Uncertainties in superscaling models: QE, 2p2h and inelastic regimes

Guillermo D. Megias

Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla



Outline

- Introduction: motivation and superscaling phenomenon
- SuSAv2 approach: SuSAv2-QE model & RMF theory
- SuSAv2-inelastic model
- SuSA-2p2h channel (Granada model)
- Conclusions

Challenge for nuclear models and cross section analysis: How to improve understanding of $\nu - A$ interactions for oscillation analysis?

Improve theory: FSI, ground states, PB and shell effects (E_b) , nuclear potentials, etc. Use external constraints (e, e'), (e, e'p) to characterize nuclear effects and improve model selection in ν event generators

Compare several nuclear models with *e*- and nu - A inclusive (*lepton* detection), semi-inclusive $(l + N, l + \pi)$ and more exclusive $(l + p + \pi)$ cross section measurements

Uncertainties in nuclear models, reweighting and tuning

Event reweighting and tuning of parameters is a key analysis technique to study the effect of neutrino interaction model uncertainties. However, these procedure should be theoretically motivated. Otherwise, they could lead to results in contradiction with electron and/or neutrino analyses.



2p2h tuning from NOvA Collaboration Eur. Phys. J. C 80, 1119 (2020)

Uncertainties in nuclear models, reweighting and tuning



SuperScaling Approach (SuSA)

SuperScaling

The analysis of the large amount of existing (e, e') data at different kinematics is a solid benchmark to test the validity of theoretical models for neutrino reactions as well as to study the nuclear dynamics. The **SuperScaling Approach** exploits universal features of lepton-nucleus scattering to connect the two processes.

In inclusive QE scattering we can observe:





$$f(\psi) \equiv f(q, \omega) \sim \frac{\sigma_{QE} \binom{\text{nuclear}}{\text{effects}}}{\sigma_{\text{single nucleon}}\binom{\text{no nuclear}}{\text{effects}}}$$

$$f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega}\right)_{exp}}{\sigma_{Mott}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$

Good superscaling behavior at $\psi' < 0$ (below QE peak). At higher kinematics (ψ'), other contributions beyond QE and IA (2p2h, Δ , etc.) can play an important role and scaling is broken.

Separate L/T scaling functions



Testing SuperScaling for ${}^{12}C(e, e')$ in different nuclear models



The SuSAv2 model

PRC90, 035501 (2014) PRD94, 013012 (2016)

SuSAv2 model: lepton-nucleus reactions adressed in the SuperScaling Approach and based on Relativistic Mean Field (RMF) theoretical scaling functions (FSI) to reproduce nuclear dynamics.

♥ RMF: Good description of the QE (e, e') data and superscaling properties $(f_{L,exp}^{ee'})$. RMF predicts $f_T > f_L$ (~ 20%) as a pure relativistic effect (FSI with the residual nucleus). Strong RMF potentials at high q_3 are corrected by RPWIA and q-dependent blending function.

$$f(\psi) \equiv f(q,\omega) \sim \frac{\sigma_{QE}(\underset{\text{effects}}{\text{nuclear}})}{\sigma_{\text{single nucleon}}(\underset{\text{effects}}{\text{no nuclear}})} \quad ; \quad f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega}\right)_{exp}}{\sigma_{Mott}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$

RMF/RPWIA transition effects in SuSAv2

0.6

0.8

0.4

ω (GeV)

0



0.2 0.40.6 0.8 ω (GeV)



RMF/RPWIA transition effects in SuSAv2



Inclusive ¹²C(e, e') cross sections PRD 94, 013012 (2016)



(e, e') JLab data on ¹²C, ⁴⁰Ar, ⁴⁸Ti vs. SuSAv2-MEC. PRC99, 042501(R), 2019

 $E_i = 2.2 \text{ GeV}, \ \theta_e = 15.5^o$



Comparison with CC0 $\pi \nu_{\mu}$ -nucleus data



SuSAv2 implementation in GENIE: prd101, 033003 (2020), prd103, 113003 (2021)

Implementation of RMF and SuSAv2 models in generators

- Ist step: Implementing SuSAv2 hadron tensor W^{μν}(q, ω) + LFG on the top and comparison with original SuSAv2 model. DONE
- 2nd step: Introducing RMF nucleon momentum distribution in generators to fully test factorization approach. Work in progress with L. Munteanu and S. Dolan
- ▶ **3**^{*rd*} **step:** Adding SuSAv2 formulas, parameters and parametrization of scaling functions into generators to speed up simulations and to allow reweighting $(M_A^{QE}, p_F, E_b, \text{ etc.})$ **TO BE DONE**
- 4th step: Implement full RMF semi-inclusive model in generators and the full inelastic regime TO BE DONE



PB effects in comparison with neutrino CC0pi data



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Uncertainties on vector (EM) form factors



Uncertainties on the axial form factor: PRC 101, 025501 (2020)



Different options: Dipole, z-expansion, two-component model

2-comp. model: The pion electroproduction data can be interpreted in different theory frameworks to extract the form factor. We consider here the Soft Pion and the Partially Conserved Axial Current approximation (PCAC).

Uncertainties in CS smaller than 5% in general (up to 10% in some cases).



Low-energy effects at T2K CC0 π 0p >500 MeV/c arXiv:1905.08556

Low-energy effects and scaling violations are only appreciable at very forward angles (low q_3 , q_0 values). RMF is more accurate than SuSAv2 at these kinematics.



Low-energy nuclear effects and its proper description can have an important effect in the C to O extrapolation, which is essential for T2K and HK.

T2K CC0 $\pi \nu_{\mu}$ -H₂O cross sections



RMF models could reveal C/O differences due to different binding energy and shell effects, mass of the residual nucleus, FSI and Coulomb distortions, etc.



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ED-RMF and ROP reproduces QE data, as SuSAv2 but improving low ω region.

Scattered Nucleon Description

Regarding the scattered nucleon, we can consider several situations:

- Relativistic Plane-Wave Impulse Approximation (RPWIA): the ejected nucleon is considered a plane-wave (i.e, there are not final state interactions)
- Energy-Dependent Relativistic Mean Field (ED-RMF): W.F. solution of the Dirac equation in the continuum using the same RMF potential that describes the initial state times a phenomenological function that weakens the potentials at high energies.
- Relativistic Optical Potential (ROP): The scattered nucleon travels under the influence of a
 phenomenological relativistic optical potential fitted to elastic proton-nucleus scattering data.

Cross sections vs proton kinematics: T2K and MINERvA



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Cross sections vs transverse kinematic imbalances



Cross sections vs MicroBooNE nu_mu -> 40Ar CC0piNp



We can also add uncertainties in the nuclear potential parameters, SF profile, binding energies, occupancy, transparency, etc. Error bands included in MicroBooNE plots for reference.

RMF SF vs. RMF SM



RPWIA = RMF-noFSI // RMF(shell model) vs. RMF (Rome SF)We can also calculate separate results for the s and p shells. Nuclear recoil effects can be switched on/off

SuSAv2 model for inelastic neutrino-nucleus scattering



- Quasielastic region.
- 2p-2h excitations.
- Δ resonance, other resonances and DIS.



• TrueDIS (Deep inelastic scattering)

SuSAv2-inelastic model describes the full inelastic spectrum (Δ , • other res. And DIS)[G. D. Megias, PhD Thesis (2017), M. B. Barbaro et al., Phys. Rev. C 69, 035502 (2004), J. Gonzalez-Rosa et al., Phys. Rev. D 105, 093009 (2022)]. Good agreement with (e,e') data.

 $R_{inel}^{K}(\kappa,\tau) = \frac{N}{\eta_F^2 \kappa} \xi_F \int_{\mu_X^{min}}^{\mu_X^{max}} d\mu_X f^{model}(\psi_X') U^k$

- **SuSAv2 model for QE** uses RMF scaling function to model nuclear dependence. Similar approach is done for **inelastic regime**.

 Inelastic hadron tensor includes: RES (DCC model) + DIS (Bodek-Ritchey/Bosted-Christy/PDFs) + soft DIS (merge).

- **SuSAv2 inelastic** can be implemented in NEUT or GENIE to predict lepton kinematics and shortly for nucleon kinematics

- Comparisons with **NEUT DCC (RFG)** in collaboration with Hayato-san *et al.* are under way.
- This approach can incorporate other inelastic models.

 $W_x^{min} = 2.1 \ GeV; \ W_x^{max} = m_N + \omega - E_s$

Bodek-Ritchie/ Bosted-Christy/ Parton Distribution Function

RES (Resonances)

 $W_x^{min} = m_N + m_\pi; \ W_x^{max} = 2.1 \ GeV$

Dynamical Coupled Channels

SoftDIS (Deep inelastic scattering in the resonance region)

 $W_x^{min} = m_N + m_\pi; \ W_x^{max} = 2.1 \ GeV$

Dynamical Coupled Channels and Bodek-Ritchie/Bosted-Christy

Inelastic Nuclear Responses & SuSAv2-inelastic model

Inelastic structure functions

Inclusive ${}^{12}C(e, e')$ double differential cross section



<u>Bodek-Ritchie</u>: poor description of the resonance region.

Bosted-Christy: Good description of the resonant structures observed in (e,e') reactions.

<u>GRV98</u>: No resonant structures (average) and poor description at $Q^2 \lesssim 1 \text{ GeV}^2$.

Validation of SuSAv2-inelastic vs elecron data: arXiv:2306.12060 [nucl-th]





SuSAv2-inelastic model & low-energy nuclear effects: arXiv:2306.12060 [nucl-th]



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SuSAv2 model for inelastic neutrino-nucleus scattering

Results: MINERvA





More strength seems to be needed in **RES** channel to compare with **MINERvA** and **MnvGENIE**, unlike **ArgoNEUT** (similar E_{ν}) and **T2K** (lower E_{ν})

MINERvA CC $u_{\mu \prime} < E_{
u_{\mu}} > \sim 3.5~GeV$ (Low)

MINERvA CC $u_{\mu \prime} < E_{
u_{\mu}} > {\sim}6.0~GeV$ (Medium)

 $\chi^2 = 6.5959$

20

θ. (Deg.)

30

10

Results: ArgoNEUT



ArgoNEUT CC $\nu_{\mu} < E_{\nu_{\mu}} > \sim 9.6 \ GeV; \ CC \bar{\nu}_{\mu} < E_{\nu_{\mu}} > \sim 3.6 \ GeV$

See Phys. Rev. D 105, 093009 (2022) for details.

Analysis with NOvA results are under way.

Uncertainties in nuclear models, reweighting and tuning



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2p-2h MEC for (e, e') and CC ν reactions PRD91, 073004 (2015)

Other 2p2h models neglect direct/exchange interference terms \Rightarrow strongly affects np/pp ratio by a factor ~ 2 (PRC94:054610,2016) \Rightarrow Implications in nucleon multiplicity and hadron E_{reco}



O Accurate implementation of np/pp pairs for the 2p2h channel using separate hadron tensors for *np* and *pp* pairs.

So The numerical evaluation of the hadronic tensor $W_{2p2h}^{\mu\nu}(R_K^{2p2h})$ is performed in the RFG model in a fully relativistic way without any approximation. It can be easily extended to all nuclei.

Separation into pp, nn and np pairs in the FS \Rightarrow also valid for $N \neq Z$ (⁴⁰Ar, ⁵⁶Fe, ²⁰⁸Pb)

arXiv:1905.08556

Comparison of SuSAv2-MEC^{Genie} with Nieves^{Genie} 2p2h



Differences in np/pp separation are mostly related to the treatment of 2p2h direct/exchange interference terms (absent in Nieves model) \rightarrow strongly affects np/pp ratio by a factor \sim 2 (PRC94:054610,2016) \Rightarrow Implications in nucleon multiplicity and hadron E_{reco}

Uncertainty in 2p2h MEC Response

Three theoretical Models:

- 1. Relativistic Mean Field (RMF): Computed with nucleon effective mass M* = 0.8 and vector energy $E_V = 141$ MeV.
- 2. Relativistic Fermi Gas (RFG): Using the full Delta propagator and separation energy $E_s = 40$ MeV.
- 3. RFG with Real Delta Propagator and $E_s = 40$ MeV.

—The comparison of these three models in (e, e') highlights the uncertainty in the 2p2h MEC response.

 The choice of theoretical framework for the 2p2h MEC response can significantly impact the model predictions



From Martinez-Consentino et al., Phys. Rev. C**104** (2021) 025501

2p2h full/real Δ prop. vs MINERvA CC0 $\pi \bar{\nu}$ -CH data



2p2h full/real Δ prop. vs T2K CC0 $\pi \nu$ -¹²C data



Δ propagator in 2p2h and Δ decay width

 \supset Our **2p2h-MEC model** can produce **semi-inclusive** (e, e') and CC0 π results. Work in progress.

⊃ An open question in 2p2h models (also in π production) is the treatment of the Δ propagator in 2-body currents and the Δ decay width. Taking only the real part of the propagator and assuming free Δ decay width is an approach taken by several 2p2h models and in our case it has resulted in a good empirical approach in very good agreement with (*e*, *e'*) and CC0 π data.

⊃ Next step (Solution?): Joint analysis of the 2p2h-MEC model (full propagator) and the ED-RMF (1p1h + π production) in comparison with (*e*, *e'*) data to infer a proper value of the Δ decay width with medium modifications.

$$\begin{split} & \int_{p_a}^{p_1'} \int_{k_2}^{p_2'} \int_{p_2}^{p_2'} \int_{p_1}^{p_1'} \int_{k_2}^{p_2'} \int_{p_2}^{p_2'} \int_{p_1}^{p_1'} \int_{p_2}^{p_2'} \int_{p_1}^{p_1'} \int_{p_2}^{p_2'} \int_{p_1}^{p_1'} \int_{p_2}^{p_2'} \int_{p_1}^{p_1'} \int_{p_2}^{p_2'} \int_{p_1}^{p_2'} \int_{p_2}^{p_2'} \int_{p_1}^{p_2'} \int_{p_2}^{p_2'} \int_{p_1}^{p_2'} \int_{p_2}^{p_2'} \int_{p_2}^{p_2'$$

2p2h full/real Δ prop. vs JLab $(e, e')^{12}$ C data



BACKUP SLIDES:

SuSAv2 is an inclusive model where scaling violations and low-energy effects present in RMF (semi-inclusive) are not fully included.

♥ Strong q-dependence of RMF vector and scalar potentials at high kinematics is addressed in SuSAv2 with a blending function to introduce RPWIA (no FSI). To have a more consistent model and preserve orthogonality, unitarity and dispersion relations \Rightarrow Solution: ED-RMF (both inclusive and semi-inclusive for ¹²C, ¹⁶O, ⁴⁰Ar, etc.)

Solution: ED-RMF model (both inclusive and semi-inclusive for ¹²C, ¹⁶O, ⁴⁰Ar, etc.) introduces an Energy-Dependent potential (based on SuSAv2) to the RMF to keep strength for low p_N while making RMF potential softer for increasing p_N . (PRC 100, 045501 (2019), PRC 101, 015503 (2020))

RMF and ED-RMF are available for 1p1h and SPP, easily extendable to all nuclei. See also PRC100, 045501 (2019), PRC101, 015503 (2020), and A. Nikolakopoulos' talk.

Model	Low kin.	Intermediate kin.	High kin.	(e,e')	CC0pi	CC1pi	Inelastic
RMF	1	1	Х	~	1	~	X
RPWIA	X	~	1	~	1	~	Х
SuSAv2	~	✓	1	~	1	*	*
ED-RMF	1	1	1	1	1	1	Х

*Under some approaches

SuSAv2-MEC 2p2h model is based on RFG microscopic calculations as most 2p2h models (Valencia, Martini, etc.)

Relev



Figure 6.6: Comparison of RMF and RPWIA contributions in the QE regime. Also shown for reference the predictions of the total QE-SuSAv2 model (long-dashed red line) and the total inclusive contribution (solid blue line). The y-axis represents $d^2\sigma/d\Omega/d\omega$ in nb/GeV/sr.

Uncertainties on the nuclear potentials and FSI effects



Description of the initial state:

- Pure shell model (first approximation): missing energy profile is given by a Dirac delta per shell
- Realistic model, i.e. Rome (Benhar spectral function) used in electron exclusive processes: short- and long-range correlations included



MINERvA Medium Flux $0.075 < p_T/GeV < 0.15$ $\chi^2 = 131.87$ (SuSAv2); 40.73 (RFG)



Some MC tuning or model approaches in the RES and SoftDIS (Background) channels do not seem consistent between electrons and neutrinos, being too large for neutrinos. GENIE Mnv RES approach is based on a single-nucleon Rein-Sehgal pion production model with lepton mass corrections and other modifications to include nuclear effects. But, it looks similar to a low-pF RFG model. Can affect RES and SoftDIS tuned models to the reconstruction and the CS data? Check with e-A



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Uncertainties in nuclear models, reweighting and tuning

da/dp_T (cm²/GeV/c/Nucleon)

- MC tuning, reweighting or model approaches in the RES and SoftDIS (Background) channels for neutrinos should be validated against electron scattering data.
- GENIE Mnv RES approach is based on a single-nucleon Rein-Sehgal pion production model with lepton mass corrections and other modifications to include nuclear effects. But, it looks similar to a low-pF RFG model.





Scaling violations and low-energy effects present in RMF are not fully included in the SuSAv2-MEC model.

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♥ The ED-RMF model introduces an Energy-Dependent potential (based on SuSAv2) to the RMF that keeps the strength for slow nucleons but makes the RMF potential softer for increasing nucleon momenta. See PRC 100, 045501 (2019), PRC 101, 015503 (2020) for details

SuSAv2 is a pure inclusive model. Solution: ED-RMF (both inclusive and semi-inclusive for ¹²C, ¹⁶O, ⁴⁰Ar, etc.)



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Uncertainties in the SuSA-2p2h model



Solution Delta De

The calculation is performed in the RFG model in which Lorentz covariance can be maintained.

- Form factors
- Description of the Delta propagator (full vs. Real)
- Fermi momentum, Eshift(~binding energy)
- Relativistic vs. Non-relat. approaches
- Nuclear model: RFG (Granada(SuSA), Valencia), Empirical, RMF(Ghent, Granada), etc
- Diagrams and interference diagrams considered
- New 2p2h semi-inclusive model from SuSA (Granada): arXiv:2310.12642 [hep-ph]
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