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Renormalized Chiral EFT for $0\nu\beta\beta$ and (radiative corrections to) β decays

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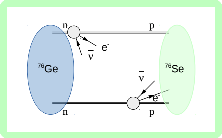
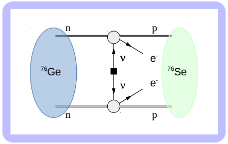
with G. Chamber-Walls, V. Cirigliano, W. Dekens, J. de Vries, S. Gandolfi, M. Graesser, M. Hoferichter, J. Lieffers, G. King, S. Novario, S. Pastore, M. Piarulli, B. van Kolck, B. Wiringa, F. Wu.

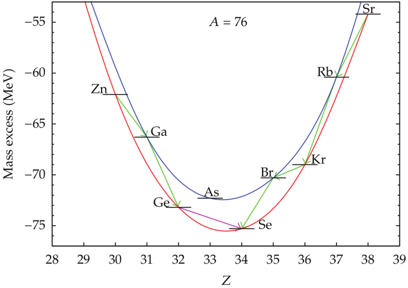
Chiral EFT: New Perspectives.

March 20th, 2025

Neutrinoless double beta decay







- Double beta decay is a rare doubly weak process
- lepton-number-conserving 2ν channel

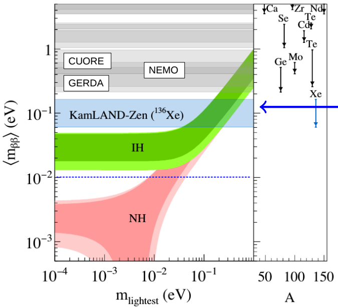
$$^{76}_{32}\mathrm{Ge} \rightarrow ^{76}_{34}\mathrm{Se} + e^- e^- \bar{\nu}_e \bar{\nu}_e \qquad T^{2\nu}_{1/2} = 1.9 \times 10^{21}\ \mathrm{yr}$$

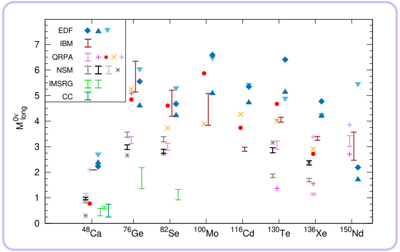
rarest observed nuclear process

- if neutrinos are Majorana, a lepton-number-violating 0ν channel becomes possible

$$^{76}_{32}\mathrm{Ge} \rightarrow ^{76}_{34}\mathrm{Se} + e^- e^- \qquad T^{0\nu}_{1/2} > 1.8 \times 10^{26}\ \mathrm{yr}$$

Neutrinoless double beta decay





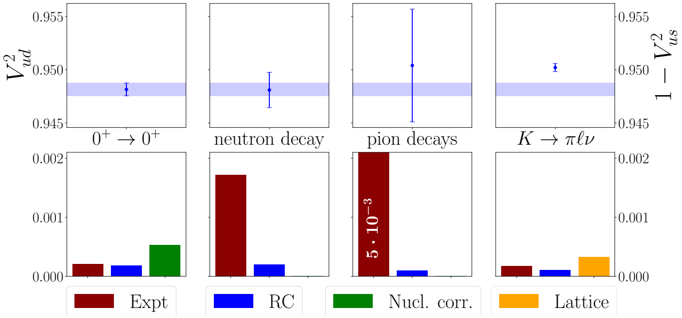
M. Agostini, G. Benato, J. Detwiler, J. Menendez, F. Vissani '22

- observation clear signal of BSM physics but interpretation needs theory!
- E.g. in "standard scenario", rate is prop. to the neutrino Majorana mass $\times$ nuclear matrix element

$$\Gamma \propto \sum_i U_{ei}^2 m_i \times M^{0\nu}$$

nuclear theory crucial to extract $m_{\beta\beta}$ & interplay with oscillation experiments

CKM unitarity tests



$V_{ud} = 0.97367 \pm 0.00032$
$V_{us}(K_{\ell 3}) = 0.2233 \pm 0.0005$

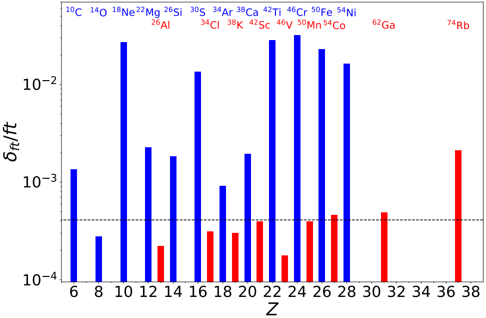
- the SM predicts $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- re-evaluation of "inner radiative correction" and precise lattice QCD calculation of $f_+(0)$

  C. Y. Seng, M. Gorchtein, H. Patel, M. Ramsey-Musolf, '18 A. Czarnecki, W. Marciano, A. Sirlin, '19; A. Bazavov *et al*, '18
- led to $\sim 2\sigma$-3σ tension in CKM unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -(1.65 \pm 0.73) \cdot 10^{-3}$$

BSM physics in the 5-10 TeV range?

V_{ud} from $0^+ \to 0^+$ decays



$T_i^z = -1,\ T_f^z = 0$

$T_i^z = 0,\ T_f^z = 1$

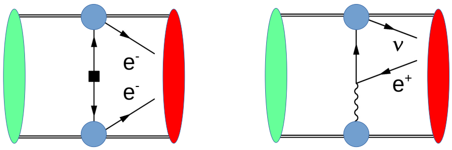
$$\frac{1}{t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} \left[C_{\text{eff}}^{(g_V)}(\mu)\right]^2 \times [1 + \bar{\delta}'_R(\mu)]\,(1 + \bar{\delta}_{\mathrm{NS}})\,(1 - \bar{\delta}_C)\,\bar{f}(\mu)$$

- 15 $0^+ \to 0^+$ transitions with $\delta ft \leq 3 \cdot 10^{-3}$, from ^{10}C to ^{74}Rb
- affected by nuclear-structure-dependent corrections δ_{NS} and δ_C
- δ_{NS} dominates the uncertainty in the "canonical" extraction of J. Hardy and I. Towner

$$[V_{ud}]_{\mathrm{HT}} = 0.97373(5)_{\mathrm{exp}}(10)_{\Delta_R}(4)_{\delta_{\mathrm{C}}}(6)_{\delta'_{\mathrm{R}}}(27)_{\delta_{\mathrm{NS}}}$$

- can we validate with EFT + *ab initio* methods?

$0\nu\beta\beta$ and electromagnetic corrections to Fermi decays. Similarities.



1. mediated by two weak or electromagnetic currents
2. receive contributions from exchange of weak particles (photon, electron, neutrinos) with different energy/momentum, sensitive to different nuclear features
3. in particular, "potential" modes are important

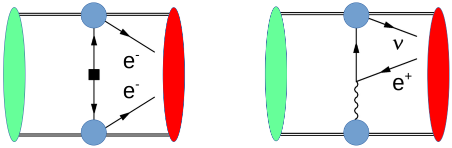
$$(k_0, \mathbf{k}) \sim (0, \gamma), \qquad \gamma \gg (E_n - E_i)$$

lead to two-body transition operators independent of nuclear intermediate states

4 at the two-nucleon level, act in the $^1S_0$ wave

$$nn(^1S_0) \to pp(^1S_0)e^- e^- \qquad \text{vs} \qquad nn(^1S_0) \to np(^1S_0)e^-\nu, \quad np(^1S_0) \to pp(^1S_0)e^-\nu$$

$0\nu\beta\beta$ and electromagnetic corrections to Fermi decays. Similarities.



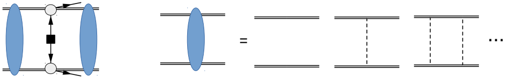
$$\langle f|\mathcal{O}_w|i\rangle$$

1. what is the leading order transition operator $\mathcal{O}_w$?
2. what happens beyond leading order?
3. what do we need to reliably estimate uncertainties?

I will focus entirely on the transition operator.
Need input from Lattice QCD and many-body methods for full predictions!

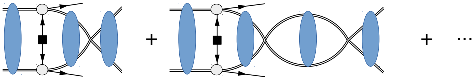
LO "neutrino potential" in S-wave

A

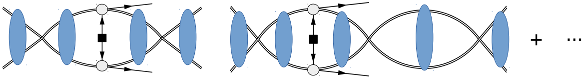


$$H_{\text{eff}} = 2G_F^2 V_{ud}^2 m_{\beta\beta} \bar{e}_L^T C e_L \left(V_{\nu L} + V_{\nu S} \right)$$

B



C



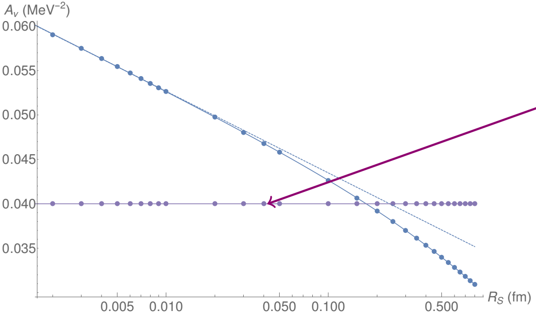
- the long range potential has a Coulomb-like form

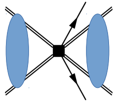
$$V_{\nu L} = \frac{\tau^{(1)+}\tau^{(2)+}}{\mathbf{q}^2} \left[1 - \frac{2}{3} g_A^2 \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} \left(1 + \frac{m_\pi^4}{2(\mathbf{q}^2 + m_\pi^2)^2} \right) - \frac{g_A^2}{3} S^{(12)} \left(1 - \frac{m_\pi^4}{(\mathbf{q}^2 + m_\pi^2)^2} \right) \right]$$

- is the scattering amplitude finite?

$$\mathcal{A}_\nu(nn \to ppe^- e^-) = \langle pp | V_{\nu L} | nn \rangle$$

$0\nu\beta\beta$ in renormalized chiral EFT





$$g_\nu^{\mathrm{NN}} = \left(\frac{m_N \tilde{C}_{1S_0}}{4\pi}\right)^2 \tilde{g}_\nu^{\mathrm{NN}}$$

$$\tilde{g}_\nu^{\mathrm{NN}} = \tilde{g}_\nu^{(0)\mathrm{NN}} - \frac{1}{2}(1 + 2g_A^2)\log R_S$$

V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, S. Pastore, U. van Kolck, '18

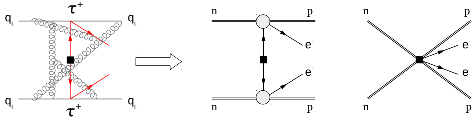
- the matrix element of the long-range neutrino potential is UV divergent
- need to promote the N^2LO counterterm to LO

$$V_{\nu S} = -2 g_\nu^{\mathrm{NN}} \tau^{+(1)} \tau^{(+)2}$$

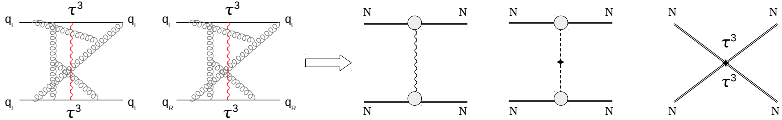
same argument for the enhancement of D_2 driven by OPE D. Kaplan, M. Savage, M. Wise, '96
related argument for the counting of contact range currents M. Pavon Valderrama, D. Phillips, '07

Chiral invariant form of the counterterm

$e_L C e_L \frac{g^{\mu\nu}}{k^2}$



$\frac{g^{\mu\nu}}{k^2}$



- using spurion techniques, we can related $g_\nu^{\rm NN}$ to an electromagnetic CIB operator, $g_\nu^{\rm NN} = \mathcal{C}_1$

$$\mathcal{L}^{NN}_{|\Delta L|=2} = \left(2\sqrt{2} G_F V_{ud}\right)^2 m_{\beta\beta}\, \bar{e}_L C \bar{e}_L^T\, \frac{g_\nu^{\rm NN}}{4} \left[\bar{N}\mathcal{Q}_L^{\rm w} N\, \bar{N}\mathcal{Q}_L^{\rm w} N - \frac{1}{6}{\rm Tr}\left(\mathcal{Q}_L^{\rm w}\, \mathcal{Q}_L^{\rm w}\right) \bar{N}\boldsymbol{\tau}N \cdot \bar{N}\boldsymbol{\tau}N\right]$$

$$\mathcal{L}^{NN}_{e^2} = \frac{e^2}{4}\left\{\bar{N}\mathcal{Q}_L^{\rm em} N\, \bar{N}(\mathcal{C}_1 \mathcal{Q}_L^{\rm em} + \mathcal{C}_2 \mathcal{Q}_R^{\rm em}) N - \frac{1}{6}{\rm Tr}\left[\mathcal{Q}_L^{\rm em}(\mathcal{C}_1 \mathcal{Q}_L^{\rm em} + \mathcal{C}_2 \mathcal{Q}_R^{\rm em})\right] \bar{N}\boldsymbol{\tau}N \cdot \bar{N}\boldsymbol{\tau}N\right\}$$

$$\mathcal{Q}_L = u Q_L u^\dagger, \qquad \mathcal{Q}_R = u^\dagger Q_R u \qquad u = 1 + i\frac{\boldsymbol{\pi}\cdot\boldsymbol{\tau}}{2F_\pi} - \frac{\boldsymbol{\pi}^2}{8F_\pi^2} + \dots$$

- data driven extraction of g_ν^{NN} from $a_{pp}(^1S_0) + a_{nn}(^1S_0) - 2a_{np}(^1S_0)$ is prevented by $\mathcal{C}_2$

Relation to CIB. How many independent counterterms?

- most direct extraction of $g_\nu^{\rm NN}$ will be a LQCD calculation of $\mathcal{A}(nn \to ppe^- e^-)$
- followed by "Cottingham" approach — V. Cirigliano *et al*, '21, see Sebastian's talk

- the RGEs for the dimensionless couplings

$$\frac{d}{d\log\mu}\tilde{\mathcal{C}}_1 = \frac{1}{2}(1+2g_A^2)$$

$$\frac{d}{d\log\mu}\tilde{\mathcal{C}}_2 = \frac{1}{2}\left(1+2g_A^2 - 4g_A^2(1-Z_\pi)\right), \qquad Z_\pi = \frac{1}{2}\frac{\delta m_\pi^2}{e^2 F_\pi^2} \sim 0.8$$

- a particular combination of $\tilde{\mathcal{C}}_{1,2}$ is RG-invariant at LO

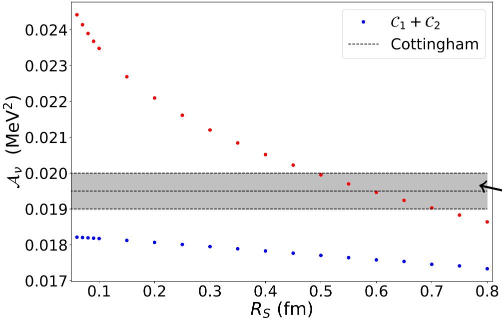
$$\frac{d}{d\log\mu}\left[\frac{\tilde{\mathcal{C}}_1}{1+2g_A^2} - \frac{\tilde{\mathcal{C}}_2}{1+2g_A^2-4g_A^2(1-Z_\pi)}\right] = 0 \Longrightarrow \mathcal{O}\left(\frac{F_\pi^2}{\Lambda^2}\right)?$$

- with this assumption, we could reduce the number of independent counterterms

$$\tilde{\mathcal{C}}_1 = \frac{\tilde{\mathcal{C}}_1+\tilde{\mathcal{C}}_2}{2}\left[1 - \frac{2g_A^2}{1+2g_A^2}(1-Z_\pi)\right]$$

- agrees with large N_c considerations — T. Richardson, S. Pastore, M. Schindler, R. Springer, '21.

Relation to CIB. How many independent counterterms?



$p_i = 25$ MeV, $p_f = 30$ MeV

see Sebastian's talk

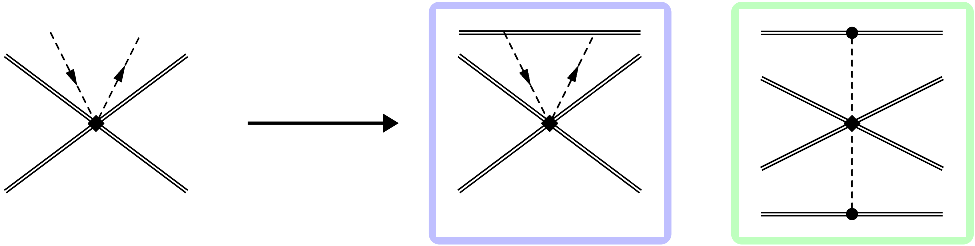
- using this relation, in dim. reg.

$$\mathcal{C}_1(\mu) = 2.13 - 2.11 \log \frac{m_\pi}{\mu} \qquad \text{vs} \qquad \mathcal{C}_1(\mu)|_{\text{Cottingham}} = 1.3(6) - 2.11 \log \frac{m_\pi}{\mu}$$

- require "alignment" of finite pieces of the counterterms and logs

is this reasonable from the QCD point of view?

Determination of counterterms. Isospin breaking many-body forces



- the isospin breaking operators $\mathcal{C}_{1,2}$ differ when considering multiple pions
- this leads to isospin-breaking three- and four-body forces
  e.g.

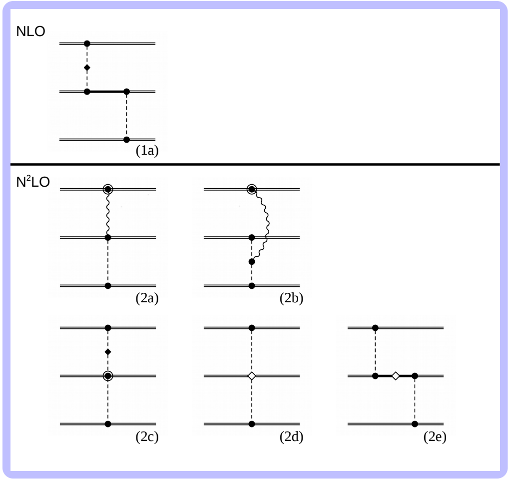
$$\mathrm{CIB}_{3\mathrm{b}} \propto \frac{e^2}{F_\pi^2}(3\mathcal{C}_1 + \mathcal{C}_2)N^\dagger N \left(N^\dagger \tau_3 N\, N^\dagger \tau_3 N - \frac{1}{3} N^\dagger \vec{\tau} N \cdot\, N^\dagger \vec{\tau} N \right) \times \mathcal{O}\left(\frac{\pi m_\pi}{(4\pi F_\pi)^2} \right)$$

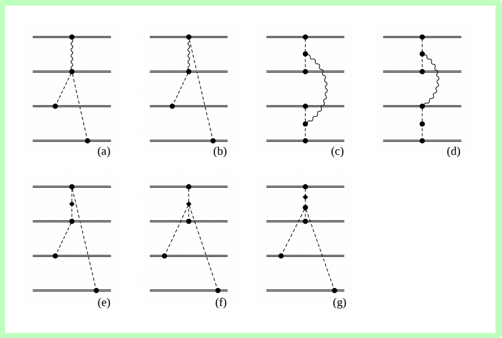
- suppressed by two powers in Friar's (Weinberg's) PC

$$\frac{\mathrm{CIB}_{3\mathrm{b}}}{\mathrm{CIB}_{\mathrm{LO}}} = \pi \times \mathcal{O}\left(\frac{Q}{\Lambda_\chi}\right)^{3(4)} \qquad \frac{\mathrm{CIB}_{4\mathrm{b}}}{\mathrm{CIB}_{\mathrm{LO}}} = \mathcal{O}\left(\frac{Q}{\Lambda_\chi}\right)^{2(4)}$$

- are these forces visible in $\Delta I = 2$ observables? E.g. $B(^6\mathrm{He}) + B(^6\mathrm{Be}) - 2B(^6\mathrm{Li})$

Isospin breaking many-body forces

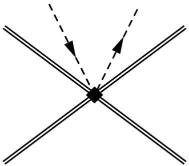


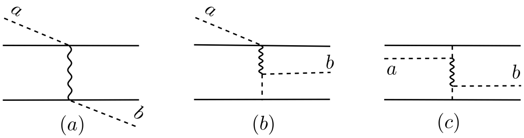


thanks to J. Lieffers

- there is a lot of background at this order...

Determination of counterterms. Isospin-breaking in pion-nucleus scattering





thanks to Feng Wu!

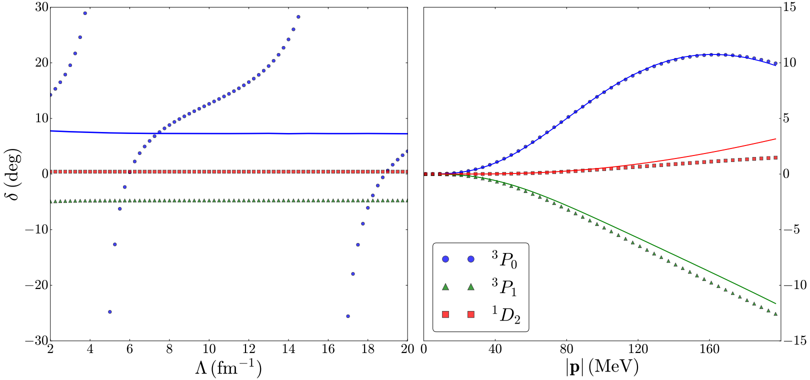
- look for isospin symmetry in pion-nucleus elastic scattering

$$\Delta a = \sum_{\alpha=0,\pm} \left(a_{\pi^\alpha A(T_z=-1)} + a_{\pi^\alpha A(T_z=1)} - 2a_{\pi^\alpha A(T_z=0)} \right)$$

- or in pion double-charge exchange $\pi^+ + {}^{Z}_{N}A \to \pi^- + {}^{Z+2}_{N-2}A$
- Δa is proportional to $3\mathcal{C}_1 + \mathcal{C}_2$, and renormalization analysis consistent with $0\nu\beta\beta$
- receive contrib. from D_2, E_2, which would need to be fixed by isospin invariant scattering length

are there data to do this analysis?

LO neutrino potential in higher partial waves

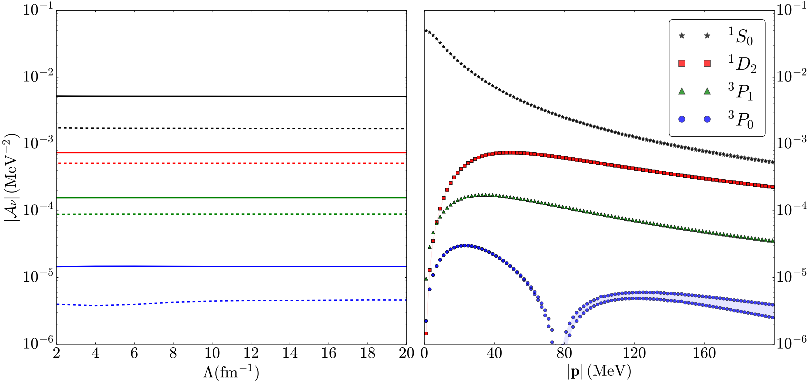


$$V_{\nu L}^{^1D_2} = V_{\nu L}^{^1S_0} = (V_F + 3g_A^2 V_{GT}), \qquad V_{\nu L}^{^3P_J} = V_F - g_A^2 V_{GT} + a_J g_A^2 V_T, \quad a_J = \{4, -2, 2/5\}$$

- $^3P_0$ phase shifts require renormalization
- after the renormalization of the phase shift, the LO LNV amplitude is finite

no need for *P*- and *D*-wave counterterms at LO

LO neutrino potential in higher partial waves



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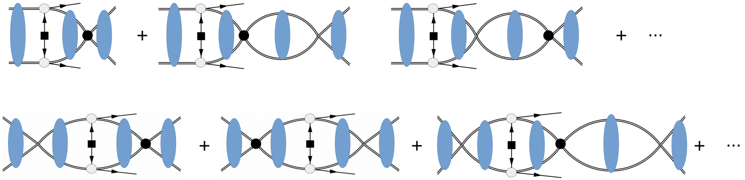
Neutrino potential at NLO

NLO



- at NLO in renormalized chiral EFT

$$\mathcal{L}_\chi^{(1)} = +\frac{1}{8} C_2 \left(N^T \vec{P}_{^1S_0} \overleftrightarrow{\nabla}^2 N \right) \cdot \left(N^T \vec{P}_{^1S_0} N \right)^\dagger + \mathrm{H.c.}, \qquad \text{with} \qquad C_2 = \mathcal{O}\left(\frac{4\pi}{m_N Q^2 \Lambda_\chi} \right)$$

- does this induce a derivative LNV counterterm?
- performing the analysis in perturbation theory, the NLO amplitude

$$\begin{aligned} \mathcal{A}_\nu^{\mathrm{NLO}} = \; & \mathcal{A}_A + \chi^+_{\mathbf{p}'}(\mathbf{0}) \left(K_{E'} + K_{E'}^{(1)} \right) \mathcal{A}_B + \bar{\mathcal{A}}_B \left(K_E + K_E^{(1)} \right) \chi^+_{\mathbf{p}}(\mathbf{0}) + \chi^+_{\mathbf{p}'}(\mathbf{0}) \, K_{E'} \, \mathcal{A}_B^{(1)} + \bar{\mathcal{A}}_B^{(1)} \, K_E \, \chi^+_{\mathbf{p}}(\mathbf{0}) \\ & + \chi^+_{\mathbf{p}'}(\mathbf{0}) \left(K_{E'} + K_{E'}^{(1)} \right) \left(\mathcal{A}_C + \frac{2 g_\nu^{\mathrm{NN}}}{C^2} \right) \left(K_E + K_E^{(1)} \right) \chi^+_{\mathbf{p}}(\mathbf{0}) + \chi^+_{\mathbf{p}'}(\mathbf{0}) \, K_{E'} \, \mathcal{A}_C^{(1)} \, K_E \, \chi^+_{\mathbf{p}}(\mathbf{0}) \, , \end{aligned}$$

Neutrino potential at NLO

- $K_E^{(1)}$ is the NLO scattering amplitude, cut-off independent if

$$\frac{d}{d\log\Lambda}\frac{C_2}{C^2} = 0 \Longrightarrow C_2 \sim \frac{1}{\Lambda^2}$$

- the one-loop NLO correction $\mathcal{A}_B$ is convergent

$$\mathcal{A}_B^{(1)} = \frac{C_2}{2C} m_N \int \frac{d^3k}{(2\pi)^3} \frac{1+3g_A^2}{(\mathbf{p}-\mathbf{k})^2} \exp\left[-\left(\frac{\mathbf{k}^2}{\Lambda^2}\right)^2\right] \longrightarrow \frac{1}{\Lambda} \times \Lambda$$

- the two-loop correction has a divergent momentum-independent correction, reabsorbed by g_ν^{NN}

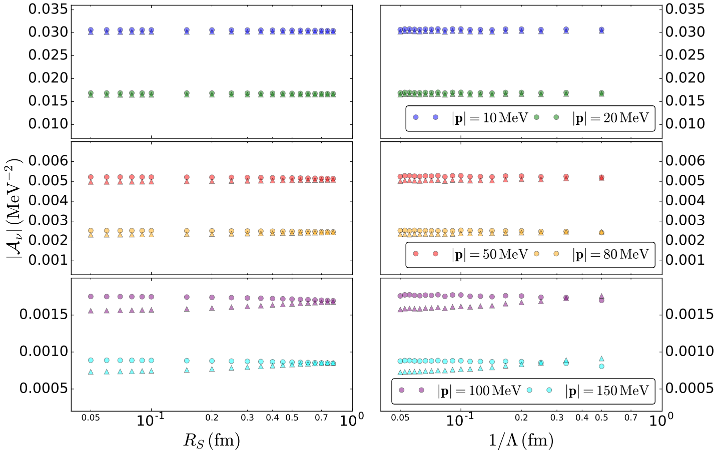
$$\mathcal{A}_C^{(1)} = \frac{2g_\nu^{\mathrm{NN}\,(1)}}{C^2} + 2\left(C_2\frac{g_\nu^{\mathrm{NN}}}{C^2} + \frac{g_{2\nu}^{\mathrm{NN}}}{C}\right)\delta I_0 - \frac{C_2}{C} I_2(\mathbf{0},\mathbf{0}),$$

- and a momentum dependent correction

$$\mathcal{A}_C^{(1)} = \left(-\frac{4g_\nu^{\mathrm{NN}}}{C^2}\frac{C_2}{C} + \frac{2g_{2\nu}^{\mathrm{NN}}}{C^2}\right)\frac{\mathbf{p}^2+\mathbf{p}'^2}{2} - \frac{C_2}{C}\left(I_2(\mathbf{p}^2,\mathbf{p}'^2) - I_2(\mathbf{0},\mathbf{0})\right) \longrightarrow \frac{\mathbf{p}^2+\mathbf{p}'^2}{2}\frac{\log\Lambda}{\Lambda}$$

no need for $g_{2\nu}^{\mathrm{NN}}$ at NLO; $g_{2\nu}^{\mathrm{NN}}$ expected at N^{2}LO

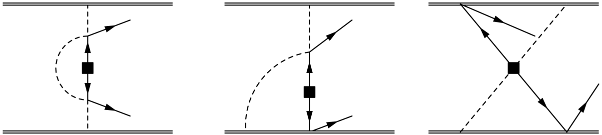
NLO neutrino potential



V. Cirigliano *et al*, ‘19

- same holds in chiral EFT, NLO amplitudes do not need further renormalization

$0\nu\beta\beta$ at N^{2}LO



At N^{2}LO $\mathcal{O}(\vec{q}^2/\Lambda_\chi^2)$

1. correction to the one-body currents (magnetic moment, radii, . . .)

$$g_A(\vec{q}^2) = g_A\left(1 - r_A^2\frac{\vec{q}^2}{6} + \dots\right) \qquad r_A = 0.47(7)\mathrm{fm}$$

2. pion-neutrino loops & local counterterms

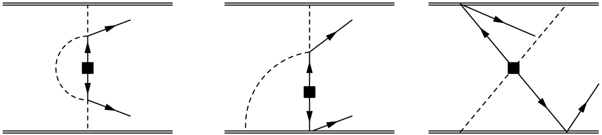
V. Cirigliano, W. Dekens, EM, A. Walker-Loud, '17

3. three-body neutrino potential (actually NLO in Friar's counting. . .)

J. Menendez, D. Gazit, A. Schwenk, '11, G. Chambers-Wall, J. Lieffers *et al*, *in progress*

no renormalization analysis of the N^{2}LO two- and three-body amplitudes yet

$0\nu\beta\beta$ at N^{2}LO



At N^{2}LO $\mathcal{O}(\vec{q}^2/\Lambda_\chi^2)$

1. correction to the one-body currents (magnetic moment, radii, ...)

$$g_A(\vec{q}^2) = g_A\left(1 - r_A^2\frac{\vec{q}^2}{6} + \dots\right) \qquad r_A = 0.47(7)\mathrm{fm}$$

2. pion-neutrino loops & local counterterms

V. Cirigliano, W. Dekens, EM, A. Walker-Loud, '17

3. three-body neutrino potential (actually NLO in Friar's counting...)

J. Menendez, D. Gazit, A. Schwenk, '11, G. Chambers-Wall, J. Lieffers *et al*, *in progress*

no renormalization analysis of the N^{2}LO two- and three-body amplitudes yet

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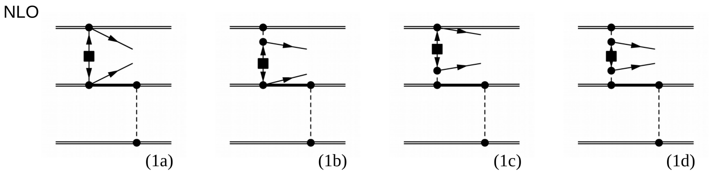

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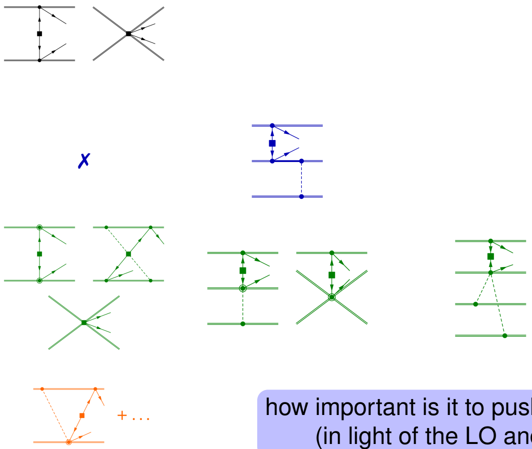
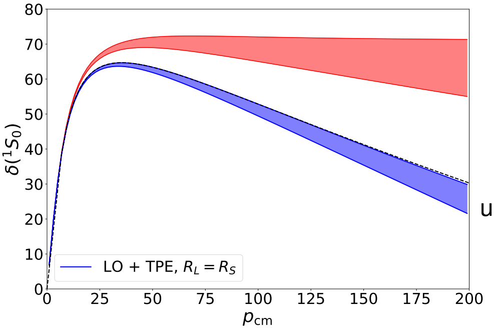
Systematically improving the neutrino potential

	2N potential	3N potential	4N potential
✓ **LO** $\mathcal{O}(G_F^2)$			
NLO $\mathcal{O}(G_F^2 Q/\Lambda_\chi)$	✗		
N^{2}LO $\mathcal{O}(G_F^2 Q^2/\Lambda_\chi^2)$			
N^{3}LO $\mathcal{O}(G_F^3 Q^2/\Lambda_\chi^3)$	$+\dots$		



how important is it to push the calculation to high orders?
(in light of the LO and many-body uncertainties)

Alternative power counting. TPE at LO?



using LO TPE + Δ, with long-range regulator $C_{\rm R_L}$

M. Piarulli, *et al*, '14

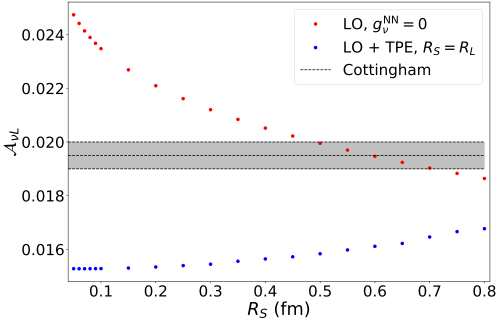
- including two-pion-exchange at LO leads to a renormalizable theory

C. Mishra, A. Ekström, G. Hagen, T. Papenbrock, L. Platter, '21

$$V_{\rm strong}(r) = \tilde{C}_{^1S_0}\, \delta^{(3)}_{R_S}(\vec{r}) + \frac{g_A^2 m_\pi^2}{16\pi F_\pi^2} \frac{e^{-m_\pi r}}{4\pi r} + v_{\rm TPE}(r) C_{R_L}(r)$$

- fit $\tilde{C}_0$ to the scattering length yields good phase shifts

Alternative power counting. TPE at LO?



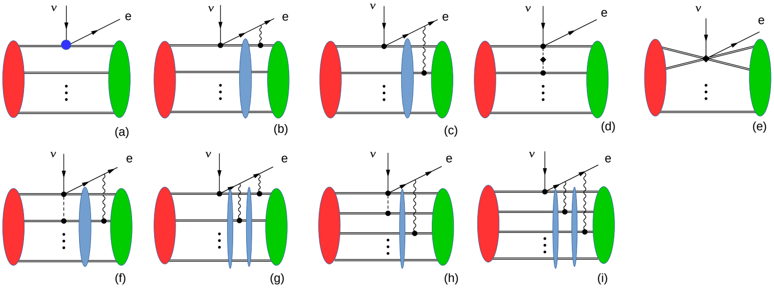
- the scale dependence of $\mathcal{A}_\nu$ is reduced, and converges for $R_S \to 0$
- a very preliminary calculation at $R_S = 0.15$ fm

$$a_C|_{\rm TPE} = -8.7 \text{ fm} \qquad a_{nn}|_{\rm TPE} = -21.2 \text{ fm}$$
$$a_C|_{\rm exp} = -7.8 \text{ fm} \qquad a_{nn}|_{\rm exp} = -18.9 \text{ fm}$$

suggesting we still need contacts to fix $a_{\rm CIB}$

how do we construct consistent weak operators?

Radiative corrections to β decays in EFT



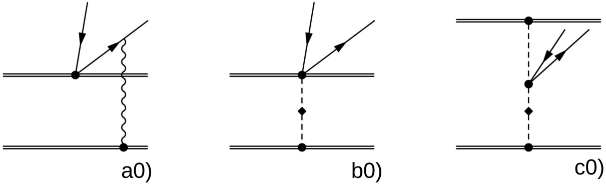
- photon probes different energy-momentum regions

  "usoft" : $E_\gamma \sim |\vec{k}_\gamma| \sim \gamma^2/m_N \sim \mathcal{Q}$

  "soft": $E_\gamma \sim |\vec{k}_\gamma| \sim \gamma$

  "potential": $E_\gamma \sim \vec{k}_\gamma^2/m_N \sim \gamma^2/m_N$ only present in $A \gtrsim 2$
- similar to $0\nu\beta\beta$, but relative importance of regions is different
- "traditional" EM corrections (outer, inner, $\delta_{\rm NS}$ ...) can be understood as coming from specific photon modes

Two-body operators for $\delta_{\rm NS}$



V. Cirigliano *et al*, '24

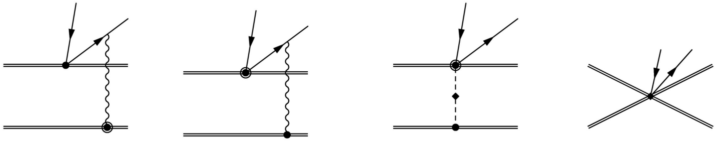
- in the EFT formalism, $\bar{\delta}_{\rm NS}$ is the matrix element of two- and higher-nucleon operators

$$\bar{\delta}_{\rm NS} = \frac{2\langle f^{(0)}|\mathcal{V}|i^{(0)}\rangle}{g_V(\mu)M_{\rm F}^{(0)}} = \alpha\left[\sum_{n=1} a_n\left(\frac{m_\pi}{\Lambda_\chi}\right)^n + E_0 R\sum_{n=0} b_n(E_e, m_e)\left(\frac{m_\pi}{\Lambda_\chi}\right)^n + \alpha\sum_{n=0} c_n\left(\frac{m_\pi}{\Lambda_\chi}\right)^n \dots\right]$$

- LO vertices lead to an operator that depends on external energies

$$\mathcal{V}_E^0 = \frac{1}{3}\left(\frac{1}{2} + \frac{4E_e}{E_0}\right)\mathcal{V}_E \qquad \mathcal{V}_E = g_V\sum_{j<k} e^2\frac{1}{\mathbf{q}^4}\left(\tau^{+(j)}P_p^{(k)} + P_p^{(j)}\tau^{+(k)}\right).$$

Two-body operators for δ_{NS}



V. Cirigliano *et al*, '24

- at NLO, magnetic and recoil corrections generate energy-independent corrections

$$\mathcal{V}_0^{\mathrm{mag}}(\mathbf{q}) = \sum_{j<k} \frac{e^2}{3} \frac{g_A}{m_N} \frac{1}{\mathbf{q}^2} \left(\boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\sigma}^{(k)} + \frac{1}{2} S^{(jk)} \right) \left[(1+\kappa_p)\tau^{+(j)} P_p^{(k)} + \kappa_n \tau^{+(j)} P_n^{(k)} + (j \leftrightarrow k) \right],$$

$$\mathcal{V}_0^{\mathrm{rec}}(\mathbf{q}, \mathbf{P}) = \sum_{j<k} \left[-i \frac{e^2 g_A}{4 m_N} \frac{\tau^{+(j)} P_p^{(k)}}{\mathbf{q}^4} ((\mathbf{P}_j - \mathbf{P}_k) \times \mathbf{q}) \cdot \boldsymbol{\sigma}^{(j)} - \frac{Z_\pi e^2 g_A^2}{m_N} \frac{\tau^{+(j)} \tau_3^{(k)}}{(\mathbf{q}^2 + m_\pi^2)^2} \boldsymbol{\sigma}^{(j)} \cdot \mathbf{q}\, \boldsymbol{\sigma}^{(k)} \cdot \mathbf{P}_j + (j \leftrightarrow k) \right],$$

- the magnetic moments induce a Coulomb-like potential in $^1S_0$ . . .
  need isospin 1 and 2 contact interactions

$$\mathcal{V}_0^{\mathrm{CT}} = e^2 \left(g_{V1}^{NN} O_1 + g_{V2}^{NN} O_2 \right), \qquad O_1 = \sum_{j \neq k} \tau^{+(j)} \mathbb{1}_k, \quad O_2 = \sum_{j<k} \left[\tau^{+(j)} \tau_3^{(k)} + (j \leftrightarrow k) \right].$$

Counterterms for β decays

- introduce the dimensionless couplings $\tilde{g}^{NN}_{V1,V2}$ as

$$g^{NN}_{V1,V2} = \frac{1}{m_N}\left(\frac{m_N C_{^1S_0}}{4\pi}\right)^2 \tilde{g}^{NN}_{V1,V2},$$

- the LO RGE

$$\frac{d\tilde{g}^{NN}_{V1}}{d\log\mu} = -g_A(1+\kappa_p+\kappa_n) = -1.12, \qquad \frac{d\tilde{g}^{NN}_{V2}}{d\log\mu} = -g_A(1+\kappa_p-\kappa_n) = -5.99,$$

- to limit the number of fit parameters, if needed we could impose

$$\tilde{g}_{V1} = \tilde{g}_{V2}\frac{1+\kappa_p+\kappa_n}{1+\kappa_p-\kappa_n} \ll \tilde{g}_{V2}$$

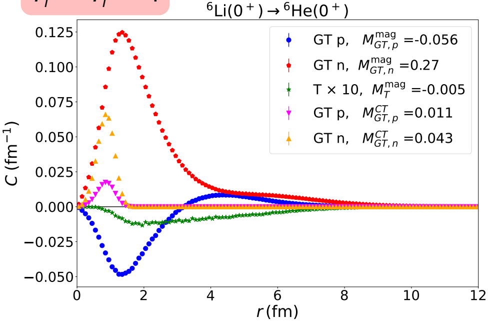
How to determine the counterterms?

1. Cottingham
   important to understand interplay with the dispersive method M. Gennari, M. Gorchtein, C. Y. Seng *et al*, '24
2. fit V_{ud}, g^{NN}_{V1} and g^{NN}_{V2} to superallowed data

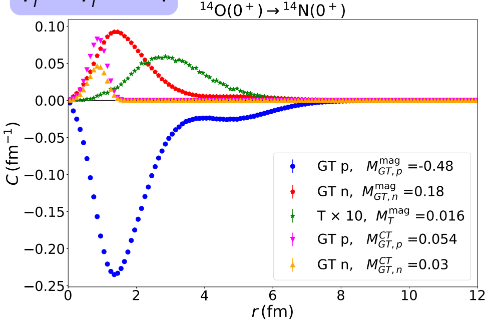
in this case we have data! opportunity to check power counting

Ab initio calculations of $\bar{\delta}_{\rm NS}$

$T_f^z - T_i^z = 1$



$T_f^z - T_i^z = -1$

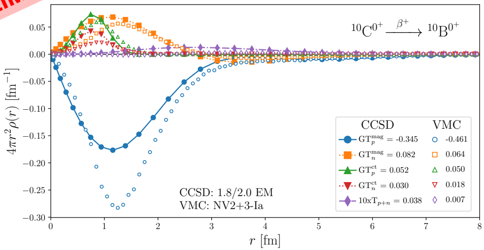


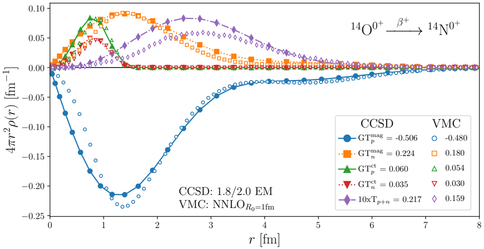
thanks to S. Gandolfi V. Cirigliano *et al*, '24

- "hybrid" calculations (Weinberg's PC for nuclear potentials, "renormalized" PC for $\delta_{\rm NS}$ operators)
- VMC calculation for ^{14}O, with N^{2}LO local chiral potential of A. Gezerlis *et al*, '14
- VMC calculation for ^{10}C with the Norfolk potentials — G. King, *in progress*
- CC calculation for ^{10}C, ^{14}O and ^{18}Ne with 1.8/2.0 EM interaction — S. Novario, *in progress*

Ab initio calculations of $\bar{\delta}_{\rm NS}$

PRELIMINARY





thanks to S. Novario!

- need a detailed study of the dependence on the interaction and on the regulator
- and the construction of the $\delta_{\rm NS}$ operator at subleading orders

what do we need for a believable error on $\bar{\delta}_{\rm NS}$?

Conclusions

- establishing the correct PC is important for BSM searches

we need to know if we are underestimating the theory error and often cannot hide our ignorance in fitting data

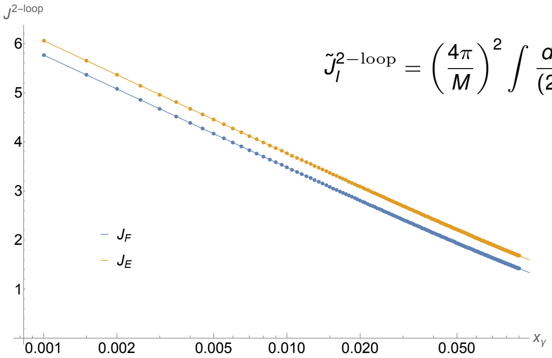
- weak/EM processes mediated by two-body operators provide an opportunity to test the PC

- for phenomenology, renormalization has been used as a “diagnostic” tool
- to build improved electroweak operators which are then fed into hybrid calculations

can we/ do we need to do better?

Backup

Alternative power countings. Relativistic chiral EFT



$$\tilde{J}_I^{2-\text{loop}} = \left(\frac{4\pi}{M}\right)^2 \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} G_0^I(\mathbf{k}_1, \mathbf{p}_i) \frac{1}{|\mathbf{k}_1 - \mathbf{k}_2|^2} G_0^I(\mathbf{k}_2, \mathbf{p}_f),$$

$$x_\gamma = \frac{\gamma}{M_N}$$

- replace the free Green's function with

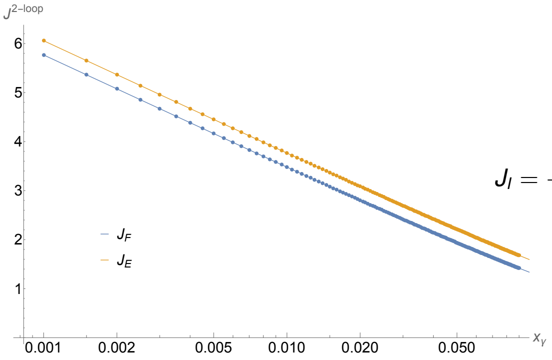
Y. L. Yang and P. W. Zhao, '23

$$G(\mathbf{k}, \mathbf{p}) = \frac{1}{\mathbf{k}^2 - \mathbf{p}^2 - i\varepsilon} \Longrightarrow G_0^F(\mathbf{k}, \mathbf{p}) = \frac{1}{2} \frac{M_N^2}{\mathbf{k}^2 + M_N^2} \frac{1}{\sqrt{\mathbf{k}^2 + M_N^2} - \sqrt{\mathbf{p}^2 + M_N^2} - i\epsilon},$$

- this makes the two-loop integral $\mathcal{A}^C$ formally convergent

$$\log \Lambda \longrightarrow \log m_N$$

Alternative power counting. Relativistic chiral EFT



$$J_l = -\log x_\gamma + a_l + b_l x_\gamma + \dots$$

$$a_F - a_E = 0.29$$

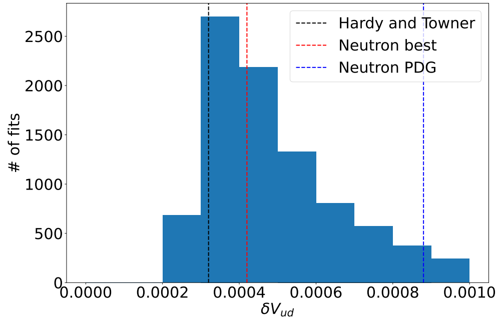
- however, the "relativistic" extension is not unique e.g. one could use — R. M. Woloshyn and A. D Jackson, '73

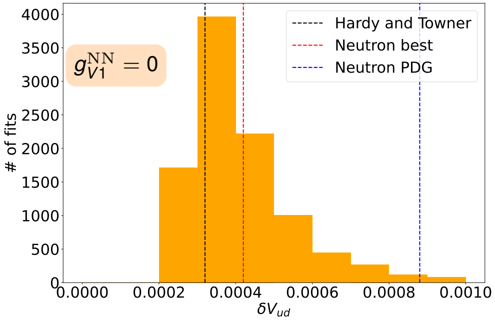
$$G_0^E(\mathbf{k}, \mathbf{p}) = \frac{M^2}{\sqrt{\mathbf{k}^2 + M^2}} \frac{1}{\mathbf{k}^2 - \mathbf{p}^2 - i\epsilon},$$

- the coefficient of the log is universal, the finite pieces depend on the choice of Green's function

integral sensitive to $|\mathbf{k}| \sim M_N$ where this EFT does not hold...

Fitting g_{V1} and g_{V2} to superallowed data





- are we absorbing BSM physics in the fits? consider ^{10}C, ^{14}O, ^{34}Cl, ^{38}K, ^{42}Sc and ^{46}V (in reach for CC)
- use VMC for ^{10}C and ^{14}O
- for the remaining nuclei, use Hardy and Towner's shell model results for the long-range part & randomly pick $M^{CT}_{GT,p}$ and $M^{CT}_{GT,n}$ between 0 and 0.05
- fit V_{ud}, g_{V1}^{NN} and g_{V2}^{NN} & repeat