Ab Initio Nuclear Structure and Reactions in the Context of Neutron Skin

Alexis Mercenne
Louisiana State University
Constraining the Neutron Skin

Electron scattering experiments (PREX, CREX)
Provide the most accurate measurement of the nucleon distribution. Upcoming MREX for neutron radius of $^{208}$Pb, but not before 2030.


Hadronic probes (FRIB)
Extraction of neutron skin is model-dependent, suffers from large and uncontrolled theoretical uncertainties.

- Optical potential models fitted on experimental data.
- Models reproduce binding energies and charge radii for a variety of nuclei.
- Very accurate representation of the experimental cross sections.
- Yet, wide range of values for the neutron-skin thickness.

Ab Initio Calculations

First Principles

Realistic Interactions ($\chi$EFT)

Many-body Dynamics

Symmetry-Adapted No-Core Shell Model (SA-NCSM)

Properties of Nuclei
Ab Initio Low Energy Nuclear Physics

Two energy regimes:
- High energy nuclear physics (quarks, ~ GeV).
- Low energy nuclear physics (nucleons, ~ MeV).

Interactions from Chiral Effective Field Theory ($\chi$EFT): Effective Lagrangians consistent with symmetries of QCD – expansion in $Q/\Lambda$.

Only nucleons as degree of freedom.

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<thead>
<tr>
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<th>NN</th>
<th>3N</th>
<th>4N</th>
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<tr>
<td>LO ($Q/\Lambda_c)^0$</td>
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<td>NLO ($Q/\Lambda_c)^2$</td>
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<td>NNLO ($Q/\Lambda_c)^3$</td>
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<td>N$^2$LO ($Q/\Lambda_c)^4$</td>
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A plethora of many-body methods:
- In-medium Similarity Renormalization Group
- Coupled Cluster methods
- Self-consistent Green’s Functions
- Configuration Interaction Approaches (No-core Shell Model, Symmetry-adapted No-core Shell Model)
- Quantum Monte Carlo
- Lattice Effective Field Theory
Symmetry-adapted No-core Shell Model (SA-NCSM)

Based on traditional No-core Shell Model (NCSM):
- Spherical harmonic oscillator basis.
- Configuration interaction.
- Ab initio (no restrictions for interactions …NN, NNN, non-local,… χEFT)

But brings new features:
- SU(3)-coupled basis states or Sp(3,R)-coupled basis states.
- Selected model space (truncation) – physically relevant + exact center-of-mass factorization!
- Equal to NCSM in complete-N<sub>max</sub> model space.
Symmetry-adapted No-core Shell Model

- Few basis states contribute.
- Practically exact calculations.
- Important shapes capture relevant correlations.
- Allow to keep non-negligible contributions only:
  I. Manageable model space.
  II. Center-of-mass/relative motion can be exactly factorized.
SA-NCSM Observables


PRC 91, 023326 (2015)

Form factor in $^6$Li

PRC 91, 023326 (2015)
Pushing Ab Initio Calculations Up to the Calcium Region

- Applied to many nuclei up to calcium region.
- Successful in calculating: energy spectrum, rms radius, electromagnetic transitions … etc.

SA-NCSM wave function

SA-NCSM Results for Neutron Rich Nuclei

$^{28}\text{Mg}$ yrast band

![Graph showing $^{28}\text{Mg}$ yrast band]

Williams et al., PRC 100, 014322 (2019)
SA-NCSM Heaviest Systems $A = 48$

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**Charge Longitudinal Form Factor**

$^{48}\text{Ca}$

$^{48}\text{Ti}$, Q$(2^+)$ [e fm$^2$]

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Experiment........ -17.7

10 shells........... -19.3

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**Proton/neutron density**

$^{48}\text{Ca}$

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Pushing Ab Initio Calculations Up to the Calcium Region

- Applied to many nuclei up to calcium region.
- Successful in calculating: energy spectrum, rms radius, electromagnetic transitions ...etc.

Next step: use this physically relevant many-body basis to nuclear reactions.
- Support FRIB experiments for exotic nuclei.
- Provide reliable inputs for astrophysical studies.
Plugging SA-NCSM Wavefunctions Into Reaction Calculations

Coupled-channel Framework

\[
(T_c - (E - E_c))u_c(r) + \sum_{c'=1}^{N_c} \int dr' V_{cc'}(r', r) u_{c'}(r') = 0
\]

Solution

Can be solved with calculable R-matrix.

A basis for reactions: channel index gathers partitions, quantum numbers of projectile and target, total angular momentum of composite …etc

\[c = \{ A \ I_T; a \ I_p; n \ell j; J \}\]

Threshold energy

Scattering energy

Kinetic part

Coupling potential

Low energies/light systems: the coupled-channel equation can be treated exactly; all thresholds are included; Non-local potential couples all channels with each other.

Higher energies/heavier systems: too many reaction thresholds; use of optical potential instead.

a.k.a optical potential, inter-cluster interaction, nucleon-nucleus potential …etc

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Alexis Mercenne – Neutron Rich Matter on Heaven and Earth
• $u_c(r)$ describes the relative motion between the two clusters.
• Asymptotic of $u_c(r)$ gives cross section for specific channel.
• Requires internal wave functions and NN interaction.
• Microscopic: full antisymmetrization + cluster correlations.
• Can be generalized to any number of clusters.

\[ |\Psi\rangle = \sum_c \int dr \frac{u_c(r)}{r} r^2 A \{ \text{Cluster} \} \]

\[ |\Phi\rangle = \sum_i C_i \]

\[ c = \{ A IT; a I_p; n\ell j; J \} \]

\[ \{ \text{Cluster} \} \overset{\text{H}}{\rightarrow} \{ \text{Cluster} \} \]

\[ (T_c - (E - E_c))u_c(r) + \sum_{c'=1}^{N_c} \int dr' V_{cc'}(r', r)u_{c'}(r') = 0 \]
Symmetry-adapted RGM:
- *Ab initio* single-nucleon projectile reactions in a coupled-channel framework.
- First results studied the influence of selected model space on non-local potentials.
Optical Potentials Derived from SA-NCSM Calculations

- $^4$He wave function calculated with SA-NCSM.
- Optical potential describing neutron scattering on $^4$He constructed with Green’s function method.

Matthew Burrows and Kristina Launey, preliminary results

![Graph and diagrams showing optical potentials derived from SA-NCSM calculations]
Uncertainties in Ab Initio Calculations

Two main sources:
- Model space truncations.
- Uncertainties from the underlying $\chi$EFT


G. H. Sargsyan et al. arxiv: 2210.08843
Summary

• Practically exact ab initio calculations are now performed up to the calcium region.

• Symmetry-adapted No Core Shell Model (SA-NCSM) makes use of a physically relevant basis that allows a good reproduction of spectra, B(E2) values, quadrupole moments, form factors … etc.

• SA-NCSM wave functions can be implemented into reaction formalisms:
  • For low-energy reactions in multiple coupled-channels framework.
  • At higher energies with the construction of effective ab initio nucleon-nucleus potentials (optical potentials).

• Applications of SA-NCSM in neutron-rich nuclei can help to evaluate the neutron skin thickness.

• Compared to traditional methods extracting $R_{\text{skin}}$ from proton scattering, theoretical uncertainties will now be better controlled.

And also: M. Ploszajczak (GANIL), Y. Alhassid (Yale), J. Escher (LLNL), N. Michel (Institute of Modern Physics, Lanzhou).