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Theoretical and Astrophysical Constraints to the Zero-Temperature Equation of State

Melissa Mendes

TU Darmstadt

European Research Council

Established by the European Commission

SFB 1245

QCD phase diagram

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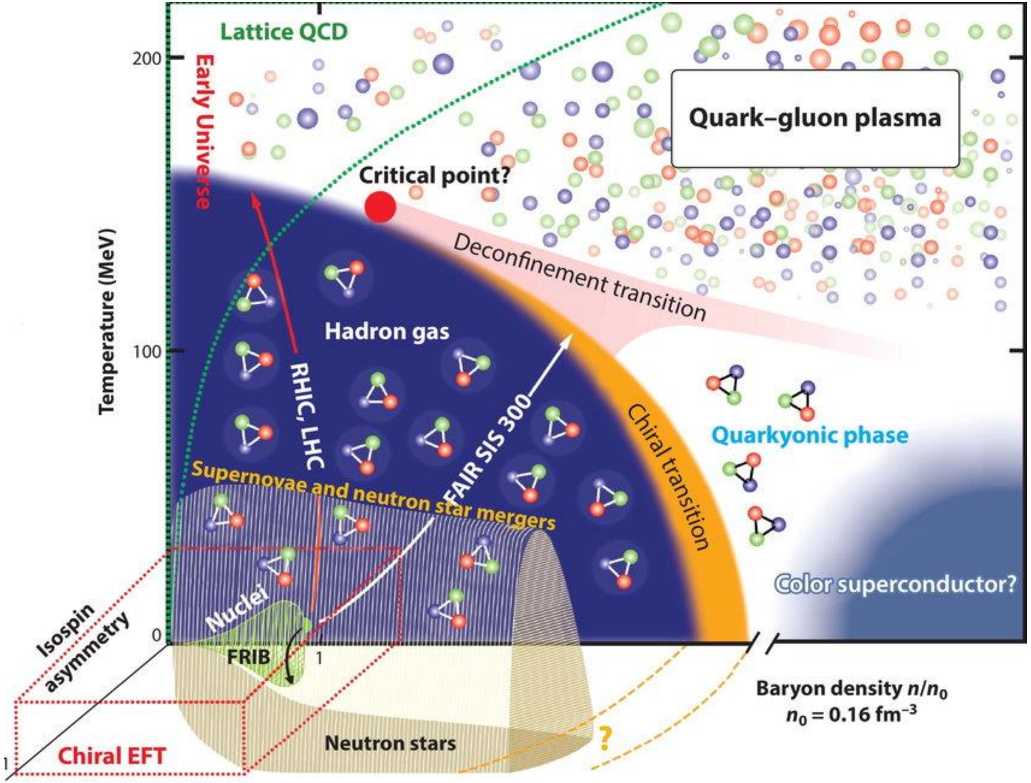

Neutron stars are interesting systems to investigate the nuclear matter equation of state

But observables currently only set constraints to the T=0 EOS

Figure from Drischler et al, Annu. Rev. Nucl. Part. Sci. (2021), arxiv:2101.01709

How can we use $EOS_{T=0}$ constraints to inform $EOS_{T\neq 0}$?

How can we use $EOS_{T=0}$ constraints to inform $EOS_{T\neq0}$?

not a rhetorical question
I need your input

Outline

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- (Very brief) primer on observational techniques
- Recent χEFT and perturbative quantum chromodynamic (pQCD) calculations
- Bayesian framework and EOS construction
- Implications for dense matter EOS and inference of EOS parameters
- More EOS information from luminosity observations

Based on **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

Raaijmakers, …, **MM**, … et al, 2025 (JOSS), https://doi.org/10.21105/joss.06003

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Rutherford, **MM**, Svensson et al, 2024 (ApJL), arxiv: 2407.06790

MM et al, 2024 (PRD), arxiv: 2408.05287

MM et al, 2022 (ApJ), arxiv: 2208.04262

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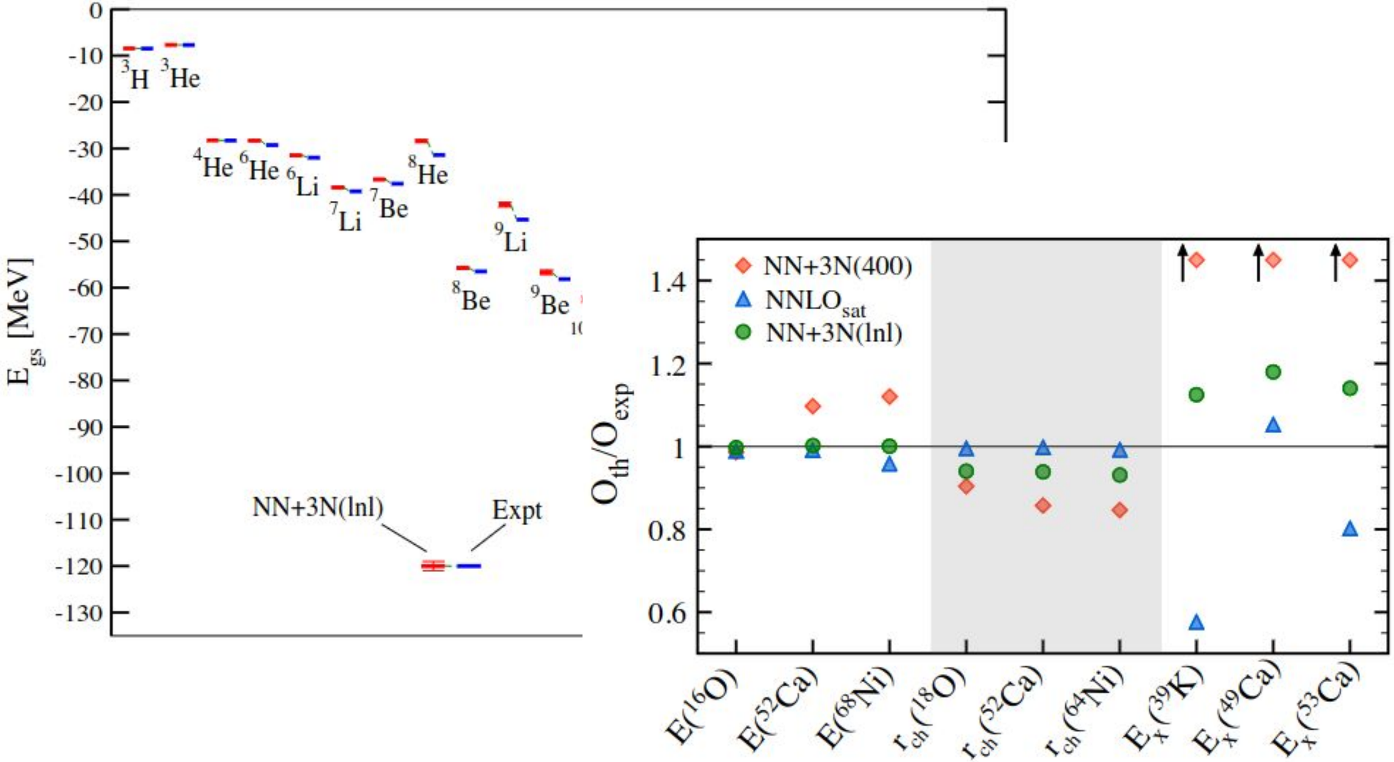
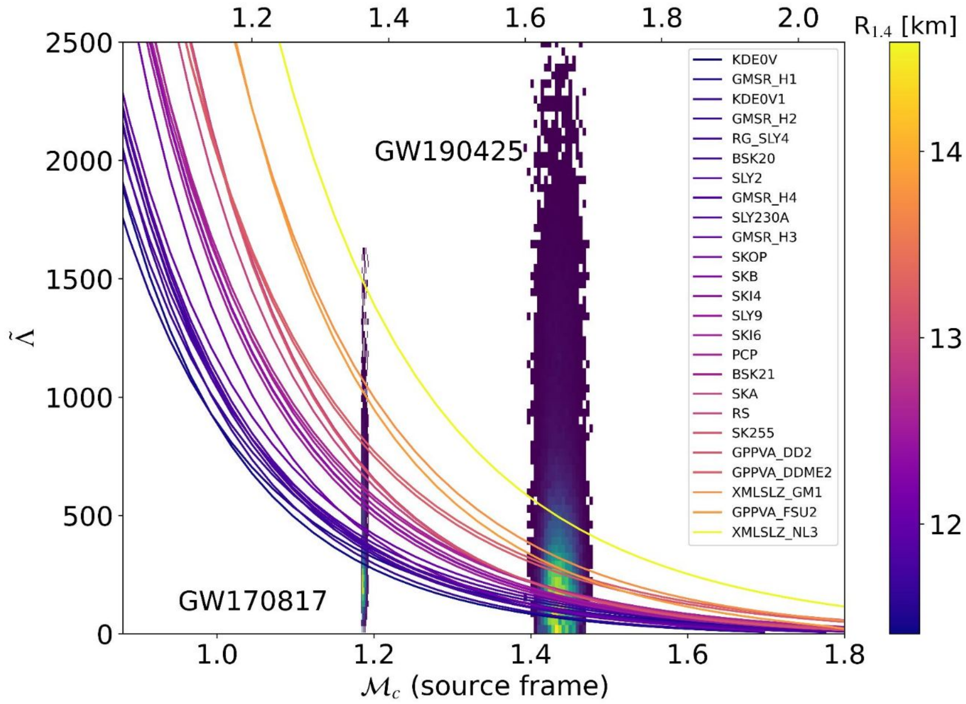
Particle accelerators



Provide us measurements of nuclei properties (for example, scattering rates, charge radii, etc) used to construct and/or check low density EOS models

Figures from Somà et al, PRC (2020), arxiv:1907.09790

BNS Gravitational Waves



GW170817 much more constraining

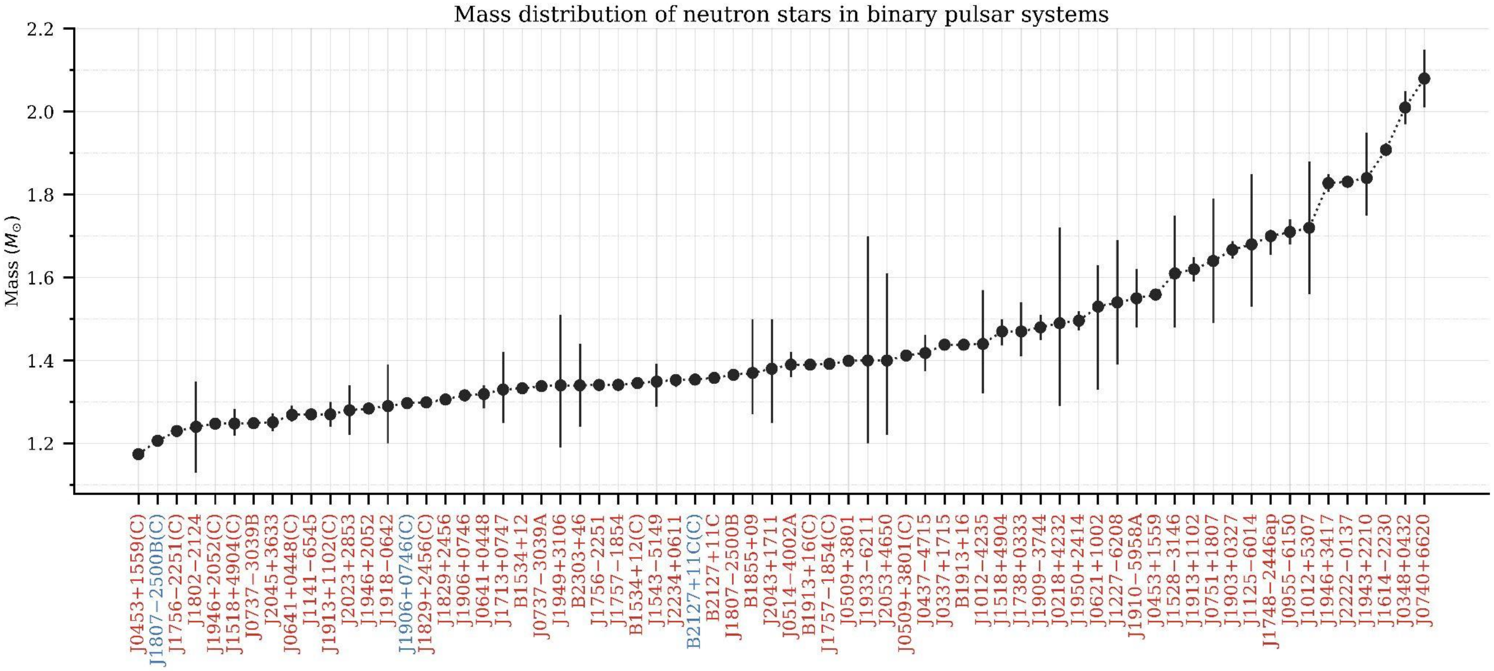
$$\tilde{\Lambda} = 300^{+420}_{-230}$$

GW190425

$$\tilde{\Lambda} \leq 600$$

Figure from J. Read, L. Suleiman's slides on behalf of LVK collaboration at Nuclei in the Cosmos 2025 - https://indico.icc.ub.edu/event/341

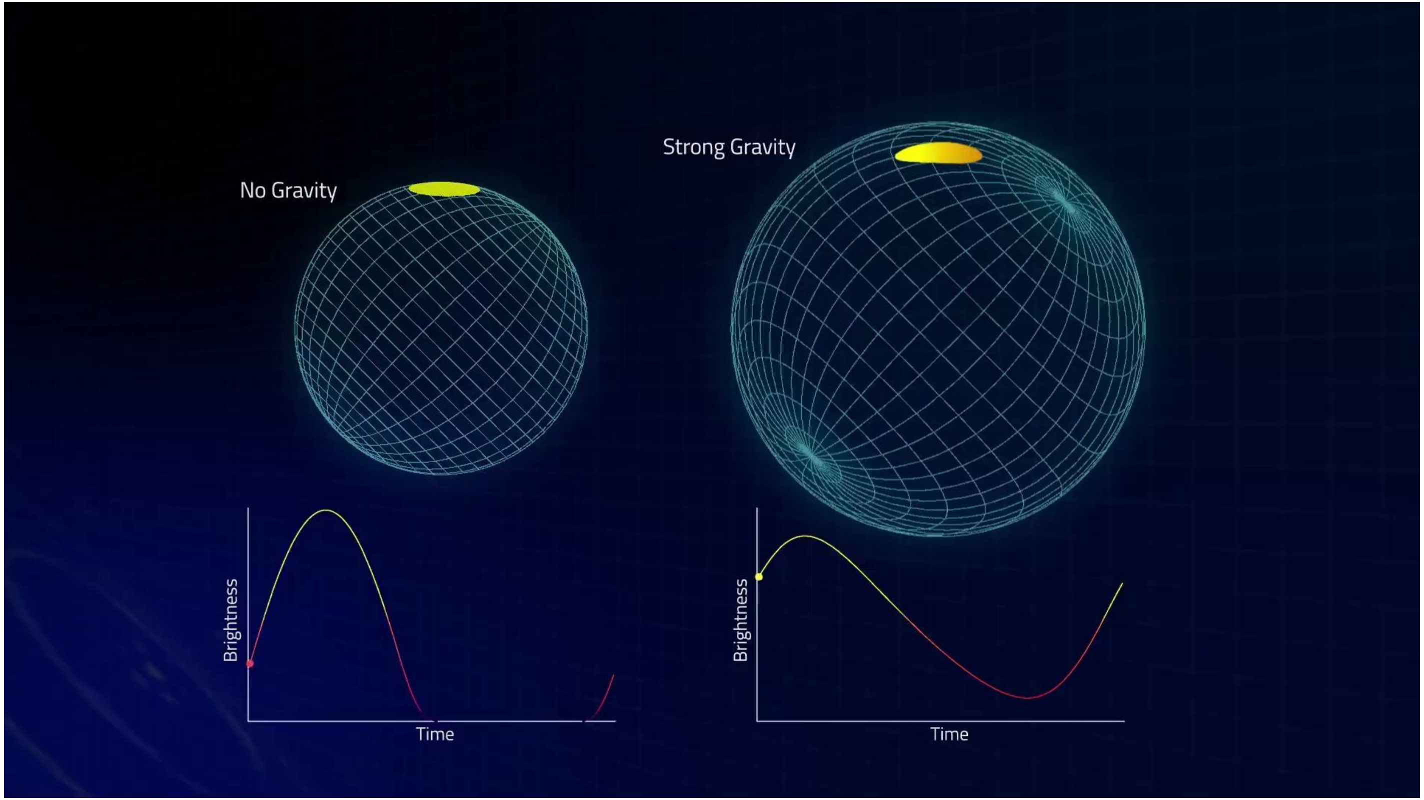
Chandra and XMM



Have been fundamental in measuring NS masses, especially of rotation-powered pulsars

Figure from https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Pulse profile modeling - NICER



Proportional effect to the stars' compactness (M/R)

Simultaneous constraint on neutron star mass and radius

But non-trivial hot-spot modelling

Animation from https://svs.gsfc.nasa.gov/vis/a020000/a020200/a020268/Lensing.1080.mp4

Recent data from NICER

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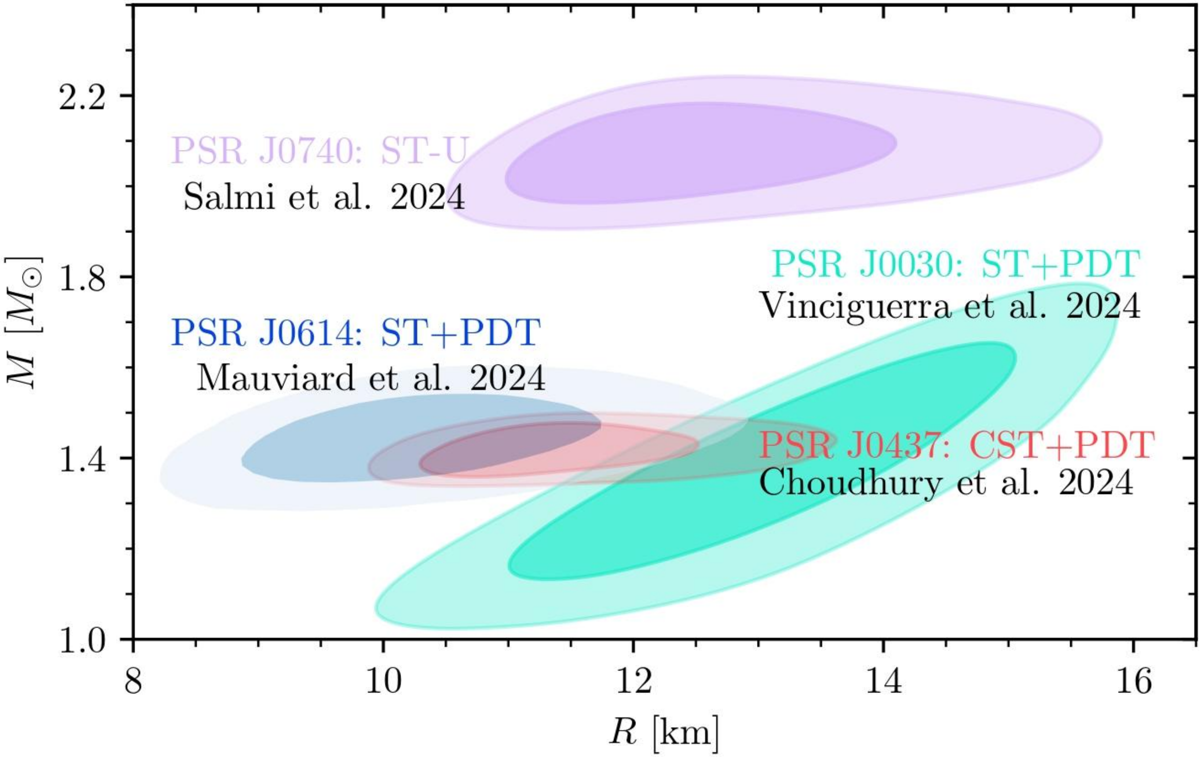

Already including mass constraints from radio observations and background constraints (when available)

Modified figure from Mauviard, …, **MM**, … et al, ApJ (2025), arxiv: 2506.14883

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MM et al, 2024 (PRD), arxiv: 2408.05287

MM et al, 2022 (ApJ), arxiv: 2208.04262

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Recent χEFT EOS implementation

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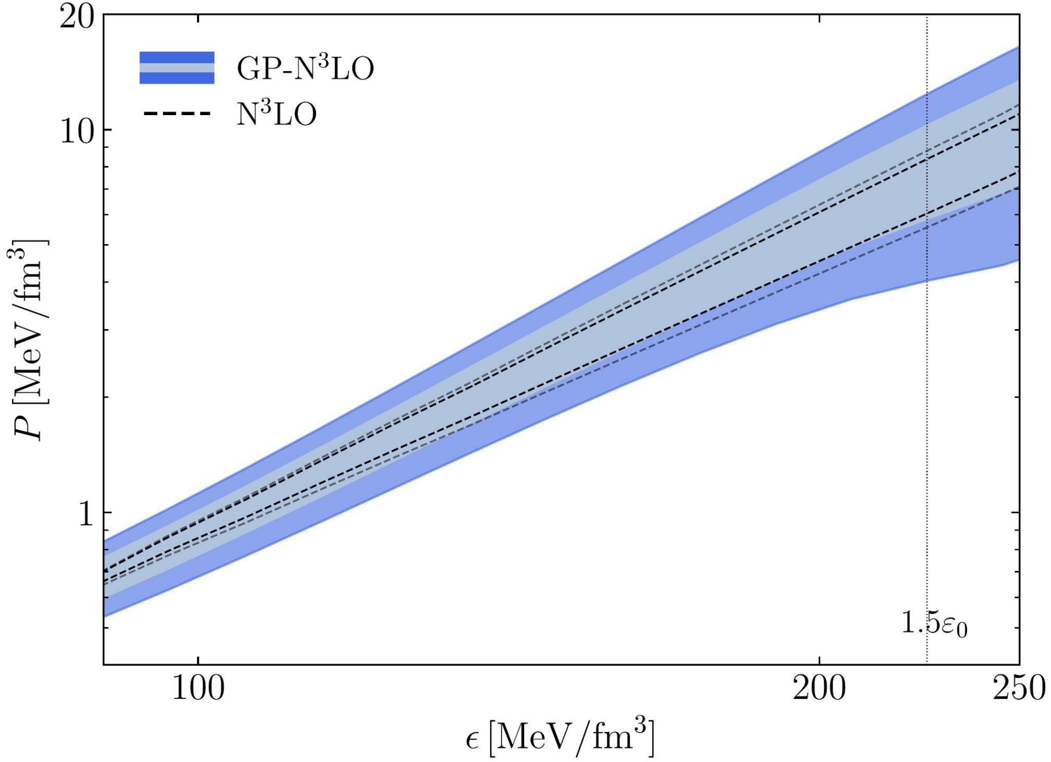

Broader pressure range covered with χEFT EOS with uncertainty quantification (GP-N^{3}LO)

Modified figure from **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

pQCD implementation

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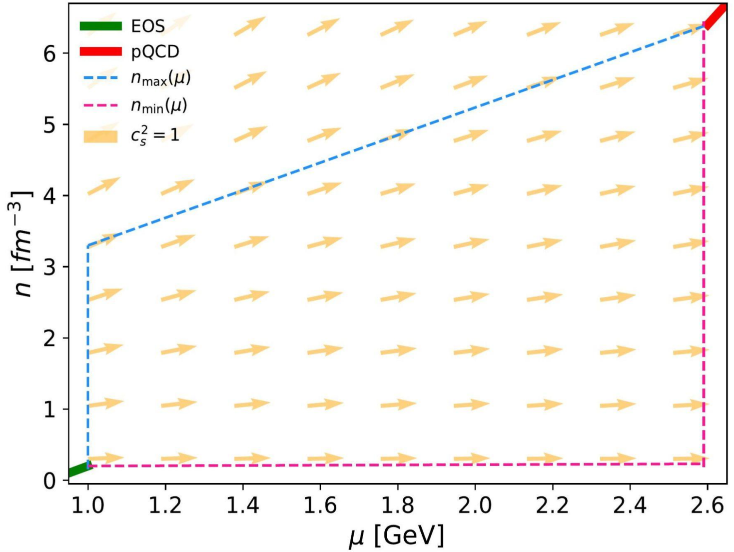
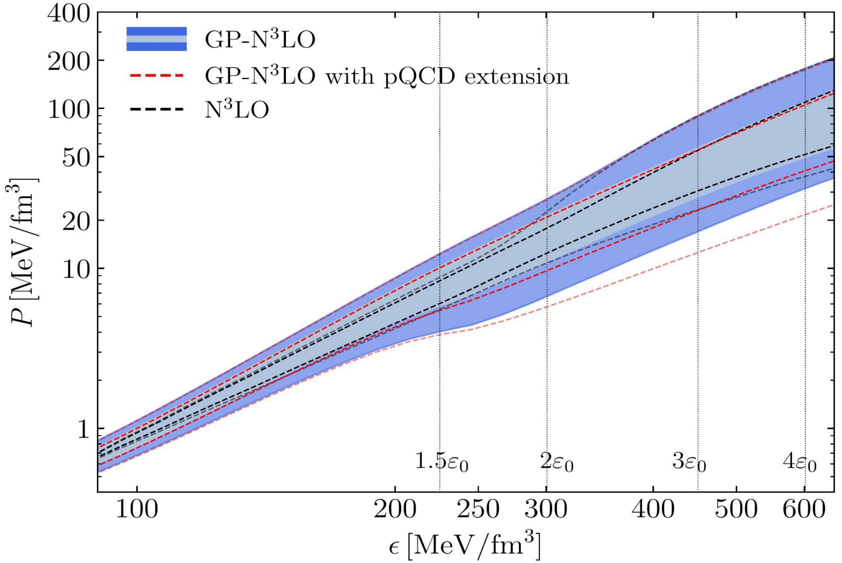
Extension procedure built by A. Hensel



Modified figure from Komoltsev and Kurkela, PRL (2022), arxiv:2111.05350

EOS extensions built to respect pQCD constraints

Further expands prior space, especially at high densities



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Bayesian framework

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Following previous works, (Raaijmakers et al, 2020; 2019), posterior distributions of all EOS parameters (θ) and central energy density (ε):

$$\overbrace{p(\theta, \varepsilon \mid d, \mathbb{M})}^{\textbf{Posterior}} \propto \overbrace{p(\theta \mid \mathbb{M})\; p(\varepsilon \mid \theta, \mathbb{M})}^{\textbf{Prior}} \times \prod_i p(\Lambda_{1,i}, \Lambda_{2,i}, q_i \mid \mathcal{M}_c, d_{\mathrm{GW},i}) \times \times \prod_l p_{\mathrm{new}}(M_l, R_l \mid d_{\mathrm{NICER(+radio)},l}),$$

with radio mass measurements included through NICER M-R likelihoods when available

See NEoST: **https://xpsi-group.github.io/neost/overview.html**

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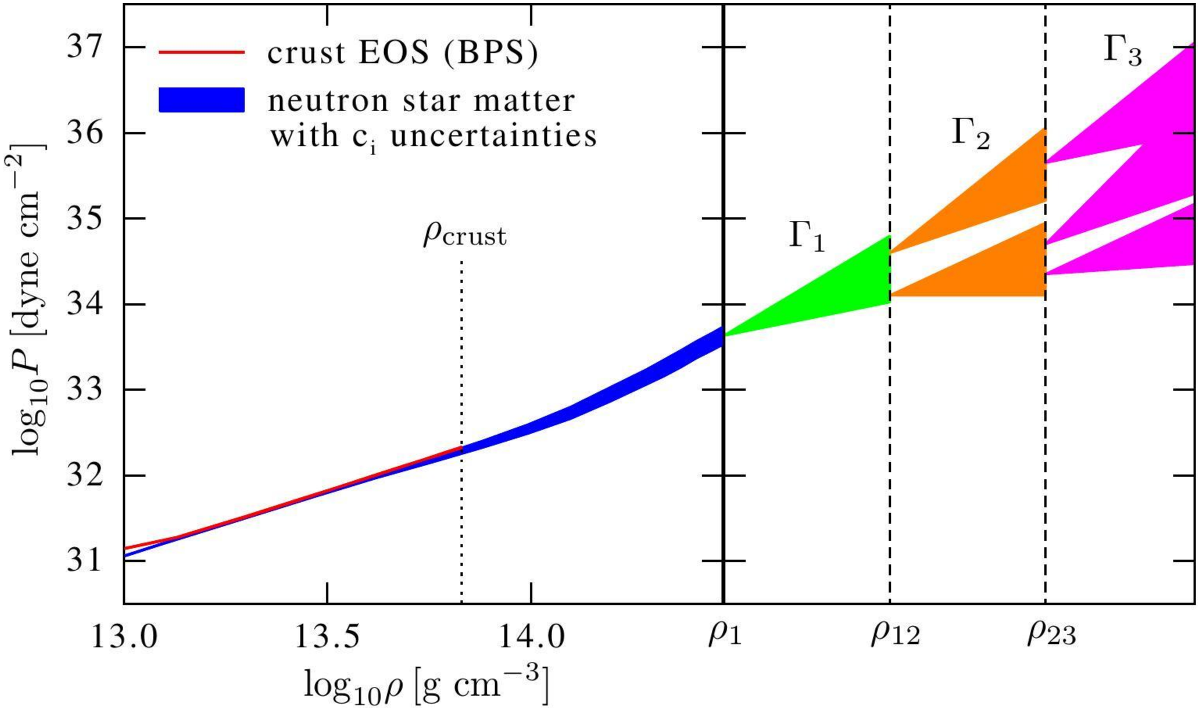
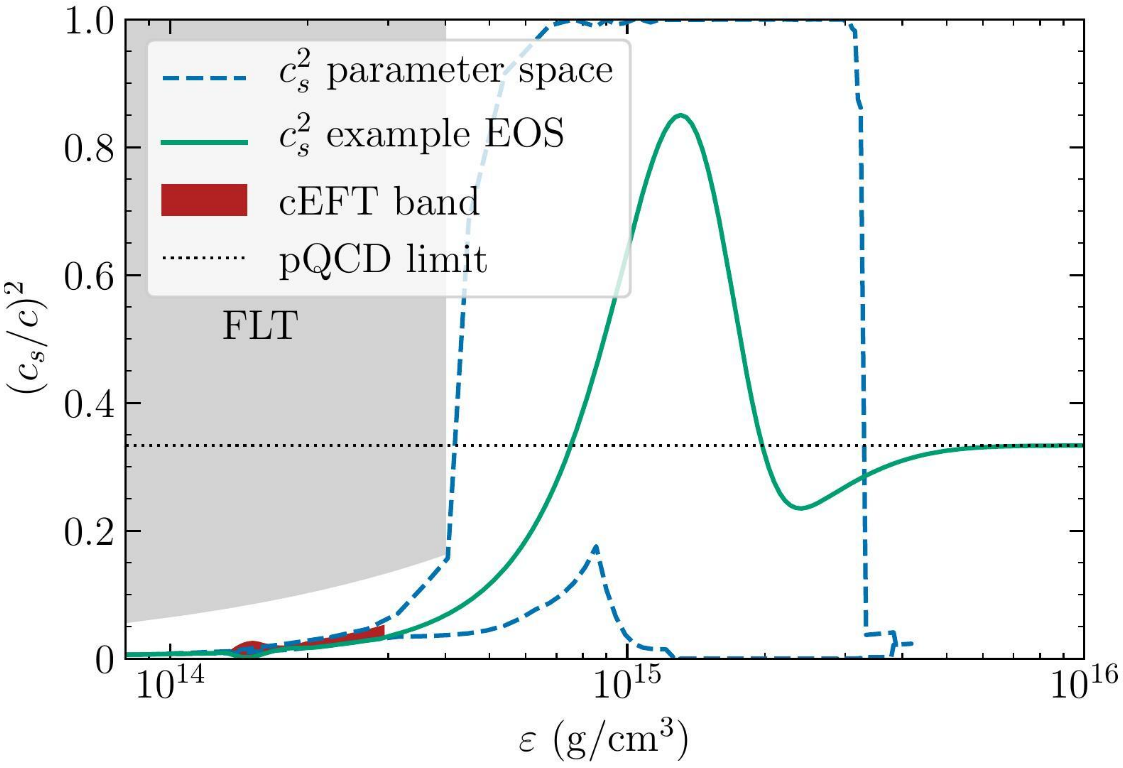
Piecewise polytropic parametrization (PP)



3 independent polytropes, respecting causality, following Read et al, PRD (2009), arxiv: 0812.2163

Figure from Hebeler et al, ApJ (2013) arxiv:1303.4662

Speed of sound parametrization (CS)



Analytical expression to speed of sound, limited by

- Fermi liquid theory
- causality
- $\lim_{n \geq 50\, n0} c_s^2 \to 1/3$

From Greif et al, MNRAS (2019)
arxiv: 1812.08188

Semiparametric with GP

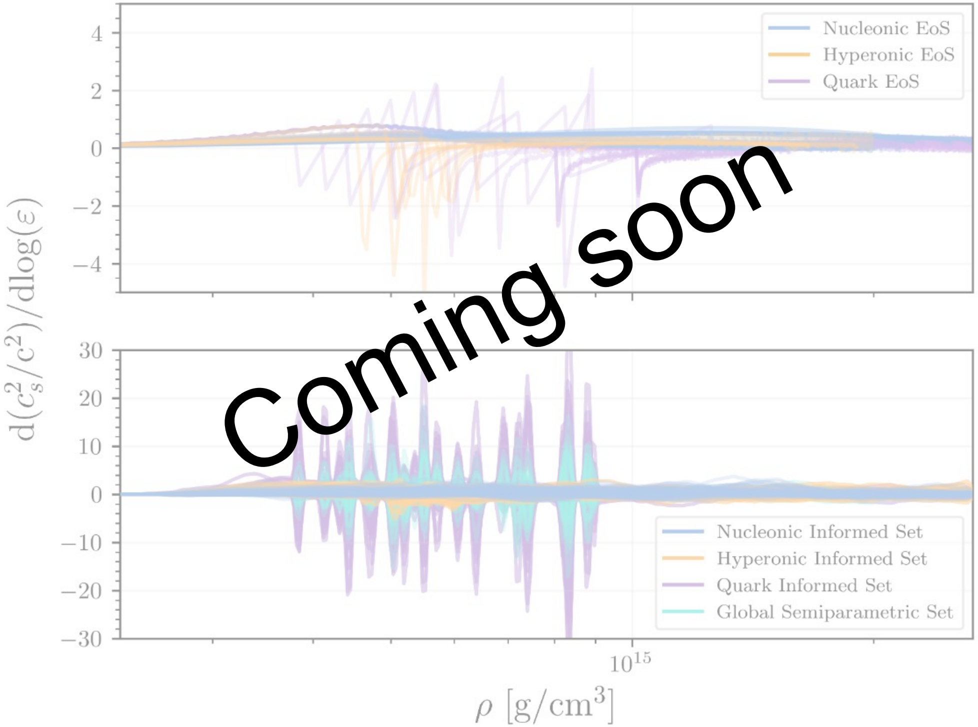
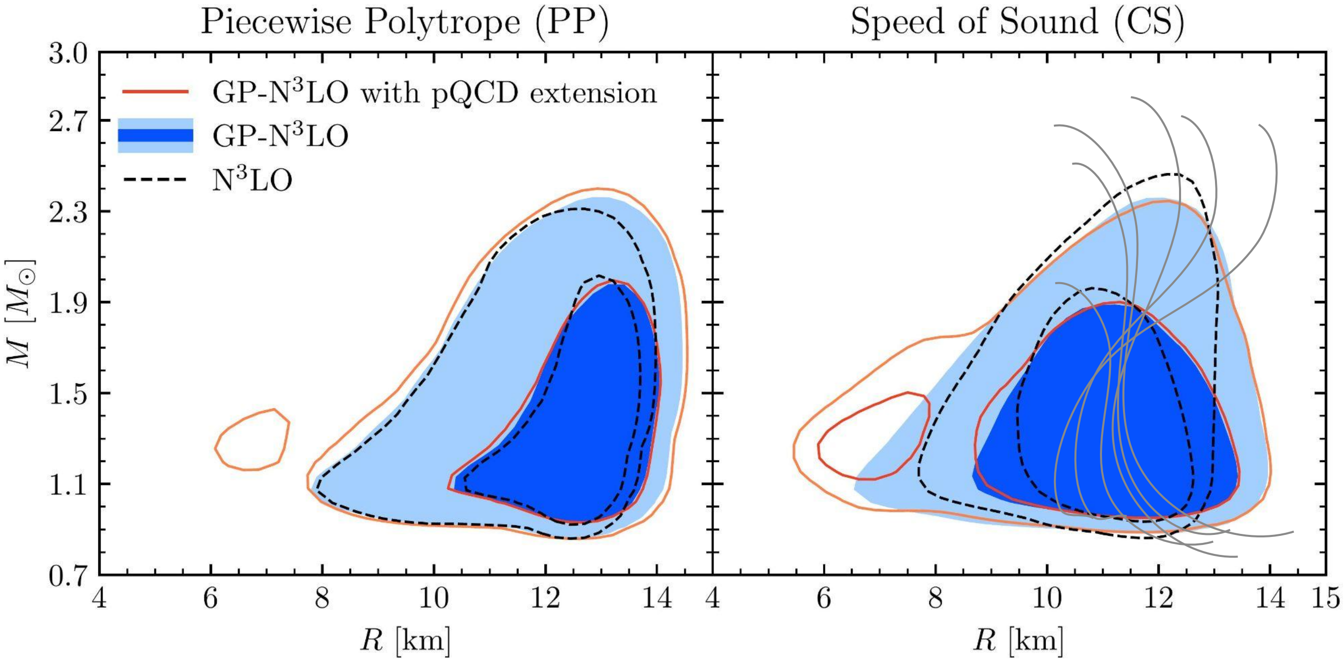


Figure from Sunny et al, Class.Quant.Grav. (2025), arxiv: 2507.032032

Mass-Radius (MR) priors



Radii around 10-14 km

Both the new treatment of the cEFT EOS and pQCD extensions expand MR priors

Modified figure from **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

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MM et al, 2024 (PRD), arxiv: 2408.05287

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With pQCD constraints

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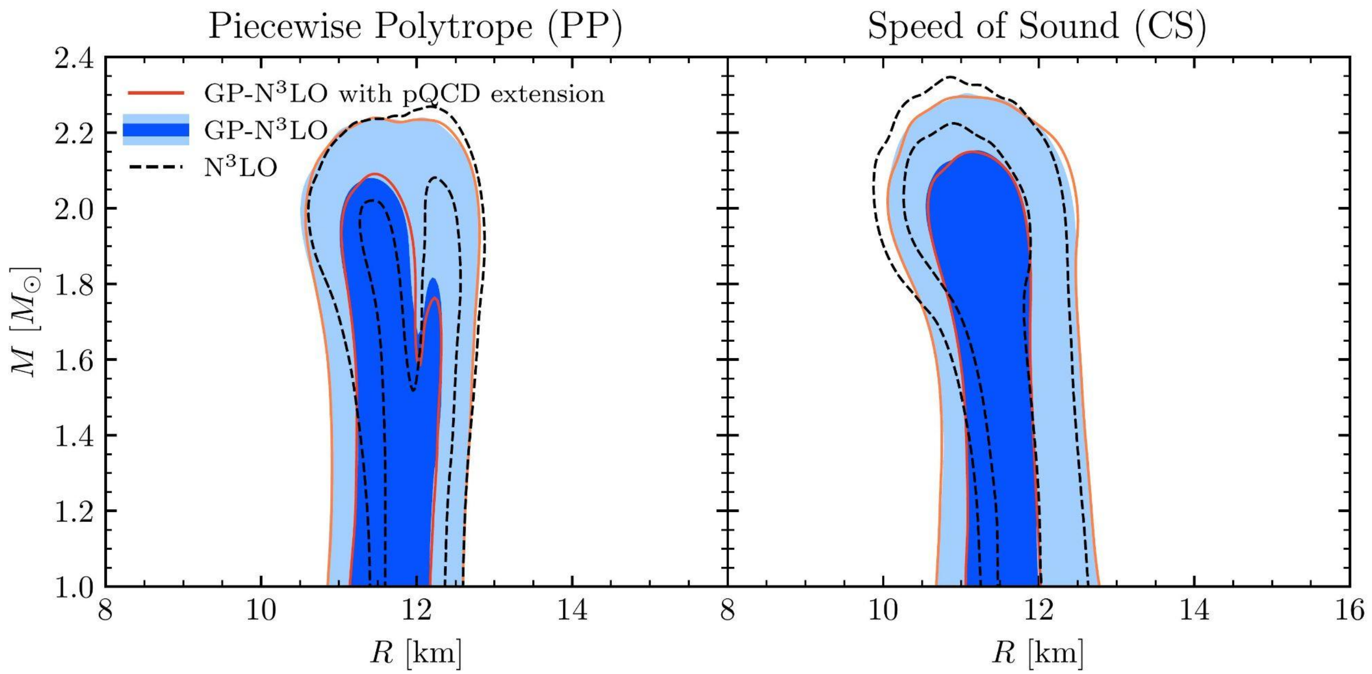
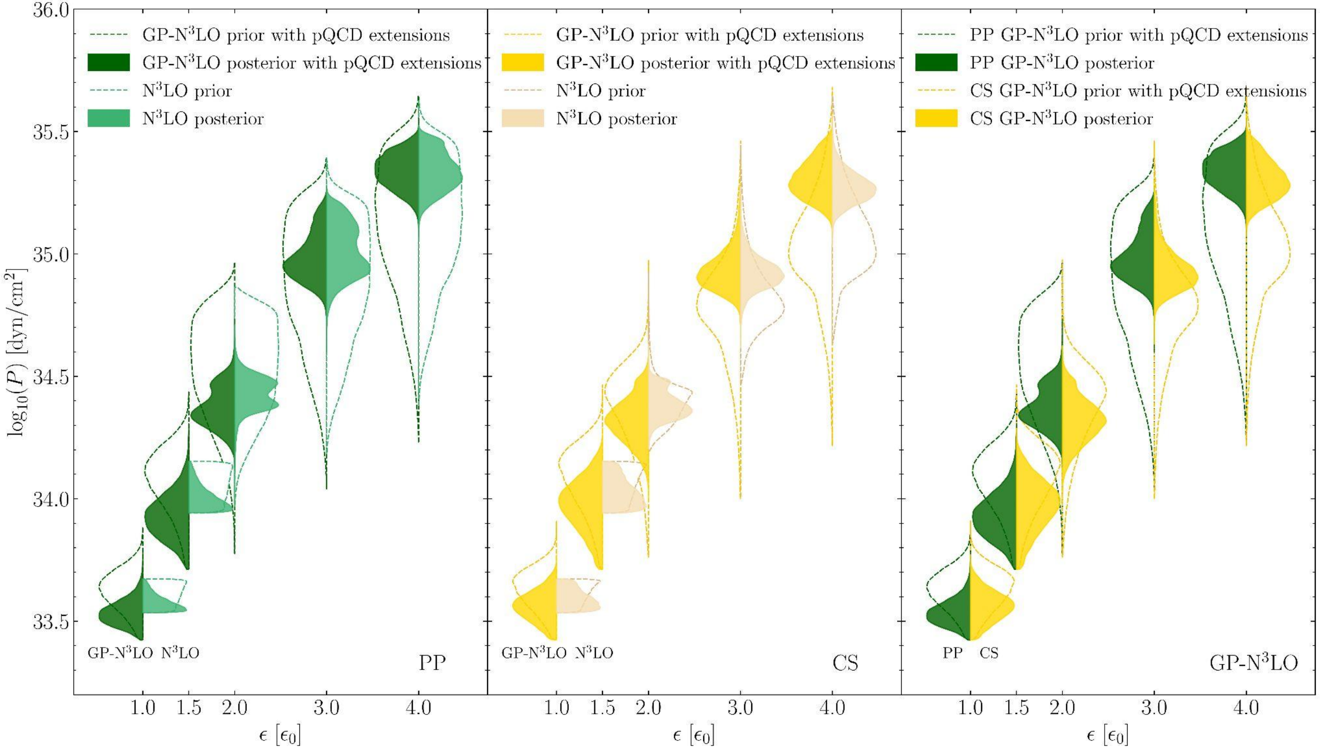


New cEFT implementation broadens the radius, especially for low mass stars

Minor differences to the posterior without pQCD constraints

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

PxE



Broadening of pressure posterior beyond cEFT range

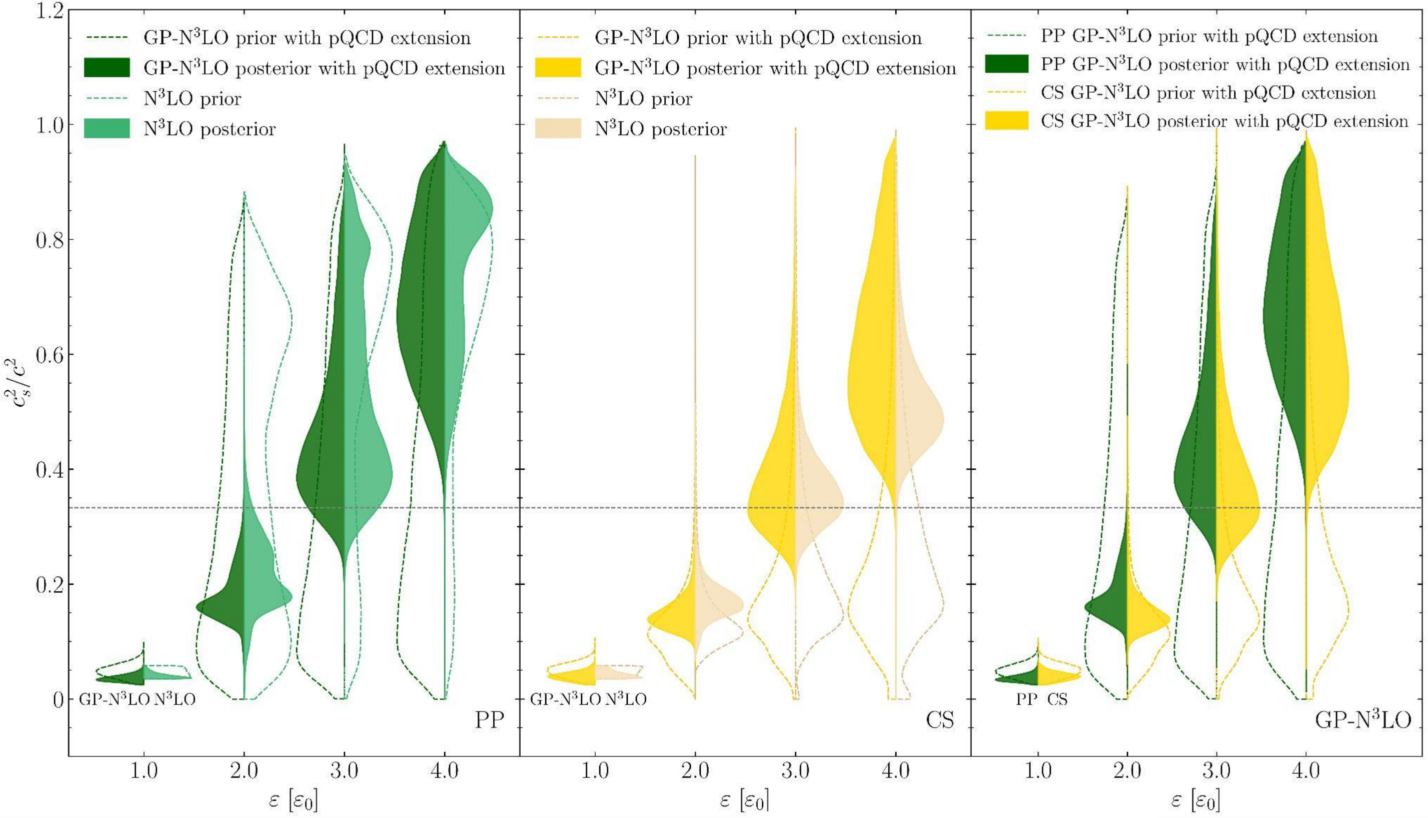
Minimal impact of pQCD extension

Consistent results for PP and CS parameterizations

Bimodal-like tendency for all posteriors

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

c_s^2 x E



Potentially useful for investigations of phase transitions

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

EOS parametrization

$$E(n,\delta) = E_{\mathrm{SNM}}(n) + E_{\mathrm{sym}}(n)\cdot\delta^2 + \mathcal{O}\left(\delta^4\right), \;\; \delta = (n_{\mathrm{n}} - n_{\mathrm{p}})/n$$

with symmetry energy:

$$E_{\mathrm{sym}}(n) = J + L\left(\frac{n-n_0}{3n_0}\right) + \frac{1}{2}K_{\mathrm{sym}}\left(\frac{n-n_0}{3n_0}\right)^2 + \cdots$$

such that $L = 3P_{PNM}/n_0$

We explore the correlation between β-equilibrated matter and pure neutron matter to infer L

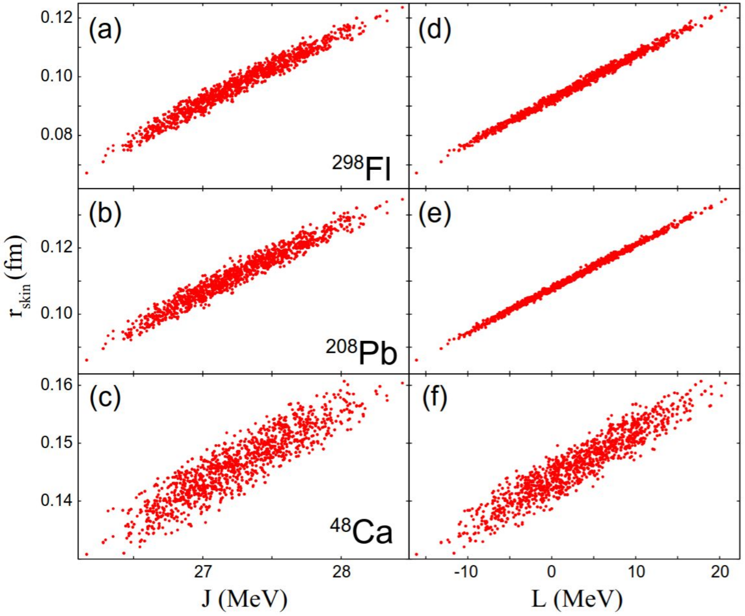


Figure from Reinhard and Nazarewicz, PRC (2016), arxiv:1601.06324

L parameter inference

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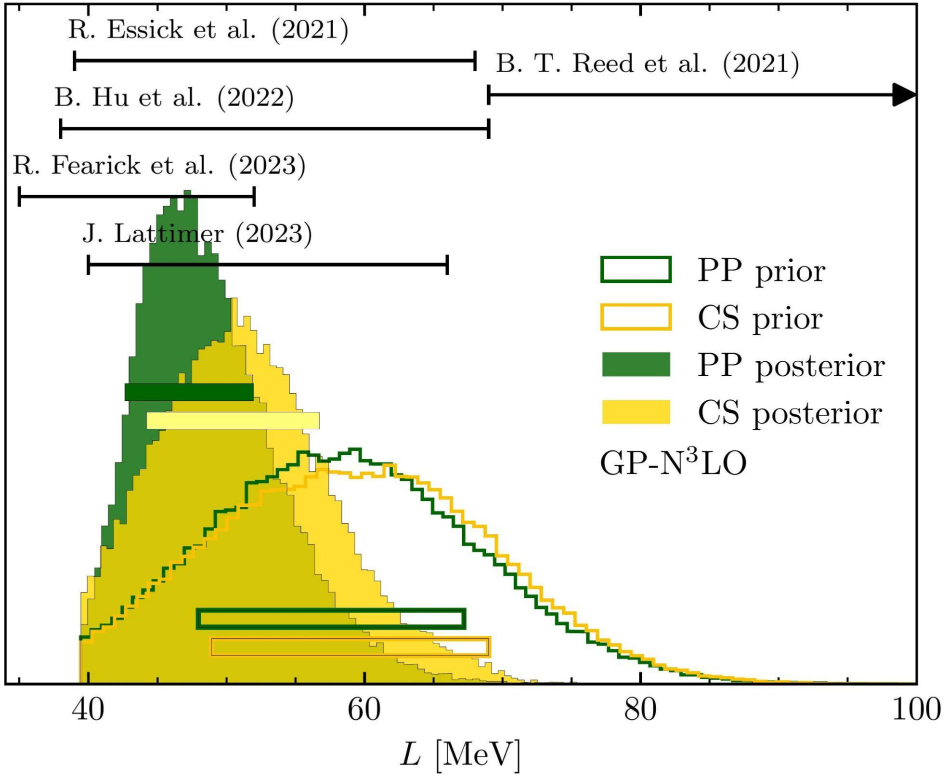

Astrophysical data informs the value of L, the slope of symmetry energy

$L_{68\%\ \textbf{Prior}\ \mathrm{PP}}$ = 48.0 - 67.2 MeV

$L_{68\%\ \mathrm{PP}}$ = 42.6 - 52.0 MeV

$L_{68\%\ \textbf{Prior}\ \mathrm{CS}}$ = 48.9 - 69.0 MeV

$L_{68\%\ \mathrm{CS}}$ = 44.2 - 56.7 MeV

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MM et al, 2024 (PRD), arxiv: 2408.05287

MM et al, 2022 (ApJ), arxiv: 2208.04262

NEoST

Investigating EOS with NS luminosities

See also Jain et al, PRL (2025) and Ong et al, PRC (2024)

Isolated neutron stars

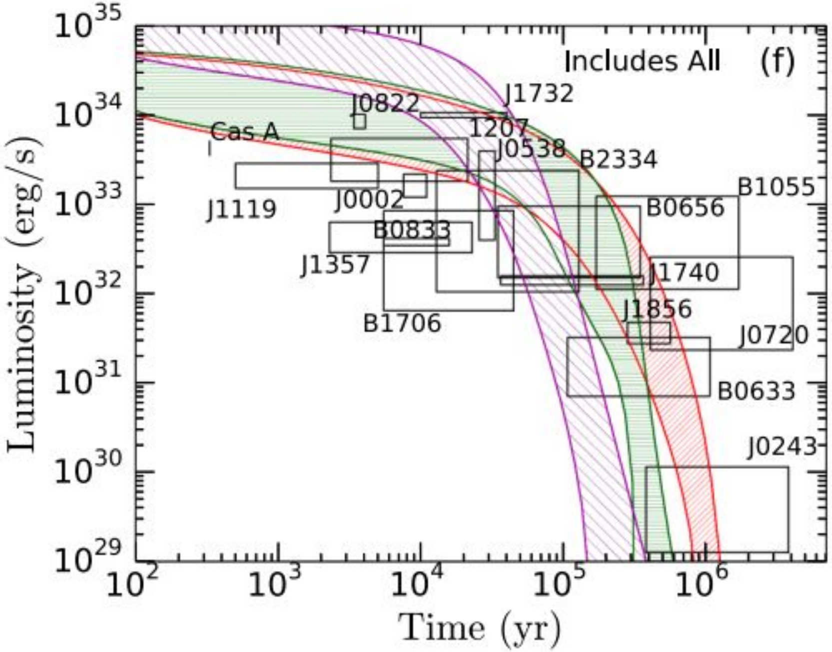

Figure from Beloin et al, 2018 (PRC), arxiv:1612.04289

Transiently-accreting neutron stars

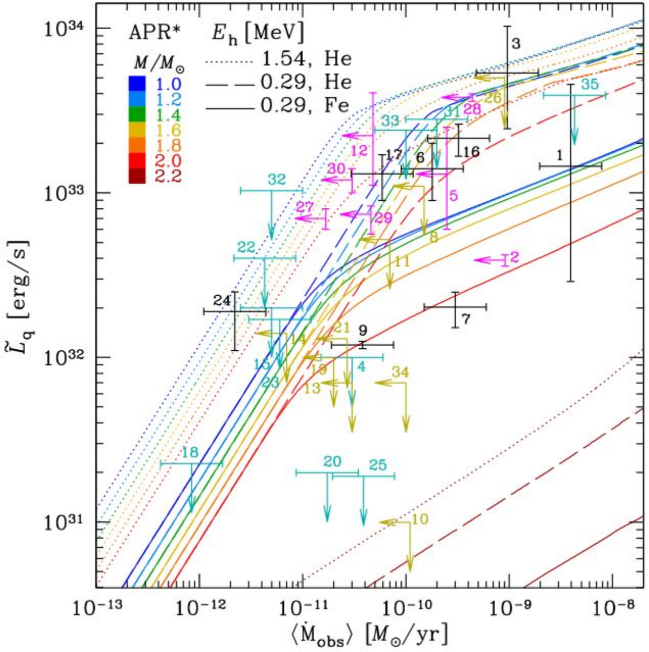

Figure from Potekhin et al, 2023 (MNRAS), arxiv:2303.08716

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NS cooling primer

Neutron stars cool by the emission of photons and neutrinos, in reactions like β-decay and e- capture:

$$n \rightarrow p + l + \bar{\nu}_l, \quad p + l \rightarrow n + \nu_l$$

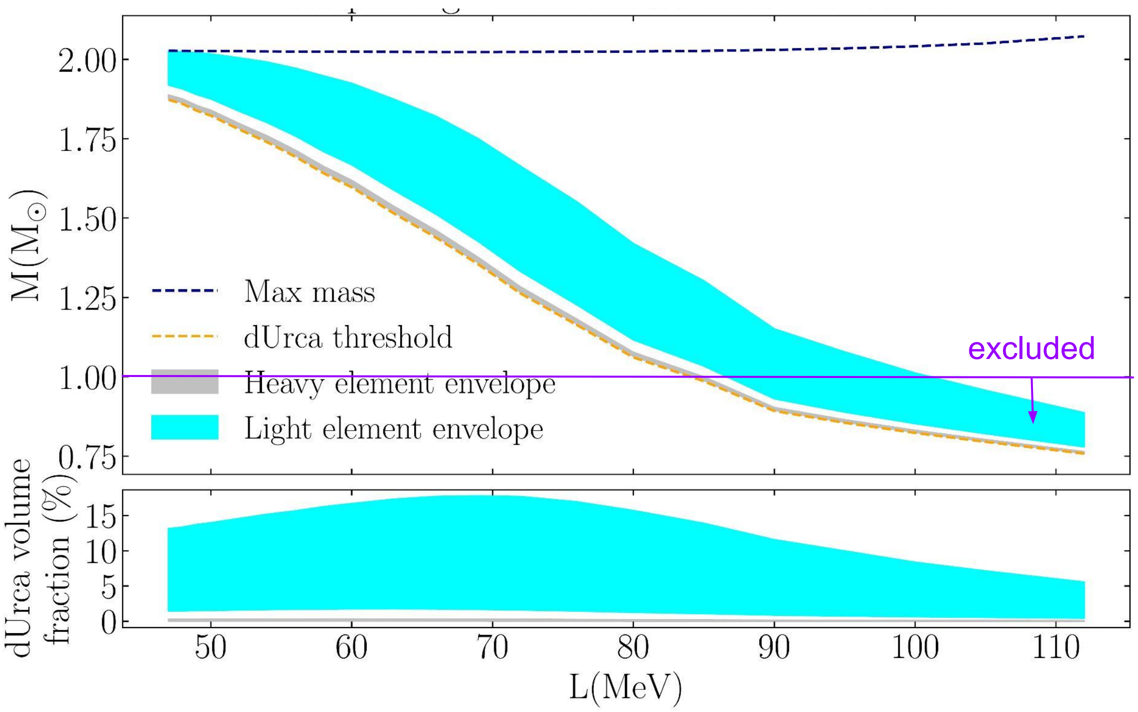
which only happen when energy and momentum are conserved, that is,

$$P_{Fn} \leq P_{Fp} + P_{Fe} \Rightarrow \quad Y_p \geq \left[(Y_n)^{1/3} - (Y_e)^{1/3} \right]^3$$

direct Urca (dUrca) threshold

Overall, luminosity data constrains microphysical EOS ingredients (Y_p, effective nuclear masses, etc) through thresholds and emissivities

For example



NS luminosity data and cooling simulations can eliminate EOS as viable candidates

but results are very model-dependent

Figure from **MM** et al, 2022 (ApJ), arxiv: 2208.04262

Summary

- ❑ **Astrophysical data significantly constrains the EOS in high density, low temperature region** (especially if combined: mass, radius, tidal, luminosities…)

- ❑ **Ab initio calculation updates can easily be integrated into EOS inference frameworks, including uncertainty quantification of cEFT EOS**

- ❑ **Inference of EOS parameters, such as L, statistically meaningful and astro informed**

- ❑ **Amount and precision of observations only increases! More astrophysical and laboratory data expected soon**

Special thanks to A. Hensel, H. Göttling, I. Svensson, K. Hebeler, A. Schwenk

Thank you!

melissa.mendes@tu-darmstadt.de

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Additional slides

NICER telescope primer

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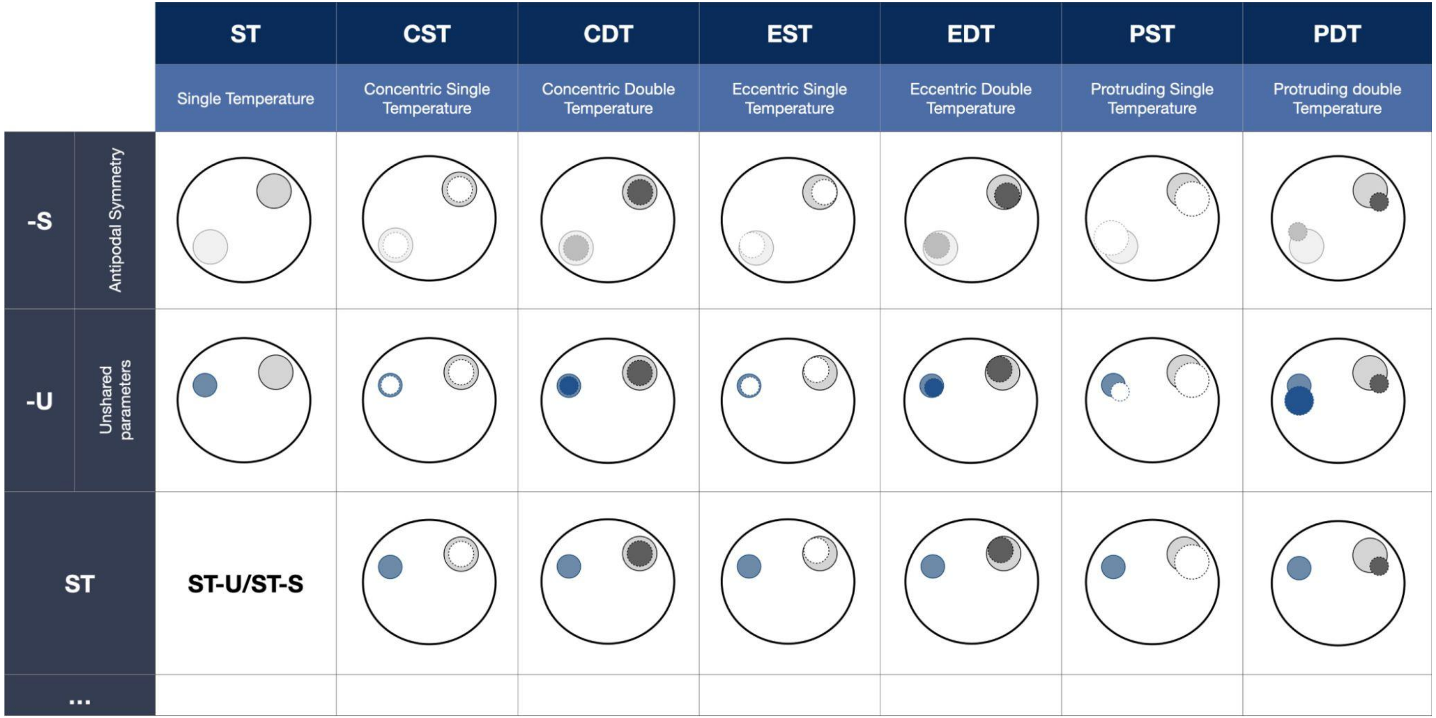
Neutron star Interior Composition ExploreR (NICER)

- X-ray telescope, aboard the International Space Station since 2017
- Needs rotating sources with f > 100 Hz
- Inferences depend on model for the surface emission, with free parameters:
    - mass, radius, spin rate, temperature distribution, beaming function or atmosphere model, observer inclination, distance, interstellar hydrogen column density and a background model

Targets:

- 5-10% precision level for PSR J0437-4715, PSR J0030+0451, PSR J1231+1411
- Plus PSR J0740+6620, PSR J0614-3329, PSR J1614-2230

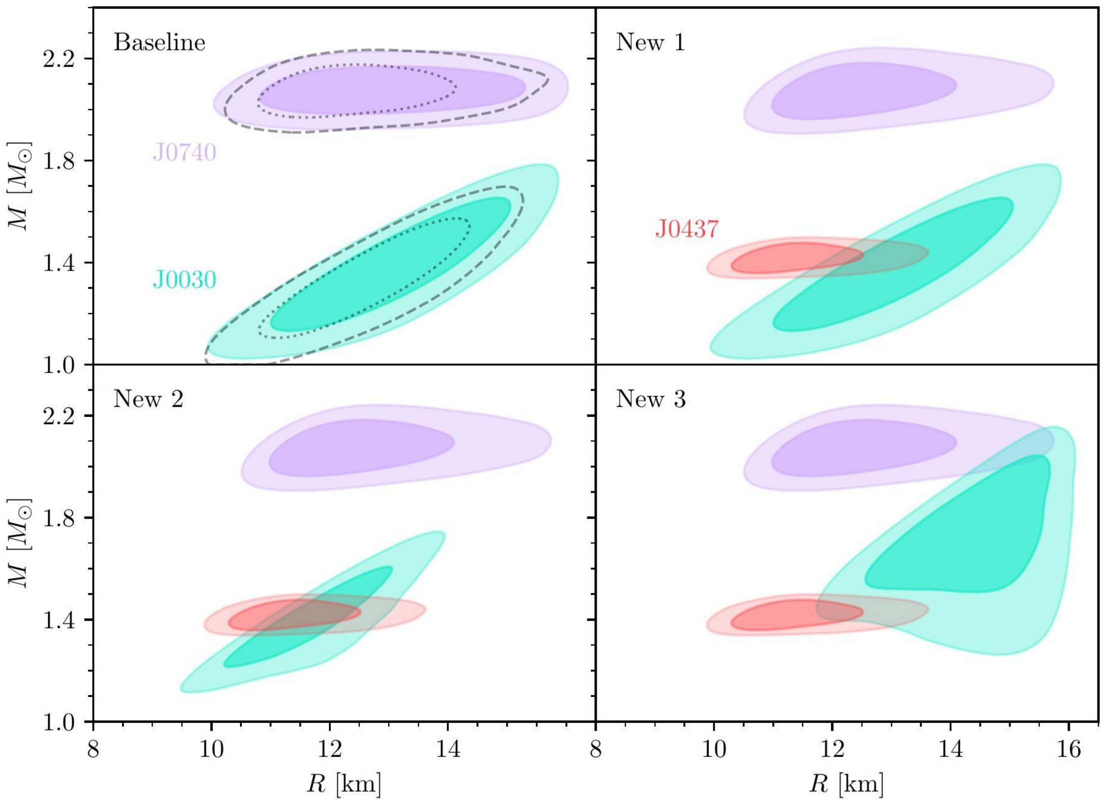
Non-trivial hot-spot modeling



Large number of parameters needed to describe hot spots

Figure from Vinciguerra et al, APJ (2023), arxiv:2308.08409

Rutherford et al NICER data

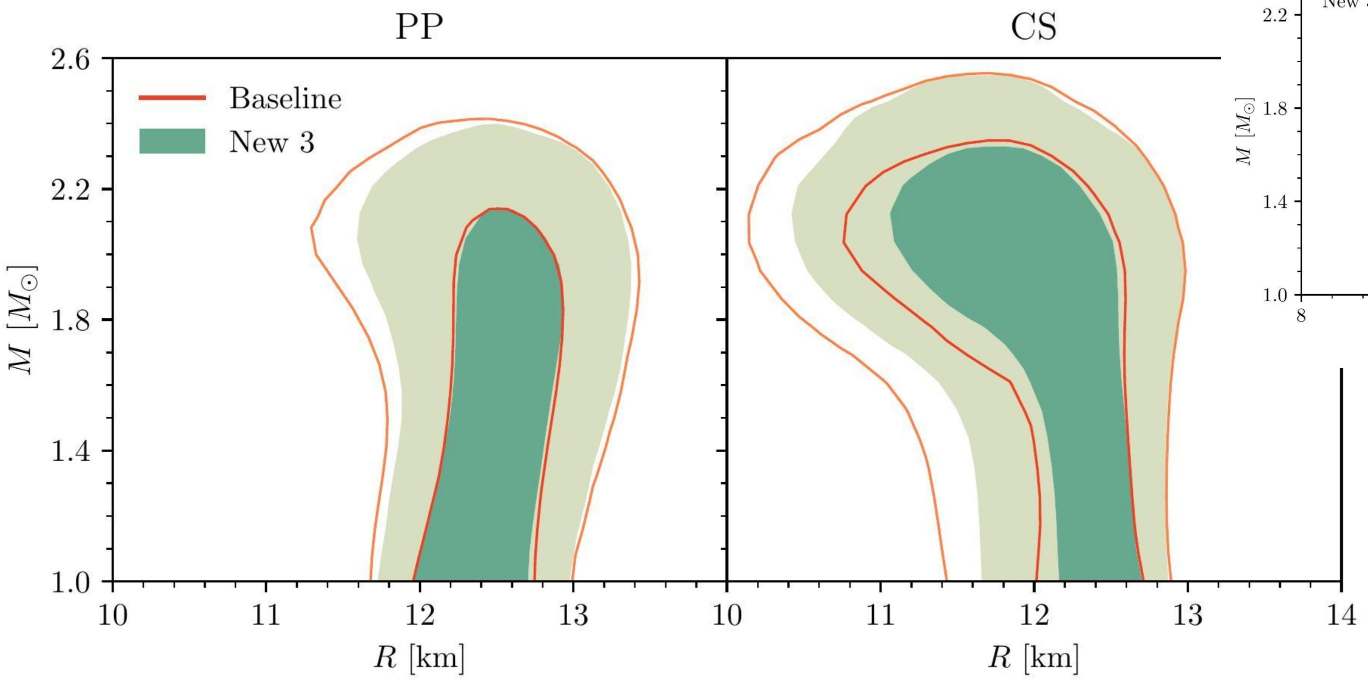


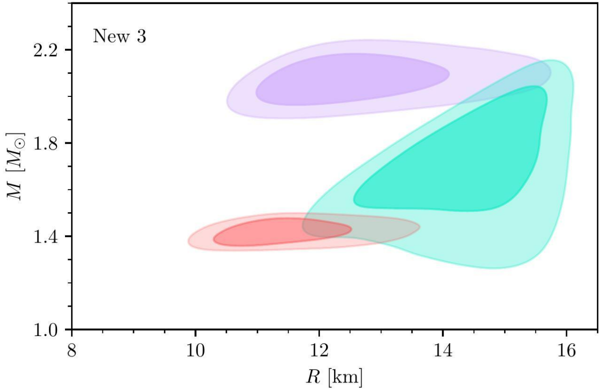
Some possible MR data contours depending on hot spot modelling

Plus GW170817 and GW190425

From Rutherford, **MM**, Svensson et al, ApJL (2024), arxiv:2407.06790

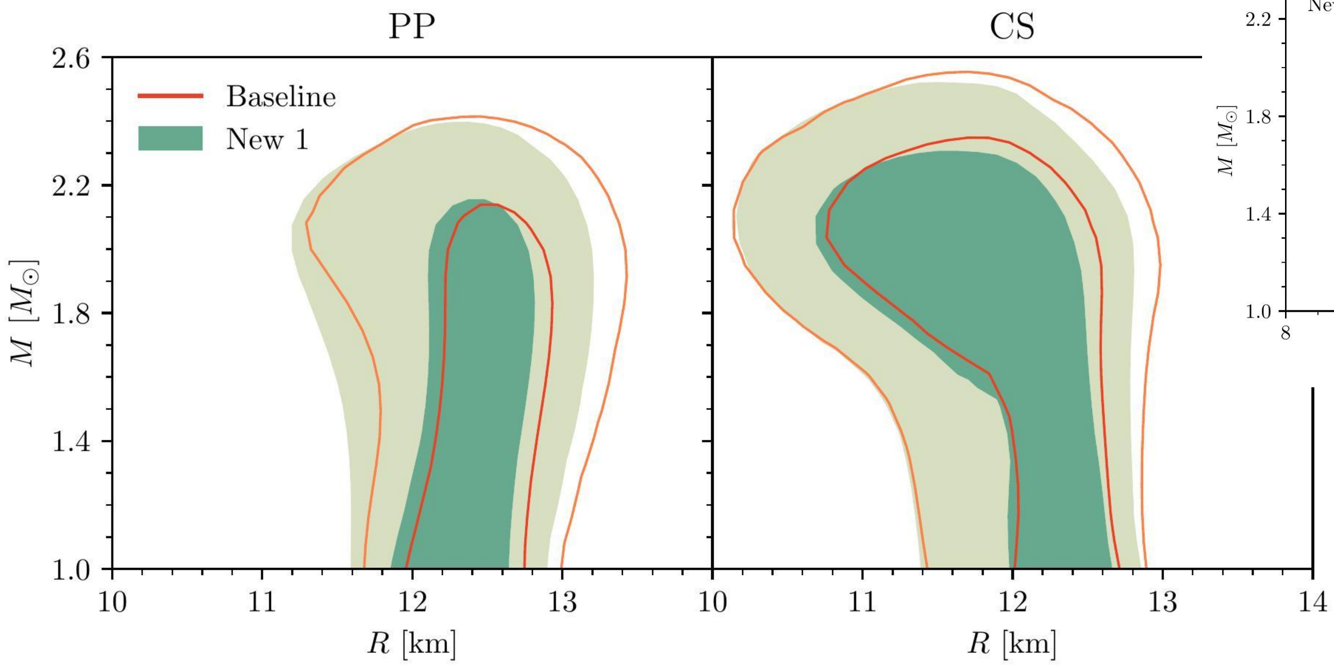
New 3 contours

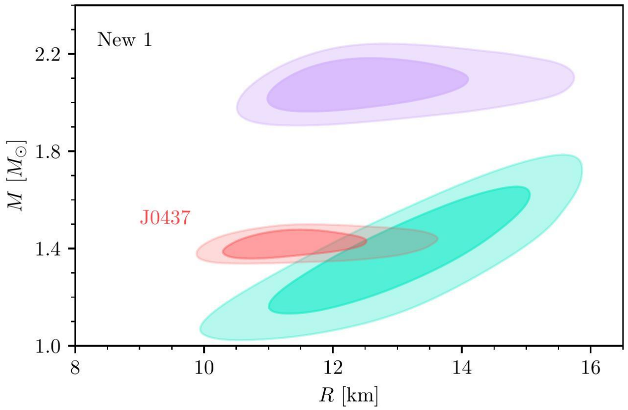




From Rutherford, **MM**, Svensson et al, ApJL (2024) arxiv:2407.06790

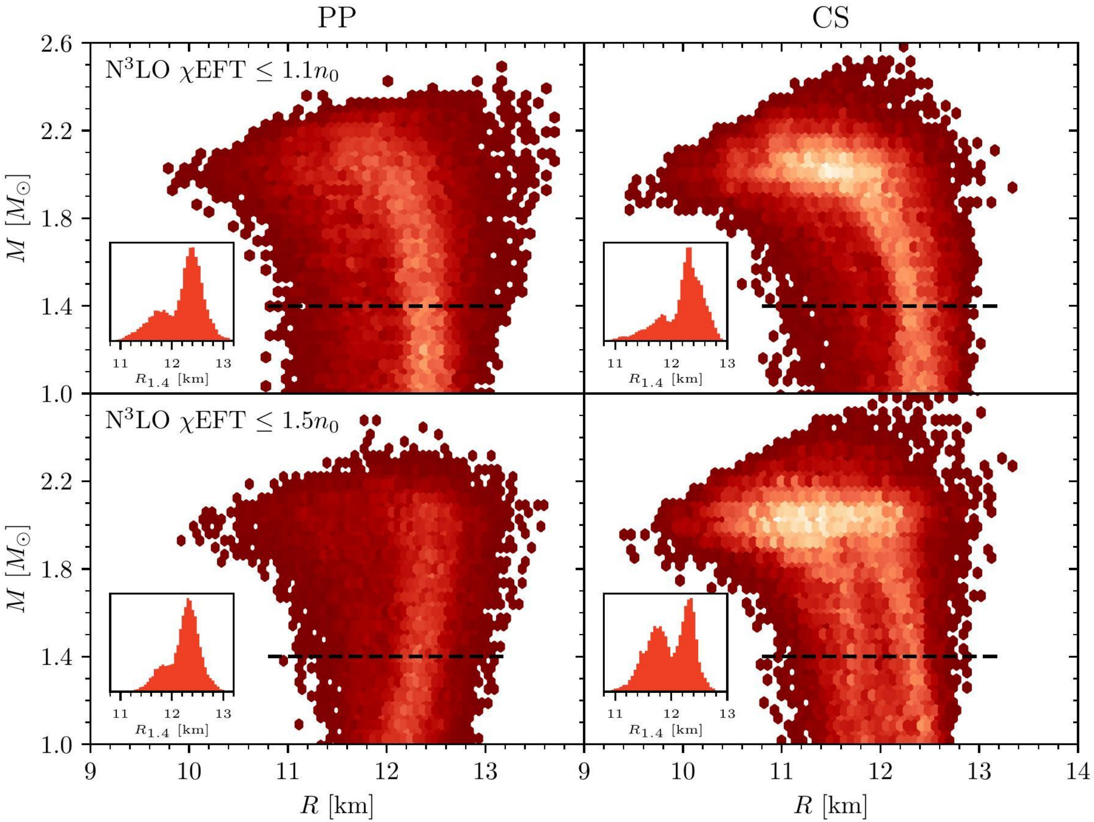
Other hot spot scenarios

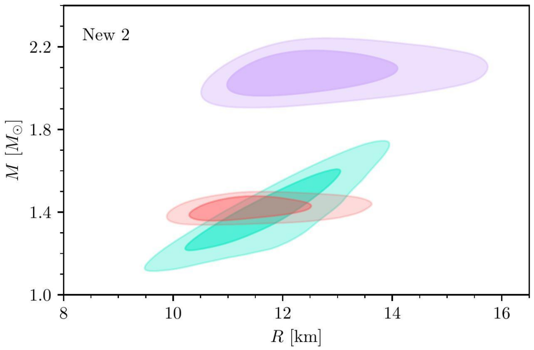




From Rutherford, **MM**, Svensson et al, ApJL (2024), arxiv:2407.06790

Most likely M-R



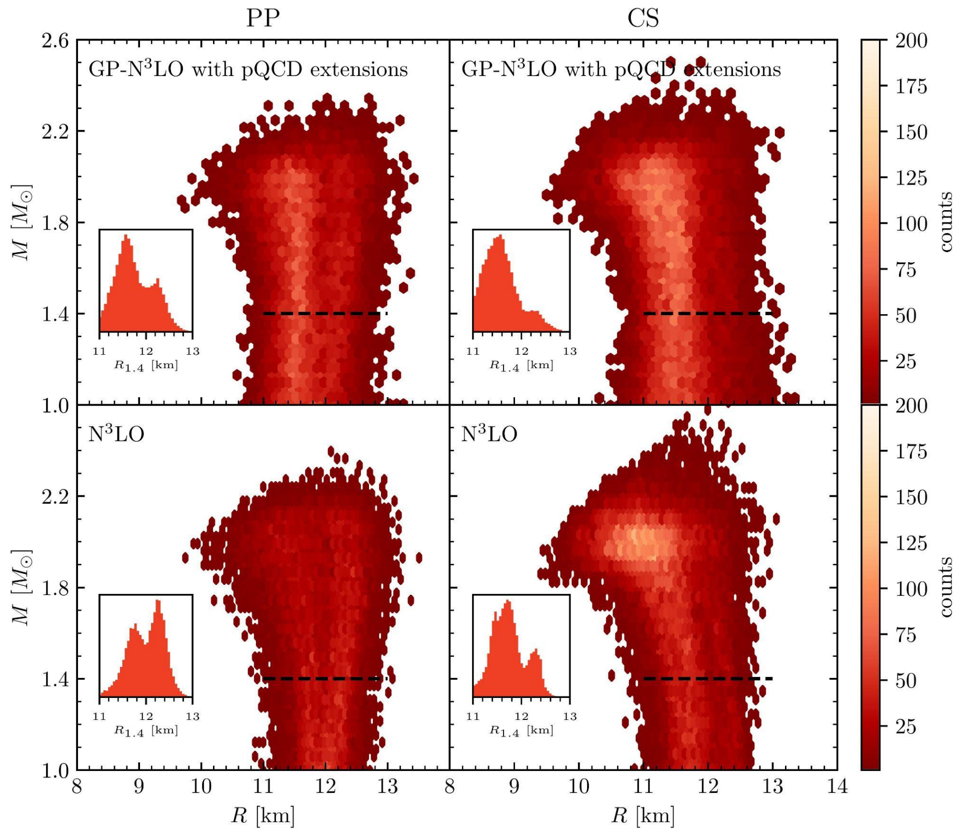


Consistent results for PP and CS parameterizations

Bimodal-like tendency for all posteriors

From Rutherford, **MM**, Svensson et al, ApJL (2024), arxiv:2407.06790

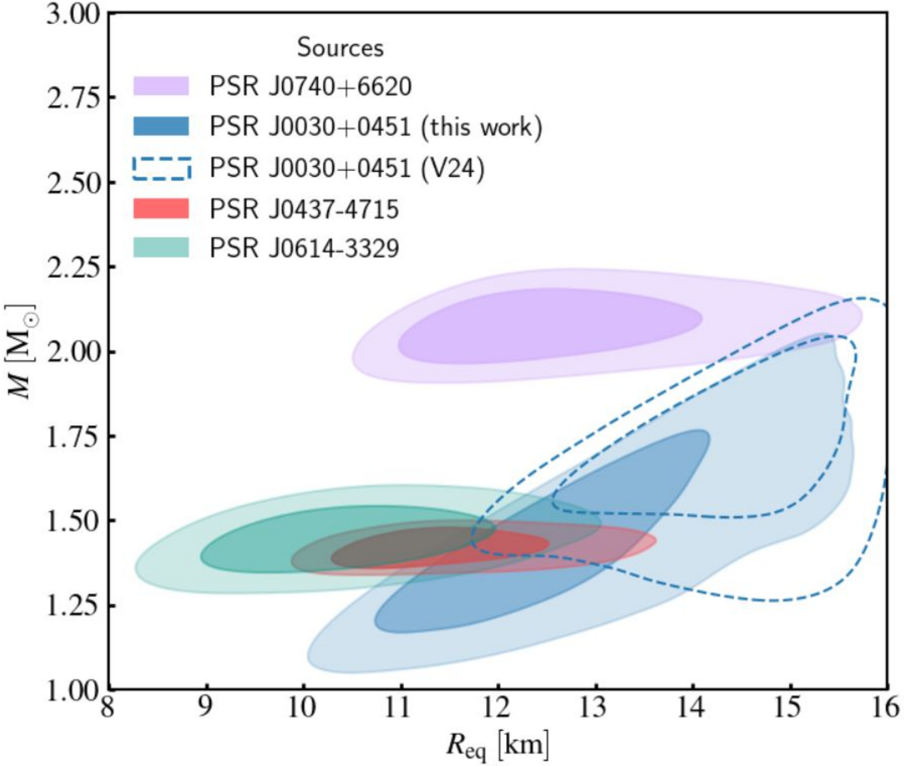
Most likely M-R



Bimodal-like tendency still present with new cEFT implementation and pQCD extensions

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

Recent data from NICER



Reanalysis with more accumulated data

Figure from Kini et al (2026), arxiv: 2602.23743

Other NICER released data

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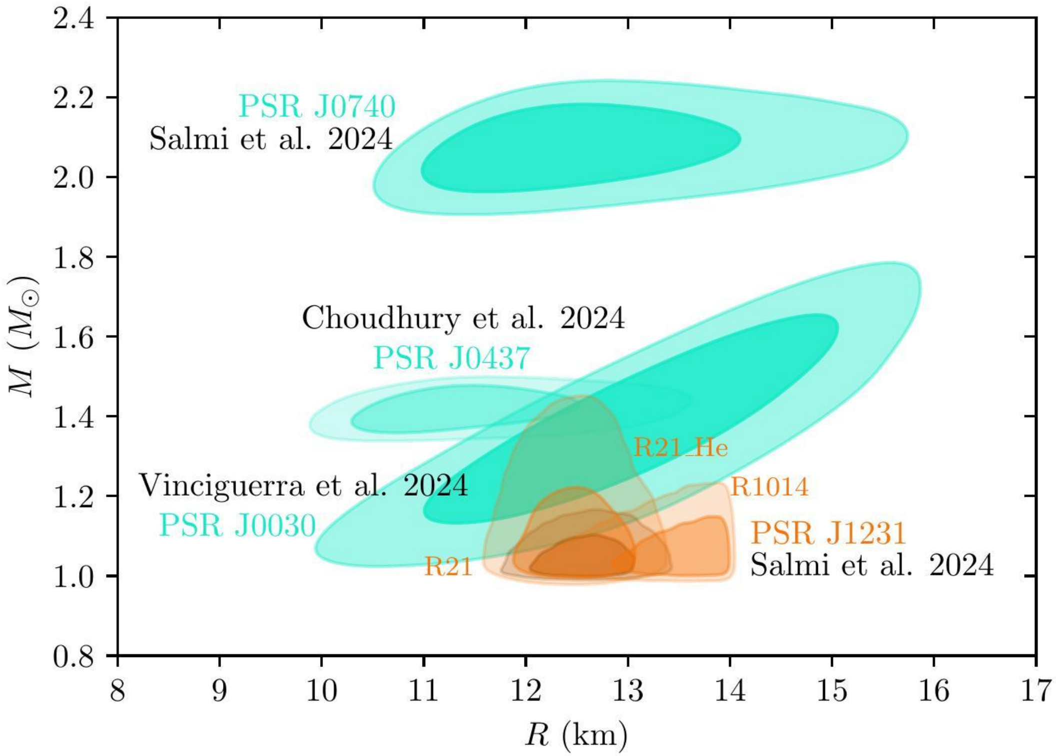

Other hot spot models for J0614

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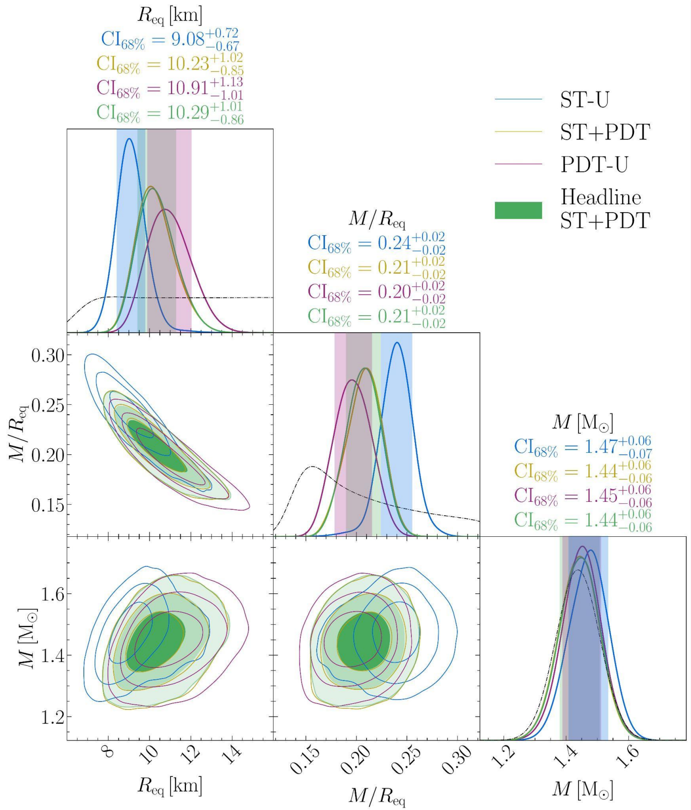

Figure from Mauviard, … **MM**, … et al, ApJ (2025), arxiv:2506.14883

TOV equations I

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$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\mathcal{E}(r)}{c^2}\frac{Gm(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2 r}\right]^{-1} ,$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \frac{\mathcal{E}(r)}{c^2} ,$$

$$\frac{\mathrm{d}y(r)}{\mathrm{d}r} = -\frac{y(r)^2}{r} - \frac{y(r)}{r}F(r) - rQ(r), \text{where}$$

$$F(r) = \left[1 - 4\pi r^2(\mathcal{E}(r) - P(r))\right]\left[1-\frac{2m(r)}{r}\right]^{-1} ,$$

$$Q(r) = 4\pi\left[5\mathcal{E}(r) + 9P(r) + \frac{\mathcal{E}(r)+P(r)}{c_s^2(r)} - \frac{6}{r^2}\right]\left[1-\frac{2m(r)}{r}\right]^{-1}$$

$$-\frac{4m^2(r)}{r^4}\left[1+\frac{4\pi r^3 P(r)}{m(r)}\right]^2\left[1-\frac{2m(r)}{r}\right]^{-2}$$

TOV equations II

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With $y(R) = y_R$, calculate the tidal Love number $k_2(\xi, y_R)$,

$$k_2(\xi, y_R) = \frac{1}{20}\xi^5(1-\xi)^2\left[(2-y_R)+(y_R-1)\xi\right]\left\{\left[(6-3y_R)+\frac{3}{2}(5y_R-8)\xi\right]\xi + \right.$$
$$+\frac{1}{2}\left[(13-11y_R)+\frac{1}{2}(3y_R-2)\xi+\frac{1}{2}(1+y_R)\xi^2\right]\xi^3 +$$
$$\left. +3\left[(2-y_R)+(y_R-1)\xi\right](1-\xi)^2\ln(1-\xi)\right\}^{-1},$$

where $\xi = 2GM/Rc^2$. The dimensionless tidal deformability is

$$\Lambda = \frac{2}{3}k_2\xi^{-5} = \frac{2}{3}k_2\left(\frac{Rc^2}{2GM}\right)^5$$

but the actual observable is the mass-weighted tidal deformability:

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1+12M_2)M_1^4\Lambda_1+(M_2+12M_1)M_2^4\Lambda_2}{(M_1+M_2)^5}.$$

PP parametrization equation

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Each polytrope given by $p(\rho) = K \rho^{\Gamma}$

such that $\varepsilon(p) = (1 + a)(p/K)^{1/\Gamma} + p/(\Gamma-1)$

Parameter ranges are:

For χ_{EFT} up to $1.1n_0$,

Γ_1: [1, 4.5], Γ_2: [0, 8], Γ_3: [0.5, 8], ρ_{12}: [1.5, 8.3], ρ_{23}: [1.5, 8.3]

For χ_{EFT} up to $1.5n_0$,

Γ_1: [0, 8], Γ_2: [0, 8], Γ_3: [0.5, 8], ρ_{12}: [2, 8.3], ρ_{23}: [2, 8.3]

CS parametrization equation

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High density EOS given by

$$c_s^2(x)/c^2 = a_1 e^{-\frac{1}{2}(x-a_2)^2/a_3^2} + a_6 + \frac{\frac{1}{3} - a_6}{1 + e^{-a_5(x-a_4)}},$$

a_6 to match χ_{EFT} band.

Fermi liquid theory (FLT) limit is c_s^2 FLT(1.5 n_0)/$c^2 \leq 1/m_N^2\ (3\pi^2 n)^{2/3}$

With a_1:[0.1,1.5], a_2:[1.5,12], a_3:[0.05,2], a_4:[1.5,37], a_5:[0.1,1]

a_1 outside this range fails maximum mass constraint or causality

Crust and outer core

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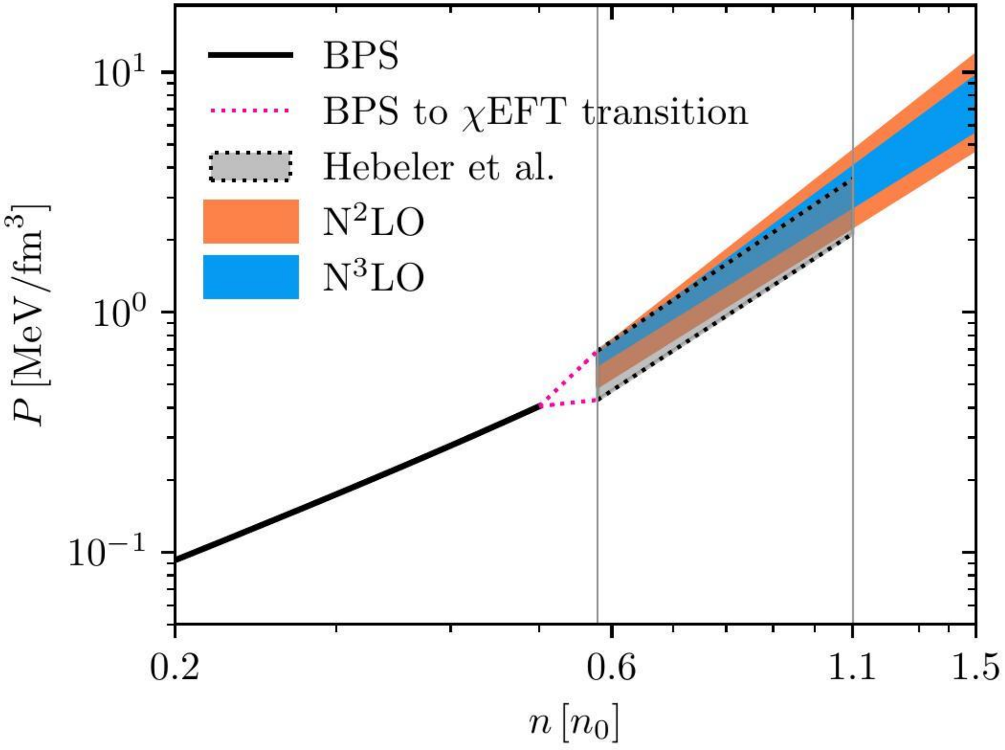

BPS EOS to model crust and polytropic fit ($P = K n^{\Gamma}$) for χEFT

Extended to both $1.1n_0$ and $1.5\ n_0$

From Rutherford, **MM**, Svensson et al, ApJL (2024), arxiv:2407.06790

pQCD implementation details

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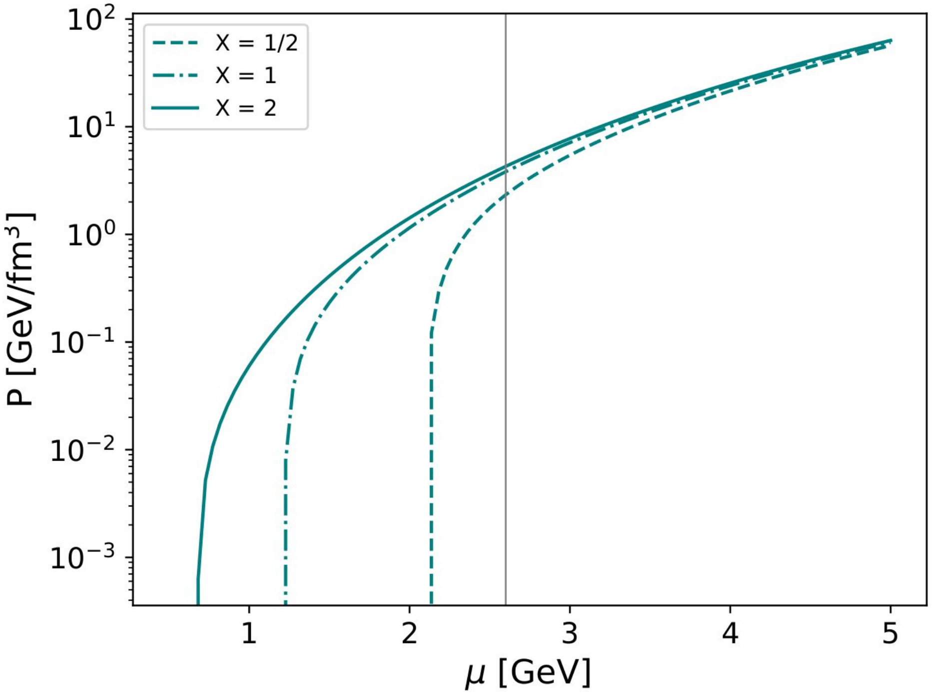


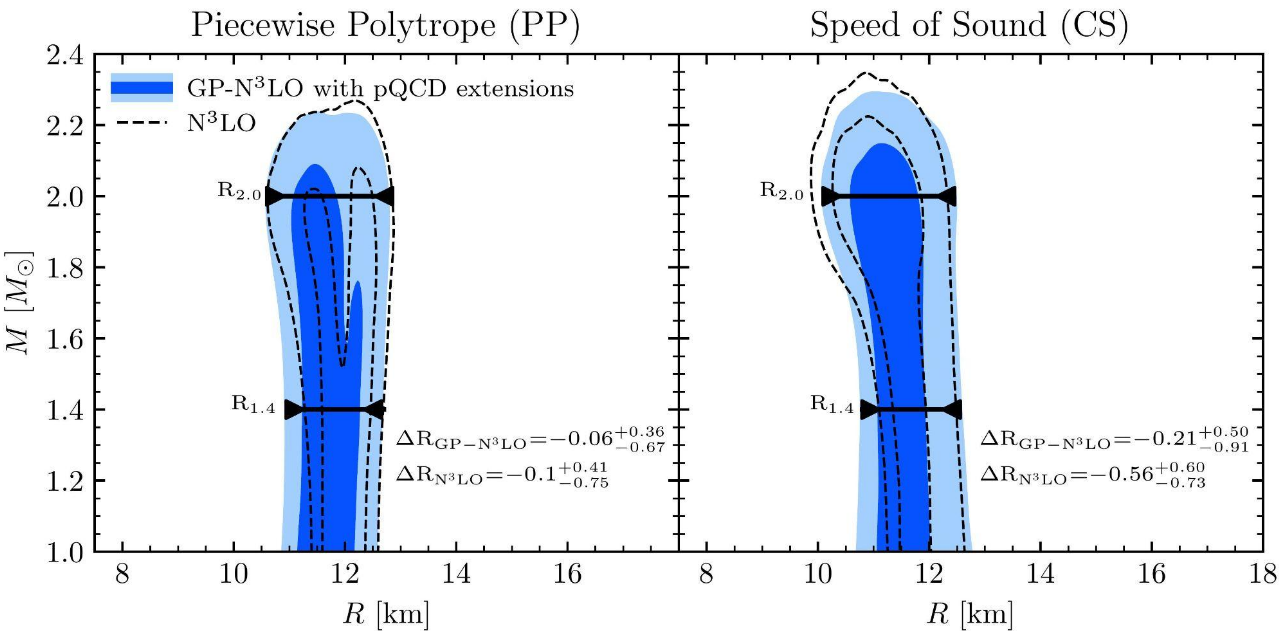
where $X \equiv 3\Lambda/(2\mu_B)$

with Λ renormalization scale

In NEoST, LogX uniformly sampled from [0.5, 2]

Figure from A. Hensel Master's thesis

Most likely M-R with GP-N3LO



We calculate

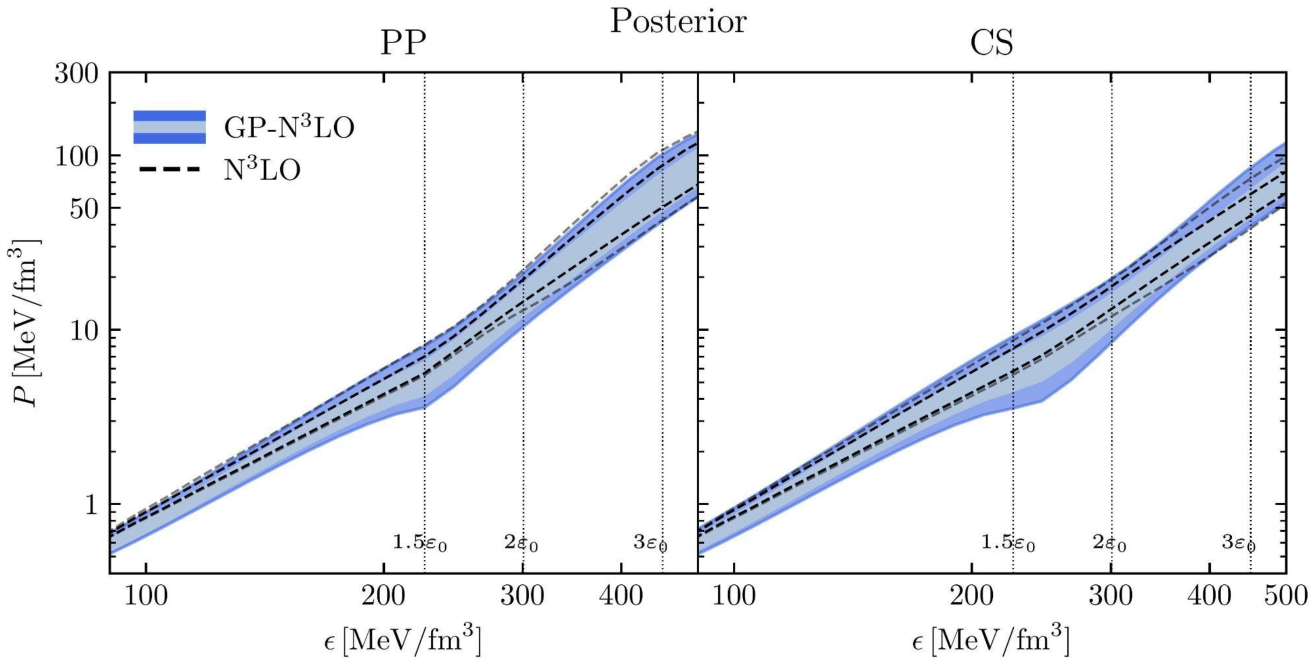
$\Delta R = R_{2.0} - R_{1.4}$

$\Delta R > 0$ suggests a stiffening of the EOS

Small dependence on high-density extension

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

Corresponding P-E curves

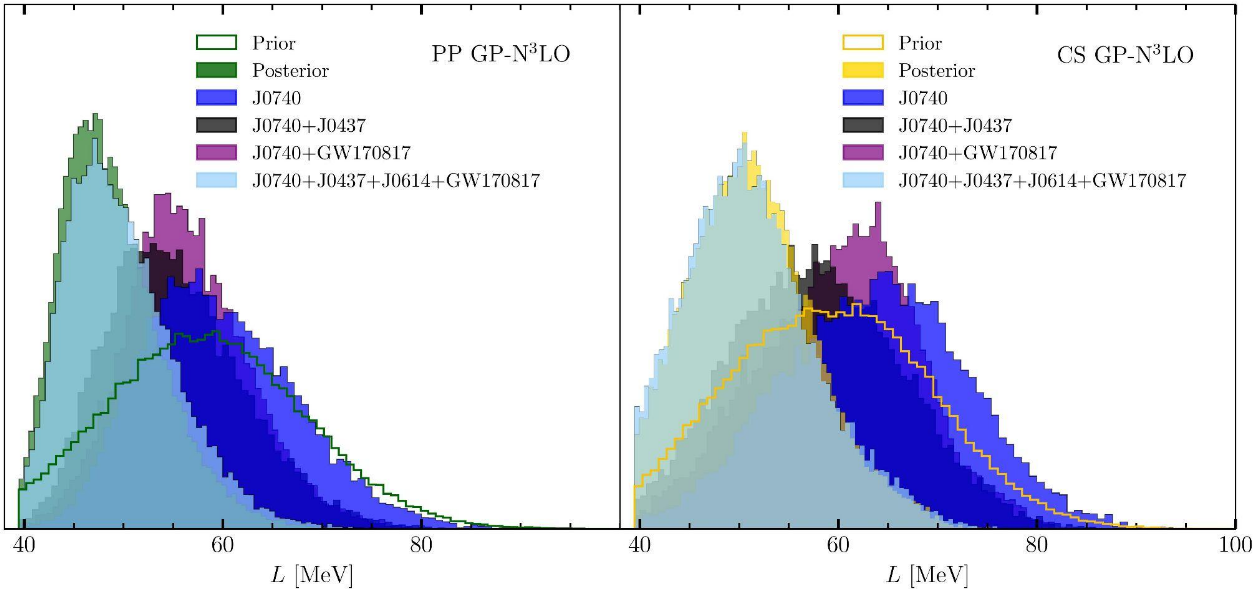


Broadening of pressure posterior beyond cEFT range

Minimal impact of pQCD extension

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

L parameter inference

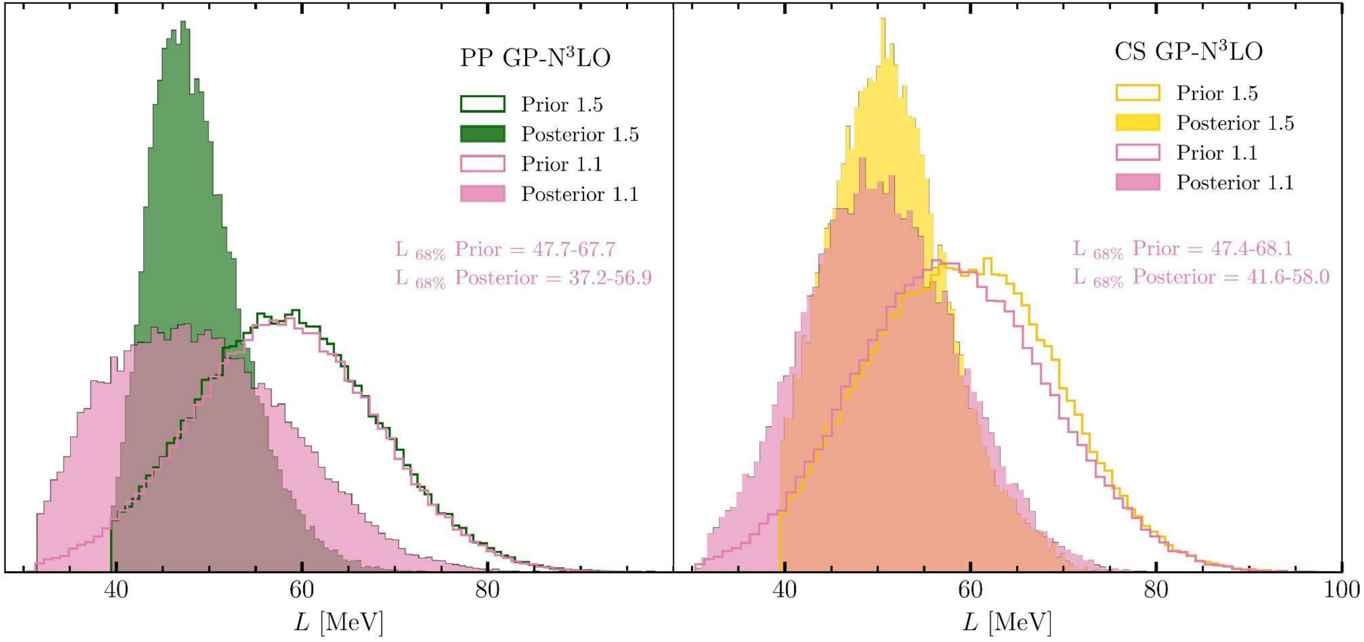


Posterior suggests that L is not prior dominated

GW data is crucial!

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

L parameter inference 1.1



Low pressure cut not so sharp if cEFT extended until 1.1 n_0 with L≳30.4 MeV

From **MM,** HG et al, 2026 (Submitted to ApJ), arxiv: 2605.18560

How can we use $EOS_{T=0}$ constraints to inform $EOS_{T\neq0}$?

- ❑ $EOS_{T\neq0}$ models need to be consistent with $EOS_{T=0}$ models in limit

  -constraints to particle content onset?

  -constraints to hybrid star formation mechanisms?

- ❑ $EOS_{T=0}$ as "final condition" for proto-neutron star cooling calculations?

  -continuous gravitational waves from supernova as a potential observable?

- ❑ cEFT $EOS_{T\neq0}$ models as input to CCSN and merger simulations

- ❑ …?