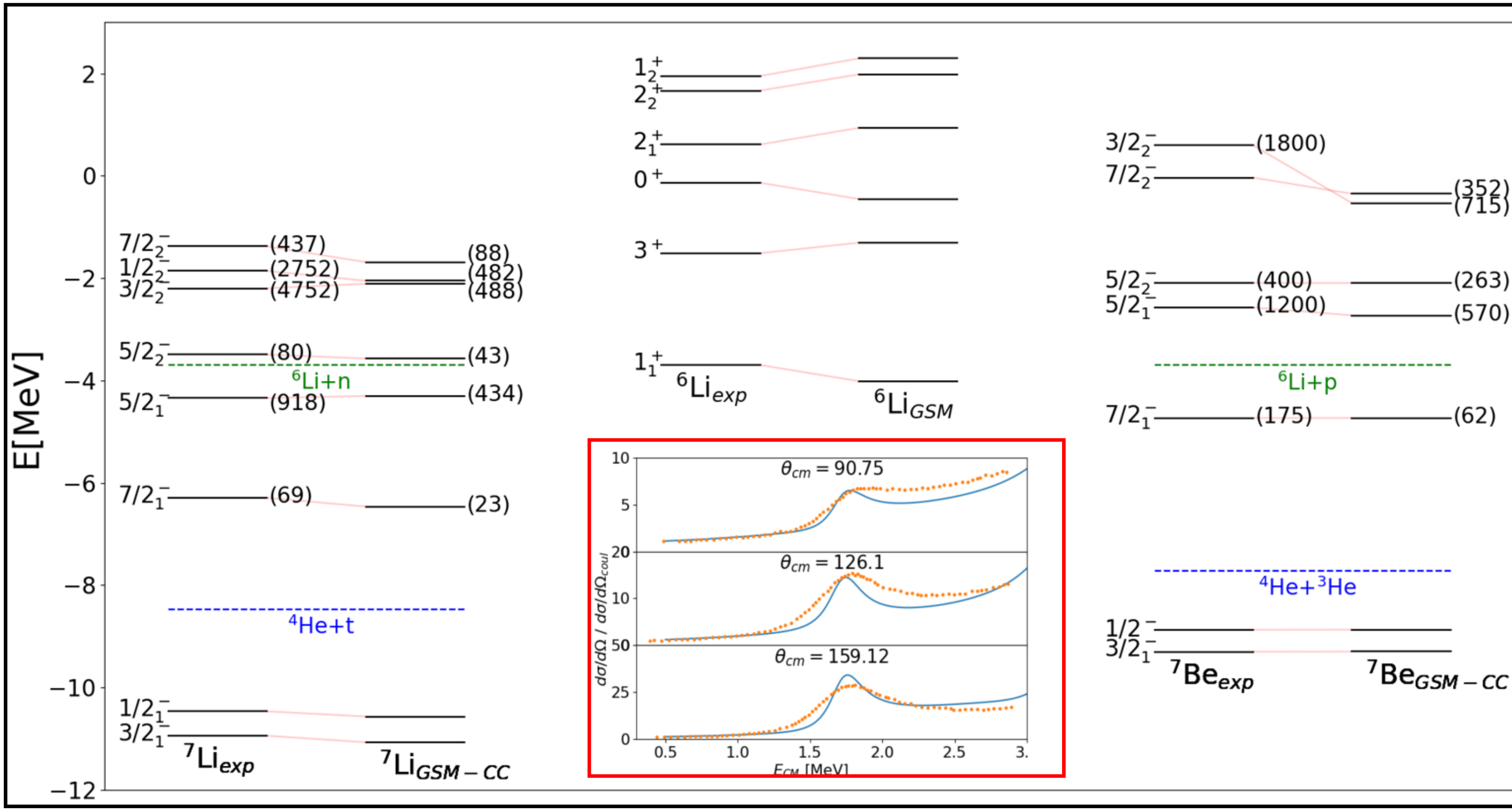
No excitation from the ^{40}Ca core

A. Mercenne, N. Michel, J. P. Linares Fernández, M. Płoszajczak

Phys. Rev. C **107**, L011603 (2023)

GSM-CC applications

^3He and ^3H projectiles



Valence nucleons above an alpha core

FHT interaction between valence nucleons

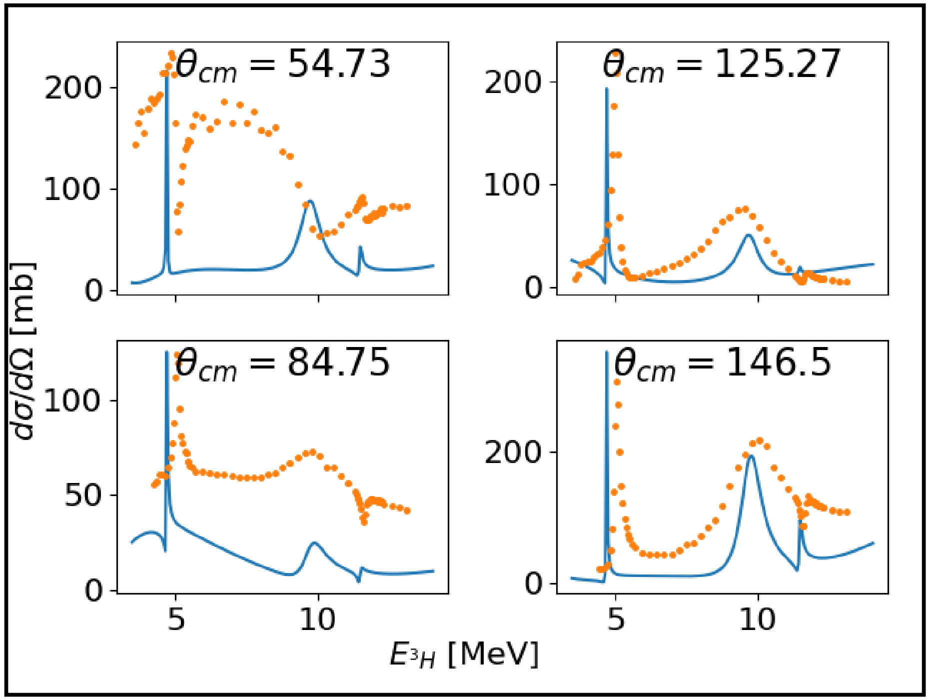
Spectrum well reproduced with GSM-CC

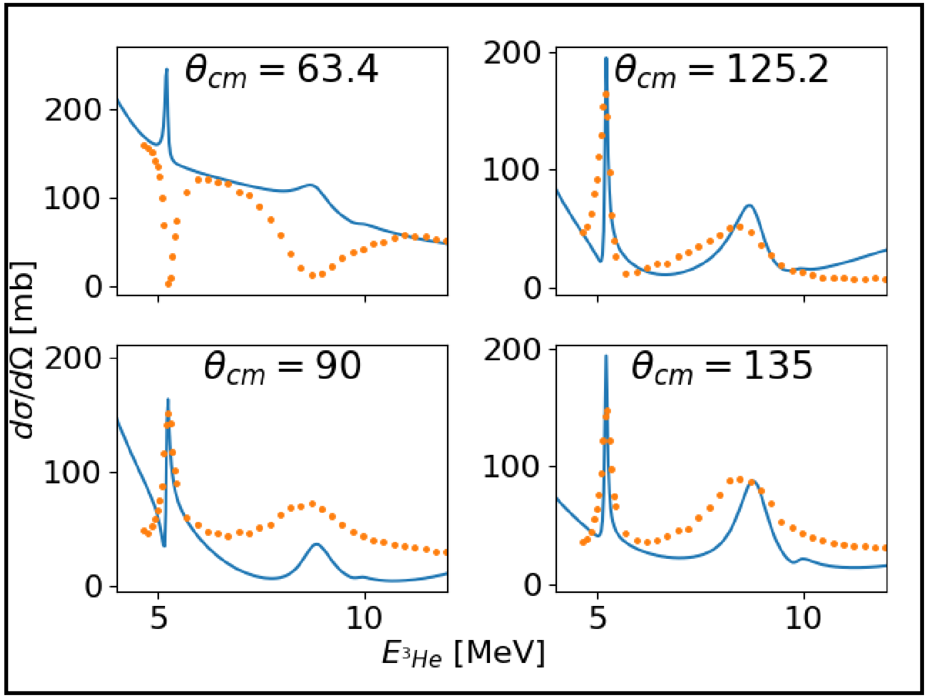
Cross sections of p + ^{6}Li reactions well reproduced

J. P. Linares Fernandez, N. Michel, M. Płoszajczak, A. Mercenne, Phys. Rev. C **108**, 044616 (2023)

GSM-CC applications

^{3}He and ^{3}H projectiles





FHT interaction between valence nucleons

N^{3}LO interaction for ^{3}H and ^{3}He projectile structure

Direct cross sections well reproduced in average

Results not satisfactory for small angles

Transfer reactions (not shown) too small

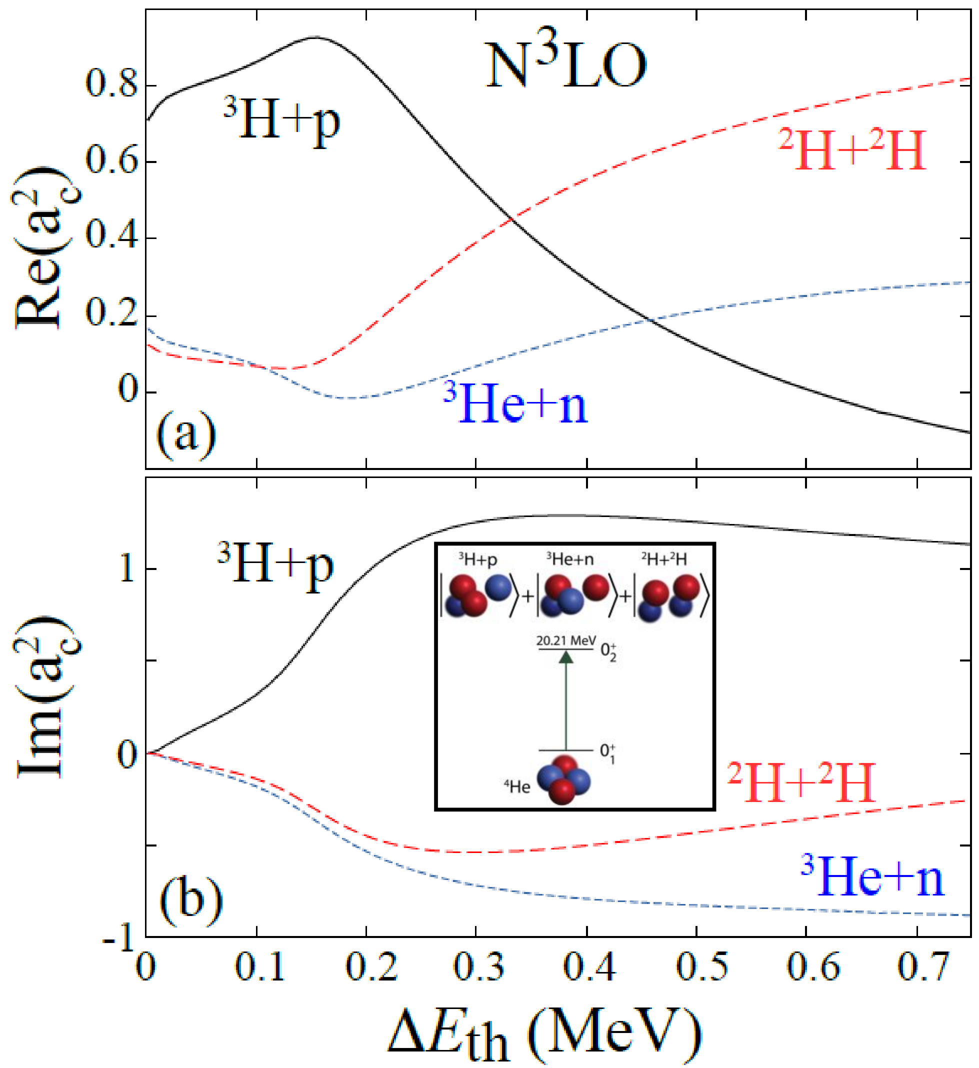
Multi-nucleon projectiles demand theoretical progress

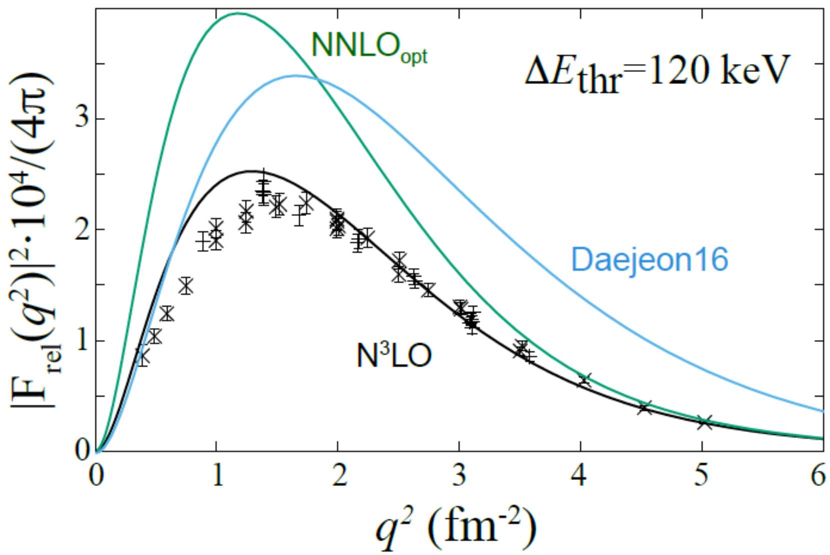
Imaginary part of potentials to determine

J. P. Linares Fernandez, N. Michel, M. Płoszajczak, A. Mercenne, Phys. Rev. C **108**, 044616 (2023)

GSM-CC applications

Proton resonance state of ^{4}He





Strong influence of closed channels with threshold energy

Aligned with ^{3}H + p at small threshold energy

$(a_c)^2$: complex probability to occupy a basis channel

Channel inversion when threshold energy increases

Large imaginary parts occur as well.

Similar conclusions found with hyperspherical harmonics

M.Viviani et al. Phys. Rev. C. **102**, 034007 (2020)

N. Michel, M. Płoszajczak, W. Nazarewicz , Phys. Rev. Lett. **131**, 242502 (2023)

Outlook

Current status

GSM: configuration interaction including the continuum

Realistic interactions in no-core GSM : A=3-5

Effective interactions with a core : A > 5

Eigenvector continuation for broad resonances and no-core GSM

Hypernuclei with a core

GSM-CC : Reaction and structure observables in the same framework

Unification of nuclear structure and reaction

Perspectives

Transfer and radiative capture reactions with multi-nucleon clusters

Systematic spectroscopy of hypernuclei with core and no-core GSM

Reactions of interest for nuclear astrophysics

(t, γ), (t,n), (t, d), (t,α) (α, γ), systematics

Deformed nuclei at drip-lines in the Berggren framework beyond the rotor model

Ternary mass partitions in GSM-CC for many-body nuclear states

Effective interaction at drip-lines with a core
Spectra, correlation densities, charge radii ($^{6-8}$He, ^{6}Li)
Two-neutron and two-proton halos (^{31}F, ^{17}Ne)
Deformation at drip-lines (^{11}Be,^{39}Mg)

GSM

Ab-initio GSM
^{5}He, ^{3}n , ^{4}n

Reactions with GSM-CC
^{7}Be(p, γ), ^{14}N(p, γ), ^{19}F(p, γ),
^{7}Li(n, γ), ^{8}Li(n, γ),
^{13,14}O(p,p) ,^{17,18}Ne(p,p)
^{4}He(d,d), ^{4}He(t,t), ^{4}He(^{3}He, ^{3}He)