Neutrino Cross Sections from Lattice QCD

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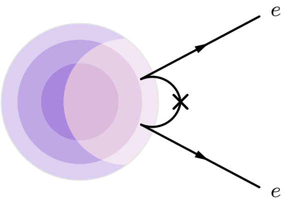
Outline

- Tests of BSM physics in nuclear systems
- Neutrino Cross Sections
- Quasielastic Scattering from Experiment
  - Constraints from Deuterium Scattering
- LQCD Applications
  - Survey of $F_A(Q^2)$ Calculations
  - T2K/DUNE Implications
- Future Prospects for BSM

Note: all references in online slides are hyperlinked

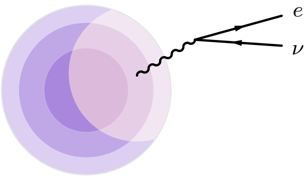
Introduction

Tests of BSM



Neutrinoless double beta decay

Superallowed beta decay



Enormous scope of BSM physics – cherry picked examples to focus on

Neutrinoless Double Beta Decay

Majorana neutrinos $\implies$ lepton number-violating ($\Delta L = 2$) nuclear decays

$$(A, Z) \to (A, Z+2) + 2e^- + Q_{\beta\beta}$$

Nuclear matrix elements for $0\nu\beta\beta$ decay not directly measureable

$\implies$ obtained from theory

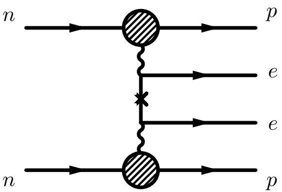
Matrix elements uncertain – nuclear many-body problem

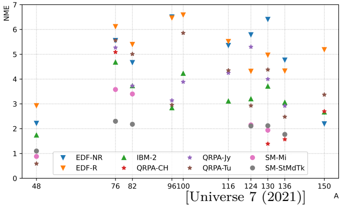
$\implies$ $O(1)$ disagreements on NME

$$\left[T_{1/2}\right]^{-1} \propto \left|\mathcal{M}\right|^2$$

Evidence that short-range contributions comparable to long-range:

[Phys.Rev.Lett.119 (2017)]





[Universe 7 (2021)]

Superallowed β Decay Lifetime

Superallowed ($0^+ \to 0^+$) beta decay to predict $V_{ud} \to$

$$\left|V_{ud}\right|^2 = \frac{2984.43\text{sec}}{\mathcal{F}t(1+\Delta_R^V)}$$

Recent experiment–theory comparison:

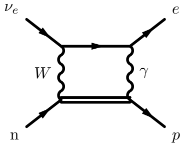
Theory:	$\left\|V_{ud}\right\| = 0.97395(21)_{\mathcal{F}t}(10)_{\text{RC}}$	[Phys.Rev.D 100 (2019)]
PDG:	$\left\|V_{ud}\right\| = 0.97420(10)_{\mathcal{F}t}(18)_{\text{RC}}$	

Theory prediction leads to a 2.2σ tension for first-row CKM unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9989(5)$$

Dominant uncertainty from $W\gamma$ box diagram contribution to Δ_R^V:

$\Delta_R^V \to$



BSM to Hadrons & Lattice QCD

Common thread in scenarios listed – hadronic matrix elements

$\implies$ Generically beneficial to constrain inputs for nuclear models

$\implies$ LQCD especially useful when matrix elements inaccessible to expt

Nuclear matrix elements are cutting-edge for LQCD:

- Exponential signal-to-noise problem for baryons
- Small state energy splittings, many states
- Developing formalism to carry out calculations
- First explorations with problems

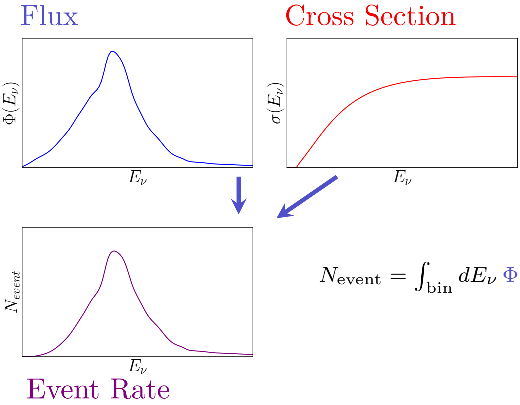
Discuss application of LQCD to neutrino quasielastic scattering

$\implies$ Weak interaction matrix element on free neutron at high energy

Circle back to applications to BSM, future prospects

Neutrino Cross Sections

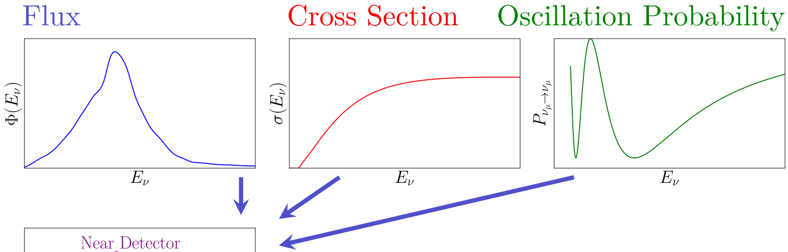
Measuring Oscillation Probability

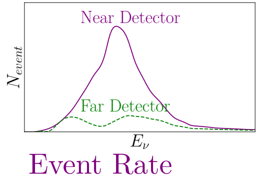


$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu)\sigma(E_\nu)$$

Event rate from convolution over E_ν. Broad flux & distribution of event E_ν

Measuring Oscillation Probability



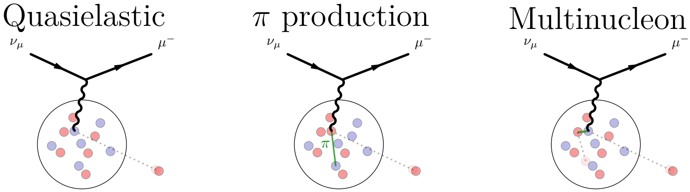


$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu) \sigma(E_\nu) \underbrace{\left[\times P(L/E_\nu) \right]}_{\text{far detector only}}$$

$$\left[P(L/E_\nu) \right]_{\text{bin}} \approx \left[\frac{N_{\text{event}}[\text{FAR}]}{N_{\text{event}}[\text{NEAR}]} \right]_{\text{bin}}$$

Event rate from convolution over E_ν. Broad flux & distribution of event E_ν far/near $\implies$ oscillation probability, but picture too simplified...

Neutrino Event Topologies

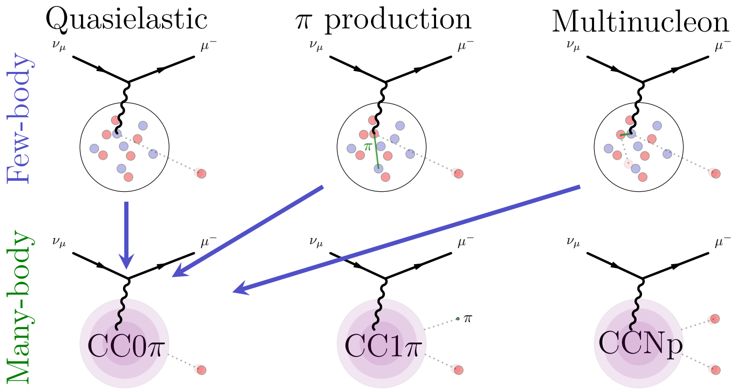


Many allowed interaction channels, reinteractions within nucleus

Particle kinematics change in nuclear medium

Only particles that escape are detectable

Neutrino Event Topologies



Many allowed interaction channels, reinteractions within nucleus

Particle kinematics change in nuclear medium

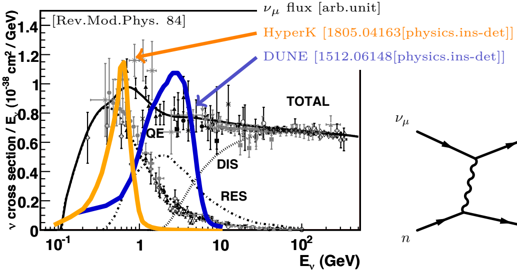
Only particles that escape are detectable

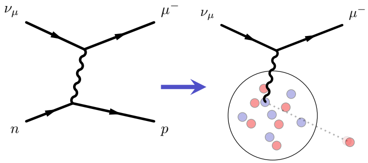
Mismatch between *nucleon* amplitudes & *nuclear* cross sections...

$\Longrightarrow$ Event-by-event E_ν measurements are not possible

$\Longrightarrow$ Reconstruct E_ν distributions from measured event rates

Neutrino Oscillation and Quasielastic





Compute *nucleon* amplitudes, ingredients for *nuclear* models

Quasielastic is lowest E_ν, simplest $\implies$ most important

Question:

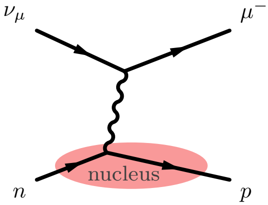
How well do we know nucleon quasielastic cross section from elementary target sources?

- Deuterium scattering
- Lattice QCD

QE Experimental Constraints

Quasielastic Form Factors

Quasielastic (QE) scattering assumes quasi-free nucleon inside nucleus



$$\mathcal{M}_{\text{nucleon}} = \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle N' | \mathcal{J}_\mu | N \rangle$$

$$\langle N'(p') | (V-A)_\mu(q) | N(p) \rangle$$
$$= \bar{u}(p') \Big[\gamma_\mu F_1(q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \Big] u(p)$$

- F_1, F_2: constrained by eN scattering
- F_P: subleading in cross section,
  $\propto F_A$ from pion pole dominance constraint

Axial form factor F_A is leading contribution to nucleon cross section uncertainty

Form Factor Parameterizations

Most common in experimental literature: dipole ansatz —

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by both experimental and LQCD data (revisit later)
- Inconsistent with QCD, requirements from unitarity bounds
- Motivated by $Q^2 \to \infty$ limit, data restricted to low Q^2

Model independent alternative: z expansion [Phys.Rev.D 84 (2011)] —

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\rm cut}) = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} - t_0}} \qquad t_{\rm cut} \leq (3M_\pi)^2$$

- Rapidly converging expansion
- Controlled procedure for introducing new parameters

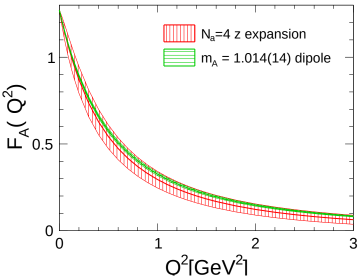
Deuterium Constraints on F_A

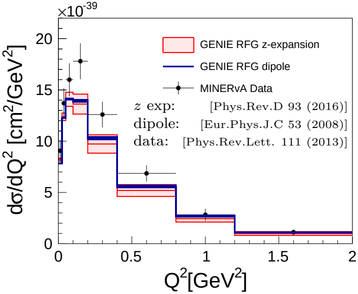
- Outdated bubble chamber experiments:
  - Total $O(10^3)$ ν_μQE events
  - Digitized event distributions only
  - Unknown corrections to data
  - Deficient deuterium correction
- Dipole overconstrained by data
  underestimated uncertainty $\times O(10)$
- Prediction discrepancies could be from
  nucleon and/or nuclear origins

Coming soon:

MINERνA $\bar{\nu}_\mu p \to \mu^+ n$ dataset
& updated form factor fits

See [Nature 614 (2023)]





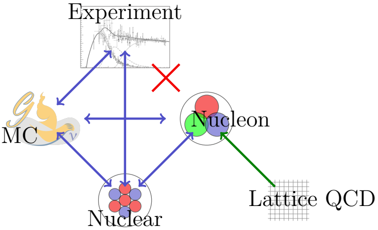
LQCD as Disruptive Technology

How can we improve precision?

Ideal: Modern high stats ν-D_2 scattering bubble chamber experiment

$\implies$ LQCD as a alternative/complement to experiment, especially with experimentally inaccessible quantities

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



Build from the ground up:

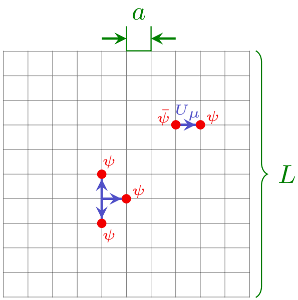
Nucleon amplitudes from first principles

Robust uncertainty quantification

Well motivated theory inputs to nuclear models/EFTs

LQCD Survey & Implications

Lattice QCD Formalism



Numerical evaluation of path integral
Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\, \mathcal{D}U \, \exp(-S)\, \mathcal{O}_\psi\, [U]$$

Parameters: $am_{(u,d),\text{bare}}$
$am_{s,\text{bare}}$
$\beta = 6/g^2_{\text{bare}}$

Matching: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
1 per parameter

Results — first principles predictions from QCD,
gluons to all orders

"Complete" error budget $\implies$ extrapolation in a, L, M_π guided by EFT, FVχPT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\text{phys}}$ (chiral limit)

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Status of Lattice QCD Determination of Nucleon Form Factors and Their Relevance for the Few-GeV Neutrino Program

Annual Review of Nuclear and Particle Science

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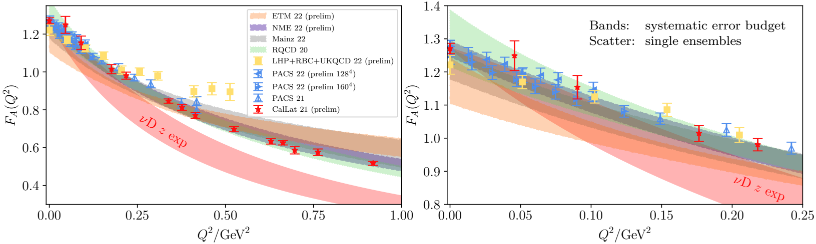
Abstract

Calculations of neutrino–nucleus cross sections begin with the neutrino–nucleon interaction, making the latter critically important to flagship neutrino oscillation experiments despite limited measurements with poor statistics. Alternatively, lattice quantum chromodynamics (LQCD) can be used to determine these interactions from the Standard Model with quantifiable theoretical uncertainties. Recent LQCD results of g_A are in excellent agreement with data, and results for the (quasi-)elastic nucleon form factors with full uncertainty budgets are expected within a few years. We review the status of the field and LQCD results for the nucleon axial form factor, $F_A(Q^2)$, a major source of uncertainty in modeling sub-GeV neutrino–nucleon interactions. Results from different LQCD calculations are consistent but collectively disagree with existing models, with potential implications for current and future neutrino oscillation experiments. We describe a road map to solidify confidence in the LQCD results and discuss future calculations of more complicated processes, which are important to few-GeV neutrino oscillation experiments.

Expected final online publication date for the *Annual Review of Nuclear and Particle Science*, Volume 72 is September 2022. Please see http://www.annualreviews.org/page/journal/pubdates for revised estimates.

Nucleon Axial Form Factor

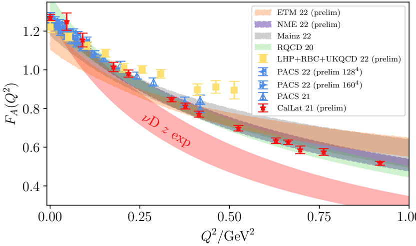


LQCD results maturing:

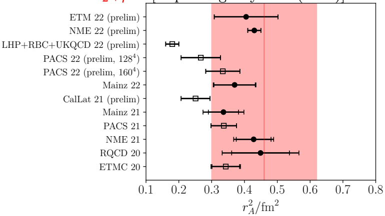
- Many results, all physical M_π: *independent data & different methods*
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Nontrivial consistency checks from PCAC

Evidence of slow Q^2 falloff, situation unlikely to change drastically

Axial Radius (r_A^2)



Filled circle: full error budget
Open square: incomplete
$D_2+\mu H$: [Rept.Prog.Phys. 81 (2018)]



Radius related to slope: $r_A^2 = -\frac{6}{g_A}\frac{dF_A}{dQ^2}\Big|_{Q^2=0}$

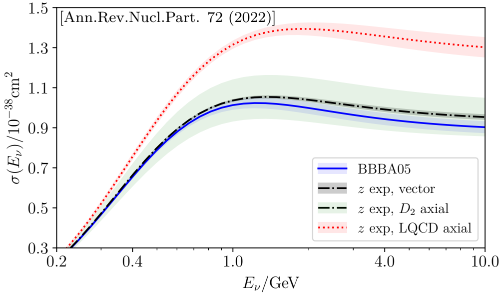
$(r_A^2 \sim 0.47\ \mathrm{fm}^2 \implies m_A \sim 1.0\ \mathrm{GeV})$

Good agreement with r_A^2 from experiment, poor agreement with large Q^2

Fixing radius to agree at large Q^2 would bring radius down to $r_A^2 \sim 0.25\ \mathrm{fm}^2$

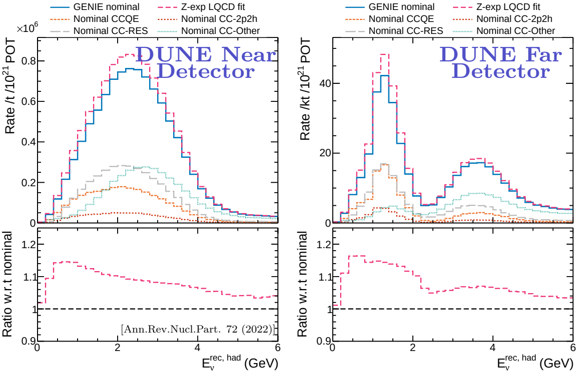
$\implies$ Incompatible with dipole ansatz

Free Nucleon Cross Section



- LQCD prefers 30-40% enhancement of ν_μ CCQE cross section
- recent Monte Carlo tunes require 20% enhancement of QE
  [Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]
  similar trend with continuum Schwinger function methods
  [Phys.Rev.D 105 (2022)] [2206.12518 [hep-ph]]
- With improved precision, sensitive to vector FF tension (black vs blue)
  [Phys.Rev.D 102 (2020)] vs [Nucl.Phys.B Proc.Suppl. 159 (2006)]

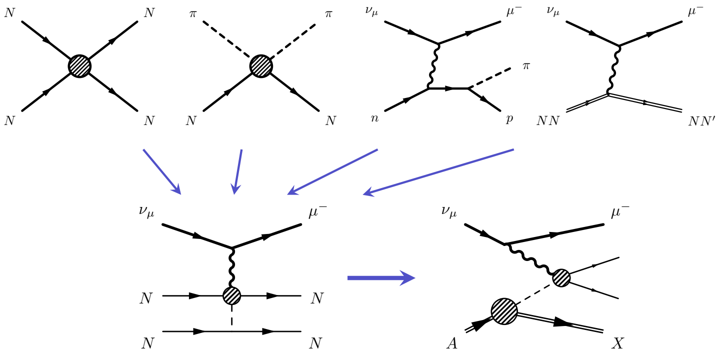
DUNE Implications



- Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)
- QE enhancements produce 10–20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning makes more detailed comparisons complicated
  $\Longrightarrow$ All channels are adjusted to compensate for QE changes
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction

Future Directions

Roadmap To Nuclear



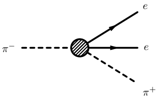
Build from the ground up:

- LQCD calculations of scattering, current interactions
- constraints of LECs for χPT, EFT
- computations of nuclear matrix elements for larger nuclei

Neutrinoless Double Beta Decay

Work is already underway!

Focused on meson contribution – two nucleon contributions are difficult, noisy

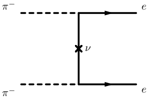


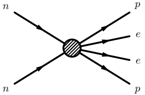
$\pi^- \to \pi^+ ee$ short-distance contributions

[Phys.Rev.Lett.121 (2018)]

$\pi^- \to \pi^+ ee$ long-distance contributions

[Phys.Rev.D.100 (2019)] [2004.07404 [hep-lat]]



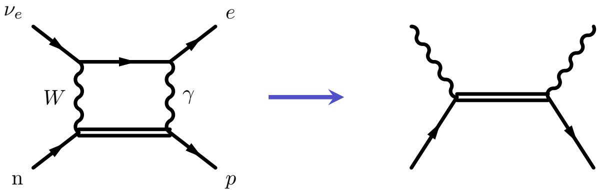


Formal developments

[Phys.Rev.Lett.126 (2021)] [Phys.Rev.D.94 (2016)]

Many ongoing computations of NN spectrum, prerequisite to NN contributions

Tackling the $W\gamma$ box diagram



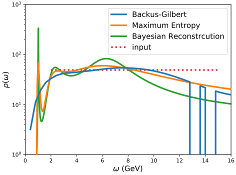
LQCD calculations to obtain unknown matrix elements

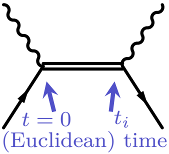
Two main issues:

- Matching of finite volume scattering amplitudes to infinite volume
  c.f. [Phys.Rev.D.101 (2020)]
- Euclidean $\rightarrow$ Minkowski time, inverse problem

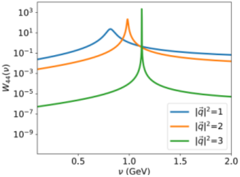
LQCD Inverse Problem

(toy data)
[Phys.Rev.D.101 (2020)]





(LQCD data)
[priv.comm. K.F.Liu]



$$\underbrace{W^{\mathrm{E}}(\vec{p},\vec{q},t)}_{\text{Euclidean}} = \int \underbrace{W^{\mathrm{M}}(\vec{p},\vec{q},\nu)}_{\text{Minkowski}} e^{-\nu t} d\nu$$

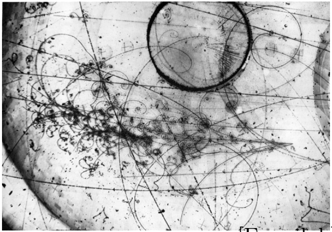
Ill-posed inverse problem, common in other applications:

- Backus-Gilbert: [Geo.Journal.Intl.16 (1968)] [Phys.Rev.D.96 (2017)]
- Maximum Entropy: [Prog.Part.Nucl.Phys.46 (2001)]
- Bayesian Reconstruction: [Phys.Rev.Let.111 (2013)]

Some work to compute inclusive hadronic tensor matrix elements

Concluding Remarks

Outlook



[Fermilab]

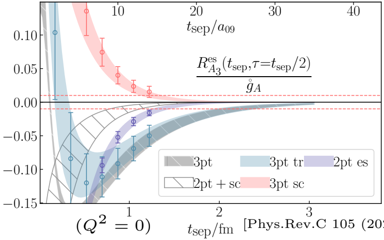
- LQCD to be key player for testing BSM scenarios in nuclear systems:
  $\implies$ proxy for missing experimental data
- Making waves in ν oscillation physics:
  - *Nucleon* form factor uncertainty significantly underestimated
  - Mounting evidence that ν QE cross section significantly underestimated
    $\implies$ Attention needed to avoid biased results
- Significant progress being made in LQCD technology
  $\implies$ Many computations enabled during past few years
- LQCD as a tool to provide insight in nuclear systems
  $\implies$ valuable inputs for other nuclear methods

Exciting times are ahead!

Thank you for your attention!

Backup

Excited States



$(Q^2 = 0)$ [Phys.Rev.C 105 (2022)]

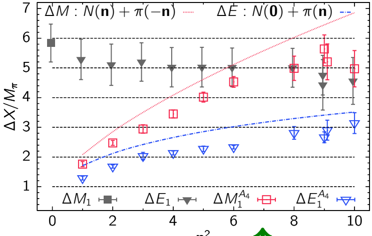
Compare fit to correlator data ratio

Remnant contamination
dominated by "transition" states
($m \to n$, violet)

Statistically significant until 2 fm
typical data $\lesssim 1$ fm

Excited states still present in
practically achievable large time limit

$(Q^2 \geq 0)$



[Phys.Rev.Lett.124(2020)]

NOTE: expect only approx
agreement between data/curves

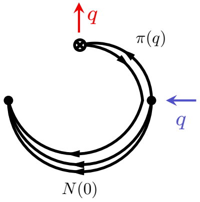
NME collab:

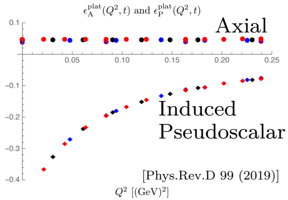
Q^2 contamination from $N \to N\pi$

Dominant contribution agrees
with χPT expectation

$N\pi$ is important for $F_A(Q^2)$

Excited States - χPT and $N\pi$





Contamination primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Correlator fits without axial current not sensitive to $N\pi$
$\implies$ need simultaneous fits including axial matrix elements
[Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Alternate fit strategies to remove $N\pi$ (are they comparable?):

- Kinematic constraints ($F_P = 0$)
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- explicit $N\pi$ operators

Prediction from χPT: [Phys.Rev.D 99 (2019)]

First demonstration by NME: [Phys.Rev.Lett. 124 (2020)]

χPT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]

LQCD $g_A(Q^2 = 0)$

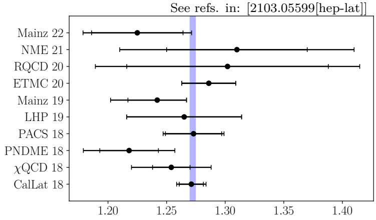
g_A is benchmark for
nucleon matrix elements in LQCD

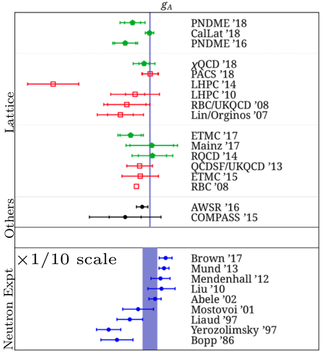
Status circa 2018 summarized by
USQCD white paper
[Eur.Phys.J.A 55 (2019)]

See also: FLAG review
[Eur.Phys.J.C 80 (2020)]

Historically g_A low compared to expt
excited states (+other...)

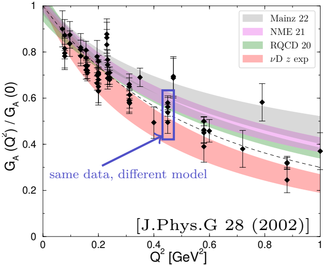
Lots of activity since 2018,
consistent agreement with PDG
full error budgets available





[Eur.Phys.J.A 55 (2019)]

Electro Pion Production



- Large model uncertainty, not included in world averages
- Valid only in $M_\pi \to 0$, $q \to 0$ limits
- Expansion to $O(M_\pi^2, Q^2)$:
  - restricted Q^2 validity
  - lacks shape freedom in Q^2
- Predates Heavy Baryon χPT, no systematic power counting

Modern experiments do not report $F_A(Q^2) \implies$ averages out of date

Possible argument for comparing to r_A^2 from low Q^2; high Q^2 untrustworthy

Effort needed to update prediction from photo/electro pion production