

# Test and correlations of calculated neutrinoless $\beta\beta$ decay nuclear matrix elements

**Javier Menéndez**

**University of Barcelona  
Institute of Cosmos Sciences**

**INT program**

**“New physics searches at the precision frontier”**

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**UNIVERSITAT DE  
BARCELONA**



# Collaborators



J. P. Linares, **P. Soriano**



**L. Jokiniemi**



C. Peña-Garay, **B. Romeo**



N. Shimizu



**R. Weiss**



A. Lovato, B. Wiringa



**C. Brase**, A. Schwenk

# Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

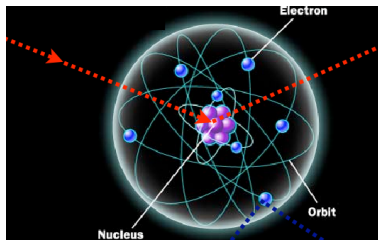
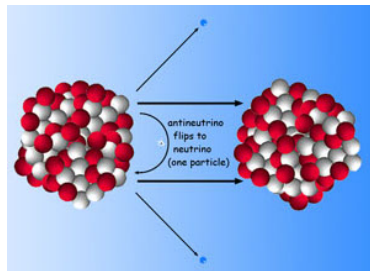
Nuclear structure physics encoded in nuclear matrix elements  
key to plan, fully exploit experiments

$$0\nu\beta\beta: \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{dq^2} \propto \left|\sum_i c_i \zeta_i \mathcal{F}_i\right|^2$$

$$\text{CE}\nu\text{NS: } \frac{d\sigma_{\nu\mathcal{N}}}{dq^2} \propto \left|\sum_i c_i \zeta_i \mathcal{F}_i\right|^2$$

$M^{0\nu\beta\beta}$ : Nuclear matrix element  
 $\mathcal{F}_i$ : Nuclear structure factor



# Creation of matter in nuclei: $0\nu\beta\beta$ decay

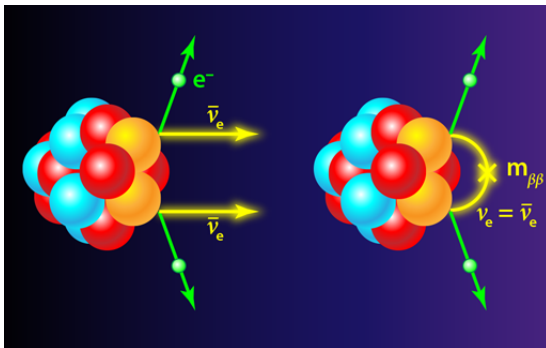
Lepton number is conserved  
in all processes observed:

single  $\beta$  decay,  
 $\beta\beta$  decay with neutrino emission...

Uncharged massive particles  
like Majorana neutrinos ( $\nu$ )  
allow lepton number violation:

neutrinoless  $\beta\beta$  decay  
two matter particles (electrons) created

Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. in press, arXiv:2202.01787

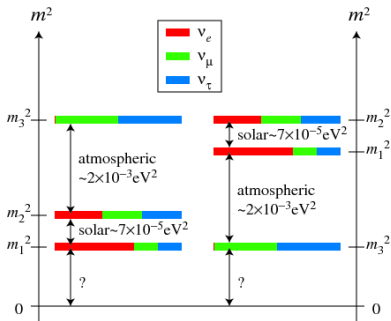


# Next generation experiments: inverted hierarchy

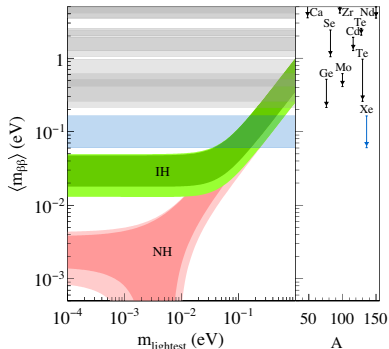
Decay rate sensitive to  
neutrino masses, hierarchy

$$m_{\beta\beta} = \left| \sum U_{ek}^2 m_k \right|$$

$$T_{1/2}^{0\nu\beta\beta} (0^+ \rightarrow 0^+)^{-1} = G_{0\nu} g_A^4 |M^{0\nu\beta\beta}|^2 \left( \frac{m_{\beta\beta}}{m_e} \right)^2$$

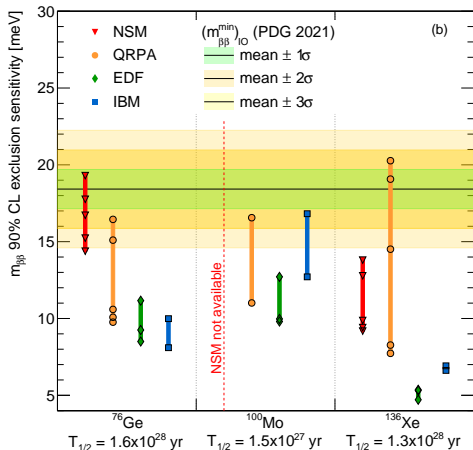


Matrix elements assess if  
next generation experiments  
fully explore "inverted hierarchy"



KamLAND-Zen, PRL117 082503(2016)

# Uncertainty in physics reach of $0\nu\beta\beta$ experiments



Nuclear matrix element theoretical uncertainty critical to anticipate  $m_{\beta\beta}$  sensitivity of future experiments

Current uncertainty in  $m_{\beta\beta}$  prevents to foresee if next-generation experiments will fully cover parameter space of "inverted" neutrino mass hierarchy

Uncertainty needs to be reduced!

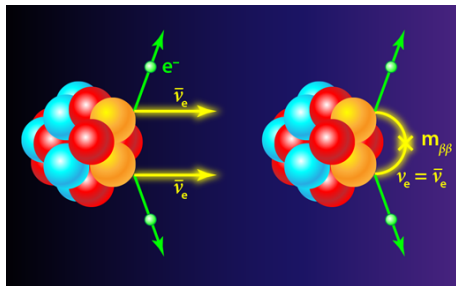
Agostini, Benato, Detwiler, JM, Vissani  
Phys. Rev. C 104 L042501 (2021)

# Nuclear matrix elements

## Nuclear matrix elements needed in low-energy new-physics searches

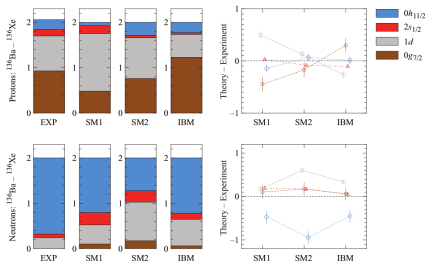
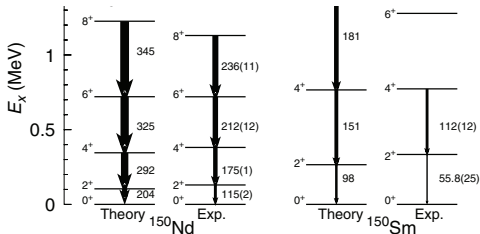
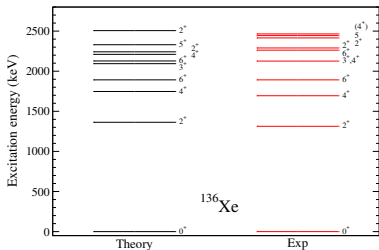
$$\langle \text{Final} | \mathcal{L}_{\text{leptons-nucleons}} | \text{Initial} \rangle = \langle \text{Final} | \int dx j^\mu(x) J_\mu(x) | \text{Initial} \rangle$$

- Nuclear structure calculation of the initial and final states:  
Shell model, QRPA, IBM, Energy-density functional  
Ab initio many-body theory  
QMC, Coupled-cluster, IMSRG...
- Lepton-nucleus interaction:  
Hadronic current in nucleus:  
phenomenological,  
effective theory of QCD



# Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...



Schiffer et al. PRL100 112501(2009)

Kay et al. PRC79 021301(2009)

...

Szwec et al., PRC94 054314 (2016)

Rodríguez et al. PRL105 252503 (2010)

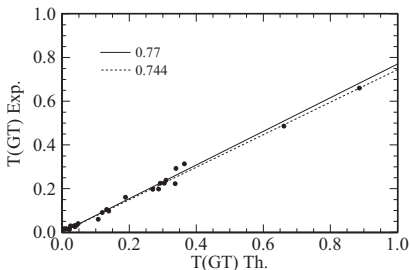
...

Vietze et al. PRD91 043520 (2015)

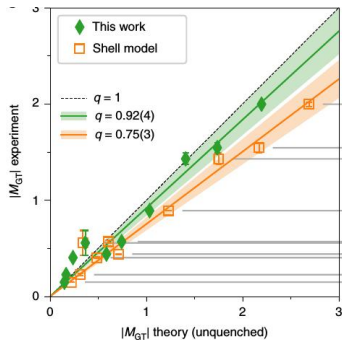


# $\beta$ -decay Gamow-Teller transitions: “quenching”

$\beta$  decays ( $e^-$  capture): phenomenology vs ab initio



Martinez-Pinedo et al. PRC53 2602(1996)

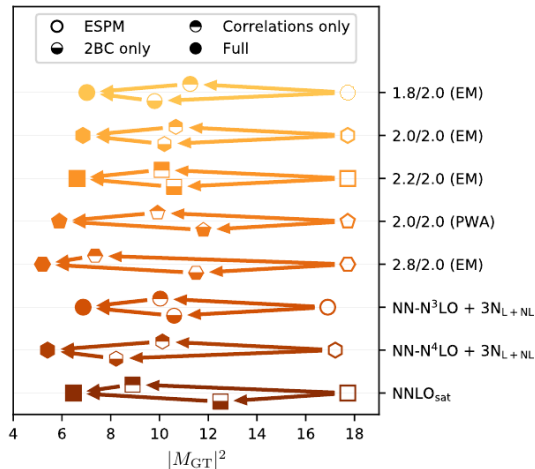


Gysbers et al. Nature Phys. 15 428 (2019)

Ab initio calculations including meson-exchange currents and additional nuclear correlations do not need any “quenching”

# Origin of $\beta$ decay “quenching”

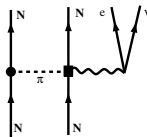
Which are main effects missing in conventional  $\beta$ -decay calculations?  
Test case: GT decay of  $^{100}\text{Sn}$



Relatively similar  
and complementary  
impact of

- nuclear correlations
- meson-exchange currents

Gysbers et al.  
Nature Phys. 15 428 (2019)



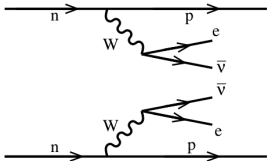
# Two-neutrino $\beta\beta$ decay, $2\nu$ ECEC

$2\nu\beta\beta$  decay same initial, final states, similar operator ( $\sigma\tau$ ) as  $0\nu\beta\beta$   
 Comparison of predicted  $2\nu\beta\beta$  decay vs data

Shell model  
 reproduce  $2\nu\beta\beta$  data  
 including “quenching”

Prediction previous to  
 $^{48}\text{Ca}$  measurement!

Caurier, Poves, Zuker  
 PLB 252 13(1990)



$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

**Table 2**

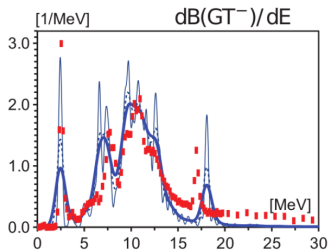
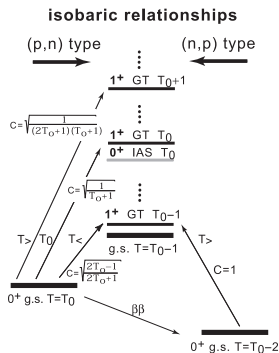
The ISM predictions for the matrix element of several  $2\nu$  double beta decays (in  $\text{MeV}^{-1}$ ). See text for the definitions of the valence spaces and interactions.

	$M^{2\nu}(\text{exp})$	$q$	$M^{2\nu}(\text{th})$	INT
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	$0.047 \pm 0.003$	0.74	0.047	kb3
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	$0.047 \pm 0.003$	0.74	0.048	kb3g
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	$0.047 \pm 0.003$	0.74	0.065	gxfp1
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	$0.140 \pm 0.005$	0.60	0.116	gcn28:50
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	$0.140 \pm 0.005$	0.60	0.120	jun45
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	$0.098 \pm 0.004$	0.60	0.126	gcn28:50
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	$0.098 \pm 0.004$	0.60	0.124	jun45
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	$0.049 \pm 0.006$	0.57	0.059	gcn50:82
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	$0.034 \pm 0.003$	0.57	0.043	gcn50:82
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	$0.019 \pm 0.002$	0.45	0.025	gcn50:82

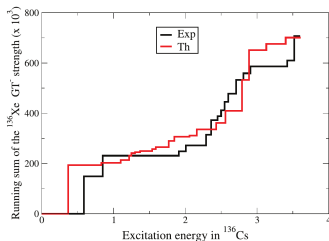
Caurier, Nowacki, Poves, PLB 711 62 (2012)

# Fix “quenching” in phenomenological calculations

GT strength distribution complements  $\beta$ -decay beyond Q-value region



Iwata et al. JPSCP 6 03057 (2015)



Caurier et al. PLB 711 62 (2012)

$$\frac{d\sigma}{d\Omega}(\theta = 0) \propto \sum_i \sigma_i \tau^\pm$$

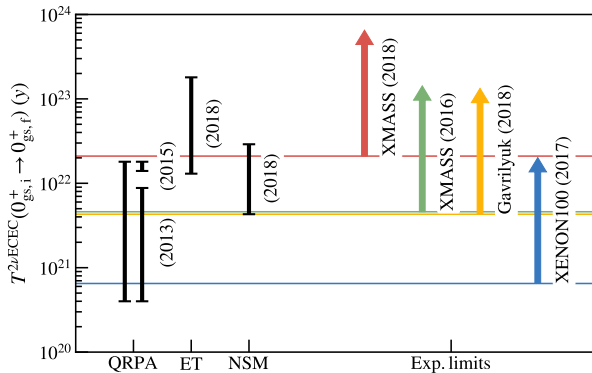
$$\langle 1_f^+ | \sum_i g_A^{\text{eff}} \sigma_i \tau_i^\pm | 0_{\text{gs}}^+ \rangle, \quad g_A^{\text{eff}} \sim 0.57 g_A \text{ for } ^{136}\text{Xe}$$

Frekers et al.  
NPA916 219 (2013)

Similar “quenching”  $q = 0.57$  needed in GT decays in xenon mass region  
Smaller “quenching”  $q = 0.42$  needed in  $2\nu\beta\beta$  of  $^{136}\text{Xe}$

# Two-neutrino ECEC of $^{124}\text{Xe}$

Predicted  $2\nu$ ECEC half-life:  
shell model error bar largely dominated by “quenching” uncertainty

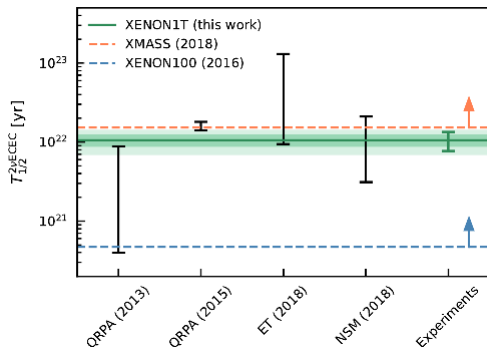


Suhonen  
JPG 40 075102 (2013)  
Pirinen, Suhonen  
PRC 91, 054309 (2015)  
Coello Pérez, JM, Schwenk  
PLB 797 134885 (2019)

Shell model, QRPA and Effective theory (ET) predictions  
suggest experimental detection close to XMASS 2018 limit

# Two-neutrino ECEC of $^{124}\text{Xe}$

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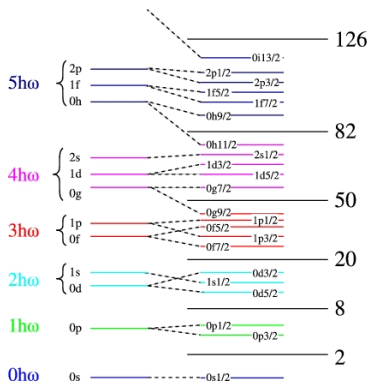


Suhonen  
JPG 40 075102 (2013)  
Pirinen, Suhonen  
PRC 91, 054309 (2015)  
Coello Pérez, JM, Schwenk  
PLB 797 134885 (2019)

XENON1T  
Nature 568 532 (2019)  
PRC106, 024328 (2022)

Shell model, QRPA and Effective theory (ET) predictions  
good agreement with XENON1T measurement of  $2\nu$ ECEC!

# Nuclear shell model



Nuclear shell model configuration space  
only keep essential degrees of freedom

- High-energy orbitals: always empty
- Valence space: where many-body problem is solved
- Inert core: always filled

$$H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{eff}|\Psi\rangle_{eff} = E|\Psi\rangle_{eff}$$

$$|\Psi\rangle_{eff} = \sum_{\alpha} c_{\alpha} |\phi_{\alpha}\rangle, \quad |\phi_{\alpha}\rangle = a_{i_1}^+ a_{i_2}^+ \dots a_{i_A}^+ |0\rangle$$

Shell model diagonalization:

$\sim 10^{10}$  Slater dets. Caurier et al. RMP77 (2005)

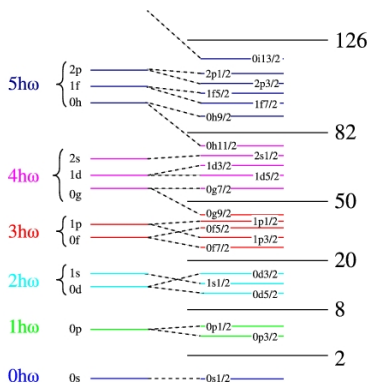
$\gtrsim 10^{24}$  Slater dets. with Monte Carlo SM

Otsuka, Shimizu, Y.Tsunoda  
Phys. Scr. 92 063001 (2017)

$H_{eff}$  includes effects of

- inert core
- high-energy orbitals

# QRPA method



QRPA configuration space comprises 18–25 single-particle orbitals with no core in the calculation

Intermediate states in odd-odd nuclei described as proton-neutron quasiparticles from ground states of initial and final nuclei

More limited nuclear correlations than nuclear shell model

Some adjustable parameters:  
especially particle-particle channel  $g_{pp}$  (isoscalar pairing)  
critical for a good description of  $\beta\beta$  decays

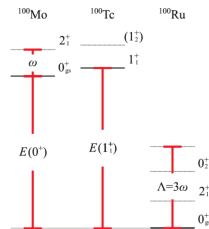
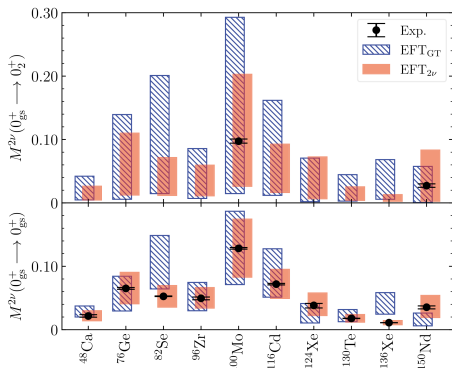
Vogel, Zirnauer, PRL 57, 3148 (1986), Engel, Vogel, Zirnauer, PRC 37 3148 (1988)



# Effective theory of $\beta\beta$ decay

Effective theory (ET) for  $\beta\beta$  decay:  
spherical core coupled to one nucleon

Couplings adjusted to experimental data,  
uncertainty given by effective theory  
(breakdown scale, systematic expansion)



Use  $\beta$ -decay data  
to predict  $2\nu\beta\beta$  decay  
Coello-Pérez, JM, Schwenk  
PRC 98, 045501 (2018)

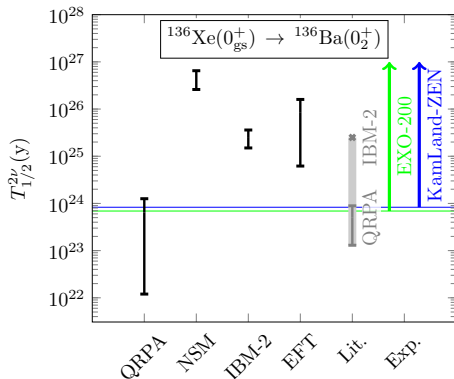
Use  $2\nu\beta\beta$ -decay data  
predict  $2\nu\beta\beta$  to excited states  
Jokiniemi, Romeo, Brase, Kotila et al.  
PLB 838 137689 (2023)

Good agreement, large error  
(leading-order in ET)

# $2\nu\beta\beta$ decay of $^{136}\text{Xe}$ to $^{136}\text{Ba } 0_2^+$

Current experiments sensitive to two-neutrino  $\beta\beta$  of  $^{136}\text{Xe}$  to  $^{136}\text{Ba } 0_2^+$

EXO-200, KamLAND-Zen



Nuclear shell model  
QRPA, EFT and IBM  
very different predictions!

Barea et al.  
PRC 91 034304 (2015)

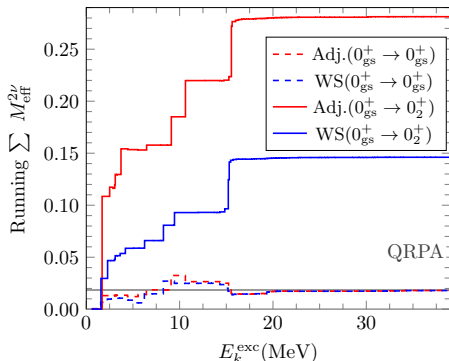
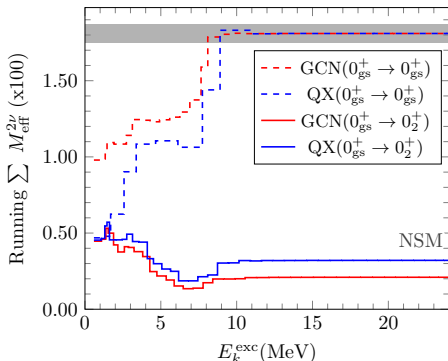
Pirinen, Suhonen  
PRC 91, 054309 (2015)

Jokiniemi, Romeo, Brase, Kotila et al.  
PLB 838 137689 (2023)

Very good test of theoretical calculations!

# $^{136}\text{Xe} \rightarrow ^{136}\text{Ba } 0_2^+$ running sums

Subtle cancellation NME running sum, depends on many-body method



Jokiniemi, Romeo, Brase, Kotila et al. PLB 838 137689 (2023)

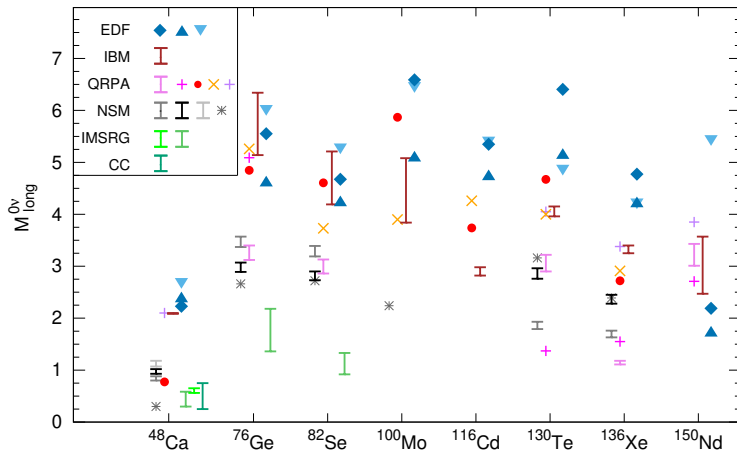
Shell-model running sum shows cancellations in decay to ground state

QRPA running sum shows cancellations in decay to excited state

Since ground-state decay fitted to data, very different decay to excited state

# $0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor  $\sim 3$



Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. in press, arXiv:2202.01787

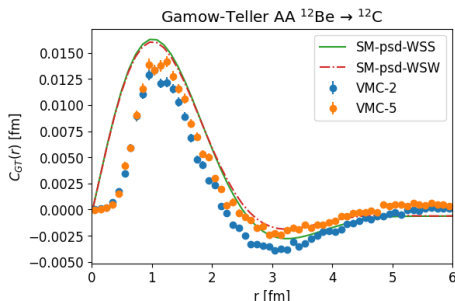
# Shell model vs quantum Monte Carlo: correlations

Compare  $\beta\beta$  transition densities in nuclear shell model and quantum Monte Carlo calculations in light nuclei

$$4\pi r^2 \rho_{GT}(r) = \langle \Psi_f | \sum_{a < b} \delta(r - r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+ | \Psi_i \rangle,$$

$$M_{GT}^{0\nu} = \int_0^\infty dr C_{GT}^{0\nu},$$

Agreement at long distances, missing short-range correlations in shell model



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Similar findings in Wang et al. PLB 798 134974 (2019)

# Generalized contact formalism (GCF)

Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

Separation of scales: wf, transition density factorize for two nearby nucleons

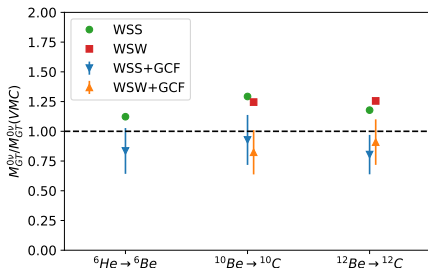
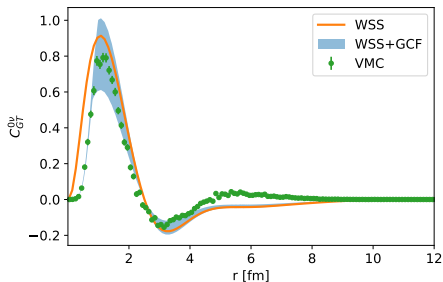
$$\Psi \xrightarrow{r_{ij} \rightarrow 0} \sum_{\alpha} \varphi^{\alpha}(\mathbf{r}_{ij}) A^{\alpha}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j}), \quad \rho_{GT}(r) \xrightarrow{r \rightarrow 0} -3|\varphi^0(r)|^2 C_{pp,nn}^0(f, i)$$

with  $\varphi(r)$  the solution of the two-nucleon Schrödinger equation

The contact  $C^0(f, i) = \frac{A(A-1)}{2} \langle A^{\alpha}(f) | A^{\beta}(i) \rangle$  is model dependent

Replace shell-model by QMC contact

to improve transition density and nuclear matrix element

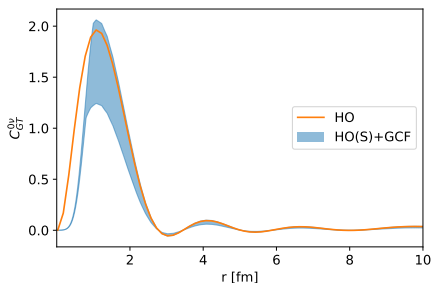
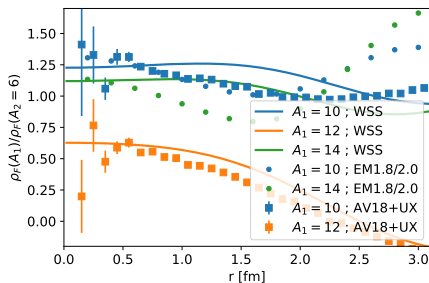


# GCF: model independence of ratios

**Generalized contact formalism** Weiss, Bazak, Barnea PRL 114 012501 (2015)

The contact  $C^0(f, i) = \frac{A(A-1)}{2} \langle A^\alpha(f) | A^\beta(i) \rangle$  is model dependent (shell model, quantum Monte Carlo, no-core shell model...)

but for two nuclei the ratio  $C_{pp,nn}^0(X)/C_{pp,nn}^0(Y)$  relatively model independent: combine QMC calculation in light nuclei with two shell model calculations:



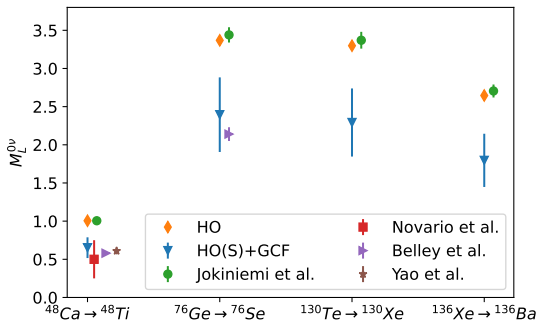
Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Yao, Belley et al. PRC 103, 014315 (2021)

# Shell model + Generalized contact formalism: NMEs

GCF builds QMC short-range correlations to shell model transitions densities can be extended to heavy nuclei where shell model calculations are possible

Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)



Short-range correlations included by GCF reduce  $0\nu\beta\beta$  NMEs moderately

~ 30% reduction in general consistent with ab initio NMEs in  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$

Good agreement in benchmark NMEs in light nuclei with ab initio calculations



# Light-neutrino exchange: contact operator

Contact operator suggested to contribute to light-neutrino exchange  
absorb cutoff depend. of two-nucleon decay amplitude: high-energy neutrinos

$$T_{1/2}^{-1} = G_{01} g_A^4 (M_{\text{long}}^{0\nu} + M_{\text{short}}^{0\nu})^2 \frac{m_{\beta\beta}^2}{m_e^2}, \quad \text{Cirigliano et al. PRL120 202001(2018)}$$

$$M_{\text{short}}^{0\nu} \equiv \frac{1.2A^{1/3} \text{ fm}}{g_A^2} \langle 0_f^+ | \sum_{n,m} \tau_m^- \tau_n^- \mathbb{1} \left[ \frac{2}{\pi} \int j_0(qr) 2g_{\nu}^{\text{NN}} g(p/\Lambda) p^2 dp \right] | 0_i^+ \rangle,$$

$$M_{\text{GT}}^{0\nu} \simeq \frac{1.2A^{1/3} \text{ fm}}{g_A^2} \langle 0_f^+ | \sum_{n,m} \tau_m^- \tau_n^- \sigma_1 \cdot \sigma_2 \left[ \frac{2}{\pi} \int j_0(qr) \frac{1}{p^2} g_A^2 f^2(p/\Lambda_A) p^2 dp \right] | 0_i^+ \rangle$$

Unknown value (and sign) of the hadronic coupling  $g_{\nu}^{\text{NN}}$ !

Lattice QCD calculations can obtain value of  $g_{\nu}^{\text{NN}}$

Davoudi, Kadam, Phys. Rev. Lett. 126, 152003 (2021), PRD105 094502('22)

or match  $nn \rightarrow pp + ee$  amplitude calculated with approximate QCD methods

Cirigliano et al. PRL126 172002 (2021), JHEP 05 289 (2021)

or charge-independence breaking of nuclear Hamiltonians

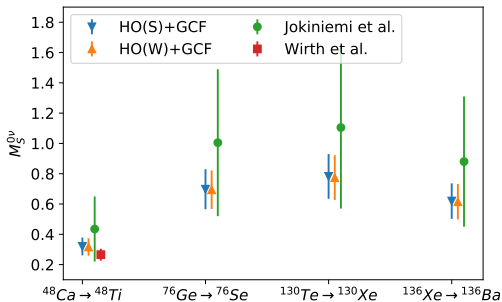
Cirigliano et al. PRC100, 055504 (2019)

# Short-range NME: GCF + shell model

Shell model with short-range correlations from QMC using the GCF give consistent contribution of new term  $M_S$

~ 25% impact of short-range NME in GCF + shell model obtained with  $g_{\nu}^{NN}$  from AV18 CIB term

consistent with 43% effect in IM-GCM for  $^{48}\text{Ca}$  using synthetic data on  $nn \rightarrow pp + ee$  decay Wirth et al. PRL127 242502 (2021)



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021): **L. Jokiniemi's talk Thursday**

# Systematic shell-model calculations

Explore systematic shell-model matrix elements  
in configuration spaces relevant for  $0\nu\beta\beta$  decay searches

- $^{46-58}\text{Ca}$ ,  $^{50-58}\text{Ti}$ , and  $^{54-60}\text{Cr}$   
in pf-shell with KB3G and GXPF1B interactions
- $^{72-76}\text{Ni}$ ,  $^{74-80}\text{Zn}$ ,  $^{76-82}\text{Ge}$ , and  $^{82,84}\text{Se}$   
in  $1p_{3/2}$ ,  $0f_{5/2}$ ,  $1p_{1/2}$ , and  $0g_{9/2}$  configuration space  
with GCN2850, JUN45, and JJ4BB interactions
- $^{124-132}\text{Sn}$ ,  $^{130-134}\text{Te}$ , and  $^{134,136}\text{Xe}$   
in  $1d_{5/2}$ ,  $0g_{7/2}$ ,  $2s_{1/2}$ ,  $1d_{3/2}$ , and  $0h_{11/2}$  configuration space  
with the GCN5082 and QX interactions

Overall,  $\sim 20 - 40$  different calculations for each configuration space

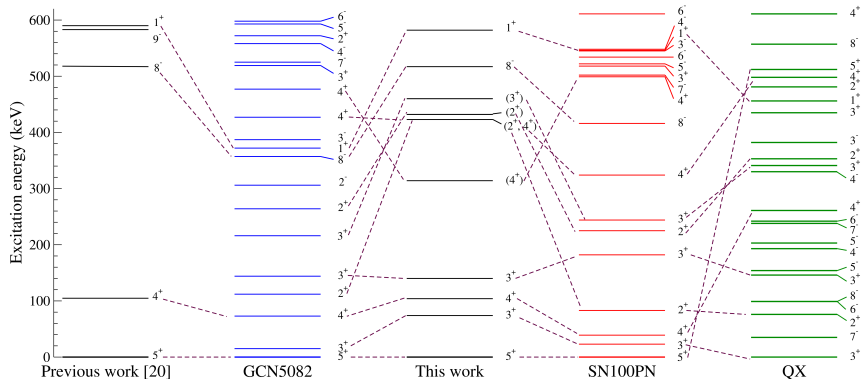
Complementary approach to randomly varying nuclear interaction

Horoi et al. PRC 106, 054302 (2022), PRC 107, 045501 (2023)

# $^{136}\text{Cs}$ experimental spectrum

While all these interactions are well, tested recent data on  $^{136}\text{Cs}$  suggests GCN5082 results agree better with experiment than QX

Rebeiro, Triambak et al. arXiv:2301.11371



QX gives systematically smaller  $^{136}\text{Xe}$   $0\nu\beta\beta$ -decay nuclear matrix elements

# Double Gamow-Teller strengths and $\beta\beta$ decay

Measurement of Double Gamow-Teller (DGT) resonance  
in double charge-exchange reactions  $^{48}\text{Ca}(pp,nn)^{48}\text{Ti}$  proposed in 80's

Auerbach, Muto, Vogel... 1980's, 90's

Recent experimental plans in RCNP, RIKEN ( $^{48}\text{Ca}$ ), INFN Catania

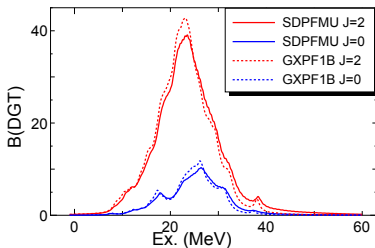
Takaki et al. JPS Conf. Proc. 6 020038 (2015)

Capuzzello et al. EPJA 51 145 (2015), Takahisa, Ejiri et al. arXiv:1703.08264

Promising connection to  $\beta\beta$  decay,  
two-particle-exchange process,  
especially the (tiny) transition  
to ground state of final state

Shell model calculation

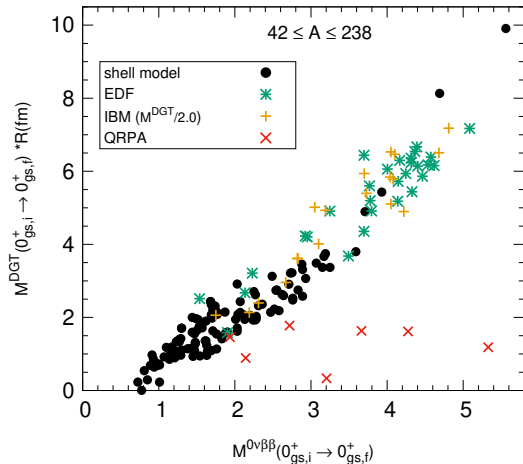
Shimizu, JM, Yako, PRL120 142502 (2018)



$$B(DGT^-; \lambda; i \rightarrow f) = \frac{1}{2J_i + 1} \left| \left\langle {}^{48}\text{Ti} \left| \left[ \sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^- \right]^{(\lambda)} \right| \left| {}^{48}\text{Ca}_{\text{gs}} \right\rangle \right|^2$$

# Correlation of $0\nu\beta\beta$ decay to DGT transitions

Double GT transition to ground state  
good linear correlation with  $0\nu\beta\beta$  decay NMEs



Double Gamow-Teller  
correlation with  
 $0\nu\beta\beta$  decay holds  
across nuclear chart  
Shimizu, JM, Yako  
PRL120 142502 (2018)

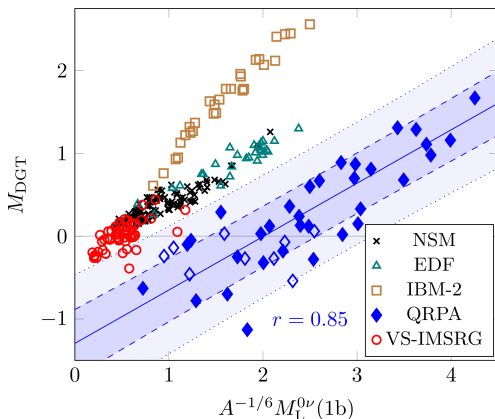
Common to shell model  
energy-density functionals  
interacting boson model,  
disagreement to QRPA  
Also correlation in  
VS-IMSRG (but weaker)  
Yao et al. PRC106 014315(2022)

Experiments at  
RIKEN, INFN, RCNP?  
access DGT transitions

# Correlation of $0\nu\beta\beta$ decay to DGT in QRPA

In QRPA,  $g_{pp}$  parameter typically fitted to reproduce  $2\nu\beta\beta$  half-life of measured transitions but actually some tension between  $g_{pp}$  values to reproduce single- $\beta$  decays

Faessler et al., J. Phys. G 35, 075104 (2008)



Jokiniemi, JM, PRC 107 044316 (2023)

Perform QRPA calculations with range of  $g_{pp} = (0.6 - 0.9)$

Correlation between DGT and  $0\nu\beta\beta$  NMEs! but different than for other many-body methods

Partially caused by relevance of  $J > 1$  intermediate states in QRPA compared to eg shell model

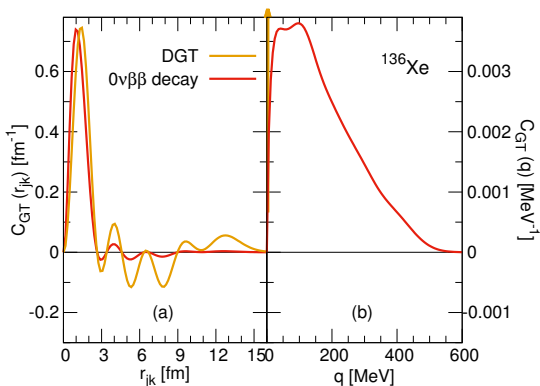
Ejiri et al. Phys. Rept. 797 1 (2019)

Horoi et al, PRC 93, 044334 (2016)

# Short-range character of DGT, $0\nu\beta\beta$ decay

Correlation between DGT and  $0\nu\beta\beta$  decay matrix elements explained by transition involving low-energy states combined with dominance of short distances between exchanged/decaying neutrons

Bogner et al. PRC86 064304 (2012)



$0\nu\beta\beta$  decay matrix element limited to shorter range

Short-range part dominant in double GT matrix element due to partial cancellation of mid- and long-range parts

Long-range part dominant in QRPA DGT matrix elements

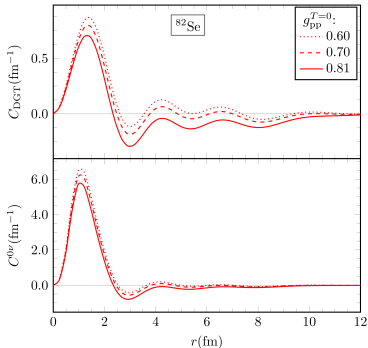
Shimizu, JM, Yako,  
PRL120 142502 (2018)



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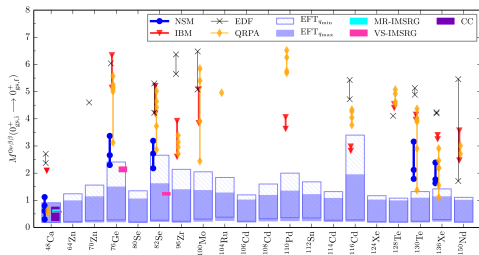
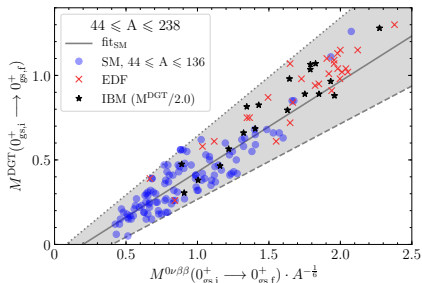
Long-range part dominant in QRPA DGT matrix elements

Shimizu, JM, Yako, PRL120 142502 (2018)

# $0\nu\beta\beta$ decay NMEs in ET of $\beta$ decay

Effective theory of  $\beta$  decay can calculate DGT with uncertainties  
(similar to calculation of  $2\nu\beta\beta$ , no energy denominator)  
DGT vs  $0\nu\beta\beta$  correlation  $\Rightarrow$  predict  $0\nu\beta\beta$  NMEs with uncertainties

Because ET couplings fitted to  $\beta$  decay and GT strengths  
correct shell model DGT NMEs in correlation  
by “quenching” factor for these observables:  $q = 0.42 - 0.65$

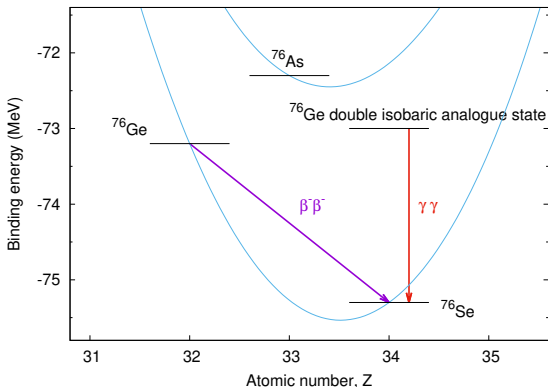


Brase, JM, Coello Pérez, Schwenk, PRC106, 034309 (2022)

# $\gamma\gamma$ decay of the DIAS of the initial $\beta\beta$ nucleus

Explore correlation between  $0\nu\beta\beta$  and  $\gamma\gamma$  decays, focused on double-M1 transitions

$$M_{M1 M1}^{\gamma\gamma} = \sum_k \frac{\langle 0_f^+ | \sum_n (g_n^I I_n + g_n^S \sigma_n)^{IV} | 1_k^+ (\text{IAS}) \rangle \langle 1_k^+ (\text{IAS}) | \sum_m (g_m^I I_m + g_m^S \sigma_m)^{IV} | 0_i^+ (\text{DIAS}) \rangle}{E_k - (E_i + E_f)/2}$$



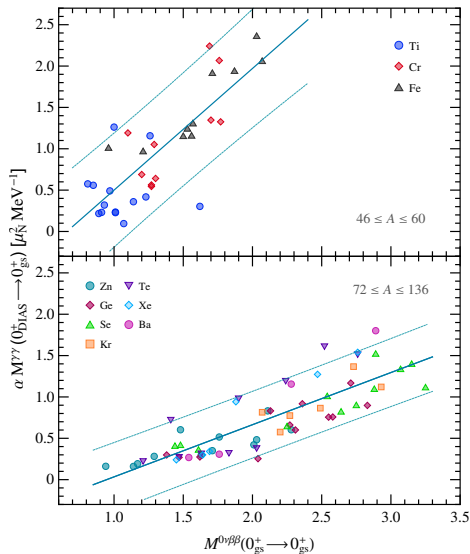
Similar initial and final states but both in same nucleus for electromagnetic transition

M1 and GT operators similar, physics of spin operator  
M1 also angular momentum

Different energy denominator

Romeo, JM, Peña-Garay  
PLB 827 136965 (2022)

# Correlation between $M1M1$ and $0\nu\beta\beta$ NMEs



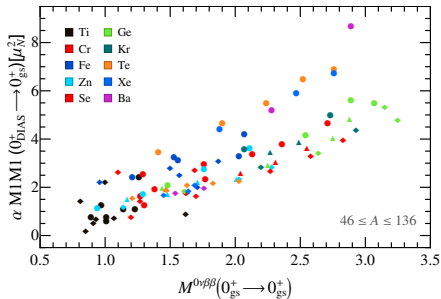
Good correlation between  $M1M1$  same-energy photons and shell-model  $0\nu\beta\beta$  NMEs

A dependence:  
energy denominator  
dominant states at higher energy in heavier nuclei

Overall, study  $\sim 50$  transitions  
several nuclear interactions  
for each of them

Romeo, JM, Peña-Garay  
PLB 827 136965 (2022)

# Intermediate states of the $M1M1$ transition

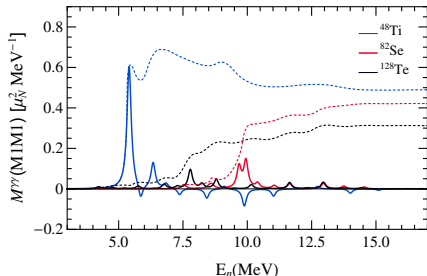


Dominant intermediate states  
lower energies for lighter nuclei,  
otherwise similar energies

One or few intermediate states  
typically dominate the transition

When energy denominators are  
(artificially) removed, same correlation  
across the nuclear chart

Romeo, JM, Peña-Garay  
PLB 827 136965 (2022)

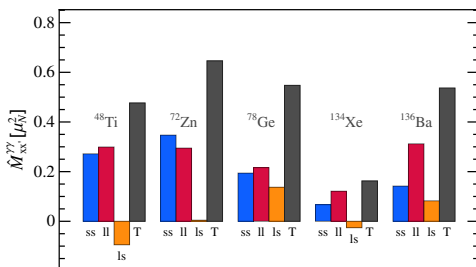


# Spin, angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{\gamma\gamma} = \hat{M}_{ss} + \hat{M}_{ll} + \hat{M}_{ls}$$

spin, angular momentum and interference components



Spin, angular momentum terms  
strikingly similar,  
always carry same sign

Interference term  
can cancel the other two  
but always much smaller

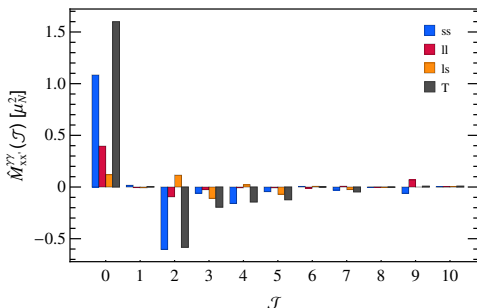
Romeo, JM, Peña-Garay  
PLB 827 136965 (2022)

# Total angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{\gamma\gamma}(\mathcal{J}) = \hat{M}_{ss}(\mathcal{J}) + \hat{M}_{ll}(\mathcal{J}) + \hat{M}_{ls}(\mathcal{J})$$

spin, angular momentum and interference components  
and total angular momentum of the nucleons involved in the transition



Dominance of  $\mathcal{J} = 0$  terms  
for spin and orbital contributions  
just like in  $0\nu\beta\beta$  decay

Cancellation from  $\mathcal{J} > 0$  terms  
less pronounced in orbital part  
Explains similar behaviour of spin  
and orbital components:

$$s_1 s_2 = S^2 - 3/2 < 0$$

$$l_1 l_2 = L^2 - l_1^2 - l_2^2 < 0$$

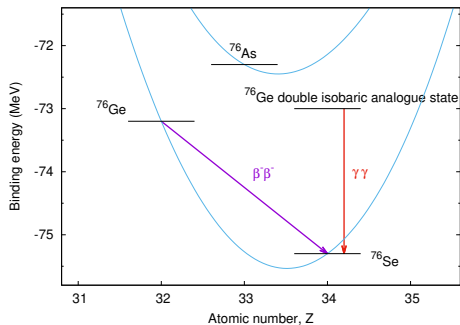
Romeo et al. PLB 827 136965 (2022)

# Experimental feasibility of $\gamma\gamma$ decay?

$\gamma\gamma$  decays are very suppressed with respect to  $\gamma$  decays  
just like  $\beta\beta$  decays are much slower than  $\beta$  decays

$\gamma\gamma$  decays have been observed recently  
in competition with  $\gamma$  decays

Waltz et al. Nature 526, 406 (2015), Soderstrom et al. Nat. Comm. 11, 3242 (2020)



Outlook:

Study in detail leading  
decay channels for  $M1M1$  decay  
in DIAS of  $\beta\beta$  nuclei

Particle emission  $M1$ ,  $E1$  decay:  
 $\text{BR} \sim 10^{-7} - 10^{-8}$

Experimental proposal for  $^{48}\text{Ti}$   
by Valiente-Dobón et al.

Valiente-Dobón, Romeo et al., in prep



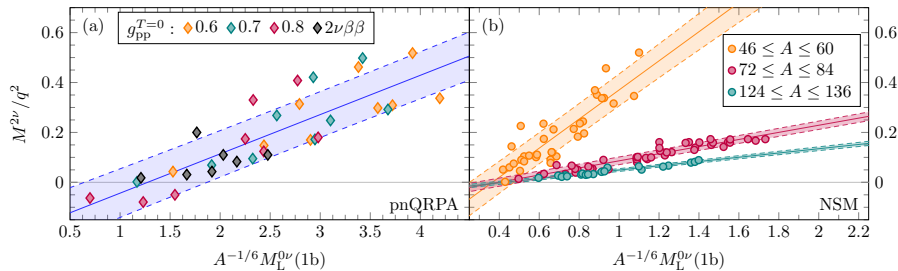
# Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between  $2\nu$  and  $0\nu$  modes of  $\beta\beta$  decay in nuclear shell model (systematic calculations of different nuclei) and QRPA calculations (decays of  $\beta\beta$  emitters with different  $g_{pp}$  values)

Similar but not common correlation, depends on mass for shell model

$0\nu\beta\beta - 2\nu\beta\beta$  correlation also observed in  $^{48}\text{Ca}$ ,  $^{136}\text{Xe}$

Horoi et al. PRC 106, 054302 (2022), PRC 107, 045501 (2023)



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

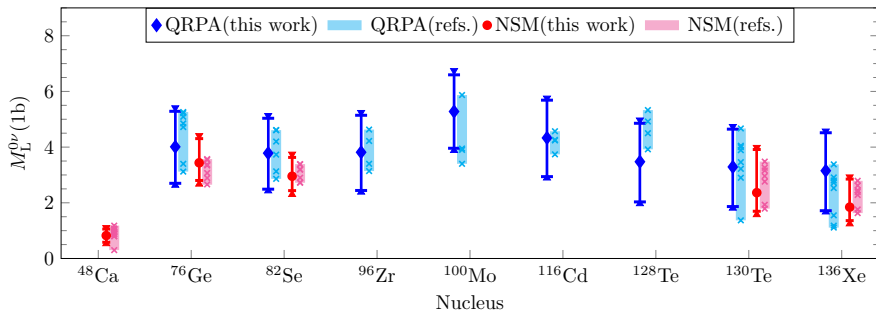
Use  $2\nu\beta\beta$  data to predict  $0\nu\beta\beta$  NMEs!

# $0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves  
systematic calculations covering dozens of nuclei and interactions  
error of each calculation (eg quenching) and experimental  $2\nu\beta\beta$  error

Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

# 2b currents in $0\nu\beta\beta$ decay

In  $0\nu\beta\beta$  decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current normal ordering with respect to a Fermi gas

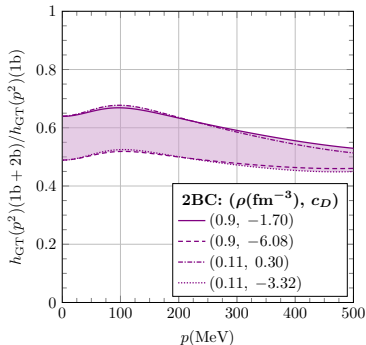
JM, Gazit, Schwenk, PRL107 062501(2011)

Normal-ordering approximation works remarkably well for  $\beta$  decay ( $q = 0$ )

Gysbers et al. Nature Phys. 15 428 (2019)

Some reduction of quenching due to 2b currents at  $p \sim m_\pi$  relevant for  $0\nu\beta\beta$  decay

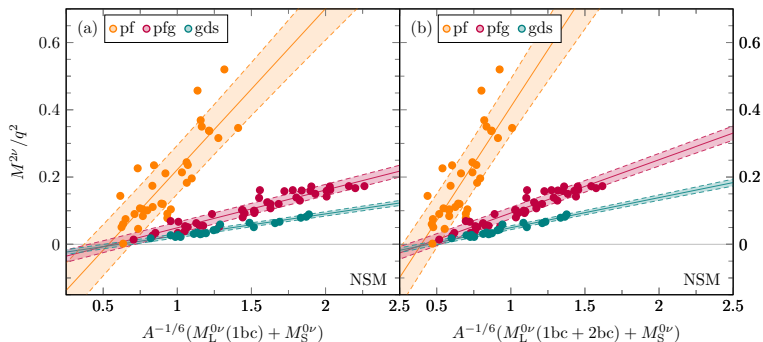
Hoferichter, JM, Schwenk PRD102 074018 (2020)



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

# Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$ : general case

A good correlation between  $2\nu\beta\beta$  and  $0\nu\beta\beta$  also appears when we include to the calculation of  $0\nu\beta\beta$  NMEs 2b currents and the short-range nuclear matrix element



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Use  $2\nu\beta\beta$  data to predict  $0\nu\beta\beta$  NMEs with 2b currents, short-range NME

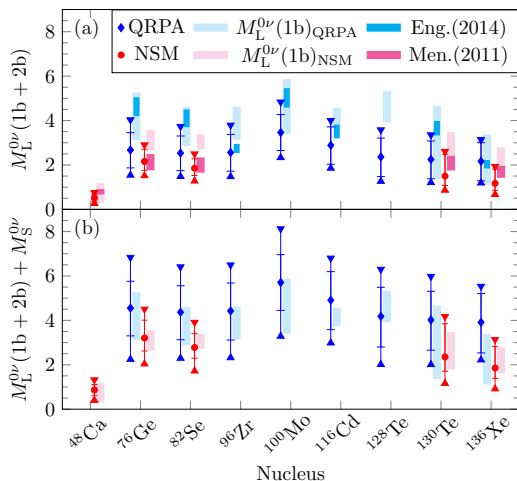
# $0\nu\beta\beta$ NMEs from correlation: 2bc, short-range

$0\nu\beta\beta$  NMEs including 2b currents and short-range NME obtained from  $0\nu\beta\beta - 2\nu\beta\beta$  correlation and  $2\nu\beta\beta$  data

Theoretical uncertainty due to correlation, calculation uncertainties: quenching, 2bc, short-range NME coupling (dominant uncertainty)

First complete estimation of  $0\nu\beta\beta$  nuclear matrix elements with theoretical uncertainties

Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)



# Summary

Calculations of  $0\nu\beta\beta$  NMEs challenge nuclear many-body methods, searches demand reliable NMEs

Ab initio results suggest reduced NMEs due to nuclear correlations (eg via GCF) and two-body currents  
Likely enhancement by short-range NME

Double Gamow-Teller transitions, electromagnetic  $M1M1$  decay of DIAS good correlation with  $0\nu\beta\beta$  NMEs

Good  $0\nu\beta\beta - 2\nu\beta\beta$  correlation exploit  $2\nu\beta\beta$  data to obtain  $0\nu\beta\beta$  NMEs with theoretical uncertainties

