Los Alamos NATIONAL LABORATORY

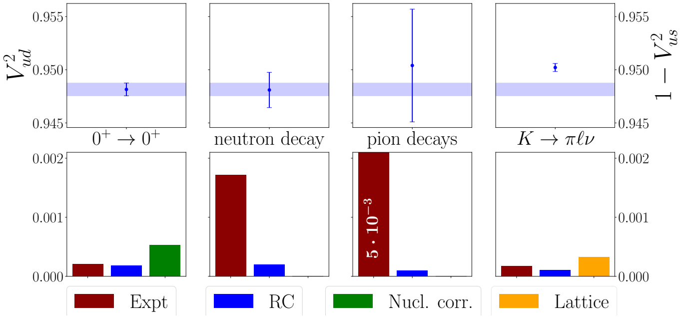
Effective Field Theory approach for radiative corrections to superallowed β decays.

E. Mereghetti

with G. Chambers-Wall, V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, S. Gandolfi, G. King, S. Novario, S. Pastore

Testing the Standard Model in Charged-Weak Decays
INT, Seattle, January 14nd, 2026

CKM unitarity tests and the Cabibbo anomaly



$V_{ud} = 0.97367 \pm 0.00032$
$V_{us}(K_{\ell 3}) = 0.2233 \pm 0.0005$

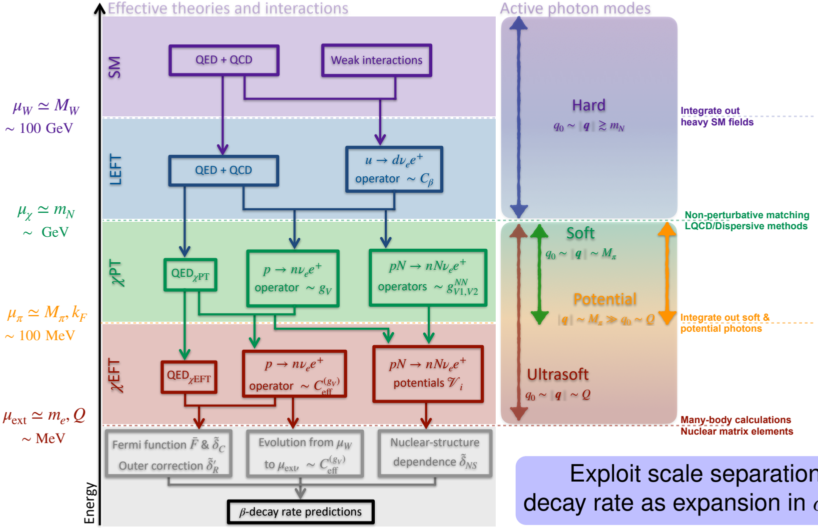
- the SM predicts $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- re-evaluation of "inner radiative correction" and precise lattice QCD calculation of $f_+(0)$

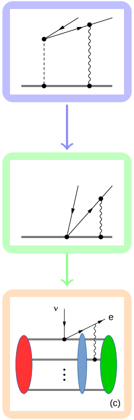
  C. Y. Seng, M. Gorchtein, H. Patel, M. Ramsey-Musolf, '18 A. Czarnecki, W. Marciano, A. Sirlin, '19; A. Bazavov *et al*, '18
- led to $\sim 2\sigma$-3σ tension in CKM unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -(1.65 \pm 0.73) \cdot 10^{-3}$$

BSM physics in the 5-10 TeV range? Can we trust the nuclear corrections?

Why EFTs? Scales in β decays





Exploit scale separation to set up decay rate as expansion in α and $(\mu_i/\mu_j)^n$

V. Cirigliano, W. Dekens, J. de Vries, S. Gandolfi, M. Hoferichter, EM, '24

EFTs and the factorization of the half-life

$$\frac{1}{t} = \frac{G_F^2|V_{ud}|^2 m_e^5}{\pi^3 \log 2} \left[C_{\rm eff}^{(g_V)}(\mu)\right]^2 \bar{f}(\mu) \times [1 + \bar{\delta}'_R(\mu)] \, (1 + \bar{\delta}_{\rm NS} - \bar{\delta}_C)$$

$\left[C_{\rm eff}^{(g_V)}(\mu)\right]^2$: hard and soft photons — expansion in α, α_s, k_F/m_N

- matching at electroweak scale, RG evolution from m_W to $\mu \sim m_N$ [pQCD]
- 1-body matching Fermi theory to χPT at $\mu \sim m_N$ [dispersive + LQCD methods]
- running from m_N to $\mu \sim k_F$ and matching at k_F [pQED]

$\bar{\delta}_{\rm NS}$ and $\bar{\delta}_C$: potential and soft photons, sensitive to nuclear details — expansion in $\alpha, k_F/m_N$, $\mathcal{Q}/k_F$

- derivation of EW operators [χPT]
- two- and three-nucleon matrix elements [QMC, ab initio methods, shell model . . .]
- low-energy constants [dispersive + LQCD methods, fit to data]

$\bar{\delta}'_R$, $\bar{f}$: ultrasoft photons — expansion in α, $\mathcal{Q}/k_F$

- sensitive to global nuclear properties, e.g. charge, radii [pQED]

- each object depends on a factorization scale μ — resum large logs and estimate higher orders

EFTs and the factorization of the half-life

$$\frac{1}{t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} (1 + \Delta_R^V)(1 + \delta_R')(1 + \delta_{\rm NS} - \delta_C) \times f$$

J. C. Hardy and I. Towner, '20

- very similar to the "standard" half-life

$\Delta_R^V \propto g_V(m_N), \;\; \delta_R' \to C_{\rm eff}^{(g_V)}(\mu)$

1. better factorization of the nuclear and low-energy dynamics
2. resummation of $\log \mathcal{Q}/k_F$ in addition to $\log \mathcal{Q}/m_N$

$\delta_{\rm NS} \to \bar\delta_{\rm NS}$

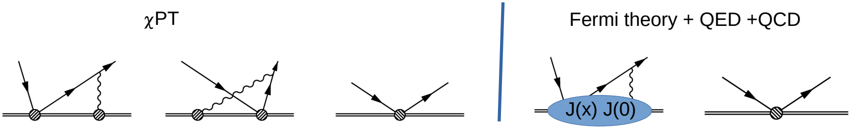
3. systematically improvable definition of EW operators

$\delta_R' \to \bar\delta_R'$

4. higher order pQED calculation in consistent scheme

Hard Modes

Hard modes: matching onto χPT



$$
\begin{aligned}
\mathcal{L}_{\rm Fermi} &= -2\sqrt{2} G_F V_{ud} C_\beta \bar{e}_L \gamma^\mu \nu_L \, \bar{q} \tau^+ \gamma_\mu q + \mathcal{L}_{\rm QED} \\
&\Longrightarrow -\sqrt{2} G_F V_{ud} \, \bar{e}_L \gamma^0 \nu_L \left\{ g_V \bar{N} \tau^+ N + g_{V1}^{NN} \bar{N} \tau^+ N \, \bar{N} N + g_{V2}^{NN} \bar{N} \tau^+ N \, \bar{N} \tau_3 N + \dots \right\} + \mathcal{L}^{\chi}_{\rm QED}
\end{aligned}
$$

- C_β encodes contributions of photons and gluons from m_W to Λ_χ
- photons with $|\vec{q}| \sim \Lambda_\chi$ are not in the low-energy EFT
- their contribution is hidden in EFT low-energy constants

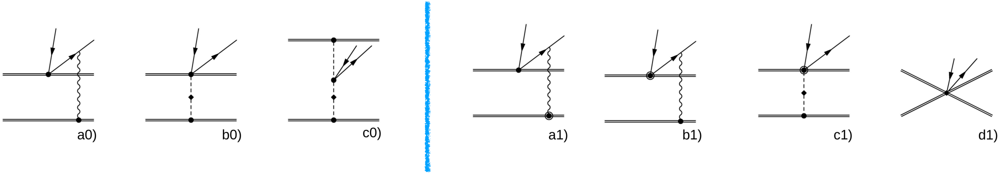
$$
g_V = 1 + \sum_n \left(\frac{\alpha}{4\pi}\right)^n g_V^{(n)}, \qquad g_{\rm Vi}^{NN} = \sum_n \left(\frac{\alpha}{4\pi}\right)^n g_{\rm Vi}^{(n)}
$$

- g_V is common to neutron and nuclear decays

see Vincenzo's and Misha's talks

Potential and Soft Modes

Potential modes. $\bar{\delta}_{\rm NS}$ and two-body weak currents



- the second important regime is $q_0 \ll |\mathbf{q}| \sim k_F$ "potential"
- photon emission puts the nuclear intermediate state far off-shell
- can neglect nuclear excitation energy in energy denominators & sum over intermediate states

similar to "closure approximation" in $0\nu\beta\beta$ (made more systematic in EFT)

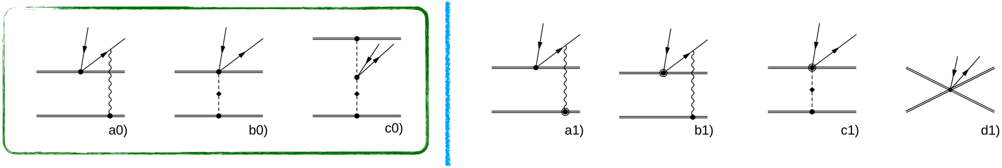
- $\bar{\delta}_{\rm NS}$ given by matrix element of a two- or higher-body current $\mathcal{V}^0$

$$\boxed{\bar{\delta}_{\rm NS} = \sqrt{2}\langle f|\mathcal{V}^0|i\rangle,}$$

- $\mathcal{V}^0$ has an expansion in α and Q/Λ_χ as "standard" vector & axial 2- and 3-body currents

$$\mathcal{V}^0 = \alpha \sum_{n,m} \mathcal{V}_{n,m}\, \alpha^n \, \varepsilon_\chi^m \qquad \varepsilon_\chi = \frac{Q}{\Lambda_\chi}$$

$\bar{\delta}_{\rm NS}$ at $\mathcal{O}(\alpha\mathcal{Q}/k_F)$

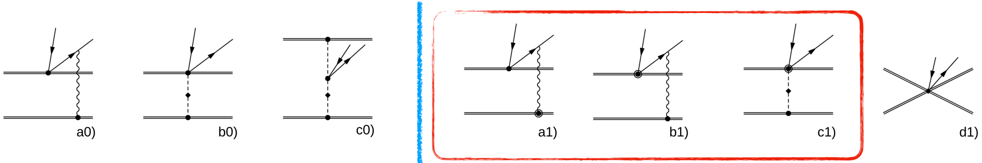


- diagrams with LO weak and EM vertices are proportional to external momenta

$$\mathcal{V}_E^0 = \left(\frac{E_0}{6} + \frac{4}{3}E_e\right)\sum_{j<k}\frac{e^2}{\mathbf{q}^4}\left(\tau^{-(j)}P_p^{(k)} + j \leftrightarrow k\right) \longrightarrow -\alpha\,(R_A E_0)\left(\frac{1}{6} + \frac{4E_e}{3E_0}\right)\sum_{j<k}\frac{r_{jk}}{2R_A}\left(\tau^{-(j)}P_p^{(k)} + j \leftrightarrow k\right)$$

- correction suppressed by E_e/k_F, still very important as it grows with Z
- replaces corrections linear in $R = \sqrt{5/3\langle r^2_{\rm ch}\rangle}$ in shape factor C and finite size corrections L_0
- scaling is somewhat similar to $\delta_{NS}(E)$ in M. Gorchtein, ‘18, but we were not able to exactly map it

$\bar{\delta}_{\rm NS}$ at $\mathcal{O}(\alpha k_F/m_N)$



- diagrams with NLO EM vertices do not require external momenta $\mathcal{O}(\alpha\varepsilon_\chi)$
- long range corrections mediated by nucleon magnetic moments and recoil corrections*

* thanks to S. Novario and C.-Y. Seng for spotting two mistakes in our original paper

$$\mathcal{V}_0^{\rm mag}(\mathbf{q}) = \sum_{j<k} \frac{e^2}{3}\frac{g_A}{m_N}\frac{1}{\mathbf{q}^2}\left(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{\sigma}^{(k)} + \frac{1}{2}S^{(jk)}\right)\left[(1+\kappa_p)\tau^{-(j)}P_p^{(k)} + \kappa_n\tau^{-(j)}P_n^{(k)} + (j\leftrightarrow k)\right],$$

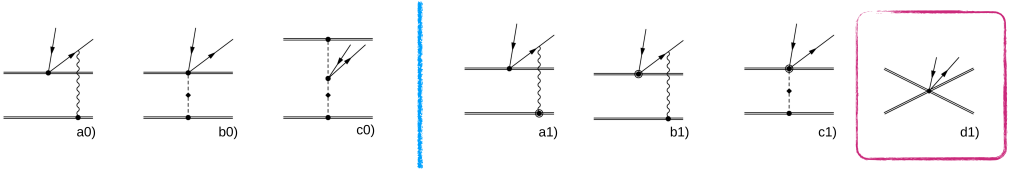
$$\mathcal{V}_0^{\rm rec}(\mathbf{q},\mathbf{P}) = \sum_{j<k} i\frac{e^2 g_A}{4m_N}\left[\frac{\tau^{-(j)}P_p^{(k)}}{\mathbf{q}^4}(\mathbf{P}_k\times\mathbf{q})\cdot\boldsymbol{\sigma}^{(j)} - (j\leftrightarrow k)\right]$$

very similar to I. S. Towner, '92,

- $\mathcal{V}_0^{\rm mag}$ is a Coulomb-like potential $\Longrightarrow$ induces UV sensitivity when acting in $^1S_0$ channel†

† same as m_π dependence of nuclear force & $0\nu\beta\beta$ D. Kaplan, M. Savage, M. Wise, '96 V. Cirigliano, *et al*, '18

$\bar{\delta}_{\rm NS}$ at $\mathcal{O}(\alpha k_F/m_N)$



- to absorb cut-off dependence, need

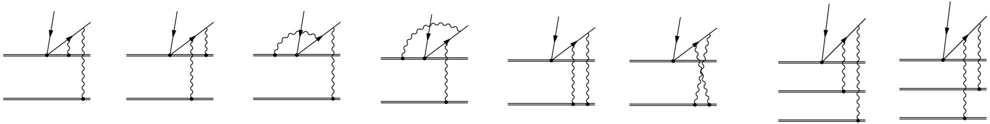
$$\mathcal{V}_0^{\rm CT} = e^2\left(g_{V1}^{NN} O_1 + g_{V2}^{NN} O_2\right), \qquad O_1 = \sum_{j\neq k} \tau^{-(j)} \mathbb{1}_k, \quad O_2 = \sum_{j<k} \left[\tau^{-(j)} \tau_3^{(k)} + (j \leftrightarrow k)\right].$$

Extraction of g_{V1}^{NN} and g_{V2}^{NN}:

1. lattice QCD — might take a long time...
2. calculate/model $W\gamma$ Compton tensor at large Q^2 — "Cottingham"
3. match "EFT" and "dispersive" $\delta_{\rm NS}$ calculations in light nuclei
4. fit them to superallowed transitions, with V_{ud} — model independent, inflate errors?

- when needed for illustration, assume "improved NDA" $g_V^{NN} \sim 1/m_N \times 1/(2F_\pi)^2$

$\mathcal{O}(\alpha^2)$ corrections to $\delta_{\rm NS}$

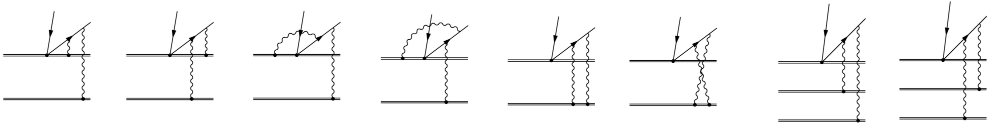


- at $\mathcal{O}(\alpha^2)$, the Fermi function and the nuclear-structure-independent correction δ_2 depend logarithmically on R A. Sirlin and R. Zucchini, '86, W. Jaus and G. Rasche, '87

$$f = 1 - \frac{\alpha Z}{\beta} + \alpha^2 Z^2 \left(\frac{\pi^2}{3\beta^2} + \frac{9}{4} - \log E_e R + \ldots \right), \qquad \delta_2 \propto \alpha^2 Z \log E_e R$$

can we understand large logs in terms of scale separation?

$\mathcal{O}(\alpha^2)$ corrections to $\delta_{\rm NS}$



- at $\mathcal{O}(\alpha^2)$, the Fermi function and the nuclear-structure-independent correction δ_2 depend logarithmically on R

A. Sirlin and R. Zucchini, '86, W. Jaus and G. Rasche, '87

$$f = 1 - \frac{\alpha Z}{\beta} + \alpha^2 Z^2 \left(\frac{\pi^2}{3\beta^2} + \frac{9}{4} - \log E_e R + \ldots \right), \qquad \delta_2 \propto \alpha^2 Z \log E_e R$$

can we understand large logs in terms of scale separation?

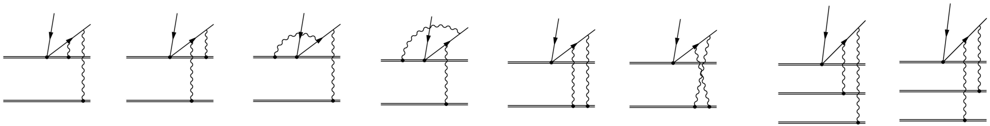
- soft and potential photons induce two more 2- and 3-body operators

$$\mathcal{V}^0 = C_\delta(\mu, \Lambda)\, \mathcal{V}_\delta + C_\delta^{\rm 3b}(\mu, \Lambda)\, \mathcal{V}_\delta^{\rm 3b} + C_+ \mathcal{V}_+(\Lambda) + C_+^{\rm 3b} \mathcal{V}_+^{\rm 3b}(\Lambda)$$

- $\mathcal{V}_\delta$ and $\mathcal{V}_\delta^{3b}$ have trivial matrix elements

$$\langle f | \mathcal{V}_\delta | i \rangle = Z M_F^{(0)}, \qquad \langle f | \mathcal{V}_\delta^{\rm 3b} | i \rangle = Z(Z-1) M_F^{(0)},$$

$\mathcal{O}(\alpha^2)$ corrections to $\delta_{\rm NS}$



- and matching coefficients that reproduce the log in the Fermi function and δ_2 correction

$$C_\delta(\mu) = -\alpha^2 \left(\frac{1}{2} \log \frac{\mu^2}{\Lambda^2} - \frac{13}{16} + \gamma_E \right) \qquad C_\delta^{\rm 3b}(\mu) = -\alpha^2 \left(\frac{1}{4} \log \frac{\mu^2}{\Lambda^2} + \frac{\gamma_E}{2} - \frac{3}{8} \right)$$

C are scale and scheme dependent!

- $\mathcal{V}_+$ and $\mathcal{V}_+^{3b}$ have a logarithmic dependence on internucleon distances

$$C_+ \mathcal{V}_+(\mathbf{r}, \Lambda) = -\alpha^2 \sum_{j \neq k} \log(r_{jk}\Lambda) \tau^{-(j)} P_p^{(k)}, \quad C_+^{3b} \mathcal{V}_+^{3b}(\mathbf{r}, \Lambda) = -\frac{\alpha^2}{2} \sum_{i \neq j \neq k} \log \left[\frac{\Lambda}{2} \left(r_{ij} + r_{ik} + r_{jk} \right) \right] \tau^{-(i)} P_p^{(j)} P_p^{(k)}$$

- nuclear matrix elements of $\mathcal{V}_+$, $\mathcal{V}_+^{3b}$ provide an operator definition of R

can relate to integrals over the weak and charge distributions?

$\delta_{\rm NS}$ summary: integrating out potential and soft modes

- we can interpret this procedure as matching onto a theory with *nuclear* fields $\mathcal{B}_{i,f}$

$$\mathcal{L}^{(0)}_{e^+} = \mathcal{B}_f^\dagger \, (iv\cdot D + \Delta)\, \mathcal{B}_f + \mathcal{B}_i^\dagger \, iv\cdot D \mathcal{B}_i - \frac{2G_F}{\sqrt{2}} V_{ud} \left(C_V \mathcal{B}_f^\dagger v_\mu \mathcal{B}_i \, \bar\nu \gamma^\mu P_L e + \mathrm{h.c.}\right) + \mathcal{O}(E/k_F).$$

can be extended to higher order, to include nuclear radii . . .
starting point for usoft loops & to reinterpret finite size corrections

- at the matching scale, the nuclear vector coupling C_V is

$$C_V(\bar\mu_\chi = \mu_\pi) = M_F^{(0)} C_{\rm eff}^{(g_V)}(\mu_\pi) \left(1 - \frac{1}{2}\delta_C + \frac{1}{2}\delta_{\rm NS}^{(0)}\right).$$

- integrating out the soft/potential modes shifts the single nucleon coupling with Z-dep. terms

$$g_V \to C_{\rm eff}^{(g_V)} = g_V \left[1 + Z\left(C_\delta - C_\delta^{3b}\right) + Z^2 C_\delta^{3b}\right]$$

- we can resume large logs by using high orders anomalous dimensions K. Borah, R. Hill and R. Plestid, '24

$$\frac{dC_{\rm eff}^{(g_V)}(\mu)}{d\log\mu} = \gamma^{(g_V)} C_{\rm eff}^{(g_V)}(\mu), \quad \gamma^{(g_V)} = \frac{\alpha}{\pi}\tilde\gamma_0 + \left(\frac{\alpha}{\pi}\right)^2 \tilde\gamma_1 + \left[\sqrt{1-\alpha^2 Z(Z\mp 1)} - 1\right] + \frac{\alpha^3}{4\pi} Z^2 \left(6 - \frac{\pi^2}{3}\right),$$

$\delta_{\rm NS}$ summary: integrating out potential and soft modes

- energy dependent and independent contributions

$$\delta_{\rm NS}^{(0)} = \frac{2}{g_V(\mu_\pi) M_{\rm F}^{(0)}} \sum_{N=n,p} \left[\alpha \left(M_{{\rm GT},N}^{\rm mag} + M_{{\rm T},N}^{\rm mag} + M_{{\rm GT},N}^{\rm CT} + M_{{\rm LS},N} \right) + \alpha^2 \left(M_{{\rm F},N}^{+} + M_{{\rm F},N}^{+,3{\rm b}} \right) \right],$$

$$\overline{\delta_{\rm NS}^{E}} = \mp \alpha \frac{2}{g_V(\mu_\pi) M_{\rm F}^{(0)}} R_A E_0 \sum_{N=n,p} \tilde{f}_E M_{{\rm F},N}^{E} \qquad \tilde{f}_E = \frac{1}{E_0} \left(\frac{4}{3} \langle E_e \rangle + \frac{1}{6} E_0 + \frac{1}{2} \left\langle \frac{m_e^2}{E_e} \right\rangle \right)$$

- all the nuclear physics input contained in two- and three-body matrix elements

$$M_{i,N}^{(\mathcal{X})} = \int_0^\infty dr\, C_{i,N}^{(\mathcal{X})}(r) = \langle f | \mathcal{V}_{i,N}^{(\mathcal{X})} | i \rangle,$$

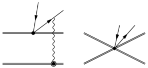
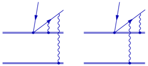
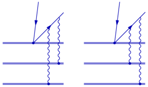
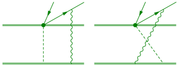
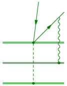
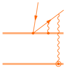
- with Fermi, Gamow-Teller, Tensor and spin-orbit (LS) components

$$\mathcal{V}_{{\rm F},N}^{\mathcal{X}} = \sum_{j<k} h_{{\rm F},N}^{\mathcal{X}}(r_{jk}) \left[\tau^{-(j)} P_N^{(k)} + j \leftrightarrow k \right], \qquad \mathcal{V}_{{\rm GT},N}^{\mathcal{X}} = \sum_{j<k} h_{{\rm GT},N}^{\mathcal{X}}(r_{jk})\, \boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\sigma}^{(k)} \left[\tau^{-(j)} P_N^{(k)} + j \leftrightarrow k \right],$$

$$\mathcal{V}_{{\rm T},N}^{\mathcal{X}} = \sum_{j<k} h_{{\rm T},N}^{\mathcal{X}}(r_{jk}) S^{(jk)}(\hat{\mathbf{r}}) \left[\tau^{-(j)} P_N^{(k)} + j \leftrightarrow k) \right], \qquad \mathcal{V}_{{\rm LS},N} = \sum_{j<k} h_{{\rm LS},N}(r_{jk}) \left[\tau^{-(j)} P_N^{(k)} (\mathbf{L}_{jk} - \mathbf{L}_{jk}^{\rm CM}) \cdot \boldsymbol{\sigma}^{(j)} + j \leftrightarrow k \right],$$

- and different radial dependence $h(r)$

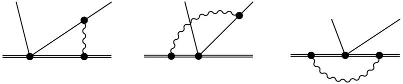
Extension of the δ_{NS} operator to higher orders

	2N current	**3N current**	**4N current**
NLO $\mathcal{O}(G_F\alpha\epsilon_\chi)$			
NLO$_\alpha$ $\mathcal{O}(G_F\alpha^2)$			
N^{2}LO $\mathcal{O}(G_F\alpha\epsilon_\chi^2)$			
N^{2}LO$_\alpha$, N^{3}LO $\mathcal{O}(G_F\alpha^3, G_F\alpha^2\epsilon_\chi, G_F\alpha\epsilon_\chi^2)$		$\dots$	$\dots$

derive $\mathcal{V}^0$ at higher orders
test convergence and theory errors

Ultrasoft Modes

Usoft modes. The Fermi and Sirlin functions



- last step of the calculation involves photon momentum modes $|q_0| \sim |\vec{q}| \sim E_e \ll R^{-1}$

  Fermi & Sirlin functions and "outer corrections"
- for Fermi function, use $\overline{\mathrm{MS}}$ calculation of R. Hill and R. Plestid, '23

$$\bar{F}(\beta,\mu) = \frac{4\eta}{(1+\eta)^2}\frac{2(1+\eta)}{\Gamma(2\eta+1)^2}|\Gamma(\eta+iy)|^2 e^{\pi y}\left(\frac{2|\mathbf{p}_e|}{\mu}e^{1/2-\gamma_E}\right)^{2(\eta-1)}, \qquad \eta=\sqrt{1-\alpha^2Z^2} \quad y=\mp Z\alpha/\beta$$

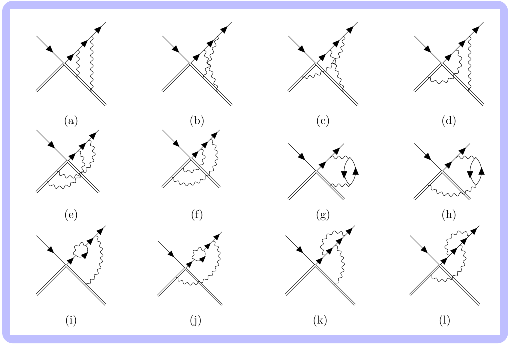
- recover the standard Fermi function by $\mu \to R^{-1}e^{1/2-\gamma_E}$
- for the "outer corrections"

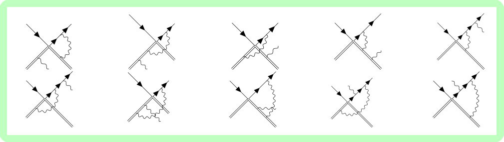
$$\tilde{\delta}'_R(E_e,\bar{\mu}) = \alpha\,\hat{g}^{(1)}(\beta,\bar{E},\bar{\mu}) + \alpha^2 Z\,\hat{g}^{(2)}(\beta,\bar{\mu}) + \dots.$$

- $\hat{g}^{(1)}(\beta,\bar{E},\bar{\mu})$ is the standard Sirlin function in HBχPT

$$\hat{g}^{(1)}(\beta,\bar{E},\bar{\mu}) = g_{\mathrm{Sirlin}} + \frac{5}{4} + \frac{3}{2}\log\frac{\bar{\mu}^2}{m_p^2}$$

Usoft modes. $\mathcal{O}(\alpha^2 Z)$ corrections





Ò. L. Crosas, EM, '25

- $\mathcal{O}(\alpha^2 Z)$ are also needed, in same scheme as the rest of the calculation
- computed diagrams in low-energy theory with nuclear dof
- diagrams are UV and IR divergent, IR cancels with real-virtual
- UV is renormalized by $C_{\rm eff}^{(g_V)}$

Usoft modes. $\mathcal{O}(\alpha^2 Z)$ corrections

$$
\begin{aligned}
\hat{g}^{(2)}(\beta,\bar{\mu}) = \pm\Bigg\{ &-\left(\frac{1}{3\beta}+\frac{1}{2}\right)L_\mu + \frac{9-\beta^2}{2\beta}\,\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right) + \frac{-5+\beta^2}{\beta}\,\mathrm{Li}_2\left(\sqrt{\frac{1-\beta}{1+\beta}}\right) \\
&-\frac{1}{\beta}\,\mathrm{Li}_2\left(\left(\frac{1-\beta}{1+\beta}\right)^2\right) + \frac{3-\beta^2}{2\beta}\frac{\pi^2}{6} - \frac{4-5\beta+\beta^3}{16\beta^2}\log^2\frac{1+\beta}{1-\beta} + \frac{\beta^2-2}{\beta^2}\log\left(1+\sqrt{\frac{1-\beta}{1+\beta}}\right) \\
&-\frac{2\beta^2+2}{\beta^2}\log\frac{1+\beta}{2} + \frac{1}{12\beta^2}\log\frac{1+\beta}{1-\beta}\left(-6+10\beta+3\beta^2+3\beta^3\right) - \frac{4}{\beta} \\
&+\left(1-\sqrt{\frac{1-\beta}{1+\beta}}\right)^3\frac{(1+\beta)^2}{144\beta^4}\left(\sqrt{\frac{1-\beta}{1+\beta}}(430-220\beta-39\beta^2+48\beta^3)\right. \\
&\left.+434-652\beta+327\beta^2-96\beta^3\right)\Bigg\},
\end{aligned}
$$

- not much worse than the Sirlin function ...
- $L_\mu = \log \mu^2/m_e^2$ absorb scale dependence of $\alpha_{\overline{\mathrm{MS}}}$ & $C_{\mathrm{eff}}^{(g_V)}$

$\mathcal{O}(\alpha^2 Z)$ corrections

- in the limit $\beta \to 1$

$$\hat{g}^{(2)}(\beta, \bar{E}) \xrightarrow[\beta\to 1]{} \pm \left[-\frac{5}{6} \log \frac{\mu^2}{4E_e^2} - \frac{131}{36} + \frac{\pi^2}{6} \right]$$

agrees with Z. Chao, R. Hill, R. Plestid, P. Vander Griend, '25

- log agree with Sirlin and Zucchini, if $\mu = m_N$
- diagrams $(g) - (l)$ are UV finite also in the heavy particle theory $\Longrightarrow$ finite part agrees with Sirlin
- diagrams $(a) - (f)$ are UV divergent

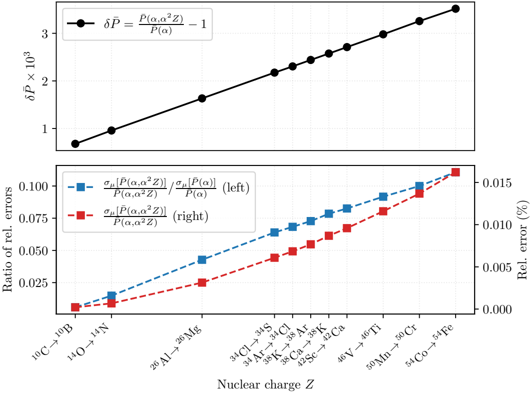
$$\hat{g}^{(2)}(\beta, \bar{E})\Big|_{(a)-(f)} \xrightarrow[\beta\to 1]{} \pm \left[-\frac{15}{4} + \frac{\pi^2}{3} \right] \qquad vs \qquad g^{(2)}_{\rm Sirlin} = \pm \left[-\frac{5}{2} + \frac{\pi^2}{6} \right]$$

- difference is most likely due to the different UV regulator (m_N vs. dim. reg.)
- regulator and scheme dependence should be absorbed by $C_{\rm eff}^{(g_V)}$ and $\alpha^2 Z$ potentials
- numerically $\sim 5 \cdot 10^{-4}$ for $Z = 26$.

$\mathcal{O}(\alpha^2 Z)$ corrections

$$\bar{P} = \left| C_{\text{eff}}^{(g_V)}(\mu) \right|^2 \bar{f}(\mu)(1 + \bar{\delta}'_R(\mu))$$

analogous to $(1 + \Delta_R^V) f (1 + \delta'_R)$



- effect of the correction is large, $1 \cdot 10^{-3} - 3 \cdot 10^{-3}$
- estimate missing pQED corrections ($\mathcal{O}(\alpha^2, \alpha^3 Z^2, \ldots)$) by varying $\mu \in [E_0, 4E_0]$
- scale variation decreases by factor of 10, to $\lesssim 1.5 \cdot 10^{-4}$

(Partial) Summary: ingredients for V_{ud}

$$\frac{1}{t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} \left[C_{\rm eff}^{(g_V)}(\mu)\right]^2 \bar{f}(\mu) \times [1 + \bar{\delta}'_R(\mu)] \, (1 + \bar{\delta}_{\rm NS} - \bar{\delta}_C)$$

$\left[C_{\rm eff}^{(g_V)}(\mu)\right]^2$:

- matching and running at NLL in α and α_s ✓
- single nucleon γ-W box ✓
- matching and running from m_N to $\mu \sim k_F$

$\bar{\delta}_{\rm NS}$

- operators at $\mathcal{O}(\alpha\varepsilon_\chi)$, $\mathcal{O}(\alpha\varepsilon_\pi)$ ✓
- low-energy constants [need to be determined]
- higher order terms & UQ [work in progress]

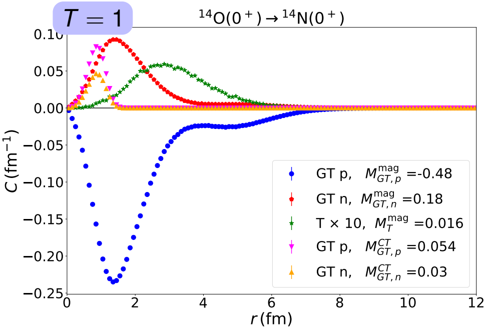
$\bar{\delta}'_R$, $\bar{f}$:

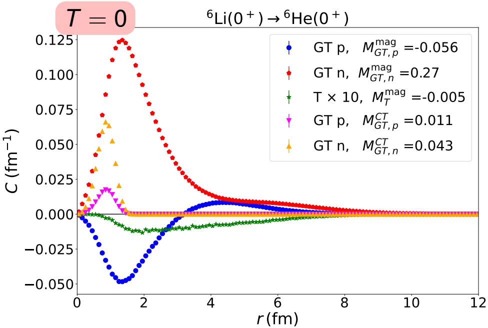
- $\mathcal{O}(\alpha^2 Z)$ ✓
- extend to subleading power in $E \times R$ & interplay with traditional finite size [yet to begin. . .]

Nuclear Matrix Elements

more in G. King's talk

Nuclear matrix elements in ^{14}O





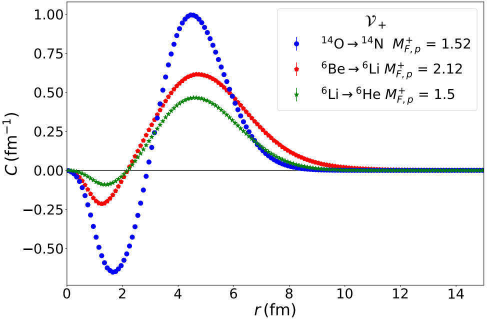
V. Cirigliano, W. Dekens, J. de Vries. **S. Gandolfi**, M. Hoferichter, EM, '24

- nuclear matrix elements in $A = 6$ (for benchmarking) and $A = 14$

$$h^{\rm mag}_{{\rm GT},p}(r) = 4h^{\rm mag}_{T,p}(r) = \frac{g_A}{3m_N}\frac{1+\kappa_p}{r}, \quad h^{\rm CT}_{{\rm GT},p}(r) = -\frac{4\pi}{3}(g^{NN}_{V1} + g^{NN}_{V2})\delta_{R_S}(r)$$

- VMC calculation with N^{2}LO local chiral potential at $R_0 = 1$ fm — A. Gezerlis, I. Tews *et al*, '14
- for local terms, Gamow-Teller component dominates, tensor small.
- contact can be sizable (depending on LECs)

NME in ^{14}O: energy independent components.



$$h^+_F(r) = -\log r/R_A$$

- after including both **L** and $\mathbf{L}_{\mathrm{CM}}$ spin-orbit term is very small
- the α^2 $\mathcal{V}_+$ potential vanishes for $\Lambda = (1.02R)^{-1}$, with $R^2 = 5/3\langle r^2\rangle$

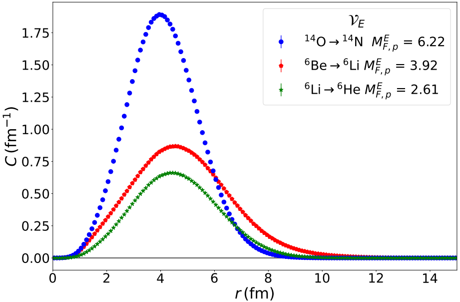
  standard choice $\mu = R$ works pretty well for ^{14}O!
- putting everything together [after fixing LS]

$$\bar{\delta}^{(0)}_{\mathrm{NS}} = -\left[\, 3.1|^{\mathrm{mag}}_{\mathrm{GT}} - 0.2|_{\mathrm{T}} + 0.2|_{\mathrm{LS}} \pm 0.9|_{\mathrm{LEC}} + 0.1|_{\alpha^2} \right] \cdot 10^{-3} = -(3.1 \pm 0.9) \cdot 10^{-3}$$

- larger than Towner and Hardy $\delta^{(B)}_{NS} = -1.96(50) \cdot 10^{-3}$

J. Hardy and I. Towner, ‘20

NME for ^{14}O. Energy dependent pieces



$$\mathcal{V}_E \propto \alpha \sum_{j<k} \frac{r_{jk}}{2R_A} \left(\tau^{+(j)} P_p^{(k)} + \tau^{+(k)} P_p^{j}\right)$$

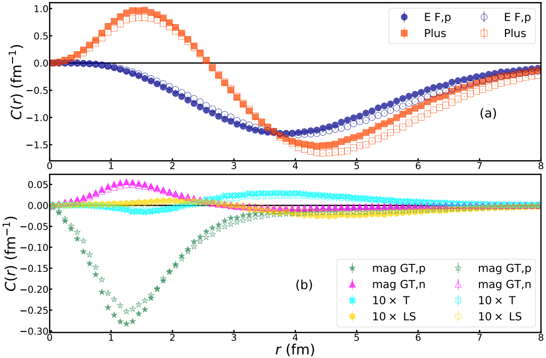
- the energy dependent operator leads to (neglecting small π-exchange corrections)

$$\bar{\delta}^E_{\rm NS} = \mp \frac{2}{g_V M_{\rm F}^{(0)}} \, \langle f^{(0)} | \mathcal{V}_E | i^{(0)} \rangle \left(\frac{E_0 + 8E_e}{6} + \frac{m_e^2}{2E_e} \right)$$

- corrections matches closely $\mathcal{O}(\alpha Z R E)$ "finite size" corrections if

$$M^E_{F,p} = M_F^{(0)} Z \frac{R}{2R_A} \sim 5.6 \Longrightarrow 10\% \text{ too small}$$

QMC calculations of δ_{NS} in ^{10}C

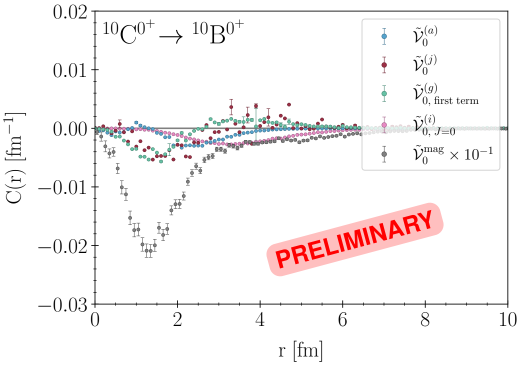


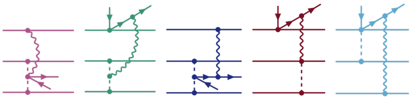
Model	Method	$\delta_{\mathrm{NS}}^{(0)}$	$\overline{\delta_{\mathrm{NS}}^{E}}$
NV2+3-Ia	VMC	$-(4.03 + 0.25 \pm 0.69) \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$
	GFMC	$-(4.43 + 0.21 \pm 0.77) \cdot 10^{-3}$	$0.97 \cdot 10^{-3}$
NV2+3-Ia*	VMC	$-(4.18 + 0.31 \pm 0.62) \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$
	GFMC	$-(4.25 + 0.31 \pm 0.66) \cdot 10^{-3}$	$1.08 \cdot 10^{-3}$
AV18+UX	VMC	$-(4.48 + 0.27 \pm 0.67) \cdot 10^{-3}$	$1.02 \cdot 10^{-3}$
	GFMC	$-(4.06 + 0.40 \pm 0.48) \cdot 10^{-3}$	$1.17 \cdot 10^{-3}$
AV18+IL7	VMC	$-(4.55 + 0.26 \pm 0.61) \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$
	GFMC	$-(4.32 + 0.29 \pm 0.64) \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$

G. King *et al*, '25

- Quantum Monte Carlo calculation of $\bar{\delta}_{\mathrm{NS}}$ in ^{10}C, with 3 different interactions.
- about 5% spread from different interactions (first step towards full UQ)
- long-range part of the result agrees well with dispersive result of M. Gennari *et al.*, '24

Higher order corrections to δ_{NS}

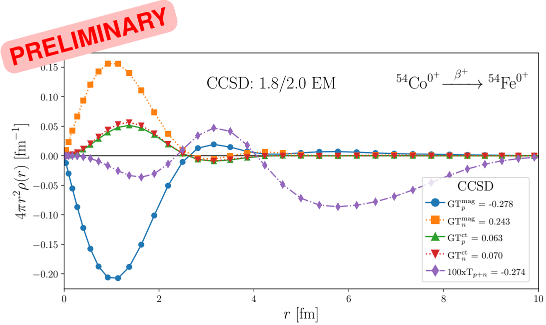


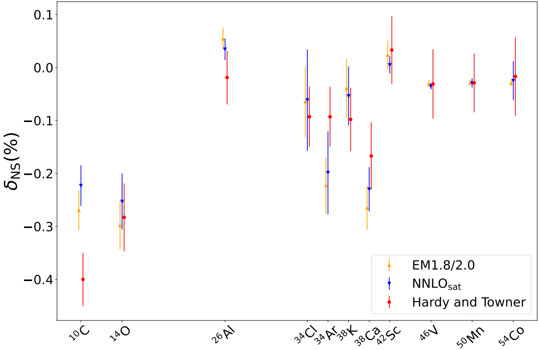


thanks to G. Chambers-Wall

- Graham Chambers-Wall has started derivation & implementation of 3-body operators
- calculation is not complete, but so far corrections look in line with χEFT power counting
- parallel work at UW on loop corrections $\Longrightarrow$ see M. d'Souza talk

Coupled cluster calculations in medium mass nuclei

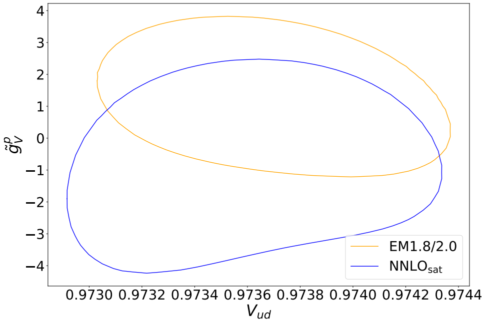


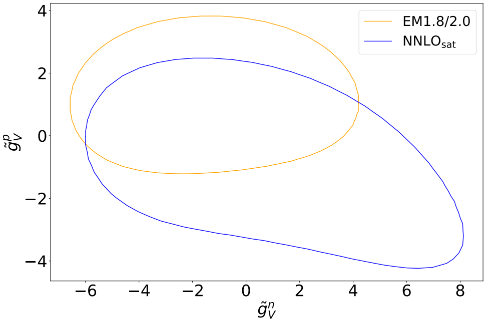


thanks to **S. Novario** (WashU)

- S. Novario computed δ_{NS} for 11 transitions with 2 interactions (EM1.8/2.0 and NNLO$_{\mathrm{sat}}$)
- error includes many-body truncation error (N_{max}) and CC approximations (triple/no triple correlations)
- still missing Hamiltonian and operator truncation
- long-distance part of δ_{NS} agrees well with Hardy and Towner adopeted values

Coupled cluster calculation in medium mass nuclei





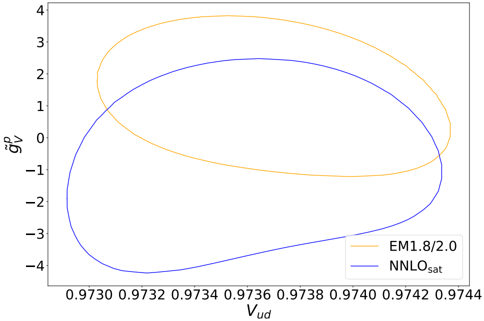
- missing a 3-body $\mathcal{O}(\alpha^2)$ ME to do full EFT analysis ✗

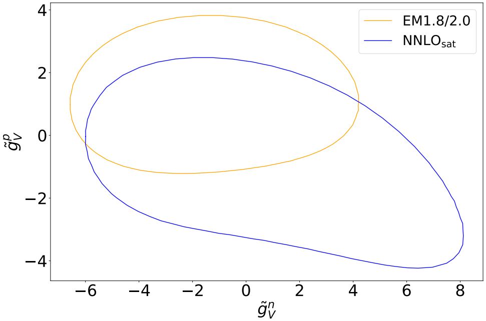
  thinking of alternative strategies to extract them
- missing δ_C with same interactions ✗
- **if** we take *ft*, δ'_R, Δ_R^V and δ_C from Hardy and Towner & fit V_{ud}, and 2 LECs

$$\begin{array}{lll} \text{EM1.8/2.0} & V_{ud} = 0.97371 \pm 0.00045 & \chi^2/dof = 1.3 \\ \text{NNLO}_{\rm sat} & V_{ud} = 0.97366 \pm 0.00048 & \chi^2/dof = 1.1 \end{array}$$

only $\delta_{\rm NS}$ error!

Coupled cluster calculation in medium mass nuclei





- compatible with Hardy and Towner, '20, with larger error

$$\delta V_{ud}|_{\delta_{\rm NS}} = 4.5 \cdot 10^{-4} \quad vs \quad \delta V_{ud}|_{\delta_{\rm NS}} = 2.9 \cdot 10^{-4}$$

- not much sensitivy to LECs, which are compatible with zero and enlarge error

not fully consistent EFT analysis yet!

Conclusion

- precision β decays are important probes of BSM physics

  ...but they require precise theoretical input!
- EFTs provide a systematic organizational tool for assessment/re-evalution of th. uncertainties

- "refactored" half-life formula according to photon virtuality
- derived lowest-order two-body operators for $\bar{\delta}_{\rm NS}$ in chiral EFT
- clarified some nuclear-structure dependence of nuclear-structure-independent corrections
- calculated $\mathcal{O}(\alpha^2 Z)$ corrections in heavy nucleus theory
- performed first QMC and CC calculations in ^{10}C, ^{14}O and medium mass nuclei

TO DO:

- higher order corrections to $\delta_{\rm NS}$ and "outer corrections"
- more rigorous assessment of nuclear theory error on the NMEs

  multiple chiral Hamiltonians, different cut-offs, multiple methods
- calculate δ_C with the same method