The Mass Decomposition of Hadrons in QCD

(Andreas Metz, Temple University)

- Motivation
- Energy momentum tensor (EMT)
- Mass decompositions (sum rules)
- Numerics for proton mass decomposition
- Summarizing comparison

Based on: S. Rodini, A.M., B. Pasquini, JHEP 09 (2020) 067, arXiv:2004.03704
A.M., B. Pasquini, S. Rodini, PRD 102 (2020) 114042, arXiv:2006.11171
C. Lorcé, A.M., B. Pasquini, S. Rodini, JHEP 11 (2021) 121, arXiv:2109.11785

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Motivation

- Different hadron mass sum rules in QCD $\rightarrow$ How do they compare to each other ?
- Example 1: 4-term decomposition (Ji, 1994, 1995, with small re-arrangement)

$$\mathcal{H}_{q[\mathrm{Ji}]} = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_{R[\mathrm{Ji}]} \qquad \text{(quark kinetic plus potential energy)}_{[\mathrm{Ji}]}$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R \qquad \text{quark mass term}$$

$$\mathcal{H}_{g[\mathrm{Ji}]} = \tfrac{1}{2}(E^2 + B^2)_{R[\mathrm{Ji}]} \qquad \text{(gluon energy)}_{[\mathrm{Ji}]}$$

$$\mathcal{H}_a = \tfrac{1}{4}\Big(\gamma_m\,(m\bar{\psi}\psi)_R + \tfrac{\beta}{2g}(F^2)_R\Big) \qquad \text{anomaly contribution}$$

- Example 2: 3-term decomposition (Rodini, AM, Pasquini, 2020 / AM, Rodini, Pasquini, 2020)

$$\mathcal{H}_q = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R \qquad \text{quark (kinetic plus potential) energy}$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R \qquad \text{quark mass term}$$

$$\mathcal{H}_g = \tfrac{1}{2}(E^2 + B^2)_R \qquad \text{gluon energy}$$

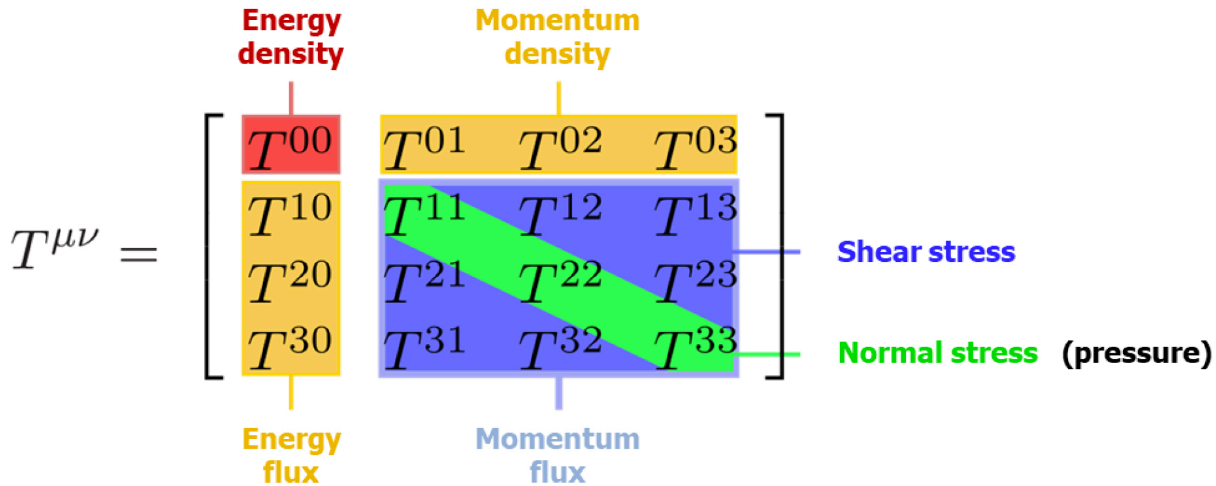
(i) either, operators identical but at least one group made a mistake concerning $\mathcal{H}_a$

(ii) or, meaning of two operators $(\mathcal{H}_q, \mathcal{H}_g)$, generally, is different ($\rightarrow$ this talk)

(but still: derivation of operators ? / interpretation of parton energy terms ?)

Energy Momentum Tensor

- Interpretation of EMT



(courtesy, C. Lorcé)

- Symmetric (gauge invariant) EMT in QCD

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T_q^{\mu\nu} = \frac{i}{4}\,\bar{\psi}\,\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\,\psi \qquad \left(\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}} = \gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)$$

$$T_g^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{g^{\mu\nu}}{4}\,F^2$$

  - $T_q^{\mu\nu}$ contains gluon field through $\overleftrightarrow{D}^{\mu} = \overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu} - 2igA_a^{\mu}\,T_a$
  - Total EMT not renormalized, but $T_i^{\mu\nu}$ require renormalization

- Trace (anomaly) of EMT in QCD
  (Collins, Duncan, Joglekar, 1977 / Nielsen, 1977 / ...)

$$T^{\mu}{}_{\mu} = \underbrace{(m\bar{\psi}\psi)_R}_{\text{classical trace}} + \underbrace{\gamma_m\,(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R}_{\text{trace anomaly}}$$

  – $T^{\mu}{}_{\mu}$, classical trace (quark mass term), and trace anomaly are UV-finite

- Quark and gluon contribution to trace of EMT (Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)

$$T^{\mu}{}_{\mu} = (T_{q,R})^{\mu}{}_{\mu} + (T_{g,R})^{\mu}{}_{\mu}$$

$$(T_{q,R})^{\mu}{}_{\mu} = (1+y)(m\bar{\psi}\psi)_R + x\,(F^2)_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - x\right)(F^2)_R$$

  x and y related to finite parts of renormalization constants $\rightarrow$ scheme dependence

- Different scheme choices (Rodini, AM, Pasquini, 2020 / AM, Pasquini, Rodini, 2020)
  – MS scheme / $\overline{\text{MS}}$ scheme (Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)
  – D1 scheme: $x = 0$, $y = \gamma_m$
  – D2 scheme: $x = y = 0$

  D-type schemes look natural

EMT and Proton Mass

- Forward matrix element of total EMT (for spin-0 and spin-$\frac{1}{2}$)

$$\langle\, T^{\mu\nu}\,\rangle \equiv \langle P|\, T^{\mu\nu}\,|P\rangle = 2P^\mu P^\nu$$

- Relation to proton mass ($n = \frac{1}{2M}$, depends on normalization of state)

$$M = n\,\langle\, T^{\mu}{}_{\mu}\rangle = n\,\langle\, T^{00}\,\rangle\Big|_{\mathbf{P}=0} = \frac{\langle\, H_{\rm QCD}\,\rangle}{\langle P|P\rangle}\bigg|_{\mathbf{P}=0} \qquad \left(\int d^3\mathbf{x}\, T^{00} = H_{\rm QCD}\right)$$

- Forward matrix element of $T^{\mu\nu}_{i,R}$ (Ji, 1996)

$$\langle\, T^{\mu\nu}_{i,R}\,\rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu}\overline{C}_i(0)$$

  – $A_i(0),\ \overline{C}_i(0)$ are gravitational FFs at $t = 0$
  – conservation of (full) EMT implies

$$A_q(0) + A_g(0) = 1 \qquad \overline{C}_q(0) + \overline{C}_g(0) = 0$$

  – in forward limit, matrix elements of EMT fully determined by two numbers only (emphasized also in Lorcé, 2017)

2-Term Sum Rule by Hatta, Rajan, Tanaka

(Hatta, Rajan, Tanaka, JHEP 12 (2018) 008 / Tanaka, JHEP 01 (2019) 120)

- Sum rule based on decomposition of $T^{\mu}{}_{\mu}$

$$M = \overline{M}_q + \overline{M}_g = n \Big(\langle \, (T_{q,R})^{\mu}{}_{\mu} \, \rangle + \langle \, (T_{g,R})^{\mu}{}_{\mu} \, \rangle \Big)$$

- Recall operators

$$(T_{q,R})^{\mu}{}_{\mu} = (1 + y)(m\bar{\psi}\psi)_R + x\,(F^2)_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \Big(\tfrac{\beta}{2g} - x\Big)(F^2)_R$$

- Using D-type schemes

$$(T_{q,R})^{\mu}{}_{\mu}\big|_{\mathrm{D1}} = (1 + \gamma_m)(m\bar{\psi}\psi)_R \qquad (T_{g,R})^{\mu}{}_{\mu}\big|_{\mathrm{D1}} = \tfrac{\beta}{2g}\,(F^2)_R$$

$$(T_{q,R})^{\mu}{}_{\mu}\big|_{\mathrm{D2}} = (m\bar{\psi}\psi)_R \qquad (T_{g,R})^{\mu}{}_{\mu}\big|_{\mathrm{D2}} = \gamma_m(m\bar{\psi}\psi)_R + \tfrac{\beta}{2g}\,(F^2)_R$$

2-Term Sum Rule by Lorcé

(Lorcé, EPJC 78, 120 (2018))

- Sum rule based on decomposition of T^{00}

$$M = U_q + U_g = n\Big(\langle\, T^{00}_{q,R}\,\rangle + \langle\, T^{00}_{g,R}\,\rangle\Big)$$

- Renormalized operators (in dimensional regularization) (Rodini, AM, Pasquini, 2020)

$$T^{00}_{q,R} = (m\bar{\psi}\psi)_R + (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R \qquad \text{total quark energy}$$

$$T^{00}_{g,R} = \tfrac{1}{2}(E^2 + B^2)_R \qquad \text{gluon energy}$$

- Interpretation looks clean (component of EMT, and operator form)

- Relation of parton energies to EMT form factors

$$U_i = M\big(A_i(0) + \overline{C}_i(0)\big)$$

- Measurement of U_i requires two observables ("indirect")
  - $A_i(0) = \langle x_i \rangle$ (parton momentum fractions)
  - information about $\overline{C}_i(0)$ from EMT trace

3-Term Sum Rule

(Rodini, AM, Pasquini, JHEP 09 (2020) 067 / AM, Rodini, Pasquini, PRD 102 (2020) 114042)

- Sum rule based on decomposition of T^{00}

$$M = M_q + M_m + M_g = n\Big(\langle\mathcal{H}_q\rangle + \langle\mathcal{H}_m\rangle + \langle\mathcal{H}_g\rangle\Big)$$

- Renormalized operators

$$\mathcal{H}_q = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R \qquad \text{quark (kinetic plus potential) energy}$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R \qquad \text{quark mass term}$$

$$\mathcal{H}_g = \tfrac{1}{2}(E^2 + B^2)_R \qquad \text{gluon energy}$$

- 3-term sum rule can be considered refinement of 2-term sum rule by Lorcé

$$M_q + M_m = U_q \qquad\qquad M_g = U_g$$

  – M_m is UV finite, has a clear interpretation, and has been studied frequently

- Interpretation looks clean

4-Term Sum Rule by Ji

(Ji, PRL 74, 1071 (1995) and PRD 52, 271 (1995))

- Sum rule based on decomposition of T^{00} into traceless part and trace part

$$T^{\mu\nu} = \underbrace{\left(T^{\mu\nu} - \hat{T}^{\mu\nu}\right)}_{\text{traceless part}} + \underbrace{\hat{T}^{\mu\nu}}_{\text{trace part}}$$

$$\hat{T}^{\mu\nu} = \tfrac{1}{4}\, g^{\mu\nu}\, T^{\alpha}_{\ \alpha} \qquad \overline{T}^{\mu\nu} = T^{\mu\nu} - \hat{T}^{\mu\nu}$$

- Motivation: $\hat{T}^{\mu\nu}$ and $\overline{T}^{\mu\nu}$ are UV finite

- (Consequence of) virial theorem
  (Ji, 1995 / Ji, Liu, Schäfer, 2021 / Lorcé, AM, Pasquini, Rodini, 2021 / ...)

$$M = E_T + E_S = \tfrac{3}{4}\, M + \tfrac{1}{4}\, M \qquad (E_T \leftrightarrow \overline{T}^{00} \qquad E_S \leftrightarrow \hat{T}^{00})$$

  decomposition follows from $\langle\, T^{\mu\nu}\,\rangle = 2P^{\mu}P^{\nu}$

- Final 4-term sum rule obtained by
  (i) decomposition of $\overline{T}^{00}$ and $\hat{T}^{00}$ into quark and gluon contributions
  (ii) re-arrangement in quark sector (re-shuffling between traceless and trace part)

- 4-term decomposition of T^{00}

$$M = M_{q[\mathrm{Ji}]} + M_m + M_{g[\mathrm{Ji}]} + M_a = n\Big(\langle\mathcal{H}_{q[\mathrm{Ji}]}\rangle + \langle\mathcal{H}_m\rangle + \langle\mathcal{H}_{g[\mathrm{Ji}]}\rangle + \langle\mathcal{H}_a\rangle\Big)$$

- Renormalized operators (Ji, 1995)

$$\mathcal{H}_{q[\mathrm{Ji}]} = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_{R[\mathrm{Ji}]} \qquad \text{(quark kinetic plus potential energy)}_{[\mathrm{Ji}]}$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R \qquad \text{quark mass term}$$

$$\mathcal{H}_{g[\mathrm{Ji}]} = \tfrac{1}{2}(E^2 + B^2)_{R[\mathrm{Ji}]} \qquad \text{(gluon energy)}_{[\mathrm{Ji}]}$$

$$\mathcal{H}_a = \tfrac{1}{4}\Big(\gamma_m\,(m\bar{\psi}\psi)_R + \tfrac{\beta}{2g}(F^2)_R\Big) \qquad \text{anomaly contribution}$$

  – compared to 3-term decomposition, $\mathcal{H}_a$ comes in addition

- Comparison with our renormalized operators

$$\begin{aligned}\mathcal{H}_{g[\mathrm{Ji}]} &= \mathcal{H}_g - \tfrac{1}{4}(T_{g,R})^\mu{}_\mu \\ &= \tfrac{1}{2}(E^2 + B^2)_R + \tfrac{y-\gamma_m}{4}(m\bar{\psi}\psi)_R - \tfrac{1}{4}\big(\tfrac{\beta}{2g} - x\big)(F^2)_R\end{aligned}$$

  – similar discussion holds for $\mathcal{H}_{q[\mathrm{Ji}]}$
  – interpretation of (operator of) $\mathcal{H}_{g[\mathrm{Ji}]}$ and $\mathcal{H}_{q[\mathrm{Ji}]}$?
  – also, interpretation of $\mathcal{H}_{g[\mathrm{Ji}]}$, $\mathcal{H}_{q[\mathrm{Ji}]}$ due to pressure terms ? (Lorcé, 2017)

- More recent result in dimensional regularization (Ji, Liu, Schäfer, 2021)

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R$$

$$\mathcal{H}_a = \frac{1}{4}\Big(\gamma_m\,(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R\Big)$$

$$(\mathcal{H}_q + \mathcal{H}_g)_{[\mathrm{JLS}]} = \Big(\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi + \frac{2-2\varepsilon}{4-2\varepsilon}\,\boldsymbol{E}^2 + \frac{2}{4-2\varepsilon}\,\boldsymbol{B}^2\Big)_R$$

  - this expression differs from original operator form (Ji, 1995)
  - we find exact agreement with our result by using (Lorcé, AM, Pasquini, Rodini, 2021)

$$-\frac{\varepsilon}{4}(\boldsymbol{E}^2 - \boldsymbol{B}^2) = \frac{\varepsilon}{8}F^2 = -\frac{1}{4}\Big(\gamma_m\,(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R\Big) \quad \text{leading to}$$

$$(\mathcal{H}_q + \mathcal{H}_g)_{[\mathrm{JLS}]} = \mathcal{H}_q + \mathcal{H}_g - \mathcal{H}_a \quad \text{implying}$$

$$\mathcal{H}_m + \mathcal{H}_a + (\mathcal{H}_q + \mathcal{H}_g)_{[\mathrm{JLS}]} = \mathcal{H}_m + \mathcal{H}_q + \mathcal{H}_g \quad \text{(our result)}$$

Numerical Results

- First input: parton momentum fractions $\langle x_i \rangle$, related to traceless parton operators

$$\frac{3}{2}\,M^2\,a = \langle\,\overline{T}^{00}_{q,R}\,\rangle \qquad \frac{3}{2}\,M^2\,(1-a) = \langle\,\overline{T}^{00}_{g,R}\,\rangle \qquad \Big(a = \langle x_q \rangle \quad 1-a = \langle x_g \rangle\Big)$$

- Second input: quark mass term

$$2M^2\,b = (1+\gamma_m)\,\langle\,(m\bar{\psi}\psi)_R\,\rangle \;\rightarrow\; 2M^2\,(1-b) = \frac{\beta}{2g}\,\langle\,(F^2)_R\,\rangle$$

  – to the extent we know b, we know $\langle (F^2)_R \rangle$, and vice versa

- Example: 3-term sum rule in terms of a and b

$$M_q = \frac{3}{4}\,M\,a + \frac{1}{4}\,M\left(\frac{(y-3)\,b}{1+\gamma_m} + x(1-b)\frac{2g}{\beta}\right)$$

$$M_m = M\,\frac{b}{1+\gamma_m}$$

$$M_g = \frac{3}{4}\,M\,(1-a) + \frac{1}{4}\,M\left[\frac{(\gamma_m - y)\,b}{1+\gamma_m} + \left(1 - x\frac{2g}{\beta}\right)(1-b)\right]$$

- Momentum fractions from CT18NNLO parameterization (at $\mu = 2\,\mathrm{GeV}$)

$$a = 0.586 \pm 0.013 \qquad 1 - a = 0.414 \pm 0.013$$

- Quark mass term from sigma terms

$$\sigma_u + \sigma_d = \sigma_{\pi N} = \frac{\langle P|\, \hat{m}\, (\bar{u}u + \bar{d}d)\, |P\rangle}{2M} \quad \sigma_s = \frac{\langle P|\, m_s\, \bar{s}s\, |P\rangle}{2M} \quad \sigma_c = \frac{\langle P|\, m_c\, \bar{c}c\, |P\rangle}{2M}$$

  - Scenario A: sigma terms from phenomenology
    (Alarcon et al, 2011, 2012 / Hoferichter et al, 2015)

$$\sigma_{\pi N}\big|_{\mathrm{ChPT}} = (59 \pm 7)\,\mathrm{MeV} \qquad \sigma_s\big|_{\mathrm{ChPT}} = (16 \pm 80)\,\mathrm{MeV}$$

  - Scenario B: sigma terms from lattice QCD
    (Alexandrou et al, 2019)

$$\sigma_{\pi N}\big|_{\mathrm{LQCD}} = (41.6 \pm 3.8)\,\mathrm{MeV} \qquad \sigma_s\big|_{\mathrm{LQCD}} = (39.8 \pm 5.5)\,\mathrm{MeV}$$
$$\sigma_c\big|_{\mathrm{LQCD}} = (107 \pm 22)\,\mathrm{MeV}$$

  - main difference between scenarios: including or not σ_c

- Dependence on EMT renormalization scheme, for 3-term sum rule ($\mu = 2\,\mathrm{GeV}$, numbers in units of GeV)

		MS	$\overline{\mathrm{MS}}_1$	$\overline{\mathrm{MS}}_2$	D1	D2
Scenario A	M_q	0.309 ± 0.044	0.194 ± 0.033	0.178 ± 0.032	0.362 ± 0.045	0.357 ± 0.051
	M_m	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080
	M_g	0.555 ± 0.036	0.669 ± 0.047	0.686 ± 0.048	0.502 ± 0.035	0.507 ± 0.029
Scenario B	M_q	0.234 ± 0.006	0.135 ± 0.003	0.120 ± 0.003	0.286 ± 0.006	0.272 ± 0.008
	M_m	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023
	M_g	0.517 ± 0.017	0.617 ± 0.020	0.631 ± 0.020	0.465 ± 0.017	0.479 ± 0.015

  - considerable numerical scheme dependence (similar for 2-term sum rules)
  - scheme dependence no new phenomenon
  - no scheme dependence for 4-term sum rule
  - contribution of M_m is $\sim 8\%$ for Scenario A, $\sim 20\%$ for Scenario B
  - quark mass term for heavy quarks significant
    (Shifman, Vainshtein, Zakharov, 1978 / AM, Pasquini, Rodini, 2020 / Liu, 2021 /...)

Further Comparison of Mass Sum Rules

- Number of independent terms, and required input parameters (a, b)
  - 2 terms $T^{\mu}{}_{\mu}$ $\quad M = \overline{M}_q + \overline{M}_g \quad \rightarrow$ 1 indep. term (b)
  - 2 terms T^{00} $\quad M = U_q + U_g \quad \rightarrow$ 1 indep. term (a, b)
  - 3 terms T^{00} $\quad M = M_q + M_m + M_g \quad \rightarrow$ 2 indep. terms (a, b)
  - 4 terms T^{00} $\quad M = M_{q[\mathrm{Ji}]} + M_m + M_{g[\mathrm{Ji}]} + M_a \quad \rightarrow$ 2 indep. terms only (a, b)

$$M_{q[\mathrm{Ji}]} - \frac{3\gamma_m}{4+\gamma_m} M_m + M_{g[\mathrm{Ji}]} - 3M_a = 0 \qquad \text{(additional relation)}$$

- Relation to experiment
  - $M_{g[\mathrm{Ji}]}$ directly related to $\langle x_g \rangle = 1 - a$
  - $M_{q[\mathrm{Ji}]}$ not directly related to $\langle x_q \rangle = a$ (admixture from b, "indirect" measurement)
    $\rightarrow$ hardly any advantage of 4-term sum rule over other sum rules
  - "side-remark": measuring $\langle F^2 \rangle$ (at the EIC) relevant for all sum rules (further constraint on b)

- Dependence on scheme (x and y)
  - 2-term and 3-term sum rules: operators don't change, numbers may change
  - 4-term sum rule: numbers don't change, operators may change

- Closest agreement in D2 scheme ($x = y = 0$)
  - relation between quark contribution to trace and quark mass term

$$\overline{M}_q^{\mathrm{D2}} = M_m$$

  - relation between parton energies

$$M_q^{\mathrm{D2}} = M_{q[\mathrm{Ji}]} \qquad\qquad M_g^{\mathrm{D2}} = M_{g[\mathrm{Ji}]} + M_a$$

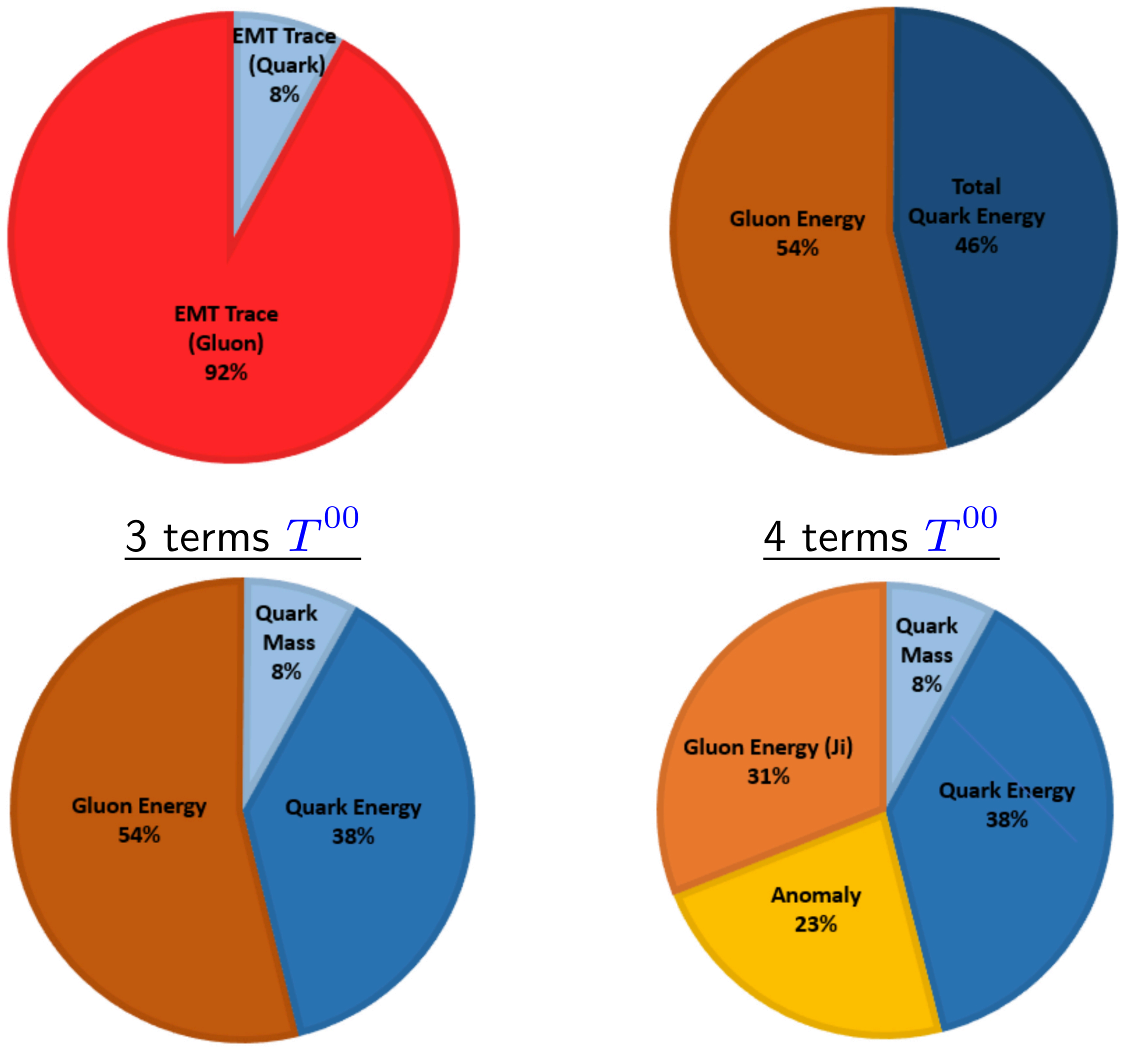
    Two different perspectives:
    (i) $M_{g[\mathrm{Ji}]}$ has no clear interpretation (operator form, components of EMT)
    $\rightarrow$ M_a must be added to get meaningful quantity (our view)
    (ii) anomaly contribution M_a hidden in M_g^{D2} (Ji, 2021)

- Scale dependence
  - "simple" for 4-term sum (given by scale dependence of A_i)
  - generally, more complicated (but known) for other sum rules (due $\bar{C}_i$)
  - in D2 scheme, scale dependence equally "simple" for all sum rules

- Numerical comparison in D2 scheme (u,d,s in quark mass term)

2 terms $T^{\mu}{}_{\mu}$ 2 terms T^{00}

3 terms T^{00} 4 terms T^{00}



$$\overline{M}_q^{\mathrm{D2}} = M_m \qquad M_q^{\mathrm{D2}} = M_{q[\mathrm{Ji}]} \qquad M_g^{\mathrm{D2}} = M_{g[\mathrm{Ji}]} + M_a$$