Suppression of Color Superconductivity in Strongly Interacting Dense QGP

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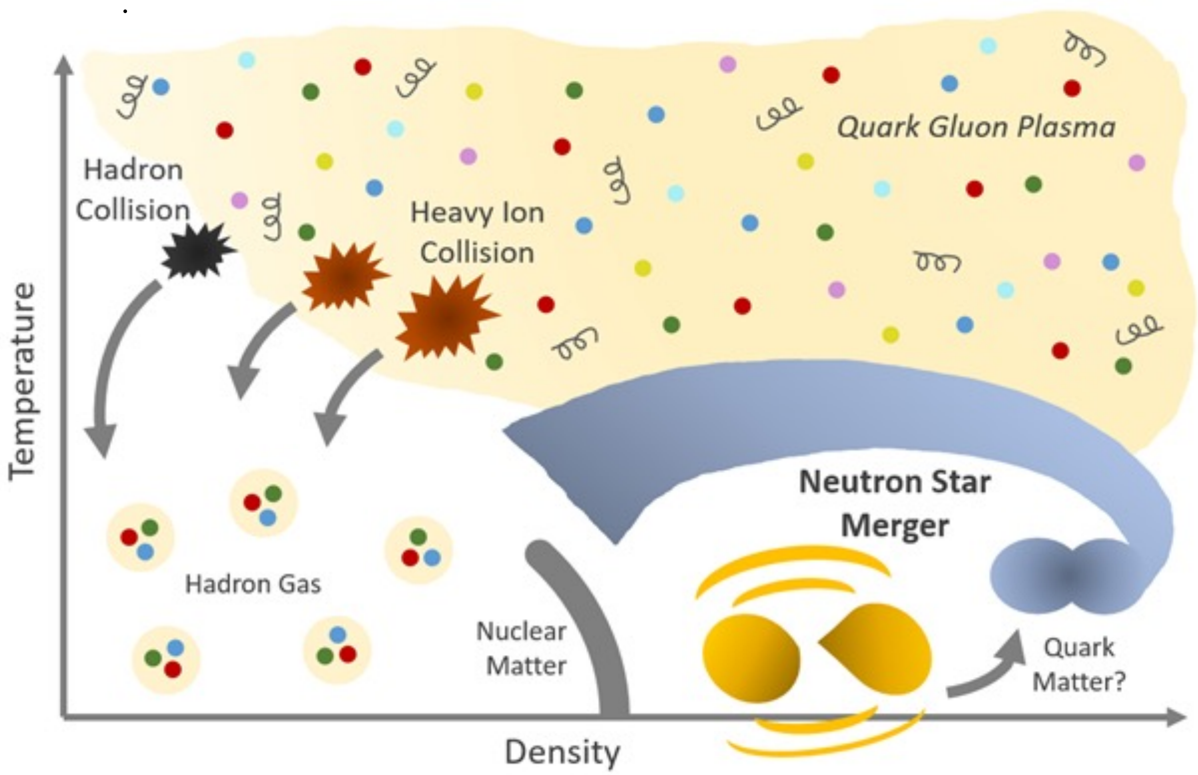
Understanding QCD matter



[M. Schwaiger-Haber, WUSTL, & A. Haber, University of Southampton]

- $T > T_c^{\chi}$: still strongly coupled QGP
  - Near perfect fluid
  - Strong jet quenching
  - Rapid heavy-quark thermalization

  [E. Shuryak Rev.Mod.Phys 89 2017]

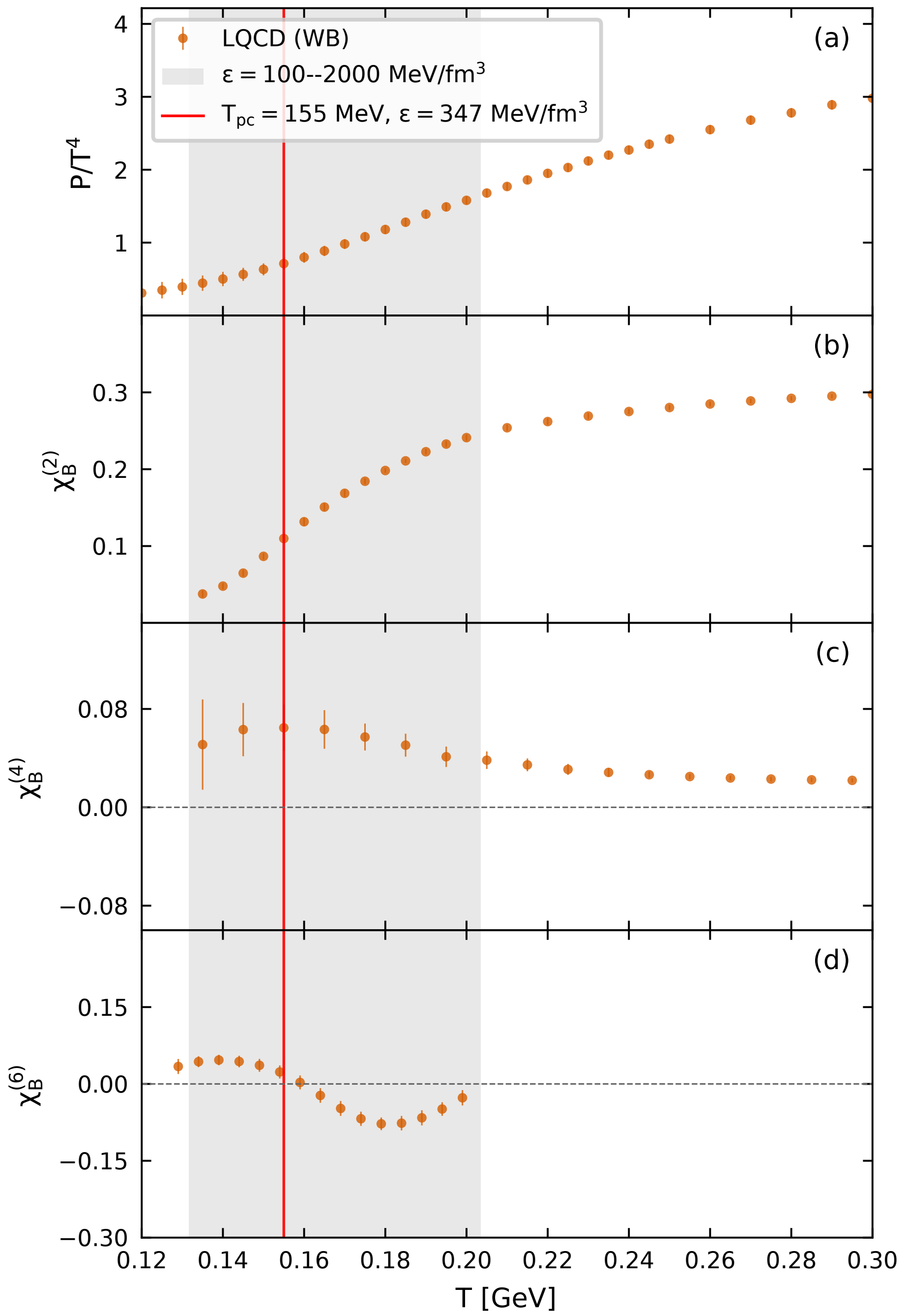
- Features of a "good" model of QGP:
  - Description of lattice QCD results at zero and low μ_B/T
  - Crossover at $\mu_B = 0$ – transition of dominant degrees of freedom

Lattice QCD results

- Ab initio calculation at $\mu_B = 0$, fermion sign problem at finite μ_B
- Can extract pressure and conserved charge susceptibilities

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial \left(\frac{\mu_B}{T}\right)^n}$$

- Typical neutron-star energy densities $\sim T < 200\ \mathrm{MeV}$
- Is there a description of the transition region using many-body theory?

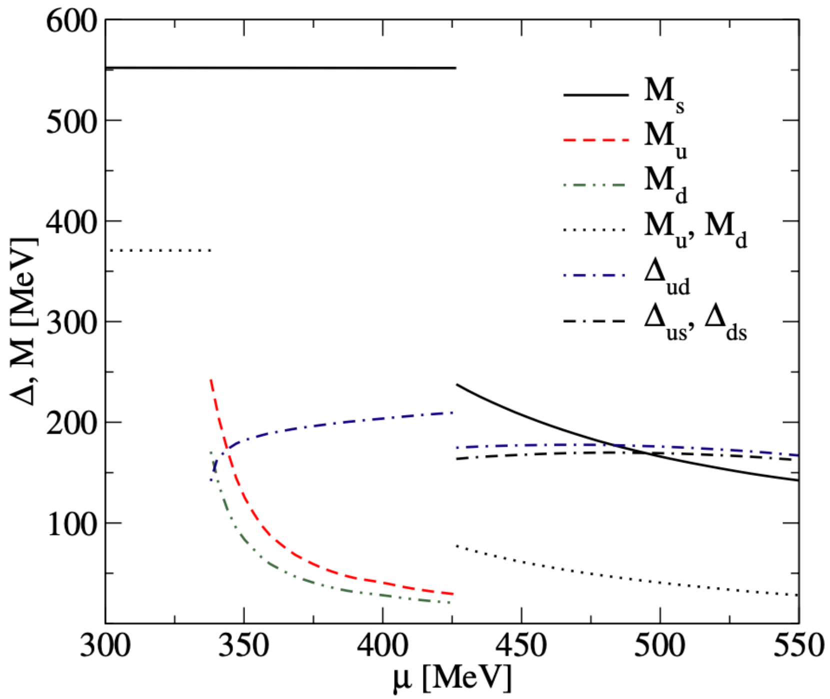


Color superconductivity

- Interaction is **attractive in the $\bar{3}$ channel** ⇒ **BCS instability** of the Fermi surface
- Lives at large μ_B, low T → **neutron-star cores**; phases **2SC / CFL**, Δ ~ 10–100 MeV

[R. Rapp et al. PRL 81 (1998)]

[Alford et al. Rev. Mod. Phys. 80 (2008)]



[Blaschke et al. PRD 72 (2005)]

- Standard treatment is **mean-field** (NJL / weak coupling) — gap equation:

$$\Delta = -G\,\frac{2}{\pi}\int p^2 dp\,\frac{\Delta}{2E_p}\tanh\frac{E_p}{2T}$$

$$E_p = \sqrt{(\varepsilon_p - \mu)^2 + \Delta^2}$$

- Critical temperature (BCS):

$$T_c \approx 0.57\,\Delta$$

- Beyond mean field: **T-matrix / Thouless criterion**

Green's functions and spectral functions

- Central objects: $G(\omega, \vec{p})$
  - Green's function
  - Spectral function:

$$\rho(\omega, p) = -\frac{1}{\pi}\mathrm{Im}G(\omega, p)$$

- Non-interacting:

$$G_0(\omega, p) = \frac{1}{\omega - (E_p - \mu) + i0}$$

- Interacting:

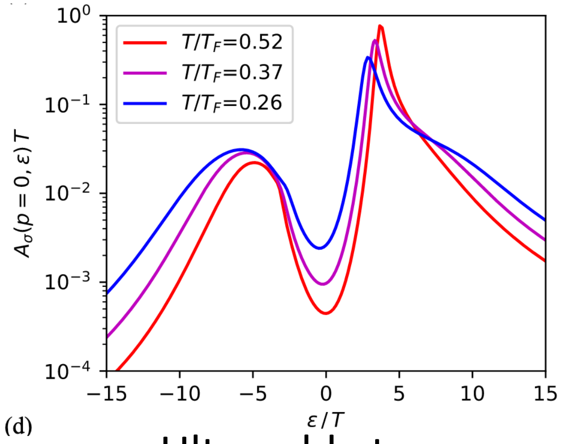
$$G(\omega, p) = \frac{1}{G_0^{-1}(\omega, p) - \Sigma(\omega, p)}$$

- Choice of diagrams in Σ ↔(largely) many-body framework

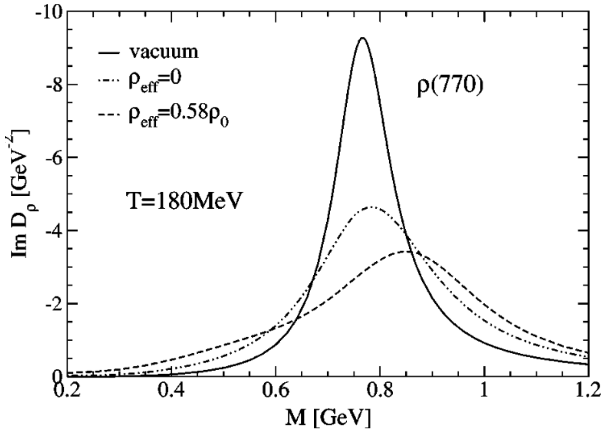
$$A(\omega, p) = \frac{-\frac{1}{\pi}\mathrm{Im}[\Sigma(\omega, p)]}{(\omega - E_p - \mathrm{Re}[\Sigma(\omega, p)])^2 + (\mathrm{Im}[\Sigma(\omega, p)])^2}$$

- If we are lucky: $\Gamma \simeq -2\mathrm{Im}\Sigma(E_p^*, p) \to 0$,
$\rho(\omega, q) = \delta(\omega - E_p^*)$

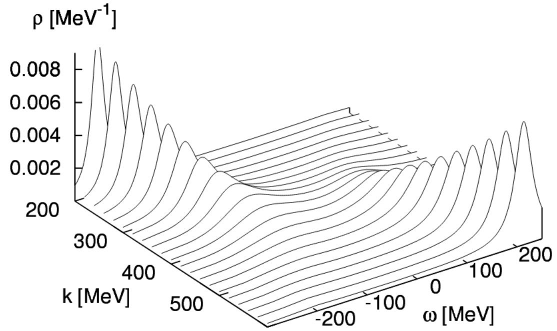
- If not:



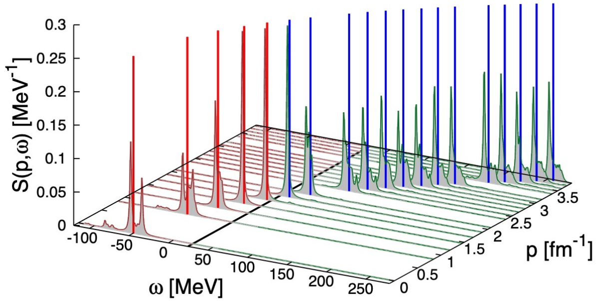
Ultracold atoms
[T. Enns Phys.Rev.A 109 (2024)]



ρ-meson melting
[R.Rapp Phys.Rev.C 63 (2001)]



Hot quark matter
[M.Kitazawa et al. Phys.Lett. B631 (2005)]

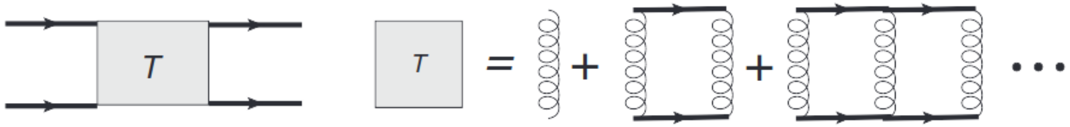


Nuclear matter
[Barbieri & Carbone "Self-Consistent Green's Function Approaches"]

T-matrix approach to strongly interacting QGP

- Basic d.o.f. – partons: $q, \bar{q}, g; N_f = 3$ (degenerate)
- Start with $\hat{H}$ with 2-body interaction $V(\vec{q}, \vec{q}')$ in various color channels
- Ladder resummation: T-matrix

$$T(E,P;q,q') = V + VG_2(E,\ldots)T$$



- Symbolically

$$T = \frac{V}{1 - VG_2} \equiv \frac{V}{1 - J}$$

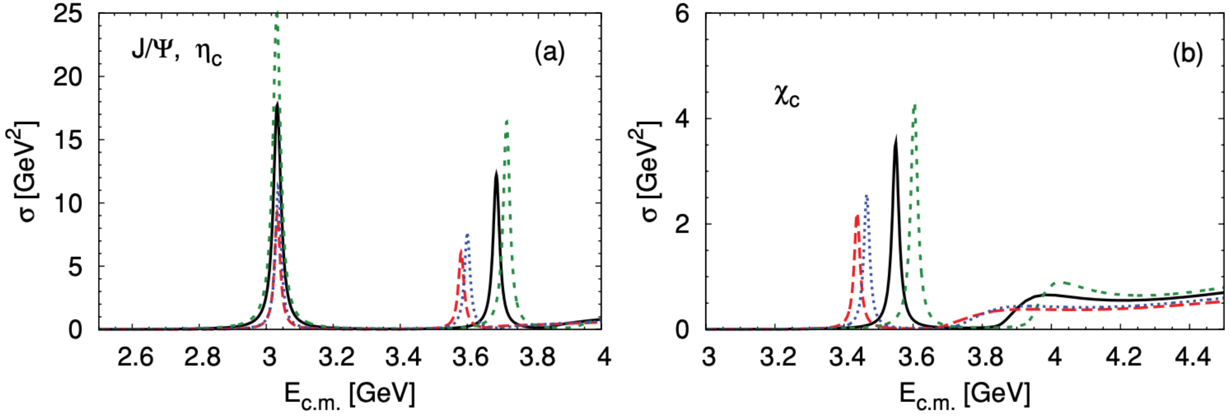
- Poles below $2m$ in T-matrix $\leftrightarrow$ bound states

Origin: heavy quarkonia $\bar{b}b, \bar{c}c$

[M. Mannarelli, R. Rapp PRC72 (2005), D. Cabrera, R. Rapp PRD76 (2007), F. Riek, R. Rapp PRC82 (2010)]

Screened Cornell potential:

$$V \sim -\frac{4}{3}\alpha_s \frac{e^{-m_d r}}{r} + \frac{\sigma}{m_s}(1 - e^{-m_s r})$$

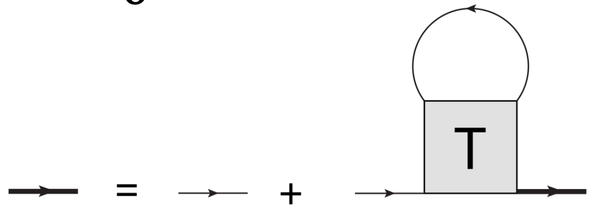


$$\sigma(E_{c.m.}) \sim \mathrm{Im}\, T(E_{c.m.})$$

In-medium: self-consistency

- Interactions ↔ self-energies

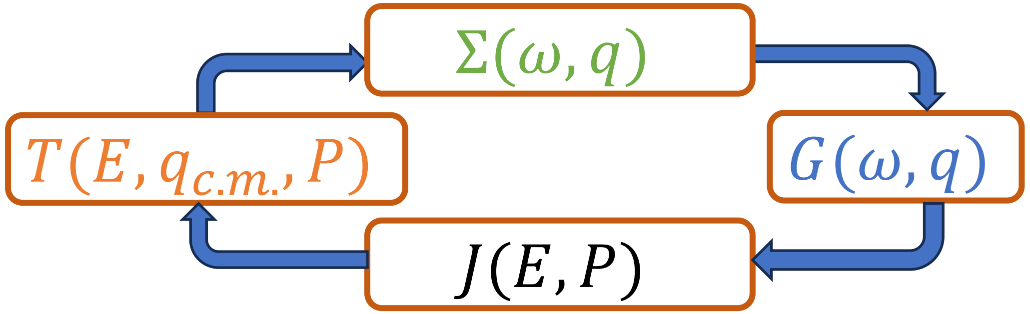
$$G = G_0 + G_0 \Sigma G, \qquad \Sigma \sim TG$$



Real-frequency formulation:

$$\mathrm{Im}\Sigma(\omega, q) \sim \int d\vec{p}_2\, d\omega_2\, \mathrm{Im}T(\omega + \omega_2, \vec{p}_1, \vec{p}_2)$$
$$\times \rho(\omega_2, \vec{p}_2) \times [n_B(\omega + \omega_2) + n_F(\omega)]$$

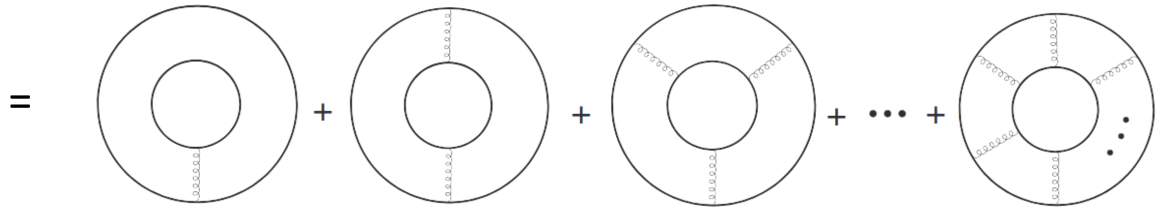
- Self-consistent scheme



Equation of state

$$\Omega(\{\mu\}, T) = \mp T \sum_n \underbrace{\mathrm{Tr}\{\ln(-G^{-1}) + \Sigma G\}}_{\text{1-body}} \pm \underbrace{\Phi[G]}_{\text{2-body}}$$

Luttinger-Ward Functional (LWF) $\Phi[G]$ =



- Self-consistent many-body framework

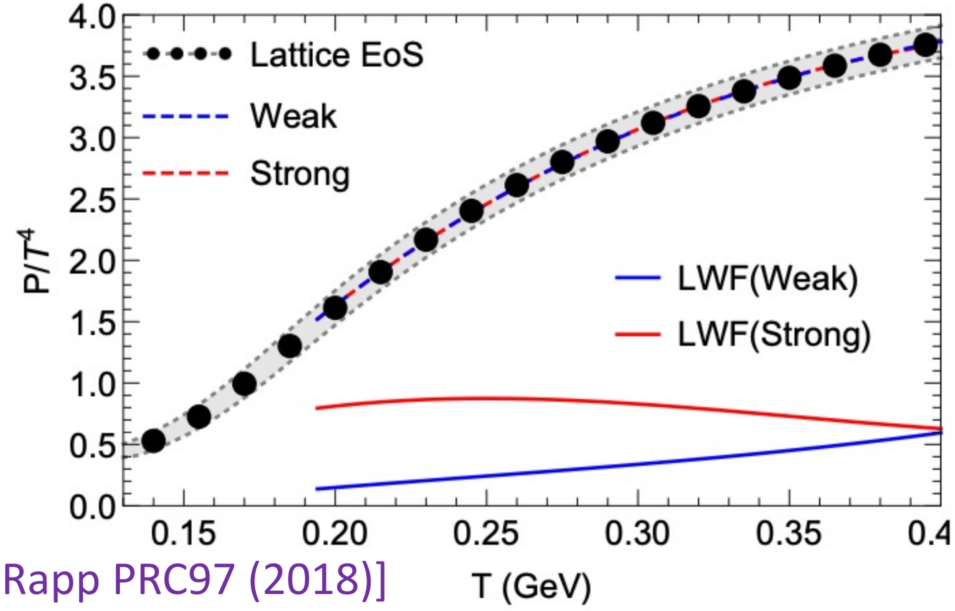
$$\Sigma[\mathrm{G}] = \frac{\delta \Phi[G]}{\delta G}$$

- Same setup as SCGF
- Conserving 2PI approximation

[G. Baym PR 127 (1962)]

Weakly/strongly coupled scenarios

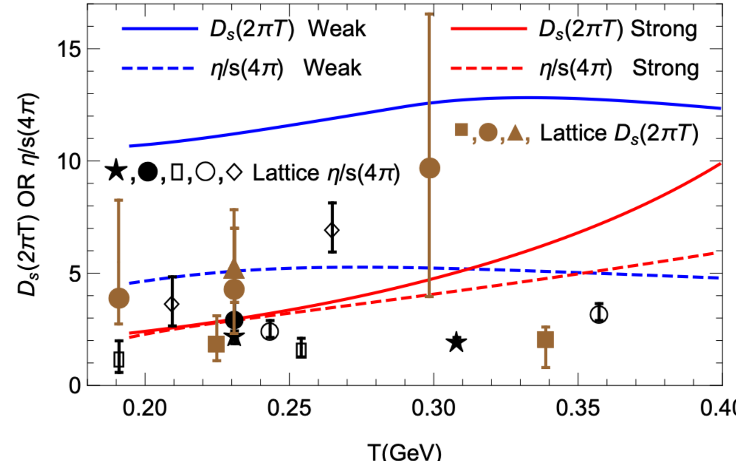
- Parton masses and interaction screening fit lQCD data:
  - $P(T, \mu_B = 0)$
  - Heavy-quark free energy $F(\vec{r})$
  - Heavy-quark correlators
- … due to remnants of the confining force

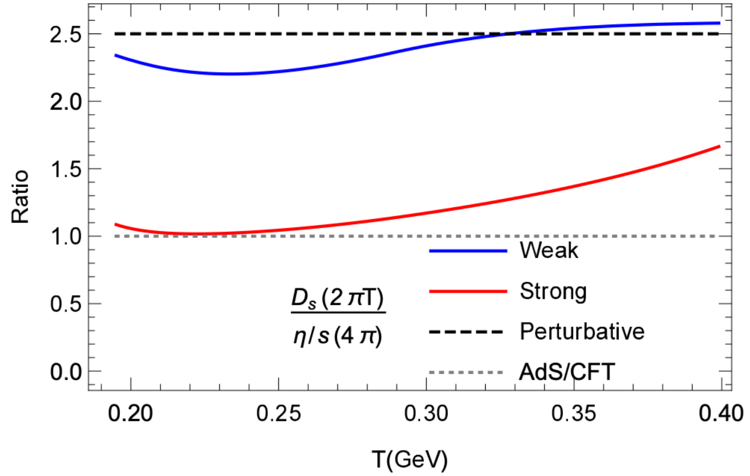


[S. Liu, R. Rapp PRC97 (2018)]

- Same EoS, significant difference in transport properties

[S. Liu, R. Rapp EPJA 56 (2020)]





This work: separable interactions

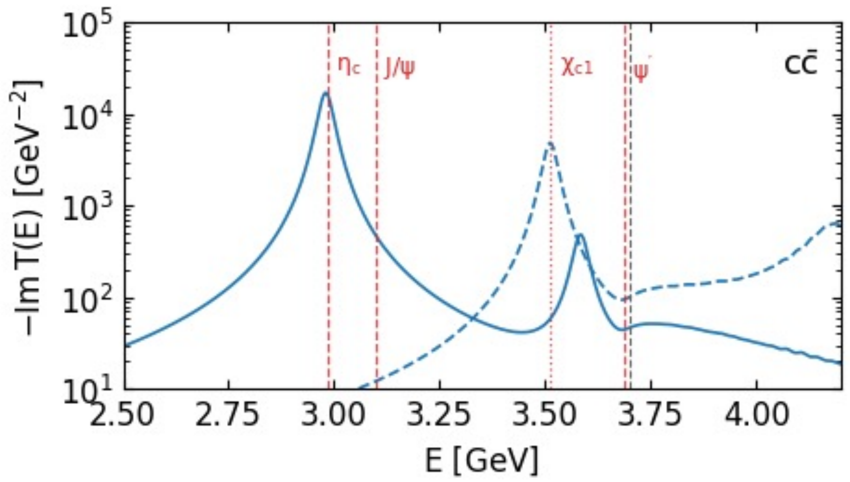
Technical simplification

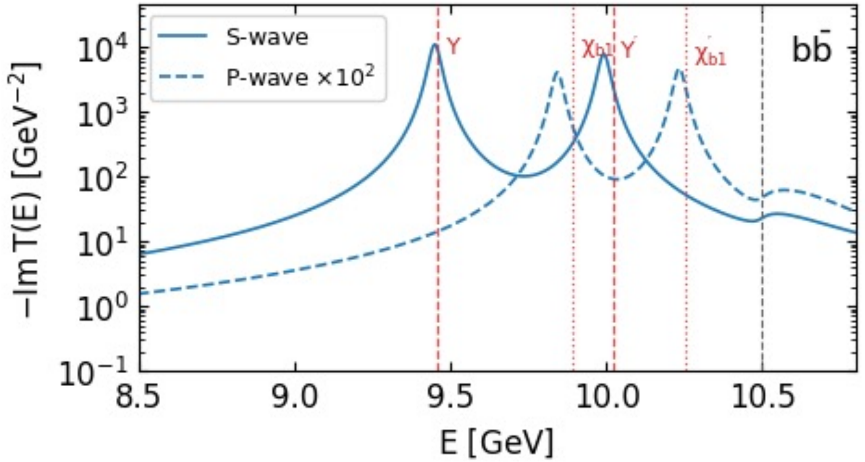
- Solves 3D-reduced BSE and allows to resum LWF semi-analytically
  $$\mathrm{Im}J(E,P) \sim \int d^3q\, d\omega\, v(q_{cm})v(q_{cm})\rho(\omega)\rho(E-\omega)$$
- Rank-3 separable interaction in s- and p-waves:
- $V(q,q') = G_S v_S(q)v_S(q') + G_c v_C(q)v_C(q') + G_P v_P(q)v_P(q')$
- $v_S^{l=0}(q) = \frac{\Lambda_S^4}{(q^2+\Lambda_S^2)^2}, v_C^{l=0}(q) = e^{-q^2/\Lambda_C^2}, v_p^{l=0}(q) = \frac{\Lambda_P^2}{(q-q_0)^2+\Lambda_P^2}$

× relativistic corrections

Can we mimic the strongly coupled QGP scenario with this simplified interaction?

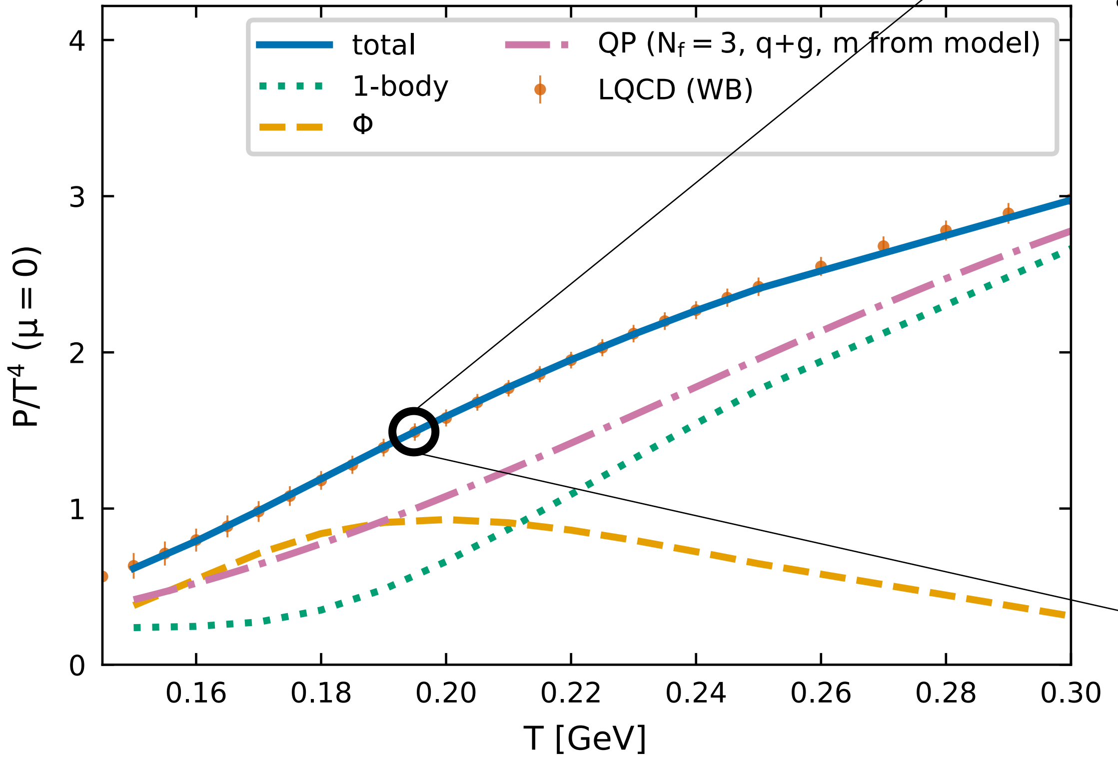
- $\Lambda_S \sim 0.5$ GeV, $\Lambda_C \sim 2$ GeV, $\Lambda_p \sim 1$ GeV
- Parameters tuned to reasonably describe quarkonium spectroscopy



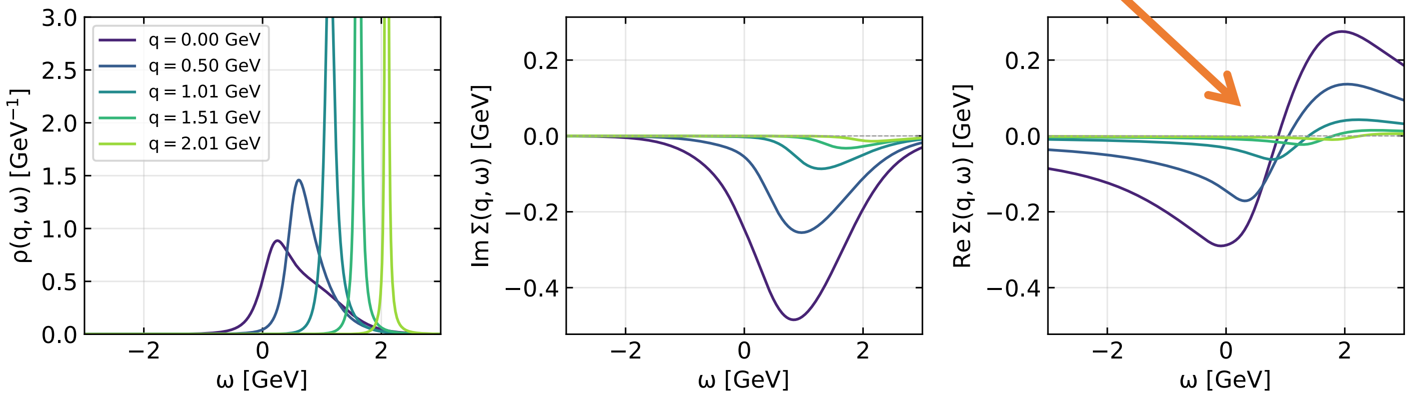


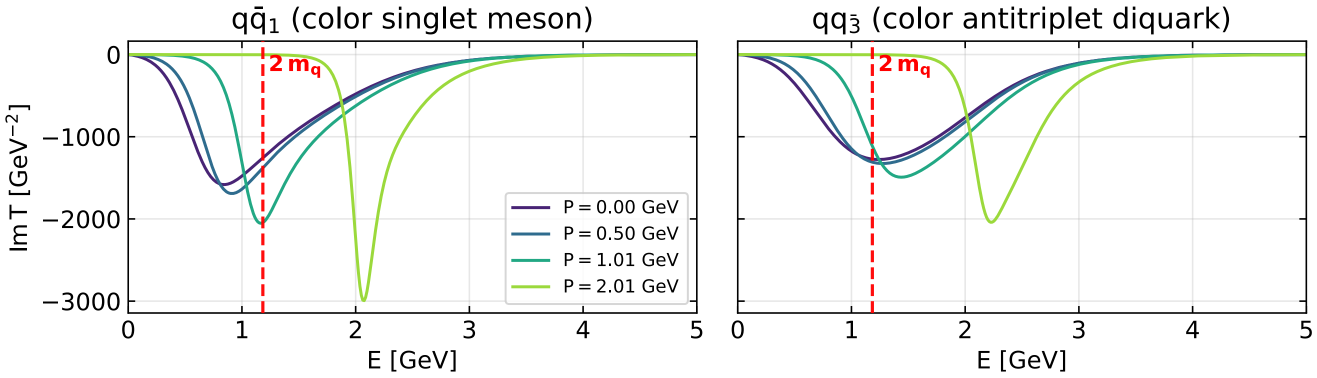
EoS at $\mu_B = 0$

- Parton masses fitted to lattice QCD pressure at $\mu_B = 0$
  [Borsanyi et al. Phys.Lett.*B* 730 (2014)]



$$\mathrm{Re}\Sigma(\omega, q) = -\frac{1}{\pi}\int d\omega' \frac{\mathrm{Im}\Sigma(\omega', q)}{\omega - \omega'}$$





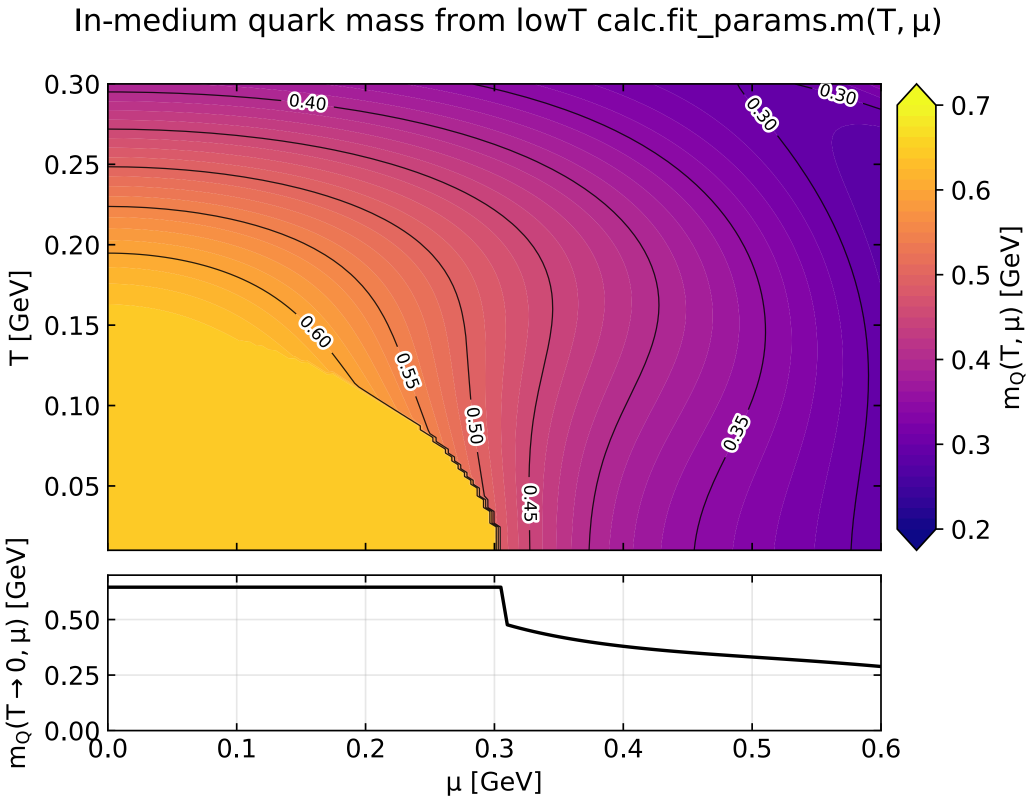
- Main contribution:
  Φ at low T → partons at large T
- 1-body contribution ≠ QP case

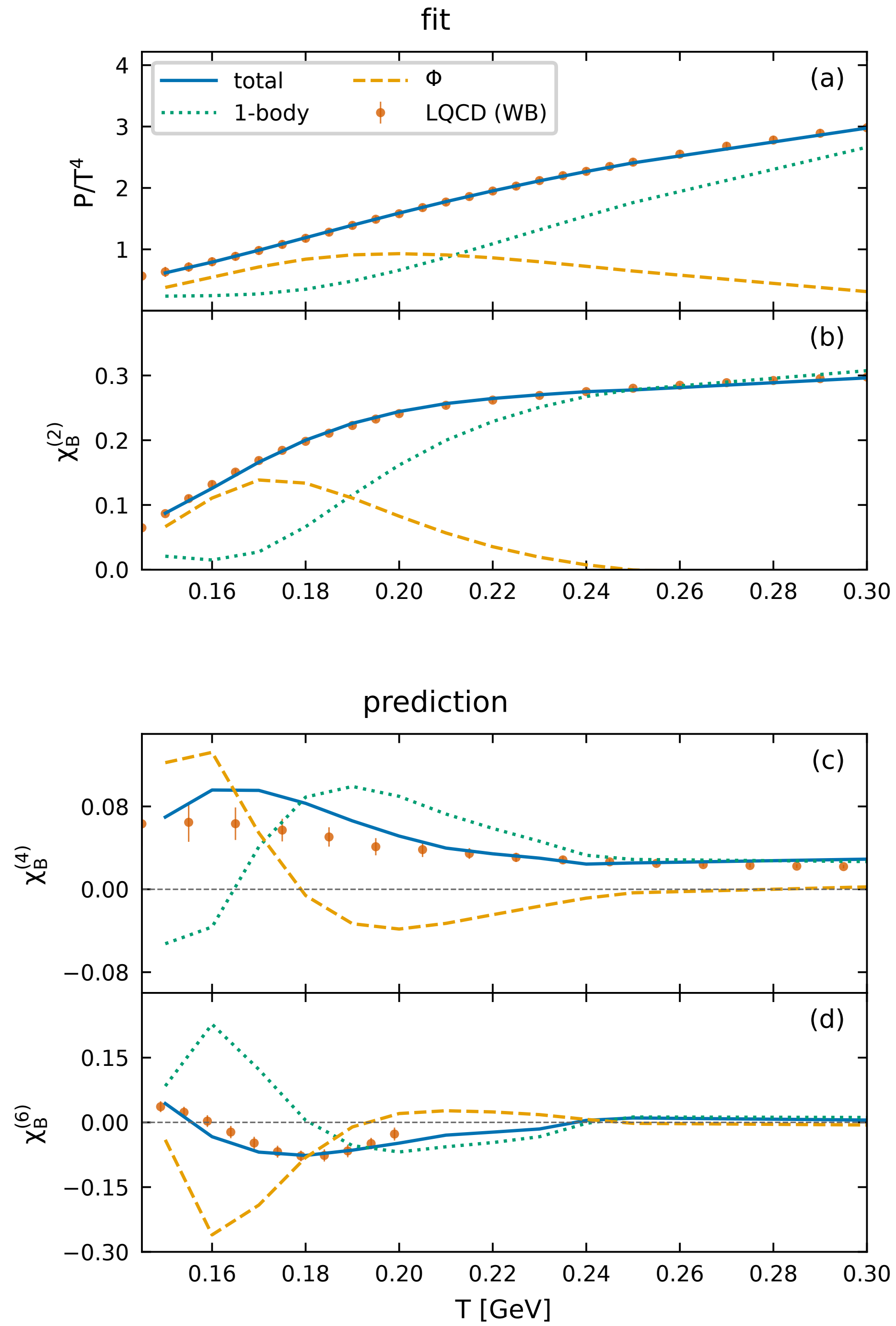
\+ p-wave
\+ 34 2-body channels:

qq	$q\bar{q}$	$(q/\bar{q})g$	gg
(1/2, 3)	(1, 1)	(9/8, 3)	(9/4, 1)
(−1/4, 6)	(−1/8, 8)	(3/8, 6)	(9/8, 16)
		(−3/8, 15)	(−3/4, 27)

Susceptibilities

- Polynomial fit to get susceptibilities
  - $P(\mu_B) = P_0 + \frac{\chi_B^2}{2}\mu_B^2 + \frac{\chi_B^4}{4!}\mu_B^4 + \frac{\chi_B^6}{6!}\mu_B^6 + \cdots$
- Fitted to P and $\chi_B^{(2)}$
- Predictive power up to χ_6
- Extrapolation of the quark masses to low T:





Thouless criterion

- BCS gap equation at T_c: $\Delta = 0$
- $\Delta = -G\frac{2}{\pi}\int p^2 dp \frac{\Delta}{2(\varepsilon_p - \mu_q)} \tanh\frac{\varepsilon_p - \mu_q}{2T}$
- Many-body equivalent:

$$[\mathrm{Re}T(E=0, P=0)]^{-1} = 0$$

bound state appears at zero energy ↔ condensation of Cooper pairs

- QP approximation:

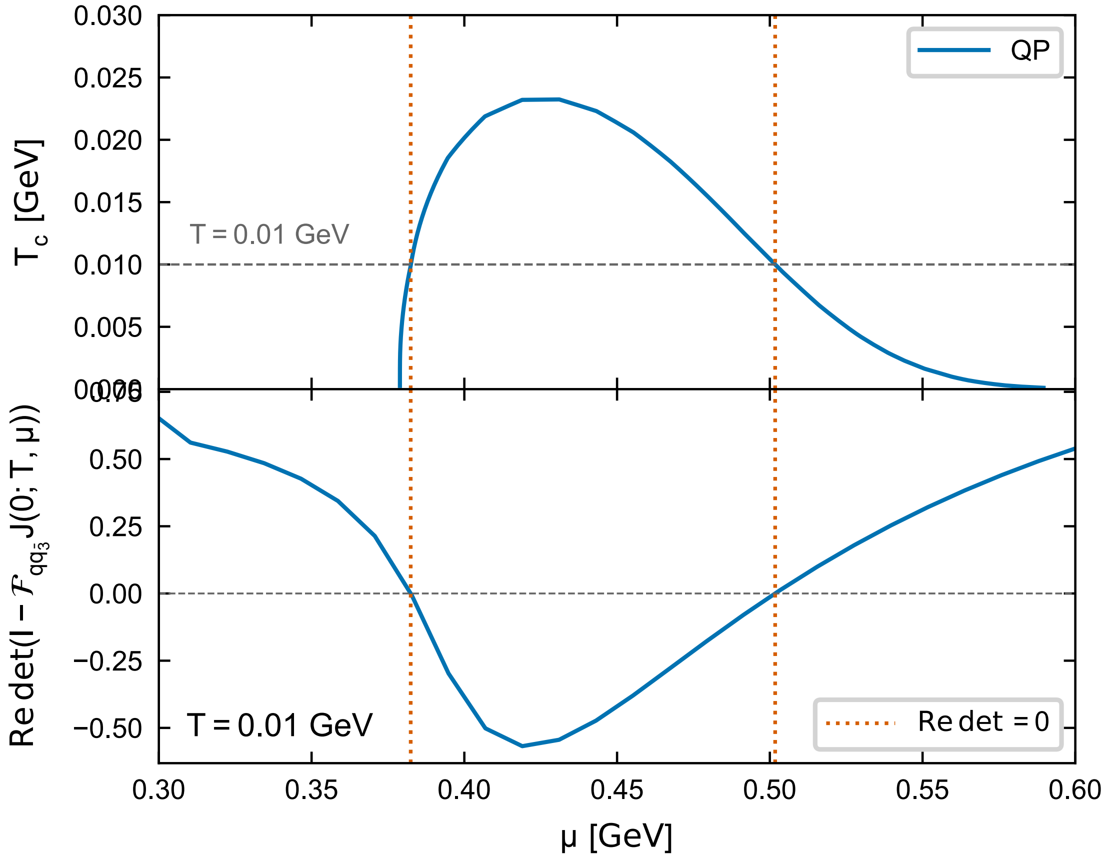
$$G_2(E, \vec{P}=0) = \frac{1}{E - 2\,(\varepsilon_p - \mu_q) + i0}$$

- $1 - V \star G_2 = 0 \leftrightarrow$

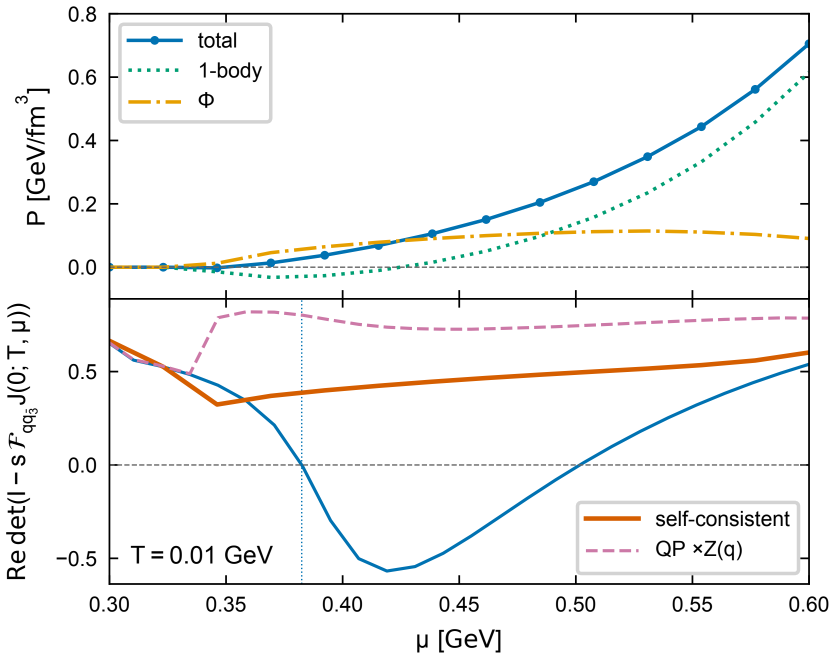
$$1 = -G\frac{2}{\pi}\int q^2 dq \frac{v^2(q)}{2(\varepsilon_p - \mu_q)} \tanh\frac{\varepsilon_p - \mu_q}{2\,T}$$

Reasons for the small gap:

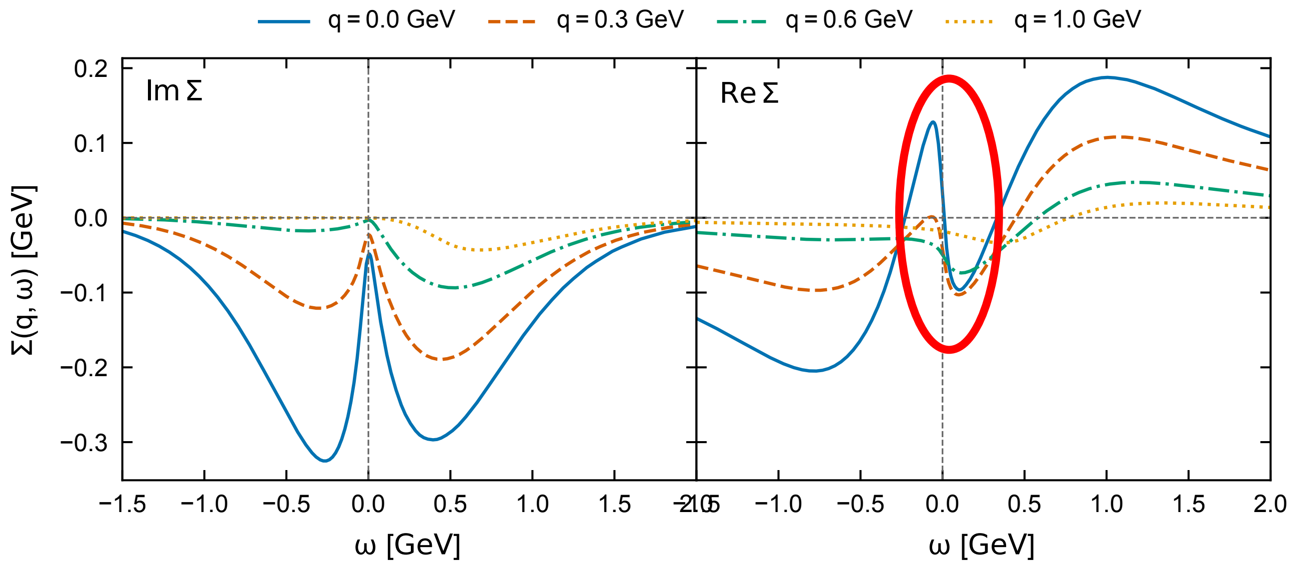
- At large μ_q - separable interaction
- Calibration to $\chi_2(T)$ - $qq_{\bar{3}}$ can't be too much bound

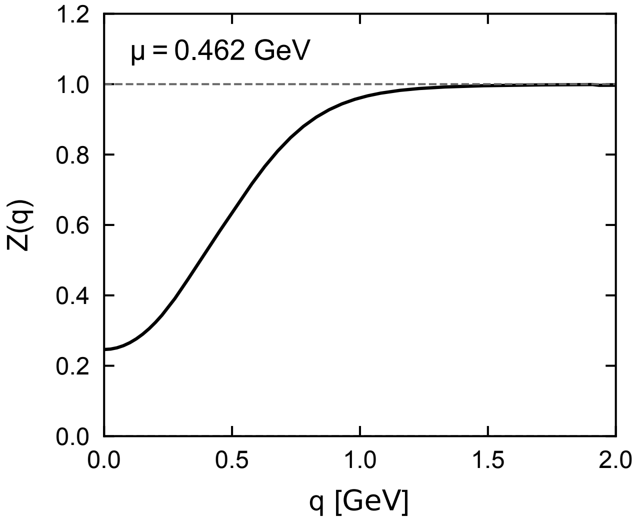


Self-consistent solution



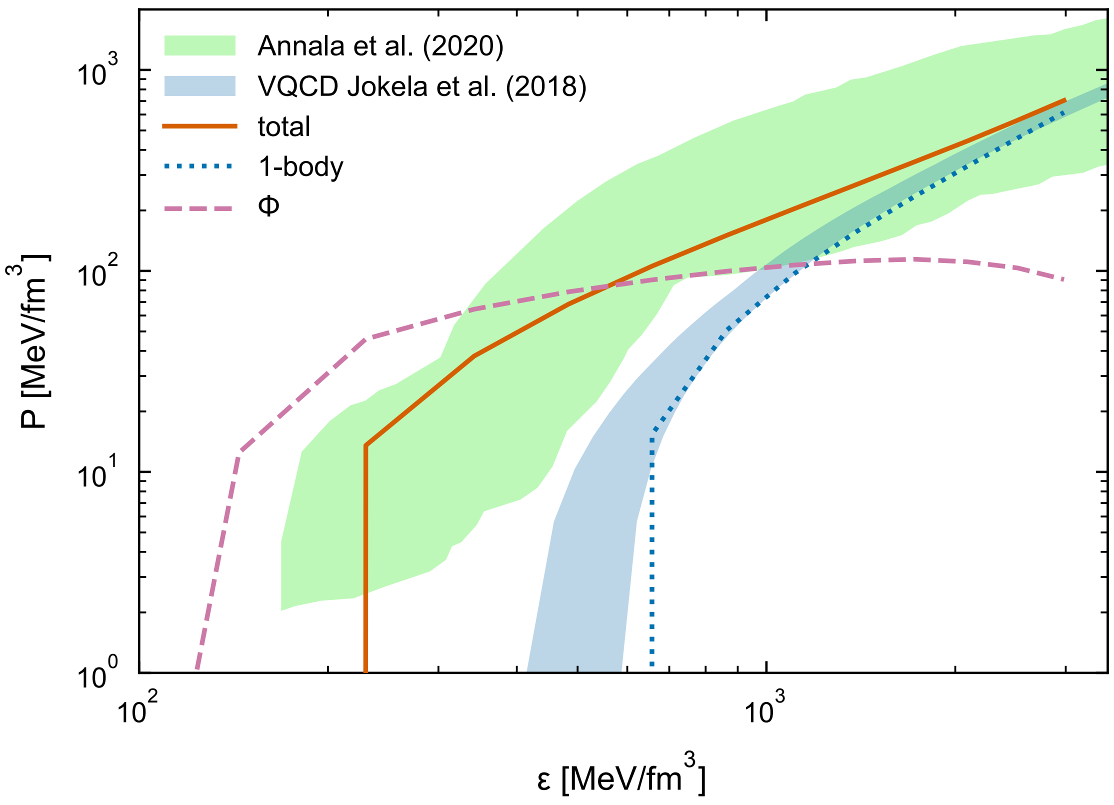
- Small μ: pressure from unconsensed fluctuations in diquark channel
- Thouless criterion not satisfied down to $T = 5$ MeV
- Spectral features:
  - $\mathrm{Im}\Sigma(\omega = 0) = O(T^2)$
  - $\mathrm{Im}\Sigma_{peak} \sim 300\ \mathrm{MeV}$
- Analyticity ⇒ ReΣ has a wiggle
- $Z(q) = \left(1 - \frac{\partial Re\Sigma}{\partial\omega}\Big|_{\omega=0}\right)^{-1} < 1$





Thermodynamics

- What do we have instead?
- 1-body contribution typical for quark matter EoS
- Pressure from precursors of CSC: correlated $qq_{\bar{3}}$ pairs!
- Two-body contribution pushes total EoS towards "typically hadronic" region
- No self-bound strange quark stars?



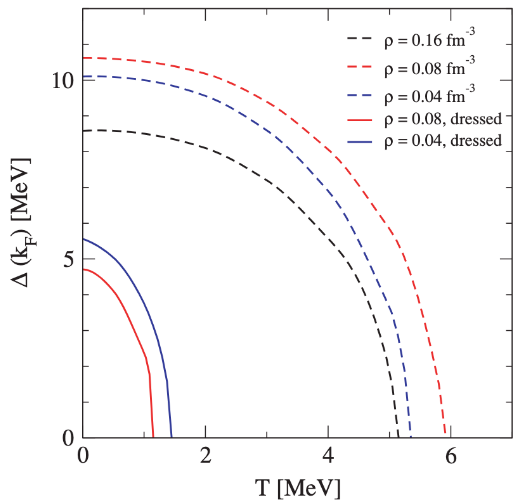
Summary $\mu_B = 0$:

- T-matrix model: beyond mean-field approach to description of strongly interacting QGP
- Explicit treatment of 2-body mesonic/diquark/glueball correlations
- Calibrated to describe $\mu_B = 0$ lattice QCD EoS and susceptibilities

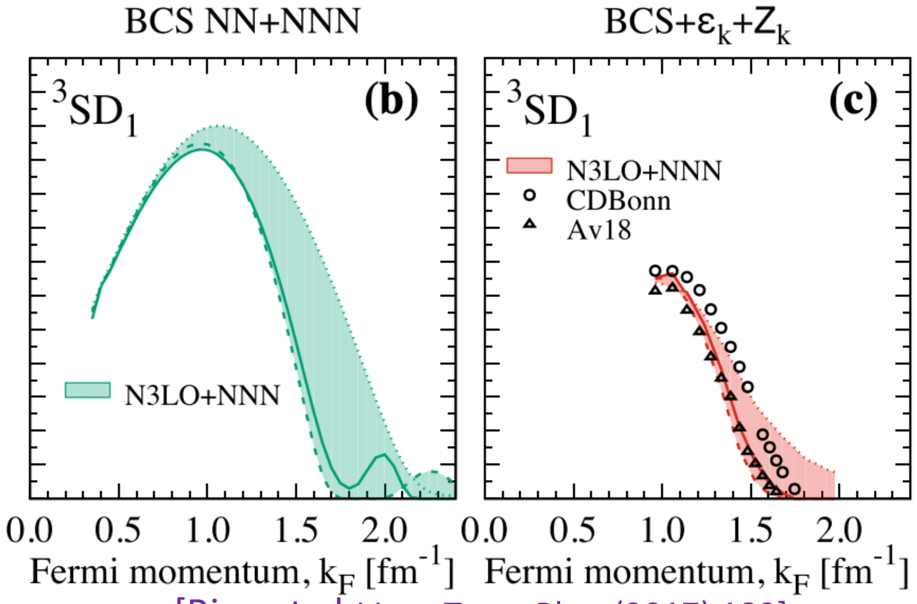
Large μ_B, low T

- Low quasiparticle T_c due to calibration to susceptibility
- The very same strong interaction responsible for condensation significantly decreases T_c due to the collisional widths and redistribution of spectral weight
- Pressure from 2-body diquark excitations brings $P(\mu)$ into typically hadronic region

Similar results



[Müther Dickhoff PRC72 (2005)]



[Rios et al J Low Temp Phys (2017) 189]

Relativistic corrections to $T-$ matrix formalism

- The Bethe-Salpeter equation has to be 3D-reduced to make tractable
- Thompson scheme:

$$G_2 \sim \frac{1}{E - E_{\vec{k}+\frac{\vec{P}}{2}} - E_{\vec{k}-\frac{\vec{P}}{2}} + i0} \times \frac{m^2}{E_{\vec{k}+\frac{\vec{P}}{2}} E_{\vec{k}-\frac{\vec{P}}{2}}}$$

- Previously were attributed to the potential as $v(q) = \frac{m}{E_q}\tilde{v}(q)$, but for finite P kinematics must be a part of G_2

- Explicit Lorentz invariance in the vacuum:

$$J(E,P) = \frac{4m^2}{\pi\sqrt{s}} k_{cm} v^2(k_{cm}),$$

$$k_{cm} = \sqrt{\frac{s}{4} - m^2}$$

Color superconductivity

- Thouless criterion for superconductivity:

$$\mathrm{Re}\Big[\, T_{qq_{\overline{3}}}(E=0, P=0)\Big]^{-1} = 0$$

$\vec{p}_1 = -\vec{p}_2$ in the medium frame

- For the separable interaction

$$\det\nolimits_{\mathrm{sep}}[1 - \mathcal{F}_a\, J_{NN'}(E=0, P=0)] = 0$$

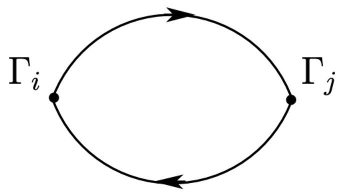
$$\mathcal{F}_a^{qq_{\overline{3}}} = 1/2$$

[T. Alm, G. Röpke et al. PRC 53 (1996)]

- Example: NJL model

$$S_D(\omega, \vec{q}) = \frac{1}{1\ - 2\, G_D\, \Pi_D(\omega, \vec{q})}$$

$\Pi(\omega, \vec{q}) =$



$= i\,\mathrm{Tr} \int_k [G\, \Gamma_i\, G \Gamma_j]$

- On-shell fermions:
  Thouless criterion $1 - 2G_D\Pi_D(0{,}0) = 0$ is equivalent to mean-field gap equation at $\Delta = 0$

$$1 = 8\, N_f G_D \int \frac{d^3p}{(2\pi)^3} \frac{1 - 2n_F(\varepsilon_p - \mu, T)}{2\varepsilon_p - 2\mu}$$